Foundations for statistical inference - Confidence intervals

If you have access to data on an entire population, say the opinion of every adult in the United States on whether or not they think climate change is affecting their local community, it's straightforward to answer questions like, "What percent of US adults think climate change is affecting their local community?". Similarly, if you had demographic information on the population you could examine how, if at all, this opinion varies among young and old adults and adults with different leanings. If you have access to only a sample of the population, as is often the case, the task becomes more complicated. What is your best guess for this proportion if you only have data from a small sample of adults? This type of situation requires that you use your sample to make inference on what your population looks like.

Setting a seed: You will take random samples and build sampling distributions in this lab, which means you should set a seed on top of your lab. If this concept is new to you, review the lab on probability.

Getting Started

Load packages

In this lab, we will explore and visualize the data using the **tidyverse** suite of packages, and perform statistical inference using **infer**.

Let's load the packages.

```
library(tidyverse)
library(openintro)
library(infer)
```

The data

A 2019 Pew Research report states the following:

To keep our computation simple, we will assume a total population size of 100,000 (even though that's smaller than the population size of all US adults).

> Roughly six-in-ten U.S. adults (62%) say climate change is currently affecting their local community either a great deal or some, according to a new Pew Research Center survey.
>
> **Source:** Most Americans say climate change impacts their community, but effects vary by region

In this lab, you will assume this 62% is a true population proportion and learn about how sample proportions can vary from sample to sample by taking smaller samples from the population. We will first create our population assuming a population size of 100,000. This means 62,000 (62%) of the adult population think climate change impacts their community, and the remaining 38,000 does not think so.

```
us_adults <- tibble(
  climate_change_affects = c(rep("Yes", 62000), rep("No", 38000))
)
```

The name of the data frame is `us_adults` and the name of the variable that contains responses to the question *"Do you think climate change is affecting your local community?"* is `climate_change_affects`.

We can quickly visualize the distribution of these responses using a bar plot.

```
ggplot(us_adults, aes(x = climate_change_affects)) +
  geom_bar() +
  labs(
    x = "", y = "",
    title = "Do you think climate change is affecting your local community?"
  ) +
  coord_flip()
```

We can also obtain summary statistics to confirm we constructed the data frame correctly.

```
us_adults %>%
  count(climate_change_affects) %>%
  mutate(p = n /sum(n))
```

```
## # A tibble: 2 x 3
##   climate_change_affects     n     p
##   <chr>                  <int> <dbl>
## 1 No                     38000  0.38
## 2 Yes                    62000  0.62
```

In this lab, you'll start with a simple random sample of size 60 from the population.

```
set.seed(1992)
n <- 60
samp <- us_adults %>%
  sample_n(size = n)
```

1. What percent of the adults in your sample think climate change affects their local community? **Hint:** Just like we did with the population, we can calculate the proportion of those **in this sample** who think climate change affects their local community.

Answer:

```
samp %>%
  count(climate_change_affects) %>%
  mutate(sp = n /sum(n))
```

```
## # A tibble: 2 x 3
##   climate_change_affects     n    sp
##   <chr>                  <int> <dbl>
## 1 No                        22 0.367
## 2 Yes                       38 0.633
```

According to the sample of a size 60 we have in total 63.3 percent of the people who think the climate changes is affecting their local community.

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1. Would you expect another student's sample proportion to be identical to yours? Would you expect it to be similar? Why or why not?

Answer:

The chances of getting similar result is very low but not impossible mainly because the sample size is too small compared to 100k. Adding to that, due the randomness in the sampling process, i did run the code for sample a few times without setting the seed and i got different answer every time.

Confidence intervals

Return for a moment to the question that first motivated this lab: based on this sample, what can you infer about the population? With just one sample, the best estimate of the proportion of US adults who think climate change affects their local community would be the sample proportion, usually denoted as $\hat{p}$ (here we are calling it `p_hat`). That serves as a good **point estimate**, but it would be useful to also communicate how uncertain you are of that estimate. This uncertainty can be quantified using a **confidence interval**.

One way of calculating a confidence interval for a population proportion is based on the Central Limit Theorem, as $\hat{p} \pm z^\star SE_{\hat{p}}$ is, or more precisely, as

$$\hat{p} \pm z^\star \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Another way is using simulation, or to be more specific, using **bootstrapping**. The term **bootstrapping** comes from the phrase "pulling oneself up by one's bootstraps", which is a metaphor for accomplishing an impossible task without any outside help. In this case the impossible task is estimating a population parameter (the unknown population proportion), and we'll accomplish it using data from only the given sample. Note that this notion of saying something about a population parameter using only information from an observed sample is the crux of statistical inference, it is not limited to bootstrapping.

