Introduction

A common task in remote sensing is estimating velocity of a given object being tracked. The initial Euclidean coordinate system is geodetics (ϕ, λ, ρ). However, estimating velocity in the coordinate system can be rather cumbersome. An easier way of estimate velocity is to convert geodetic to geocentric coordinates, which is (x, y, z). In this excerpt we will outline one possible ways of estimating velocity of a given object being tracked.

Euclidean Coordinates Transformation

Convert (ϕ, λ, ρ) to (x, y, z) can be rather trivial. The restrictions on (ϕ, λ, ρ) is as follow

$$\phi \in [\frac{-\pi}{2}, \frac{\pi}{2}]$$

$$\lambda \in [-\pi, \pi]$$

$$\rho > 0$$

Converting the geodetics coordinates to Earth Center Earth Fixed is shown below

$$x = \rho \cos\phi \cos\lambda$$

$$y = \rho \cos\phi \sin\lambda$$

$$x = (\frac{b^2}{a^2} N + h) \sin\phi$$

for $\rho = N + h$, h is altitude, and N radius of curvature (meters). See 'Datum Transformations of GPS Positions' to compute N.

Estimating Velocity

Calculating a derivative at a given point x_i is defined as

$$\frac{df(x_i)}{dx_i} = \lim_{h \to 0} \frac{f(x_i + h) - f(x_i)}{h}$$

However, since this is a continues process we can estimate the derivative as follow

$$\frac{df(x_i)}{dx_i} \approx \frac{f(x_i \pm h) - f(x_i)}{h}$$

given some $h > 0$.

Estimating velocity at given points is trivial based of the about function but what about estimating the velocity function $f(x)$? Let us assume some set of points are given, that is $\{x_i, f(x_i)\}_{i=0}^{n}$ for a fixed n. Defining the Lagrange basis $l_i(x)$ as

$$m_i(x) = \prod_{j=0, j \neq i}^{n} (x - x_j)$$

$$l_i(x) = \frac{m_i(x)}{m_i(x_i)} = \begin{cases} 1 & x = x_i \\ 0 & x = x_j \end{cases}$$

. Moreover, interpolating $f(x_i)$ with $p_n(x)$ defined as

$$p_n(x) = \sum_{i=0}^{n} f(x_i) l_i(x)$$

we have

$$p_n(x_i) = f(x_i), 0 \leq i \leq n$$

. However, this is a global polynomial which has convergence issues (Runge's phenomenon). If we restrict the domain of interpolation to sub-intervals, i.e., $\Omega_i = (x_i, x_{i+1}]$ then

$$p_{2,\Omega_i}(x) = \frac{(x - x_{i+1})}{(x_i - x_{i+1})} f(x_i) + \frac{(x - x_i)}{(x_{i+1} - x_i)} f(x_{i+1})$$

Restricting our polynomial to Ω_i allows for convergence for $\Omega = \bigcup_{i \in I} \Omega_i$. This is known as piecewise linear polynomial interpolation. Therefore, if we wish to estimate the velocity given $\{(x_i, f(x_i)\}_{i=0}^{n}$

1. Estimate the velocities at the nodal points.
2. Generate the piecewise polynomial to interpolate $\{(x_i, f'(x_i))\}_{i=0}^{n}$
3. Compute $v(x)$ for some $x \in \Omega$ where $v(x)$ is the estimated velocity.

Experimental Design and Results

Let consider testing our algorithms and code with the following data provided in the SciTec code problem data csv file. The steps below shows how the plots and numeric table is generated.

1. $(\phi(t), \lambda(t), \rho(t)) \rightarrow \vec{x}(t) = [x(t), y(t), z(t)]^T$.
2. Estimate $\vec{v}(t_i)$ for $\{(t_i, \vec{x}_i)\}_{i=0}^{n}$.
3. Compute $\|\vec{x_i}\|_{l2}$ and $\|\vec{v_i}\|_{l2}$.

Note: Only a small sample size of the data was used to generate fig 1, 2, 3, and the table. This allows for easier visuals and reading.

*Fig two and three legend:

- Red dots is ecef position data computed.
- Black arrows is ecef estimated velocity.
- Orange dots is interpolated ecef position at requested time.

