PDA

$q_0 \xrightarrow{a, Z_0 / aZ_0} q_1$

$\delta(q_0, a, Z_0) = (q_1, aZ_0)$

q_1: next new state; aZ_0: push operation

$\delta(q_0, \epsilon, Z_0) = (q_1, Z_0)$

ϵ: empty I/P string; Z_0: initial symbol of stack

$\delta(q_1, a, Z_0) = (q_2, \epsilon)$

ϵ: pop operation

$\delta(q_0, a, a) = (q_1, aa) \Rightarrow$ push

$\delta(q_0, a, Z_0) = (q_0, aZ_0) \Rightarrow$ push

$\delta(q_0, b, Z_0) = (q_0, bZ_0) \Rightarrow$ push.

$\delta(q_1, c, Z_0) = (q_2, \epsilon)$

$\delta(q_1, a, a) = (q_2, \epsilon)$

$\delta(q_1, b, b) = (q_2, \epsilon)$

$\delta(q_1, \epsilon, Z_0) = (q_2, Z_0)$

$q_1 \xrightarrow{c, Z_0 / \epsilon} q_2$ pop.

$L = \{ wcw^R \mid w \in (a+b)^* \}$ Check if palindrome or not.

$\delta(q_0, abbcbba, Z_0)$

Till middle of string, push the symbol on top of stack

$\delta(q_0, abbcbba, aZ_0)$

$\delta(q_0, bcbba, baZ_0)$

$\delta(q_0, cbba, bbaZ_0)$

$\delta(q_1, bba, bbaZ_0)$

$\delta(q_1, ba, baZ_0)$

$\delta(q_1, a, aZ_0)$

$\delta(q_1, \epsilon, Z_0)$

Transition Function

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, b, Z_0) = (q_0, bZ_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, a, b) = (q_0, ab)$

$\delta(q_0, b, a) = (q_0, ba)$

$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, c, Z_0) = (q_1, Z_0)$

$\delta(q_1, a, Z_0) = (q_1, Z_0)$

$\delta(q_1, b, Z_0) = (q_1, Z_0)$

PDA is used to design CFG

$\delta(q_1, a, a) = (q_1, \epsilon)$
$\delta(q_1, b, b) = (q_1, \epsilon)$
$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$

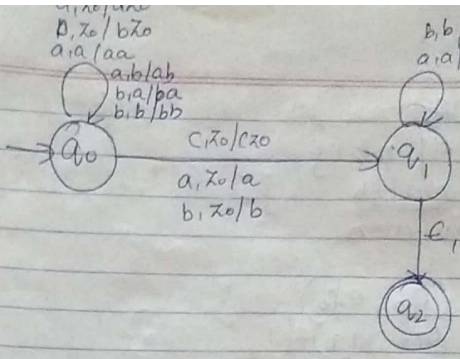
* GNF always starts with terminal

TO accept the string

final state — empty stack (don't show z_0 explicitly in the last transition function)

(q_2, z_0)
$\downarrow$
(q_2, ϵ)

$\delta(q_0, baacaab, z_0) \vdash (q_0, aacaab\ bz_0)$
$\vdash (q_0, acaab\ abz_0)$
$\vdash (q_0, caab\ aabz_0)$
$\vdash (q_1, aab\ aabz_0)$
$\vdash (q_1, ab\ abz_0)$
$\vdash (q_1, b\ bz_0)$
$\vdash (q_1, \epsilon\ z_0)$
$\vdash (q_2, z_0)$



$\delta(q_0, aabcab, z_0) \vdash (q_0, abcab\ az_0)$
$\vdash (q_0, bcab\ baz_0)$
$\vdash (q_0, cab\ baaz_0)$
$\vdash (q_1, ab\ baaz_0)$
$\vdash (q_1, b\ aaz_0)$
$\vdash (q_1, \epsilon\ az_0)$

Rejected.

