Lecture 1

CHAPTER 1

Physics and Measurement

Lecture-1 Outline

- What is Physics
- Physical Quantities, Units
- Measurements
- Significant figures
- Vectors
- Vectors addition, multiplication
- Example solve Problems

Physics: An Introduction

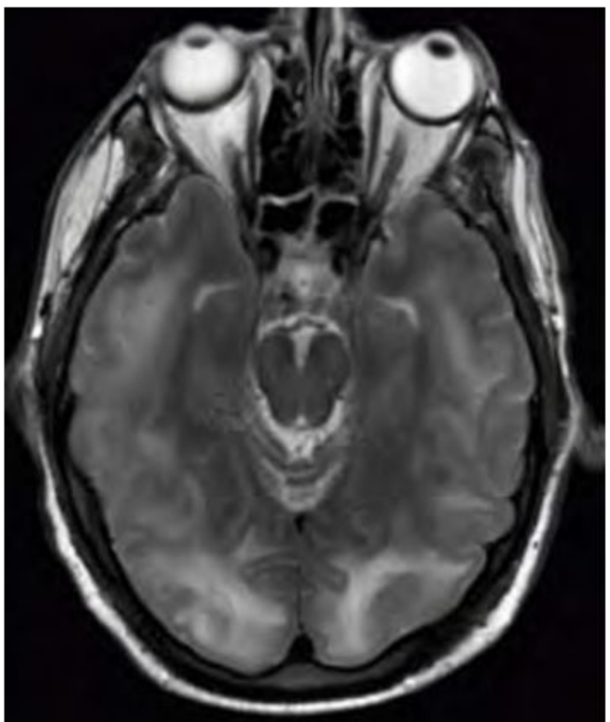
The physical universe is enormously complex in its detail. Every day, each of us observes a great variety of objects and phenomena. Over the centuries, the curiosity of the human race has led us collectively to explore and catalog a tremendous wealth of information. From the flight of birds to the colors of flowers, from lightning to gravity, from quarks to clusters of galaxies, from the flow of time to the mystery of the creation of the universe, we have asked questions and assembled huge arrays of facts. In the face of all these details, we have discovered that a surprisingly small and unified set of physical laws can explain what we observe. As humans, we make generalizations and seek order. We have found that nature is remarkably cooperative—it exhibits the *underlying order and simplicity* we so value.

Applications of Physics

Physics is the foundation of many important disciplines and contributes directly to others. Chemistry, for example—since it deals with the interactions of atoms and molecules—is rooted in atomic and molecular physics. Most branches of engineering are applied physics. In architecture, physics is at the heart of structural stability, and is involved in the acoustics, heating, lighting, and cooling of buildings. Parts of geology rely heavily on physics, such as radioactive dating of rocks, earthquake analysis, and heat transfer in the Earth. Some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines

The laws of physics help us understand how common appliances work. For example, the laws of physics can help explain how microwave ovens heat up food, and they also help us understand why it is dangerous to place metal objects

in a microwave oven

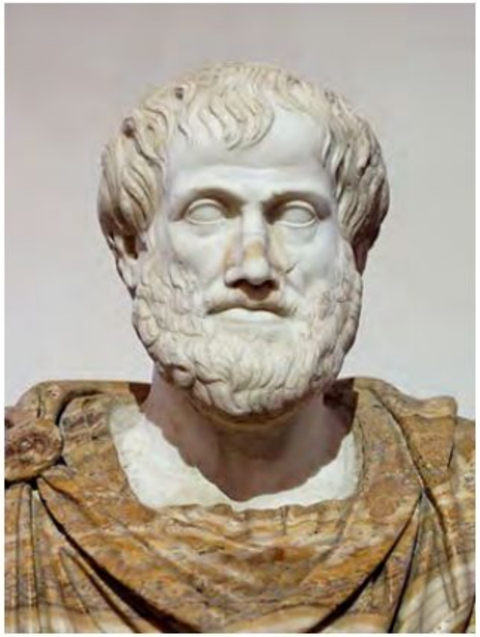


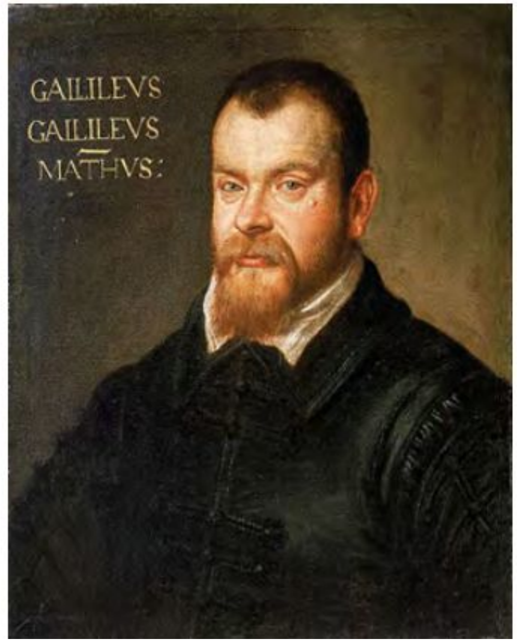
These two applications of physics have more in common than meets the eye. Microwave ovens use electromagnetic waves to heat food. Magnetic resonance imaging (MRI) also uses electromagnetic waves to yield an image of the brain, from which the exact location of tumors can be determined

Sir Isaac Newton

Isaac Newton (1642–1727) was very reluctant to publish his revolutionary work and had to be convinced to do so. In his later years, he stepped down from his academic post and became exchequer of the Royal Mint. He took this post seriously, inventing reeding (or creating ridges) on the edge of coins to prevent unscrupulous people from trimming the silver off of them before using them as currency

Marie Curie (1867–1934) sacrificed monetary assets to help finance her early research and damaged her physical well-being with radiation exposure. She is the only person to win Nobel prizes in both physics and chemistry. One of her daughters also won a Nobel Prize





The Greek philosopher **Aristotle** (384–322 B.C.) wrote on a broad range of topics including physics, animals, the soul, politics, and poetry.

Galileo Galilei (1564–1642) laid the foundation of modern experimentation

Niels Bohr (1885–1962) made fundamental contributions to the development of quantum mechanics, one part of modern physics.

Physical Quantities and Units

We define a **physical quantity** either by *specifying how it is measured* or by *stating how it is calculated* from other measurements. For example, we define distance and time by specifying methods for measuring them, whereas we define *average speed* by stating that it is calculated as distance traveled divided by time of travel.

Measurements of physical quantities are expressed in terms of **units**, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in units of meters (for sprinters) or kilometers (for distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way.

SI Units: Fundamental and Derived Units

Table 1.1 Fundamental SI Units

Length	Mass	Time	Electric Current
meter (m)	kilogram (kg)	second (s)	ampere (A)

Units of Time, Length, and Mass: The Second, Meter, and Kilogram

The Second

In 1967 the second was redefined as the time required for 9,192,631,770 of these vibrations.

