Signal Processing (SP) Advanced Topics

Exercises report AA. 2021/22

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course held by

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Ex.1 - Bases and Discrete Transforms

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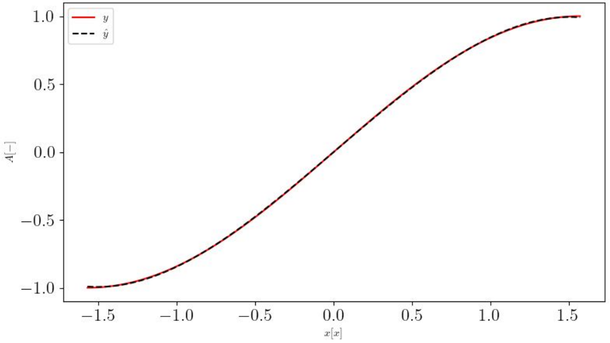
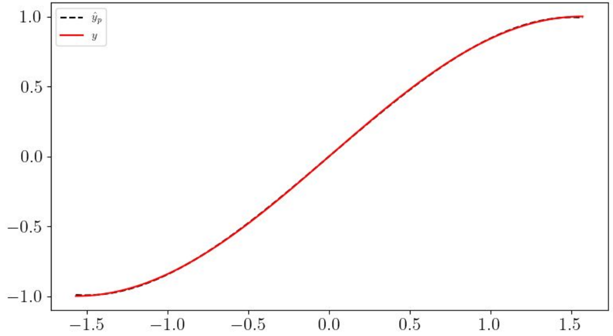
1a. Function approximation with data

$$\bar{b} = [1, x, x^2, x^3]^T$$

polyfit





ϵ_{rms}	2.00E-4	2.00E-4
c_0	-2.97E-16	-8.46E-18
c_1	**9.88E-01**	**9.88E-01**
c_2	2.44E-16	0.0E+00
c_3	**-1.45E-01**	**-1.45E-01**

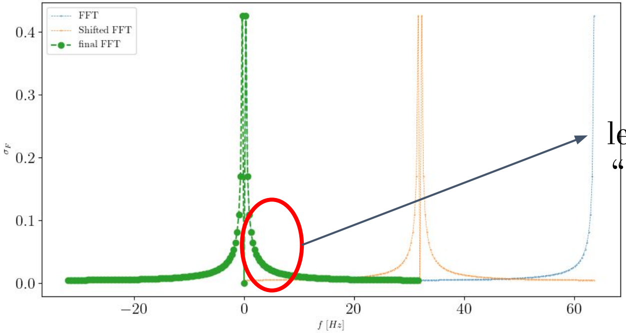
The basis is <u>**not**</u> orthogonal,
i.e. also off-diagonal values of the matrix

$$B(B^\dagger B)^{-1}$$

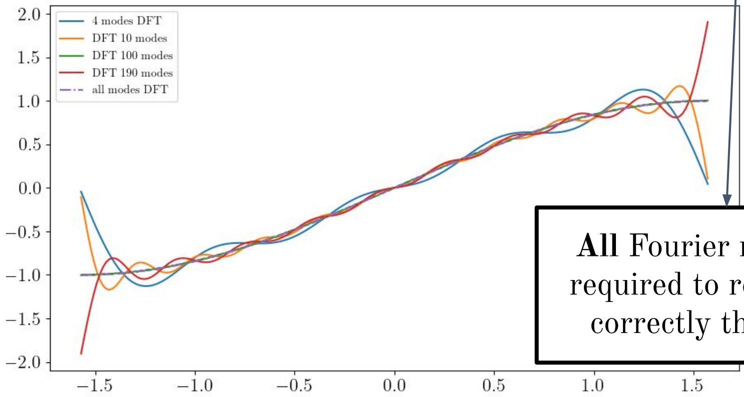
are different from zero.