Q) Construct PDA

$L = \{a^n b^n \mid n \geq 1\}$

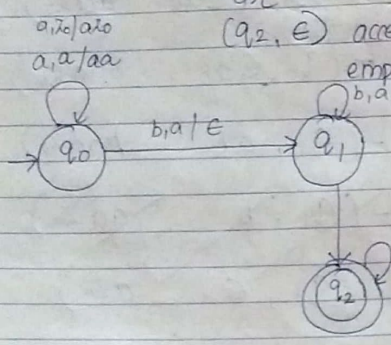
$L = \{ab, aabb, aaabbb \ldots\}$

Sol Transition Function

$\delta(q_0, a, z_0) = (q_0, az_0)$
$\delta(q_0, a, a) = (q_0, aa)$
$\delta(q_0, b, a) = (q_1, \epsilon)$
$\delta(q_1, b, a) = (q_1, \epsilon)$
$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$

or

(q_2, ϵ) acceptence by empty stack



Acceptance

$\delta(q_0, aaabbb, z_0) \vdash (q_0, aabbb, az_0)$
$\vdash (q_0, abbb, aaz_0)$
$\vdash (q_0, bbb, aaaz_0)$
$\vdash (q_1, bb \quad aaz_0)$
$\vdash (q_1, b \quad az_0)$
$\vdash (q_1, \epsilon, z_0)$
$\vdash (q_2, z_0) \Rightarrow$ Acceptance.

Rejection

$\delta(q_0, aabbb, z_0) \vdash (q_0, abbb, az_0)$
$\vdash (q_0, bbb, aaz_0)$
$\vdash (q_1, bb, az_0)$
$\vdash (q_1, b, z_0)$

⇓

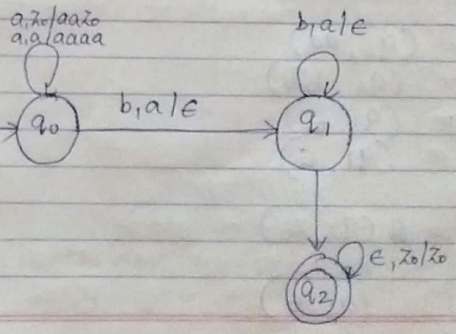
Rejected.

Q) $L = \{a^n b^{2n} \mid n \geq 1\}$ For each a, push two a's on to the stack

$L = \{abb, aabbbb \ldots\}$

Transition Function

$\delta(q_0, a, z_0) = (q_0, aaz_0)$
$\delta(q_0, a, a) = (q_0, aaaa)$
$\delta(q_0, b, a) = (q_1, \epsilon)$
$\delta(q_1, b, a) = (q_1, \epsilon)$
$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$



$\delta(q_0, aabbbb, z_0) \vdash (q_0, abbbb, aaz_0)$

$\vdash (q_0, bbbb, aaaaz_0)$

$\vdash (q_1, bbb\ aaaz_0)$

$\vdash (q_1, bb\ aaz_0)$

$\vdash (q_1, b, az_0)$

$\vdash (q_1, \epsilon, z_0)$

$\vdash (q_2, z_0)$

⇓

Accepted.

$\delta(q_0, baab, z_0) \vdash (q_0, aab, bz_0)$

$\vdash (q_1, ab\ z_0)$

$\vdash (q_1, b\ az_0)$

$\vdash (q_1, \epsilon, z_0)$

$\vdash (q_2, z_0)$

⇓

Acceptance

$\alpha_1 = \{w \mid w \in (a,b) \quad n_a(w) > n_b(w)\}$

$\alpha_2 = \{w \mid w \in (a,b) \quad n_b(w) > n_a(w)\}$

$\alpha_1 = \{aab, aaab, aaabb, aaa$

(3) Draw PDA for acceptance by final state

$\alpha = \{w \mid w \in (a,b) \quad n_a(w) = n_b(w)\}$

$\alpha = \{ab, ba, baab, abab \ldots\}$

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, b, z_0) = (q_0, bz_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, a, b) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$

or

(q_2, ϵ)

) Construct PDA for the given lang
$L = \{ a^n b^m c^n \mid n, m \geq 1 \}$

$L = \{ abc, \ldots \}$

$\delta(q_0, a, z_0) = \delta(q_0, az_0)$ } push
$\delta(q_0, a, a) = (q_0, aa)$
$\delta(q_0, b, a) = (q_1, a)$ } no change
$\delta(q_1, b, a) = (q_1, a)$
$\delta(q_1, c, a) = (q_1, \epsilon)$ } pop
$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$

(For each a pop c)

$L = \{ a^n b^m c^m d^n \mid n, m \geq 1 \}$

a - push
b - push
c - pop
d - pop

* CFG to PDA

① Convert the given grammar to GNF form

CNF
$A \to BC$ } NT
or
$A \to a$ } T

GNF
$A \to a\alpha$
↓
NT*

(should start with terminal later can have any number of NT)

② Let q_0 be the start state without consuming any input symbol push the start symbol on to the top of stack & change state from q_0 to q_1.

