
4.1 Undirected Graphs Glossary

Definitions.

A **graph** is a set of vertices and a collection of edges that each connect a pair of vertices.

A **self-loop** is an edge that connects a vertex to itself.

Two edges that connect the same pair of vertices are **parallel**.

When there is an edge connecting two vertices, we say that the vertices are **adjacent** to one another and that the edge is **incident** to both vertices.

The **degree of a vertex** is the number of edges incident to it.

A **subgraph** is a subset of a graph's edges (and associated vertices) that constitutes a graph.

A **path** in a graph is a sequence of vertices connected by edges.

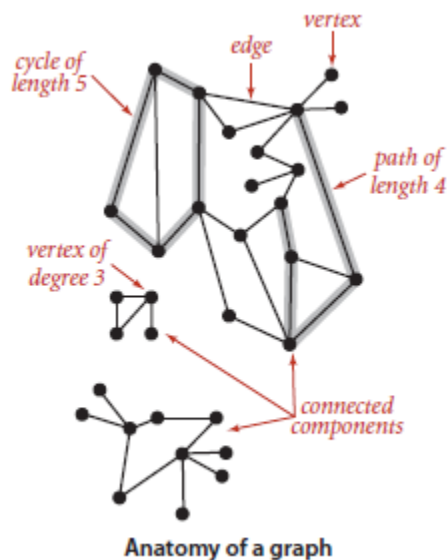
A **simple path** is one with no repeated vertices.

A **cycle** is a path with at least one edge whose first and last vertices are the same.

A **simple cycle** is a cycle with no repeated edges or vertices.

We say that one vertex is **connected** to another if there exists a path that contains both of them.

A **graph is connected** if there is a path from every vertex to every other vertex in the graph. A graph that is not connected consists of a set of **connected components**, which are maximal connected subgraphs.



An **acyclic graph** is a graph with no cycles.

A **tree** is an acyclic connected graph.

A disjoint set of trees is called a **forest**.

A **spanning tree** of a connected graph is a subgraph that contains all of that graph's vertices and is a single tree.

A **spanning forest** of a graph is the union of spanning trees of its connected components.



Graph G with V vertices is a tree if and only if it satisfies any of the following five conditions:

- G has $V-1$ edges and no cycles.
- G has $V-1$ edges and is connected.
- G is connected, but removing any edge disconnects it.
- G is acyclic, but adding any edge creates a cycle.
- Exactly one simple path connects each pair of vertices in G .

The **density of a graph** is the proportion of possible pairs of vertices that are connected by edges.

A **bipartite graph** is a graph whose vertices we can divide into two sets such that all edges connect a vertex in one set with a vertex in the other set.