An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of better than a microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image is looking down from the top of an atomic fountain nearly 30 feet tall!

The Meter

In 1983, the meter was given its present definition (partly for greater accuracy) as the distance light travels in a vacuum in 1/299,792,458 of a second.

Light travels a distance of 1 meter in 1/299,792,458 seconds

The Kilogram

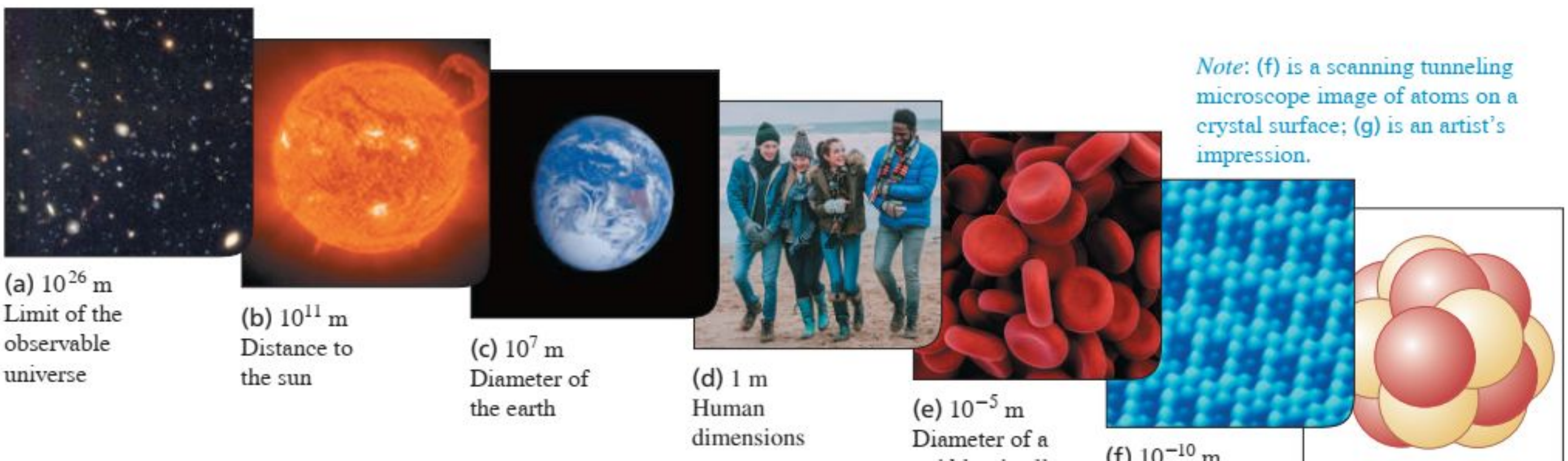
Until recently the unit of mass, the **kilogram** (abbreviated kg), was defined to be the mass of a metal cylinder kept at the International Bureau of Weights and Measures in France (**Fig. 1.4**). This was a very inconvenient standard to use. Since 2018 the value of the kilogram has been based on a fundamental constant of nature called *Planck's constant* (symbol h), whose defined value $h = 6.62607015 * 10^{-34}$ kg m^2/s is related to those of the kilogram, meter, and second. Given the values of the meter and the second, the masses of objects can be experimentally determined in terms of h.

platinum-iridium

TABLE 1.1 Some Units of Length, Mass, and Time

Length	Mass	Time
1 nanometer = 1 nm = 10^{-9} m (a few times the size of the largest atom)	1 microgram = 1 μg = 10^{-6} g = 10^{-9} kg (mass of a very small dust particle)	1 nanosecond = 1 ns = 10^{-9} s (time for light to travel 0.3 m)
1 micrometer = 1 μm = 10^{-6} m (size of some bacteria and other cells)	1 milligram = 1 mg = 10^{-3} g = 10^{-6} kg (mass of a grain of salt)	1 microsecond = 1 μs = 10^{-6} s (time for space station to move 8 mm)
1 millimeter = 1 mm = 10^{-3} m (diameter of the point of a ballpoint pen)	1 gram = 1 g = 10^{-3} kg (mass of a paper clip)	1 millisecond = 1 ms = 10^{-3} s (time for a car moving at freeway speed to travel 3 cm)
1 centimeter = 1 cm = 10^{-2} m (diameter of your little finger)		
1 kilometer = 1 km = 10^{3} m (distance in a 10 minute walk)		

Figure **1.5** Some typical lengths in the universe.



(a) 10^{26} m Limit of the observable universe

(b) 10^{11} m Distance to the sun

(c) 10^{7} m Diameter of the earth

(d) 1 m Human dimensions

(e) 10^{-5} m Diameter of a red blood cell

(f) 10^{-10} m Radius of an atom

(g) 10^{-14} m Radius of an atomic nucleus

Note: (f) is a scanning tunneling microscope image of atoms on a crystal surface; (g) is an artist's impression.

USING AND CONVERTING UNITS

We use equations to express relationships among physical quantities, represented by algebraic symbols. Each algebraic symbol always denotes both a number and a unit. For example, d might represent a distance of 10 m, t a time of 5 s, and v a speed of 2 m/s.

An equation must always be **dimensionally consistent.** You can't add apples and automobiles; two terms may be added or equated only if they have the same units. For example, if an object moving with constant speed v travels a distance d in a time t, these quantities are related by the equation

$$d = vt$$

If d is measured in meters, then the product vt must also be expressed in meters. Using the above numbers as an example, we may write

$$10\ \mathrm{m} = \left(2\ \frac{\mathrm{m}}{\cancel{\mathrm{s}}}\right)(5\ \cancel{\mathrm{s}})$$

UNCERTAINTY AND SIGNIFICANT FIGURES

In many cases the uncertainty of a number is not stated explicitly. Instead, the uncertainty is indicated by the number of meaningful digits, or **significant figures,** in the measured value. We gave the thickness of the cover of the book as 2.91 mm, which has three significant figures. By this we mean that the first two digits are known to be correct, while the third digit is uncertain. The last digit is in the hundredths place, so the uncertainty is about 0.01 mm. Two values with the *same* number of significant figures may have *different* uncertainties; a distance given as 137 km also has three significant figures, but the uncertainty is about 1 km. A distance given as 0.25 km has two significant figures (the zero to the left of the decimal point doesn't count); if given as 0.250 km, it has three significant figures.

When you use numbers that have uncertainties to compute other numbers, the computed numbers are also uncertain. When numbers are multiplied or divided, the result can have no more significant figures than the factor with the fewest significant figures has. For example, $3.1416 \times 2.34 \times 0.58 = 4.3$. When we add and subtract numbers, it's the location of the decimal point that matters, not the number of significant figures. For example, $123.62 + 8.9 = 132.5$. Although 123.62 has an uncertainty of about 0.01, 8.9 has an uncertainty of about 0.1. So their sum has an uncertainty of about 0.1 and should be written as 132.5, not 132.52. **Table 1.2** summarizes these rules for significant figures.

