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				(Autonomous Institute, affillated to VIO) DEPARTMENT OF INFORMATION SCIENCE & ENGINEERING	mous Institute, al INFORMATION S	CIENCE & EN	GINEERING	do: ISFR1	2	
,		Term:	31 ⁴¹ Jan to 22 nd May, 2019	1ay, 2019			Course Code:	+	B, C	
	1	Course:	Machine Learning	8			Max Marks:	rks: 30		
•		Test Portio	Test Portioner 13-03-2019 9.30 to 10.30am Petanismer any two full Qs. Mobiles are banned.	14 125019	9.30 to 10.30am	s: Answer	any two full	as. Mobil	es are ban	ned.
	SI. #		TO# HOSSAT #OT-	Ollection				Marks	Bloom s Level	3
	13	Apply Naive	Apply Naive Bayes Classifier to the following dataset to evaluate your activity	to the followir	ng dataset to	evaluate yo	our activity	9	EJ	001
		for tonight	for tonight given that you have deadlines looming - but none of them are ureent there is no narty on tonight and you are feeling lazy.	have deadlines	s looming - b	ut none o	f them are			
			Deadline? [18	Is there a party?	y? Lazy?	Activity				
7			_	Yes ON		Party Study				
			Near	χς Χ	Yes No	Party Party				
			None	Yes	χς. Σο	Party			,	
			Near	o c Z Z	Yes	TV				
A			Near Urgent	$_{ m No}^{ m Yes}$	Yes	Party				
1					2000000	two of th	e foll:	4	77	001
	1 b	Explain with	Explain with an example each the difference between any was a comment of the comm	th the differenc	e Detween an					
		a) KOC an	KOC and AUC Curves Bias and Variance of the ML Model	e ML Model						
			Variance and Covariance of the ML Model	e of the ML Mo	del					
		d) Overfit	Overfitting and Underfitting of the ML Model	tting of the ML	Model				2	000
	1	What is the	What is the need of 'Perceptron Activation Function'? What are the types?	ron Activation	Function'? W	nat are the	types?	٠	ż	
	Contract of the		Andread of the Angres of the	The transport of the same	Section Co.					100
	2a	A Spam Ma	Snam Mail Filter Classification Model was developed to filter out	cation Model	was develope	d to filter	out spam	10	F4	00
		where 1000	where 1000 email samples were	were examine	examined. The number of IP = 100, FP	er of IP = ion Model	100, FP			
		30,TN = 700,	30,TN = 700, FN = 170. Do the following for this Classification income.	e tollowing for	IIIIs cadasilicad					
			Draw the Confusion Matrix Colombia the Accuracy value & Error Rate	rix alue & Error Ra	te					
		b) Calculate	Assess the Sensitivity & Specificity Value	specificity Value	c)					
			Determine the Precision & Recall Value	& Recall Value						
		e) Compute	Compute the F-Measure	4-11.0	Joham 'Ilaan	luctify yo	III answer			
			Is this Model a 'High Precision' or High Recall Model: Justin from Commerced one?"	cision or High ole given to vou	a Balanced o	r an Imbala	anced one?			
		g) Evaluate Justify yo	Evaluate II the data surry Justify your answer							
	4	Consider the	consider the following set of points: {(-2, -1), (1, 1), (3, 2)}	points: {(-2, -1	(1, 1), (3,	2)}		2	ខា	CO2
		a) Find the	Find the least square regression line for the given data points.	gression line fo	r the given da	ta points.	-			
			plot the given points and the regression line in the same rectangular	and the regres	ssion line in t	ne same	ectangular			
		system	system of axes.							
		07. 00.	and the way mean by Imbalanced Datasets? Why does Accuracy fail	alanced Datas	ets? Why do	es Accurac	y fail as a	4	14	CO1
	39	performance	what do you metrics for these datasets?	se datasets?						
	36	What do yo	What do you understand by the following terms?	y the followin		Answer any	any two. Give	9	77	CO 100
		relevant examples:	nples:	2019122210 (0.44	olud acit					
			Maximum A Posterior (IVIAP) Classification rule	VIAP) CIASSIIICA	non vale					
		b) Conditional Pr c) Bayesian Rule	Conditional Probability Bayesian Rule							
	36	Write and de	Write and describe the Perceptron Algorithm. What is the difference between	eptron Algorith	ım. What is th	e differen	se between	S	77	C02
		the 'Recall' a	the 'Recall' and 'Training' phase?	ise?						

