



EECS 280

Programming and Introductory Data Structures

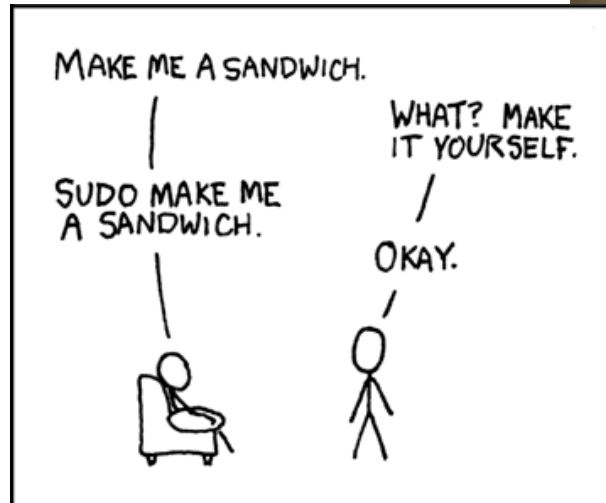
Procedural Abstraction and Recursion

Abstraction

- Abstraction helps manage complexity
- Abstraction hides details

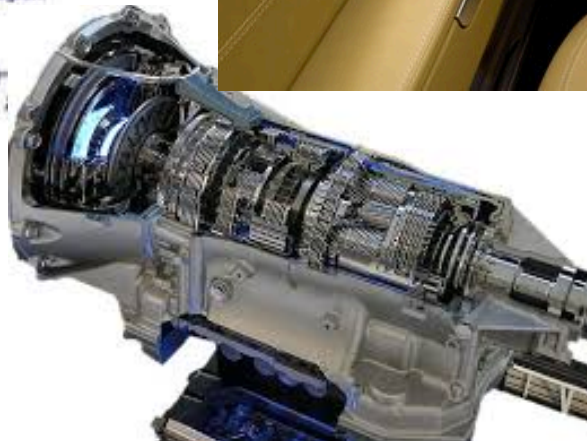
Abstraction example

- **How** make a PBJ sandwich?
- You just need to know **what** the ingredients are combine them
- You don't need to know **how** to make the ingredients
- The processes for making the ingredients have been abstracted away



Abstraction example

- Steering wheel, gear shift and pedals are an abstraction
- Hides the details of rack-and-pinion steering, transmissions, and internal combustion engine

