

## Signals and systems Lab 03

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### Task a

Handwritten mathematical derivation showing the calculation of the average value of a sine wave over one period. The derivation starts with the formula for the average value of a function  $y(t)$  over a period  $T$ , which is  $y(t) = y(0) + \int_0^T x(\lambda) d\lambda$ . This is then simplified to  $\int_0^T x(\lambda) d\lambda = \int_0^T A \sin(2\pi f_0 t) dt$ . The integral is evaluated as  $\int_0^T \sin(2\pi t) dt = \left[ -\frac{\cos(2\pi t)}{2\pi} \right]_0^T = \left( -\frac{\cos(2\pi T)}{2\pi} \right) - \left( -\frac{\cos(2\pi \cdot 0)}{2\pi} \right) = \frac{-\cos(2\pi T) + 1}{2\pi}$ . Since  $\cos(2\pi T) = 1$ , the result is  $\frac{1 - \cos(2\pi T)}{2\pi}$ . Using the identity  $1 - \cos(2\theta) = 2\sin^2(\theta)$ , the final result is  $\frac{2\sin^2(\pi T)}{2\pi} = \frac{\sin^2(\pi T)}{\pi}$ .

$$y(t) = y(0) + \int_0^T x(\lambda) d\lambda$$
$$\int_0^T x(\lambda) d\lambda = \int_0^T A \sin(2\pi f_0 t) dt$$
$$\int_0^T \sin(2\pi t) dt$$
$$= \left[ -\frac{\cos(2\pi t)}{2\pi} \right]_0^T$$
$$= \left( -\frac{\cos(2\pi T)}{2\pi} \right) - \left( -\frac{\cos(2\pi \cdot 0)}{2\pi} \right)$$
$$= \frac{-\cos(2\pi T) + 1}{2\pi}$$
$$= \frac{1 - \cos(2\pi T)}{2\pi}$$
$$= \frac{1 - \cos(2\pi T)}{2\pi}$$
$$= \frac{1 - (\cos^2(\pi T) - \sin^2(\pi T))}{2\pi}$$
$$= \frac{1 - (1 - 2\sin^2(\pi T))}{2\pi}$$
$$= \frac{2\sin^2(\pi T)}{2\pi}$$
$$= \frac{\sin^2(\pi T)}{\pi}$$

### **Task b**

The value of  $y_t$  obtained is the same as calculated in part(a). This explains the use of the `int` function and how we can integrate a function and set parameters in that as well

### **Task c:**

```
clc
clear

T = 0:0.01:3;

xt = sin(2*pi*T);
yt = (sin(pi*T).^2)/pi;

figure;

plot(T, xt, 'b-', 'LineWidth', 2);

grid on;

hold on;

plot(T, yt, 'r-', 'LineWidth', 2);

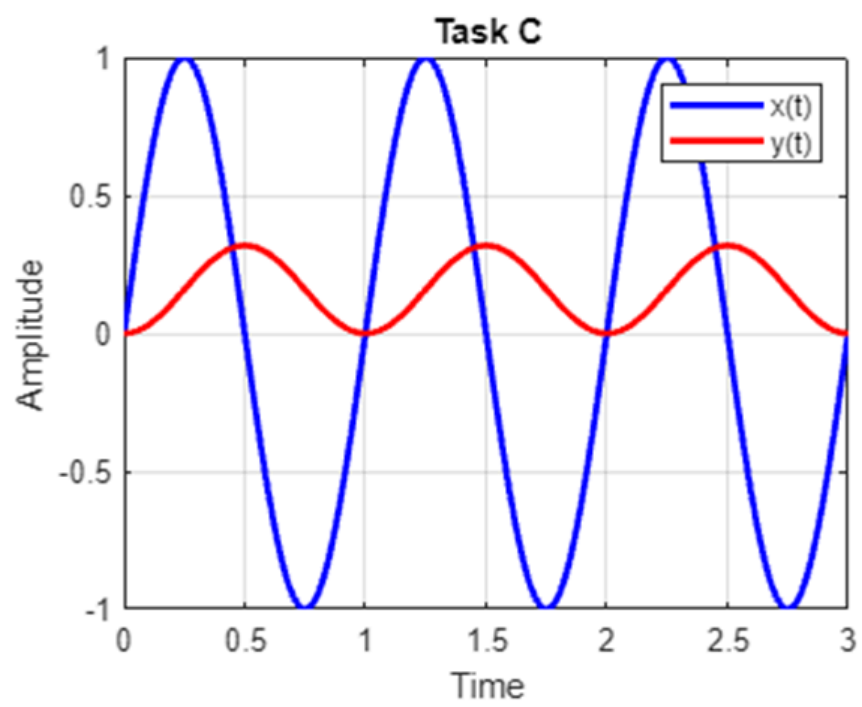
title('Task C');

xlabel('Time');

ylabel('Amplitude');

legend('x(t)', 'y(t)');

hold off;
```



