Stat4DS Homework 02 – Due Wednesday, November 02, 2016, 04:00 PM on Moodle

General Instructions

For the exercises involving `R`, I expect you to upload a **running** `R Markdown` file (`.rmd`), named with your group ID, to Moodle. You will give the commands to answer each question in its own code block, which will also produce plots that will be automatically embedded in the output file. Your responses must be supported by both textual explanations and the code you generate to produce your results. *Just examining your various objects in the "Environment" section of RStudio is insufficient – you must use scripted commands and functions.*

For the other exercises, those with possibly no `R` involved, you may scan your **readable** manuscript or use any other solution you like (even `R Markdown` itself if you know how to write math formulas in LaTex) as long as, in the end, you will be able to upload on Moodle this part also.

R Markdown Test

To be sure that everything is working fine, start `RStudio` and create an empty project called `HW1`. Now open a new `R Markdown` file (`File > New File > R Markdown...`); set the output to `HTML mode`, press `OK` and then click on `Knit HTML`. This should produce a web page with the knitting procedure executing the default code blocks. You can now start editing this file to produce your homework submission.

Please Notice

For more info on `R Markdown` check the slides on moodle and/or support webpage that explains the main steps and ingredients: R Markdown from RStudio.

Part I: Exercises from the blue book

Remark: In the following, if you need to evaluate quantiles or tail probabilities, use `R` not book–tables.

0. **Old Topics: Chapters 3–4**
   a. **Do**: Ch.3, page 39, exercise 3.16.
1. **Topic: Continuous random variables – Chapter 5**
   a. *Warm–up*: read Ch.5 + solved–exercises at page 68–70: 5.4, 5.5, 5.12, 5.13.
   b. **Do**: page 68–70: 5.1, 5.3, 5.7, 5.11, 5.14.
2. **Topic: Expectation and variance – Chapter 7**
   a. *Warm–up*: read Ch.7 + solved–exercises at page 68–70: 7.2, 7.5, 7.6, 7.10, 7.11.
   b. **Do**: page 68–70: 7.3, 7.4, 7.7.
3. **Topic: Joint distributions and independence – Chapter 9**
   a. *Warm–up*: read Ch.9 + solved–exercises at page 127–134: 9.2, 9.6, 9.9, 9.11.
   b. **Do**: page 127–134: 9.1, 9.3, 9.7, 9.12.
4. **Topic: Covariance and Correlation – Chapter 10**
   a. *Warm–up*: read Ch.10 + solved–exercises at page 144–150: 10.1, 10.9, 10.10, 10.14.
   b. **Do**: page 144–150: 10.3, 10.8, 10.15.

Part II: Exercises from outer space

1. Suppose that X has a Unif(0,1) distribution. Construct a random variable $Y = g(X)$ for some deterministic function $g(\cdot)$ such that its PDF will be:

$$f_Y(y) = \begin{cases} \frac{3}{8}y^2 & \text{for } y \in (0,2), \\ 0 & \text{otherwise.} \end{cases}$$

2. In a *die–coin experiment*, a fair die is rolled and then a fair coin is tossed the number of times showing on the die. Let N denote the die score and X the number of heads. Find $\mathbb{E}(X)$ and $\mathbb{V}ar(X)$.

3. Let Ω be the sample space and A and B two events in Ω. Although we did not say this explicitly, we can define covariance and correlation for events too. The trick is to attach to each event a suitable random variable, and then apply the definition. The obvious choice for these random variable are indicator functions, that is, given an event $A \subset \Omega$, define

$$\mathbf{1}_A = \begin{cases} 1 & \text{if the event } A \text{ occurs;} \\ 0 & \text{otherwise, that is, if } A^{\mathsf{c}} \text{ occurs.} \end{cases}$$

   Since A is random, $\mathbf{1}_A$ is also random. Based on this little trick, answer the following questions:

   a. What is the distribution of the random variable $Y = \mathbf{1}_A$?
   b. Based on the previous answer, show that

$$\mathbb{C}ov(A, B) = \mathbb{P}(A \cap B) - \mathbb{P}(A) \cdot \mathbb{P}(B).$$

   c. Give a formula for the correlation coefficient $\rho_{A,B}$.
