(1.9)

$x = [1, 2, 0, -1, 0, 0, 0]$

$g = [1, 0, 0, 2, 1, 0, 0]$

$x \xrightarrow{DFT} X$

$g \xrightarrow{DFT} G$

$X(\omega) * G(\omega) \xrightarrow{IDTFT} (x * g)(n)$

$$(x * g)(n) = \sum_{k=0} x(n-k) \cdot g(k)$$

$g(k)$ 1 0 0 2 1 0 0

$x(n-k)$

$k = 0$

0, −1, 0, 2, 1

0 0 −1, 0, 2, 1

0 2 1

−1 0 2 1

0 −1 0 2 1

0 0 −1 0 2 1

0 0 0 −1 0 2 1

~~0 2 0 0 −1 2 0 2~~

~~−1 2 0~~

0 0 0 −1 0 2

0 0 0 −1 0

−1

0

k	output
0	1
1	2
2	0
3	1
4	5
5	2
6	−2
7	−1
	0
	0
	0

~~Convert~~ ~~to~~ Computing circular convolution

1, 2, 0, 1, 5, 2, −2, −

$+$ −1 0 0 0 0

$y = [0, 2, 0, 1, 5, 2, -2]$

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1.37

Signal	DFT
1	4
2	6
3	1
4	2
5	8
6	7
7	3
8	5

1.1

$x = [20, 5]$

$$X_k = \sum_{n=0}^{1} x_n \cdot e^{-j\frac{2\pi}{N}k \cdot n}$$

$$X_0 = (20)\, e^{-j\frac{2\pi}{2}\cdot 0 \cdot 0} + (5)\, e^{-j\frac{2\pi}{2}(0)\cdot(1)}$$

$$= 20 + 5$$

$$= 25$$

$$X_1 = (20)\cdot e^{-j\frac{2\pi}{2}\cdot(1)\cdot(0)} + (5)\cdot e^{-j\frac{2\pi}{2}(1)(1)}$$

$$= 20 - 5$$

$$= 15$$

$$X_K = [25 \quad 15]$$

1.2

$x = [3, 2, 5, 1]$

$$X_d = \sum_{n=0}^{3} x_n \cdot e^{-j\frac{2\pi}{N}nk}$$

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(1.2)

$X_0 = 3 \cdot e^{-j\frac{2\pi}{4}(0)(0)} + 2\, e^{-j\frac{2\pi}{4}(0)(1)} + 5 \cdot e^{-j\frac{2\pi}{4}(0)\cdot(2)}$
$+ 1 \cdot e^{-j\frac{2\pi}{4}(0)(3)}$

$= 3 + 2 + 5 + 1 = 11$

$X_1 = 3 \cdot e^{-j\frac{2\pi}{4}(1)\cdot(0)} + 2 \cdot e^{-j\frac{2\pi}{4}(1)\cdot(1)} + 5 \cdot e^{-j\frac{2\pi}{4}(1)\cdot(2)}$
$+ 1 \cdot e^{-j\left(\frac{2\pi}{4}\right)(1)(3)}$

$= 3 + 2\cdot(-j) + (5)\cdot(-1) + (1)(j) = -2-j$

$X_2 = 3 \cdot e^{-j\frac{2\pi}{4}(2)\cdot(0)} + 2 \cdot e^{-j\frac{2\pi}{4}(2)(1)} + 5 \cdot e^{-j\frac{2\pi}{4}(2)\cdot(2)}$
$+ 1\, e^{-j\frac{2\pi}{4}(2)(3)}$

$= 3 + 2(-1) + 5(1) + 1\cdot(-1) = 5$

$X_3 = 3 \cdot e^{-j\frac{2\pi}{4}(3)(0)} + 2 \cdot e^{-j\frac{2\pi}{4}(3)(1)} + 5 \cdot e^{-j\frac{2\pi}{4}(3)(2)} + 1 \cdot e^{-j\frac{2\pi}{4}(3)(3)}$