### **Task d:**

```
clc;  
clear all;  
close all;  
  
% Creating a vector  
a = [1 2 3 4 5]
```

```
a = 1×5  
    1    2    3    4    5
```

```
% Calculating the cumsum of a  
b = cumsum(a)
```

```
b = 1×5  
    1    3    6   10   15
```

### ***Observation:***

It adds the individual members of the array, one by one, giving the cumulative sums.

### **Task e:**

#### **Code:**

```
clc  
  
A = 1;  
  
f0 = 1;
```

```
dt = 0.01;

t = 0:dt:3;

xt = A*sin(2*pi*f0*t);

yt = cumsum(xt) * dt;

figure;

plot(t, xt, 'b-', 'LineWidth', 2);

hold on;

plot(t, yt, 'r--', 'LineWidth', 2);

grid on;

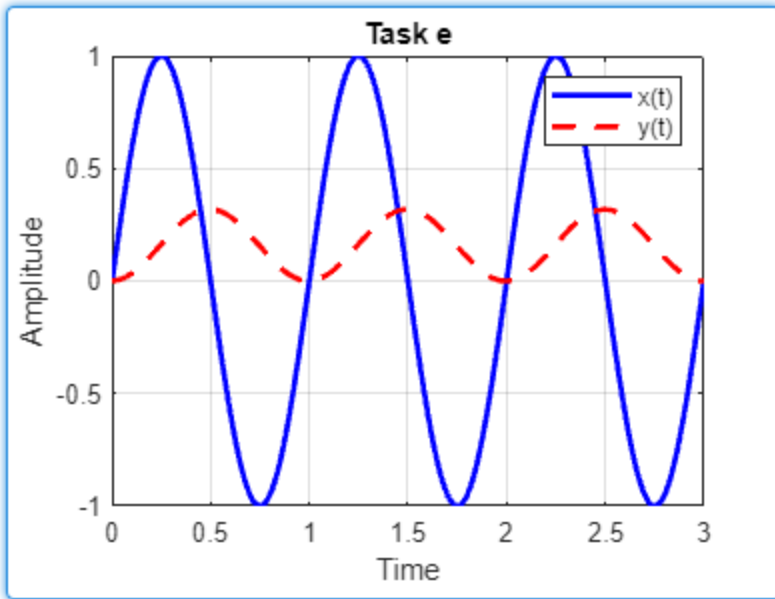
xlabel('Time');

ylabel('Amplitude');

title('Plot of x(t) and y(t)');

legend('x(t)', 'y(t)');

hold off;
```



### **Task f:**

Utilizing the built-in cumsum operation for numerical integration gives the same result as symbolic integration using the built-in int function. This conclusion is supported by the identical graphs obtained in part C, derived from expressions in parts A and B, and part E

### **Task g:**

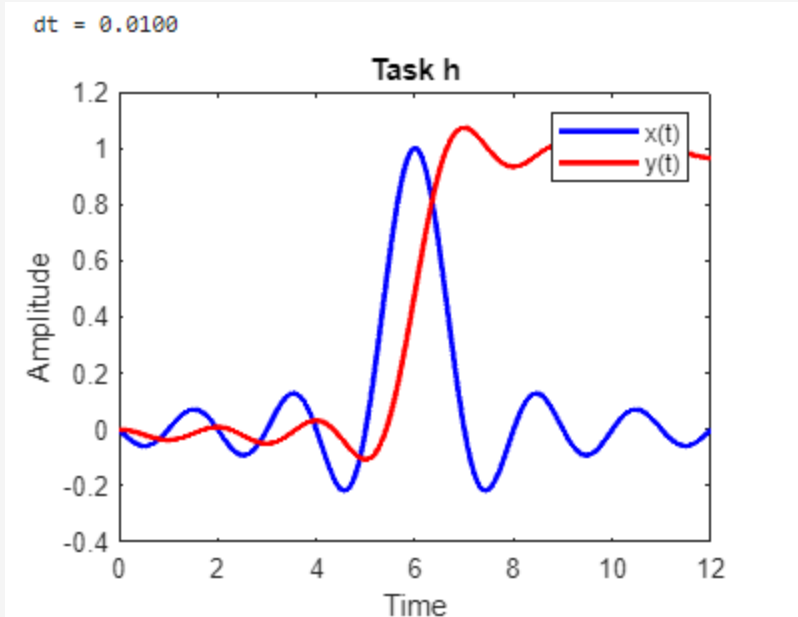
A key advantage is evident when working with real-world data, like sensor inputs. As these inputs are discrete and may lack a precise mathematical function, numerical integration allows for direct processing and analysis, making it highly practical for real-world applications

