Lecture 10

Energy of a System

CHAPTER

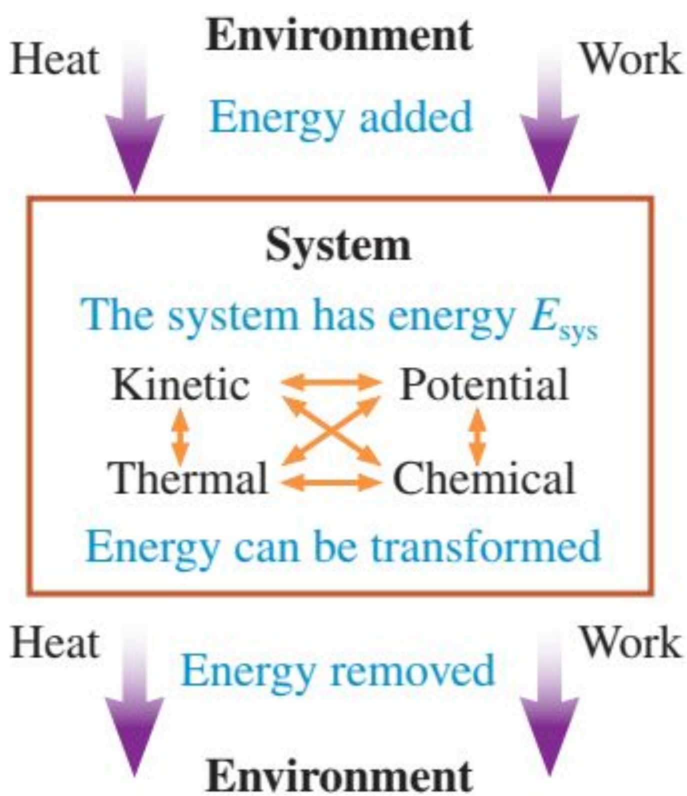
7

Lecture-10 Outline

- System and Environment
- Work done by constant Force
- Work done by Varying Force
- Work-Kinetic Energy Theorem
- Gravitational Potential Energy
- Conservation of Energy
- Examples

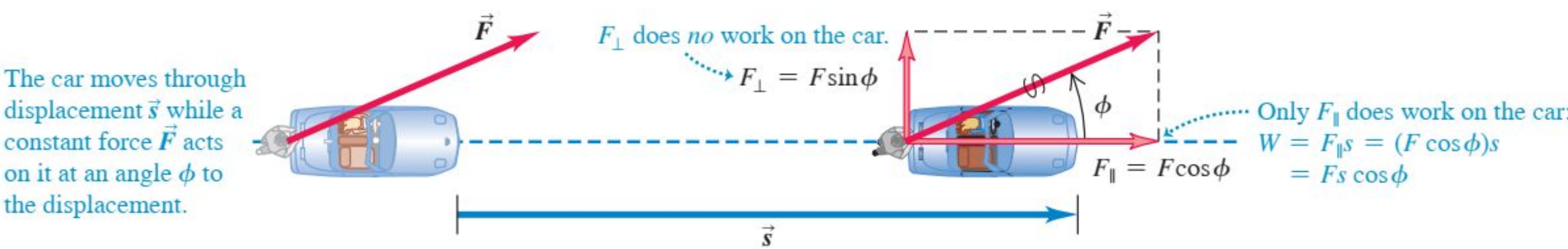
7.1 Systems and Environments

- may be a single object or particle
- may be a collection of objects or particles
- may be a region of space (such as the interior of an automobile engine combustion cylinder)
- may vary with time in size and shape (such as a rubber ball, which deforms upon striking a wall)



7.2 Work Done by a Constant Force

Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.



Work done on a particle by **constant force $\vec{F}$** during **straight-line displacement $\vec{s}$**

$$W = Fs\cos\phi$$

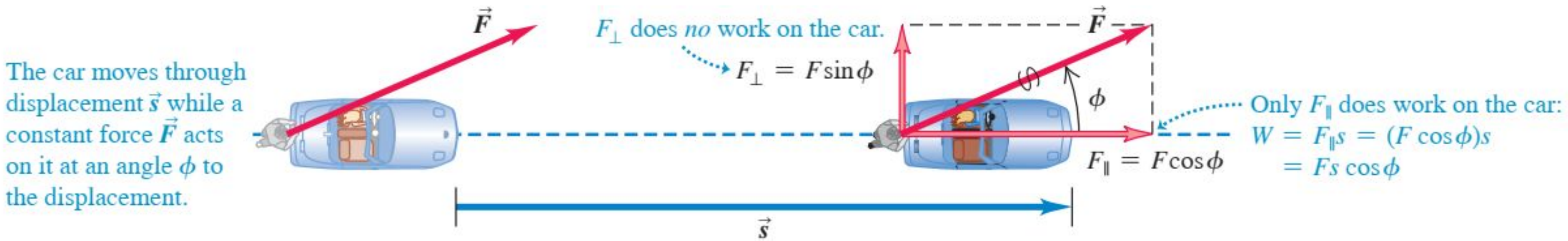
Magnitude of $\vec{F}$; Magnitude of $\vec{s}$; Angle between $\vec{F}$ and $\vec{s}$

