A New Mathematical Model for the Deterministic Crop Rotation Planning Problem

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Abstract – **The objective of the crop rotation planning problem addressed in this paper is to maximize the total net return. An integer linear programming model is formulated which optimizes the crop rotation plans using the branch and bound algorithm. The model considers the maximum allowable frequency for planting each crop during the rotation cycle in each plot. The proposed model incorporates a novel timing preference constraint, along with the agronomic, water availability and seasonal demand restrictions. An analysis has been performed to simultaneously visualize the temporal and spatial variations during the rotation cycle.**

Keywords - **Agricultural production planning, Crop Rotation Planning, Integer Programming**

I. INTRODUCTION

The agri-food supply chain problem is facing major challenges in satisfying global demand and preserving natural resources. It includes many activities starting from planting the planned crops, moving to the harvesting at the correct timing until delivering the final fresh/processed products to the customers. The problem could be divided into cropping (i.e. farming decisions), transportation and processing of the targeted crop.

Recent reviews of the agri-food supply chain could be found in [1, 2]. Cropping decisions include: deciding on the allocated area to each crop, timing of planting and harvesting, and the utilization of the available resources for planting the crops. Improper crop planning affects the agri-food supply chain activities; hence, farming decisions play a crucial role in shaping the whole agri-chain. Dury *et al.* [3] classified the farming decisions into crop planning and crop rotation, where this paper tackles the latter one.

Crop rotation is defined as the cyclic pattern of planting different crops in the same plot. It increases the organic matter, nitrogen and soil properties, and hence it preserves the soil quality and increase yields [4]. It helps in regulating the irrigation water, reducing diseases and agricultural pests, as well as minimizing the use of synthetic fertilizers and pesticides. The highest profitable crop during the rotation cycle is called "the main crop". The rotation length is defined by replanting the main crop in the plot, and the cycle is usually named for the main crop. For example, the wheat rotation cycle in Egypt extends for three years with two crops harvested per year.

In the analysis of the strategic crop rotation planning problem, the main objective is to select the optimum crop rotation plans. The determination of the optimum sequence and schedule while optimizing single or multiple objectives is a challenging task, where a set of pre-defined constraints should be considered. The limitations include the availability of resources (e.g. water, and cultivated area), and the production time of each crop [5]. Crop rotation planning problem is an NP-hard optimization problem. It involves both sequencing and resource allocation to find the allocations of the restricted number of agriculture plots optimally amongst the various crops each season. For example, Fig.1 shows a schematic layout for the crops' succession problem, wherein each season several crops could be planted in the available plots. The planted crops will affect the decision of which crop to cultivate in the next season. This structure makes the crop rotation schedules, a complex combinatorial task with many decision variables and constraints.

The objective of this article is to examine the crop rotation problem by formulating an integer linear programming model, in order to support the farmer's decision in determining the optimal crop distribution across the plots. In section II, some related works for the problem are addressed. The problem description is presented in section III, and the proposed mathematical model for the crop rotation problem is derived in section IV. Section V presents the primary computational results and finally, Section VI gives the conclusions and future work.

II. LITERATURE REVIEW

The recent research on the crop rotation planning problem was motivated by the work of Detlefsen and Jensen [6] who used a simple network model to describe the problem. Many analytical and computational models have been proposed for solving the crop rotation problem, which can be divided into two main areas: economics and agronomy. The general objective of the current work is generating an optimal performance related to economic decisions, where agronomic constraints should be respected. A recent review that covered the mathematical models of crop planning were presented in [7].

