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HW 8

1.13

DFT of Real Signal with circular symmetry is real and circular symmetric.

(a) [1, 1, 1, 0, 0, 0, 1, 1] } Real DFT.

(d) [0, 1, 1, 0, 0, 0, 1, 1] }

DFT of real signal with circular anti-symmetry is purely imaginary

(c) [0, 1, 1, 0, 0, 0, −1, −1] } imaginary DFT.

DFT of real signal without any symmetry is circular conjugate symmetric.

(b) [1, 1, 0, 0, 0, 0, −1, −1]. } Circular complex conjugate DFT.

1.39

Signal	DFT.
1	4
2	6
3	3
4	7
5	1
6	9
7	2
8	8
9	5

1.23

$$x(n) = \cos\left(\frac{2\pi}{N}n + \theta\right) \qquad n = 0, 1, 2, \ldots, (N-1)$$

$$x(n) = \frac{e^{j\left(\frac{2\pi}{N}n+\theta\right)} + e^{-j\left(\frac{2\pi}{N}n+\theta\right)}}{2}$$

$$X(k) = \sum_{n=0}^{(N-1)} x(n) \cdot e^{-j\frac{2\pi}{N}kn}$$

$$= \frac{1}{2}\sum_{n=0}^{(N-1)}\left[e^{j\left(\frac{2\pi}{N}n+\theta\right)} + e^{-j\left(\frac{2\pi}{N}n+\theta\right)}\right] e^{-j\frac{2\pi}{N}kn}$$

$$= \frac{1}{2}\sum_{n=0}^{(N-1)}\left[W_N^{n} \cdot e^{j\theta} + W_N^{-n} \cdot e^{-j\theta}\right] e^{-j\left(\frac{2\pi}{N}\right)kn}$$

$$x(n) = \frac{1}{N}\left[\frac{N}{2} \cdot e^{j\left(\frac{2\pi n}{N}+\theta\right)} + \frac{N}{2} e^{-j\left(\frac{2\pi n}{N}+\theta\right)}\right]$$

$$= \frac{1}{N}\left[\frac{N}{2} e^{j\left(\frac{2\pi}{N}n+\theta\right)} + \frac{N}{2} e^{j\left(\frac{2\pi}{N}(N-1)-\theta\right)}\right]$$

$$= \frac{1}{N}\left[\frac{N}{2} e^{j\left(\frac{2\pi}{N}n\right)} \cdot e^{j\theta} + \frac{N}{2} e^{j\left(\frac{2\pi}{N}(N-1)\right)} \cdot e^{-j\theta}\right]$$

$$= \frac{1}{N}\sum_{k=0}^{(N-1)} X(k) \cdot e^{j\left(\frac{2\pi}{N}k\right)}$$

1.23

$x(1) = e^{j\theta}$ $\qquad$ $x(N-1) = e^{-j\theta}$

$$x = [0, e^{j\theta}, \underbrace{0, \ldots\ldots}_{(N-3)\ \text{Zeros}}, e^{-j\theta}]$$

1.43 $FFT([0, 1, 1, 1, 1, 1]) = FFT(x)$

$x_1 = [1, 1, 1, 1, 1, 1]$

$x_2 = [1, 0, 0, 0, 0, 0]$

$$x = x_1 - x_2$$

Taking DFT on both sides

by linearity property

$DFT(x) = DFT(x_1) - DFT(x_2)$

$DFT(x_1) = [6, 0, 0, 0, 0, 0]$

$DFT(x_2) = [1, 1, 1, 1, 1, 1]$

$DFT(x) = [6, 0, 0, 0, 0, 0] - [1, 1, 1, 1, 1, 1]$

$DFT(x) = [5, -1, -1, -1, -1, -1]$

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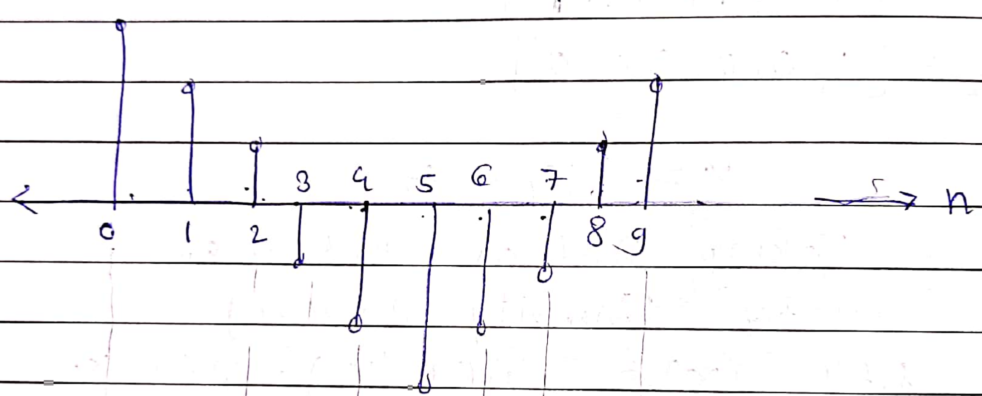
1.30

