

ISEB1 (Machine Learning) T1 - Answer Scheme

1a) Answer as below (6 marks):

- Correct statement of Conditional Probability Equations : **1 Mark**
- Calculation of each Conditional Probability : **4 Marks**
- Writing the final inference : **1 Mark**

- $P(\text{Party}) \times P(\text{Near} | \text{Party}) \times P(\text{No Party} | \text{Party}) \times P(\text{Lazy} | \text{Party})$
- $P(\text{Study}) \times P(\text{Near} | \text{Study}) \times P(\text{No Party} | \text{Study}) \times P(\text{Lazy} | \text{Study})$
- $P(\text{Pub}) \times P(\text{Near} | \text{Pub}) \times P(\text{No Party} | \text{Pub}) \times P(\text{Lazy} | \text{Pub})$
- $P(\text{TV}) \times P(\text{Near} | \text{TV}) \times P(\text{No Party} | \text{TV}) \times P(\text{Lazy} | \text{TV})$

Using the data above these evaluate to:

$$P(\text{Party} | \text{near (not urgent) deadline, no party, lazy}) = \frac{\frac{5}{10} \times \frac{2}{5} \times \frac{9}{5} \times \frac{3}{5}}{\frac{5}{10} \times \frac{2}{5} \times \frac{9}{5} \times \frac{3}{5} + \frac{3}{10} \times \frac{1}{3} \times \frac{3}{3} \times \frac{1}{3} + \frac{1}{10} \times \frac{0}{1} \times \frac{1}{1} \times \frac{1}{1} + \frac{1}{10} \times \frac{1}{1} \times \frac{1}{1} \times \frac{1}{1}} = \frac{5}{10}$$

$$P(\text{Study} | \text{near (not urgent) deadline, no party, lazy}) = \frac{\frac{3}{10} \times \frac{1}{3} \times \frac{3}{3} \times \frac{1}{3}}{\frac{5}{10} \times \frac{2}{5} \times \frac{9}{5} \times \frac{3}{5} + \frac{3}{10} \times \frac{1}{3} \times \frac{3}{3} \times \frac{1}{3} + \frac{1}{10} \times \frac{0}{1} \times \frac{1}{1} \times \frac{1}{1} + \frac{1}{10} \times \frac{1}{1} \times \frac{1}{1} \times \frac{1}{1}} = \frac{1}{10}$$

$$P(\text{Pub} | \text{near (not urgent) deadline, no party, lazy}) = \frac{\frac{1}{10} \times \frac{0}{1} \times \frac{1}{1} \times \frac{1}{1}}{\frac{5}{10} \times \frac{2}{5} \times \frac{9}{5} \times \frac{3}{5} + \frac{3}{10} \times \frac{1}{3} \times \frac{3}{3} \times \frac{1}{3} + \frac{1}{10} \times \frac{0}{1} \times \frac{1}{1} \times \frac{1}{1} + \frac{1}{10} \times \frac{1}{1} \times \frac{1}{1} \times \frac{1}{1}} = 0$$

$$P(\text{TV} | \text{near (not urgent) deadline, no party, lazy}) = \frac{\frac{1}{10} \times \frac{1}{1} \times \frac{1}{1} \times \frac{1}{1}}{\frac{5}{10} \times \frac{2}{5} \times \frac{9}{5} \times \frac{3}{5} + \frac{3}{10} \times \frac{1}{3} \times \frac{3}{3} \times \frac{1}{3} + \frac{1}{10} \times \frac{0}{1} \times \frac{1}{1} \times \frac{1}{1} + \frac{1}{10} \times \frac{1}{1} \times \frac{1}{1} \times \frac{1}{1}} = \frac{1}{10}$$

So based on this you will be watching TV tonight.

