ECE 203 Formula Sheet

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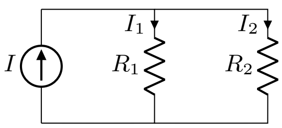
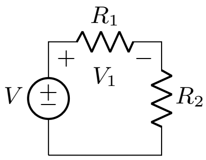
Review



Current division

$$I_1 = \frac{R_2}{R_1 + R_2} I$$



Voltage division

$$V_1 = \frac{R_1}{R_1 + R_2} V$$

Capacitors and Inductors

$$q(t) = Cv(t) \qquad i_c(t) = C\frac{dv(t)}{dt} \qquad v_c(t) = v_c(t_0) + \frac{1}{C}\int_{t_0}^{t} i_c(x)\ dx$$

$$p_c(t) = v_c(t)i_c(t) \qquad E_c(t) = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$$

$$L = \frac{\lambda(t)}{i_L(t)} \qquad v_L(t) = L\frac{di(t)}{dt} \qquad i_L(t) = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t} i_L(x)\ dx$$

$$p_L(t) = v_L(t)i_L(t) \qquad E_L(t) = \frac{1}{2}L[i_L(t)]^2$$

$$C_s = \left(\sum_{i=1}^{N} \frac{1}{C_i}\right)^{-1} \qquad C_p = \sum_{i=1}^{N} C_i \qquad L_s = \sum_{i=1}^{N} L_i \qquad L_p = \left(\sum_{i=1}^{N} \frac{1}{L_i}\right)^{-1}$$

General Solution of First-Order Circuits

$\dot{x}(t) + ax(t) = A$, where $x(t) = K_1 + K_2 e^{-t/\tau}$

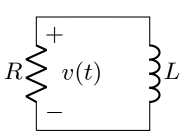
$$K_1 = \frac{A}{a} \qquad \tau = \frac{1}{a} \qquad K_2 = x(0) - K_1$$

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \qquad i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

Compute R_{Th} from view of energy storage device to find τ.

Transient Behaviour of First-Order Circuits

Discharging RL Circuit

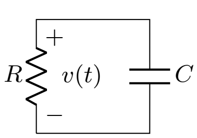


$$L\frac{di(t)}{dt} + Ri(t) = 0 \qquad v_c(0^-) = v_c(0^+)$$

$$i(t) = i(0^+)e^{-t/\tau} \qquad \tau = L/R$$

$$E_L(0^+) = \frac{1}{2}L\left(\frac{V_s}{R}\right)^2 \qquad E_L(\infty) = 0$$

Discharging RC Circuit



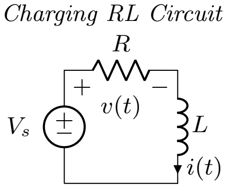
$$C\frac{dv(t)}{dt} + \frac{v(t)}{R} = 0 \qquad i_L(0^-) = i_L(0^+)$$

$$v(t) = v(0^+)e^{-t/\tau} \qquad \tau = RC$$

$$E_C(0^+) = \frac{1}{2}Cv(0^+) \qquad E_C(\infty) = 0$$

Step Response of First-Order Circuits

Charging RL Circuit

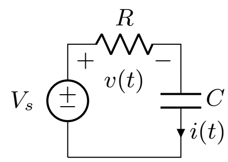


$$L\frac{di(t)}{dt} + Ri(t) = V_s$$

$$i(t) = \frac{V_s}{R}\left(1 - e^{-t/\tau}\right) \qquad \tau = L/R$$

$$v_L(t) = V_s e^{-t/\tau}$$

Charging RC Circuit



$$C\frac{dv(t)}{dt} + \frac{v(t) - V_s}{R} = 0$$

$$i(t) = \frac{V_s}{R}\left(1 - e^{-t/\tau}\right) \qquad \tau = RC$$

$$v(t) = V_s\left(1 - e^{-t/\tau}\right)$$

General Solution of Second-Order Circuits

$\ddot{x}(t) + a_1\dot{x}(t) + a_2 x(t) = A$, where $x(t) = x_p(t) + x_c(t)$

$$x_p(t) = \frac{A}{a_2} \qquad x_c(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

$$x(0) = \frac{A}{a_2} + K_1 + K_2 \qquad \dot{x}(0) = K_1 s_1 + K_2 s_2$$

Obtain $\dot{x}(0)$ from ODE of circuit

Characteristic eq: $s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$

$$\zeta > 1: \ x_c(t) = K_1 e^{-\omega_0(\zeta - \sqrt{\zeta^2 - 1})t} + K_2 e^{-\omega_0(\zeta + \sqrt{\zeta^2 - 1})t}$$

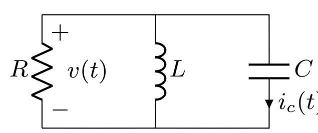
$$\zeta = 1: \ x_c(t) = (B_1 + B_2 t)e^{-\zeta\omega_0 t}$$

$$\zeta < 1: \ x_c(t) = e^{-\sigma t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$$

$$\sigma = \omega_0\zeta, \quad \omega_d = \omega_0\sqrt{1 - \zeta^2}$$

Second-Order Circuit w/o Source

Parallel RLC Circuit

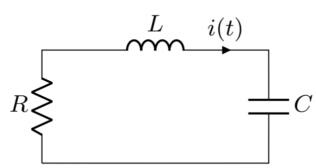


$$v(0) = K_1 + K_2$$

$$\frac{dv(0)}{dt} = \frac{1}{C}i_c(t) = K_1 s_1 + K_2 s_2$$

$$\frac{d^2 v_c(t)}{dt^2} + \frac{1}{RC}\frac{dv_c(t)}{dt} + \frac{1}{LC}v_c(t) = 0$$

Series RLC Circuit



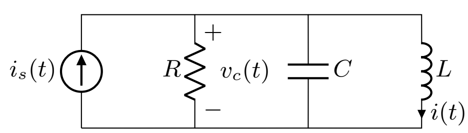
$$i(0) = K_1 + K_2$$

$$\frac{di(0)}{dt} = \frac{v_L(0)}{L} = K_1 s_1 + K_2 s_2$$

$$\frac{d^2 i_L(t)}{dt^2} + \frac{R}{L}\frac{di_L(t)}{dt} + \frac{1}{LC}i_L(t) = 0$$

