DSP exercise

10.14 $\quad y(n) = x(n) + \left(\frac{2}{3}\right)\cdot y(n-1)$

Taking Z.T.

$$Y(z) = X(z) + (2/3)\cdot z^{-1}\cdot Y(z)$$

$$Y(z)\left[1-(2/3)z^{-1}\right] = X(z)$$

$$H(z) = \frac{1}{1-(2/3)\cdot z^{-1}}$$

Back word filtering

$$H(z^{-1}) = \frac{1}{1-(2/3)(z^{-1})^{-1}} = \frac{1}{1-(2/3)(z)}$$

Forward- Backward Filter. ~~Resu~~ Transfer function

$$H_2(z) = H(z)\cdot H(z^{-1})$$

$$= \frac{1}{1-(2/3)z^{-1}}\ \frac{1}{1-(2/3)z}$$

$$= \frac{1}{1-(2/3)z-(2/3)z^{-1}+(4/9)} = \frac{1}{(-2/3)z^2+(13/9)z-(2/3)}$$

$$= \frac{(-3/2)}{z^2-(13/6)z+1} = \left(\frac{-3}{2}\right)\frac{1}{(z-2/3)(z-3/2)}$$

$$= \left(\frac{-3}{2}\right)\left[\frac{-6/5}{(z-2/3)} + \frac{6/5}{z-3/2}\right]$$

$$= \left(\frac{-3}{2}\right)\left(\frac{-6}{5}\right)\left[\frac{1}{(z-2/3)} - \frac{1}{(z-3/2)}\right]$$

$$H_2(z) = (9/5)\left[\frac{1}{(z-2/3)} - \frac{1}{z-3/2}\right]$$

Taking stable inverse z transform.

$$h(n) = \left(\frac{9}{5}\right)\left[\left(\frac{2}{3}\right)^n \cdot u[n] + \left(\frac{2}{3}\right)^n \cdot u[-n-1]\right]$$

Scanned with CamScanner

(10.14)

Frequency Response.

~~H2(ejω)~~ =

$$H2(e^{j\omega}) = H(e^{j\omega}) \cdot H(e^{-j\omega})$$

$$Mag(H2(e^{j\omega})) = [H(e^{j\omega})]^2$$

$$|H2(e^{j\omega})| = \left|\frac{1}{1-(2/3)e^{-j\omega}}\right|^2$$

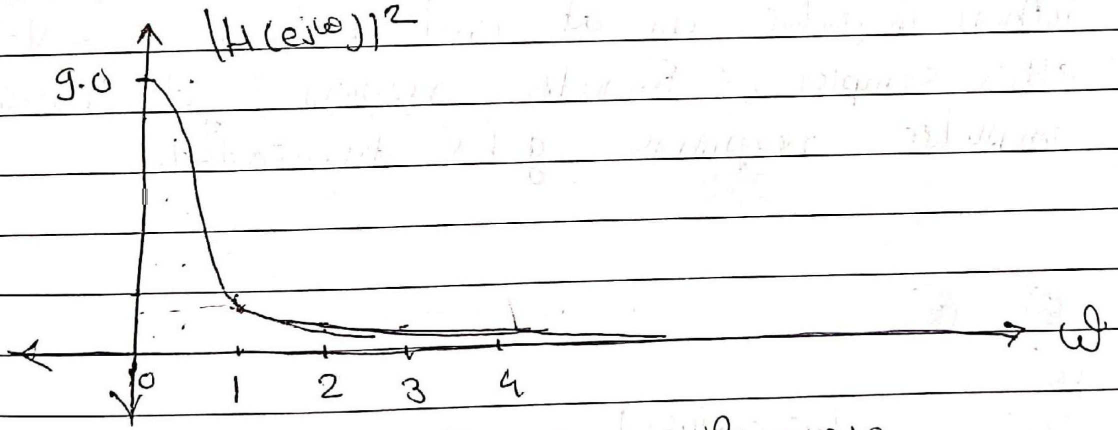
$$0 = \omega_0 = |H2(e^{j0})_{\omega=0}|^2 = (3)^2 = 9$$

$\omega = 1$ $\quad |H2(e^{1j})| = (1.1752)^2 = 1.3810$

$\omega = 2$ $\quad |H2(e^{2j})| = (0.7072)^2 = 0.5002$

$\omega = 3$ $\quad |H2(e^{3j})| = (0.6014)^2 = 0.3617$

$\omega = 4$ $\quad |H2(e^{4j})| = (0.6571)^2 = 0.4318$



Frequency Response.

