LA
Unit III
page 1

Q: Is T invertible. If so find T^{-1}. PT TT^{-1} is Identity LT.

Given $T(x, y, z)$

$$T(x_1, x_2, x_3) = (2x_1 + x_2 + x_3 \quad x_1 + x_2 + 2x_3 \quad x_1 - 2x_3)$$

Soln Given LT $T(x_1, x_2, x_3) = (2x_1 + x_2 + x_3, x_1 + x_2 + 2x_3, x_1 - 2x_3)$

Matrix of LT $= A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & -2 \end{bmatrix}$

$\det(A) = -1 \Rightarrow A$ is non-singular

$\therefore A^{-1}$ exists $\Rightarrow T^{-1}$ exists $\Rightarrow T$ is invertible

To find T^{-1}, $A^{-1} = \begin{bmatrix} 2 & -2 & -1 \\ -4 & 5 & 3 \\ 1 & -1 & -1 \end{bmatrix}$

$\therefore T^{-1}(x_1, x_2, x_3) = (2x_1 - 2x_2 - x_3, -4x_1 + 5x_2 + 3x_3, x_1 - x_2 - x_3)$

To PT TT^{-1} is identity LT.

Consider $TT^{-1}(x_1, x_2, x_3) = T(\underbrace{2x_1 - 2x_2 - x_3}_{X_1}, \underbrace{-4x_1 + 5x_2 + 3x_3}_{X_2}, \underbrace{x_1 - x_2 - x_3}_{X_3})$

$= (2X_1 + X_2 + X_3, \quad X_1 + X_2 + 2X_3 \quad X_1 - 2X_3)$

$= \big(2(2x_1 - 2x_2 - x_3) + (-4x_1 + 5x_2 + 3x_3) + (x_1 - x_2 - x_3), \ (2x_1 - 2x_2 - x_3) + (-4x_1 + 5x_2 + 3x_3) + 2(x_1 - x_2 - x_3), \ (2x_1 - 2x_2 - x_3) - 2(x_1 - x_2 - x_3)\big) = (x_1, x_2, x_3)$

$\Rightarrow TT^{-1}(x_1, x_2, x_3) = (x_1, x_2, x_3)$

$\Rightarrow TT^{-1}$ is identity LT

Algebra of L.T.

Q Let T_1, T_2 be linear Operators defined on $\mathbb{R}^2$,

$$T_1(x_1, x_2) = (x_2, x_1)$$

$$T_2(x_1, x_2) = (x_1, 0)$$

PT $T_1 T_2 \neq T_2 T_1$

Soln Given $T_1(x_1, x_2) = (x_2, x_1)$

$$T_2(x_1, x_2) = (x_1, 0)$$

Consider $\cancel{T_1 T_2(x_1, x_2) = T_1(x_1, x_2) = (x_2, x_1)}$

$$T_1 T_2(x_1, x_2) = T_1(x_1, 0) = (0, x_1) \rightarrow ①$$

$$T_2 T_1(x_1, x_2) = T_2(x_2, x_1) = (x_2, 0) \rightarrow ②$$

$\therefore$ $T_1 T_2(x_1, x_2) \neq T_2 T_1(x_1, x_2)$

$\Rightarrow$ $T_1 T_2 \neq T_2 T_1$ [Recaptilate $AB \neq BA$ in matrices]

Algebra of LT

LA
Unit III
Page 13

Q If $T(a,b,c) = (3a,\ a-b,\ 2a+b+c)$

$\forall (a,b,c) \in R^3$ is a linear Operator

P.T $(T^2 - I)(T - 3I) = \hat{O} \longrightarrow$ operator

Def^n ZERO operator
$T(x,y,z) = O = (0,0,0)$

Sol^n $(T-3I)(a,b,c) = T(a\ b\ c) - 3I(a,b,c)$

$= (3a,\ a-b,\ 2a+b+c) - (+3a,\ +3b,\ 3c)$

$\cancel{T}(T-3I)(a,b,c) = (0,\ a-4b\ \ 2a+b-2c)$

$(T-3I)(a,b,c) = (x,\ y,\ z) \rightarrow$ say

Now $\boxed{(T^2 - I)((T-3I)(a,b,c))} = (T^2 - I)(x,y,z)$

$= T^2(x,y,z) - I(x\ y,z)$ (Identity)

$\therefore \boxed{} = T(T(x,y,z)) - (x,y,z)$

$= T(3x,\ x-y,\ 2x+y+z)$

$= T(3(0),\ (0)-(a-4b),\ 2(0)+(a-4b)+2a+b-2c) - (x\ y\ z)$

$= T(\underset{A}{0},\ \underset{B}{-a+4b},\ \underset{C}{3a-3b-2c}) - (x\ y\ z)$

$= (3A\ \ \ A-B\ \ \ 2A+B+C) - (x,y,z)$

$$= (0 \quad a-4b \quad 2a+b+2c) - (0, a-4b, 2a+b-2c)$$

$$= (0 \quad 0 \quad 0)$$

$$\therefore (T^2 - I)(T-3I)(a,b,c) = (0,0,0)$$

Kernel of Linear Transformation