### **Task H:**

clc

clear

```
dt=0.01  
t=0:dt:12;  
xt=sinc(t-6);  
yt = cumsum(xt) * dt;  
figure;  
plot(t, xt, 'b-', 'LineWidth', 2)  
hold on;  
plot(t, yt, 'r-', 'LineWidth', 2)  
xlabel('Time');  
ylabel('Amplitude');  
title('Task h');  
legend('x(t)', 'y(t)');  
hold off;
```



**Task i:**

b =

1

1

1

1

calculates differences between adjacent elements of X along the first array dimension whose size does not equal 1: If X is a vector of length m , then  $Y = \text{diff}(X)$  returns a vector of length m-1 . The elements of Y are the differences between adjacent elements of X .



**Task j:**

$$x(t) = \frac{d(y(t))}{dt}, \text{ where } y = 2te^{-t}$$

$$x(t) = \frac{d}{dt} (2te^{-t}) \quad \begin{array}{ll} u = 2t & u' = 2 \\ v = e^{-t} & v' = -e^{-t} \end{array}$$

$$x(t) = vu' + uv'$$

$$x(t) = \cancel{2t} 2e^{-t} - 2te^{-t}$$

$$x(t) = 2e^{-t} (1-t)$$

**Task k:**

**Code:**

```
dt = 0.01;  
  
t = 0: dt: 10;  
  
xt = (2 * exp(-t)) .* (1-t);  
  
yt = 2 * (t .* exp(-t));  
  
figure; hold on; grid on;  
  
plot(t, xt, 'Color', 'Red', 'linewidth', 3)  
  
plot(t, yt, 'Color', 'Cyan', 'linewidth', 3)  
  
title('Plotting x(t) and y(t)');
```

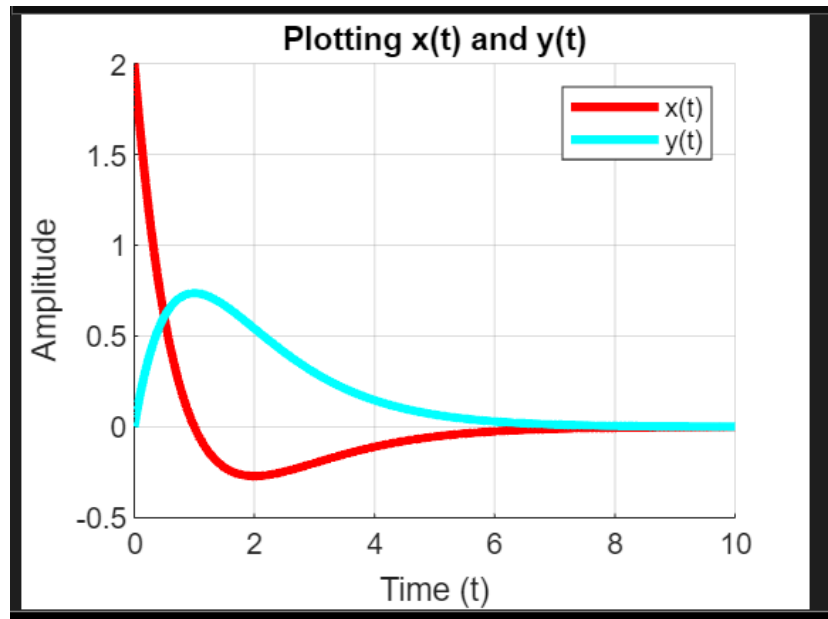
```
xlabel('Time (t)');
```

```
ylabel('Amplitude');
```

```
legend('x(t)', 'y(t)');
```

```
hold off
```

