Decision making under uncertainty

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Part 2: Multiagent Frameworks

MAS Course Leuven
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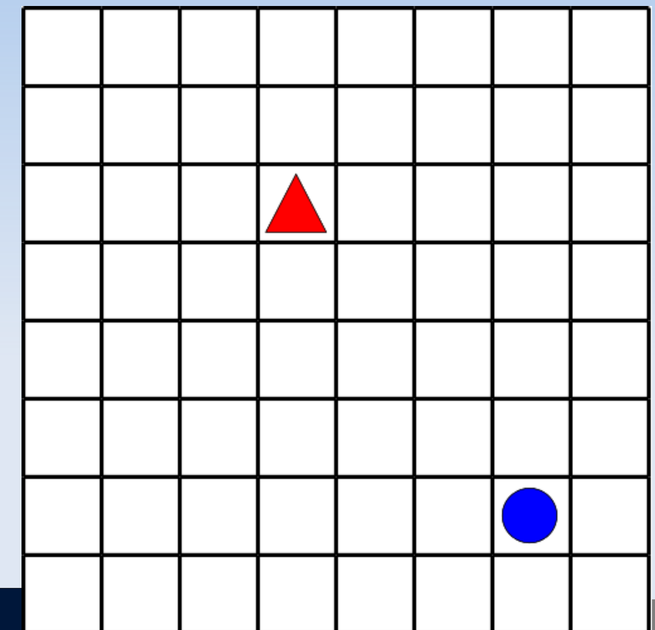
Multiagent Systems (MASs)

Why MASs?

- If we can make intelligent agents, soon there will be many...
- Physically distributed systems: centralized solutions expensive and brittle.
- can potentially provide [Vlassis, 2007,Sycara, 1998]
  - Speedup and efficiency
  - Robustness and reliability (‘graceful degradation’)
  - Scalability and flexibility (adding additional agents)

Example: Predator-Prey Domain

- Predator-Prey domain – still single agent!
  - 1 agent: the predator (blue)
  - prey (red) is part of the environment
  - on a torus ('wrap around world')



- Formalization:
  - states
  - actions
  - transitions
  - rewards

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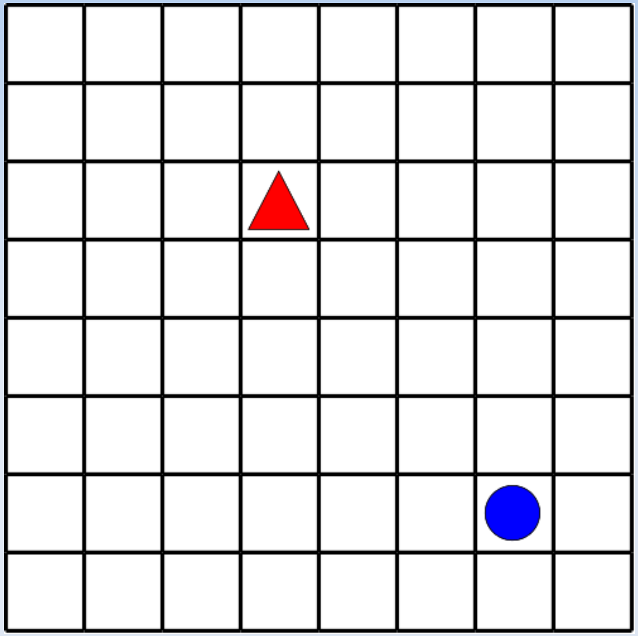


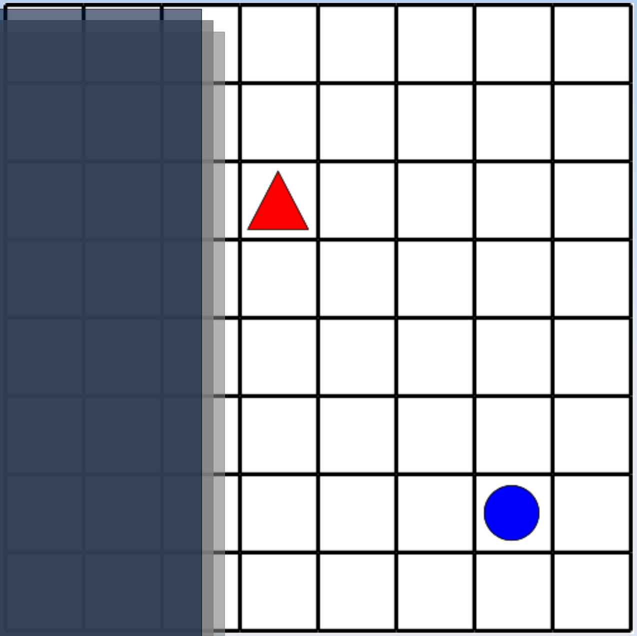
- Formalization:
  - states (-3,4)
  - actions N,W,S,E
  - transitions probability of failing to move, prey moves
  - rewards reward for capturing

Example: Predator-Prey Domain

- Predator-Prey domain

Markov decision process (MDP)

[redacted]



[redacted] prey moves

- rewards reward for capturing

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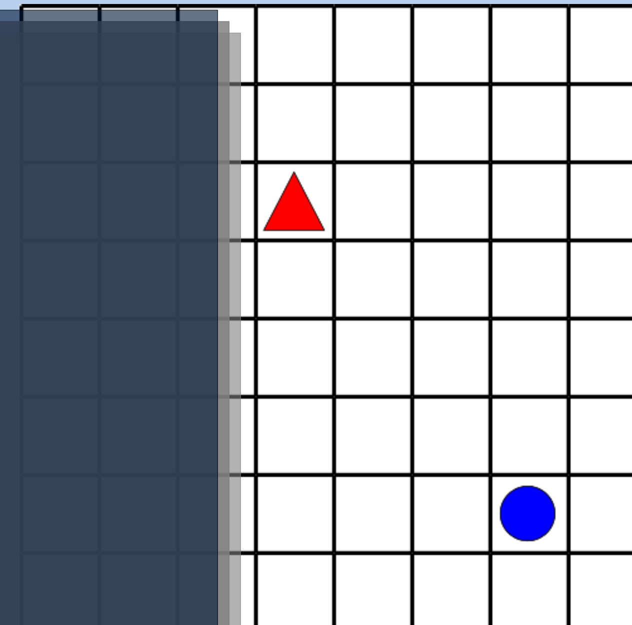


Markov decision process (MDP)

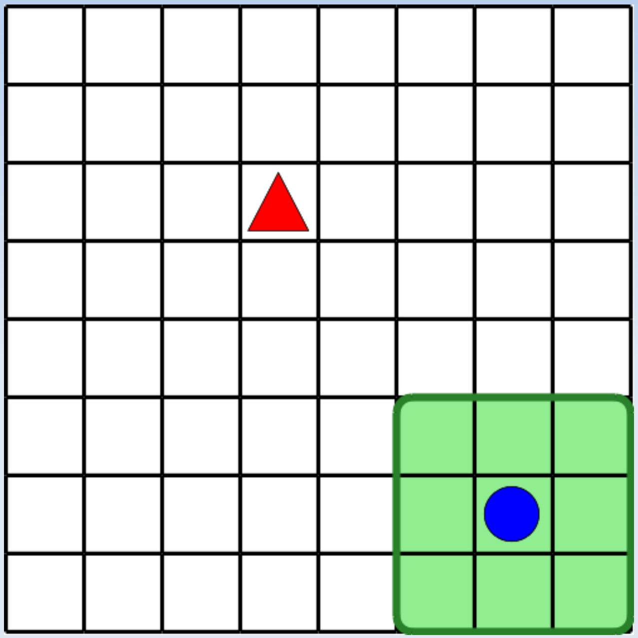
- Markovian state *s*...
- ...which is observed
- policy π maps states → actions
- Value function Q(s,a)
- Value iteration: way to compute it.

prey moves

- rewards   reward for capturing

Partial Observability

- Now: partial observability
  - E.g., limited range of sight
- MDP + observations
  - explicit observations
  - observation probabilities
    - noisy observations (detection probability)



$o = \text{'}nothing\text{'}$

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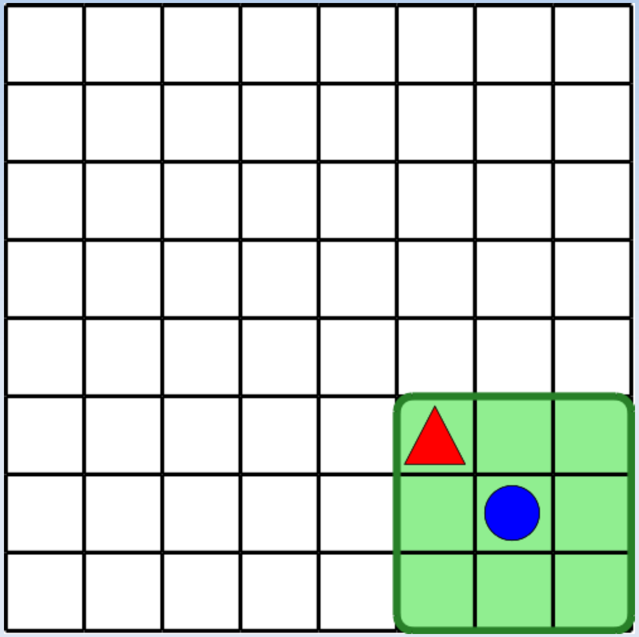

$o = (-1, 1)$

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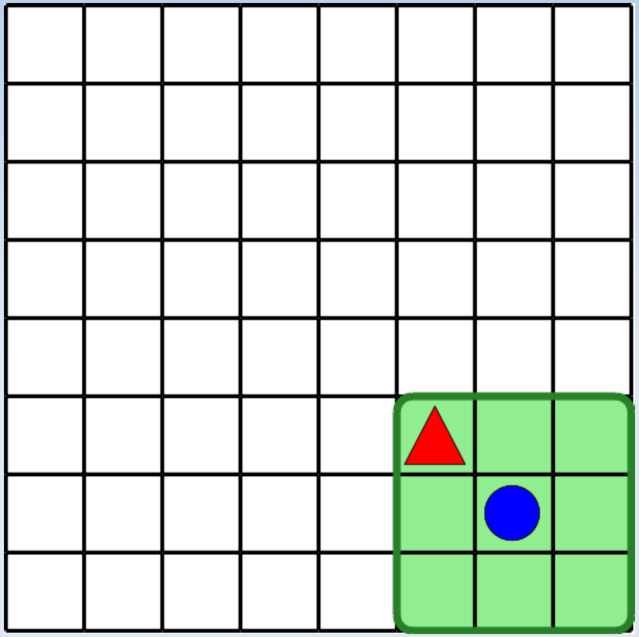

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Can not observe the state
→ Need to maintain a belief over states *b(s)*
→ Policy maps beliefs to actions $\pi(b)=a$

Partial Observability

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Partially Observable MDP (POMDP)

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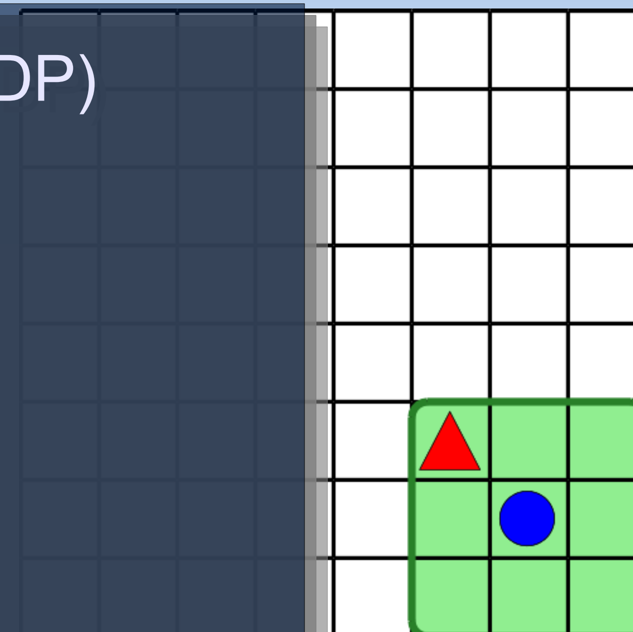

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Partial Observability

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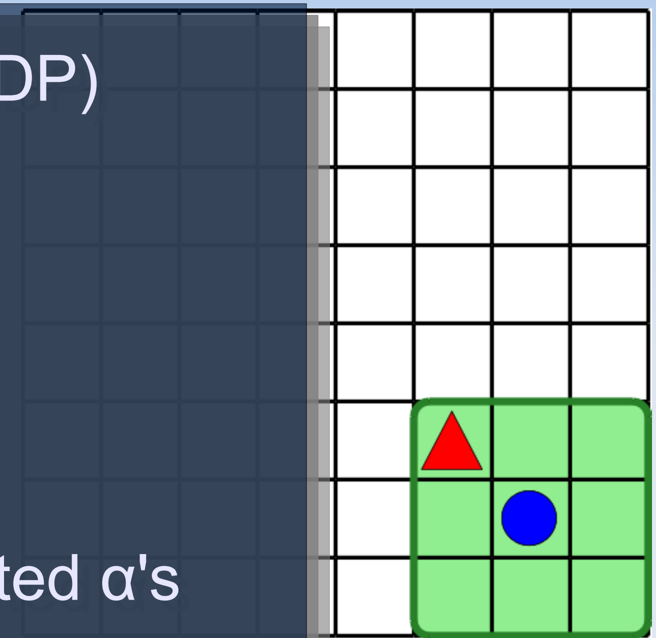
Partially Observable MDP (POMDP)

- reduction → continuous state MDP (in which the belief **is** the state)
- Value iterations:
  - make use of α-vectors (correspond to complete policies)
  - perform pruning: eliminate dominated α's



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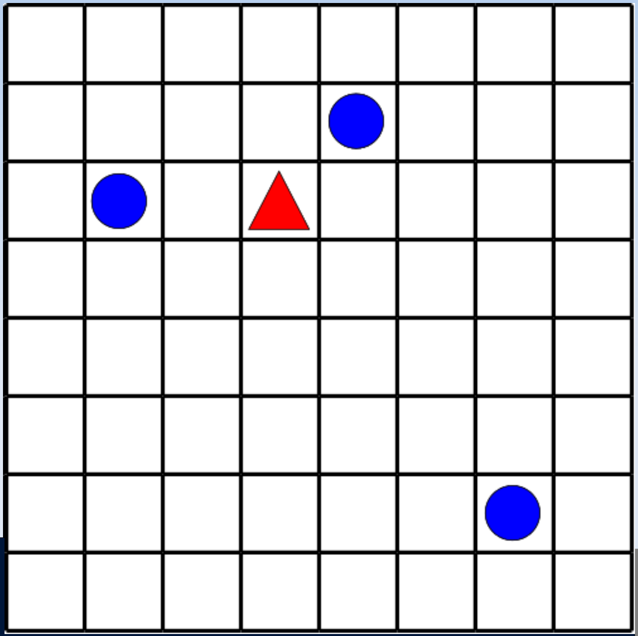
Multiple Agents

- Now: multiple agents
  - fully observable



- Formalization:
  - states
  - actions
  - **joint** actions
  - transitions
  - rewards

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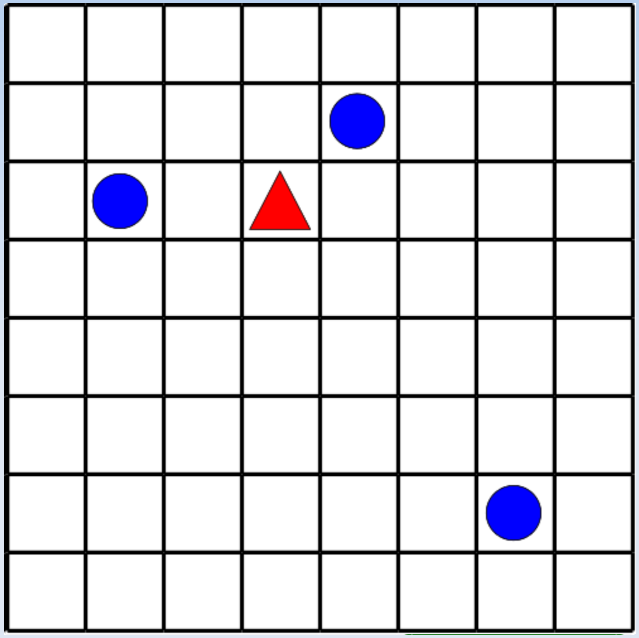
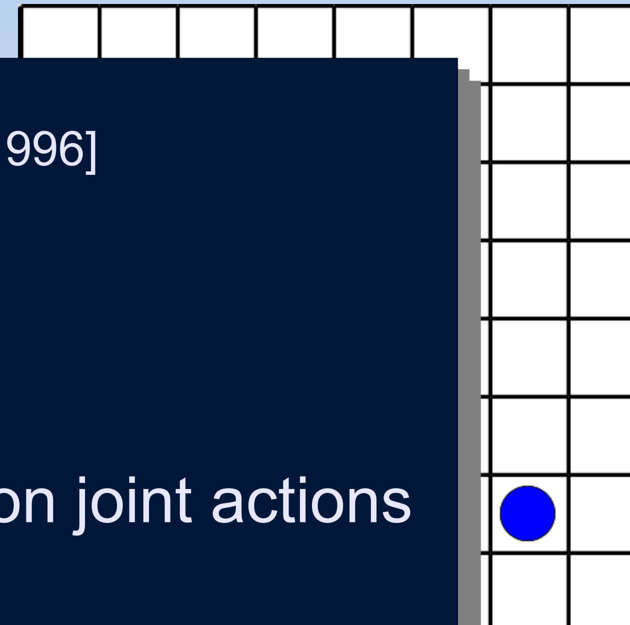


- Formalization:
  - states ((3,-4), (1,1), (-2,0))
  - actions {N,W,S,E}
  - **joint** actions {(N,N,N), (N,N,W),...,(E,E,E)}
  - transitions probability of failing to move, prey moves
  - rewards reward for capturing jointly

Multiple Agents

- Now: multiple agents
- Fo



Multiagent MDP [Boutilier 1996]

- Differences with MDP
  - *n* agents
  - joint actions $a = \langle a_1, a_2, \ldots, a_n \rangle$
  - transitions and rewards depend on joint actions
- Solution:
  - Treat as normal MDP with 1 'puppeteer agent'
  - Optimal policy $\pi(s) = a$
  - Every agent executes its part

es

- rewards reward for capturing jointly

Multiple Agents

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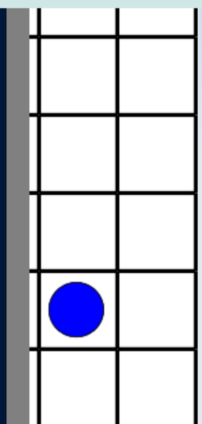
Catch: ...?



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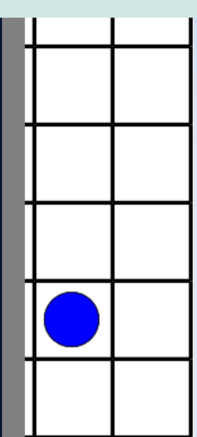
Catch: number of joint actions is **exponential**!
(but other than that, conceptually simple.)

- Fo



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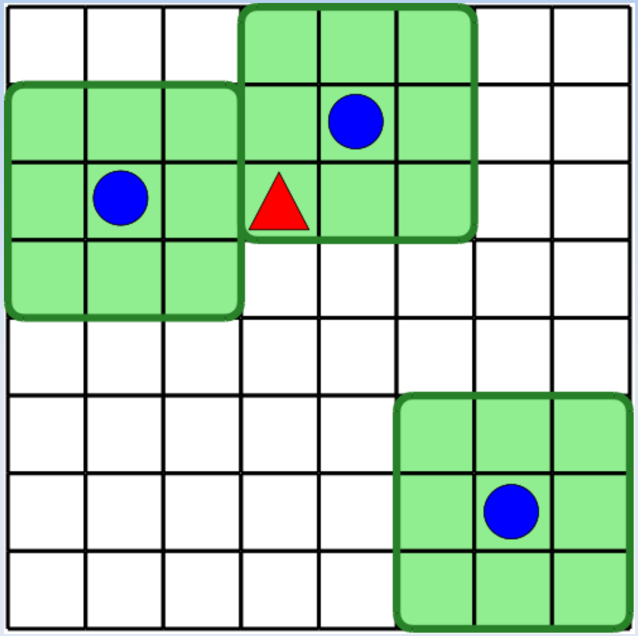
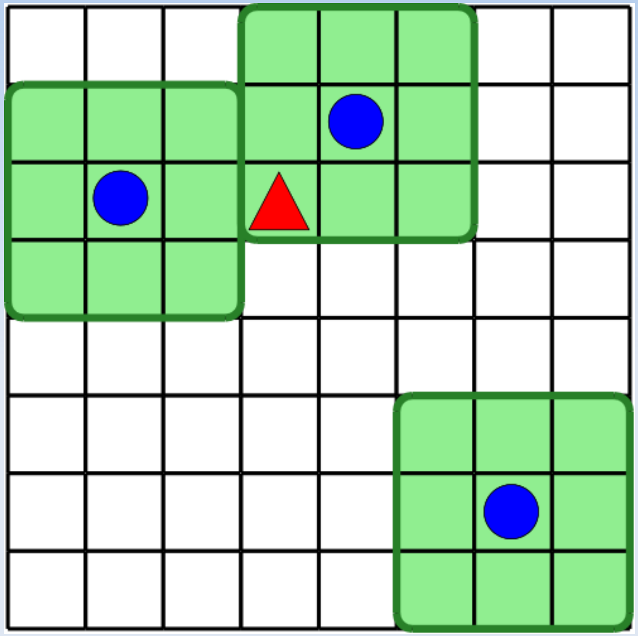
Multiple Agents & Partial Observability

- Now: Both
  - partial observability
  - multiple agents



Multiple Agents & Partial Observability

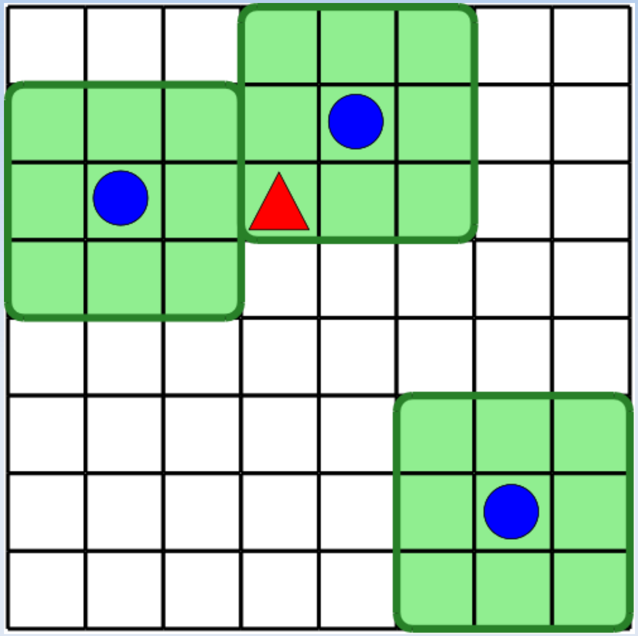
- Now: Both
  - partial observability
  - multiple agents
- Decentralized POMDPs (Dec-POMDPs) [Bernstein et al. 2002]
- both
  - joint actions and
  - joint observations



Multiple Agents & Partial Observability

- Again we can make a reduction...

  any idea?

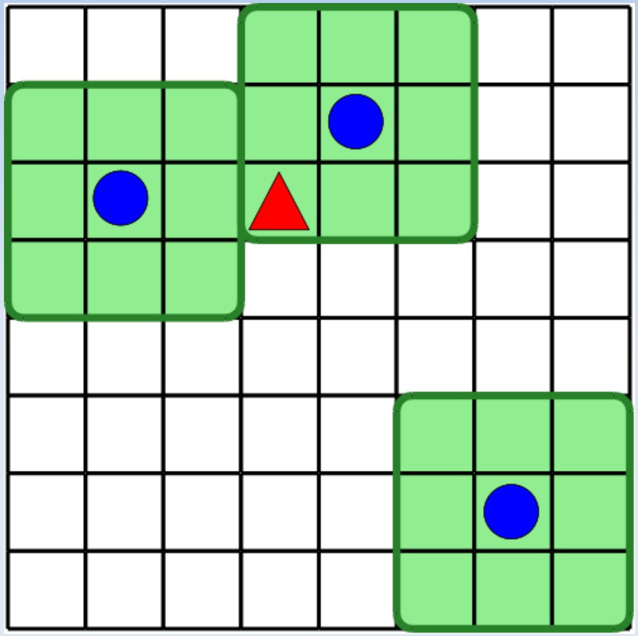


Multiple Agents & Partial Observability

- Again we can make a reduction...

  Dec-POMDPs → MPOMDP

  (multiagent POMDP)

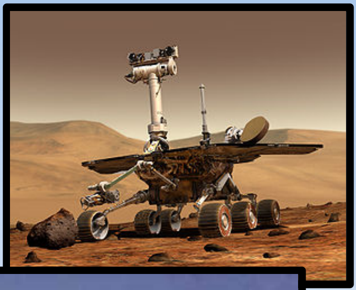


- 'puppeteer' agent that
  - receives joint observations
  - takes joint actions
- requires broadcasting observations!
  - instantaneous, cost-free, noise-free communication → optimal [Pynadath and Tambe 2002]
- Without such communication: no easy reduction.

The Dec-POMDP Model

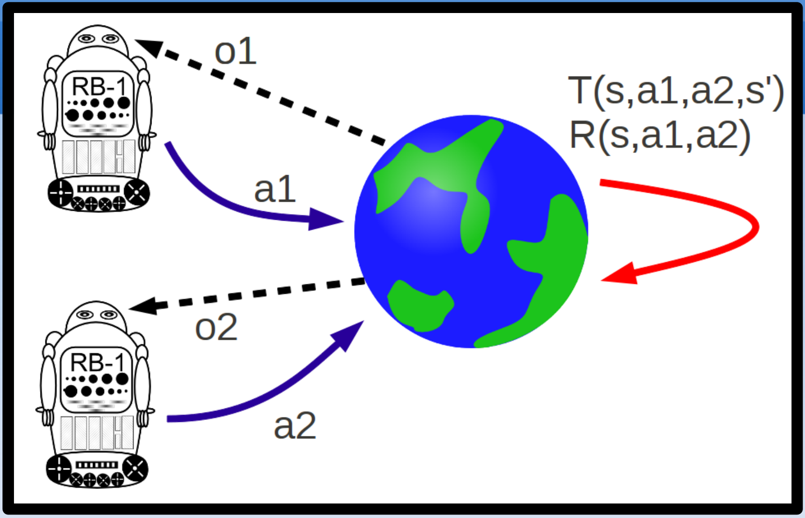
Acting Based On Local Observations

- MPOMDP: Act on global information
- Can be impractical:
  - communication not possible
  - significant cost (e.g battery power)
  - not instantaneous or noise free
  - scales poorly with number of agents!



- Alternative: act based only on local observations
  - Other side of the spectrum: no communication at all
  - (Also other intermediate approaches: delayed communication, stochastic delays)

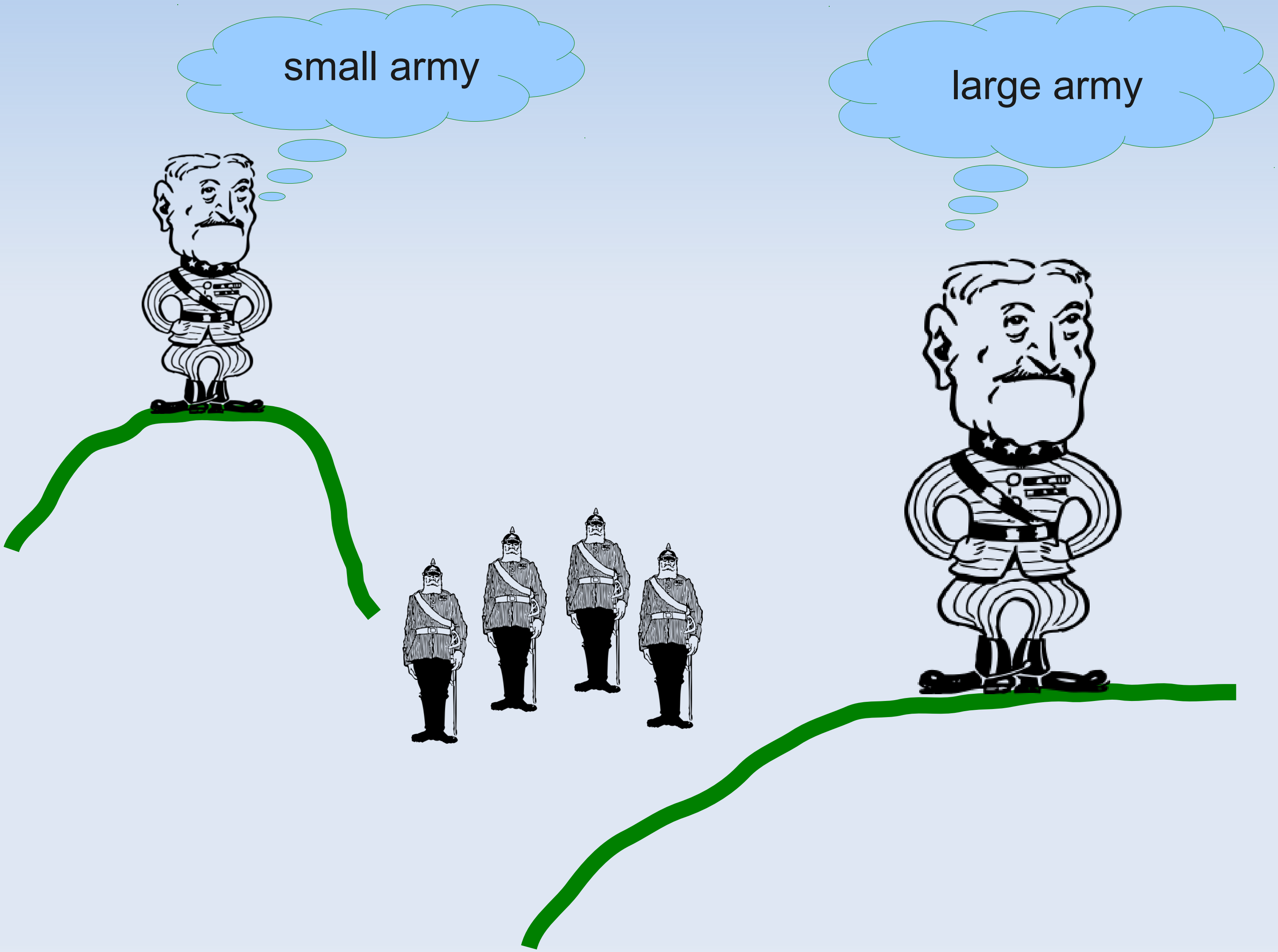
Formal Model



- A Dec-POMDP
  - $\langle S, A, P_T, O, P_O, R, h \rangle$
  - n agents
  - S – set of states
  - A – set of **joint** actions $\quad a = \langle a_1, a_2, \ldots, a_n \rangle$
  - P_T – transition function $\quad P(s' | s, a)$
  - O – set of **joint** observations $\quad o = \langle o_1, o_2, \ldots, o_n \rangle$
  - P_O – observation function $\quad P(o | a, s')$
  - R – reward function $\quad R(s, a)$
  - h – horizon (finite)

Running Example

- 2 generals problem



Running Example

- 2 generals problem

S – { s_L, s_S }

A_i – { (O)bserve, (A)ttack }

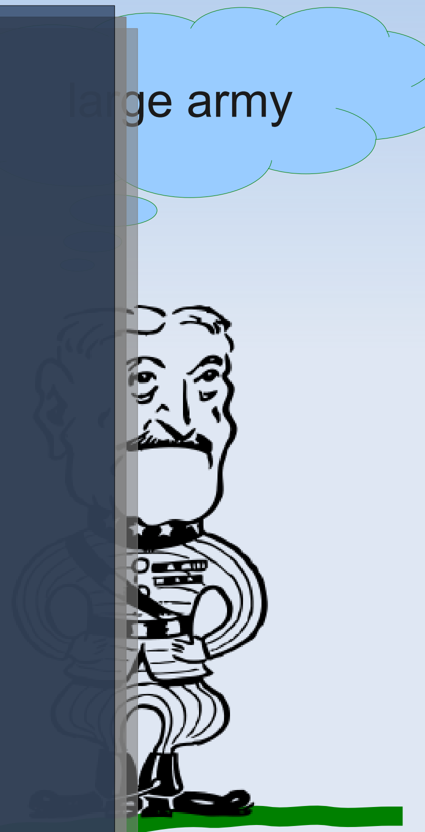
O_i – { (L)arge, (S)mall }

Transitions

- Both Observe: no state change
- At least 1 Attack: reset with 50% probability

Observations

- Probability of correct observation: 0.85
- E.g., P(<L, L> | s_L) = 0.85 * 0.85 = 0.7225



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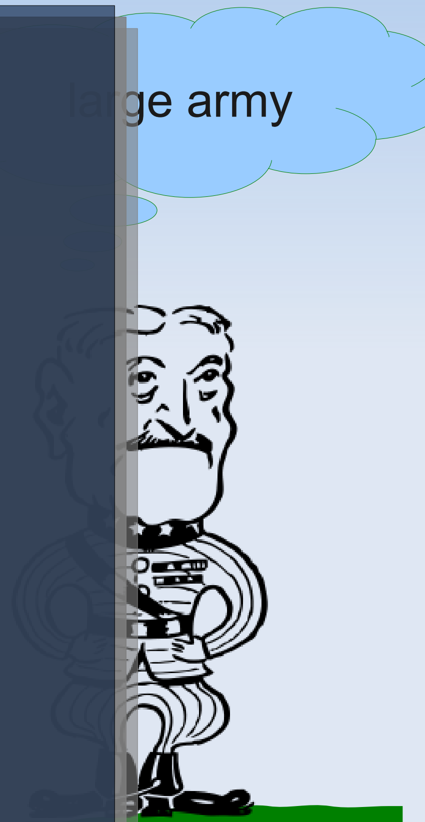
Rewards

- 1 general attacks: he loses the battle
  - R(*,<A,O>) = -10
- Both generals Observe: small cost
  - R(*,<O,O>) = -1
- Both Attack: depends on state
  - R(s_L,<A,A>) = -20
  - R(s_R,<A,A>) = +5



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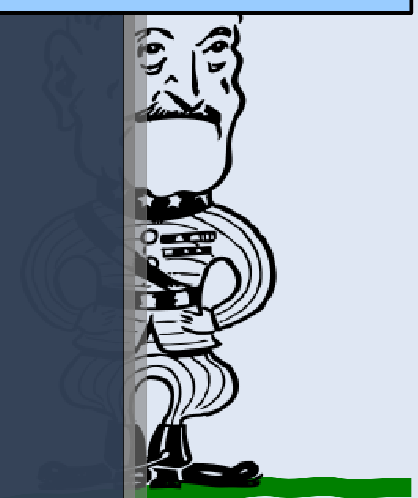
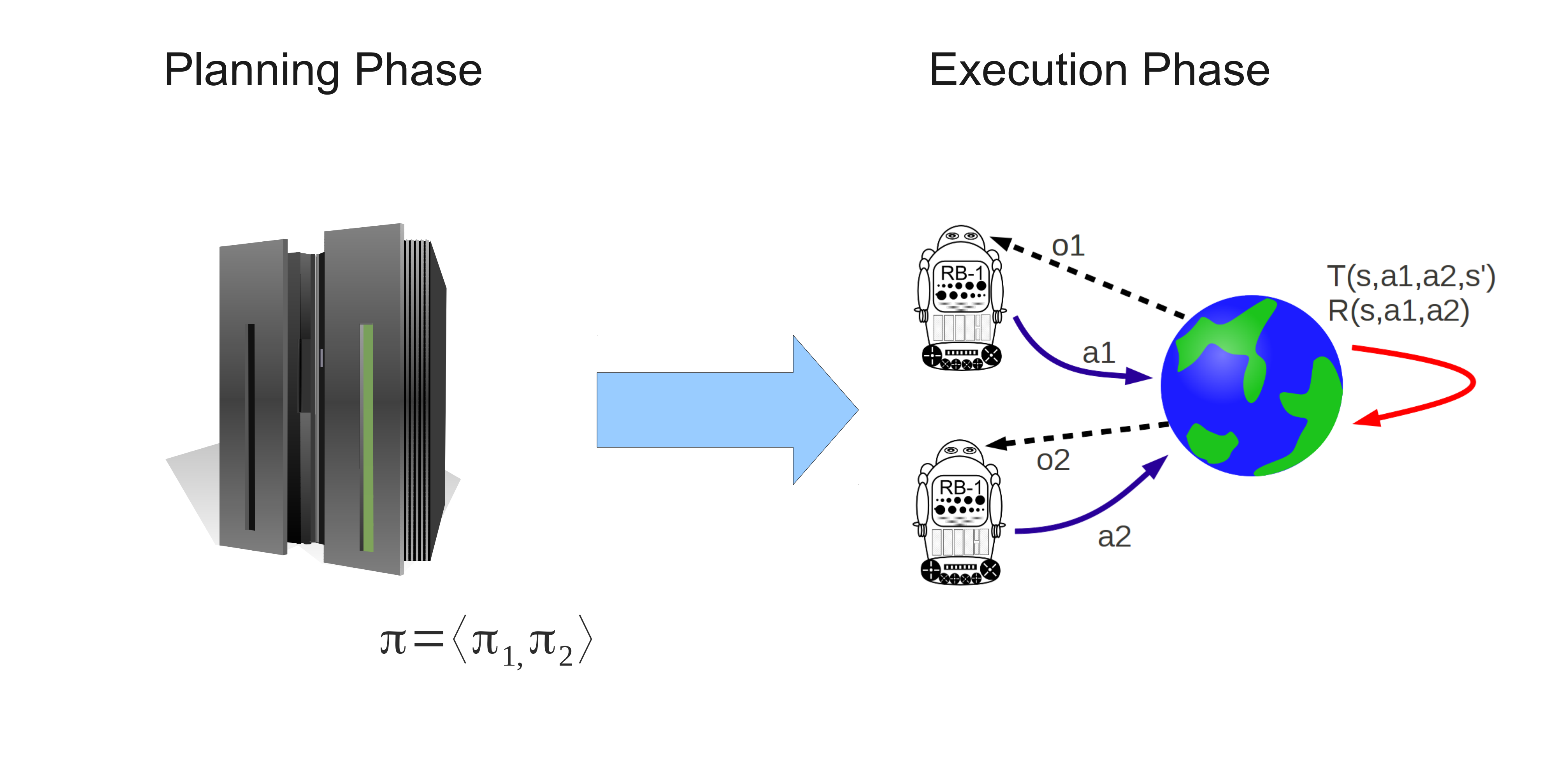
suppose h=3,
what do you think is optimal in this problem?



