ECE 210 Formula Sheet
Eddie Guo

Number Systems

$N_{10} \to N_R$: Do N/R until 0 then build back using remainder.

$(0 < N_{10} < 1) \to N_R$: Do $N \times R$ until number of terms sufficient and build sequentially using the ones digit.

1's complement: add last carry to right $\qquad \overline{N} = (2^n - 1) - N$

2's complement: flip bits then add 1 $\qquad N^* = 2^n - N$

Boolean Algebra

Identities	
Identities	$X + 0 = X$
	$X \cdot 0 = 0$
	$X + 1 = 1$
	$X \cdot 1 = X$
Idempotency	$X + X = X$
	$X \cdot X = X$

Negation	
Complementarity	$X + X' = 1$
	$X \cdot X' = 0$
Involution	$(X')' = X$

Laws	
Commutativity	$A \cdot B = B \cdot A$
	$A + B = B + A$
Associativity	$A \cdot (B \cdot C) = (A \cdot B) \cdot C$
	$A + (B + C) = (A + B) + C$
Distributivity	$A \cdot (B + C) = A \cdot B + A \cdot C$
	$A + B \cdot C = (A + B)(A + C)$

De Morgan's Laws	
$(X \cdot Y)' = X' + Y'$	$\left(\prod_{i=1}^{n} X_i\right)' = \sum_{i=1}^{n} X_i'$
$(X + Y)' = X' \cdot Y'$	$\left(\sum_{i=1}^{n} X_i\right)' = \prod_{i=1}^{n} X_i'$

Theorems	
Uniting	$XY + XY' = X$
	$(X + Y)(X + Y') = X$
Absorption	$X + XY = X$
	$X(X + Y) = X$
Elimination	$X + X'Y = X + Y$
	$X(X' + Y) = XY$
Consensus	$XY + X'Z + YZ = XY + X'Z$
	$(X + Y)(X' + Z)(Y + Z) = (X + Y)(X' + Z)$
Other	$(X + Y)(X' + Z) = XZ + X'Y$

Binary Codes

Be careful about multiple ways to form a number!

Dec	BCD	6311	XS-3	2/5	Gray
0	0000	0000	0011	00011	0000
1	0001	0001	0100	00101	0001
2	0010	0011	0101	00110	0011
3	0011	0100	0110	01001	0010
4	0100	0101	0111	01010	0110
5	0101	0111	1000	01100	0111
6	0110	1000	1001	10001	0101
7	0111	1001	1010	10010	0100
8	1000	1011	1011	10100	1100
9	1001	1100	1100	11000	1101

Look Up Tables

n inputs $\implies$ 2^n entries/states $\implies$ 2^{2^n} possible fns

Caution: Write out the formula (e.g., 2^{2^n}) for full marks

If $n = 6$, how many fns take 2 or fewer inputs?

- Exactly 0 inputs: $2^{2^0} = 2$ fns
- Exactly 1 input: $2^{2^1} - 2^{2^0} = 2$ fns

  Choose 1 of 6 inputs: $N = \binom{6}{1} \times 2 = 6 \times 2 = 12$
- Exactly 2 inputs: $2^{2^2} - 2^{2^1} - 2^{2^0} = 10$ fns

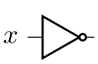
  Choose 2 of 6 inputs: $N = \binom{6}{2} \times 2 = 15 \times 10 = 150$
- $N = 2 + 12 + 150 = 164$ fns total

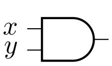
SOP and POS

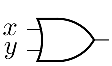
SOP: multiply out $\qquad$ if one term is 1, whole expr is 1

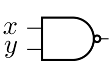
POS: factor $\qquad$ if one term is 0, whole expr is 0

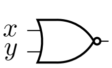
Logic Gates

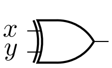
NOT $\quad x$  x'

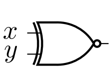
AND $\quad x, y$  xy

OR $\quad x, y$  $x + y$

NAND $\quad x, y$  $(xy)' = x' + y'$

NOR $\quad x, y$  $(x + y)' = x'y'$

XOR $\quad x, y$  $x \oplus y = x'y + xy'$

XNOR $\quad x, y$  $x \otimes y = (x \equiv y) = (x \oplus y)' = xy + x'y'$

Logic Gate Heuristics

NAND is inverted OR $\quad f = (xy)' = x' + y'$

NOR is inverted AND $\quad f = (x+y)' = x'y'$

NAND returns 0 $\iff$ all inputs 1

NOR returns 1 $\iff$ all inputs 0

XOR returns 1 $\iff$ inputs have odd number of 1s

XNOR returns 1 $\iff$ all inputs equivalent

XOR Properties	
$X \oplus 0 = X$	$X \oplus 1 = X'$
$X \oplus X = 0$	$X \oplus X' = 1$
$X \oplus Y = Y \oplus X$	
$(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z)$	
$X(Y \oplus Z) = XY \oplus XZ$	
$(X \oplus Y)' = X \oplus Y' = X' \oplus Y = XY + X'Y' = (X \equiv Y)$	

