DALHOUSIE
UNIVERSITY

PROJECT REPORT

VRP SOLUTION FOR FRESHROUTES LOGISTICS

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Executive Summary

This report presents an in-depth analysis of the distribution management challenges faced by FreshRoutes Logistics, a Toronto-based company specializing in the delivery of fresh produce and groceries. The primary objective of this study was to optimize the company's delivery routes using the Vehicle Routing Problem with Time Windows (VRPTW) model to enhance efficiency, reduce costs, and improve customer satisfaction.

The project involved constructing a model based on a dataset of 20 customers, representing a subset of the company's broader customer base. The model incorporated key parameters such as customer locations, time windows, vehicle capacities, and travel times. The Gurobi solver, integrated with Excel, was used to identify the optimal routes for a fleet of five homogeneous vehicles.

Key findings from the study indicate that the VRPTW model effectively automates and optimizes delivery operations, ensuring that all customers are served within their specified time windows while minimizing total delivery time. Although the model was simplified for this project, it successfully demonstrated the potential benefits of route optimization for FreshRoutes Logistics.

The report also explores various scenarios to assess the impact of changing parameters on the model's performance. The insights gained from this analysis suggest that, while the current model meets the company's needs, further enhancements could be made to address additional complexities, such as vehicle breakdowns or dynamic order cancellations.

In conclusion, the implementation of the VRPTW model has proven advantageous for FreshRoutes Logistics. Moving forward, the company could consider scaling the model to accommodate a heterogeneous fleet or expanding the scope to include multiple depots as the business grows.

Introduction

In today's fast-paced and highly competitive logistics industry, efficient distribution management is critical to maintaining service quality and customer satisfaction. FreshRoutes Logistics, a Toronto-based company, has established itself as a key player in the delivery of fresh produce and groceries to a diverse customer base, including households, restaurants, hospitals, and supermarkets. As the company continues to grow and expand its service area, particularly with increasing orders from locations outside its regular network, it faces significant challenges in optimizing its delivery routes.

The primary challenge for FreshRoutes Logistics lies in managing a growing volume of orders while ensuring that deliveries are completed within specified time windows, a critical factor for maintaining customer satisfaction. To address this challenge, the company is exploring the use of the Vehicle Routing Problem with Time Windows (VRPTW) model, a well-established optimization technique in the logistics industry. The VRPTW model aims to minimize total delivery time while adhering to constraints such as vehicle capacity and delivery time windows.

This report focuses on the application of the VRPTW model to a pilot scenario involving 20 customers within the Toronto suburb, serviced by a fleet of 5 homogeneous vehicles. The objective of this study is to verify the effectiveness of the model in optimizing delivery routes and to explore potential enhancements for future scalability.

The report is structured to provide a detailed overview of the problem context, the dataset and parameters used, the formulation and solving of the optimization model, and an analysis of the results. By systematically addressing the complexities of route optimization, this study seeks to offer valuable insights that can be applied to improve the efficiency and effectiveness of FreshRoutes Logistics' distribution operations, supporting the company's continued growth and competitive advantage in the market.

Project Description/Business Need

As FreshRoutes Logistics continues to expand its operations in Toronto, the company is experiencing a significant increase in the volume of orders. These orders originate from various locations, including those outside the company's regular service network, creating new challenges in maintaining efficient and timely deliveries. The delivery locations are geographically dispersed, and each customer has specific time windows within which their orders must be delivered. This growing demand is putting pressure on FreshRoutes Logistics' existing distribution network, potentially impacting service quality and customer satisfaction.

To address these challenges, FreshRoutes Logistics is seeking a route optimization solution that is both cost-effective and capable of improving overall operational efficiency. The company aims to minimize delivery times while ensuring that all deliveries are made within the designated time windows. Given the complexity of this task, FreshRoutes Logistics is implementing a pilot project using the Vehicle Routing Problem with Time Windows (VRPTW) model. This model will be tested with a limited set of customers and vehicles to validate its effectiveness before scaling it to meet the full scope of the company's operations.

The VRPTW model considers multiple factors, including the need to serve 20 customers within specific time frames, the use of a homogeneous fleet of 5 vehicles, and the constraints imposed by vehicle capacities and travel times. FreshRoutes Logistics utilizes temperature-controlled trucks to ensure product freshness, and it is crucial that these trucks operate within optimized routes to meet customer expectations and minimize operational costs.

