MATLAB Assignment 2

Optimum and Adaptive Filters

- **This is an individual assignment.**
- **Use data files (.mat) that are attached as instructed throughout the assignment.**
- **Submission guidelines**

Submission document	**Submission method**	**Notes**
Report	Upload the softcopy to Moodle	Should include observations and discussions with relevant plots to support your answers.
MATLAB scripts	Upload a single ZIP file including all the .m files to Moodle	Name each script according to the question number. Also, include necessary comments on the scripts for better readability.

Introduction

This assignment involves implementing the Weiner filter, as well as adaptive noise cancellers based on the LMS and RLS algorithms.

1. Weiner filtering

Data construction

- Ideal signal $y_i(n)$: idealECG.mat. Sampling frequency = 500 Hz
- Input signal $x(n) = y_i(n) + \eta(n)$
- Where $\eta(n) = \eta_{wg}(n) + \eta_{50}(n)$
- $\eta_{wg}(n)$: white Gaussian noise such that SNR is 10 dB with respect to $y_i(n)$
- $\eta_{50}(n) = 0.2\sin(2\pi 50n)$

1.1 Discrete time-domain implementation of the Wiener filter

Consider the two cases given in Part 1 and Part 2 below. For each case, do the following:

a) Calculate the optimum weight vector $\mathbf{w}_0 = (\Phi_{\mathbf{Y}} + \Phi_{\mathbf{N}})^{-1}\Theta_{\mathbf{Y}y}$ for an arbitrary filter order.
b) Find the optimum filter order and its coefficients. Plot the magnitude response
c) Obtain the filtered signal $\hat{y}(n)$ using the filter obtained above
d) Plot the spectra of $y_i(n), \eta(n), x(n), \hat{y}(n)$ on the same plot
e) Interpret the spectra and the magnitude response of the filter. given the signals and parameter in the following Part 1 and Part 2 given below. For each case, find the following.

Part 1:

- Desired signal: A single beat of $y_i(n)$
- Noise signal: Extract the signal segment from the T wave of any chosen ECG beat to the P wave of the next ECG beat (isoelectric segment) of $x(n)$

Part 2:

- Desired signal: Construct a linear model have the same length and a comparable morphology to that of a single ECG beat of the ideal signal $y_i(n)$. For an example, see Figure 1.
- Noise signal: Extract the signal segment from the T wave of any chosen ECG beat to the P wave of the next ECG beat (isoelectric segment) of $x(n)$

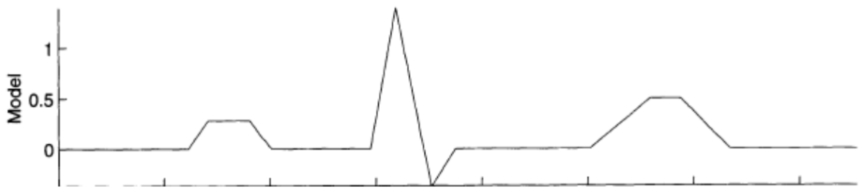

Figure 1

1.2 Frequency domain implementation of the Wiener filter

Consider the equation to calculate the weights of the Wiener filter in the frequency domain given by:

$$W(f) = \frac{S_{YY}(f)}{S_{YY}(f) + S_{NN}(f)}$$

Where, $S_{ZZ}(f)$ is the PSD of the signal $z(n)$. Alternatively, $S_{ZZ}(f)$ is the power of the fourier transform ($\mathcal{F}\{.\}$) of the template $z(n)$. Therefore, $S_{ZZ}(f) = |\mathcal{F}\{z(n)\}|^2$.

a) Implement the frequency domain version of the Wiener filter $S_{\hat{Y}\hat{Y}}(f) = W(f)S_{XX}(f)$ for the two cases given in Part 1 and Part 2 above.
b) Compare the filtered signal with Part 1 and Part 2 with the aid of a plot and the mean squared error with respect to the ideal signal $y_i(n)$

1.3 Effect on non-stationary noise on Wiener filtering

Now consider the case where the 50 Hz noise is changed to 100 Hz halfway through the signal duration (T).

$$\eta(n) = \begin{cases} \eta_{wg}(n) + \eta_{50}(n) & 0 \le n < T/2 \\ \eta_{wg}(n) + \eta_{100}(n) & T/2 \le n < T \end{cases}$$

a) Apply one of the Wiener filters derived in sections 1.1 or 1.2 on the new $x(n)$. Do NOT recalculate the weights.
b) Plot the filtered time-domain signal and interpret the result.