Ex.1 - Bases and Discrete Transforms

$I \in [-\pi/2, /pi/2]$ ***not** periodic signal*



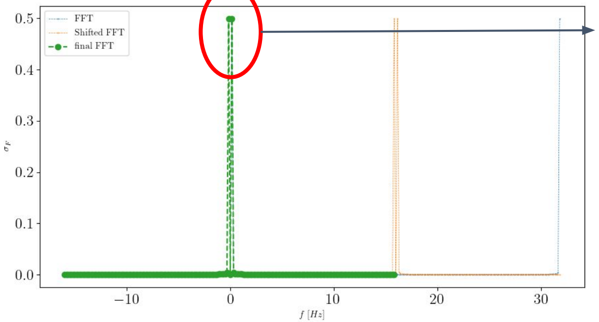
Approximating as harmonic a not-periodic signal leads to **leakage** - i.e. “spilling” signal over close frequencies



All Fourier modes are required to reconstruct correctly the signal!

$I \in [-\pi, /pi]$ *periodic signal*

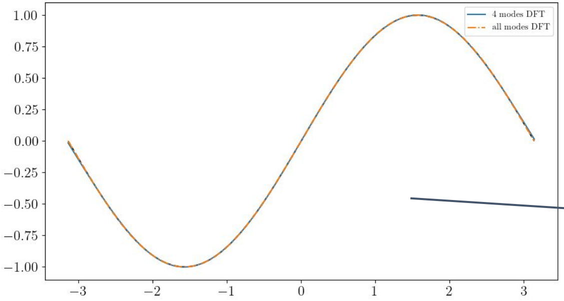
Frequency content of the signal successfully captured



Amplitude of **single mode** is correctly

$$\sigma_F \approx 0.5$$

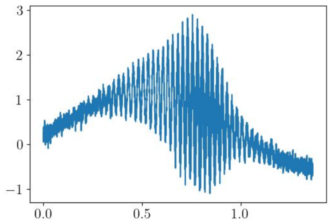
i.e. the correct amplitude divided by half (complex conj.)



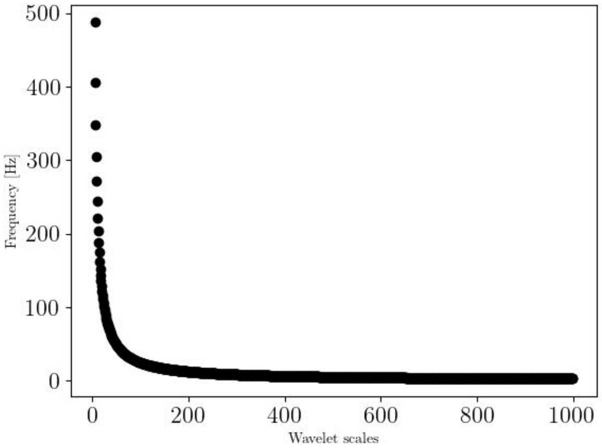
Using the first four modes only does not lose a lot of resolution!

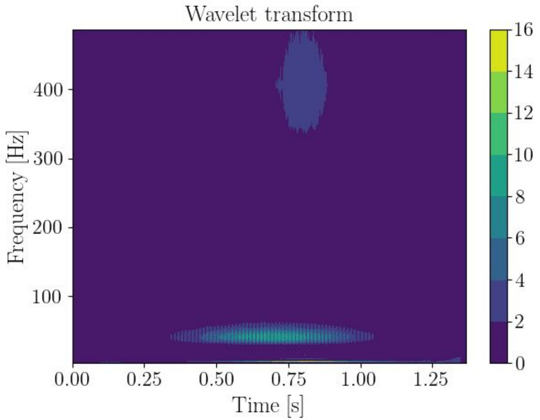
Ex.2 - Time-Frequency Analysis

Which of the introduced harmonics is harder to localize in time? Why?



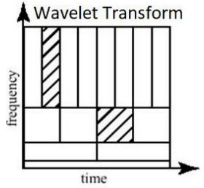
Step 0. A note on the "<u>resolution</u>" of the chosen wavelet: *Morlet wavelet.*
With the current scaling we capture both $f = 400\ [Hz]$ and $f = 40\ [Hz]$
However, the current scaling tends asymptotically to $f = 2.4 [Hz]$ with the current scaling: [5, 1E3]





The <u>harmonic with the lowest frequency</u> is the hardest to localize.