By solving this routing problem, FreshRoutes Logistics aims to enhance its distribution network's efficiency, reduce operational costs, and ultimately improve customer satisfaction. The successful implementation of this model could lead to its adoption on a larger scale, potentially accommodating a more extensive customer base and a more complex distribution network in the future.

Literature Review

Optimization is a critical concept in various fields, particularly within the logistics and transportation industries. At its core, optimization involves identifying the most efficient and cost-effective solution to a given problem, often under a set of specific constraints. In the context of logistics, one of the most studied optimization problems is the Vehicle Routing Problem (VRP). The VRP seeks to determine the optimal set of routes for a fleet of vehicles to deliver goods to a set of customers, minimizing the total cost, which can include distance, time, or other resources.

The VRP was first formulated by Dantzig and Ramser in 1959 and has since been a subject of extensive research due to its practical importance and computational complexity. Over the years, numerous variants of the VRP have been developed to address different real-world scenarios. These include the Capacitated VRP (CVRP), VRP with Time Windows (VRPTW), VRP with Pickup and Delivery (VRPPD), and the VRP with Backhauls (VRPBH), among others. Each variant introduces specific constraints and objectives that reflect the complexities of real-life logistics operations. Furthermore, Clarke and Wright's savings algorithm (1964) and Solomon's insertion heuristics (1987) are notable early contributions that laid the groundwork for more advanced approaches.

The VRPTW, in particular, is an extension of the basic VRP that incorporates time windows within which deliveries must be completed. This variant is especially relevant for industries where timely deliveries are crucial to maintaining customer satisfaction and operational efficiency. The VRPTW has been extensively studied, with various algorithms and heuristics developed to solve it. In recent years, significant advancements have been made in solving large-scale instances of VRPTW, leveraging sophisticated techniques such as metaheuristics, branch-and-bound algorithms, and integer programming. The work of Potvin and Rousseau (1993) on genetic algorithms, as well as the contributions of Desrochers, Desrosiers, and Solomon (1992) on exact algorithms, have been particularly influential in this area.

For this project, the VRPTW model was chosen as the most appropriate tool for optimizing the delivery routes of FreshRoutes Logistics, a company that must navigate the complexities of delivering fresh produce and groceries within tight time windows. The literature provides a strong foundation for applying the VRPTW model in this context, offering insights into effective methods for minimizing total delivery time while adhering to time and capacity constraints.

Among the key references reviewed for this project, the paper titled "The VRP with Time Windows" by J.F. Cordeau and Guy Desaulniers stands out as a significant contribution to the field. This paper provides a comprehensive survey of the research on VRPTW, discussing various algorithms and their applicability to different problem sizes and complexities. It has served as a valuable resource in understanding the formulation of VRPTW and its practical applications in optimizing delivery operations.

