(3.16)

$$y(n) = \left(\frac{1}{N}\right) \sum_{k=0}^{(N-1)} (-1)^k \cdot x(n-k)$$

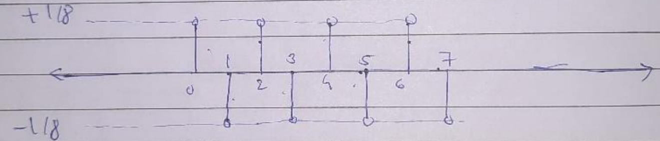
$N = 8$

(a)

$$y(n) = \frac{1}{8} \sum_{k=0}^{7} (-1)^k \cdot x(n-k)$$

$x(n) = \delta$

$$h(n) = \frac{1}{8} \Big[(-1)^0 \cdot \delta(n-0) + (-1)^1 \cdot \delta(n-1)$$
$$+ (-1)^2 \cdot \delta(n-2) + (-1)^3 \cdot \delta(n-3) + (-1)^4 \cdot \delta(n-4)$$
$$+ (-1)^5 \cdot \delta(n-5) + (-1)^6 \cdot \delta(n-6) + (-1)^7 \cdot \delta(n-7) \Big]$$



(b)

DC gain $H^f(0)$

$$H(1) = \sum h(n) = \left(\frac{1}{8}\right) [1-1+1-1+1-1+1-1]$$
$$= 0$$

(c) $$y(n) = \left(\frac{1}{N}\right) \sum_{k=0}^{N-1} \cos(\pi \cdot k) \cdot x(n-k)$$
$$= (1/N) \sum_{k=0}^{N-1} \frac{(e^{-j\pi k} + e^{j\pi k}) \cdot x(n-k)}{2}$$

$N = 8$

$$= \left(\frac{1}{16}\right) \left[\sum_{k=0}^{7} e^{-j\pi k} \cdot x(n-k) + \sum_{k=0}^{7} e^{j\pi k} \cdot x(n-k) \right]$$

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3.16

$$= \left(\frac{1}{8}\right)[x(n) - x(n-1) + x(n-2) - x(n-3) + x(n-4) - x(n-5) + x(n-6) - x(n-7)]$$

$$H(Z) = \left(\frac{1}{8}\right)[1 - Z^{-1} + Z^{-2} - Z^{-3} + Z^{-4} - Z^{-5} + Z^{-6} - Z^{-7}]$$

$$= \left(\frac{1}{8}\right)[(1 + Z^{-2} + Z^{-4} + Z^{-6}) - (Z^{-1} + Z^{-3} + Z^{-5} + Z^{-7})]$$

(3.17)

$$y(n) = (1/N) \sum_{k=0}^{(N-1)} x(n-k) \qquad \boxed{N=6}$$

$$H(z) = \left(\frac{1}{N}\right)\left[1+z^{-1}+z^{-2}+z^{-3}+z^{-4}+z^{-5}\right]$$

(a) cascading the system.

$$\rightarrow \boxed{H(z)} \rightarrow \boxed{H(z)} \rightarrow$$

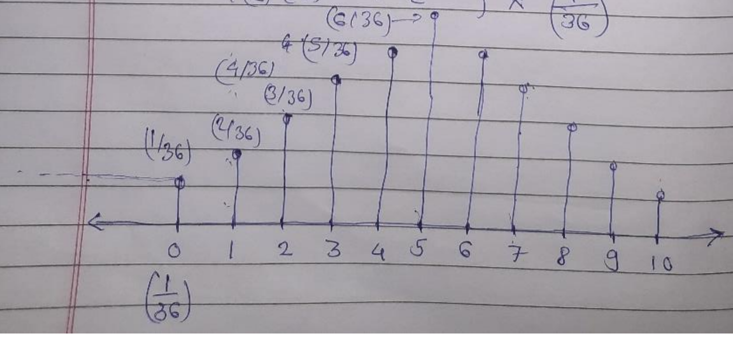
Transfer function of total system

$$H'(z) = H(z)\cdot H(z)$$

$$= \left(\frac{1}{N}\right)^2 \left[1+z^{-1}+z^{-2}+z^{-3}+z^{-4}+z^{-5}\right]^2$$