In essence, bootstrapping assumes that there are more of observations in the populations like the ones in the observed sample. So we "reconstruct" the population by resampling from our sample, with replacement. The bootstrapping scheme is as follows:

- **Step 1.** Take a bootstrap sample - a random sample taken **with replacement** from the original sample, of the same size as the original sample.
- **Step 2.** Calculate the bootstrap statistic - a statistic such as mean, median, proportion, slope, etc. computed on the bootstrap samples.
- **Step 3.** Repeat steps (1) and (2) many times to create a bootstrap distribution - a distribution of bootstrap statistics.
- **Step 4.** Calculate the bounds of the XX% confidence interval as the middle XX% j knof the bootstrap distribution.

Instead of coding up each of these steps, we will construct confidence intervals using the **infer** package.

Below is an overview of the functions we will use to construct this confidence interval:

Function	Purpose
`specify`	Identify your variable of interest
`generate`	The number of samples you want to generate
`calculate`	The sample statistic you want to do inference with, or you can also think of this as the population parameter you want to do inference for
`get_ci`	Find the confidence interval

This code will find the 95 percent confidence interval for proportion of US adults who think climate change affects their local community.

```
set.seed(1992)
samp %>%
  specify(response = climate_change_affects, success = "Yes") %>%
  generate(reps = 1000, type = "bootstrap") %>%
  calculate(stat = "prop") %>%
  get_ci(level = 0.95)
```

```
## # A tibble: 1 x 2
##   lower_ci upper_ci
##      <dbl>    <dbl>
## 1    0.517     0.75
```

- In `specify` we specify the `response` variable and the level of that variable we are calling a `success`.
- In `generate` we provide the number of resamples we want from the population in the `reps` argument (this should be a reasonably large number) as well as the type of resampling we want to do, which is `"bootstrap"` in the case of constructing a confidence interval.
- Then, we `calculate` the sample statistic of interest for each of these resamples, which is `prop`ortion.

Feel free to test out the rest of the arguments for these functions, since these commands will be used together to calculate confidence intervals and solve inference problems for the rest of the semester. But we will also walk you through more examples in future chapters.

To recap: even though we don't know what the full population looks like, we're 95% confident that the true proportion of US adults who think climate change affects their local community is between the two bounds reported as result of this pipeline.

Confidence levels

1. In the interpretation above, we used the phrase "95% confident". What does "95% confidence" mean?

Answer:

The 95% confidence level means that, we are 95% confident that the true proportion of US adults who thinks that climate change is affecting the local community is between 55-78.3%.

In this case, you have the rare luxury of knowing the true population proportion (62%) since you have data on the entire population.

1. Does your confidence interval capture the true population proportion of US adults who think climate change affects their local community? If you are working on this lab in a classroom, does your neighbor's interval capture this value?

Answer:

Yes my confidence interval did capture the true population proportion. Since we know the true populations proportion for the entire population is 62% and our range of 55% to 78,3% actually covers that.

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1. Each student should have gotten a slightly different confidence interval. What proportion of those intervals would you expect to capture the true population mean? Why?

Answer:

Each student should get a slightly different confidence interval due to different samples of US adults that each one will select, but one would expect at least 95% of students (if not all of them) to capture the true population mean. This is because as mentioned above, there is just a slight difference in confidence interval each one will get, and as we are all working in 95% level, we are all 95% confident that the true population proportion is contained in our confidence interval, that's, I would expect at least 95% of those intervals to capture the true population.

In the next part of the lab, you will collect many samples to learn more about how sample proportions and confidence intervals constructed based on those samples vary from one sample to another.

- Obtain a random sample.
- Calculate the sample proportion, and use these to calculate and store the lower and upper bounds of the confidence intervals.
- Repeat these steps 50 times.

Doing this would require learning programming concepts like iteration so that you can automate repeating running the code you've developed so far many times to obtain many (50) confidence intervals. In order to keep the programming simpler, we are providing the interactive app below that basically does this for you and created a plot similar to Figure 5.6 on OpenIntro Statistics, 4th Edition (page 182).

1. Given a sample size of 60, 1000 bootstrap samples for each interval, and 50 confidence intervals constructed (the default values for the above app), what proportion of your confidence intervals include the true population proportion? Is this proportion exactly equal to the confidence level? If not, explain why. Make sure to include your plot in your answer.

Answer:

96% of my confidence intervals included true population proportion. It is not exactly equal to the confidence level but in fact differ by a mere percent and rightly so because the confidence level of 95% means that i have chance of 5% to be wrong but not necessarily to be wrong exactly 5% so that is why in this particular scenario I'm only four percent wrong with 95% of confidence level.

