Frequency Response Matlab Exercise

ECE-GY 6113

Consider a causal discrete-time LTI system implemented using the difference equation,

$$y(n) = 0.1\,x(n) - 0.12\,x(n-1) + 0.1\,x(n-2) + 1.7\,y(n-1) - 0.8\,y(n-2)$$

The frequency response of the system is denoted $H^f(\omega)$.

1. What is the transfer function $H(z)$ of the system? (Not a Matlab question!)

2. Plot the magnitude of the frequency response $H^f(\omega)$ of the system using the Matlab function `freqz`:

```
>> [H,om] = freqz(b,a);
>> plot(om,abs(H));
```

   where `b` and `a` are appropriately defined.

3. When the input signal is

$$x(n) = \cos(0.1\pi n)u(n) \tag{1}$$

   find the output signal $y(n)$ for $-10 \le n \le 100$ using `filter`. Make stem plots of the input and output signals. (Use subplot in Matlab.) Comment on your observations.

4. Find the exact value of $H^f(\omega)$ at $\omega = 0.1\pi$. First, express $H^f(0.1\pi)$ as

$$H^f(0.1\pi) = \frac{B(\mathrm{e}^{\mathrm{j}0.1\pi})}{A(\mathrm{e}^{\mathrm{j}0.1\pi})}$$

   where $B(z)$ and $A(z)$ are polynomials. Second, use `exp` to find the complex number $z = \mathrm{e}^{\mathrm{j}0.1\pi}$. Third, evaluate $B(z)$ and $A(z)$ at the complex value $z = \mathrm{e}^{\mathrm{j}0.1\pi}$ using `polyval` in Matlab.

5. Recall that when the impulse response h is real,

$$\cos(\omega_o n) \quad \longrightarrow \quad \boxed{h(n)} \quad \longrightarrow \quad |H^f(\omega_o)| \cos\big(\omega_o n + \angle H^f(\omega_o)\big)$$

   Note this input signal starts at $n = -\infty$. (There is no $u(n)$ term.) However, we are interested here in the case where the input signal starts at $n = 0$, namely the input given by Equation (1). In this case, we have

$$\cos(\omega_o n)u(n) \quad \longrightarrow \quad \boxed{h(n)} \quad \longrightarrow \quad |H^f(\omega_o)| \cos\big(\omega_o n + \angle H^f(\omega_o)\big)u(n) \ + \ \text{Transients}$$

   The transients decay to zero, and the *steady-state* output signal is simply

$$s(n) = |H^f(\omega_o)| \cos\big(\omega_o n + \angle H^f(\omega_o)\big)u(n). \tag{2}$$

In Matlab, use `abs` and `angle` to compute $|H^f(0.1\pi)|$ and $\angle H^f(0.1\pi)$. Create a stem plot of the steady-state output signal

$$s(n) = |H^f(0.1\pi)| \cos(0.1\pi n + \angle H^f(0.1\pi)).$$

Compare your plot of $s(n)$ with your plot of $y(n)$ obtained using `filter`. You can plot both on the same graph using `plot(n,y,n,s)`. You should find that $s(n)$ and $y(n)$ agree after a while. After the transient response decays to zero, the steady-state output signal $s(n)$ remains.

6. When the input signal is

$$x(n) = \cos(0.3\pi n) u(n)$$

find the output signal $y(n)$ for $-10 \le n \le 100$ using `filter`. How could $y(n)$ be predicted from the frequency response of the system? What is the steady-state signal? Relate your explanation to the concept described in the previous question.

7. Plot the poles and zeros of the transfer function using `zplane(b, a)` in Matlab. The shape of the frequency response magnitude $|H^f(\omega)|$ can be predicted from the pole-zero diagram.

To submit: The plots you produced, your Matlab commands to produce the plots, and discussion.

The following is to do but not to be submitted:

1. Simple Systems: For the causal discrete-time LTI systems implemented by each of the following difference equations, plot the frequency response magnitude $|H^f(\omega)|$, the pole-zero diagram, and the impulse response.

$$y(n) = x(n) + 1.8\, y(n-1) - 0.9\, y(n-2) \tag{3}$$

$$y(n) = x(n) + 1.6\, y(n-1) - 0.72\, y(n-2) \tag{4}$$

$$y(n) = x(n) + 1.53\, y(n-1) - 0.9\, y(n-2) \tag{5}$$

$$y(n) = x(n) + 1.4\, y(n-1) + 0.72\, y(n-2) \tag{6}$$

$$y(n) = x(n) - 0.85\, y(n-1) \tag{7}$$

$$y(n) = x(n) - 0.95\, y(n-1) \tag{8}$$

Comment on your observations: How can you predict from the pole-zero diagram what the frequency response will look like and what the impulse response will look like?

Suppose you are given a diagram of each of the pole-zero diagrams, frequency responses, and impulse responses, but that they are out of order. Can you match each diagram to the others (without doing any computation)?