$$x1 = \cos\left(\frac{[0:9]}{9} 2\pi\right)$$

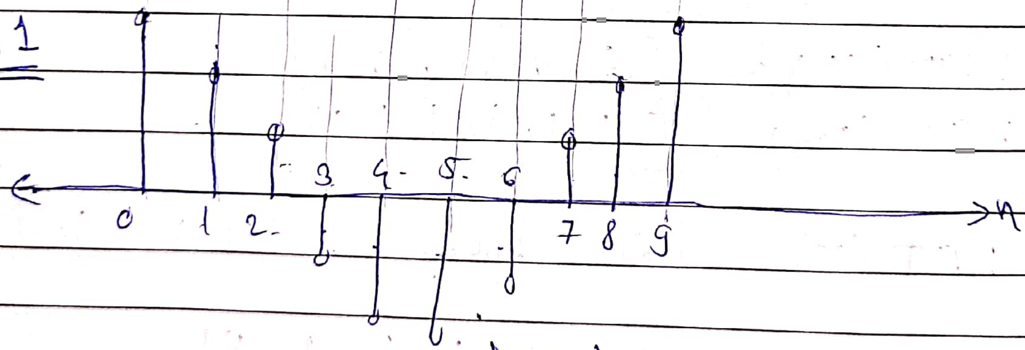
$$x2 = \cos\left(\frac{[0:9]}{10} \cdot 2\pi\right)$$

(a)

X2



X1



Difference between X1 & X2

X2 signal completes one cycle in given range of samples. X1 completes one cycle and ~~excess~~ ...onal samples ~~is~~ are present

(b) X2 signal completes exactly one cycle in 10 samples Therefore

$X_2(K) = 1$ at $k=1$, ~~Real signal~~

DFT of Real signal is circular symmetric

Scanned with CamScanner

1.30

$\cancel{k=1}$ $X2(1) = X(\langle 10-1\rangle_{10}) = X2(9)$.

Signal ~~x~~. x1 has higher frequency than signal x2 but not integer multiple. therefore its frequeny does not ~~the~~ coincide with frequency bins its frequency content leak in DFT

DFT	Graph
X1	C
X2	A

1.49

$$W_N = e^{j\frac{2\pi}{N}}$$

For $N=6$

$$W_6 = e^{j\left(\frac{2\pi}{6}\right)}$$

$$W_6^3 = e^{\left(j\frac{2\pi}{6}\right)\times 3} = (-1)$$

$$(-1)^{[0:5]} = (W_6^3)^{[0:5]}$$

$$G = X\cdot(W_6^3)^k$$

$x[\langle n-m\rangle_N]$ $\quad W_N^{-mk}\cdot X(k)$

circular shift property

$$g = [3, 2, 1, 6, 5, 4]$$

1.64

$F_s = 500\,Hz$, $N = 980$

$F_1 = 120\,Hz$

(a)

$$F_k' = \frac{2\pi k}{N} \times F_s \quad (\text{Radians})$$

$$F_k = \frac{k}{N} \times F_s \quad (Hz)$$

$$F_1 = \frac{k}{(980)} \times 500$$

$$\frac{120 \times 980}{500} = k$$

$$k = 235.2$$

Nearest integer value $\underline{k = 235}$

corresponding F_k

$$F_k = \frac{235}{980} \times 500 = 119.89\ Hz$$

(b)

$$120 = \frac{k}{N} \times 500 \ , \quad \frac{k}{N} = \frac{12\cancel{0}}{50\cancel{0}} = \frac{6}{25}$$

new $N = 25 \times 40 = 1000$ (> 980)

new $k = 06 \times 40 = 240$.

Zeros needed $= 1000 - 980 = 20$

$k = 240$ corresponds to $120\,Hz$.