Dos Santos *et al.* [8] developed a binary optimization model for vegetable farming, intending to determine the crop rotation schedules for multiple plots with adjacency constraints. Also, the model considers the planting of legume crops and fallow periods for each plot during each rotation. The model was solved by means of a heuristic based on column generation, and

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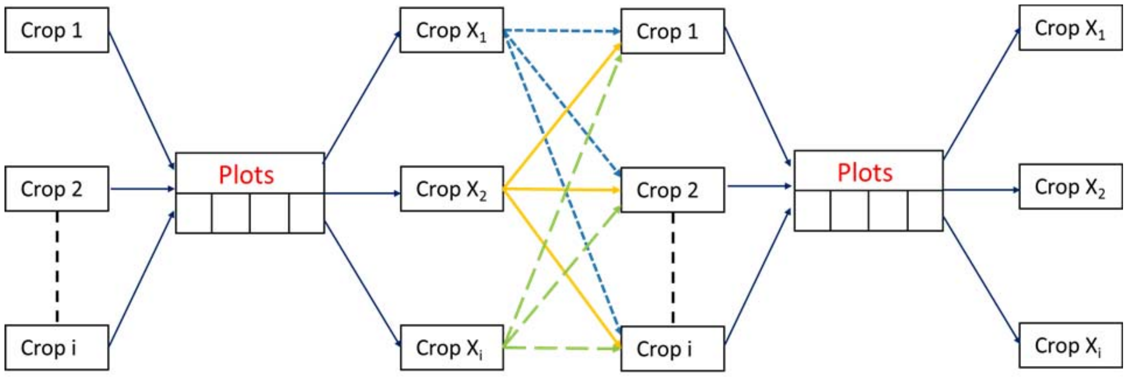

Fig. 1. Schematic representation of the crop rotation problem

compared to the default branch-and-cut approach of CPLEX. Dos Santos *et al.* [9] have extended their original model to take into account the effects of production constraints, and the heterogeneity of each available plot. Although they used the same solution approach with plot size as a decision variable, two additional heuristics were added to refine the obtained results. The first one (the plot elimination heuristic) guarantees that tiny plot sizes are eliminated, while the second one (the plot-minimization) ensures that demand is met with the minimum number of plots. Similarly, Dos Santos *et al.* [10] developed a Branch-and-Price-and-Cut (BPC) algorithm that involves many improved features to optimally solve the master crop rotation problem.

Another BPC algorithm for the crop rotation planning problem was proposed adapting restricted branching rules [11]. Their original model involved a binary crop rotation planning model, with an objective of minimizing the needed area during the time horizon and covering the seasonal demand [12]. On the other hand, the stochastic nature of agriculture systems was handled in [13], where a dynamic stochastic programming model is presented to investigate the risks associated with production. The model showed that higher risks in the crop's yield and prices occurred in the presence of longer rotations; hence, longer crop rotations are unfavourable to farmers.

A linear programming model called CropRota, generates typical crop rotations with the percentage share of crops in rotation as the decision variables [14]. The model derives the crop rotations by using an agronomic score matrix and compares the obtained results with the observed crop data from the Austrian region. A four-year crop rotation schedule was developed using a MILP model for an organic farm in Pennsylvania [15]. The developed model considers crop yield requirements, required harvested quantity, irrigation type, weed control, and other rotation principles.

III. PROBLEM DESCRIPTION

Since 1987, the Egyptian farmers were freely choosing the crops to plant, and the area allocated to each crop. As a result, the farmers took the decisions to plant the most profitable crops (e.g. wheat, maize, rice) regardless of the negative consequences. The consequent negative effects involve excessive use of water due to the expansion of the annual rice area and the decline of soil fertility and properties due to mono-cropping. The main objective of this work is to utilize the crop rotation concepts and achieve a reasonable income for the farmer while respecting the agronomic constraints. The optimal crop rotation plan should contain the following conditions:

- Each crop has its own production time, planting dates, deterministic demand, and the total expected profit of selling the harvested quantity.
- Different crop species require varying amounts of water for irrigation.
- Each Crop belongs to a known family. Crops from the same family have succession constraints (cannot be planted in succession). The crop cannot be repeated yearly in the same land, and it is controlled by the crop frequency.
- Every cycle contains a legume crop (e.g. beans and soybeans), which affects the soil contents positively and hence, the crop's productivity [16].
- One or more fallow periods should be allowed to restore the soil's moisture and fertility content.
- Design the rotation in a way that achieves a reasonable profit by allocating a sufficient area for the main cash crop.

