

Welcome Test

Stat4DS2+DS (key: *tardella2017*)

<https://elearning2.uniroma1.it/course/view.php?id=4951>

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0. Do you know how to use an RMarkdown file to answer these questions with text and R code?

Yes, I Know

1. Fix the *seed* to start the simulation of a single random variable from a (standard) uniform distribution on $[0,1]$. Use your *matricola* number as seed. Call the simulated value **lambda**

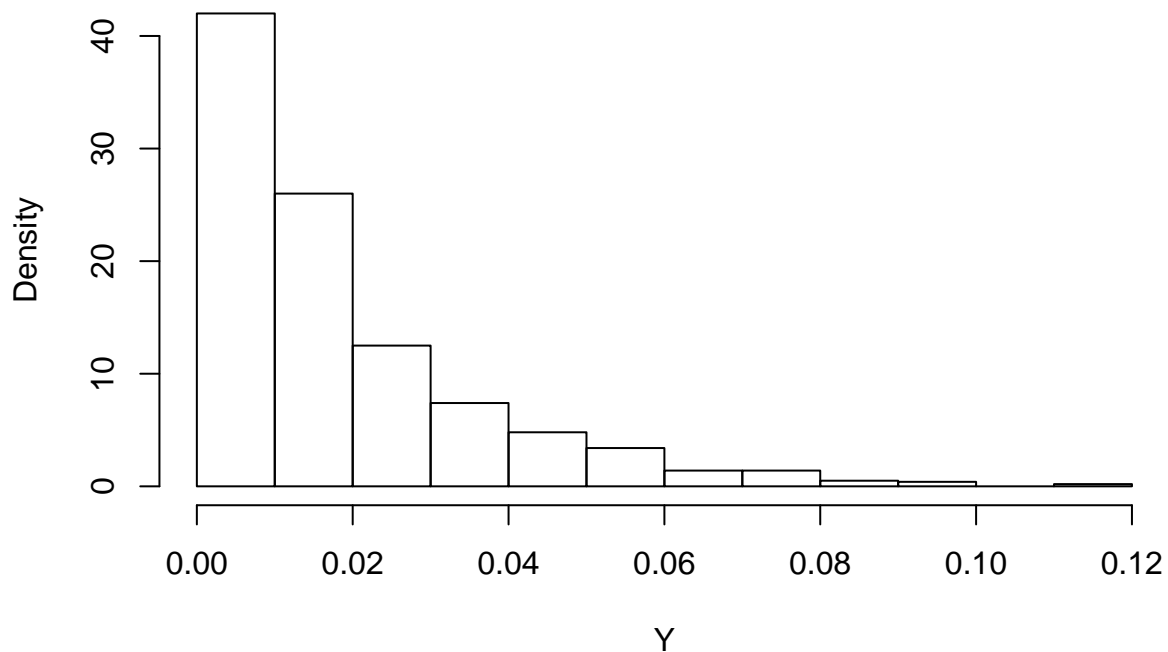
```
set.seed(1388371)
lambda = runif(1)
lambda
```

```
## [1] 0.01909172
```

2. Simulate 1000 random variables using the following code

```
X = runif(1000)
Y = - lambda * log(X)
hist(Y, freq=FALSE)
```