$$= \frac{1}{N^2}\Big[1+z^{-1}+z^{-2}+z^{-3}+z^{-4}+z^{-5}$$
$$+z^{-1}+z^{-2}+z^{-3}+z^{-4}+z^{-5}+z^{-6}$$
$$+z^{-2}+z^{-3}+z^{-4}+z^{-5}+z^{-6}+z^{-7}$$
$$+z^{-3}+z^{-4}+z^{-5}+z^{-6}+z^{-7}+z^{-8}$$
$$+z^{-4}+z^{-5}+z^{-6}+z^{-7}+z^{-8}+z^{-9}$$
$$+z^{-5}+z^{-6}+z^{-7}+z^{-8}+z^{-9}+z^{-10}\Big]$$

$$= \left(\frac{1}{6}\right)^2 \Big(1+(2)\cdot z^{-1}+(3)\cdot z^{-2}+(4)\cdot z^{-3}+(5)\cdot z^{-4}$$
$$+(6)\, z^{-5}+(5)\cdot z^{-6}+(4)\cdot z^{-7}+(3)\cdot z^{-8}$$
$$+(2)(z)^{-9}+z^{-10}\Big) \times \left(\frac{1}{36}\right)$$



3.17

$h(n) = \{ \underline{1/36}, 2/36, 3/36, 4/36, 5/36, 6/36, 5/36, 4/36, 3/36, 2/36, 1/36 \}$

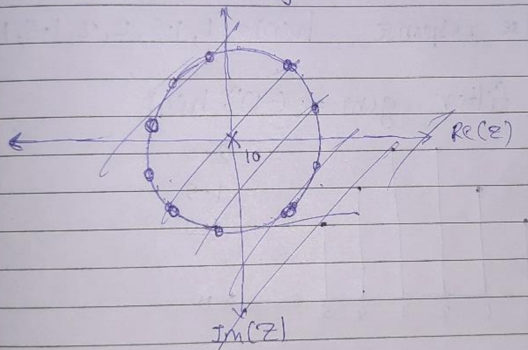
(b) DC gain of the total filter $H_f(0)$.

$= \sum h(n)$

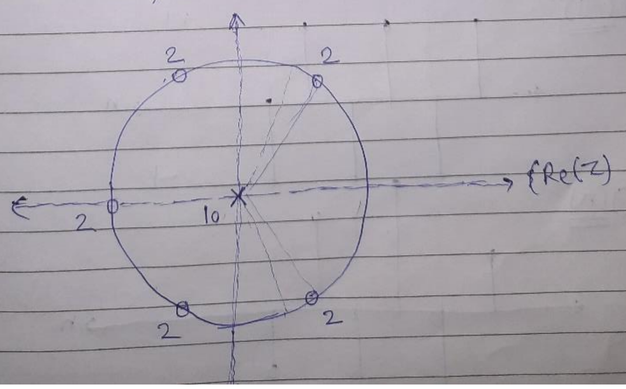
$= 1/36 \, (1+2+3+4+5+6+5+4+3+2+1)$

$= 1/36 \, (36) = 1 = \text{DC gain}$

(c) Pole-Zero diagram.

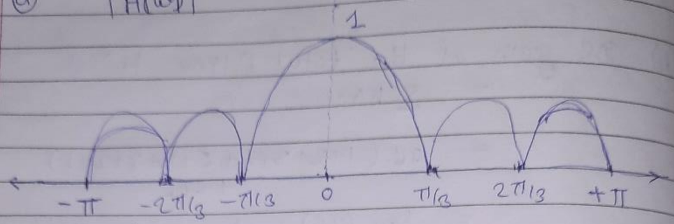


(d)



(3.17)

(a) $|H(\omega)|$

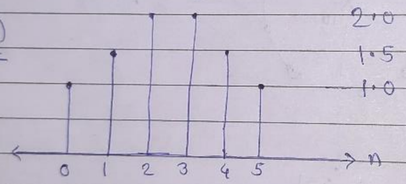


(3.6)