Work done on a particle by **constant force $\vec{F}$** during **straight-line displacement $\vec{s}$**

$$W = \underbrace{\vec{F}\cdot\vec{s}}$$

Scalar product (dot product) of vectors $\vec{F}$ and $\vec{s}$

To calculate the work a force does on an object as the object moves through some displacement, we use only the force component along the object's displacement. The force component perpendicular to the displacement does zero work.



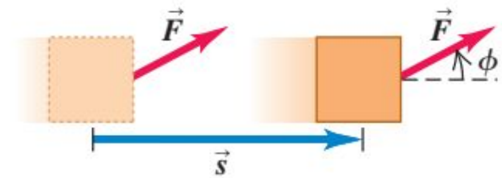
The unit of work, that of force multiplied by distance, is the N m. Recall that $1\ \mathrm{N} = 1\ \mathrm{kg\,m/s^2}$. Thus

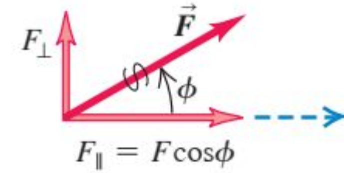
$$1\ \mathrm{N\,m} = 1\ (\mathrm{kg\,m/s^2})\,\mathrm{m} = 1\ \mathrm{kg\,m^2/s^2} = 1\ \mathrm{J}$$

Work: Positive, Negative, or Zero

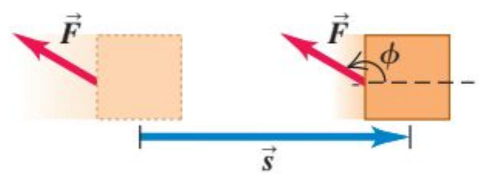
Direction of Force (or Force Component) | **Situation** | **Force Diagram**

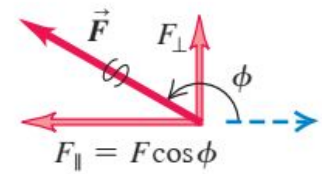
(a) **Force $\vec{F}$ has a component in direction of displacement:**
$W = F_{\parallel}s = (F\cos\phi)s$
Work is *positive*.



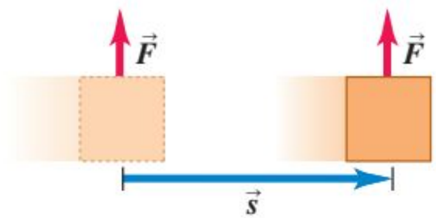


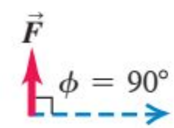
(b) **Force $\vec{F}$ has a component opposite to direction of displacement:**
$W = F_{\parallel}s = (F\cos\phi)s$
Work is *negative* (because $F\cos\phi$ is negative for $90° < \phi < 180°$).

