Signals And Systems LAB : 08

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TASK 01:

SNS : Lab : 08 Date

Task : 01

$$x(t) = \begin{cases} a\cos\left(\frac{\pi t}{2T'}\right) & -T' < t < T' \\ 0 & T' < t < T - T' \end{cases}$$

$$a_0 = \frac{1}{T}\int_{-T'}^{T'} a\cos\left(\frac{\pi t}{2T'}\right) dt = \frac{a}{T} \times \frac{1}{\frac{\pi}{2T'}} \times \left(\sin\left(\frac{\pi t}{2T'}\right)\right)\Big|_{-T'}^{T'}$$

$$a_0 = \frac{a}{T} \times \frac{2T'}{\pi} \times \left(\sin\left(\frac{\pi T'}{2T'}\right) - \sin\left(\frac{-\pi T'}{2T'}\right)\right)$$

$$= \frac{2aT'}{T\pi} \times \left(\sin\frac{\pi}{2} - \left(-\sin\frac{\pi}{2}\right)\right) = \frac{2aT'}{\pi T} \times \left(1 - (-1)\right) = \frac{2aT'}{\pi T} \times 2$$

$$\boxed{a_0 = \frac{4aT'}{\pi T}}$$

$$a_k = \frac{2}{T}\int_{-T'}^{T'} a\cos\left(\frac{\pi t}{2T'}\right) \times \cos(k\omega_0 t)\, dt$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

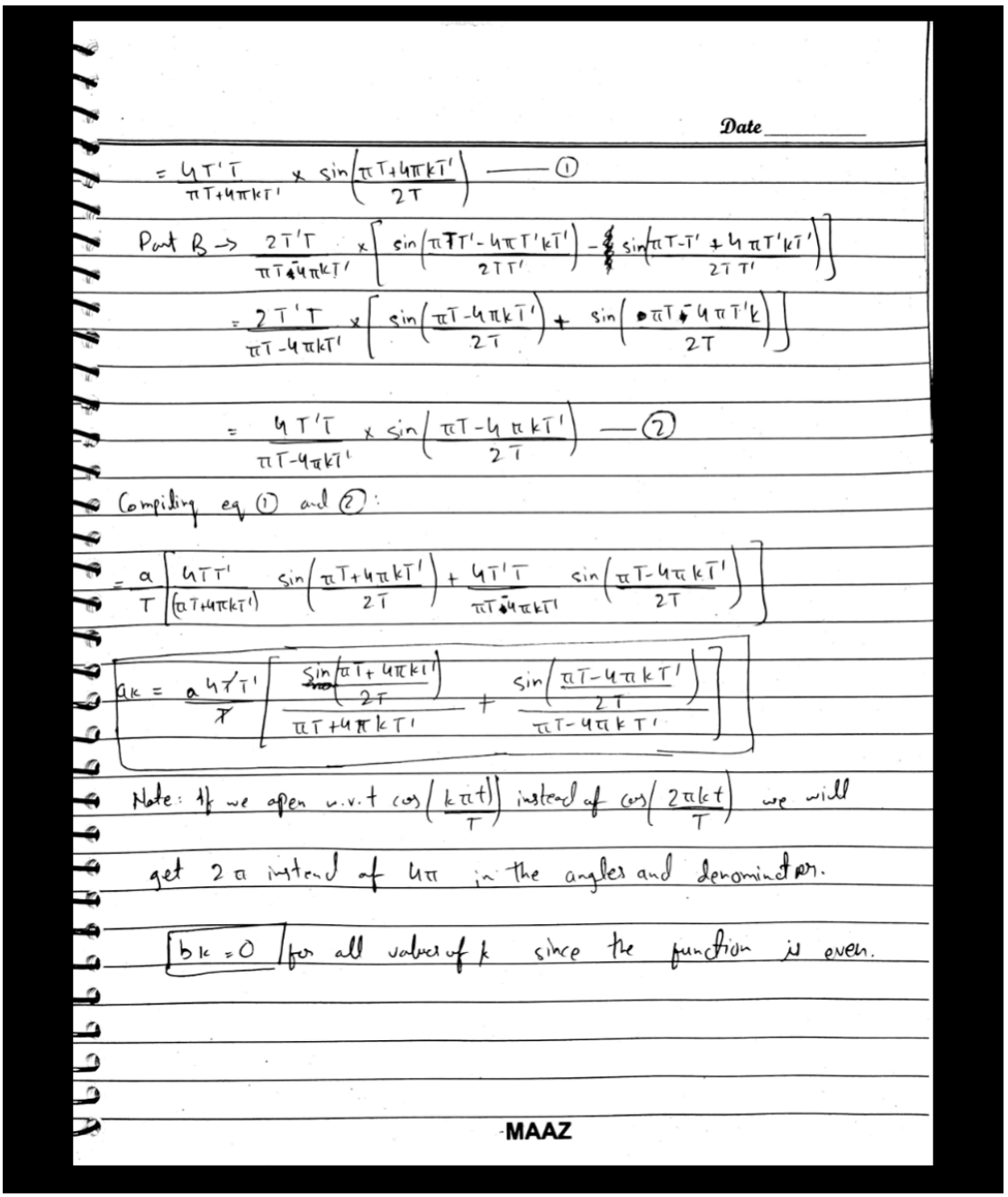
$$a_k = \frac{2a}{2T}\int_{-T'}^{T'} \left(\cos\left(\frac{\pi t}{2T'} + \frac{2\pi tk}{T}\right) + \cos\left(\frac{\pi t}{2T'} - \frac{2\pi tk}{T}\right)\right) dt$$

