

# AI1103-Assignment 2

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Download latex-tikz codes from

<https://github.com/Umesh-k26/AI-1103/blob/main/Assignment2/assignment2.tex>

and python codes from

<https://github.com/Umesh-k26/AI-1103/blob/main/Assignment2/codes/assignment2.py>

## QUESTION

Let the probability density function of a random variable X be

$$f_X(x) = \begin{cases} x & 0 \leq x < \frac{1}{2} \\ c(2x-1)^2 & \frac{1}{2} < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

then, the value of c is equal to \_\_\_\_.

## SOLUTION

For a probability density function of a continuous random variable,

$$\int_{-\infty}^{\infty} f_X(x) dx = 1 \quad (0.0.1)$$

$$\int_{-\infty}^{\infty} f_X(x) dx = \int_0^{1/2} f_X(x) dx + \int_{1/2}^1 f_X(x) dx \quad (0.0.2)$$

$$= \frac{1}{2}(x)(x) \Big|_{x=\frac{1}{2}} + \int_{1/2}^1 c(2x-1)^2 dx \quad (0.0.3)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + c \left( \frac{4x^3}{3} - 2x^2 + x \right) \Big|_{1/2}^1 \quad (0.0.4)$$

$$= \frac{1}{8} + c \left( \frac{1}{3} - \frac{1}{6} \right) \quad (0.0.5)$$

$$= \frac{1}{8} + \frac{c}{6} \quad (0.0.6)$$

from (0.0.1) and (0.0.6) we get

$$1 = \frac{1}{8} + \frac{c}{6} \quad (0.0.7)$$

$$\therefore c = \frac{21}{4} \quad (0.0.8)$$

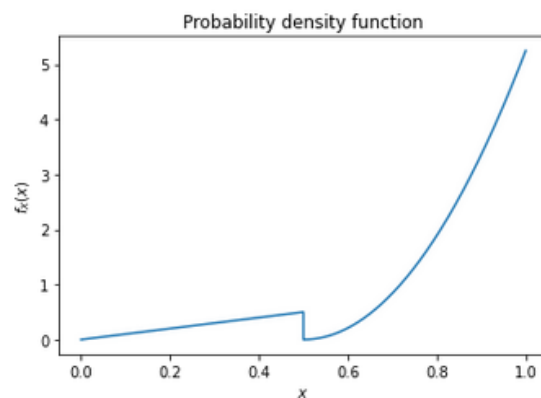


Fig. 0: Graph of  $f_X(x)$

$$F_X(x) = \begin{cases} \int_{-\infty}^0 f_X(x) dx & x \leq 0 \\ \int_{-\infty}^{1/2} f_X(x) dx & 0 \leq x \leq \frac{1}{2} \\ \int_{-\infty}^1 f_X(x) dx & \frac{1}{2} \leq x \leq 1 \\ \int_{-\infty}^{\infty} f_X(x) dx & x > 1 \end{cases}$$

$$F_X(x) = \begin{cases} 0 & x \leq 0 \\ \frac{x^2}{2} & 0 \leq x \leq \frac{1}{2} \\ \frac{1}{8} + \frac{7}{8}(2x-1)^3 & \frac{1}{2} \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

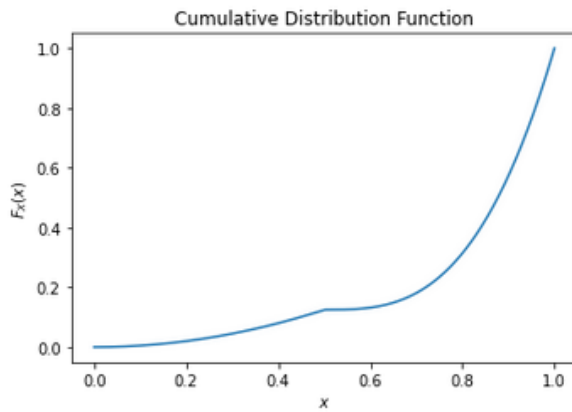


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