Off-line / On-line phases

- off-line planning, on-line execution is decentralized

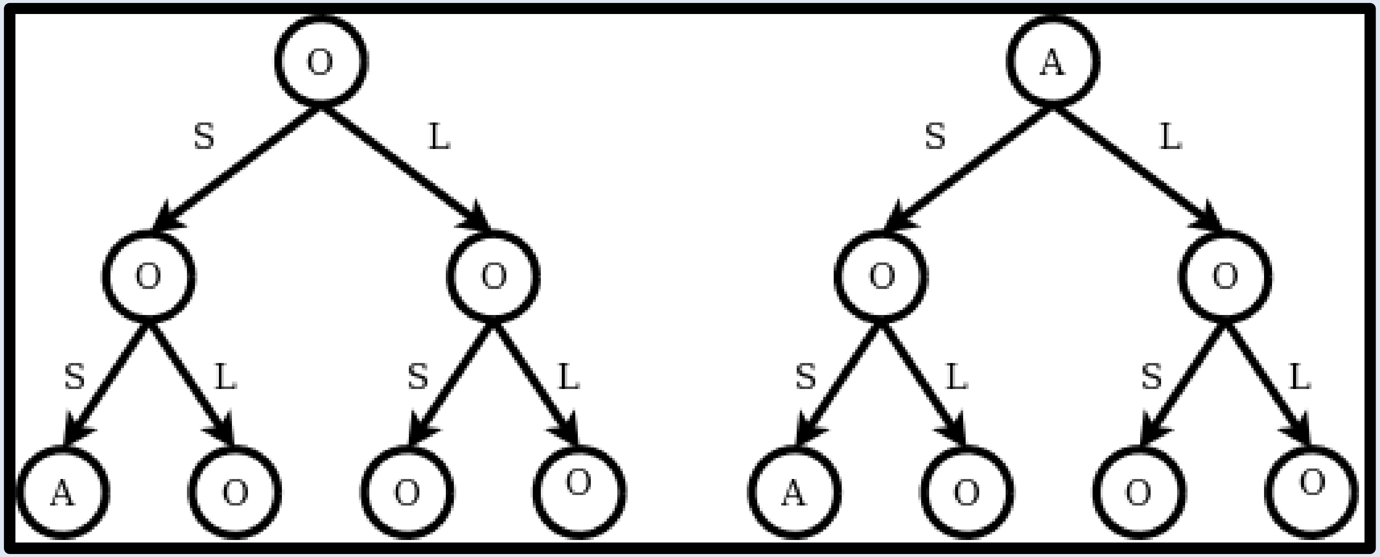


$$\pi = \langle \pi_1, \pi_2 \rangle$$

Policy Domain

- What do policies look like?
  - In general histories → actions
  - before: more compact representations...
- Now, this is difficult: no such representation known!

  → So we will be stuck with histories



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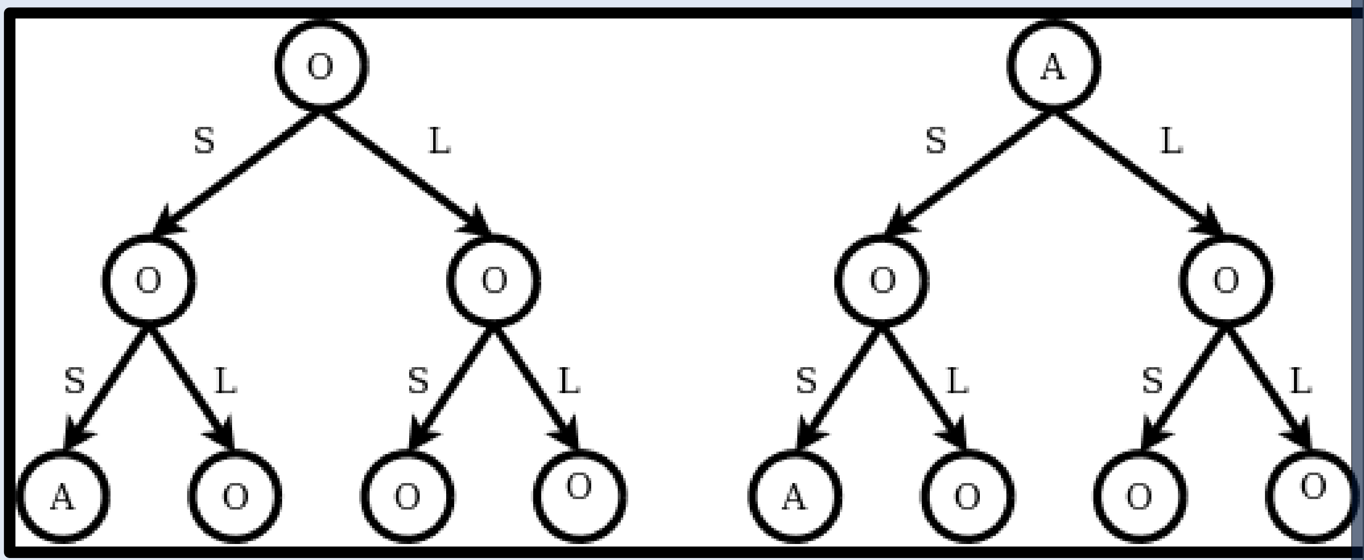

Most general, AOHs:

$$\left(a_i^0, o_i^1, a_i^1, \ldots, a_i^{t-1}, o_i^t\right)$$

But: can restrict to deterministic policies → only need OHs:

$$\vec{o}_i = \left(o_i^1, \ldots, o_i^t\right)$$

No Compact Representation?

There are a number of types of beliefs considered

- **Joint Belief**, *b(s)* (as in MPOMDP) [Pynadath and Tambe 2002]
  - compute b(s) using joint actions and observations
  - Problem: ?

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- **Joint Belief**, *b(s)* (as in MPOMDP) [Pynadath and Tambe 2002]
  - compute b(s) using joint actions and observations
  - Problem: agents do not know those during execution
- **Multiagent belief**, $b_i(s,q_{-i})$ [Hansen et al. 2004]
  - belief over (future) policies of other agents
  - Need to be able to predict the other agents!
    - for belief update $P(s'|s,a_i,a_{-i})$, $P(o|a_i,a_{-i},s')$, and prediction of $R(s,a_i,a_{-i})$
  - form of those other policies? most general: $\pi_j : \vec{o}_j \rightarrow a_j$
  - if they use beliefs? → infinite recursion of beliefs!

Goal of Planning

- **Find** the **optimal** joint policy $\pi^* = \langle \pi_{1,} \pi_2 \rangle$
  - where individual policies map OHs to actions $\pi_i : \vec{O}_i \rightarrow A_i$

- What is the optimal one?
  - Define **value** as the expected sum of rewards:

$$V(\pi) = \boldsymbol{E}\left[\sum_{t=0}^{h-1} R(s,a) \mid \pi, b^0\right]$$

  - optimal joint policy is one with maximal value (can be more that achieve this)

Goal of Planning

Optimal policy for 2 generals, h=3

value=-2.86743

() --> observe
(o_small) --> observe
(o_large) --> observe
(o_small,o_small) --> attack
(o_small,o_large) --> attack
(o_large,o_small) --> attack
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conceptually:

what should policy optimize to allow for good coordination (thus high value)

?

Coordination vs. Exploitation of Local Information

- Inherent trade-off

  coordination vs. **exploitation of local information**

  - Ignore own observations → 'open loop plan'
    - E.g., “ATTACK on 2nd time step”

      + maximally predictable
      - low quality
  - Ignore coordination
    - E.g., compute an individual belief $b_i(s)$ and execute the MPOMDP policy

      $$b_i(s) = \sum_{q_{-i}} b(s, q_{-i})$$

      + uses local information
      - likely to result in mis-coordination
  - Optimal policy π^* should balance between these.

Planning Methods

Brute Force Search

- We can compute the value of a joint policy $V(\pi)$
  - using a Bellman-like equation [Oliehoek 2012]
- So the **stupidest algorithm** is:
  - compute $V(\pi)$, for all π
  - select a π with maximum value
- Number of joint policies is huge!
  (doubly exponential in horizon h)
- Clearly intractable...

h	num. joint policies
1	4
2	64
3	16384
4	1.0737e+09
5	4.6117e+18
6	8.5071e+37
7	2.8948e+76
8	3.3520e+153

Brute Force Search

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No easy way out...

The problem is
NEXP-complete [Bernstein et al. 2002]

most likely (assuming EXP != NEXP)
doubly exponential time required.

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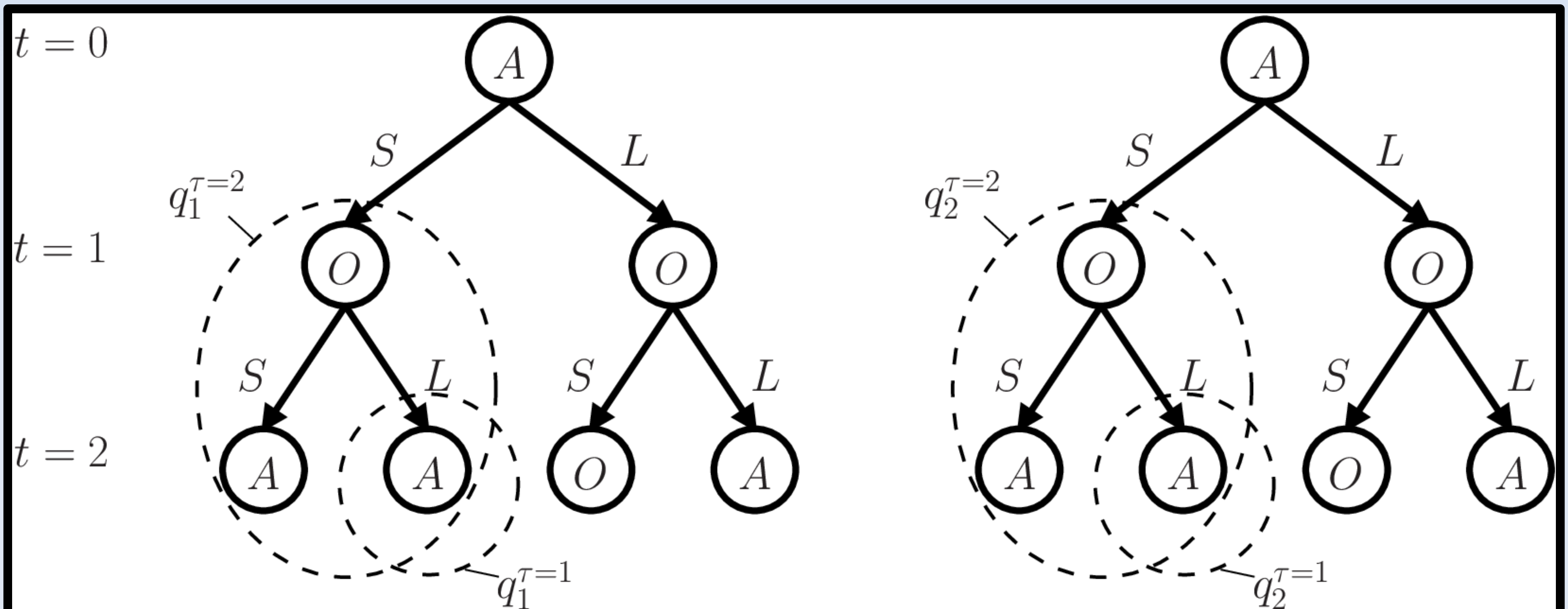
- Clearly intracta

- Still, there are better algorithms that work better for at least some problems...
- Useful to understand what optimal really means!
  (trying to compute it helps understanding)

Dynamic Programming – 1

- Generate all policies in a special way:
  - from 1 stage-to-go policies $Q^{\tau=1}$
  - construct all 2-stages-to-go policies $Q^{\tau=2}$, etc.



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Exhaustive backup operation

Dynamic Programming – 1

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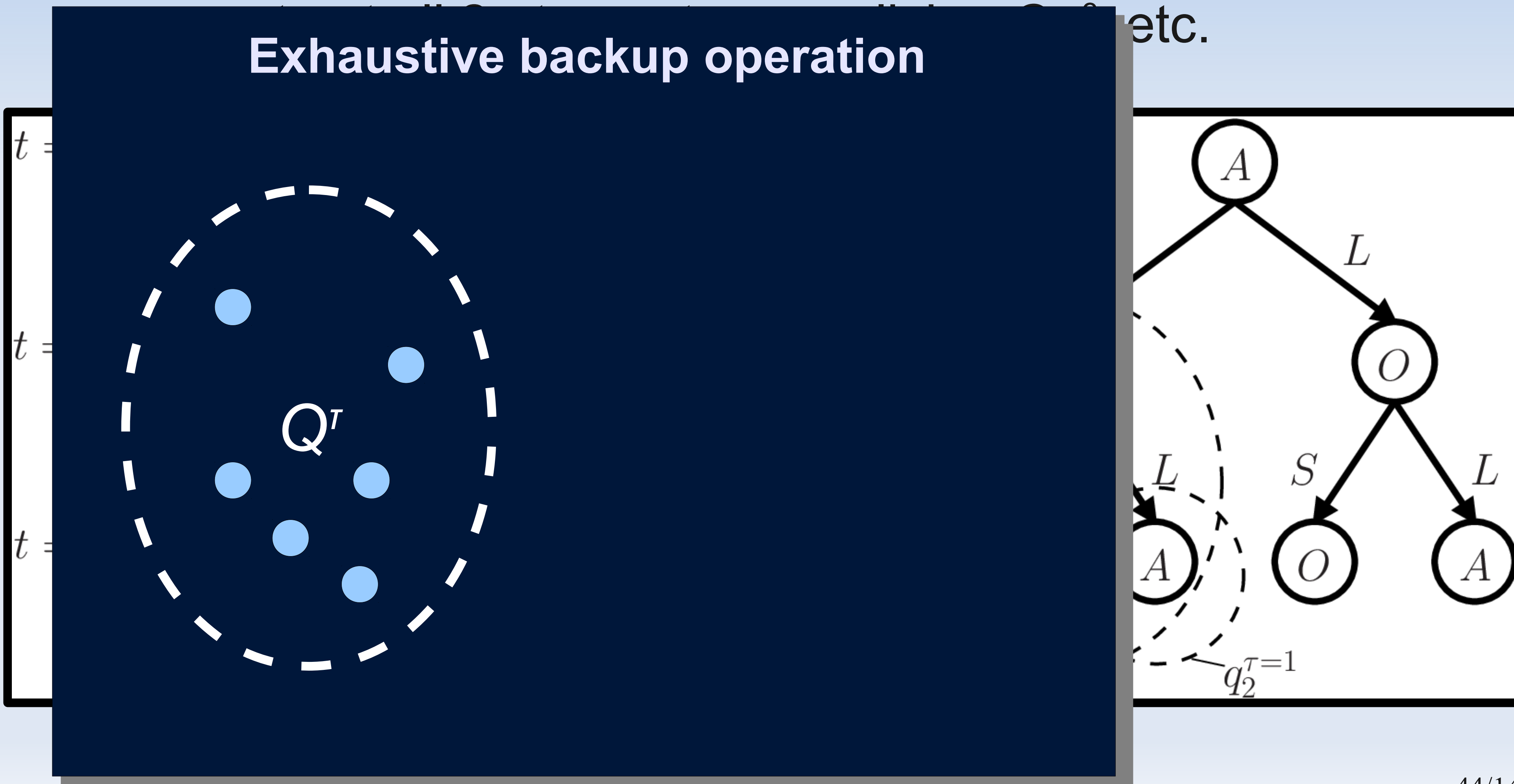

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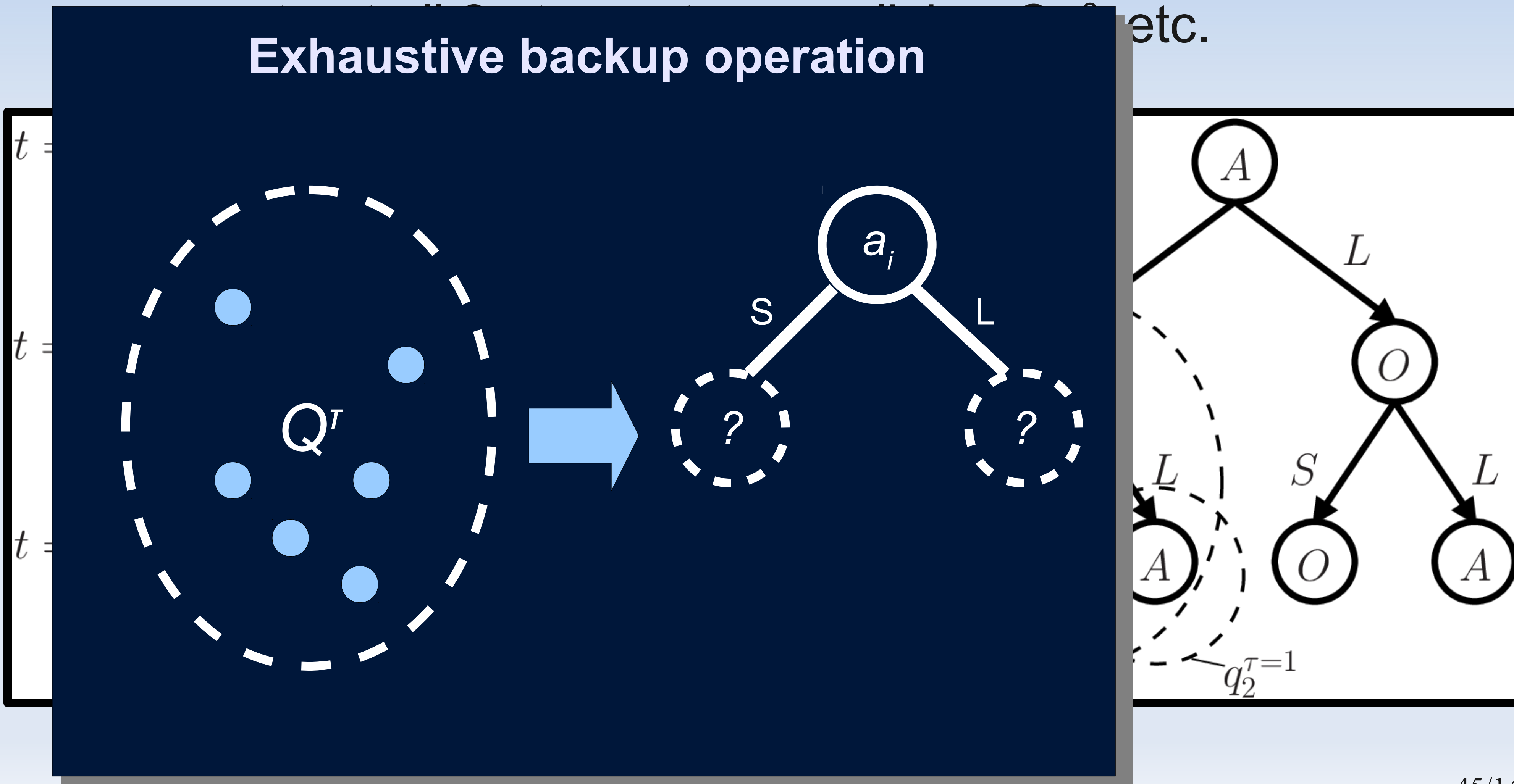

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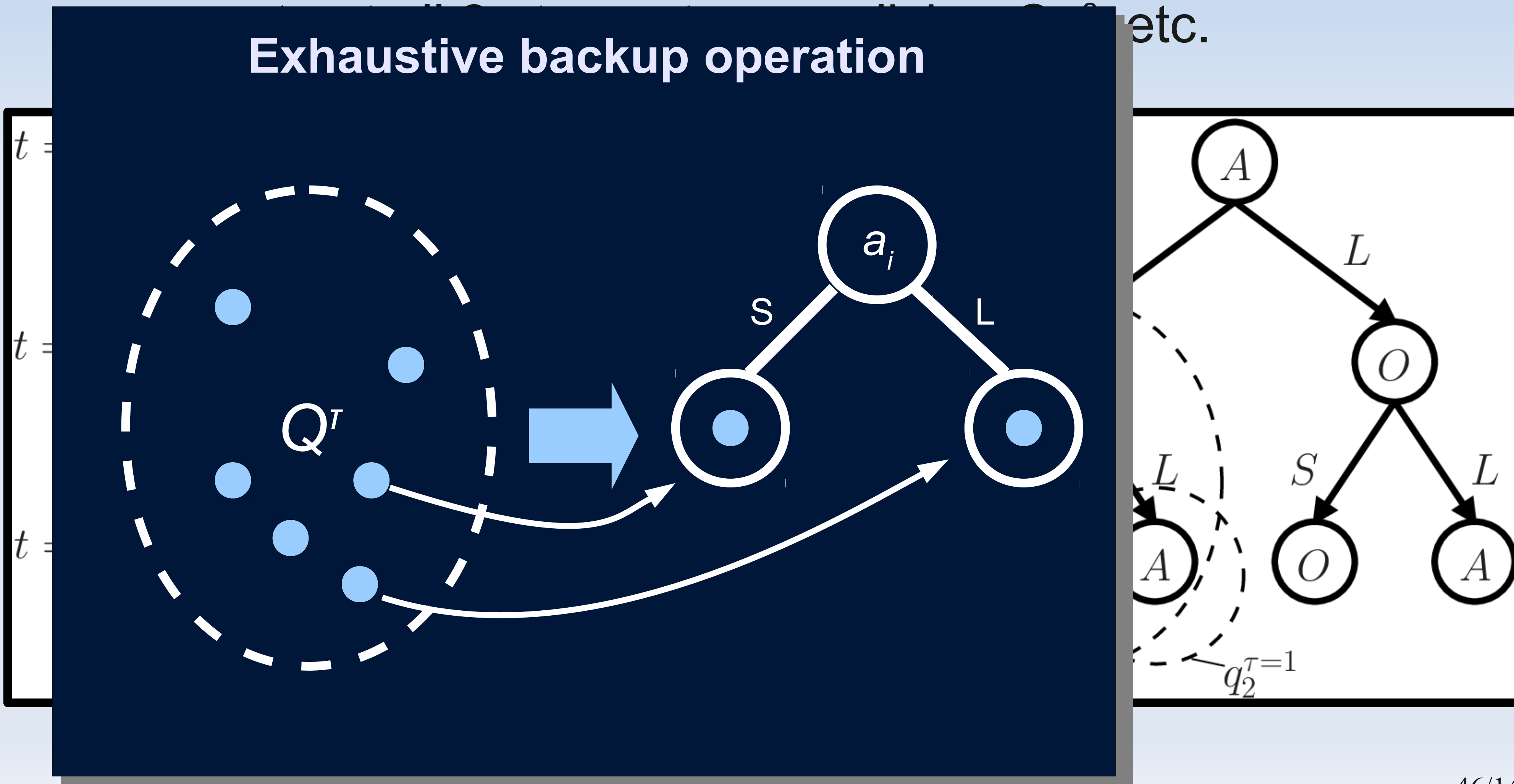

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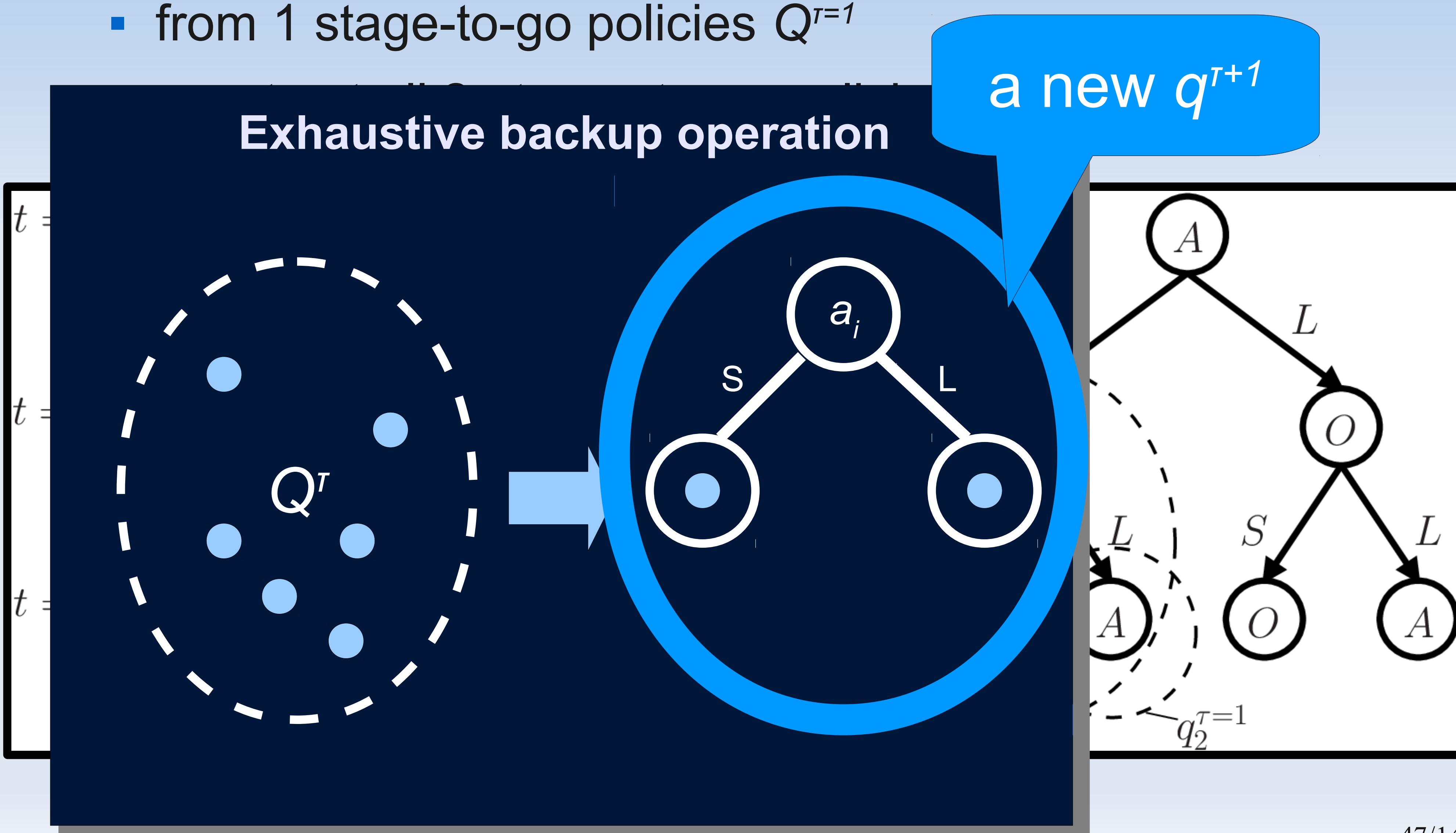

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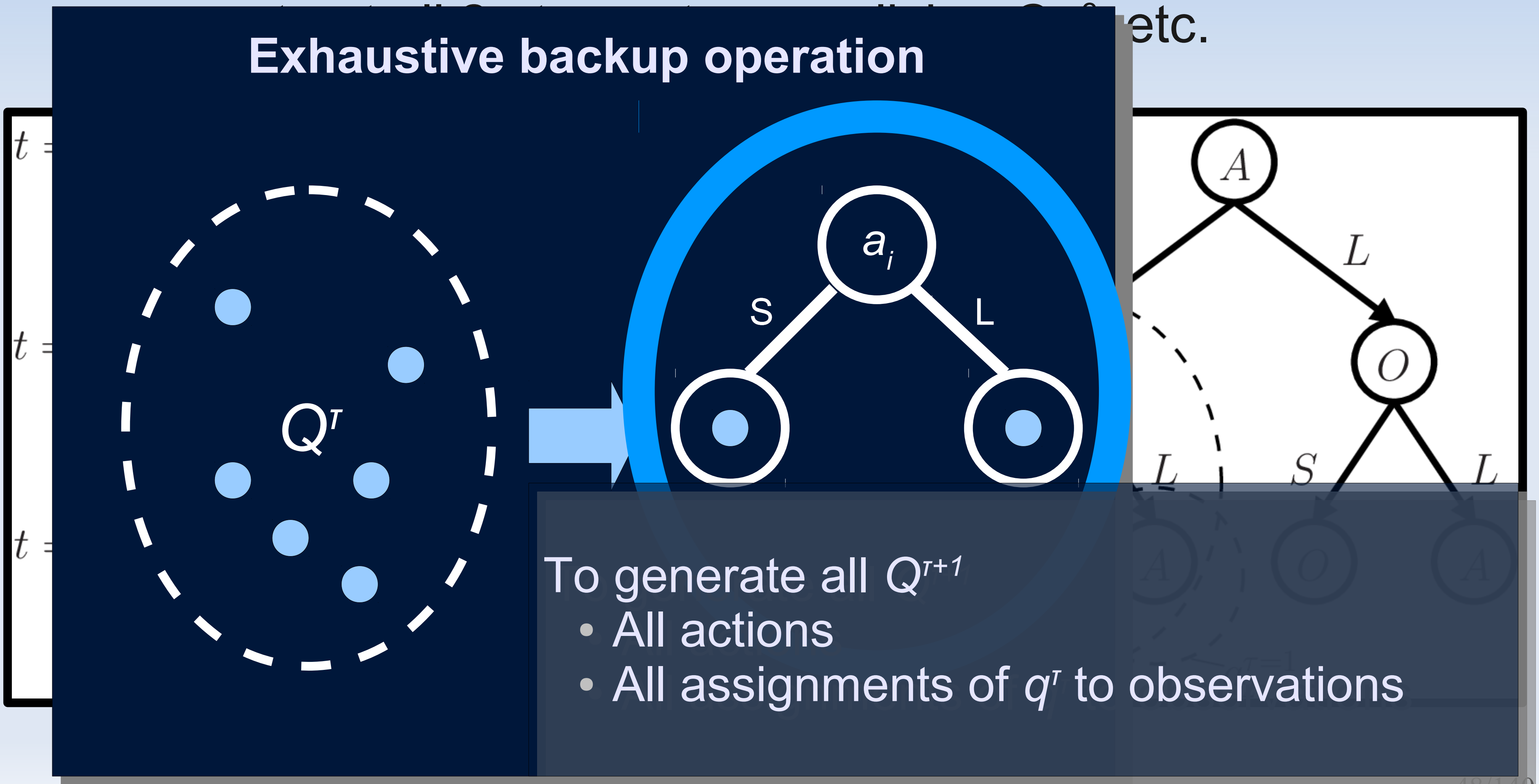
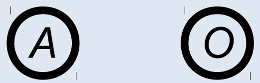
- Generate all policies in a special way:
  - from 1 stage-to-go policies $Q^{\tau=1}$
  - etc.



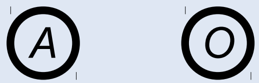
Dynamic Programming – 2

- (obviously) this scales very poorly...

$Q_1^{\tau=1}$



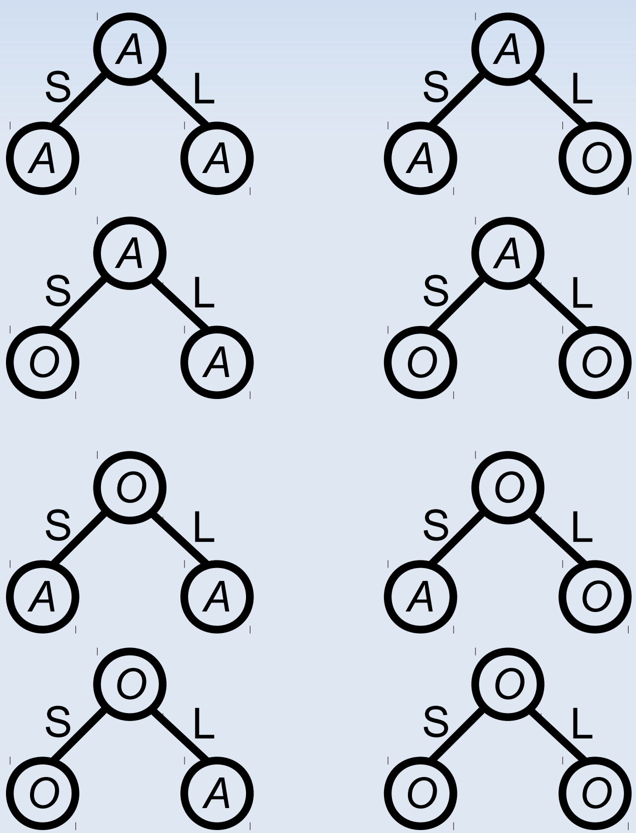
$Q_2^{\tau=1}$



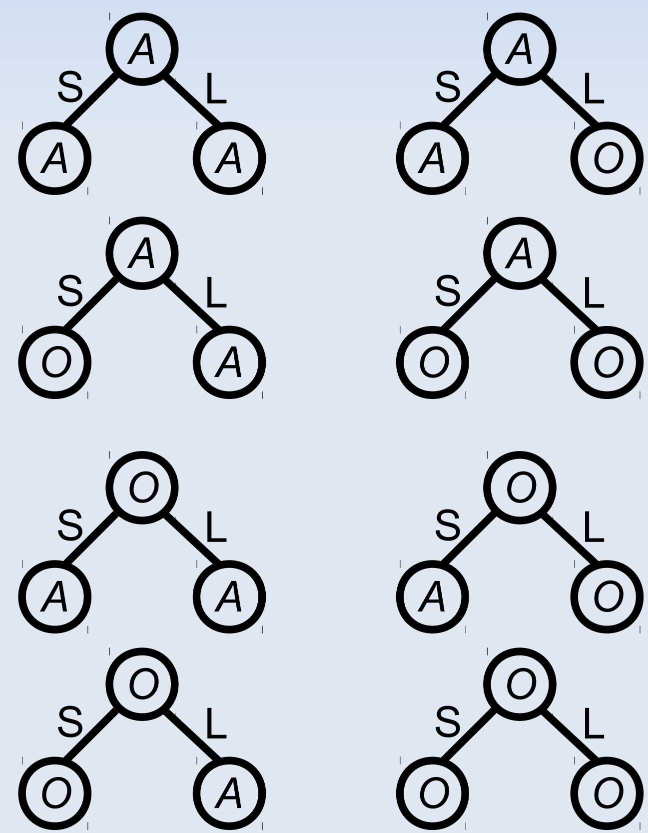
Dynamic Programming – 2

- (obviously) this scales very poorly...

$Q_1^{\tau=2}$



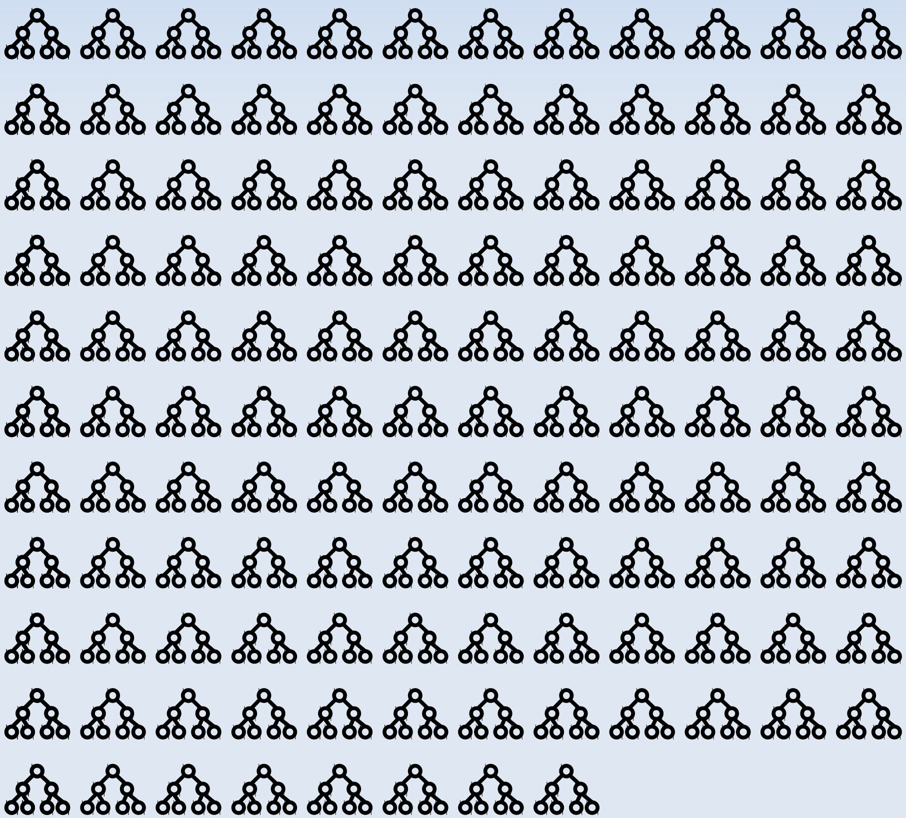
$Q_2^{\tau=2}$



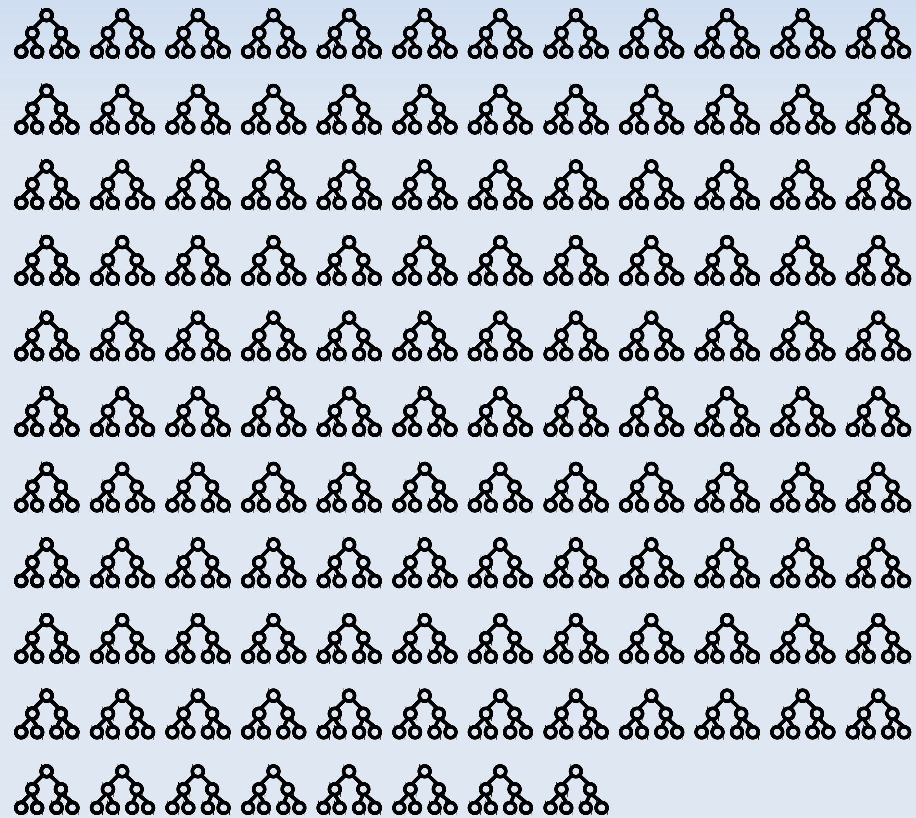
Dynamic Programming – 2

- (obviously) this scales very poorly...