- Orange arrows is interpolated ecef velocity at requested time.

t_i	$\|\vec{x}_i\|_{l2}$	$\|\vec{v}_i\|_{l2}$
$1.53233285904 \cdot 10^9$	$6.370509371917 \cdot 10^6$	0.00000000000000
$1.53233295904 \cdot 10^9$	$6.509412577562 \cdot 10^6$	3,404.42632700000000
$1.53233305904 \cdot 10^9$	$6.953293623168 \cdot 10^6$	6,151.80005100000000
$1.53233315904 \cdot 10^9$	$7.435744049139 \cdot 10^6$	5,510.69957300000000
$1.53233325904 \cdot 10^9$	$7.856655834451 \cdot 10^6$	4,959.87600900000000
$1.53233335904 \cdot 10^9$	$8.22196706618 \cdot 10^6$	4,480.74588300000000
$1.53233345904 \cdot 10^9$	$8.536162862272 \cdot 10^6$	4,061.86474300000000
$1.53233355904 \cdot 10^9$	$8.80268133549 \cdot 10^6$	3,696.63169800000000
$1.53233365904 \cdot 10^9$	$9.024174621481 \cdot 10^6$	3,382.04836400000000
$1.53233375904 \cdot 10^9$	$9.202682336704 \cdot 10^6$	3,117.97667900000000
$1.53233385904 \cdot 10^9$	$9.339749306474 \cdot 10^6$	2,906.54141100000000
$1.53233395904 \cdot 10^9$	$9.43650605386 \cdot 10^6$	2,751.39558800000000
$1.532334 \cdot 10^9$	$9.464681650431 \cdot 10^6$	2,705.06513000000000
$1.53233405404 \cdot 10^9$	$9.491793201845 \cdot 10^6$	2,659.90535700000000
$1.53233415404 \cdot 10^9$	$9.51186492852 \cdot 10^6$	2,625.60206000000000
$1.53233425404 \cdot 10^9$	$9.492982244419 \cdot 10^6$	2,656.14871800000000
$1.53233435404 \cdot 10^9$	$9.435000116386 \cdot 10^6$	2,750.30737400000000
$1.53233445404 \cdot 10^9$	$9.337467557392 \cdot 10^6$	2,904.77492200000000
$1.53233455404 \cdot 10^9$	$9.199608007912 \cdot 10^6$	3,115.38562000000000
$1.53233465404 \cdot 10^9$	$9.020284173395 \cdot 10^6$	3,378.44490400000000
$1.53233475404 \cdot 10^9$	$8.797943317803 \cdot 10^6$	3,691.78493100000000
$1.53233485404 \cdot 10^9$	$8.530536248667 \cdot 10^6$	4,055.49251500000000
$1.53233495404 \cdot 10^9$	$8.215398826047 \cdot 10^6$	4,472.49740300000000
$1.53233505404 \cdot 10^9$	$7.849077343675 \cdot 10^6$	4,949.30029400000000
$1.53233515404 \cdot 10^9$	$7.427065646905 \cdot 10^6$	5,497.18938100000000
$1.53233525404 \cdot 10^9$	$6.943396046492 \cdot 10^6$	6,134.48889700000000
$1.532335268 \cdot 10^9$	$6.870527178763 \cdot 10^6$	6,232.07989400000000
$1.53233534904 \cdot 10^9$	$6.419464686127 \cdot 10^6$	6,848.37799600000000

