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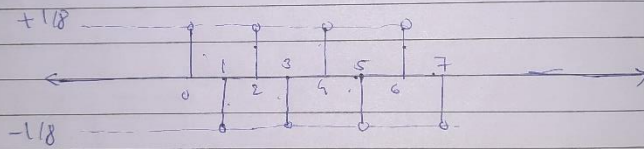
$$y(n) = \left(\frac{1}{N}\right) \sum_{k=0}^{N-1} (-1)^k \cdot x(n-k)$$

$$N=8$$

$$a) \quad y(n) = \frac{1}{8} \sum_{k=0}^7 (-1)^k \cdot x(n-k)$$

$$x(n) = \delta$$

$$h(n) = \frac{1}{8} \left[(-1)^0 \cdot \delta(n-0) + (-1)^1 \cdot \delta(n-1) + (-1)^2 \cdot \delta(n-2) + (-1)^3 \cdot \delta(n-3) + (-1)^4 \cdot \delta(n-4) + (-1)^5 \cdot \delta(n-5) + (-1)^6 \cdot \delta(n-6) + (-1)^7 \cdot \delta(n-7) \right]$$



b)

DC gain $H^f(0)$

$$H(0) = \sum h(n) = \left(\frac{1}{8}\right) [1 - 1 + 1 - 1 + 1 - 1 + 1 - 1] = 0$$

$$c) \quad y(n) = \left(\frac{1}{N}\right) \sum_{k=0}^{N-1} \cos(\pi \cdot k) \cdot x(n-k)$$

$$= \left(\frac{1}{N}\right) \sum_{k=0}^{N-1} \frac{(e^{j\pi k} + e^{-j\pi k})}{2} \cdot x(n-k)$$

$$N=8$$

$$= \left(\frac{1}{16}\right) \left[\sum_{k=0}^7 e^{j\pi k} \cdot x(n-k) + \sum_{k=0}^7 e^{-j\pi k} \cdot x(n-k) \right]$$

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$$= \left(\frac{1}{8} \right) [x(n) - x(n-1) + x(n-2) - x(n-3) + x(n-4) - x(n-5) + x(n-6) - x(n-7)]$$

$$H(z) = \left(\frac{1}{8} \right) [1 - z^{-1} + z^{-2} - z^{-3} + z^{-4} - z^{-5} + z^{-6} - z^{-7}]$$

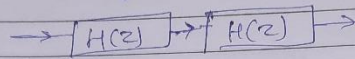
$$= \left(\frac{1}{8} \right) [(1 + z^{-2} + z^{-4} + z^{-6}) - (z^{-1} + z^{-3} + z^{-5} + z^{-7})]$$

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$$v(n) = \left(\frac{1}{N}\right) \sum_{k=0}^{(N-1)} x(n-k) \quad [N=6]$$

$$H(z) = \left(\frac{1}{N}\right) [1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5}]$$

① cascading the system.



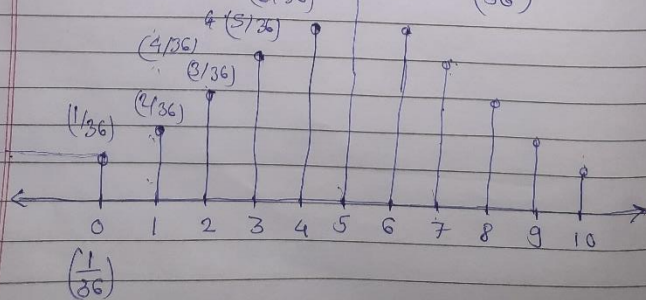
Transfer function of total system

$$H'(z) = H(z) \cdot H(z)$$

$$= \left(\frac{1}{N}\right)^2 [1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5}]^2$$

$$= \frac{1}{N^2} [1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6} + z^{-7} + z^{-3} + z^{-4} + z^{-5} + z^{-6} + z^{-7} + z^{-8} + z^{-4} + z^{-5} + z^{-6} + z^{-7} + z^{-8} + z^{-9} + z^{-5} + z^{-6} + z^{-7} + z^{-8} + z^{-9} + z^{-10}]$$

$$= \frac{1}{36} (1 + (2) \cdot z^{-1} + (3) \cdot z^{-2} + (4) \cdot z^{-3} + (5) \cdot z^{-4} + (6) \cdot z^{-5} + (5) \cdot z^{-6} + (4) \cdot z^{-7} + (3) \cdot z^{-8} + (2) \cdot z^{-9} + z^{-10}) \times \left(\frac{1}{36}\right)$$



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$$h(n) = \left\{ \frac{1}{36}, \frac{2}{36}, \frac{3}{36}, \frac{4}{36}, \frac{5}{36}, \frac{6}{36}, \frac{5}{36}, \frac{4}{36}, \frac{3}{36}, \frac{2}{36}, \frac{1}{36} \right\}$$

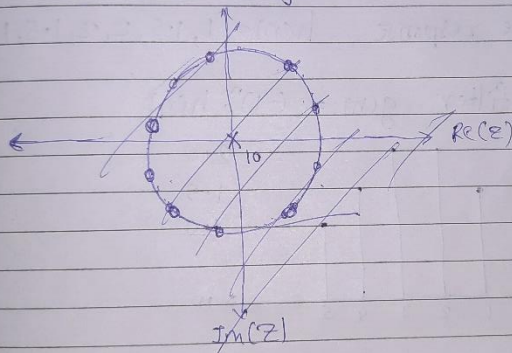
(b) DC gain of the total filter $Hf(0)$.

