Contents

1 Cognition, computation, computers, programs

1.1 Cognition and computation

1. Cognitive Science: mind/brain is analogous to software/hardware.

1.1. Various names: "the computational view of mind," "information processing psychology," "the computational theory of mind," or simply "computationalism."

1.2. The analogy is usually far from strict and there are many varieties.

1.3. For instance: "mind/cognition is computation" versus "mind/cognition is computable."

2. Why we need computationalism?

2.1. Compare the tasks:[1]

- Predicting the trajectory of celestial bodies, say the motion of the earth in the next six hours.
- Predicting the next move of a chess player at a given state of the game.

2.2. Crane (2003:103): "The planets do not 'compute' their orbits from the input they receive: they just move."

2.3. In the case of chess, physical description and physical laws are helpless. The source of helplessness is twofold: (i) The state of a chess game has infinitely many physical realizations, and whether a physical state is a game state is dependent on whether or not some cognitive agents interpret the "scene" as a chess game or not. Therefore a function from physical description to game state is at best extremely complex. (ii) Even if we find a way from physical description to game state, the physical description of rule-based behavior would be a function of the particular realization function (the mapping from physics-to-chess states) at that particular occasion (Pylyshyn 1984).

3. Computation and cognition share an essential property: both operate on rules and representations, yet are based on (instantiated by) physical causal systems. What is happening in a computer and mind/brain are quite similar.

Levels (more on this below):

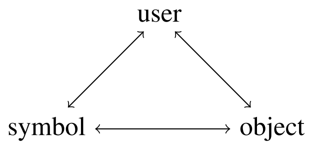
i. physical (device)
i. symbolic (syntactic)
iii. intentional (semantic)

[1] We gloss over a potential objection to the legitimacy of this comparison, given the huge difference between the well-definedness of the questions.

1.2 What is (symbolic) computation?

1. Symbols (or signs) and signification are central concepts in language, logic, and computation.

1.1. Take signification as a three-part relation:



- A user uses a sign to **refer** to an object.
- A sign **denotes** an object.
- A user has certain **intentions** about an object.

1.2. A sequence of binary digits (bits) is a symbolic representation:

0010110101001110000000010

2. A computational process is a sequence of manipulations performed over symbolic representations.

2.1. Example: the process by which you flip the digits of the representation above, one at a time is a computational process.

1.3 Abstract machines (optional)

- A Turing Machine (TM) is specified as a quintuple $\langle \Sigma, Q, q_0, q_H, \omega \rangle$, where

Σ	is an alphabet of admissible symbols (including a symbol for blank cells);
Q	is a finite set of internal states;
$q_0 \in Q$	is the starting state;
$q_H \in Q$	is the halting state;
ω	is the state transition function of the form:

$$\langle q, \sigma \rangle \mapsto \langle q', \sigma', M \rangle$$

where q and q' are states, σ and σ' are symbols, and $M \in \{-1, 0, 1\}$, for 'move left', 'don't move', and 'move right', respectively.

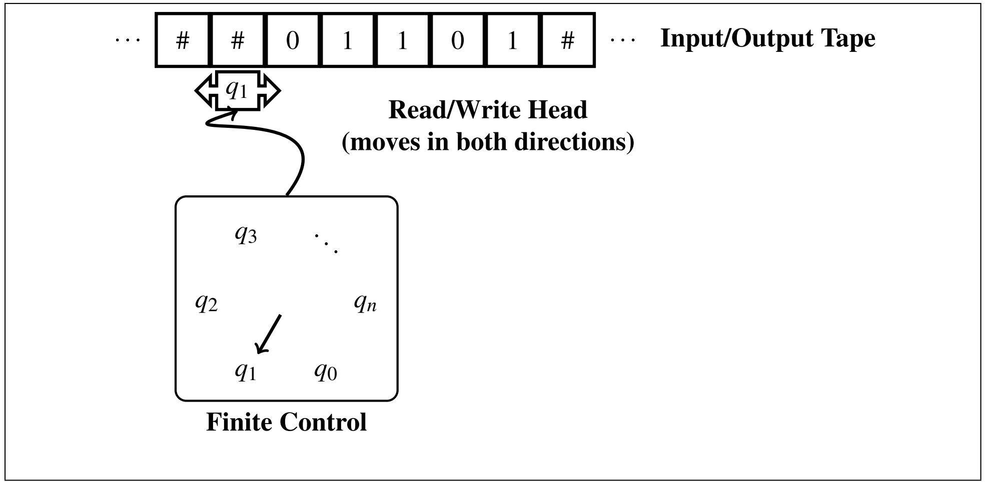


Figure 1: A Turing Machine

Example 1.1
A TM that decides whether its input is a palindrome. Let # be the blank symbol, q_0 the initial, and q_8 the halting state.

$$\begin{array}{lcl@{\qquad}lcl}
\langle q_0,\#\rangle & \mapsto & \langle q_8,1,0\rangle & \langle q_4,\#\rangle & \mapsto & \langle q_8,1,0\rangle \\
\langle q_0,0\rangle & \mapsto & \langle q_1,\#,1\rangle & \langle q_4,0\rangle & \mapsto & \langle q_7,\#,-1\rangle \\
\langle q_0,1\rangle & \mapsto & \langle q_2,\#,1\rangle & \langle q_4,1\rangle & \mapsto & \langle q_5,\#,-1\rangle \\
\langle q_1,\#\rangle & \mapsto & \langle q_3,\#,-1\rangle & \langle q_5,\#\rangle & \mapsto & \langle q_0,\#,1\rangle \\
\langle q_1,0\rangle & \mapsto & \langle q_1,0,1\rangle & \langle q_5,0\rangle & \mapsto & \langle q_5,0,-1\rangle \\
\langle q_1,1\rangle & \mapsto & \langle q_1,1,1\rangle & \langle q_5,1\rangle & \mapsto & \langle q_5,1,-1\rangle \\
\langle q_2,\#\rangle & \mapsto & \langle q_4,\#,-1\rangle & \langle q_6,\#\rangle & \mapsto & \langle q_0,\#,1\rangle \\
\langle q_2,0\rangle & \mapsto & \langle q_2,0,1\rangle & \langle q_6,0\rangle & \mapsto & \langle q_6,0,-1\rangle \\
\langle q_2,1\rangle & \mapsto & \langle q_2,1,1\rangle & \langle q_6,1\rangle & \mapsto & \langle q_6,1,-1\rangle \\
\langle q_3,\#\rangle & \mapsto & \langle q_8,1,0\rangle & \langle q_7,\#\rangle & \mapsto & \langle q_8,0,0\rangle \\
\langle q_3,0\rangle & \mapsto & \langle q_5,\#,-1\rangle & \langle q_7,0\rangle & \mapsto & \langle q_7,\#,-1\rangle \\
\langle q_3,1\rangle & \mapsto & \langle q_7,\#,-1\rangle & \langle q_7,1\rangle & \mapsto & \langle q_7,\#,-1\rangle
\end{array}$$

$\square$

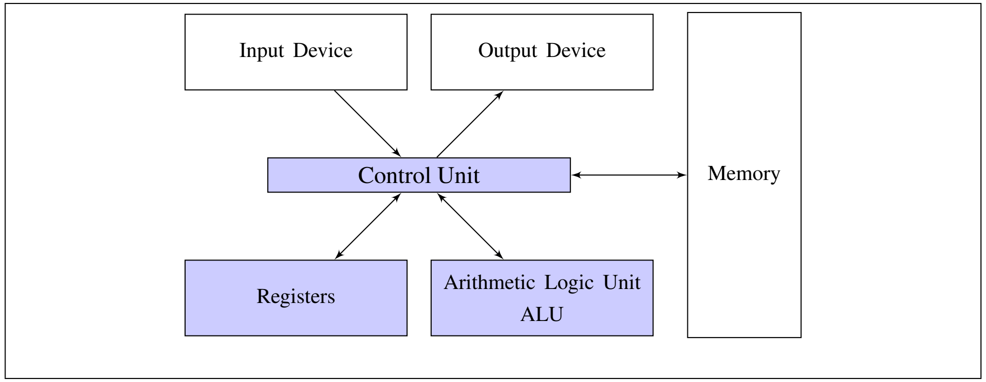


Figure 2: Von Neumann architecture

1.4 Hardware/software

1. Most computers we have around are based on von Neumann[2] architecture.

2. VNA consists of **memory**, **central processing unit**, and I/O devices.

3. Memory consists of a sequence of **cells** (also called **words**).

3.1. Each cell is of a fixed capacity.

3.1.1. What we mean by capacity is how many binary digits (bits) a cell can hold.

3.1.2. Bits are thought of in groups of 8's; 8 bits = 1 **byte**.

3.1.3. A single byte can hold 2^8 different values, or symbols, if you like.

3.2. Each cell of memory has a unique **address**, itself a binary number.

4. The two basic components, CPU and memory, communicate through three channels:

   i. A collection of wires called **address bus**;

   ii. Another collection of wires called **data bus**;

   iii. A single wire called **R/W line**, the status of which signals whether CPU wants to write to or read from the memory.

