4.23 FIR impulse responses.

(A) $h1 = [1, j, 2, j, 1]$

Frequency response = DFT (h1) = H1

$$X(k) = \sum_{n=0}^{(N-1)} x(n) \cdot e^{-j\frac{2\pi}{N}kn}$$

$$H1 = \sum_{n=0}^{(N-1)} h1(n) \cdot e^{-j\frac{2\pi}{N}kn}$$

$$= 1 \cdot e^{-j\frac{2\pi}{5}k(0)} + j \cdot e^{-j\frac{2\pi}{5}k(1)} + 2 \cdot e^{-j\frac{2\pi}{5}k(2)}$$

$$+ j \cdot e^{-j\frac{2\pi}{5}(k)(3)} + 1 \cdot e^{-j\frac{2\pi}{5}k(4)}$$

$k=0$ $\quad H1(0) = 1 + j + 2 + j + 1$

$$= 4 + 2j$$

$k=1$ $\quad H(1) = j + (0.9511 + 0.309j) + (-1.618 - 1.1756j)$

$$+ (-0.5878 - 0.8090j) + (0.309 + 0.9511j)$$

$$= (0.0543 - 0.7245j)$$

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4.23

$k=2 \quad H(2) = \cancel{2} + (0.5878 - 0.809j) + (0.6180 + 1.9021j)$
$+ (1.9021 + 0.6180j) + (-1.6180 + 1.1756j)$
$= 2.3479 + 1.9899j$

$K=3 \quad H(3) = j + \cancel{(-1.1756 - 1.618j)} + (0.5878 - 0.8090j)$
$+ (0.6180 - 1.9021j) + (-0.9511 + 0.3090j) + (-0.8090 - 0.5878j)$
$= -0.7299 - 2.9899j$

$K=4 \quad H(4) = 1 + (-0.9511 + 0.3090j) + (-1.618 + 1.1756j)$
$+ (0.5878 - 0.8090j) + (0.3090 - 0.9511j)$
$= -0.6723 - 0.2755j$

$H1(K) = [4+2j, 0.0543 - 0.7245j, 2.3479 + 1.9899j,$
$-0.7299 - 2.89j, -0.6723 - 0.2755j]$

(B) $h2 = [1, j, 2, -j, 1]$

$H2 = 1 \cdot e^{-j\frac{2\pi}{5}k(0)} + j \cdot e^{-j\frac{2\pi}{5}k(1)} + 2 \cdot e^{-j\frac{2\pi}{5}k(2)} - j \cdot e^{-j\frac{2\pi}{5}k(3)} + 1 \cdot e^{-j\frac{2\pi}{5}k(4)}$

$k=0, H2(0) = 1 + j + 2 - j + 1$
$= 4$

$k=1, H2(1) = 1 + (0.9511 + 0.3090j) + (-1.6180 - 1.1756j)$
$+ (-0.5878 - 0.8090j) + (0.3090 + 0.9511j) = 1.2299 + 0.8035j$

$K=2, H2(2) = \cancel{2} + \cancel{(0.810j)} + (0.1620) \cancel{+ (0.0131)} \cancel{+ (0.0017)}$

$= 1 + (-1.6180 - 1.1756j) + (0.6180 + 1.9021j)$
$+ (0.6180 - 1.9021j) + (-1.6180 + 1.1756j) = 0.4457 + 1.3719j$

Q.23

$K = 3,\ H2(3) = 1(1) + (j)(-0.809 + 0.5878j)$
$+ 2(0.3090 - 0.9511j) - (j)(0.3090 + 0.9511j) + (-0.8090 - 0.5878j)$
$= 1.1723 - 3.6079j$

$K = 4,\ H2(4)$
$= 1(1) + (j)(0.3090 + 0.9511j) + (2)(-0.8090 + 0.5878j)$
$+ (-j)(-0.8090 - 0.5878j) + (0.3090 - 0.9511j)$
$= -1.8479 + 1.3425j$

$H2 = [4,\ 1.2299 + 0.8935j,\ 0.4457 + 1.3719j,$
$1.1723 - 3.6079j,\ -1.8479 + 1.3425j]$

