# AI1103-Assignment 2

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Download latex-tikz codes from

https://github.com/Umesh-k26/AI-1103/blob/main/Assignment2/assignment2.tex

and python codes from

https://github.com/Umesh-k26/AI-1103/blob/main/Assignment2/codes/assignment2.py

## Question

Let the probability density funciton of a random variable X be

$$f_X(x) = \begin{cases} x & 0 \le x < \frac{1}{2} \\ c(2x-1)^2 & \frac{1}{2} < x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

then, the value of c is equal to _____.

## Solution

For a probability density function of a continuous random variable,

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1 \tag{0.0.1}$$

$$\int_{-\infty}^{\infty} f_X(x)\,dx = \int_{0}^{1/2} f_X(x)\,dx + \int_{1/2}^{1} f_X(x)\,dx \tag{0.0.2}$$

$$= \frac{1}{2}(x)(x)\Big|_{x=\frac{1}{2}} + \int_{1/2}^{1} c(2x-1)^2\,dx \tag{0.0.3}$$

$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} + c\left(\frac{4x^3}{3} - 2x^2 + x\right)\Big]_{1/2}^{1} \tag{0.0.4}$$

$$= \frac{1}{8} + c\left(\frac{1}{3} - \frac{1}{6}\right) \tag{0.0.5}$$

$$= \frac{1}{8} + \frac{c}{6} \tag{0.0.6}$$

from (0.0.1) and (0.0.6) we get

$$1 = \frac{1}{8} + \frac{c}{6} \tag{0.0.7}$$

$$\therefore c = \frac{21}{4} \tag{0.0.8}$$

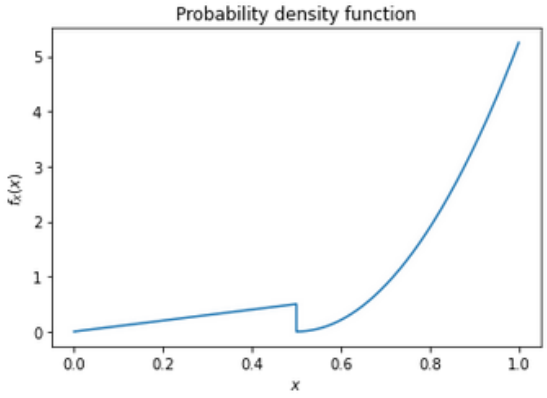
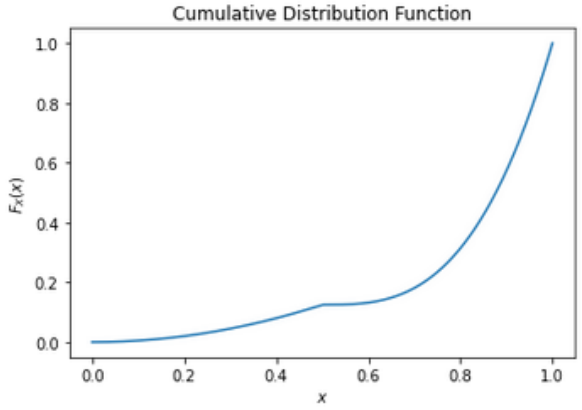



Fig. 0: Graph of $f_X(x)$

$$F_X(x) = \begin{cases} 0 & x \leq 0 \\ \frac{x^2}{2} & 0 \leq x \leq \frac{1}{2} \\ \frac{1}{8} + \frac{7}{8}(2x-1)^3 & \frac{1}{2} \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

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