T. 11

$$H_0: \xi \sim p_0(x) = 1\{(0;1)\}$$

$$H_1: \xi \sim p_1(x) = \frac{e}{e-1} \cdot e^{-x}\{(0;1)\}$$

a) $(n=1)$

$$\ell = \frac{L_1}{L_0} = \frac{\frac{e}{e-1} \cdot e^{-x}}{1} \geq C = const$$

$$e^{-x} \geq B \Rightarrow x \leq A \text{ - усл. } G$$

$$P(x \leq A / H_0) = \alpha = \int_0^A 1 \cdot dx = A \Rightarrow G: x \leq \alpha$$

$$\Rightarrow \alpha_1 = \alpha$$

$$W = P(x \leq A / H_1) = \int_0^\alpha \frac{e}{e-1} \cdot e^{-x} dx = \frac{e}{e-1}\left(1 - e^{-\alpha}\right)$$

$$\alpha_2 = 1 - W = 1 - \frac{e}{e-1}\left(1 - e^{-\alpha}\right)$$

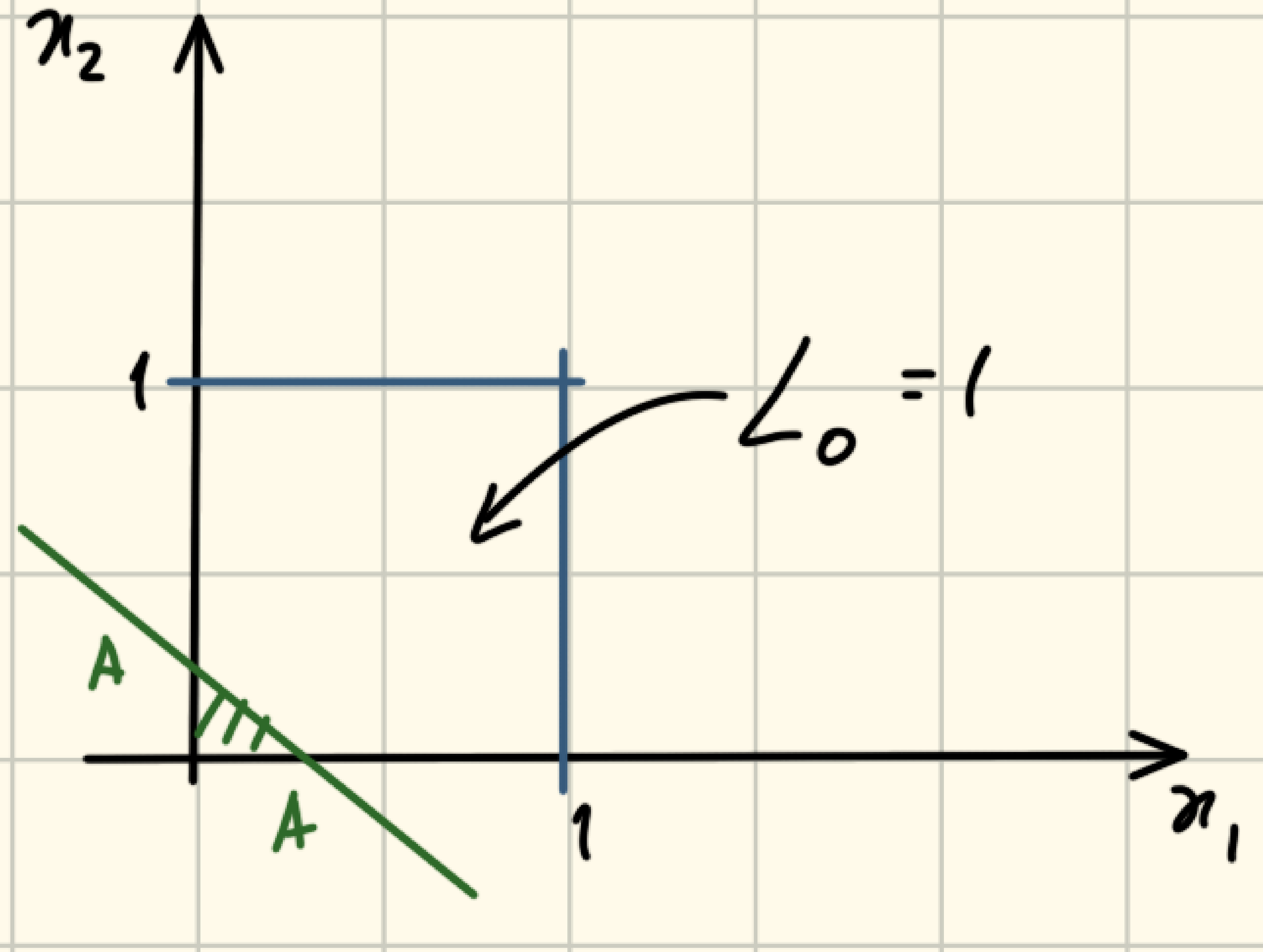
b)