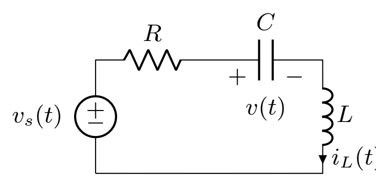
Second-Order w/ Source

Parallel RLC Circuit with Current Source



$$\frac{d^2 v_c(t)}{dt^2} + \frac{1}{RC}\frac{dv_c(t)}{dt} + \frac{1}{LC}v_c(t) = \frac{1}{C}\frac{di_s(t)}{dt}$$

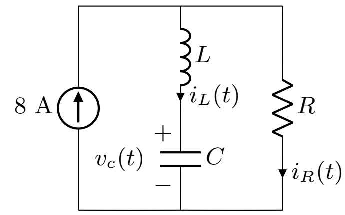
Series RLC Circuit with Voltage Source



$$\frac{d^2 i_L(t)}{dt^2} + \frac{R}{L}\frac{di_L(t)}{dt} + \frac{1}{LC}i_L(t) = \frac{1}{L}\frac{dv_s(t)}{dt}$$

Tricky Questions

Set up the 2nd-order eq for $v_c(t)$.



$8 = i_L(t) + i_R(t)$

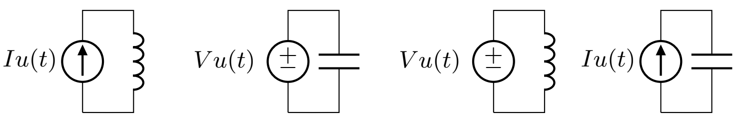
$\implies i_R(t) = 8 - i_L(t)$

$L\dfrac{di_L(t)}{dt} + v_c(t) - R(8 - i_L(t)) = 0$

But $i_L(t) = i_c(t) = C\dfrac{dv_c(t)}{dt}$

Therefore, $LC\dfrac{d^2v_c(t)}{dt^2} + v_c(t) + RC\dfrac{dv_c(t)}{dt} = 8R$

Which circuits do not violate any v-i terminal relationships?



Not allowed   Not allowed   Allowed   Allowed

Generalized Phasor

$x(t) = \mathrm{Re}\{Xe^{j\theta}e^{st}\}, \quad s = \sigma + j\omega$

- $s = 0 \implies$ dc.
- $s = \sigma \implies$ pure exponential.
- $s = j\omega \implies$ pure sinusoidal.
- $s = \sigma + j\omega \implies$ damped sinusoidal.
- σ in Np/s, ω n rad/s, f in cycles/s.

$\sin \mapsto \cos: \ \sin\theta = \cos(\theta - 90^\circ)$

$\cos \mapsto \sin: \ \cos\theta = \sin(\theta + 90^\circ)$

$x(t) = 4e^{-3t}\sin(6t + 10^\circ) \implies s = -3 + 6j, \ \overline{x} = 4\angle -80^\circ$

	R	L	C
$\overline{Z}(s)$	R	sL	$1/sC$
$\overline{Y}(s)$	$1/R$	$1/sL$	sC

Transfer Functions

$H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{\text{o/p}}{\text{i/p}} = \dfrac{\text{forced response}}{\text{forcing fn}}$

$\overline{Y}_p = H(s)\overline{X}(s)|_s \implies y_p(t) = \mathrm{Re}\{\overline{Y}_p e^{st}\}$

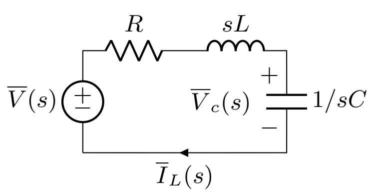
s is complex freq of excitation source.

Find $H_1(s) = \dfrac{\overline{V}_c(s)}{\overline{V}(s)}$

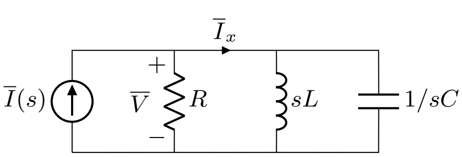
$H_1(s) = \dfrac{\frac{1}{sC}}{R + sL + \frac{1}{sC}}$

Find $H_2(s) = \dfrac{\overline{I}(s)}{\overline{V}(s)}$.

$H_2(s) = \dfrac{s/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$



Pole plot: need poles (X) and zeros (O) of $H(s)$, i.e., critical freqs.



$$\frac{\overline{V}}{\overline{I}} = \frac{1}{\frac{1}{R} + \frac{1}{sL} + sC} = \frac{sL}{s^2LC + \frac{sL}{R} + 1} = \frac{sL}{\left(\frac{s}{\omega_0}\right)^2 + \left(\frac{s}{\omega_0}\right)\frac{1}{Q} + 1}$$

$\omega_0 = \dfrac{1}{\sqrt{LC}}$ $Q = \dfrac{\omega_0}{\beta} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

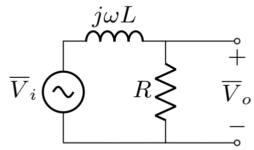
Frequency Response Analysis

For $x_i(t) = X_i\cos(\omega t + \theta_i)$ and transfer fn $H(j\omega) = |H(j\omega)|\angle\theta_H(j\omega)$, the output is $\overline{X}_0 = X_i|H(j\omega)|\angle(\theta_i + \theta_H(j\omega))$.

Freq response plot consists of magnitude plot, $|H(j\omega)|$ vs ω, and phase plot, $\angle\theta_H(j\omega)$ vs ω.