③ Introduce the transition function for each and every production in given grammar

④ Change the state from q_1 to q_f where q_1 is an accepting state

① GNF
② $(q_0, \epsilon, z_0) = (q_1, Sz_0)$
③ $A \to a\alpha$
$\delta(q_1, a, A) = (q_1, \alpha)$

⑤ Convert the given grammer to its equivalent PDA

$S \to aABB$
$A \to aBB$
$B \to bAA$
$C \to a$

$\delta(q_0, \epsilon, z_0) = (q_1, Sz_0)$

$S \to aABB$
A a α

$\delta(q_1, a, S) = (q_1, ABB)$
$\delta(q_1, a, A) = (q_1, BB)$
$\delta(q_1, b, B) = (q_1, AA)$
$\delta(q_1, a, C) = (q_1, \epsilon)$
$\delta(q_1, \epsilon, z_0) = (q_f, z_0)$

Deterministic / Non-deterministic PDA

2) $S \to aSa / aa$
$S \to bSb / bb$

$S \to aSA / aA$
$S \to bSB / bB$

$\delta(q_0, \epsilon, Z_0) = (q_1, SZ_0)$

$S \to aSA$
$A \to a \quad \alpha \to a$

$\delta(q_1, a, S) = (q_1, SA)$
$\delta(q_1, a, S) = (q_1, A)$
$\delta(q_1, b, S) = (q_1, SB)$
$\delta(q_1, b, S) = (q_1, B)$
$\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$

3) Grammar for
$\alpha = \{a^n b^n \mid n \geq 1\}$

$S \to aSb / ab$
⇓
Not in GNF.

Converting
$S \to aSB / aB$
$B \to b$

$\delta(q_0, \epsilon, Z_0) = (q_1, SZ_0)$
$\delta(q_1, a, S) = (q_1, SB)$
$\delta(q_1, a, S) = (q_1, B)$
$\delta(q_1, b, B) = (q_1, \epsilon)$
$\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$

4) $\alpha = \{n_a(w) = n_b(w) \quad w \in (a+b)\}$

$S \to aSb / bSa$
$S \to \epsilon$
$S \to SS$

~~Convert to GNF~~

~~$S \to aSB$~~
~~B~~

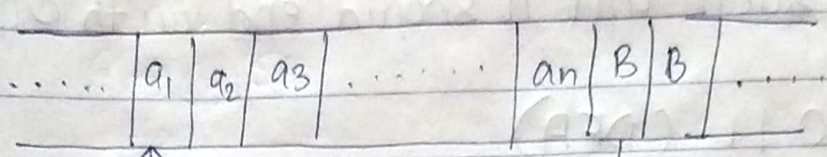
5) $S \to aSb / bSa$ → not in GNF
$S \to aA$ → GNF
$S \to bB$ → GNF

convert
$S \to aSB / bSA$
$B \to b$
$A \to a$
$S \to aA$
$S \to bB$

$\delta(q_0, \epsilon, Z_0) = (q_1, SZ_0)$
$\delta(q_1, a, S) = (q_1, SB)$
$\delta(q_1, b, S) = (q_1, SA)$
$\delta(q_1, a, S) = (q_1, A)$
$\delta(q_1, b, B) = (q_1, B)$
$\delta($

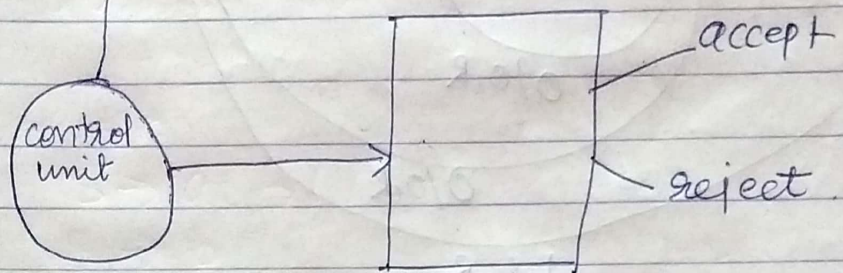
TURING MACHINE

def, working model, problems - DFA.
short notes
↳ Transducers
↳ multitape ~~rote~~ machines.
↳ programming techniques.