L.C.M $\rightarrow \frac{\pi T - 4\pi tkT'}{2T'T}$

$$= \frac{a}{T} \times \underbrace{\left[\frac{1}{\frac{\pi T + 4\pi kT'}{2T'T}} \times \sin\left(\frac{\pi tT + 4\pi tkT'}{2T'T}\right)\right]_{-T'}^{T'}}_{A} + \frac{1}{\frac{\pi T + 4\pi kT'}{2T'T}} \times \sin\left(\frac{\pi tT - 4\pi tkT'}{2T'T}\right)\Big|_{-T'}^{T'}$$

$$= \frac{2T'T}{\pi T + 4\pi kT'} \times \left[\sin\left(\frac{\pi TT' + 4\pi T'kT'}{2T'T}\right) - \sin\left(\frac{-\pi T'T - 4\pi T'kT'}{2T'T}\right)\right] \rightarrow \text{Part (A)}$$

$$= \frac{2T'T}{\pi T + 4\pi kT'} \times \left[\sin\left(\frac{\pi T + 4\pi kT'}{2T}\right) + \sin\left(\frac{\pi T + 4\pi kT}{2T}\right)\right] \quad \because \sin\theta + \sin\theta = 2\sin\theta$$



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$$= \frac{4T'\bar{T}}{\pi T + 4\pi k\bar{T}'} \times \sin\left(\frac{\pi T + 4\pi k\bar{T}'}{2T}\right) \quad \text{——} ①$$

Part B → $\frac{2\bar{T}'T}{\pi\bar{T} \pm 4\pi kT'} \times \left[\sin\left(\frac{\pi T\bar{T}' - 4\pi T'k\bar{T}'}{2T\bar{T}'}\right) - \sin\left(\frac{\pi T - \bar{T}' + 4\pi T'k\bar{T}'}{2\bar{T}T'}\right)\right]$

$= \frac{2\bar{T}'T}{\pi\bar{T} - 4\pi kT'} \times \left[\sin\left(\frac{\pi\bar{T} - 4\pi k\bar{T}'}{2\bar{T}}\right) + \sin\left(\frac{\pi\bar{T} \pm 4\pi T'k}{2T}\right)\right]$

$= \frac{4T'\bar{T}}{\pi\bar{T} - 4\pi k\bar{T}'} \times \sin\left(\frac{\pi T - 4\pi k\bar{T}'}{2\bar{T}}\right) \quad \text{——} ②$

Compiling eq ① and ②:

$= \frac{a}{T}\left[\frac{4T\bar{T}'}{(\pi T + 4\pi k\bar{T}')} \sin\left(\frac{\pi T + 4\pi k\bar{T}'}{2\bar{T}}\right) + \frac{4T'\bar{T}}{\pi\bar{T} \pm 4\pi kT'} \sin\left(\frac{\pi T - 4\pi k\bar{T}'}{2T}\right)\right]$

$$a_k = \frac{a\,4T'\bar{T}'}{T}\left[\frac{\sin\left(\frac{\pi T + 4\pi k\bar{T}'}{2T}\right)}{\pi T + 4\pi kT'} + \frac{\sin\left(\frac{\pi\bar{T} - 4\pi kT'}{2T}\right)}{\pi\bar{T} - 4\pi kT'}\right]$$

Note: If we apply u.v.t $\cos\left(\frac{k\pi t}{T}\right)$ instead of $\cos\left(\frac{2\pi kt}{T}\right)$ we will get 2π instead of 4π in the angles and denominators.