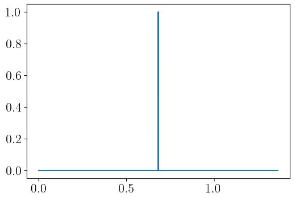
In fact, the larger the scaling factor, the more expanded the wavelet, bad time resolution!



<u>Heisenberg uncertainty principle</u>

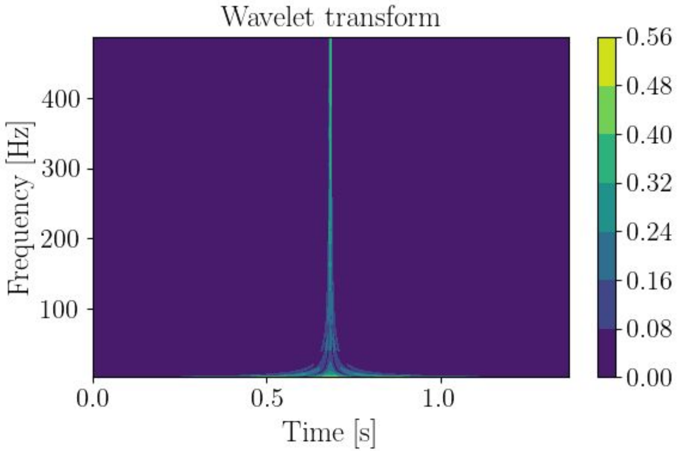
At $f = 40\ [Hz]$ the transform (high scale) shows that identify the component in time is more difficult (well defined in the frequency domain means bad definition in the time domain).

At $f = 400\ [Hz]$ the scales are small and hence the wavelets shrunks, allowing to a better time resolution and worse frequency resolution

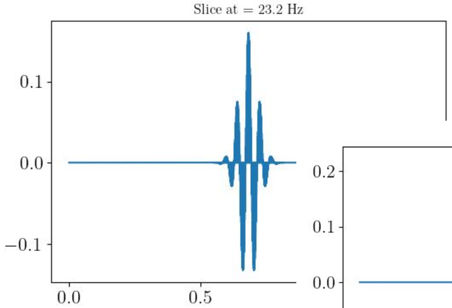


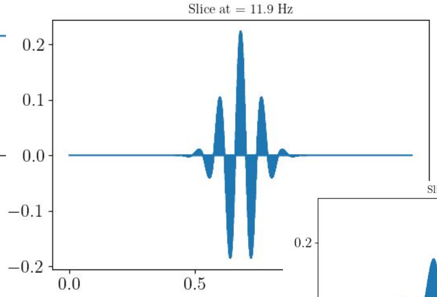
Ex.2 - Time-Frequency Analysis

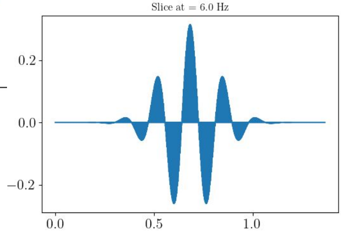
Time-frequency analysis of a delta function



- At <u>high scales</u> (low frequencies), <u>the wavelet dot product with the delta function, is non-zero earlier</u> - i.e. the wavelet recognizes the presence of the delta sooner (in terms of time bins)
- At smaller scales (higher frequencies) the dot product of the signal is non-zero only very close to the signal to be captured.







- Slicing the plot at a given frequency will <u>return a wavelet</u>, because we are doing a dot product of the wavelet and an impulse with unitary length that yields back the wavelet itself.
  - Slicing at <u>different frequencies</u> will affect the <u>scale</u> of the wavelet

