Institute of Business & Information Technology

University of the Punjab

Quaid-e-Azam Campus, Lahore

Sheet No. _____

Solution

Assignment 03

Spring Term 2017

Code	IT-466	**Degree**	BBIT
Title	Analysis of Algorithm	**Batch**	F14-IT Specialization
Due Date:	April 06, 2017 before 1100hr	**Marks**	75

Instructions:

1. Do not forget to pray before starting to attempt the paper. Trust me it helps.
   Remember! SOMEONE is always with you (Be Relaxed), and HE is also watching you (Be Honest)
2. **Question Paper is SELF EXPLANATORY. Understanding the Question Paper is part of Solution.**
3. Nothing Beyond the Finish Line will be Evaluated. Back Side of Pages is Beyond Finish Line.
4. For Calculations etc. Use the back side of the pages.
5. Error in Question will be advantageous to Student.
6. Read the Questions carefully before attempting.
7. Solve your paper using Black/Blue Pen only.
8. Attempt All Questions in a Precise Fashion.
9. Switch Off your Cellular Phones.
10. Manage Your Time.

GOOD LUCK

Reg. No: _____ **Name:** _____

Class: IT Specialization **Date:** Monday, April 3, 2017

Section: _____ **Signature:** _____

Q 01.	Q 02.	Q 03.	Q 04.	Q 05.	Q 06.	Q 07.	Q 08.	Q 09.	Q 10.	Total
										75

Assistant's Signature

Examiner's Signature

Do Not Open until YOU are Advised by the Invigilator.

Question No 01.

Find the Time Complexity of the Following Recursive Algorithms using Master Theorem

a. $T(n) = 3\left(\frac{n}{2}\right) + nlgn$ 3

$a = 3$; $b = 2$; $K = 1$; $P = 1$

$3 \quad 2^1$

$3 > 2$

$a > b^K$

① $T(n) = O(n^{\log_b a}) = O(n^{\log_2 3})$

$T(n) = O(n^{\log_2 3})$

b. $T(n) = 5T\left(\frac{n}{5}\right) + \frac{n}{lgn}$ 3

$a = 5$; $b = 5$; $K = 1$; $P = -1$

$5 = 5^1$

$a = b^K$; $T(n) = O(n^{\log_b a} \log \log n)$

$T(n) = O(n^{\log_5 5} \log \log n)$

c. $T(n) = 4T\left(\frac{n}{2}\right) + n^2\sqrt{n}$ $T(n) = 4T\left(\frac{n}{2}\right) + n^2(n)^{1/2}$ $\therefore n^{2+\frac{1}{2}} = n^{\frac{4+1}{2}} = n^{5/2}$ 3

$a = 4$; $b = 2$; $K = 2.5$; $P = 0$

$4 \quad 2^{2.5}$

$4 < 5.5$

$a < b^K$; $T(n) = O(n^K \log^P n)$; $T(n) = O(n^{2.5} \log^0 n)$

d. $T(n) = 3T\left(\frac{n}{3} + 5\right) + \frac{n}{2}$ 3

$a = 3$; $b = 2$; $K = 1$; $P = 0$

$3 = 3^1$

$a = b^K$; $T(n) = O(n^{\log_b a} \log^{P+1} n)$

$T(n) = O(n^{\log_3 3} \log^{0+1} n)$

$T(n) = O(n^1 \log^1 n)$

e. $T(n) = T\left(\frac{n}{2}\right) + 1$ 3

$a = 1$; $b = 2$; $P = 0$; $K = 0$

$1 \quad 2^0$

$1 = 1$

$a = b^K$ $(P > -1)$; $T(n) = O(n^{\log_b a} \log^{P+1} n)$

$T(n) = O(n^{\log_2 1} \log^{0+1} n)$

$T(n) = O(n^{\log_2 1} \log^1 n)$

f. $T(n) = 2T\left(\frac{n}{2} + 17\right) + n$ 3

$a = 2$; $b = 2$; $K = 1$; $P = 0$

$2 = 2^1$

$a = b^K$

$T(n) = O(n^{\log_b a} \log^{P+1} n)$

$T(n) = O(n^{\log_2 2} \log^{0+1} n)$

$T(n) = O(n^{(1)} \log^1 n)$; $T(n) = O(n \log^1 n)$

g. $T(n) = 4T\left(\frac{n}{3}\right) + n$ 3

$a = 4$; $b = 3$; $K = 1$; $P = 0$

$a > b^K$; $4 > 3$

$T(n) = O(n^{\log_b a})$

$T(n) = O(n^{\log_3 4})$

h. $T(n) = 4T\left(\frac{n}{2}\right) + n^2$ 3

$a = 4$; $b = 2$; $K = 2$; $P = 0$

$4 = 2^2$

$4 = 4$

$a = b^K$; $T(n) = O(n^{\log_b a} \log^{P+1} n)$

$T(n) = O(n^{\log_2 4} \log^{0+1} n)$

$T(n) = O(n^{\log_2 2} \log^1 n) = O(n^{2\log_2 2} \log n)$

$T(n) = O(n^2 \log n)$.

i. $T(n) = 3T(\sqrt{n}) + \lg n$ 3

$a = 3$; $b = 1$; $K = 1$

For master theorem b must be greater than 1 so master theorem is not applicable.