$b_k = 0$ for all values of k since the function is even.

MAAZ

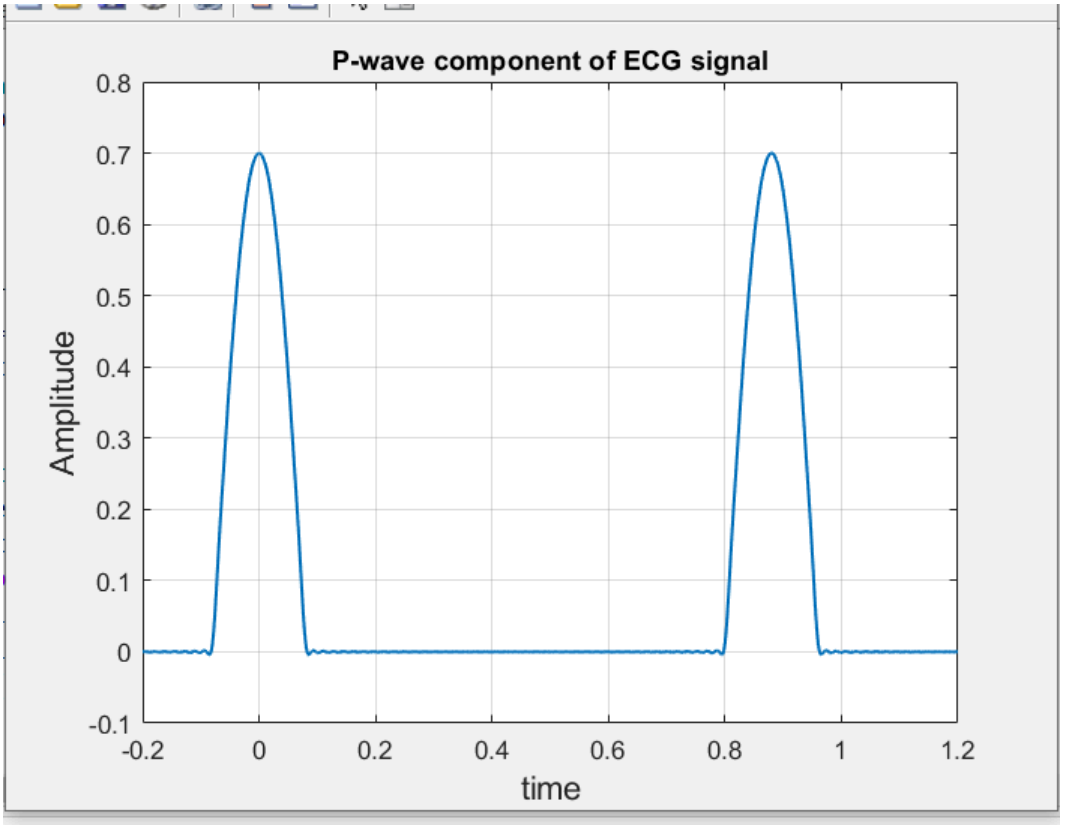
TASK 02:

CODE:

```
%% TASK2
clc
clear all
close all
%Time period and Amplitude
T0 =0.16;
T = 0.88;
a = 0.35;
%DC term
a0 = (4*a*T0)/(pi*T);
```

```
%Number of terms
N = 50;
%TFS coeffiecients
k = 1:N;
ak = (4*a*T0).*( (sin( (pi.*T -2.*k.*pi*T0)./(2*T) )./(pi.*T -2.*k.*pi.*T0))
+ (sin( (pi*T +2.*k.*pi.*T0)./(2*T) )./(pi*T +2.*k.*pi.*T0))  );
% Create a vector of time instants
t = -0.2: 1/N/10 :1.2;
% Set the fundamental frequency.
omega = 2*pi/T;
xtFIFTY = a0;
for i=1:N
   xtFIFTY = xtFIFTY + ak(i)*cos(i*omega*t);
end
% Plot the harmonic approximations
figure;
plot(t, xtFIFTY, 'linewidth',1);
grid on;
xlabel('time','Fontsize',13);
ylabel('Amplitude','Fontsize',13)
title('P-wave component of ECG signal');
```