Based on the above-discussed conditions, the generation of optimal crop rotation schedules while targeting the highest income from the suggested rotations is a hard-combinatorial optimization task. In this paper, an integer linear programming model is formulated for the crop rotation planning problem. Fig.2 provides an initial base that shapes the decision-making problem and identifies the nature of factors involved in the cropping-plan decision-making process. The proposed mathematical model can facilitate taking the decisions related to defining the crops sequence in each plot within the time horizon, estimating the needed area for each crop to satisfy the seasonal demand, and obtaining an operational plan for different agricultural activities during the rotation cycle. To the best of our knowledge, no previous study has considered the maximum frequency of planting the desired crop within the rotation cycle, while considering the necessary conditions (e.g. water, demand) to implement the

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proper rotation schedules successfully. This study integrates all the required economic and agronomic constraints through a binary crop rotation model.

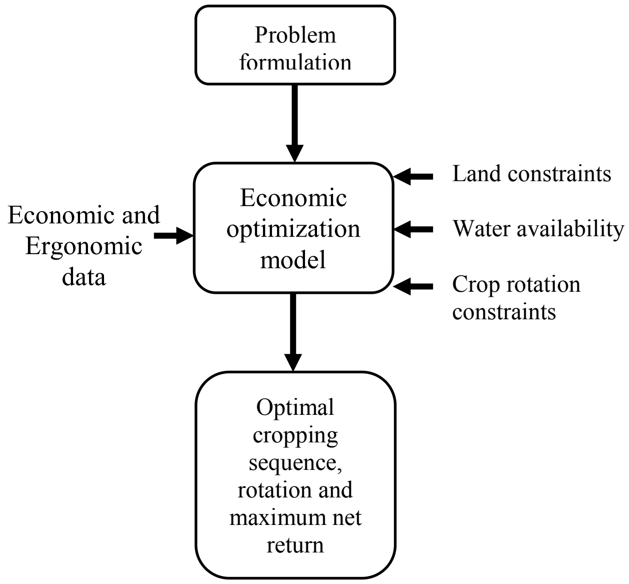

Fig. 2. The analytical framework for the crop rotation problem.

IV. THE PROPOSED MODEL

a. *The Model Assumptions*

- There is no limitation on the seasonal demand of crops.
- The field consists of homogenous plots with standard sizes; hence the obtained yield for the desired crop is the same regardless of the plot used to plant that crop.
- The expected crop yield and irrigation water do not depend on the preceding crop, but on the present crop. Both values are known and deterministic.
- The size of each plot equals to one feddan, which is subdivided according to the rotation cycle.

b. *Model Nomenclature*

SETS AND INDICES

$Crops = \{1,\ldots\ldots I, I+1 \ldots\ldots M, M+1\}$, is the set of crops including legumes and fallow. Where $\{1,\ldots I\}$ are the crops, $\{I+1,\ldots M\}$ are the legumes, and $M+1$ is the fallow (i.e. unplanted period).
i is the index over the set of $crops$
$Families = \{1,\ldots\ldots F\}$, is the set of crop families
f is the index over the set of $Families$
$Time\ periods = \{1,\ldots\ldots J\}$, is the set of time periods
j is the index over the set of $Time\ periods$
$Years = \{1,..T\}$, is the set of years in the rotation cycle
$Plots = \{1,..L\}$, is the set of Plots
l is the index over the set of $Plots$

PARAMETERS

- C_i The net return of crop i per plot
- s_i Starting period of the planting of crop i
- t_i Required production time of crop i
- E_i Ending period of harvesting of crop i
- Y_i Yield of crop i
- D_i Demand for crop i
- B_i Maximum number of plots assigned to crop i during the rotation cycle
- W_i Required water for irrigating crop i
- AW Total available water during the rotation cycle
- f_i Frequency of crop i
- N_i Frequency modification factor for crop i
- T Rotation cycle in years
- F_i The reciprocal of frequency of crop i
- Y Total yearly time slots