(d) Forward-Backward Filtering can be used to implement linear ~~or~~ (Zero) phase filtering from nonlinear phase (IIR) filter.

(e) It is easier and possible to design FIR filter as linear phase

(f) filtfilt function in MATLAB implements forward-backward filtering

Least squares in Signal Processing

(5.1)

$$\min_{x} \|y - Ax\|_2^2 + \lambda \|b - x\|_2^2$$

$$J(x) = \|y - Ax\|_2^2 + \lambda \|b - x\|_2^2$$

$$J(x) = (y - Ax)^T \cdot (y - Ax) + \lambda \cdot (b - x)^T \cdot (b - x)$$

$$= (y^T - x^T \cdot A^T) \cdot (y - Ax) + \lambda (b^T - x^T) \cdot (b - x)$$

$$= y^T \cdot y - y^T \cdot Ax - x^T \cdot A^T \cdot y + x^T \cdot A^T \cdot A \cdot x + \lambda (b^T \cdot b - b^T x - x^T \cdot b + x^T \cdot x)$$

$$= y^T \cdot y - 2 \cdot y^T \cdot Ax + x^T \cdot A^T \cdot A \cdot x + \lambda (b^T b - 2 b^T x + x^T \cdot x)$$

~~minimizing x~~

$$\cancel{\min} \frac{\partial}{\partial x} J(x) = (-2 \cdot y^T A)^T + 2 A^T \cdot A \cdot x + \lambda (-2b + 2x)$$

Setting $\frac{\partial}{\partial x} J(x) = 0$

$$-2 A^T \cdot y + (2 \cdot A^T A) \cdot x + (-2 \cdot \lambda \cdot b) + (2 \cdot \lambda) \cdot x = 0$$

$$x (2 A \cdot A^T + 2\lambda) = (2 A^T y + 2 \lambda \cdot b)$$

$$x = (A \cdot A^T \cdot \lambda)^{-1} (A^T \cdot y + \lambda \cdot b)$$

15.2

$$J(x) = \lambda_1 \cdot \|b_1 - A_1 x\|_2^2 + \lambda_2 \|b_2 - A_2 x\|_2^2 + \lambda_3 \|b_3 - A_3 x\|_2^2$$

$$= (\lambda_1)(b_1 - A_1 x)^T \cdot (b_1 - A_1 x) + (\lambda_2)(b_2 - A_2 x)^T (b_2 - A_2 x) + (\lambda_3)(b_3 - A_3 x)^T \cdot (b_3 - A_3 x)$$

$$= \lambda_1 (b_1^T \cdot b_1 - b_1^T \cdot A_1 x - x^T A_1^T \cdot b_1 + x^T A_1^T A_1 x) + \lambda_2 (b_2^T b_2 - b_2^T A_2 x - x^T A_2^T b_2 + x^T A_2^T A_2 x) + \lambda_3 (b_3^T b_3 - b_3^T A_3 x - x^T A_3^T b_3 + x^T A_3^T A x)$$

$$= \lambda_1 (b_1^T \cdot b_1 - 2 b_1^T A_1 x + x^T \cdot A_1^T \cdot A_1 \cdot x) + \lambda_2 (b_2^T \cdot b_2 - 2 b_2^T A_2 x + x^T \cdot A_2^T \cdot A_2 x) + \lambda_3 (b_3^T \cdot b_3 - 2 b_3^T A_3 A_3 x + x^T A_3^T \cdot A_3 x)$$

Setting $\frac{\partial}{\partial x} J(x) = 0$.

$$\lambda_1 (-2 \cdot A_1^T b_1 + 2 A_1^T \cdot A_1 x) + \lambda_2 (-2 A_2^T \cdot b_2 + 2 (A_2^T A_2 x) + \lambda_3 (-2 A_3^T b_3 + 2 A_3^T \cdot A_3 x) = 0$$

$$2x (\lambda_1 A_1^T \cdot A_1 + \lambda_2 A_2^T \cdot A_2 + \lambda_3 A_3^T \cdot A_3) + (-2)(\lambda_1 A_1^T b_1 + \lambda_2 A_2^T b_2 + \lambda_3 A_3^T \cdot b_3) = 0$$

$$x (\lambda_1 A_1^T \cdot A_1 + \lambda_2 A_2^T \cdot A_2 + \lambda_3 A_3^T \cdot A_3) = (\lambda_1 A_1^T \cdot b_1 + \lambda_2 \cdot A_2^T \cdot b_2 + \lambda_3 \cdot A_3^T \cdot b_3)$$

$$x = (\lambda_1 A_1^T \cdot A_1 + \lambda_2 \cdot A_2^T A_2 + \lambda_3 A_3^T \cdot A_3)^{-1} \cdot (\lambda_1 A_1^T \cdot b_1 + \lambda_2 A_2^T \cdot b_1 + \lambda_3 \cdot A_3^T \cdot b_3)$$

