Dsp Assignment

1) A discrete time signal $x(n)$ is defined as $x(n) = \begin{cases} 1+\frac{n}{3}, & -3 \le n \le -1 \\ 1, & 0 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$

(a) Determine its value and sketch the signal $x(n)$

(i) $1+\frac{n}{3}$, $-3 \le n \le -1 \Rightarrow$ at $n = -3$ 0

$n = -2$ $\frac{1}{3}$

from $0 \le n \le 3$ 1 $n = -1$ $\frac{2}{3}$

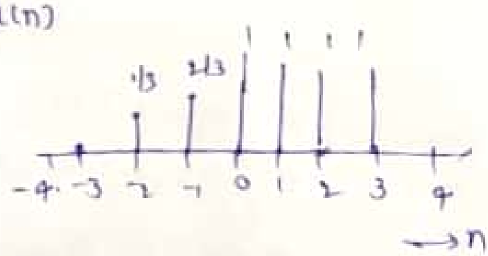
else 0

so $x(n) = \{ \ldots\ldots \; 0, \frac{1}{3}, \frac{2}{3}, \underset{\uparrow}{1}, 1, 1, 1, 0, \ldots \}$

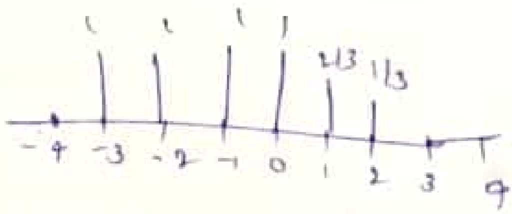
(b) Sketch its values and the signals that result if we:

i) First fold $x(n)$ and then delay the resulting by 4 samples

$x(n)$



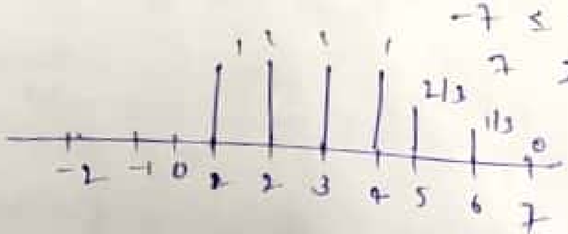
folding $x(n) = x(-n)$



$x(-n+4) \rightarrow$ after delaying $(-3 \le n+4 \le 2)$

$-7 \le -n \le -2$

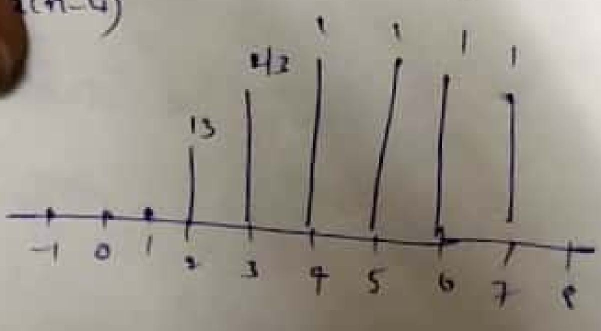
$7 \ge n \ge -2$

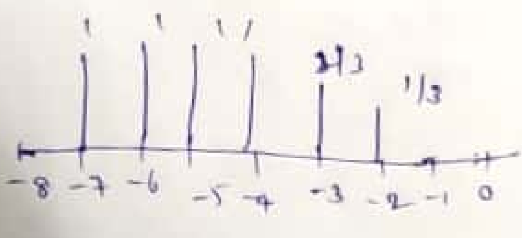


(2)

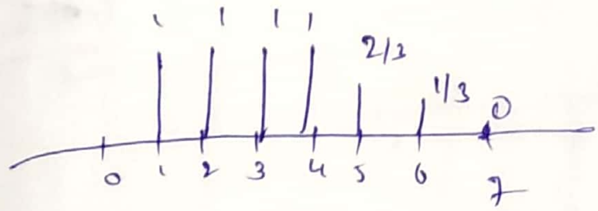
$x(n) \rightarrow$ delay by 4 samples

$x(n-4)$





Sketch the Signal $x(-n+4)$



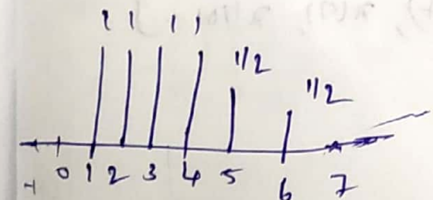
(d) Compare the result in parts (b) and (c) and derive a rule for obtaining the signal $x(-n+k)$ from $x(n)$.

By Comparing results in parts (b) and (c) we can say that to get $x(-n+k)$ from $x(n)$ first we need to find $x(n)$ which results in $x(-n)$ and then we need shift by k samples to right if $k>0$ (or) to left if $k<0$ result in $x(-n+k)$

(e) yes ; $x(n) = \frac{1}{3}\delta(n-2) + \frac{2}{3}\delta(n-1) + u(n) - u(n-4)$

(2)

(a) $x(n-2)$

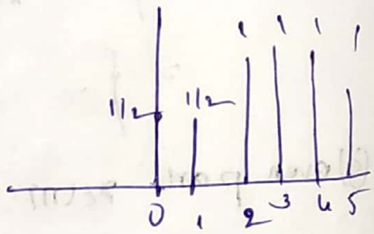


(b) $x(4-n)$

$-1 \le 4-n \le 4$

$-5 \le -n \le 0$

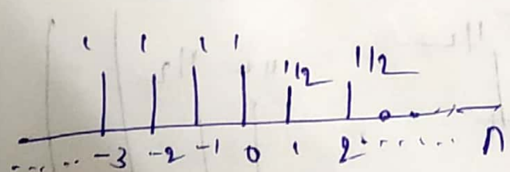
$5 \ge n \ge 0$



(c) $x(n+2)$

$-1 \le n+2 \le 4$

$-3 \le n < 2$

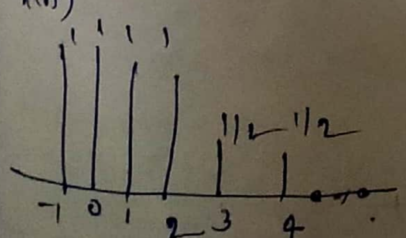


(d) $x(n)\,u(2-n)$

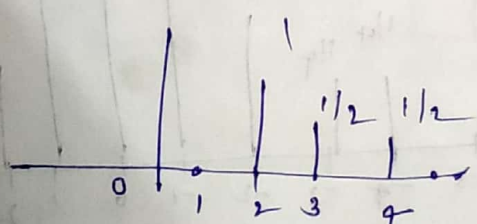
(1) $u(2-n) \Rightarrow 1 \quad 2-n>0$

$-n \ge -2$

$n \ge 2$



$\Rightarrow$



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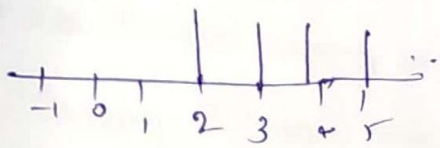
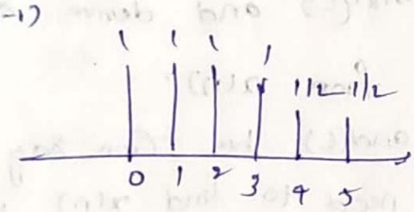