5. The address bus and the R/W line are one way channels. The value is always dictated by the CPU. The data bus is a two-way channel.

6. The two basic interactions between CPU and Memory go as follows:

   - CPU sets the address bus and R/W line to W. In that case it also sets the data bus. Obviously, this amounts to dictating to write the data to the given address in memory.
   - CPU sets the address bus and R/W line to R. In this case it is the memory that sets the data bus in accord with the data located at the address provided by CPU; this is reading from memory.

7. CPU itself also has some local memory slots. These are called **registers**. Access to registers is faster than access to memory. But there is a reason to keep the size of the CPU small, therefore there are a limited number of registers, which are used to store intermediate results of computations and some frequently used information.

8. The computation unit of CPU is **arithmetic logic unit** (ALU, for short.) ALU is responsible for arithmetic and logical operations.

9. CPU feeds on **instructions**. Some typical types of instructions are:

   i. Store the number at the register X at the memory address Y;

   ii. Fetch the number at the address X and store it at the register Y;

   iii. Add the number at address X to the number at address Y, and store the result at address Z;

   iv. Compute the bitwise *and* of the numbers at addresses X and Y, and store the result at the address Z;

   v. If the contents of the register X and Y are not identical, go to address Z;

   vi. Jump to the address X;

   vii. and so on.

10. **Stored-program** concept

   a. Not only data but also instructions are represented as numbers;

   b. Programs are stored in memory; they can be read and written just like data.

11. A central aspect of a computational system is **flow of control**. Some instructions have *go to* or *jump*, which sends the control to another instruction. But other instructions lack such a mechanism for flow of control.

12. There is a special register called **program counter**, where the address of the next instruction is kept.

[2] Named after the mathematician and physicist John von Neumann.

13. CPU operates through a sequence of cycles. Each cycle begins by fetching a binary description of what to do, an **instruction** from an address in Memory. Following this, CPU understands, or technically speaking, **decodes** the instruction. The next step is to **execute** the instruction. This completes a single cycle, which is followed by reading the next instruction from Memory.[3]

1.5 Levels of programming

1. Computers operate on numbers, in the sense that the functional architecture of the machines get their instructions as binary numbers organized into expressions of **machine code**.

2. Given a functional architecture, say VNA, a straightforward specification would be to code the memory byte-by-byte (writing **machine code**); so that CPU "knows" what to do in each possible state of the process.

3. Example of an addition instruction:

   `000000 10001 10010 01000 00000 1000000`

4. A machine code instruction is organized into **fields** with different meanings.

5. Some levels up the machine code is the **assembly** language. The above machine code takes the form:

   ```
   add $s1,$t1,$t
   ```

6. Assembly level lies between high-level languages and machine code.

   A simple while loop in C:

   ```
   while (save[i] == k) {
           i = i + 1;
           }
   ```

   Assuing that `i` and `k` are stored in registers `$s3` and `$s5`, and the array `save` starts at the memory address stored in `$s6`, the above C code is **compiled** to the following assembly code (for MIPS):

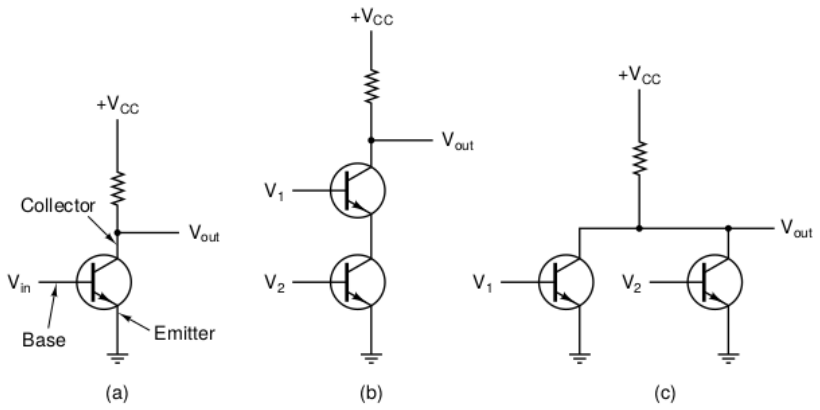
[3]Cycles are called 'fetch-decode-execute' or 'instruction cycle'.

```
Loop:  sll $t1,$s3,2
       add $t1,$t1,$s6
       lw  $t0,0($t1)
       bne $t0,$s5,Exit
       add $s3,$s3,1
       j   Loop
Exit:
```

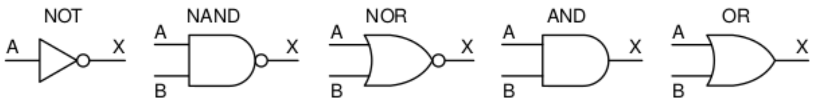
7. A computational process can be specified at various levels.

1.6 More into hardware

1. The basic building block of modern computer is the transistor.[4] Transistors are organized into logic gates.



[4]The figures in this subsection are taken from Tanenbaum 1999.



A	X
0	1
1	0

(a)

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

(b)

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

(c)

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

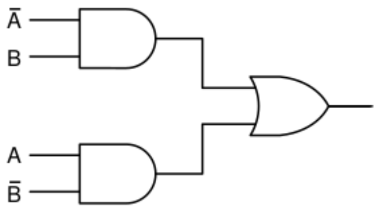
(d)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

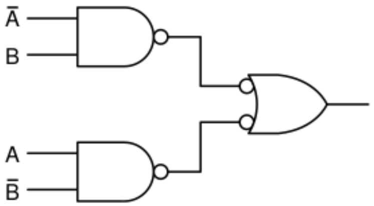
(e)

A	B	XOR
0	0	0
0	1	1
1	0	1
1	1	0

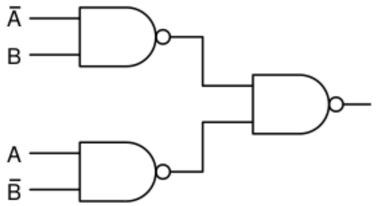
(a)



(b)



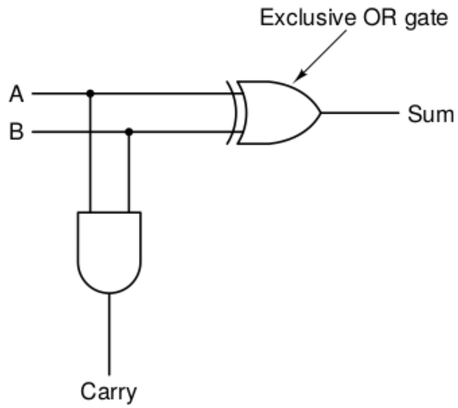
(c)



(d)

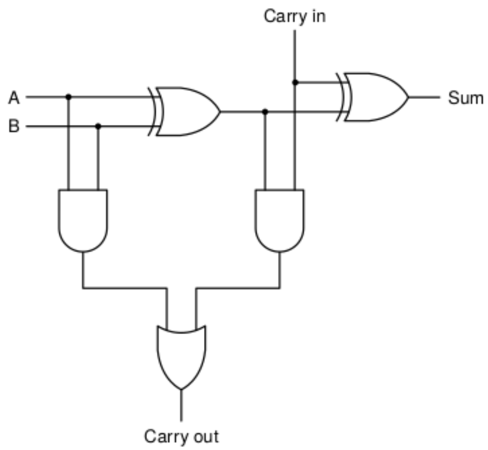
2. Mechanical addition with circuits:

A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



A	B	Carry in	Sum	Carry out
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

(a)



(b)

1.7 Further reading

Pohl and Shaw (1981) is an introduction to computer science, for non-computer scientists. Tanenbaum (1999) (or a newer edition) is an accessible book on computer architecture. It also provides a short and nice history of computers. Pylyshyn (1984) is a classic on cognition and computation, from the representational camp. Consult Churchland and Sejnowski (1992) for a connectionist approach to the same topic. The last two books are NOT introductory level. For a general introduction, see Crane (2003).

2 Conses and Lists

2.1 Building blocks

1. We start with three building blocks: **symbols**, **numbers**, **addresses** and memory locations called **cells**.

1.1. Numbers include, but are not limited to, integers $\{\ldots -3, -2, -1, 0, 1, 2, 3\ldots\}$.

1.2. Symbols include, but are not limited to, combinations of numbers, alphabetical characters and the '-', which are not numbers.

