

Introduction

random asn	random sampling		
		Y	N
	Y	both	causal inferences
	N	population inferences	neither

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} \quad s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} = \frac{1}{n-1} \left[\sum_{i=1}^n x_i^2 - \frac{\left(\sum_{i=1}^n x_i \right)^2}{n} \right]$$

$$\text{relative frequency} = \frac{f}{\sum f} \quad \% = \text{relative frequency} \times 100\%$$

$$\text{IQR} = Q_3 - Q_1$$

- Mild outliers 1.5 IQR below Q_1 or above Q_3 (inner fence)
- Extreme outliers 3 IQR below Q_1 or above Q_3 (outer fence)
- EXCLUDE median when finding quartiles for odd n

Probability: Two Set Operations

Complement law: $P(A') = 1 - P(A)$

$$(A \cup B)' = A' \cap B' \quad (A \cap B)' = A' \cup B'$$

$$P(A \cup B) = 1 - P(A' \cap B')$$

Mult law: $P(A \cap B) = P(B) \times P(A|B) = P(A) \times P(B|A)$

Add law: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Disjoint/mutually exclusive: $P(A \cup B) = P(A) + P(B)$

Conditional prob: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Indep events: $P(A \cap B) = P(A) \times P(B)$ given $P(A|B) = P(A)$

$$P(A) = P(A \cap B) + P(A \cap B') \\ = P(B) \times P(A|B) + P(B) \times P(A|B')$$

Probability: Three Set Operations

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) \\ - P(A \cap B) - P(A \cap C) - P(B \cap C) \\ + P(A \cap B \cap C)$$

$$P(A \cap B' \cap C') = P(A) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C) \\ = P(A \cup B \cup C) - P(B) - P(C) + P(B \cap C)$$

$$P(A|B \cap C) = P(A|(B \cap C)) = \frac{P(A \cap B \cap C)}{P(B \cap C)}$$

Deck of Cards

- 52 cards total w/ 13 cards in each suit
- 26 black, 26 red
- Each rank has 4 cards ($4/52 = 1/13$)
- Face cards: J, Q, K 12 face cards ($12/52 = 3/13$)
- Diamonds (R) < clubs (B) < hearts (R) < spades (B)
- Black/red and face are independent

Counting Theory

Expt with k steps, and step i has n_i outcomes:

$$\text{Total no. of outcomes} = \prod_{i=1}^k n_i$$

Permutations: $P_k^n = \frac{n!}{(n-k)!}$ (order impt)

$$P_0^n = 1$$

$$P_1^n = n$$

$$P_n^n = n!$$

Combinations: $C_k^n = \binom{n}{k} = \frac{n!}{k!(n-k)!}$ (order not impt)

$$\binom{n}{0} = \binom{n}{n} = 1$$

$$\binom{n}{1} = n$$

Discrete Random Variables

Probability mass function (pmf)

$$1. f(x_i) = P(X = x_i)$$

$$2. 0 \leq f(x_i) \leq 1$$

$$3. \sum_{i=1}^n f(x_i) = 1$$

Cumulative distribution function (cdf)

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$$

$$1. 0 \leq F(x) \leq 1$$

$$2. \text{ If } x \leq y, \text{ then } F(x) \leq F(y)$$

$$3. P(a \leq X \leq b) = F(b) - F(a-1)$$

Expectations

$$\mu = \mathbb{E}(X) = \sum_{i=1}^n x_i p_i$$

$$\sigma^2 = \text{Var}(X) = \mathbb{E}(X^2) - [\mathbb{E}(X)]^2 = \sum_{i=1}^n (x_i - \mu)^2 p_i = \sum_{i=1}^n x_i^2 p_i - \mu^2$$

Continuous Random Variables

Probability density function (pdf)

$$1. f(x) \geq 0$$

$$2. \int_{-\infty}^{\infty} f(x) dx = 1 \quad (\text{generic limits here})$$

$$3. P(a \leq X \leq b) = \int_a^b f(x) dx$$

$$4. P(X = x) = 0 \Rightarrow P(a < X < b) = P(a \leq X \leq b)$$

Cumulative distribution function (cdf)