j. $T(n) = 2T\left(\frac{n}{2}\right) + n^4$ 3

$a = 2$; $b = 2$; $K = 4$; $P = 0$

$a < b^K$; $2 < 2^4$; $2 < 16$

3 (a) $T(n) = O(n^K \log^P n) = O(n^4 \log^0 n)$

$T(n) = O(n^4 \log^0 n)$

k. $T(n) = T\left(\frac{7n}{10}\right) + n$ 3

$a = 1$; $b = 10/7$; $K = 1$; $P = 0$

$1 < 1.42$

$a < b^K$ $(P \geq 0)$; $T(n) = O(n^K \log^P n)$

$T(n) = O(n^1 \log^0 n)$; $T(n) = O(n \log^0 n)$

l. $T(n) = 7T\left(\frac{n}{4}\right) + n^2$ 3

$a = 7$; $b = 4$; $K = 2$; $P = 0$

$x(\because 4^{1/2} = (2^2)^{1/2} = 2)x$

$7 \quad 4^2$; $T(n) = O(n^K \log^P n)$

$7 < 16$ $\quad T(n) = O(n^2 \log^0 n)$

$a < b^K$ $\quad T(n) = O(n^2 \log^0 n)$

m. $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$ 3

$a = 2$; $b = 4$; $K = \frac{1}{2} = 0.5$; $P = 0$ $\because 4^{1/2} = 2$

$2 \quad (4)^{1/2}$; $2 = 2$; $a = b^K$

$T(n) = O(n^{\log_b a} \log^{P+1} n)$

$T(n) = O(n^{\log_4 2} \log^{0+1} n)$

$T(n) = O(n^{\log_4 2} \log^1 n)$

n. $T(n) = 2T\left(\frac{n}{2}\right) + \theta(n)$ 3

$a = 2$; $b = 2$; $K = 1$; $P = 0$

$2 = 2^1$; $a = b^K$

$T(n) = O(n^{\log_b a} \log^{P+1} n)$

$T(n) = O(n^1 \log^{0+1} n)$

$T(n) = O(n \log^1 n)$

o. $T(n) = 8T\left(\frac{n}{2}\right) + \theta(n^2)$ 3

$a = 8$; $b = 2$; $K = 2$; $P = 0$

$8 \quad 2^2$; $8 > 4$; $a > b^K$

$T(n) = O(n^{\log_b a})$

$T(n) = O(n^{\log_2 8})$

p. $T(n) = \frac{7}{2}T\left(\frac{n}{2}\right) + \theta(n^3)$ 3

$a = 3.5$; $b = 2$; $K = 3$; $P = 0$

$3.5 \quad 2^3$

$3.5 < 8$

$a < b^K$ $(P \geq 0)$ $\quad T(n) = O(n^K \log^P n)$

$T(n) = O(n^3 \log^0 n)$

q. $T(n) = 3T\left(\frac{n}{4}\right) + n \log n$ 3

$a = 3$; $b = 4$; $K = 1$; $P = 1$

$3 < 4^1$; $a < b^K$; $T(n) = O(n^K \log^P n)$

$T(n) = O(n^1 \log^1 n)$

r. $T(n) = 2T\left(\frac{n}{2}\right) + n^2 \log n$ 3

$a = 2$; $b = 2$; $K = 2$; $P = 1$

$2 \quad 2^2$

$2 < 4$

$a < b^K$ $(P \geq 0)$; $T(n) = O(n^K \log^P n)$

$T(n) = O(n^1 \log^2 n)$

s. $T(n) = 7T\left(\frac{n}{2}\right) + \theta(n^2)$ 3

$a = 7$; $b = 2$; $K = 2$; $P = 1$

$7 \quad 2^2$

$7 > 4$

$a > b^K$; $T(n) = O(n^{\log_b a})$

$T(n) = O(n^{\log_2 7})$

t. $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$ 3

$a = 2$; $b = 4$; $K = 1/2$; $P = 0$ $\because (4)^{1/2} = (2^2)^{1/2} = 2$

$2 \quad (4)^{1/2}$

$2 = 2$

$a = b^K$; $T(n) = O(n^{\log_b a} \log^{P+1} n)$

$T(n) = O(n^{\log_4 2} \log^{0+1} n)$; $T(n) = O(\log_4^2 \log^1 n)$

u. $T(n) = 4T\left(\frac{n}{4}\right) + n^2$ 3

$a = 4$; $b = 4$; $K = 2$; $P = 0$

$4 \quad 4^2$

$4 < 16$

$a < b^K$

$T(n) = O(n^K \log^P n)$

$T(n) = O(n^2 \log^0 n)$

v. $T(n) = 3T\left(\frac{n}{3}\right) + 1$ 3

$a = 3$; $b = 3$; $K = 0$; $P = 0$

$3 \quad 3^0$

$3 \quad 1$

$3 > 1$

$a > b^K \quad T(n) = O(n^{\log_b a}) = O(n^{\log_2 3})$

$T(n) = O(n^{\log_2 3})$

w. $T(n) = 3T\left(\frac{n}{4}\right) + n$ 3

$a = 3 \; ; \; b = 4 \; ; \; K = 1 \; ; \; P = 0$

$3 \quad 4^1$

$3 < 4$

$a < b^K \quad (P \geq 0) \; ; \; T(n) = O(n^K \log^P n)$

$T(n) = O(n^1 \log^0 n) \; ; \; T(n) = O(n \log n)$

x. $T(n) = 4T\left(\frac{n}{2}\right) + n^2 lgn$ 3

$a = 4 \; ; \; b = 2 \; ; \; K = 2 \; ; \; P = 1$

$4 \quad 2^2$

$4 = 4$

$a = b^K \; ; \; (P > -1) \; ; \; T(n) = O(n^{\log_b a} \log^{P+1} n)$

$= O(n^{\log_2 4} \log^2 n) = O(n^2 \log^2 n)$

Finish Line

Nothing beyond this line will be evaluated