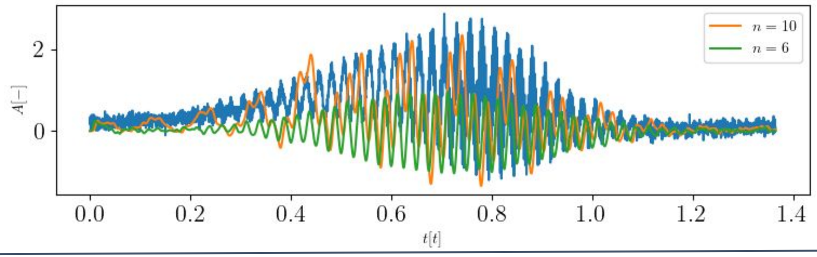
Ex. 3 - Filtering

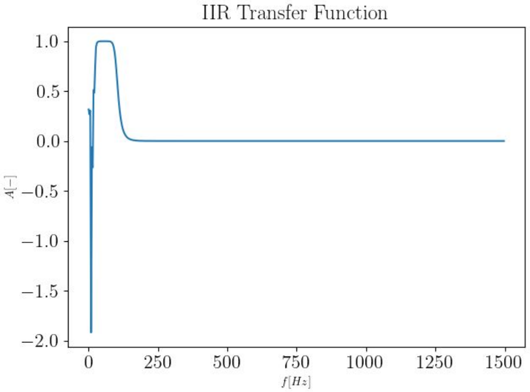
Warning: order of the filter means different things in FIR and IIR

Ex. 3 - Filtering

Comparison between IIR and FIR

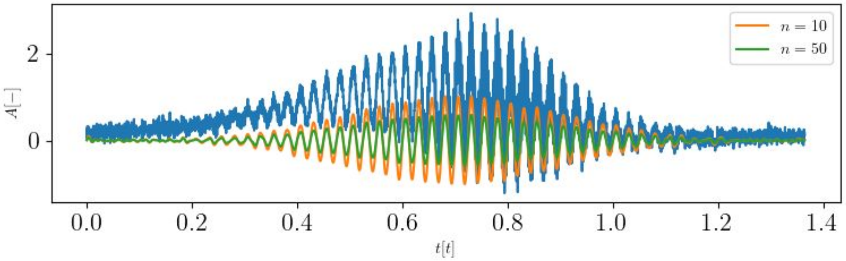
IIR

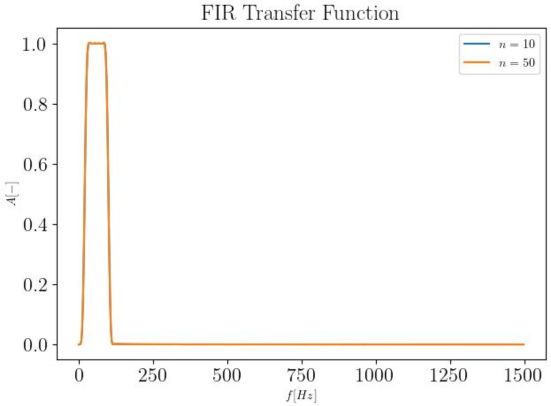




The IIR cannot be applied by convolution because it has an infinite impulse response!

FIR





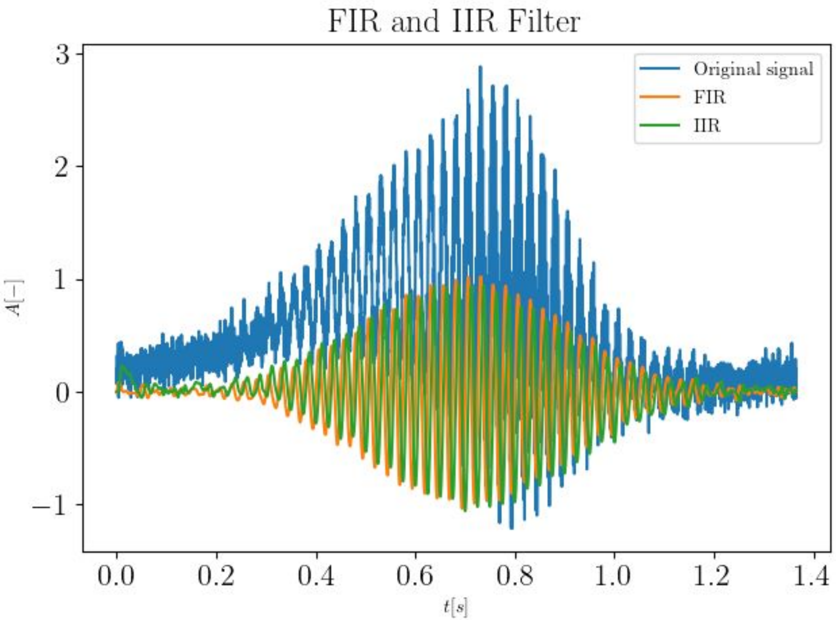
FIR filters can be applied by convolution!

