# AI1103-Assignment 4

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Download latex-tikz codes from

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https://github.com/Umesh−k26/AI−1103/blob/main
    /Assignment4/assignment4.tex
```

and python codes from

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https://github.com/Umesh−k26/AI−1103/tree/main/
    Assignment4/codes
```

UGC/MATH 2019, Q.50

Let $X_1, X_2, X_3, X_4, X_5$ be *i.i.d.* random variables having a continuous distribution function. Then

$$\Pr\left(X_1 > X_2 > X_3 > X_4 > X_5 | X_1 = max(X_1, X_2, X_3, X_4, X_5)\right)$$

equals ____.

$(A)$ $\frac{1}{4}$

$(B)$ $\frac{1}{5}$

$(C)$ $\frac{1}{4!}$

$(D)$ $\frac{1}{5!}$

## Solution

Required probability

$$= \Pr\left(X_1 > X_2 > X_3 > X_4 > X_5 | X_1 = max\left(X_1, X_2, X_3, X_4, X_5\right)\right) \tag{0.0.1}$$

$$= \Pr\left(X_2 > X_3 > X_4 > X_5\right) \tag{0.0.2}$$

$$= \Pr\left(X_2 = max(X_2, X_3, X_4, X_5)\right) \times \Pr\left(X_3 = max(X_3, X_4, X_5)\right) \times \Pr\left(X_4 = max(X_4, X_5)\right) \tag{0.0.3}$$

Since $X_1$, $X_2$, $X_3$, $X_4$ and $X_5$ are identical and independently distributed random variables, required probability is

$$= \int_{-\infty}^{\infty} \Pr\left(X_3, X_4, X_5 < x | X_2 = x\right) \times \int_{-\infty}^{\infty} \Pr\left(X_4, X_5 < x | X_3 = x\right) \times \int_{-\infty}^{\infty} \Pr\left(X_5 < x | X_4 = x\right) \tag{0.0.4}$$

$$= \int_{-\infty}^{\infty} F_{X_3}(x) F_{X_4}(x) F_{X_5}(x) f_{X_2}(x)\, dx \times \int_{-\infty}^{\infty} F_{X_4}(x) F_{X_5}(x) f_{X_3}(x)\, dx \times \int_{-\infty}^{\infty} F_{X_5}(x) f_{X_4}(x)\, dx \tag{0.0.5}$$

$$= \int_{-\infty}^{\infty} (\Phi(x))^3 \phi(x)\, dx \times \int_{-\infty}^{\infty} (\Phi(x))^2 \phi(x)\, dx \times \int_{-\infty}^{\infty} \Phi(x) \phi(x)\, dx \tag{0.0.6}$$

Substituting,

$$u = \Phi(x) \tag{0.0.7}$$

$$du = \phi(x)\, dx \tag{0.0.8}$$

Hence, the required probability is

$$= \int_0^1 u^3\,du \times \int_0^1 u^2\,du \times \int_0^1 u\,du \tag{0.0.9}$$

$$= \left.\frac{u^4}{4}\right]_0^1 \times \left.\frac{u^3}{3}\right]_0^1 \times \left.\frac{u^2}{2}\right]_0^1 \tag{0.0.10}$$

$$= \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \tag{0.0.11}$$

$$= \frac{1}{4!} \tag{0.0.12}$$

Here $\phi(x)$ and $\Phi(x)$ represent the PDF and CDF respectively of random variables $X_1, X_2, X_3, X_4, X_5$.

The correct option is **Option (C)**.