Percent Uncertainty $\% \text{ unc} = \dfrac{\delta A}{A} \times 100\%.$

TABLE 1.2 Using Significant Figures

Multiplication or division:
Result can have no more significant figures than the factor with the fewest significant figures:

$$\frac{0.745 \times 2.2}{3.885} = 0.42$$

$$1.32578 \times 10^7 \times 4.11 \times 10^{-3} = 5.45 \times 10^4$$

Addition or subtraction:
Number of significant figures is determined by the term with the largest uncertainty (i.e., fewest digits to the right of the decimal point):

$$27.153 + 138.2 - 11.74 = 153.6$$

EXAMPLE 1.3 Significant figures in multiplication

The rest energy E of an object with rest mass m is given by Albert Einstein's famous equation $E = mc^2$, where c is the speed of light in vacuum. Find E for an electron for which (to three significant figures) $m = 9.11 \times 10^{-31}$ kg. The SI unit for E is the joule (J); $1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$.

IDENTIFY and SET UP Our target variable is the energy E. We are given the value of the mass m; from Section 1.3 (or Appendix F) the speed of light is $c = 2.99792458 \times 10^8$ m/s.

EXECUTE Substituting the values of m and c into Einstein's equation, we find

$$\begin{aligned} E &= (9.11 \times 10^{-31} \text{ kg})(2.99792458 \times 10^8 \text{ m/s})^2 \\ &= (9.11)(2.99792458)^2(10^{-31})(10^8)^2 \text{ kg} \cdot \text{m}^2/\text{s}^2 \\ &= (81.87659678)(10^{[-31+(2\times 8)]}) \text{ kg} \cdot \text{m}^2/\text{s}^2 \\ &= 8.187659678 \times 10^{-14} \text{ kg} \cdot \text{m}^2/\text{s}^2 \end{aligned}$$

Since the value of m was given to only three significant figures, we must round this to

$$E = 8.19 \times 10^{-14} \text{ kg} \cdot \text{m}^2/\text{s}^2 = 8.19 \times 10^{-14} \text{ J}$$

EVALUATE While the rest energy contained in an electron may seem ridiculously small, on the atomic scale it is tremendous. Compare our answer to 10^{-19} J, the energy gained or lost by a single atom during a typical chemical reaction. The rest energy of an electron is about 1,000,000 times larger! (We'll discuss the significance of rest energy in Chapter 37.)

KEYCONCEPT When you are multiplying (or dividing) quantities, the result can have no more significant figures than the quantity with the fewest significant figures.

Because significant figures measures uncertainty relative to the size of the number

Suppose you take a measurement of something and it comes out to be 0.002 meters.

You then measure something else and it comes to 345 meters.

You know that 0.002 means 0.002 ± 0.0005 and 345 means between 345 ± 0.5.

The uncertainty in the numbers here are 0.0005 and 0.5, respectively.

Notice how the difference between 345 and 0.5 is much greater than 0.002 and 0.0005.

345 is 690 times bigger than 0.5. 0.002 is only 4 times bigger than 0.0005.

Thus, 345 is a more precise relative to its size – 2 more digits precise. :)

A grocery store sells 5-lb bags of apples. You purchase four bags over the course of a month and weigh the apples each time. You obtain the following measurements:

- Week 1 weight: $4.8\ \text{lb}$
- Week 2 weight: $5.3\ \text{lb}$
- Week 3 weight: $4.9\ \text{lb}$
- Week 4 weight: $5.4\ \text{lb}$

You determine that the weight of the 5-lb bag has an uncertainty of $\pm 0.4\ \text{lb}$. What is the percent uncertainty of the bag's weight?

Strategy

First, observe that the expected value of the bag's weight, A, is 5 lb. The uncertainty in this value, δA, is 0.4 lb. We can use the following equation to determine the percent uncertainty of the weight:

$$\%\ \text{unc} = \frac{\delta A}{A} \times 100\%. \tag{1.9}$$

Solution

Plug the known values into the equation:

$$\%\ \text{unc} = \frac{0.4\ \text{lb}}{5\ \text{lb}} \times 100\% = 8\%. \tag{1.10}$$

EXAMPLE 1.4 An order-of-magnitude estimate

You are writing an adventure novel in which the hero escapes with a billion dollars' worth of gold in his suitcase. Could anyone carry that much gold? Would it fit in a suitcase?