- Reducing the order of the IIR filter results in a reduced ill-behavior at the boundaries, but also reduces overall amplitude of filtered signal: **trade-off**
- IIR filter with 20 poles is unstable, and hence it has not been reported in the interest of understanding

The bandpass filtering has been implemented using two low-pass filters

Ex. 3 - Filtering

Comparison between best IIR and FIR:



filtfilt: filtering forward and backward

- De-phasing
- <u>Boundaries</u>: better behavior of FIR, IIR instabilities might be mitigated with zero-padding
- **FIR** is more computationally-expensive (higher order required) but is **always stable**

Ex. 4 - Data Driven Modal Analysis

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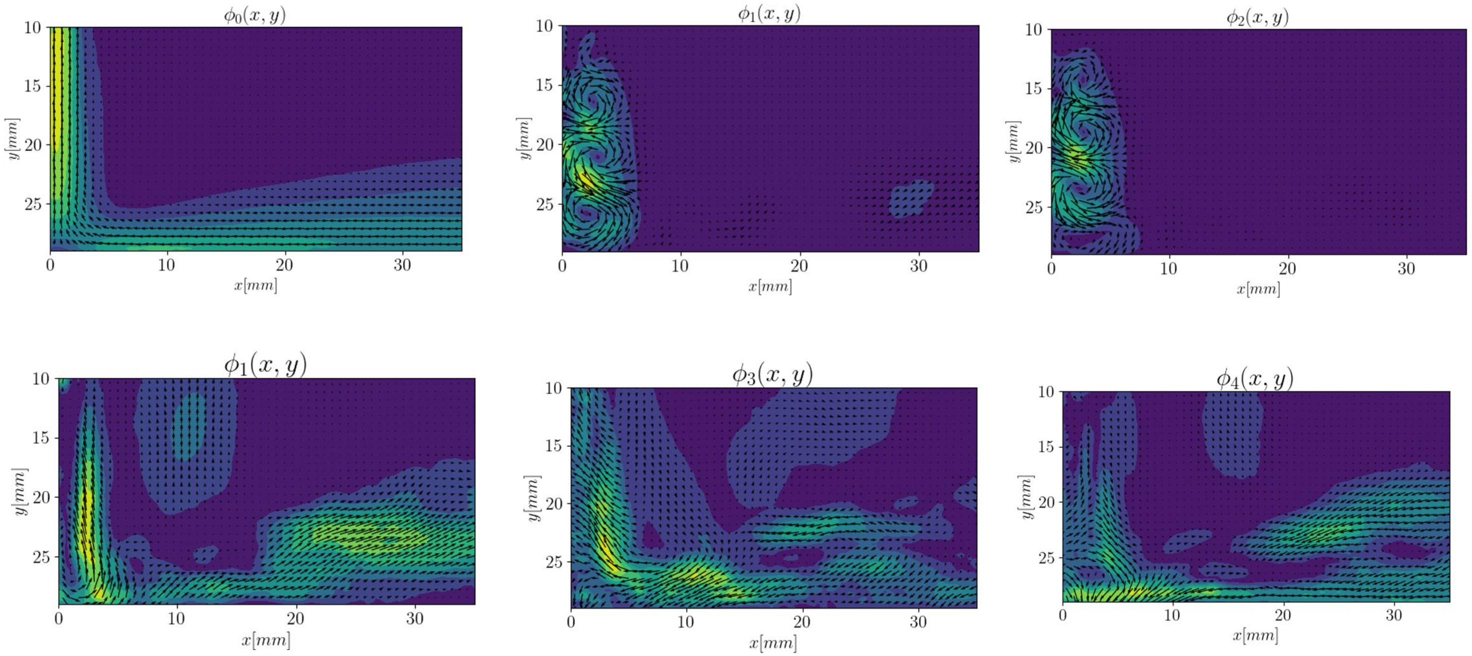
Comparison of the first three modes of DFT and POD in the space domain