1b) Answer any two (2 x 2 = 4 Marks : Any two valid points of difference)

- ROC and AUC Curves
- Bias and Variance of the ML Model
- Variance and Covariance of the ML Model
- Overfitting and Underfitting of the ML Model

2a) Total: 10 Marks

a) 1 mark

Folder			
Email	Spam Folder	Inbox	
	Spam	Not spam	
Spam	100 True positives	170 False Negatives	
Not spam	30 False Positives	700 True Negatives	

b) 2 x 1 = 2 Marks

$$\text{Accuracy} = \frac{100 + 700}{1000} = 80\%$$

$$\text{Error Rate} = 1 - \text{Accuracy} = 0.2 \text{ or } 20\%$$

c) 2 x 1 = 2 Marks

$$\text{Sensitivity} = \frac{\#TP}{\#TP + \#FN}$$

$$\text{Specificity} = \frac{\#TN}{\#TN + \#FP}$$

$$\text{Precision} = \frac{\#TP}{\#TP + \#FP}$$

$$\text{Recall} = \frac{\#TP}{\#TP + \#FN}$$

- Sensitivity = $100 / (100 + 170) = 0.3703$
- Specificity = $700 / (30 + 700) = 0.9589$

d) 2 x 1 = 2 Marks

2b) Total : 5 Marks

Let us organize the data in a table.

x	y	xy	x ²
-2	-1	2	4
1	1	1	1
3	2	6	9
$\Sigma x = 2$	$\Sigma y = 2$	$\Sigma xy = 9$	$\Sigma x^2 = 14$

- Writing the above values correctly: **1 Mark**
- Calculating the least square regression: **2 Marks**
- Plotting the graph : **1 Mark**

$$\text{Precision} = \frac{100}{100 + 30} = 76.9\%$$

$$\text{Recall} = \frac{100}{100 + 170} = 37\%$$

e) 1 mark

$$\text{F1 Score} = \frac{2 \times 76.9 \times 37}{76.9 + 37} = 49.96\%$$

f) 1 mark

High Precision

Justification: FPR must be as minimum as possible for the Spam Mail Filter system given

g) 1 mark

Imbalanced

Justification: The number of True Positives are very less (Only 100: 10% of the samples) compared to the number of True Negatives (700 : 70% of the samples)

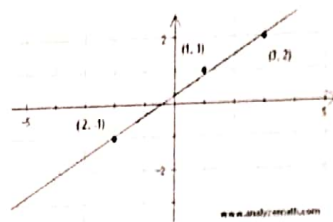
We now use the above formula to calculate a and b as follows

$$a = (n \sum x y - \sum x \sum y) / (n \sum x^2 - (\sum x)^2) = (3 \cdot 9 - 2 \cdot 2) /$$

$$(3 \cdot 14 - 2^2) = 23/38$$

$$b = (1/n)(\sum y - a \sum x) = (1/3)(2 - (23/38) \cdot 2) = 5/19$$

b) We now graph the regression line given by $y = a x + b$ and the given points.



3a)

When TPR / TNR is very low because the number of TP samples are very less, the dataset is imbalanced.

Accuracy fails as a performance metrics because it tends to give a value from the majority sample (TN) which is very high and will fail to detect the minority sample (TP).

3b)

Answering any two below: 3 Marks

- a) Maximum A Posteriori (MAP) Classification Rule
- b) Conditional Probability
- c) Bayesian Rule

Explanation : 1 Mark

One example : 2 Marks

3c)

The Perceptron Algorithm

• Initialisation

- set all of the weights w_{ij} to small (positive and negative) random numbers

• Training

- for T iterations or until all the outputs are correct:
 - for each input vector:
 - compute the activation of each neuron j using activation function g

$$y_j = g \left(\sum_{i=0}^m w_{ij} x_i \right) = \begin{cases} 1 & \text{if } \sum_{i=0}^m w_{ij} x_i > 0 \\ 0 & \text{if } \sum_{i=0}^m w_{ij} x_i \leq 0 \end{cases} \quad (3.4)$$

- update each of the weights individually using:

$$w_{ij} \leftarrow w_{ij} - \eta (y_j - t_j) \cdot x_i \quad (3.5)$$

• Recall

- compute the activation of each neuron j using:

$$y_j = g \left(\sum_{i=0}^m w_{ij} x_i \right) = \begin{cases} 1 & \text{if } w_{ij} x_i > 0 \\ 0 & \text{if } w_{ij} x_i \leq 0 \end{cases} \quad (3.6)$$

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DEPARTMENT OF INFORMATION SCIENCE & ENGINEERING

Term:	31 st Jan to 22 nd May, 2019	Course Code:	ISEB1
Course:	Machine Learning	Semester:	6 A, B, C
CIE Details:	Test – II	10-04-2019	9.30 to 10.30am
		Max Marks:	30

Test Portions: Lesson #14-36 **Instructions to Candidates:** Answer any two full Qs. Mobiles are banned.

