AI1103-Assignment 4

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Download latex-tikz codes from

https://github.com/Umesh−k26/AI−1103/blob/main/Assignment4/assignment4.tex

and python codes from

https://github.com/Umesh−k26/AI−1103/tree/main/Assignment4/codes

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Let X_1, X_2, X_3, X_4, X_5 be *i.i.d.* random variables having a continuous distribution function. Then

$$\Pr(X_1 > X_2 > X_3 > X_4 > X_5 | X_1 = max(X_1, X_2, X_3, X_4, X_5))$$

equals ____.

(A) $\frac{1}{4}$

(B) $\frac{1}{5}$

(C) $\frac{1}{4!}$

(D) $\frac{1}{5!}$

Solution

Required probability

$$= \Pr(X_1 > X_2 > X_3 > X_4 > X_5 | X_1 = max(X_1, X_2, X_3, X_4, X_5)) \tag{0.0.1}$$

$$= \Pr(X_2 > X_3 > X_4 > X_5) \tag{0.0.2}$$

$$= \Pr(X_2 = max(X_2, X_3, X_4, X_5)) \times \Pr(X_3 = max(X_3, X_4, X_5)) \times \Pr(X_4 = max(X_4, X_5)) \tag{0.0.3}$$

Since X_1, X_2, X_3, X_4 and X_5 are identical and independently distributed random variables,

$$\Pr(X_2 = max(X_2, X_3, X_4, X_5))$$

$$= \int_{-\infty}^{\infty} \Pr(X_3, X_4, X_5 < x | X_2 = x) \tag{0.0.4}$$

$$= \int_{-\infty}^{\infty} F_{X_3}(x) F_{X_4}(x) F_{X_5}(x) f_{X_2}(x)\, dx \tag{0.0.5}$$

$$= \int_{-\infty}^{\infty} (\Phi(x))^3 \phi(x)\, dx \tag{0.0.6}$$

Substituting

$$u = \Phi(x) \tag{0.0.7}$$

$$du = \phi(x)\, dx \tag{0.0.8}$$

and similarly solving other two terms of the product of (0.0.3), we get required probability

$$= \int_0^1 u^3\, du \times \int_0^1 u^2\, du \times \int_0^1 u\, du \tag{0.0.9}$$

$$= \left.\frac{u^4}{4}\right]_0^1 \times \left.\frac{u^3}{3}\right]_0^1 \times \left.\frac{u^2}{2}\right]_0^1 \tag{0.0.10}$$

$$= \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \tag{0.0.11}$$

$$= \frac{1}{4!} \tag{0.0.12}$$

Here $\phi(x)$ and $\Phi(x)$ represent the pdf and cdf of respectively of random variables X_1, X_2, X_3, X_4, X_5.

The correct answer is **Option (C)**.