(ii) Given $\boxed{T(a\ b\ c) = (2b+c,\ a-4b,\ 3a)\ *}$

Base $= B' = \{(1,1,1)\ (1,1,0)\ (1,0,0)\}$ — f_1, f_2, f_3

<u>Step 1</u> Find $T(f_1) = T(1\ 1\ 1)$ $T(f_2) = T(1\ 1\ 0)$ $T(f_3) = T(1\ 0\ 0)$ using $*$

in $*$ put $(a\ b\ c) = (1,1,1)$

$T(1\ 1\ 1) = (3\ \ -3\ \ 3)$

llly $T(1\ 1\ 0) = (2\ \ -3\ \ 3)$

$T(1\ 0\ 0) = (0\ \ 1\ \ 3)$

<u>Step 2</u> Express each of $T(1,1,1)$ $T(1,1,0)$ $T(1,0,0)$ in terms of f_1 f_2 f_3 as

$x(1,1,1) + y(1,1,0) + z(1,0,0) = (3, -3, +3)$

" $= (2, -3, 3)$

$= (0, 1, 3)$

		OR	OR
$\Rightarrow$ $x+y+z$	$= 3$	2	0
$x+y$	$= -3$	-3	1
x	$= +3$	3	3

1st	similarly	
$x+y+z = 3$	$x+y+z = 2$	$x+y+z = 0$
$x+y = -3$	$x+y = -3$	$x+y = 1$
$x = +3$	$x = 3$	$x = 3$
$\Rightarrow$ $x = +3$	$\Rightarrow$ $x = 3$	$\Rightarrow$ $x = 3$
$y = -6$	$y = -6$	$y = -2$
$z = 6$	$z = \cancel{6}\ 5$	$z = -1$

Matrix of LT with Basis B' $= [T]_{B'} = [A]_{B'} = \cancel{\begin{bmatrix} 3 & -6 & 6 \\ 3 & -6 & 5 \\ 3 & -2 & -1 \end{bmatrix}} = \begin{bmatrix} 3 & 3 & 3 \\ -6 & -6 & -2 \\ 6 & 5 & -1 \end{bmatrix}$

Q Prove that T is invertible and find ~~A~~ T^{-1} given

$$\boxed{T(x_1, x_2, x_3) = (3x_1 + x_3,\ -2x_1 + x_2,\ -x_1 + 2x_2 + 4x_3)} \quad (1)$$

Soln wrt Std Base = {(1 0 0), (0 1 0), (0 0 1)}

Matrix of $\boxed{}$ (1)

is $A = \begin{bmatrix} 3 & 0 & 1 \\ -2 & 1 & 0 \\ -1 & 2 & 4 \end{bmatrix}$

$|A| = 9 \neq 0$

$\Rightarrow A^{-1}$ exists

$\Rightarrow A$ is invertible

$\Rightarrow T$ is invertible

$A^{-1} = \frac{1}{9}\begin{bmatrix} 4 & 2 & -1 \\ 8 & 13 & -2 \\ -3 & -6 & 3 \end{bmatrix}$

$A^{-1}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 4 & 2 & -1 \\ 8 & 13 & -2 \\ -3 & 6 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$T^{-1}(x, y, z)$

$T^{-1}(x, y, z) = \left(\frac{4x + 2y - z}{9}, \frac{8x + 13y - 2z}{9}, \frac{-3x + 6y + 3z}{9}\right)$

Matrix of LT in different Basis

Q Find the matrix representation of the LT (OR operators) T on R^2.
(means $T: R^2 \to R^2$) relative to standard or usual basis $\{e_1, e_2\} \equiv \{(1,0)\ (0,1)\}$

① $T(x\ y) = (2y, 3x-y)$

② $T(x, y) = (3x-4y\ \ x+5y)$

Sol^n ① Give $T: R^2 \to R^2$, $T(x, y) = (2y, 3x-y)$
$= (0x+2y, 3x-y)$

Matrix of LT T is $A = \begin{bmatrix} 0 & 2 \\ 3 & -1 \end{bmatrix}$

---

② $T(x, y) = (3x-4y, x+5y)$ $A = \begin{bmatrix} 3 & -4 \\ 1 & 5 \end{bmatrix}$

Q Find the matrix representation of LT ~~on~~ given previous example (page 8) wrt basis $\{f_1, f_2\} = \{(1,3), (2,5)\}$

Soln Given Two L.T. $T: R^2 \to R^2$

(i) $T(x, y) = (2y, 3x-y)$

(ii) $T(x, y) = (3x-4y, x+5y)$

Basis $\{f_1, f_2\} = \{(1,3), (2,5)\} = B$ (say)

(i) Consider $\boxed{T(x, y) = (2y, 3x-y)}$ → ①

To find matrix of T wrt Base $\{f_1, f_2\} = B$

Step 1 In ① find $T(1,3)$ $T(2,5)$

$T(1, 3) = (6, 0)$

$T(2, 5) = (10, 1)$

Step 2 Express $T(1,3)$ ~~ang~~ $T(2,5)$ interms of f_1 f_2

as $T(f_1) = (6, 0) = x f_1 + y f_2$

$T(f_2) = (10, 1) = x f_1 + y f_2$ → ②

$\Rightarrow$ $(6, 0) = x(1,3) + y(2,5)$

$(10, 1) = x(1,3) + y(2,5)$

$x + 2y = 6$
$3x + 5y = 0$

$x + 2y = 10$
$3x + 5y = 1$

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$$S = \left[\begin{array}{cc|cc} 1 & 2 & 6 & 10 \\ 3 & 5 & 0 & 1 \end{array}\right]$$

$R_2 - 3R_1$

$$\sim \left[\begin{array}{cc|cc} 1 & 2 & 6 & 10 \\ 0 & -1 & -18 & -29 \end{array}\right]$$

$$\boxed{\begin{array}{l} x + 2y = 6 \\ -y = -18 \end{array} \quad \text{and} \quad \begin{array}{l} x + 2y = 10 \\ -y = -29 \end{array}}$$

OR

$$\boxed{\begin{array}{l} y = 18 \\ x = -30 \end{array} \qquad \begin{array}{l} y = 29 \\ x = -48 \end{array}}$$

Putt on ②
$T(f_1) = T(1,3) = (6, 0) = -30 f_1 + 18 f_2$
$T(f_2) = T(2,5) = (10, 1) = -48 f_1 + 29 f_2$

Matrix of $[T]_f = \begin{pmatrix} -30 & -48 \\ 18 & 29 \end{pmatrix}$

Similarly Try
(ii) For $T(x, y) = (3x - 4y,\ x + 5y)$

$$A = \begin{pmatrix} 77 & 124 \\ -43 & -69 \end{pmatrix}$$