AI1103-Assignment 5

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Download latex-tikz codes from

https://github.com/Umesh−k26/AI−1103/blob/main/Assignment5/assignment5.tex

CSIR UGC NET EXAM (June 2012), Q.52

Question

Suppose X and Y are independent random variables where Y is symmetric about 0. Let $U = X + Y$ and $V = X - Y$. Then

1) U and V are always independent.
2) U and V have the same distribution.
3) U is always symmetric about 0.
4) V is always symmetric about 0.

Solution

Since Y is symmetric about 0,

$$\phi_Y(-t) = \phi_Y(t) \tag{0.0.1}$$

Since X and Y are independent random variables,

$$\phi_{X+Y}(t) = \phi_X(t)\,\phi_Y(t) \tag{0.0.2}$$
$$\phi_{X-Y}(t) = \phi_X(t)\,\phi_Y(-t) \tag{0.0.3}$$
$$\phi_{X-Y}(t) = \phi_X(t)\,\phi_Y(t) \qquad \text{(from(0.0.1))} \tag{0.0.4}$$

Let $U = X + Y$ and $V = X - Y$.

$$\phi_U(t) = \phi_X(t)\,\phi_Y(t) \tag{0.0.5}$$
$$\phi_V(t) = \phi_X(t)\,\phi_Y(t) \tag{0.0.6}$$
$$\phi_U(t)\,\phi_V(t) = \phi_X^2(t)\,\phi_Y^2(t) \tag{0.0.7}$$
$$\phi_{U+V}(t) = \phi_{2X}(t) = \phi_X(2t) \tag{0.0.8}$$

Examining each option :

1) If U and V are independent, then

$$\phi_{U+V}(t) = \phi_U(t)\,\phi_V(t)$$

But from (0.0.7) and (0.0.8),

$$\phi_{U+V}(t) \neq \phi_U(t)\,\phi_V(t) \tag{0.0.9}$$

Hence, **Option 1 is incorrect.**

2) From (0.0.5) and (0.0.6),

$$\phi_U(t) = \phi_V(t)$$

$\implies$ U and V have same distribution.
Hence, **Option 2 is correct.**

3)

$$\phi_U(-t) = \phi_X(-t)\,\phi_Y(-t) \tag{0.0.10}$$
$$\phi_U(-t) = (-\phi_X(t))\;(\phi_Y(t)) \tag{0.0.11}$$
$$\phi_U(-t) = -\phi_X(t)\,\phi_Y(t) \tag{0.0.12}$$
$$\phi_U(-t) = -\phi_U(t) \tag{0.0.13}$$

$\implies$ $\phi_U(-t) \neq \phi_U(t)$
$\implies$ U is not symmetric about 0.
Hence, **Option 3 is incorrect.**

4) Since U and V have the same distribution, V is also not symmetric about 0.
Hence, **Option 4 is incorrect.**