PHYS 281 Formula Sheet

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Electric Fields

Coulomb's law: $\mathbf{E} = \frac{kq}{r^2}\hat{\mathbf{r}}$ $\quad \mathbf{F} = \frac{kq_1q_2}{r^2}\hat{\mathbf{r}}$ $\quad \hat{\mathbf{r}} = \frac{\mathbf{r_0} - \mathbf{r}}{|\mathbf{r_0} - \mathbf{r}|}$

$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9\ \text{Nm}^2/\text{C}^2$ $\quad \epsilon_0 = 8.85 \times 10^{-12}\ \text{C}/(\text{Nm})^2$

Curve: $\mathbf{E} = k\int_C \frac{\hat{\mathbf{r}}}{r^2}\,dq = k\lambda\int_C \frac{\mathbf{r_0} - \mathbf{r}}{|\mathbf{r_0} - \mathbf{r}|^3}\,ds$

$dq = \lambda\,ds = \lambda\frac{d|\mathbf{r}|}{dt}\,dt = \lambda\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}\,dt$

Surface: $\mathbf{E} = \frac{k}{r^2}\iint_S \hat{\mathbf{r}}\,dq = k\sigma\iint_S \frac{\mathbf{r_0} - \mathbf{r}}{|\mathbf{r_0} - \mathbf{r}|^3}\,dS$

$dq = \sigma\,dS = \sigma\,|\mathbf{r_u} \times \mathbf{r_v}|\,du\,dv$

Volume: $\mathbf{E} = \frac{k}{r^2}\iiint_V \hat{\mathbf{r}}\,dq = k\rho\iiint_V \frac{\mathbf{r_0} - \mathbf{r}}{|\mathbf{r_0} - \mathbf{r}|^3}\,dV$

$dq = \rho\,dV = \rho\,dS\,ds$

$\mathbf{r_0}$ is arbitrary position $\quad$ $\mathbf{r}$ is position on curve/surface/volume

Electric Flux

Gauss' law: $\Phi_E = \frac{q_{\text{enc}}}{\epsilon_0} = \oiint_S \mathbf{E}\cdot d\mathbf{S} = \oiint_S E\,dS\cos\theta = \iiint_V \nabla\cdot\mathbf{E}\,dV$

$q_{\text{enc}} = \iiint_V \rho\,dV$ $\quad \nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_0}$

Cylinder: $\Phi_E = 2\pi rhE(r)$ $\quad$ Sphere: $\Phi_E = 4\pi r^2E(r)$

$E(r)$ = electric field $\quad$ r = d from origin $\quad$ h = cylinder length

Electric Potential

Work: $W = |\mathbf{F}|d = q|\mathbf{E}|d = q\mathbf{E}\cdot\mathbf{d} = \int_C q\mathbf{E}\cdot d\mathbf{r}$ $\quad U + W = 0$

Electric potential energy: $U = -\int_C q\mathbf{E}\cdot d\mathbf{r}$

Electric potential: $\Delta V = U/q = -\int_C \mathbf{E}\cdot d\mathbf{r}$

$\nabla\times\mathbf{E} = 0 \implies \oint_C \mathbf{E}\cdot d\mathbf{r} = 0$ $\quad \mathbf{E} = -\nabla V$

For a test charge q: $\quad \mathbf{F} = q\mathbf{E}$ $\quad U = qV$

Capacitors

$C = \left|\frac{q}{\Delta V}\right|$ $\quad C_{\text{parallel}} = \sum_{i=1}^{n} C_i$ $\quad C_{\text{series}} = \left(\sum_{i=1}^{n}\frac{1}{C_i}\right)^{-1}$

Parallel plate: $C = \frac{\epsilon_0 A}{d}$ $\quad$ area A, separated by d

Cylindrical: $C = \frac{\epsilon_0 2\pi L}{\ln(b/a)}$ $\quad$ length L, inner radius a, outer radius b

Spherical: $C = 4\pi\epsilon_0\frac{ab}{b-a}$ $\quad$ inner radius a, outer radius b

E stored by capacitor: $U = \frac{q^2}{2C} = \frac{1}{2}CV^2$

$U = \iiint_V u\,dV = \iiint_V \frac{1}{2}\epsilon_0|\mathbf{E}|^2\,dV$

Current, Resistance, and Power

$i = \frac{dq}{dt} = \iint_S \mathbf{J}\cdot d\mathbf{S}$ $\quad 1\ \text{A} = 1\ \text{C/s}$ $\quad \mathbf{J}$ = current density