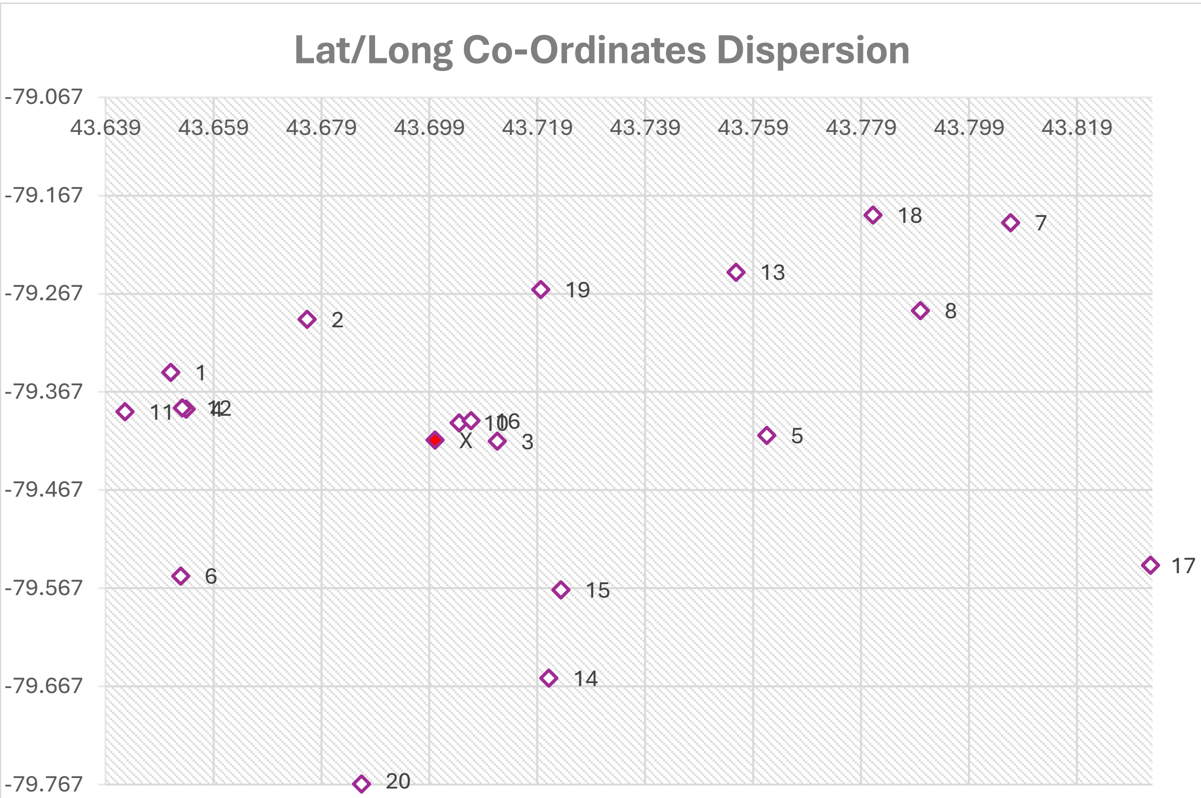
The literature also highlights the importance of considering additional real-world factors when applying VRPTW models, such as traffic conditions, vehicle breakdowns, and dynamic order changes. These factors can introduce additional layers of complexity, requiring further enhancements to the basic VRPTW model. Recent studies have explored extensions to the VRPTW, such as the Electric Vehicle Routing Problem (EVRP) and the Green Vehicle Routing Problem (GVRP), which incorporate environmental considerations into route optimization.

In conclusion, the literature on VRPTW and its variants provides a robust framework for addressing the distribution challenges faced by FreshRoutes Logistics. The insights gained from this body of research have been instrumental in guiding the formulation and solving of the optimization model used in this project. As FreshRoutes Logistics continues to expand its operations, the application of advanced VRP models, potentially incorporating additional complexities, will be critical to maintaining and enhancing its distribution efficiency.

Dataset and Variables

A dataset was generated to facilitate the formulation and solving of this model. The model was constructed for 20 customers. A significant portion of FreshRoutes Logistics' customers are restaurants, hospitals, and supermarkets. Therefore, 20 such geographic locations were identified within the Toronto suburb. The driving distance between each of the 20 customers and the FreshRoutes Logistics depot was calculated using the Bing Maps API, based on latitude and longitude coordinates. The resulting distance matrix is presented in Table

Lat/Long Co-Ordinates			
Delivery locations	X	Y	Delivery Locations
1	43.65107	-79.347015	Downtown Toronto (Near CN Tower)
2	43.676357	-79.293031	Greektown
3	43.711566	-79.416907	Yonge and Eglinton
4	43.653908	-79.384293	Distillery District
5	43.761539	-79.411079	Yorkdale Mall
6	43.652917	-79.554357	Etobicoke (Royal York Rd)
7	43.806686	-79.194353	Scarborough (STC area)
8	43.789998	-79.28412	Scarborough Bluffs
9	43.63889	-79.43035	Liberty Village
10	43.704521	-79.398568	Casa Loma
11	43.64257	-79.387057	Harbourfront
12	43.653226	-79.383184	Queen Street West
13	43.755798	-79.244873	East York
14	43.721112	-79.658277	Vaughan (near Wonderland)
15	43.723374	-79.568631	North York (Downsview Park)
16	43.706685	-79.396171	Forest Hill
17	43.832619	-79.543367	Thornhill (Markham area)
18	43.781188	-79.18679	Rouge National Urban Park
19	43.71962	-79.262657	Riverdale Park
20	43.686412	-79.76639	Brampton (Mount Pleasant area)
X	43.70011	-79.4163	FreshRoutes Logistics



