AI1103-Assignment 4

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Let X_1, X_2, X_3, X_4, X_5 be *i.i.d.* random variables having a continuous distribution function. Then

$$\Pr(X_1 > X_2 > X_3 > X_4 > X_5 | X_1 = max(X_1, X_2, X_3, X_4, X_5))$$

equals _____.

1) $\dfrac{1}{4}$

2) $\dfrac{1}{5}$

3) $\dfrac{1}{4!}$

4) $\dfrac{1}{5!}$

SOLUTION

Since X_1, X_2, X_3, X_4 and X_5 are identical and independently distributed random variables, they can be represented by a single random variable X.
Let

$$\{x_1, x_2, x_3, x_4, x_5\} \in X$$

Required probability,

$$= \Pr(X_1 > X_2 > X_3 > X_4 > X_5 | X_1 = max(X_1, X_2, X_3, X_4, X_5)) \tag{0.0.1}$$

$$= P_X(x_1 > x_2 > x_3 > x_4 > x_5 | x_1 = max(x_1, x_2, x_3, x_4, x_5)) \tag{0.0.2}$$

Required probability,

$$= P_X(x_2 > x_3 > x_4 > x_5) \tag{0.0.3}$$

$$= P_X(x_2 = max(x_2, x_3, x_4, x_5)) \times P_X(x_3 = max(x_3, x_4, x_5)) \tag{0.0.4}$$

$$\times P_X(x_4 = max(x_4, x_5))$$

$$= \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \tag{0.0.5}$$

$$= \frac{1}{4!} \tag{0.0.6}$$

The correct answer is **Option 3**.