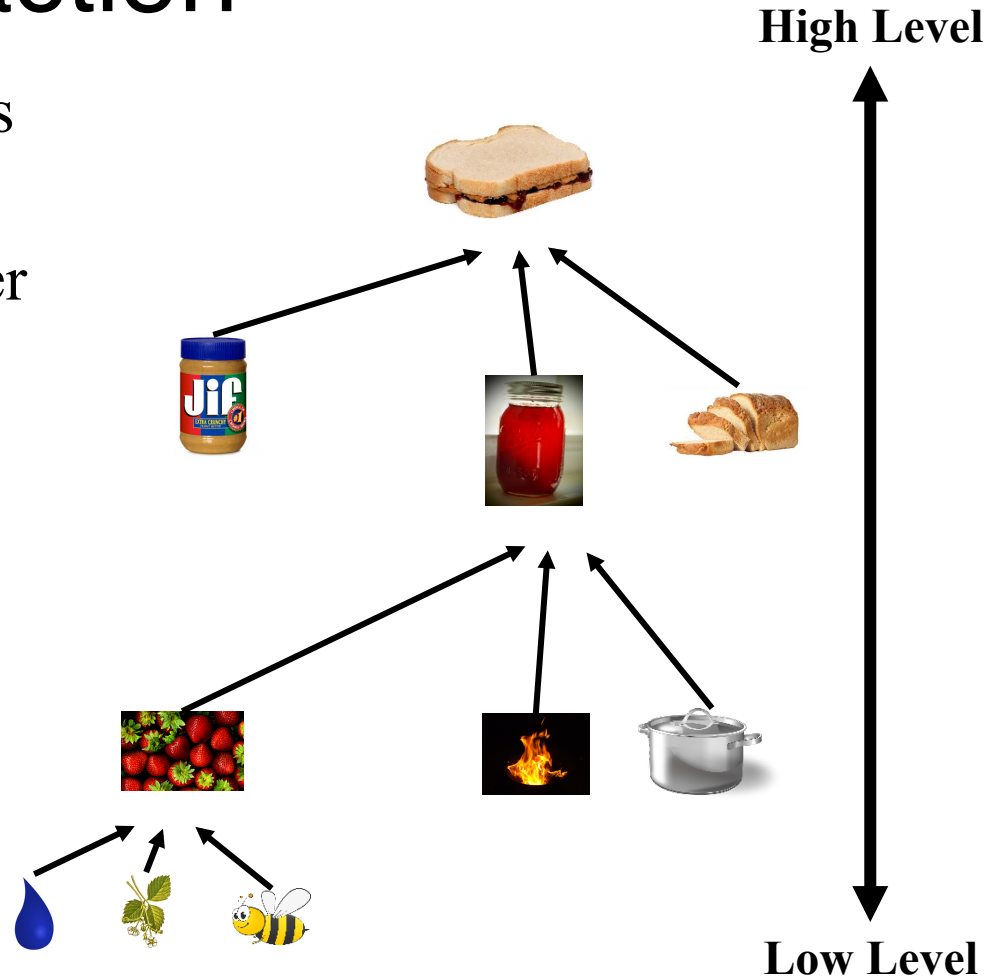


Abstraction exercise

- Try to tell your neighbor how to do something **without** using abstraction
- Examples:
 - How to ride a bike
 - How to bake a cake
 - How to submit to the autograder

Layers of Abstraction

- Abstractions of abstractions
- You don't need to worry about the “**how**” from lower layers



Abstraction in computer programs

- Abstraction lets us separate *what* code does from *how* it works
- Abstraction helps us model complex phenomena
 - Like statistical methods in project 1
- Abstraction makes programs easier to maintain and modify
 - You can change the implementation and no users of the code can tell
- We'll cover two kinds of abstraction in this class
 - Procedural abstraction
 - Data abstraction

Abstraction in computer programs

- **Procedural abstraction** lets us separate *what* a procedure does from *how* it is implemented
- In C++, we use functions to implement procedural abstraction

Example

- Here's an example from project 1
- *What* the functions do, but not *how*
- *How* the functions work
- Finally, we use our functions

```
p1_library.h
```

```
p1_library.cpp
```

```
main.cpp
```

Example

- Works well when you have multiple programmers

- Prof. DeOrio and student Alice agree on an abstraction



`p1_library.h`

- Prof. DeOrio codes `p1_library.cpp`
 - Implements procedural abstraction



`p1_library.cpp`

- Alice codes `main.cpp`
 - Uses procedural abstraction



`main.cpp`



p1_library.h

Example

```
//EFFECTS: extracts one column of data from a tab
// separated values file (.tsv)
// Prints errors to stdout and exits with non-zero
// status on errors
std::vector<double> extract_column(
    std::string filename, std::string column_name);
```

You can understand what the function does by reading
p1_library.h



main.cpp

Example

```
#include "p1_library.h"
int main() {
    //...
    std::vector<double> v = extract_column(
        filename, column_name);
    // do something with v
}
```

- You can use the `extract_column` function in your `main.cpp` without ever knowing how it works!



p1_library.h

Building an abstraction

- Describe abstraction using function inputs, outputs, and "what it's supposed to do"
- Function signature include inputs and outputs

```
std::vector<double> extract_column(  
    std::string filename, std::string column_name);
```

- Need a comment to describe "what it's supposed to do"

```
//EFFECTS: extracts one column of data from a tab  
// separated values file (.tsv)  
// Prints errors to stdout and exits with non-zero  
// status on errors
```

Building an abstraction



p1_library.h

- Anatomy of a *specification comment*
- Need to answer three questions:
 - What pre-conditions must hold to use the function?
 - Does the function change any inputs (even implicit ones)? If so, how?
 - What does the procedure actually do?



p1_library.h

Building an abstraction

- Anatomy of a *specification comment*
- Need to answer three questions:
 - What pre-conditions must hold to use the function?
REQUIRES: the pre-conditions that must hold, if any
 - Does the function change any inputs (even implicit ones)? If so, how?
MODIFIES: how inputs are modified, if any
 - What does the procedure actually do?
EFFECTS: what the procedure computes given legal inputs.
- You can omit REQUIRES or MODIFIES if the function doesn't require or modify anything

MODIFIES example

- Here's another example from `p1_library.h`

```
//MODIFIES: v  
//EFFECTS: sorts v  
void sort(std::vector<double> &v);
```