### Graph:



### Observation:

The MATLAB code plots two functions,  $x(t)$  and  $y(t)$ , over the time interval  $[0, 10]$  with a time step of 0.01. The functions are defined as follows:

$$x(t) = 2e^{-t}(1 - t)$$

$$y(t) = 2te^{-t}$$

The code uses the `plot` function to draw the graphs of the functions, with different colors and line widths for each function. The code also adds a title, labels for the axes, a legend, and a grid to the plot. The plot shows how the functions behave over time, and how they compare to each other.

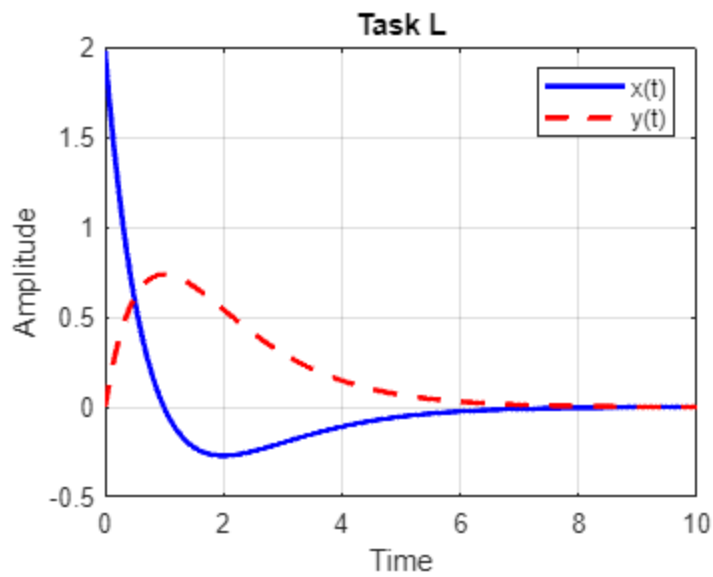
### **Task L:**

```
clc
dt = 0.01;
t = 0:dt:10;
yt = 2 * t .* exp(-t);
xt = diff(yt) / dt;
figure;
plot(t(1:end-1), xt, 'b-', 'LineWidth', 2);
hold on;
plot(t, yt, 'r--', 'LineWidth', 2);
grid on;
xlabel('Time');
ylabel('Amplitude');
title('Task L');
legend( 'x(t)', 'y(t)');
hold off;
```

### ***Observation:***

The MATLAB code plots two functions,  $x(t)$  and  $y(t)$ , over the time interval  $[0, 10]$  with a time step of 0.01.

The code uses the `diff` function to approximate the derivative of  $y(t)$  with respect to  $t$ , and the `plot` function to draw the graphs of the functions, with different colors and line styles for each function. The code also adds a title, labels for the axes, a legend, and a grid to the plot. The plot shows how the functions behave over time, and how they compare to each other.



### **Task m:**

```
dt = 0.001;
```

```
t = 0: dt: 10;
```

```
yt = sinc(t - 6);
```

```
xt = diff(yt) / dt;
```

```
xt = [0 xt];
```

```
% xt = [nan xt]; % This command may also be used to parameterize the plot restraint for  
theresultant function %
```

```
figure; hold on; grid on;
```

```
plot(t, xt,'Color','Red', 'linewidth', 3)
```

