Trigonometry

Sine law: $\frac{\sin \mathrm{A}}{\sin \mathrm{B}} = \frac{a}{b}$ Cosine law: $c^2 = a^2 + b^2 - 2ab\cos \mathrm{C}$

Vectors

$\mathbf{F} = \mathbf{F_x} + \mathbf{F_y} + \mathbf{F_z} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k} = F(\cos\alpha\mathbf{i} + \cos\beta\mathbf{j} + \cos\gamma\mathbf{k})$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Position vector: $\mathbf{r}_{AB} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$

Unit position vector: $\mathbf{u} = \frac{\mathbf{r}}{||\mathbf{r}||}$

$\mathbf{F} = F\mathbf{u}$ ($\mathbf{u}$ is in the direction of $\mathbf{F}$)

Dot Product

$\langle \mathbf{A}, \mathbf{B} \rangle = \mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z = AB\cos\theta$

$\cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{\mathrm{AB}}$

Vector directed along a line: $A_{||} = A\cos\theta = \mathbf{A}\cdot\mathbf{u}$

Determinants

Second order:

$$\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1b_1 - a_2b_2$$

Third order:

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2\begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

Cross Product

Magnitude of cross product: $C = AB\sin\theta$

$\mathbf{C} = \mathbf{A}\times\mathbf{B} = (AB\sin\theta)\mathbf{u_c}$ $\mathbf{u_c} \perp \mathbf{A}, \mathbf{B}$ by the RHR

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Moments

$M = Fd$ (scalar form) $\mathbf{M} = \mathbf{r}\times\mathbf{F}$ (vector form)

Moment about a specified axis:

- Scalar form: $M = Fd$
- Vector form: $\mathbf{M_a} = [\mathbf{u_a}\cdot(\mathbf{r}\times\mathbf{F})]\mathbf{u_a}$

Shear Force and Bending Moment Diagrams (SFD & BMD)

Shear and load: $\frac{dV}{dx} = -w(x)$ $\int_a^b dV = \int_a^b -w(x)dx$

Moment and shear: $\frac{dM}{dx} = V(x)$ $\int_a^b dM = \int_a^b V(x)dx$

Concentrated loads: +ve if point load ↑ -ve if point load ↓

Couple moments: +ve if ↺ (CW) -ve if ↻ (CCW)

Friction

Max static friction force: $F_{\max} = \mu_s N$

Kinetic friction force: $F = \mu_k N$

Angle of static friction: $\tan\phi_s = \frac{\mathbf{F}_s}{\mathbf{N}} = \mu_s$

Center of Gravity and Mass

Center of Gravity (Uniform Body)

$$\bar{x} = \frac{\int \tilde{x}\, dW}{\int dW} \quad \bar{y} = \frac{\int \tilde{y}\, dW}{\int dW} \quad \bar{z} = \frac{\int \tilde{z}\, dW}{\int dW}$$

Center of Gravity (Non-Uniform Body)

$$\bar{x} = \frac{\int \tilde{x}\gamma\, dV}{\int \gamma\, dV} \quad \bar{y} = \frac{\int \tilde{y}\gamma\, dV}{\int \gamma\, dV} \quad \bar{z} = \frac{\int \tilde{z}\gamma\, dV}{\int \gamma\, dV}$$

$$\gamma = \rho g = \frac{mg}{V} = \frac{W}{V} \quad \Rightarrow \quad dW = \gamma\, dV$$

Center of Mass (Uniform Body)

$$\bar{x} = \frac{\int \tilde{x}\, dm}{\int dm} \quad \bar{y} = \frac{\int \tilde{y}\, dm}{\int dm} \quad \bar{z} = \frac{\int \tilde{z}\, dm}{\int dm}$$

Center of Mass (Non-Uniform Body)

$$\bar{x} = \frac{\int \tilde{x}\rho\, dV}{\int \rho\, dV} \quad \bar{y} = \frac{\int \tilde{y}\rho\, dV}{\int \rho\, dV} \quad \bar{z} = \frac{\int \tilde{z}\rho\, dV}{\int \rho\, dV}$$

$$\rho = \frac{m}{V} \quad \Rightarrow \quad dm = \rho\, dV$$

Centroids

Centroid of a Volume

$$\bar{x} = \frac{\int_V \tilde{x}\, dV}{\int_V dV} \quad \bar{y} = \frac{\int_V \tilde{y}\, dV}{\int_V dV} \quad \bar{z} = \frac{\int_V \tilde{z}\, dV}{\int_V dV}$$

Centroid of an Area

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} \quad \bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA} \quad \bar{z} = \frac{\int_A \tilde{z}\, dA}{\int_A dA}$$

Centroid of a Line

$$\bar{x} = \frac{\int_L \tilde{x}\, dL}{\int_L dL} \quad \bar{y} = \frac{\int_L \tilde{y}\, dL}{\int_L dL} \quad \bar{z} = \frac{\int_L \tilde{z}\, dL}{\int_L dL}$$

Centroids by Composite Bodies

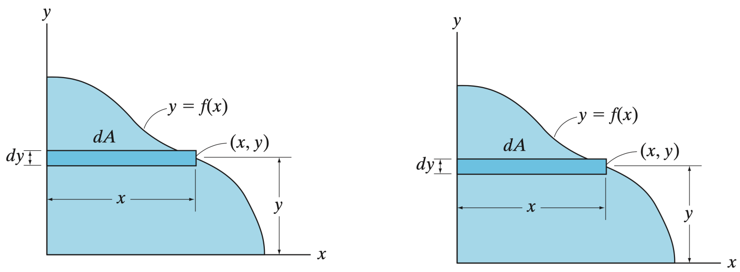
$$\bar{x} = \frac{\sum \tilde{x}W}{\sum W} \quad \bar{y} = \frac{\sum \tilde{y}W}{\sum W} \quad \bar{z} = \frac{\sum \tilde{z}W}{\sum W}$$

Set $A < 0$ for holes.

section	$\tilde{x}$	A	$\tilde{x}A$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
Sum	n/a	$\cdots$	$\cdots$

. . . .

Area Moments of Inertia



(a) $I_x = \int_A y^2 \, dA$ **(b)** $I_y = \int_A x^2 \, dA$

Figure 1: Length of dA should be *parallel* to axis about which I is computed.

Polar moment of inertia: $J_0 = I_x + I_y = \int_A r^2 \, dA$

Product moment of inertia: $I_{xy} = \int_A xy \, dA$

Radii of gyration:

$$k_x = \sqrt{\frac{I_x}{A}} \quad k_y = \sqrt{\frac{I_y}{A}} \quad k_0 = \sqrt{\frac{I_0}{A}}$$

Parallel axis theorem:

- $I_x = \bar{I}_{x'} + Ad_y^2$ $\qquad I_y = \bar{I}_{y'} + Ad_x^2$
- $J_0 = \bar{J}_c + Ad^2$ $\qquad I_{xy} = \bar{I}_{x'y'} + Ad_x d_y$

. . . .

Area Moments of Inertia by Composite Bodies

Find the appropriate centroids. Set $A < 0$ for holes.

section	$\tilde{I}_{x'}$	$d = \tilde{x} - x$	A	$\bar{I}_x = Ad^2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Sum	$\cdots$	n/a	n/a	$\cdots$

Geometric Properties of Line and Area Elements