- `v` is passed by reference
- `sort` will modify `v`
- Modifying global state would also go here
- We say a function that modifies things has *side effects*

REQUIRES example

- Here's an example from project 1's `stats.h`

```
//REQUIRES: v is not empty
//EFFECTS: returns the sum of the numbers in v
double sum(std::vector<double> v);
```

- This function **REQUIRES** that the input is not empty
- `sum` doesn't make any sense if there aren't any numbers!
- The function can safely assume that `v` will have size ≥ 1
- Functions with **REQUIRES** clauses are called *partial*
- Functions without **REQUIRES** clauses are called *complete*

Checking the requires clause

- A function implementation is free to assume that another programmer hasn't violated the REQUIRES clause
- It's a good habit to check yourself any ways

assert()

```
assert ( /*EXPRESSION*/ );
```

- `assert()` is a programmer's friend for debugging
- Does nothing if statement `EXPRESSION` is true
- Exits and prints an error message if `EXPRESSION` is false

```
#include <cassert>
```

```
int main () {
```

```
    assert(true); // does nothing
```

```
    assert(false); // crash with debug message
```

```
}
```

```
Assertion failed: (false), function main, file test.cpp, line 5.
```

Checking the requires clause

- We can use `assert()` to check a `requires` clause
- GOOD HABIT!!!

```
double sum(std::vector<double> v) {  
    assert(!v.empty()) ;  
    // ...  
}
```

another way to do it

```
double sum(std::vector<double> v) {  
    assert(v.size() > 0) ;  
    // ...  
}
```

Properties of procedural abstraction

- Local

The implementation of an abstraction can be understood without examining any other abstraction implementation

- Substitutable

You can replace one (correct) implementation of an abstraction with another (correct) one, and no callers of that abstraction will need to be modified

Example: substitutable

- Here's the current implementation in `p1_library.cpp`

```
void sort(std::vector<double> &v) {  
    std::sort(v.begin(), v.end());  
}
```

- And let's say your `mode()` function in `stats.cpp` uses `sort()`:

```
double mode(std::vector<double> v) {  
    assert(!v.empty());  
    sort(v);  
    // ...  
}
```

- If the staff changes the implementation of `sort()`, do you need to change your `mode` function?

Example: substitutable

- If the staff changes the implementation of `sort()`, do you need to change your mode function?
- No! As long as no one changes the *abstraction* in `pl_library.h`, your code in `stats.cpp` still works!
- This is a big benefit of abstraction

The stack

- Some questions we'd like to answer:
- Each function has it's own local variables, how can the program keep track of them all?
- How does the flow of control happen between callers and callees?

The stack

- When a function is called, data for the execution of that function is stored as an **activation record**
 - e.g. local variables, parameters, return address, etc.
- Activation records are typically stored in a **stack**
- A stack is a container with the Last-In-First-Out (**LIFO**) property
 - You can add/remove things from the “top”¹ of the stack
 - You can’t take them off the bottom or out of the middle
 - Naturally leads to LIFO

¹ Note: stacks are sometimes drawn growing downward, so “top” is a relative term

The stack

- When a function is called, an activation record is created for it and added to the top of the stack
- Activation records are often called stack frames

```
int plus_one(int x) {  
    return (x+1);  
}  
  
int plus_two(int x) {  
    return (1 + plus_one(x));  
}  
  
int main() {  
    int result = 0;  
    result = plus_one(0);  
    result = plus_two(result);  
    cout << result; //3  
    return 0;  
}
```

The stack

Since environments are lexically scoped, `plus_one` cannot see `plus_two`'s `x`. Instead, a copy of `plus_two`'s `x` is passed to `plus_one`, and stored in `plus_one`'s `x`

```
int plus_one(int x) {  
    return (x+1);  
}  
  
int plus_two(int x) {  
    return (1 + plus_one(x));  
}  
  
int main() {  
    int result = 0;  
    result = plus_one(0);  
    result = plus_two(result);  
    cout << result; //3  
    return 0;  
}
```

Function Calls

1. Make a new stack frame
2. Pause the original function
3. Run the called function
4. Restart the original function where it left off
5. Destroy the stack frame

Stack example

```
main  
result: ??  
paused: (running)  
return: ??
```

Stack



```
int plus_one(int x) {  
1.  return (x+1);  
}  
  
int plus_two(int x) {  
1.  return (1 + plus_one(x));  
}
```

```
int main() {  
1.  int result = 0;  
2.  result = plus_one(0);  
3.  result = plus_two(result);  
4.  cout << result; //3  
5.  return 0;  
}
```