$Q_1^{\tau=3}$



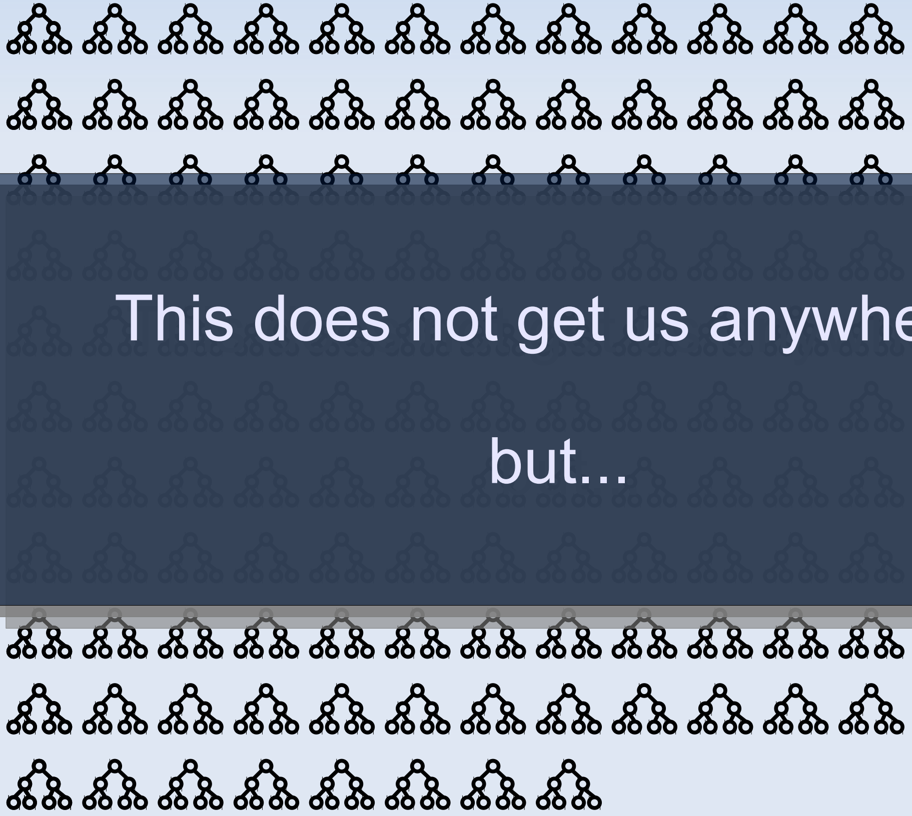
$Q_2^{\tau=3}$



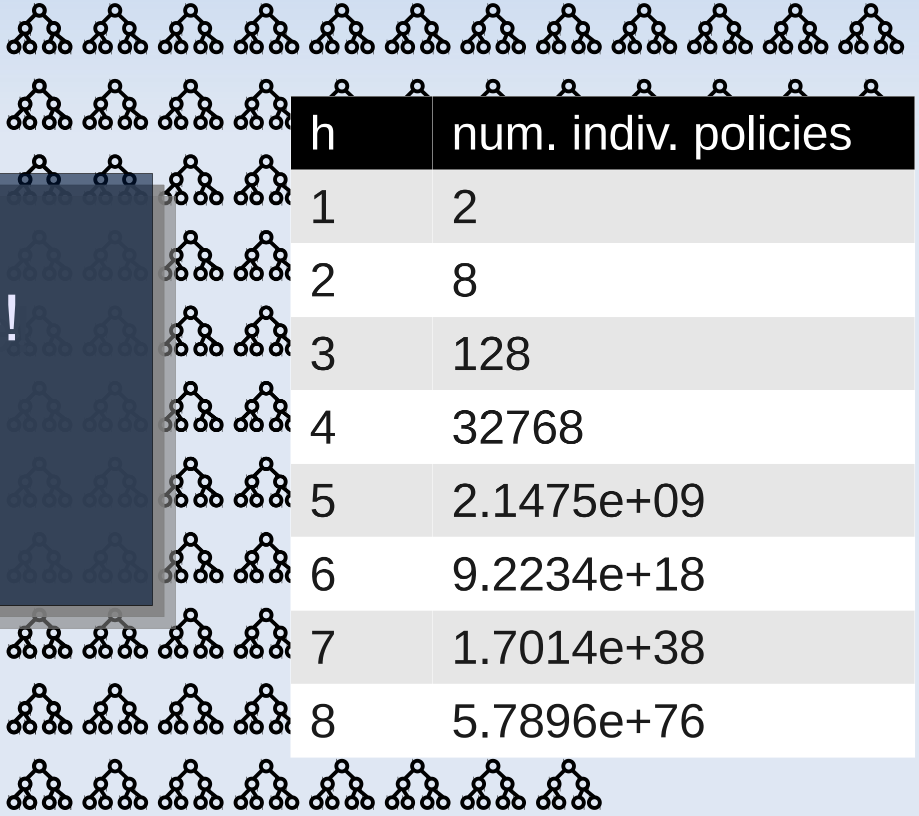
Dynamic Programming – 2

- (obviously) this scales very poorly...

$Q_1^{\tau=3}$



$Q_2^{\tau=3}$



This does not get us anywhere!

but...

h	num. indiv. policies
1	2
2	8
3	128
4	32768
5	2.1475e+09
6	9.2234e+18
7	1.7014e+38
8	5.7896e+76

Dynamic Programming – 3

- Perhaps not all those Q_i^τ are useful!
  - Perform **pruning** of 'dominated policies'!

- Algorithm [Hansen et al. 2004]

$$Q_i^{\tau=1} = A_i$$

```
Initialize Q1(1), Q2(1)
for tau=2 to h
  Q1(tau) = ExhaustiveBackup(Q1(tau-1))
  Q2(tau) = ExhaustiveBackup(Q2(tau-1))
  Prune(Q1,Q2,tau)
end
```

