Welcome Test

*Stat4DS2+DS (key: **`tardella2017`**)*
https://elearning2.uniroma1.it/course/view.php?id=4951

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0. Do you know how to use an RMarkdown file to answer these questions with text and R code?

Yes, I Know

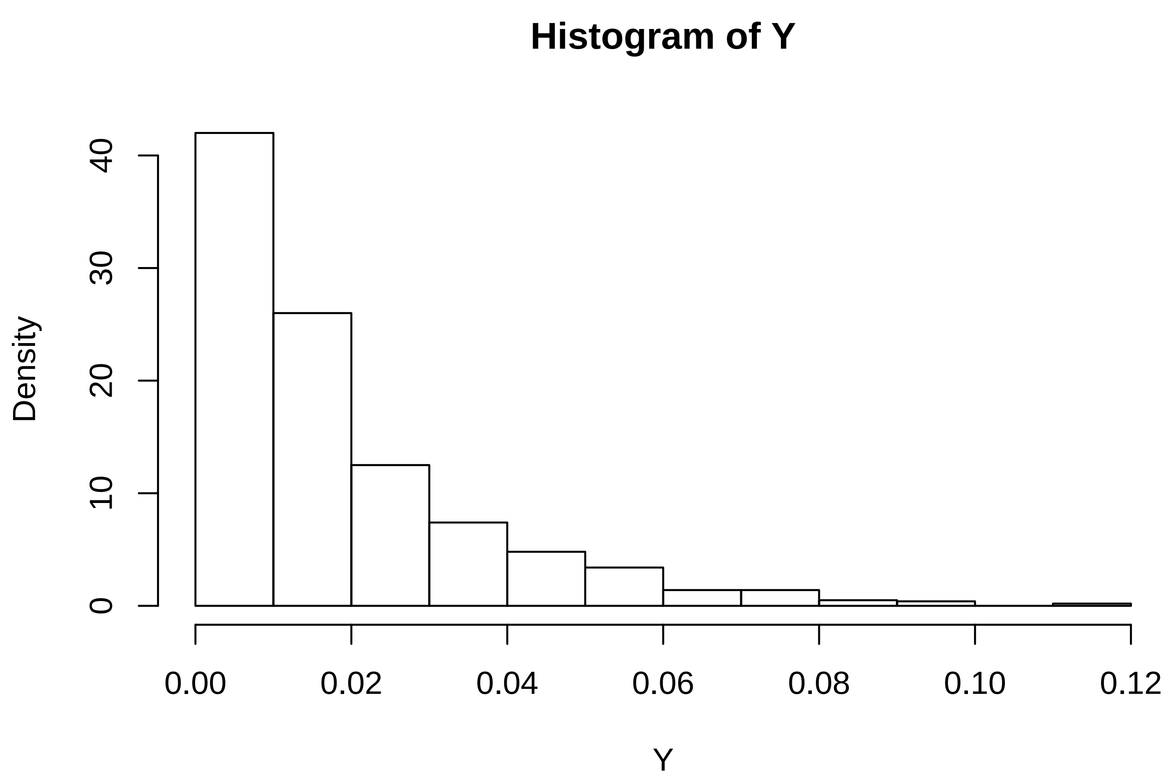
1. Fix the *seed* to start the simulation of a single random variable from a (standard) uniform distribution on [0,1]. Use your *matricola* number as seed. Call the simulated value `lambda`

```
set.seed(1388371)
lambda  = runif(1)
lambda
```

```
## [1] 0.01909172
```

2. Simulate 1000 random variables using the following code

```
X = runif(1000)
Y = - lambda * log(X)
hist(Y, freq=FALSE)
```