$= (3) + (2)\cdot(+j) + (5)(-1) + (1)(-j)$

$= -2 + j$

$X_d = [11,\ -2-j,\ 5,\ -2+j]$

(3)

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1.5

$g = [1, 2, 1, -1]$

$h = [0, 1/3, -1/3, 1/3]$

$$y(n) = \sum_{k=0}^{n-1} h(n-k)\, g(k)$$

$1, 2, 1, -1$	
$1/3, -1/3, 1/3, 0$	0
$1/3, -1/3, 1/3 \quad 0$	$1/3$
$1/3, -1/3, 1/3, 0$	$1/3$
$1/3 \; -1/3, 1/3, \; 0$	0
$1/3, -1/3, 1/3, 0$	0
$1/3 \; -1/3 \; 1/3, \; 0$	$2/3$
$1/3 \; -1/3 \; 1/3, \; 0$	$-1/3$

Linear convolution output $= \left[0, \frac{1}{3}, \frac{1}{3}, 0, 0, \frac{2}{3}, \frac{-1}{3}\right]$

Circular Convolution

$$\begin{array}{rl} = & 0, \; 1/3, \; 1/3, \; 0 \\ + & 0 \quad 2/3, \; -1/3, \\ \hline & 0, \; 1, \; 0, \; 0 \end{array}$$

Circular Convolution $= [0, 1, 0, 0]$

1.6

$x = [1, 2, 3, 0, 0, 0, 0]$

$h = [1, 2, 3, 0, 0, 0, 0]$

conv. $= \sum_{k=0}^{2l-1} h(n-k) \cdot x(k)$.

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1.6

			1,	2,	3,	0,	0,	0,	0		
0,0,0,0,	3,	2,	1								1
0 0 0 0	3	2	1								4
0 0 0	0	3	2	1							10
0 0	0	0	3	2	1						12
0	0	0	0	3	2	1	0	0			9
	0 0	0	0	0	3	2	1	0			0
		0	0	0	0	3	2	1			0
											0

linear conv. = 1, 4, 10, 12, 9, 0, 0, 0, 0, 0, 0, 0, 0.

Circular convolution = 1, 4, 10, 12, 9, 0, 0

1.18

$X(k) = [0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0]$, $k = 0, 1, \ldots, 12$

$$x[n] = \text{IDFT}[X(k)]$$

$$= \frac{1}{N} \sum_{k=0}^{(N-1)} X(k) \cdot e^{+j\left(\frac{2\pi}{N}\right) \cdot nk}$$

$$x(n) = \frac{1}{13} \sum_{k=0}^{12} X(k) \cdot e^{+j\left(\frac{2\pi}{13}\right)(n)k}$$

$$x(n) = \frac{1}{13}\left[e^{+j\left(\frac{2\pi}{13}\right)(n)(2)} + e^{+j\left(\frac{2\pi}{13}\right)(n)(11)} \right]$$

n	x[n]	n	x[n]	n	x[n]
0	0.1538	6	0.1362	12	0.0873
1	0.0873	7	0.1362		
2	−0.0545	8	0.0185		
3	−0.1493	9	−0.1151		
4	−0.1151	10	−0.1493		
5	0.0185	11	−0.0546		

1.18

$x[n] = [0.1538, 0.0873, -0.545, -0.1493, -0.1151, 0.085, 0.1362, 0.1362, 0.0185, -0.1151, -0.1493, -0.545, 0.0873]$

$$x(n) = \frac{1}{13}\left[e^{-j\left(\frac{4\pi}{13}\right)n} + e^{-j\left(\frac{22\pi}{13}\right)n}\right]$$

$\langle -22 \rangle_{13} = \underline{+4}$

$$= \frac{1}{13}\left[e^{-j\left(\frac{4\pi}{13}\right)n} + e^{+j(4\pi/13)n}\right]$$

$$x(n) = \left(\frac{2}{13}\right)\cdot \cos\left(\frac{4\pi}{13}n\right)$$

1.21

$$X(k) = [0, \underbrace{e^{j\theta}, 0, 0, 0, \ldots\ldots}_{(N-3)\text{ zeros}}, e^{-j\theta}]$$