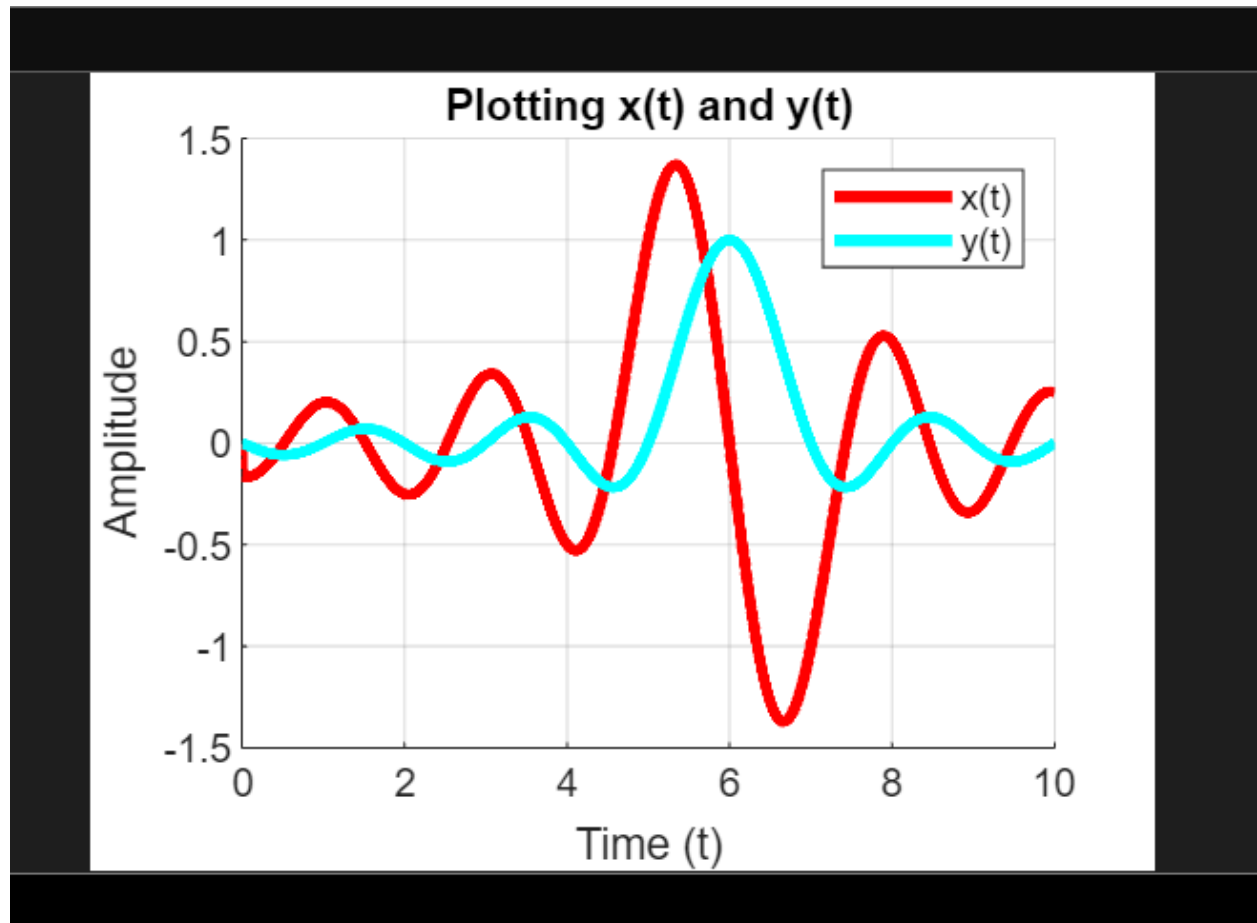
```
plot(t, yt,'Color', 'Cyan', 'linewidth', 3)
```

```
title('Plotting x(t) and y(t)');
```

```
xlabel('Time (t)');  
  
ylabel('Amplitude');  
  
legend('x(t)', 'y(t)')  
  
hold off
```

### **Observation:**

The MATLAB code plots two functions,  $x(t)$  and  $y(t)$ , over the time interval  $[0, 10]$  with a time step of 0.001. The code uses the `diff` function to approximate the derivative of  $y(t)$  with respect to  $t$ , and the `plot` function to draw the graphs of the functions, with different colors and line widths for each function. The code also adds a title, labels for the axes, a legend, and a grid to the plot. The plot shows how the functions behave over time, and how they compare to each other



### ***Task n***

```
clc; clear; close all;
```

```
dt = 0.001;
```

```
a = 0.001;
```

```
t = -10:dt:10;
```

```
Unit_Impulse_Function = (1 ./ (a * sqrt(pi))) .* exp((-1 .* (t.^2)) ./ (a ^ 2));
```