1.3. An address is a binary number (typically 32bit) that points to a cell, where symbols and numbers are stored.

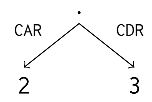
2. A fourth type of entity is a **cons**, a concatenation of two addresses, which is stored in a cell.

2.1. Therefore, an address, by virtue of uniquely identifying a cell, can point to a symbol, a number, or a cons.

2.2. Or, equivalently, cells host symbols, numbers and conses.

2.3. The two parts forming a cons are called `CAR` and `CDR`, respectively.

2.4. A cons is used to pair two objects residing in the cells pointed to by the `CAR` and the `CDR` of the cons:[5]



2.2 Lists

1. List is the central data structure in LISP, which stands for Lis(t) P(rocessor).

1.1. Lists are represented as sequences of objects separate by white space and enclosed in parentheses:

```
(1 3 9)
(RED GREEN BLUE)
(1 GREEN)
()
(7)
```

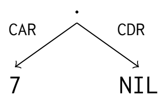
[5]Think of it as an ordered pair in set theory.

2. We call an object in a list its element.

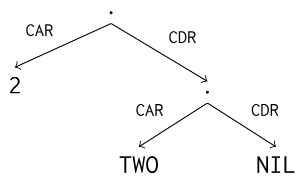
3. The empty list is represented in two equivalent ways: `NIL` and `()` .

4. Lists are represented internally as conses. The `CAR` of the cons points to the first element of the list, while the `CDR` of the cons points to the list but its first element.

4.1. The single element list `(7)` is represented as:



4.2. The two element list `(2 TWO)` is represented as:

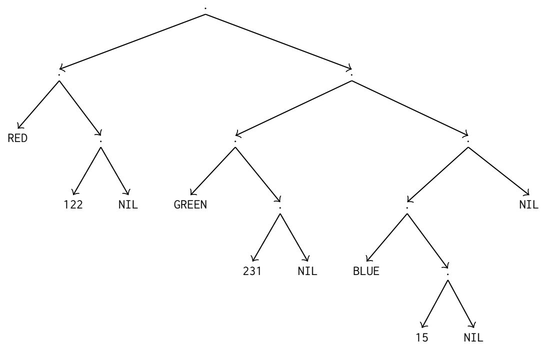


and so on. . .

4.3. The only exception to cons-based representation of lists is the empty list, which is the object `NIL`, which is not a cons.

4.4. Lists can be nested (= lists as elements of lists).

4.4.1. Take for example `((RED 122) (GREEN 231) (BLUE 15))`:[6]



[6] From here on we adopt the convention that the left branch leaving a cons is `CAR` and the right branch is `CDR`, to be able to omit the labels on the edges.

5. Two basic constructors for lists: `CONS` and `LIST`.

5.1. Make sure you totally understand how they differ.

5.2. See Touretzky 1990:59 on how `LIST` works.

3 Evaluation

1. Open up `SBCL` by typing `rlwrap sbcl` in the command-line and hit return.

2. If you see the prompt `*`, you are at the **top-level**.

3. Whatever you enter at the top-level will be **read**, **evaluated** and the result of the evaluation will be **printed**. This is called REPL, read-evaluate-print-loop.

4. `LISP` has strict rules on how to evaluate what you give to it.

4.1. Numbers, strings (things that start and end with `"`), and the symbols `NIL` and `T` evaluate to themselves – you get what you give.

4.2. Lists, on the other hand, are evaluated as follows:

4.2.1. The first element of the list is treated as naming a function – `LISP` will bitterly complain if it fails to find a function with that name.[7]

4.2.2. If the first step is successful, `LISP` will evaluate the rest of the elements, from left to right.

4.2.3. If all goes well, it will apply the function stored in the first element to the *values* obtained by the evaluation of the rest.

5. What do you expect from the following – think before trying?

```
(+ 2 4)
(- (+ 2 4) 7)
(max (+ 2 4) 7)
(max (+ 2 4) a)
(1 2 3)
(car (1 2 3))
(cdr (a b c))
(list a b c)
```

6. Can you see why some of these expressions put you in trouble at the top-level?

[7] This "name" business is slightly more complicated than stated here, we will get back to it later.

7. Sometimes you mean a list to be just a list and would not want LISP to evaluate that expression as a function call. The way to tell LISP not to evaluate an expressions is, as in natural language, putting the expression in quotes. The following would behave well:

```
(car (quote (1 2 3)))
(cdr (quote (a b c)))
(list (quote a) (quote b) (quote c)))
```

8. As one needs so frequently to quote expressions, there is a shorthand for QUOTE:

```
(car '(1 2 3))
(cdr '(a b c))
(list 'a 'b 'c)
```

9. Note that `'(a b c)` is not the same as `('a 'b 'c)`, make sure that you understand why.

4 Defining functions

1. Assume you want to square a number (=multiply it with itself), say 3. Why not try,

```
(square 3)
```

2. Unfortunately, LISP does not understand the word `square`. Here is how to define it:

```
(defun square (n)
    (* n n))
```

3. Now you can use SQUARE as the first element of a list to invoke the square function:

```
(square 7)
```

4. The definition in 2 is a recipe telling what SQUARE should do to its one and only **argument**.

5. Notice that our function definition is also a list, like everything else in LISP. Without the indents, it looks more like so:

```
(defun square (n) (* n n))
```

5.1. Does DEFUN obey the rule of evaluation in 4.2.?

6. At this point it is natural to think that SQUARE is a symbol (or name) of a function, so giving it to top-level should return some representation of this function; after all LISP should be familiar with the name SQUARE. Try it. And see that it doesn't work. You will understand very well why in the coming weeks.

5 Making decisions

1. Predicates are expressions that take arguments and return T or NIL, that is "true" or "false".

2. Built-in predicates usually end in letter `p`: SYMBOLP, LISTP, ZEROP; but there are exceptions like ATOM and NULL. You're strongly advised to follow the `p` convention when naming your own predicates.

3. Testing for equality: there are more than one ways for testing equality, but to keep things simple for now, use the two argument predicate EQUAL. You may encounter EQL, EQUALP and = in examples and solutions to exercises. For the time being you can think of them as equivalent to EQUAL.

4. Comparison predicates: <, >, <=, >=.

5. A predicate applied to its arguments is a **test**. E.g. `(< 3 2)` is a test that evaluates to NIL.

6. The simplest structure to make a decision is IF which takes three arguments:

6.1. **test** – a predicate applied to its arguments;

6.2. **success** – an expression that will be *evaluated and returned* in case the test succeeds;

6.3. **failure** – an expression that will be *evaluated and returned* in case the test fails.

7. NULL checks whether its argument evaluates to NIL or not, here is a function to return human readable indications of whether a list is empty or not:

```
(defun emptyp (lst)
  (if (null lst)
    'empty
    'not-empty))
```

What happens if you try this with a non-list argument, say the number 4?

Here is another way to test the emptiness of a list; `ENDP` is similar to `NULL`, except it gives an error if you want to test something other than a list. Therefore `ENDP` is preferable to `NULL` in cases where you want the argument always to be a list; this will help you catch the presence of a non-list argument and correct the bug.

```
(defun emptyp (lst)
  (if (endp lst)
    'empty
    'not-empty))
```

Now try this one with an empty and non-empty list; why does it work?

```
(defun emptyp (lst)
  (if lst
    'not-empty
    'empty))
```

8. In `LISP`, failure means `NIL`; but success is not limited to `T`: any value that is not `NIL` is success.

9. The third argument of `if` is optional; if you do not provide it, `NIL` is returned if the test fails.

10. Let us now handle a more complex decision task. Let `SET-ADD-NUMBER` be a function that takes a number and set as arguments, and adds the number to the set. There are two things you need to be sure about: (1) that the first argument is really a number (you can test this with `NUMBERP`); (2) that the given number is not already in the set – remeber that sets do not allow repetitions of elements.[8]

    It's always good to plan things, before starting to write the code. Here is the plan:

[8] We will trust the users of this function in providing a set for the second argument; therefore no tests about that – we already have enough to worry about.

(i) if we discover that the first argument is not a number, we will return an error message – see the code below for how;

(ii) if we are given a number but the number is already in the set, we will return the set unaltered;

(iii) if everything is OK, we will cons the number to the front of the set and return this new set.

