F-Measures

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For: Machine Learning Elective Class
Target Audience: Sem 6 Students
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Performance Measures

Classification:

- Simple Accuracy
- Precision
- Recall
- F-beta measure
- ROC (and AUC)

Regression:

- Sum of Squares Error
- RMS Error
- Mean Absolute Error

Accuracy as a Performance Measure

- What is 95% accuracy?
  - Classification: 95 / 100 shoes correctly classified
  - Regression:Predict 95/100 house prices correctly

$600,000

$400,000

$599,999

Limitations of Simple Accuracy

$$Accuracy = \frac{No.Samples\ Predicted\ Correctly}{Total\ No.of\ Samples}$$

What is wrong with this ?

9,990 Non-Nike

10 Nike

```
def classifier(shoe):
    return False
```

$$Accuracy = \frac{9,990}{10,000} = 99.9\%$$

Limitation with Accuracy

Is this tumor cancerous?

very few positive examples

most are negative examples

Class Imbalance Problem

	Diagnosed Sick	Diagnosed Healthy
Sick	True positive	False Negative
Healthy	False Positive	True Negative

$$\text{Accuracy} = \frac{1{,}000 + 8{,}000}{10{,}000} = 90\%$$

Confusion Matrix

10,000
Patients

(Predicted) Diagnosis

(Actual) Patients

	Diagnosed sick	Diagnosed Healthy
Sick	1000 True positives	200 False Negatives
Healthy	800 False Positives	8000 True Negatives

	Sent to Spam Folder	Sent to Inbox
Spam	True Positives 	False Negatives 
Not Spam	False Positives 	True Negatives 

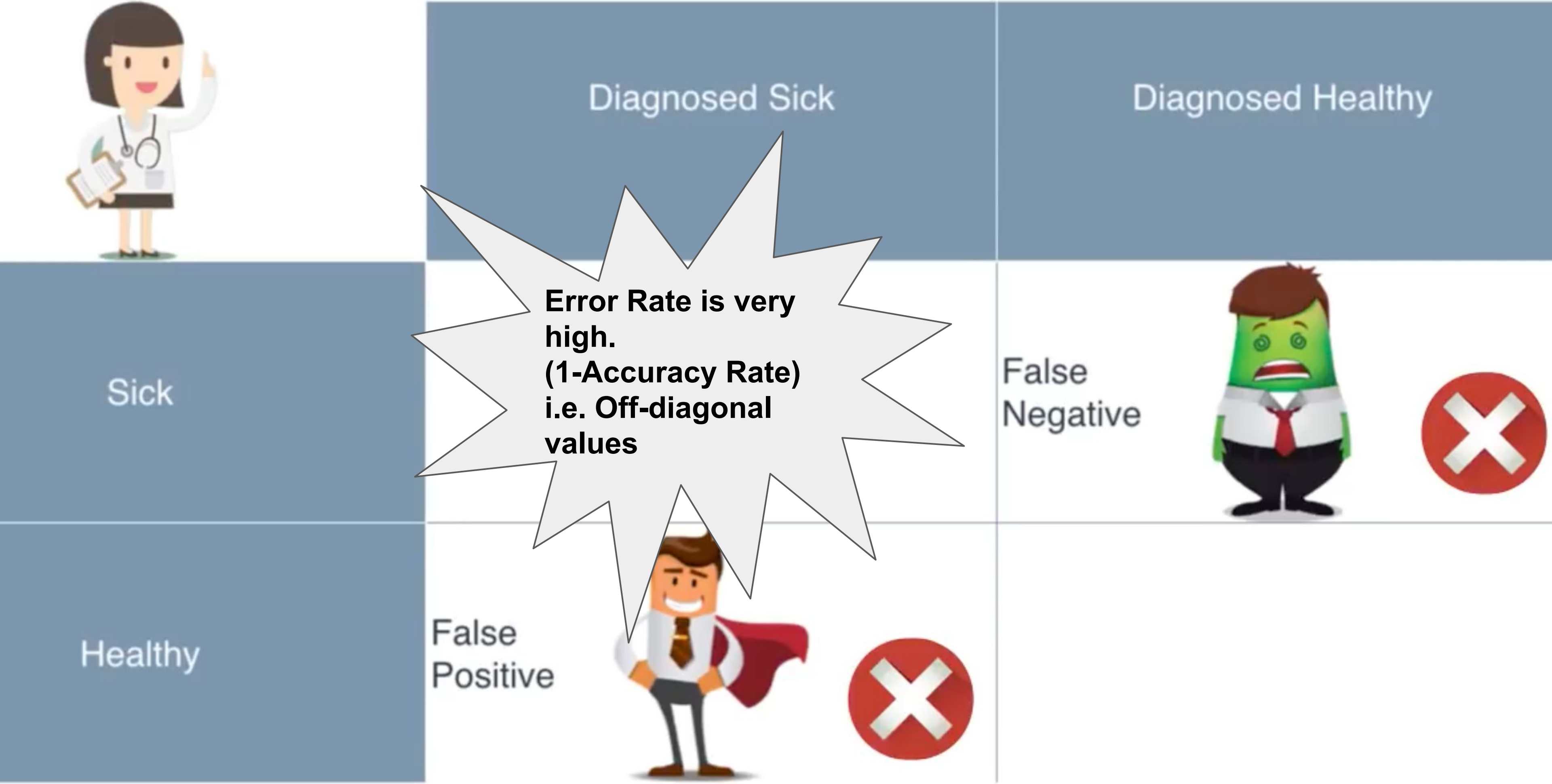
$$\text{Accuracy} = \frac{100 + 700}{1000} = 80\%$$