			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Depot
		X	43.65107	43.676357	43.711566	43.65391	43.76154	43.65292	43.80669	43.79	43.63889	43.704521	43.64257	43.65323	43.7558	43.72111	43.72337	43.70669	43.83262	43.78119	43.71962	43.68641	43.70011
	X	Y	-79.347015	-79.293031	-79.416907	-79.3843	-79.4111	-79.5544	-79.1944	-79.2841	-79.4304	-79.398568	-79.387057	-79.3832	-79.2449	-79.6583	-79.5686	-79.3962	-79.5434	-79.1868	-79.2627	-79.7664	-79.4163
1	43.65107	-79.347015	0	5	9	3	13	17	21	16	7	7	3	3	14	26	20	7	26	19	10	34	8
2	43.676357	-79.293031	5	0	11	8	13	21	17	13	12	9	8	8	10	30	23	9	27	14	5	38	10
3	43.711566	-79.416907	9	11	0	7	6	13	21	14	8	2	8	7	15	19	12	2	17	20	12	28	1
4	43.653908	-79.384293	3	8	7	0	12	14	23	17	4	6	1	0	16	23	17	6	24	21	12	31	6
5	43.761539	-79.411079	13	13	6	12	0	17	18	11	14	6	13	12	13	20	13	6	13	18	13	30	7
6	43.652917	-79.554357	17	21	13	14	17	0	34	27	10	14	14	14	27	11	8	14	20	33	25	18	12
7	43.806686	-79.194353	21	17	21	23	18	34	0	7	27	20	24	23	7	39	32	20	28	3	11	48	21
8	43.789998	-79.28412	16	13	14	17	11	27	7	0	21	13	18	17	5	31	24	13	21	8	8	41	15
9	43.63889	-79.43035	7	12	8	4	14	10	27	21	0	8	4	4	20	21	15	8	23	25	16	28	7
10	43.704521	-79.398568	7	9	2	6	6	14	20	13	8	0	7	6	14	21	14	0	18	19	11	30	2
11	43.64257	-79.387057	3	8	8	1	13	14	24	18	4	7	0	1	17	24	17	7	25	22	13	31	7
12	43.653226	-79.383184	3	8	7	0	12	14	23	17	4	6	1	0	16	23	17	6	24	21	12	31	6
13	43.755798	-79.244873	14	10	15	16	13	27	7	5	20	14	17	16	0	34	26	13	25	5	4	43	15
14	43.721112	-79.658277	26	30	19	23	20	11	39	31	21	21	24	23	34	0	7	21	15	39	32	10	20
15	43.723374	-79.568631	20	23	12	17	13	8	32	24	15	14	17	17	26	7	0	14	12	31	25	16	13
16	43.706685	-79.396171	7	9	2	6	6	14	20	13	8	0	7	6	13	21	14	0	18	19	11	30	2
17	43.832619	-79.543367	26	27	17	24	13	20	28	21	23	18	25	24	25	15	12	18	0	29	26	24	18
18	43.781188	-79.18679	19	14	20	21	18	33	3	8	25	19	22	21	5	39	31	19	29	0	9	48	21
19	43.71962	-79.262657	10	5	12	12	13	25	11	8	16	11	13	12	4	32	25	11	26	9	0	41	13
20	43.686412	-79.76639	34	38	28	31	30	18	48	41	28	30	31	31	43	10	16	30	24	48	41	0	28
Depot	43.70011	-79.4163	8	10	1	6	7	12	21	15	7	2	7	6	15	20	13	2	18	21	13	28	0

The above distance matrix serves as the foundation for the model formulation. We have 20 customers, which corresponds to 20 jobs. For the purpose of modeling, time windows were randomly generated in terms of minutes, ranging from 420 to 960, which corresponds to a clock time of 7 a.m. to 4 p.m. The average duration and width of these time windows are set at 10 minutes.

The model considers a homogeneous fleet of 4 identical vehicles, all with the same capacities and starting from the same depot. The average speed is assumed to be 60 km/h, or 1 km/min, and the vehicle cost is calculated at 200 minutes. The maximum truck capacity is set at 50 units.