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1.66

$F_s = 50\,Hz$, max. allowed frequency $= \frac{F_s}{2} = 25\,Hz$.

signal max freq. $= 30\,Hz$.

signals aliased from $= 25\,Hz$ to $30\,Hz$.

$$25 = \frac{k_1 \times 50}{1000} = 500 = k_1$$

$$30 = \frac{k_2 \times 50}{1000} = 600 = k_2$$

by circular symmetry of Real signals

$$k_1^* = (1000 - 500)_{1000} = 500$$

$$k_2^* = (1000 - 600)_{1000} = 400$$

Aliased frequency from $k = 400$ to $k = 600$

Remaining DFT coefficients $x_d(k)$ with $k = 0$ to 399 & 601 to 999 are free of aliasing.

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...pulse Response	Frequency Response	Zero Dia...
A		
B		
C		
D	C	D

...lse ...ponse	Zero Diagram	Frequency Response
	C	B
	A	D
	B	A
	D	C

...near-phase FIR filter.
...(0) = 1, h(1) = 3, h(2) = -2

Type I : Symmetric Odd length
$h = \{1, 3, -2, 3, 1\}$

Type II : Symmetric even length
$h = \{1, 3, -2, -2, 3, 1\}$

...) Type III : Anti-Symmetric odd length.
$h = \{1, 3, -2, 0, 0, -3, -1\}$

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4.2

$h_1(n)$	$h_2(n)$	$h_1(n) * h_2(n)$
I	I	I
I	II	II
I	III	III
I	IV	IV
II	II	II
II	III	III
II	IV	III
III	III	III
III	IV	III
IV	IV	IV

Type	Zeros	
II	$\omega = \pi$	No highpass filter, only Low or Bandpass
III	$\omega = 0, \pi$	No Lowpass, No Highpass
IV	$\omega = 0$	No Lowpass

4.2 $Z = \{ 1, 0.5\, e^{j\pi/3}, -5, j \}$

(a)