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5.2

Window Function	DTFT $W(\omega)$
A	$W_2(\omega)$
B	$W_1(\omega)$
C	$W_3(\omega)$

5.3 $h(n) = d(n) \cdot w(n)$

Sinc Function	Window	Freq. Response
1	1	2
1	2	4
1	3	3
2	1	6
2	2	5
2	3	1

5.5

Window length $= N$ (odd)
largest side lobe $= (2\pi / N)$ at freq.
Smallest side lobe at freq. (π) [$W^f(\omega)$ is 2π periodi

$$\left|W^f(\omega)\right|_{\omega=\pi} = \frac{\sin(N\omega/2)}{\sin(\omega/2)} \quad \text{at } \boxed{\omega = \pi}$$

$\omega = \pi$

$$= \frac{\sin(N\pi/2)}{\sin(\pi/2)} = 1$$

$$\left|W^f(0)\right| = \frac{\sin(\cancel{0}N\omega/2)}{\sin(\omega/2)} = N$$

$$\frac{|W^f(\pi)|}{|W^f(0)|} = \frac{1}{N}$$

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(5.6)

No. of samples = 128

$$x(n) = \sin\left(2\pi \cdot \frac{6.3}{128} n\right) + (0.001) \cdot \sin\left(2\pi \cdot \frac{56}{128} n\right)$$

(A)
For Rectangular window smallest side lobe is at $\theta = \pi$ and have value $1/N$ of main lobe second sinusoid has amplitude of (0.001), so second sinusoid gets completely masked by sidelobes

(b) Hann window sidelobe amplitudes decay much faster than side lobe amplitudes of Hamming window.
Hann window is better for detecting second component.

5/7

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5.8

$N = 50$

$x(n) = A_1 \cdot \sin(\omega_1 n) + A_2 \cdot \sin(\omega_2 n) + A_3 \cdot \sin(\omega_3 n)$

(A) $x(n) = (10)\cdot \sin(0.31\pi n) + (11)\cdot \sin(0.32\pi n) + (9)\sin(0.33 \cdot \pi n)$.

(i) $\omega_2 - \omega_1 = (0.32\pi - 0.31\pi) = (0.01)\pi = \frac{\pi}{100}$

$\frac{\pi/100}{\pi/N} = \frac{\pi/100}{\pi/50} = \frac{\cancel{\pi}}{100} \times \frac{50}{\cancel{\pi}} =$

(ii) $\omega_3 - \omega_2 = (0.33\pi - 0.32\pi) = (0.01)\pi$

Highest amplitude for $\omega = (0.32\pi)$

width of main lobe $= \frac{4\pi}{N} = \frac{4\pi}{50} = (0.08)\pi$

both secondary sinusoids gets masked by main lobe

(B)

$x(n) = (0.01)\cdot\sin(0.2\pi n) + (0.011)\cdot\sin(0.5\pi n) + (0.012)\cdot\sin(0.8\pi n)$

main lobe width $= 4\pi/N = (0.08)\pi$

(i) $\omega_3 - \omega_2 = (0.8\pi - 0.5\pi) = (0.3\pi)$

$\frac{(3/10)\pi}{\pi/50} = \frac{3\cancel{\pi}}{1\cancel{0}} \times \frac{5\cancel{0}}{\cancel{\pi}} = 15$ odd multiple of (π/N)

(ii) $\omega_3 - \omega_1 = (0.8\pi - 0.2\pi) = (0.6\pi)$

$\frac{6/10\,\pi}{\pi/50} = \frac{6\cancel{\pi}}{10} \times \frac{50}{\cancel{\pi}} = 30$

In Signal (A) it is more difficult to estimate frequency content of signal as smaller sinusoids gets masked by biggest sinusoid main lobe in FFT.

5.11 (A)

$$x(n) = 1 \cdot \sin(0.31 \cdot \pi \cdot n) + 1 \cdot \sin(0.33 \cdot \pi \cdot n) + (0.1) \cdot \sin(0.7 \cdot \pi \cdot n)$$

No. of points $N = 50$

Main lobe width in FFT $= \frac{4\pi}{N} = \frac{4 \cdot \pi}{50} = (0.08\pi)$

$$\Delta f_1 = \omega_2 - \omega_1 = (0.33 - 0.31)\pi = (0.02\pi)$$
$$\Delta f_2 = \omega_3 - \omega_2 = (0.33 + 0.7)\pi = (0.37\pi)$$
$$\Delta f_3 = \omega_3 - \omega_1 = (0.7 - 0.31)\pi = (0.39\pi)$$

(B)

$$x(n) = (10) \cdot \sin(0.31\pi n) + 1 \cdot \sin(0.33\pi n) + (1) \cdot \sin(0.7\pi n)$$

$$\Delta f_1 = \omega_2 - \omega_1 = (0.33 - 0.31)\pi = (0.02\pi)$$

Both signals have ~~frequency~~ 3 sinusoids with same frequencies but different amplitude.