Model Formulation

The problem is initially formulated as a VRPTW model and then the capacity constraints are added at a later stage. This is a minimization problem.

$$Minimize = \sum_{i=1}^{n+m} \sum_{j=1}^{n+m} Cij\ Xij \qquad (5.1)$$

S.T:

$$\sum_{i=1}^{n+m} Xij = 1\,, \qquad \forall\ j = 1, \ldots, n+m, \qquad (5.2)$$

$$\sum_{j=1}^{n+m} Xij = 1\,, \qquad \forall\ i = 1, \ldots, n+m, \tag{5.3}$$

$$xij \in 0{,}1 \quad \forall i, j = 1, \ldots, n+m \tag{5.4}$$

$$\mathrm{S}i + di + tij - sj \le Mij\ 1 - xij \qquad \forall\ i = 1, \ldots, n;\ j = 1, \ldots, n \tag{5.5}$$

$$bi \le si \le ei \qquad \forall\ i = 1, \ldots, n$$

$$ui + tij - uj \le Mij\ 1 - xij \qquad \forall\ i = 1, \ldots, n+m-1;\ j = 1, \ldots, n+m$$

(5.6) $0 \le ui \le G \quad \forall\ i = 1, \ldots, n$

$$ui = 0 \qquad \forall\ i = n+1, \ldots, n+m$$

$$ri + pi - rj \le M\ 1 - xij \qquad \forall\ i = 1, \ldots, n;\ j = 1, \ldots, n \tag{5.7}$$

$$0 \le ri \le Q - p\mathrm{i} \qquad \forall\ i = 1, \ldots, n$$

Parameters: n = number of points (1 - depot, 2, … , n - customers)

d_{ij} = distance from point i to point j

p_i = demand of client i

Q = capacity of each truck

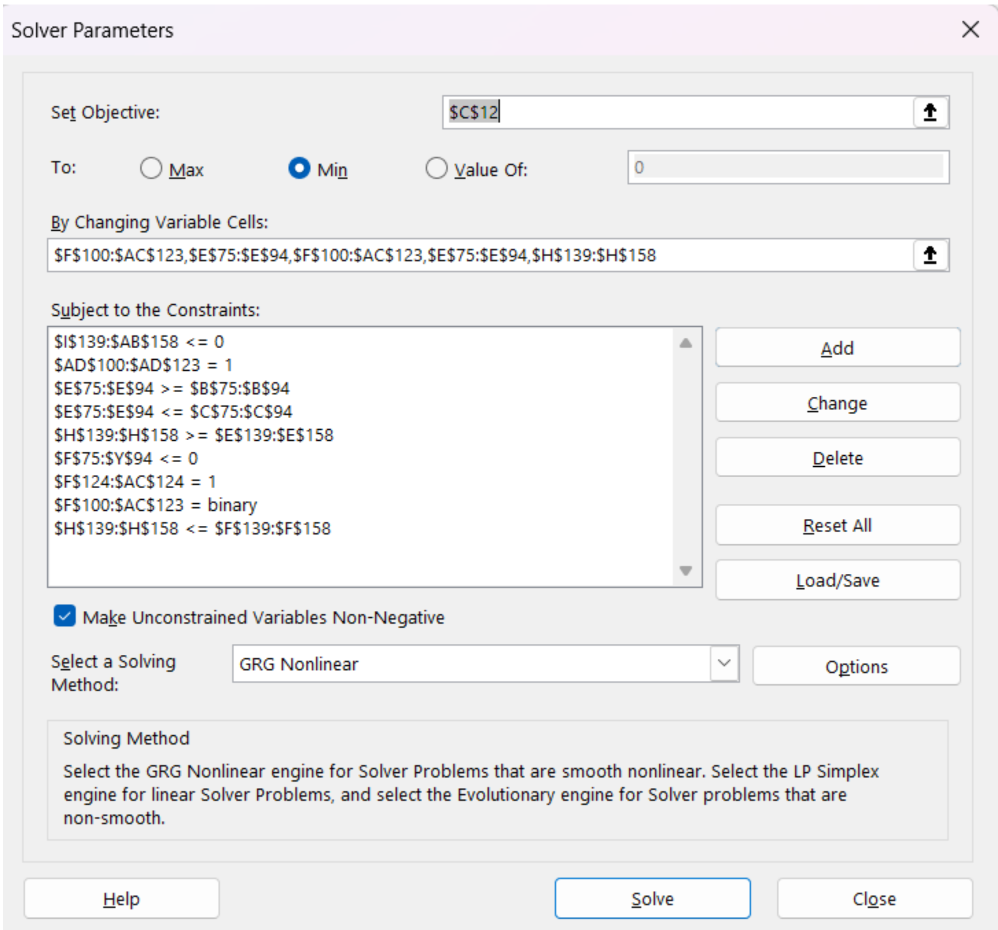
Constraints:

(5-1) Represents the cost of traveling from node i to node j
(5-2) & (5-3) These constraints ensure that exactly one arc enters and exactly one arc leaves each vertex associated with a customer.