ISEB1 (Machine Learning) T1 - Answer Scheme

1a) Answer as below (6 marks):

- Correct statement of Conditional Probability Equations: 1 Mark
- Calculation of each Conditional Probability: 4 Marks
- Writing the final inference : 1 Mark
- $P(Party) \times P(Near \mid Party) \times P(No(Party \mid Party) \times P(Lazy \mid Party)$
- $P(Study) \times P(Near \mid Study) \times P(No Party \mid Study) \times P(Lazy \mid Study)$
- $P(Pub) \times P(Near \mid Pub) \times P(No|Party \mid Pub) \times P(Lazy \mid Pub)$
- $P(TV) \times P(Near \mid TV) \times P(No Party \mid TV) \times P(Lazv \mid TV)$

Using the data above these evaluate to:

So based on this you will be watching TV tonight

P(Party near (not urgent) deadline, no party, lazy)	Best/	$\frac{5}{10} \times \frac{2}{5} \times \frac{9}{5} \times \frac{3}{5}$
$P(\operatorname{Study} \operatorname{nese}$ (not urgent) deadline, no party, lazy)	1,32	$\frac{3}{10} \times \frac{1}{3} \times \frac{3}{3} \times \frac{1}{3}$
	10.00	301
P(Pub near (not urgent) deadline, no party, lazv)	LAUP THE REAL PROPERTY.	$\frac{1}{10} \times \frac{0}{1} \times \frac{1}{1} \times \frac{1}{1}$
P(TV near (not urgent) deadline, no party, lazy)	with	$\frac{1}{10} \times \frac{1}{1} \times \frac{1}{1} \times \frac{1}{1}$
	no.	10

1b) Answer any two (2 x 2 = 4 Marks : Any two valid points of difference)

- a) ROC and AUC Curves
- b) Bias and Variance of the ML Model
- c) Variance and Covariance of the ML Model
- d) Overfitting and Underfitting of the ML Model

2a) Total: 10 Marks

a) 1 mark

Confusion Matrix Folder Shirm Folder Shirm Folder Shirm Folder 100 170 True positives False Negatives 30 700 False Positives Title Negatives

b) 2 x 1 = 2 Marks

Accuracy =
$$\frac{100 + 700}{1000} = 80\%$$

Error Rate = 1 - Accuracy = 0.2 or 20%

c) $2 \times 1 = 2$ Marks

Sensitivity =
$$\frac{\#TP}{\#TP + \#FN}$$
Specificity =
$$\frac{\#TN}{\#TN + \#FP}$$
Precision =
$$\frac{\#TP}{\#TP + \#FP}$$
Recall =
$$\frac{\#TP}{\#TP + \#FN}$$

- Sensitivity = 100 / (100+170) = 0.3703
- Specificity = 700/(30 + 700) = 0.9589

<u>d)2 x 1 = 2 Marks</u>

2b) Total: 5 Marks

Let us organize the data in a table.

x	У	хy	x ²
-2	-1	2	4
1	1	1	1
3	2	6	9
$\Sigma x = 2$	Σy = 2	$\Sigma xy = 9$	$\Sigma x^2 = 14$

- Writing the above values correctly: 1
 Mark
- Calculating the least square regression: 2 Marks
- Plotting the graph: 1 Mark

Recall =
$$\frac{100}{100 + 170}$$
 = 37%

<u>e) 1 mark</u>

F1 Score =
$$\frac{2 \times 76.9 \times 37}{76.9 + 37}$$
 = 49.96%

f) 1 mark

High Precision

<u>Justification</u>: FPR must be as minimum as possible for the Spam Mail Filter system given

g) 1 mark

Imbalanced

<u>Justification</u>: The number of True Positives are very less (Only 100: 10% of the samples) compared to the number of True Negatives (700: 70% of the samples)

We now use the above formula to calculate a and b

as follows

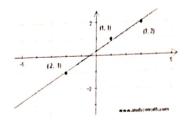
$$a = (n\Sigma x y - \Sigma x \Sigma y) / (n\Sigma x^2 - (\Sigma x)^2) = (3*9 - 2*2) /$$

$$(3*14 - 2^2) = 23/38$$

b =
$$(1/n)(\Sigma y - a \Sigma x) = (1/3)(2 - (23/38)^2) = 5/19$$

b) We now graph the regression line given by y = a

x + b and the given points.