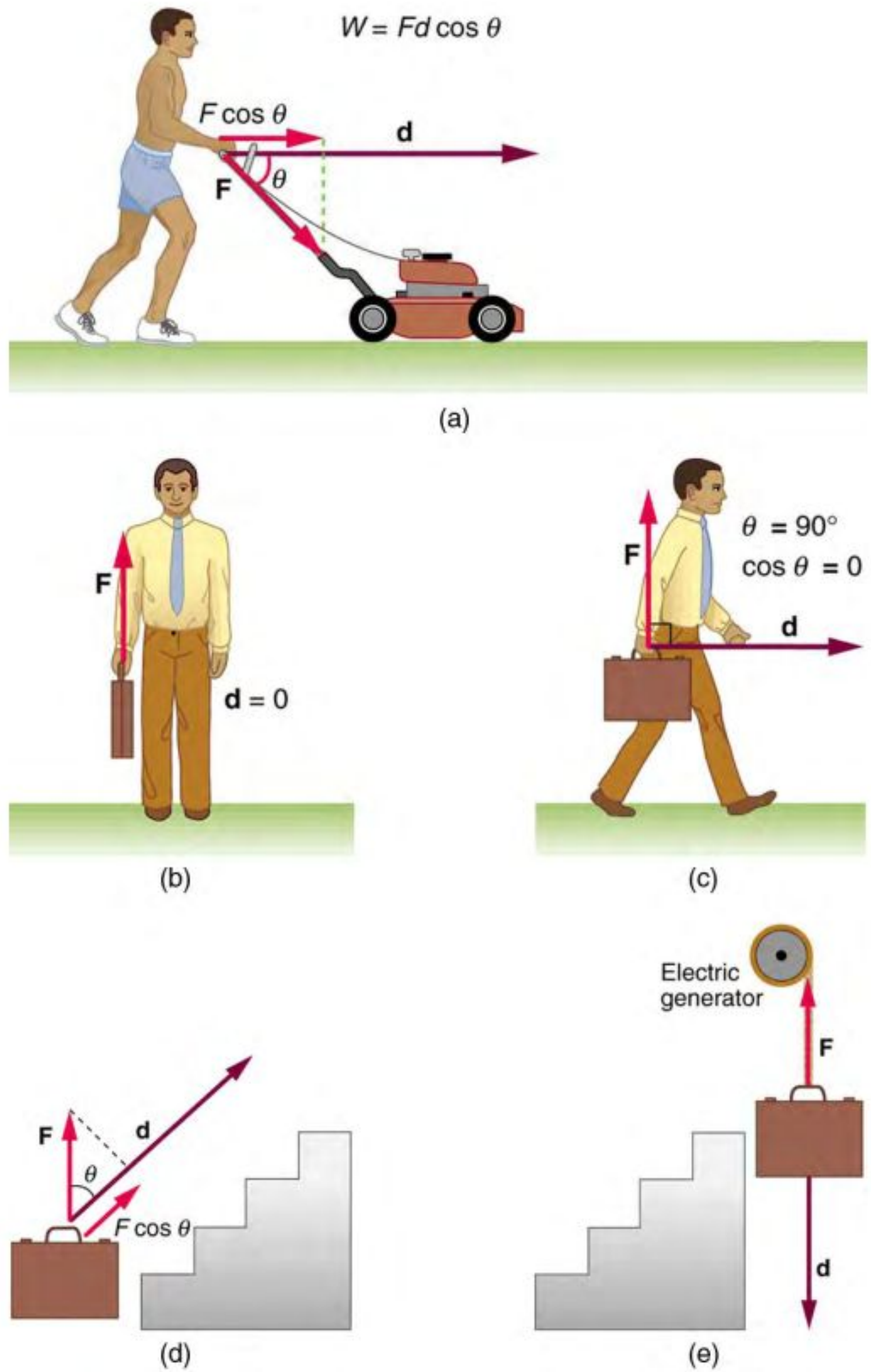




(c) **Force $\vec{F}$ (or force component $F_{\perp}$) is perpendicular to direction of displacement:** The force (or force component) does *no* work on the object.



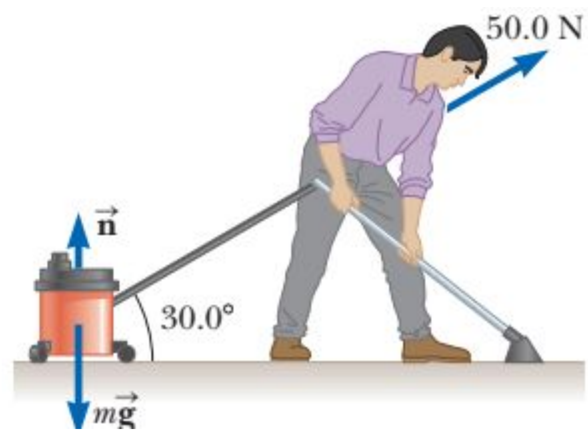




W = Fd cos θ
F cos θ
d
θ
F
(a)
F
d = 0
(b)
θ = 90°
cos θ = 0
F
d
(c)
F
d
θ
F cos θ
(d)
Electric
generator
F
d
(e)