Low Pass Filter (LPF)

RL Implementation



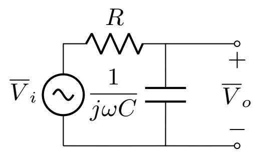
$H(j\omega) = \dfrac{\overline{V}_0}{\overline{V}_i} = \dfrac{R}{R + j\omega L} = \dfrac{1}{1 + j\omega\tau_c}$

$\tau_c = L/R$

$|\overline{V}_0| \approx |\overline{V}_i| \iff \omega\tau_c \ll 1$

$|\overline{V}_0| \ll |\overline{V}_i| \iff \omega\tau_c \gg 1$

RC Implementation



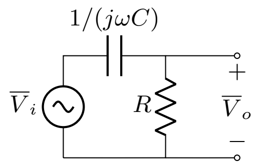
$H(j\omega) = \dfrac{\overline{V}_0}{\overline{V}_i} = \dfrac{R}{R + \frac{1}{j\omega C}} = \dfrac{1}{1 + j\omega\tau_c}$

$\tau_c = RC$

$|H(j\omega)| = \dfrac{1}{\sqrt{(\omega\tau_c)^2 + 1}}$ $\angle H(j\omega) = \arctan(-\omega\tau_c)$

High Pass Filter (HPF)

RC Implementation



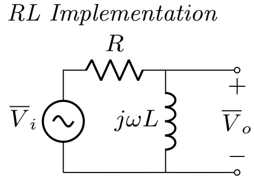
$H(j\omega) = \dfrac{\overline{V}_0}{\overline{V}_i} = \dfrac{R}{R + \frac{1}{j\omega C}} = \dfrac{1}{1 + \frac{1}{j\omega\tau_c}}$

$\tau_c = RC$

$|\overline{V}_0| \approx |\overline{V}_i| \iff \omega\tau_c \gg 1$

$|\overline{V}_0| \ll |\overline{V}_i| \iff \omega\tau_c \ll 1$

RL Implementation

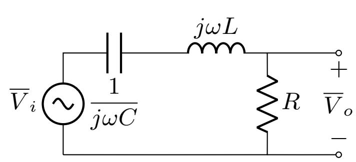


$H(j\omega) = \dfrac{\overline{V}_0}{\overline{V}_i} = \dfrac{j\omega L}{R + j\omega L} = \dfrac{1}{1 + \frac{1}{j\omega\tau_c}}$

$\tau_c = L/R$

$|H(j\omega)| = \dfrac{1}{\sqrt{\left(\frac{1}{\omega\tau_c}\right)^2 + 1}}$ $\angle H(j\omega) = \arctan\left(\dfrac{1}{\omega\tau_c}\right)$

Band Pass Filter (BPF)



$\tau_{c1} = RC$ $\tau_{c2} = L/R$

$$H(j\omega) = \frac{\overline{V}_0}{\overline{V}_i} = \frac{R}{(j\omega C)^{-1} + j\omega L + R} = \frac{1}{(j\omega\tau_{c1})^{-1} + j\omega\tau_{c2} + 1}$$

$$= \frac{j\omega\tau_{c1}}{1 + 2\zeta(j\omega\tau_0) + (j\omega\tau_0)^2}$$

$\tau_0 = \sqrt{\tau_{c1}\tau_{c2}} = \sqrt{LC}$

$\zeta = \dfrac{\tau_{c1}}{2\tau_0} = \dfrac{R}{2}\sqrt{\dfrac{C}{L}} = \dfrac{1}{2Q}$ $\omega = \dfrac{1}{\sqrt{LC}} \implies |\overline{V}_0| \approx |\overline{V}_i|$

$\omega \ll 1/\tau_{c1} \implies |\overline{V}_0| \ll |\overline{V}_i|$ $\omega \gg 1/\tau_{c2} \implies |\overline{V}_0| \ll |\overline{V}_i|$

$$|H(j\omega)| = \frac{1}{\sqrt{\left(\omega\tau_{c2} - \frac{1}{\omega\tau_{c1}}\right)^2 + 1}}$$

$$\angle H(j\omega) = \arctan\left(\frac{1}{\omega\tau_{c1}} - \omega\tau_{c2}\right)$$

Cutoff Frequency

$|H(j\omega_c)| = \dfrac{H_{\max}}{\sqrt{2}}$ $\qquad H_{\max} = 1$

LPF: $|H(j\omega_c)| = \dfrac{1}{\sqrt{(\omega\tau_c)^2+1}} = \dfrac{1}{\sqrt{2}} \implies \omega_c = \dfrac{1}{\tau_c}$

HPF: $|H(j\omega_c)| = \dfrac{1}{\sqrt{\left(\frac{1}{\omega\tau_c}\right)^2+1}} = \dfrac{1}{\sqrt{2}} \implies \omega_c = \dfrac{1}{\tau_c}$

RL: $f_c = \dfrac{R}{2\pi L}$, RC: $f_c = \dfrac{1}{2\pi RC}$, LPF and HPF have same f_c

Parameters for Series BPF

$\omega_0 = \dfrac{1}{\sqrt{LC}}$ $\qquad Z_c = Z_L$ at $\omega = \omega_0$

$\beta = \omega_{c2} - \omega_{c1} = \dfrac{R}{L}$ $\qquad Q = \dfrac{\omega_0}{\beta} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

$$\omega_{c1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2}$$

$$= \omega_0\left(-\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right) = -\frac{1}{2\tau_{c2}} + \sqrt{\left(\frac{1}{2\tau_{c2}}\right)^2 + \frac{1}{\tau_{c1}\tau_{c2}}}$$

$$\omega_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2}$$

$$= \omega_0\left(\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right) = \frac{1}{2\tau_{c2}} + \sqrt{\left(\frac{1}{2\tau_{c2}}\right)^2 + \frac{1}{\tau_{c1}\tau_{c2}}}$$