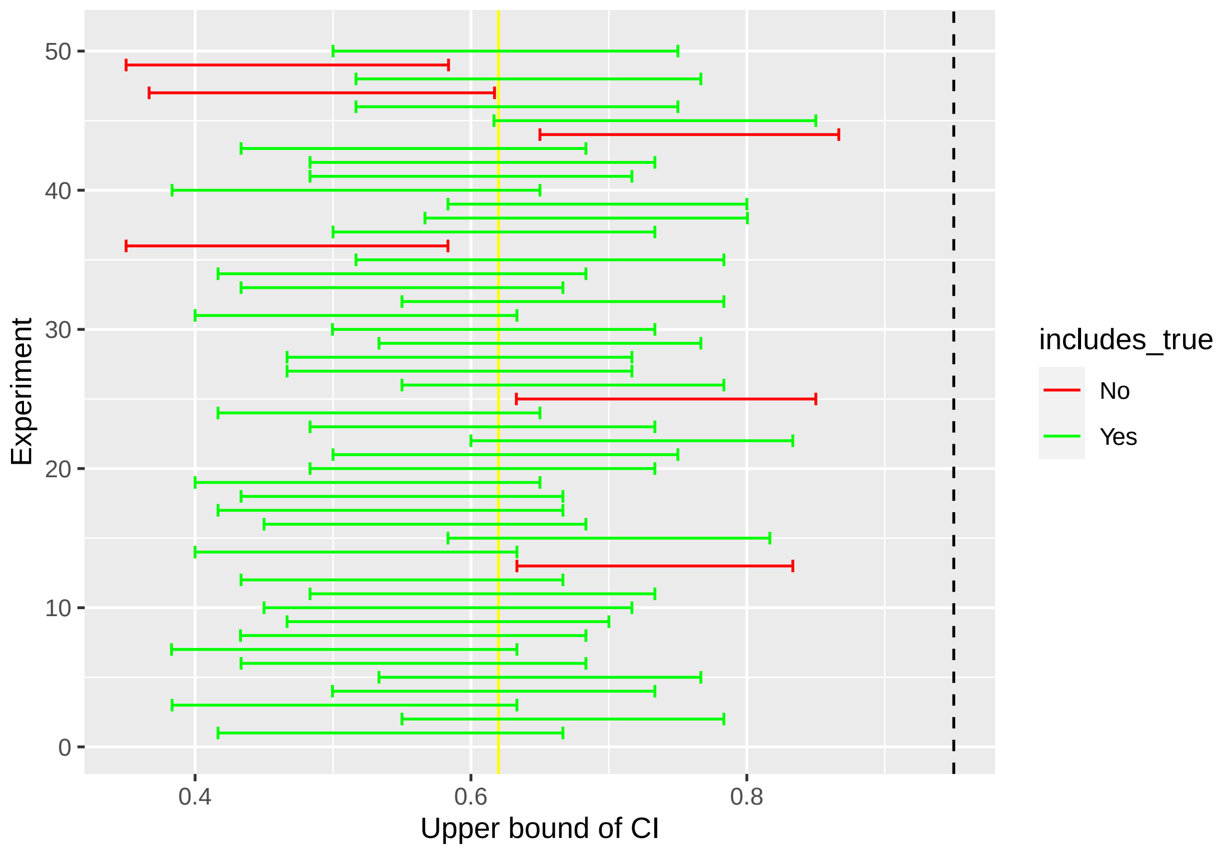
```r
set.seed(1333)

size <- 60
n_boots <- 1000
p_true <- us_adults %>%
  filter(climate_change_affects == "Yes") %>%
  summarize(p = n()/nrow(us_adults)) %>%
  pull()

results <- tibble()
for (i in 1:50) {
  samp <- us_adults %>%
    sample_n(size)
  ci <- samp %>%
    specify(response = climate_change_affects, success = "Yes") %>%
    generate(reps = n_boots, type = "bootstrap") %>%
    calculate(stat = "prop") %>%
    get_ci(level = 0.95)
  result <- tibble(experiment = i,
                   lower_ci = ci$lower_ci,
                   upper_ci = ci$upper_ci,
                   p_true = p_true)
  results <- bind_rows(results, result)
}

results <- results %>%
  mutate(includes_true = if_else(p_true >= lower_ci & p_true <= upper_ci, "Yes", "No"))
prop_includes_p_true <- results %>%
  summarize(prop_includes_true = mean(includes_true == "Yes"))


library(ggplot2)
ggplot(results, aes(x = experiment, y = upper_ci)) +
  geom_hline(yintercept = p_true, color = "yellow") +
  geom_hline(yintercept = 0.95, linetype = "dashed")+
  geom_errorbar(aes(ymin = lower_ci, ymax = upper_ci, color = includes_true)) +
  scale_color_manual(values = c("No" = "red", "Yes" = "green")) +
  labs(x = "Experiment", y = "Upper bound of CI") +
  coord_flip()
```



More Practice

1. Choose a different confidence level than 95%. Would you expect a confidence interval at this level to me wider or narrower than the confidence interval you calculated at the 95% confidence level? Explain your reasoning.