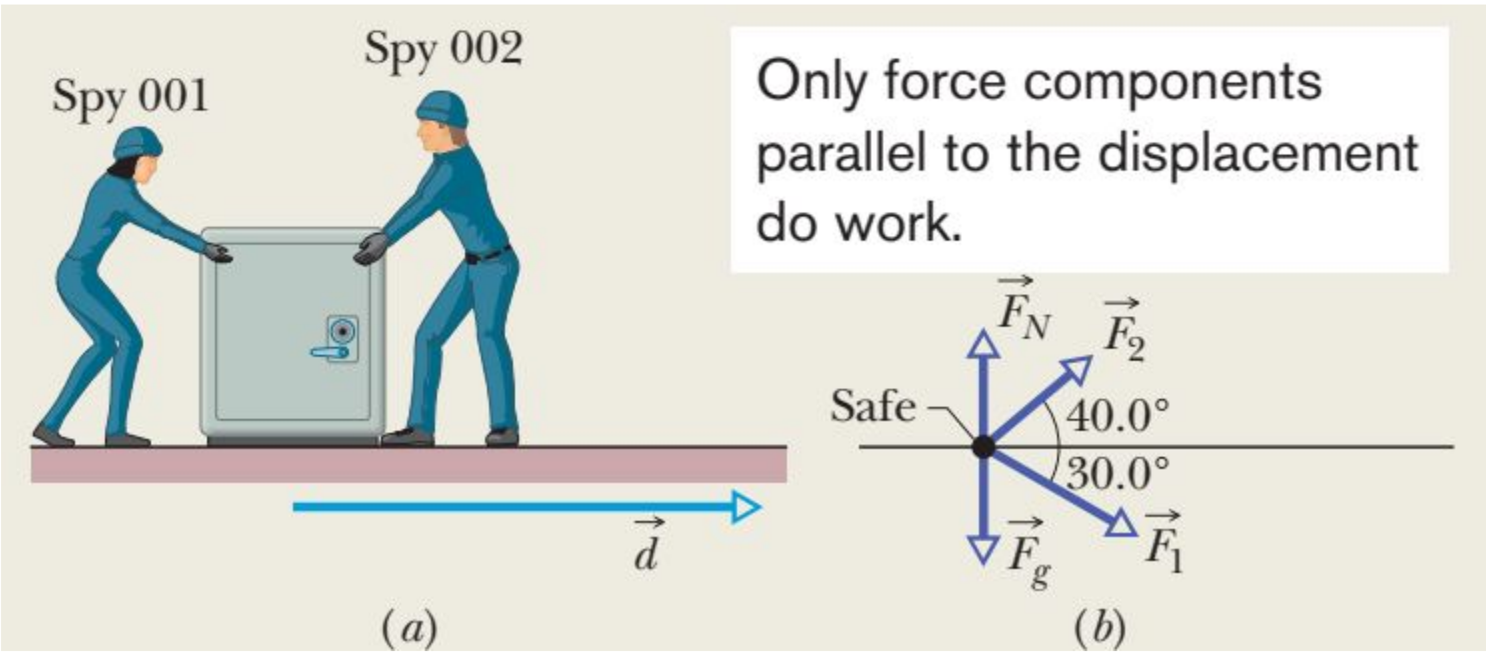
Example 7.1 Mr. Clean

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude $F = 50.0$ N at an angle of 30.0° with the horizontal (Fig. 7.5). Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3.00 m to the right.



Work done by two constant forces, industrial spies

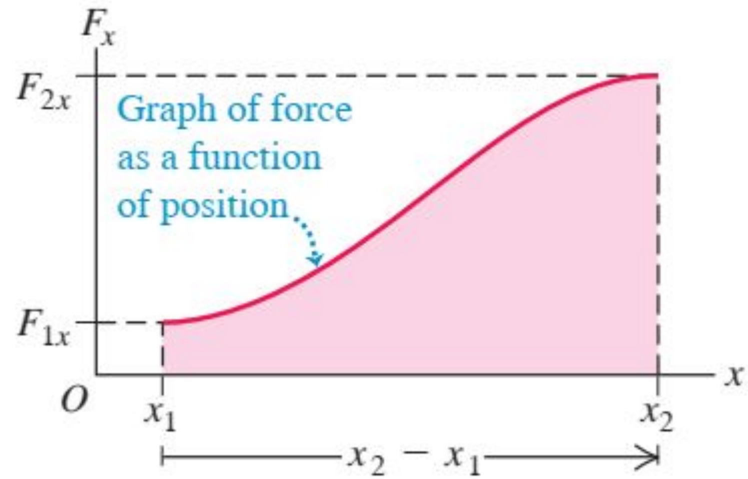
Figure 7-4a shows two industrial spies sliding an initially stationary 225 kg floor safe a displacement $\vec{d}$ of magnitude 8.50 m. The push $\vec{F}_1$ of spy 001 is 12.0 N at an angle of 30.0° downward from the horizontal; the pull $\vec{F}_2$ of spy 002 is 10.0 N at 40.0° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.