$0 \le k \le N-1$

$$x[n] = \frac{1}{N}\sum_{k=0}^{(N-1)} X(k)\cdot e^{+(j2\pi/N)kn}$$

$$= \frac{1}{N}\left[e^{j\theta}\cdot e^{j\left(\frac{2\pi}{N}\right)(1)\cdot n} + e^{-j\theta}\cdot e^{j\left(\frac{2\pi}{N}\right)(N-1)\cdot n}\right]$$

$= \cancel{\frac{1}{N}[}$ $\langle (N-1) \rangle_N = (N-1)$

$$= \frac{1}{N}\left[e^{j\left(\frac{2\pi n}{N}+\theta\right)} + e^{j\left(\frac{2\pi(N-1)\cdot n}{N}-\theta\right)}\right]$$

$$= \frac{1}{N}\left[e^{j\left(\frac{2\pi n}{N}+\theta\right)} + e^{j\frac{2\pi N}{N}}\cdot e^{-j\frac{2\pi n}{N}}\cdot e^{-\theta j}\right]$$

$$= \frac{1}{N}\left[e^{j\left(\frac{2\pi}{N}+\theta\right)} + e^{+j\left(2\pi - \frac{2\pi n}{N} - \theta\right)}\right]$$

(1.21)

$$= \left(\frac{1}{N}\right)\left[e^{j\left(\frac{2\pi}{N}+\theta\right)} + e^{j\left(2\pi - \left(\frac{2\pi}{N}+\theta\right)\right)}\right]$$

$$e^{j\left[2\pi - \left(\frac{2\pi}{N}+\theta\right)\right]} = e^{-j\left(\frac{2\pi}{N}+\theta\right)}$$

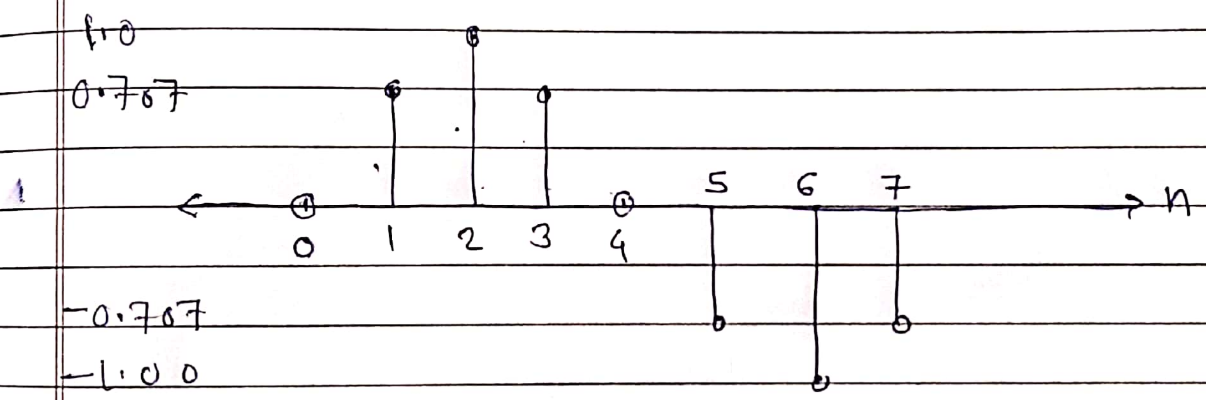
$$= \left(\frac{1}{N}\right)\left[e^{j\left(\frac{2\pi}{N}+\theta\right)} + e^{-j\left(\frac{2\pi}{N}+\theta\right)}\right]$$

$$x[n] = \left(\frac{2}{N}\right) \cdot \cos\left(\frac{2\pi}{N}+\theta\right)$$

(1.24)

x= sin (2 x pi x [0:7]/8);