(e) $x(n-1)\,\delta(n-3)$

(A) $x(n-1)$



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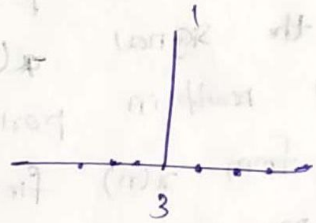
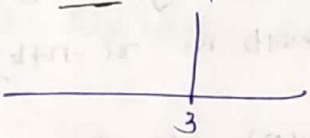


$\delta(n-3)$



$\Rightarrow$ $x(n-1)\,\delta(n-3)$

(f) $x(n^2)$

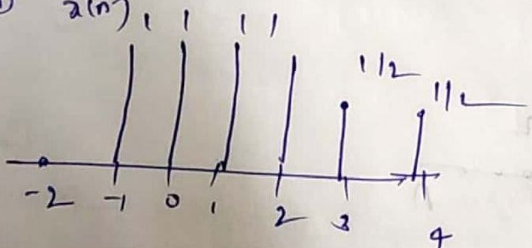
(A) $x(n) = \{x(-2), x(-1), x(0), x(1), x(2), x(3), x(4), \dots\}$

$x(n^2) = \{\dots x(4), x(1), x(0), x(1), x(4), x(9), x(16) \dots\}$

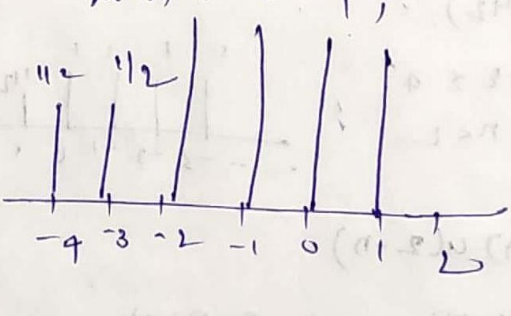
$= \{\dots \frac{1}{2}, 1, 1, 1, \frac{1}{2}, 0, 0 \dots\}$

(g) even part $x_e(n) = \dfrac{x(n) + x(-n)}{2}$

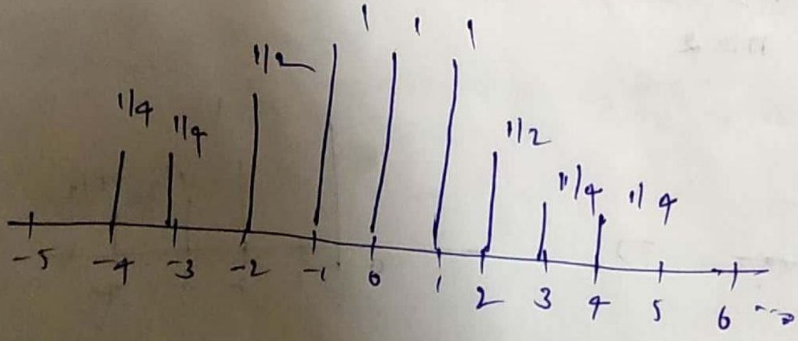
(A) $x(n)$



$x(-n)$

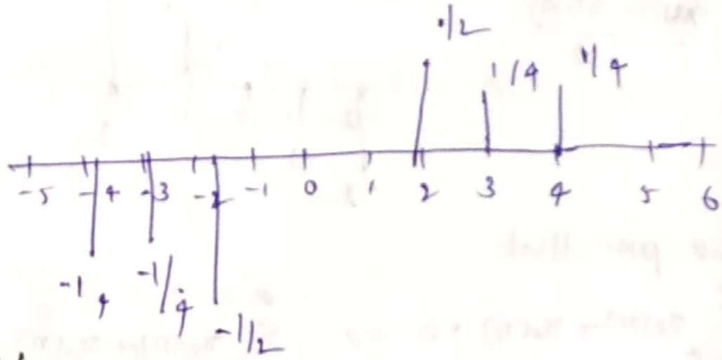


$\dfrac{x(n) + x(-n)}{2}$



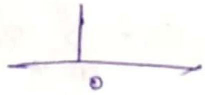
odd part:

$$\frac{x(n) - x(-n)}{2}$$



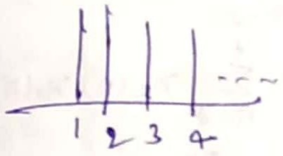
3) show that $\delta(n) = u(n) - u(n-1)$

① w.k.t $\delta(n) =$



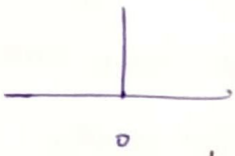
$u(n) =$

$u(n-1) = (n=1)$



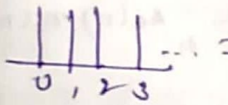
$\therefore \delta(n) = u(n) - u(n-1)$

$u(n) = u(n-n)$



(ii) $u(n) = \sum_{k=-\infty}^{\infty} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$

② $u(n) =$



$\Rightarrow \sum_{k=-\infty}^{\infty} \delta(k) = u(n) = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$

$\sum_{k=-\infty}^{\infty} \delta(n-k) = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$

4)

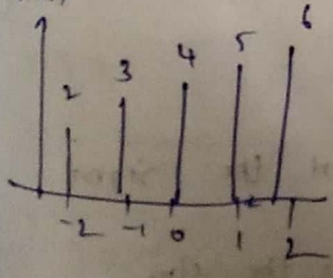
$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$

$x_e(n) = \frac{x(n) + x(-n)}{2}$; $x_e(n) = x_e(-n)$

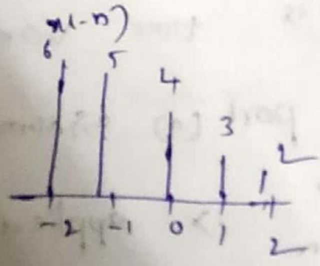
$x_o(n) = -x_o(-n)$

$x(n) = x_e(n) + x_o(n)$

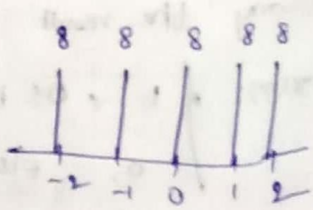
$x(n)$



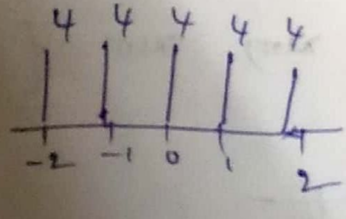
$x(-n)$



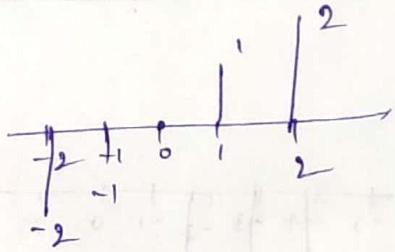
$x(n) + x(-n)$



$x_e(n) = \frac{x(n) + x(-n)}{2} \Rightarrow$



$x_o(n) = \frac{x(n) - x(-n)}{2}$



(5) first prove that

$\sum_{n=-\infty}^{\infty} x_e(n) + x_o(n) = 0 \Rightarrow \sum_{n=-\infty}^{\infty} x_e(n) + x_o(n) = \sum_{m=-\infty}^{\infty} x_e(-m)\, x_o(-m)$

$= - \sum_{m=-\infty}^{\infty} x_e(m)\, x_o(m)$

$= - \sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n)$

$= \sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n)$

$= 0$

Energies (power)