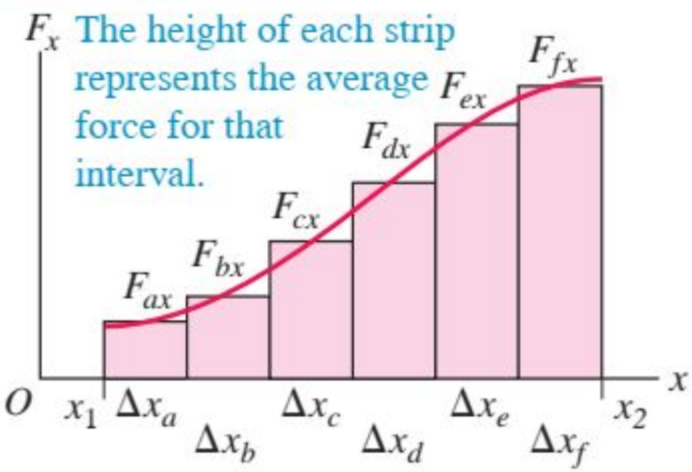
7.4 Work Done by a Varying Force

(a) A particle moves from x_1 to x_2 in response to a changing force in the x-direction.

(b) The force F_x varies with position x ...



(c) ... but over a short displacement Δx, the force is essentially constant.



$$W = F_{ax}\Delta x_a + F_{bx}\Delta x_b + \cdots$$

$$W \approx \sum_{x_i}^{x_f} F_x \, \Delta x$$

Work done on a particle by a **varying x-component of force F_x** during **straight-line displacement** along x-axis

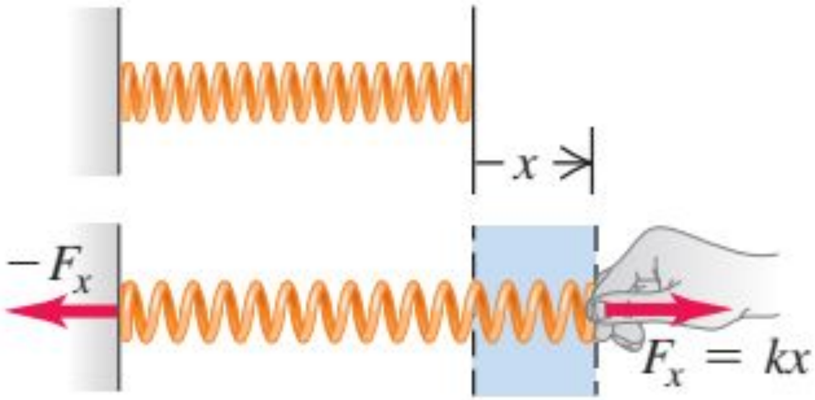
$$W = \int_{x_1}^{x_2} F_x \, dx$$

Upper limit = final position

Integral of x-component of force

Lower limit = initial position

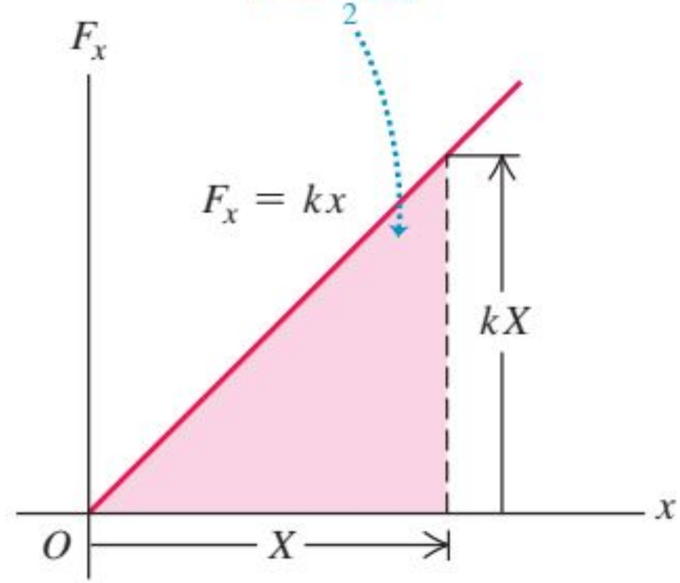
Work Done by a Spring

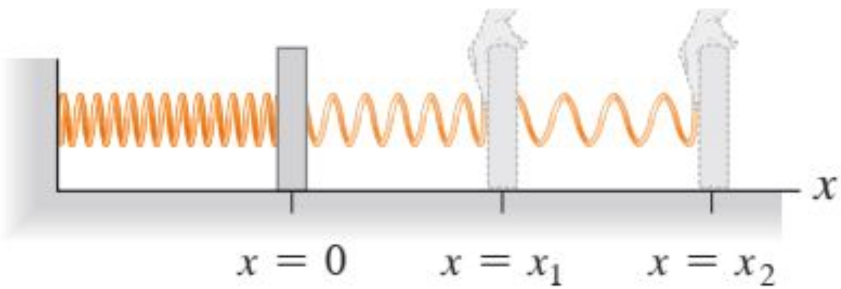


$$W = \int_0^X F_x\,dx = \int_0^X kx\,dx = \tfrac{1}{2}kX^2$$

The area under the graph represents the work done on the spring as the spring is stretched from $x = 0$ to a maximum value X:

