University of Moratuwa

Department of Electronic and Telecommunication Engineering

BM4152

Bio-signal Processing

Assignment 3 - Continuous and Discrete Wavelet Transforms

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1.2. Wavelet properties

i) $m(t) = -\frac{d^2}{dt^2} g(t)$

$$g(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}$$

when $\mu = 0$ and $\sigma = 1$;

$$g(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2}$$

$$\frac{dg}{dt} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} \times -\frac{1}{2} \times 2t$$

$$= -\frac{1}{\sqrt{2\pi}} \left(e^{-\frac{1}{2}t^2}\right) t$$

$$\frac{d^2g}{dt^2} = -\frac{1}{\sqrt{2\pi}} \left\{ e^{-\frac{1}{2}t^2} \times 1 + t\, e^{-\frac{1}{2}t^2} \times -\frac{1}{2} \times 2t \right\}$$

$$= -\frac{1}{\sqrt{2\pi}} \left\{ e^{-\frac{1}{2}t^2} - t^2 e^{-\frac{1}{2}t^2} \right\}$$

$$\frac{d^2g}{dt^2} = -\frac{1}{\sqrt{2\pi}} (1-t^2) e^{-\frac{1}{2}t^2}$$

$$m(t) = -\frac{d^2}{dt^2} g(t)$$

$$= \frac{1}{\sqrt{2\pi}} (1-t^2) e^{-\frac{1}{2}t^2}$$

ii) $E = \int_{-\infty}^{\infty} m^2(t)\, dt$

If normalizing factor $= c$ (a positive real nomber)

$$\int_{-\alpha}^{\alpha} \frac{m^2(t)}{c^2} dt = 1$$

$$\frac{1}{2\pi} \int_{-\alpha}^{\alpha} (1-t^2)^2 e^{-t^2} dt = c^2$$

$$\frac{1}{2\pi} \int_{-\alpha}^{\alpha} \left[1 - 2t^2 + t^4 \right] e^{-t^2} dt = c^2$$

$$\frac{1}{2\pi} \left(\int_{-\alpha}^{\alpha} e^{-t^2} dt - 2 \int_{-\alpha}^{\infty} t^2 e^{-t^2} dt + \int_{-\alpha}^{\alpha} t^4 e^{-t^2} dt \right] = c^2$$

from; $\int_{-\alpha}^{\alpha} e^{-t^2} dt = \sqrt{\pi}$ and $\int_{-\alpha}^{\alpha} t^{2n} e^{-\alpha t^2} dt = \sqrt{\frac{\pi}{\alpha}} \frac{(2n-1)!!}{(2\alpha)^n}$

$n=1 \quad \alpha=1 \qquad n=2 \quad \alpha=1$

$$\frac{1}{2\pi} \left(\cancel{\sqrt{\pi}} - \frac{\cancel{2}\cancel{\sqrt{\pi}}}{\cancel{2}} + \sqrt{\pi} \frac{3!!}{2^2} \right)$$

$$\frac{1}{2\sqrt{\pi}} \times \frac{3\cancel{\sqrt{\pi}}}{4} = \frac{3}{8\sqrt{\pi}} = c^2 \quad //$$

normalizing factor $= c = \dfrac{\sqrt{3}}{2\sqrt{2}(\pi)^{1/4}}$

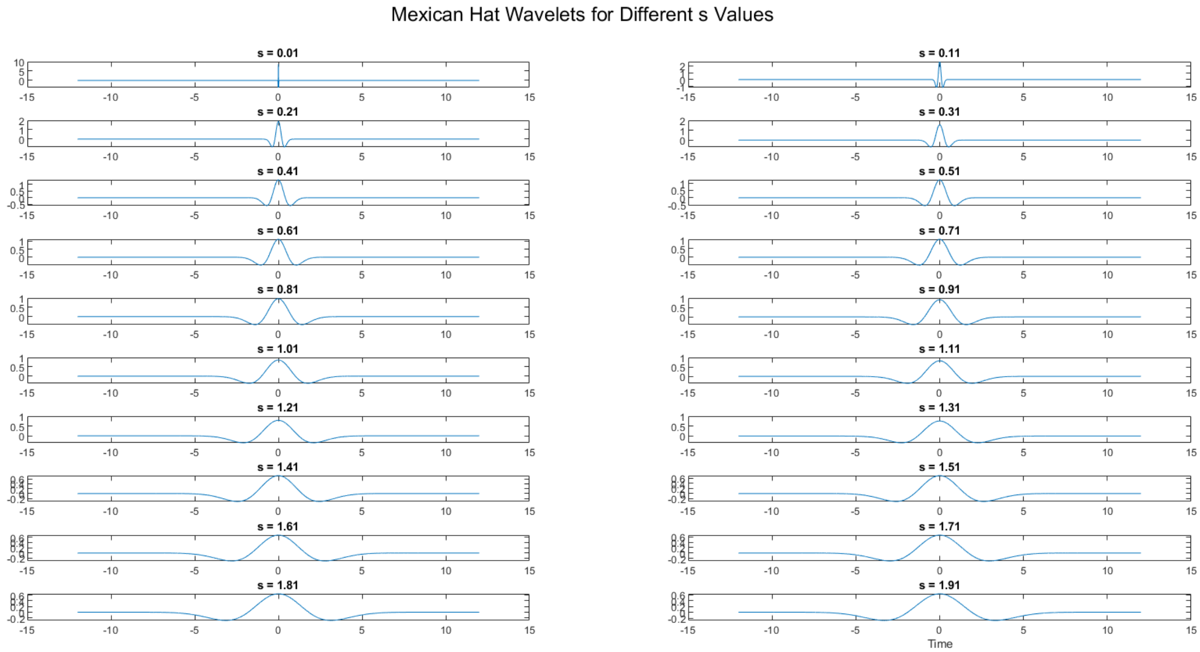
$$\therefore \psi(t) = \frac{1}{c} m(t)$$

$$= \frac{2\sqrt{\cancel{2}}(\pi)^{1/4}}{\sqrt{3}} \times \frac{1}{\sqrt{\cancel{2}\pi}} (1-t^2) e^{-\frac{1}{2}t^2}$$

$$= \frac{2}{\sqrt{3}\pi^{1/4}} (1-t^2) e^{-\frac{1}{2}t^2}$$

$$\psi_s(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t}{s}\right)$$

iii)



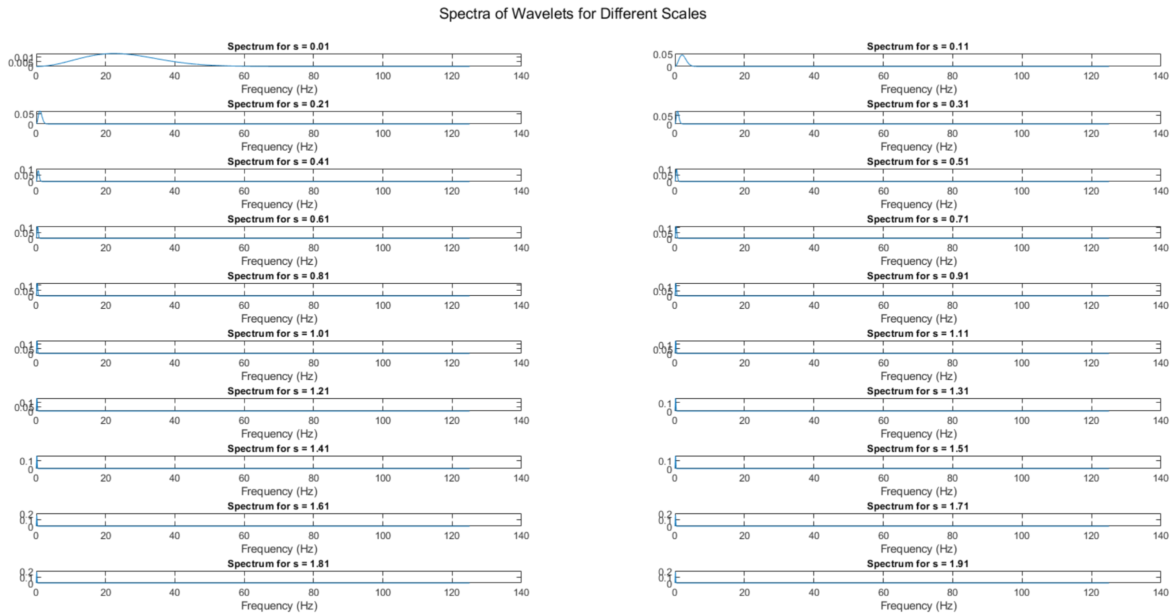
iv)