Stack example

```
main  
result: ??  
paused: line 2  
return: ??
```

```
plus_one  
x: 0  
paused: (running)  
return: ??
```

Stack



```
int plus_one(int x) {  
1.  return (x+1);  
}  
  
int plus_two(int x) {  
1.  return (1 + plus_one(x));  
}  
  
int main() {  
1.  int result = 0;  
2.  result = plus_one(0);  
3.  result = plus_two(result);  
4.  cout << result; //3  
5.  return 0;  
}
```

Stack example

```
main  
result: ??  
paused: line 2  
return: ??
```

```
plus_one  
x: 0  
paused: (running)  
return: 0 + 1
```

Stack



```
int plus_one(int x) {  
1.  return (x+1);  
}  
  
int plus_two(int x) {  
1.  return (1 + plus_one(x));  
}  
  
int main() {  
1.  int result = 0;  
2.  result = plus_one(0);  
3.  result = plus_two(result);  
4.  cout << result; //3  
5.  return 0;  
}
```

Stack example

```
main  
result: 1  
paused: (resumed)  
return: ??
```

Stack



```
int plus_one(int x) {  
1.  return (x+1);  
}  
  
int plus_two(int x) {  
1.  return (1 + plus_one(x));  
}  
  
int main() {  
1.  int result = 0;  
2.  result = plus_one(0);  
3.  result = plus_two(result);  
4.  cout << result; //3  
5.  return 0;  
}
```


Stack example

```
main  
result: 1  
paused: line 3  
return: ??
```

```
plus_two  
x: 1  
paused: (running)  
return: ??
```

Stack



```
int plus_one(int x) {  
1.  return (x+1);  
}  
  
int plus_two(int x) {  
1.  return (1 + plus_one(x));  
}  
  
int main() {  
1.  int result = 0;  
2.  result = plus_one(0);  
3.  result = plus_two(result);  
4.  cout << result; //3  
5.  return 0;  
}
```

Stack example

```
main  
result: 1  
paused: line 3  
return: ??
```

```
plus_two  
x: 1  
paused: line 1  
return: ??
```

```
plus_one  
x: 1  
paused: (running)  
return: ??
```

Stack



```
int plus_one(int x) {  
1.  return (x+1);  
}  
  
int plus_two(int x) {  
1.  return (1 + plus_one(x));  
}
```

```
int main() {  
1.  int result = 0;  
2.  result = plus_one(0);  
3.  result = plus_two(result);  
4.  cout << result; //3  
5.  return 0;  
}
```

Stack example

```
main  
result: 1  
paused: line 3  
return: ??
```

```
plus_two  
x: 1  
paused: line 1  
return: ??
```

```
plus_one  
x: 1  
paused: (running)  
return: 1 + 1
```

Stack



```
int plus_one(int x) {  
1.  return (x+1);  
}  
  
int plus_two(int x) {  
1.  return (1 + plus_one(x));  
}  
  
int main() {  
1.  int result = 0;  
2.  result = plus_one(0);  
3.  result = plus_two(result);  
4.  cout << result; //3  
5.  return 0;  
}
```

Stack example

```
main  
result: 1  
paused: line 3  
return: ??
```

```
plus_two  
x: 1  
paused: (resumed)  
return: 1 + 2
```

Stack



```
int plus_one(int x) {  
1.  return (x+1);  
}  
  
int plus_two(int x) {  
1.  return (1 + plus_one(x));  
}  
  
int main() {  
1.  int result = 0;  
2.  result = plus_one(0);  
3.  result = plus_two(result);  
4.  cout << result; //3  
5.  return 0;  
}
```

Stack example

```
main  
result: 1  
paused: line 3  
return: ??
```

```
plus_two  
x: 1  
paused: (resumed)  
return: 3
```

Stack



```
int plus_one(int x) {  
1.  return (x+1);  
}  
  
int plus_two(int x) {  
1.  return (1 + plus_one(x));  
}  
  
int main() {  
1.  int result = 0;  
2.  result = plus_one(0);  
3.  result = plus_two(result);  
4.  cout << result; //3  
5.  return 0;  
}
```