$W = \frac{1}{2}kX^2$





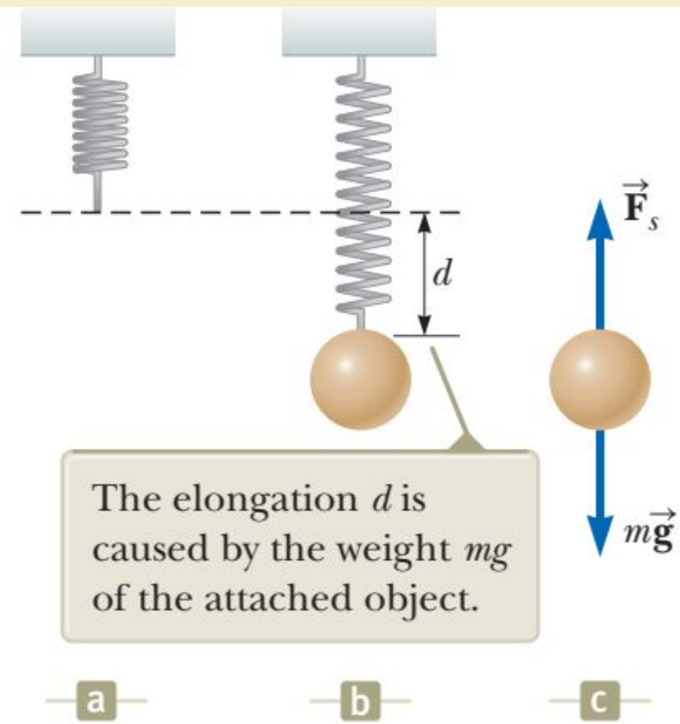
$$W = \int_{x_1}^{x_2} F_x\,dx = \int_{x_1}^{x_2} kx\,dx = \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2$$

Example 7.5 Measuring k for a Spring AM

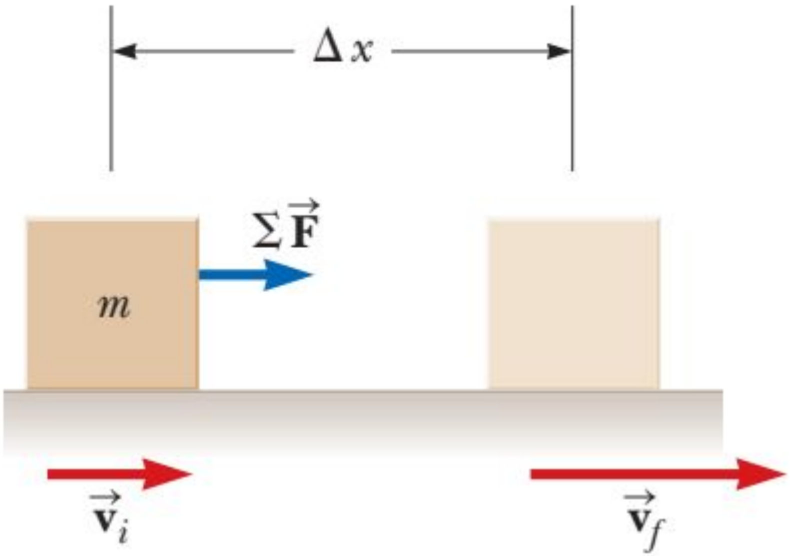
A common technique used to measure the force constant of a spring is demonstrated by the setup in Figure 7.11. The spring is hung vertically (Fig. 7.11a), and an object of mass m is attached to its lower end. Under the action of the "load" mg, the spring stretches a distance d from its equilibrium position (Fig. 7.11b).

(A) If a spring is stretched 2.0 cm by a suspended object having a mass of 0.55 kg, what is the force constant of the spring?

(B) How much work is done by the spring on the object as it stretches through this distance?