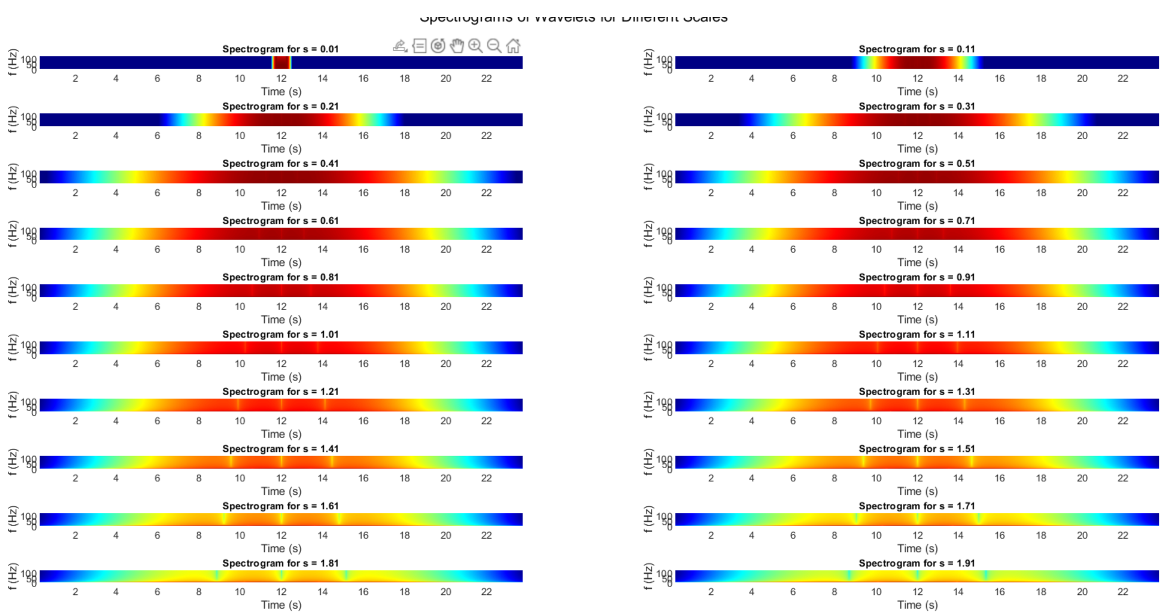
```
Mean of s = 0.01 is -6.5837e-19
Energy of s = 0.01 is 1
Mean of s = 0.11 is 7.4817e-19
Energy of s = 0.11 is 1
Mean of s = 0.21 is -1.1868e-18
Energy of s = 0.21 is 1
Mean of s = 0.31 is 5.9901e-18
Energy of s = 0.31 is 1
Mean of s = 0.41 is -2.7943e-18
Energy of s = 0.41 is 1
Mean of s = 0.51 is 7.0989e-18
Energy of s = 0.51 is 1
Mean of s = 0.61 is 7.6729e-18
Energy of s = 0.61 is 1
Mean of s = 0.71 is -1.3054e-17
Energy of s = 0.71 is 1
Mean of s = 0.81 is 5.8726e-18
Energy of s = 0.81 is 1
Mean of s = 0.91 is -7.352e-19
Energy of s = 0.91 is 1
```

```
Mean of s = 1.01 is 1.7049e-17
Energy of s = 1.01 is 1
Mean of s = 1.11 is 1.287e-17
Energy of s = 1.11 is 1
Mean of s = 1.21 is 5.091e-18
Energy of s = 1.21 is 1
Mean of s = 1.31 is -5.3379e-18
Energy of s = 1.31 is 1
Mean of s = 1.41 is 1.2045e-16
Energy of s = 1.41 is 1
Mean of s = 1.51 is 1.3519e-14
Energy of s = 1.51 is 1
Mean of s = 1.61 is 5.854e-13
Energy of s = 1.61 is 1
Mean of s = 1.71 is 1.332e-11
Energy of s = 1.71 is 1
Mean of s = 1.81 is 1.8261e-10
Energy of s = 1.81 is 1
Mean of s = 1.91 is 1.6728e-09
Energy of s = 1.91 is 1
```

From the above outputs it can be observed that the mean w.r.t to different scaling values are equal to unity, and the mean is also very close to zero(almost). From the plots we can see, most of the energy(considerable amplitudes) is concentrated within a few samples, and the maximum sample range in the largest scale observed here is also 10. This indicates the compact support of wavelets. Overall these indicate that the Mexican hat wavelet abides by the main properties of a wavelet.

v)



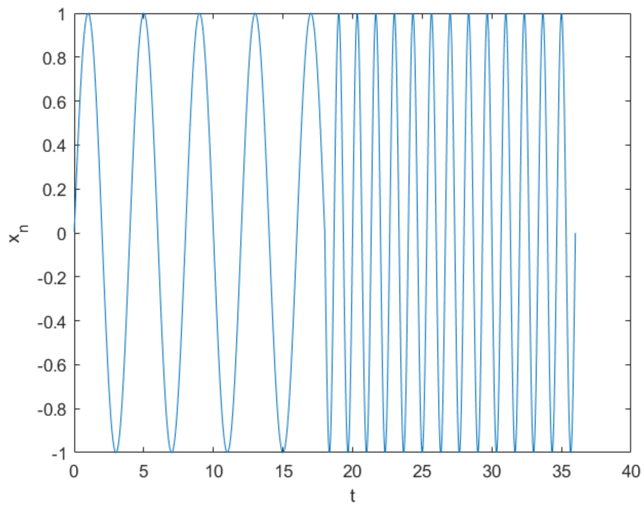


Due to the properties of the Fourier transform, when the signal is in a time limited region, it is spread out more in the frequency domain due to the uncertainty. Therefore vice versa, we can see as the scale increases, it is constricted to a smaller region in the frequency domain.

Also, since with the increasing scale, the highest frequency component in the signal is reducing, we can see that the main bandwidth lobe is moving towards lower frequencies, alongside the center frequency of that lobe.

1.3. Continuous Wavelet Decomposition

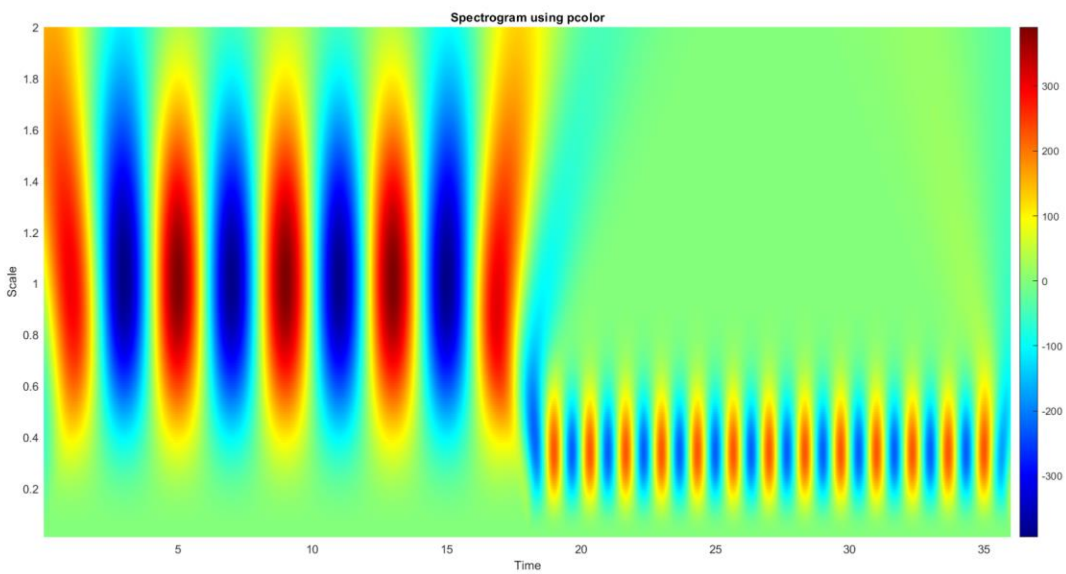
i) Time domain plot of x[n]:



ii)

iii)

Spectrogram after using the Mexican hat wavelet function:



iv)

Here, we can see that the CWT has identified the peaks and valleys of the signal w.r.t to the correct scales. During the first half, it has identified the lower frequency signal, corresponding to using a larger scale. Similarly, the higher frequency components have been identified with a lower scale of the wavelet. The color change shift signifies the positive and negative amplitude peaks associated with the particular frequencies.