Resistance: $R = \frac{V}{i}$ $\quad$ Resistivity: $\rho = \left|\frac{\mathbf{E}}{\mathbf{J}}\right|$ $\quad$ Conductivity: $\sigma = \frac{1}{\rho}$

$\mathbf{J} = \sigma\mathbf{E}$ $\quad \mathbf{E} = \rho\mathbf{J}$

Power: $P = \frac{dU}{dt} = iV$ $\quad$ Power loss: $P = i^2R = \frac{V^2}{R}$

Circuits

Series: $R = \sum_{i=1}^{n} R_i$ $\quad$ Parallel: $R = \left(\frac{1}{R_i}\right)^{-1}$

$q_{\text{eq}} = C\epsilon$ and $\tau = RC$

Charging RC circuit: $\epsilon - iR - \frac{q}{C} = 0 \implies R\frac{dq}{dt} + \frac{1}{C}q = \epsilon$

$q(t) = q_{\text{eq}}(1 - e^{-t/\tau})$ $\quad i(t) = \frac{dq}{dt} = \frac{q_{\text{eq}}}{\tau}e^{-t/\tau} = \frac{\epsilon}{R}e^{-t/\tau}$

Discharging RC circuit: $iR + \frac{q}{C} = 0 \implies R\frac{dq}{dt} + \frac{1}{C}q = 0$

$q(t) = q_{\text{eq}}e^{-t/\tau}$ $\quad i(t) = \frac{dq}{dt} = -\frac{q_{\text{eq}}}{\tau}e^{-t/\tau} = -\frac{q_{\text{eq}}}{RC}e^{-t/\tau}$

Magnetic Fields

$\mathbf{F}_B = q\mathbf{v}\times\mathbf{B}$ $\quad |\mathbf{B}| = \frac{|\mathbf{F}_B|}{qv_\perp} = \frac{|\mathbf{F}_B|}{qv\sin\phi},$ $\quad \phi$ is angle from $\mathbf{v}$ to $\mathbf{B}$

Biot-Savart law: $d\mathbf{B} = \left(\frac{\mu_0 i}{4\pi r^2}\right)d\mathbf{s}\times\hat{\mathbf{r}}$

$\mathbf{B} = \frac{\mu_0 i}{4\pi}\int_C \frac{d\mathbf{s}\times\hat{\mathbf{r}}}{r^2} = \frac{\mu_0 i}{4\pi}\int_a^b \frac{d\mathbf{s}}{dt}\times\frac{\mathbf{r_0} - \mathbf{r}}{|\mathbf{r_0} - \mathbf{r}|^3}\,dt$

$\mathbf{r_0}$ is arbitrary position $\quad$ $\mathbf{r}$ is position on curve

Ampere's law: $\nabla\times\mathbf{B} = \mu_0\mathbf{J}_{\text{enc}}$

$\int_C \mathbf{B}\cdot d\mathbf{S} = \mu_0 i_{\text{enc}} = \mu_0\iint_S \mathbf{J}_{\text{enc}}\,dA$

Induction and Inductance

$\varepsilon = -\frac{d}{dt}\iint_S \mathbf{B}\cdot d\mathbf{S}$ $\quad \Phi_B = \iint_S \mathbf{B}\cdot d\mathbf{S} \implies \varepsilon = -\frac{d\Phi_B}{dt}$

Lenz's law: direction of i induces $\mathbf{B}$ to oppose change in Φ_B

Induced current: $i = \frac{\varepsilon}{R} = \frac{BLv}{R}$

Circuit has speed v $\quad$ length L perp to mvmt of circuit

$P_{\text{in}} = Fv = iLB_0v = \frac{B_0^2L^2v^2}{R}$ $\quad P_{\text{out}} = i^2R = \frac{B_0^2L^2v^2}{R}$

Faraday's law: $\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$

Solenoid: $L = \frac{N\Phi_B}{i} = \mu_0\frac{N^2}{l}\pi R^2 = \mu_0 n^2\pi R^2 l$ $\quad n = N/l$

N = number of turns, l = length, R = radius

$\varepsilon_L = -L\frac{di}{dt}$ $\quad$ Inductance units: $1\ \text{H} = 1\ \text{T m}^2/\text{A}$