Resonance for BPF

	Series RLC	*Parallel RLC*
ω_0	$1/\sqrt{LC}$	$1/\sqrt{LC}$
Q	$\dfrac{\omega_0 L}{R} = \dfrac{1}{\omega_0 RC} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$	$\dfrac{R}{\omega_0 L} = \omega_0 RC = R\sqrt{\dfrac{C}{L}}$
β	R/L	$1/(RC)$
Other	$\lvert\overline{V}_L\rvert = Q\lvert\overline{V_s}\rvert$ $\lvert\overline{V}_c\rvert = Q\lvert\overline{V_s}\rvert$ $\overline{V}_L + \overline{V}_c = 0$ $\implies$ Resonant V rise	$\lvert\overline{I}_L\rvert = Q\lvert\overline{I_s}\rvert$ $\lvert\overline{I}_c\rvert = Q\lvert\overline{I_s}\rvert$ $\overline{I}_L + \overline{I}_c = 0$ $\implies$ Resonant I rise

Bode Plots

Magnitude: $20\log|H(j\omega)|$ vs $\log\omega$

Phase: $\angle H(j\omega)$ vs $\log\omega$

Examples

$H(j\omega) = K \implies H_{db} = 20\log K, \quad \angle H(j\omega) = 0°$

$H(j\omega) = (j\omega)^{\pm n} \implies H_{db} = \pm 20n\log\omega, \quad \angle H(j\omega) = \pm n90°$

1st Order Magnitude Response: $H_{\text{dB}} = \pm 20\log|1+j\omega|$

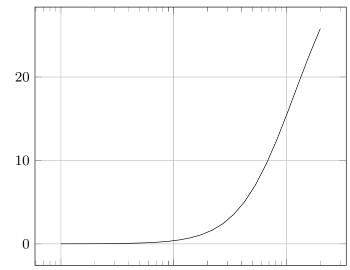
Standard form (zero): $c(1+j\omega\tau)$ $\qquad c$ is gain

Standard form (pole): $\dfrac{1}{1+j\omega\tau}$ $\qquad$ Corner freq: $\omega = \dfrac{1}{\tau}$

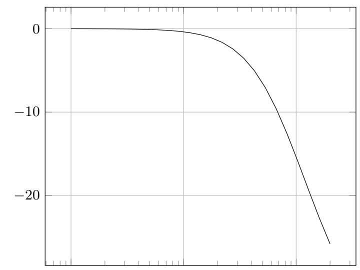
Max error (3 dB) at corner freq

	zero	pole
$\omega\tau < 1$	0 dB	0 dB
$\omega\tau = 1$	0 dB	0 dB
$\omega\tau > 1$	$20\log\omega\tau$	$-20\log\omega\tau$

Zero



Pole

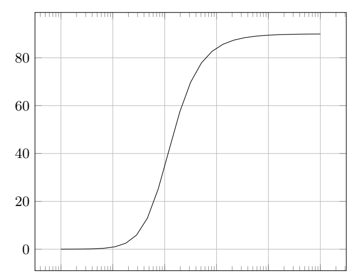


1st Order Phase Response: $\angle H(j\omega) = \pm\angle(1+j\omega\tau) = \arctan(\omega\tau)$

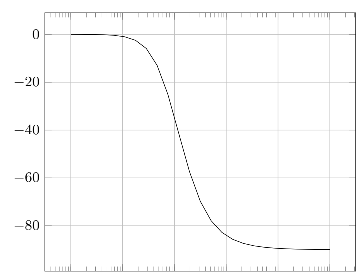
Connect $\dfrac{1}{10\tau} < \omega < \dfrac{10}{\tau}$ w/ straight line.

	zero	pole
$\omega\tau \leq 0.1$	0°	0°
$\omega\tau = 1$	+45°	−45°
$\omega\tau > 10$	+90°	−90°
$0.1 < \omega\tau < 10$	+45 °/dec	−45 °/dec

Zero



Pole



2nd Order Magnitude Response: $H_{\text{dB}} = \pm 20\log|1-(\omega\tau)^2 + j2\zeta(\omega\tau)|$

Standard form: $H(j\omega) = 1 + 2\zeta(j\omega\tau) + (j\omega\tau)^2$

$$s^2 + 2\zeta\omega_0 s + \omega_0^2 = \left(\frac{s}{\omega_0}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right) + 1 = 0, \quad s = j\omega$$

Corner freq: $\omega_0 = 1/\tau \implies H_{\text{dB}} = \pm 20\log(2\zeta)$

	Complex conj zeros	Complex conj poles
$\omega\tau \ll 1$	0 dB	0 dB
$\omega\tau = 1$	$20\log 2\zeta$	$-20\log 2\zeta$
$\omega\tau \gg 1$	+40 dB/dec	−40 dB/dec
Btw regions	Depends on ζ	Depends on ζ

2nd Order Phase Response: $\angle H(j\omega) = \arctan\left(\dfrac{2\zeta(\omega\tau)}{1-(\omega\tau)^2}\right)$

	Complex conj zeros	Complex conj poles
$\omega\tau \ll 1$	0°	0°
$\omega\tau = 1$	+90°	−90°
$\omega\tau \gg 1$	+180°	−180°
Btw regions	Depends on ζ	Depends on ζ

Laplace Transforms

$$\mathcal{L}[f(t)] = \int_{0^-}^{\infty} f(t)e^{-st}\, dt = F(s)$$

$$\mathcal{L}^{-1}[F(s)] = \frac{1}{2\pi j}\int_{\sigma_1 - j\infty}^{\sigma_1 + j\infty} F(s)e^{st}\, ds = f(t)$$

$$\mathcal{L}[f'] = sF(s) - f(0) \qquad \mathcal{L}[f''] = s^2F(s) - sf(0) - f'(0)$$

$$\mathcal{L}[u(t)] = \frac{1}{s} \qquad \mathcal{L}[u(t-t_0)] = \frac{e^{st_0}}{s} \qquad \mathcal{L}[p(t)] = \frac{1-e^{st_0}}{s}$$

$$\mathcal{L}[\delta(t)] = 1 \qquad \mathcal{L}[\delta(t-t_0)] = e^{st_0}$$

IVT: $\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s)$ FVT: $\lim_{t\to\infty} = \lim_{s\to 0} sF(s)$

Poles of $F(s)$ on LHS of s-plane (except poles at $s = 0$)