Specifically, we can see that the frequency during the first region is identified with the scale close to 1. In the plot, of s = 1.01, we can see that it fits with the time domain sample count of half the sin wave in x[n]. Similarly, a lower scale is identified in the next region of x[n].

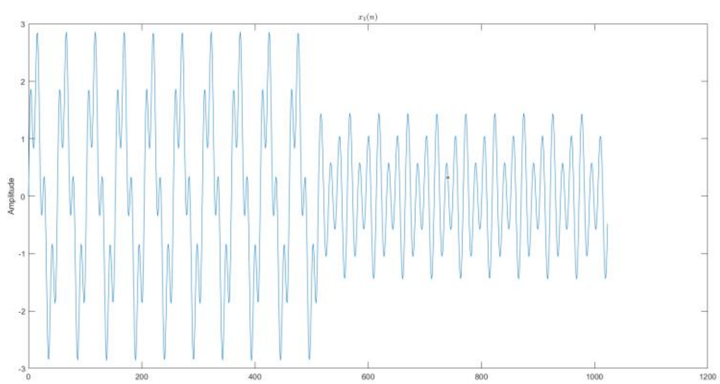
Also, considering that this is CWT, we can see a smoother spectrogram with wavelet coefficients calculated for each point of the spectrogram. Therefore, this contains redundant information.

II. Discrete Wavelet Transform

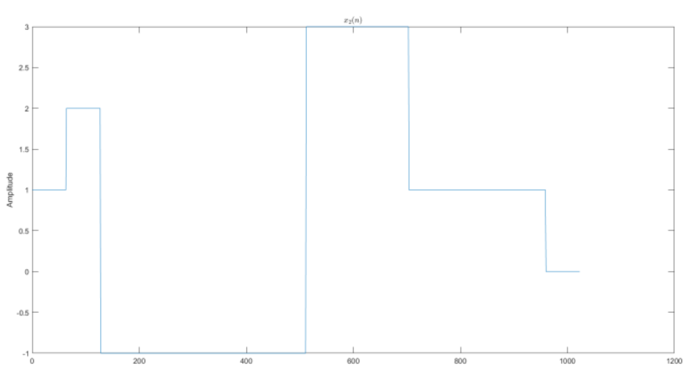
2.2. Applying DWT with the Wavelet Toolbox in MATLAB

i)

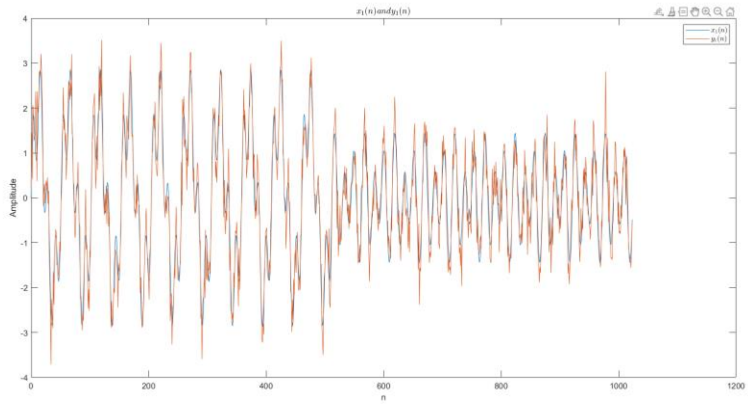
$x_1[n]$:



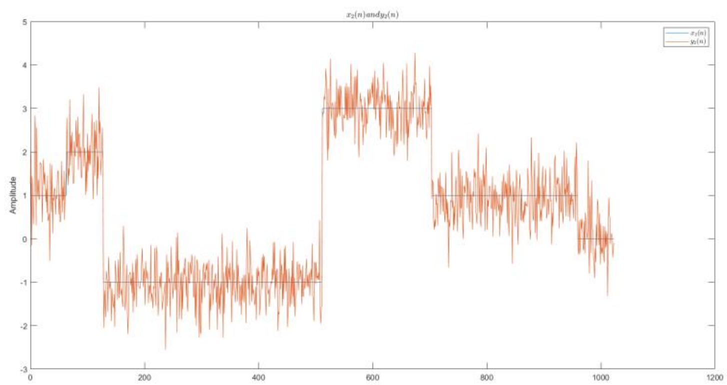
$x_2[n]$:



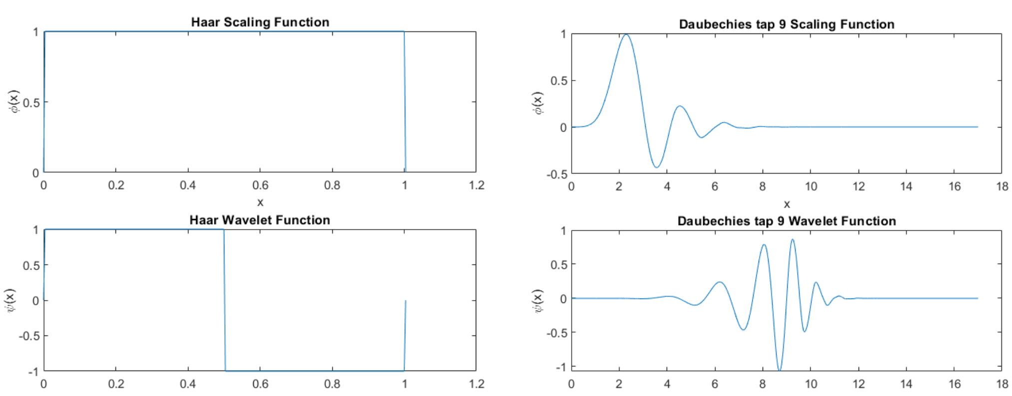
$y_1[n]$:



$y_2[n]$:

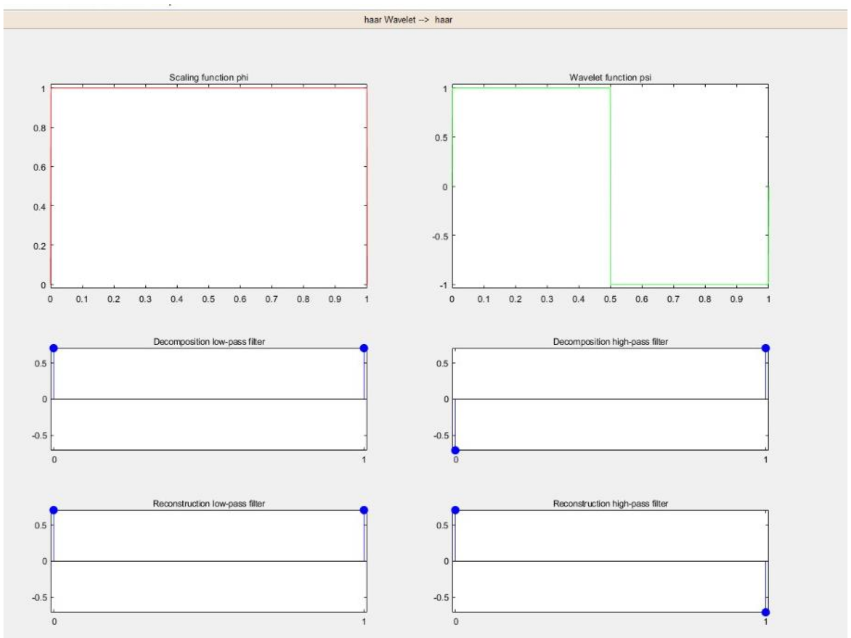


ii) Wavefun():