Now we can express these cases in `LISP` using `IF`. One `IF` will not be enough as we need to make a three way decision; therefore we need to *nest* an `IF` under another:

```
(defun set-add-number (n numbers)
  (if (not (numberp n))
    (error "Sorry, not a number")
    (if (member n numbers)
      numbers
      (cons n numbers))))
```

11. `COND` is helpful with decisions involving more than two cases. Its syntax may appear a little complicated in comparison to `IF`, but one gets used to it after several times. Here is the same function as above:

```
(defun set-add-number (n numbers)
  (cond ((not (numberp n)) (error "Sorry, not a number"))
        ((member n numbers) numbers)
        (t (cons n numbers))))
```

12. Everything that can be done by `COND` can also be done by `IF` and `PROGN` (which you haven't seen yet), but `COND` makes life easier for complex decisions.

13. `AND` and `OR` form sequences of expressions with special evaluation algorithms:

13.1. `AND`: Evaluate the expressions from left to right until you reach either the end or an expression that evaluates to `NIL`; return the value of the last expression.

13.2. `OR`: Evaluate the expressions from left to right until you reach either the end or an expression that evaluates to something other than `NIL`; return the value of the last expression.

```
(defun custom-if (test succ fail) ; wrong!
  (or (and test succ) fail))
```

But it is unsatisfactory in one case – can you see which? Can you write a better function which avoids this failure (Touretzky 1990)?

6 Iteration

1. Computing frequently involves repeating an action, each time with different inputs and/or conditions. There are mainly three perspectives on how to do this: (i) iterative, (ii) recursive, (iii) applicative. Which perspective is best mainly depends on the task at hand.

2. Let's take a list and print each element to the screen, no matter what.

3. But first we need to learn how to print stuff to screen; that's what we mean by "saying". Try this:

```
(print 'hello)
(print 8)
(print (* 2 4))
```

3.1. You'll see everything printed twice to screen. Why is that?

3.2. A list headed by `PRINT` is evaluated specially. The one and only argument to `PRINT` gets evaluated and is returned by `PRINT`. Besides this, `PRINT` also prints this value to the screen. Every `LISP` expression returns a value. `PRINT` not only returns a value, but also does something else – it has a **side effect**. We will talk more on this.

4. Now we are ready to write our function:

```
(defun printer (lst)
  (dolist (x lst 'done)
    (print x)))
```

5. `DOLIST` is an example of a **block structure**. Its job is to run for as many times as there are elements in `LST`. Runs are called **iterations**. In each iteration, the variable `X` holds the next element in `LST`. The symbol `'done` is what will be returned when all the elements in the list are iterated over. The rest of the `DOLIST` list, namely its `CDDR`, is a possibly empty sequence of expressions, that are evaluated in order in each iteration of the `DOLIST`.

5.1. Observe that we see each element only once on the screen, nothing is repeated. The reason is that `DOLIST` does not care about values returned by the expressions in its body; the value it returns is always the third element in its first argument (= the list where you name your iteration variable). In the body of a `DOLIST`only side-effects count, anything that doesn't have a side effect has no place in the body of a `DOLIST`.

5.2. For instance, imagine you wrote the code below to double a list, by adding each element to the end of the list in order:

```
(defun no-fun (lst)
  (dolist (x lst 'done)
    (append lst (list x))))
```

All this does is to iterate through the list; evaluate the `APPEND` form in each iteration, and finally evaluate `'DONE` and return its value, which is the symbol `DONE`. The list we sent to the function stays totally intact, nothing added, nothing removed. The reason is that `APPEND` has no side effect, it just returns a value: the newly formed list obtained by bringing together two other lists. Therefore nothing happens from the perspective of `DOLIST`, it doesn't "see" these returned values.

6. Now we go one step further and try to achieve the task of computing the length of a list. So far, what we are doing is to simply iterate and wait for something to happen in each iteration. In the present task, while iterating, we need to keep track of how many elements we have seen so far. If we can do that, we can return that number when we reach the end of the list as the computed length of the list. "Keeping track of something" means storing its value somewhere that we can look up and update the value when we need. Such places are called **variables**. What we need here is to first create a variable that has the initial value of 0; then we will **increment** the variable by one in each iteration; and then we will return the final state of the variable.

7. Creating a variable is done by a `LET` construct and changing the state of the variable is done by `SETF`. Let's go over an example, type the following to top-level:

```
(let ((x 1)) (print x) (setf x 4) (print x))
```

7.1. `LET` consists of a list of variable declarations and a number of expressions that constitutes its **body**. The list of variable declarations is a list consisting of a number of variable declarations – as many as you like – where each declaration itself is a list of two elements: a variable name and an expression whose *value* will be stored in the variable just named. In our present case, we have only one such declaration, a very simple one. After this declarations list, comes the body of `LET`. The body can have as many expressions as you like – here we have three. `LET` evaluates each expression in its body one by one, and returns the value of the expression it evaluated last. In this case, the three expressions we put in the body of `LET` all have side-effects: first 1 is printed, then `X` is made to hold 4 instead of 1; finally 4, which is the value of `X` at that point, is printed. But you see an extra 4 as well. That's the value returned by the `LET` clause itself – remember that `LET` returns the value of the last expression in its body and top-level, also called REPL, reads an expression, evaluates it and prints the value. Therefore the first 4 you see is printed by `PRINT`, the second 4 you see is printed by the top-level.

7.2. Now save the expression above to a file, say `let.lisp`, and load the file at the top-level by doing:

```
(load "let.lisp")
```

Now you do not see the second 4, but a `T` instead. Can you see why? The reason is this: the top-level prints the value of the top-most expression it evaluated. In this case the top-most expression is a `LOAD` expression – which is, by the way, just another `LISP` list, standing for a function with a side-effect; its side effect is to load the file whose name is given to it. As every other `LISP` expression, `LOAD` as well returns a value: if everything goes fine with loading the file, it returns `T`. And it is this `T` that you see instead of the second 4.

8. Now we can write our function that computes the length of a given list:

```
(defun length2 (lst)
  (let ((counter 0))
    (dolist (x lst counter)
      (setf counter (+ 1 counter)))))
```

Let us have a closer look at what's going on here:

8.1. First, you see a strange warning message when you load the program. `SBCL` warns you that you have something named `X` in your `DOLIST`, which you do not use anywhere else. This is a style warning; something that is aimed to alert the programmer that there *may* be something wrong, but doesn't have to be. In our case there is nothing wrong; we need to iterate over a list, but we are not interested in the elements; all we care to know is how many of them there are. So, ignore the warning.[9]

8.2. Second, there is no quote on `COUNTER` in the `DOLIST` variable specification, we want `DOLIST` to return the *value* of the symbol, not the symbol itself, as we did with `'DONE`.

8.3. Third, observe how we update the value of the variable `COUNTER`. `SETF` is again a special form, it does not obey the rule of evaluation, it has its own rule. It first evaluates its second argument; then stores the value obtained in its first argument; and finally *returns* the value stored. Again, like `PRINT`, it has both a side-effect (= change the value stored in a variable) and a return value (= the value itself).[10]

9. Similarly we can write a function that sums the numbers in a list:

```
(defun summer (lst)
  (let ((sum 0))
    (dolist (x lst sum)
      (setf sum (+ sum x)))))
```

10. Or, take a list of numbers and return another list that contains only the even ones in the original list:

```
(defun get-evens (lst)
  (let ((store nil))
    (dolist (k lst (reverse store))
      (if (evenp k)
        (setf store (cons k store))))))
```

We `SETF` only on the condition that the current iterated item is even; and we omit the final argument of `IF`, where `NIL` is returned when the test fails. This does no harm, because `DOLIST` does not care about return values anyway. Finally, observe that we do not return the `RESULT` itself, but its reverse as the value of `DOLIST`; otherwise we would have obtained the even numbers in the original list in the reverse order of their appearance.

11. So far we kept counters and stores that we conditionally increment or add elements along the iteration. Some problems require keeping a **flag**. A flag is a variable that holds a value that allows you to make decisions, usually boolean values like `T` and `NIL`.

11.1. Take the task of removing every odd occurrence of a symbol from a list – remove the first, third, fifth etc.

[9] There are ways to "muffle" (=make silent) such warnings; but for now let them stay, sometimes they are helpful.