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(u) du \quad f(x) = \frac{dF}{dx}$$

$$1. 0 \leq F(x) \leq 1$$

$$2. \text{ If } x \leq y, \text{ then } F(x) \leq F(y)$$

$$3. P(a \leq X \leq b) = F(b) - F(a)$$

Expectations

$$\mu = \mathbb{E}(X) = \int_{-\infty}^{\infty} x f(x) dx \quad \tilde{x} = F(x) = \frac{1}{2}$$

$$\sigma^2 = \text{Var}(X) = \mathbb{E}(X^2) - [\mathbb{E}(X)]^2 = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2$$

$$\text{If } X \text{ has pdf } f(x), \mathbb{E}[h(X)] = \int_{-\infty}^{\infty} h(x) f(x) dx$$

Linear Combinations of Random Variables

If $Y = a_1X_1 + \dots + a_nX_n + b$, then $\mathbb{E}(Y) = a_1\mathbb{E}(X_1) + a_2\mathbb{E}(X_2) + \dots + a_n\mathbb{E}(X_n) + b$

If all X_i are indep, $V(Y) = a_1^2V(X_1) + a_2^2V(X_2) + \dots + a_n^2V(X_n)$

Means	Variances
1. $\mathbb{E}(a) = a$	1. $V(a) = 0$
2. $\mathbb{E}(aX) = a\mathbb{E}(X)$	2. $V(aX) = a^2V(X)$
3. $\mathbb{E}(aX + b) = a\mathbb{E}(X) + b$	3. $V(aX + b) = a^2V(X)$
4. $\mathbb{E}(aX \pm bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$	4. $V(aX \pm bY) = a^2V(X) + b^2V(Y) \pm 2ab \operatorname{cov}(X, Y)$ If independent, $\operatorname{cov}(X, Y) = 0$

If X_i are indep rvs w/ $\mathbb{E}(X_i) = \mu$ and $V(X_i) = \sigma^2$, then $\bar{X} = \frac{\sum X_i}{n}$ is a random variable w/ $\mathbb{E}(\bar{X}) = \mu$ and $V(\bar{X}) = \frac{\sigma^2}{n}$
 \bar{X} more likely to be w/in larger sample size due to decreased variability. **NOTE:** $(X_1 + X_2) - (Y_1 + Y_2 + Y_3) \neq 2X - 3Y$

Discrete Probability Distributions

X	Interpretation of X	Pmf: $P(X = x)$	$\mathbb{E}(x)$	$Var(x)$
Binomial	No of successes x in n trials	$\binom{n}{x} p^x (1 - p)^{n - x}$	np	$np(1 - p)$
Geometric	No of trials x until 1st success	$(1 - p)^{x - 1} p$	$\frac{1}{p}$	$\frac{1 - p}{p^2}$
Negative binomial	No of trials x until r th success	$\binom{x - 1}{r - 1} (1 - p)^{x - r} p^r$	$\frac{r}{p}$	$\frac{r(1 - p)}{p^2}$
Hypergeometric	Pop size N , n draws, M success states, x successes, no replacement	$\frac{\binom{M}{x} \binom{N - M}{n - x}}{\binom{N}{n}}$	$\frac{nM}{N}$	$\left(\frac{N - n}{N - 1}\right) np(1 - p)$
Poisson	No of arrivals x in fixed interval w/ expected value λ	$\frac{e^{-\lambda} \lambda^x}{x!}$	λ	λ

Properties of Discrete Probability Distributions

- Geometric dist'n has lack of memory property:
$$P(X < t + \Delta t | X > t) = \frac{P(t < X < t + \Delta t)}{P(X > t)} = P(X < \Delta t)$$
- Geometric dist'n is special case of -ve binom dist'n w/ $r = 1$.
- Binomial \approx hypergeometric if sampling small part of pop'n.
- Poisson \approx binomial dist'n $\implies n$ is large and p is small.

	Binomial	-ve Binomial	Poisson
Trials	constant (n)	variable (X)	∞
Successes	variable (X)	constant (r)	variable (X)
$P(\text{success})$	constant (p)	constant (p)	constant ($p = \lambda/n$)