Histogram of Y

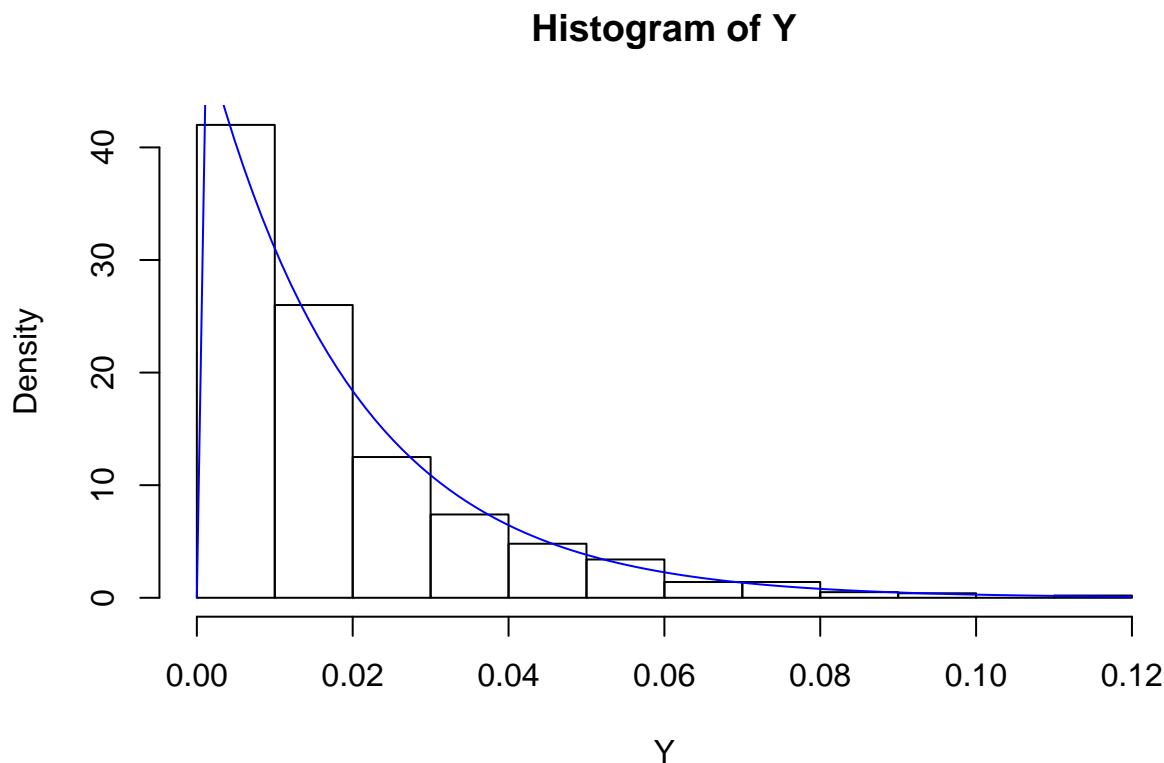


3. Can you show that the theoretical distribution of Y is exponential with rate equal to $1/\text{lambda}$? [explain or ... do it!]

The CDF of an exponential distribution is $F(x) = 1 - e^{-\lambda x}$, and $F^{-1}(x) = -\frac{1}{\lambda} \ln(1 - x)$, so we are sampling from the CDF of the exponential distro, where $\lambda = 1/\text{lambda}$.

4. Can you plot a graphical representation of the theoretical distribution?

```
hist(Y, freq=FALSE)
curve(dexp(x, rate = 1/lambda), add=TRUE, col='blue')
```



5. Can you tell what is the relation between the theoretical curve of the density of Y and the graph of the histogram?

How I demonstrated earlier, Y is a sampling of the exponential distribution with parameter $1/\text{lambda}$. So the theoretical distribution has to be Asymptotic distribution for the histogram, in fact Y_i converge on distribution on Y . ($Y_i \sim Y$)

6. Suppose we have a random vector (Z, Y) Do you know what a joint density $f_{Z,Y}(z, y)$ is? What are the properties it must satisfy?

A joint density is a function that have as domain the random vector and as codominio a number $\in \mathbb{R}^+$.

Properties are :

$$f_{Z,Y}(z, y) \geq 0 \quad \forall z, y$$

$$\int \int f_{Z,Y}(z, y) dz dy = 1$$

$$P(Z, Y \in R) = \int \int_R f_{Z,Y}(z, y) dz dy$$

7. Can you derive in formula the distribution of Z ? How it is called?

The distribution Z is the marginal distribution of the joint density and we can write resolving:

$$f_z(z) = \int f_{z,y}(z, y) dy = \int f_{z|y}(z|y) f_y(y) dy$$

8. Conditionally on each simulated $Y_i = y_i$ ($i=1,2,\dots,1000$), simulate $Z_i = y_i * W_i$ where W_i has distribution standard Normal and is independent of Y_i . What is the conditional distribution of $Z|Y = y$?

We know that :

$$f_z(z) = f_{y,w}(y, w) = f_y(y) \cdot f_w(w)$$

the last equality comes from independence. So, for all $y_i \in Y_i$:

$$f_{z|y=y_i}(w) = f_y(y_i) \cdot f_w(w) = f_y(y_i) \cdot N(0, 1) = N(0, y_i^2)$$

the last equality comes from $Z = cost \cdot W$ where:

$$E(Z) = cost \cdot E(W) = 0$$

$$Var(Z) = cost^2 \cdot Var(W) = cost^2$$

9. Can you provide a simulation from the marginal distribution of Z ?

```
sim = rep(NA, 10000)
prov=rep(NA,10000)
for(i in 1:10000){
  y_i = rexp(1, rate=1/lambda)
  prov[i]=y_i
  z = rnorm(1, sd= y_i^2)
  sim[i]=z
}
```

Where `sim` is the simulation from the marginal Z distribution because we are sampling `y_i` from an exponential distribution, so randomly, and then we are sampling from $W|Y=y$.