SNAPSHOT:

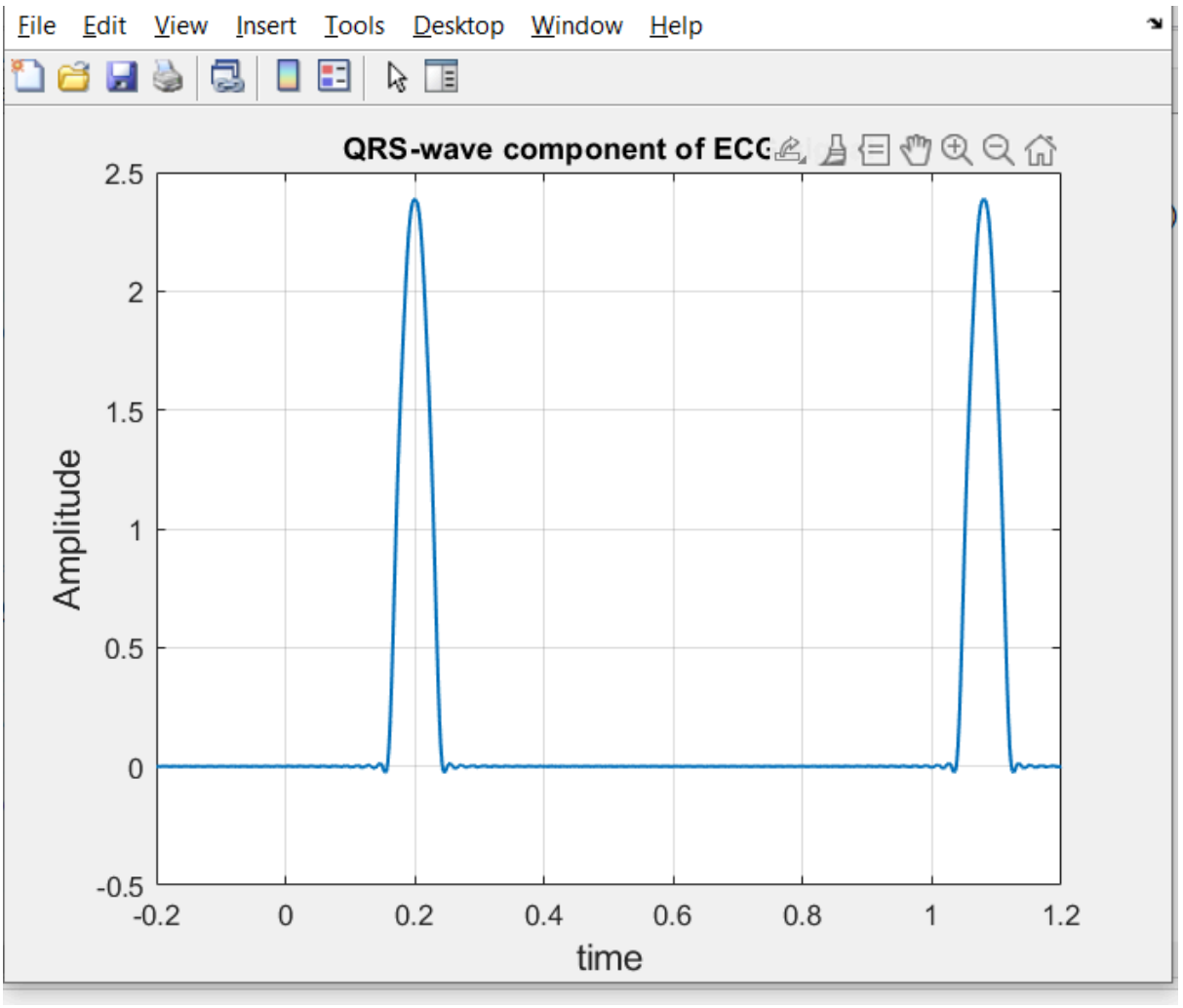


TASK 03:

CODE:

```
clc
clear all
close all
%Time period and Amplitude
T0 =0.08;
T = 0.88;
a = 1.2;
t0=0.2;
%DC term
a0 = (4*a*T0)/(pi*T);
%Number of terms
N = 50;
%TFS coeffiecients
k = 1:N;
ak = (4*a*T0).*( (sin( (pi.*T -2.*k.*pi*T0)./(2*T) )./(pi.*T -2.*k.*pi.*T0))
+ (sin( (pi*T +2.*k.*pi.*T0)./(2*T) )./(pi*T +2.*k.*pi.*T0))  );
% Create a vector of time instants
t = -0.2: 1/N/10 :1.2;
% Set the fundamental frequency.
omega = 2*pi/T;
xtQRS = a0;
for i=1:N
  xtQRS = xtQRS + ak(i)*cos(i*omega*(t-t0));
end
% Plot the harmonic approximations
figure;
plot(t, xtQRS, 'linewidth',1);
grid on;
xlabel('time','Fontsize',13);
ylabel('Amplitude','Fontsize',13)
title('QRS-wave component of ECG signal');
```

SNAPSHOT:



TASK 04:

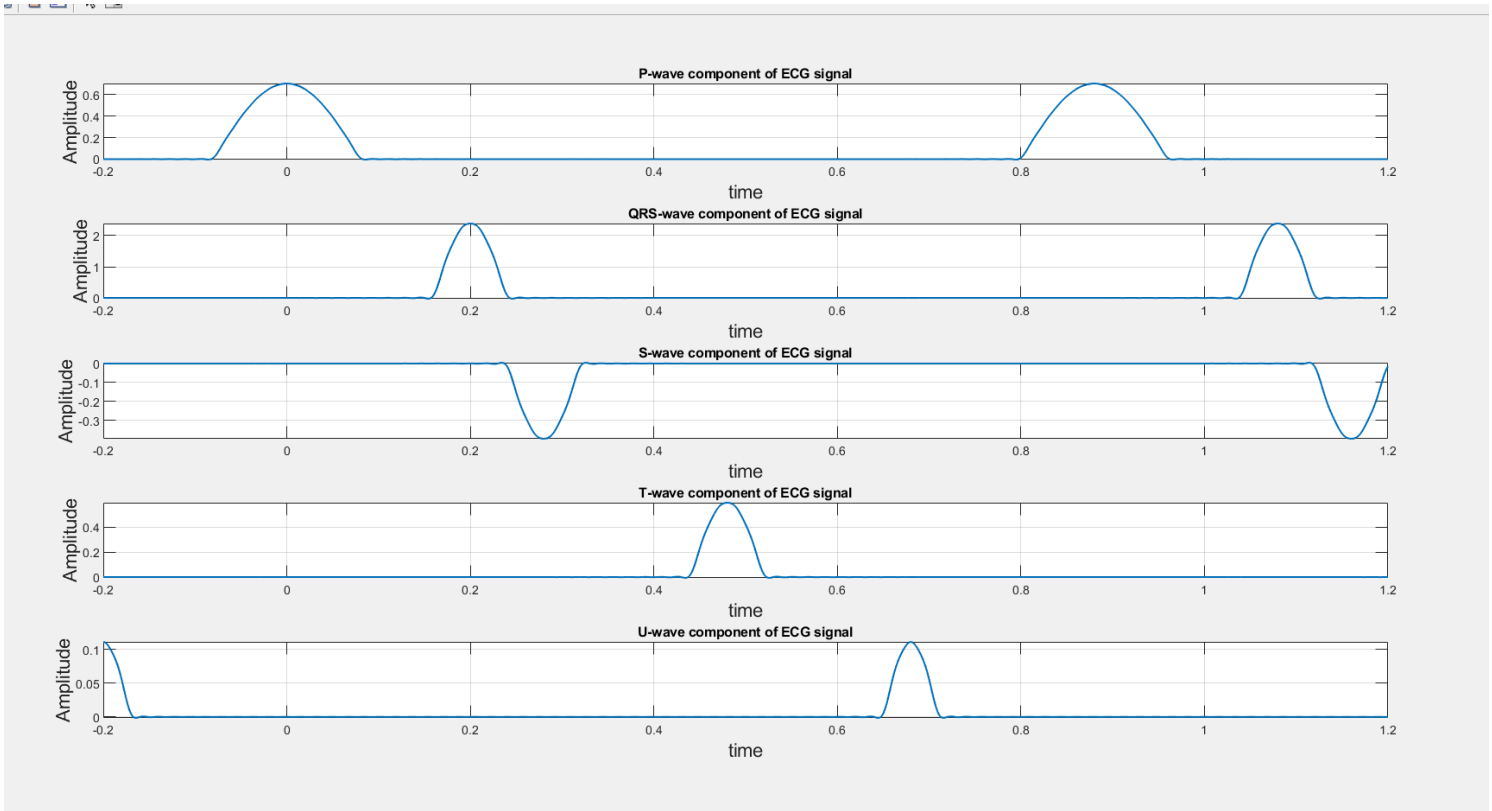
CODE:

```
% TASK04
%-------------S-WAVe-----------
%Time period and Amplitude
T0 =0.08;
T = 0.88;
a = -0.2;
t0=0.28;
%DC term
a0 = (4*a*T0)/(pi*T);
%Number of terms
N = 50;
%TFS coeffiecients
k = 1:N;
ak = (4*a*T0).*( (sin( (pi.*T -2.*k.*pi*T0)./(2*T) )./(pi.*T -2.*k.*pi.*T0))
+ (sin( (pi*T +2.*k.*pi.*T0)./(2*T) )./(pi*T +2.*k.*pi.*T0))  );
% Create a vector of time instants
t = -0.2: 1/N/10 :1.2;
% Set the fundamental frequency.
```

```
omega = 2*pi/T;
xtS = a0;
for i=1:N
  xtS = xtS + ak(i)*cos(i*omega*(t-t0));
end
% Plot the component
subplot 513
plot(t, xtS, 'linewidth',1);
grid on;
xlabel('time','Fontsize',13);
ylabel('Amplitude','Fontsize',13)