$\sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} \left[x_e(n) + x_o(n) \right]^2$

$= \sum_{n=-\infty}^{\infty} x_e^2(n) + x_o^2(n) + 2 x_e(n)\, x_o(n)$

$= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + 2 \sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n)$

$= E_e + E_o + 0$

$\boxed{E = E_e + E_e}$

(6) given $y(n) = T[x(n)] = x(n^2)$

$x(n-k) \rightarrow y_1(n) = x[(n-k)^2]$

$= x[n^2 + k^2 - 2nk]$

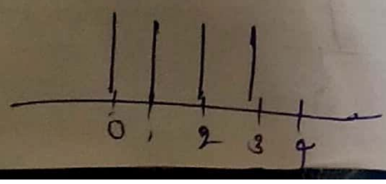
$x(n-k) \neq y(n-k)$

So, the given s/m is time varient

Carry the result in part (a) assume that the signal

$x(n) = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$ is applied into the s/m

1) sketch the s/g $x(n)$ $x(n) = \{ \cdots 0, \underset{\uparrow}{1}, 1, 1, 1, 0, 0 \cdots \}$



2) Determine and sketch the sig $y(n) = T[x(n)]$

① $y(n) = T[x(n)] = x(n^2) \Rightarrow \{x(0), x(1), x(2^2), x(3), x(4^2) \ldots\}$

$\Rightarrow \{x(0), x(1), x(4), x(9), \ldots\}$

$y(n) = x(n^2) = \{\ldots\ldots 1, 1, 0, 0, 0 \ldots\}$
$\uparrow$
0 1 2 3 4

(3) sketch the sig $y_2'(n) = y(n-2)$

① $y(n-2) = \{\ldots\ldots 0, 0, 1, 1, 0, 0, 0 \ldots\}$
$\uparrow$
0

4) Sketch the sig $x_2(n) = x(n-2)$

$x(n-2) = \{\ldots\ldots 0, 0, 1, 1, 1, 1, 0 \ldots\}$
$\uparrow$

5) Determine and sketch the sig $y_2(n) = T[x_2(n)]$

① $y_2(n) = T[x_2(n-1)] = \{x(0), x(1), x(2), x(3)\ x(4), x(5)\ x(6)\}$

$= \{\ldots\ldots 0, 1, 0, 0, 0, 1, 0 \ldots\}$
$\uparrow$

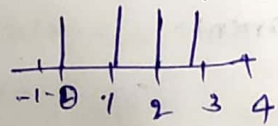
6) Compare the signals $y_2(n)$ and $y(n-2)$. What is your conclusion

$y_2(n) \neq y(n-2) \Rightarrow$ S/m is time Varient

(c) Repeat part (b) for the s/m.

$y(n) = x(n) - x(n-1)$. Can you use this stmt about the time invariance of that

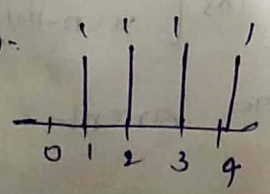
① $x(n) =$



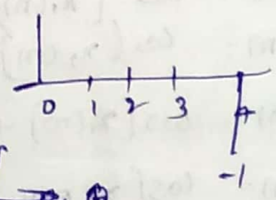
$= \{1, 1, 1, 1\}$
$\uparrow$

② $y(n) = x(n) - x(n-1)$

$x(n-1) =$



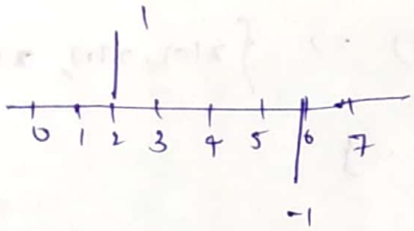
$y(n) = x(n) - x(n-1) =$



$= \{0, 1, 0, 0, 0, -1\} = y(n)$
$\uparrow$

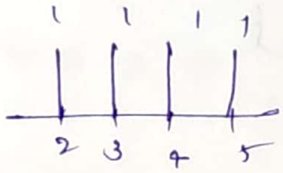
$= \{$

③ $y(n-2) =$



$\Rightarrow \{0, 0, 1, 0, 0, 0, -1\}$

④ $x(n-2)$



$\Rightarrow \{0, 0, 1, 1, 1, 1\}$

⑤ $y_2(n) = \{0, 0, 1, 0, 0, 0, -1\}$

⑥ $y_2(n) = y(n-2) \Rightarrow$ S/m is time invarient

(d) Repeat parts (b) and (c) from S/m $y(n) = T[x(n)] = n x(n)$

1) $y(n) = n x(n)$

$x(n) = \{\ldots 0, \underset{\uparrow}{1}, 1, 1, 1, 0 \ldots\}$ $n \to$ integer value from $0 \ldots$

2) $y(n) = \{\ldots \underset{\uparrow}{0}, 1, 2, 3, 4 \ldots\}$

3) $y(n-2) = \{\ldots \underset{\uparrow}{0}, 0, 0, 1, 2, 3, 4 \ldots\}$

4) $x(n-2) = \{\ldots 0, \underset{\uparrow}{0}, 0, 1, 1, 1, 1, 1 \ldots\}$

5) $y_2(n) = T[x(n-2)] = \{\ldots 0, 0, 2, 3, 4, 5 \ldots\}$

⑥ $y_2(n) \neq y(n-2) \Rightarrow$ S/m is time varrent

(7)

(a) $y(n) = \cos[x(n)]$

(i) Static (only present i/p)

(ii) $y_1(n) = \cos[x_1(n)]$

$y_2(n) = \cos(x_2(n))$

$y(n) = \cos[x_1(n)] + \cos(x_2(n))$

$y'(n) = \cos[x_1(n) + x_2(n)]$

$\Rightarrow$ non-linear

(iii) $y(n) = \cos[x(n-n_0)]$

$y'(n) = \cos[x(n-n_0)]$

$\Rightarrow$ Time Varrient

(b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

Dynamic (depends upon future values)

linear, time invarient, non causal, (also depends upon future values), unstable

(c) $y(n) = x(n)\cos(w_0 n)$

Static, linear, Time Varrent, Causal, stable

$y(n) = x(n-n_0)\cos(w_0(n-n_0))$

$y'(n) = x(n-n_0)\cos w_0 n$

(d) $y(n) = x(-n+2)$

Dynamic

at $n=0 \Rightarrow y(0) = x(2)$

future value

$y_1(n) = x_1(-n+2) + x_2(-n+2)$

$y_2(n) = (x+x_2(-n+2)) \Rightarrow x_1(-n+2) + x_2(-n+2)$

Linear

Non-causal, stable, time invariant

(e) $y(n) = \text{Trun}[x(n)]$

Static, non-linear, time invarient, Causal, stable

(f) $y(n) = \text{Round}[x(n)]$

Static, non-linear, time invarient, Causal, stable

(g) $y(n) = |x(n)|$

Static, non-linear, time invarent, causal, stable

(h) $y(n) = x(n)u(n)$

Static, Linear, time invarrent, Causal, Stable

$y(n) = x(n) + n\,x(n+1)$

Dynamic, linear, time Varrent, non-Causal, unstable

(j) $y(n) = x(2n)$

Dynamic, linear, Time varrent, non-causal, stable

(k) $y(n) = \begin{cases} x(n) & ; \text{ if } x(n) \geq 0 \\ 0 & ; \text{ if } x(n) < 0 \end{cases}$

Static, linear, time invarent, non-Causal, stable.