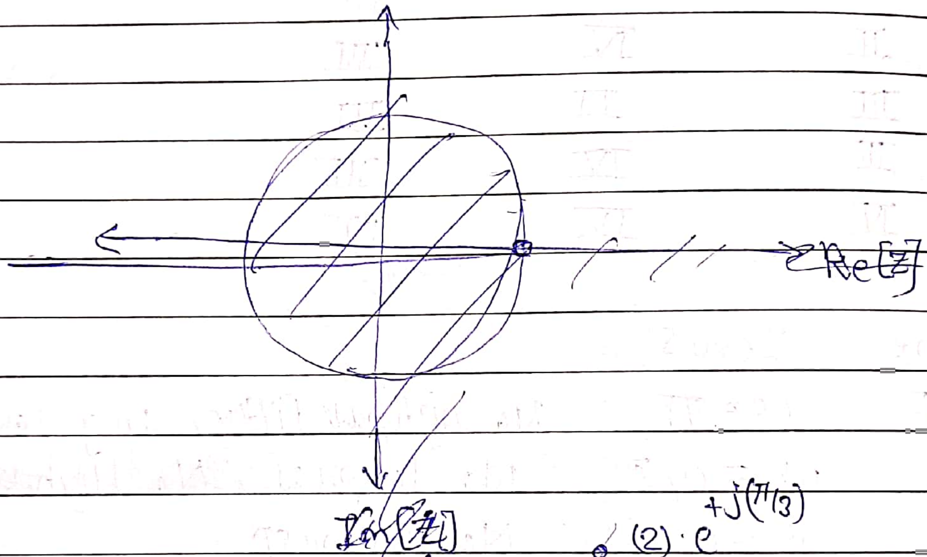
$Z_0 = 1$

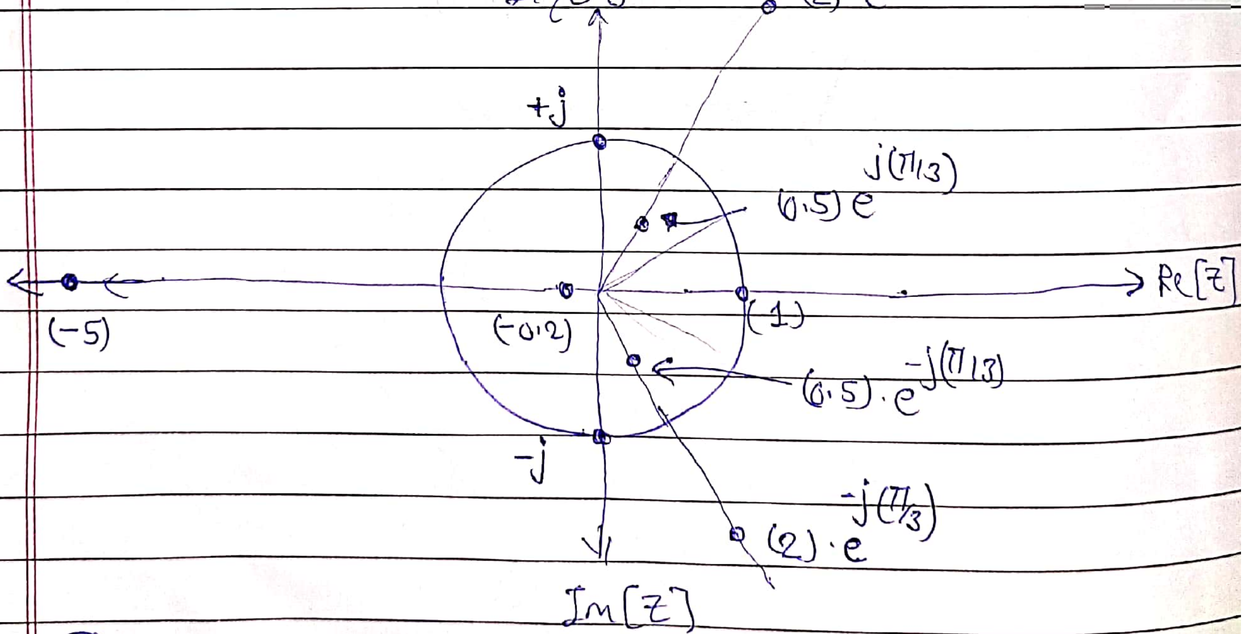
$Z_1 = (0.5)\cdot e^{j(\pi/3)}$, $1/Z_1 = 2\cdot e^{-j(\pi/3)}$, $Z_1^* = (0.5)\cdot e^{-j(\pi/3)}$

$1/Z_1^* = (2)\cdot e^{j(\pi/3)}$

$Z_2 = (-5)$, $1/Z_2 = (-0.2)$.

$Z_3 = j$, $1/Z_3 = (-j)$





(b) Order of filter = 9, length of filter = 10

(c) ~~Filter is odd~~ Filter Type = IV

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4.7

(a) linear-phase FIR filter for Bandpass filter.
(i) Type I
(ii) Type II
(iii) Type III
(iv) Type II & Type IV cascade

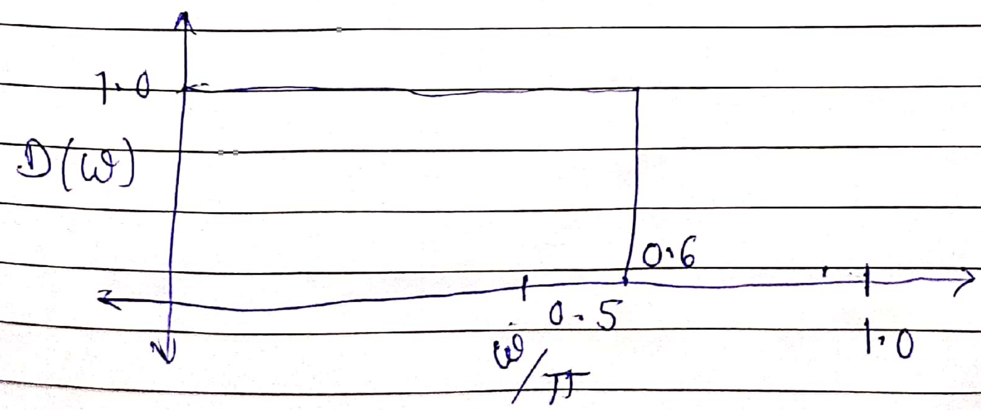
(b) linear phase FIR filter for Bandstop Filter.
(i) Type I ~~fit~~
(ii)
(iii)
(iv) Type II and Type IV ~~cascade~~ in parallel

3.28

Signal = 15 Hz =
Noise Spectrum - 20 Hz to 30 Hz
(Fs) Sampling Frequency - 50 Hz
Max. freq. without aliasing $(F_s/2)$ = 25 Hz

Signal is aliased from - $(25-(30-25))$ = 20 Hz to 25 Hz

D(w): Desired Frequency Response



Weighing Function W(ω)