Dynamic Programming – 3

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  Q2(tau) = ExhaustiveBackup(Q2(tau-1))
  Prune(Q1,Q2,tau)
end
```

Note: cannot prune independently!

- usefulness of a q_1 depends on Q_2
- and vice versa

→ **Iterated elimination** of policies

Dynamic Programming – 4

- Initialization

$Q_1^{\tau=1}$

A O

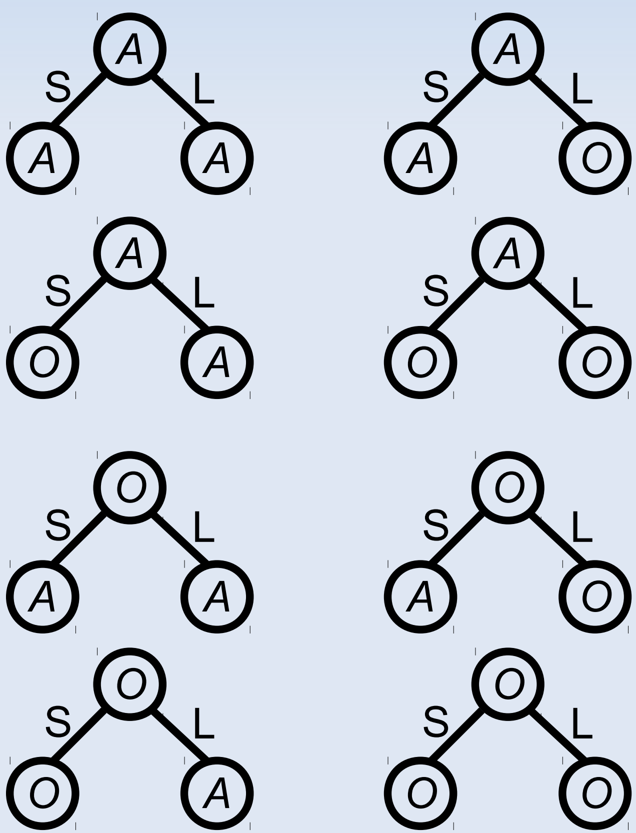
$Q_2^{\tau=1}$

A O

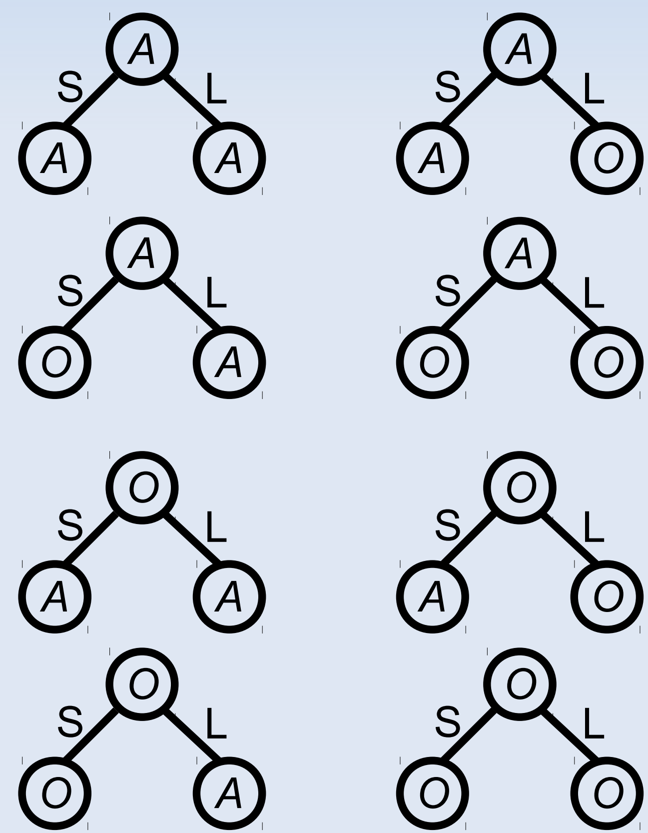
Dynamic Programming – 4

- Exhaustive Backups gives

$Q_1^{\tau=2}$



$Q_2^{\tau=2}$

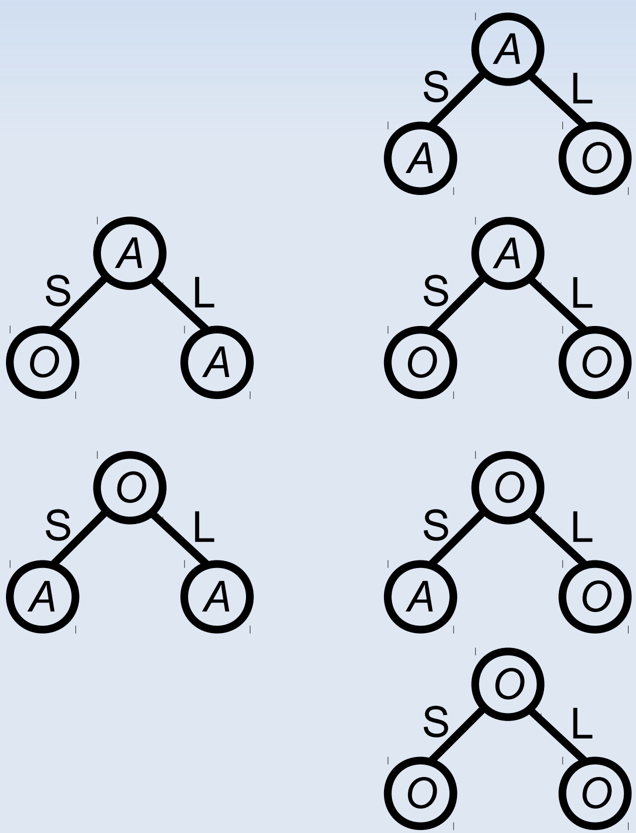


Dynamic Programming – 4

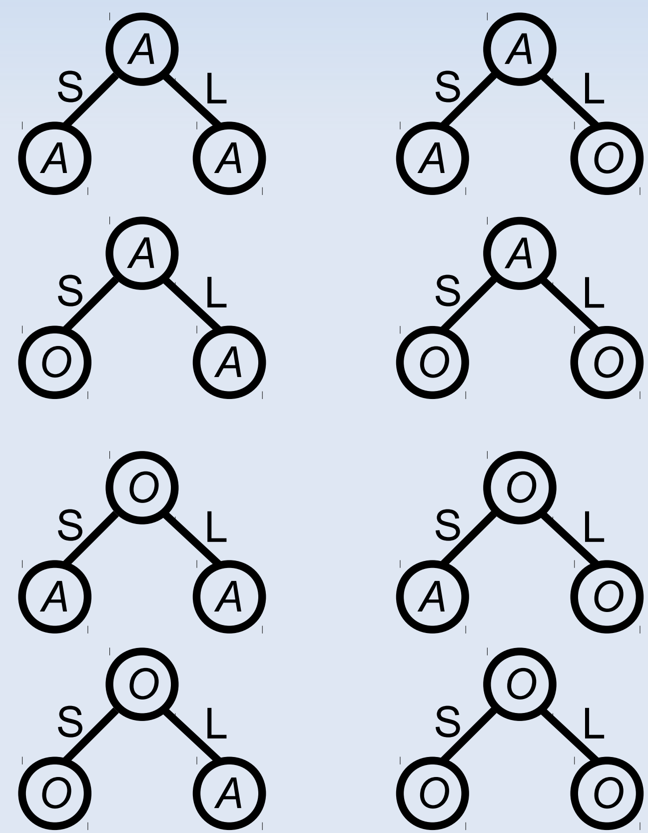
Hypothetical Pruning
(not the result of actual pruning)

- Pruning agent 1...

$Q_1^{\tau=2}$



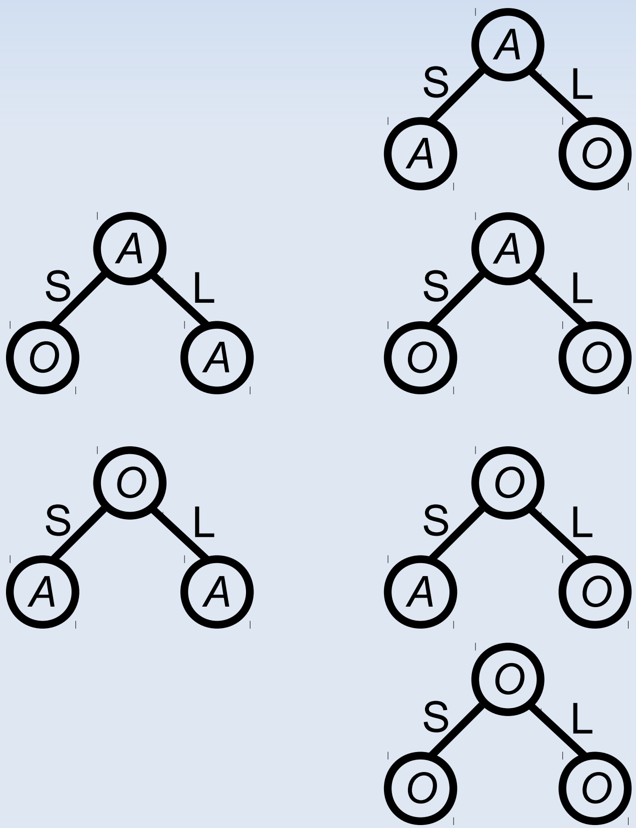
$Q_2^{\tau=2}$



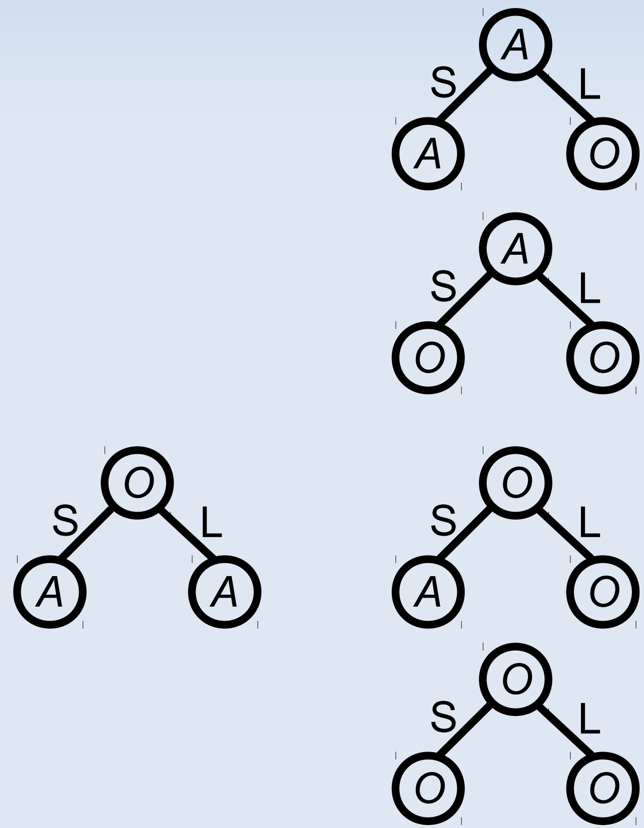
Dynamic Programming – 4

- Pruning agent 2...

$Q_1^{\tau=2}$



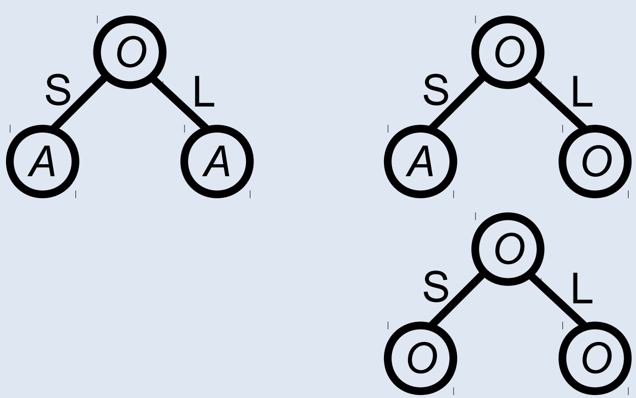
$Q_2^{\tau=2}$



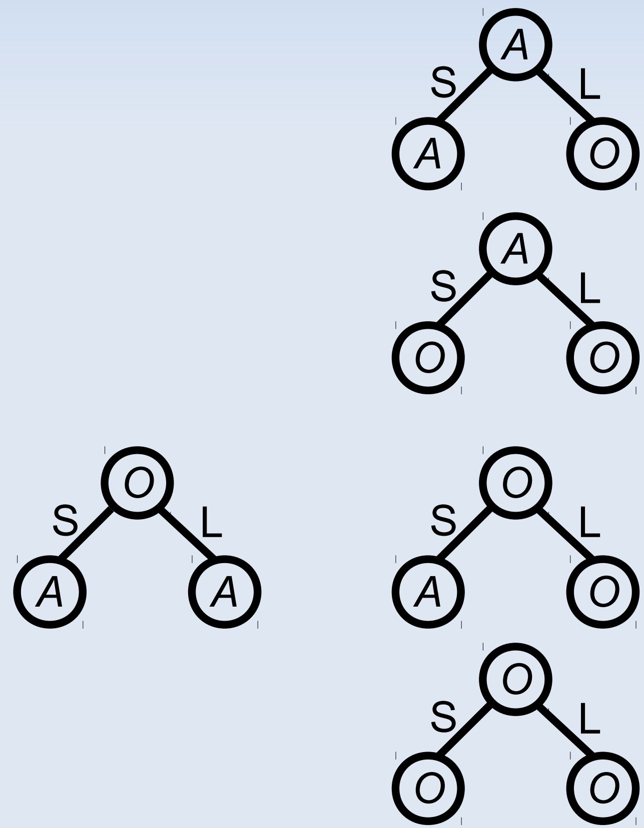
Dynamic Programming – 4

- Pruning agent 1...

$Q_1^{\tau=2}$



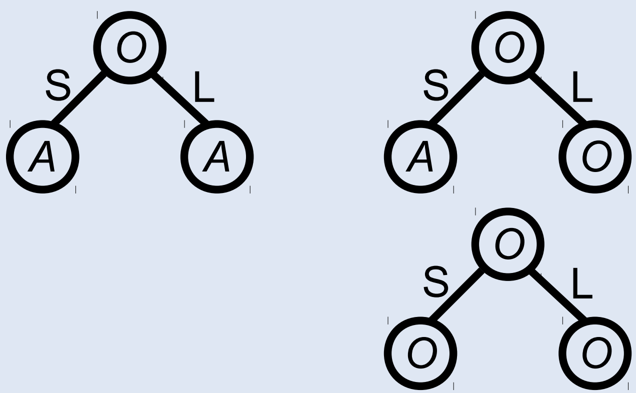
$Q_2^{\tau=2}$



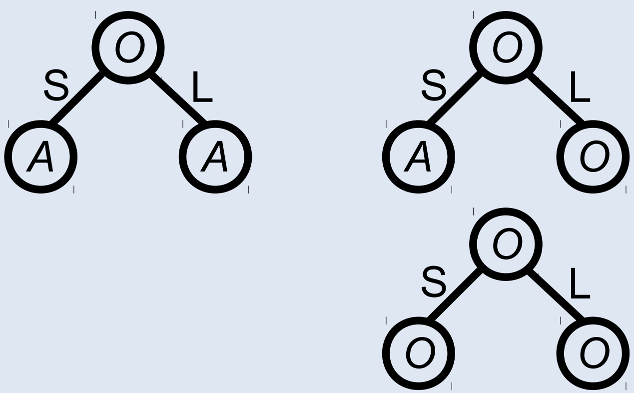
Dynamic Programming – 4

- Etc...

$Q_1^{\tau=2}$



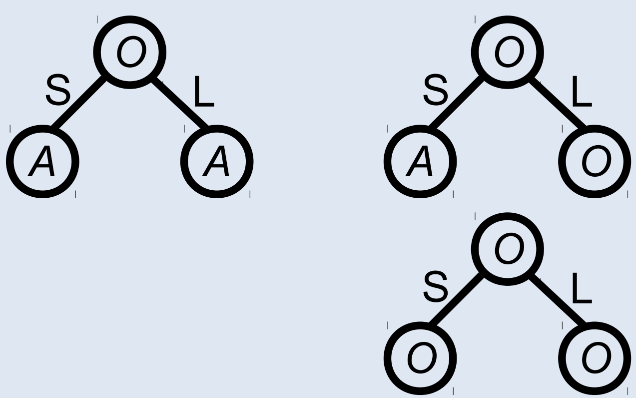
$Q_2^{\tau=2}$



Dynamic Programming – 4

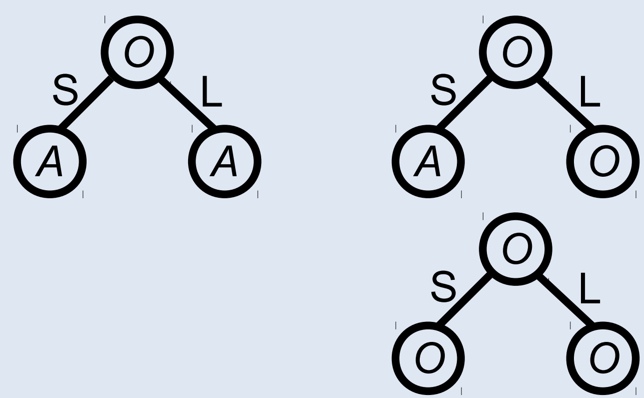
- Etc...

$Q_1^{\tau=2}$



$Q_2^{\tau=2}$

In this case: symmetric
→ but need not be in general!



Dynamic Programming – 4

- Exhaustive backups:

We **avoid** generation of many policies!

$Q_1^{\tau=3}$ $Q_2^{\tau=3}$

Dynamic Programming – 4

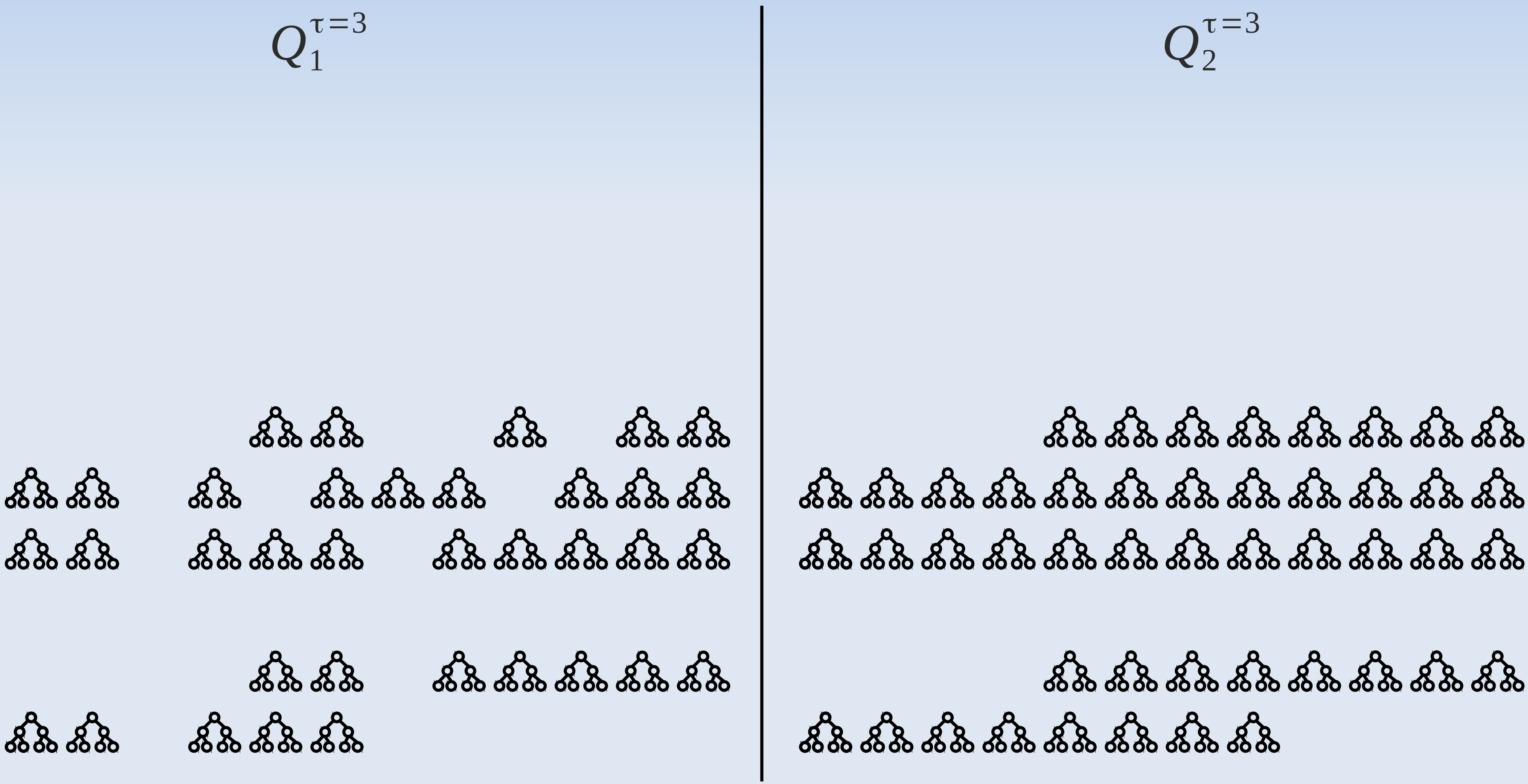
- Exhaustive backups:

$Q_1^{\tau=3}$

$Q_2^{\tau=3}$

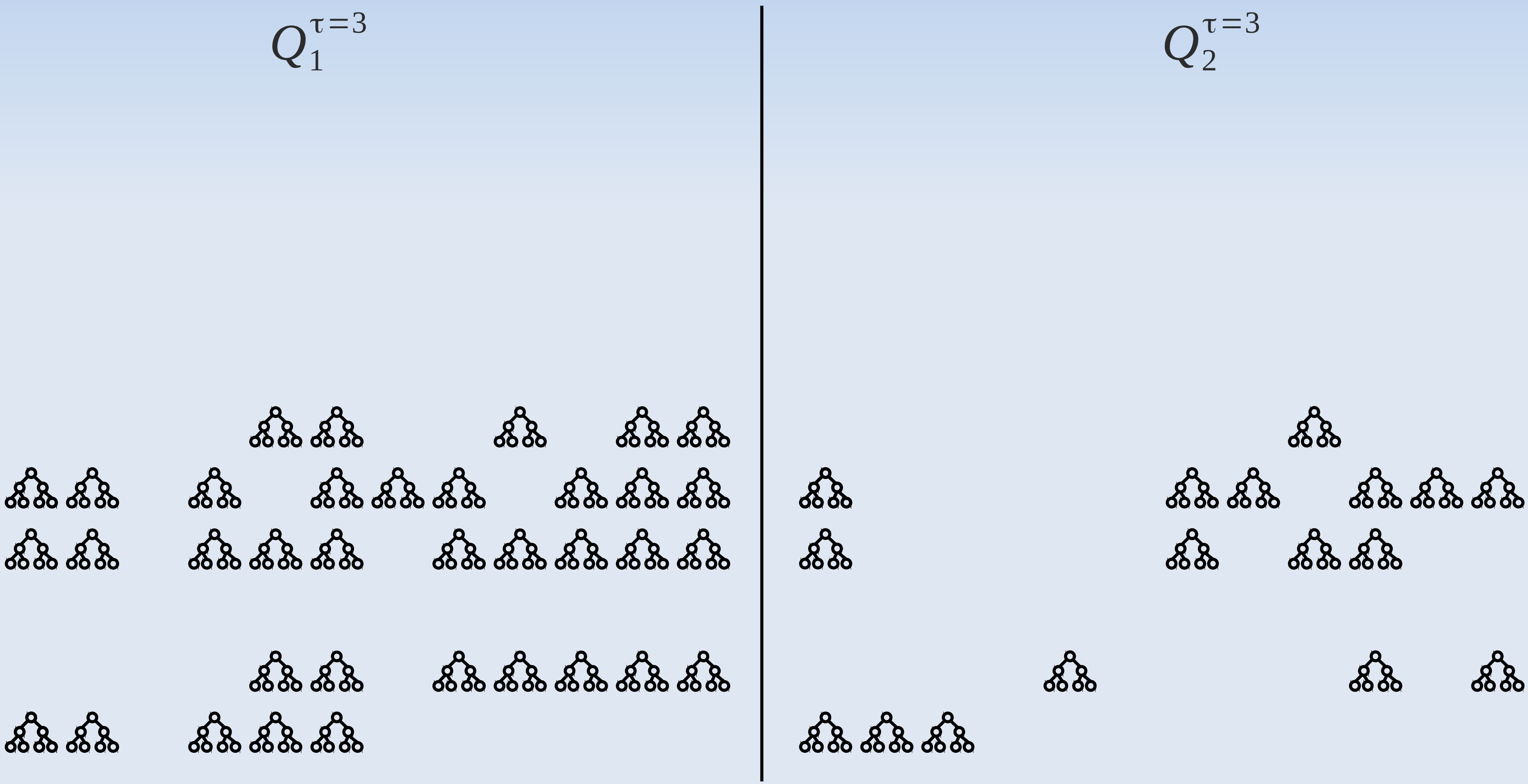
Dynamic Programming – 4

- Pruning agent 1...



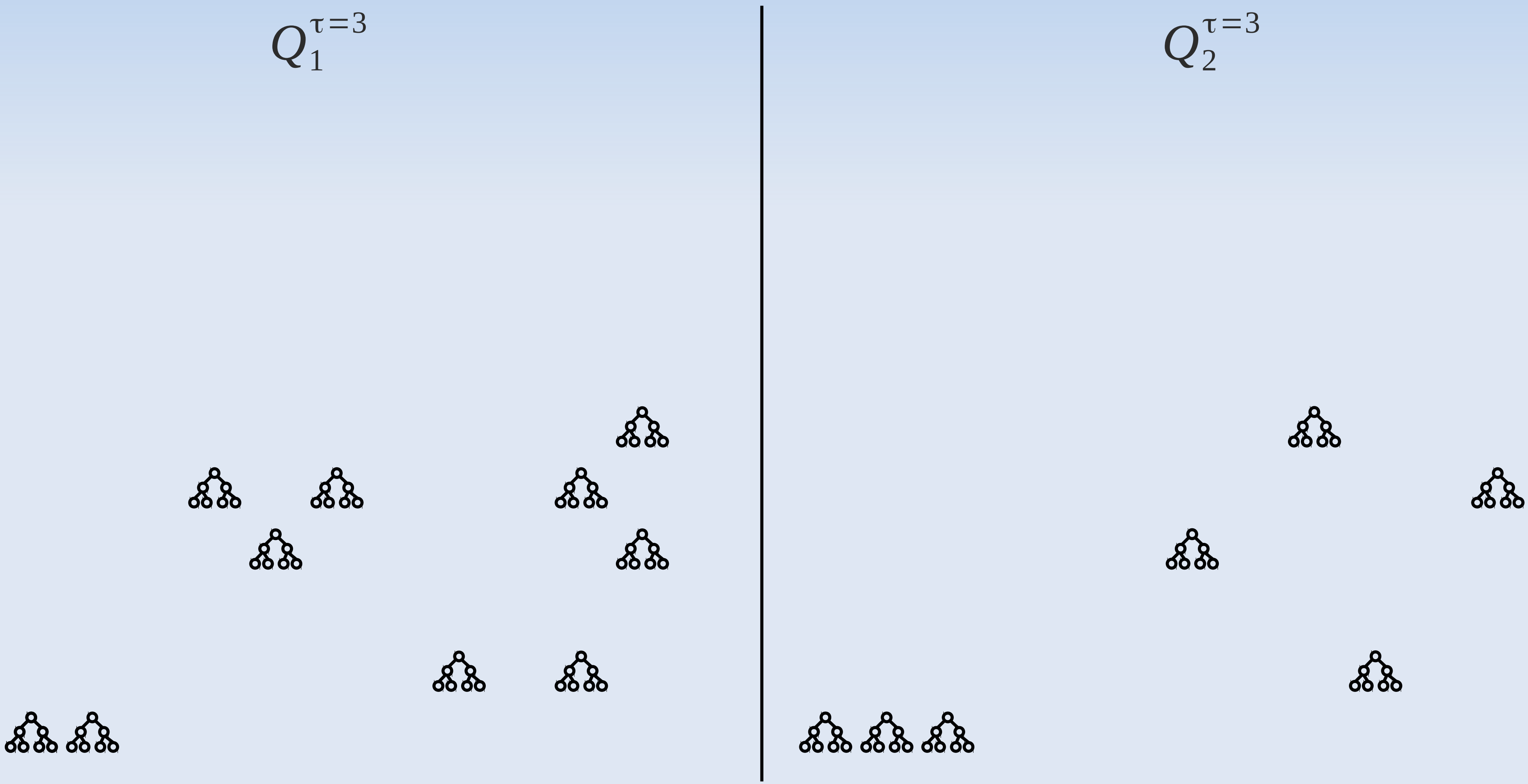
Dynamic Programming – 4

- Pruning agent 2...



Dynamic Programming – 4

- Etc...

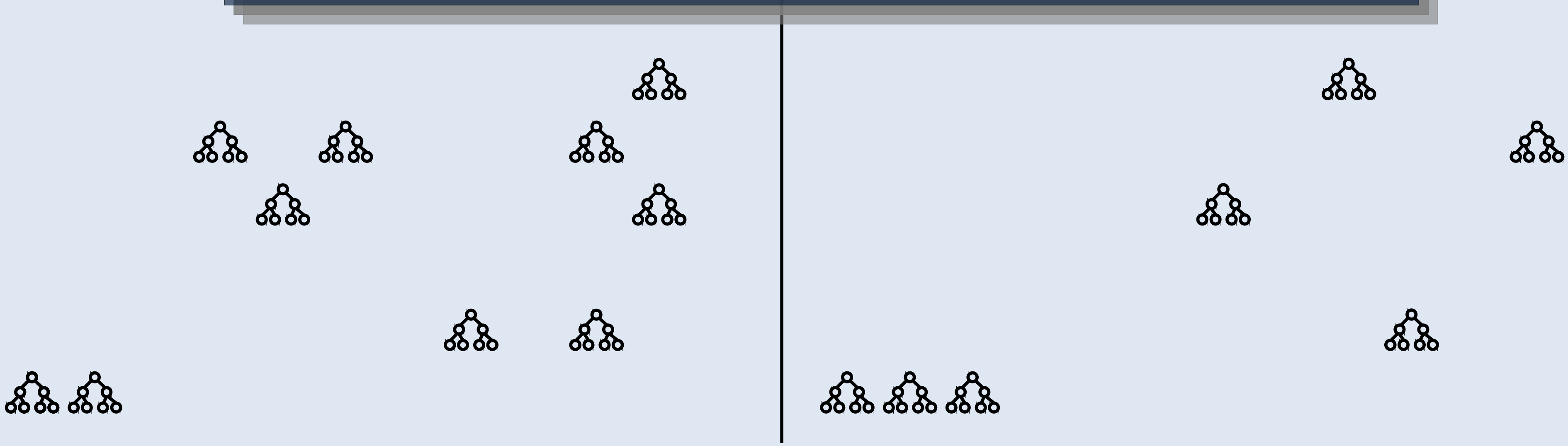


Dynamic Programming – 4

- Etc...

At the very end:

- ...?

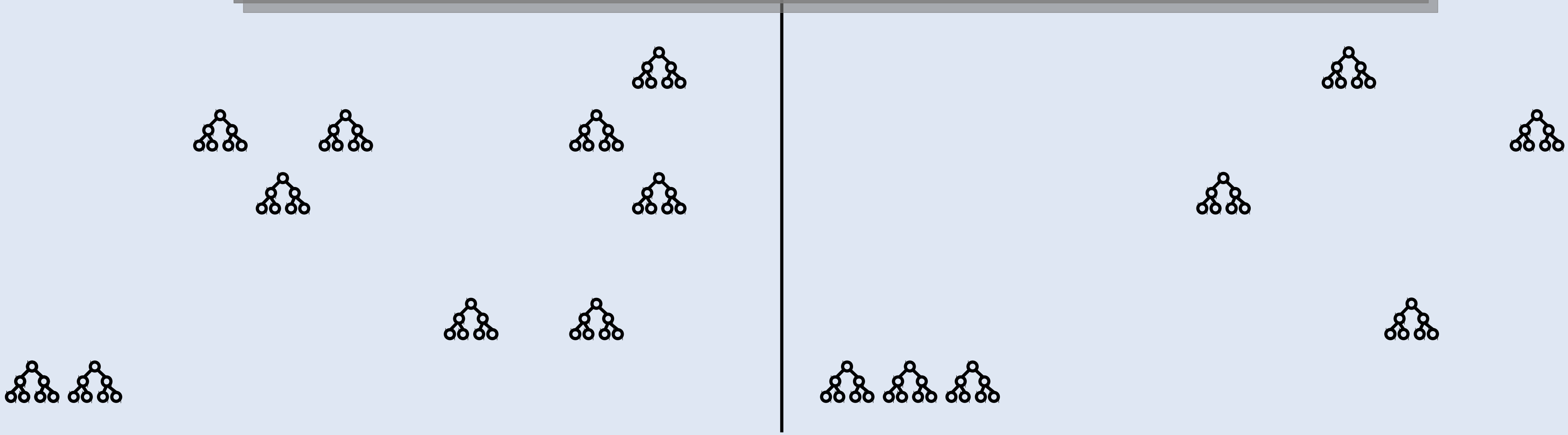


Dynamic Programming – 4

- Etc...

At the very end:

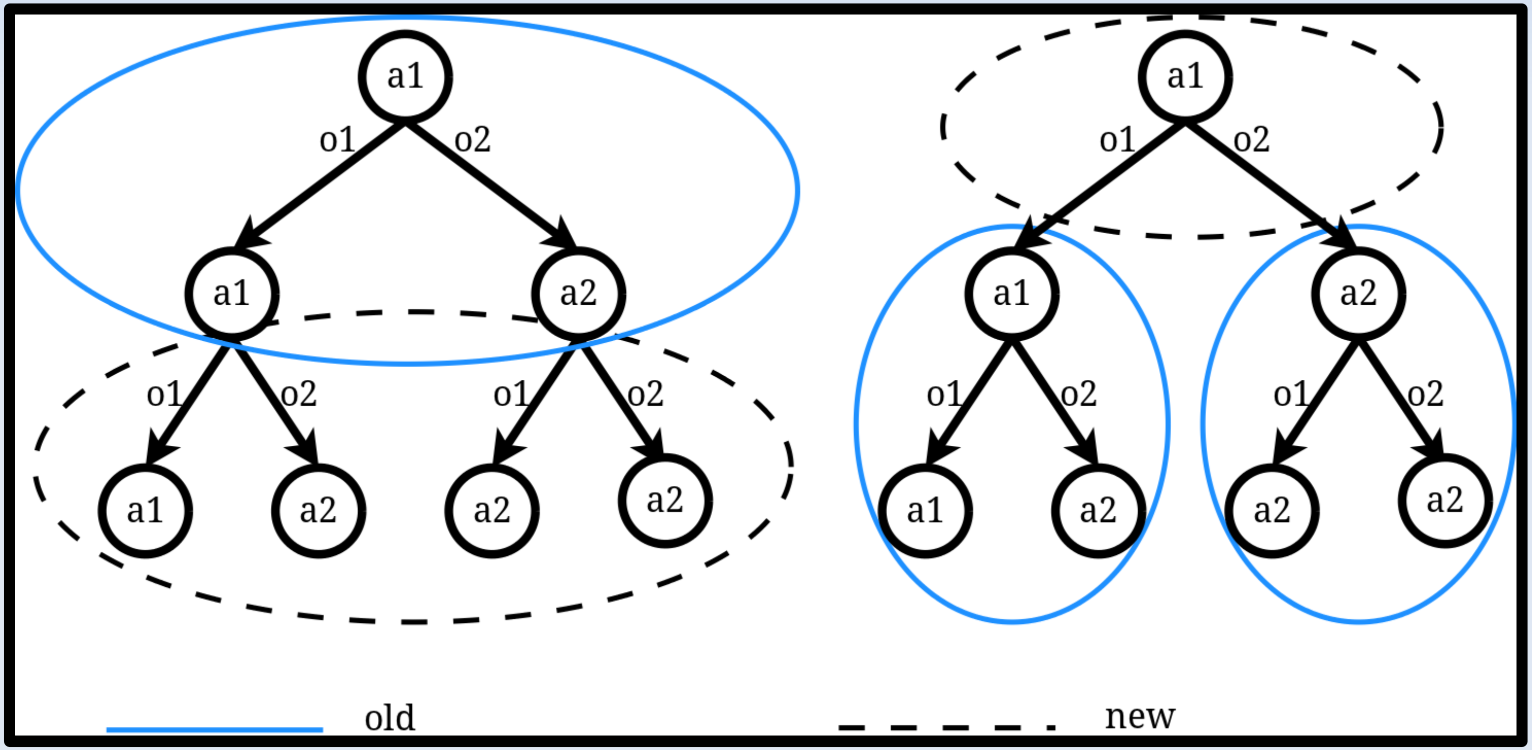
- evaluate all the remaining combinations of policies (i.e., the 'induced joint policies')
- select the best one



Bottom-up vs. Top-down

- DP constructs bottom-up
- Alternatively try and construct top down

  → leads to (heuristic) search [Szer et al. 2005, Oliehoek et al. 2008]



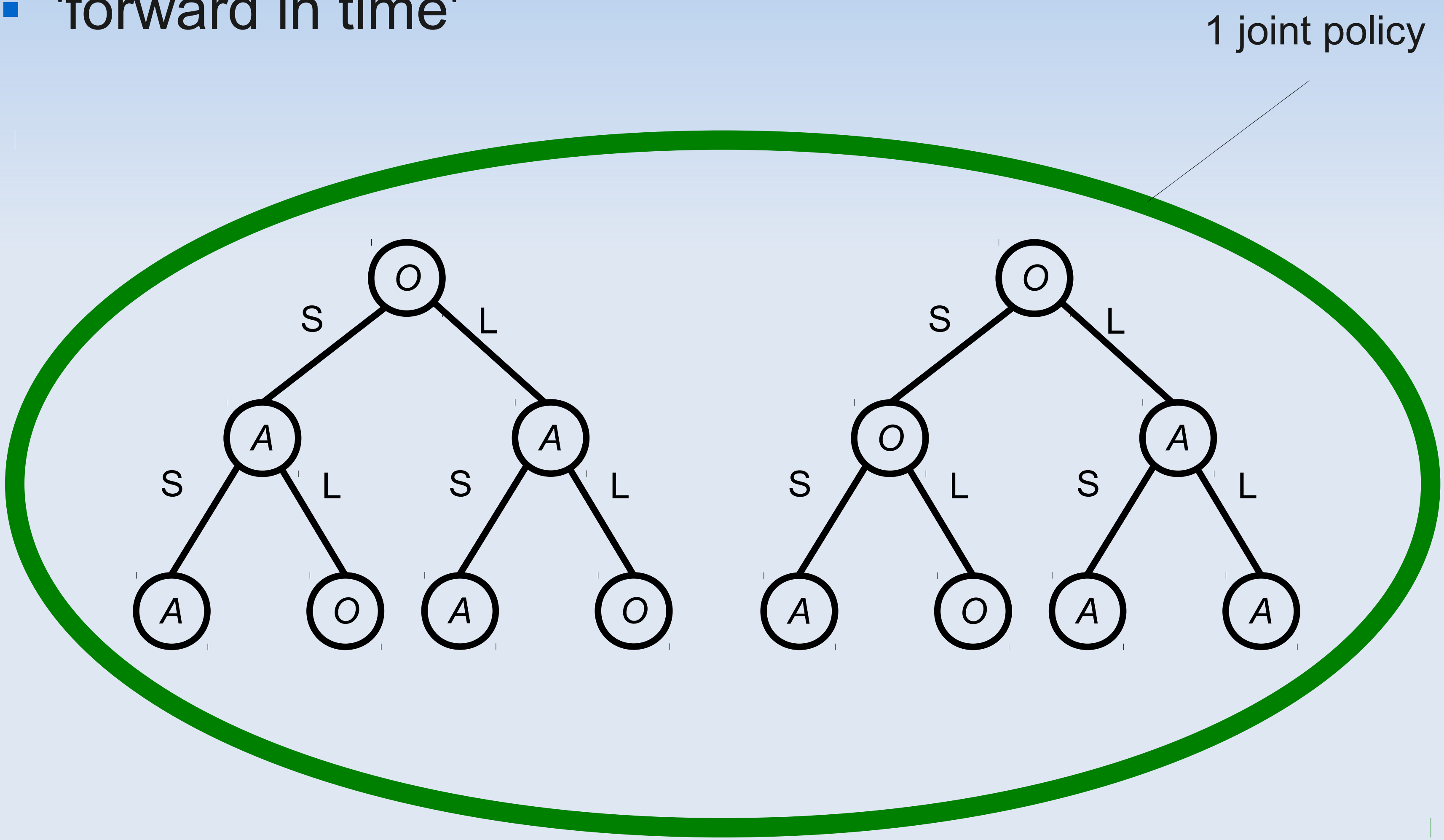
Heuristic Search – Intro

- Core idea is the same as DP:
  - incrementally construct all (joint) policies
  - try to avoid work

- Differences
  - different starting point and increments
  - use **heuristics** (rather than pruning) to avoid work

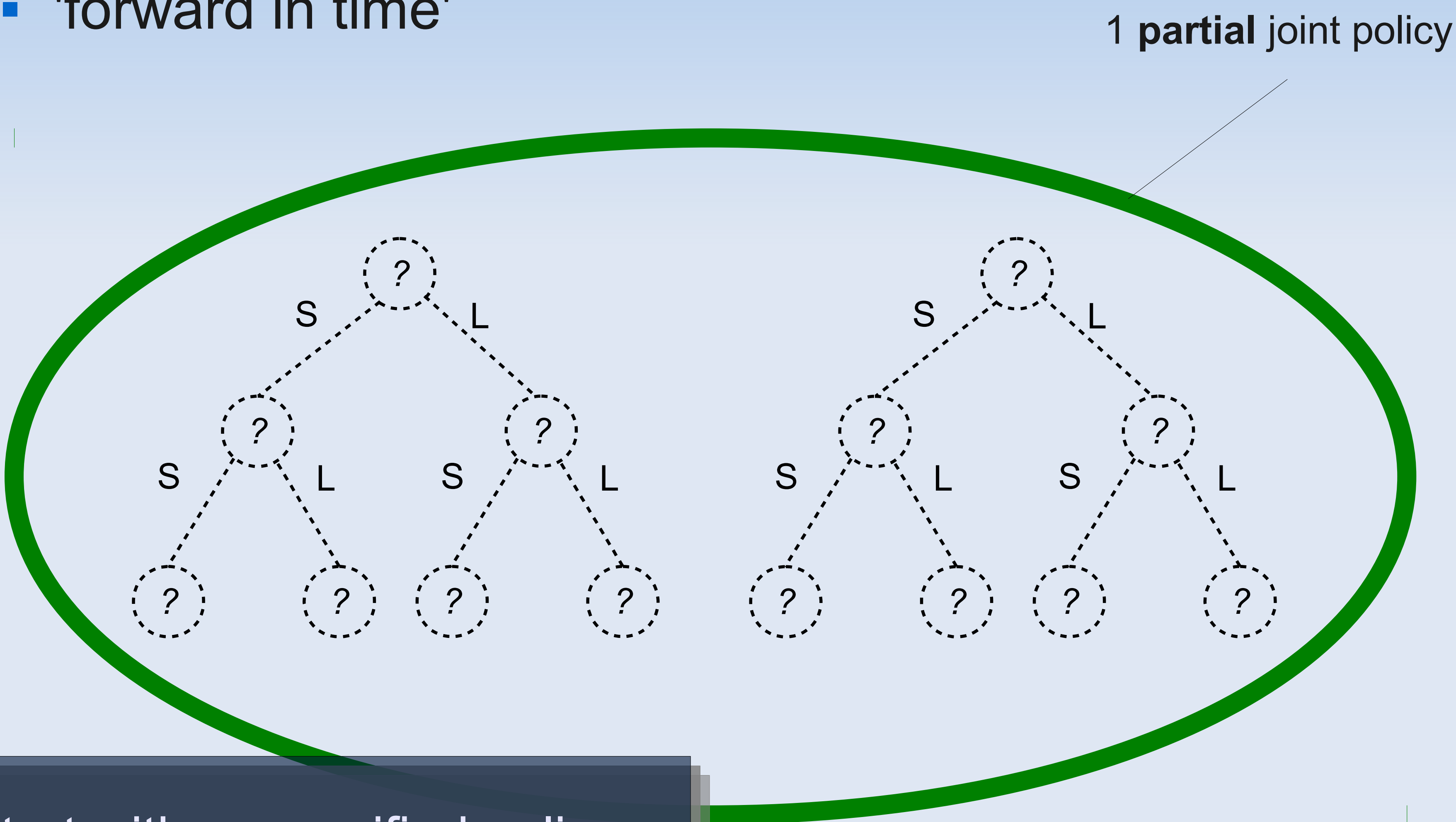
Heuristic Search – 1

- Incrementally construct all (joint) policies
  - 'forward in time'



Heuristic Search – 1

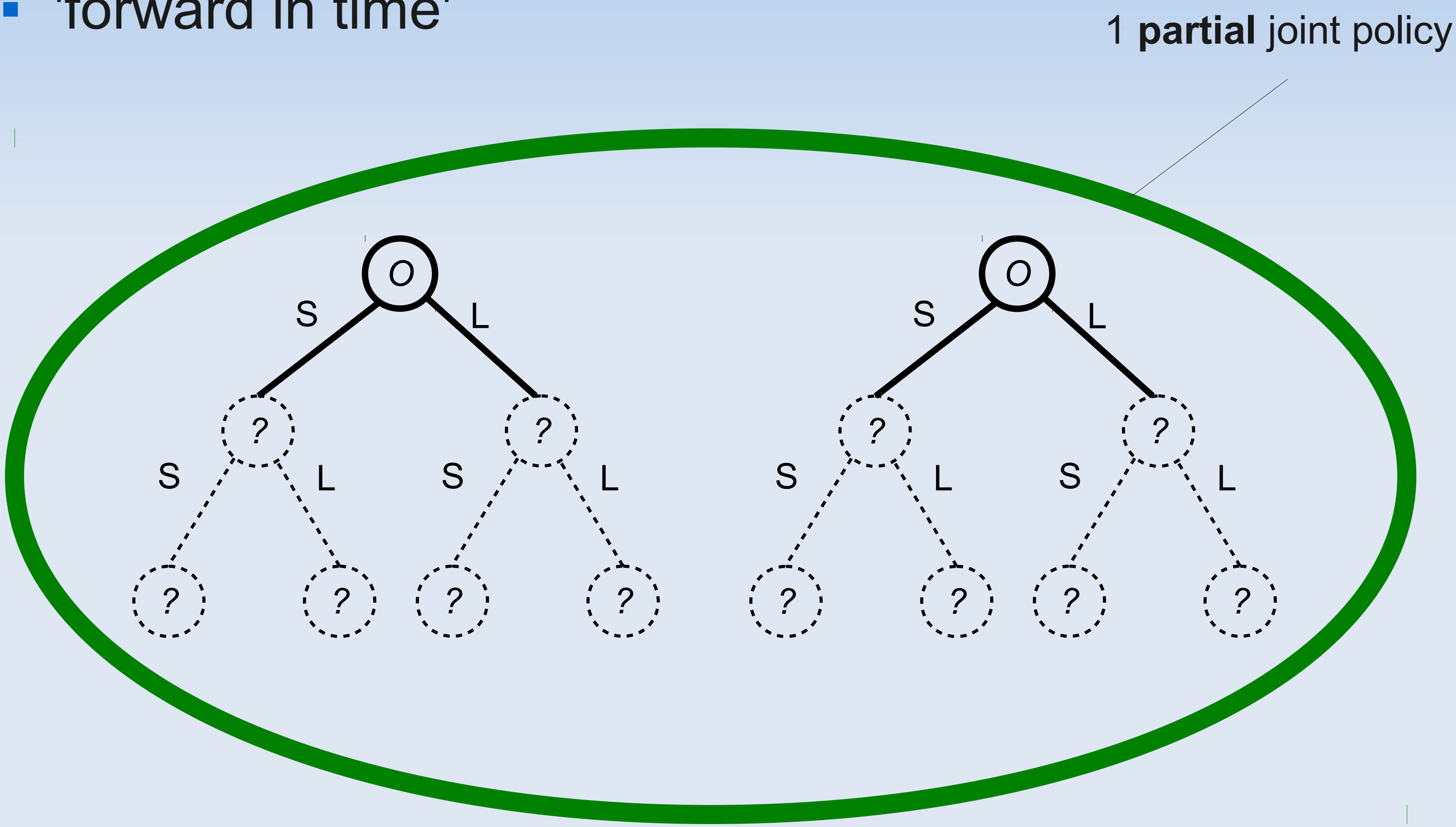
- Incrementally construct all (joint) policies
  - 'forward in time'



Start with unspecified policy

Heuristic Search – 1

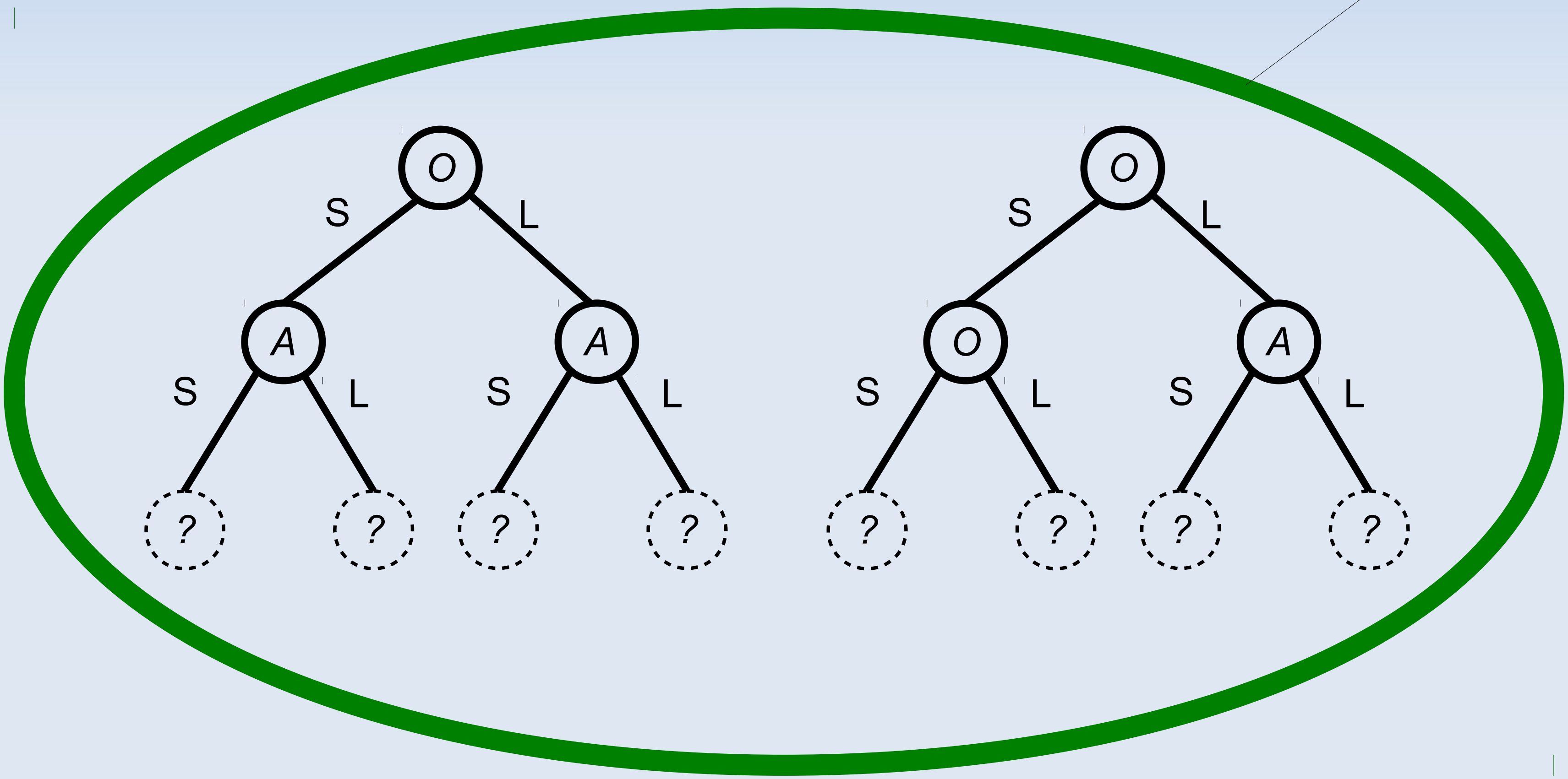
- Incrementally construct all (joint) policies
  - 'forward in time'



Heuristic Search – 1

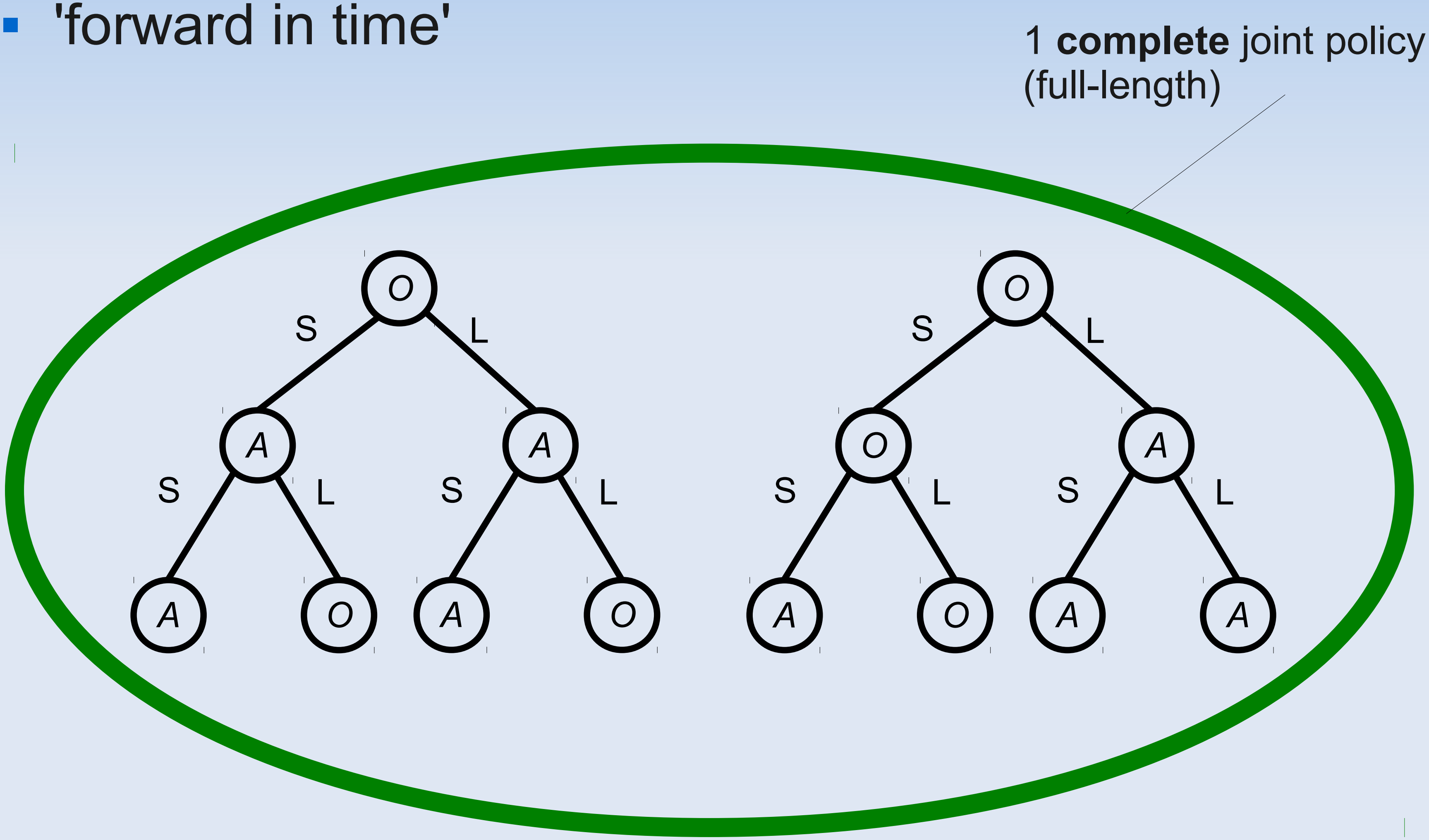
- Incrementally construct all (joint) policies
  - 'forward in time'

1 **partial** joint policy



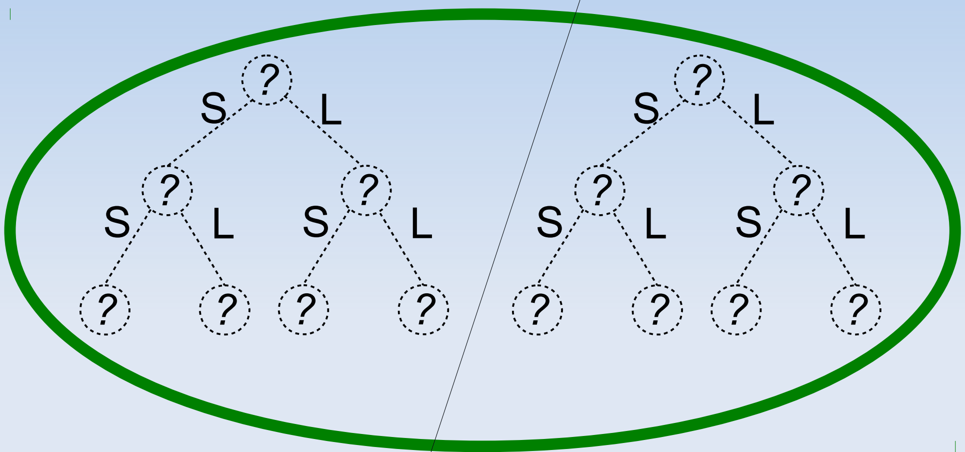
Heuristic Search – 1

- Incrementally construct all (joint) policies
  - 'forward in time'



Heuristic Search – 2

- Creating **ALL** joint policies → tree structure!

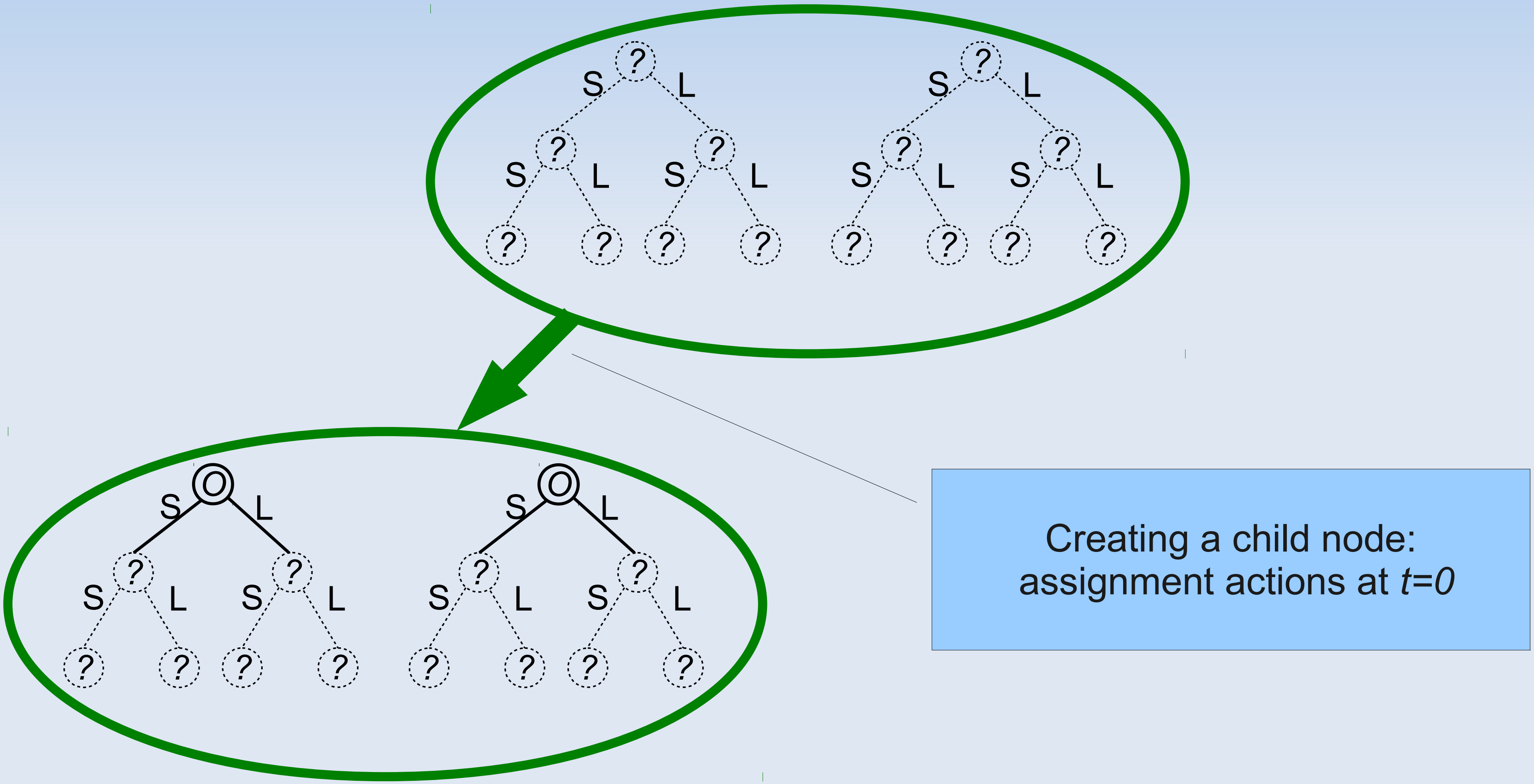


Root node:
unspecified joint policy

why?

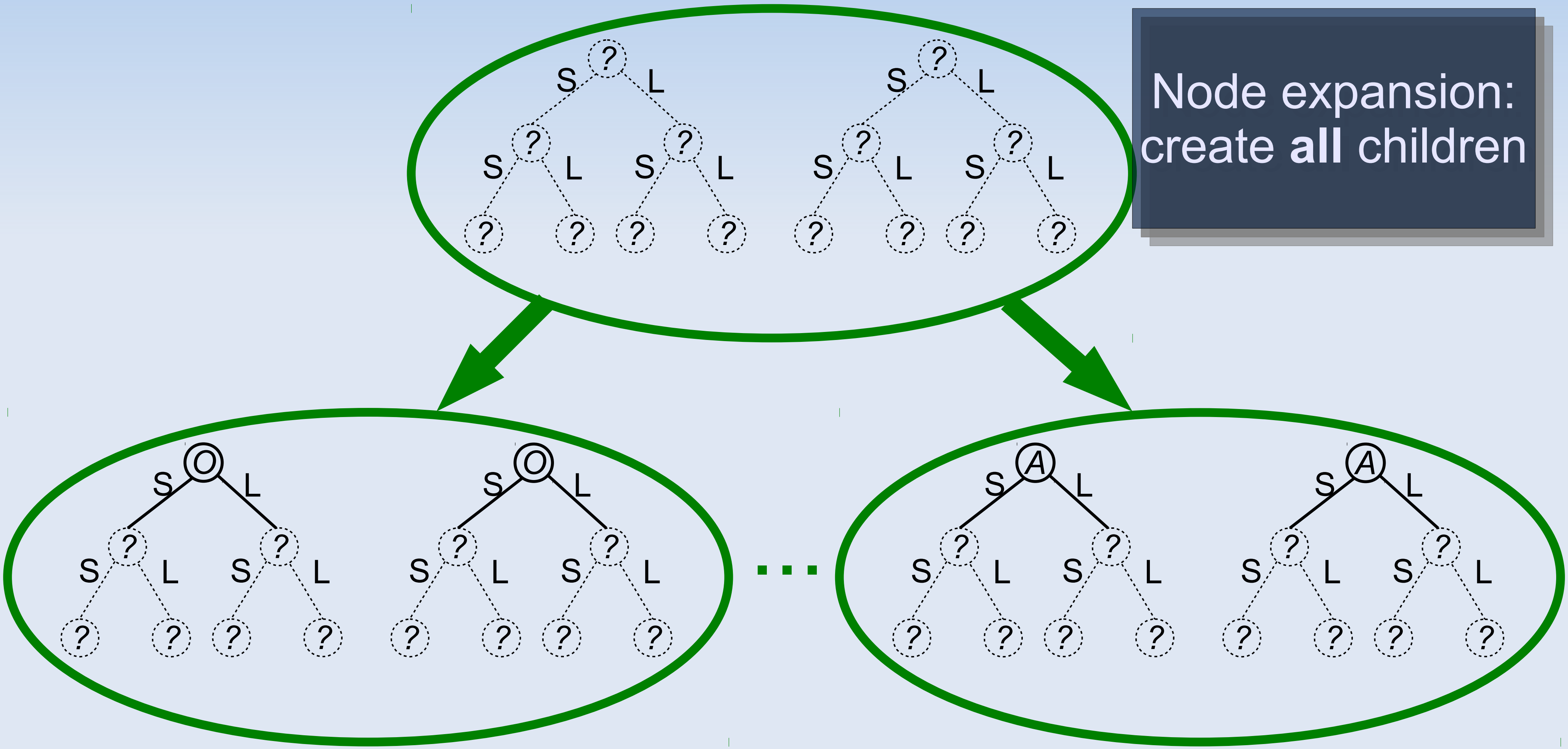
Heuristic Search – 2

- Creating **ALL** joint policies → tree structure!



Heuristic Search – 2

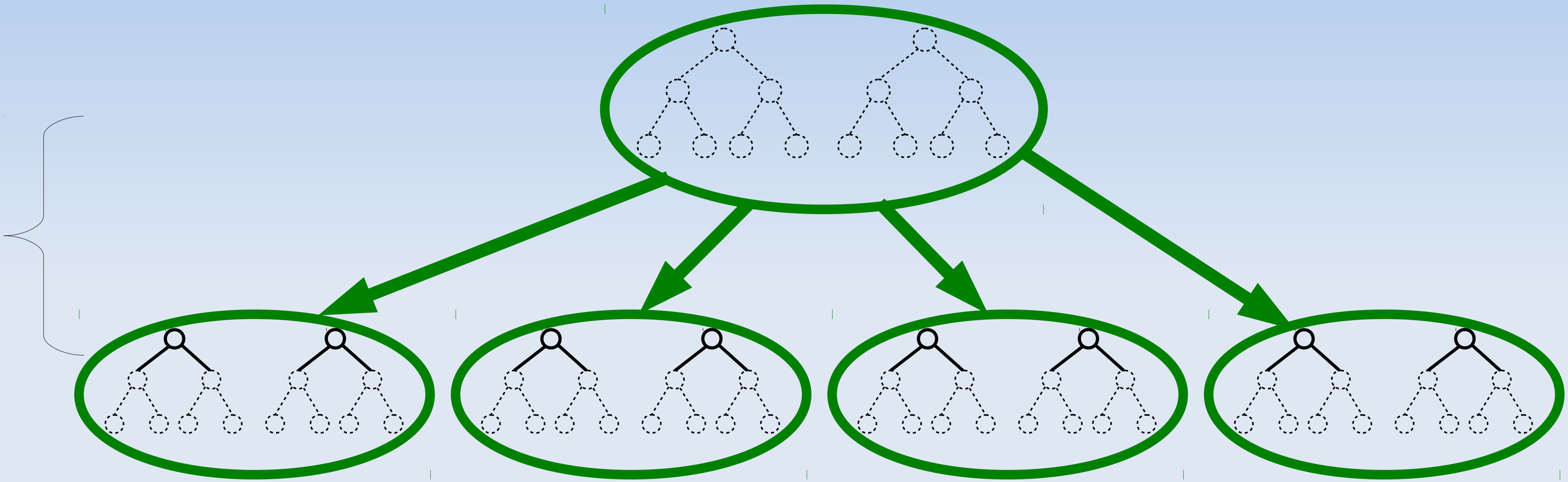