DECISION VARIABLES

$$X_{ijl} \begin{cases} 1 & if\ crop\ i \text{ is } planted\ in\ period\ j\ in\ plot\ l \\ 0 & otherwise \end{cases}$$

c. *The Mathematical Model*

$$max \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} C_i.X_{ijl} \qquad (1)$$

$$\sum_{i=1}^{M+1} X_{ijl} \le 1 \qquad \forall\, j \in J\ \forall\, l \in L \qquad (2)$$

$$X_{ijl} + \sum_{t_i \in i}\sum_{i \in f} X_{i(j+t_i)l} \le 1 \qquad \forall\, l \in L \quad \forall\, j \in J \quad \forall\, i \in F \qquad (3)$$

$$\sum_{j=s_i}^{E_i} X_{ijl} = t_i X_{i(j=s_i)l} \qquad \forall\, i \in I/M+1 \quad \forall\, l \in L \quad \forall\, s_i, E_i \in T \qquad (4)$$

$$\sum_{l=1}^{L}\sum_{j \in J/\{s_i,..,E_i\}} X_{ijl} = 0 \qquad \forall\, i \in I \qquad (5)$$

$$\sum_{l=1}^{L}\sum_{j=1}^{J} X_{ijl} \le B_i \qquad \forall\, i \in \mathrm{I}/(\mathrm{I}+1 \ldots \ldots \mathrm{M}, \mathrm{M}+1) \qquad (6)$$

$$\sum_{j=1}^{J}\sum_{I+1}^{M} X_{ijl} \ge 1 \qquad \forall\, l \in \mathrm{L} \qquad (7)$$

$$\sum_{j=1}^{J} X_{(M+1)jl} \ge 1 \qquad \forall\, l \in \mathrm{L} \qquad (8)$$

$$\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{i=1}^{M} W_i.X_{ijl} \le AW \qquad (9)$$

$$\sum_{j \in s_i} X_{ijl} \le f_i T + N_i \qquad \forall\, i \in I \qquad \forall\, j \in J \qquad (10)$$

$$\sum_{j=1}^{F_i} X_{i(s_i+(j-1)Y)l} \le 1 \qquad \forall\, i \in I\ \forall\, l \in L \qquad (11)$$

$$X_{ijl} \in \{0,1\} \qquad \forall\, i \in \mathrm{I} \qquad \forall\, j \in \mathrm{J} \qquad \forall\, l \in \mathrm{L} \qquad (12)$$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Plot				Eggplant						soybeans									Tomatoes(W)														Tomatoes(S)							

(a) Periods from 1 to 40

	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
Plot				Clover (short-cut)								Eggplant				Watermelon												Clover (long-cut)				

(b) Periods from 41 to 72

Fig. 3. Crop production schedule throughout the planning horizon for all plots.

The objective function (1) maximizes the total profit from selling the crops which grow during the rotation cycle. Constraint (2) assigns at most one crop per plot per period, while constraint (3) prevents the same crop or two crops from the same family to be planted consecutively. Constraint (4) ensures that the planted crop will occupy the plot during its production cycle. Constraint (5) guarantees that each crop is only grown in its planting period. Constraint (6) represents the required number of plots for each crop to satisfy the seasonal demand during the rotation cycle. Constraints (7) and (8) adopt the presence of at least single green manure crop and fallow for each plot during the rotation cycle, respectively. Constraint (9) limits the used water for irrigation to the total available amount of water. Constraint (10) ensures that the occurrence of each crop will be defined according to the crop's frequency during the rotation cycle. Constraint (11) ensures that there is enough time before planting the same crop again. It is worth noting that constraint (10) will be enough to represent the crops' frequencies without introducing constraint (12) in case that the rotation length is less than or equal to the frequency reciprocal of the crops [17]. Finally, the binary decision variable X_{ijl} is equal to one if and only if crop i is planted in period j in plot l.

The potential contribution of the formulated model is the integration of crops sequence with spatial and temporal variations, while considering the agronomic constraints. The model produces the crops sequence and the area allocated for each crop simultaneously using a single model.