Binomial	If a coin is tossed 20 times, what is the probability heads comes up exactly 14 times? ($x = 14$, $n = 20$, $p = 0.5$)
Geometric	If a coin is repeatedly tossed, what is the probability the first time heads appears occurs on the 8th toss? ($x = 8$, $p = 0.5$)
Negative binomial	If a coin is repeatedly tossed, what is the probability the third time heads appears occurs on the 9th toss? ($x = 9$, $r = 3$, $p = 0.5$)
Hypergeometric	5 pandas caught, tagged, and released into the pop. After mixing, a random sample of 10 animals selected. Suppose there are 25 animals in the region. What is the probability that exactly 2 of the caught animals are tagged? ($N = 25$, $n = 10$, $M = 5$, $x = 2$)
Poisson	Floods occur once every 100 years on avg in Canada. What is the probability there will be 4 floods in 100 years? ($\lambda = 1$, $x = 4$)

Continuous Probability Distributions

Distribution	Pdf: $f(x)$	$\mathbb{E}(X)$	$Var(X)$	Cdf: $F(x)$
Uniform	$\frac{1}{b-a}$	$\frac{b+a}{2}$	$\frac{(b-a)^2}{12}$	$\frac{x-a}{b-a}$
Exponential	$\lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$1 - e^{-\lambda x}$
Normal	$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	μ	σ^2	$\Phi(z) = P(Z \leq z)$

- Exponential dist'n has lack of memory property: $P(X < t + \Delta t | X > t) = P(X < \Delta t)$
- Exponential X is length of interval until next success in a Poisson process; $P(X)$ is probability of next success.
- Exponential λ is rate (i.e., per unit time); it's the SAME as that for Poisson.
- Poisson and exponential interchangeable if success hasn't happened yet ($N = 0$).
- If $N \neq 0$, use Poisson (i.e., number of successes in interval): $\lambda = \lambda_x x$ where x is the interval.
- If success already happened, must use exponential.

	Poisson	Exponential
Length of interval	constant	variable (X , until 1st success)
No of successes	variable (N)	constant (1)

$X \sim N(\mu, \sigma^2), \quad Z \sim N(0, 1)$

$Z = \frac{X - \mu}{\sigma}, \quad z = \frac{x - \mu}{\sigma}, \quad x = \mu + z\sigma$

$P(X \leq x) = P\left(\frac{X - \mu}{\sigma} \leq \frac{x - \mu}{\sigma}\right) = P(Z \leq z)$

Normal Approximation to Binomial

If $X \sim B(n, p)$, then $P(a \leq X \leq b) \approx P(a - 0.5 \leq Y \leq b + 0.5)$ where $Y \sim N(\mu = np, \sigma^2 = np(1 - p))$, $np \geq 5$ and $n(1 - p) \geq 5$

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Sampling Distributions

CLT: If n is sufficiently large, the sampling distribution of \bar{x} is approximately normal, even if the pop'n dist'n is not itself normal.

Confidence lvl: Success rate of the method used to construct the interval: $100(1 - \alpha)\%$. If $\alpha = 0.05$, 95/100 samples will capture μ .

Lower confidence = narrower interval. Higher confidence = wider interval.

$CI = \text{estimator} \pm \underbrace{(\text{critical value}) \times (\text{standard error})}_{\text{margin of error}}$

$\text{test statistic} = \frac{\text{estimate} - H_0}{\text{se}(\text{estimate})}$

Decision	Actual situation	
	H_0 true	H_0 false
	Don't reject H_0	correct type II, β
	Reject H_0	type I, α correct

- α = significance = $P(H_0 \text{ rejected} \mid H_0 \text{ true})$
- $1 - \alpha$ = confidence
- $\beta = P(H_0 \text{ not rejected} \mid H_0 \text{ false})$
- $1 - \beta$ = power of the test

Evidence against H_0 :

$\begin{cases} 0 < p < 0.01 & \text{convincing to strong} \\ 0.01 < p < 0.05 & \text{strong too moderate} \\ 0.05 < p < 0.1 & \text{moderate to suggestive but inconclusive} \\ 0.1 < p < 1 & \text{little to no} \end{cases}$

Note: you can find sample size using $V(\hat{\theta})$, where $\hat{\theta}$ is a generic estimator for θ with $\mathbb{E}(\hat{\theta}) = \mu_{\hat{\theta}}$ and $SD = \sigma_{\hat{\theta}}$.