Signal (B) is easier to estimate because frequency nearest to weakest sinusoid is of same amplitude

3.27

Bandlimited signal = 10Hz = 0.25 π

Spectrum of Noise = 20 Hz to 40 Hz = 0.5 π to π

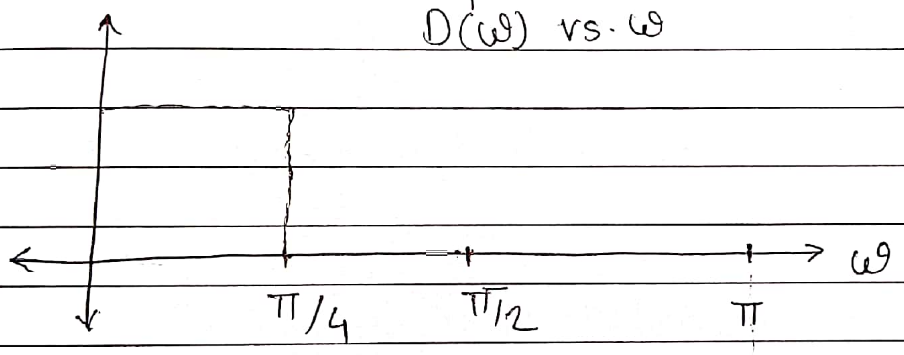
Sampling freq = 80 Hz = Fs

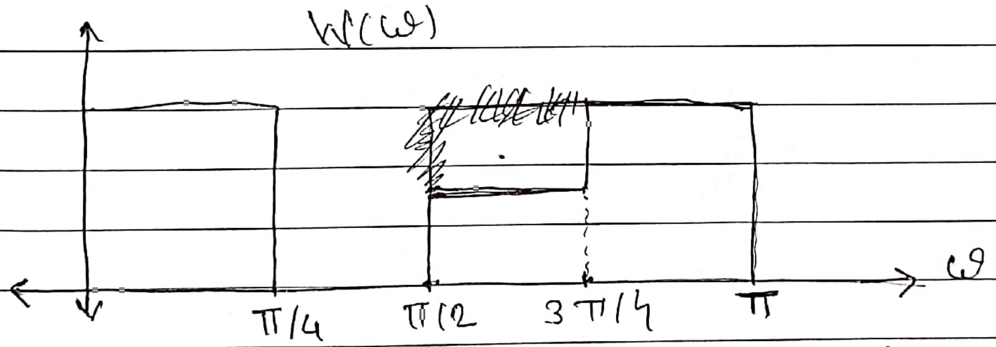
Max. freq. limited by sampling freq. = $\frac{Fs}{2}$ = 40Hz

Noise - Power	Freq.	Freq (Radians)
20Hz - 30Hz	x1	0.5π to 0.75π
30 Hz - 40 Hz	2 x x1	0.75π to π

