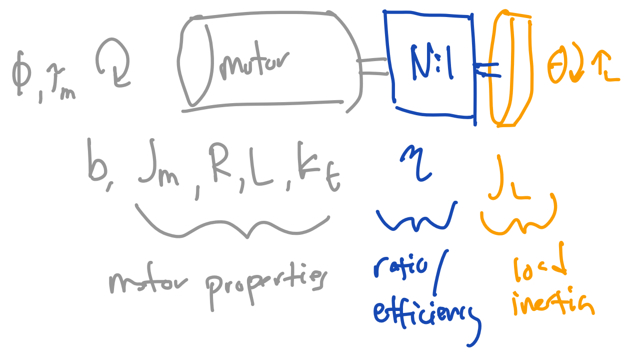

Write expressions for the current & voltage of the motor / transmission / inertia

Begin w/ Newton's 2nd law:

① $k_t i = J_m \ddot{\phi} + b\dot{\phi} + \frac{\tau_L}{\eta N}$ ← in this example the only torque is accelerating the inertia. We divide by eta·N to account for the torque through the transmission

② $\tau_L = J_L \ddot{\theta}$ ← The load torque is the inertial torque in this example. For example, it's not interacting with any springs or other torque requirements

③ $\phi = N\theta \rightarrow \theta = \frac{\phi}{N}$ ← now we can convert θ to ϕ, which accounts for the other N in the "divide by N^2"

$$k_t i = J_m \ddot{\phi} + b\dot{\phi} + \frac{J_L \frac{\ddot{\phi}}{N}}{\eta N}$$ ← Substituting ② & ③ into ①

$$\tau_m = k_t i = \left[J_m + \frac{J_L}{\eta N^2}\right]\ddot{\phi} + b\dot{\phi}$$ ← Algebra to rearrange

$$i = \frac{\left[J_m + \frac{J_L}{\eta N^2}\right]\ddot{\phi} + b\dot{\phi}}{k_t} \qquad V = iR + k_t\dot{\phi} + L\,{di}/{dt}$$ ← answers

Another way to solve is to "pattern match" & use the equivalent inertia concept we discussed when mentioning kinetic energy & reflected inertia

$$\tau_m = k_t i = [\text{equivalent inertia}]\ddot{\phi} + b\dot{\phi} + \frac{\tau_L}{N\eta}$$

This is zero since the motor isn't doing any other task

from our kinetic energy example, we know inertias on the other side of the transmission have to be divided by N^2

$$\tau_m = k_t i = \left[J_m + \frac{\frac{J_L}{N^2}}{\eta}\right]\ddot{\phi} + b\dot{\phi}$$

we divide by eta since that torque is on the other side of the transmission

In class, I did a combination of these methods (also skipping steps) which was confusing. Hopefully this example helps provide clarity