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Question 1 LINEAR ALGEBRA (MID).

= Equations from Diagram are

i) $I_1 - I_2 - I_3 = 0$

ii) $3I_1 + 4I_2 + 0 = 1$

iii) $3I_1 + 0 + 5I_3 = -2$

$$A = \left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 3 & 4 & 0 & -1 \\ 3 & 0 & 5 & -2 \end{array}\right] \quad R_2 - R_3$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 4 & -5 & 1 \\ 3 & 0 & 5 & -2 \end{array}\right] \quad R_2 \times 1/4$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & -5/4 & 1/4 \\ 3 & 0 & 5 & -2 \end{array}\right] \quad R_3 - (-3R_1)$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & -5/4 & 1/4 \\ 0 & 3 & 8 & -2 \end{array}\right] \quad R_3 - (-3R_2)$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & -5/4 & 1/4 \\ 0 & 0 & 47/4 & -5/4 \end{array}\right]$$

$\frac{3}{4} - \frac{2}{1} = \frac{3-8}{4} = -5/4$

Multiply 3rd row by 4/47.

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$I_1 \quad I_2 \quad I_3$

$$A = \left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 1 & -5/4 & 1/4 \\ 0 & 0 & 1 & -5/47 \end{array}\right]$$

$$I_3 = -5/47 \approx \boxed{-0.1063}$$

$$I_2 = 1/4 - (5/47 \times 5/4)$$

$$= 1/4 - \frac{25}{188}$$

$$\boxed{I_2 = 0.11703}$$

$$I_1 = I_2 + I_3$$

$$I_1 = 0.11703 - 0.1063$$

$$\boxed{I_1 = 0.01073}$$

So, $I_1 = 0.01073$

$I_2 = 0.11703$

$I_3 = -0.1063$

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Linear Algebra (Mid)

Q2 (a) $A = \begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 3 & 7 & 7 \end{bmatrix}$

→ Row echleon form.

i) Multiply Row1 with 1/2

$$\begin{bmatrix} 1 & 3 & 3 \\ 2 & 7 & 6 \\ 3 & 7 & 7 \end{bmatrix}$$

ii) Multiply Row1 with -2 and add in Row2

$$\begin{bmatrix} 1 & 3 & 3 \\ 0 & 1 & 0 \\ 3 & 7 & 7 \end{bmatrix}$$

iii) Multiply Row1 with -3 and add in Row 3.

$$\begin{bmatrix} 1 & 3 & 3 \\ 0 & 1 & 0 \\ 0 & -2 & -2 \end{bmatrix}$$

iv) Multiply 2nd row with 2 and add in row 3

$$\begin{bmatrix} 1 & 3 & 3 \\ 0 & 1 & 0 \\ 0 & 1 & -2 \end{bmatrix}$$

v) Multiply Row3 with $-1/2$

$$B = \begin{pmatrix} 1 & 3 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ Row echleon form

Determinent of $A = \begin{vmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 3 & 7 & 7 \end{vmatrix}$

= ~~(Det of A)~~

Reduced form = $\begin{pmatrix} 1 & 3 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

relation of row echleon form.

(Det of A) $\times$ $1/2$ $\times$ $-1/2$ = Det of B

(Det of A) $\frac{1}{-4}$ = $1 \times 1 \times 1$

(Det of A) $\times \frac{1}{-4} = 1$

so **Det of A = -4** Answer

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Adjoint of A. $\begin{pmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 3 & 7 & 7 \end{pmatrix}$

Calculate co-factors.

$C_{11} = (-1)^{1+1} \times (49-42) = 7$ $C_{12} = (-1)^{2+1} \times (14-18) = -4$ $C_{13} = -7$

$C_{21} = (-1)^{2+1} \times (42-42) = 0$ $C_{22} = (-1)^{2+2} \times (14-18) = -4$ $C_{23} = -4$

$C_{31} = (-1)^{3+1} \times (36-42) = -6$ $C_{32} = (-1)^{3+2} \times (12-12) = 0$ $C_{33} = 2$

So Minors = $\begin{pmatrix} 7 & -4 & -7 \\ 0 & -4 & -4 \\ -6 & 0 & 2 \end{pmatrix}$

Co-factors = Transpose of Minors

Adj A = $\begin{pmatrix} 7 & -4 & -7 \\ 0 & -4 & 4 \\ -6 & 0 & 2 \end{pmatrix}$ Answer

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Question 3

Distance b/w 2 planes.

$$x + 2y + 3z = 6 \rightarrow eq1$$
$$2x + 4y + 6z = 5 \rightarrow eq2$$

put $y = 0$ and $z = 0$ in eq 1

so $x = 6$ so point in Plane A : $(6, 0, 0)$

Using distance formula b/w point and plane:

$$D = \frac{|ax + by + cz + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{|2(6) + 4(0) + 6(0) + 5|}{\sqrt{4 + 16 + 36}}$$

$$= \boxed{\frac{17}{2\sqrt{14}} \text{ units}}$$

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Part b:- point = (1, 2, 0, 3)

$$i + 2j + 0k + 3l$$

$$|v| = \sqrt{1^2 + 2^2 + 0^2 + 3^2}$$

$$|v| = \sqrt{14}$$

= unit vector parallel to $\vec{v}$ =

$$U = \frac{1}{\sqrt{14}} (1, 2, 0, 3)$$

$$\boxed{U = \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, 0, \frac{3}{\sqrt{14}}}$$