Partial Fraction Expansion

Real & distinct roots: $\frac{P(s)}{Q(s)} = \frac{K_1}{s+p_1} + \cdots + \frac{K_j}{s+p_j} + \cdots + \frac{K_n}{s+p_n}$

$$K_j = \frac{P(s)}{Q(s)}(s+p_j)|_{s=-p_j}$$

$$\mathcal{L}^{-1}\left[\frac{P(s)}{Q(s)}\right] = u(t)\left(K_1e^{-p_1t} + \cdots + K_je^{-p_jt} + \cdots + K_ne^{-p_nt}\right)$$

Distinct complex roots: $\frac{P(s)}{Q(s)} = \frac{K}{s+(\alpha - j\beta)} + \frac{K^*}{s+(\alpha+j\beta)} + \cdots$

$$K = \frac{P(s)}{Q(s)}(s+\alpha-j\beta)|_{s=-(\alpha-j\beta)} = |K|\angle\theta_K$$

$$\mathcal{L}^{-1}\left[\frac{P(s)}{Q(s)}\right] = u(t)\left[2|K|e^{-\alpha t}\cos(\beta t + \theta_K) + \cdots\right]$$

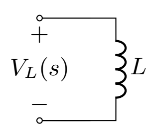
Repeated real roots: $\frac{P(s)}{Q(s)} = \frac{K_1}{(s+p)^n} + \frac{K_2}{(s+p)^{n-1}} + \cdots + \frac{K_n}{s+p}$

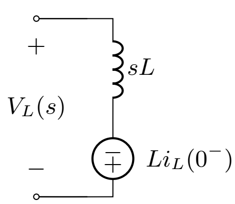
$$K_j = \frac{1}{(j-1)!}\left[\frac{d^{(j-1)}}{ds^{(j-1)}}(s+p)^nF(s)\right]_{s=-p}$$

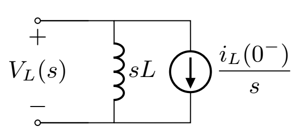
Laplace Domain Model of Inductor

$$\mathcal{L}\{v_L(t)\} = V_L(s) = L[sI_L(s) - i_L(0^-)]$$

$$\mathcal{L}\{i_L(t)\} = I_L(s) = \frac{V_L(s)}{sL} + \frac{i_L(0^-)}{s}$$



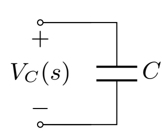


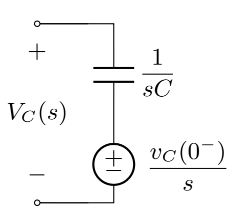


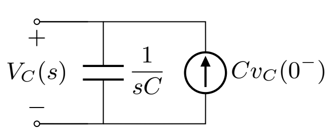
Laplace Domain Model of Capacitor

$$\mathcal{L}\{i_C(t)\} = I_C(s) = C[sV_C(s) - v_C(0^-)]$$

$$\mathcal{L}\{v_C(t)\} = \frac{I_C(s)}{sC} + \frac{v_c(0^-)}{s}$$







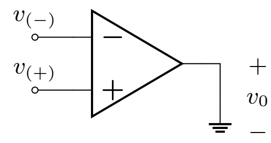
Impulse Response

$$Y(s) = H(s)X(s)$$

$$x(t) = \delta(t) \implies \mathcal{L}[\delta(t)] = 1 \implies y(t) = \mathcal{L}^{-1}[H(s)] = h(t)$$

Given $h(t)$, $y(t) = \mathcal{L}^{-1}[\mathcal{L}[h(t)]\mathcal{L}[x(t)]]$

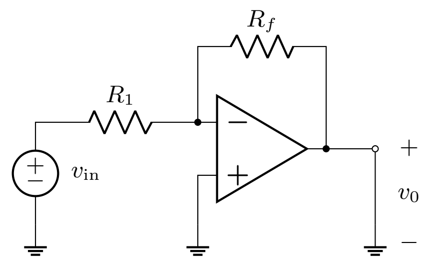
Op-Amps



Assumptions

1. $i_{(+)} = i_{(-)} = 0$
2. $v_{(+)} - v_{(-)} = 0$
3. Gain $A \approx \infty \implies v_{(+)} = v_{(-)}$
4. $\beta \approx \infty \implies A = \text{const}$

Inverting Amplifer

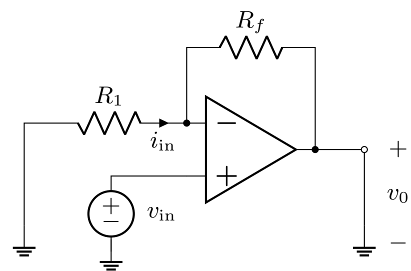


$$\frac{v_0}{v_{\text{in}}} = -\frac{R_f}{R_1} = A_f$$

$$v_0 = A_f v_{\text{in}}$$

A_f called closed-loop (feedback) gain

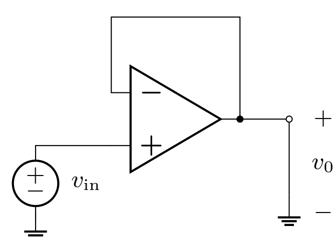
Non-Inverting Amplifier



$$\frac{0 - v_{\text{in}}}{R_1} = \frac{v_{\text{in}} - v_0}{R_f}$$

$$\frac{v_0}{v_{\text{in}}} = 1 + \frac{R_f}{R_1} = A_f$$

Voltage Follower (Unity Gain Buffer)