POD

DFT



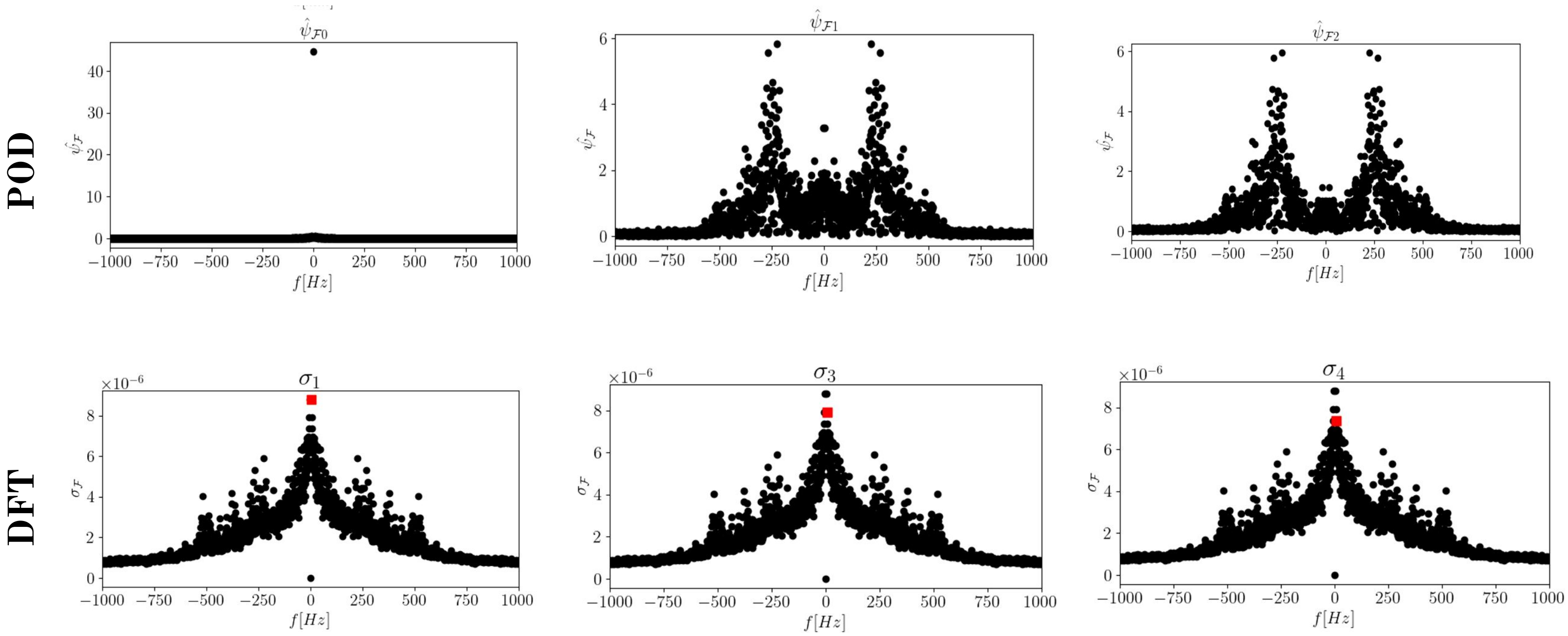
Looking for **coherent** structures in space. Using **energetically optimal** base

Investigating the effect of **wavenumbers** (length scales). Using **harmonic temporal structures**

DFT is not "data-driven" i.e. its base does not change according to the data analysed

Ex. 4 - Data Driven Modal Analysis

Comparison of the first three modes of DFT and POD in the frequency domain



DFT identifies only **one frequency per mode;** POD identifies multiple (coherent) frequencies per mode

DFT can show convergence issue, while the POD guarantees the best possible convergence (in an "energetic" sense)