Confusion Matrix

1,000
e-mails

(Predicted) Folder

E-mail (Actual)	Spam Folder	Inbox
Spam	100 True positives	170 False Negatives
Not spam	30 False Positives	700 True Negatives



Diagnosed Sick
Diagnosed Healthy
Sick
Healthy
False Negative
False Positive
Error Rate is very high.
(1-Accuracy Rate)
i.e. Off-diagonal values

	Sent to Spam Folder	Sent to Inbox
Spam		False Negatives 
Not Spam	False Positives 	

Error Rate is very high.
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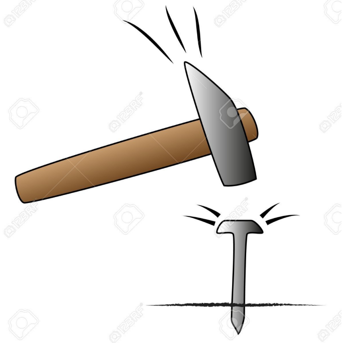


- Simple Accuracy is excellent when we have a Balanced Data Set

- It fails when the Dataset is "Imbalanced".

Precision and Recall as Performance Measure

EVALUATION METRICS

	p' (Predicted)	n' (Predicted)
p (Actual)	True Positive	False Negative
n (Actual)	False Positive	True Negative

Medical Model

False positives ok
False negatives **NOT** ok

Find all the sick people
Ok if not all are sick

	p' (Predicted)	n' (Predicted)
p (Actual)	True Positive	False Negative
n (Actual)	False Positive	True Negative

Spam Detector

False positives **NOT** ok
False negatives ok

You don't necessarily need to find all spam
But they better all be spam

High Recall Model

High Precision Model

Precision

Folder

E-mail		Spam Folder	Inbox
	Spam	100	170
	Not spam	30	700

Precision: Out of the all the e-mails, sent to the spam inbox, how many were actually spam?

$$\text{Precision} = \frac{100}{100 + 30} = 76.9\%$$

$$\text{Precision} = \frac{\text{True positives}}{\text{True positives} + \text{False Positives}}$$

Recall

Folder

E-mail	Spam Folder	Inbox
Spam	100	170
Not spam	30	700

Recall: Out of the all the spam e-mails, how many were correctly sent to the spam folder?