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linear, Time Varrent, Causal, Stable

m) $y(n) = \text{Sign}[x(n)]$

Static, non-linear, Time invarrent, Causal, Stable

n) The Ideal sampling s/m with input $x_a(t)$ and o/p $x(n) = x_a(nT)$, $-\infty < n < \infty$

$x(n) = x_a(nT)$

Static, linear, time varrent, non-Causal, Stable

(9)

(a)

$x(n) = x(n+N) \quad \forall n \geq 0$

$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$

$y(n+N) = \sum_{k=-\infty}^{n+N} h(k)\, x(n+N-k)$

$y_1(n+N) = \sum_{k=n+1}^{n+N} h(k)\, x(n-k) + \sum_{k=-\infty}^{n} h(k)\, x(n-k)$

$y(n+N) = y(n) + \sum_{k=n+1}^{n+N} h(k)\, x(n-k)$

for BIBO s/m $\lim_{n\to\infty} |h(n)| = 0$

$\lim_{n\to\infty} \sum_{k=n+1}^{n+N} h(k)\, x(n-k) = 0$

$\lim_{n\to\infty} y(n+N) = y(N)$

$\therefore \; y(N) = y(n+N)$

(b)

$x(n) = x_0(n) + a u(n)$ $\quad x_0(n) \to$ bounded with $\lim_{n\to\infty} x_0(n) = 0$

$\Rightarrow y(n) = a \sum_{k=0}^{\infty} h(k)\, u(n-k) + \sum_{k=0}^{\infty} h(k)\, x_0(n-k) = a \sum_{k=0}^{n} h(k) + y_0($

$\Rightarrow \sum_n x_0^2(n) < \infty \Rightarrow \sum_n y_0^2(n) < \infty$

hence $\lim_{n\to\infty} |y_0(n)| = 0$ $\quad a \sum_{k=0}^{n} h(k) = \text{const}$

(c) If $x(n)$ is a energy sig, the o/p $y(n)$ will also be a energy signal?

$y(n) = \sum_k h(k)\, x(n-k)$

$$\sum_{-\infty}^{\infty} y^2(n) = \sum_{-\infty}^{\infty} \left[\sum_k h(k)\, x(n-k) \right]^2 = \sum_k \sum_l h(k)\, h(l) \sum_n x(n-k)\, x(n-l)$$

but $\sum_n x(n-k)\, x(n-l) \leq \sum_n x^2(n) / h(l))$

for BIBO stable s/m $\sum_k |h(k)| \leq m$

Hence $\mathcal{E}_y \leq m^2 \mathcal{E}_x$, so that $\mathcal{E}_y < 0$ if $\mathcal{E}_x < 0$

(9)

(10) As it is a time-invarient s/m $y_2(n)$ should have only 3 elements and $y_3(n)$ should have 4 elements. so, it is non-linear

(11)

Since $x_1(n) + x_2(n) = \delta(n)$

& S/m is linear, the impulse response of the s/m is

$y_1(n) + y_2(n) = \{0, \underset{\uparrow}{3}, -1, 2, 1\}$

if s/m were time invarient the response of $x_3(n)$ would be $\{\underset{\uparrow}{3}, 2, 1, 3, 1\}$

(12)

$y_i(n) = T[x_i(n)] : i = 1, 2, \ldots N$

(a) A linear combination of signal in the form of $x_i(n)$; $i = 1, 2, 3, \ldots N$

because if we take $i = 1, 3$

$y_1(n) = x_1(n)$

$y(n) = y_1(n) + y_3(n) = x_1(n) + x_3(n)$

$y_3(n) = x_3(n)$

$y(n) = x_1(n) + x_3(n)$

$\therefore$ linear//

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(ii) Same repeat for the s/m is invarient?

Ⓐ Any $x_i(n-k)$ where k is any integer, $i = 1, 2, \ldots N$

1st replace $n = n - n_0 \Rightarrow x_i(n - n_0 - k)$

$x(n)$. by $x(n-n_0) \Rightarrow x_i(n-k-n_0)$

[Time invarrent]

13) Show that the necessary and sufficient condition for a relaxed LTI s/m to be BIBO stable is

$$\sum_{n=-\infty}^{\infty} |h(n)| \leq M_n < \infty \quad \text{for Constant } M_n$$

Ⓐ A s/m to be BIBO stable only when bounded output produced bounded input

$$y(n) = \sum_k h(k)\, x(n-k)$$

$$|y(m)| = \sum_k |h(k)|\, |x(n-k)|$$

$$= \sum_k |x(n-k)| \leq M_n \text{ [some Constant]}$$

So $|y(n)| = M_n \sum_k |h(k)|$

$|y(n)| < \infty$ for all n, if and only if $\sum_k |h(k)| < \infty$

so $\sum_{n=-\infty}^{\infty} |y(n)|$

→ A s/m to be BIBO stable only when bounded input produced bounded o/p.

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \; ; \; n \leq n-k$$

$$|y(n)| = \sum_{k=-\infty}^{\infty} |h(k)|\, |x(n-k)|$$

as $\sum_{k=-\infty}^{\infty} |x(n-k)| \leq M_n$ for some Constant

$$|y(n)| = M_n \sum_{k=-\infty}^{\infty} |h(k)| \; ; \; n \leq n-k$$

$$k \geq 0$$

$|y(n)|$ is $< \infty$ if & only if $\sum_{k=-\infty}^{\infty} |h(k)| < \infty$

so $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$

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(14)

(a) If a s/m is causal o/p depends only on the present and past inputs as $x(n)=0$ for $n<n_0$ then $y(n)$ also becomes zero for $n<n_0$

b) A relaxed LTI s/m is causal if and only if $h(n)=0$ for $n<0$

(c) $y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$

for finite impulse response

$h(n)=0$, $n<0$ and $n \geq m$

so $y(n)$ reduces to $y(n) = \sum_{k=0}^{m-1} h(k)\, x(n-k)$

(15) show that for any real or complex constant a and any finite integer numbers m and N, we have

(a) $\sum_{n=M}^{N} a^n = \begin{cases} \dfrac{a^m - a^{N+1}}{1-a} & , \text{if } a \neq 1 \\ N-m+1 & ; \text{if } a=1 \end{cases}$

(a) for $a=1$, $\sum_{n=M}^{N} a^n = N-M+1$

for $a \neq 1$, $\sum_{n=m}^{N} a^n = \dfrac{a^m - a^{N+1}}{1-a}$

$(1-a)^N \sum_{n=m}^{N} a^n = a^m + a^{m+1} - a^{m+1} + \dots a^N - a^N - a^{N+1}$

$= a^m - a^{N+1}$

(b) for $m=0$, $|a|<1$ and $N \to \infty$ $\sum_{n=0}^{\infty} a^n = \dfrac{1}{1-a}$, $|a|<1$

(16)

(a) If $y(n) = x(n) * h(n)$ show that $\sum_y = \sum_x , \sum_h$

when $\sum_x = \sum_{n=-\infty}^{\infty} x(n)$

(a) $y(n) = \sum_k h(k) x(n-k)$

$\sum_n y(n) = \sum_n \sum_k h(k)\, x(n-k)$

$\sum_n y(n) = \sum_k h(k) \sum_{n=-\infty}^{\infty} x(n-k) \Rightarrow \sum_n y(n) = \left(\sum_k h(k)\right)\left(\sum_n x(n)\right)$