[10] `SETF` is actually capable of much more, but it's more than enough for now.

```
(defun remove-odd (elm lst)
  "remove odd occurrences of elm from lst"
  (let ((remove? t)
        (store nil))
    (dolist (a lst (reverse store))
      (if (equal a elm)
        (if remove?
          (setf remove? nil)
          (progn
            (setf store (cons a store))
            (setf remove? t)))
        (setf store (cons a store))))))
```

We keep a flag called REMOVE?, which keeps track of whether we have seen an even or an odd number of ELMs so far. When REMOVE? is T it means we have seen an even number of ELMs; therefore we start with T – zero is an even number. When REMOVE? is T, it means that we need to delete the next ELM we encounter. We first check whether the current item is ELM, and if not, we simply cons the current symbol to STORE. If the current symbol is ELM, we need to decide whether to take it into STORE or not. If our flag REMOVE? is T, all we need to do is to change the status of our flag – we could also have done (setf remove? (not remove?)); not consing the current element to STORE "deletes" it. You encounter a new construct, PROGN, in the failure clause of the internal IF. When the current item is equal to ELM and REMOVE? is NIL, we need to do two things: cons the current element to STORE and switch REMOVE? to T. As you will remember, IF is a three argument function, a test, an expression to be evaluated when the test succeeds, and an expression to be evaluated when the test fails. Therefore, there is room for only one expression for the failure case. But we have two SETFs to be evaluated. To solve this problem, we group the two expressions in a PROGN block. PROGN is a list that can have as many elements you like; what it does is to evaluate these elements one by one and return the value of the last one as the value of PROGN.

11.2. Here is a slightly simplified version of the same program.As an exercise, trace the execution of the program with pen and paper on a sample input.

```
(defun remove-odd (elm lst)
  "remove odd occurrences of elm from lst"
  (let ((remove? nil)
        (store nil))
    (dolist (a lst (reverse store))
      (if (equal a elm)
        (progn
          (setf remove? (not remove?))
          (if (not remove?)
            (setf store (cons a store))))
        (setf store (cons a store))))))
```

12. In iterative programs incrementing/decrementing a counter and updating a store variable by consing an element are quite common tasks. For this reason, LISP has some shortcuts for these operations.

    (incf counter) is equivalent to (setf counter (+ counter 1)).

    You can also specify the amount of increment by a second argument to INCF.

    (incf counter 3) is equivalent to (setf counter (+ counter 3)).

    DECF does the same thing, this time by subtracting instead of adding.

    For updates by consing elements,

    (push x y) is equivalent to (setf y (cons x y)).[11]

13. Let us now write a function that checks whether a given element is a member of a given list. Here is one way to do it,

```
(defun elementp (item lst)
  (let ((answer nil))
    (dolist (k lst answer)
      (if (equal k item)
        (setf answer t)))))
```

So nice. But there is one efficiency issue. Imagine you are given a very long list and the item you are looking for is somewhere close to the front of the list. With the above strategy, you will have to wait untill DOLIST finishes iterating through the entire list in order to get the value – T or NIL – you want to know, even the value is settled long before you see the outcome. For such kind of situations, LISP has the construct RETURN. Let us first see it in action:

[11] There is a dual of PUSH as well: POP. We will use and explain it later.

```
(defun elementp (item lst)
  (dolist (i lst)
    (if (equal i item)
      (return t))))
```

First notice that the first argument of DOLIST has two elements rather than three; the third argument, which specifies the return value of DOLIST is not given, and therfore it gets its default value NIL. This means that if DOLIST can reach the end of the list, it will return NIL. The only case that this will not happen is that the equality test of the IF in the body returns T, in which case the DOLIST iteration is aborted and the value T is returned as the value of the entire DOLIST, and thereby the value of the call to ELEMENTP. This is much shorter than the previous code; and it does not have to run till the end of the list, in case it finds the item somewhere before the end.

14. Now we will see another common programming technique. But first we need a problem that requires the use of it. Our task is to decide whether a given list is a sublist of another. In the previous tasks, we were keeping counters and flags; this time we will keep a **window**. A window is a fixed size list that is updated in each iteration. For instance, assume we are asked to see whether a three element list (let's call this the search list) is a sublist of another list (let's call this the mother list). What we need for this problem is a three element window. We will start with an empty window and will update it as we iterate through the mother list. The proper way to update the window is to add elements to the back of the window, while discarding the element at the front – otherwise our window will grow beyond the size of the search list. Apart from updating our window, we will also check whether our window equals to the search list. If we get the equality, we will (RETURN T), otherwise we will go on iterating and updating our window. If no equality is catched until the end of the mother list, we will return NIL.

14.1. A critical step in our solution is to create a window of a given size. There is a very useful function MAKE-LIST, which constructs a list of a given length, initially holding NILs.[12]

Here is the code:

```
(defun sublistp (search-list mother-list)
  (let ((window (make-list (length search-list))))
    (dolist (i mother-list)
      (setf window (append (cdr window) (list i)))
      (if (equal window search-list)
        (return t)))))
```

15. Another builtin iterative construct is DOTIMES. It's similar to DOLIST; instead of iterating a list, the iterative variable iterates over numbers from 0 up to 1 minus the given iteration parameter – where there was a list in DOLIST, there is a number.

15.1. Here is a function definition that takes its first argument to the power of its second argument.[13]

```
(defun exponent (base power)
  (let ((result 1))
    (dotimes (x power result)
      (setf result (* base result)))))
```

15.2. Here is another example with DOTIMES that computes the factorial of a given number. Factorial of an integer n is the number $n \times (n-1) \times (n-2) \times \ldots \times 1$, the factorial of 0 is 1 by definition.

```
(defun factorial (n)
  (let ((result 1))
    (dotimes (i n result)
      (setf result (* result (+ i 1))))))
```

In using DOTIMES, you need to bear in mind that the iteration starts with 0 and ends with one minus the given iteration limit.

16. **Iterating on more than one list in parallel.** One drawback of DOLIST and DOTIMES is that they cannot handle more than one list in parallel.

16.1. Assume you have two lists: one with student names and the other with grades, where lists are so nicely ordered that names and grades match position-wise. You want to merge these lists into a single list, where each element is itself a list of two elements, the first is a name and the second is the corresponding grade. To accomplish this,

[12] Our program will assume that NIL will not be the element of any search list. There is a way to avoid this shortcoming: namely telling MAKE-LIST to fill the list it constructs with symbols that are different from anything that can be in the input lists. We will see how to do this below.

[13] x to the power y, designated as x^y, is the number obtained by multiplying x by itself for y times.

we will need a way to refer to the elements of a list by their positions. The builtin for this is NTH. It takes a number argument, which is the position, and a list. You should always keep in mind that positions start with 0 in LISP: the first element sits at position 0, the second at 1, and so on.

16.2. Here is the function that solves our name-grade pairing problem (and many similar others):

```
(defun pairlists (lst1 lst2)
  (let ((store nil))
    (dotimes (i (length lst1)  store)
      (push
        (list (nth i lst1) (nth i lst2))
        store))))
```

Notice how we broke down PUSH to increase readiblity of the code. A sample interaction of the program would look like:

```
* (pairlists '(john mary ted) '(82 74 42))

((TED 42) (MARY 74) (JOHN 82))
```

16.3. There is one thing we need to be careful about while pairing lists with this method; namely we need to decide what happens when the lists to be paired are not of equal length. One possible action to take is to see which list is shorter and use the length of that list with DOTIMES. In this strategy, the excess elements in the longer list will get discarded. Let us do the job of deciding on the smaller length with a separate function. This will make our program easier to understand and test. Also let us write our shorter-length-detector function in a general way, such that it will find the minimum length not only of a pair of lists but any number of lists we provide to it. Here is a way to do it:

```
(defun min-length (list-of-lists)
  (let ((minlen (length (car list-of-lists))))
    (dolist (x (cdr list-of-lists) minlen)
      (let ((len (length x)))
        (if (< len minlen)
          (setf minlen len))))))
```

16.4. Now we can update our list pairing function as:

```
(defun pairlists2 (lst1 lst2)
  (let ((store nil))
    (dotimes (i (min-length (list lst1 lst2)) store)
      (push
        (list (nth i lst1) (nth i lst2))
        store))))
```

17. **Changing the way iteration proceeds.** Another drawback of DOLIST and DOTIMES is that how they traverse a list or increment a number is fixed, namely one-by-one from left to right and increasing. And this drawback is not as easy to remedy as the other drawback. We will see how to do it later. But if you can't wait that long, go to Graham (1996) and study how to use DO.

