AI1103-Assignment 1

Name : Umesh Kalvakuntla Roll No. : CS20BTECH11024

Download all python codes from

https://github.com/Umesh−k26/Assignment/blob/main/Assignment1/assignment.py

and latex-tikz codes from

https://github.com/Umesh−k26/Assignment/blob/main/Assignment1/assignment.tex

Question

A family has two children. What is the probability that both the children are boys given that at least one of them is a boy?

Solution

Given, a family has two children.
Let's denote girl by 'G' and boy by 'B'.
Sample space of the outcomes is given by :

$$S = [(B,B),(G,B),(B,G),(G,G)]$$

Let X denote the random variable representing number of boys.

$$X = \{0,1,2\}$$

where $n = 2$ and $p = \frac{1}{2}$

$$\Pr(X=0) = \binom{2}{0} \times q^2 = \frac{1}{4} \quad (0.0.1)$$

$$\Pr(X=1) = \binom{2}{1} \times p \times q = \frac{2}{4} \quad (0.0.2)$$

$$\Pr(X=2) = \binom{2}{2} \times p^2 = \frac{1}{4} \quad (0.0.3)$$

$$\Pr(X \geq 1) = \Pr(X=1) + \Pr(X=2) = \frac{3}{4} \quad (0.0.4)$$

To find $\Pr(X = 2 \mid X \geq 1)$.

$$\Pr(X=2 \mid X \geq 1) = \frac{\Pr(X=2)}{\Pr(X \geq 1)} \quad (0.0.5)$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}} \quad (0.0.6)$$

$$= \frac{1}{3} \quad (0.0.7)$$

$\therefore$ The probability that both the children are boys given that at least one of them is a boy is $\frac{1}{3}$.