(b) Correlations (for Checking)

(1) $x(n) = \{1, 2, 4\}$, $h(n) = \{1, 1, 1, 1, 1\}$

$y(n) = \{1, 3, 7, 7, 7, 6, 4\}$

$\sum_n y(n) = 35$; $\sum_n x(n) = 7$ $\sum_n h(n) = 5$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$= 7 \times 5 = 35$

h(n) \ x(n)	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

(2) $x(n) = \{1, 2, -1\}$, $h(n) = x(n)$

A $x(n) = \{1, 2, -1\}$, $h(n) = \{1, 2, -1\}$

$y(n) = x(n) * h(n)$

$= \{1, 4, 2, -4, 1\}$

$\sum_n y(n) = 4$; $\sum_n x(n) = 2$, $\sum_n h(n) = 2$

$\sum_n y(n) = \sum_n x(n) \sum h(n)$:

$4 = 4$

h(n) \ x(n)	1	2	-1
1	1	2	-1
2	2	4	-2
-1	-1	-2	1

(3) $x(n) = \{0, 1, -2, 3, -4\}$, $h(n) = \{1/2, 1/2, 1, 1/2\}$

A $y(n) = \{0, 1/2, -1/2, 3/2, -2, 0, -5/2, 2\}$

$\sum_n y(n) = -5$; $\sum_n x(n) = -2$, $\sum_n h(n) = 5/2$

$\sum_n y(n) = \sum_n x(n)\ h(n)$

$-5 = 5$

h(n) \ x(n)	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2
1/2	0	1/2	-1	3/2	-2
1	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2

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(4) $x(n) = \{1, 2, 3, 4, 5\}$, $h(n) = \{1\}$

$y(n) = \{1, 2, 3, 4, 5\}$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$15 = 15(1)$

$= 15$

(5) $x(n) = \{\underset{\uparrow}{1}, 2, 3\}$, $h(n) = \{0, 0, 1, 1, 1, 1\}$

(a) $y(n) = \{\underset{\uparrow}{0}, 0, 1, -1, 2, 2, 1, 3\}$

$\sum_n y(n) = 8$; $\sum_n x(n) = 2$, $\sum_n h(n) = 4$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$8 = 8$

h(n) \ x(n)	1	-2	3
0	0	0	0
0	0	0	0
1	1	-2	3
1	1	-2	2
1	1	2	3
1	1	2	3

(6) $x(n) = \{0, 0, 1, 1, 1, 1\}$, $h(n) = \{1, -2, 3\}$

(a) $y(n) = \{\underset{\uparrow}{0}, 0, 1, -1, 2, 2, 1, 3\}$

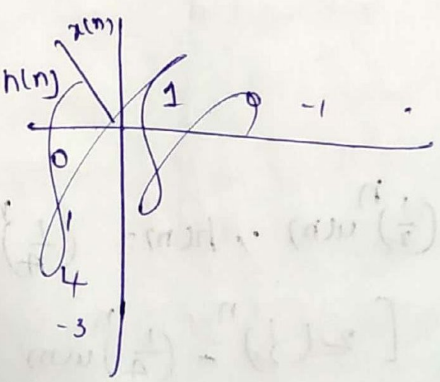
$\sum_n y(n) = 8$, $\sum_n x(n) = 4$ $\sum_n h(n) = 2$

$\sum_y (n) = \sum_n x(n) \sum_n h(m) = 8$

$8 = 8$

(7) $x(n) = \{\underset{\uparrow}{0}, 1, 4, -3\}$, $h(n) = \{\underset{\uparrow}{1}, 0, -1, -1\}$

h(n) \ x(n)	0	1	4	-3
1	0	1	4	-3
0	0	0	0	0
-1	0	-1	-4	3
-1	0	-1	-4	3



$y(n) = \{0, 1, 4, -4, -5, -1, 3\}$

$\sum_n y(n) = -2$, $\sum_n x(n) = -2$, $\sum_n h(n) = 1$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$.

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(8) $x(n) = \{\underset{\uparrow}{1}, 1, 2\}$, $h(n) = u(n)$

(A) $y(n) = \{1, 2, 4, 3, 2\}$

$\sum_n y(n) = \sum_n x(n) \; \sum h(n)$

$12 = 4 \times 3$

$12 = 12$

$h(n) \backslash x(n)$	1	1	2
1	1	1	2
1	1	1	2
1	1	1	2

(9) $x(n) = \{1, 1, 0, 1, 1\}$, $h(n) = \{1, -2, -3, 4\}$

(A) $y(n) = \{1, -1, -5, 2, 3, -5, 1, 4\}$

$\sum_n y(n) = 0 \quad \sum_n x(n) = 4 \quad \sum_n h(n) = 0$

$\sum_n y(n) = \sum_n x(n) \; \sum_n h(n)$

$0 = 0$

$h(n) \backslash x(n)$	1	1	0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
-3	-3	-3	0	-3	-3
-4	4	4	0	-4	4

(10)

$x(n) = \{1, 2, 0, 2, 1\}$, $h(n) = x(n)$

$y(n) = \{1, 4, 4, 4, 10, 4, 4, 4, 1\}$

$\sum_n y(n) = 36$, $\sum_n x(n) \; \sum_n h(n)$

$= 36$

$h(n) \backslash x(n)$	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	2
0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

(16) $x(n) = \left(\frac{1}{2}\right)^n u(n)$, $h(n) = \left(\frac{1}{4}\right)^n u(n)$

(A) $y(n) = \left[2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right] u(n)$

$\sum_n y(n) = \frac{8}{3}$, $\sum_n h(n) = \frac{4}{3}$ $\sum_n x(n) = 2$

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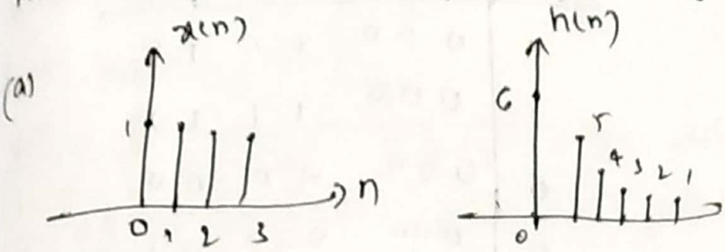
12) Compute and plot Convolution $x(n) * h(n)$ and $h(n) * x(n)$ for the pairs of signal shown below

(a)



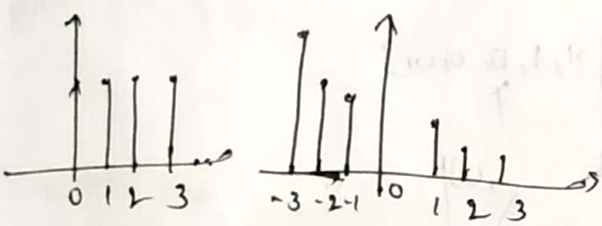
$x(n) = \{\underset{\uparrow}{1}, 1, 1, 1\}$; $h(n) = \{\underset{\uparrow}{6}, 5, 4, 3, 2, 1, 0\}$

$y(n) = x(n) * h(n)$

$= \{6, 11, 15, 18, 14, 10, 6, 3, 1\}$

h(n) \ x(n)	1	1	1
6	6	6	6
5	5	5	5
4	4	4	4
3	3	3	3
2	2	2	2
1	1	1	1

(b)