3a)

When TPR / TNR is very low because the number of TP samples are very less, the dataset is imbalanced.

Accuracy fails as a performance metrics because it tends to give a value from the majority sample (TN) which is very high and will fail to detect the minority sample (TP).

36)

Answering any two below: 3 Marks

- a) Maximum A Posterior (MAP)
 Classification Rule
- b) Conditional Probability
- c) Bayesian Rule

Explanation: 1 Mark
One example: 2 Marks

3c)

The Perceptron Algorithm

- Initialisation
 - set all of the weights will to small (positive and negative) random numbers
- Training
 - for T iterations or until all the outputs are correct:
 - · for each input vector.
 - compute the activation of each neuron j using activation function g.

$$y_j = g\left(\sum_{i=0}^m w_{ij}x_i\right) = \begin{cases} 1 & \text{if } \sum_{i=0}^m w_{ij}x_i > 0\\ 0 & \text{if } \sum_{i=0}^m w_{ij}x_i \leq 0 \end{cases}$$
 (3.4)

· update each of the weights individually using:

$$w_{ij} \leftarrow w_{ij} - \eta(q_i - t_j) \cdot x_i \tag{3.5}$$

- · Recall
 - compute the activation of each neuron j using:

$$y_j = g\left(\sum_{i=0}^m w_{ij} x_i\right) = \begin{cases} 1 & \text{if } w_{ij} x_i > 0 \\ 0 & \text{if } w_{ij} x_i \le 0 \end{cases}$$
 (3.6)

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Term: 31st Jan	to 22 nd May, 2019	THE CONTRACT CONTRACT CONTRACT OF THE CONTRACT C	Course	ISER1
The second secon			Code:	ISLD1
CIED	Learning		Semester:	6 A, B, C
CIE Details: Test - I	10-04-2019	9.30 to 10.30am	Max Marks:	30

Test Portions: Lesson #14-36Instructions to Candidates: Answer any two full Qs. Mobiles are banned.

Sl. #	Question	Marks	Bloom's Level	СО
1a	Apply MLP to train the neural network to output 0.01 and 0.99 for given inputs 0.05 and 0.10 with bias values 1 by using weights as w1=0.15, w2=0.20, w3=0.25, w4=0.30, b1=0.35, w5=0.40, w6=0.45, w7=0.50, w8=0.55 and b2=0.60.	7	L3	CO2
) 1b	 Show example transaction data where for the rule X → Y: (a) Both support and confidence are high. (b) Support is high and confidence is low. (c) Support is low and confidence is high. (d) Both support and confidence are low. 	4	L3	CO3
1c	In a two-class, two-action problem, if the loss function is $\lambda 11 = \lambda 22 = 0$, $\lambda 12 = 10$, and $\lambda 21 = 1$, write the optimal decision rule.	4	L3	CO4
				000
2a	What are the limitations of Single Layer Perceptrons? Why do we need MLP? Explain with an example how multilayer perceptron handles nonlinearity.	7	L3	CO2
2b	In PCA, obtain the Eigenvalues for the feature vector and covariance matrix given below. Which 'Eigenvalue' will you consider for PC1 and why? Original Feature Vector X1	4	L3	CO3
2c	Compare the approaches of ID3 and CART	4	L4	CO4
20				
3a	Write the Backpropagation Algorithm in MLP. Explain how MLP overcomes the limitations of single layered perceptron.	5	L3	CO2
3b	Answer any two of the following questions: (i) In LDA what is Fischer Ratio? What is its significance? (ii) Two differences between PCA and ICA (iii) Probably Approximately Correct (PAC) learning	6	L2	CO3
	Write a brief note on Classification and Regression Trees.	4	L2	CO4
3c	WITH a OTION THE CONTRACTOR OF			

