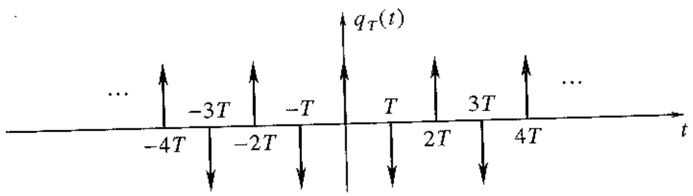

Figure 2.11 Pertaining to Problem 2.24.

2.26 The form of the Poisson sum formula given in (2.48) is

$$\sum_{n=-\infty}^{\infty} \delta(t-nT) = \frac{1}{T}\sum_{k=-\infty}^{\infty} \exp\left(j\frac{2\pi kt}{T}\right). \tag{2.159}$$

In mathematical texts, the formula is usually stated as

$$\sum_{n=-\infty}^{\infty} x(nT) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X^{\mathrm{F}}\left(\frac{2\pi k}{T}\right), \tag{2.160}$$

provided $x(t)$ is continuous at the points nT and $X^{\mathrm{F}}(\omega)$ is continuous at the points $2\pi k/T$. Show that (2.159) implies (2.160).

2.27 Compute the Fourier transform of the discrete-time signal

$$x[n] = \begin{cases} C, & n_1 \le n \le n_2, \\ 0, & \text{otherwise}, \end{cases}$$

where n_1, n_2 are integer constants, and C is a real constant.

2.28 The Fourier transform of a discrete-time signal $x[n]$ is

$$X^{\mathrm{f}}(\theta) = \cos\theta + \sin(2\theta).$$

Compute $x[n]$.

2.29 Let $x[n]$ be the signal

$$x[n] = \begin{cases} \{1,-1,-2,4,-2,-1,1\}, & -3 \le n \le 3, \\ 0, & \text{otherwise}. \end{cases}$$

Compute the following quantities without finding $X^{\mathrm{f}}(\theta)$ first.

(a) $X^{\mathrm{f}}(0)$.

(b) $\measuredangle X^{\mathrm{f}}(\theta)$.

(c) $\int_{-\pi}^{\pi} X^{\mathrm{f}}(\theta)d\theta$.

(d) $X^{\mathrm{f}}(\pi)$.

(e) $\int_{-\pi}^{\pi} |X^{\mathrm{f}}(\theta)|^2 d\theta$.

(f) $\frac{dX^{\mathrm{f}}(\theta)}{d\theta}\big|_{\theta=0}$.

2.30 Can the function

$$X^{\mathrm{f}}(\theta) = \cos(0.5\theta), \quad \theta \in \mathbb{R}$$

be the Fourier transform of a discrete-time signal $x[n]$? If so, find $x[n]$. If not, explain the reason.

2.31 This problem discusses the Fourier transform of a sequence modulated by sign alternations.

(a) If $X^{\mathrm{f}}(\theta)$ is the Fourier transform of $x[n]$, what is the Fourier transform of $(-1)^n x[n]$?

(b) Express $\sum_{n=-\infty}^{\infty}(-1)^n x[n]$ in terms of the Fourier transform of $x[n]$.

2.32 We are given two linear discrete-time systems. The response of the first to a unit impulse $\delta[n-k]$ is $h_1[n] = \sin[3(n-k)]$ and that of the second is $h_2[n] = \sin[3(n+k)]$. Is either system time invariant?

2.33 Prove Theorem 2.5.

2.34 Let $x[n]$ be a discrete-time signal with Fourier transform $X^{\mathrm{f}}(\theta)$, and let

$$Y^{\mathrm{f}}(\theta) = X^{\mathrm{f}}(\theta) + X^{\mathrm{f}}(\theta-\pi).$$

Prove that $Y^{\mathrm{f}}(\theta)$ depends only on the even-indexed signal values $x[2m]$, and is independent of the odd-indexed values $x[2m+1]$.

2.35 Prove that

$$x[n] = \cos(\theta_0 n + \phi_0)$$

is periodic if and only if $\theta_0 = 2\pi p/q$, where p and q are positive integers. Find the period N in case the condition is satisfied.

2.36 The *correlation* (or *cross-correlation*) of two continuous-time signals is defined as

$$z(t) = \{x \star y\}(t) = \int_{-\infty}^{\infty} x(t+\tau)\bar{y}(\tau)d\tau. \tag{2.161}$$

Similarly, the correlation of two discrete-time signals is

$$z[n] = \{x \star y\}[n] = \sum_{m=-\infty}^{\infty} x[n+m]\bar{y}[m]. \tag{2.162}$$

Express the correlation operation in the frequency domain, for both continuous-time and discrete-time signals.

2.37* Prove that (2.65) implies (2.66).

2.38* Suppose we have two discrete-time LTI systems connected in series, so the frequency response of the series connection is

$$H^{\mathrm{f}}(\theta) = H_1^{\mathrm{f}}(\theta)H_2^{\mathrm{f}}(\theta).$$

Recall the definition of noise gain (2.137). Let NG_1, NG_2, NG be the noise gains of the corresponding frequency response functions.

(a) Show that, in general,

$$\mathrm{NG} \neq \mathrm{NG}_1\mathrm{NG}_2.$$

(b) If $H_1^{\mathrm{f}}(\theta) = C$, where C is a real positive constant, what can you say about NG in this special case?

2.39* This problem discusses the effect of an LTI system on white noise in the covariance domain.