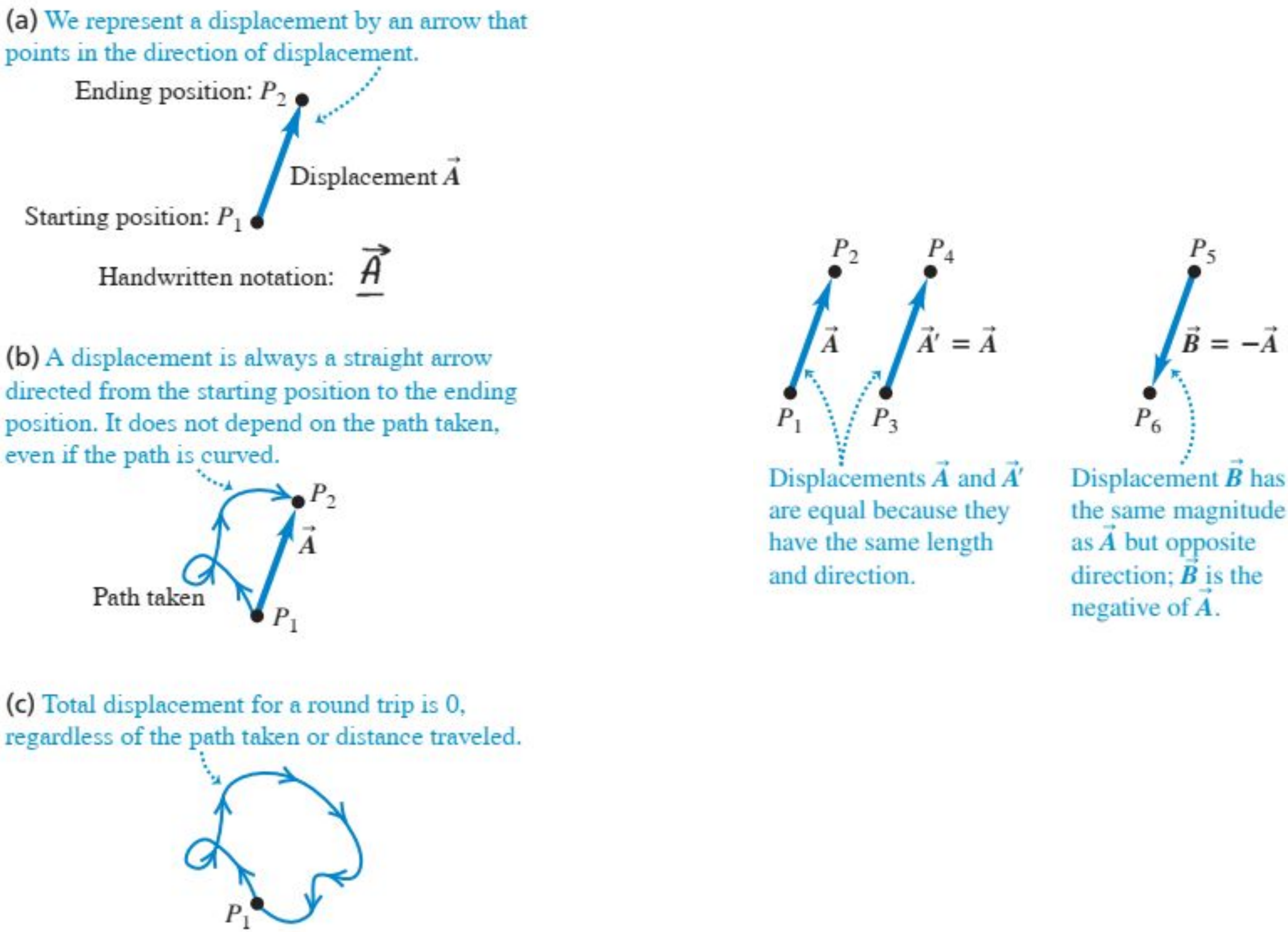
IDENTIFY, SET UP, and EXECUTE Gold sells for about \$1400 an ounce, or about \$100 for $\frac{1}{14}$ ounce. (The price per ounce has varied between \$200 and \$1900 over the past twenty years or so.) An ounce is about 30 grams, so \$100 worth of gold has a mass of about $\frac{1}{14}$ of 30 grams, or roughly 2 grams. A billion (10^9) dollars' worth of gold has a mass 10^7 times greater, about 2×10^7 (20 million) grams or 2×10^4 (20,000) kilograms. A thousand kilograms has a weight in British units of about a ton, so the suitcase weighs roughly 20 tons! No human could lift it.

Roughly what is the *volume* of this gold? The density of water is $10^3\ \text{kg/m}^3$; if gold, which is much denser than water, has a density 10 times greater, then 10^4 kg of gold fits into a volume of $1\ \text{m}^3$. So 10^9 dollars' worth of gold has a volume of $2\ \text{m}^3$, many times the volume of a suitcase.

EVALUATE Clearly your novel needs rewriting. Try the calculation again with a suitcase full of five-carat (1-gram) diamonds, each worth \$500,000. Would this work?

KEYCONCEPT To decide whether the numerical value of a quantity is reasonable, assess the quantity in terms of other quantities that you can estimate, even if only roughly.

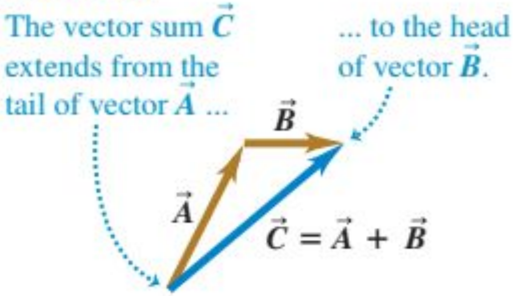
1.7 VECTORS AND VECTOR ADDITION



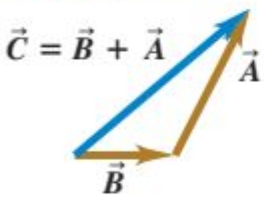
Vector Addition and Subtraction

Figure 1.11 Three ways to add two vectors.

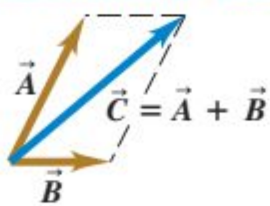
(a) We can add two vectors by placing them head to tail.



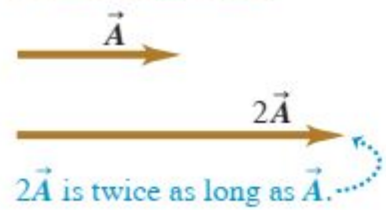
(b) Adding them in reverse order gives the same result: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$. The order doesn't matter in vector addition.



(c) We can also add two vectors by placing them tail to tail and constructing a parallelogram.



(a) Multiplying a vector by a positive scalar changes the magnitude (length) of the vector but not its direction.



(b) Multiplying a vector by a negative scalar changes its magnitude and reverses its direction.

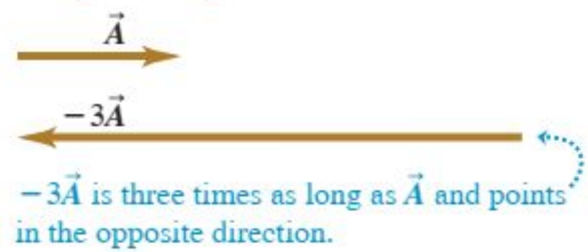


Figure 1.13 Several constructions for finding the vector sum $\vec{A} + \vec{B} + \vec{C}$.

(a) To find the sum of these three vectors ...

(b) ... add $\vec{A}$ and $\vec{B}$ to get $\vec{D}$ and then add $\vec{C}$ to $\vec{D}$ to get the final sum (resultant) $\vec{R}$ ...

(c) ... or add $\vec{B}$ and $\vec{C}$ to get $\vec{E}$ and then add $\vec{E}$ to $\vec{A}$ to get $\vec{R}$ ...

(d) ... or add $\vec{A}$, $\vec{B}$, and $\vec{C}$ to get $\vec{R}$ directly ...

(e) ... or add $\vec{A}$, $\vec{B}$, and $\vec{C}$ in any other order and still get $\vec{R}$.