7 Recursion

1. Let us start with the factorial function:

$$F(n) = \begin{cases} n = 0, & 1 \\ n \in \mathbb{Z}^+, & n \times F(n-1) \\ \text{otherwise}, & \text{undefined} \end{cases} \quad (1)$$

2. This definition can be directly turned into an algorithm:

```
function FACTORIAL(n)
    if n = 0 then
        return 1
    else
        return n* FACTORIAL(n − 1)
```

3. Here is the factorial function in LISP:

```
(defun factorial (n)
  (if (zerop n)
    1
    (* n (factorial (- n 1)))))
```

4. Here is a "safer" version:

```
(defun factorial (n)
  (cond ((or (not (integerp n)) (< n 0)) (error "Factorial undefined!"))
        ((zerop n) 1)
        (t (* n (factorial (- n 1))))))
```

5. A custom LENGTH

```
(defun c-length (lst)
  (if (endp lst) 0 (+ 1 (c-length (cdr lst))))
  )
```

6. A custom MEMBER:

```
(defun c-member (item lst)
  (cond ((endp lst) nil)
        ((equal item (car lst)) lst)
        (t (c-member item (cdr lst)))))
```

7. A power function:[14]

```
(defun power (x y)
  (cond ((zerop y) 1)
        ((= 1 y) x)
        (t (* x (power x (- y 1))))))
```

8. Range function, "unsafe" and "safe" versions:

```
(defun range (n)
  (if (zerop n)
    nil
    (append (range (- n 1)) (list (- n 1)))))

(defun s-range (n)
  (cond ((or (not (integerp n)) (< n 0)) (error "Range undefined!"))
        ((zerop n) nil)
        (t (append (range (- n 1)) (list (- n 1))))))
```

7.1 Recursion with accumulators

1. The examples we saw so far were simple recursions. A slightly more complicated recursive strategy for solving problems is to keep an accumulator (or store) that gets build as we "recurse" down (or up) the problem.

1.1. Writing a recursive length function was straightforward, a more complicated one would keep track of the count of members along recursion and return it when the recursion hits bottom. Let us see a version where the counter is expected to be provided by the user, with the default value of 0:

[14] The corresponding built-in is EXPT; another built-in EXP is a one-place function that raises Euler's constant e to the power of the given argument.

```lisp
(defun c-length (lst counter)
  (if (endp lst)
    counter
    (c-length (cdr lst) (+ counter 1))))
```

1.2. However, it would be more appropriate to have a function that expects only the list, where the counter is kept behind the scenes. One way to do it is to write a second function which is not called by the user, but invoked only by the function that the user interacts with:

```lisp
(defun count-length-user (lst)
  (count-length lst 0))
```

1.3. There is still a better way. `LISP` allows us to define optional parameters to functions, which are not necessary to be provided in function calls. In such cases the parameter gets a default value, if provided, and `NIL` otherwise. Here is how to write a length function with an optional counter parameter.

```lisp
(defun c-length (lst &optional (counter 0))
  (if (endp lst)
    counter
    (c-length (cdr lst) (+ counter 1))))
```

2. In the case of length, writing an alternative with a counter was quite unnecessary. Here is a case where an accumulator is desirable for efficiency concerns. We wrote above a function `RANGE` that produces a list starting from 0 up to one less than the given argument. This function is fine for small numbers of input. It is an extremely "low-performance" function, however. Try it with big numbers, say `(range (factorial 8))` the factorial of 8 is 40320, not a very big number, anyway; try it with `(factorial 9)`, most probably you will not be able to get a result before you run out of resources.

3. The reason `RANGE` performs so poorly is that it uses `APPEND`. Whenever you want `LISP` to append two lists it creates a copy of the first and makes its last cdr point to the cons cell of the second list rather than to `NIL`. For this reason `APPEND` should be avoided if you expect your programs to append long lists and for many times. But what could be the alternative? Here is another range function, `F-RANGE` ('f' for 'fast'), that makes use of a store:

```lisp
(defun f-range (n &optional (store nil))
    (if (zerop n)
      store
      (f-range (- n 1) (cons (- n 1) store))))
```

4. We will now see how dramatic is the improvement; try `(f-range (factorial 9))`, you should be able to get a long list of numbers. In order not to wait for the numbers to get printed on the screen, do this:

```lisp
(and (f-range (factorial 9)) t)
```

it should be quite fast to see the `T` on the screen; meaning the range is computed successfully. The function breaks down in factorial 12,[15] hitting the memory limit – no more room for a longer list.[16]

5. `SBCL` allows you to inspect the time and space resources used by your programs by the built-in `TIME`. To see the difference between two versions of the range function run these:

```lisp
(time (and (range (factorial 8)) t))
(time (and (f-range (factorial 8)) t))
```

6. Let us do more recursion exercises; first a custom reverse function operating over lists:

```lisp
(defun c-reverse (lst)
  (if (endp lst)
    nil
    (append (c-reverse (cdr lst)) (list (car lst)))))
```

7. Now, let us write the same function – same in the sense of mapping same inputs to same outputs – with an accumulator; and compare their performances.

[15] The actual limit is around 50,100,000, which is larger than factorial 11 = 39,916,800. You can gain around a 4 fold further improvement by invoking `SBCL` with the `--dynamic-space-size 2048` option.

[16] An iterative version would perform identically, therefore the limit is not related to recursion here; but it was, for the former `RANGE`.

```
(defun f-reverse (lst &optional (store nil))
  (if (endp lst)
    store
    (f-reverse (cdr lst) (cons (car lst) store))))
```

8 Symbols

1. Symbols are more complicated than they first appear:

Name
Value Binding
Function Binding
Package Binding

2. To set the Value Binding for a symbol, say `K`:

```
(setf k (random 10))
```

3. Note that `SETF` also does not obey the rule of evaluation.

4. `SETF` establishes the value binding of a symbol.

5. `DEFUN` establishes the function binding of a symbol.

6. A symbol gets evaluated to its value binding, unless it appears as the first element of an unquoted list, in which case it gets evaluated to its function-binding.

7. If you want to access the function binding of a symbol elsewhere, you need to prefix the symbol with `#'`. More on these rules below.

8. Second argument of `SETF` can make use of the first:

```
(defvar k)
(setf k 8)
(setf k (+ k 3))
```

9. At first glance `SETF` might seem to be a simple function which takes a symbol and a value and associates the value-binding of the symbol with the value. Actually it is more clever than that. Observe the following interaction with `SBCL`:

```
* (defvar k)

K
* (setf k '(A B C))

(A B C)
* (setf (car k) 'Z)

Z
* k

(Z B C)
*
```

10. `SETF` takes two *expressions* and make sure that they evaluate to the same thing.

11. Also notice that `SETF` itself has a return value – can you see what?

12. Therefore, `SETF` does two things at the same time: manipulates some bindings in the memory and returns a value. The first is called a “side effect” of `SETF` – a little bit confusing given the daily usage of the expression.

13. Knowing the side effects and return values of constructs is crucial.

13.1. Assume you want to write a function that takes a number and a list, and replaces the first element of the list with the number, in case the number is odd. Here is one way to do it:

```
(defun replace-if-odd (a-number a-list)
  (if (oddp a-number)
    (setf (car a-list) a-number)))
```

13.2. Now also assume that you have some reason to check the length of the list after the replacement. You can check the length of a list by the built-in `LENGTH`. You do this:

```
(length (replace-if-odd 3 '(1 2 3)))
```

and you get an error – can you see why?

13.3. Here is a way to make our function return the modified list, so that we can immediately check its length:

```
(defun replace-if-odd (a-number a-list)
  (if (oddp a-number)
    (cons a-number (cdr a-list))
    a-list))
```

13.4. This time, however, we lost the modified list; we will see a way to both manipulate the list (in-place) and return a value.

9 Delayed function calls

1. Assume you have a list of integers that you would want to turn into a list of their, say, factorials in a single stroke. And further assume that you would like to have a more general tool, which does the same trick not only with factorial but any unary function you provide to it, e.g. cube, square root, etc. What you need is a function that takes a function and a list as arguments, apply the function one by one to the elements of the list while storing the results in another list. Let us call this function `MAPP` – note the double 'P' not to clash with the built-in function `MAP`.

1.1. This is a task that would be straightforwardly implemented with recursion. Assume `MAPP` is given a function `f` and some list. If the list is empty, than its `MAPP` should be empty; if the list is non-empty the `MAPP` of it is simply the value obtained by applying `f` to the `CAR` of the list consed with the value obtained by calling `MAPP` with `f` and the rest of the list.

1.2. Here is a definition that attempts to achieve this:

```
(defun mapp (func lst) ;; WRONG!
  (if (endp lst)
    nil
    (cons (func (car lst)) (mapp func (cdr lst)))))
```

1.3. The problem with the above definition is that it expects `LISP` to bind the function provided by the parameter `func` to the `func` that occurs in the function definition. `LISP` is designed *not* to do this;[17] when you call this function, say with `(mapp factorial '(1 2 3 4))`, or `(mapp 'factorial '(1 2 3 4))`, the `func` in the definition will not get replaced by the parameter you provided for the argument named `func`. Such replacements are done only with non-initial elements of lists – even not for all such elements, see 1.7. below.