- Creating **ALL** joint policies → tree structure!



Heuristic Search – 2

- Creating **ALL** joint policies → tree structure!

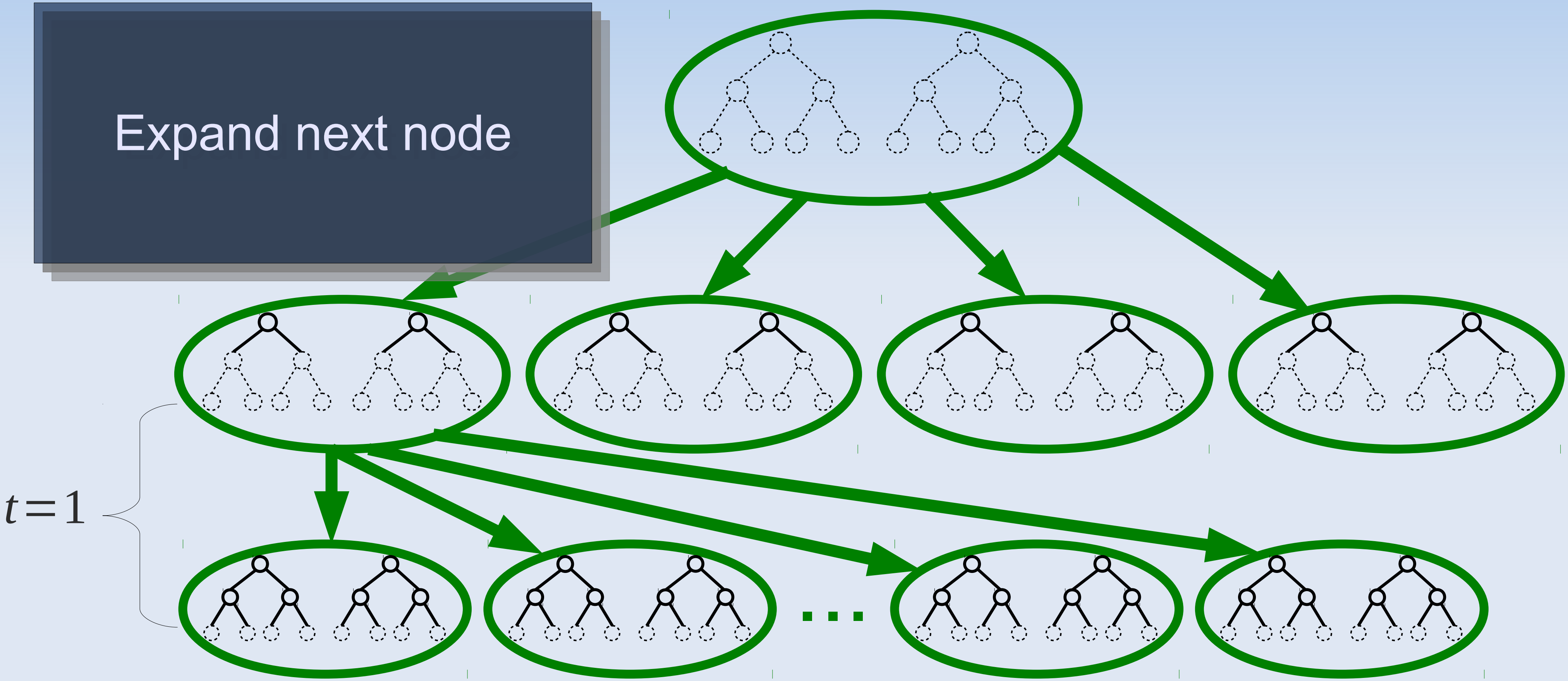
$t = 0$



Heuristic Search – 2

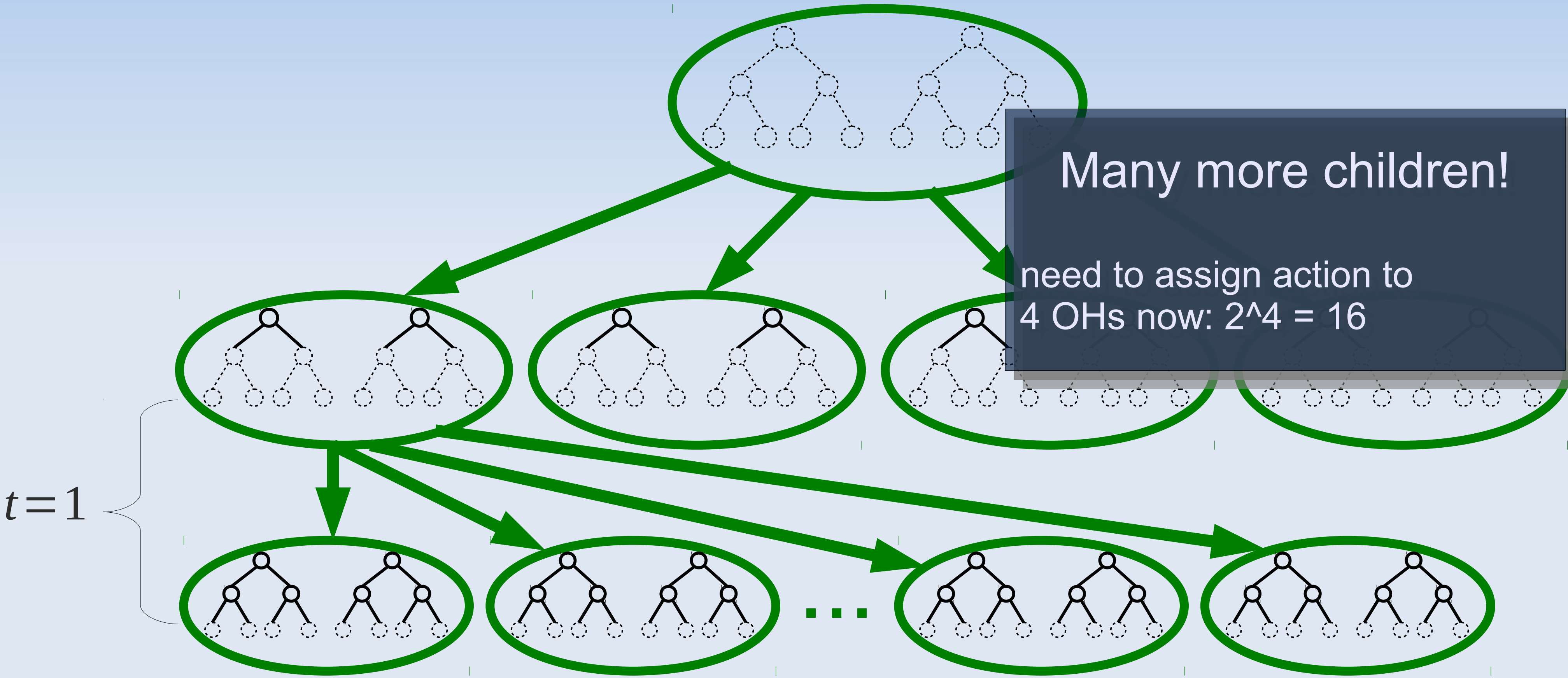
- Creating **ALL** joint policies → tree structure!

Expand next node



Heuristic Search – 2

- Creating **ALL** joint policies → tree structure!

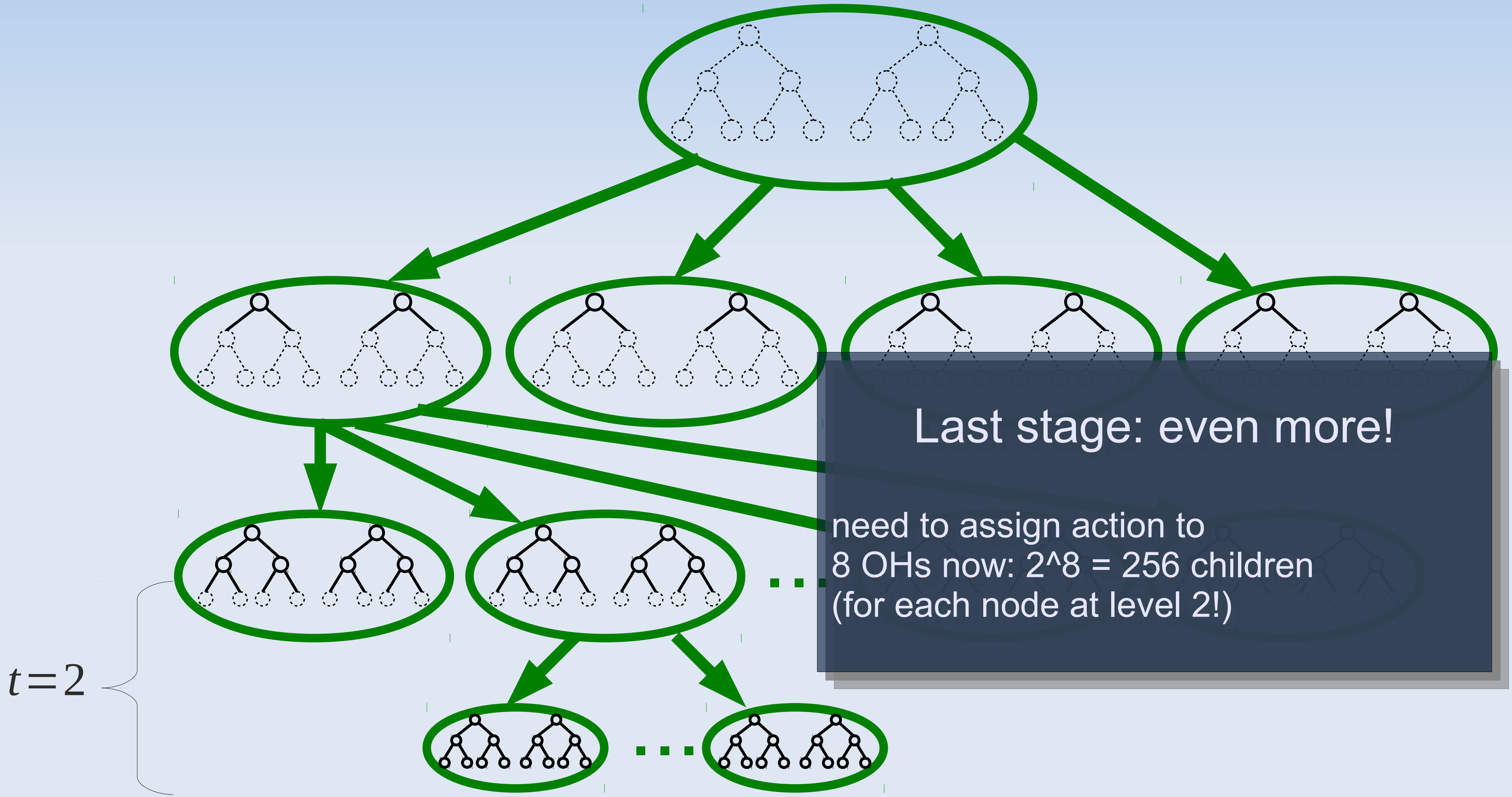


Many more children!

need to assign action to
4 OHs now: 2^4 = 16

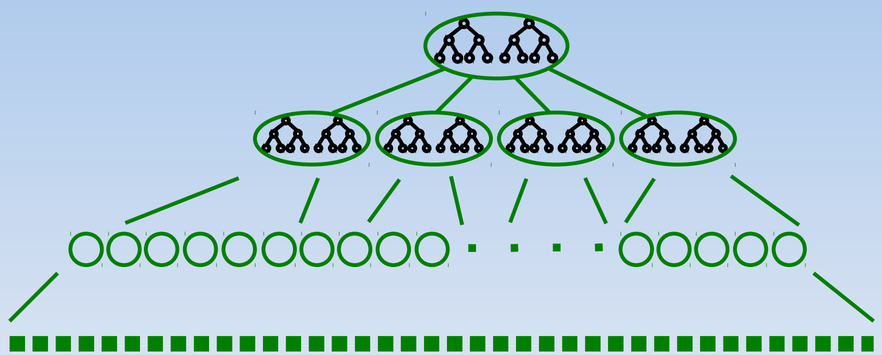
Heuristic Search – 2

- Creating **ALL** joint policies → tree structure!



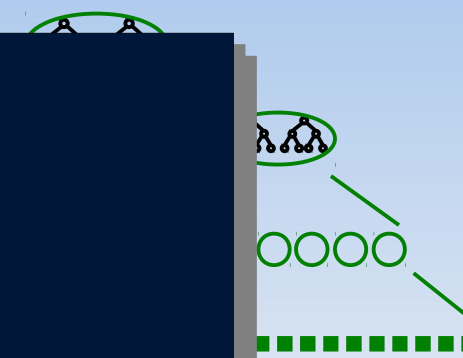
Heuristic Search – 3

- too big to create completely...
- Idea: use **heuristics**
  - avoid going down non-promising branches!
- Apply A* → **Multiagent A*** [Szer et al. 2005]

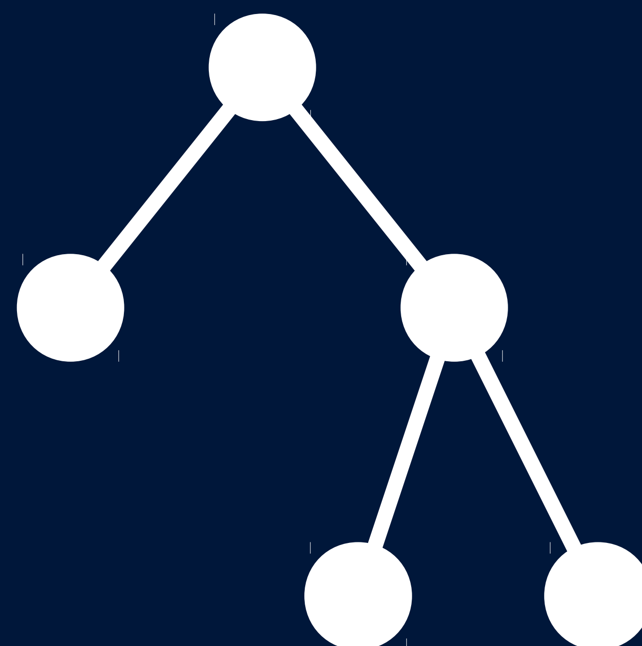


Heuristic Search – 3

- too big to create completely
- Idea:
  - avo
    nor
- Apply



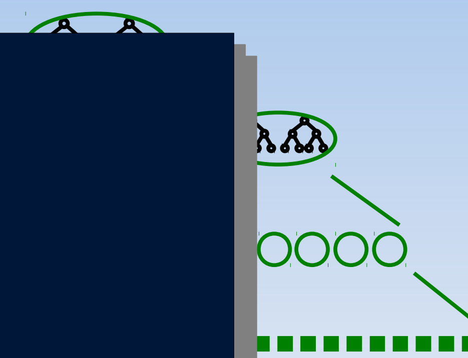
Main intuition A*



- For each node, compute F-value
- Select next node based on F-value
- More info: [Russel&Norvig 2003]

Heuristic Search – 3

- too big to create completely
- Idea:
  - avo
    nor
- Apply



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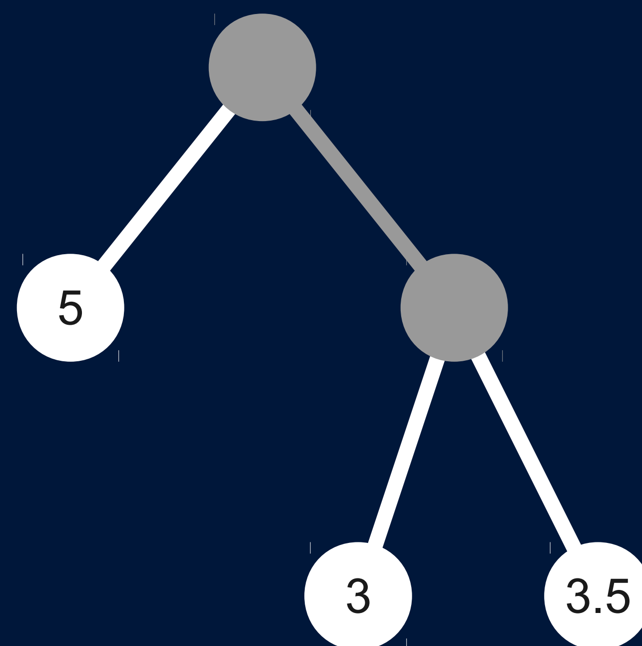

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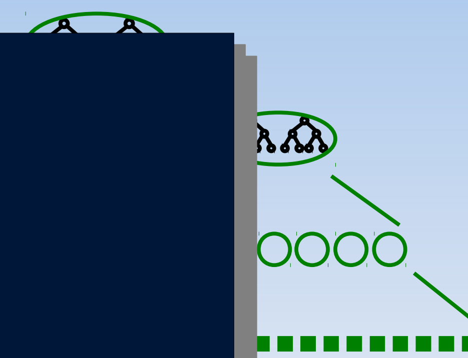
Heuristic Search – 3

- too big to create completely
- Idea:
  - avo
    non
- Apply



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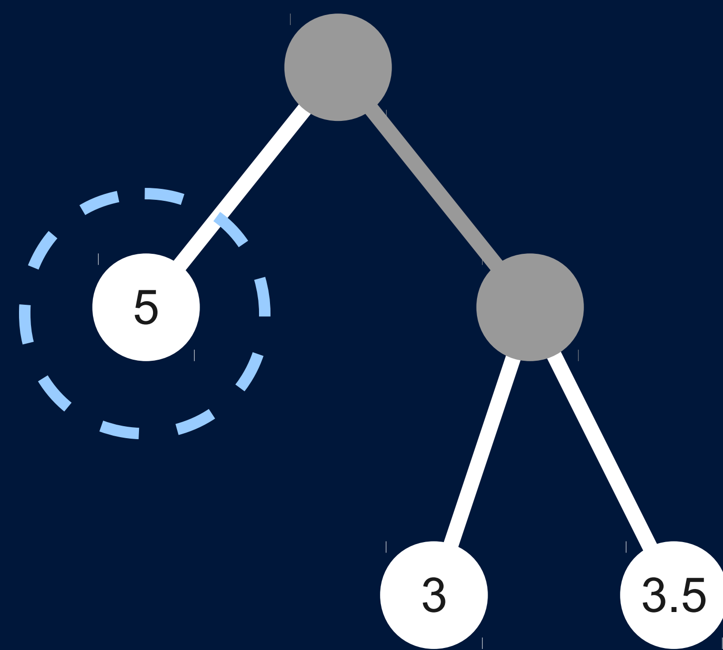
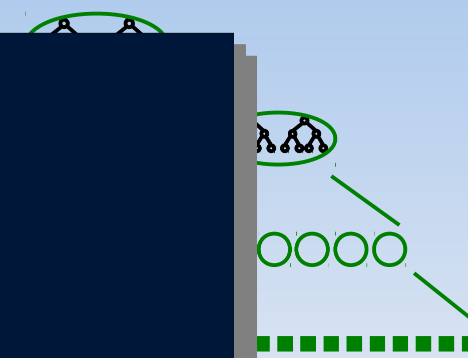


Select highest valued node & expand...

- For each node, compute F-value
- Select next node based on F-value
- More info: [Russel&Norvig 2003]

Heuristic Search – 3

- too big to create completely
- Idea:
  - avo
    nor
- Apply



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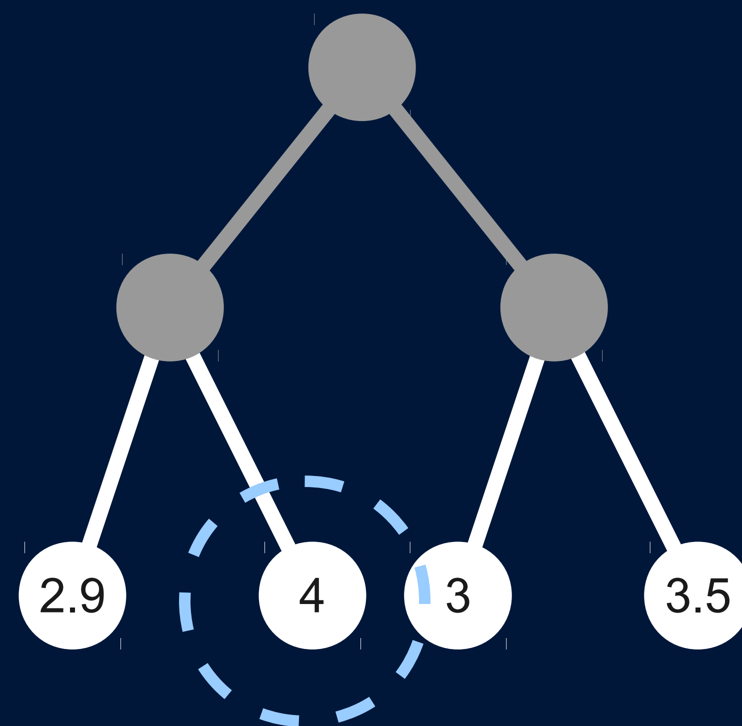

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Heuristic Search – 3

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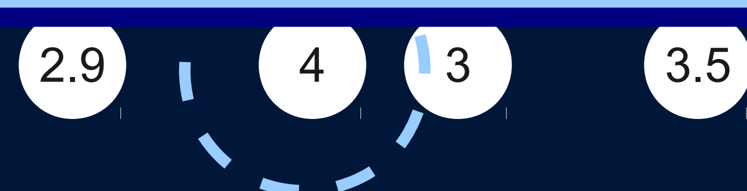
Main intuition



- For each node, compute F-value
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F-Value of a node n

- F(n) is a optimistic estimate
- I.e., F(n) >= V(n') for any descendant n' of n
- F(n) = G(n) + H(n)

G(n): reward up to n (for first *t* stages)

H(n): Optimistic estimate of reward below n (reward for stages t,t+1,...,h-1)

Further Developments

- DP
  - Improvements to exhaustive backup [Amato et al. 2009]
  - Compression of values (LPC) [Boularias & Chaib-draa 2008]
  - (Point-based) Memory bounded DP [Seuken & Zilberstein 2007a]
  - Improvements to PB backup [Seuken & Zilberstein 2007b, Carlin and Zilberstein, 2008; Dibangoye et al, 2009; Amato et al, 2009; Wu et al, 2010, etc.]
- Heuristic Search
  - No backtracking: just most promising path [Emery-Montemerlo et al. 2004, Oliehoek et al. 2008]
  - Clustering of histories: reduce number of child nodes [Oliehoek et al. 2009]
  - Incremental expansion: avoid expanding all child nodes [Spaan et al. 2011]
- MILP [Aras and Dutech 2010]

State of The Art

To get an impression...

- Optimal solutions
  - Improvements of MAA* lead to significant increases
  - but problem dependent

h	MILP	LPC	GMAA-ICE*
4	72	534.9	0.04
6		-	46.43*

dec-tiger – runtime (s)

h	MILP	LPC	GMAA-ICE*
5	25	–	<0.01
500	–	–	0.94*

broadcast channel runtime (s)

* excluding heuristic

- Approximate (no quality guarantees)
  - MBDP: linear in horizon [Seuken & zilberstein 2007a]
  - Rollout sampling extension: up to 20 agents [Wu et al. 2010b]
  - Transfer planning: use smaller problems to solve large (structured) problems (up to 1000) agents [Oliehoek 2010]

Related Areas

- Partially observable stochastic games [Hansen et al. 2004]