Desired (ideal) freq Response

D(ω) vs. ω





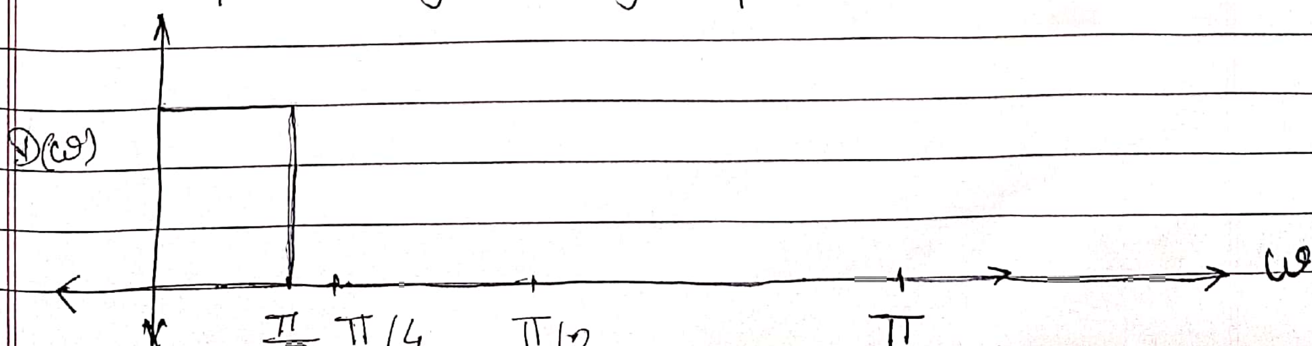
3.29

Bandlimited signal = 10Hz = (0.20)π

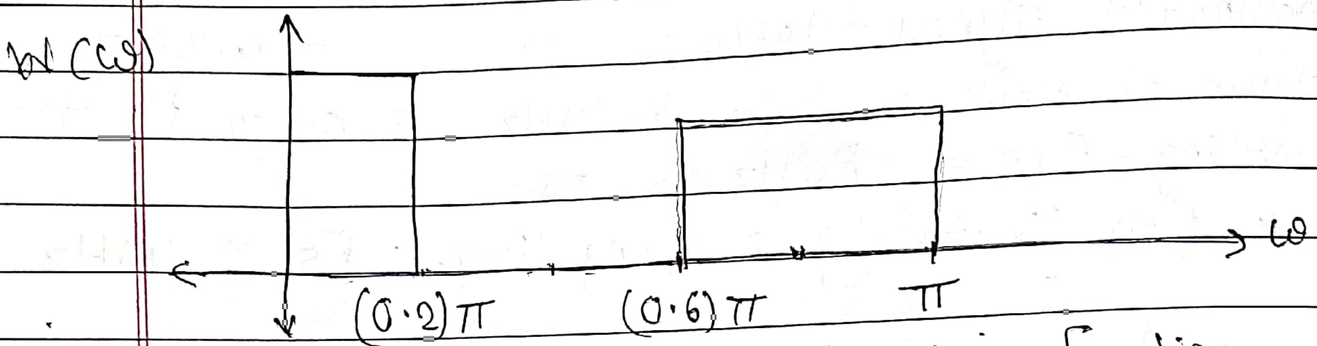
Noise Spectrum = 40Hz to 70Hz = (0.8)π to (1.4)π

Sampling Frequency = Fs = 100Hz

Max. Freq. limited by sampling freq. = Fs/2 = 50Hz



Noise spectrum $= \pi - (1.4\pi - \pi) = 0.6\pi$



6.4 Frequency Response Weighting Function

Frequency Response	Weighting Function
A	2
B	3
C	4
D	1

1.31 (a)

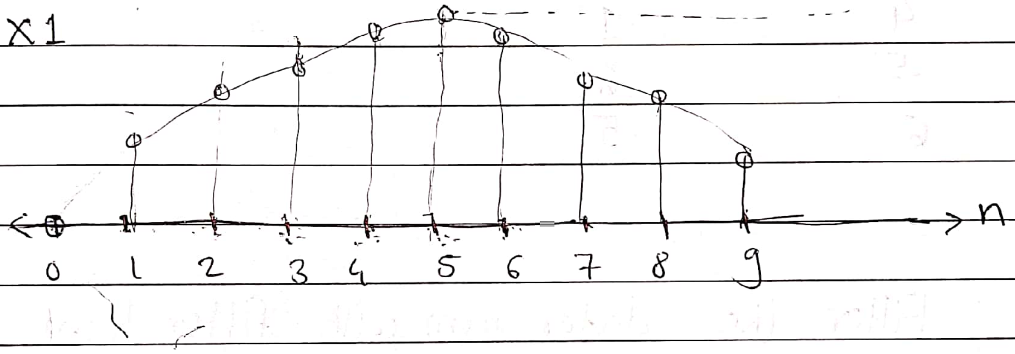
$$x1 = \sin\left(pi * \frac{[0:9]}{10}\right)$$