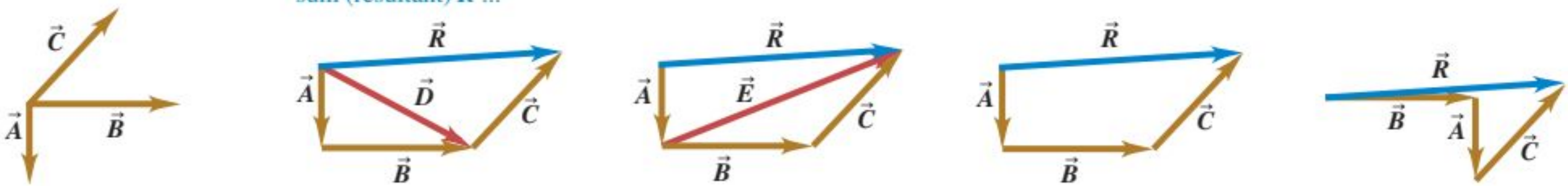
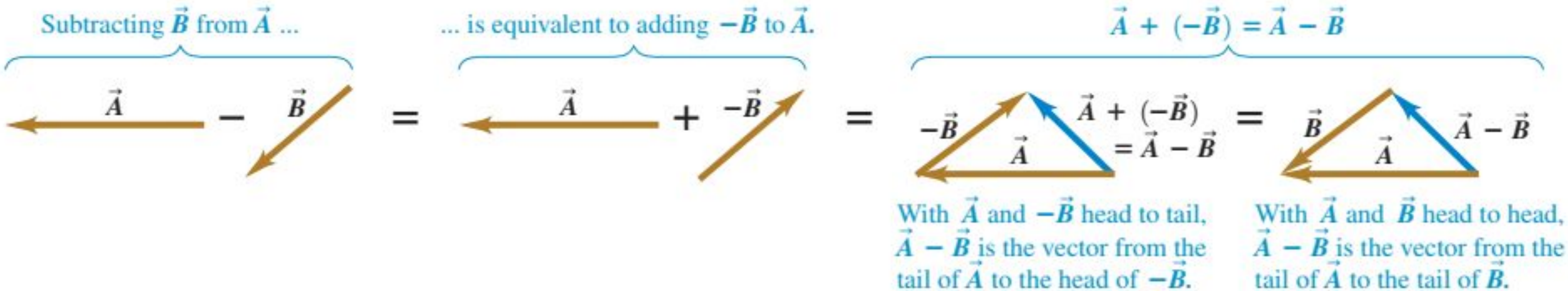


Figure 1.14 To construct the vector difference $\vec{A} - \vec{B}$, you can either place the tail of $-\vec{B}$ at the head of $\vec{A}$ or place the two vectors $\vec{A}$ and $\vec{B}$ head to head.



A cross-country skier skis 1.00 km north and then 2.00 km east on a horizontal snowfield. How far and in what direction is she from the starting point?

IDENTIFY and SET UP The problem involves combining two displacements at right angles to each other. This vector addition amounts to solving a right triangle, so we can use the Pythagorean theorem and trigonometry. The target variables are the skier's straight-line distance and direction from her starting point. **Figure 1.16** is a scale diagram of the two displacements and the resultant net displacement. We denote the direction from the starting point by the angle ϕ (the Greek letter phi). The displacement appears to be a bit more than 2 km. Measuring the angle with a protractor indicates that ϕ is about 63°.

EXECUTE The distance from the starting point to the ending point is equal to the length of the hypotenuse:

$$\sqrt{(1.00\text{ km})^2 + (2.00\text{ km})^2} = 2.24\text{ km}$$

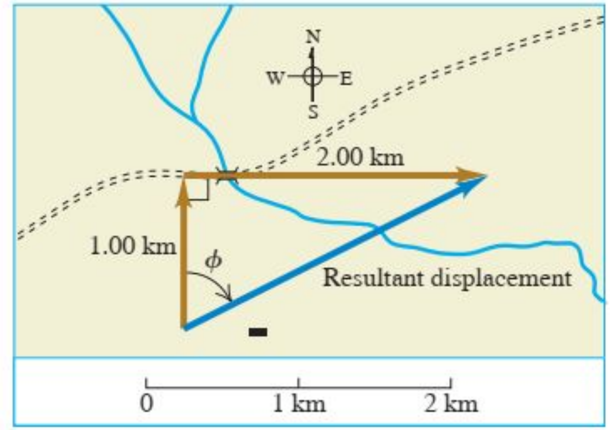
A little trigonometry (from Appendix B) allows us to find angle ϕ:

$$\tan\phi = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{2.00\text{ km}}{1.00\text{ km}} = 2.00$$

$$\phi = \arctan 2.00 = 63.4°$$

We can describe the direction as 63.4° east of north or $90° - 63.4° = 26.6°$ north of east.

Figure 1.16 The vector diagram, drawn to scale, for a ski trip.



EVALUATE Our answers (2.24 km and $\phi = 63.4°$) are close to our predictions. In Section 1.8 we'll learn how to easily add two vectors *not* at right angles to each other.

KEYCONCEPT In every problem involving vector addition, draw the two vectors being added as well as the vector sum. The head-to-tail arrangement shown in Figs. 1.11a and 1.11b is easiest. This will help you to visualize the vectors and understand the direction of the vector sum. Drawing the vectors is equally important for problems involving vector subtraction (see Fig. 1.14).

TEST YOUR UNDERSTANDING OF SECTION 1.7 Two displacement vectors, $\vec{S}$ and $\vec{T}$, have magnitudes $S = 3$ m and $T = 4$ m. Which of the following could be the magnitude of the difference vector $\vec{S} - \vec{T}$? (There may be more than one correct answer.) (i) 9 m; (ii) 7 m; (iii) 5 m; (iv) 1 m; (v) 0 m; (vi) -1 m.

1.8 COMPONENTS OF VECTORS

Figure 1.17 Representing a vector $\vec{A}$ in terms of its components A_x and A_y.

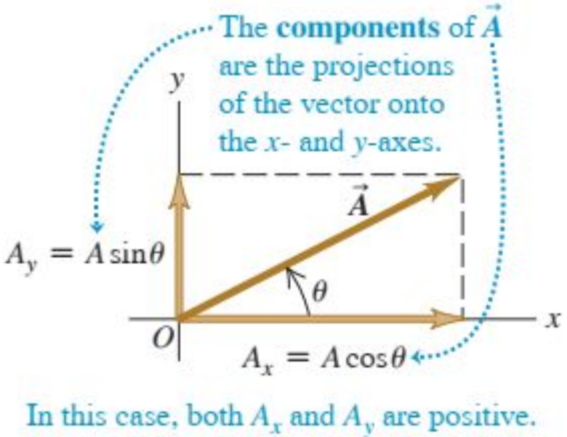
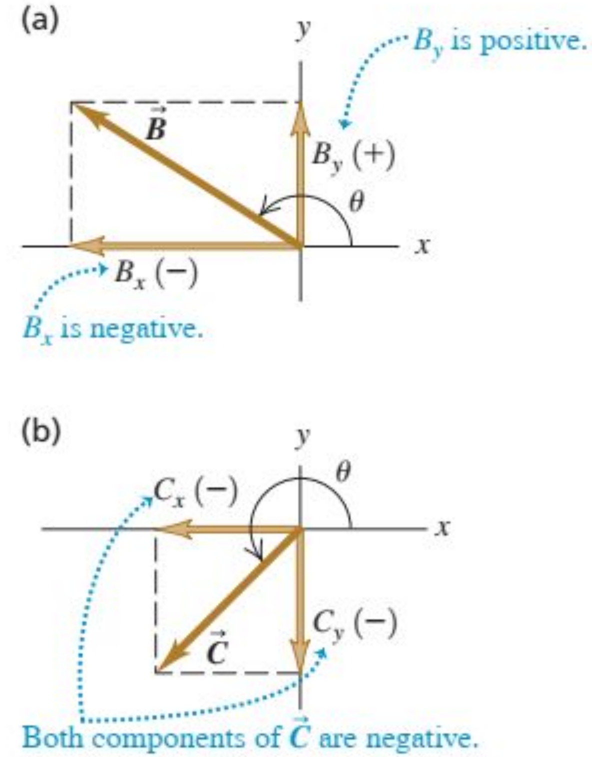


