AI1103-Assignment 2

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Download latex-tikz codes from

https://github.com/Umesh−k26/AI−1103/blob/main/Assignment2/assignment2.tex

Question

Let the probability density funciton of a random variable X be

$$f(x) = \begin{cases} x & 0 \le x < \frac{1}{2} \\ c(2x-1)^2 & \frac{1}{2} < x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

then, the value of c is equal to _____.

Solution

For a probability density function of a continuous random variable,

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{0}^{1/2} f(x)\,dx + \int_{1/2}^{1} f(x)\,dx \tag{0.0.1}$$

$$= \int_{0}^{1/2} x\,dx + \int_{1/2}^{1} c(2x-1)^2\,dx \tag{0.0.2}$$

$$= \frac{x^2}{2}\Big]_0^{1/2} + c\left(\frac{4x^3}{3} - 2x^2 + x\right)\Big]_{1/2}^{1} \tag{0.0.3}$$

$$= \frac{1}{8} + c\left(\frac{1}{3} - \frac{1}{6}\right) \tag{0.0.4}$$

$$1 = \frac{1}{8} + \frac{c}{6} \tag{0.0.5}$$

$$\therefore c = \frac{21}{4} \tag{0.0.6}$$