IA test 2
Machine Learning (ISEB1) - Scheme

One		Marks
Q. no la	Solution	2
	11 .15 w1 h1 .40 w5 o1	
	.05 .20 w2 .45 w6 .01	
	25 w3 50 w	
	12 h2 F5 w8 o2	
	.30 w4 .55 w8 .99	
	.10	
	b1.35 b2.60	
		2
	The Forward Pass	
	i. Evaluate activation function (one sample for each layer)	
	ii. Calculate the total error for the sample considered in step i	3
	The Backward Pass i. Update the weights at output layer	3
	ii. Update the weights at hidden layer	
	opanio mo morgina ai maatin ing i	
lb	Example:	4x1 =
	(a) A rule that has high support and high confidence. Answer: Milk → Bread. Such	4
	obvious rule tends to be uninteresting. (b) A rule that has reasonably high support but low confidence. Answer: Milk → Tuna.	
	While the sale of tuna and milk may be higher than the support threshold, not all	
	transactions that contain milk also contain tuna. Such low-confidence rule tends to be	
	uninteresting. (c) A rule that has low support and low confidence. Answer: Cooking oil → Laundry	
	detergent. Such low confidence rule tends to be uninteresting.	
	(d) A rule that has low support and high confidence. Answer: Vodka → Caviar. Such rule	
	tends to be interesting	
1c	In a two-class, two-action problem, if the loss function is $\lambda_{11} = \lambda_{22} = 0$,	1
	$\lambda_{12}=10$, and $\lambda_{21}=1$, write the optimal decision rule.	
	We calculate the expected risks of the two actions:	
	$P(C_1 x) = P(C_2 x) = 10P(C_2 x)$	1
	$R(\alpha_1 x) = \lambda_{11}P(C_1 x) + \lambda_{12}P(C_2 x) = 10P(C_2 x)$	1
	$R(\alpha_2 x) = \lambda_{21}P(C_1 x) + \lambda_{22}P(C_2 x) = P(C_1 x)$	
	and we choose C_1 if $R(\alpha_1 x) < R(\alpha_2 x)$, or if $P(C_1 x) > 10P(C_2 x)$,	
	and we choose C_1 if $K(C_1 X) \in K(C_2 X)$, of if $Y(C_1 X) = 10/11$. Assigning accidentally an instance of C_2 to C_1 is so	1
	bad that we need to be very sure before assigning an instance to C_1 .	1
	Dud (Max III)	
2a	Limitations of Single layer perceptron	2
Za	Need for MLP	2
	Non-linearity handling with an example	3
2b	Dimensionality reduction from 2D to 1D	2 2
	Explanation for selecting PC1 Minimum of 2 comparison between ID3 and CART	$\frac{2}{2x^2=4}$
2c	Minimum of 2 comparison between 105 and c. 1111	-/

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	The state of the s	DEPARTMENT OF INF	ORMATION SCIENCE & EN	WINEEKI VO	1
	Tar		CHAITE	Course Code: ISEB1	_
	lerm:	31 st Jan to 22 nd May, 2019		Course Couch	
	-	31 Jan to 22 May, 2019		Semester: 6 A, B, C	
	Course:	March 1 1		Semester: 071, 27	
	The state of the s	Machine Learning		20	1
	CIE Details:	(1)	9.30 to 10.30am	Max Marks: 30	_
- 1	CIE Delane.	Test III 11.05.2010	1 9 40 10 10 30 200		

Test Portions: Lesson #37-56Instructions to Candidates: Answer any two full Qs. Mobiles are banned.

