AI1103-Assignment 4

Name : Umesh Kalvakuntla Roll No. : CS20BTECH11024

Download latex-tikz codes from

and python codes from

UGC/MATH 2019, Q.50

Let X_1, X_2, X_3, X_4, X_5 be *i.i.d.* random variables having a continuous distribution function. Then

$$\Pr(X_1 > X_2 > X_3 > X_4 > X_5 | X_1 = max(X_1, X_2, X_3, X_4, X_5))$$

equals _____.

(A) $\dfrac{1}{4}$

(B) $\dfrac{1}{5}$

(C) $\dfrac{1}{4!}$

(D) $\dfrac{1}{5!}$

Solution

Required probability

$$= \Pr(X_1 > X_2 > X_3 > X_4 > X_5 | X_1 = max(X_1, X_2, X_3, X_4, X_5)) \tag{0.0.1}$$

$$= \Pr(X_2 > X_3 > X_4 > X_5) \tag{0.0.2}$$

$$= \Pr(X_2 = max(X_2, X_3, X_4, X_5)) \\ \times \Pr(X_3 = max(X_3, X_4, X_5)) \tag{0.0.3} \\ \times \Pr(X_4 = max(X_4, X_5))$$

Since X_1, X_2, X_3, X_4 and X_5 are identical and independently distributed random variables, required probability is

$$\Pr(X_2 = max(X_2, X_3, X_4, X_5))$$

$$= \int_{-\infty}^{\infty} \Pr(X_3, X_4, X_5 < x | X_2 = x) \tag{0.0.4}$$

$$= \int_{-\infty}^{\infty} F_{X_3}(x) F_{X_4}(x) F_{X_5}(x) f_{X_2}(x)\, dx \tag{0.0.5}$$

$$= \int_{-\infty}^{\infty} (\Phi(x))^3 \phi(x)\, dx \tag{0.0.6}$$

Substituting

$$u = \Phi(x) \tag{0.0.7}$$

$$du = \phi(x)\, dx \tag{0.0.8}$$

and similarly solving other two terms of the product, we get required probability

$$= \int_0^1 u^3\, du \times \int_0^1 u^2\, du \times \int_0^1 u\, du \tag{0.0.9}$$

$$= \frac{u^4}{4} \Big]_0^1 \times \frac{u^3}{3} \Big]_0^1 \times \frac{u^2}{2} \Big]_0^1 \tag{0.0.10}$$

$$= \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \tag{0.0.11}$$

$$= \frac{1}{4!} \tag{0.0.12}$$

Here $\phi(x)$ and $\Phi(x)$ represent the pdf and cdf of respectively of random variables X_1, X_2, X_3, X_4, X_5.

The correct answer is **Option (C)**.