XNOR Properties	
$X \otimes 0 = X'$	$X \otimes 1 = X$
$X \otimes X = 1$	$X \otimes X' = 0$
$X \otimes X \otimes \cdots \otimes X = \begin{cases} X, & n \text{ odd} \\ 1, & n \text{ even} \end{cases}$	

XOR and XNOR complements if even no of inputs given

$$A \oplus B \oplus C \oplus D = \overline{A \otimes B \otimes C \otimes D}$$

XOR and XOR same if odd no of inputs given

$$A \oplus B \oplus C = A \otimes B \otimes C$$

Minterms and Maxterms

Row	A B C	Minterms	Maxterms
0	0 0 0	$A'B'C'$	$A+B+C$
1	0 0 1	$A'B'C$	$A+B+C'$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
9	1 1 1	ABC	$A'+B'+C'$

Q: Find minterms and maxterms of $f(a,b,c,d) = a'b'$.

A: Since 4 inputs, $2^4 = 16$ possible outputs.

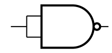
$$\begin{aligned} \text{minterms} &= a'b' \underbrace{(c+c')(d+d')}_{\text{equals 1}} \\ &= \underbrace{a'b'cd}_{0011} + \underbrace{a'b'cd'}_{0010} + \underbrace{a'b'c'd}_{0001} + \underbrace{a'b'c'd'}_{0000} \\ &= \sum m(0,1,2,3) \\ \text{maxterms} &= \prod M(4,5,\ldots,15) \end{aligned}$$

If $f = \sum m(3,4,5,6,7)$,

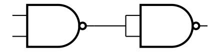
- Maxterm expansion of f is $\prod M(0,1,2)$.
- Minterm expansion of f' is $\sum m(0,1,2)$.
- Maxterm expansion of f' is $\prod M(3,4,5,6,7)$.

NAND as a Universal Gate

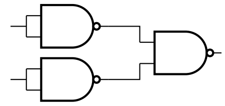
NAND as NOT



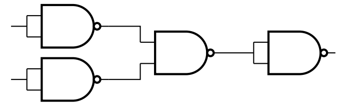
NAND as AND

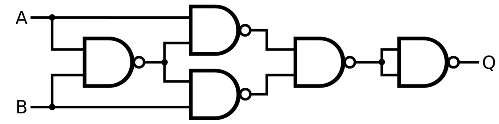


NAND as OR

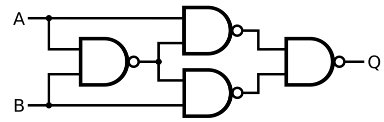


NAND as NOR





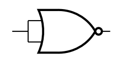
NAND as XNOR



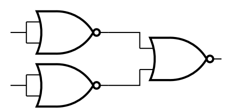
NAND as XOR

NOR as a Universal Gate

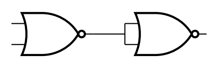
NOR as NOT



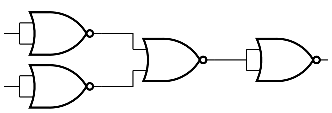
NOR as AND

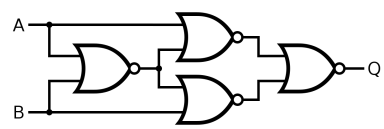


NOR as OR

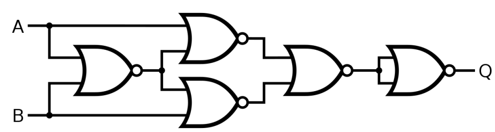


NOR as NAND



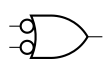
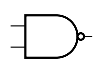


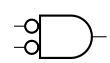
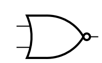
NOR as XNOR



NOR as XOR

Miscellaneous Concepts

 $\equiv$ 

 $\equiv$ 

Transistor count

- NOT: 2
- NAND, NOR: $2 \times$ inputs $\quad$ AND = NAND + NOT
- AND, OR: $2 \times \text{inputs} + 2$ $\quad$ OR = NOR + NOT

Min transistor count: $2 \times \text{inputs} + 2 \times \text{inverters}$

Karnaugh Maps

Implicant: groups of 1s on map in powers of 2 and is rectangular.

Prime implicant: largest possible implicant.

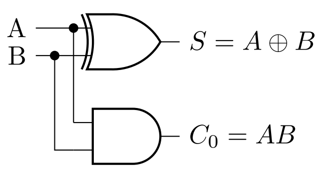
Essential prime implicant: prime implicant that contains at least 1 term not covered by another prime implicant (can overlap).

