AI1103-Assignment 1

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QUESTION

A family has two children. What is the probability that both the children are boys given that at least one of them is a boy?

SOLUTION

Given, a family has two children.
Let's denote girl by 'G' and boy by 'B'.
Sample space of the outcomes is given by :

$$S = [(B, B), (G, B), (B, G), (G, G)]$$

Let X denote the random variable representing number of boys.

$$X = \{0, 1, 2\}$$

where $n = 2$ and $p = \frac{1}{2}$

$$\Pr(X = 0) = {}^2C_0 \times p^2 \qquad = \frac{1}{4} \quad (0.0.1)$$

$$\Pr(X = 1) = {}^2C_1 \times p \times q \qquad = \frac{2}{4} \quad (0.0.2)$$

$$\Pr(X = 2) = {}^2C_2 \times q^2 \qquad = \frac{1}{4} \quad (0.0.3)$$

$$\Pr(X \geq 1) = \Pr(X = 1) + \Pr(X = 2) \quad = \frac{3}{4} \quad (0.0.4)$$

To find $\Pr(X = 2 \mid X \geq 1)$.

$$\Pr(X = 2 \mid X \geq 1) = \frac{\Pr(X = 2)}{\Pr(X \geq 1)} \qquad (0.0.5)$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}} \qquad (0.0.6)$$

$$= \frac{1}{3} \qquad (0.0.7)$$

$\therefore$ The probability that both the children are boys given that at least one of them is a boy is $\frac{1}{3}$.