Table 1

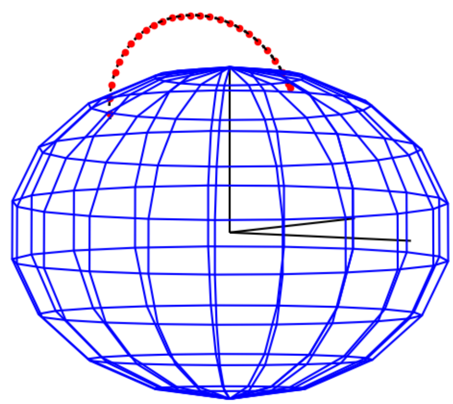

Figure 1: 3D Data Plot

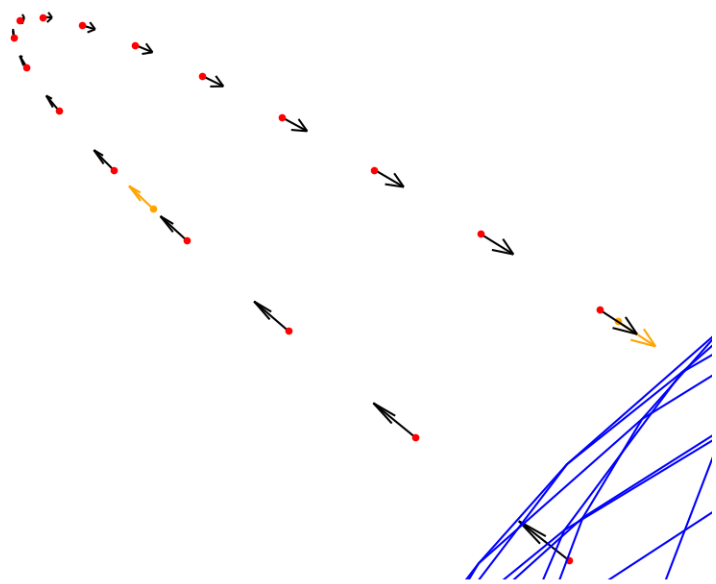

Figure 2: 3D Data Plot

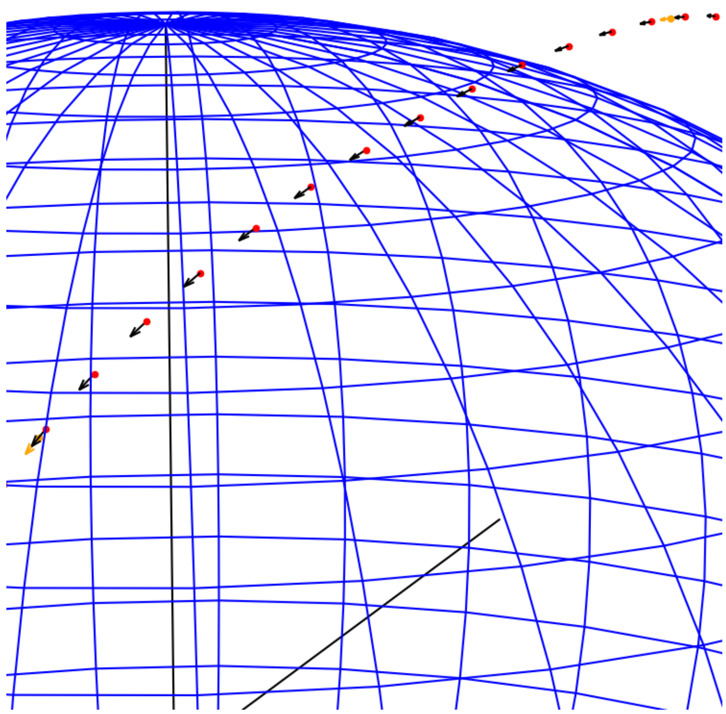

Figure 3: 3D Data Plot

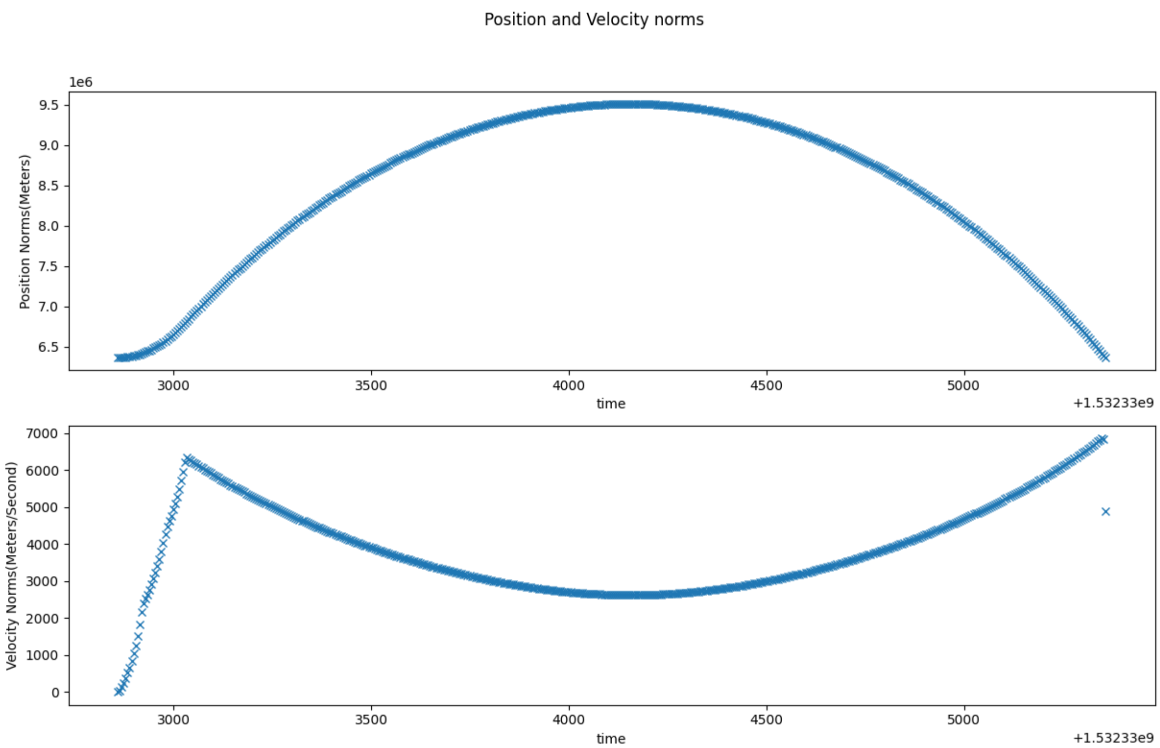

Figure 4: Position and Velocity Norms