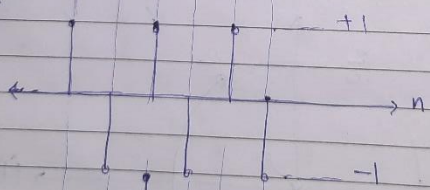
(a) Impulse response $h(n) = (\underline{1}, 1.5, 2, 2, 1.5, 1)$

New FIR filter $g(n) = (-1)^n \cdot h(n)$

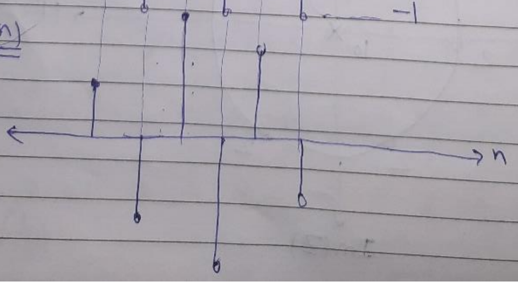
$h(n)$



$(-1)^n$



$g(n)$



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$$g(n) = (\underline{1}, -1.5, 2, -2, 1.5, -1)$$

(b)

$$g(n) = 1 - 1.5\, x(n-1) + 2\, x(n-2) - 2\, x(n-3) + 1.5\, x(n-4)$$
$$-1\, x(n-5)$$

$$G(Z) = 1 - (1.5) \cdot Z^{-1} + (2)\, Z^{-2} - (2) \cdot Z^{-3} + 1.5 \cdot Z^{-4} - 1 Z^{-5}$$

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$$y(n) = x(n) - x(n-N) + y(n-1)$$

$$y(n) - y(n-1) = x(n) - x(n-N)$$

Z.T.

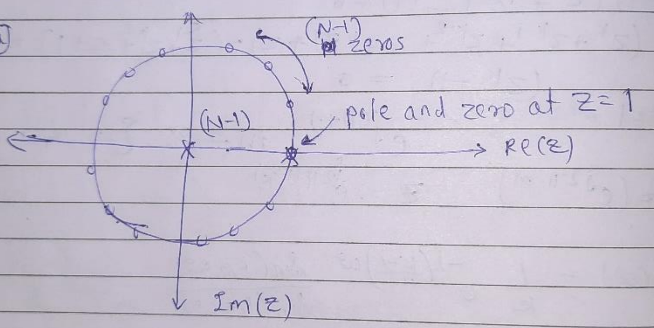
$$Y(z)\left[1 - z^{-1}\right] = X(z)\left[1 - z^{-N}\right]$$

$$H(z) = \frac{1 - z^{-N}}{1 - z^{-1}}$$

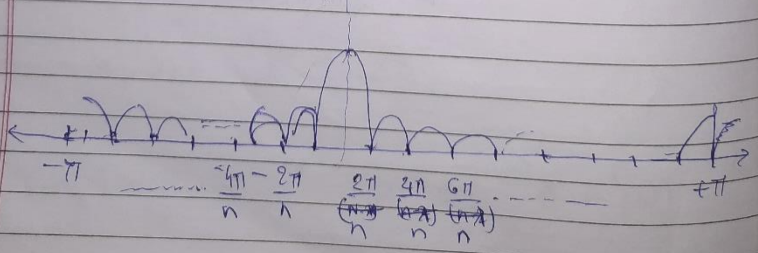
Pole at $z = 1$, $(N-1)$ Poles at origin

N equally spaced Zeros on Unit circle

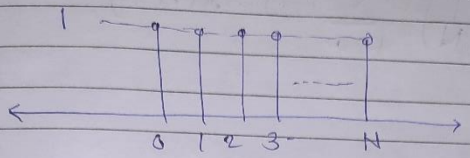
(a)



(b) Frequency Response Magnitude $|H(\omega)|$



(c) Impulse Response. $h(n)$



(d) DC gain of filter $H^f(0) = H(1)$

$$H(1) = \cancel{\frac{1-(1)^{-N}}{1-1}} \sum h(n)$$

$$= \sum_{N} 1$$

$$= N$$