title('S-wave component of ECG signal');
%-------------T-WAVe-----------
%Time period and Amplitude
T0 =0.08;
T = 0.88;
a = 0.3;
t0=0.48;
%DC term
a0 = (4*a*T0)/(pi*T);
%Number of terms
N = 50;
%TFS coeffiecients
k = 1:N;
ak = (4*a*T0).*( (sin( (pi.*T -2.*k.*pi*T0)./(2*T) )./(pi.*T -2.*k.*pi.*T0))
+ (sin( (pi*T +2.*k.*pi.*T0)./(2*T) )./(pi*T +2.*k.*pi.*T0))  );
% Create a vector of time instants
t = -0.2: 1/N/10 :1.2;
% Set the fundamental frequency.
omega = 2*pi/T;
xtT = a0;
for i=1:N
  xtT = xtT + ak(i)*cos(i*omega*(t-t0));
end
% Plot the component
subplot 514
plot(t, xtT, 'linewidth',1);
grid on;
xlabel('time','Fontsize',13);
ylabel('Amplitude','Fontsize',13)
title('T-wave component of ECG signal');
%-------------U-WAVe-----------
%Time period and Amplitude
T0 =0.06;
T = 0.88;
a = 0.055;
t0=0.68;
%DC term
a0 = (4*a*T0)/(pi*T);
%Number of terms
N = 50;
%TFS coeffiecients
k = 1:N;
```

```
ak = (4*a*T0).*( (sin( (pi.*T -2.*k.*pi*T0)./(2*T) )./(pi.*T -2.*k.*pi.*T0))
+ (sin( (pi*T +2.*k.*pi.*T0)./(2*T) )./(pi*T +2.*k.*pi.*T0))  );
% Create a vector of time instants
t = -0.2: 1/N/10 :1.2;
% Set the fundamental frequency.
omega = 2*pi/T;
xtU = a0;
for i=1:N
  xtU = xtU + ak(i)*cos(i*omega*(t-t0));
end
% Plot the component
subplot 515
plot(t, xtU, 'linewidth',1);
grid on;
xlabel('time','Fontsize',13);
ylabel('Amplitude','Fontsize',13)
title('U-wave component of ECG signal');
```