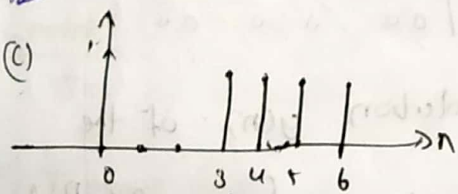
$x(n) = \{\underset{\uparrow}{1}, 1, 1, 1\}$, $h(n) = \{6, 5, 4, \underset{\uparrow}{3}, 2, 1, 0\}$

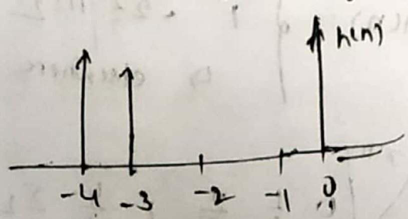
$y(n) = \{6, 11, 15, \underset{\uparrow}{18}, 14, 10, 6, 3, 1\}$

h(n) \ x(n)	1	1	1
6	6	6	6
5	5	5	5
4	4	4	4
3	3	3	3
2	2	2	2
1	1	1	1

sol

(c)





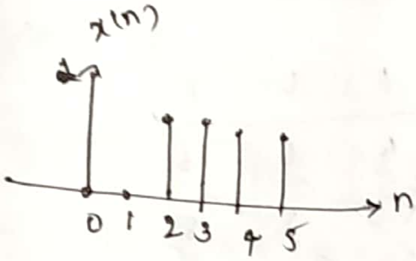
$x(n) = \{\underset{\uparrow}{0}, 0, 0, 1, 1, 1, 1\}$

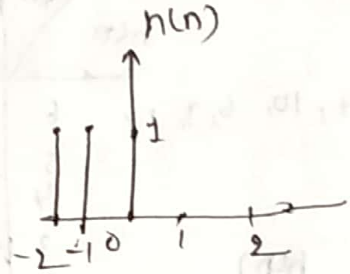
$h(n) = \{1, 1, 0, 0, 0, \underset{\uparrow}{0}\}$

$y(n) = \{0, 0, 0, 1, \underset{\uparrow}{2}, 2, 2, 1, 0, 0, 0\}$

h(n) \ x(n)	0	0	0	1	1	1	1
1	0	0	0	1	1	1	1
1	0	0	0	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

(d)





(d) $x(n) = \{0, 0, 1, 1, 1, 1\}$, $h(n) = \{1, 1, 0, 0, 0\}$
$\quad\uparrow \qquad\qquad\qquad\qquad \uparrow$

$y(n) = \{0, 0, 1, 2, 2, 2, 1, 0, 0, 0\}$
$\qquad\quad \uparrow$

h(n) \ x(n)	0	0	1	1	1	1
1	0	0	1	1	1	1
1	0	0	1	1	1	1
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

18) Determine and sketch the convolution $y(n)$ of the signals

$$x(n) = \begin{cases} \frac{1}{3}n, & 0 \le n \le 6 \\ 0, & \text{elsewhere} \end{cases} \qquad h(n) = \begin{cases} 1, & -2 \le n \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

(a) graphically

$x(n) = \{0, 1/3, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2\}$
$\qquad\quad \uparrow$

$h(n) = \{1, 1, 1, 1, 1\}$

$y(n) = \{0, \frac{1}{3}, 1, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\}$
$\qquad\qquad \uparrow$

h(n) \ x(n)	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$5/3$	2
		$1/3$	$4/3$	1	$4/3$	$5/3$	

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(b) analytically

$x(n) = \frac{1}{3} n \left[u(n) - u(n-7) \right]$

$h(n) = u(n+2) - u(n-3)$

$y(n) = \frac{1}{3} n \left[u(n) - u(n-7) \right] * u(n+2) - u(n-3)$

$= \frac{1}{3} n u(n) * u(n+2) - \frac{1}{3} n (u(n)) * u(n-3) - \frac{1}{3} n\, u(n-7) * u(n+2)$

$+ \frac{1}{3} n\, u(n-7) * u(n-3)$

(20) consider the following three operation

(a) multiply the integer numbers; 131 and 122

131 × 122 = 15982

(b) Compute the Convolution of signals ; $\{1, 3, 1\} * \{1, 2, 2\}$

$y(n) = \{1\,5, 9, 8, 2\}$

h(n) \ x(n)	1	2	2
1	1	2	2
3	3	6	6
1	1	2	2

(c) multiply the polynomial

$1 + 3z + z^2$ = and $1 + 2z + 2z^2$

Ⓐ $(1+3z+z^2)(1+2z+2z^2) = 2z^4 + z^3 + 9z^2 + 5z + 1$

(d) repeat part (a) for the numbers 1.31 and 1.22

Ⓐ 1.31 × 1.22 = 1.5982

(e) Comment:

These are different ways to perform Convolution

(21) Compute Convolution of $y(n) * h(n)$

(a) $x(n) = a^n u(n)$ $h(n) = b^n u(n)$ when $a \neq b$ and when $a = b$

Ⓐ $y(n) = x(n) * h(n)$

$= a^n u(n) * b^n u(n)$

$= \left[a^n * b^n \right] u(n)$

$y(n) = \sum_{k=0}^{n} a^k u(k)\, b^{n-k} u(n-k)$

$= b^n \sum_{k=0}^{n} a^k u(k)\, b^{-k} = b^n \sum_{k=0}^{n} (ab)^{-k}$

if $a \neq b$ then $y(n) = \frac{b^{n+1} - a^{n+1}}{b-a} u(n)$

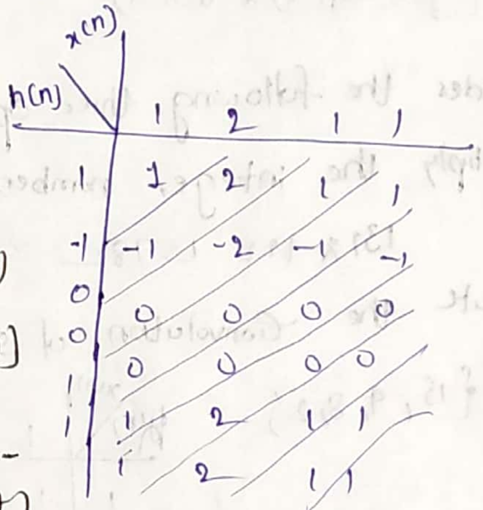
if $a = b \Rightarrow b^n (n+1) u(n)$

b) $x(n) = \begin{cases} 1 ; & n = 0, 1 \\ 2 ; & n = -1 \\ 0 ; & \text{else} \end{cases}$ $h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$