$$\ell = \frac{L_1}{L_0} = \frac{\left(\frac{e}{e-1}\right)^2 \cdot e^{-x_1} \cdot e^{-x_2}}{1 \cdot 1} \geq C$$

$$e^{-(x_1+x_2)} \geq B \Rightarrow x_1 + x_2 \leq A \text{ - усл. } G$$

$$P(x_1 + x_2 \leq A / H_0) = \alpha$$

(при малых α)



$$\iint\limits_{x_1+x_2 \leq A} 1 \cdot dx_1 \cdot dx_2 = \frac{A^2}{2} = \alpha$$

$$\Rightarrow A = \sqrt{2\alpha}$$

$$G: x_1 + x_2 \leq \sqrt{2\alpha}$$

$$\alpha_1 = \alpha$$

$$W = P(x_1 + x_2 \leq A / H_1) = \iint\limits_{x_1+x_2 \leq A} \left(\frac{e}{e-1}\right)^2 e^{-x_1 - x_2} dx_1 dx_2 =$$

$$= \left(\frac{e}{e-1}\right)^2 \int_0^A e^{-x_1} dx_1 \int_0^{A-x_1} e^{-x_2} dx_2 = \left(\frac{e}{e-1}\right)^2 \cdot \int_0^A e^{-x_1} (1 -$$

$$- e^{-A+x_1}) dx_1 = -\frac{A e^2 \cdot e^{-A}}{(e-1)^2} + \left(\frac{e}{e-1}\right)^2 \cdot (1 - e^{-A}) =$$

$$= \left(\frac{e}{e-1}\right)^2 \cdot (1 - e^{-A} - A e^{-A})$$

(c) Наиболее мощный асимптотич. критерий

$$l = \frac{L_1}{L_0} = \prod_{i=1}^n \frac{p_1(x_i)}{p_0(x_i)} \geq C$$

$$G: \ln l = \sum_{i=1}^n \underbrace{\ln \frac{p_1(x_i)}{p_0(x_i)}}_{\eta_i} \geq \ln C$$

$$\frac{\sum \eta_i - n M\eta_1}{\sqrt{n \cdot D\eta_1}} \to N(0,1)$$

← одинак. распр.

$$P(\ln l \geq \ln C | H_0) = \alpha$$

$$\eta = \ln\left(\frac{e}{e-1} \cdot e^{-x}\right) = \ln \frac{e}{e-1} - x$$

$$\ln l = \sum_{i=1}^n \ln \frac{e}{e-1} - \sum_{i=1}^n x_i \geq \ln C$$

$$G: \sum x_i \leq A$$

$$P\left(\frac{\sum x_i - nMx}{\sqrt{nDx}} \leq \frac{A - nMx}{\sqrt{nDx}} \middle| H_0\right) = \alpha$$

для норм. расп.: $\dfrac{A - n \cdot \frac{1}{2}}{\sqrt{n \cdot \frac{1}{12}}} = u_\alpha$

$$A = \frac{n}{2} + u_\alpha \sqrt{\frac{n}{12}}$$

$$G: \sum x_i \leq n \cdot \frac{1}{2} + u_\alpha \sqrt{\frac{n}{12}}$$

$$\alpha_1 = \alpha$$

$$W = P\left(\sum x_i \le A \,/\, H_1\right) =$$

$$= P\left(\frac{\sum x_i - nMx}{\sqrt{nDx}} \le \frac{A - nMx}{\sqrt{nDx}} \,\Big|\, H_1\right) \ominus$$