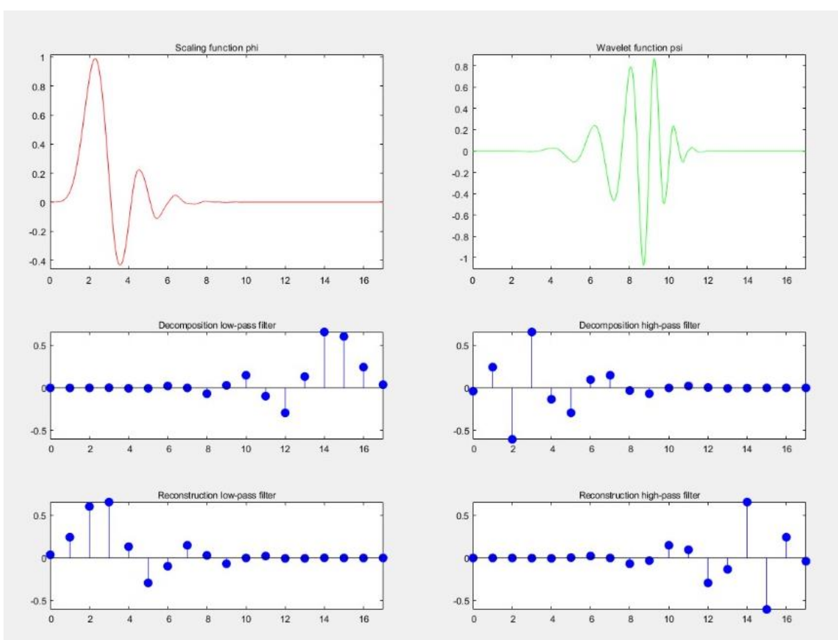


Using the waveletAnalyzer GUI:

Haar



Db9:

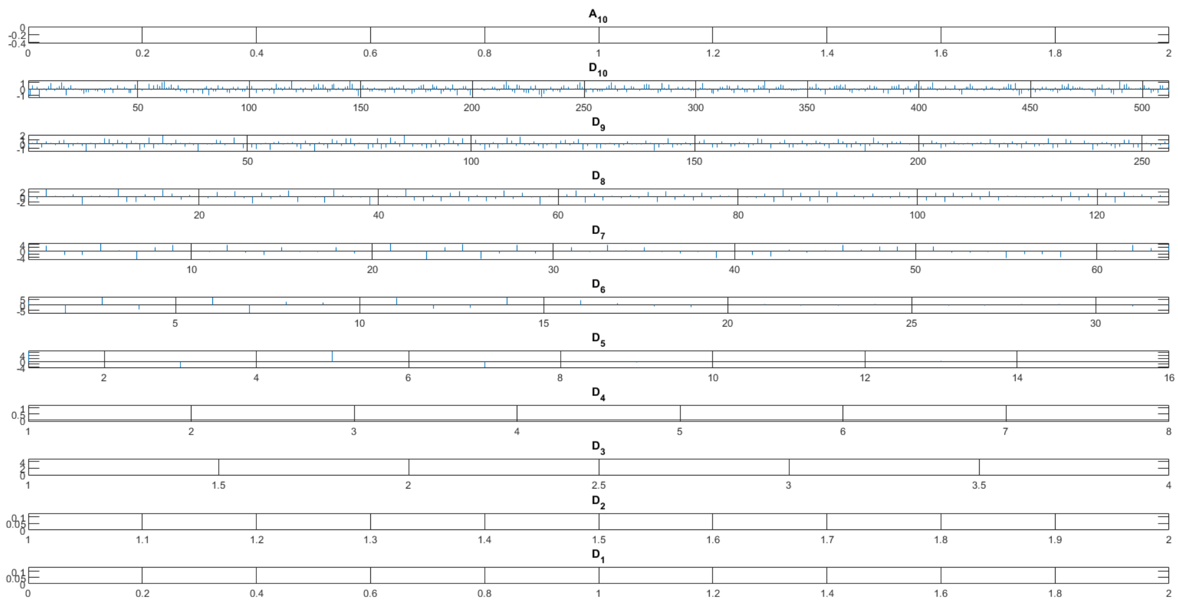


iii)

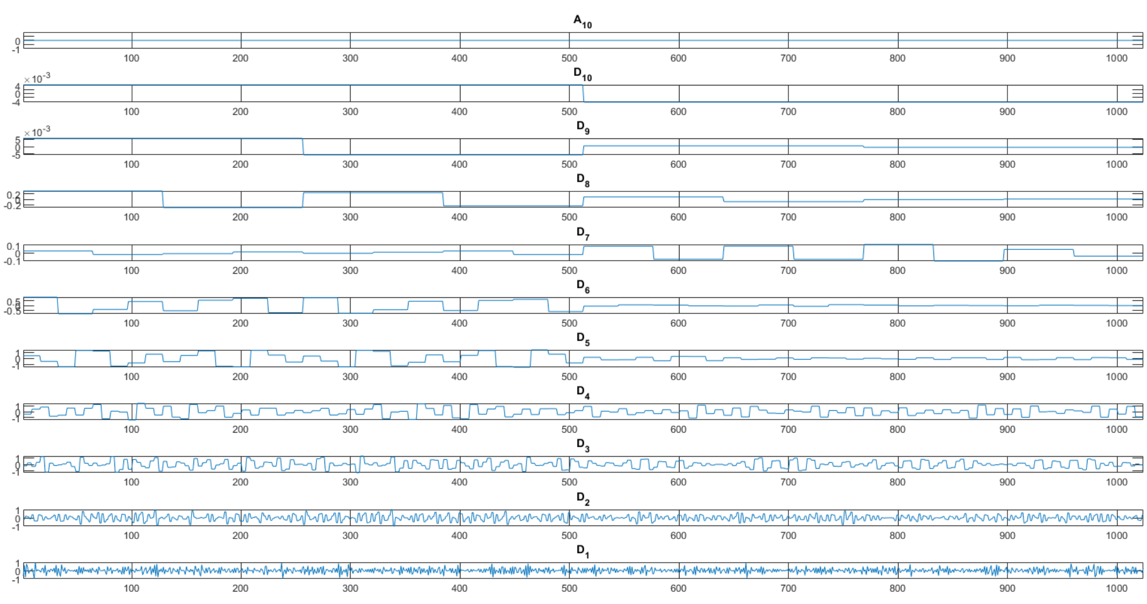
and

iv)