$h(n) = \{ \underset{\uparrow}{1}, -1, 0, 0, 1, 1 \}$

$\therefore x(n) = \{1, 2, 1, 1\}$

$y(n) = \{1, 1, \underset{\uparrow}{-1}, 0, 0, 3, 3, 2, 1\}$



(c) $x(n) = u(n+1) - u(n-4) - \delta(n-5)$

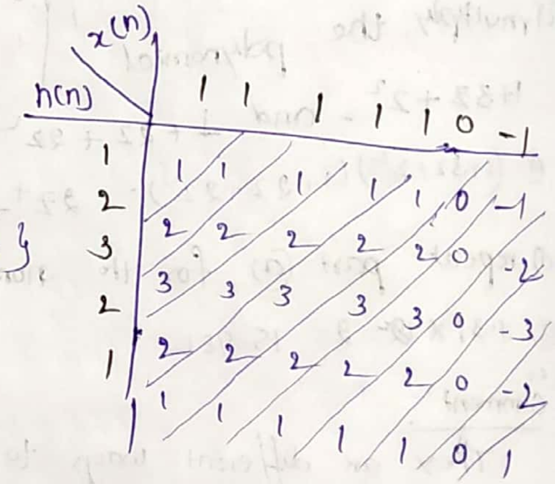
$h(n) = [u(n+2) - u(n-3)] (3 - |n|)$

(d) $h(n) = u(n-2) - u(n+8) + u(n-1) - u(n-17)$

(c) $x(n) = \{1, \underset{\uparrow}{1}, 1, 1, 1, 0, -1\}$

$h(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$

$y(n) = \{1, 3, 6, 8, 9, 8, 5, 1, -2, -2, 1\}$



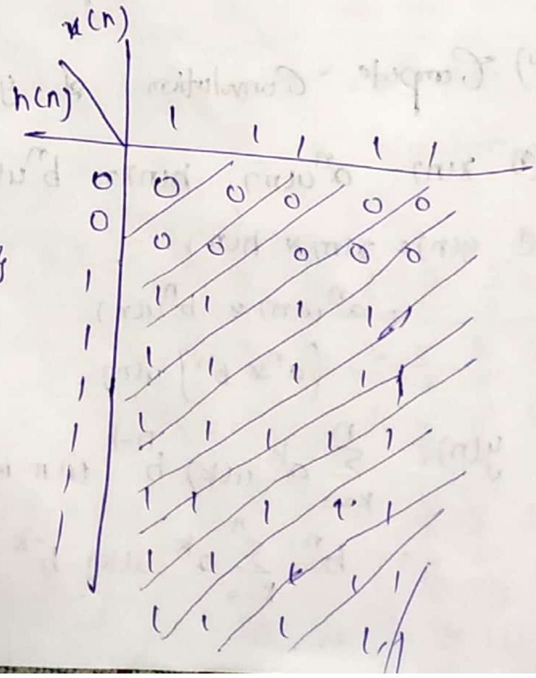
(d) $x(n) = \{ \underset{\uparrow}{1}, 1, 1, 1, 1 \}$

$h'(n) = \{ \underset{\uparrow}{0}, 0, 1, 1, 1, 1, 1, 1 \}$

$h(n) = h'(n) + h'(n-9)$

$y(n) = y'(n) + y'(n-9)$, where

$y'(n) = \{0, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2\}$



22)

(a) $x(n) = \{1, 4, 2, 3, 5, 3, 3, 4, 5, 7, 6, 9\}$

$h_1(n) = \{1, 1\}$, $h_2(n) = \{1, 2\}$, $h_3(n) = \{1/2, 1/2\}$, $h_4(n) = \{1/4, \frac{1}{2}, 1/4\}$

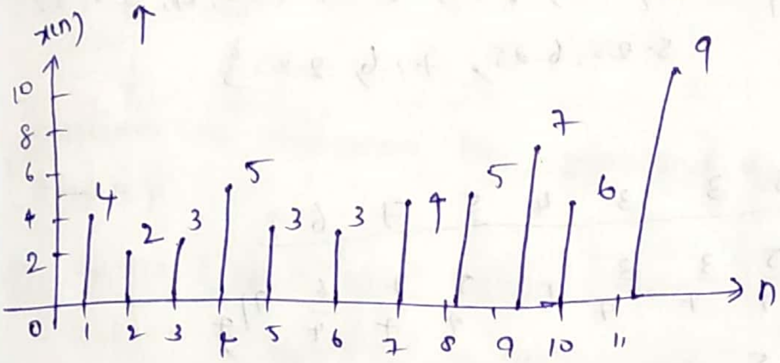
$h_5(n) = \{\frac{1}{4}, -1/2, 1/4\}$.

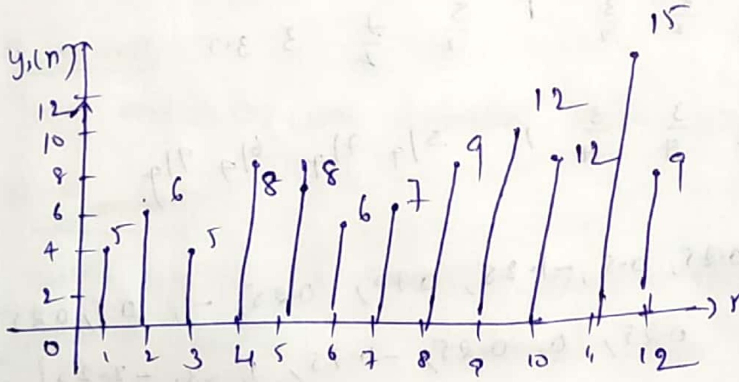
sketch $x(n)$, $y_1(n)$, $y_2(n)$, on graph & $x(n)$, $y_3(n)$, $y_4(n)$, $y_5(n)$ on another graph

(a) $y_1(n) = x(n) * h_1(n)$

h(n) \ x(n)	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9

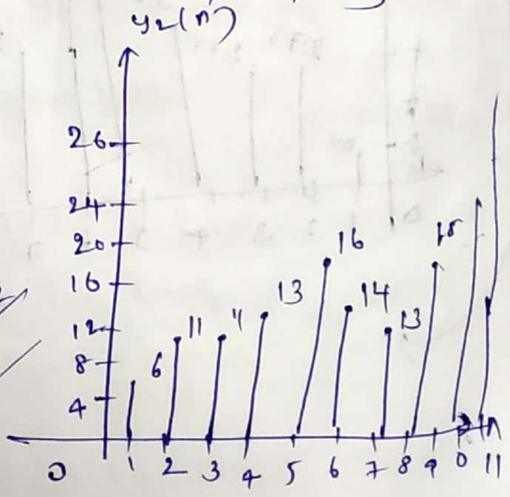
$y_1(n) = \{1, 5, 6, 5, 8, 8, 6, 7, 9, 12, 12, 15, 9\}$
$\uparrow$





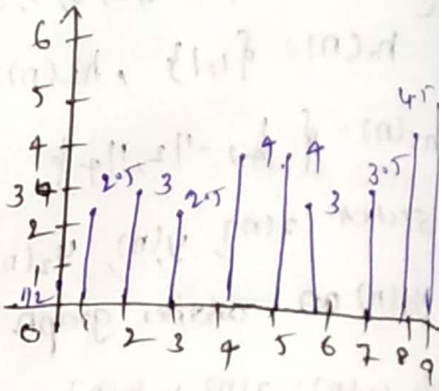
$y_2(n) = x(n) * h_2(n) = \{1, 6, 11, 11, 13, 16, 14, 13, 16, 21, 25, 28\}$

y(n) \ x(n)	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
2	2	8	4	6	10	13	13	8	10	14	12	18
1	1	4	2	3	5	3	3	4	5	7	6	9



$y_3(n) = x(n) * h_3(n) = \{1/2,\ 2.5,\ 3,\ 2.5,\ 4,\ 4,\ 3,\ 3.5,\ 4.5,\ 6,\ 6,\ 7\}$