$$= \sum h(n)$$

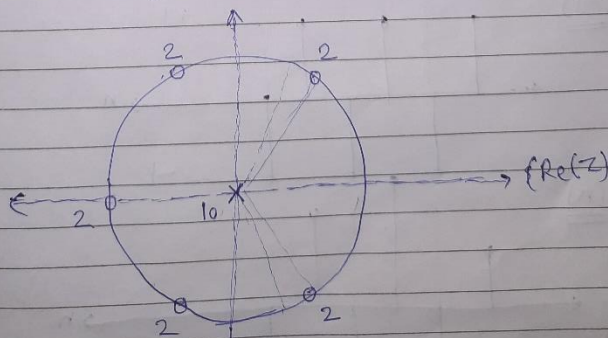
$$= \frac{1}{36} (1+2+3+4+5+6+5+4+3+2+1)$$

$$= \frac{1}{36} (36) = 1 = \text{DC gain}$$

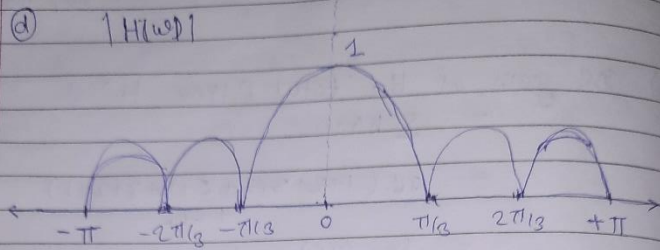
(c) Pole-Zero diagram.



(d)



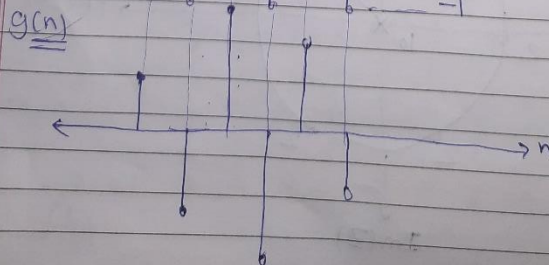
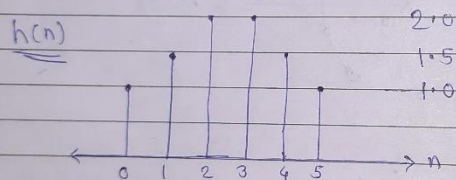
(3.17)



(3.6)

(a) Impulse response $h(n) = (1, 1.5, 2, 2, 1.5, 1)$

New FIR filter $g(n) = (-1)^n \cdot h(n)$



3.6) $g(n) = (1, -1.5, 2, -2, 1.5, -1)$

6)
$$g(n) = 1 - 1.5x(n-1) + 2x(n-2) - 2x(n-3) + 1.5x(n-4) - 1x(n-5)$$

$$G(z) = 1 - (1.5) \cdot z^{-1} + (2)z^{-2} - (2) \cdot z^{-3} + 1.5 \cdot z^{-4} - z^{-5}$$

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$$y(n) = x(n) - x(n-N) + y(n-1)$$

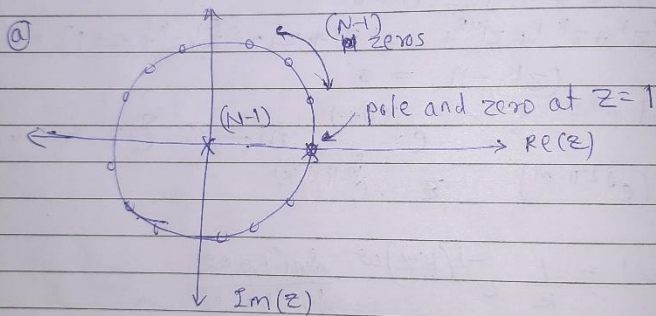
$$y(n) - y(n-1) = x(n) - x(n-N)$$

Z.T.

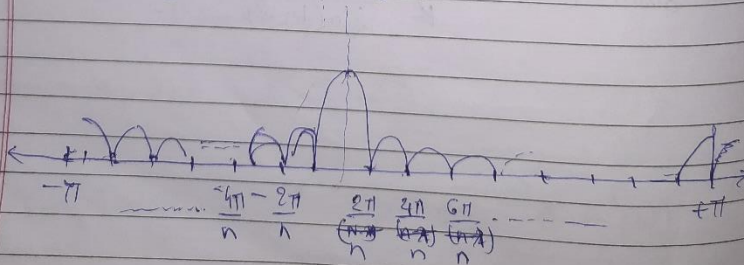
$$Y(z) [1 - z^{-1}] = X(z) [1 - z^{-N}]$$

$$H(z) = \frac{1 - z^{-N}}{1 - z^{-1}}$$

Pole at $z=1$, $(N-1)$ Poles at origin
 N equally spaced Zeros on Unit circle

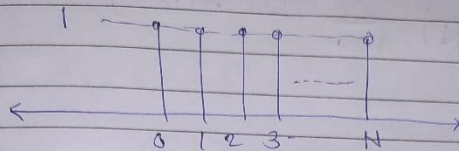


b) Frequency Response Magnitude $|H(\omega)|$



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© Impulse Response $h(n)$



d) DC gain of filter $H^f(0) = H(1)$

$$\begin{aligned} H(1) &= \sum_{n=0}^{N-1} h(n) \\ &= \sum_{n=0}^{N-1} 1 \\ &= N \end{aligned}$$