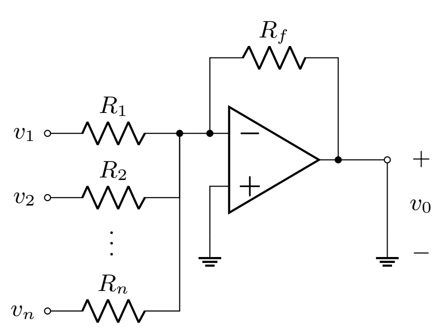


$$R_f = 0\,\Omega,\ R_1 \to \infty$$

$$v_0 = v_{\text{in}}$$

$$A_f = 1$$

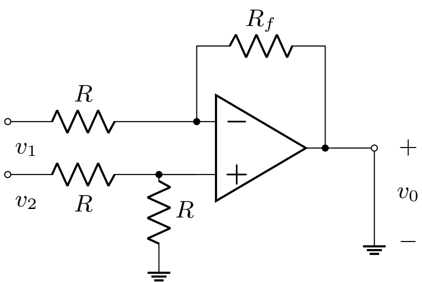
Summing Amplifier



$$\frac{v_1 - 0}{R} + \frac{v_n - 0}{R} + \cdots + \frac{v_n - 0}{R} = \frac{0 - v_0}{R_f}$$

$$v_0 = -\frac{R_f}{R}(v_1 + v_2 + \cdots + v_n)$$

Difference Amplifier



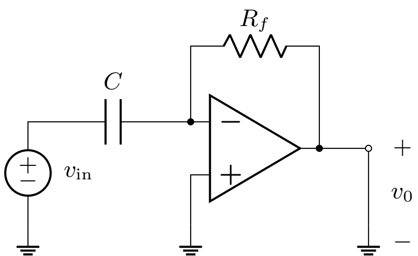
$v_{(+)} = \frac{v_2}{2} = v_{(-)}$

$i_1 = \frac{v_1 - v_2/2}{R}$

$v_0 = \frac{v_2}{2} - i_1 R$

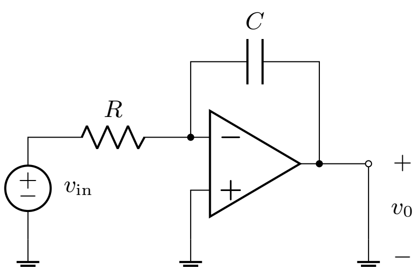
$v_0 = v_2 - v_1$

Differentiator



$$i_{\text{in}} = i_f = C\frac{dv_{\text{in}}}{dt} = \frac{0 - v_0}{R_f} \implies v_0 = -R_f C\frac{dv_{\text{in}}}{dt}$$

Integrator



$$\frac{v_{\text{in}}}{R} = -C\frac{dv_0}{dt} \implies v_0 = -\frac{1}{RC}\int_0^t v_{\text{in}}(x)\ dx$$

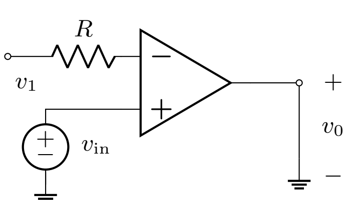
Non-Ideal Op-Amps

$$\frac{v_0}{v_s} = \frac{1}{1 + \frac{R_i}{R_0 + AR_i}}$$

$A \to \infty$ for ideal case ($v_0/v_s = 1$)

$v_0 = A(v_{(+)} - v_{(-)}) = A\Delta v,\ i_{\text{in}} = \Delta v/R_i$

Comparator



$v_0 = A(v_{\text{ref}} - v_{\text{in}})$

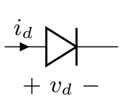
For E^+ and E^- to op-amp,

$v_{\text{ref}} > v_{\text{in}} \implies v_0 = E^+$

$v_{\text{ref}} < v_{\text{in}} \implies v_0 = E^-$

Since A large, any diff in v_{ref} and v_{in} will drive o/p to saturation.

Diode

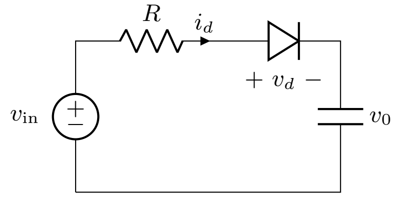


$$i_D = I_S(e^{qv_0(t)/(nkT)} - 1)$$

If $v < 0$, diode acts as open circuit ($i = 0$); diode is reverse-biased.

If $v > 0$, diode acts as short circuit ($v = 0$); diode is forward-biased.

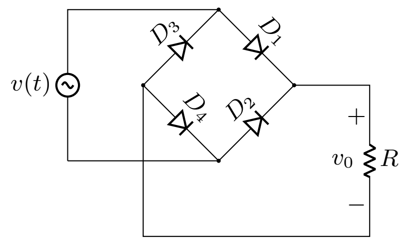
Half-Wave Voltage Rectifier



Diode conducts when $v_d = v_{\text{in}} - v_0 > 0 \implies v_{\text{in}} > v_0$

$i_{\text{d, max}} = \frac{v_{\text{in,max}} - v_0}{R}$

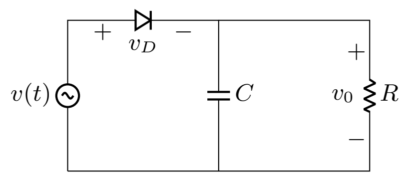
Full-Wave Voltage Rectifier



$v(t) = V_m \sin\omega t$

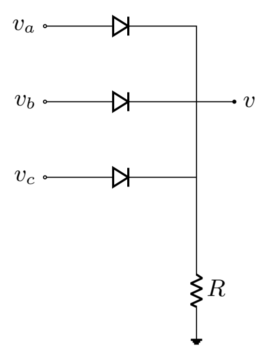
$$v_0 = \begin{cases} v_{\text{in}}, & v_{\text{in}} > 0 \\ -v_{\text{in}}, & v_{\text{in}} < 0 \end{cases}$$