[17] A dialect of `LISP`, called Scheme, does this.

1.4. What we want is achievable with the functions `FUNCALL` or `APPLY`.

1.5. `FUNCALL` wants its first argument to be something that would *evaluate* to:

i. a symbol with a function binding; or

ii. a function.

1.6. The rest of its arguments are treated as arguments of the function provided via the first argument; the function is applied to its arguments and the value is returned.

1.6.1. Assuming the definition of `FACTORIAL` is loaded,

```
(funcall factorial 8)
```

would lead to an error. The reason is that before getting fed into `FUNCALL` the argument `FACTORIAL` gets evaluated. Remember the rule of evaluation, which says that if a symbol is encountered at a non-initial position in a list, it gets evaluated to its value-binding. In this case `LISP` cannot find anything in the value-binding. The above specification of `FUNCALL` says that the first argument should be something that would *evaluate* to a symbol. Therefore the correct form is:

```
(funcall 'factorial 8)
```

This way `FUNCALL` gets a symbol – that's what `'FACTORIAL` gets evaluated to – with a function-binding. This function is retrieved and applied to the remaining arguments – we have only one in the present case.

1.6.2. You can use the built-in functions as follows:

```
(funcall '+ 8 7 29)
(funcall 'member 'a '(z c a t))
```

1.6.3. The specification of `FUNCALL` in 1.5. has a second clause, which says that one can also provide an argument that evaluates to a function. Given a symbol, you can access its function-binding in two ways:

```
(function factorial)
(symbol-function 'factorial)
```

note the quote in the second form. The first form is the frequent one and like the `QUOTE` function, it has an abbreviated form:

```
#'factorial
```

Therefore you can directly send a function as an argument to FUNCALL as,

```
* (funcall #'+ 8 7 29)
* (funcall #'factorial 9)
```

1.7. Now you might think that with the below code we could get what we want, for instance with `(mapp factorial '(1 2 3 4))` to get `(1 2 6 24)`.

```
(defun mapp (func lst) ; STILL WRONG!
  (if (endp lst)
    nil
    (cons (funcall (function func) (car lst)) (mapp func (cdr lst)))))
```

but again FUNCTION – QUOTE is no different – prevents the argument FUNC from getting bound to the parameter provided to the MAPP.
The correct code is:

```
(defun mapp (func lst)
  (if (endp lst)
    nil
    (cons (funcall func (car lst)) (mapp func (cdr lst)))))
```

and you can call the function in two ways, both are fine:

```
(mapp 'factorial '(1 2 3 4))
(mapp #'factorial '(1 2 3 4))
```

10 Applicative programming

1. The list is the basic data structure in LISP and in many other programming languages. Once you have your data in the form of a list, then you can do various transformations and/or checks on your data.

2. **Global variables.** In most applications, data is read from a file, entered by the user, or provided by a stream over a network. We will come to these topics, but for now, we will assume that data is stored in a **global** variable in our program. One way to declare and assign a global variable is DEFPARAMETER. For instance, let us assume we have a set of grades:

```
(defparameter *grades*
    '(86 98 79 45 0 75 96 83 91 90 0 70 85 82 91 47 0 70))
```

2.1. What makes a global variable global is that you can access the variable from within any function definition you have in the same file.

2.2. The asterisk characters * around the word "grades" have no special meaning for LISP. It is a convention among LISP programmers to name global variables as such; thereby you can recognize that a variable is global only by looking at its name.

3. Having all the grades stored in a list, let us write a very simple function that would compute their sum. What about this one?

```
(defun total (lst) ; WRONG
  (+ lst))
```

4. Evaluating `(total *grades*)` would give an error. The reason this is unsuccessful is that the + operator works on numbers, but we provide a list as an argument. Using FUNCALL would not help as well. The construct to use in such situations is APPLY. It is like FUNCALL with the only difference that the arguments are provided in a list.

```
(defun total (lst)
  (apply #'+ lst))
```

5. Now, evaluating `(total *grades*)` should give you the number 1188.

6. Sometimes, one needs to transform a list into another list with an equal length and with a specific correspondence between the elements of the two lists. This is a mapping. LISP provides the built-in MAPCAR for this purpose. For instance assume you would – for some strange reason – take the square root of the grades, and collect them in a list. LISP has the built-in SQRT that can be applied to numbers. To apply this wholesale on a list, say `*grades*`, just do:

```
(mapcar #'sqrt *grades*)
```

7. There is no limit to the complexity of the functions you can use with MAPCAR. Let us first define a function that gives the letter grade corresponding to the numerical value. You can easily do this by COND. Give it the name LETTER. Doing `(mapcar #'letter *grades*)` should give you:

```
(BA AA CB FF FF CB AA BB AA AA FF CC BA BB AA FF FF CC)
```

8. Or, you can define a function APPEND-LETTER, which pairs each grade with the letter grade, returning everything in a list. What you get in the end should look like:

```
((86 BA) (98 AA) (79 CB) (45 FF) (0 FF) (75 CB)
 (96 AA) (83 BB) (91 AA) (90 AA) (0 FF) (70 CC)
 (85 BA) (82 BB) (91 AA) (47 FF) (0 FF) (70 CC))
```

9. Remember that we had to use APPLY to get the sum of a list of numbers. Another way to do this is to use REDUCE. This is somewhat similar to MAPCAR. It takes a function with two arguments and a list; then it reduces the entire list to a single value, by applying the function first to the first two elements, then to the result of this application and the third element, then the result of this application and the fourth element, and so on until it reaches the end of the list. For instance, the following function calls will give you the sum and the mean of the grades, respectively.

```
(reduce #'+ *grades*)
(/ (reduce #'+ *grades*) (length *grades*))
```

10. Usually we would not want to include the 0 grades in computing the mean, especially if we know that these are coming from people who did not participate in the exam or the course. A useful pair of list filtering functions are REMOVE-IF and REMOVE-IF-NOT. To filter out zero grades, just do:

```
(remove-if #'zerop *grades*)
```

11. If we want to have a function that computes the mean of the grades, with first filtering the zero values, we might have the following:

```
(defun class-mean (grades-list)
  (float (/
          (reduce #'+ (remove-if #'zerop grades-list))
          (length (remove-if #'zerop grades-list)))))
```

12. The code is inefficient as we compute the very same thing twice. We can improve this with LET:

```
(defun class-mean (grades-list)
  (let ((real-grades (remove-if #'zerop grades-list)))
    (float (/ (reduce #'+ real-grades) (length real-grades)))))
```

13. Example: A Collatz sequence is obtained by starting with an integer n; if n is odd, add $3n+1$ to the sequence, if n is even add $n/2$ to the sequence. If you obtain 1 stop, otherwise go on as before with the lastly added integer. The Collatz Conjecture states that no matter which integer you start the sequence, you are guaranteed to reach 1 and stop after a finite number of iterations. Let us write a function that computes the Collatz sequence for a given integer.

```
(defun collatz-generate (n)
  (if (= n 1)
    '(1)
    (cons n (collatz-generate (if (evenp n)
                                  (/ n 2)
                                  (+ (* n 3) 1))))))
```

14. Let us define Collatz (sequence) length of an integer to be the number of steps needed to reach 1 from that integer. This is one less than the length of Collatz sequence. So,

```
(defun collatz-length (n)
  (- (length (collatz-generate n)) 1))
```

15. Knowing how numbers are correlated with their Collatz length would be interesting; does the length grow as the number grows, for instance? Or is there a fluctuating pattern? To do this let us first find a way of generating a sequence of integers within a given range; after this it would be easy to MAPCAR it to what we want to investigate. We already wrote a function for generating ranges; this time we will write a more sophisticated one. We will use keyword arguments in doing that. First the code, then we will look at it in detail.

```
(defun ranger (&key (start 0) (end 9) (step 1) (acc nil))
  (cond ((>= start end) (cons start acc))
        (t (ranger :acc (cons end acc)
                   :start start
                   :end (- end step)
                   :step step))))
```

this range function generates a range including its start and end points. Keyword arguments allow you to refer to the parameters by names – don't forget the ':' before the keywords – so that you do not have to remember the order of the parameters, as you should for optional parameters.[18]

16. Now we have enough machinery to list the Collatz lengths of the first one million integers:

```
(mapcar #'collatz-length (ranger :start 1 :end 1000000))
```

17. Check the maximum Collatz length in this range:

```
(apply #'max
    (mapcar #'collatz-length (ranger :start 1 :end 1000000)))
```

18. `MAX` breaks down for 1,000,000 range – it should respond 350 for 100,000. We define our own maximum function, which operates on lists:

```
(defun maxx (lst &optional (max nil))
  (cond ((endp lst) max)
        ((null max) (maxx (cdr lst) (car lst)))
        (t (maxx (cdr lst) (if (> (car lst) max)
                                (car lst)
                                max)))))
```

19. Now we can check the maximum Collatz length in a range of one million:

```
(maxx (mapcar #'collatz-length (ranger :start 1 :end 1000000)))
```

20. We can observe how the maximum Collatz length changes with the size of the range. Here is how to do it with `LAMBDA`:

```
(mapcar
    #'(lambda (range)
        (maxx (mapcar #'collatz-length
            (ranger :start 1 :end range))))
    '(10 100 1000 10000 100000 1000000))
```

giving `(19 118 178 261 350 524)`.

