Freight Train Path Finder: Mathematical Model and Documentation

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December 27, 2024

1 Project Overview

1.1 Description

The Freight Train Path Finder is an AI-enhanced tool designed to identify optimal paths for freight trains within an existing passenger train timetable. It considers infrastructure constraints, operational rules, and historical performance data to suggest feasible and efficient paths for freight services.

1.2 Benefits

- Efficient utilization of railway capacity
- Reduced planning time for freight paths
- Higher quality paths through machine learning insights
- Systematic consideration of all constraints
- Data-driven decision support

1.3 AI Integration

The system incorporates machine learning in three key areas:

1. Path Success Prediction: Uses Random Forest to predict success probability of proposed paths
2. Congestion Pattern Analysis: Employs DBSCAN clustering to identify typical congestion patterns
3. Speed Profile Optimization: Utilizes neural networks for optimal speed profile generation

1.4 Technology Stack

- Python 3.11 for core implementation
- NumPy and Pandas for data handling
- Plotly for interactive visualization
- Scikit-learn for ML components
- Custom path-finding algorithms

1.5 Implementation Approach

The system is implemented with a modular architecture:

1. Core Components:
    - Infrastructure modeling (track sections, platforms)
    - Train service definitions (passenger, freight)
    - Path representation and scheduling
2. Algorithms:
    - Path finding with variable speeds
    - Conflict detection and resolution
    - Dwell time optimization
3. Visualization:
    - Time-space diagrams
    - Interactive path display
    - Conflict visualization

2 Mathematical Model

2.1 Sets and Indices

$$
\begin{aligned}
&S = \{\text{SEC1}, \text{SEC2}, \text{SEC3}\} && \text{Set of track sections}\\
&D = \{\text{UP}, \text{DOWN}\} && \text{Set of directions}\\
&T = [t_{start}, t_{end}] && \text{Time horizon}\\
&P = \{p_1, \ldots, p_n\} && \text{Set of passenger trains}\\
&F = \{f_1, \ldots, f_m\} && \text{Set of candidate freight paths}
\end{aligned}
$$

2.2 Parameters

$$
\begin{aligned}
v_s^{max} &: \text{Maximum speed limit in section } s \in S \\
l_s &: \text{Length of section } s \in S \\
d^{min} &: \text{Minimum dwell time} \\
d^{max} &: \text{Maximum dwell time} \\
\sigma &: \text{Speed factor range } [0.6, 1.0] \\
\tau &: \text{Time window for departure } [t_0, t_0 + 5 \text{ min}]
\end{aligned}
$$

2.3 Decision Variables

$$
\begin{aligned}
v_{sf} &: \text{Speed of freight train in section } s \\
d_{sf} &: \text{Dwell time in section } s \\
t_{sf} &: \text{Entry time to section } s
\end{aligned}
$$

2.4 Path Generation Model

For each candidate path $f \in F$:

$$
\begin{aligned}
& v_{sf} = \sigma \cdot v_s^{max} && \forall s \in S \\
& d_{sf} \in [d^{min}, 1.5 \cdot d^{max}] && \forall s \in S \\
& t_{sf} = t_{s-1,f} + \frac{l_s}{v_{sf}} + d_{s-1,f} && \forall s \in S
\end{aligned}
$$

2.5 Constraints

2.5.1 Speed Constraints

$$0.6 \cdot v_s^{max} \leq v_{sf} \leq v_s^{max} \quad \forall s \in S \tag{1}$$

2.5.2 Dwell Time Constraints

$$d^{min} \leq d_{sf} \leq 1.5 \cdot d^{max} \quad \forall s \in S \tag{2}$$

2.5.3 Path Crossing Prevention

For opposing trains i, j with different directions:

$$\neg(t_{si} < t_{sj} \wedge t_{s+1,i} > t_{s+1,j}) \quad \forall s \in S \tag{3}$$

2.6 Objective Function

$$\min Z = \sum_{f \in F} \left(w_1 \cdot \text{time}_f + w_2 \cdot \text{conflicts}_f - w_3 \cdot \text{success\_prob}_f \right) \cdot y_f \tag{4}$$

where:

- time_f is the total journey time

- conflicts_f is the number of potential conflicts
- success_prob_f is the ML-predicted success probability

2.7 Constraints

2.7.1 Track Occupation

$$x_{st} + p_{st} \leq 1 \quad \forall s \in S, t \in T \tag{5}$$

2.7.2 Speed Limits

$$v_{st} \leq v_s^{max} \quad \forall s \in S, t \in T \tag{6}$$

2.7.3 Acceleration Constraints

$$v_{s,t+1} - v_{st} \leq \alpha \quad \forall s \in S, t \in T \tag{7}$$

2.7.4 Path Continuity

$$\sum_{s \in S} x_{st} = 1 \quad \forall t \in T \tag{8}$$

3 Path Finding Strategy

The system employs a strategic approach to path finding:

3.1 Fixed Time Window

- Start time window: 7:20-7:25
- Allows for flexible departure timing
- Balances predictability and flexibility

3.2 Variable Parameters

1. Speed Variation:
   - Range: 60% to 100% of maximum speed
   - Section-specific adjustments
   - Continuous speed profile
2. Dwell Times:
   - Minimum: Base dwell time
   - Maximum: 150% of standard maximum
   - Section-specific allocation
3. Path Generation:

- Multiple candidate paths
- Conflict checking
- Journey time optimization

4 Input Requirements

4.1 Infrastructure Data

- Track section definitions with lengths
- Speed limits per section
- Signal locations and types
- Passing loop locations and lengths

4.2 Timetable Data

- Passenger train schedules
- Section occupation times
- Platform usage times

4.3 Historical Data (for ML)

- Past freight paths with success/failure labels
- Historical congestion patterns
- Speed profiles of successful paths

5 Process Flow

1. Data Preprocessing:
   - Clean and structure input data
   - Generate feature vectors for ML models
   - Validate infrastructure constraints
2. ML Model Application:
   - Predict congestion patterns
   - Generate success probabilities
   - Optimize speed profiles
3. Path Generation:
   - Apply mathematical constraints
   - Generate candidate paths
   - Score and rank solutions

6 Outputs

6.1 Primary Outputs

1. Ranked list of feasible paths with:
   - Detailed timing
   - Section-by-section speed profiles
   - Success probability scores

6.2 Secondary Outputs

1. Conflict analysis
2. Robustness metrics
3. Alternative path suggestions

7 Performance Metrics

The solution quality is evaluated using:

$$Q = \frac{w_1 \cdot \text{TimeFit} + w_2 \cdot \text{SuccessProb} + w_3 \cdot \text{Robustness}}{\text{TotalPossibleScore}} \tag{9}$$

where weights w_1, w_2, w_3 are configurable based on operational priorities.