Sl. #	Question	Marks	Bloom's Level	CO																		
1a	Apply MLP to train the neural network to output 0.01 and 0.99 for given inputs 0.05 and 0.10 with bias values 1 by using weights as $w_1=0.15$, $w_2=0.20$, $w_3=0.25$, $w_4=0.30$, $b_1=0.35$, $w_5=0.40$, $w_6=0.45$, $w_7=0.50$, $w_8=0.55$ and $b_2=0.60$.	7	L3	CO2																		
1b	Show example transaction data where for the rule $X \rightarrow Y$: (a) Both support and confidence are high. (b) Support is high and confidence is low. (c) Support is low and confidence is high. (d) Both support and confidence are low.	4	L3	CO3																		
1c	In a two-class, two-action problem, if the loss function is $\lambda_{11} = \lambda_{22} = 0$, $\lambda_{12} = 10$, and $\lambda_{21} = 1$, write the optimal decision rule.	4	L3	CO4																		
2a	What are the limitations of Single Layer Perceptrons? Why do we need MLP? Explain with an example how multilayer perceptron handles nonlinearity.	7	L3	CO2																		
2b	In PCA, obtain the Eigenvalues for the feature vector and covariance matrix given below. Which 'Eigenvalue' will you consider for PC1 and why? <table><tr><th>Original Feature Vector</th><th>Covariance Matrix</th></tr><tr><td><table><tr><td>x1</td><td>x2</td></tr><tr><td>2</td><td>4</td></tr><tr><td>1</td><td>3</td></tr><tr><td>0</td><td>1</td></tr><tr><td>-1</td><td>0.5</td></tr></table></td><td><table><tr><td>1.667</td><td>2.08</td></tr><tr><td>2.08</td><td>2.727</td></tr></table></td></tr></table>	Original Feature Vector	Covariance Matrix	<table><tr><td>x1</td><td>x2</td></tr><tr><td>2</td><td>4</td></tr><tr><td>1</td><td>3</td></tr><tr><td>0</td><td>1</td></tr><tr><td>-1</td><td>0.5</td></tr></table>	x1	x2	2	4	1	3	0	1	-1	0.5	<table><tr><td>1.667</td><td>2.08</td></tr><tr><td>2.08</td><td>2.727</td></tr></table>	1.667	2.08	2.08	2.727	4	L3	CO3
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2	4																					
1	3																					
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2c	Compare the approaches of ID3 and CART	4	L4	CO4																		
3a	Write the Backpropagation Algorithm in MLP. Explain how MLP overcomes the limitations of single layered perceptron.	5	L3	CO2																		
3b	Answer any two of the following questions: (i) In LDA what is Fischer Ratio? What is its significance? (ii) Two differences between PCA and ICA (iii) Probably Approximately Correct (PAC) learning	6	L2	CO3																		
3c	Write a brief note on Classification and Regression Trees.	4	L2	CO4																		