Figure 1.18 The components of a vector may be positive or negative numbers.



$$\frac{A_x}{A} = \cos\theta \qquad \text{and} \qquad \frac{A_y}{A} = \sin\theta$$

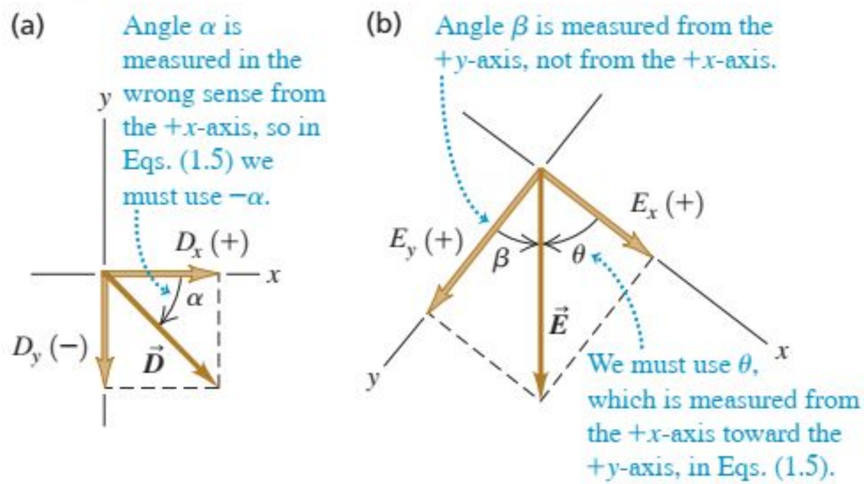
$$A_x = A\cos\theta \qquad \text{and} \qquad A_y = A\sin\theta$$

(θ measured from the $+x$-axis, rotating toward the $+y$-axis)

(a) What are the x- and y-components of vector $\vec{D}$ in **Fig. 1.19a**? The magnitude of the vector is $D = 3.00$ m, and angle $\alpha = 45°$. (b) What are the x- and y-components of vector $\vec{E}$ in Fig. 1.19b? The magnitude of the vector is $E = 4.50$ m, and angle $\beta = 37.0°$.

IDENTIFY and SET UP We can use Eqs. (1.5) to find the components of these vectors, but we must be careful: Neither angle α nor β in Fig. 1.19 is measured from the $+x$-axis toward the $+y$-axis. We estimate from the figure that the lengths of both components in part (a) are roughly 2 m, and that those in part (b) are 3 m and 4 m. The figure indicates the signs of the components.

Figure **1.19** Calculating the x- and y-components of vectors.



EXECUTE (a) The angle α (the Greek letter alpha) between the positive x-axis and $\vec{D}$ is measured toward the *negative* y-axis. The angle we must use in Eqs. (1.5) is $\theta = -\alpha = -45°$. We then find

$$D_x = D\cos\theta = (3.00\text{ m})(\cos(-45°)) = +2.1\text{ m}$$
$$D_y = D\sin\theta = (3.00\text{ m})(\sin(-45°)) = -2.1\text{ m}$$

Had we carelessly substituted $+45°$ for θ in Eqs. (1.5), our result for D_y would have had the wrong sign.

(b) The x- and y-axes in Fig. 1.19b are at right angles, so it doesn't matter that they aren't horizontal and vertical, respectively. But we can't use the angle β (the Greek letter beta) in Eqs. (1.5), because β is measured from the $+y$-axis. Instead, we must use the angle $\theta = 90.0° - \beta = 90.0° - 37.0° = 53.0°$. Then we find

$$E_x = E\cos 53.0° = (4.50\text{ m})(\cos 53.0°) = +2.71\text{ m}$$
$$E_y = E\sin 53.0° = (4.50\text{ m})(\sin 53.0°) = +3.59\text{ m}$$

EVALUATE Our answers to both parts are close to our predictions. But why do the answers in part (a) correctly have only two significant figures?

KEYCONCEPT When you are finding the components of a vector, always use a diagram of the vector and the coordinate axes to guide your calculations.

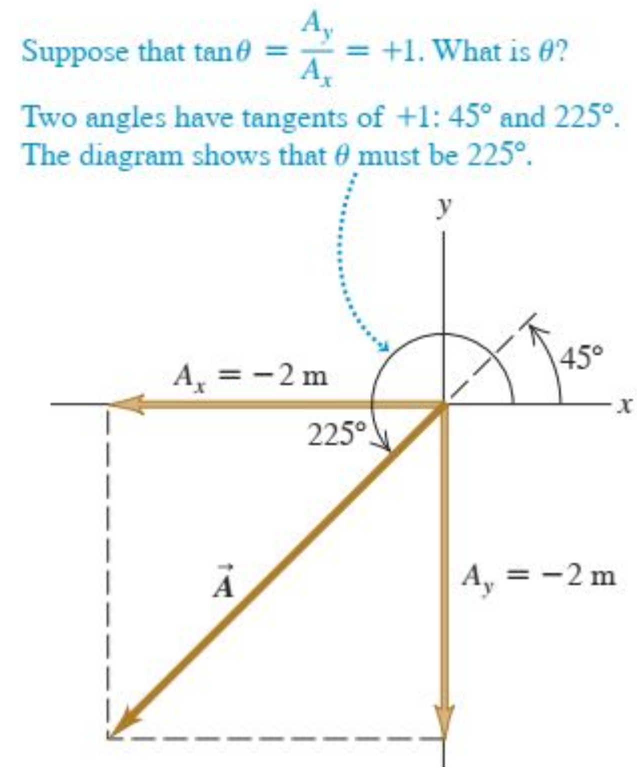
Using Components to Do Vector Calculations

Finding a vector's magnitude and direction from its components.

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\tan\theta = \frac{A_y}{A_x} \qquad \text{and} \qquad \theta = \arctan\frac{A_y}{A_x}$$

Figure 1.20 Drawing a sketch of a vector reveals the signs of its x- and y-components.



