

AI1103-Assignment 2

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Download latex-tikz codes from

<https://github.com/Umesh-k26/AI-1103/blob/main/Assignment2/assignment2.tex>

and python codes from

<https://github.com/Umesh-k26/AI-1103/blob/main/Assignment2/codes/assignment2.py>

QUESTION

Let the probability density function of a random variable X be

$$f(x) = \begin{cases} x & 0 \leq x < \frac{1}{2} \\ c(2x-1)^2 & \frac{1}{2} < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

then, the value of c is equal to ____.

SOLUTION

For a probability density function of a continuous random variable,

$$\int_{-\infty}^{\infty} f(x) dx = 1 \quad (0.0.1)$$

$$\int_{-\infty}^{\infty} f(x) dx = \int_0^{1/2} f(x) dx + \int_{1/2}^1 f(x) dx \quad (0.0.2)$$

$$= \frac{1}{2}(x)(x) \Big|_{x=\frac{1}{2}} + \int_{1/2}^1 c(2x-1)^2 dx \quad (0.0.3)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + c \left(\frac{4x^3}{3} - 2x^2 + x \right) \Big|_{1/2}^1 \quad (0.0.4)$$

$$= \frac{1}{8} + c \left(\frac{1}{3} - \frac{1}{6} \right) \quad (0.0.5)$$

$$= \frac{1}{8} + \frac{c}{6} \quad (0.0.6)$$

from (0.0.1) and (0.0.6) we get

$$1 = \frac{1}{8} + \frac{c}{6} \quad (0.0.7)$$

$$\therefore c = \frac{21}{4} \quad (0.0.8)$$

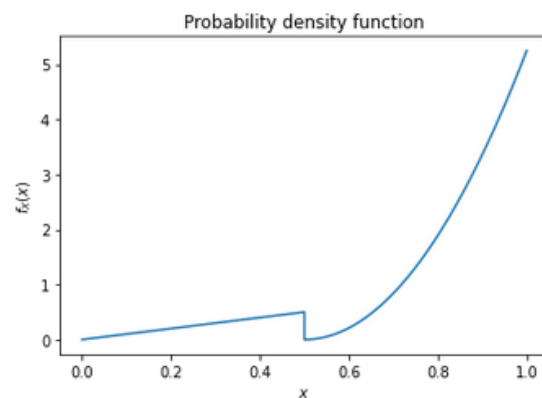


Fig. 0: PDF of $f(x)$

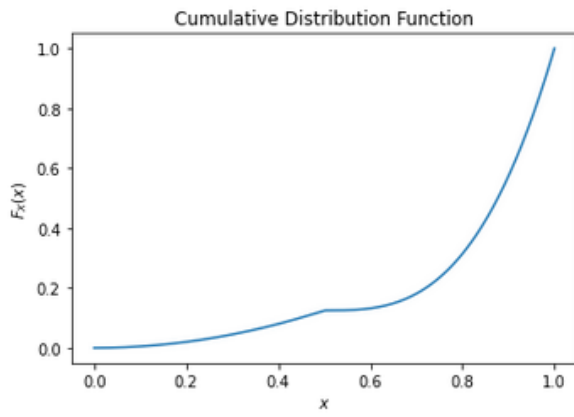


Fig. 0: CDF of $f(x)$