AI1103-Assignment 5

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Download latex-tikz codes from

https://github.com/Umesh−k26/AI−1103/blob/main/Assignment5/assignment5.tex

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Question

Suppose X and Y are independent random variables where Y is symmetric about 0. Let $U = X + Y$ and $V = X - Y$. Then

1) U and V are always independent.
2) U and V have the same distribution.
3) U is always symmetric about 0.
4) V is always symmetric about 0.

Solution

Y is symmetric about 0,

$$\implies f_Y(-y) = f_Y(y) \tag{0.0.1}$$

$$\begin{aligned} F_U(u) &= \Pr(U \le u) && (0.0.2)\\ &= \Pr(X + Y \le u) && (0.0.3)\\ &= \Pr(X \le u - Y) && (0.0.4)\end{aligned}$$

$$F_U(u) = \int_{-\infty}^{\infty} f_Y(y) \int_{-\infty}^{u-y} f_X(x)\,dx\,dy \tag{0.0.5}$$

differentiating equation (0.0.5) gives,

$$f_U(u) = \int_{-\infty}^{\infty} f_Y(y) f_X(u - y)\,dy \tag{0.0.6}$$

$$f_U(u) = \int_{-\infty}^{\infty} f_Y(-y) f_X(u + y)\,dy \qquad \left(\because \int_a^b f(x)\,dx = \int_a^b f(a + b - x)\,dx\right) \tag{0.0.7}$$

from (0.0.1),

$$f_U(u) = \int_{-\infty}^{\infty} f_Y(y) f_X(u + y)\,dy \tag{0.0.8}$$

$$\begin{aligned} F_V(v) &= \Pr(V \le v) && (0.0.9)\\ &= \Pr(X - Y \le v) && (0.0.10)\\ &= \Pr(X \le v + Y) && (0.0.11)\end{aligned}$$

$$F_V(v) = \int_{-\infty}^{\infty} f_Y(y) \int_{-\infty}^{v+y} f_X(x)\,dx\,dy \tag{0.0.12}$$

differentiating equation (0.0.12) gives,

$$f_V(v) = \int_{-\infty}^{\infty} f_Y(y) f_X(v + y)\,dy \tag{0.0.13}$$

From (0.0.8) and (0.0.13), U and V have same distribution.

$\therefore$ The correct answer is **Option 2**.