$bc \backslash a$	0	1
00	1	0
01	X	0
11	1	1
10	0	X

$$f = \sum m(0,3,7) + d(1,6) = a'b' + bc$$
$$= \prod M(2,4,5) + d(1,6) = (a'+b)(b'+c)$$

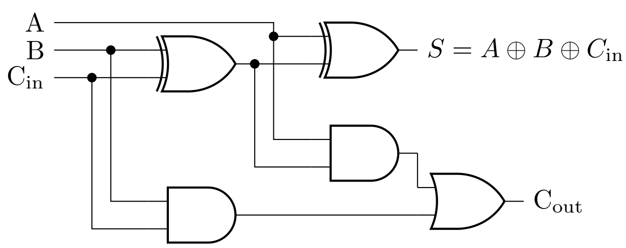
Adders and Subtractors

Half Adder



A	B	S	C_0
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

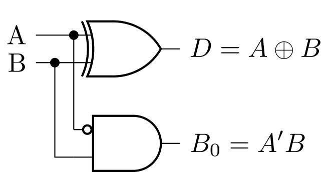
Full Adder



$C_{out} = AB + AC_{in} + BC_{in} = AB + C_{in}(A \oplus B)$

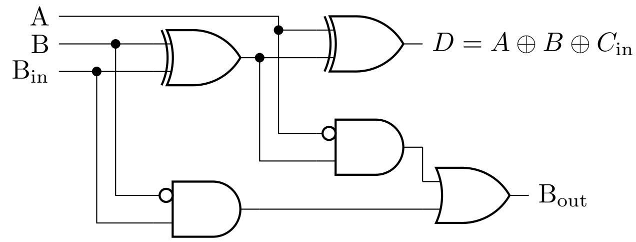
A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Half-Subtractor



A	B	D	B_0
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

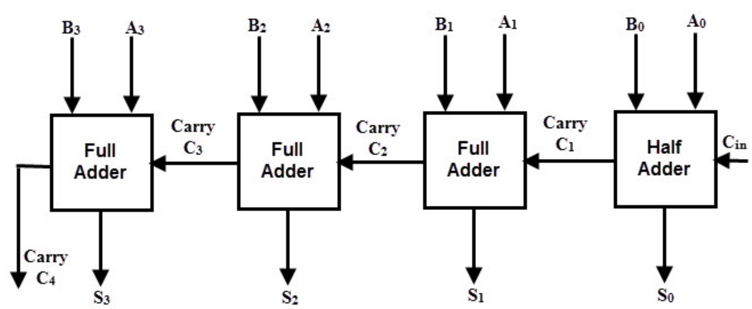
Full Subtractor



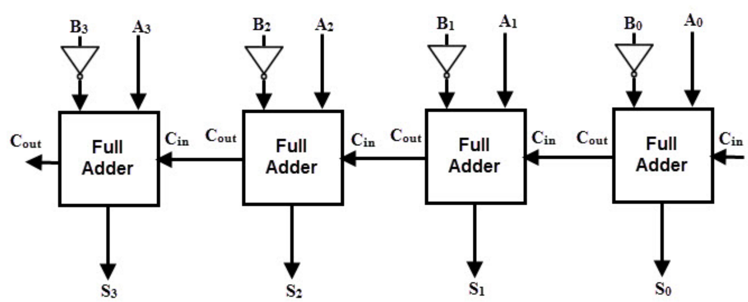
$B_{out} = A'B + A'B_{in} + BB_{in}$

A	B	B_{in}	D	B_{out}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Parallel Binary Adder

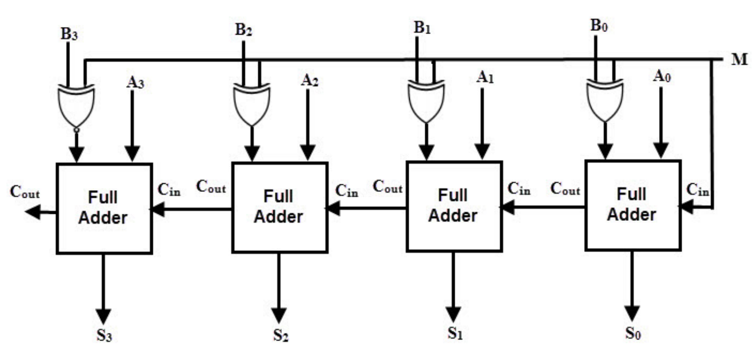


Parallel Binary Subtractor



Adder and Subtractor

M is control bit. If subtracting, M adds 1 and inverts all B bits. If adding, M is 0 (i.e., does nothing).



Design of Multi-Level NAND- and NOR-Gate Circuits

1. Design multi-lvl circuit w/ AND and OR gates.
    - Output must be an OR gate.
    - Each lvl must alternate btw AND and OR.
2. Number all lvls w/ output as lvl 1. Replace all gates w/ NAND gates.
    - Leave inputs to even lvls unchanged.
    - Invert literal inputs to odd lvls.

Procedure for NOR same except (i) output is AND and (ii) all gates replaced w/ NOR gates.