IA test 2
Machine Learning (ISEB1) - Scheme

Q. no		Marks
1a	<p style="text-align: center;">Solution</p> <p style="text-align: center;"> 1 $b1 .35$ 1 $b2 .60$ </p> <p>The Forward Pass</p> <ol style="list-style-type: none"> Evaluate activation function (one sample for each layer) Calculate the total error for the sample considered in step i <p>The Backward Pass</p> <ol style="list-style-type: none"> Update the weights at output layer Update the weights at hidden layer 	<p style="text-align: center;">2</p> <p style="text-align: center;">2</p> <p style="text-align: center;">3</p>
1b	<p>Example:</p> <p>(a) A rule that has high support and high confidence. Answer: Milk \rightarrow Bread. Such obvious rule tends to be uninteresting.</p> <p>(b) A rule that has reasonably high support but low confidence. Answer: Milk \rightarrow Tuna. While the sale of tuna and milk may be higher than the support threshold, not all transactions that contain milk also contain tuna. Such low-confidence rule tends to be uninteresting.</p> <p>(c) A rule that has low support and low confidence. Answer: Cooking oil \rightarrow Laundry detergent. Such low confidence rule tends to be uninteresting.</p> <p>(d) A rule that has low support and high confidence. Answer: Vodka \rightarrow Caviar. Such rule tends to be interesting</p>	<p style="text-align: center;">$4 \times 1 = 4$</p>
1c	<p><i>In a two-class, two-action problem, if the loss function is $\lambda_{11} = \lambda_{22} = 0$, $\lambda_{12} = 10$, and $\lambda_{21} = 1$, write the optimal decision rule.</i></p> <p>We calculate the expected risks of the two actions:</p> $R(\alpha_1 x) = \lambda_{11}P(C_1 x) + \lambda_{12}P(C_2 x) = 10P(C_2 x)$ $R(\alpha_2 x) = \lambda_{21}P(C_1 x) + \lambda_{22}P(C_2 x) = P(C_1 x)$ <p>and we choose C_1 if $R(\alpha_1 x) < R(\alpha_2 x)$, or if $P(C_1 x) > 10P(C_2 x)$, $P(C_1 x) > 10/11$. Assigning accidentally an instance of C_2 to C_1 is so bad that we need to be very sure before assigning an instance to C_1.</p>	<p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p>
2a	<p>Limitations of Single layer perceptron</p> <p>Need for MLP</p> <p>Non-linearity handling with an example</p>	<p style="text-align: center;">2</p> <p style="text-align: center;">2</p> <p style="text-align: center;">3</p>
2b	<p>Dimensionality reduction from 2D to 1D</p> <p>Explanation for selecting PC1</p>	<p style="text-align: center;">2</p> <p style="text-align: center;">2</p>
2c	Minimum of 2 comparison between ID3 and CART	$2 \times 2 = 4$

3a

MLP with backpropagation

4

- Initialisation

- initialise all weights to small (positive and negative) random values

- Training

- repeat:

- * for each input vector:

Forwards phase:

- compute the activation of each neuron j in the hidden layer(s) using:

$$h_{\zeta} = \sum_{i=0}^L x_i v_{i\zeta}$$

$$a_{\zeta} = g(h_{\zeta}) = \frac{1}{1 + \exp(-\beta h_{\zeta})}$$

- work through the network until you get to the output layer neurons which have activations (although see also Section 4.2.3):

$$h_{\kappa} = \sum_j a_j w_{j\kappa}$$

$$y_{\kappa} = g(h_{\kappa}) = \frac{1}{1 + \exp(-\beta h_{\kappa})}$$

Backwards phase:

- compute the error at the output using:

$$\delta_o(\kappa) = (y_{\kappa} - t_{\kappa}) y_{\kappa} (1 - y_{\kappa})$$

- compute the error in the hidden layer(s) using:

$$\delta_h(\zeta) = a_{\zeta} (1 - a_{\zeta}) \sum_{k=1}^N w_{\zeta k} \delta_o(k)$$

- update the output layer weights using:

$$w_{\zeta\kappa} \leftarrow w_{\zeta\kappa} - \eta \delta_o(\kappa) a_{\zeta}^{\text{hidden}}$$

- update the hidden layer weights using:

$$v_i \leftarrow v_i - \eta \delta_h(\zeta) x_i$$

(if using sequential updating) randomise the order of the input vectors so that you don't train in exactly the same order each iteration

- Recall

- use the Forwards phase in the training section above

MLP overcomes the limitations of single layer perceptron

1

3b

i. Fisher ratio:

$$J(\mathbf{w}) = \frac{\mathbf{w}^T S_B \mathbf{w}}{\mathbf{w}^T S_W \mathbf{w}}$$

Explain the significance

- ii. Minimum of 2 differences between LDA and PCA with example for each
- iii. Explain PAC learning algorithm steps

2
2+1
3

3c

Explain CART with an example

4

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Term:	31 st Jan to 22 nd May, 2019	Course Code:	ISEB1
Course:	Machine Learning	Semester:	6 A, B, C
CIE Details:	Test - III	11-05-2019	9.30 to 10.30am
		Max Marks:	30

Test Portions: Lesson #37-56 **Instructions to Candidates:** Answer any two full Qs. Mobiles are banned.

