

Expectation maximization

Ex 1

maximization
 $\arg\min_{\mu, Z} R(\mu, Z, X) = \arg\min_{\mu, Z} \|X - Z\mu\|_2^2$

$$\sum_{n=1}^N \sum_{k=1}^K \|x_n - z_{nk} \mu_k\|_2^2$$

$$\sum_{k=1}^K z_{nk} \mu_k$$

$$\sum_{n=1}^N \|x_n - \sum_{k=1}^K z_{nk} \mu_k\|_2^2 - (K-1) \sum_{n=1}^N \|x_n\|_2^2$$

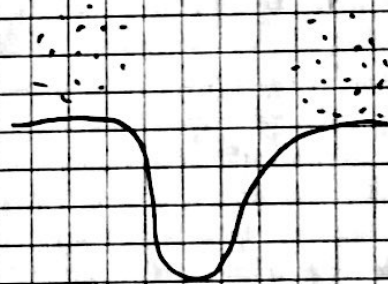
$$= \|X - Z\mu\|_2^2$$

Ex 2

$$P(x_n | \mu_i, \Sigma_i) = \mathcal{N}(x_n | \mu_i, \sigma_i^2 I)$$

$$= \frac{1}{(\sqrt{2\pi})^d (\sigma_i^2)^{d/2}} \exp\left(-\frac{1}{2} (x_n - \mu_i)^T \Sigma_i^{-1} (x_n - \mu_i)\right)$$

$$= \frac{1}{(\sqrt{2\pi})^d} \frac{1}{(\sigma_i^2)^{d/2}} \Rightarrow$$



$$\frac{1}{N} \left[\sum_{n=1}^N a_n - \frac{1}{2} \sum_{n,m=1}^N a_n a_m t_n t_m X_m^T X_n \right]$$

$$\sum_{n=1}^N a_n - \frac{1}{2} \sum_{n,m=1}^N a_n a_m t_n t_m X_m^T X_n$$

$$N - \frac{1}{2} \sum_{n,m=1}^N a_n a_m t_n t_m X_m^T X_n = 0$$

$$N = \frac{1}{2} \sum_{n,m=1}^N a_n a_m t_n t_m X_m^T X_n$$

$$\frac{\partial \mathcal{F}}{\partial w} = w - \sum_{n=1}^N \lambda_n t_n x_n = 0$$

$$w = \sum_{n=1}^N \lambda_n t_n x_n$$

$$\frac{\partial \mathcal{F}}{\partial b} = - \sum_{n=1}^N \lambda_n t_n = 0$$

$$\frac{\partial \mathcal{F}}{\partial \lambda_n} = \sum_{n=1}^N t_n w^T x_n + t_n b - 1 = 0$$

$$\sum_{n=1}^N t_n w^T x_n + t_n b = N$$

$$\sum_{n=1}^N$$

$$b = t_n - \sum_{m=1}^N a_m t_m x_m^T x_n$$

$$\sum_{n=1}^N a_n - \frac{1}{2} \sum a_m a_n t_m t_n x_m^T x_n$$

$4w_1 \rightarrow$