RL Circuits

RL charge: $\varepsilon - iR - L\dfrac{di}{dt} = 0 \Longrightarrow \dfrac{di}{dt} + \dfrac{R}{L}i = \dfrac{\varepsilon}{L}$ $\qquad i(0) = 0$

$$i(t) = \frac{\varepsilon}{R}(1 - e^{Rt/L}) = \frac{\varepsilon}{R}(1 - e^{-t/\tau_L}) \qquad \tau_L = \frac{L}{R}$$

RL discharge: $0 - iR - L\dfrac{di}{dt} = 0 \Longrightarrow \dfrac{di}{dt} + \dfrac{R}{L}i = 0$ $\qquad i(0) = i_0 = \dfrac{\varepsilon}{R}$

$$i(t) = i_0 e^{-Rt/L} = \frac{\varepsilon}{R}e^{-t/\tau_L}$$

E: $U_B = \dfrac{1}{2}Li^2$ $\qquad$ E density: $u_B = \dfrac{B^2}{2\mu_0}$

Mutual inductance: $M = \dfrac{N_2\Phi_{B2}}{i_1} = \dfrac{N_1\Phi_{B1}}{i_2}$

$$\varepsilon_2 = -M_{21}\frac{di_1}{dt} \qquad \varepsilon_1 = -M_{12}\frac{di_2}{dt}$$

Self-inductance: $L = \dfrac{N\Phi_B}{i}$ $\qquad \varepsilon = -L\dfrac{di}{dt}$

LRC Circuits

LC (no R, charging): $-L\ddot{q} - \dfrac{1}{C}q = 0$ $\qquad q(0) = q_0,\ i(0) = 0$

$$q(t) = q_0 \cos\omega t \qquad \omega = \frac{1}{\sqrt{LC}}$$

$$i(t) = -\omega q_0 \sin\omega t$$

LRC (weak R, charging): $L\ddot{q} + R\dot{q} + \dfrac{1}{C}q = 0$ $\qquad q(0) = q_0,\ i(0) = 0$

$$q(t) = q_0 e^{-t/\tau}(\cos\omega' t + \frac{1}{\omega'\tau}\sin\omega' t) = Ae^{-t/\tau}\cos(\omega' t + \phi)$$

$$A = q_0\sqrt{1 + \frac{1}{(\omega'\tau)^2}} \qquad \phi = \arctan\frac{1}{\omega'\tau}$$

$$i(t) = -\frac{q_0}{\tau}e^{-t/\tau}\left(\frac{1}{\omega'\tau} + \omega'\tau\right)\sin\omega' t$$

$$\tau = \frac{2L}{R} \qquad \omega' = \sqrt{\omega^2 - \frac{1}{\tau^2}} \qquad \omega = \frac{1}{\sqrt{LC}}$$

LRC w/ battery (weak R, charging): $L\ddot{q} + R\dot{q} + \dfrac{1}{C}q = \varepsilon$

$$q(t) = C\varepsilon\left[1 - e^{t/\tau}\left(\cos\omega' t + \frac{1}{\omega'\tau}\sin\omega' t\right)\right] \qquad q(0) = i(0) = 0$$

$$\lim_{t\to\infty} q(t) = C\varepsilon \qquad \lim_{t\to\infty} i(t) = 0$$

Alternatng Currents

$\varepsilon = \varepsilon_{\max}\sin\omega_d t$ $\qquad i = i_{\max}\sin(\omega_d t - \phi)$

AC + R: $\varepsilon - iR = 0 \Longrightarrow i(t) = \dfrac{\varepsilon_{\max}}{R}\sin\omega_d t$

AC + C: $\varepsilon - \dfrac{1}{C}q = 0$

$$q(t) = C\varepsilon_{\max}\sin\omega_d t = C\varepsilon_{\max}\omega_d\sin(\omega_d t - (-\pi/2))$$

$I = (C\omega_d)\varepsilon_m$ $\qquad$ Capacitive reactance: $X_C = \dfrac{1}{C\omega_d}$

AC + L: $\varepsilon - L\dfrac{di}{dt} = 0$

$$i(t) = -\frac{1}{L\omega_d}\varepsilon_{\max}\cos\omega_d t = \frac{1}{L\omega_d}\varepsilon_{\max}\sin(\omega_d t - \pi/2)$$