$$\text{Recall} = \frac{100}{100 + 170} = 37\%$$

$$\text{Recall} = \frac{\text{True positives}}{\text{True positives} + \text{False Negatives}}$$

Precision

Diagnosis

Patients

	Diagnosed sick	Diagnosed Healthy
Sick	1000	200
Healthy	800	8000

Precision: Out of the patients we diagnosed with an illness, how many did we classify correctly?

$$\text{Precision} = \frac{1{,}000}{1{,}000 + 800} = 55.7\%$$

$$\text{Precision} = \frac{\text{True positives}}{\text{True positives} + \text{False Positives}}$$

Recall

Diagnosis

Patients		Diagnosed Sick	Diagnosed Healthy
	Sick	1000	200
	Is Healthy	800	8000

Recall: Out of the sick patients, how many did we correctly diagnose as sick?

$$\text{Recall} = \frac{1{,}000}{1{,}000 + 200} = 83.3\%$$

$$\text{Recall} = \frac{\text{True positives}}{\text{True positives} + \text{False Negatives}}$$

Precision and Recall

Medical Model

Precision: 55.7%
Recall: 83.3%

Spam Detector

Precision: 76.9%
Recall: 37%

F-Measures as Performance Measure

- Used on imbalanced datasets
- Harmonic Mean of Precision & Recall
- Used because simple mean fails

Measuring Machine Learning Models : F1 Score

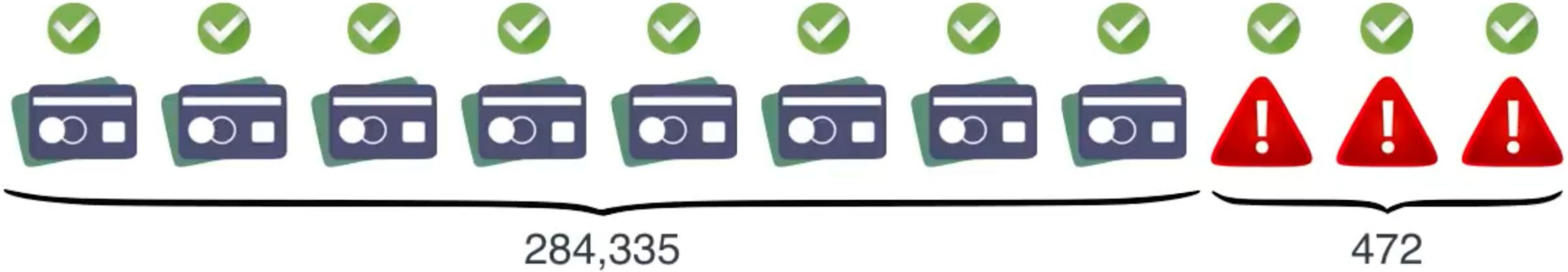
F-Measure

Precision + Recall

$$F_1 = 2 * \frac{\text{Precision} * \text{Recall}}{\text{Precision} + \text{Recall}}$$

- F_1 : evenly weighted
- F_2 : weights Recall more
- $F_{0.5}$: weights Precision more

Credit Card Fraud



Model: All transactions are good.

Precision = 100%

$$\text{Recall} = \frac{0}{472} = 0\%$$

Average = 50%

Credit Card Fraud

Model: All transactions are fraudulent.

$$\text{Precision} = \frac{472}{284{,}807} = .016\%$$

$$\text{Recall} = \frac{472}{472} = 100\%$$

Average = 50.008%

Harmonic mean

Y: Recall

y

$$\text{Arithmetic Mean} = \frac{x+y}{2}$$

$$\text{Harmonic Mean} = \frac{2xy}{x+y}$$

x

X: Precision

Precision = 1
Recall = 0
Average = 0.5
Harmonic Mean = 0

Precision = 0.2
Recall = 0.8
Average = 0.5
Harmonic Mean = 0.32

~~Arithmetic Mean(Precision, Recall)~~

F1 Score = Harmonic Mean(Precision, Recall)