Diode Rectification w/ Capacitive O/P Filter

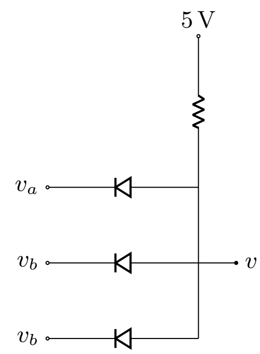


Diodes as Logic Circuits

OR gate

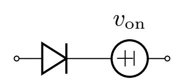


AND gate



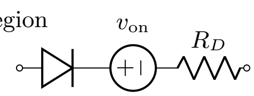
Non-Ideal Diode Models

Const-voltage model: $v_D = v_{\text{on}}$ in FB region

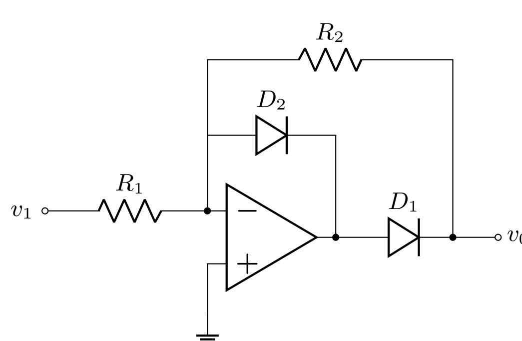


Src-resistor model: $v_D = v_{\text{on}} + i_D R_D$ in FB region

$P = iv_D + i^2 R_D$



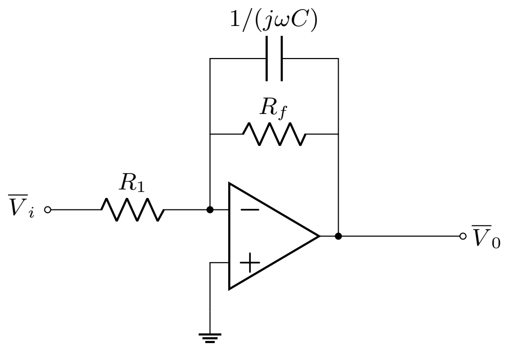
Precision Half-Wave Rectifier



$v_1 > 0$: D_1 RB and D_2 FB $\implies v_0 = 0$ (i to v_0 ground)

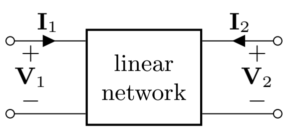
$v_1 < 0$: D_1 FB and D_2 RB $\implies v_0 = -\dfrac{R_2}{R_1} v_1$ (i to v_1 ground)

LPF



$$\frac{\overline{V}_0}{\overline{V}_i} = -\frac{R_f}{R_1}\left(\frac{1}{1 + j\omega R_f C}\right)$$

Two Port Networks



Admittance Parameters

$$\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix} = \begin{bmatrix}\mathbf{y}_{11} & \mathbf{y}_{12}\\ \mathbf{y}_{21} & \mathbf{y}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix}$$

$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} \qquad \mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

$$\mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} \qquad \mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

All units in [S]

$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2$

$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$

Can also compute by finding currents using nodal analysis.

$\mathbf{y}_{11}$: short ciruit i/p admittance

$\mathbf{y}_{22}$: short ciruit o/p admittance

$\mathbf{y}_{12}$, $\mathbf{y}_{21}$: short ciruit transfer admittance

Impedance Parameters

$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix} = \begin{bmatrix}\mathbf{z}_{11} & \mathbf{z}_{12}\\ \mathbf{z}_{21} & \mathbf{z}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}$$

$$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} \qquad \mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

$$\mathbf{z}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} \qquad \mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

All units in [Ω]

$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$

$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$

$\mathbf{z}_{11}$: short ciruit i/p impedance

$\mathbf{z}_{22}$: short ciruit o/p impedance

$\mathbf{z}_{12}$, $\mathbf{z}_{21}$: short ciruit transfer impedance

Hybrid Parameters

$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{I}_2\end{bmatrix} = \begin{bmatrix}\mathbf{h}_{11} & \mathbf{h}_{12}\\ \mathbf{h}_{21} & \mathbf{h}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{V}_2\end{bmatrix}$$

$$\mathbf{h}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} \qquad \mathbf{h}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$

$$\mathbf{h}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} \qquad \mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$

$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$

$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$

$\mathbf{h}_{11}$: short circuit input impedance [Ω]

$\mathbf{h}_{22}$: open circuit input admittance [S]

$\mathbf{h}_{12}$: open circuit input reverse V gain [pu]

$\mathbf{h}_{21}$: short circuit forward I gain [pu]

Transmission (ABCD) Parameters

$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{I}_1\end{bmatrix} = \begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{C} & \mathbf{D}\end{bmatrix}\begin{bmatrix}\mathbf{V}_2\\ -\mathbf{I}_2\end{bmatrix}$$

$$\mathbf{A} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} \qquad \mathbf{B} = -\left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0}$$

$$\mathbf{C} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} \qquad \mathbf{D} = \left.\frac{\mathbf{I}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0}$$

$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$

$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$

A: open circuit V ratio [pu]

B: -ve short circuit transfer impedance [Ω]

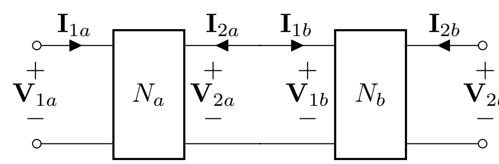
C: open circuit transfer admittance [S]

D: -ve short circuit I ratio [pu]

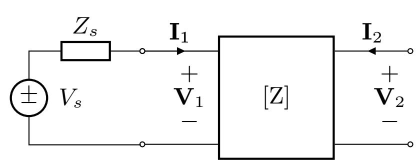
Cascading Two Port Networks

Most conveniently modelled using ABCD parameters.

$$\begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{C} & \mathbf{D}\end{bmatrix} = \begin{bmatrix}\mathbf{A}_a & \mathbf{B}_a\\ \mathbf{C}_a & \mathbf{D}_a\end{bmatrix}\begin{bmatrix}\mathbf{A}_b & \mathbf{B}_b\\ \mathbf{C}_b & \mathbf{D}_b\end{bmatrix}$$