$x = [0, 0.707, 1, 0.707, 0, -0.707, -1, \cancel{-0.7}-0.707]$



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(1.45)
$x = [6, 4, 3, 2, 1]$

(i) DFT of real signal is complex-conjugate symmetric
(ii) taking conjugate of DFT of Real signal is equivalent of circular reversal. $\Rightarrow$ $x[\langle -k \rangle_N]$
$x^*[n]$ $X^*[\langle -k \rangle_N]$
(iii) By symmetry of DFT properties.

Time reversal property $x[\langle -n \rangle N]$ $X[\langle -k \rangle N]$

$x[\langle -n \rangle N] = [6, 1, 2, 3, 4] = g$

(1.46) $x = [6, 5, 4, 3, 2, 1]$

G = Circular Reversing of DFT of input signal x.
by ~~property~~ ~~of~~ time reversal property of DFT
$x[\langle -n \rangle N]$ $X[\langle -k \rangle N]$

$g = x[\langle -n \rangle N] = [6, 1, 2, 3, 4, 5]$

(1.48) $x = [6, 5, 4, 3, 2, 1]$

$x \xrightarrow{DFT} X$

$G = X \cdot e^{-j[0:5]2\pi/6}$
Circular shift property of DFT.
$W_N^{-mk} X[k]$. $x[\langle n-m \rangle N]$

$m =$

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1.48

comparing W_N^{-mk} and $e^{-j[0:5]\frac{2\pi}{6}}$

$$k = [0:5]$$

$$W_N = e^{j\frac{2\pi}{N}} = e^{j\frac{2\pi}{6}}$$

therefore $\underline{\underline{m = 1}}$

$$g = x[\langle n-1\rangle_6] = [1, 6, 5, 4, 3, 2]$$

1.12 $x[n]$ = Real signal $0 \leq n \leq (N-1)$

DFT. $X(K)$ $0 \leq k \leq (N-1)$

(a)

DFT.

$$X(K) = \sum_{n=0}^{(N-1)} x[n] \cdot e^{-j\frac{2\pi}{N}kn}$$

$$X(0) = \sum_{n=0}^{(N-1)} x[n] \cdot \underbrace{e^{-j\left(\frac{2\pi}{N}\right)(0)\cdot(n)}}_{1}$$

$$X(0) = \sum_{n=0}^{(N-1)} x[n]$$

as $x[n]$ ~~is real No~~. $\sum_{n=0}^{(N-1)} x[n] \rightarrow$ Real
is real signal

therefore.
$X(0)$ is real No

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1.12

(b)

$$X(k) = \sum_{n=0}^{(N-1)} x[n]\, e^{-j\left(\frac{2\pi}{N}\right)(k)(n)}$$

$$k = (N/2)$$

$$X(N/2) = \sum_{n=0}^{(N-1)} x[n].\, \underbrace{e^{-j\left(\frac{2\pi}{N}\right)\left(\frac{N}{2}\right)}}_{=(-1)}{}^{(n)}$$

$$X(N/2) = \sum_{n=0}^{(N-1)} x[n].\,(-1)^n$$

as x is real signal therefore above summation is real.

X(N/2) is real Number.

1.3 DFT of Real signal is always complex conjugat symmetric

$$X(0) = 3.1$$

$$X(1) = X(8)^* = (5.5 - 8.0j)^* = 5.5 + 8.0j$$

$$X(2) = X(7)^*$$

$$X(7) = X(2)^* = 2.5 - 4.6j$$

$$X(3) = X(6)^* = 9.3 - 6.3j$$

$$X(4)^* = X(5) = -1.7 - 5.2j$$

$$X(1) = 5.5 + 8j$$

$$X(3) = 9.3 - 6.3j$$

$$X(5) = -1.7 - 5.2j$$

$$X(7) = 2.5 - 4.6j$$