Sample mean	Sample prop'n	Difference in sample means
$\mathbb{E}(\bar{X}) = \mu$ $V(\bar{X}) = \frac{\sigma^2}{n}$ Normal if $n \geq 30$ OR pop'n normal	$\mathbb{E}(\hat{p}) = p$ $V(\hat{p}) = \frac{p(1-p)}{n}$ Normal if $np \geq 5$ and $n(1-p) \geq 5$	$\mathbb{E}(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2$ $V(\bar{X}_1 - \bar{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$ Normal if $n_1 \geq 30$ & $n_2 \geq 30$ OR pop'n normal

Inferences About μ (σ known)

Asn's: $n \geq 30$ or pop'n is normal

Statistic: $z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ CI: $\bar{x} \pm z_{\alpha/2} \left(\frac{\sigma}{\sqrt{n}} \right)$

Upper bound: $\mu \leq \bar{x} + z_{\alpha} \left(\frac{\sigma}{\sqrt{n}} \right)$

Lower bound: $\bar{x} - z_{\alpha} \left(\frac{\sigma}{\sqrt{n}} \right) \leq \mu$

Inferences About μ (σ unknown)

Asn's: random sample, $n \geq 30$ or pop'n normal

Statistic: $t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$ CI: $\bar{x} \pm t_{\alpha/2, n-1} \left(\frac{s}{\sqrt{n}} \right)$

Upper bound: $\mu \leq \bar{x} + t_{\alpha, n-1} \left(\frac{s}{\sqrt{n}} \right)$

Lower bound: $\bar{x} - t_{\alpha, n-1} \left(\frac{s}{\sqrt{n}} \right) \leq \mu$

Inferences About μ_d for Paired Data

Asn's: paired and random samples, $n \geq 30$ or pop'n normal

Statistic: $t_0 = \frac{\bar{d} - \delta_0}{s_d/\sqrt{n}}$ CI: $\bar{d} \pm t_{\alpha/2, n-1} \left(\frac{s_d}{\sqrt{n}} \right)$

Inferences About $\mu_1 - \mu_2$ (indep, V known)

Statistic: $z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$ CI: $(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

Inferences About $\mu_1 - \mu_2$ (indep, V unknown, equal V)

Asn's: $n_1 \geq 30$ and $n_2 \geq 30$ or both pop'ns normal, $\frac{s_{\max}}{s_{\min}} < 2$

N.B., $n_1 \geq 15$ & $n_2 \geq 15$ to check equal variance

df = $n_1 + n_2 - 2$

Statistic: $t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \delta_0}{\text{se}(\bar{x}_1 - \bar{x}_2)}$ CI: $(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2, \text{df}} \times \text{se}(\bar{x}_1 - \bar{x}_2)$

$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$ $\text{se}(\bar{x}_1 - \bar{x}_2) = s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

Inferences About $\mu_1 - \mu_2$ (indep, V unknown, unequal V)

Asn's: $\frac{s_{\max}}{s_{\min}} > 2$ df = $\min\{n_1, n_2\} - 1$

Statistic: $t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ CI: $(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2, \text{df}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

Inferences About p

Asn's: $np_0 \geq 5$ and $n(1 - p_0) \geq 5$

Statistic: $z_0 = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}$ CI: $\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$

Upper bound: $p \leq \hat{p} + z_{\alpha} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$

Lower bound: $\hat{p} - z_{\alpha} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \leq p$

Inferences About $p_1 - p_2$

Asn's: $n_1\hat{p}_1, n_1(1 - \hat{p}_1), n_2\hat{p}_2, n_2(1 - \hat{p}_2)$ all ≥ 5

Statistic: $z_0 = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}(1 - \hat{p}) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$

$\hat{p} = \frac{\text{successes}}{\text{total}} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2}$

CI: $(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_1(1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2(1 - \hat{p}_2)}{n_2}}$

Simple Linear Regression (SLR)

Model: $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$ $Y|x \sim N(\beta_0 + \beta_1 x_i, \sigma^2)$

$\mathbb{E}(Y|x) = \mathbb{E}(\beta_0 + \beta_1 x_i + \epsilon) = \beta_0 + \beta_1 x$ $\mathbb{E}(\epsilon) = 0$

$V(Y|x) = V(\beta_0 + \beta_1 x + \epsilon) = \sigma^2$ $V(\beta_0 + \beta_1 x) = 0$

dist'n of ϵ is normal $\sigma_{\epsilon} = \text{const for all } x$

Least Squares Fitting

$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$ Residuals: $\epsilon_i = y_i - \bar{y}_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i$

$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{\text{Cov}(x, y)}{\text{Var}(x)} = r \frac{s_y}{s_x}$ $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