Sl. #	Question	Marks	Bloom's Level	CO
1a	Cluster the dataset = {2,3,4,10,11,12, 20, 25,30} using k-means algorithm. We need to group into two clusters. Assume the initial centroids as 2 and 12.	5	L3	CO5
1b	Explain Reinforcement learning cycle with suitable example.	5	L2	CO5
1c	Explain how extensions of SVM is applied to any one of the below scenarios: (i) Multi-class classification (ii) Regression	5	L2	CO4
2a	Write and explain the SOM Algorithm. Explain with proper example why does it fall under the category of ' <i>competitive learning</i> ' algorithms?	5	L2	CO5
2b	suppose that the following are the given positively labeled data points in $\square 2$: $\left\{ \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 6 \\ 1 \end{pmatrix}, \begin{pmatrix} 6 \\ -1 \end{pmatrix} \right\}$ and the following negatively labeled data points in $\square 2$: $\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right\}$ Obtain a separating hyperplane that accurately discriminates the two classes using SVM.	6	L3	CO4
2c	Compare K-means and Hierarchical clustering	4	L4	CO4
3a	Explain the following with appropriate examples: (i) Criteria for choosing the Number of Clusters (ii) Mixture Densities	5	L2	CO5
3b	With any example of your choice illustrate the use of k-Nearest Neighbour as a probabilistic learning algorithm.	5	L2	CO4
3c	Explain The Gaussian Mixture Model EM Algorithm steps and list few application of EM	5	L2	CO4

SOLUTION

1a. Cluster the dataset = {2,3,4,10,11,12, 20, 25,30} using k-means algorithm. We need to group into two clusters. Assume the initial centroids as 2 and 12.

	Centroid1 = 2	Centroid2 = 12
Iteration 1	{2,3,4}	{10,11,12,20,25,30}
	Centroid1 = 3	Centroid2 = 18
Iteration 2	{2,3,4,10}	{11,12,20,25,30}
	Centroid1 = 4.75~5	Centroid2 = 19.6~20
Iteration 3	{2,3,4,10,11,12}	{20,25,30}
	Centroid1 = 7	Centroid2 = 25
Iteration 4	{2,3,4,10,11,12}	{20,25,30}
	Centroid1 = 7	Centroid2 = 25
Iteration 5	{2,3,4,10,11,12}	{20,25,30}

Clusters in Iteration 4 = Clusters in Iteration 5 and centroids for these iteration converges. (5X1=5)

1b. Explain Reinforcement learning cycle with suitable example.

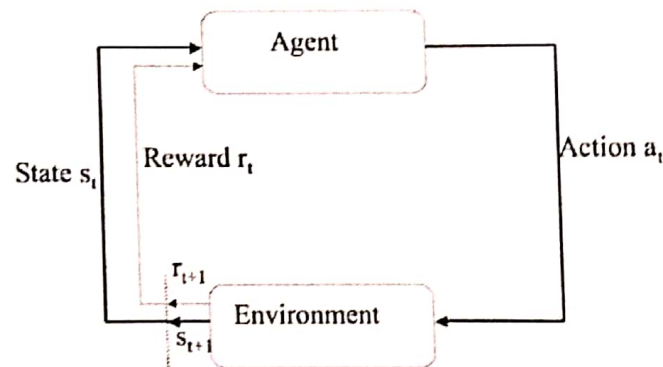


Figure explanation with example (2+3)

1c. Explain how extensions of SVM is applied to any one of the below scenarios:

- (i) Multi-class classification (2.5)
- (ii) Regression (2.5)

2a. Write and explain the SOM Algorithm. (3)

• Initialisation

- choose a size (number of neurons) and number of dimensions d for the map
- either:
 - * choose random values for the weight vectors so that they are all different OR
 - * set the weight values to increase in the direction of the first d principal components of the dataset

- Learning

- repeat:

- * for each datapoint:

- select the best-matching neuron n_b using the minimum Euclidean distance between the weights and the input,

$$n_b = \min_j \|x - w_j^T\|. \quad (14.8)$$

- * update the weight vector of the best-matching node using:

$$w_j^T \leftarrow w_j^T + \eta(t)(x - w_j^T), \quad (14.9)$$

where $\eta(t)$ is the learning rate.

- * update the weight vector of all other neurons using:

$$w_j^T \leftarrow w_j^T + \eta_n(t)h(n_b, t)(x - w_j^T), \quad (14.10)$$

Explain with proper example why does it fall under the category of 'competitive learning' algorithms?