V. MODEL DISCUSSION AND ANALYSIS

In this section, the solution of the proposed binary linear programming model is discussed, and the computational analysis is conducted to maximize the accumulated net return at the end of the rotation cycle. The model was implemented using GUROBI 8.1 and tested on a 2.20 GHz Core i7 with 6 GB RAM computer. The solver uses the branch and bound algorithm to find the optimal rotation schedules. The study selected thirty-one crops related to eleven families from old lands in Egypt. Crop data[1] involves planting and harvesting dates, average crop water requirement, and total net return. A rotation cycle length of three years is divided into weeks (J =72), three sets of plots (L = 6,12,18), were used during the computational analysis. The full optimization model consists of 78,112 variables with 11,074 linear constraints and could be solved in less than one second.

The verification of the proposed model is done by setting L =12 with no restrictions on demand; hence, the model should allocate the most profitable crop in all plots. Fig.3 shows the obtained schedule in that case, where the cash crop (i.e. tomatoes) is planted to get the highest profit. The optimal schedule consists of one legume crop (soybeans), fallow periods to utilize the crop rotation concepts and restrictions on repeating the cash crop due to the frequency limitation.

Fig.4 demonstrates how the use of different amounts of water alter the range of obtained profit for the three sets of plots. With small amounts of water, the different number of plots have the same earned benefit as a result of low utilizing of the available area regardless of the owned number of plots until reaching approximately ten thousand cubic meters. At this point, larger number of plots will continue in gaining a similar profit until a certain water level is used and so on for large-size farms. As indicated in Fig. 4, each set has a shaded area for maximum profit, which cannot be exceeded at a certain amount of irrigation water. Consequently, the projected area represents the maximum allowable profit that the farmer can obtain when utilizing the rotation methodology. Each curve reaches a constant rate corresponding to the optimum required water for irrigation (e.g. optimum needed water for eighteen plots approximately equals to 405 thousand cubic meters). Table 1 summarizes the optimum water amounts and objective values for the different number of plots.

TABLE I
PROFIT AND IRRIGATION REQUIREMENTS BY THE PROPOSED MODEL

Plots	**Water Requirements (10^3 m^3)**	**Obj. Value (Egyptian Pounds)**
6	135	451,614
12	270	903,228
18	405	1,354,842
24	540	1,806,456
30	675	2,258,070

[1] Data is available upon request due to limit paper space.

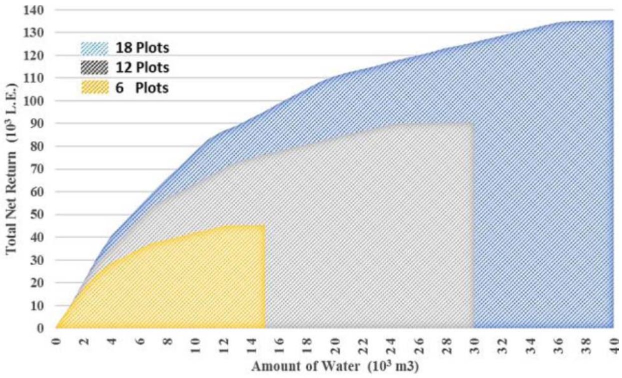

Fig. 4. Expected profit for each set of plots at different water amount

VI. CONCLUSION AND FUTURE WORK

The planning of crop rotation is used in maximizing the farmer's profit by determining the optimum crops' schedules. In this paper, a binary linear programming model capable of solving optimally the crop rotation planning problem is proposed. The combinatorial aspect arises as a result of the ecological criteria that must be respected in the crop rotation. These criteria include certain production dates for crops, restriction on growing specific crops in succession and enhancing soil ability by legumes and fallows. The planted areas are considered an essential parameter in investigating the crop rotation problems, while achieving the efficient utilization of available water resources. In this study, we have demonstrated the trade-offs between irrigation water and expected total profit. Two important factors should be included in future work in order to obtain more relevant results. First, an extension of the model to include soil properties, deficit irrigation and resource constraints such as used machinery and available workforce. Second, incorporating risk assessment with its impact on the optimal crop selection.

ACKNOWLEDGMENT

This research project was supported by the Egyptian ministry of higher education grant and the Japanese International Cooperation Agency (JICA) in the scope of the Egypt-Japan University of Science and Technology.

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