↳ Blank characters

read/write



transition function

$TM = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

- Q: set of states
- Σ: i/p alphabet
- Γ: magnetic tape symbols
- δ: transition function
- q_0: Start state
- B: Blank character
- F: Final state

$$\delta : (Q \times \Gamma) = (Q \times \Gamma \times \{L, R\})$$

magnetic tape symbol

L: left, R: Right (directions)

$\delta(q_0, 0) = (q_1, X, L)$ — $q_0 \xrightarrow{0/X,L} q_1$

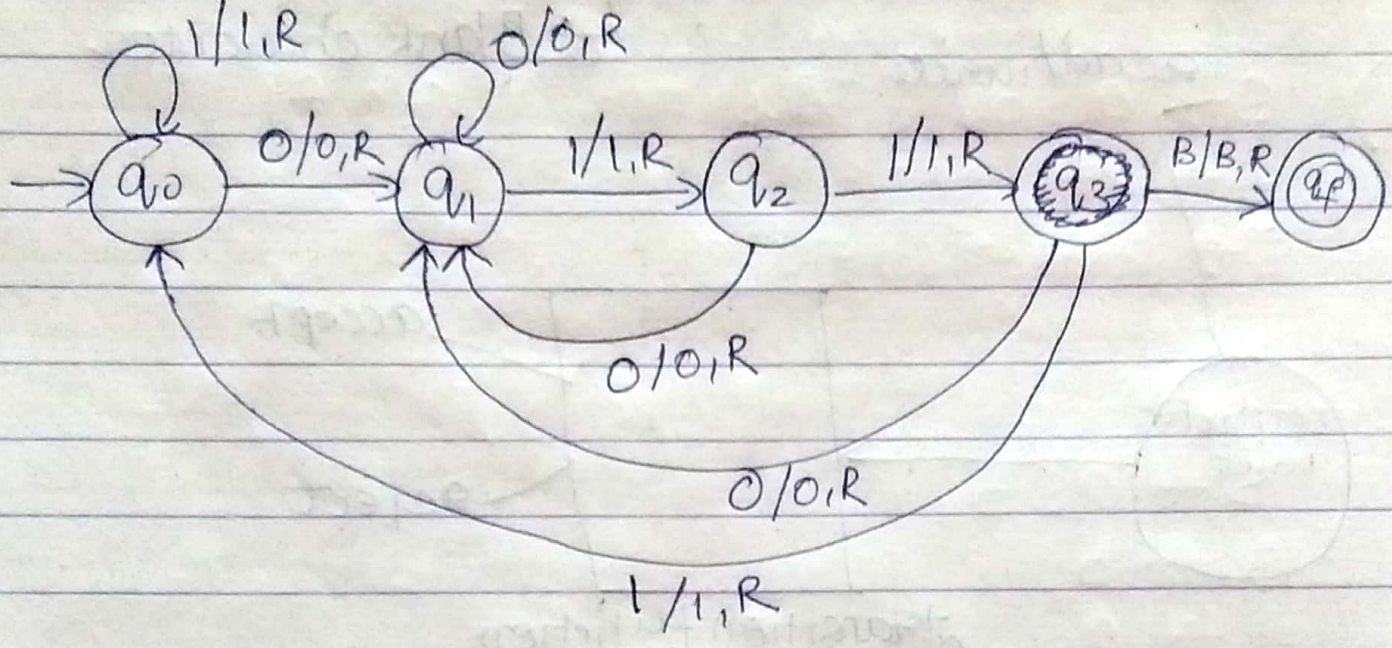
$\delta(q_1, 1) = (q_1, Y, L)$ — $q_0 \xrightarrow{1/Y,L} q_1$

$\delta(q_1, 0) = (q_2, X, R)$ — $q_1 \xrightarrow{0/X,R} q_2$

$\delta(q_2, X) = (q_f, B, R)$ — $q_2 \xrightarrow{X/B,R} q_f$

Q) Design a turing machine over an input symbol 0,1 such that it should end with 011

end with 011 (DFA)



δ	0	1	B
→ q_0	q_1	q_0	
q_1	q_1	q_2	
q_2	q_1	q_3	
q_3	q_1	q_0	
* q_f			