$$= \sin\left(2 \times pi \times \frac{[0:9]}{20}\right)$$

$$x2 = \sin\left(2 \times pi \times \frac{[0:9]}{9}\right)$$

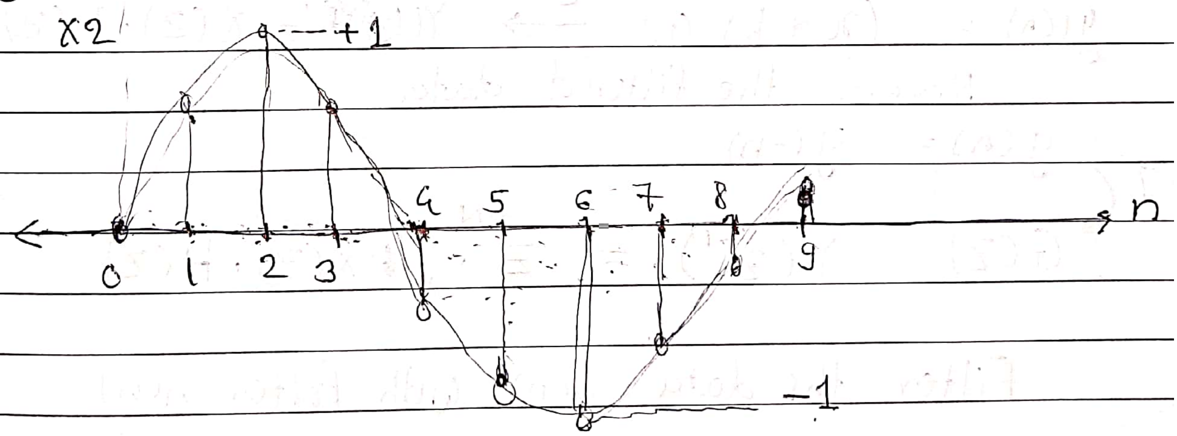
$$x3 = \sin\left(2 \times pi \times \frac{[0:9]}{10}\right)$$

(i) x1

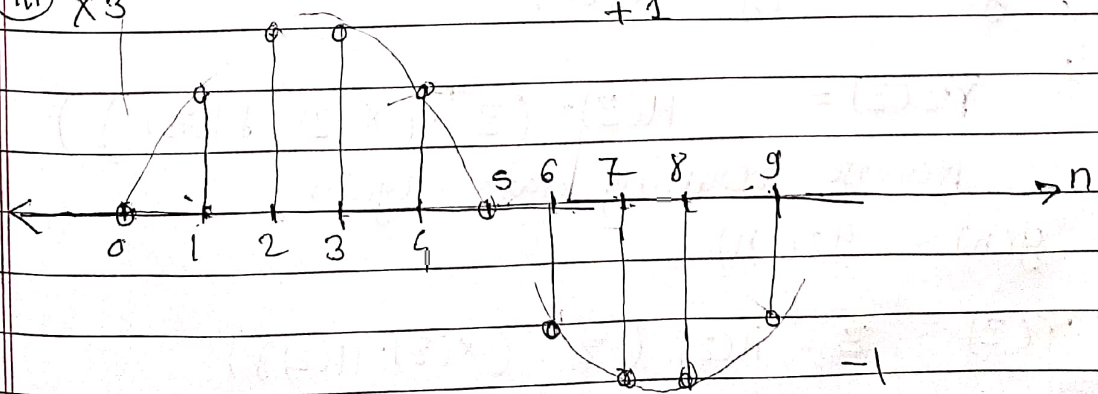


(ii)

x2



(iii) x3



1.81

(b)

DFT	Graph
X1	C
X2	H
X3	B

1.55

CODE	FIGURE
1	2
2	4
3	3
4	1
5	6
6	5

10.11

Filter the datar $x(n)$ with filter $h(n)$

$$y_1(n) = (x * h)(n) \xrightarrow{Z.T.} Y_1(z) = X(z) \cdot H(z)$$

Reverse the filtered data

$$g(n) = y(-n)$$

Z.T

$$G(z) = Y(z^{-1}) = z^{-N} \cdot (X(z) \cdot H(z))$$

Filter the data $g(n)$ with filter $h(n)$

$$y_2(n) = (g * h)(n)$$

Z.T.

$$Y_2(z) = H(z) \cdot (z^{-N} \cdot (X(z) \cdot H(z)))$$

Reverse resulting data again

$$y(n) = y_2(-n)$$

$$Y(z) = z^{-N} \cdot H(z)\,(z^{-N} \cdot (X(z) \cdot H(z)))$$

Rearranging Using associativity property

10.11

$= \underbrace{z^{-2N}} \cdot (X(z) \cdot H^2(z))$

equivalent to reversing circularly twice yields same original signal.

$Y(z) = H^2(z) \cdot X(z)$

$Y(z) = H(z) \cdot H^*(z) \cdot X(z)$

Frequency Response. $\cancel{z:k} = e^{j\omega}$.

$Y(e^{j\omega}) = H(e^{j\omega}) \cdot H(e^{-j\omega}) \cdot X(e^{j\omega})$

$= \underbrace{|H^2(e^{j\omega})|} \cdot X(e^{j\omega})$

Phase Response term is '0'