SI.	Question	Marks	Bloom's Level	СО
# 1a	Cluster the dataset = $\{2,3,4,10,11,12,20,25,30\}$ using k-means algorithm. We	5	L3	CO5
	need to group into two clusters. Assume the initial centroids as 2 and 12.	5	L2	CO5
1b	Explain Reinforcement learning cycle with suitable example.	5	L2	CO4
1c	Explain how extensions of SVM is applied to any one of the below scenarios: (i) Multi-class classification			
	(ii) Regression			
2a	Write and explain the SOM Algorithm. Explain with proper example why	5	L2	CO5
2b	does it fall under the category of 'competitive learning' algorithms? suppose that the following are the given positively labeled data points in □2:	6	L3	CO4
	$\left\{ \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 6 \\ 1 \end{pmatrix}, \begin{pmatrix} 6 \\ -1 \end{pmatrix} \right\}$ and the following negatively labeled data points in $\square 2$:			
	$\left\{ \left(\begin{array}{c} 1\\0 \end{array}\right), \left(\begin{array}{c} 0\\1 \end{array}\right), \left(\begin{array}{c} 0\\-1 \end{array}\right), \left(\begin{array}{c} -1\\0 \end{array}\right) \right\}$			
	Obtain a separating hyperplane that accurately discriminates the two			
	alagge using SVM	4	L4	CO4
2c	Compare K-means and Hierarchical clustering			
3a	Explain the following with appropriate examples: (i) Criteria for choosing the Number of Clusters	5	L2	CO5
3b	(ii) Mixture Densities With any example of your choice illustrate the use of k-Nearest Neighbour as a	5	L2	CO4
3c	probabilistic learning algorithm. Explain The Gaussian Mixture Model EM Algorithm steps and list few	5	L2	CO4
30	application of EM			

SOLUTION

1a. Cluster the dataset = {2,3,4,10,11,12, 20, 25,30} using k-means algorithm. We need to group into two clusters. Assume the initial and the

clusters. Assume the init	ial centroids as 2 and 12.	(12) or 12
	Centroid1 = 2	Centroid2 = 12
Iteration 1	{2,3,4}	{10,11,12,20,25,30}
	Centroid $1 = 3$	Centroid2 = 18
Iteration 2	{2,3,4,10}	{11,12,20,25,30}
	Centroid $1 = 4.75 \sim 5$	Centroid2 = 19.6~20
Iteration 3	{2,3,4,10,11,12}	{20,25,30}
	Centroid1 = 7	Centroid2 = 25
Iteration 4	{2,3,4,10,11,12}	{20,25,30}
	Centroid1 = 7	Centroid2 = 25
Iteration 5	{2,3,4,10,11,12}	{20,25,30}

Clusters in Iteration 4 = Clusters in Iteration 5 and centroids for these iteration converges. (5X1=5)

1b. Explain Reinforcement learning cycle with suitable example.

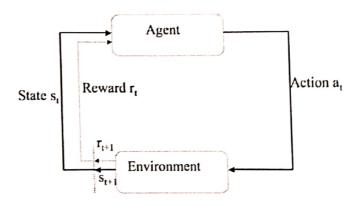


Figure explanation with example

(2+3)

1c. Explain how extensions of SVM is applied to any one of the below scenarios:

(i) Multi-class classification

(2.5)

(ii) Regression

(2.5)

2a. Write and explain the SOM Algorithm. (3)

Initialisation

- choose a size (number of neurons) and number of dimensions d for the map
- either:
 - * choose random values for the weight vectors so that they are all different OR
 - st set the weight values to increase in the direction of the first d principal components of the dataset



- repeat:
 - * for each datapoint:
 - select the best-matching neuron n_b using the minimum Euclidean distance between the weights and the input,

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$$n_b = \min_{j} \|\mathbf{x} - \mathbf{w}_j^T\|.$$
 (14.8)

update the weight vector of the best-matching node using:

$$\mathbf{w}_{i}^{T} \leftarrow \mathbf{w}_{i}^{T} + \eta(t)(\mathbf{x} - \mathbf{w}_{i}^{T}), \tag{14.9}$$

where $\eta(t)$ is the learning rate.

* update the weight vector of all other neurons using:

$$\mathbf{w}_j^T \leftarrow \mathbf{w}_j^T + \eta_n(t)h(n_b, t)(\mathbf{x} - \mathbf{w}_j^T), \tag{14.10}$$

Explain with proper example why does it fall under the category of 'competitive learning' algorithms?