10. Can you provide an approximation of the marginal distribution of Z ? Please justify your answer.

Because $Z = Y \cdot W$ and for the independence of Y and W :

$$f_z(z) = \int f_w(w) \cdot f_y(z/w) \cdot \frac{1}{|w|} dw$$

in our case:

$$f_z(z) = \int \frac{1}{\sqrt{2\pi}} e^{-\frac{w^2}{2}} \cdot \lambda e^{-\lambda \frac{z}{w}} \cdot \frac{1}{|w|} dw$$

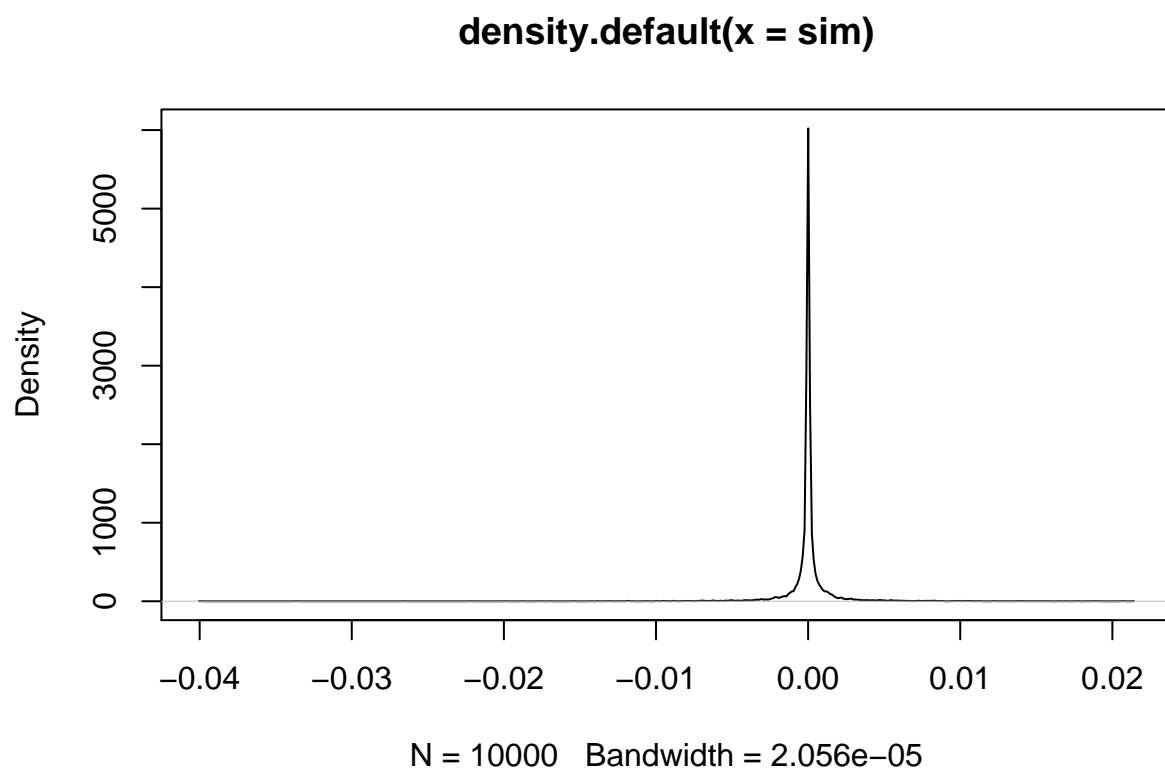
But we should also approximate it by the marginal condition, using:

$$f_z(z) = \int f_{z,y}(z, y) dy = \int f_{z|y}(z|y) f_y(y) dy$$

so...

$$f_z(z) = \int \frac{1}{\sqrt{2y^2\pi}} e^{-\frac{z^2}{2y^2}} \cdot \lambda e^{-\lambda y} dy$$

```
plot(density(sim) )
```



11. How could you derive/write the joint distribution of (Z, Y) ?

From the marginal properties we know that:

$$f_{z,y}(z, y) = f_{z|y}(z) \cdot f_y(y)$$

so...

$$f_{z,y}(z, y) = \frac{1}{\sqrt{2y^2\pi}} e^{-\frac{z^2}{2y^2}} \cdot \lambda e^{-\lambda y}$$