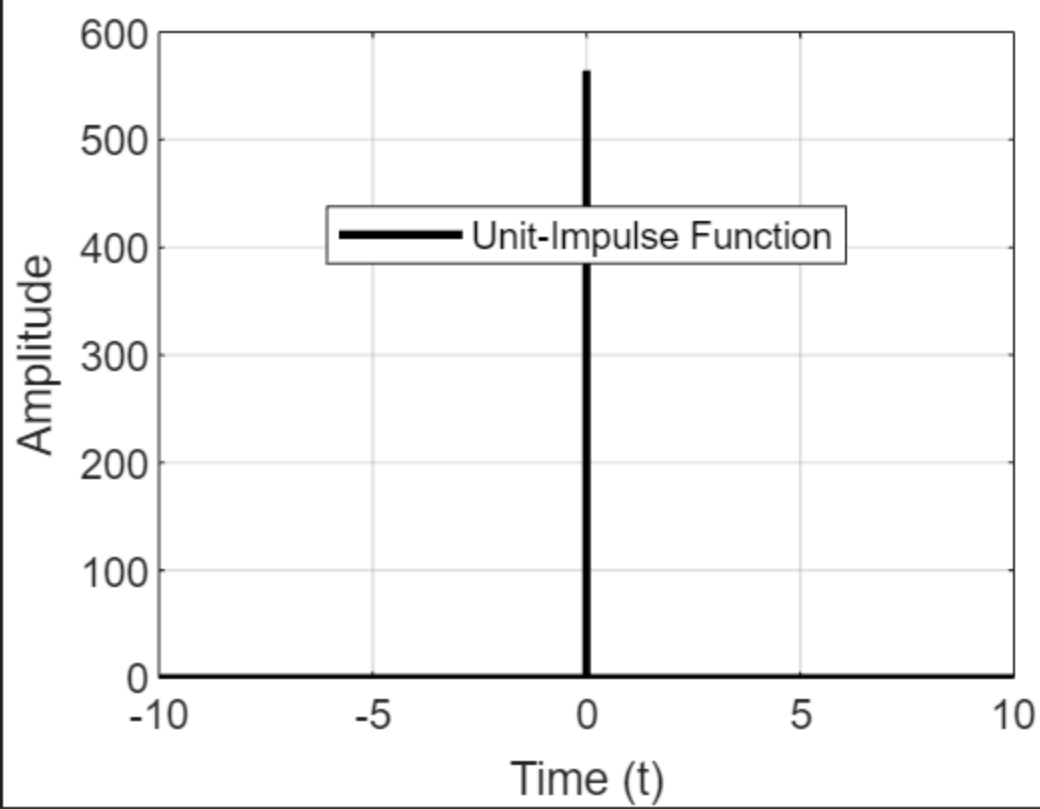
```
figure;
```

```
plot(t, Unit_Impulse_Function, 'Color', 'black', 'linewidth', 2); grid on;  
  
xlabel('Time (t)');  
  
ylabel('Amplitude');  
  
legend('Unit-Impulse Function', 'location', 'best');
```

### ***Observation:***

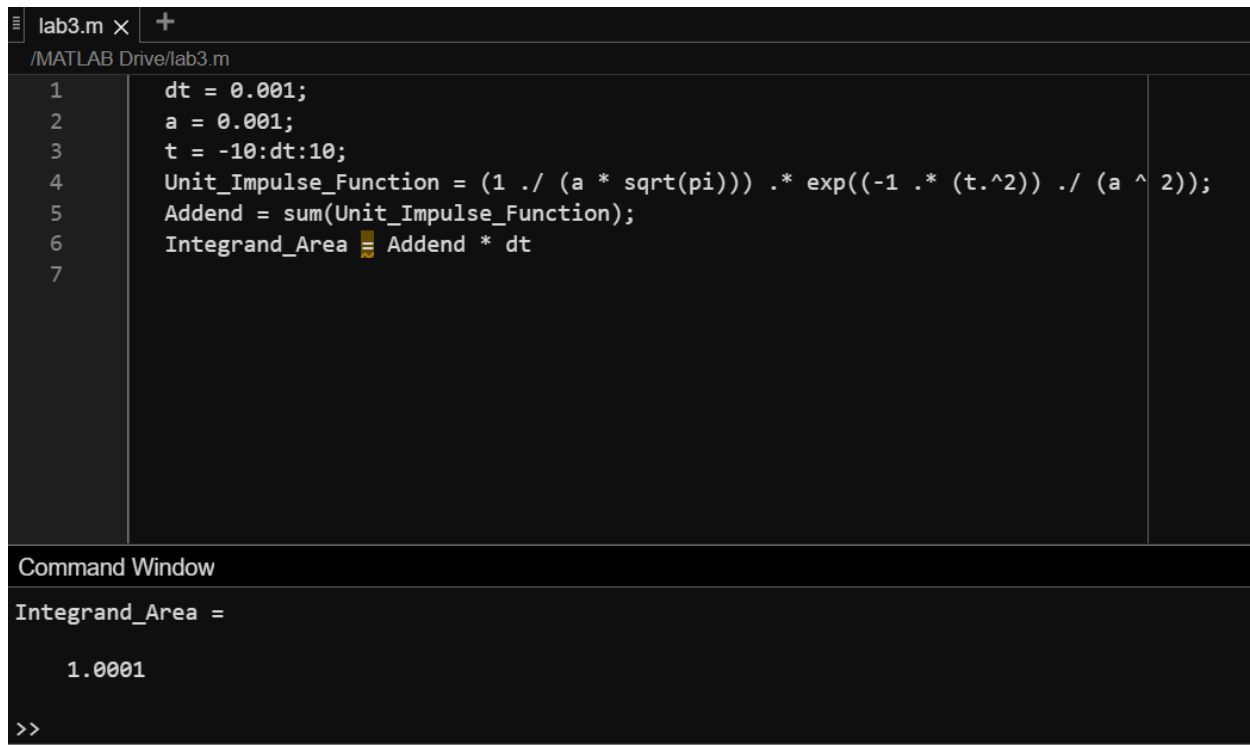
The MATLAB code plots the function  $y(t)$  over the time interval  $[-10, 10]$  with a time step of 0.001.