Thevenin Equivalent System



$$\begin{cases}\mathbf{V}_1 = \mathbf{I}_1\mathbf{z}_{11} + \mathbf{I}_2\mathbf{z}_{12}\\ \mathbf{V}_2 = \mathbf{I}_1\mathbf{z}_{21} + \mathbf{I}_2\mathbf{z}_{22}\\ \mathbf{V}_s = \mathbf{V}_1 + \mathbf{I}_1\mathbf{z}_s\end{cases} \implies \mathbf{I}_1 = \frac{\mathbf{V}_s - \mathbf{I}_2\mathbf{z}_{12}}{\mathbf{z}_s + \mathbf{z}_{11}}$$

$$\mathbf{V}_2 = \underbrace{\frac{\mathbf{z}_{21}}{\mathbf{z}_{11} + \mathbf{z}_s}\mathbf{V}_s}_{\mathbf{V}_{\text{th}}} + \underbrace{\left(\mathbf{z}_{22} - \frac{\mathbf{z}_{12}\mathbf{z}_{21}}{\mathbf{z}_{11} + \mathbf{z}_s}\right)}_{\mathbf{z}_{\text{th}}}\mathbf{I}_2$$

LT Properties

Operation	Time domain	Laplace domain
Scaling	$f(at), \quad a \geq 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3f}{dt^2}$	$s^3F(s) - s^2f(0^-) - sf'(0^-) - f''(0^-)$
Time integration	$\int_{0^-}^{t} f(x)\ dx$	$\frac{1}{s}F(s)$
	$\int_{-\infty}^{t} f(x)\ dx$	$\frac{1}{s}F(s) + \frac{1}{s}\int_{-\infty}^{0^-} f(t)\ dt$
Time shift	$f(t-t_0)u(t-t_0)$	$F(s)e^{-st_0}, \quad t_0 \geq 0$
Frequency shift	$f(t)e^{s_0 t}$	$F(s - s_0)$
Frequency differentiation	$-tf(t)$	$\frac{dF(s)}{ds}$
Initial value thm	$f(0^+)$	$\lim_{s\to\infty} sF(s)$
Final value thm	$\lim_{t\to\infty} f(t)$	$\lim_{s\to 0} sF(s)$
		(poles of $sF(s)$ in LHP)

Unilateral LT of Important Functions

	$f(t)$	$F(s)$		$f(t)$	$F(s)$
1	$\delta(t)$	1	7	$t^n e^{\lambda t}u(t)$	$\frac{n!}{(s-\lambda)^{n+1}}$
2	$u(t)$	$\frac{1}{s}$	8	$\cos(bt)u(t)$	$\frac{s}{s^2+b^2}$
3	$tu(t)$	$\frac{1}{s^2}$	9	$\sin(bt)u(t)$	$\frac{b}{s^2+b^2}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$	10	$e^{-at}\cos(bt)u(t)$	$\frac{s+a}{(s+a)^2+b^2}$
5	$e^{\lambda t}u(t)$	$\frac{1}{s-\lambda}$	11	$e^{-at}\sin(bt)u(t)$	$\frac{b}{(s+a)^2+b^2}$
6	$te^{\lambda t}u(t)$	$\frac{1}{(s-\lambda)^2}$			

Parameter Conversion

$\Delta \equiv$ determinant

$\begin{bmatrix}\mathbf{z}_{11} & \mathbf{z}_{12}\\ \mathbf{z}_{21} & \mathbf{z}_{22}\end{bmatrix}$	$\dfrac{1}{\Delta Y}\begin{bmatrix}\mathbf{y}_{11} & -\mathbf{y}_{12}\\ -\mathbf{y}_{21} & \mathbf{y}_{11}\end{bmatrix}$	$\dfrac{1}{\mathbf{C}}\begin{bmatrix}\mathbf{A} & \Delta T\\ \mathbf{1} & \mathbf{D}\end{bmatrix}$	$\dfrac{1}{\mathbf{h}_{22}}\begin{bmatrix}\Delta H & \mathbf{h}_{12}\\ -\mathbf{h}_{21} & 1\end{bmatrix}$
$\dfrac{1}{\Delta Z}\begin{bmatrix}\mathbf{z}_{22} & -\mathbf{z}_{12}\\ -\mathbf{z}_{21} & \mathbf{z}_{11}\end{bmatrix}$	$\begin{bmatrix}\mathbf{y}_{11} & \mathbf{y}_{12}\\ \mathbf{y}_{21} & \mathbf{y}_{22}\end{bmatrix}$	$\dfrac{1}{\mathbf{B}}\begin{bmatrix}\mathbf{D} & -\Delta T\\ -1 & \mathbf{A}\end{bmatrix}$	$\dfrac{1}{\mathbf{h}_{11}}\begin{bmatrix}1 & -\mathbf{h}_{12}\\ \mathbf{h}_{21} & \Delta H\end{bmatrix}$
$\dfrac{1}{\mathbf{z}_{21}}\begin{bmatrix}\mathbf{z}_{11} & \Delta Z\\ 1 & \mathbf{z}_{22}\end{bmatrix}$	$-\dfrac{1}{\mathbf{y}_{21}}\begin{bmatrix}\mathbf{y}_{22} & 1\\ \Delta Y & \mathbf{y}_{11}\end{bmatrix}$	$\begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{C} & \mathbf{D}\end{bmatrix}$	$-\dfrac{1}{\mathbf{h}_{21}}\begin{bmatrix}\Delta H & \mathbf{h}_{11}\\ \mathbf{h}_{22} & 1\end{bmatrix}$
$\dfrac{1}{\mathbf{z}_{22}}\begin{bmatrix}\Delta Z & \mathbf{z}_{12}\\ -\mathbf{z}_{21} & 1\end{bmatrix}$	$\dfrac{1}{\mathbf{y}_{11}}\begin{bmatrix}1 & -\mathbf{y}_{12}\\ \mathbf{y}_{21} & \Delta Y\end{bmatrix}$	$\dfrac{1}{\mathbf{D}}\begin{bmatrix}\mathbf{B} & \Delta T\\ -1 & \mathbf{C}\end{bmatrix}$	$\begin{bmatrix}\mathbf{h}_{11} & \mathbf{h}_{12}\\ \mathbf{h}_{21} & \mathbf{h}_{22}\end{bmatrix}$