21. It may be hard to count 0's in specifying various sizes of ranges – it is easy to err. Let us handle that task with mapcar as well.

```
(mapcar #'(lambda (x) (expt 10 x)) '(1 2 3 4 5 6))
```

Now we can have

```
(mapcar #'(lambda (range)
            (maxx (mapcar #'collatz-length
                          (ranger :start 1 :end range))))
        (mapcar #'(lambda (x)
                    (expt 10 x)) '(1 2 3 4 5 6)))
```

22. We have gained some knowledge on Collatz sequences, but we still do not know about individual numbers. We can pair numbers with their Collatz lengths:

```
(mapcar #'(lambda (x) (list x (collatz-length x)))
    (ranger :start 1 :end 1000000))
```

23. Let us find the number with the maximum Collatz length. `MAXX` on its own is not helpful here; it justs finds the maximum number in a list of numbers. What we need is a way to find the pair(s) with the maximum second element. One way is to write a function similar to `MAXX`. Let us do this. But let us do this in a way that will have a general use. We will parametrize `MAXX` in such a way that, the caller/user of the function will decide where `MAXX` should look for items to compare. In the above version it looks at top most elements, by checking the `CAR` of the list in every recursion. Here is our new, more "customizable" `MAXX`.

[18]The function has a flaw; it may not work as expected for some ranges with step different than 1, can you see why? Any ideas to fix it?

```
(defun maxx (lst &key
                 (max nil)
                 (hook #'(lambda (x) x)))
  (cond ((endp lst) max)
        ((null max) (maxx (cdr lst) :max (car lst) :hook hook))
        (t (maxx
             (cdr lst)
             :max (if (> (funcall hook (car lst))
                         (funcall hook max))
                      (car lst)
                      max)
             :hook hook))))
```

24. With our new MAXX in our hands we can now find the number with the largest Collatz length in a given range, say 1000.

```
(maxx (mapcar #'(lambda (x)
                  (list x (collatz-length x)))
              (ranger :start 1 :end 1000)) :hook #'cadr)
```

25. Finally, we would like to know the numbers in a given range that have the same Collatz length. The solution is left as an exercise.

11 Association lists

1. Table lookup, which consists in associating a "key" with a "value", is a basic component of programming. The most straightforward way of this in LISP is to store the key-value pairs in a list. For instance you can keep the grade thresholds for letter grades in such form:

```
(defparameter *letter-table*
    '((AA 90)
      (BA 85) (BB 80)
      (CB 75) (CC 70)
      (DC 65) (DD 60)
      (FD 55) (FF 1)
      (NA 0)))
```

2. If you keep your data in such form, then you can search your data on the basis of keys. One way is to use the following LAMBDA function, which, when applied to a letter grade, gives the threshold.

```
(lambda (y) (find-if #'(lambda (x) (equal (car x) y))
    *letter-table*))
```

3. LISP has a build-in function ASSOC for such tasks. In its simplest usage ASSOC takes an object and a list of conses and returns the first cons in the list with the object as its car, if such a cons exists. For instance,

```
(assoc 'CC *letter-table*)
```

would give you (CC 70).

4. Any list of conses is called an **association list** and ASSOC will work for such a list.

5. What happens when you want to search by the second component of the pairs rather than the first. Let's say you want to see whether 70 is a threshold for any letter. One obvious way is to do

```
(assoc 70 (mapcar #'reverse *letter-table*))
```

But always remember that REVERSE is costly and, therefore, needs to be avoided if possible.

6. LISP provides RASSOC for searching by the cdr of each cons in the association list. Here, each cdr is the list of the threshold. Therefore the correct way to search for 70 is:

```
(rassoc '(70) *letter-table* :test #'equal)
```

Here we tell LISP to look at the cdr's in our association list and using EQUAL as the way of testing for matches.

7. In working with association lists, in order not to deal with the extra complication resulting from cdrs being lists themselves, one usually uses dotted pairs, instead of lists. A dotted pair is a cons where the cdr is not a list but an element. For instance (cons 'aa 90) will give you (AA . 90) rather than (AA 90), where the former pairs two symbols and the latter pairs a symbol and a list.

8. Let's keep our letter grade threshold information as a list of dotted pairs instead of lists, we change the name as well:

```
(defparameter *letters*
    '((AA . 90)
      (BA . 85) (BB . 80)
      (CB . 75) (CC . 70)
      (DC . 65) (DD . 60)
      (FD . 55) (FF . 1)
      (NA . 0)))
```

Now we can directly do

```
(rassoc 70 *letters*)
```

and get what we want.

9. How can we use `*LETTERS*` to find the letter grade of a given numerical grade? Obviously, doing `(rassoc 83 *letters*)` would give `NIL`. We need to specify a test criterion different from equality. For the present case the right criteria is the "greater than or equal to" relation. If we call `RASSOC` with this relation as the test parameter, we will find the cons with the correct letter. As this will be a cons, we will need to take out the letter by a car in the end. Here is the `LAMBDA` function that gives the letter grade for a given numerical grade. Test it and completely understand how it works. One critical thing is that `ASSOC` and `RASSOC` return the *first* match they find. We made use of this property in computing the correct letter grade.

```
(lambda (x) (car (rassoc x *letters* :test #'>=)))
```

12 User interaction

1. Here is a basic user interface:

```
(defun basic-ui ()
  (format t "Please enter a number:~%")
  (let ((input (read)))
    (if (equal 'q input)
      (format t "OK, bye!")
      (progn
        (format t "What do you mean by ~A?~%" input)
        (basic-ui)))))
```

2. Two things are new here: `FORMAT` and `PROGN`.

3. Think of `PROGN` exactly as the cdr of a `COND` clause whose car is `T`. All the elements in a `PROGN` list are evaluated one-by-one, and the final value is returned as the value of `PROGN`.

4. `FORMAT` is much more complicated.

4.1. The second element, `T`, specifies the stream where the output will be sent. `T` means standard output, the screen by default.

4.2. The third element is a string – enclosed by double quotes – that will be the output.

4.3. The string element has **format directives**, character sequences starting with `~`. The two we will use here are `~A` and `~%`. The first is a place holder that will be replaced by the value obtained by evaluating the expression following the string element. You can have as many such directives as you like, as long as you have an equal number of expressions following the string element. The other directive, `~%` stands for a newline.

12.1 Reading spreadsheets

1. First you need to save the spreadsheet in **tab-separated format**. This is usually done by making `Save as` in your office program and selecting "Comma separated values (.csv)" option, and specifying `Tab` as the delimiter and double quotes as text quoting method, making sure that cells are quoted – this appears as a checkbox in LibreOffice.

2. After that, you will create a pathname and send it to the following function:

```
(defun file-to-lists (path)
  (labels ((text-to-list (text)
                         (read-from-string
                           (concatenate 'string "(" text ")")))
           (read-lines (str &optional acc)
                       (let ((line (read-line str nil 'eof)))
                         (if (equal line 'eof)
                           acc
                           (read-lines
                             str
                             (append
                               acc
                               (list (text-to-list line))))))))
    (with-open-file (input-stream
                     (make-pathname :name path)
                     :direction :input)
      (read-lines input-stream))))
```

References

Churchland, P. S. and Sejnowski, T. J. (1992). *The Computational Brain*. MIT Press.

Crane, T. (2003). *The Mechanical Mind*. Routledge, New York.

Pohl, I. and Shaw, A. (1981). *The Nature of Computation: An Introduction to Computer Science*. Computer Science Press, Rockville, Maryland.

Pylyshyn, Z. (1984). *Computation and Cognition: Toward a foundation for Cognitive Science*. MIT Press, Cambridge, MA.

Tanenbaum, A. S. (1999). *Structured Computer Organization*. Prentice Hall, NJ, 4th edition.

Touretzky, D. S. (1990). *COMMON LISP: A Gentle Introduction to Functional Programming*. Benjamin/Cummings Publisihing Co., CA.