The code uses the `plot` function to draw the graph of the function, with black color and line width of 2. The code also adds a title, labels for the axes, a legend, and a grid to the plot. The plot shows how the function behaves over time. It is a unit impulse function as it approaches zero.





Task o:



The image shows a MATLAB script editor with a file named 'lab3.m'. The script contains the following code:

```
1 dt = 0.001;
2 a = 0.001;
3 t = -10:dt:10;
4 Unit_Impulse_Function = (1 ./ (a * sqrt(pi))) .* exp((-1 .* (t.^2)) ./ (a ^ 2));
5 Addend = sum(Unit_Impulse_Function);
6 Integrand_Area = Addend * dt
7
```

Below the script editor is the Command Window, which displays the result of the calculation:

```
Integrand_Area =
    1.0001
>>
```

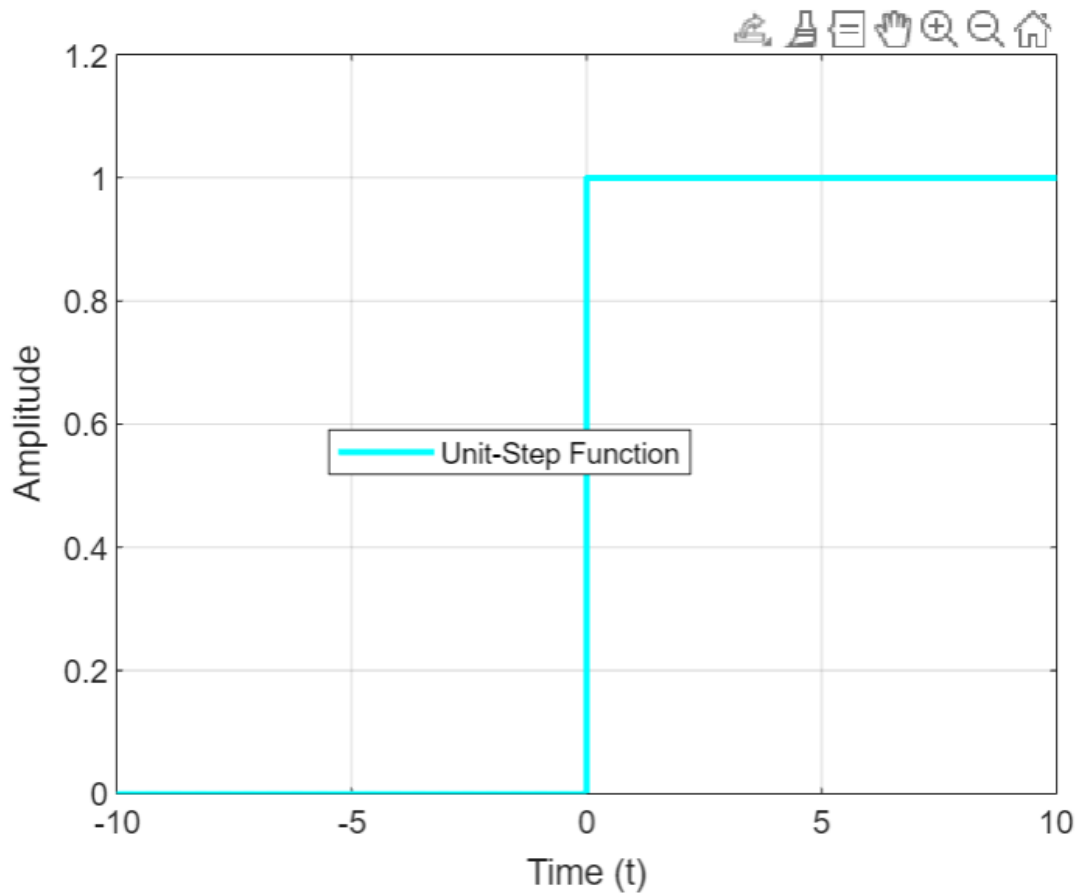
```
dt = 0.001;
a = 0.001;
t = -10:dt:10;
Unit_Impulse_Function = (1 ./ (a * sqrt(pi))) .* exp((-1 .* (t.^2)) ./ (a ^ 2));
Addend = sum(Unit_Impulse_Function);
Integrand_Area = Addend * dt
```