Approximation and Detail Coefficients of y1 with haar wavelet

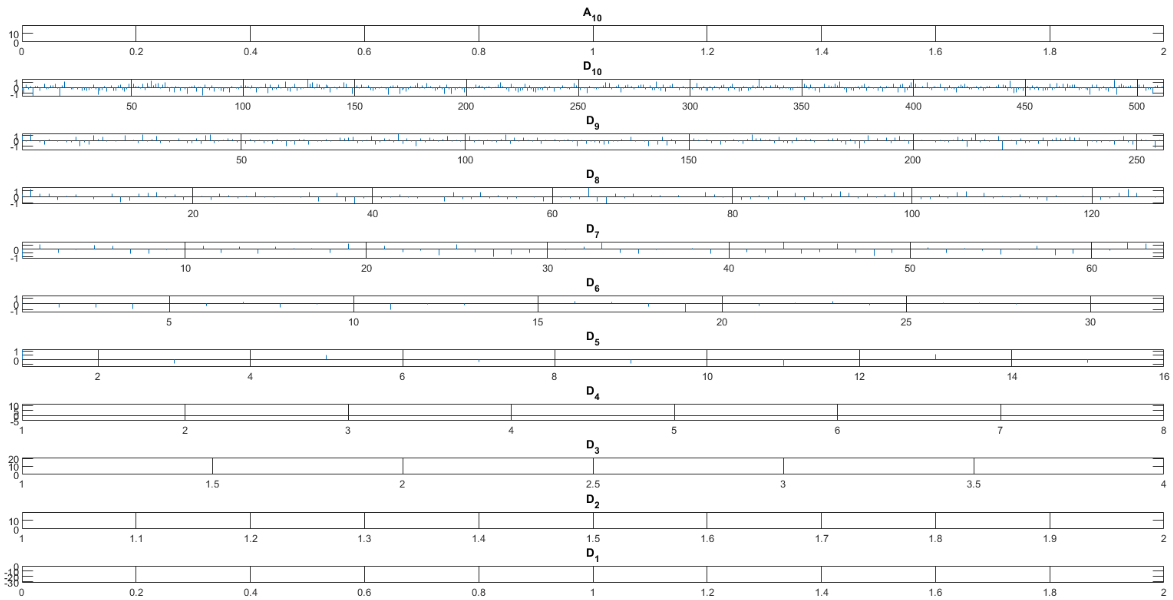


Reconstructed Approximation and Detail Signals of y1 with haar wavelet

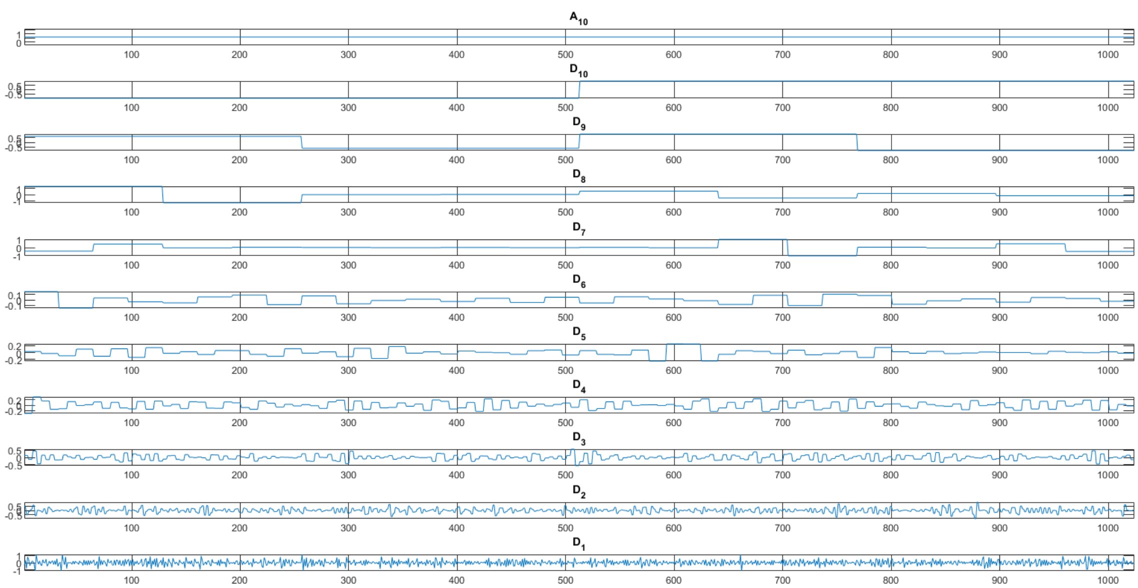


Energy difference in y1 with haar: -2.1605e-12

Approximation and Detail Coefficients of y2 with haar wavelet

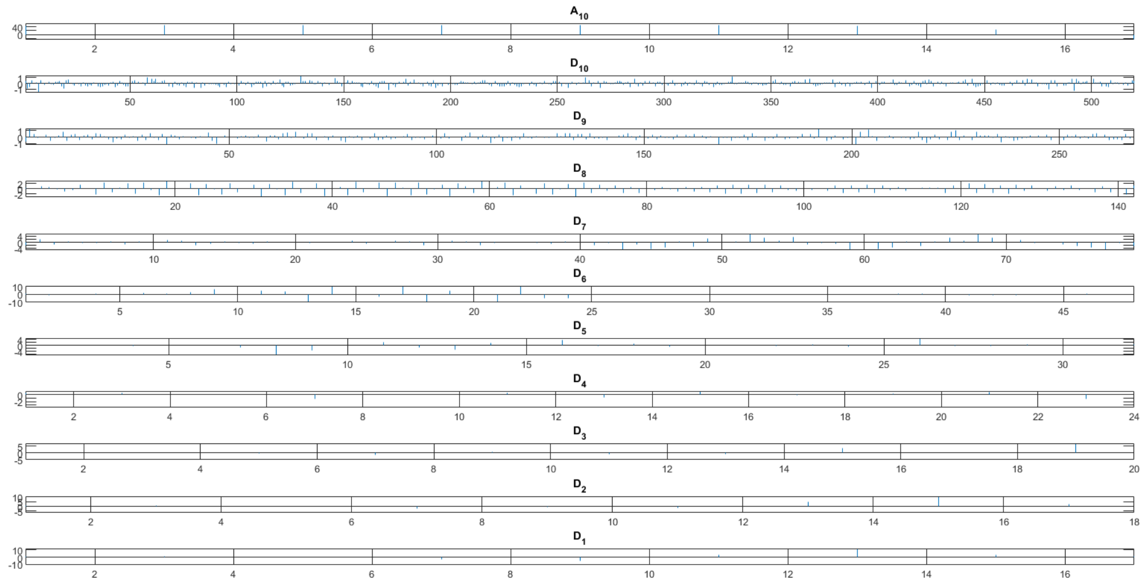


Reconstructed Approximation and Detail Signals of y2 with haar wavelet

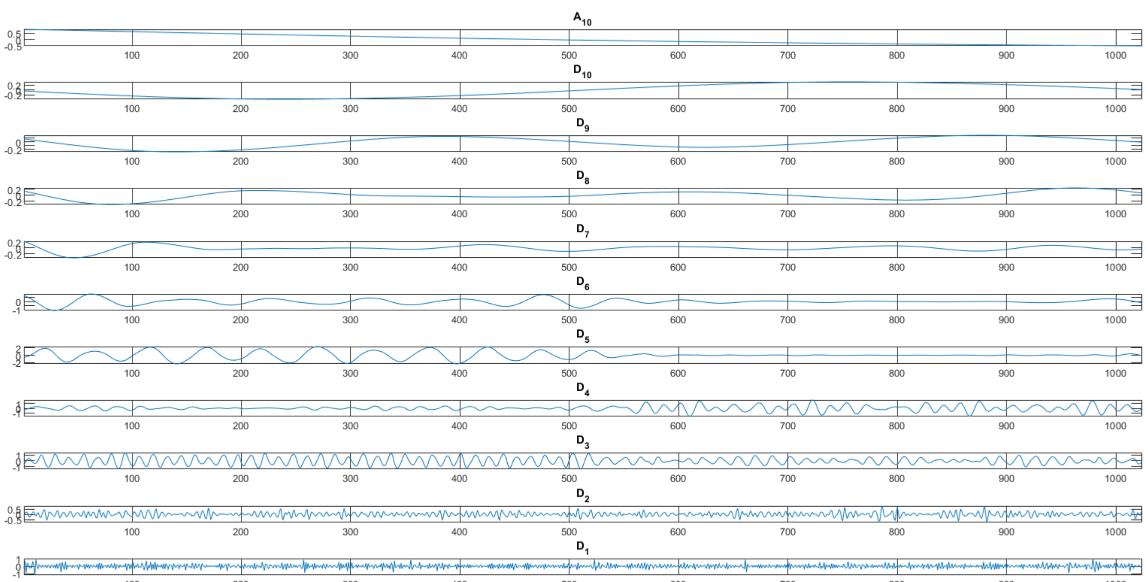


Energy difference in y2 with haar: -7.2449e-12

Approximation and Detail Coefficients of y1 with db9 wavelet

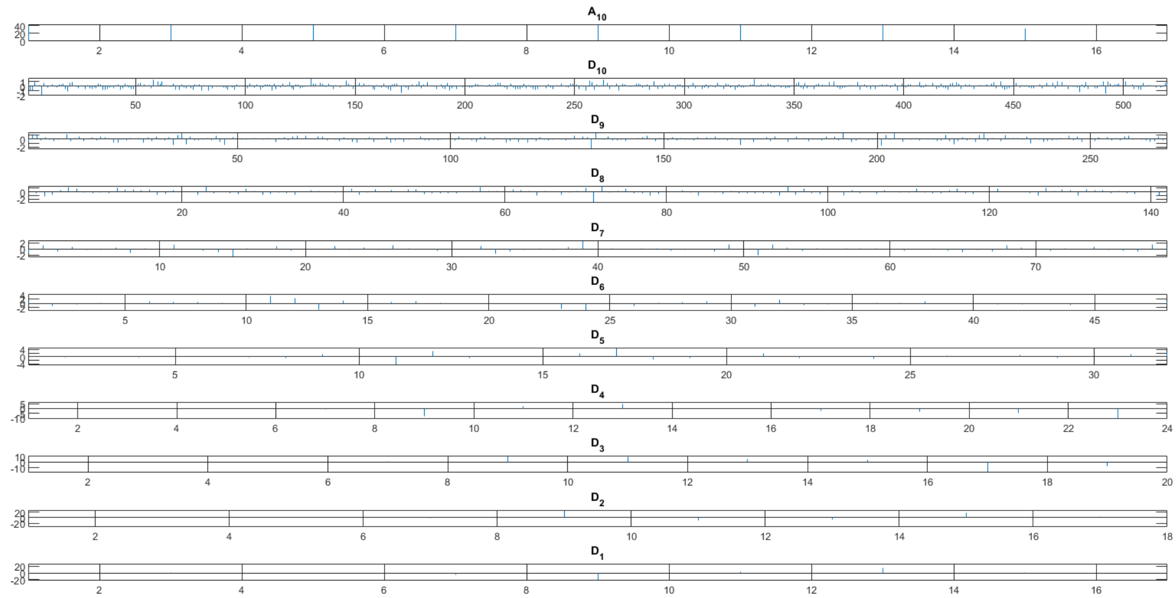


Reconstructed Approximation and Detail Signals of y1 with db9 wavelet

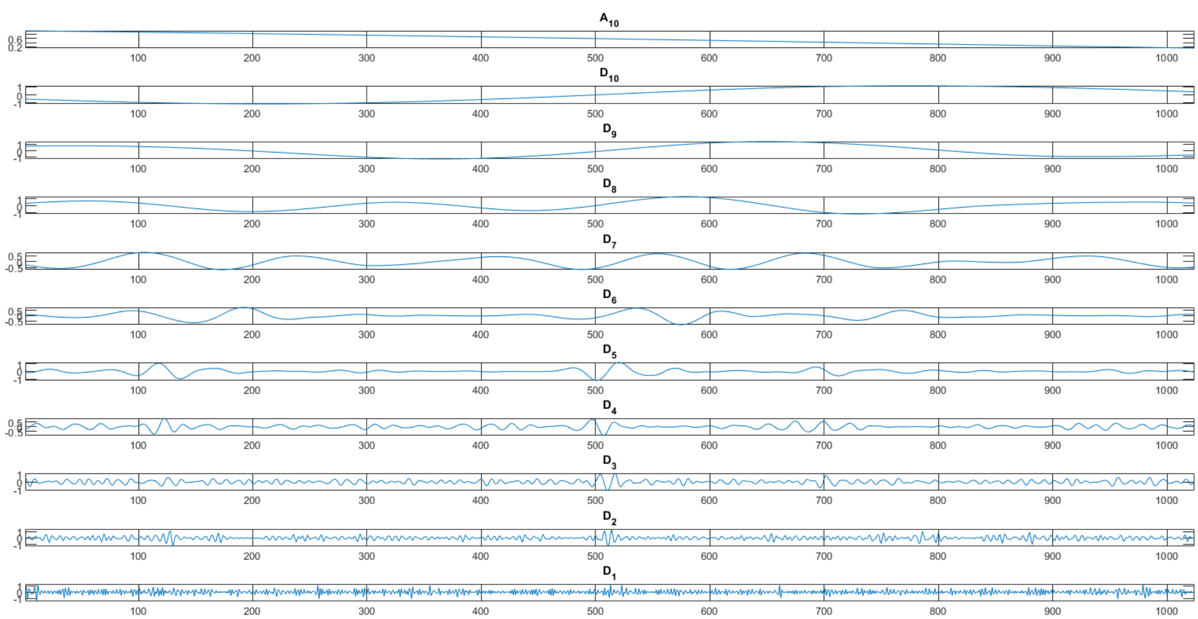


Energy difference in y1 with db9: 3.1147e-07

Approximation and Detail Coefficients of y2 with db9 wavelet



Reconstructed Approximation and Detail Signals of y2 with db9 wavelet



Energy difference in y2 with db9: 5.0957e-07

Explanation for iv):

The energy differences are almost zero indicating that $y = \Sigma D^i + A$.

