Homework set 4 TDA231 Algorithms for Machine Learning & Inference

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1 SVM

1.1 Question 1

In figure 1 we have drawn, by inspection, the hyper-plane that separates the two classes. The parameters ω and b will be ω=[1,1] and $b = -3$ for this particular hyper-plane of the form $\omega_1 x_1 + \omega_2 x_2 + b = 0$. To calculating the margin 2γ we make use of equation 1 as given in the exercise. In table 1 the distances of all the support vectors have been calculated to also prove that we indeed have found the correct hyper-plane. Now that we have γ from table 1 we can calculate the margin as $2\gamma = \frac{1}{\sqrt{2}} * \frac{1}{\sqrt{2}} = \sqrt{2}$

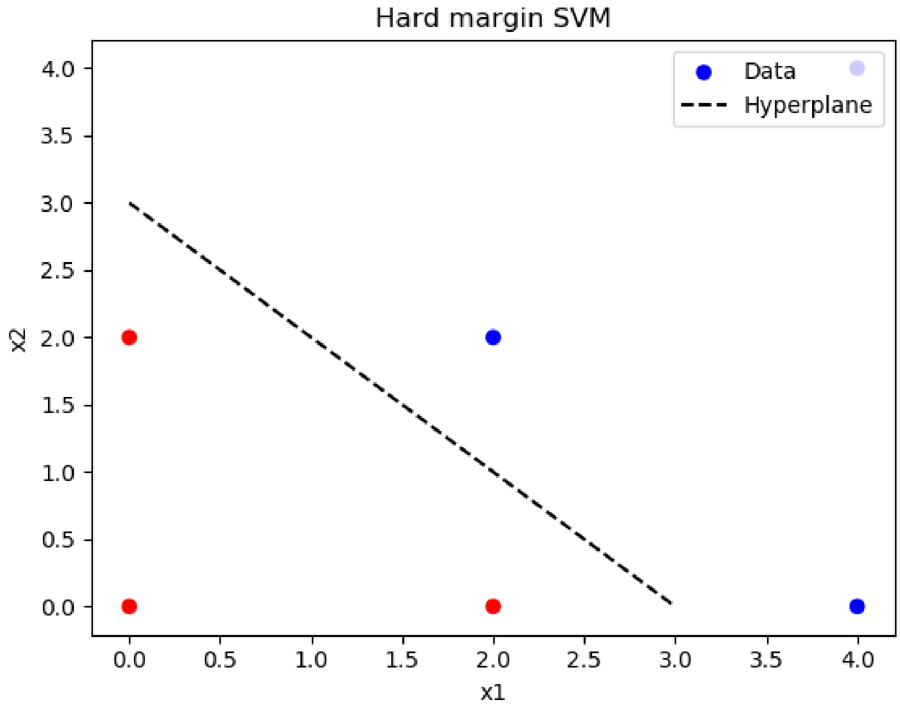

Figure 1: Caption

Point	γ
(2,0)	$\frac{1}{\sqrt{2}}$
(0.2)	$\frac{1}{\sqrt{2}}$
(2,2)	$\frac{1}{\sqrt{2}}$
(4,0)	$\frac{1}{\sqrt{2}}$

Table 1: Caption

$$\frac{|\omega_1 a_1 + w_2 x_2 - b|}{\sqrt{\omega_1^2 + \omega^2}} \tag{1}$$

1.2 Question 2

1.2.1 Primal formulation

$$\begin{aligned}
\min_{w_1,w_2} \frac{1}{2}(w_1^2 + w_2^2) \\
\text{subj. to} \\
2w_1 + 2w_2 + b >= 1 \\
4w_1 + 4w_2 + b >= 1 \\
4w_1 + b >= 1 \\
-b >= 1 \\
-2w_1 - b >= 1 \\
-2w_2 - b >= 1
\end{aligned}$$

1.2.2 Optimal primal solution

Using the `FindMinimum` function in *Mathematica* with the above formulation, we obtain:

$$\begin{aligned}
w_1 &= 1 \\
w_2 &= 1 \\
b &= -3
\end{aligned}$$

It matches the results obtain in Question 1.

1.2.3 Dual formulation

We used the `quadprog` function in *Matlab* to resolve the quadratic programming problem corresponding to that formulation. It expects a formulation of the following form:

$$\begin{aligned}
\min_{\alpha} \frac{1}{2}\alpha^T H\alpha + f^T\alpha \\
\text{subj. to} \\
A\alpha \leq b \\
A_{eq}\alpha = b_{eq}
\end{aligned}$$

Below is the adequate dual formulation:

$$
\begin{aligned}
H &= (t \circ x)^T (t \circ x) \\
f &= -1_N \\
A &= -I_{N \times N} \\
b &= 0_N \\
A_{eq} &= t \\
b_{eq} &= 0
\end{aligned}
$$

Where $\circ$ is the Hadamard product (element-wise matrix multiplication).

1.2.4 Optimal dual solution

The optimal solution found is:

$$
\alpha = \begin{pmatrix} 0.766708008374335 \\ 0.000000000003356 \\ 0.233291991666217 \\ 0.000000000003083 \\ 0.733291991689050 \\ 0.266708008351774 \end{pmatrix}
$$

Which gives us the following parameters for the hyperplane:

$$
w = \begin{pmatrix} 1.000000000048861 \\ 1.000000000058543 \end{pmatrix}
$$

2 Practical problems

See the completed Jupyter notebook for the code and results.