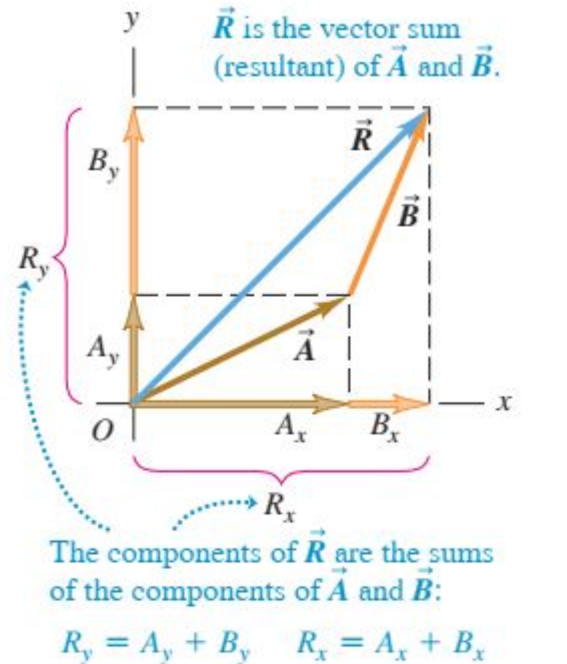
CAUTION **Finding the direction of a vector from its components** There's one complication in using Eqs. (1.7) to find θ: Any two angles that differ by 180° have the same tangent. For example, in **Fig. 1.20** the tangent of the angle θ is $\tan\theta = A_y/A_x = +1$. A calculator will tell you that $\theta = \tan^{-1}(+1) = 45°$. But the tangent of $180° + 45° = 225°$ is also equal to +1, so θ could also be 225° (which is actually the case in Fig. 1.20). *Always* draw a sketch like Fig. 1.20 to determine which of the two possibilities is correct.

2. **Multiplying a vector by a scalar.** If we multiply a vector $\vec{A}$ by a scalar c, each component of the product $\vec{D} = c\vec{A}$ is the product of c and the corresponding component of $\vec{A}$:

$$D_x = cA_x, \qquad D_y = cA_y \quad \text{(components of } \vec{D} = c\vec{A}\text{)} \tag{1.8}$$

3. **Using components to calculate the vector sum (resultant) of two or more vectors.** **Figure 1.21** shows two vectors $\vec{A}$ and $\vec{B}$ and their vector sum $\vec{R}$, along with the x- and y-components of all three vectors. The x-component R_x of the vector sum is simply the sum $(A_x + B_x)$ of the x-components of the vectors being added. The same is true for the y-components. In symbols,

Figure **1.21** Finding the vector sum (resultant) of $\vec{A}$ and $\vec{B}$ using components.

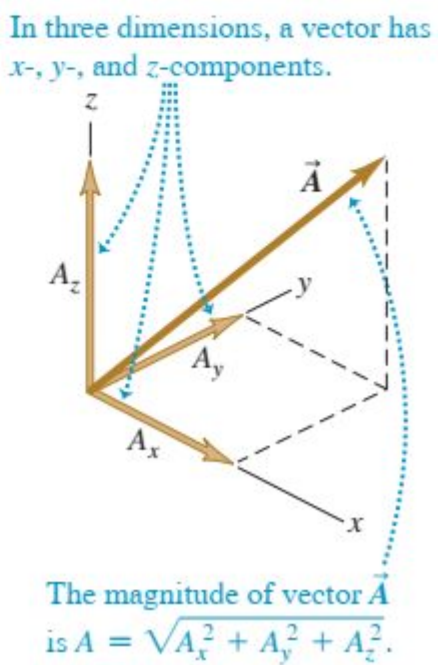


Each **component of $\vec{R} = \vec{A} + \vec{B}$** ...

$$R_x = A_x + B_x, \qquad R_y = A_y + B_y \tag{1.9}$$

... is the sum of the corresponding components of $\vec{A}$ and $\vec{B}$.

Figure **1.22** A vector in three dimensions.



Three players on a reality TV show are brought to the center of a large, flat field. Each is given a meter stick, a compass, a calculator, a shovel, and (in a different order for each contestant) the following three displacements:

$\vec{A}$: 72.4 m, 32.0° east of north

$\vec{B}$: 57.3 m, 36.0° south of west

$\vec{C}$: 17.8 m due south

The three displacements lead to the point in the field where the keys to a new Porsche are buried. Two players start measuring immediately, but the winner first *calculates* where to go. What does she calculate?

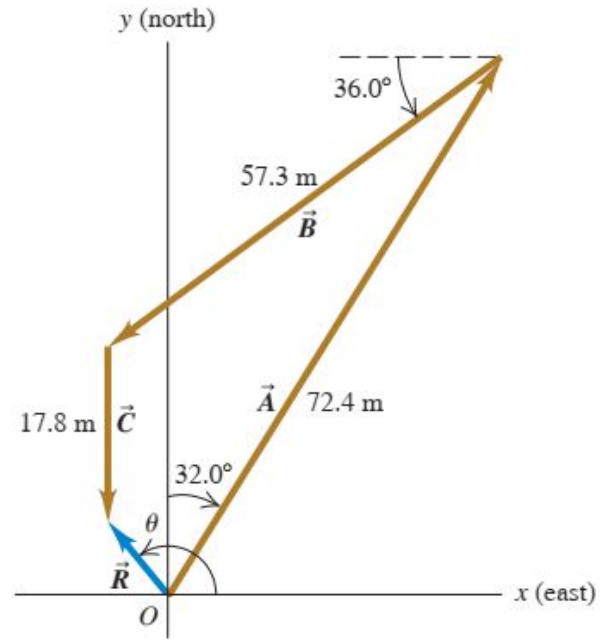
IDENTIFY and SET UP The goal is to find the sum (resultant) of the three displacements, so this is a problem in vector addition. See **Fig. 1.23**. We have chosen the $+x$-axis as east and the $+y$-axis as north. We estimate from the diagram that the vector sum $\vec{R}$ is about 10 m, 40° west of north (so θ is about 90° plus 40°, or about 130°).

EXECUTE The angles of the vectors, measured from the $+x$-axis toward the $+y$-axis, are $(90.0° - 32.0°) = 58.0°$, $(180.0° + 36.0°) = 216.0°$, and 270.0°, respectively. We may now use Eqs. (1.5) to find the components of $\vec{A}$:

$$A_x = A\cos\theta_A = (72.4\text{ m})(\cos 58.0°) = 38.37\text{ m}$$
$$A_y = A\sin\theta_A = (72.4\text{ m})(\sin 58.0°) = 61.40\text{ m}$$

We've kept an extra significant figure in the components; we'll round to the correct number of significant figures at the end of our calculation. The table at right shows the components of all the displacements, the addition of the components, and the other calculations from Eqs. (1.6) and (1.7).

Figure 1.23 Three successive displacements $\vec{A}$, $\vec{B}$, and $\vec{C}$ and the resultant (vector sum) displacement $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.