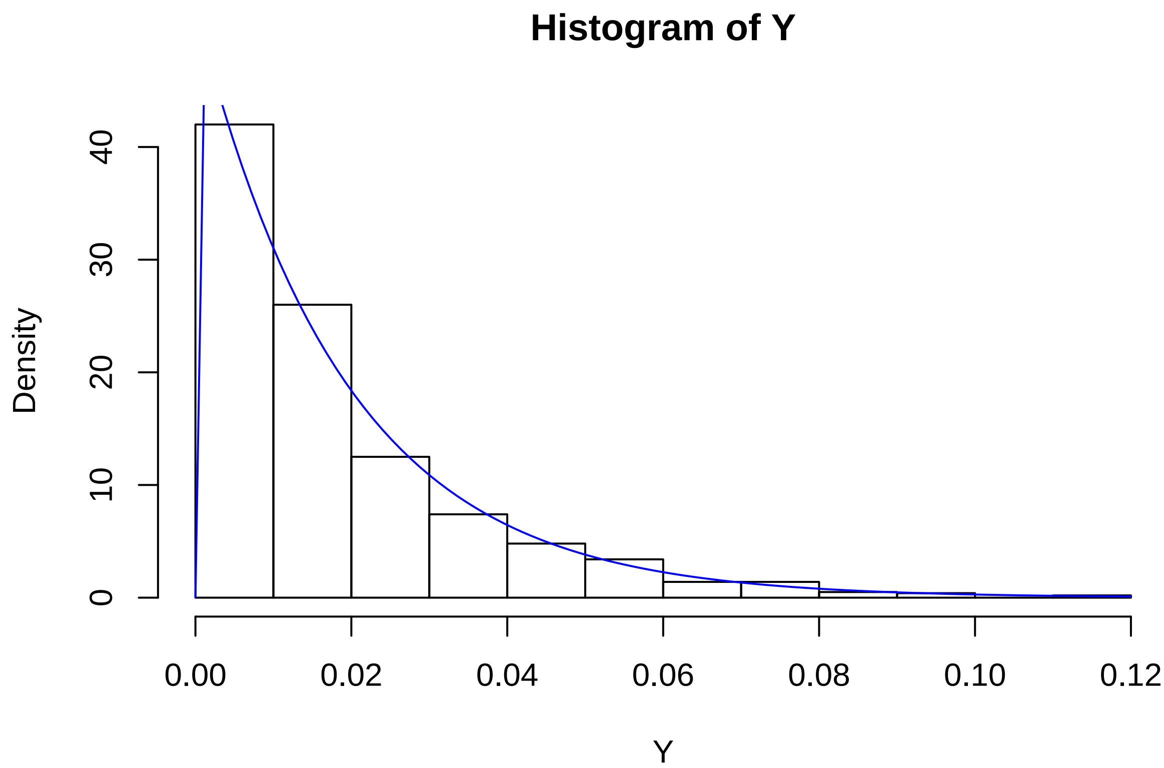


3. Can you show that the theoretical distribution of Y is exponential with rate equal to `1/lambda`? [explain or ... do it!]

The CDF of an exponential distribution is $F(x) = 1 - e^{\lambda x}$, and $F^{-1}(x) = -\frac{1}{\lambda}ln(1-x)$, so wee are sampling from the CDF of the exponential distro, where $\lambda =$`1/lambda`.

4. Can you plot a graphical representation of the theoretical distribution?

```
hist(Y, freq=FALSE)
curve(dexp(x, rate = 1/lambda), add=TRUE, col='blue')
```



5. Can you tell what is the relation between the theoretical curve of the density of Y and the graph of the histogram?

How I demostrated earlier, Y is a sampling of the eponential distribution with parameter `1/lambda`. So the theoretical distribution has to be Asymptotic distribution for the histogram, infact Y_i converge on distribution on Y. ($Y_i \sim Y$)

6. Suppose we have a random vector (Z, Y) Do you know what a joint density $f_{Z,Y}(z, y)$ is? What are the properties it must satisfy?

A joint density is a function that have as domain the random vector and as codominio a number $\in R^+$.

Properties are :

$$f_{Z,Y}(z, y) \geq 0 \quad \forall z, y$$

$$\int\int f_{Z,Y}(z, y)dzdy = 1$$

$$P(Z, Y \in R) = \int\int_R f_{Z,Y}(z, y)dzdy$$

7. Can you derive in formula the distribution of Z? How it is called?

The distribution Z is the marginal distribution of the joint density and we can write resolving:

$$f_z(z) = \int f_{z,y}(z,y)dy = \int f_{z|y}(z|y)f_y(y)dy$$

8. Conditionally on each simulated $Y_i = y_i$ (i=1,2,...,1000), simulate $Z_i = y_i * W_i$ where W_i has distribution standard Normal and is independent of Y_i. What is the conditional distribution of $Z|Y = y$?

We know that :

$$f_z(z) = f_{y,w}(y,w) = f_y(y) \cdot f_w(w)$$

the last equality comes from indipendence. So, for all $y_i \in Y_i$:

$$f_{z|y=y_i}(w) = f_y(y_i) \cdot f_w(w) = f_y(y_i) \cdot N(0,1) = N(0, y_i^2)$$

the last equality cames from $Z = cost \cdot W$ where:

$$E(Z) = cost \cdot E(W) = 0$$

$$Var(Z) = cost^2 \cdot Var(W) = cost^2$$

9. Can you provide a simulation from the marginal distribution of Z?

```
sim = rep(NA, 10000)
prov=rep(NA,10000)
for(i in 1:10000){
  y_i = rexp(1, rate=1/lambda)
  prov[i]=y_i
  z = rnorm(1, sd= y_i^2)
  sim[i]=z
}
```

Where sim is the simulation from the marginal Z distribution because we are sampling y_i from a exponential distribution, so randomly, and the we are sampling from W|Y=y.

