Machine learning

Post in complex system to find group.

Problem Set 1

$P_{test\ Posetiv\ Infected} = 0{,}99$

$P_{test\ negativ\ susept} = 0{,}99$

Sick	True Posetiv	False negative
Healthy	False Posetiv	True Negative

$P_{sick}(sick \mid Test\ sick$

$$P(Sick \mid Test\ Posetive) = P(Test\ Posetiv \mid Sick) \cdot \frac{P(Sick)}{P(Posetiv\ test)}$$

$$P(Posetiv\ test) = P_{test\ Posetiv\ infect} \cdot Infection\ rate +$$

$$(1 - P_{test\ negativ\ not\ infect}) \cdot Infection\ rate^{-1}$$

Let x be a continus variable uniformly distributed on $[-1,1]$ and let $Y=x^2$ clearly Y is not independent of x in fact Y is uniquly determand by x

Show that $\mathrm{cov}(x,Y)=0$

$$\mathrm{cov}(x,Y)=E[xY]-E[x]E[Y]$$

$E[xY]=E[x^3]$

$E[x]=0$ sins x is uniform on $[-1,1]$

$E[Y]=E[x^2]$

x^3 is uniform on $[-1,1]$

wich lead to $E[x^3]=E[xY]=0$

thus $\mathrm{cov}(x,Y)=0-0\cdot E[Y]=0$

x^2 is uniform on $(0,1] \Rightarrow$

$E[Y]=E[x^2]=\frac{1}{2}$

Generate 1000 points from 2D multivar normal distribution having mean $\mu \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and cov $\Sigma = \begin{bmatrix} 0{,}1 & -0{,}05 \\ -0{,}05 & 0{,}2 \end{bmatrix}$

Define the function

$$f(x,r) := \frac{(x-\mu)^T \cdot \Sigma^{-1} \cdot (x-\mu)}{2} - r$$

on a single plot show the folowing

The level sets $f(x,r)=0$ $r = 1,2,3$

Scatter plot of randomly generated points with points lying outsid $f(x,3)=0$

$x = (2,1000)$ matrix

aplye f on every single point

$$0 = \frac{(x-\mu)^T \cdot \Sigma^{-1} \cdot (x-\mu)}{2} - r$$

$$E[\theta] = \frac{a}{a+b} = m \quad \Rightarrow \quad \frac{a(1-m)}{m} = b$$

$$Var[\theta] = \frac{ab}{(a+b)^2(a+b+1)} = V =$$

$$\frac{a^2 \frac{(1-m)}{m}}{\left(a+\frac{a(1-m)}{m}\right)^2\left(a+\frac{a(1-m)}{m}+1\right)} = \frac{\frac{a^2}{m}(1-m)}{\frac{a^2}{m^2}\left(\frac{a+m}{m}\right)} =$$

$$= \frac{m^2(1-m)}{a+m} \Rightarrow \frac{m^2(1-m)}{V} = a+m \Rightarrow \frac{m^2(1-m)-Vm}{V} = a$$

$$\Rightarrow b = \frac{m^2(1-m)-Vm}{V} \cdot \frac{(1-m)}{m} = \frac{m(1-m)^2+V(m-1)}{V}$$

Test $m=1 \quad V=1 \qquad m=\frac{1}{2} \quad V=\frac{1}{2}$

$$a = \frac{1^2(1-1)-1\cdot 1}{1} = -1 \qquad \frac{\frac{1}{2}\cdot\frac{1}{4}+\frac{1}{2}\left(-\frac{1}{2}\right)}{1/2} = \frac{\frac{1}{8}-\frac{1}{4}}{1/2} = -\frac{1}{4}$$

$$b) \quad \frac{1\cdot(1-1)^2-1(1-1)}{1}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\right)-\frac{1}{4}}{1/2} = -\frac{1}{4}$$

$$\frac{-\frac{1}{4}}{-1/2} = \frac{1}{2}$$