(15.4) $J(x) = \|(Ax - b)\|_2^2$

$J(x) = (Ax-b)^T \cdot (Ax-b) = x^T A^T A x - x^T A^T b - b^T A x + b^T b$

$J(x) = x^T A^T A^T A x - 2 b^T A \cdot x + b^T b$

$\frac{\partial \{ J(x)}{\partial x} = 2(A^T A \cdot x) - 2(A^T b)$

$= 2(A^T A \cdot x - A^T b)$

$= 2A^T(Ax - b)$

(16.2)

$\cancel{x = ((I + \lambda \cdot D^T D)^{-1} y}$, $\lambda \geq 0$

~~$D \rightarrow$ first order difference~~

(15.3) Matrix is invertible when its determinant is not equal to '0' (zero) by loading diagonals. of $H^T H$ its determinent becomes non-zero.

15.5 Large value λ can smooth out even large crest, and trough of signal along with noise in the data.
In an ECG signal large value of λ can destroy useful information in signal.

15.6 To check effect of higher order difference. ~~on~~ in smoothing demo amplitude of noise is increased.
Compare to second order third order output produces more curves. i.e. it tries to match the estimate with noise.

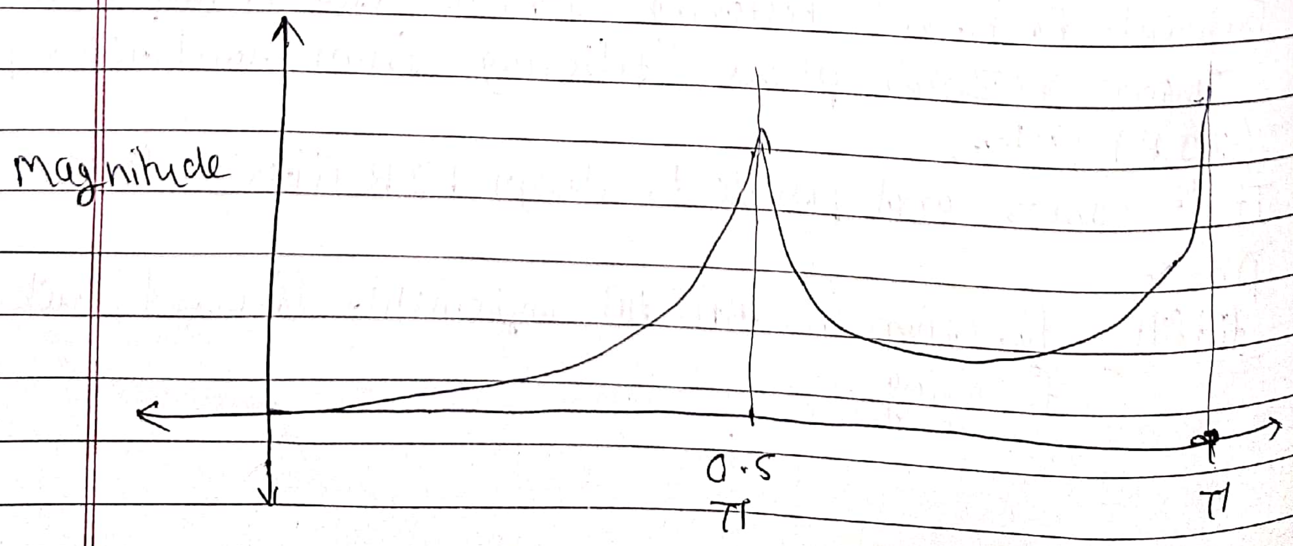
16.1 When impulse is at end $n_0 = 0$ or $n_0 = N-1$ other samples of impulse response get cut-off. impulse response gets truncated.

~~16.2~~

~~(a)~~ : ~~(b)~~

~~(a)~~

16.3 (b) $|G(e^{j\omega})| =$



e.g →

16.3

a)

$$G(z) = \frac{1}{H(z)}$$

$$G(e^{j\omega}) = \quad H(e^{-j\omega})$$