10. Can you provide an approximation of the marginal distribution of Z? Please justify your answer.

Because $Z = Y \cdot W$ and for the independence of Y and W:

$$f_z(z) = \int f_w(w) \cdot f_y(z/w) \cdot \frac{1}{|w|} dw$$

in our case:

$$f_z(z) = \int \frac{1}{\sqrt{2\pi}} e^{-\frac{w^2}{2}} \cdot \lambda e^{-\lambda \frac{z}{w}} \cdot \frac{1}{|w|} dw$$

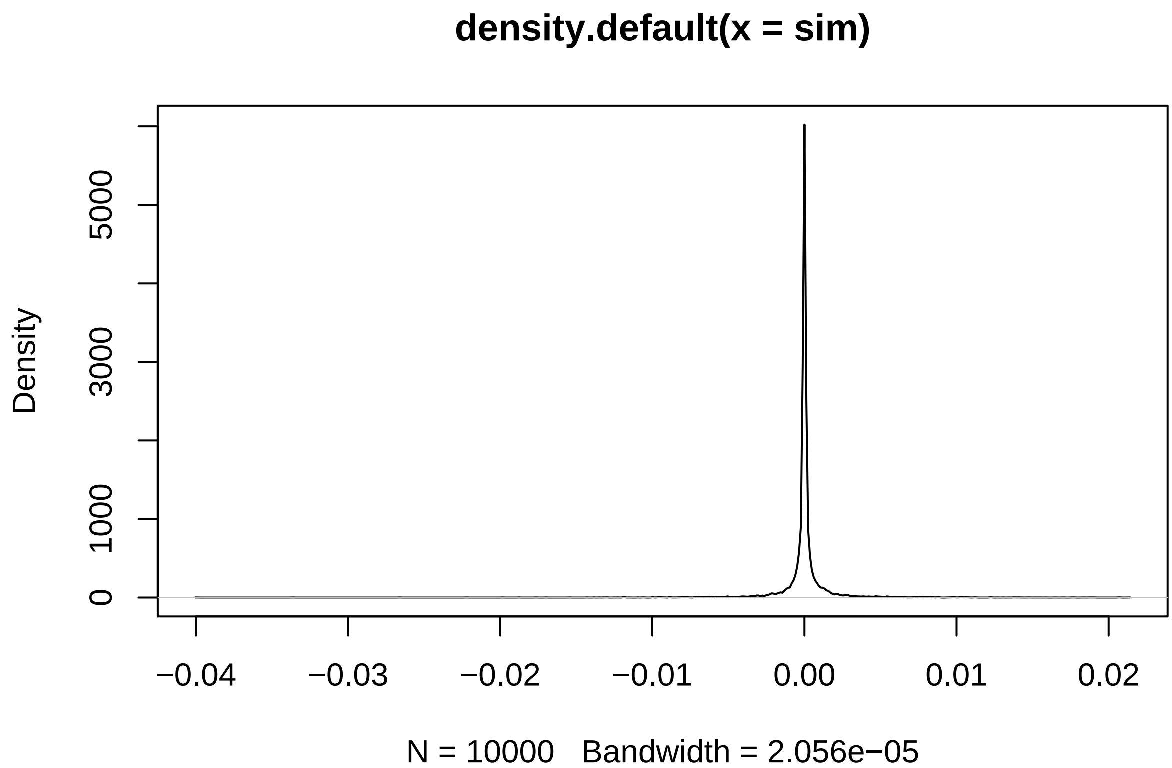
But we should also approximate it by the marginal condition, using:

$$f_z(z) = \int f_{z,y}(z,y)dy = \int f_{z|y}(z|y)f_y(y)dy$$

so...

$$f_z(z) = \int \frac{1}{\sqrt{2y^2\pi}} e^{-\frac{z^2}{2y^2}} \cdot \lambda e^{-\lambda y} dy$$

```
plot(density(sim) )
```



11. How could you derive/write the joint distribution of (Z, Y)?

From the marginal properties we know that:

$$f_{z,y}(z,y) = f_{z|y}(z) \cdot f_y(y)$$

so. . .

$$f_{z,y}(z,y) = \frac{1}{\sqrt{2y^2\pi}} e^{-\frac{z^2}{2y^2}} \cdot \lambda e^{-\lambda y}$$