(2)

2b. Obtain a separating hyperplane that accurately discriminates the two classes using SVM.

$$\left\{ \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 6 \\ 1 \end{pmatrix}, \begin{pmatrix} 6 \\ -1 \end{pmatrix} \right\} \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right\} \quad (1+1+2+1+1)$$

Soln:

$$\left\{ s_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, s_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, s_3 = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \right\}$$

SV hyperplane representation

$$\begin{aligned} \alpha_1 \Phi(s_1) \cdot \Phi(s_1) + \alpha_2 \Phi(s_2) \cdot \Phi(s_1) + \alpha_3 \Phi(s_3) \cdot \Phi(s_1) &= -1 \\ \alpha_1 \Phi(s_1) \cdot \Phi(s_2) + \alpha_2 \Phi(s_2) \cdot \Phi(s_2) + \alpha_3 \Phi(s_3) \cdot \Phi(s_2) &= +1 \\ \alpha_1 \Phi(s_1) \cdot \Phi(s_3) + \alpha_2 \Phi(s_2) \cdot \Phi(s_3) + \alpha_3 \Phi(s_3) \cdot \Phi(s_3) &= +1 \end{aligned}$$

Substitute values for s_1, s_2 and s_3 and solve for α

$$\begin{aligned} 2\alpha_1 + 4\alpha_2 + 4\alpha_3 &= -1 \\ 4\alpha_1 + 11\alpha_2 + 9\alpha_3 &= +1 \\ 4\alpha_1 + 9\alpha_2 + 11\alpha_3 &= +1 \end{aligned} \quad \alpha_1=3.5; \alpha_2=0.75 \text{ and } \alpha_3=0.75$$

$$y = wx + b \text{ with } w = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } b = -2.$$

2c. Compare K-means and Hierarchical clustering (minimum 2 differences)

3a. Explain the following with appropriate examples: (2x2.5 = 5)

- (i) Criteria for choosing the Number of Clusters
- (ii) Mixture Densities

3b. With any example of your choice illustrate the use of k-Nearest Neighbour as a probabilistic learning algorithm.

Explain working of algorithm with example (3+2=5)

3c. Explain The Gaussian Mixture Model EM Algorithm steps and list few application of EM(4+1=5)
GMM EM algorithm steps

- Initialisation

- set $\hat{\mu}_1$ and $\hat{\mu}_2$ to be randomly chosen values from the dataset
- set $\hat{\sigma}_1 = \hat{\sigma}_2 = \frac{1}{N} \sum_{i=1}^N (y_i - \bar{y})^2$ (where \bar{y} is the mean of the entire dataset)
- set $\hat{\pi} = 0.5$

- Repeat until convergence:

- (E-step) $\hat{\gamma}_i = \frac{\hat{\pi} \phi(y_i; \hat{\mu}_1, \hat{\sigma}_1)}{\hat{\pi} \phi(y_i; \hat{\mu}_1, \hat{\sigma}_1) + (1 - \hat{\pi}) \phi(y_i; \hat{\mu}_2, \hat{\sigma}_2)}$ for $i = 1 \dots N$

- (M-step 1) $\hat{\mu}_1 = \frac{\sum_{i=1}^N (1 - \hat{\gamma}_i) y_i}{\sum_{i=1}^N (1 - \hat{\gamma}_i)}$

- (M-step 2) $\hat{\mu}_2 = \frac{\sum_{i=1}^N \hat{\gamma}_i y_i}{\sum_{i=1}^N \hat{\gamma}_i}$

- (M-step 3) $\hat{\sigma}_1 = \frac{\sum_{i=1}^N (1 - \hat{\gamma}_i) (y_i - \hat{\mu}_1)^2}{\sum_{i=1}^N (1 - \hat{\gamma}_i)}$

- (M-step 4) $\hat{\sigma}_2 = \frac{\sum_{i=1}^N \hat{\gamma}_i (y_i - \hat{\mu}_2)^2}{\sum_{i=1}^N \hat{\gamma}_i}$

- (M-step 5) $\hat{\pi} = \frac{\sum_{i=1}^N \hat{\gamma}_i}{N}$

List any 2 applications of EM