Assignment

January 27, 2024

CBSE QUESTION PAPER MATHEMATICS 2018

1. Find the acute angle between the planes $\overrightarrow{r} \cdot \left(\hat{i} - 2\hat{j} - 2\hat{k}\right) = 1$ and $\overrightarrow{r} \cdot \left(3\hat{i} - 6\hat{j} + 2\hat{k}\right) = 0$.

2. Find the length of the intercept, cut off by the plane $2x + y - z = 5$ on the x-axis.

3. If $y = log(\cos e^x)$,then find $\dfrac{dy}{dx}$.

4. A is a square matrix with $| A |= 4$.Then find the value of $| A.(adjA) |$.

5. From the differential equation representing the family of curves $y = A \sin x$, by eliminating the arbitrary constant A.

6. Find :
$$\int x.\tan^{-1} x dx$$

7. Find:
$$\int \frac{dx}{\sqrt{5 - 4x - 2x^2}}$$

8. Solve the following differential equation:
$$\frac{dy}{dx} + y = \cos x - \sin x$$

9. Find:

$$\int_0^{\pi/4} \frac{1+\tan x}{1-\tan x}\, dx$$

10. Let $*$ be an operation defined as $* : R \times R \to R$ such that $a * b = 2a + b, a, b \in R$.Check if $*$ is a binary operation.If yes,find if it is associative too.

11. X and Y are two points with position vectors $3\vec{a}+\vec{b}$ and $\vec{a}-3\vec{b}$ respectively.Write the position vector of a point Z which divides the lines segment XY in the ratio $2:1$ externally.

12. Let $\vec{a} = \hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b} = 3\hat{i}-\hat{j}+2\hat{k}$be two vectors. Show that the vectors $\left(\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-\vec{b}\right)$ are perpendicular to each other.

13. Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.

14. In a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

15. The probabilities of solving a specific problem independently by A and B are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. If both try to solve the problem independently, find the probability that the problem is solved.

16. For the matrix $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$,find $(A + A')$ and verify that is a symmetric matrix.

17. A ladder 13m long is leaning against a vertical wall. The bottom of the ladder is dragged away from the wall along the ground at the rate of 2 cm/sec.How fast is the height of the wall decreasing when the foot of the ladder is 5m away from the wall?

18. Prove that:

$$\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$$

19. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, and hence evaluate $\int_0^1 x^2(1-x)^n dx$.

20. If $x = \sin t$, $y = \sin pt$, prove that $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + p^2y = 0$.

21. Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$with respect to $\cos^{-1} x^2$.

22. Integrate the function $\dfrac{\cos(x+a)}{\sin(x+b)}$ w.r.t x.

23. Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f : A \to B$ is a function defined by $f(x) = \dfrac{x-1}{x-2}$, show that f is one-one and onto.Hence, find f^{-1}.

24. Show that the relation S in the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by $S = \{(a,b) : a, b \in Z, |a-b| \text{ is divisible by } 3\}$ is an equivalence relation.

25. Solve the differential equation $\dfrac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1$ when $x = 0$.

26. Find the particular solution of the differential equation $\dfrac{dy}{dx} = \dfrac{xy}{x^2+y^2}$,given that $y = 1$ when $x = 0$.

27. Using properties of determinants, find the value of x for which

$$\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$

28. Find the vector equation of the plane which contains the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0, \vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$.

29. Find the value of x such that the four points with position vectors,$A\left(3\hat{i} + 2\hat{j} + \hat{k}\right)$,$B\left(4\hat{i} + x\hat{j} + 5\hat{k}\right)$,$C\big($ and $D\left(6\hat{i} + 5\hat{j} - \hat{k}\right)$ are coplanar.

30. If $y = (log x)^x + x^{log x}$, find $\dfrac{dy}{dx}$.

31. Find the vector equation of a line passsing through the point $(2, 3, 2)$ and parallel to the line $\vec{r} = \left(-2\hat{i} + 3\hat{j}\right) + \lambda\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$.Also, find the distance between these two lines.

32. Find the coordinates of the foot of perpendicular Q drawn from $P(3, 2, 1)$ to the plane $2x - y + z + 1 = 0$.Also,find the distance PQ and the image of the point P treating this plane as a mirror.

33. Using elementary row transformsations, find the inverse of the matrix $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$.

34. Using matrices, solves the following system of linear equations :

$$\begin{aligned} x + 2y - 3z &= -4 \\ 2x + 3y + 2z &= 4 \\ 3x - 3y - 4z &= 11 \end{aligned}$$

35. Using integraion, find the area of the region bounded by the parabola $y^2 = 4x$ and the circle $4x^2 + 4y^2 = 9$.

36. Using the method of the integration, find the area of the region bounded by the lines $3x - 2y + 1 = 0$, $2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$.

37. An insurance company insured 3000 cyclists, 6000 scooter drivers and 9000 car drivers. The probability of an accident involving a cyclist, a scooter driver and a car driver are 0.3, 0.05 and 0.02 respectively. One of the insured persons meets with an accident. What is the probability that he is a cyclist ?

38. Using integration, find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.

39. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹ 35 per package of nuts and ₹ 14 per package of bolts. How many packages of each should be poduced each day so as to maxmise his profit, if he operates each machine for atmost 12 hours a day ? Convertit into a LPP and solve graphically.