Observation: The MATLAB code computes the area under the curve of the function  $y(t)$  over the time interval  $[-10, 10]$  with a time step of  $0.001$ .  $T$

The code uses the `sum` function to add up the values of  $y(t)$  at each time step, and then multiplies the result by  $dt$  to get the approximate area. The code also assigns the area value to the variable `Integrand_Area`. The function  $y(t)$  is an approximation of the unit impulse function as  $a$  approaches zero. Therefore, the code is essentially calculating the area of the unit impulse function, which should be equal to one. The code returns the following value for `Integrand_Area`:

```
Integrand_Area = 1.0001
```

Task p:



```
dt = 0.001;
a = 0.001;
t = -10:dt:10;
Unit_Impulse_Function = (1 ./ (a * sqrt(pi))) .* exp((-1 .* (t.^2)) ./ (a ^ 2));
Integrand = cumsum(Unit_Impulse_Function);
Unit_Step_Function = Integrand * dt;
figure;
plot(t, Unit_Step_Function, 'Color', 'cyan', 'linewidth', 2); grid on;
xlabel('Time (t)');
ylabel('Amplitude');
legend('Unit-Step Function', 'location', 'best');
```

Observation: The MATLAB code plots the function  $y(t)$  over the time interval  $[-10, 10]$  with a time step of 0.001.

The code uses the `'cumsum'` function to approximate the integral of the unit impulse function, and the `'plot'` function to draw the graph of the function, with cyan color and line width of 2. The code also adds a title, labels for the axes, a legend, and a grid to the plot. The plot shows how the function behaves over time, and how it is a unit step function as it approaches zero.

Task q:

```
dt = 0.001;
a = 0.001;
t = -10:dt:10;
Unit_Impulse_Function = (1 ./ (a * sqrt(pi))) .* exp((-1 .* (t.^2)) ./ (a ^ 2));
Unit_Step_Function = cumsum(Unit_Impulse_Function) * dt;
Delta_Function = diff(Unit_Step_Function) / dt;
Delta_Function = [0 Delta_Function];
% Delta_Function = [nan Delta_Function]; % This command may also be used to
% parametrize the plot restraint for the resultant Unit Impulse Function %
figure;
plot(t, Delta_Function, 'Color', 'black', 'linewidth', 2); grid on;
xlabel('Time (t)');
ylabel('Amplitude');
legend('Delta Function / Unit-Impulse Function', 'location', 'best');
```

Observation: The MATLAB code plots the function  $y(t)$  over the time interval  $[-10, 10]$  with a time step of  $0.001$ .

The code uses the `diff` and `cumsum` functions to approximate the derivative and the integral of the unit impulse function, and the `plot` function to draw the graph of the function, with black color and line width of 2. The code also adds a title, labels for the axes, a legend, and a grid to the plot. The plot shows how the function behaves over time, and how it resembles a delta function or a unit impulse function as  $a$  approaches zero.