F1 Score

Medical Model

Precision = 55.7%

Recall = 83.3%

Average = 69.5%

$$\text{F1 Score} = \frac{2 \times 55.7 \times 83.3}{55.7 + 83.3} = 66.76\%$$

$$F_1 = 2 * \frac{\text{Precision} * \text{Recall}}{\text{Precision} + \text{Recall}}$$

F1 Score

Spam Detector
Model

Precision = 76.9%

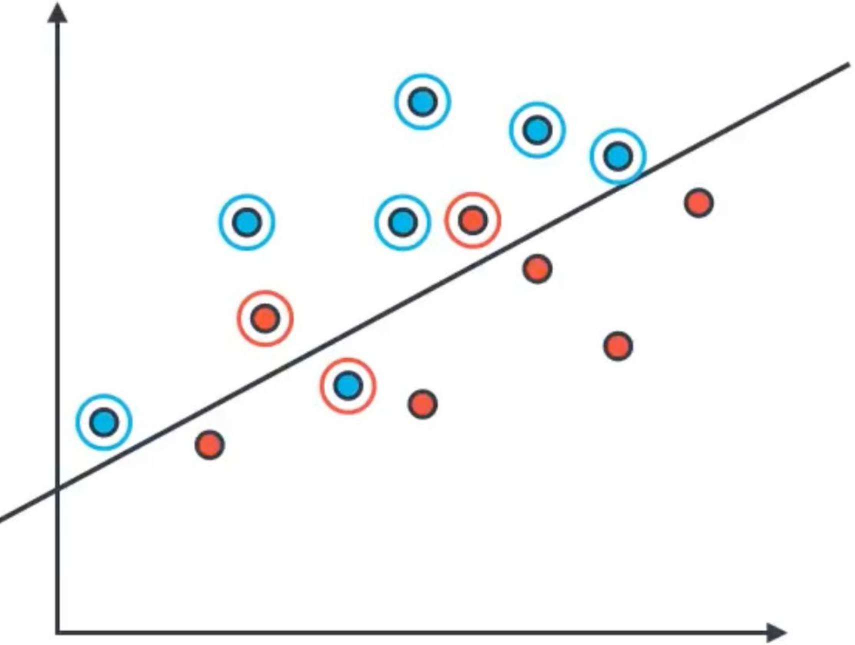
Recall = 37%

Average = 56.95%

$$\text{F1 Score} = \frac{2 \times 76.9 \times 37}{76.9 + 37} = 49.96\%$$

$$F_1 = 2 * \frac{\text{Precision} * \text{Recall}}{\text{Precision} + \text{Recall}}$$

F1 Score



Precision = 75%

Recall = 85.7%

Average = 80.35

$$\text{F1 Score} = \frac{2 \times 75 \times 85.7}{75 + 85.7} = 80\%$$

F_β Score

Precision **$F_{0.5}$ Score** **F_1 Score** **F_2 Score** **Recall**

Comparing Systems

System 1

- Precision: 70%
- Recall: 60%

?

System 2

- Precision: 80%
- Recall: 50%

$$F_\beta = \frac{1}{\beta \times \frac{1}{Precision} + (1-\beta) \times \frac{1}{Recall}}$$

- Greater β, Greater importance to Precision

Comparing Systems

System 1

- Precision: 70%
- Recall: 60%

?