(5-4) Represents the integrality constraint, ensuring that the decision variables are integers.
(5-5) Represents the time window constraints, ensuring that no time window requirements are violated.
(5-6) Represents the travel time between the vehicles and the travel time from the depot.
(5-7) Represents the capacity constraints, ensuring that the vehicle capacities are not exceeded.

Solution Approach

The approach to solving the distribution challenges faced by FreshRoutes Logistics involved leveraging the Vehicle Routing Problem with Time Windows (VRPTW) model to optimize delivery routes. The model was designed to balance the need for timely deliveries with the operational constraints of vehicle capacity and service time. By incorporating real-world data, such as customer locations and time-sensitive delivery windows, the model aimed to create an efficient routing plan that minimized total travel time while ensuring high service quality. The optimization process was executed using the Gurobi solver in Excel, which enabled the handling of complex constraints and provided actionable insights into how FreshRoutes Logistics could enhance its delivery operations. This pilot implementation not only validated the model's effectiveness for a smaller subset of customers but also laid the groundwork for scaling up to more extensive scenarios as the company continues to grow.



Result

A model was constructed for the above dataset, constraints, and variables. The solution is as follows:

X:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
3	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
4	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
6	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1
7	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
8	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1
10	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
11	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
12	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
13	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
18	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
19	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
22	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
23	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
24	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	

We can observe that all five vehicles are in use, with each vehicle completing a route that connects back to itself. All 20 customers (nodes) have been successfully served, with each customer being visited exactly once. Furthermore, all constraints have been adhered to, ensuring that no violations occurred in the solution.

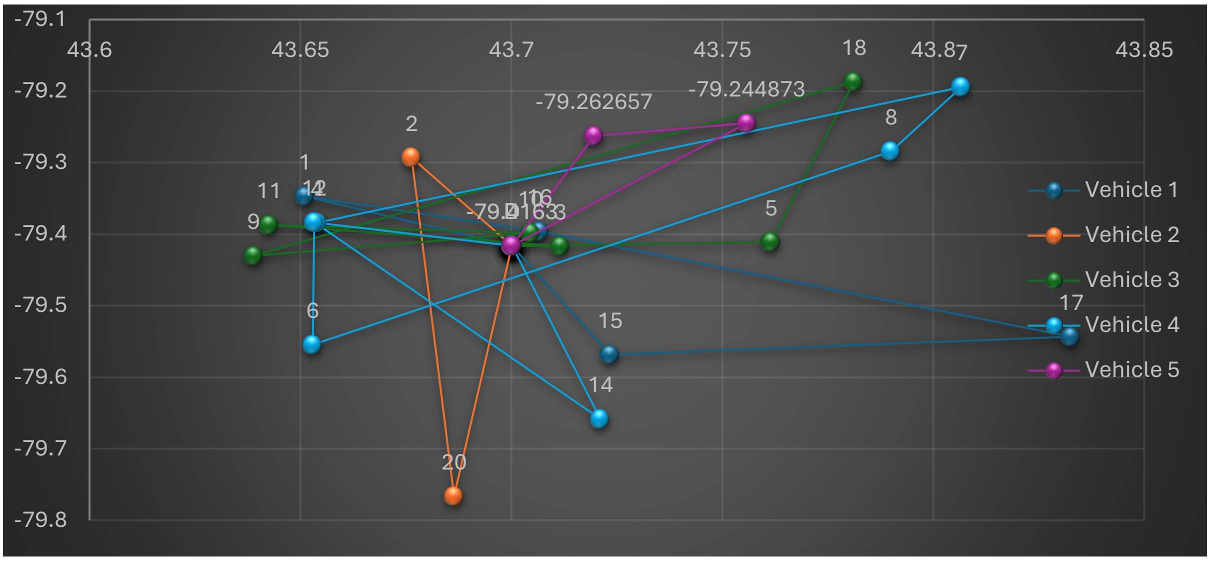
The primary objective of this model is to minimize the total time required to complete all deliveries to the 20 customers located throughout the Toronto suburb:

TOTAL OBJECTIVE 1131

By analyzing the output, we can determine the optimal route for each of the five vehicles, including identifying the first and last customers served by each vehicle:

Vehicle	Order of Deliveries						
Vehicle 1	1	16	17	15			
Vehicle 2	2	20					
Vehicle 3	3	11	10	9	18	5	
Vehicle 4	4	7	8	6	12	14	
Vehicle 5	19	13					

By further visualizing the optimal routes, we can observe the delivery paths of all five vehicles across all nodes. Even though some routes may appear suboptimal visually, all nodes, including outliers, are covered effectively. It's important to note that the time windows were randomly selected, and capacity requirements were incorporated into the problem, with no constraints being violated. Therefore, we can conclude that the solution is indeed optimal, with the time windows being the dominant factor in determining the routes.



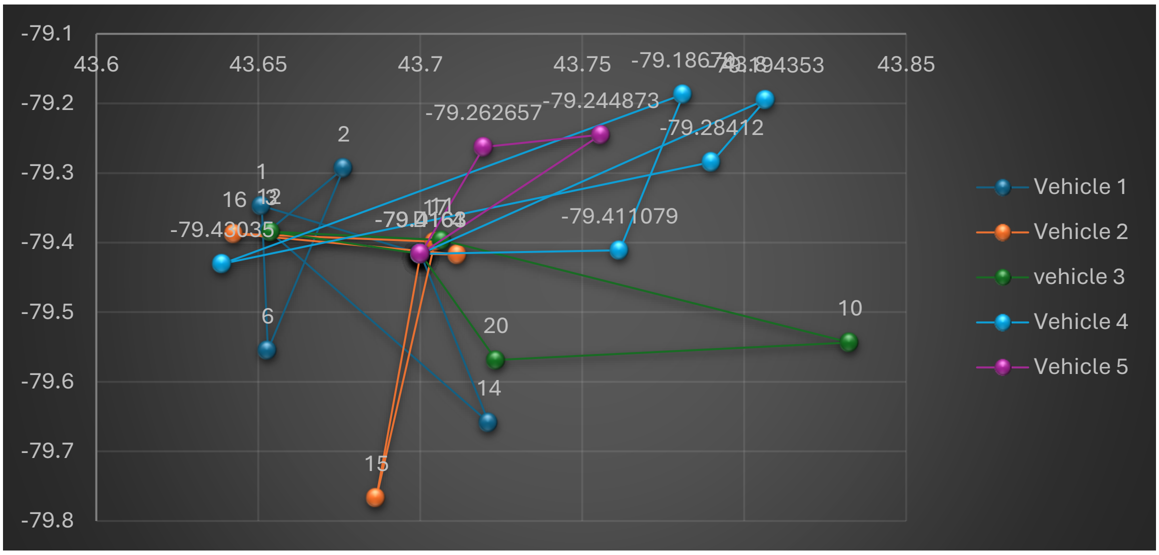
Scenarios and Analysis

Additional scenarios were explored to assess how altering certain parameters would impact the results.

Scenario 1: Increased Vehicle Capacity

Change the vehicle capacity to 80 units, which is 30 units higher than the previous problem. Upon solving, the total objective value found is 1087 and number of vehicles used are 5. The results are summarized below –