  - Non-identical payoff
- Interactive POMDPs [Gmytrasiewicz & Doshi 2005, JAIR]
  - Subjective view of MAS
- Imperfect information extensive form games
  - Represented by game tree
  - E.g., poker [Sandholm 2010, AI Magazine]

Some Further Topics

Overview:

- On-line planning
- Communication
- Factored Models
  - Single Agent
  - Multiple agents

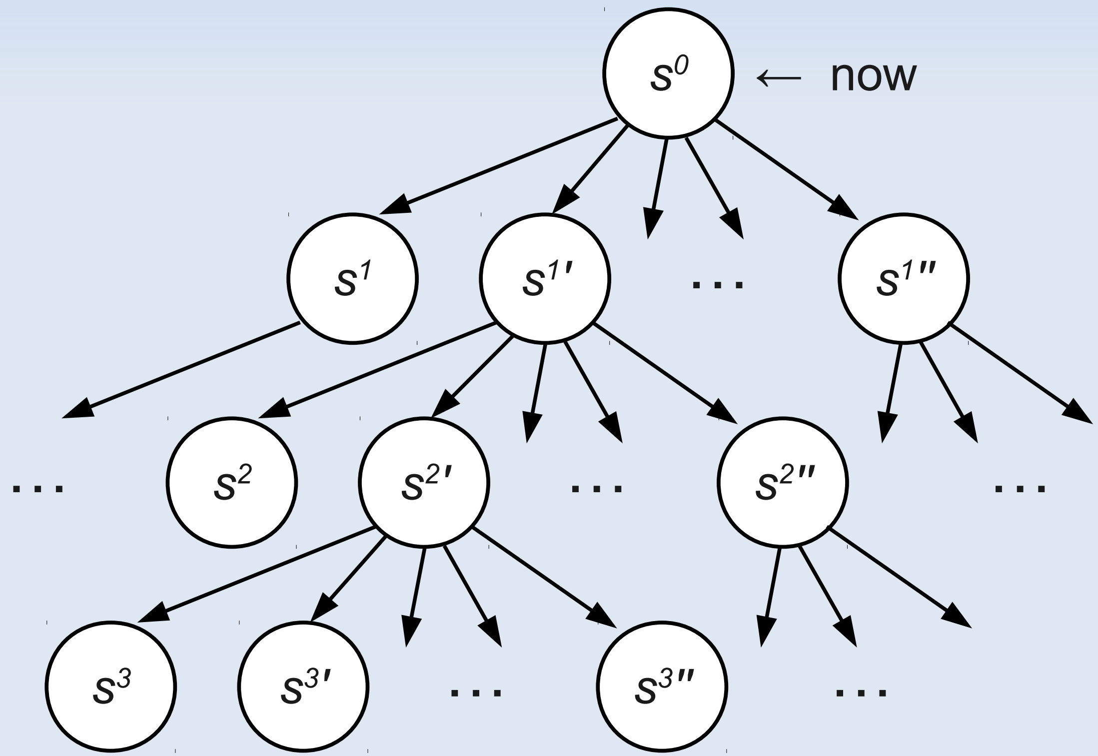
- Goal: present an overview of some high-level ideas

On-line Planning

- So far: planning in a separate off-line phase
- However: could also consider performing the planning during execution!
  - do not plan over entire space, but only those reachable in the (near) future!
  - but: need to plan at every step.
- In control theory 'receding horizon control' or 'model predictive control' (but details different)

Lookahead Planning

- Main idea: plan ahead for T stages
- Construct a tree of all possibilities and perform dynamic programming over this tree



Lookahead Planning

- Main idea: plan ahead for T stages
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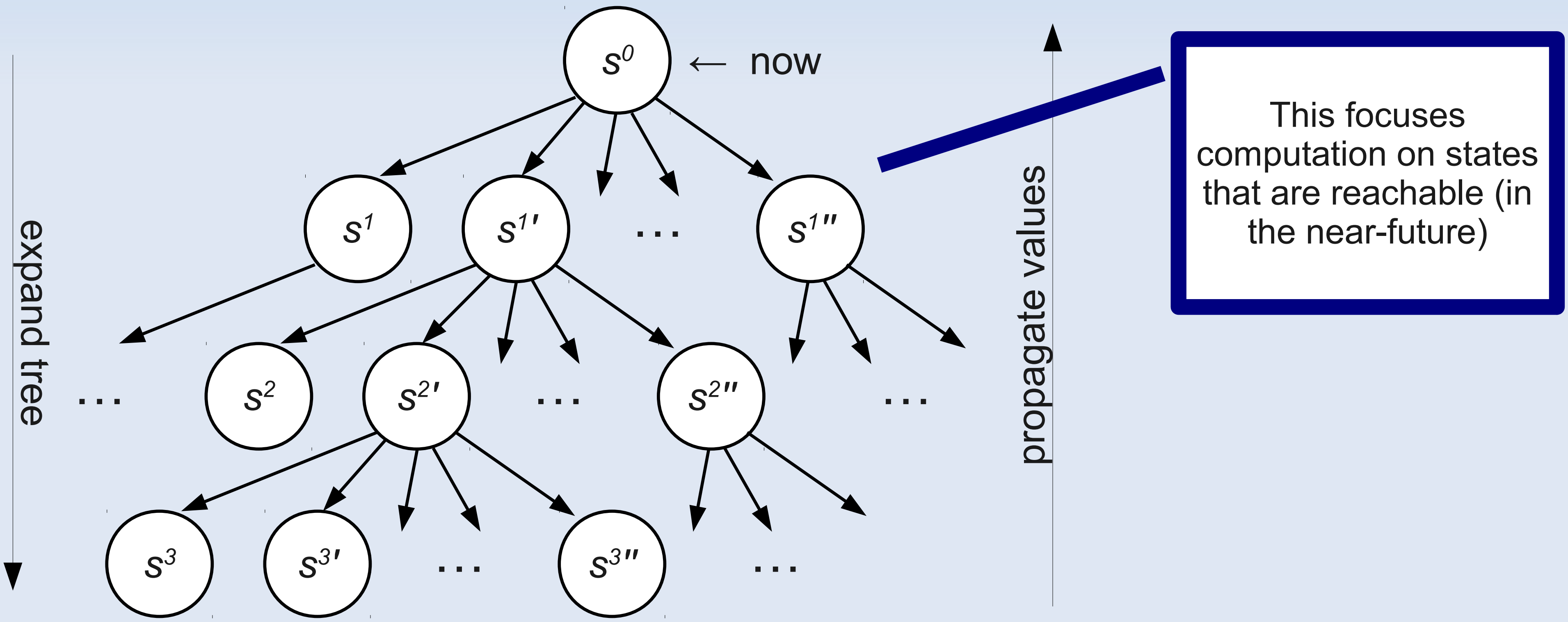

Lookahead Planning

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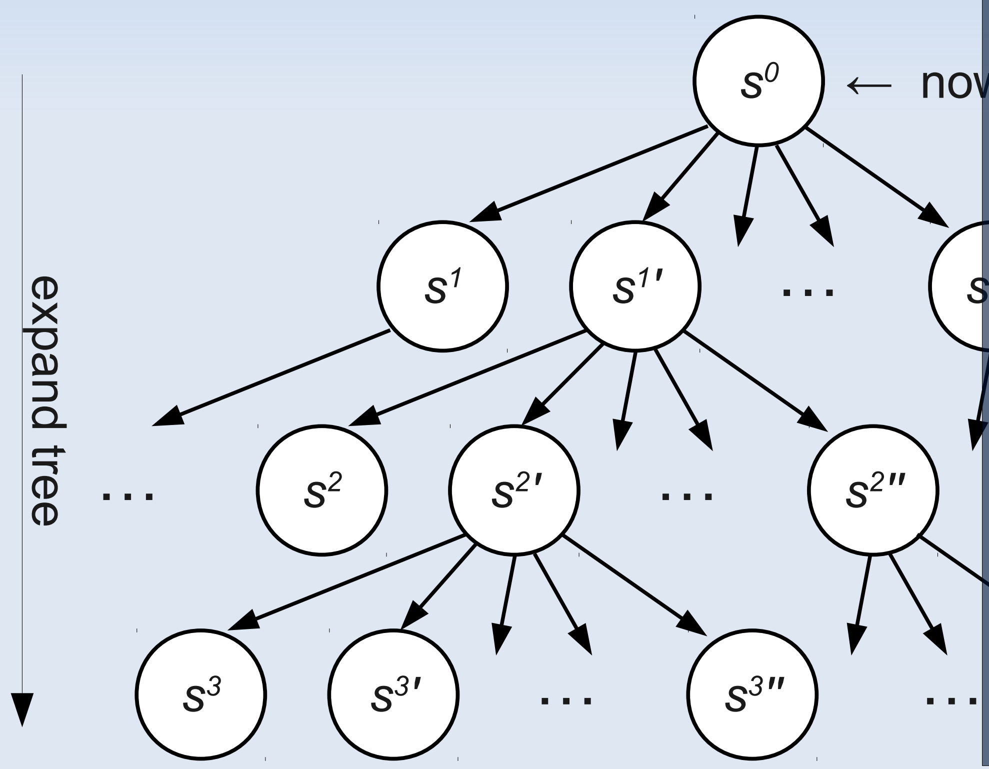
Expanding all possible next states
→ tree is huge...



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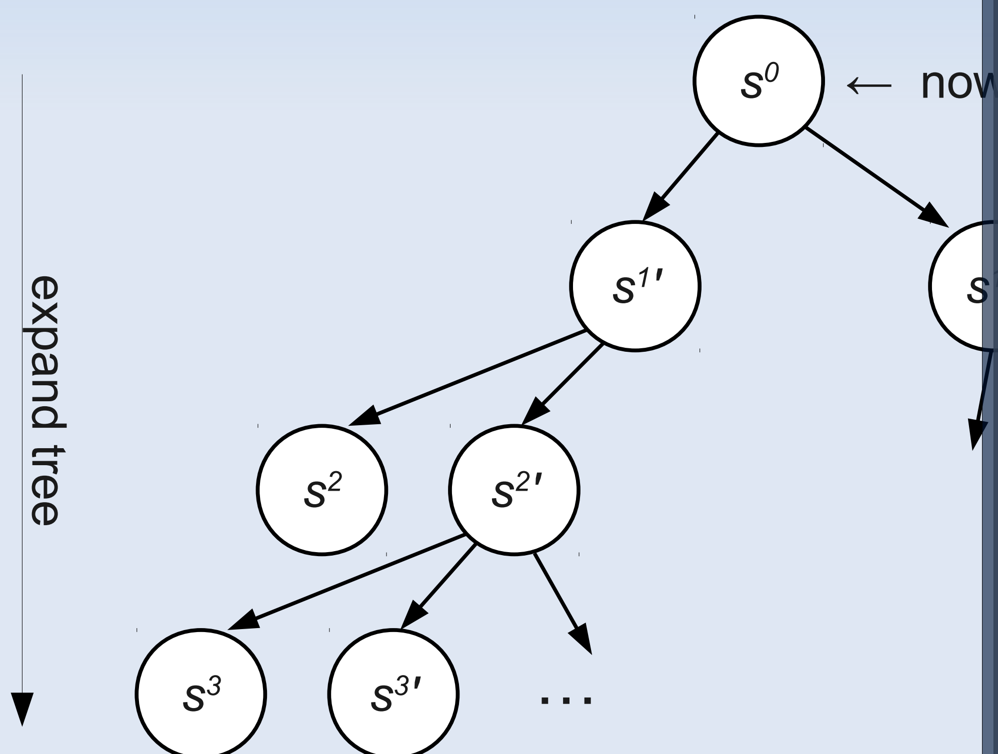

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- one idea: Sample!
- That works pretty good: bound independent of number of states [Kearns et al. 2002 ML]

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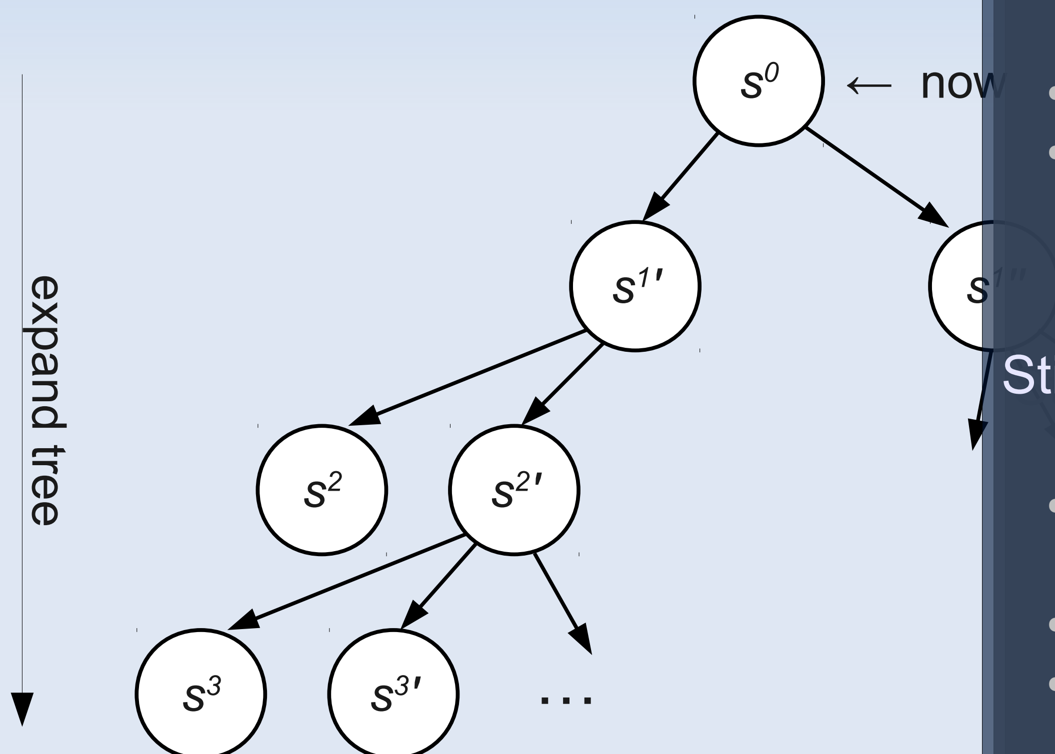


Expanding all possible next states
→ tree is huge...

- one idea: Sample!
- That works pretty good: bound independent of number of states [Kearns et al. 2002 ML]

Still very big...

- Further idea: avoid expanding non-promising branches.
- Use upper confidence bounds
- UCT [Kocsis & Szepesvári, 2006 ECML]

Some Further Topics

Overview:

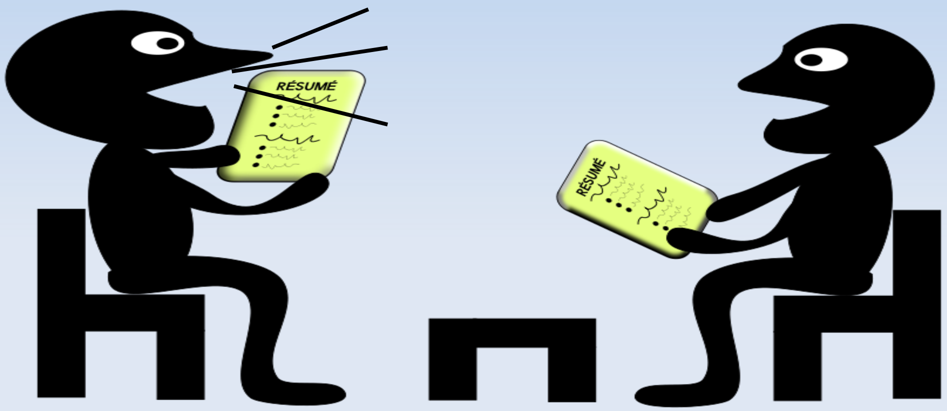
- On-line planning
- **Communication**
- Factored Models
  - Single Agent
  - Multiple agents

Communication

- Already discussed:
  instantaneous cost-free and noise-free communication
  - Dec-MDP → multiagent MDP (MMDP)
  - Dec-POMDP → multiagent POMDP (MPOMDP)
- but in practice:
  - probability of failure
  - delays
  - costs
- Also: implicit communication!
  (via observations and actions)

Implicit Communication

- Encode communications by actions and observations



- Embed the **optimal meaning** of messages by finding the optimal plan [Goldman and Zilberstein 2003, Spaan et al. 2006]

Implicit Communication

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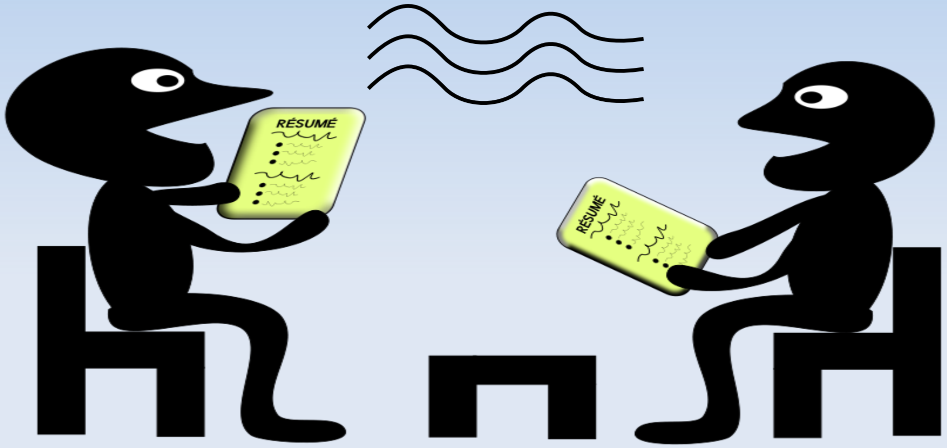

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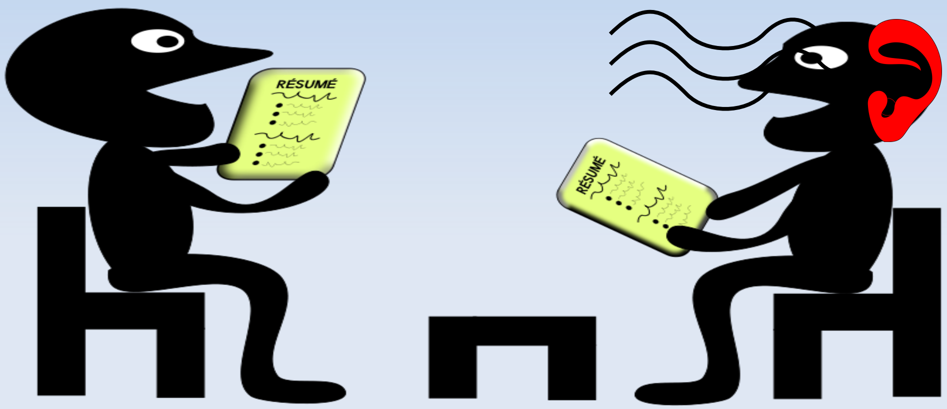


- Embed the **optimal meaning** of messages by finding the optimal plan [Goldman and Zilberstein 2003, Spaan et al. 2006]
- E.g. communication bit
  - doubles the #actions and observations!
  - Clearly, useful... but intractable for general settings
    (perhaps for analysis of very small communication systems)

Explicit Communication

- perform a particular information update (e.g., sync) as in the MPOMDP:
  - each agent broadcasts its information, and
  - each agent uses that to perform joint belief update

- Other approaches:
  - Communication cost [Becker et al. 2005]
  - Delayed communication [Hsu 1982, Spaan 2008, Oliehoek 2012]
  - communicate every k stages [Goldman & Zilberstein 2008]

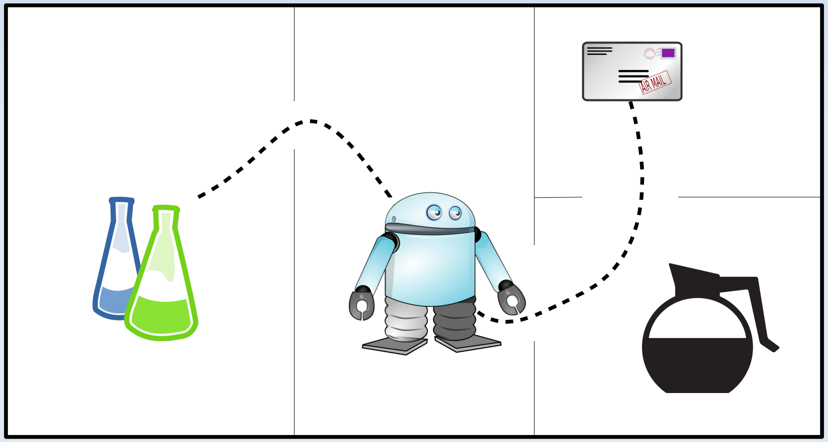
Some Further Topics

Overview:

- On-line planning
- Communication
- **Factored Models**
  - Single Agent
  - Multiple agents

Factored MDPs

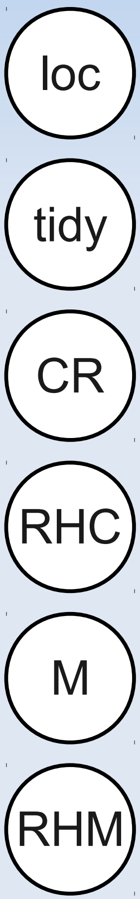
- So far: used 'states'
- But in many problems states are **factored**