2. Adaptive filtering

Data construction

- Sampling frequency $f_s = 500$ Hz
- Number of samples $N = 5000$
- $y_i(n)$: Sawtooth waveform with a width of 0.5
- $\eta(n)$: Non-stationary noise, same as in section 1.3
- $r(n) = a(\eta_{wg}(n) + \sin(2\pi 50n + \phi_1) + \sin(2\pi 100n + \phi_2))$
  where a, ϕ_1, ϕ_2 are arbitrary constants

Use the following code to generate signals

```
N = 5000; % Number of points
t = linspace(0,5,N)'; % Time vector with fs = 500 Hz
s = sawtooth(2*pi*2*t(1:N,1),0.5);  % Sawtooth signal
n1 = 0.2*sin(2*pi*50*t(1:N/2,1)-phi);   % Sinusoid at 50 Hz
n2 = 0.3*sin(2* pi*100*t(N/2+1:N,1)-phi);   % Sinusoid at 100 Hz
nwg = s - awgn(s,snr,'measured'); % Gaussian white noise
```

2.1 LMS method

a) Implement the LMS method as per the equation

$$\mathbf{w}(n+1) = \mathbf{w}(n) + 2\mu e(n)\mathbf{R}^T(n)$$

b) Plot the signals $y_i(n)$, $x(n)$, $e(n) = \hat{y}(n)$ and the absolute error $|y_i(n) - \hat{y}(n)|$. Your plots should be similar to Figure 2 given below.

c) Explore the rate of adaptation by varying the rate of convergence μ and the order of the adaptive filter $(M-1)$. To quantify you may calculate the mean squared error with respect to the desired signal.

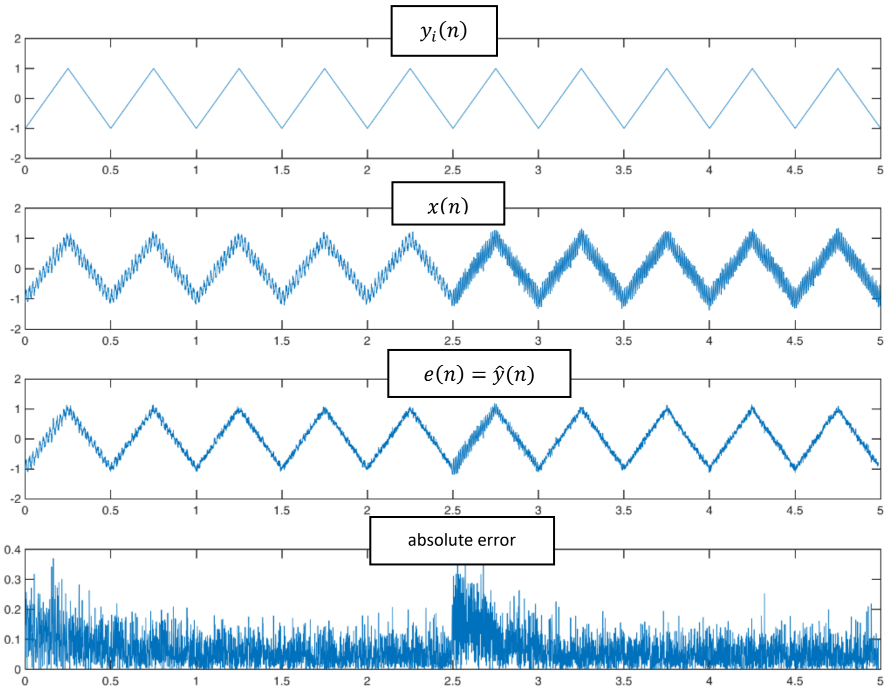

Figure 2

2.2 RLS method

a) Implement the RLS method as per the equations:

$$\mathbf{k}(n) = \frac{\lambda^{-1}\mathbf{P}(n-1)\mathbf{r}(n)}{1+\lambda^{-1}\mathbf{r}^T(n)\mathbf{P}(n-1)\mathbf{r}(n)}$$

$$\mathbf{P}(n) = \lambda^{-1}\mathbf{P}(n-1) - \lambda^{-1}\mathbf{k}(n)\mathbf{r}^T(n)\mathbf{P}(n-1)$$

$$\alpha(n) = x(n) - \mathbf{w}^T(n-1)\mathbf{r}(n)$$

$$\mathbf{w}(n) = \mathbf{w}(n-1) + \mathbf{k}(n)\alpha(n)$$

$$\alpha(n) = \hat{s}(n) = x(n) - \mathbf{w}^T(n)\mathbf{r}(n)$$

Initializations:
$\mathbf{P}(0) = \delta^{-1}I$ where δ is a small constant and I is the identity matrix
$\mathbf{w}(0) = 0$ or a random weight vector if necessary

b) Compare the performance of LMS and RLS algorithms using a plot similar to Figure 2.

c) Explore the rate of adaptation by varying the forgetting factor λ and the order of the filter $(M-1)$. To quantify you may calculate the mean squared error with respect to the desired signal.

d) Now test LMS and RLS algorithms using $y_i(n)$ as the idealECG.mat (noise $\eta(n)$ remains the same as before)