Distance	Angle	x-component	y-component
$A = 72.4$ m	58.0°	38.37 m	61.40 m
$B = 57.3$ m	216.0°	−46.36 m	−33.68 m
$C = 17.8$ m	270.0°	0.00 m	−17.80 m
		$R_x = -7.99$ m	$R_y = 9.92$ m

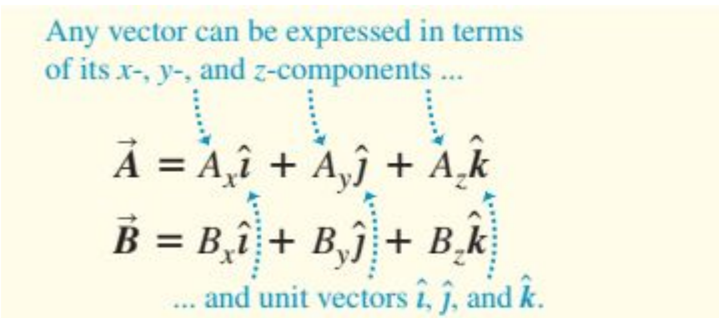
$$R = \sqrt{(-7.99\text{ m})^2 + (9.92\text{ m})^2} = 12.7\text{ m}$$
$$\theta = \arctan\frac{9.92\text{ m}}{-7.99\text{ m}} = -51°$$

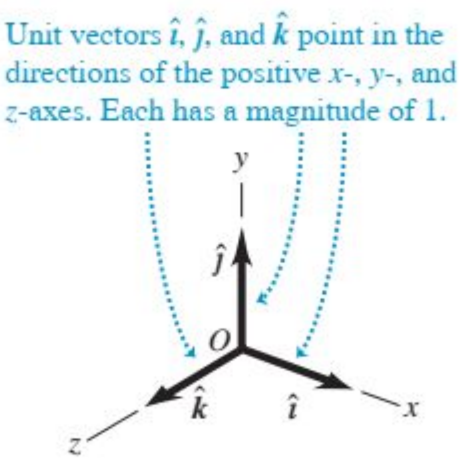
Continued

Comparing to angle θ in Fig. 1.23 shows that the calculated angle is clearly off by 180°. The correct value is $\theta = 180° + (-51°) = 129°$, or 39° west of north.

EVALUATE Our calculated answers for R and θ agree with our estimates. Notice how drawing the diagram in Fig. 1.23 made it easy to avoid a 180° error in the direction of the vector sum.

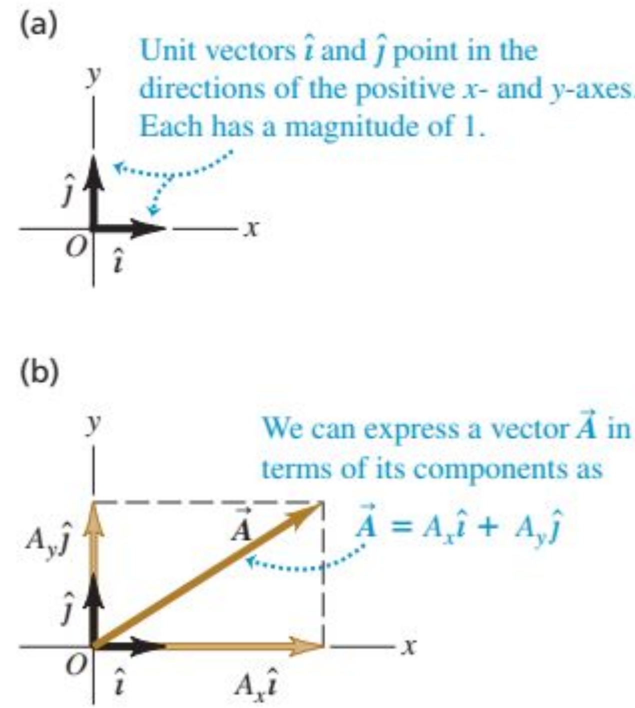
1.9 UNIT VECTORS





KEYCONCEPT When you are adding vectors, the x-component of the vector sum is equal to the sum of the x-components of the vectors being added, and likewise for the y-component. Always use a diagram to help determine the direction of the vector sum.

Figure 1.24 (a) The unit vectors $\hat{\imath}$ and $\hat{\jmath}$. (b) Expressing a vector $\vec{A}$ in terms of its components.



Given the two displacements

$$\vec{D} = (6.00\hat{\imath} + 3.00\hat{\jmath} - 1.00\hat{k})\text{ m} \quad \text{and}$$
$$\vec{E} = (4.00\hat{\imath} - 5.00\hat{\jmath} + 8.00\hat{k})\text{ m}$$

find the magnitude of the displacement $2\vec{D} - \vec{E}$.

IDENTIFY and SET UP We are to multiply vector $\vec{D}$ by 2 (a scalar) and subtract vector $\vec{E}$ from the result, so as to obtain the vector $\vec{F} = 2\vec{D} - \vec{E}$. Equation (1.8) says that to multiply $\vec{D}$ by 2, we multiply each of its components by 2. We can use Eq. (1.15) to do the subtraction; recall from Section 1.7 that subtracting a vector is the same as adding the negative of that vector.

EXECUTE We have

$$\begin{aligned}\vec{F} &= 2(6.00\hat{\imath} + 3.00\hat{\jmath} - 1.00\hat{k})\text{ m} - (4.00\hat{\imath} - 5.00\hat{\jmath} + 8.00\hat{k})\text{ m}\\ &= [(12.00 - 4.00)\hat{\imath} + (6.00 + 5.00)\hat{\jmath} + (-2.00 - 8.00)\hat{k}]\text{ m}\\ &= (8.00\hat{\imath} + 11.00\hat{\jmath} - 10.00\hat{k})\text{ m}\end{aligned}$$

From Eq. (1.11) the magnitude of $\vec{F}$ is

$$\begin{aligned}F &= \sqrt{F_x^2 + F_y^2 + F_z^2}\\ &= \sqrt{(8.00\text{ m})^2 + (11.00\text{ m})^2 + (-10.00\text{ m})^2}\\ &= 16.9\text{ m}\end{aligned}$$

EVALUATE Our answer is of the same order of magnitude as the larger components that appear in the sum. We wouldn't expect our answer to be much larger than this, but it could be much smaller.

KEYCONCEPT By using unit vectors, you can write a single equation for vector addition that incorporates the x-, y-, and z-components.

1.72 ••• Ricardo and Jane are standing under a tree in the middle of a pasture. An argument ensues, and they walk away in different directions. Ricardo walks 26.0 m in a direction 60.0° west of north. Jane walks 16.0 m in a direction 30.0° south of west. They then stop and turn to face each other. (a) What is the distance between them? (b) In what direction should Ricardo walk to go directly toward Jane?