  - state is an assignment of variables $s=<f_1,f_2,...,f_k>$
  - *factored MDP* [Boutilier et al. 99 JAIR]



Examples:

- Predator-prey: x, y coordinate!
- Robotic P.A.
  - location of robot (lab, hallway, kitchen, mail room), tidiness of lab, coffee request, robot holds coffee, mail present, robot holds mail, etc.
  - Actions: move (2 directions), pickup coffee/mail, deliver coffee/mail

Factored States & Transitions



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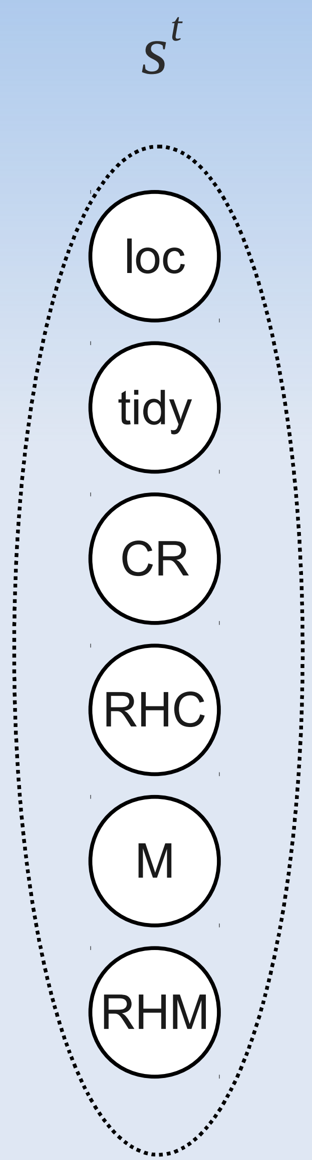

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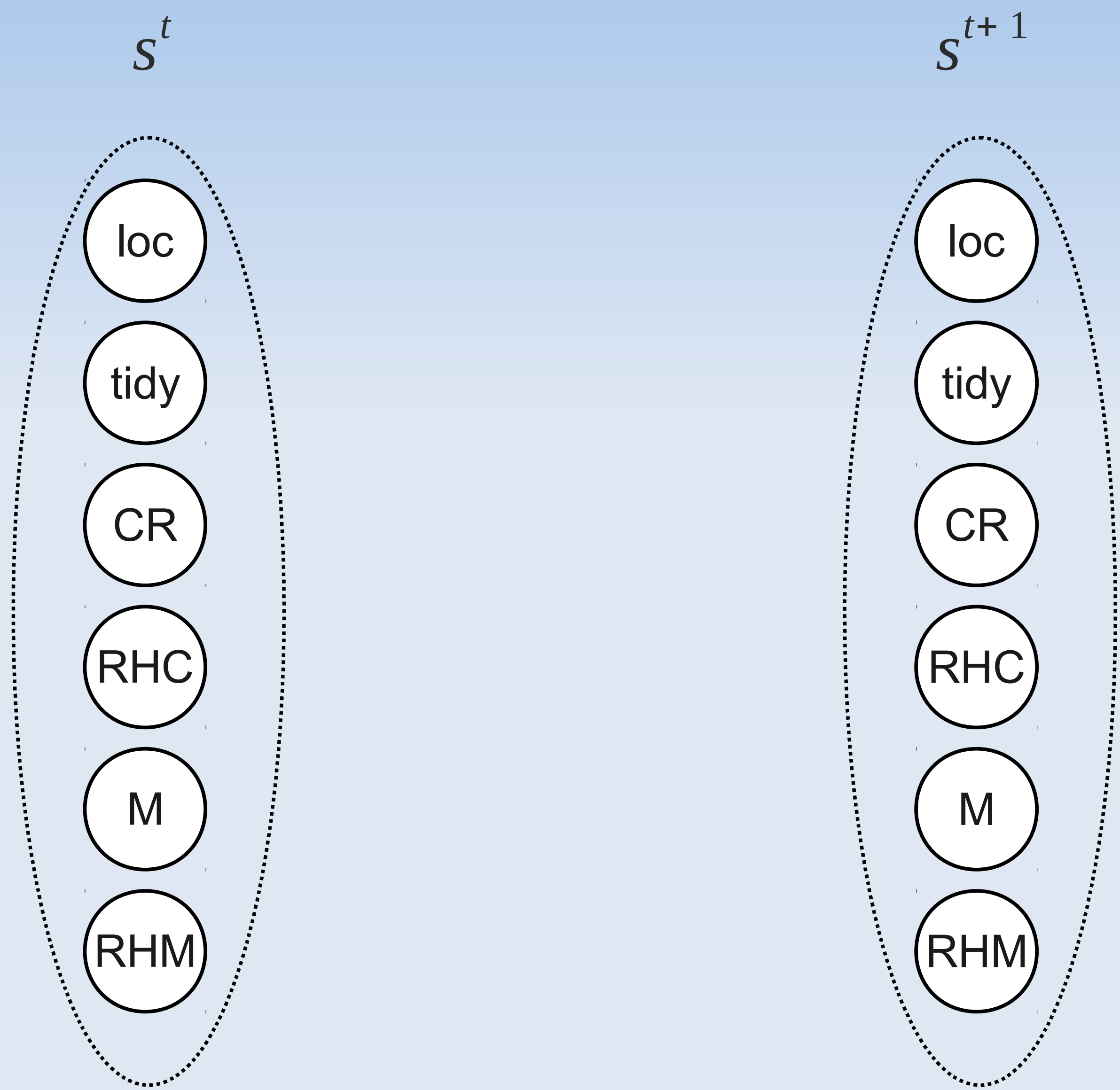

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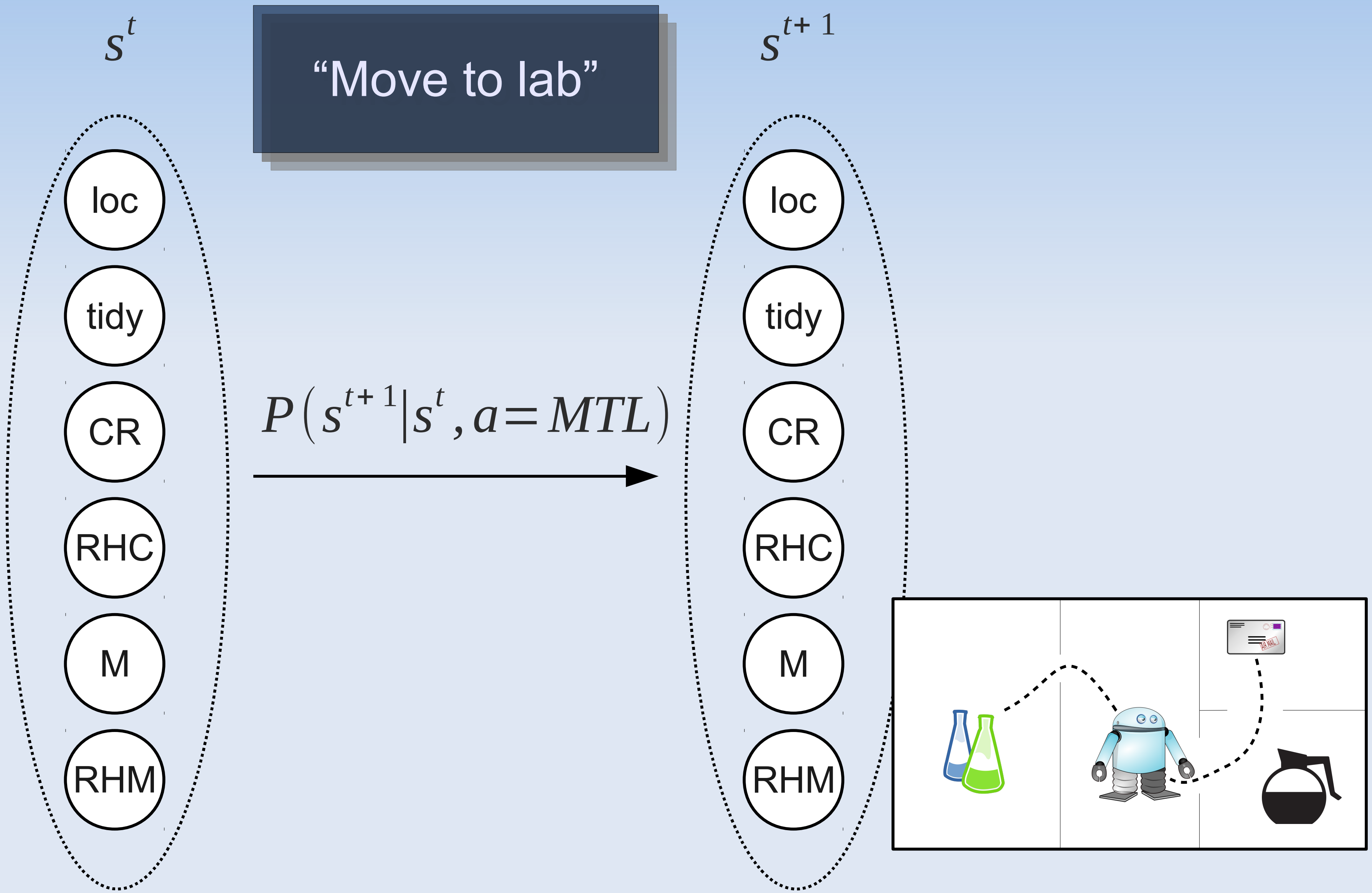

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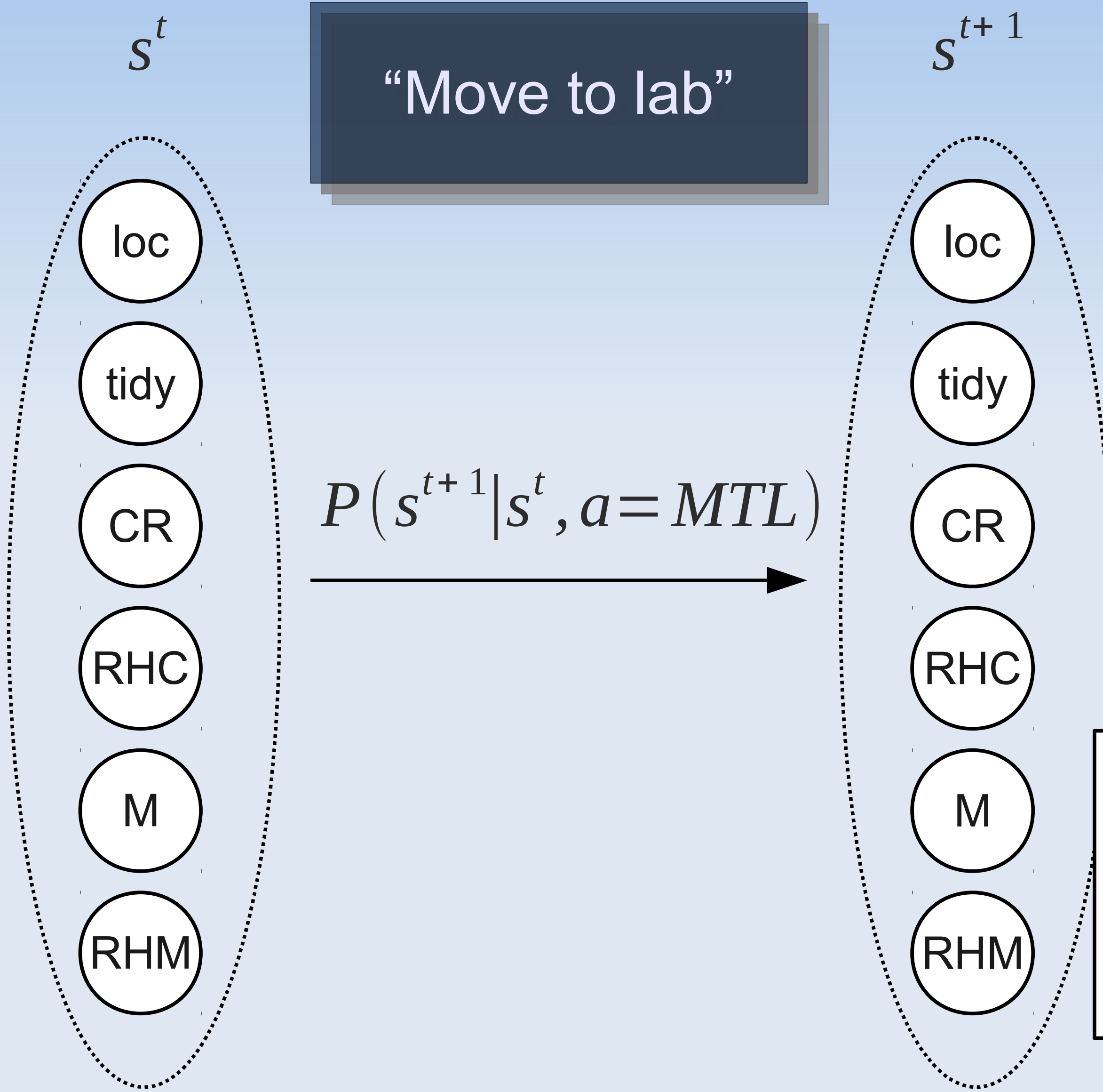
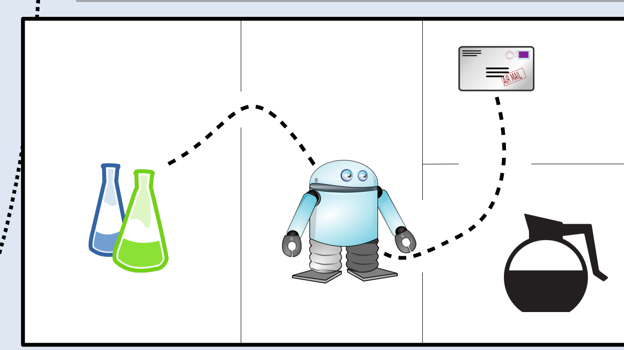


But there is conditional independence!

E.g.: 'M' does not influence 'loc'



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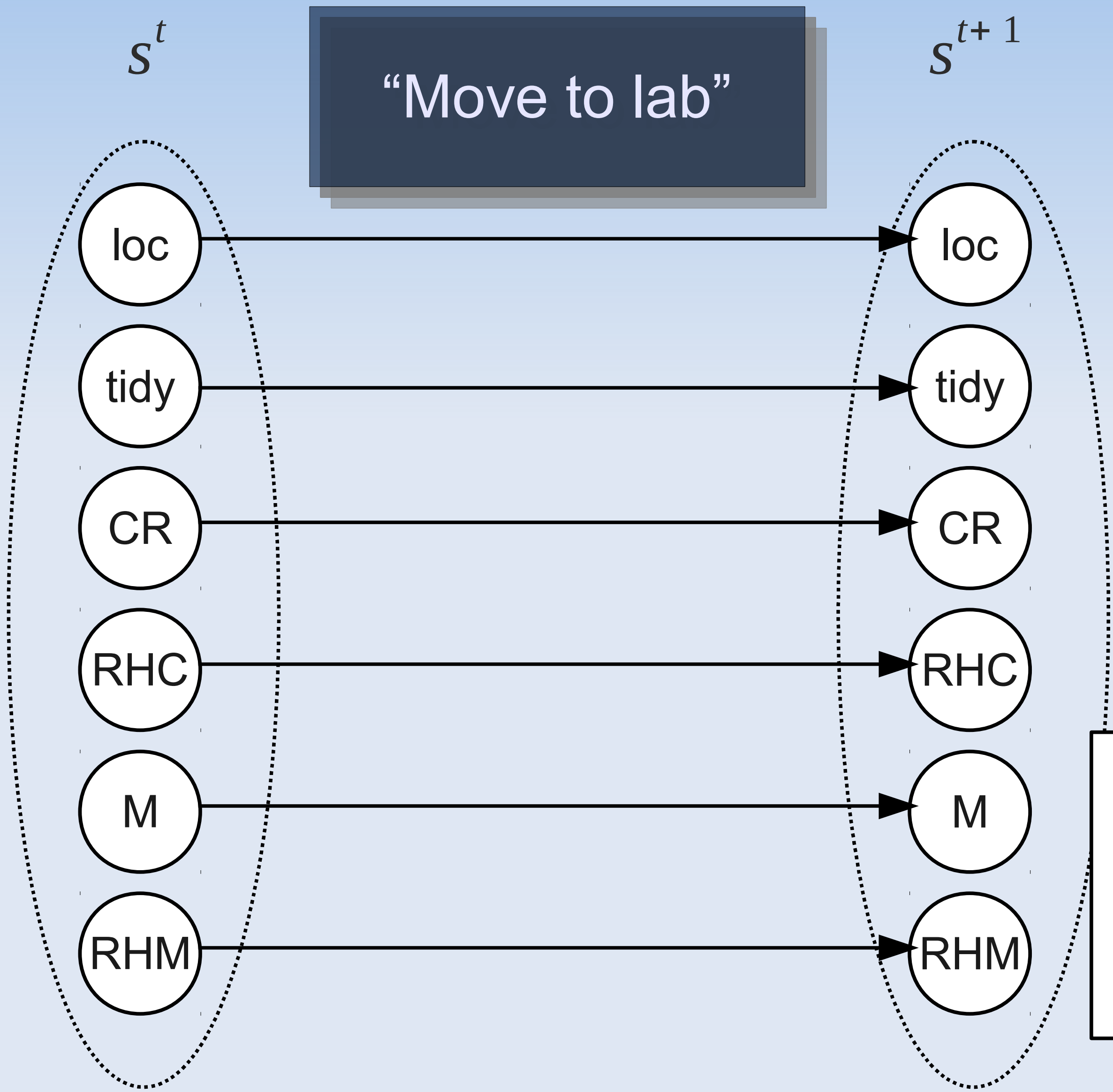

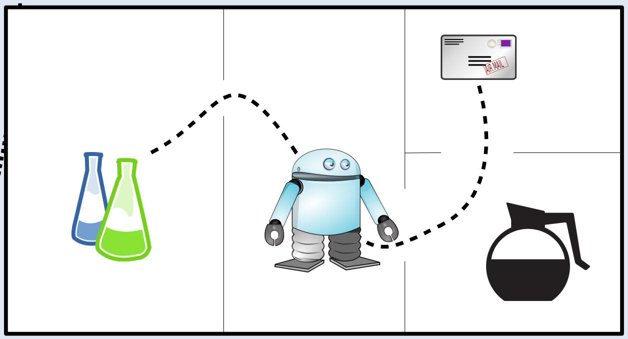

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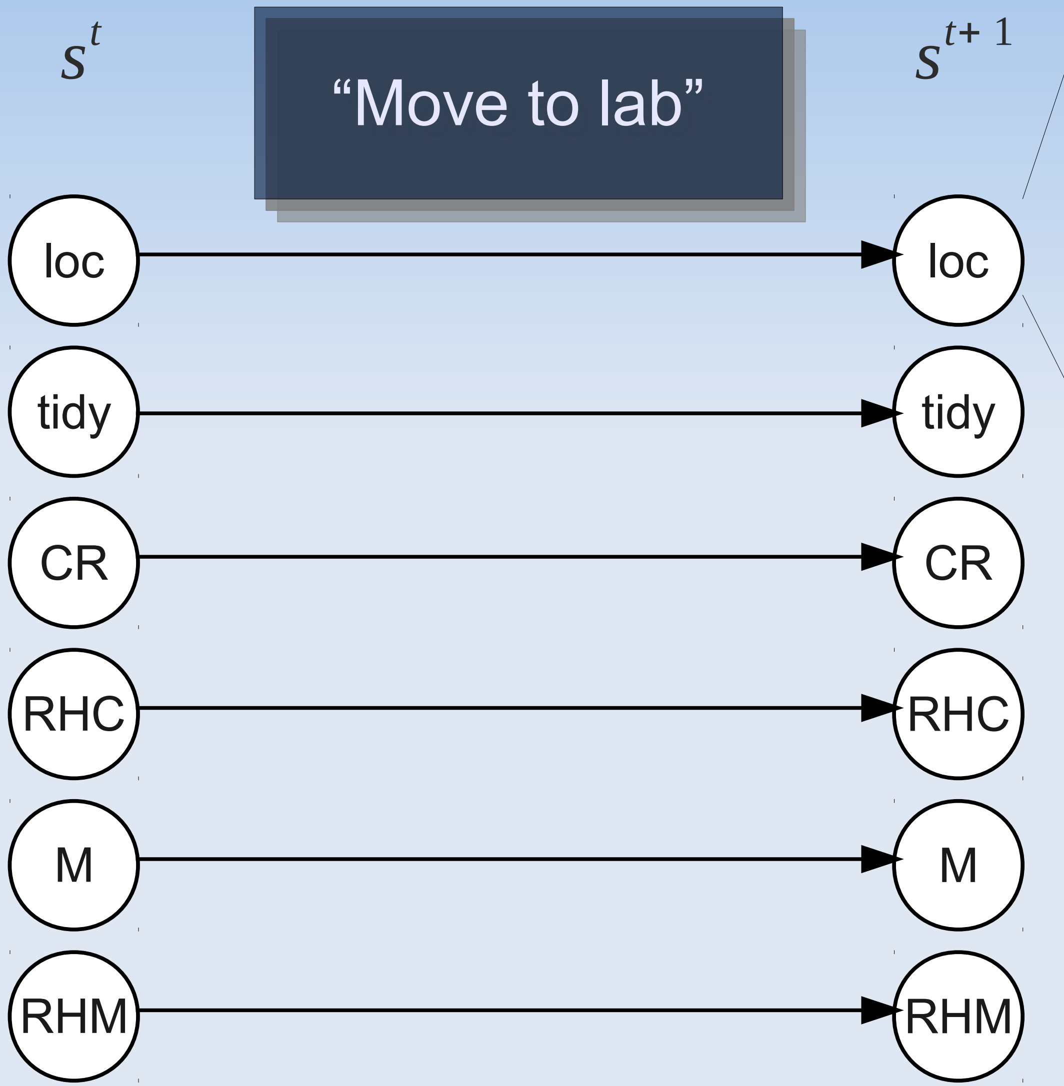
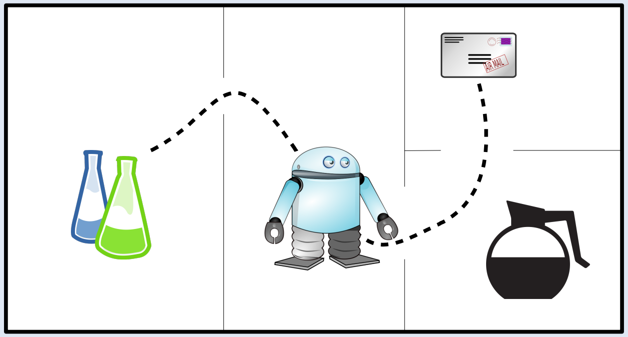


		L	H	K	M
	L	1	.9	0	0
t+1	H	0	.1	.9	0
	K	0	0	.1	.9
	M	0	0	0	.1

conditional probability table (CPT)



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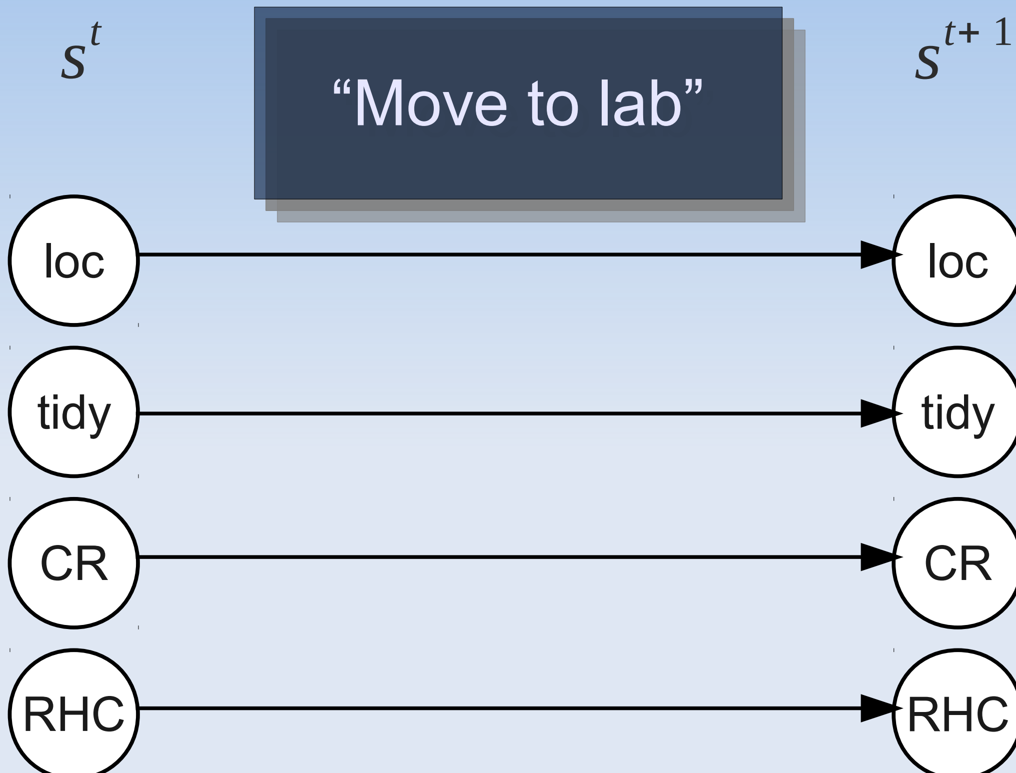


		L	H	K	M
	L	1	.9	0	0
t+1	H	0	.1	.9	0
	K	0	0	.1	.9
	M	0	0	0	.1

conditional probability table (CPT)

- Each next-stage variable has a CPT
- This allows for a much more compact representation!
- “Two-stage dynamic Bayesian network” (2DBN)

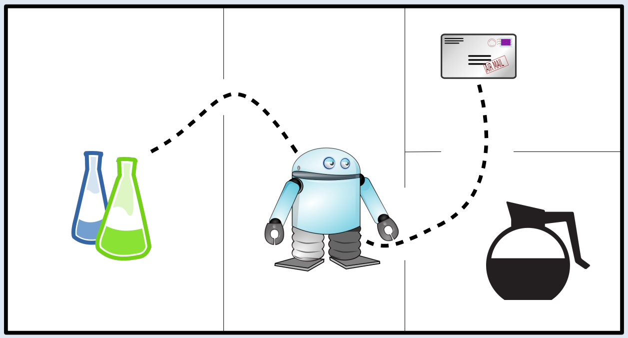
Factored States & Transitions

		L	H	K	M
	L	1	.9	0	0
t+1	H	0	.1	.9	0
	K	0	0	.1	.9
	M	0	0	0	.1

conditional probability table (CPT)

Do we always have so much independence?

(what about other actions?)



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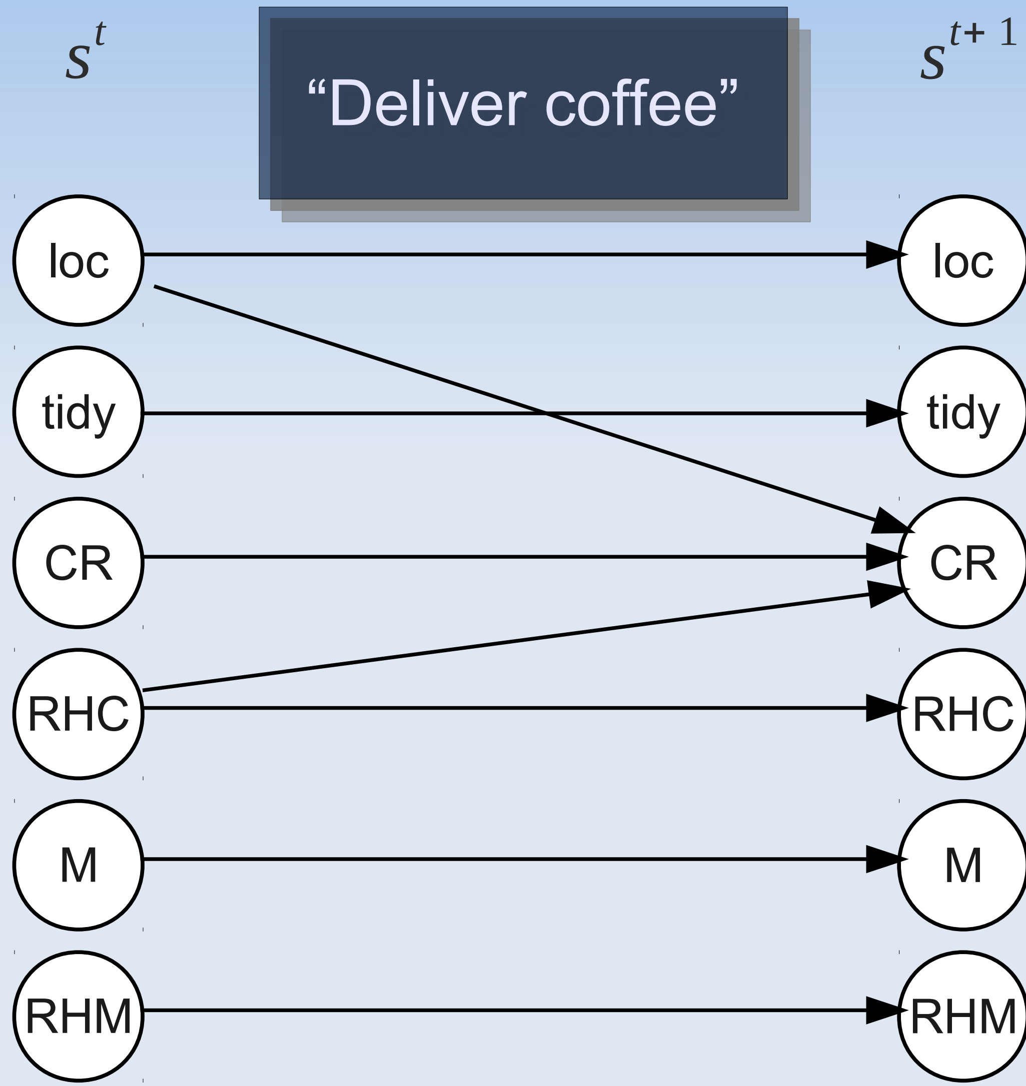
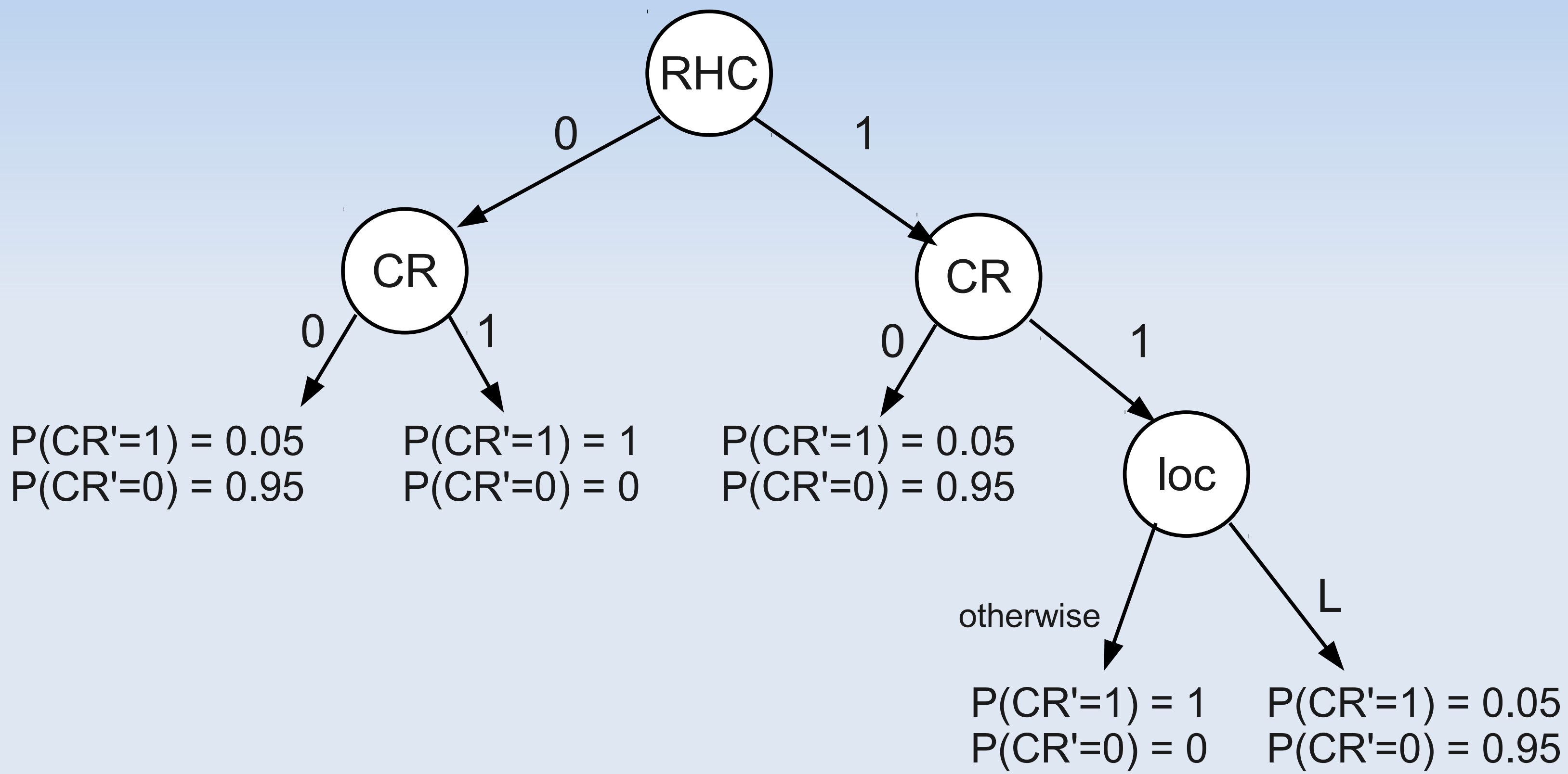


CPT encodes that IF
- loc=lab
- CR=1

→ high probability of CR becoming 0

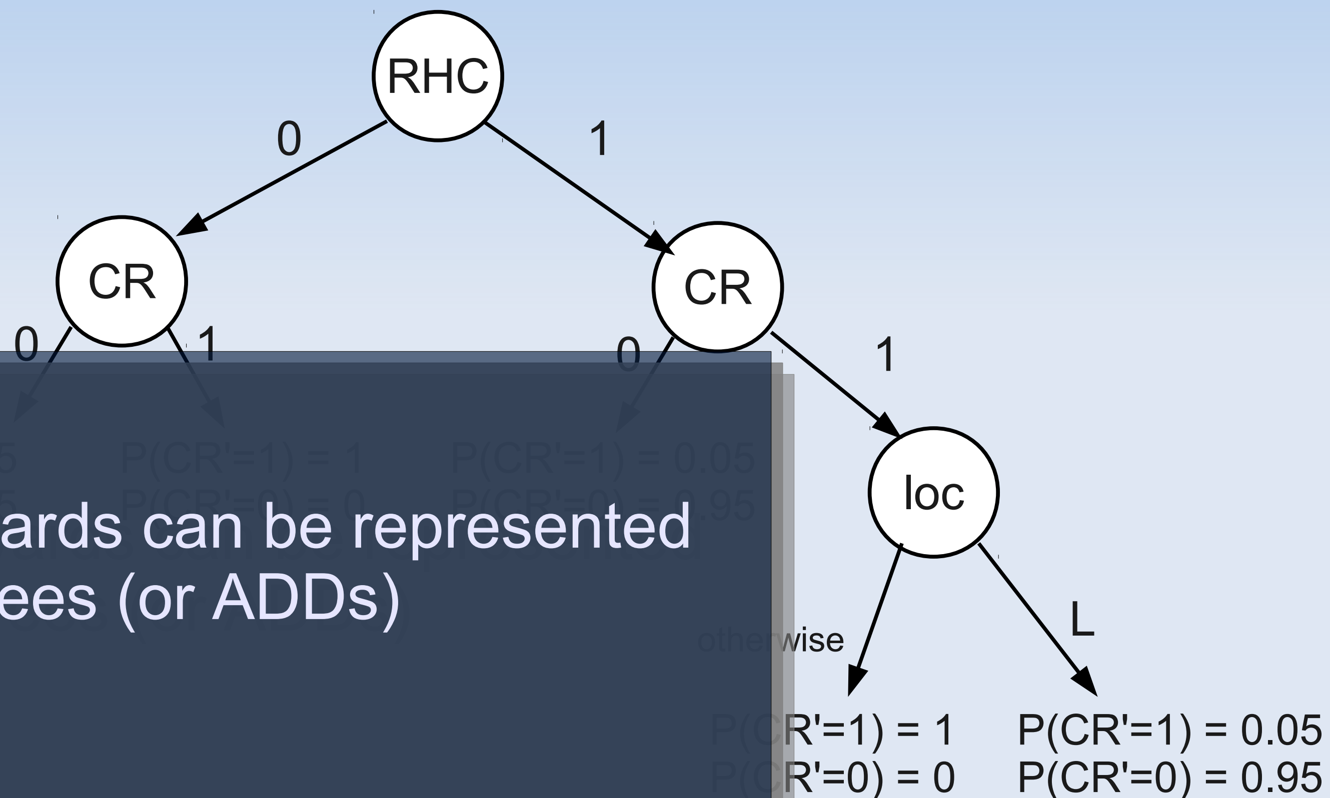
Solving Factored MDPs

- CPT also representable as a decision tree



Solving Factored MDPs

- CPT also representable as a decision tree

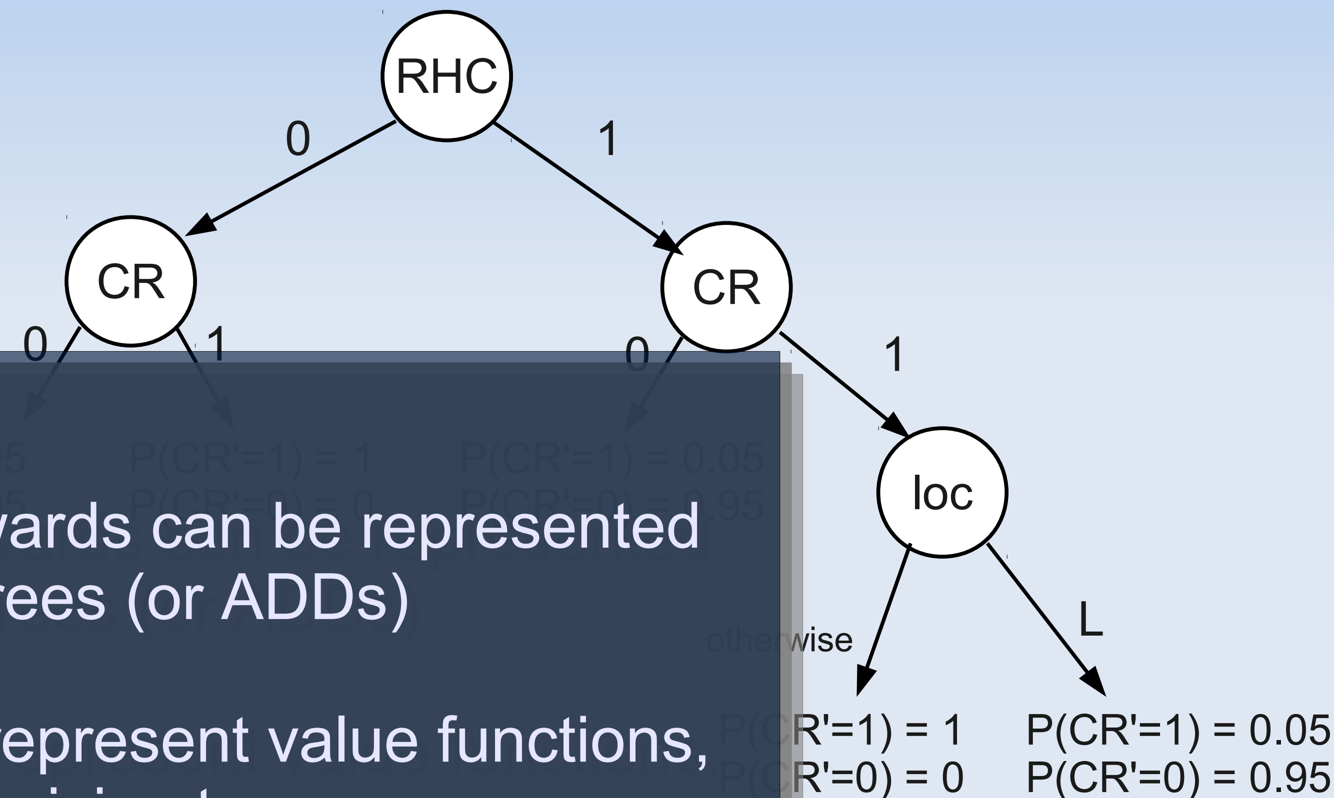


Similarly: rewards can be represented as decision trees (or ADDs)

→ So…?

Solving Factored MDPs

- CPT also representable as a decision tree



Similarly: rewards can be represented as decision trees (or ADDs)

→ Can also represent value functions, policies as decision trees [Boutilier et al 99]

Factored POMDPs

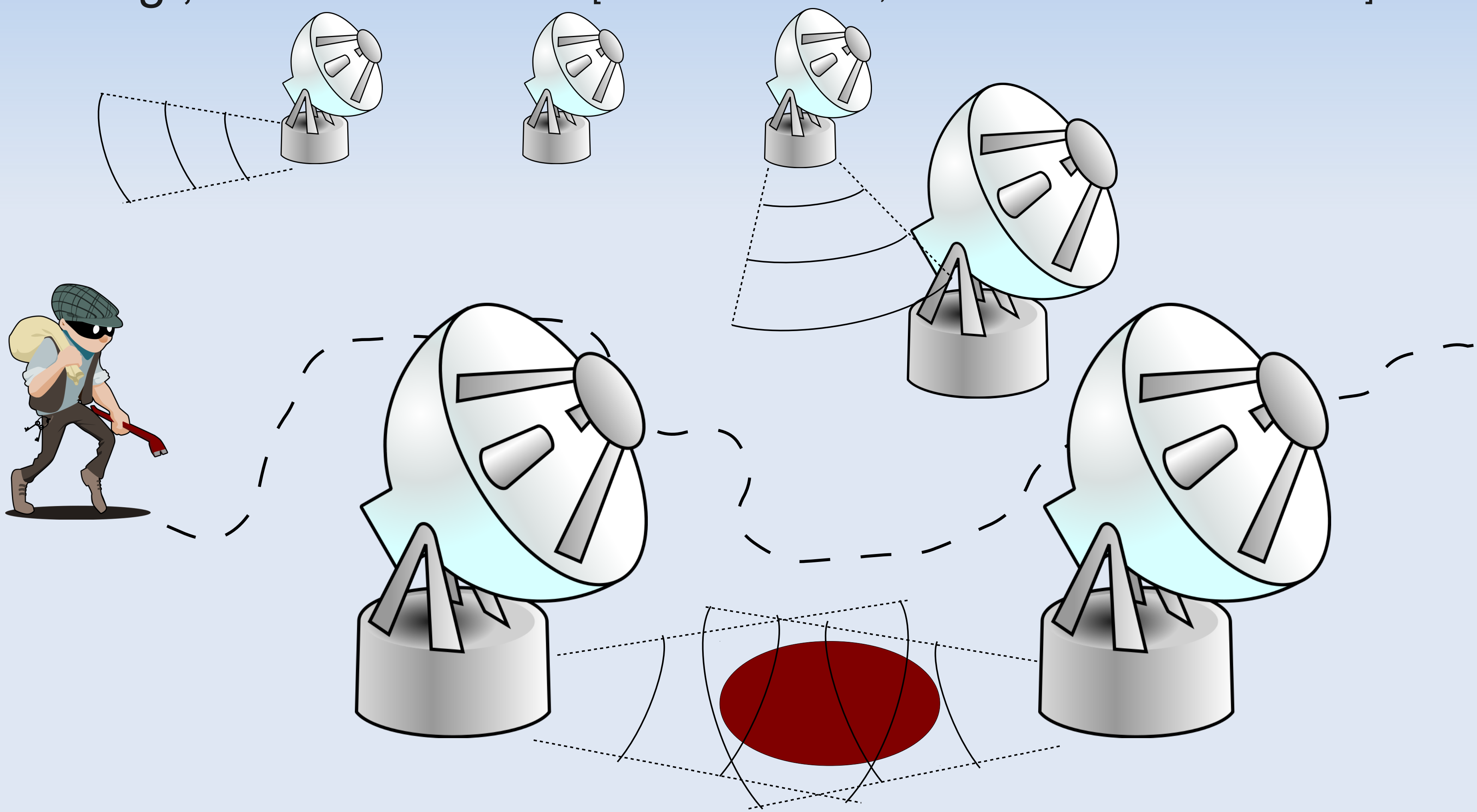
- Of course POMDP models can also be factored
- Similar ideas applied [Hansen & Feng 2000, Poupart 2005, Shani et al. 2008]
  - α-vectors represented by ADDs
  - beliefs too.
- This does not solve all problems:
  - over time state factors get more and more correlated, so representation grows large.

Factored Multiagent Models

- Of course multiagent models can also be factored!
- Work can be categorized in a few directions:
  - Trying to execute the factored (PO)MDP policy
    [Roth et al. 2007, Messias et al. 2011]
  - Trying to execute independently as much as possible
    [Spaan & Melo 2008, Melo & Veloso 2011]
  - Exploiting graphical structure between agents
    (ND-POMDPs, Factored Dec-POMDPs)
  - Influence-based abstraction of policies of other agents
    (TOI-Dec-MDPs, TD-POMDPs, IBA for POSGs)

Graphical Structure between Agents

- Exploit (conditional) independence between agents
  - E.g., sensor networks [Nair et al '05 AAAI, Varakantham et al. '07 AAMAS]



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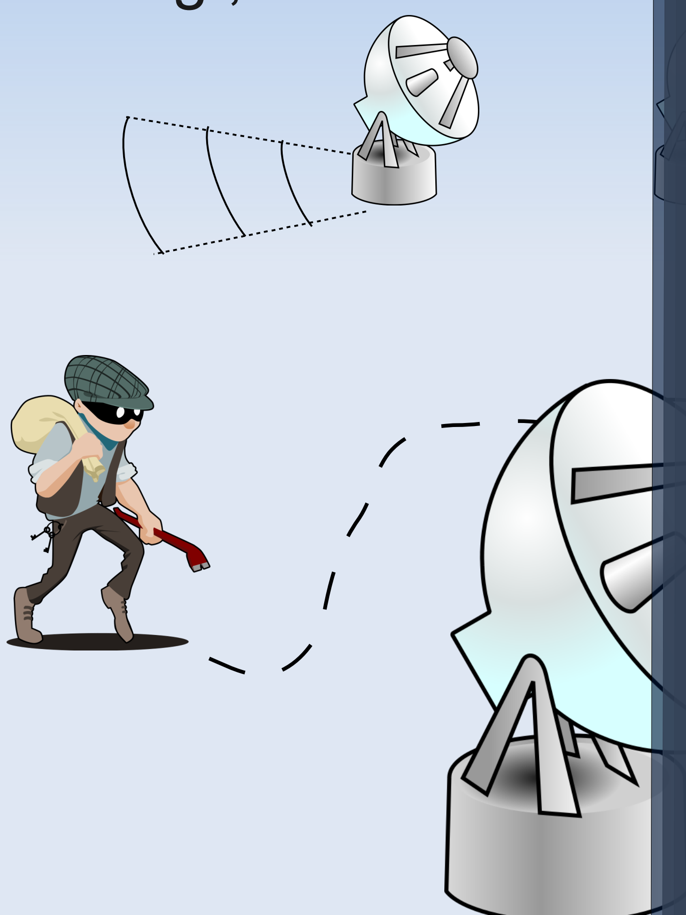

These problems have
- State that cannot be influenced
