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HOMEWORK

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$$\text{Mean square Error} = \frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2$$

$$\varepsilon = \frac{1}{N} \sum_{i=1}^N (y_i - (ax_i + b))^2$$

$$\frac{d\varepsilon}{da} = \frac{\partial}{\partial a} \left[\frac{1}{N} \sum_{i=1}^N (y_i - (ax_i + b))^2 \right]$$

$$= \frac{1}{N} \sum_{i=1}^N \frac{\partial (y_i - ax_i - b)^2}{\partial a}$$

$$= \frac{1}{N} \sum_{i=1}^N 2(y_i - ax_i - b) \cdot -x_i$$

$$0 = \frac{1}{N} \sum_{i=1}^N [-2x_i y_i + 2ax_i - 2bx_i]$$

$$a = \frac{\sum_{i=1}^N x_i y_i - b \sum_{i=1}^N x_i}{\sum_{i=1}^N x_i^2}$$

$$\frac{\partial \varepsilon}{\partial b} = \frac{1}{N} \sum_{i=1}^N \frac{\partial (y_i - ax_i - b)^2}{\partial b}$$

$$= \frac{1}{N} \sum_{i=1}^N 2(y_i - ax_i - b) \cdot -1$$

$$0 = \frac{1}{N} \sum_{i=1}^N [-2y_i + 2ax_i + 2b]$$

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$$= \sum_{i=1}^N [-y_i + ax_i + b]$$

$$= -\sum_{i=1}^N y_i + a \sum_{i=1}^N x_i + b \rightarrow b = \sum_{i=1}^N y_i - a \sum_{i=1}^N x_i$$

SUBSTITUTING A IN ABOVE EQUATION

$$b = \sum_{i=1}^N y_i - a \sum_{i=1}^N x_i$$

$$b = \sum_{i=1}^N y_i - \left(\frac{\sum_{i=1}^N x_i y_i - b \sum_{i=1}^N x_i}{\sum_{i=1}^N (x_i)^2} \right) \sum_{i=1}^N x_i$$

$$\sum_{i=1}^N x_i^2 b = \sum_{i=1}^N y_i \sum_{i=1}^N (x_i)^2 - \sum_{i=1}^N x_i y_i \sum_{i=1}^N x_i + b \sum_{i=1}^N x_i \sum_{i=1}^N x_i$$

$$b \sum_{i=1}^N (x_i)^2 - b \sum_{i=1}^N x_i \sum_{i=1}^N x_i = \sum_{i=1}^N y_i \sum_{i=1}^N (x_i)^2 - \sum_{i=1}^N x_i y_i \sum_{i=1}^N x_i$$

$$b \left(\sum_{i=1}^N x_i^2 - \sum_{i=1}^N x_i \sum_{i=1}^N x_i \right)$$

$$b = \frac{\sum_{i=1}^N y_i \sum_{i=1}^N x_i^2 - \sum_{i=1}^N x_i y_i \sum_{i=1}^N x_i}{\sum_{i=1}^N x_i^2 - \sum_{i=1}^N x_i \sum_{i=1}^N x_i}$$

$$b = \frac{\sum_{i=1}^N y_i \sum_{i=1}^N x_i^2 - \sum_{i=1}^N x_i y_i \sum_{i=1}^N x_i}{\sum_{i=1}^N x_i^2 - \left(\sum_{i=1}^N x_i \right)^2}$$

$$a = \frac{\sum_{i=1}^N x_i y_i - b \sum_{i=1}^N x_i}{\sum_{i=1}^N x_i^2}$$