2b. Obtain a separating hyperplane that accurately discriminates the two classes using SVM.

$$\left\{ \left(\begin{array}{c} 3\\1 \end{array}\right), \left(\begin{array}{c} 3\\-1 \end{array}\right), \left(\begin{array}{c} 6\\1 \end{array}\right), \left(\begin{array}{c} 6\\-1 \end{array}\right) \right\} \ \left\{ \left(\begin{array}{c} 1\\0 \end{array}\right), \left(\begin{array}{c} 0\\1 \end{array}\right), \left(\begin{array}{c} 0\\-1 \end{array}\right), \left(\begin{array}{c} -1\\0 \end{array}\right) \right\}_{(1+1+2+1+1)}$$

$$\left\{s_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, s_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, s_3 = \begin{pmatrix} 3 \\ -1 \end{pmatrix}\right\}$$

SV hyperplane representation

SV hyperplane representation
$$\alpha_1 \Phi(s_1) \cdot \Phi(s_1) + \alpha_2 \Phi(s_2) \cdot \Phi(s_1) + \alpha_3 \Phi(s_3) \cdot \Phi(s_1) = -1$$

$$\alpha_1 \Phi(s_1) \cdot \Phi(s_2) + \alpha_2 \Phi(s_2) \cdot \Phi(s_2) + \alpha_3 \Phi(s_3) \cdot \Phi(s_2) = +1$$

$$\alpha_1 \Phi(s_1) \cdot \Phi(s_3) + \alpha_2 \Phi(s_2) \cdot \Phi(s_3) + \alpha_3 \Phi(s_3) \cdot \Phi(s_3) = +1$$

Substitute values for s1, s2 and s3 and solve for α

$$\begin{array}{rcl} 2\alpha_{1}+4\alpha_{2}+4\alpha_{3}&=&-1\\ 4\alpha_{1}+11\alpha_{2}+9\alpha_{3}&=&+1\\ 4\alpha_{1}+9\alpha_{2}+11\alpha_{3}&=&+1\\ &\alpha_{1}=3:5;\,\alpha_{2}=0:75\,\,\mathrm{and}\,\,\alpha_{3}=0:75 \end{array}$$

$$y = wx + b$$
 with $w = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $b = -2$.

2c. Compare K-means and Hierarchical clustering (minimum 2 differences)

3a. Explain the following with appropriate examples: (2x2.5 = 5)

- (i) Criteria for choosing the Number of Clusters
- (ii) Mixture Densities

3b. With any example of your choice illustrate the use of k-Nearest Neighbour as a probabilistic learning algorithm.

Explain working of algorithm with example

(3+2=5)

3c. Explain The Gaussian Mixture Model EM Algorithm steps and list few application of EM(4+1=5) GMM EM algorithm steps

- · Initialisation
 - set $\hat{\mu}_1$ and $\hat{\mu}_2$ to be randomly chosen values from the dataset

– set
$$\hat{\sigma}_1=\hat{\sigma}_2=\sum\limits_{i=1}^N(y_i-\bar{y})^2/N$$
 (where \bar{y} is the mean of the entire dataset)

- $\sec \hat{\pi} = 0.5$
- · Repeat until convergence:

– (E-step)
$$\hat{\gamma}_i = \frac{\hat{\pi}\phi(y_i;\hat{\mu}_1,\hat{\sigma}_1)}{\hat{\pi}\phi(y_i;\hat{\mu}_1,\hat{\sigma}_1) + (1-\hat{\pi})\phi(y_i;\hat{\mu}_2,\hat{\sigma}_2)}$$
 for $i = 1 \dots N$

- (M-step 1)
$$\hat{\mu}_1 = \frac{\sum\limits_{t=1}^{N} (1-\hat{\gamma}_t) y_t}{\sum\limits_{t=1}^{N} (1-\hat{\gamma}_t)}$$

- (M-step 2)
$$\hat{\mu}_2 = \frac{\sum\limits_{i=1}^{N} \hat{\gamma}_i y_i}{\sum\limits_{i=1}^{N} \hat{\gamma}_i}$$

- (M-step 3)
$$\hat{\sigma}_1 = \frac{\sum_{i=1}^{N} (1 - \hat{\gamma}_i) (y_i - \hat{\mu}_1)^2}{\sum_{i=1}^{N} (1 - \hat{\gamma}_i)}$$

- (M-step 4)
$$\tilde{\sigma}_2 = \frac{\sum\limits_{i=1}^N \hat{\gamma}_i (y_i - \hat{\mu}_2)^2}{\sum\limits_{i=1}^N \hat{\gamma}_i}$$

- (M-step 5)
$$\hat{\pi} = \sum_{i=1}^{N} \frac{\hat{\gamma}_i}{N}$$

List any 2 applications of EM