Stack example

```
main  
result: 3  
paused: (resumed)  
return: ??
```

Stack



```
int plus_one(int x) {  
1.  return (x+1);  
}  
  
int plus_two(int x) {  
1.  return (1 + plus_one(x));  
}  
  
int main() {  
1.  int result = 0;  
2.  result = plus_one(0);  
3.  result = plus_two(result);  
4.  cout << result; //3  
5.  return 0;  
}
```

Stack example

```
main  
result: 3  
paused: line 4  
return: ??
```

```
cout  
...
```

Stack



```
int plus_one(int x) {  
1.  return (x+1);  
}  
  
int plus_two(int x) {  
1.  return (1 + plus_one(x));  
}  
  
  
int main() {  
1.  int result = 0;  
2.  result = plus_one(0);  
3.  result = plus_two(result);  
4.  cout << result; //3  
5.  return 0;  
}
```

Stack example

```
main  
result: 3  
paused: (resumed)  
return: ??
```

Stack



```
int plus_one(int x) {  
1.  return (x+1);  
}  
  
int plus_two(int x) {  
1.  return (1 + plus_one(x));  
}  
  
int main() {  
1.  int result = 0;  
2.  result = plus_one(0);  
3.  result = plus_two(result);  
4.  cout << result; //3  
5.  return 0;  
}
```


Stack example

```
main  
result: 3  
paused: (resumed)  
return: 0
```

Stack



```
int plus_one(int x) {  
1.  return (x+1);  
}  
  
int plus_two(int x) {  
1.  return (1 + plus_one(x));  
}  
  
int main() {  
1.  int result = 0;  
2.  result = plus_one(0);  
3.  result = plus_two(result);  
4.  cout << result; //3  
5.  return 0;  
}
```

Stack example

Stack



```
int plus_one(int x) {  
1.  return (x+1);  
}  
  
int plus_two(int x) {  
1.  return (1 + plus_one(x));  
}  
  
  
int main() {  
1.  int result = 0;  
2.  result = plus_one(0);  
3.  result = plus_two(result);  
4.  cout << result; //3  
5.  return 0;  
}
```

Recursion

- Functions can call other functions
- Can a function call itself?
- Let's try it

```
void countToInfinity(int x) {  
    cout << start << endl;  
    countToInfinity(x + 1);  
}
```

```
int main() {  
    countToInfinity(0);  
}
```

Solving problems with recursion

- Two features of problems make recursion an attractive solution:
- Subproblems that are:
 - Similar
 - “Smaller” (closer to a base case)
- A Base Case
 - Can be solved without recursion
- The next few lectures, and most of project 2, is all about solving problems using recursion

Base case

- You have to stop somewhere

```
void countToTen(int x) {  
    cout << start << endl;  
    if (x == 10) return;  
    countToTen(x + 1);  
}
```

```
int main() {  
    countToTen(0);  
}
```

Recursive step

- Solve a “smaller” problem
- One that’s closer to the base case

```
void countToTen(int x) {  
    cout << start << endl;  
    if (x == 10) return;  
    countToTen(x + 1);  
}
```

```
int main() {  
    countToTen(0);  
}
```

Exercise: factorial

- Recall the factorial function from math class:
 - $0! = 1$
 - $n! = n * (n-1)!$
- Try to write `factorial` using recursion

```
// REQUIRES: n >= 0
// EFFECTS: computes and returns n!
int factorial(int n){

}
```

Writing recursive functions

- Don't try to do it all in your head
- Instead, treat it like an inductive proof
- Identify the “trivial” base case and write it explicitly
- For all other cases
 - Assume there is a function that can solve smaller versions of the same problem
 - Figure out how to get from the smaller solution to the bigger one

Solution: factorial

- Recall the factorial function from math class:
 - $0! = 1$
 - $n! = n * (n-1)!$
- Try to write an implementation for factorial using recursion

```
// REQUIRES: n >= 0
// EFFECTS: computes and returns n!
int factorial(int n){
    if (n == 0) { // BASE CASE
        return 1;
    }
    else{
        return n * factorial(n-1); // RECURSIVE CASE
    }
}
```

Solution: factorial

- Another correct solution

```
// REQUIRES: n >= 0
// EFFECTS: computes and returns n!
int factorial(int n){
    if (n == 0) return 1;
    return n * factorial(n-1);
}
```

Exercise: draw the call stack

```
int main() {  
    int x;  
    x = factorial(3);  
    return 0;  
}
```

```
int factorial (int n) {  
    if (n == 0) return 1;  
    return n * factorial(n-1);  
}
```