AI1103-Assignment 4

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Download latex-tikz codes from

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https://github.com/Umesh-k26/AI-1103/blob/main
    /Assignment4/assignment4.tex
```

and python codes from

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https://github.com/Umesh-k26/AI-1103/tree/main/
    Assignment4/codes
```

UGC/MATH 2019, Q.50

Let X_1, X_2, X_3, X_4, X_5 be *i.i.d.* random variables having a continuous distribution function. Then

$$\Pr\left(X_1 > X_2 > X_3 > X_4 > X_5 | X_1 = max(X_1, X_2, X_3, X_4, X_5\right)$$

equals ____.

1) $\frac{1}{4}$
2) $\frac{1}{5}$
3) $\frac{1}{4!}$
4) $\frac{1}{5!}$

Solution

Since X_1, X_2, X_3, X_4 and X_5 are identical and independently distributed random variables, they can be represented by a single random variable X.
Let

$$\{x_1, x_2, x_3, x_4, x_5\} \in X$$

Required probability,

$$= \Pr\left(X_1 > X_2 > X_3 > X_4 > X_5 | X_1 = max(X_1, X_2, X_3, X_4, X_5)\right) \quad (0.0.1)$$

$$= \Pr\left(x_1 > x_2 > x_3 > x_4 > x_5 | x_1 = max(x_1, x_2, x_3, x_4, x_5)\right) \quad (0.0.2)$$

Required probability,

$$= \Pr\left(x_2 > x_3 > x_4 > x_5\right) \quad (0.0.3)$$

$$= \Pr\left(x_2 = max(x_2, x_3, x_4, x_5)\right) \times \Pr\left(x_3 = max(x_3, x_4, x_5)\right) \times \Pr\left(x_4 = max(x_4, x_5)\right) \quad (0.0.4)$$

Since x_2, x_3, x_4, x_5 are distinct, only one of them can be maximum. And the probability that x_2 being maximum is $\frac{1}{4}$.

Similarly, the probabilities of other two terms of the equation (0.0.4) are $\frac{1}{3}$ and $\frac{1}{2}$ respectively.

$\therefore$ Required probability is,

$$= \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \quad (0.0.5)$$

$$= \frac{1}{4!} \quad (0.0.6)$$

The correct answer is **Option 3**.