Answer:

If we choose a confidence level more than 95% the confidence interval widens and if we go below the 95% the confidence interval gets narrower. The reason behind is that when imagine a situation in which trying to solve a mathematical problem with in you mind and you're 95% confident that the correct answer lies between 15 and 20. Then I ask you to give me your confidence for it falling between 17 and 18. The correct answer is less likely to fall within the narrower interval, so your confidence naturally decreases.

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1. Using code from the **infer** package and data fromt the one sample you have (`samp`), find a confidence interval for the proportion of US Adults who think climate change is affecting their local community with a confidence level of your choosing (other than 95%) and interpret it.

Answer:

```r
set.seed(1992)
samp %>%
  specify(response = climate_change_affects, success = "Yes") %>%
  generate(reps = 1000, type = "bootstrap") %>%
  calculate(stat = "prop") %>%
  get_ci(level = 0.85)
```

```
## # A tibble: 1 x 2
##   lower_ci upper_ci
##      <dbl>    <dbl>
## 1    0.533    0.717
```

I'm 85% confident that the proportion of true population of US adults that believe that climate changes is affecting their communities is between 55-71.7%

---

1. Using the app, calculate 50 confidence intervals at the confidence level you chose in the previous question, and plot all intervals on one plot, and calculate the proportion of intervals that include the true population proportion. How does this percentage compare to the confidence level selected for the intervals?

Answer:

I tried it on app twice and I got 85% and 87% which lies with in the range of 85% confidence.

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1. Lastly, try one more (different) confidence level. First, state how you expect the width of this interval to compare to previous ones you calculated. Then, calculate the bounds of the interval using the **infer** package and data from `samp` and interpret it. Finally, use the app to generate many intervals and calculate the proportion of intervals that are capture the true population proportion.

*Answer:**

This time i will try to drop down my confidence level to 40% and then lets see if the interval gets narrower or not.

```r
set.seed(1992)
samp %>%
  specify(response = climate_change_affects, success = "Yes") %>%
  generate(reps = 1000, type = "bootstrap") %>%
  calculate(stat = "prop") %>%
  get_ci(level = 0.40)
```

```
## # A tibble: 1 x 2
##   lower_ci upper_ci
##      <dbl>    <dbl>
## 1      0.6    0.667
```

As we can see that the interval got narrower and sits around 60-66.7%. Which confirms our previous explanations.

According to the shiny app 46% of the confidence interval capture the true population proportion.

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1. Using the app, experiment with different sample sizes and comment on how the widths of intervals change as sample size changes (increases and decreases).

Answer:

With the increment in sample size the interval actually decreases and vice versa. The reason being that with we are getting closer the total population and also the spread drops so does the standard error.

The shiny app confirms that as soon as i increase the sample to 1000 the x axis got narrower, indicating that the interval got narrower. We can confirm that by the code below:

```
set.seed(1992)
n <- 1000
sam1000 <- us_adults %>%
  sample_n(size = n)

sam1000 %>%
  specify(response = climate_change_affects, success = "Yes") %>%
  generate(reps = 1000, type = "bootstrap") %>%
  calculate(stat = "prop") %>%
  get_ci(level = 0.95)
```

```
## # A tibble: 1 x 2
##   lower_ci upper_ci
##      <dbl>    <dbl>
## 1    0.604    0.661
```

As soon as the sample size went up to 1000 so the interval got narrower to 60.4-66.1%.

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1. Finally, given a sample size (say, 60), how does the width of the interval change as you increase the number of bootstrap samples. **Hint:** Does changing the number of bootstap samples affect the standard error?

Answer:

```
set.seed(1992)
samp %>%
  specify(response = climate_change_affects, success = "Yes") %>%
  generate(reps = 100000, type = "bootstrap") %>%
  calculate(stat = "prop") %>%
  get_ci(level = 0.95)
```

```
## # A tibble: 1 x 2
##   lower_ci upper_ci
##      <dbl>    <dbl>
## 1    0.517     0.75
```

Increasing the number of bootstrap slightly effects the interval and makes it narrower if the bootstraps are increase it can be confirmed by the code above. But again since there a other factor like sample size and confidence that effect the interval too so the effect might not be that great and it can be confirmed by the shiny app too.

Bootstrapping is not a tool to make standard errors smaller. It is a tool to, among other things to give an estimate of standard error. * * *