SNAPSHOT:



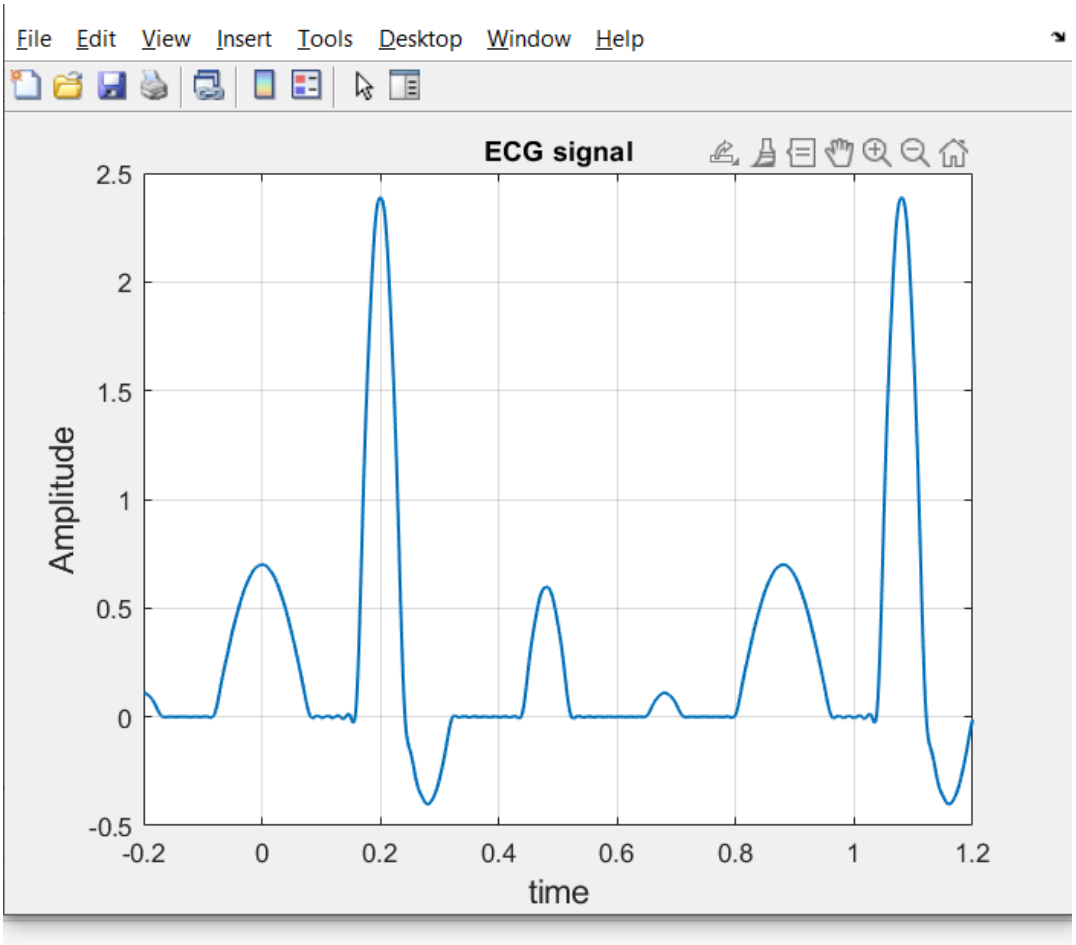
TASK : 05

CODE:

```
% TASK 05
ECG = xtP + xtQRS +xtS + xtT + xtU;
figure
plot(t, ECG, 'linewidth',1);
grid on;
```

```
xlabel('time','Fontsize',13);
ylabel('Amplitude','Fontsize',13)
title('ECG signal');
```

SNAPSHOT:



TASK : 06

$\int uv' = uv - \int vu'$

Task : 6 Date

$$a_0 = \frac{1}{T}\left[\int_{-T'}^{0} a\times\left(\frac{t}{T'}+1\right)dt + \frac{1}{T}\int_{0}^{T'} a\times\left(\frac{-t}{T'}+1\right)dt\right]$$

$$= \frac{4a}{T}\times\frac{1}{1/T'}\times\left(\frac{t^2}{2T'}+t\Big|_{-T'}^{0}\right) + \frac{a}{T}\times\frac{1}{-1/T'}\times\left(\frac{-t^2}{2T'}+t\Big|_{0}^{T'}\right)$$

$$= \frac{aT'}{T}\times\left[-\left(\frac{(-T')^2}{2T'}+T'\right) - \left(\frac{-(T')^2}{2T'}+T'\right)\right]$$

$$= \frac{aT'}{T}\times\left[-\frac{(T')^2}{2T'} - T' + \frac{(T')^2}{2T'} - T'\right] = \frac{aT'}{T}(-2T')$$

$$= \frac{-2a(T')^2}{T}$$

$$a_k = \underbrace{\frac{2}{T}\int_{-T'}^{0} a\left(\frac{t}{T'}+1\right)\cos(k\omega_0 t)}_{①} + \underbrace{\frac{2}{T}\int_{0}^{T'} a\left(\frac{-t}{T'}+1\right)\sin(k\omega_0 t)dt}_{②}$$

For ① →

$$\frac{2a}{T}\int_{-T'}^{0} \underbrace{\cos\left(\frac{k2\pi t}{T}\right)}_{u}\times\underbrace{\frac{t}{T'}}_{v''} + \underbrace{\cos\left(\frac{k2\pi t}{T}\right)}_{b}dt \quad \text{——(A)}$$