$I = \dfrac{\varepsilon_m}{L\omega_d}$ $\qquad$ Inductive reactance: $X_L = \dfrac{1}{L\omega_d}$

AC + LRC: $L\ddot{q} + R\dot{q} + \dfrac{1}{C}q = \varepsilon_{\max}\sin\omega_d t$

$$q(t) = A^{-t/\tau}\cos(\omega' t + \phi_H) + B\cos(\omega_d t - \phi_p)$$

$$B = -\frac{\varepsilon_{\max}}{\omega_d\sqrt{(X_L - X_C)^2 + R^2}} \qquad \phi = \arctan\frac{X_L - X_C}{R}$$

$$\lim_{t\to\infty} q(t) = B\cos(\omega_d t - \phi) \qquad \lim_{t\to\infty} i(t) = -B\omega_d\sin(\omega_d t - \phi)$$

Impedance: $Z = \sqrt{(X_L - X_C)^2 + R^2}$

Power in AC Circuits

$$\langle P_{\text{in}}\rangle = \langle P_{\text{out}}\rangle = \frac{\varepsilon_{\max}^2 R}{2Z^2}$$

$$\langle P_{\text{in}}\rangle = \frac{1}{T}\int_0^T P_{\text{in}}\ dt \qquad \langle P_{\text{out}}\rangle = \frac{1}{T}\int_0^T P_{\text{out}}\ dt$$

$$P_{\text{in}} = i\varepsilon = \left(\frac{\varepsilon_{\max}}{Z}\sin(\omega_d t - \phi)\right)(\varepsilon_{\max\sin\omega_d t})$$

$$P_{\text{out}} = i^2 R = \left(\frac{\varepsilon_{\max}}{Z}\right)^2 R\sin^2(\omega_d t - \phi)$$

$$\varepsilon_{\text{rms}} = \frac{\varepsilon_{\max}}{\sqrt{2}} \qquad i_{\text{rms}} = \frac{i_{\max}}{\sqrt{2}} = \frac{\varepsilon_{\max}}{Z\sqrt{2}}$$

$$\langle A\cos\omega_t\rangle_{\text{rms}} = \langle A\sin\omega_t\rangle_{\text{rms}} = \frac{A}{\sqrt{2}}$$

$$\langle f\rangle_{\text{rms}} = \left[\frac{1}{b-a}\int_a^b [f(t)]^2\ dt\right]^{1/2}$$

Transformers: $\dfrac{N_p}{N_s} = \dfrac{V_p}{V_s} = \dfrac{i_s}{i_p}$

Maxwell's Equations

Integral Form

Gauss' law for electricity: $\displaystyle\oiint_S \mathbf{E}\cdot d\mathbf{S} = \frac{q_{\text{enc}}}{\epsilon_0}$

Gauss' law for magnetism: $\displaystyle\oiint_S \mathbf{B}\cdot d\mathbf{S} = 0$

Faraday's law for induction: $\displaystyle\oint_C \mathbf{E}\cdot d\mathbf{r} = -\frac{\partial\Phi_B}{\partial t}$

Ampere-Maxwell law: $\displaystyle\oint_C \mathbf{B}\cdot d\mathbf{r} = \mu_0\epsilon_0\frac{\partial\Phi_E}{\partial t} + \mu_0 i_{\text{enc}}$

Differential Form

Gauss' law for electricity: $\nabla\cdot\mathbf{E} = \dfrac{\rho}{\epsilon_0}$

Gauss' law for magnetism: $\nabla\cdot\mathbf{B} = 0$

Faraday's law for induction: $\nabla\times\mathbf{E} = -\dfrac{\partial\Phi_B}{\partial t}$

Ampere-Maxwell law: $\nabla\times\mathbf{B} = \mu_0\epsilon_0\dfrac{\partial\mathbf{E}}{\partial t} + \mu_0\mathbf{J}$

Maxwell's Equations in a Vacuum

$$\nabla\cdot\mathbf{E} = 0 \qquad \nabla\cdot\mathbf{B} = 0 \qquad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad \nabla\times\mathbf{B} = \mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t}$$

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 2.998\times 10^7\ \text{m/s}$$

Maxwell's Equations w/ Constant Fields

$$\nabla\cdot\mathbf{E} = 0 \qquad \nabla\cdot\mathbf{B} = 0 \qquad \nabla\times\mathbf{E} = \mathbf{0} \qquad \nabla\times\mathbf{B} = \mathbf{0}$$