System 2

- Precision: 80%
- Recall: 50%

$$F_\beta = \frac{1}{\beta \times \frac{1}{Precision} + (1-\beta) \times \frac{1}{Recall}}$$

$\beta = 0.95$

0.6942 < 0.7766

$\beta = 0.5$

$$F_\beta = \frac{1}{0.5 \times \frac{1}{0.7} + (1-0.5) \times \frac{1}{0.6}} = 0.6461$$

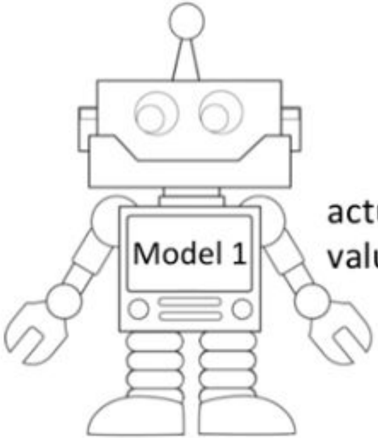
>

$\beta = 0.5$

$$F_\beta = \frac{1}{0.5 \times \frac{1}{0.8} + (1-0.5) \times \frac{1}{0.5}} = 0.6153$$

F-Measure

F1 Score on imbalanced data

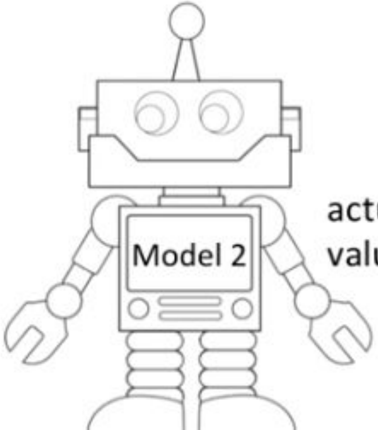


predictions →

actual values ↓

	A	B	C	D
A	100	80	10	10
B	0	9	0	1
C	0	1	8	1
D	0	1	0	9

F1 Score = **0.601** accuracy = 0.547



predictions →

actual values ↓

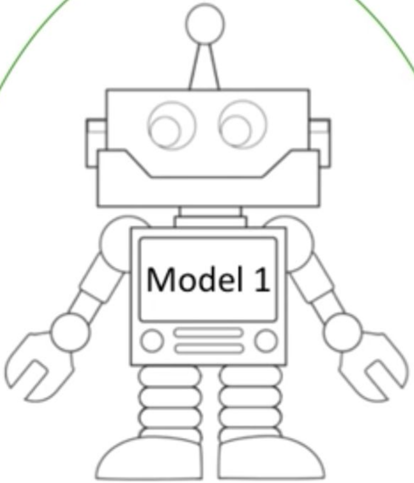
	A	B	C	D
A	198	2	0	0
B	7	1	0	2
C	0	8	1	1
D	2	3	4	1

F1 Score = **0.342** accuracy = 0.87

F1 Score on imbalanced data

Which to choose ?

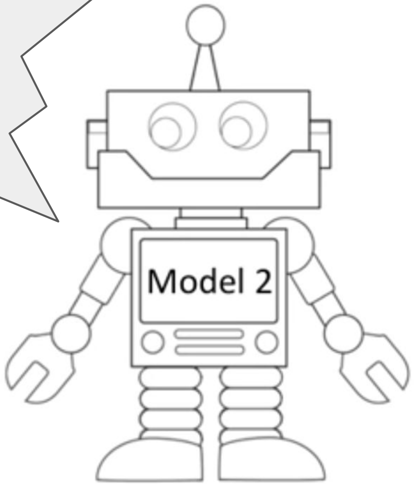
Model 1 could be KNN



F1 Score = **0.601**

accuracy = 0.547

Model 2 could be SVM



F1 Score = **0.342**

accuracy = 0.87

Model 1 predicts well on multiple class classification on imbalanced given data, and F1 score is the metric to quantify its performance.

QUIZ

In each of the following scenarios which choice of F_1, $F_{0.5}$ or F_2 be the best choice of metric.

#1: *FPR* must be reduced - *Precision* must be high
$F_ß$ where ß must be high. **So F_2**

1. Cancer Detection: If someone is falsely diagnose we may do some extra tests. If someone who actually has cancer is not diagnosed they may die.

2. Convicting to Prison: People are innocent until proven guilty by US Law. We want to avoid false convictions. But we also want criminals to not run free.

#2: *FNR* must be reduced - *Recall* must be high
$F_ß$ where ß must be low. **So $F_{0.5}$**