↓a

a → $= \cos\left(\frac{k2\pi t}{T}\right) * -\int \frac{T}{2\pi k}\times\sin\left(\frac{k2\pi t}{T}\right)\Big|_{-T'}^{0}$

$$= \cos\left(\frac{k2\pi t}{T}\right) + \frac{T^2}{(2\pi k)^2}\cos\left(\frac{k2\pi t}{T}\right)\Big|_{-T'}^{0}$$

b → $\int_{-T'}^{0}\cos\left(\frac{k2\pi t}{T}\right)dt = \frac{T}{2\pi k}\sin\left(\frac{k2\pi t}{T}\right)\Big|_{-T'}^{0}$

$$= \frac{T}{2\pi k}\left(\sin\left(\frac{k2\pi T'}{T}\right)\right)$$

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Eq (1) → a_k →

$$= \frac{1+T^2}{(2\pi k)^2} - \left[\cos\left(k2\pi\frac{T'}{T}\right) + \frac{T^2}{(2\pi k)^2} \cos\left(\frac{2\pi k T'}{T}\right) \right]$$

$$= 1 + \frac{T^2}{(2\pi k)^2} - \cos\left(\frac{2\pi k T'}{T}\right) - \frac{T^2}{(2\pi k)^2}\left(\cos\left(\frac{2\pi k T'}{T}\right)\right)$$

Eq (1) →

$$\frac{2a}{T}\left[1 + \frac{T^2}{4\pi k^2} - \cos\left(\frac{2\pi k T'}{T}\right) - \frac{T^2}{4\pi^2 k^2} \cos\left(\frac{2\pi k T'}{T}\right) + \frac{T}{2\pi k} \sin\left(\frac{2\pi k T'}{T}\right) \right]$$

For ② → $\frac{2a}{T}\int_0^{T'} \underbrace{\sin\left(k\frac{2\pi t}{T}\right)\left(-\frac{t}{T'}\right)}_{c} + \underbrace{\sin\left(k\frac{2\pi t}{T}\right)}_{d} dt$

c → $= -\left[\sin\left(k\frac{2\pi t}{T}\right) - \int \frac{T}{2\pi k} \cos\left(k\frac{2\pi t}{T}\right) dt \Big|_0^{T'} \right]$

$$= -\left[\sin\left(k\frac{2\pi t}{T}\right) - \frac{T^2}{(2\pi k)^2} \sin\left(k\frac{2\pi t}{T}\right) \Big|_0^{T'} \right]$$

$$= -\left[\sin\left(k\frac{2\pi T'}{T}\right) - \frac{T^2}{(2\pi k)^2} \sin\left(\frac{2\pi k T'}{T}\right) \right]$$

d → $\int_0^{T'} \sin\left(k\frac{2\pi t}{T}\right) dt = -\cos\left(k\frac{2\pi t}{T}\right) \times \frac{T}{k2\pi} \Big|_0^{T'}$

$$= -\frac{T}{k2\pi}\left[\cos\left(k\frac{2\pi T'}{T}\right) - 1 \right] = \frac{T}{2\pi k} - \frac{T}{2\pi k} \cos\left(k\frac{2\pi T'}{T}\right)$$

② →

$$\frac{2a}{T}\left[\frac{T^2}{4\pi^2 k^2} \sin\left(\frac{2\pi k T'}{T}\right) - \sin\left(\frac{2\pi k T'}{T}\right) + \frac{T}{2\pi k} - \frac{T}{2\pi k} \cos\left(\frac{2\pi k T'}{T}\right) \right]$$

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$$a_k = \frac{2a}{T}\left[1 + \frac{T^2}{4\pi k^2} - \cos\left(\frac{2\pi k T'}{T}\right) - \frac{T^2}{4\pi^2 k^2} \cos\left(\frac{2\pi k T'}{T}\right) + \frac{T}{2\pi k} \sin\left(\frac{2\pi k T'}{T}\right) + \frac{T^2}{4\pi^2 k^2} \sin\left(\frac{2\pi k T'}{T}\right) - \sin\left(\frac{2\pi k T'}{T}\right) + \frac{T}{2\pi k} - \frac{T}{2\pi k} \cos\left(\frac{2\pi k T'}{T}\right) \right]$$

$\boxed{b_k = 0}$ since the function is even.