$$Mx = \int_0^1 x \cdot \frac{e}{e-1} e^{-x} dx = \frac{e}{e-1}\left(-xe^{-x}\Big|_0^1 - \int_0^1 e^{-x} dx\right) =$$

$$= \frac{e}{e-1}\left(-e^{-1} + (1 - e^{-1})\right) = \frac{e-2}{e-1}$$

$$Mx^2 = \int_0^1 \frac{x^2 \cdot e^{1-x}}{e-1} dx = \frac{e}{e-1} \cdot \left(-\int_0^1 x^2 de^{-x}\right) =$$

$$= \frac{e}{e-1}\left(-x^2 \cdot e^{-x}\Big|_0^1 + \int_0^1 2xe^{-x}dx\right) = \frac{e}{e-1}\left(-e^{-1} - 2e^{-1} + 2 - \right.$$
$$\left. - 2e^{-1}\right) = \frac{2e-5}{e-1}$$

$$\Rightarrow Dx = \frac{e^2 - 3e + 1}{(e-1)^2}$$

$$\ominus \int_{-\infty}^{B} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}} dx$$, где

$$B = \frac{\frac{n}{2} + u_\alpha \sqrt{\frac{n}{12}} - n\frac{e-2}{e-1}}{\sqrt{n \cdot \frac{e^2-3e+1}{(e-1)^2}}}$$

$$\alpha_2 = 1 - W$$

$$B \xrightarrow[n \to \infty]{} +\infty$$

т.о. критерий явл. состоят.

d) $G: x_{min} \le C$ $\qquad H_0: \xi \sim R(0,1)$

$$P(\vec{x}_n \in G \,/\, H_0) = \alpha$$

если $\xi \sim F_0(x)$, то $\xi_{min} \sim 1 - (1 - F_0(x))^n$

где $\xi_{\overline{1,n}}$ – незав.

$$P(x_{min} \le C) = 1 - (1 - F(C))^n = \alpha$$

$$\alpha - 1 = -(1-c)^n \Rightarrow C = 1 - \sqrt[n]{1-\alpha}$$

$\alpha = \alpha_1$

$$W = P(\vec{x_n} \in G \mid H_1) = P(x_{min} \le C \mid H_1) =$$

$$= 1 - (1 - F_1(C))^n =$$

где $F_1 = \int\limits_0^x \frac{e}{e-1} \cdot e^{-t}\, dt = \frac{e}{e-1}(1 - e^{-x})$

$$W = 1 - \left(1 - \frac{e}{e-1}\left(1 - e^{-1+\sqrt[n]{1-\alpha}}\right)\right)^n =$$

$$= 1 - \left(1 - \frac{e}{e-1} + \frac{e^{\sqrt[n]{1-\alpha}}}{\cancel{e}} \cdot \frac{\cancel{e}}{e-1}\right)^n \cancel{\xrightarrow[n\to\infty]{} 1}$$

$\blacktriangleright$

рассмотрим $e^{\sqrt[n]{1-\alpha}} = \exp\left(e^{\frac{1}{n}\cdot \ln(1-\alpha)}\right) \overset{\text{ф. Тейлора}}{=}$

$$= \exp\left(1 + \frac{1}{n}\cdot \ln(1-\alpha) + o\left(\frac{1}{n}\right)\right) \overset{\text{ф. Тейлора}}{=} e \cdot \left(1 + \frac{1}{n}\cdot \ln(1-\alpha) + o\left(\frac{1}{n}\right)\right)$$

$$\Rightarrow W = 1 - \left(1 + \frac{\cancel{e} + \frac{e}{n}\cdot \ln(1-\alpha) + o\left(\frac{1}{n}\right)}{e-1} - \cancel{\frac{e}{e-1}}\right)^n =$$

$$= 1 - \left(1 + \frac{e \cdot \ln(1-\alpha)}{e-1} \cdot \frac{1}{n} + \underset{\to 0}{o\left(\frac{1}{n}\right)}\right)^n \longrightarrow \exp\left(\frac{e}{e-1}\ln(1-\alpha)\right) =$$

$$= 1 - (1-\alpha)^{\frac{e}{e-1}} \ne 1, \text{ т.к. } (1-\alpha)^{\frac{e}{e-1}} \ne 0 \quad \blacksquare$$

! критерий не явл. состоят. !