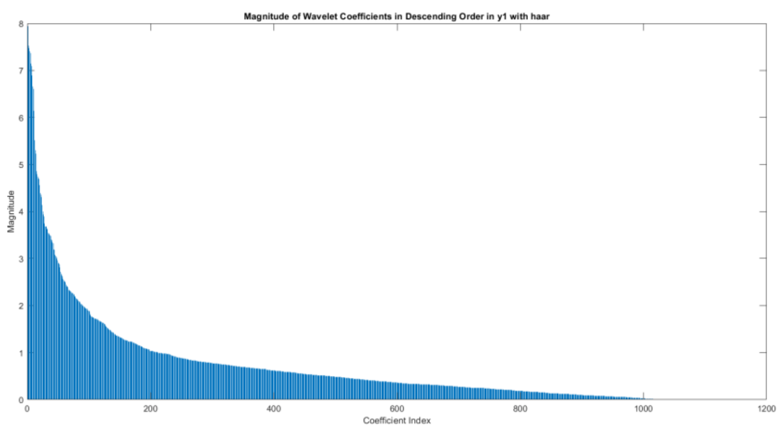
[C, L] = wavedec(signal, 10, wavelet); This returns the wavelet coefficients. The first is A10, and the detailed coefficients are then in it in the descending order. L specifies the lengths of each. Different lengths are due to the fact that the coefficients are downsampled in each step due to the fact that the highest frequency is halved at each step. (or can be halved by shifting). The separate coefficients are extracted using the appcoef and detcoef functions. To reconstruct each coefficient using inverse DWT, the wrcoef command can be used by indicating the level of decomposition associated with that coefficient. By taking the sum of all such reconstructed signals, the initial signal can be reconstructed back. It was done for both y1 and y2 with the two wavelets haar and db9 for each.

2.3. Signal Denoising with DWT

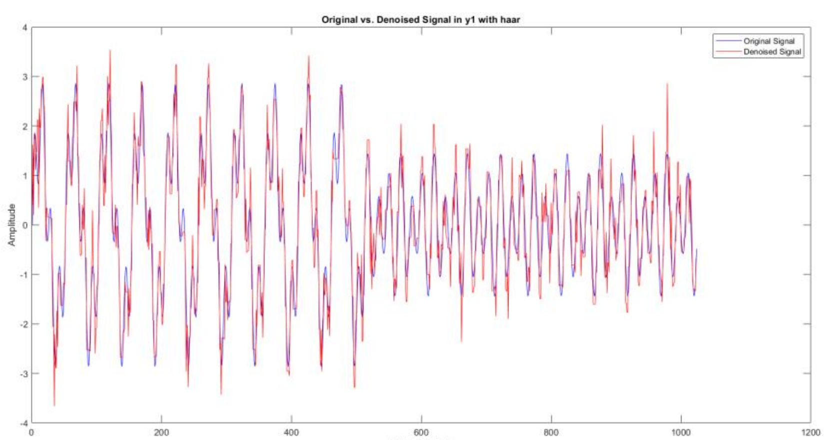
i) , ii) , iii) and iv)

y1 with haar:

Magnitude of wavelet coefficients in descending order:

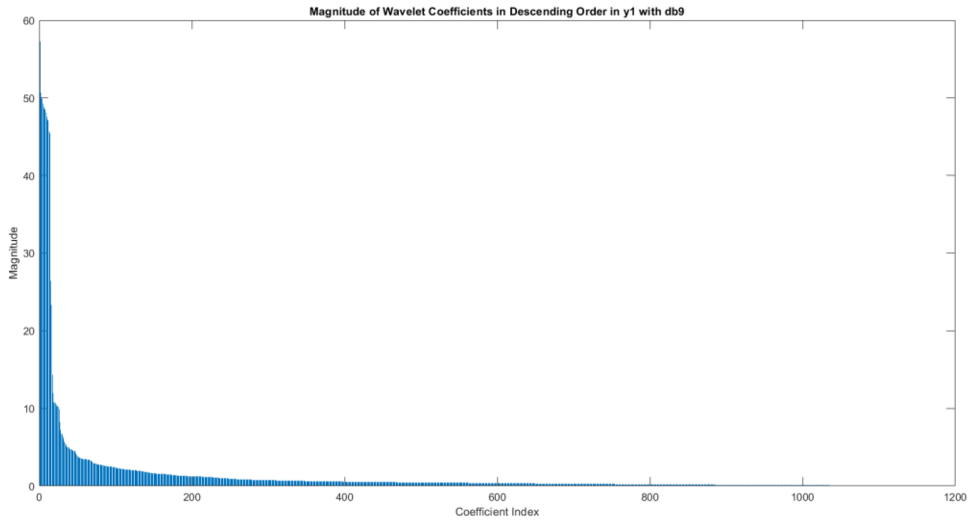


Selected threshold from observation: 0.5

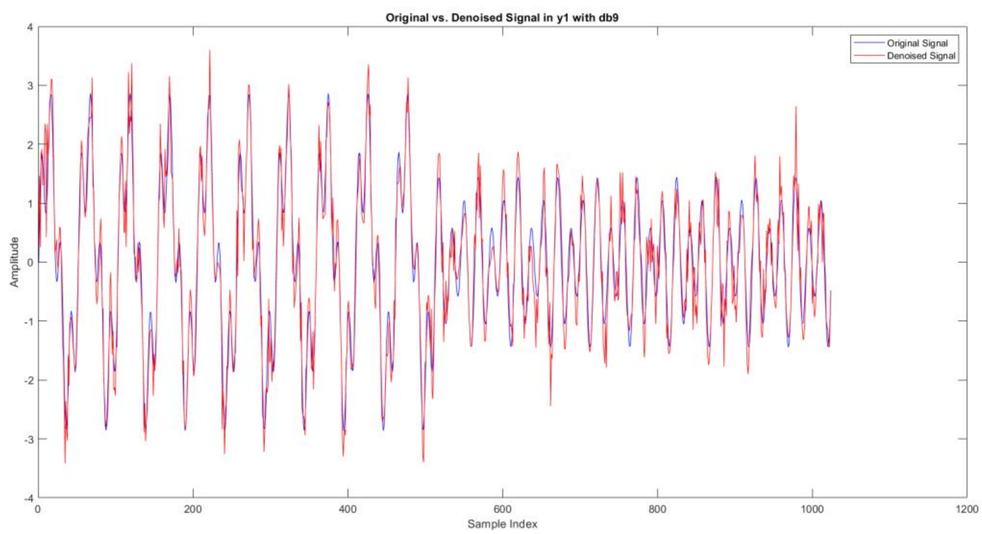


y1 with db9:

Magnitude of wavelet coefficients in descending order:

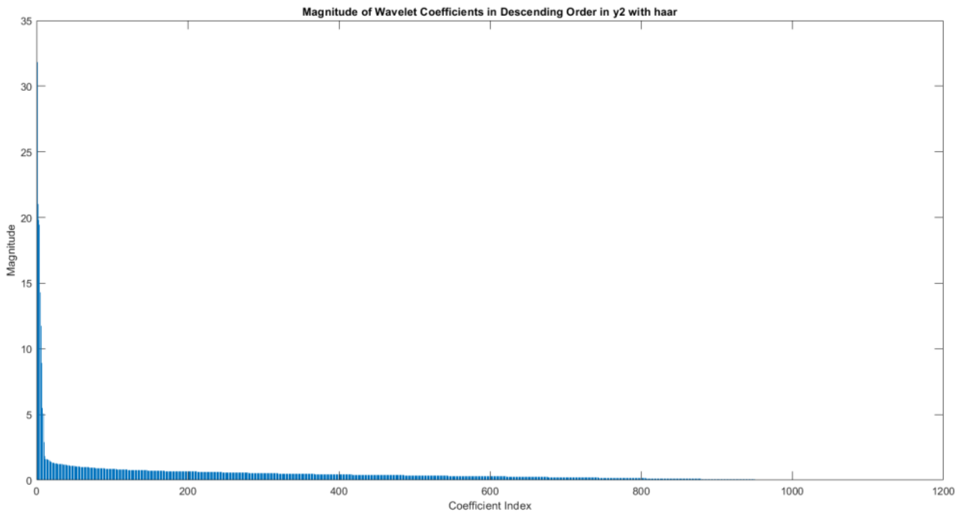


Selected threshold from observation: 0.5

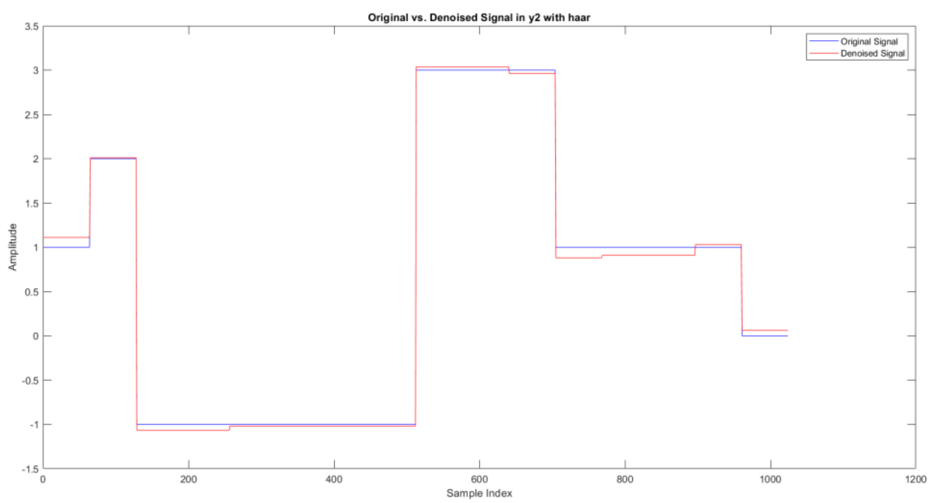


y2 with haar:

Magnitude of wavelet coefficients in descending order:

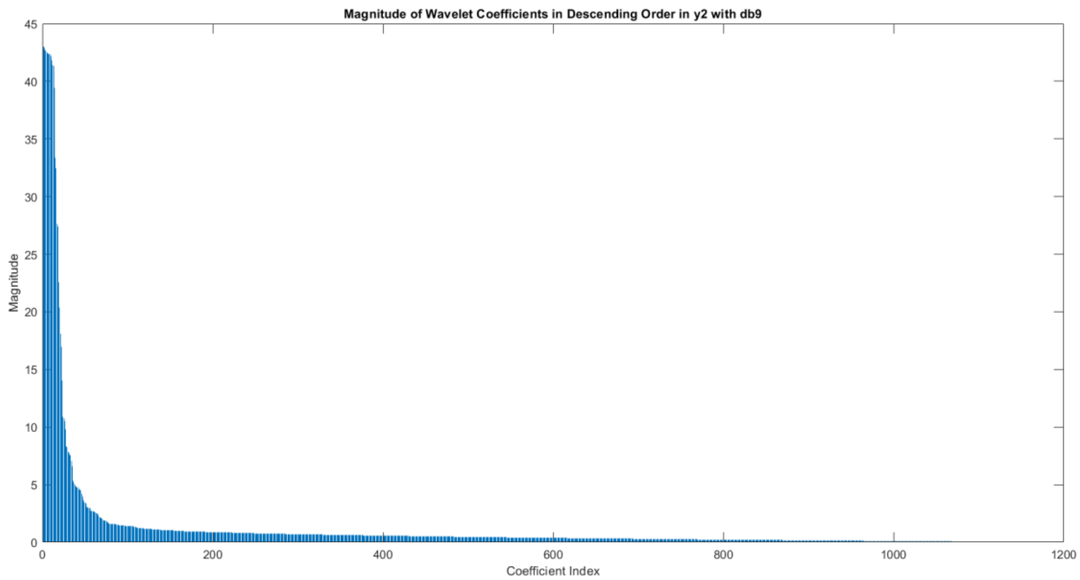


Selected threshold from observation: 2

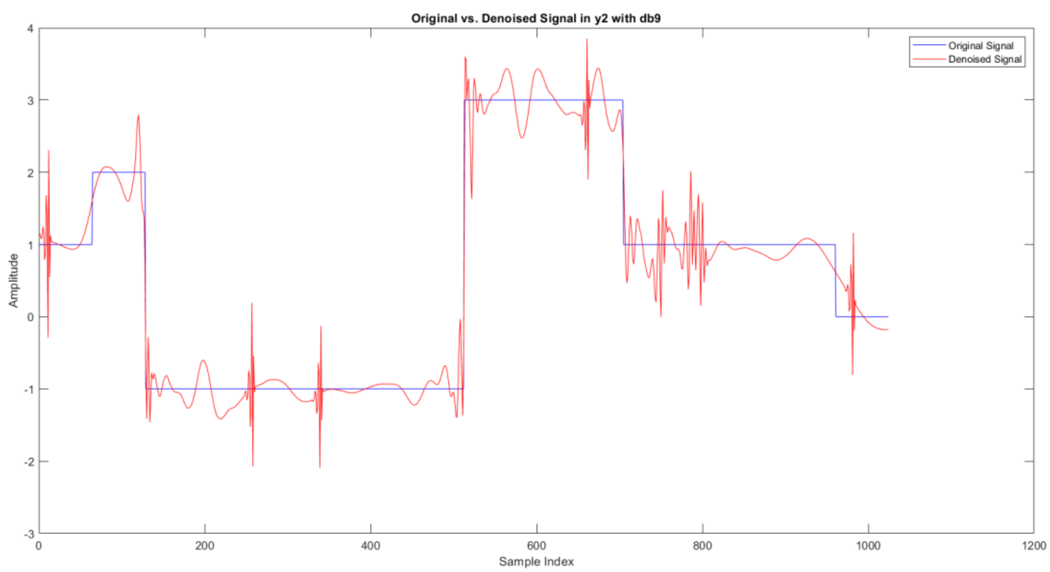


y2 with db9:

Magnitude of wavelet coefficients in descending order:



Selected threshold from observation: 1.5

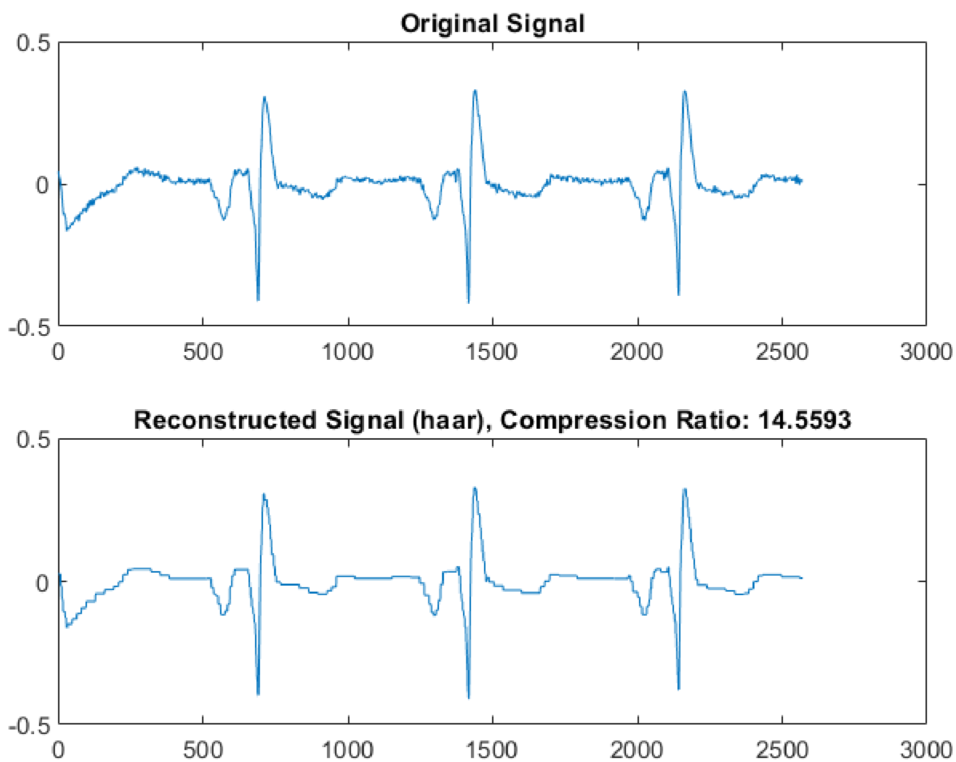



```
RMSE, y1 with haar: 0.39573
RMSE, y1 with db9: 0.34503
RMSE, y2 with haar: 0.062462
RMSE, y2 with db9: 0.27379
```

v) When comparing the RMSE, we can see that a better denoising has been done for the y1 signal when db9 was used, and in the y2 signal, the haar wavelet has done a better denoising. This is because for any task done with wavelet, it is necessary that the wavelet morphology matches with the general characteristics of the signal for optimum use of it. When the morphologies match, the wavelet is more effective at localizing signal energy in fewer coefficients, which enables efficient thresholding. Since the initial x1 signal has better similarity with the db9, the signal is easily captured. On the other hand, since x2 has more similarity with the haar wavelet, it has performed better here in a similar fashion.

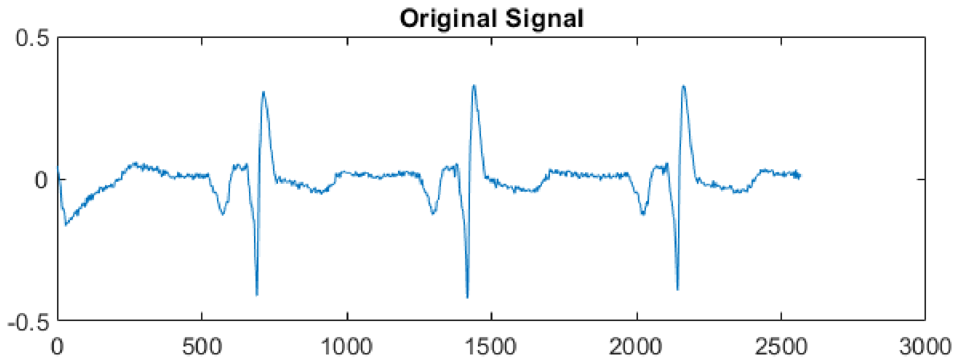
2.4. Signal Compression with DWT

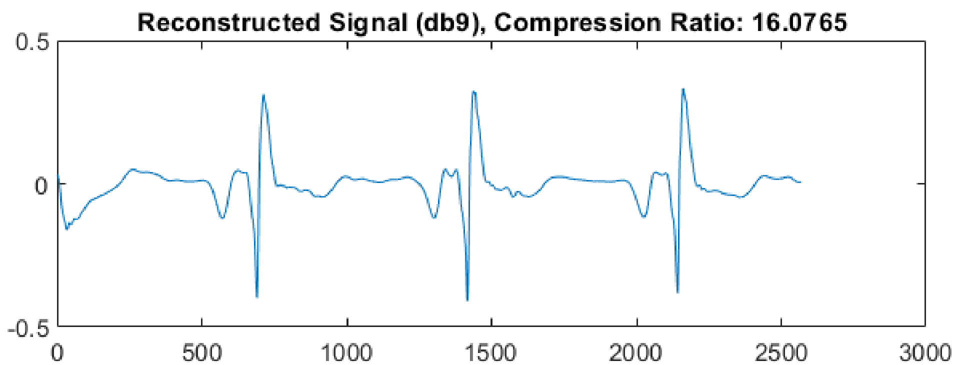
Using haar



No. of coefficients: 177 out of 2577 coefficients.

Using db9:





No. of coefficients: 170 out of 2733 coefficients.

$$Compression\ ratio\ = \frac{No.of\ total\ coefficients}{No.of\ coefficients\ needed\ to\ get\ 99\%\ energy}$$

Due to the shape of the haar, we can se that the compressed, and reconstructed signal has sharper edges in a similar fashion. On the other hand, in using the db9 wavelet, more smoother reconstruction has occurred. However, it seems that Db9 has captured more energy with lesser no. of coefficients, indicating that for this particular signal, the db9 wavelet has been slightly superior at localizing the signal energy with fewer coefficients. This indicates that the db9 wavelet has a comparatively better morphology with resembles with the ecg signal that is compressed.

Therefore, a higher compression ratio has been achieved with db9.