- $S_{xx} = \sum (x_i - \bar{x})^2 = \sum x^2 - \frac{1}{n} \left(\sum x \right)^2$
- $S_{yy} = \sum (y_i - \bar{y})^2 = \sum y^2 - \frac{1}{n} \left(\sum y \right)^2$
- $S_{xy} = \sum (x_i - \bar{x})(y_i - \bar{y}) = \sum xy - \frac{1}{n} \sum x \sum y$

ANOVA for SLR

$F_0 = \frac{\text{SSR}/1}{\text{SSE}/(n - 2)} = \frac{\text{MSR}}{\text{MSE}}$ $(t_{n-2})^2 = F_{1, n-2}$

$\sum_{i=1}^n \overbrace{(y_i - \bar{y})^2}^{SST} = \sum_{i=1}^n \overbrace{(\hat{y}_i - \bar{y})^2}^{SSR} + \sum_{i=1}^n \overbrace{(y_i - \hat{y}_i)^2}^{SSE}$

MSR = SSR/1 MSE = SSE/($n - 2$)

	df	SS	MS	F	p-value
Regression	1	SSR	MSR	$\frac{\text{MSR}}{\text{MSE}}$	$P(F_{1, n-2} > F_0)$
Residual	$n - 2$	SSE	MSE		
Total	$n - 1$	SST			

Inferences in Regression

Statistic: $t_0 = \frac{\hat{\beta}_1 - b_1}{\text{se}(\hat{\beta}_1)} \sim t_{n-2}$ $\text{se}(\hat{\beta}_1) = \sqrt{\frac{\hat{\sigma}^2}{S_{xx}}}$

CI (slope): $\hat{\beta}_1 \pm t_{\alpha/2, n-2} \times \text{se}(\hat{\beta}_1)$ $\hat{\beta}_1 \sim N(\beta_1, \sigma^2/S_{xx})$

Error/model variance: $\hat{\sigma}^2 = \frac{\text{SSE}}{n - 2}$

Inferences on Regression Line

CI: $(\hat{\beta}_0 + \hat{\beta}_1 x^*) \pm t_{\alpha/2, n-2} \times \hat{\sigma} \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{S_{xx}}}$

PI: $(\hat{\beta}_0 + \hat{\beta}_1 x^*) \pm t_{\alpha/2, n-2} \times \hat{\sigma} \sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{S_{xx}}}$

Correlation

Coefficient of determination: $R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$

Pearson's sample correlation: $r = \frac{1}{n-1} \sum z_x z_y = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$

$$\hat{\beta}_1 = r \frac{s_y}{s_x}$$
$$z_x = \frac{x_i - \bar{x}}{s_x} \qquad z_y = \frac{y_i - \bar{y}}{s_y}$$
$$R^2 = r^2 \text{ for single-variable regression}$$

One-Way ANOVA

$$H_0 : \mu_1 = \mu_2 = \dots = \mu_k \qquad SST = SSTr + SSE$$
$$F_0 = \frac{SSTr/(k-1)}{SSE/(N-k)} = \frac{MSTr}{MSE} \sim F_{k-1, N-k}$$
$$F_0 = \frac{\text{MS btw groups}}{\text{MS w/in groups}}$$

Assumptions:

- 1. Samples from diff pop'ns are random and indep.
- 2. Pop'ns all normally distributed.
- 3. Pop'ns all have same sd.

$N = kn_i$ k is n(groups); n_i is n(obs in each group)

	df	SS	MS	F	p-value
Tx	$k - 1$	SSTr	MSTr	$\frac{MSTr}{MSE}$	$P(F_{1,n-2} > F_0)$
Error	$N - k$	SSE	MSE		
Total	$N - 1$	SST			

Sample Size for Desired Margin of Error

One proportion: $n = \left(\frac{z^*}{ME}\right)^2 p^*(1-p^*)$

One mean: $n = \left(\frac{z^*}{ME}\right)^2 \hat{\sigma}^2$

Tips

On the TI-36X Pro, `{distn}cdf` computes $P(X \leq x)$

Use `quartile.inc()` and `tdist()` on Excel

Round DOWN in the t -table

1-sided test: use $\alpha = \frac{1}{2}(1 - \text{confidence}) \iff$ one-sided CI

2-sided test: use $\alpha = 1 - \text{confidence} \iff$ two-sided CI

Lower-tailed test \iff upper bound

Upper-tailed test \iff lower bound