7.5 Kinetic Energy and the Work–Kinetic Energy Theorem



$$W_{\text{ext}} = \int_{x_i}^{x_f} \sum F\,dx$$

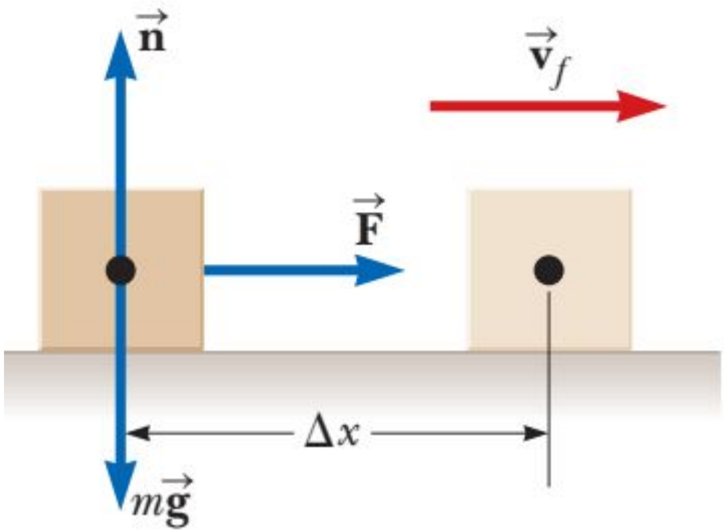
$$W_{\text{ext}} = \int_{x_i}^{x_f} ma\,dx = \int_{x_i}^{x_f} m\frac{dv}{dt}\,dx = \int_{x_i}^{x_f} m\frac{dv}{dx}\frac{dx}{dt}\,dx = \int_{v_i}^{v_f} mv\,dv$$

$$W_{\text{ext}} = \tfrac{1}{2}mv_f^{\,2} - \tfrac{1}{2}mv_i^{\,2}$$

$$W_{\text{ext}} = K_f - K_i = \Delta K$$

Example 7.6 A Block Pulled on a Frictionless Surface AM

A 6.0-kg block initially at rest is pulled to the right along a frictionless, horizontal surface by a constant horizontal force of magnitude 12 N. Find the block's speed after it has moved through a horizontal distance of 3.0 m.



Conceptual Example 7.7 Does the Ramp Les

A man wishes to load a refrigerator onto a truck using a ramp at angle θ as shown in Figure 7.14. He claims that less work would be required to load the truck if the length L of the ramp were increased. Is his claim valid?

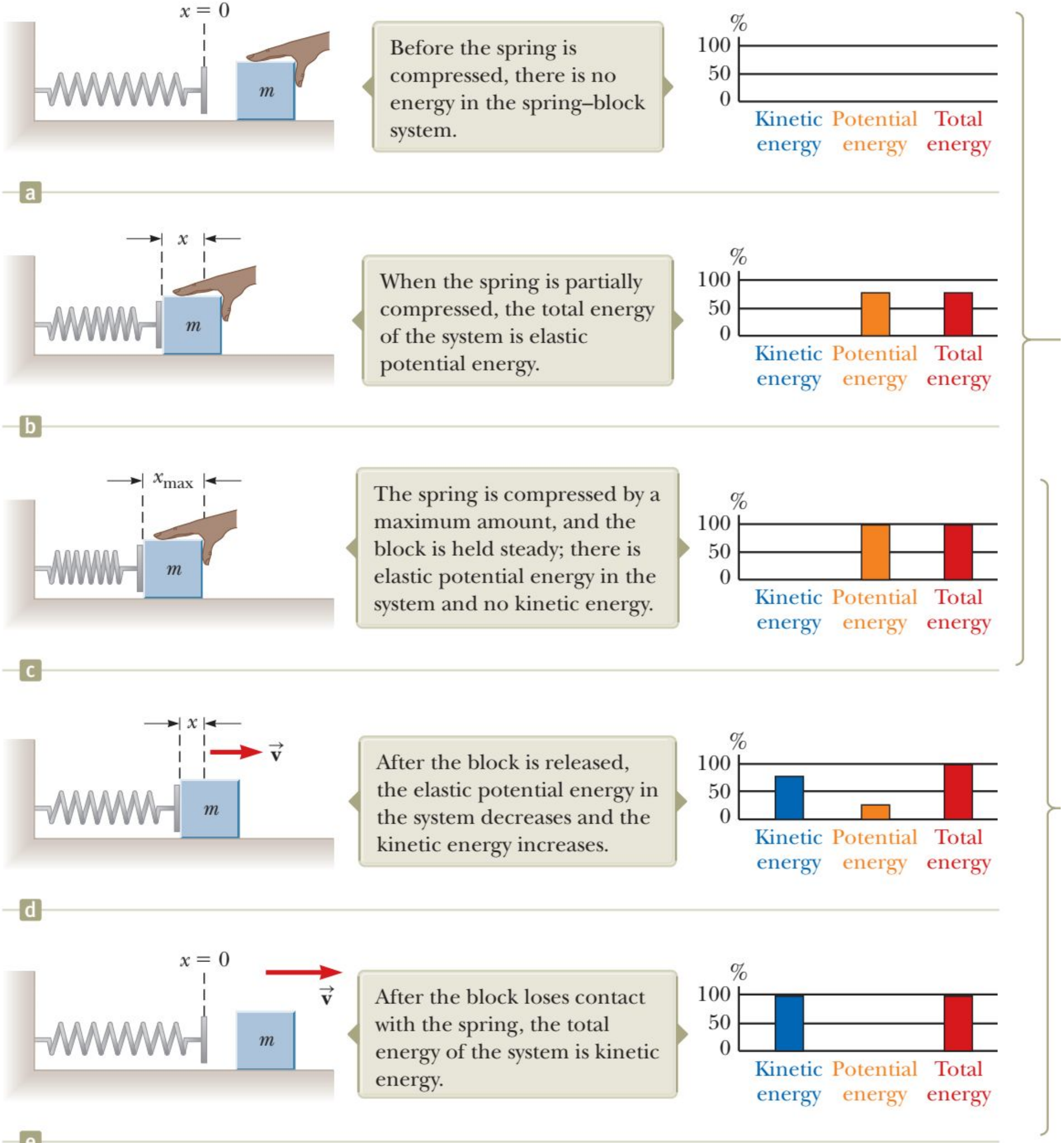
Example 7.8 The Proud Athlete and the Sore Toe