- Factored reward function

$$R(s,a)=\sum_e R_e(s,a_e)$$

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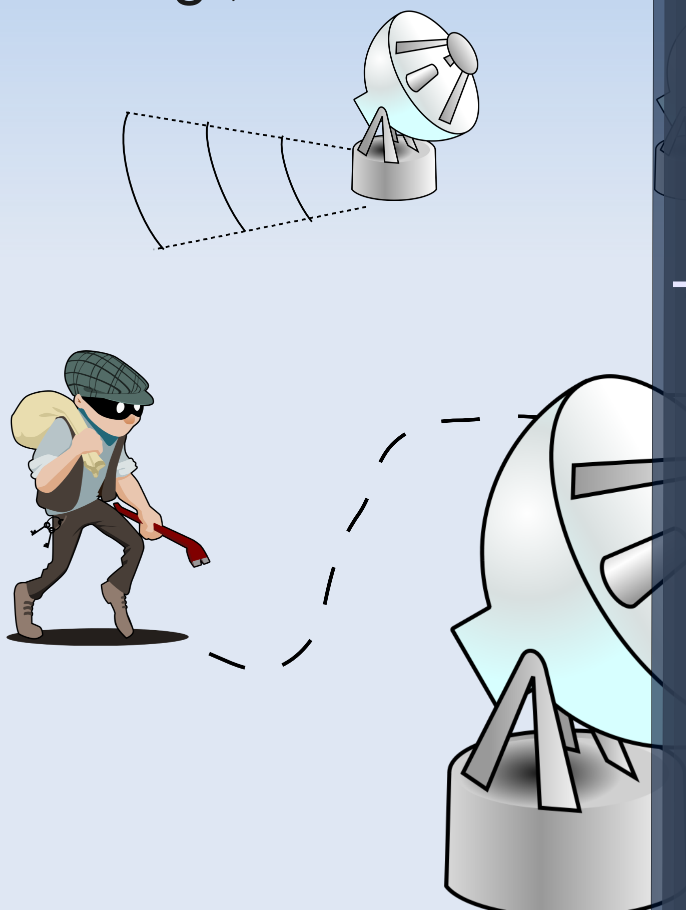

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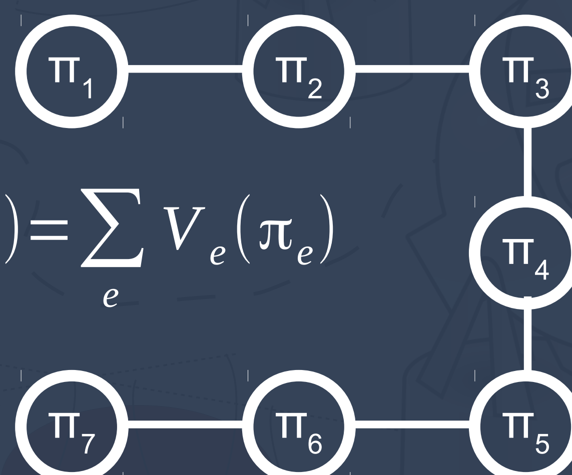
$$R(s,a)=\sum_e R_e(s,a_e)$$

This allows a reformulation as a (D)COP



$$V(\pi)=\sum_e V_e(\pi_e)$$

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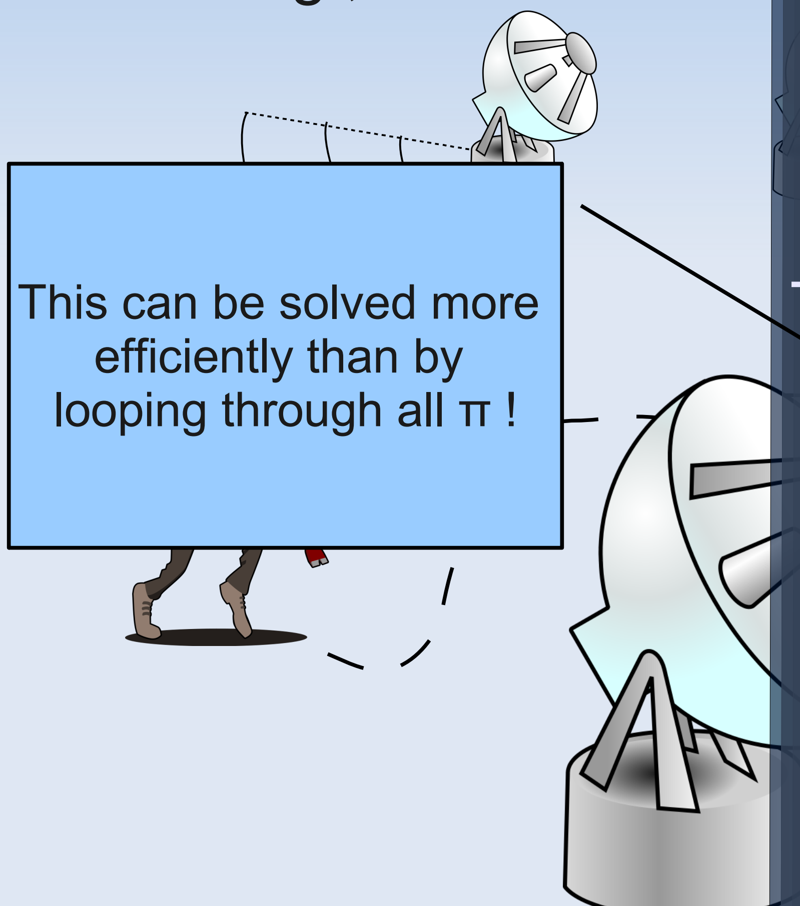

This can be solved more efficiently than by looping through all π !

These problems have

- State that cannot be influenced
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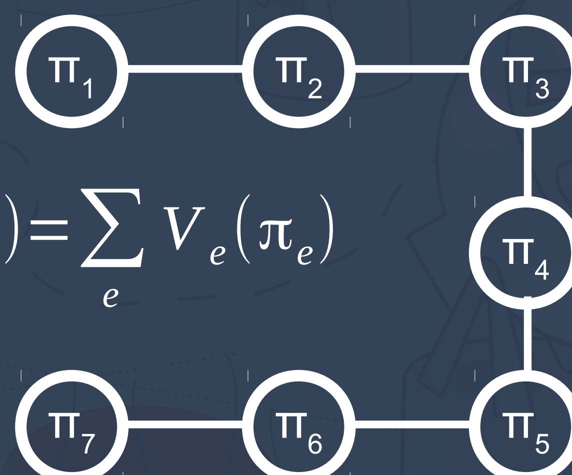
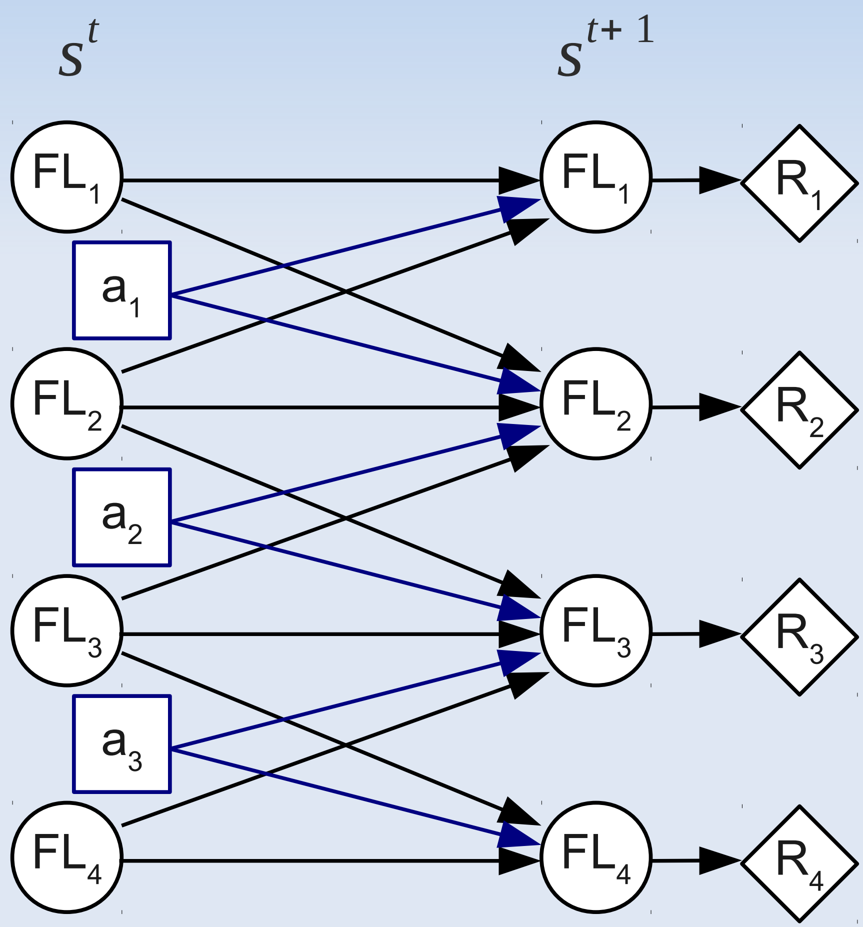
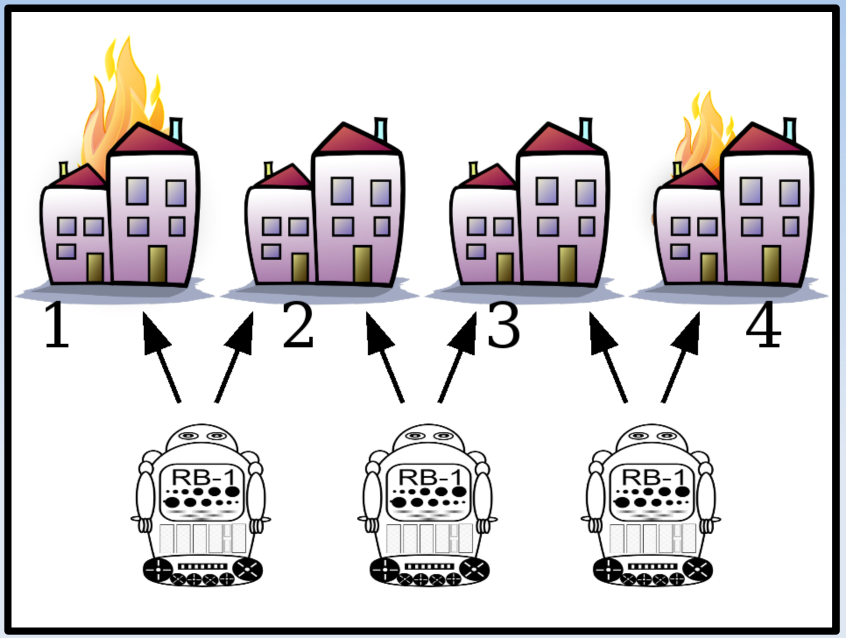


Graphical Structure between Agents

- Factored Dec-POMDPs
  [Oliehoek et al. 2008 AAMAS]

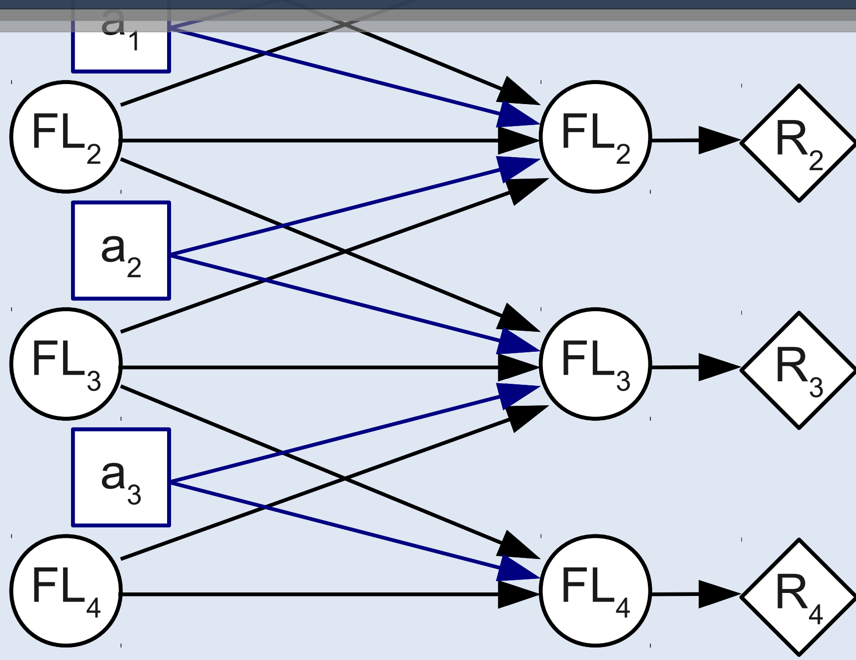


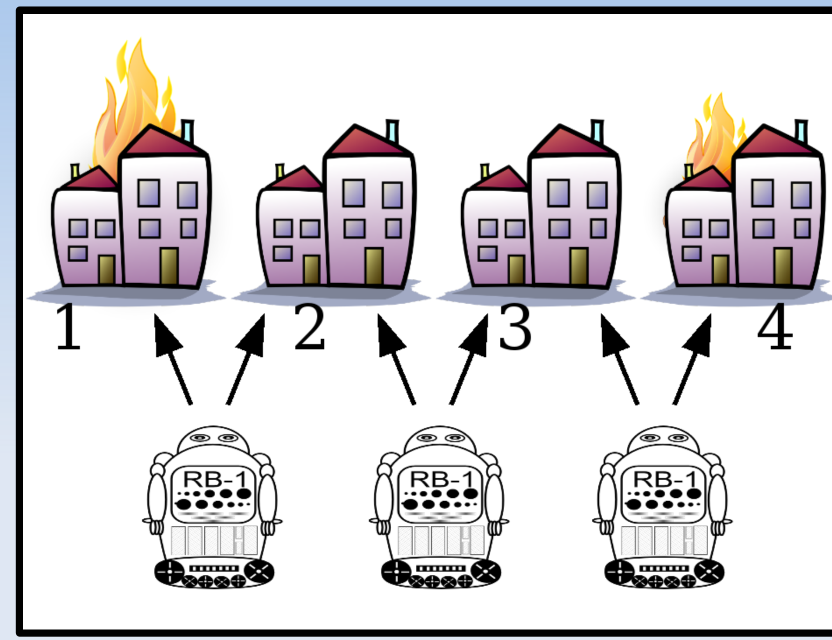


Graphical Structure between Agents

Can't we use the previous methods (reduction to DCOP) directly...

- Why ?





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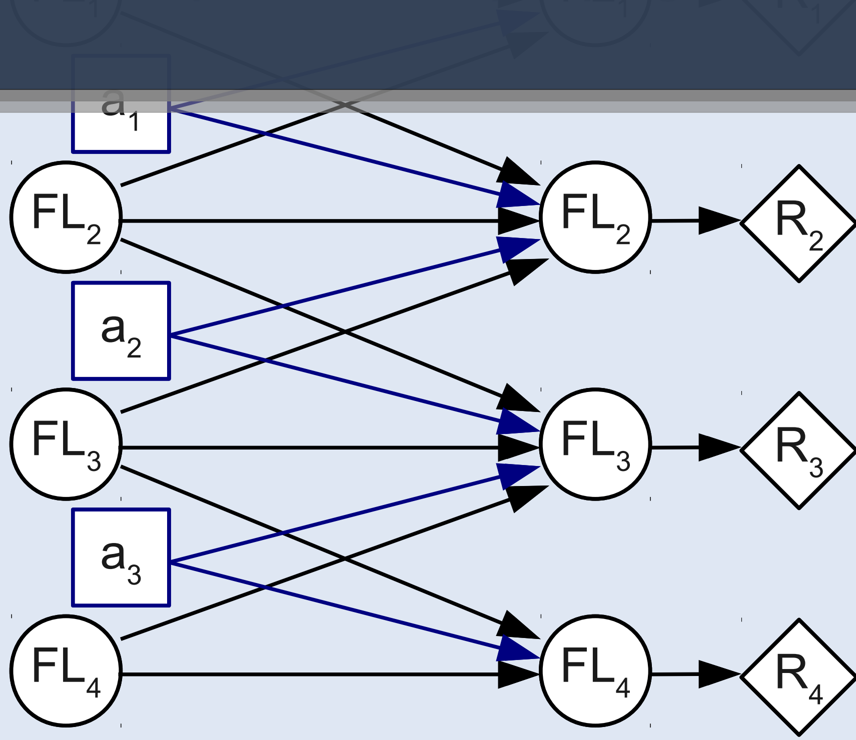
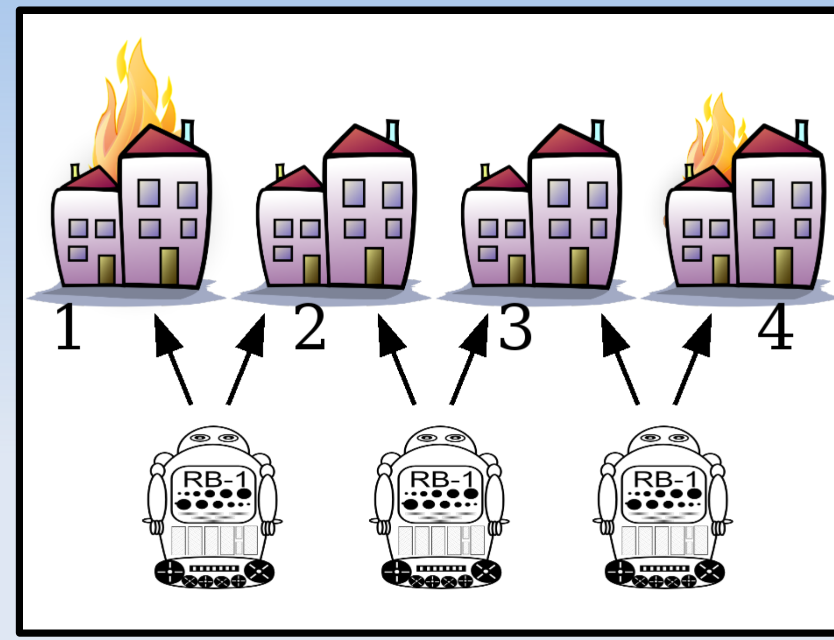
  → dependence propagates!





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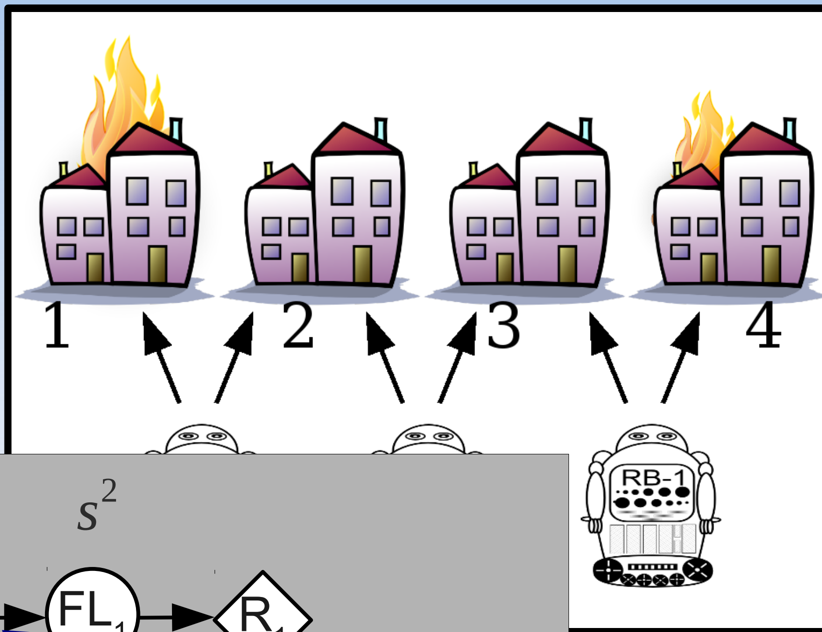

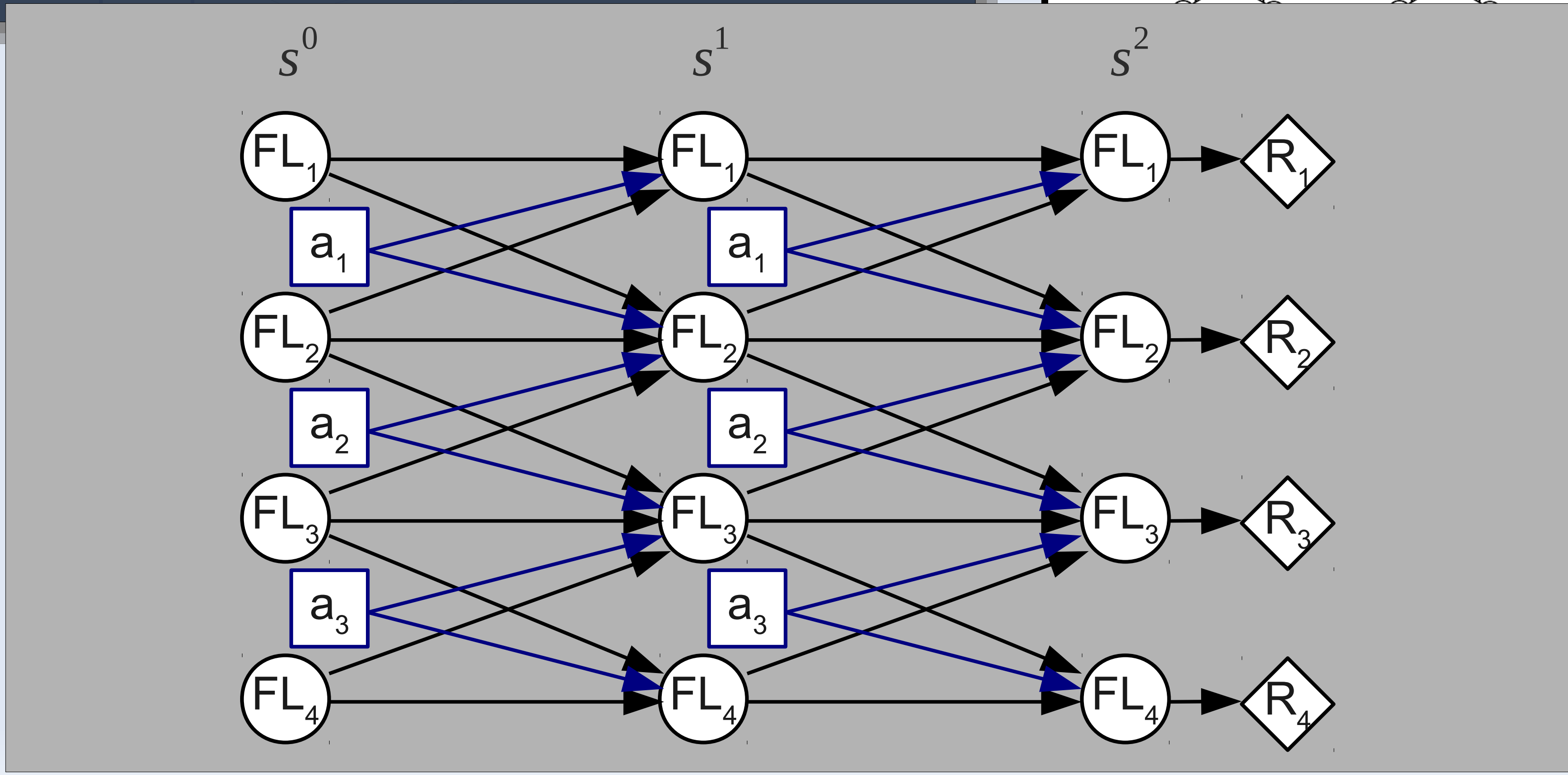

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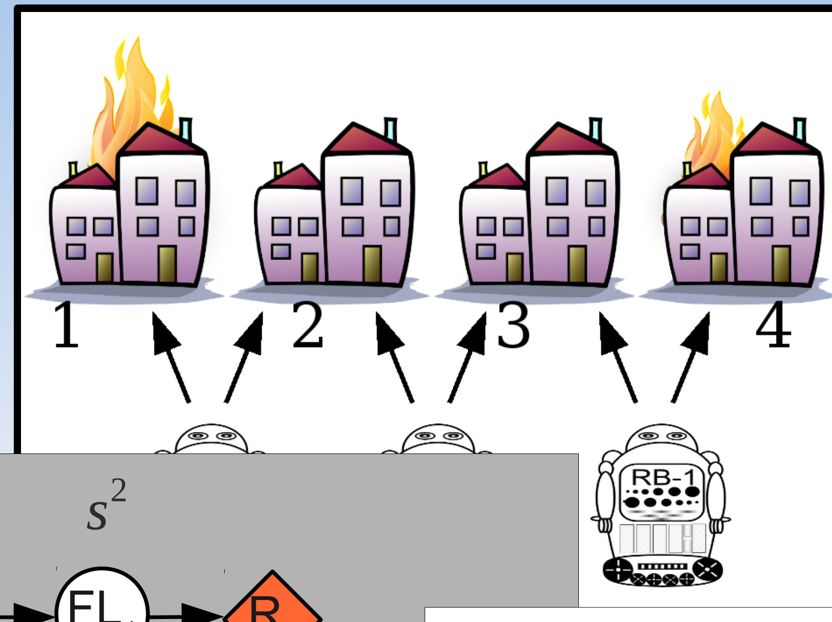

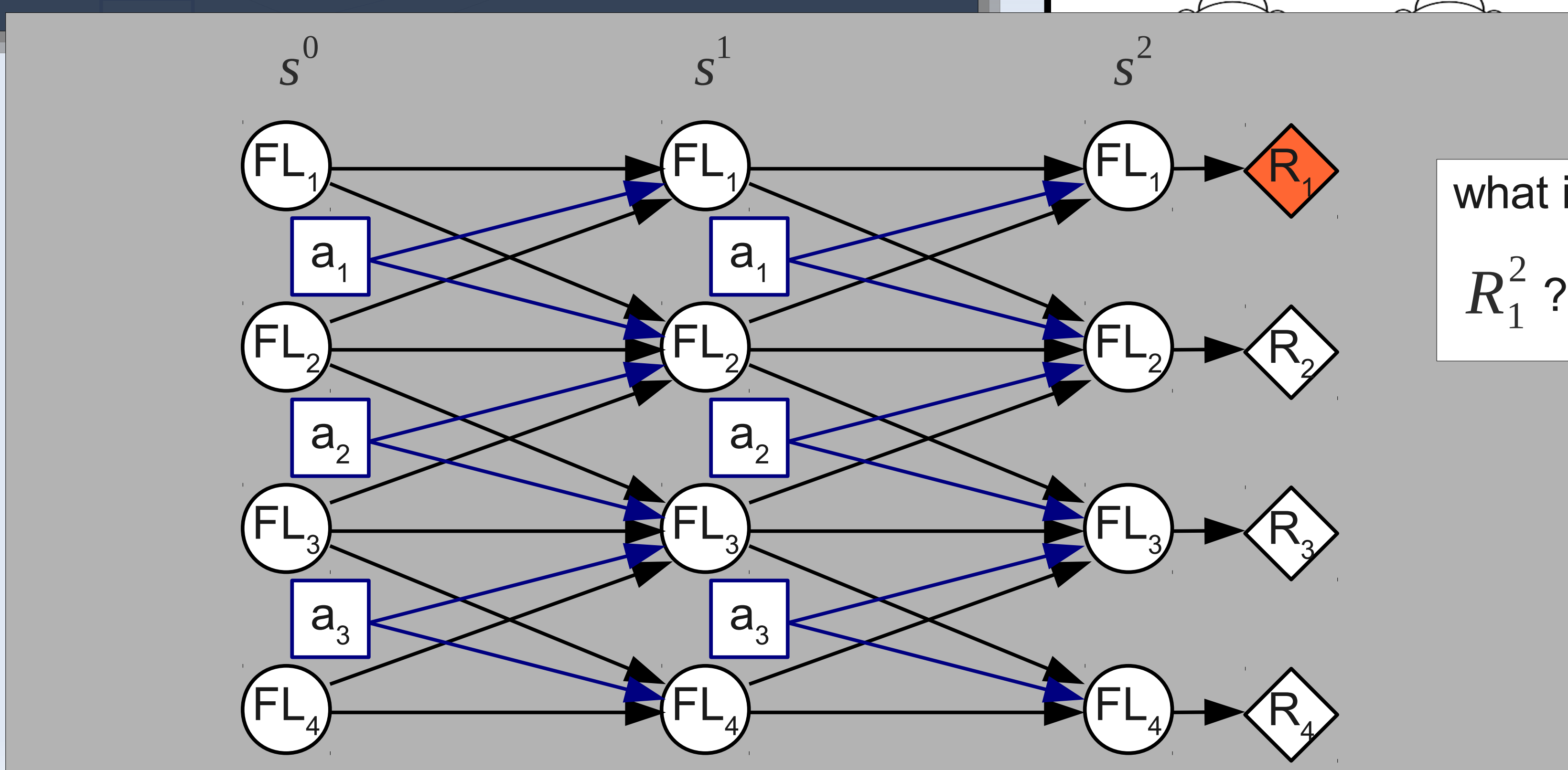

what influences R_1^2 ?

Graphical Structure between Agents

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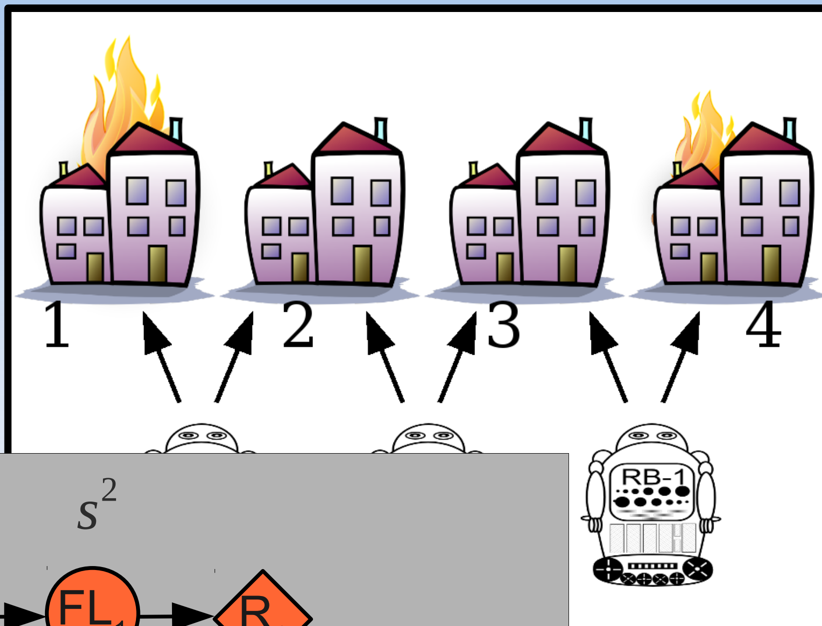

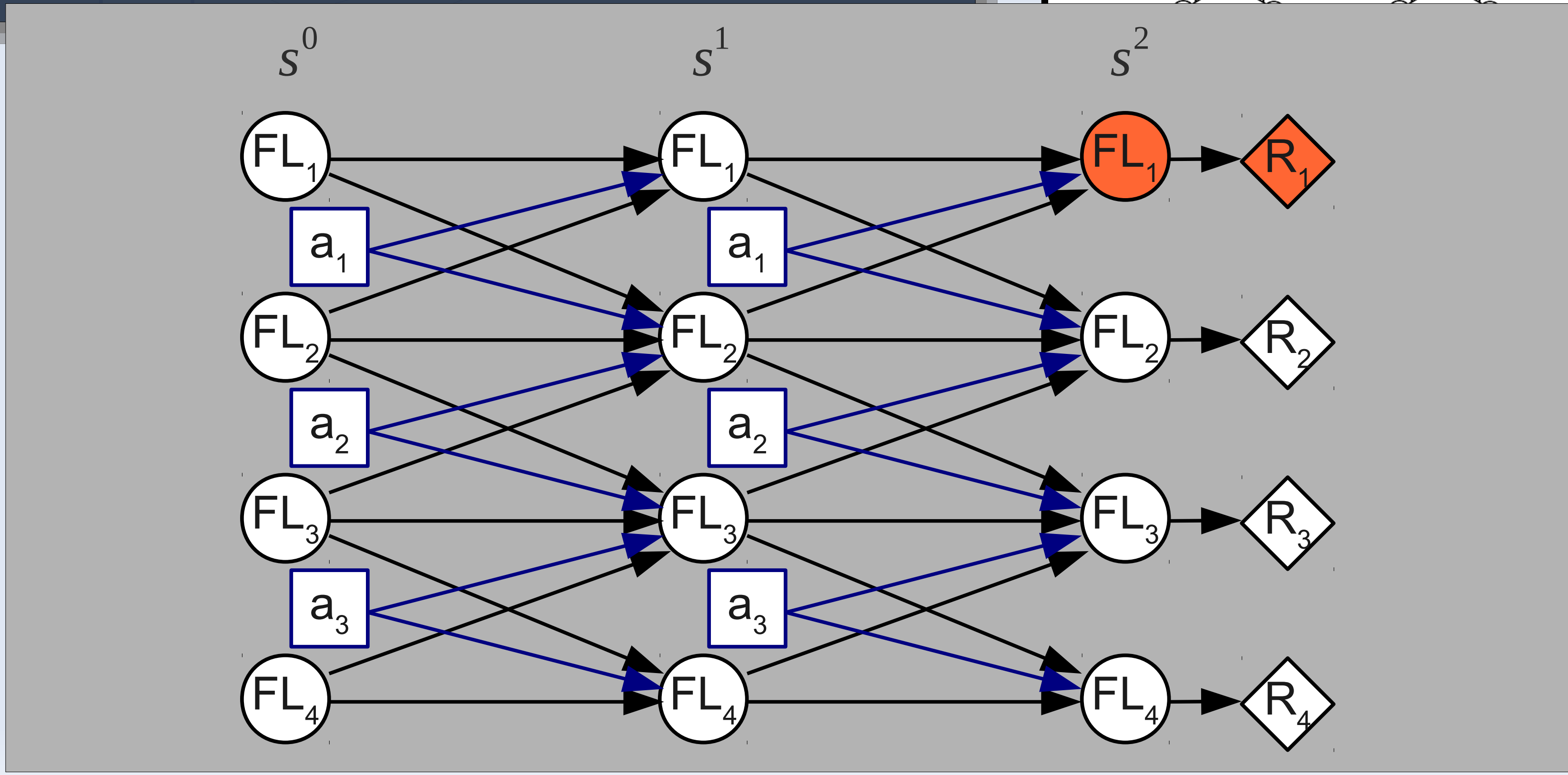

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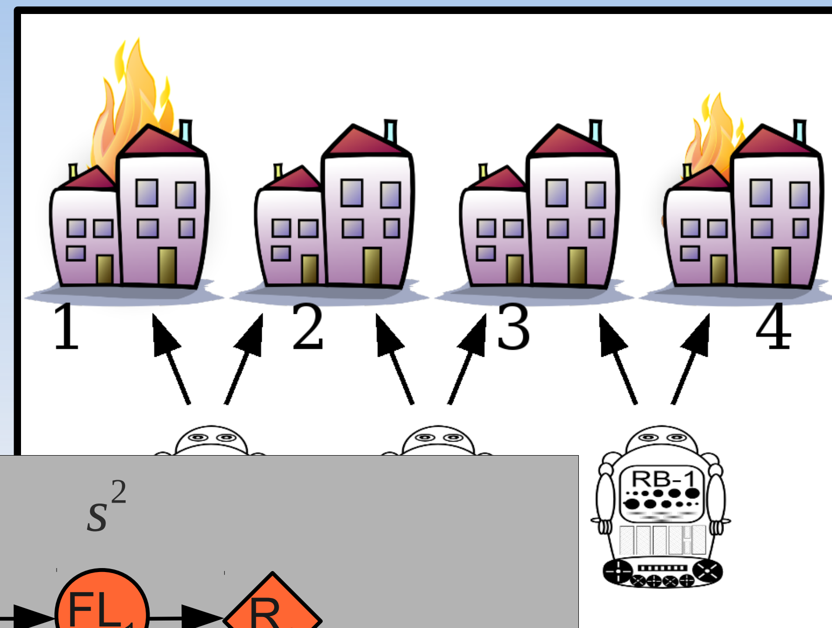

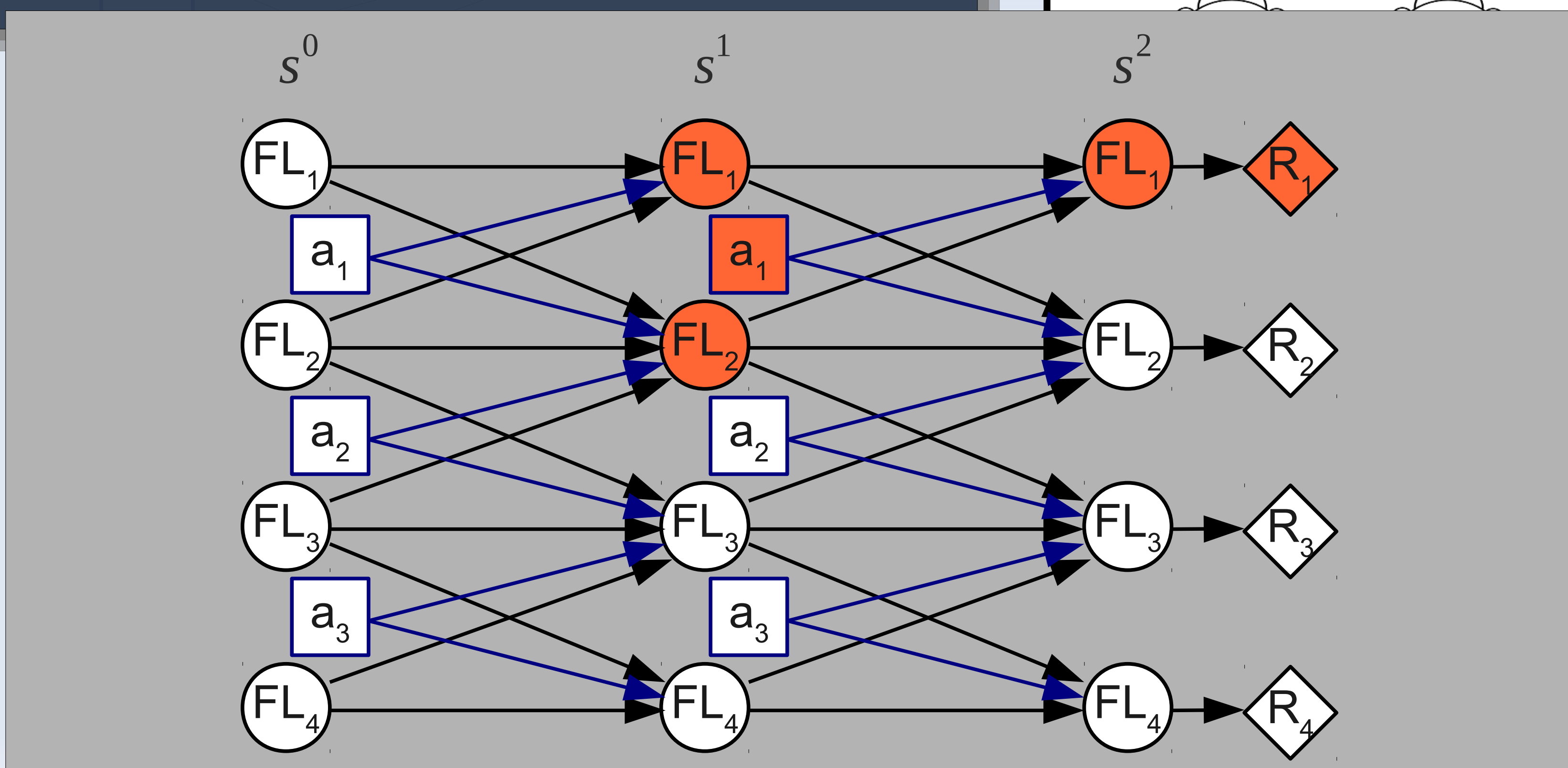

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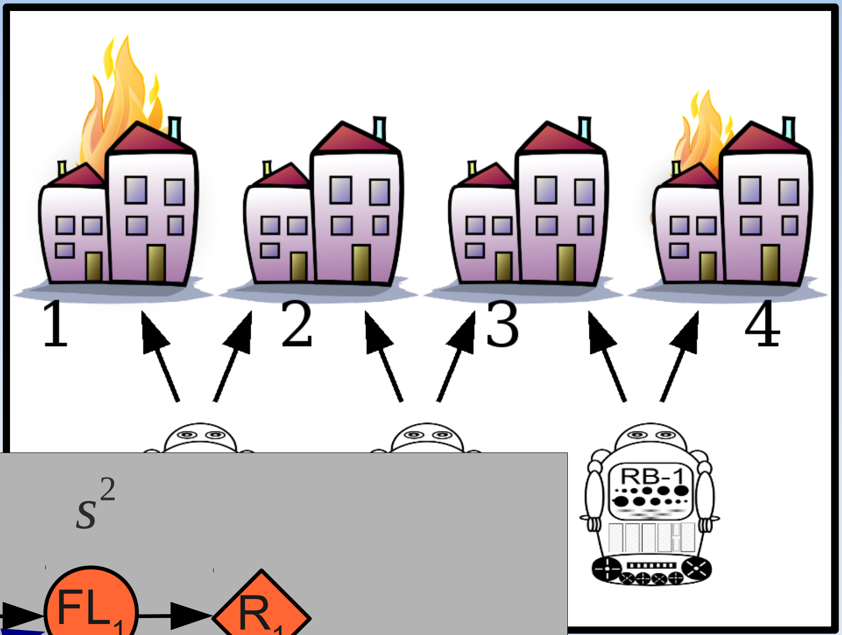
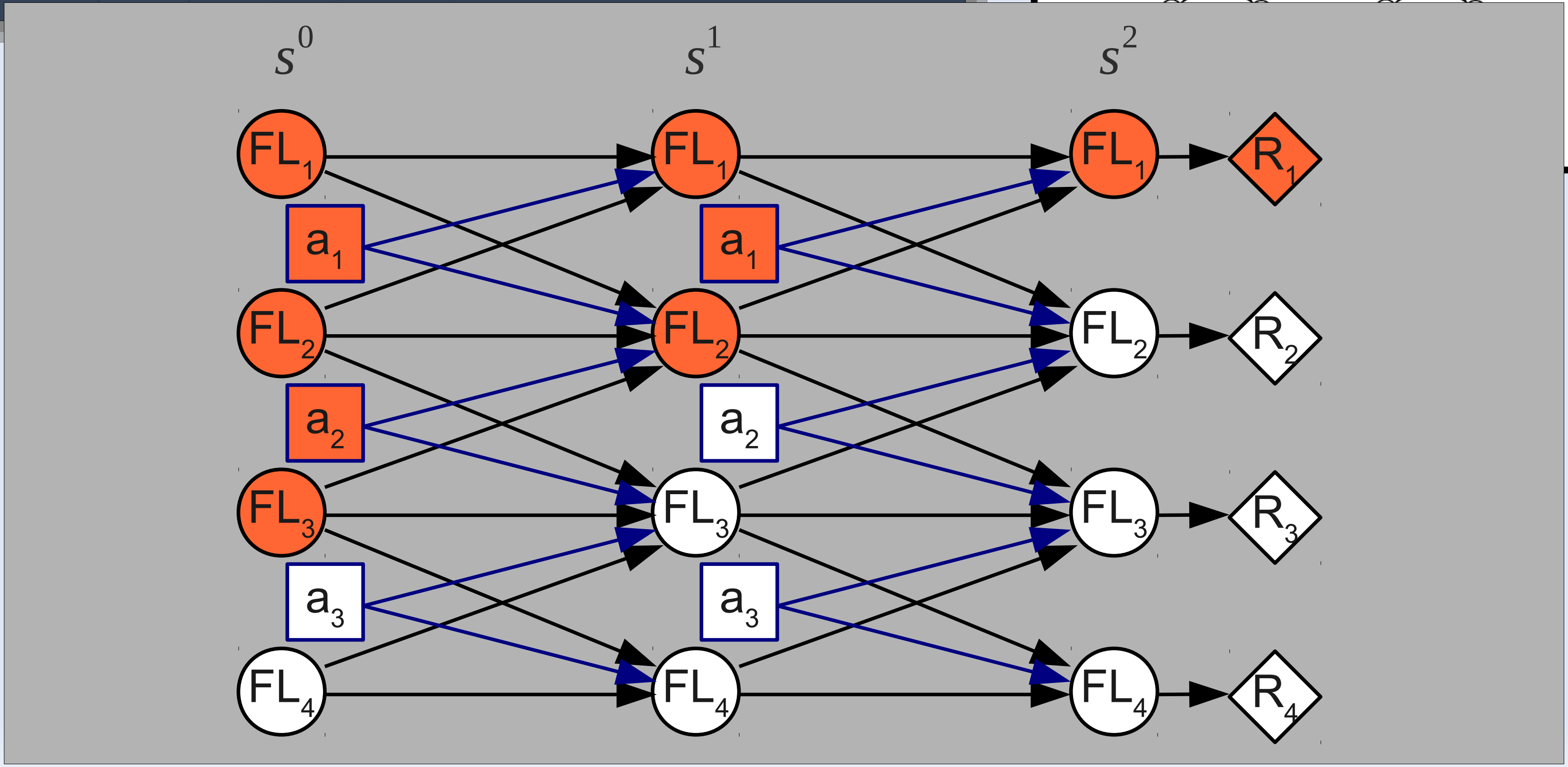
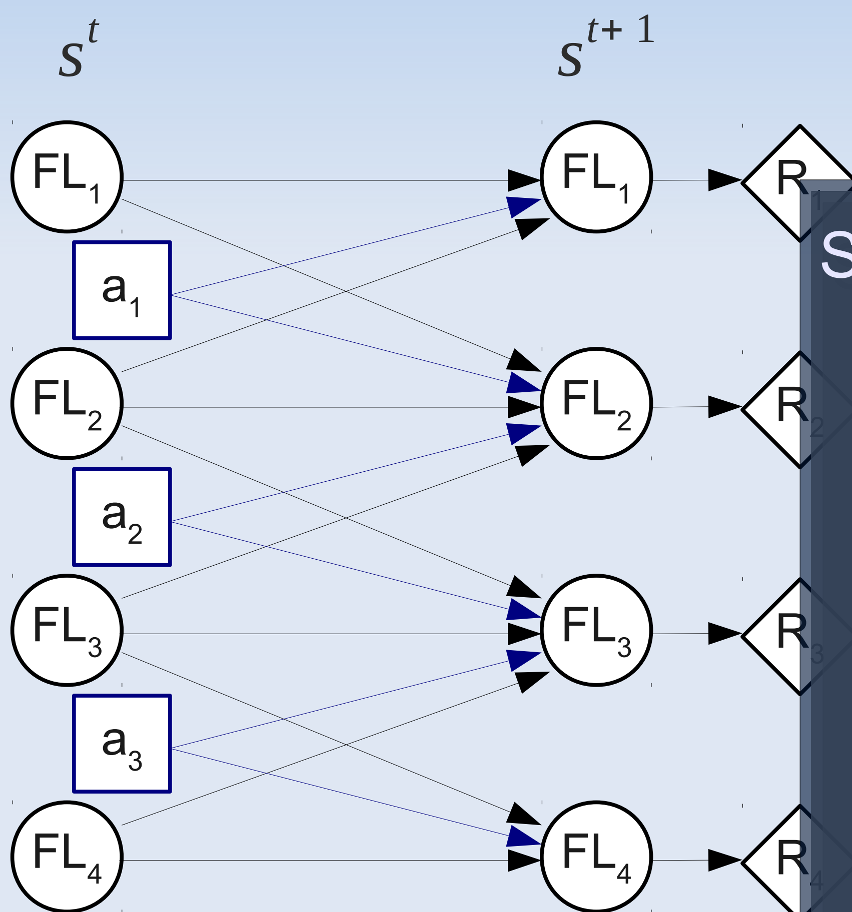
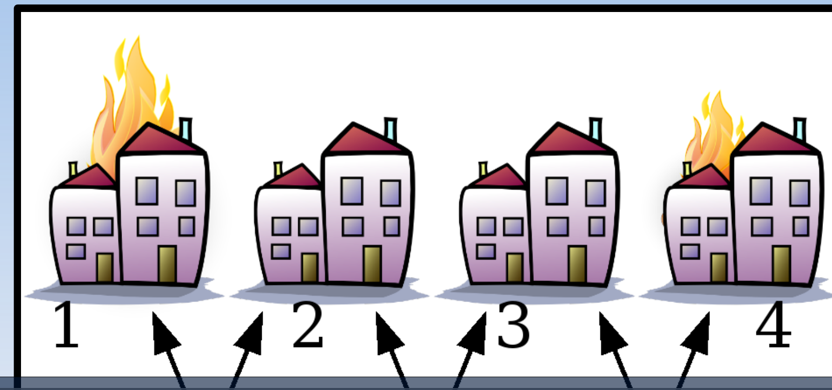




Graphical Structure between Agents

- Factored Dec-POMDPs
  [Oliehoek et al. 2008 AAMAS]





Solution Methods

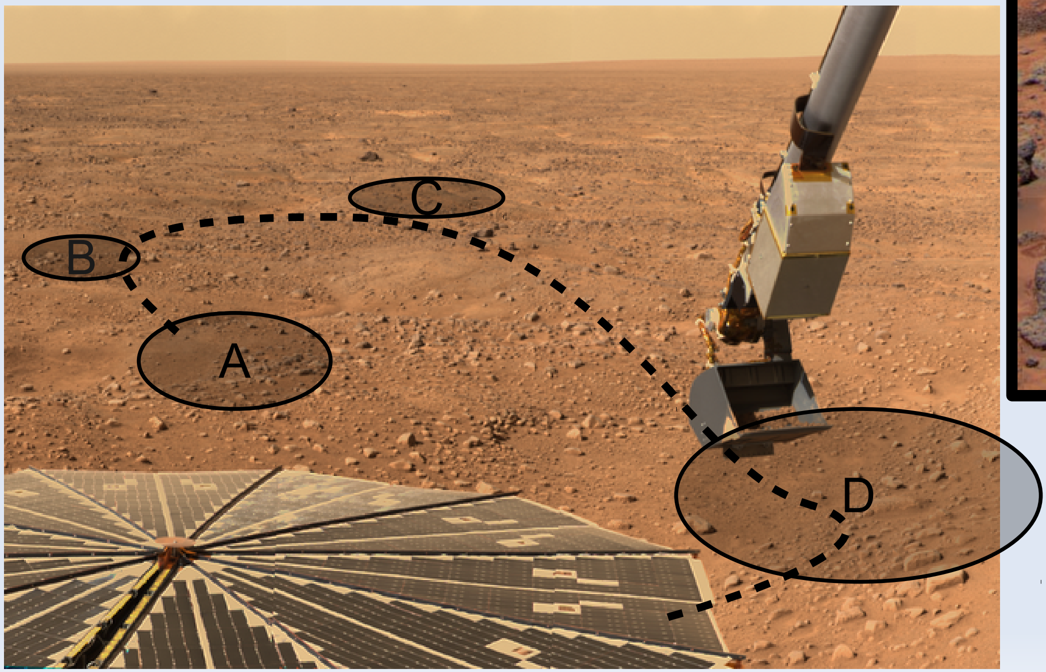
- reduction to a type of COP
- but now: one for each stage!

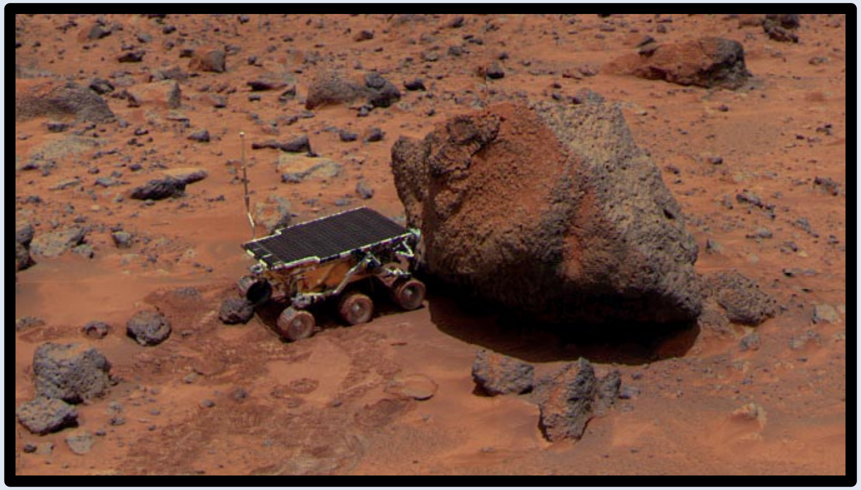
- δ is a decision rule
  (part of policy for 1 stage t)

→ leads to factored form of heuristic search
[Oliehoek 2010 PhD]

Influence-Based Abstraction

- Try to define agents' **local state**
- Analyze how policies of other agents affect it
  - find compact description for this **influence**
- Example: Mars Rovers [Becker et al. 2004 JAIR]
  - 2 rovers collect data at 4 sites

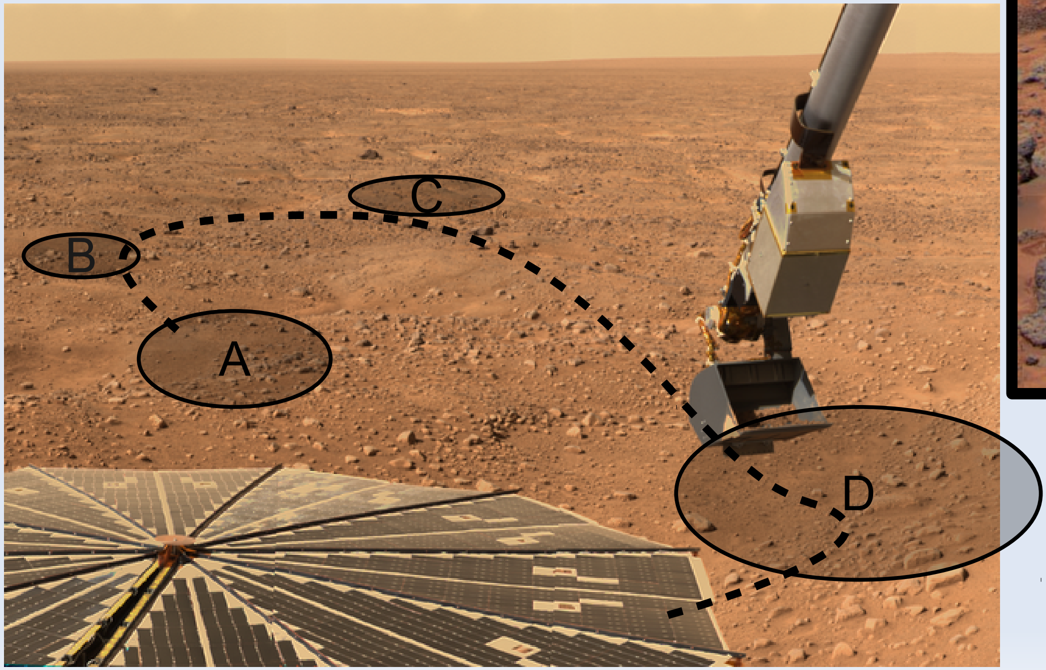


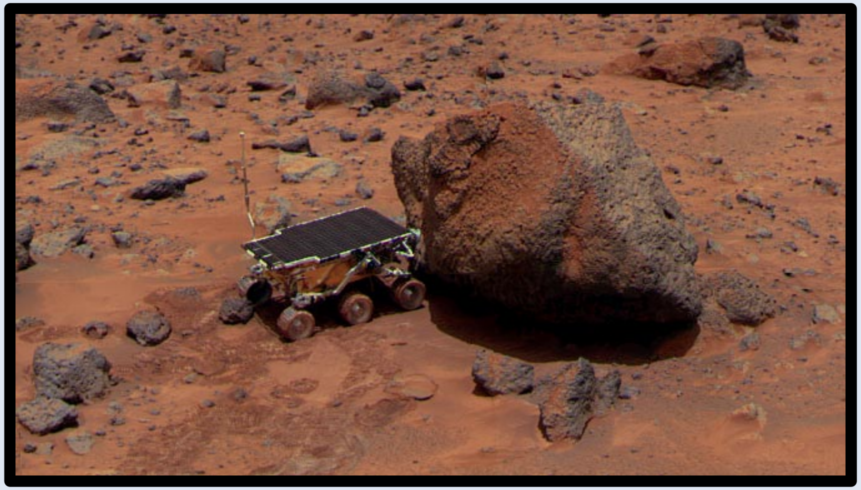


Influence-Based Abstraction

Transitions **independent**: Rovers drive independently
Rewards are **dependent**:
- 2 same soil samples of same site not so useful (sub additive)
- 2 pictures of (different sides) of same rock is useful (super additive)

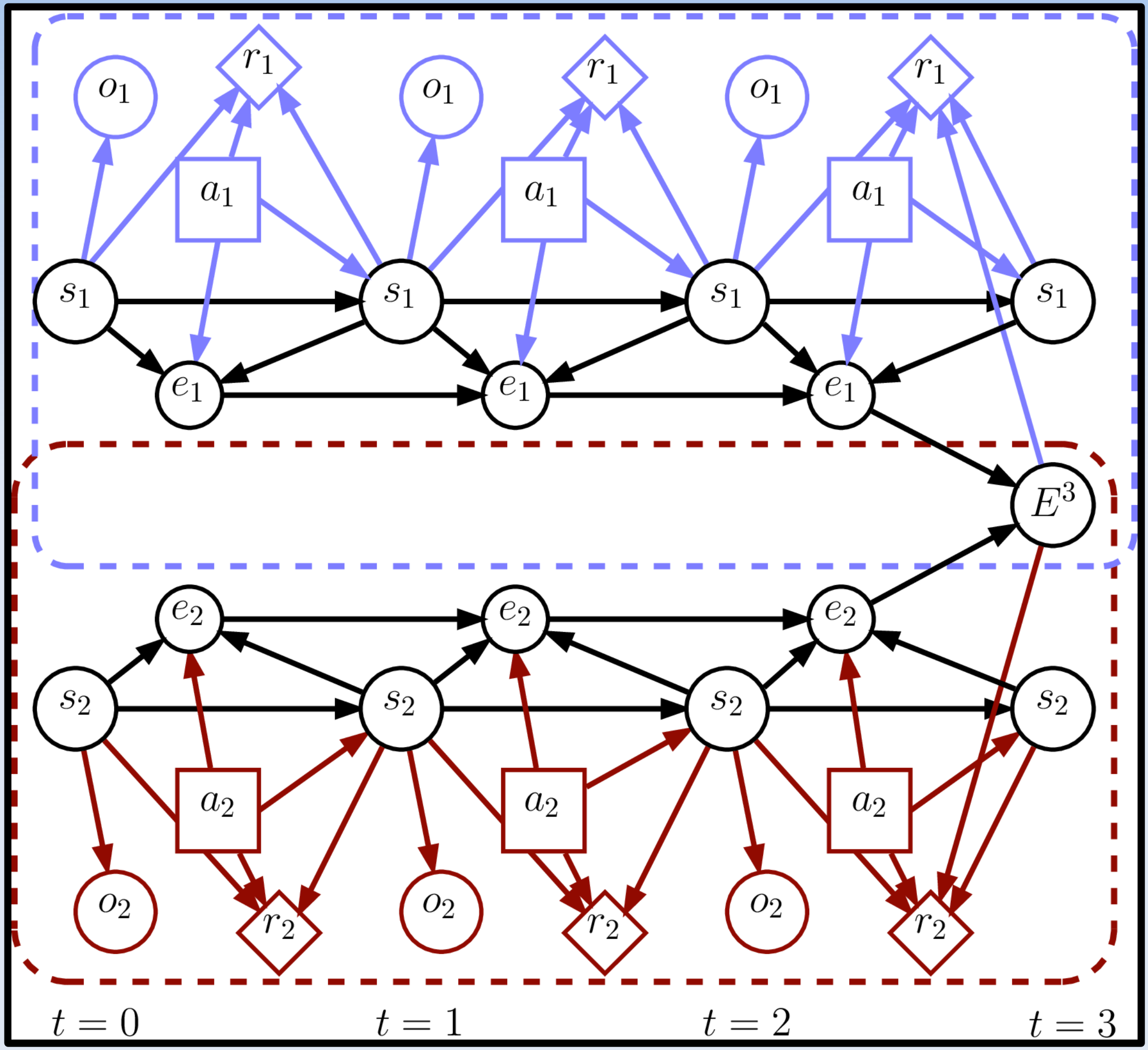
- Example: Mars Rovers [Becker et al. 2004 JAIR]
  - 2 rovers collect data at 4 sites





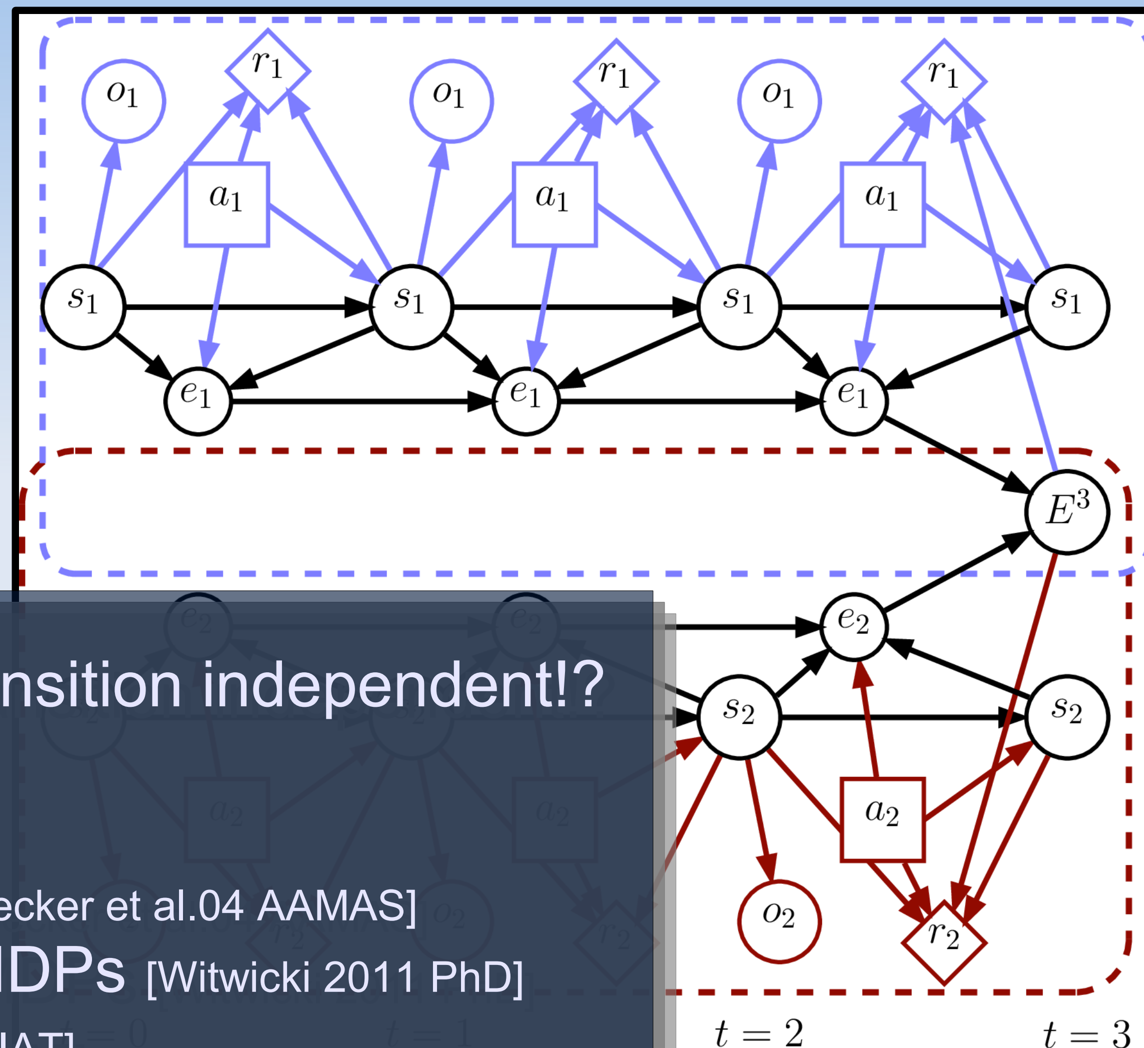
Influence-Based Abstraction

- TI Dec-MDP
- extra reward (or penalty) at the end **if** 'joint event' happens
- joint event E=<e_1,e_2>
- From agent i's perspective: if it realizes e_i → extra reward with probability $P(e_j)$



Influence-Based Abstraction

- TI Dec-MDP
- extra reward (or penalty) at the end **if** 'joint event' happens
- joint event E=<e_1,e_2>



But most problems are not transition independent!?

Much further research, e.g.:

- Event-driven Dec-MDPs [Becker et al.04 AAMAS]
- Transition-decoupled POMDPs [Witwicki 2011 PhD]
- EDI-CR [Mostafa & Lesser 2009 WIIAT]
- IBA for Factored POSGs [Oliehoek et al. 2012 AAAI]