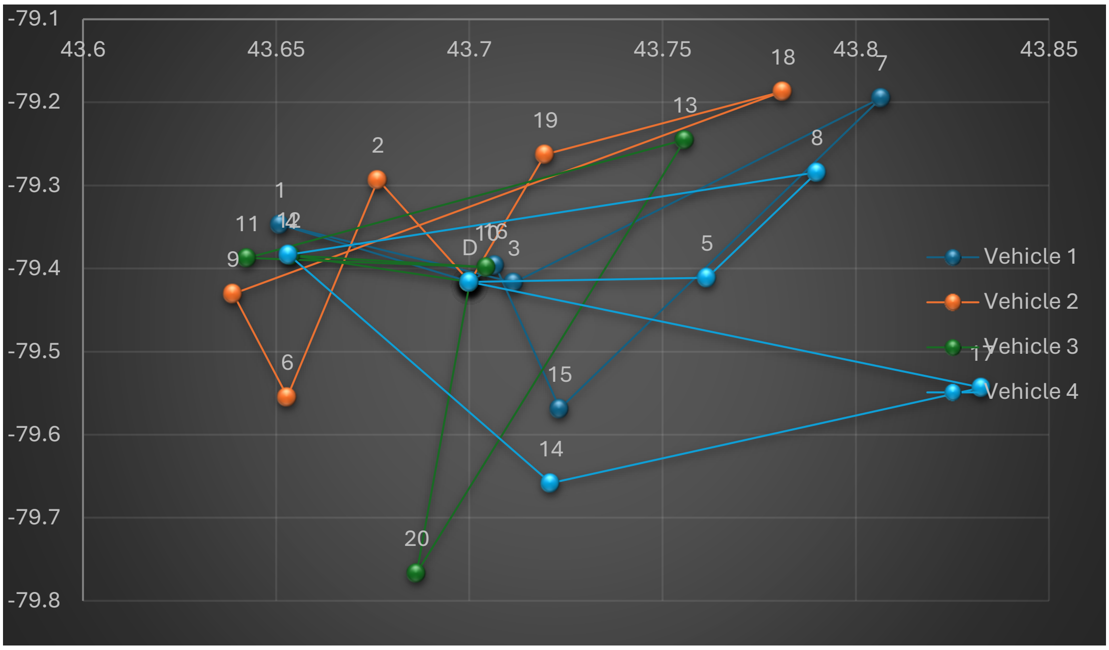
TOTAL OBJECTIVE 1087



Scenario 2: Pre-allocated Time Windows

This time the 'Time windows' are not selected randomly, instead the earliest start times are pre-allocated by increment of 10 mins from 440 to 630 for the 20 customers. Similarly, the width is averaged at 15 mins. However, the capacity remains the same at 50 units/vehicle. Upon solving, the total objective value found is 1129 and the number of vehicles used is 4.

TOTAL OBJECTIVE 1129


-79.1
-79.2
-79.3
-79.4
-79.5
-79.6
-79.7
-79.8
43.6
43.65
43.7
43.75
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Vehicle 1
Vehicle 2
Vehicle 3
Vehicle 4
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Discussion

The above two scenarios suggest that any change in parameter such as vehicle capacity or time windows will influence the total objective and the fleet size. Particularly, in scenario 1, we can observe that by increasing the truck capacity to 30 units, the total objective has reduced to 1087 as opposed to the primary problem in which the total objective is 1131. The increased capacity allowed each vehicle to carry more, but the total objective value still required the use of all 5 vehicles, suggesting limited impact on overall fleet efficiency.

In scenario 2 too, it is apparent that any change in time window or width has an impact on the solution. In this case, where the earliest start times are not randomly selected instead allocated in increment of 10 minutes gap. We can observe that there has not been any significant change in the total objective, but the number of vehicles has decreased by 1. The delivery paths also appear relatively improvised and desirable. This is a likely case if the time slots of deliveries are decided by the supplier and not by the customers. Pre-allocating time windows led to a slightly higher total objective value but reduced the fleet size to 4 vehicles, indicating better route efficiency despite more rigid timing constraints

Conclusion

The proposed model was solved in Excel using the Gurobi solver. The problem is relatively small, involving only 20 customers and 5 vehicles. This limited set of customers was selected to ensure that the model functions correctly for the identified customers across the Toronto suburb. The decision to use 4 vehicles aimed to produce a solution with multiple routes. In a real-life scenario, FreshRoutes Logistics receives hundreds of orders with various time slots, necessitating a large delivery fleet to ensure all customers are served promptly.

In conclusion, the VRPTW model has proven beneficial to FreshRoutes Logistics by automating and optimizing their delivery system. Although the problem was simplified for the purposes of this project, real-world scenarios would likely require the incorporation of additional complexities and constraints. These could include vehicle delays due to traffic congestion, vehicle breakdowns, or dynamic order cancellations, where the concept of 'penalties' would become relevant. Moreover, this problem serves as a foundation for further enhancements. In the future, this model could be upgraded to accommodate a heterogeneous fleet and a multi-depot system, especially if the company expands its operations to other provinces.

Reference

1. **Bing Maps API**:

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