A trophy being shown off by a careless athlete slips from the athlete's hands and drops on his foot. Choosing floor level as the $y = 0$ point of your coordinate system, estimate the change in gravitational potential energy of the trophy–Earth system as the trophy falls. Repeat the calculation, using the top of the athlete's head as the origin of coordinates.

Elastic Potential Energy

$$W_{\text{ext}} = \tfrac{1}{2}kx_f^{\,2} - \tfrac{1}{2}kx_i^{\,2}$$

$$\boxed{U_s \equiv \tfrac{1}{2}kx^2}$$

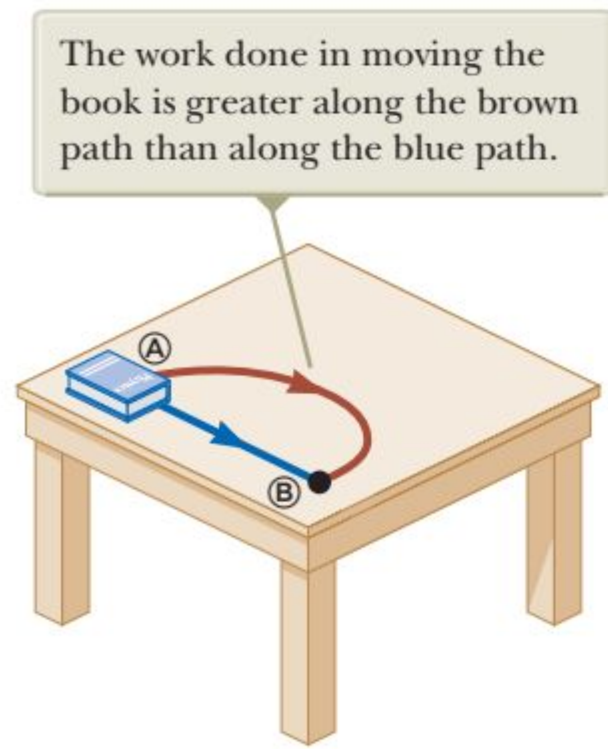


x = 0
m
Before the spring is compressed, there is no energy in the spring–block system.
%
100
50
0
Kinetic energy
Potential energy
Total energy
a
x
m
When the spring is partially compressed, the total energy of the system is elastic potential energy.
%
100
50
0
Kinetic energy
Potential energy
Total energy
b
x_max
m
The spring is compressed by a maximum amount, and the block is held steady; there is elastic potential energy in the system and no kinetic energy.
%
100
50
0
Kinetic energy
Potential energy
Total energy
c
x
v
m
After the block is released, the elastic potential energy in the system decreases and the kinetic energy increases.
%
100
50
0
Kinetic energy
Potential energy
Total energy
d
x = 0
v
m
After the block loses contact with the spring, the total energy of the system is kinetic energy.
%
100
50
0
Kinetic energy
Potential energy
Total energy
e

7.7 Conservative and Nonconservative Forces

Conservative forces have these two equivalent properties:

1. The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.
2. The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one for which the beginning point and the endpoint are identical.)



Example 7.8 Using Conservation of Mechanical Energy to Calculate the Speed of a Toy Car

A 0.100-kg toy car is propelled by a compressed spring, as shown in **Figure 7.12**. The car follows a track that rises 0.180 m above the starting point. The spring is compressed 4.00 cm and has a force constant of 250.0 N/m. Assuming work done by friction to be negligible, find (a) how fast the car is going before it starts up the slope and (b) how fast it is going at the top of the slope.