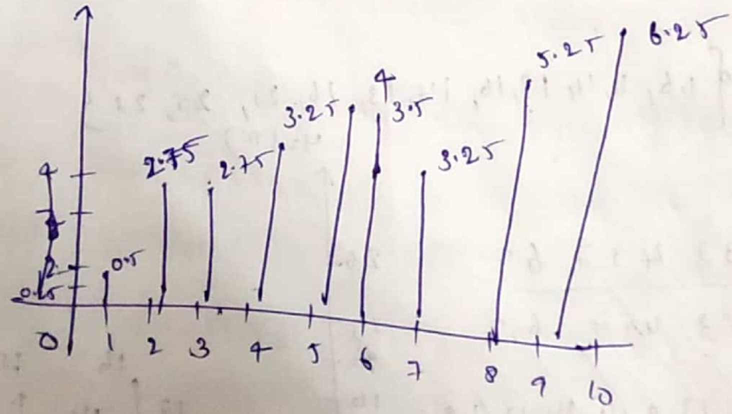
h(n) \ x(n)	1	4	2	3	5	3	3	4	5	7	6	9
$\frac{1}{2}$	$\frac{1}{2}$	2	1	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	2	$\frac{5}{2}$	$\frac{7}{2}$	3	9/2
1/2	$\frac{1}{2}$	2	1	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	2	$\frac{5}{2}$	$\frac{7}{2}$	3	9/2

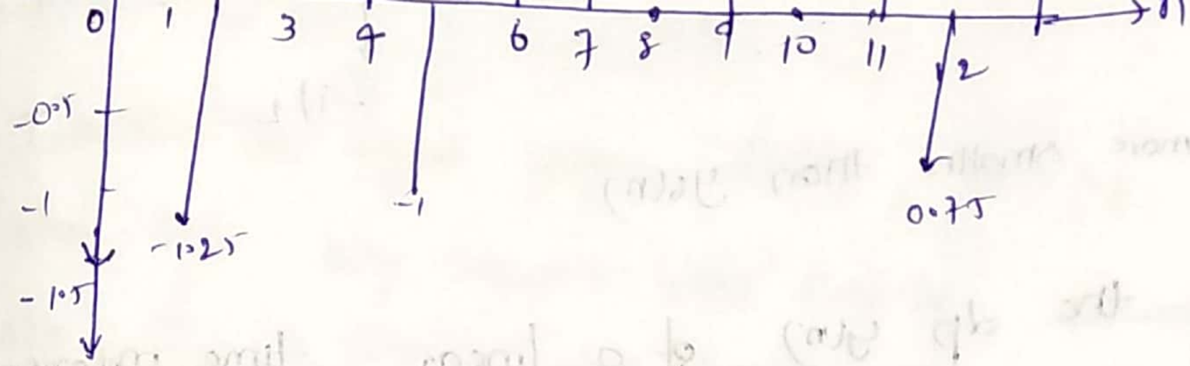


$y_4(n) = x(n) * h_4(n) = \{0.25,\ 1.5,\ 2.75,\ 2.75,\ 3.25,\ 4,\ 3.5,\ 3.25,$
$5.25,\ 6.25,\ 7,\ 6,\ 2.25\}$

h(n) \ x(n)	1	4	2	3	5	3	3	4	5	7	6	9
$\frac{1}{4}$	$\frac{1}{4}$	1	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{6}{4}$	9/4
1/2	$\frac{1}{2}$	2	1	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	1	$\frac{5}{2}$	$\frac{7}{2}$	3	3.5
1/4	$\frac{1}{4}$	1	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1	5/4	7/4	6/4	9/4

$y_5(n) = x(n) * h_5(n) = \{0.25,\ 0.5,\ -1.25,\ 0.75,\ 0.25,\ -1,\ 0.5,\ 0.25,$
$0.25,\ 0,\ 0.25,\ -0.75,\ 1,\ -3,\ -2.25\}$





(b) What is the difference b/w $y_1(n)$ and $y_2(n)$ and b/w $y_3(n)$ & $y_4(n)$

(A) $y_3(n) = \frac{1}{2} y_1(n)$; $\therefore h_3(n) = \frac{1}{2} h_1(n)$

$y_4(n) = \frac{1}{4} y_2(n)$ $\therefore h_4(n) = \frac{1}{4} h_2(n)$

c) Comments

$y_2(n)$ and $y_4(n)$ are smoother than $y_1(n)$ bcz of smaller scalar factor.

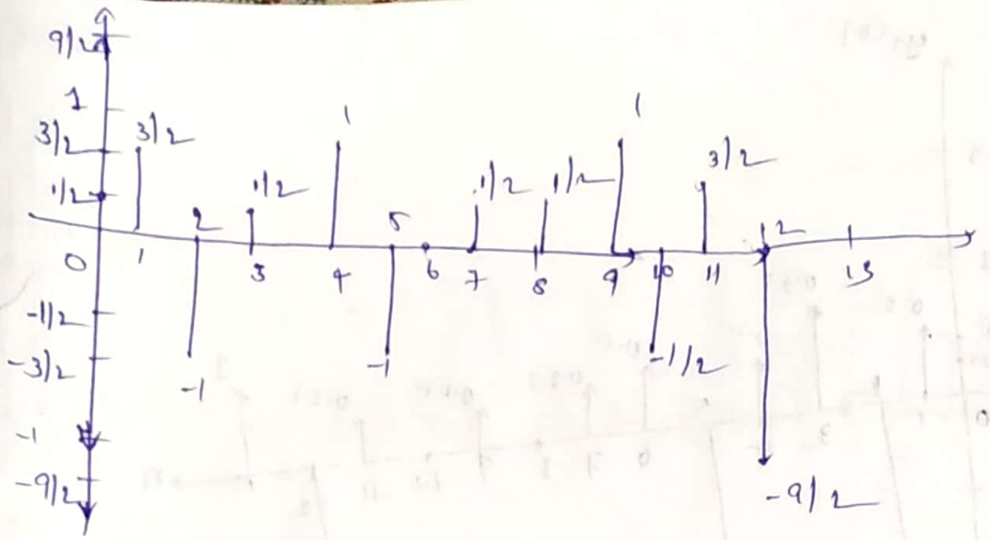
(d) Comments

$y_4(n)$ results in smoother outputs than $y_5(n)$. The negative value of $h_5(0)$ is responsible for the non-smooth characteristics of $y_5(n)$

(e) let $h_5(n) = \{1/2, -1/2\}$, compute $y_6(n)$. Sketch $x(n)$, $y_2(n)$ and $y_6(n)$ on the same figure and comment on the results.

(A) $y_6(n) = x(n) * h_6(n)$

$y_6(n) = \{ \frac{1}{2}, 3/2, -1, 1/2, 1, -1, 0, 1/2, 1/2, 1, -1/2, 3/2, -9/2 \}$



$y_2(n)$ is more smaller than $y_6(n)$

23) Express the o/p $y(n)$ of a linear time invarient s/m with impulse response $h(n)$ in terms of its step response $s(n) = h(n) * u(n)$ and the input $x(n)$

Ⓐ We can Express $\delta(n) = u(n) - u(n-1)$

$h(n) = h(n) * \delta(n)$

$= h(n) * [u(n) - u(n-1)]$

$= h(n) * u(n) - h(n) * u(n-1)$

$= s(n) - s(n-1)$

then $y(n) = h(n) * x(n)$

$= [s(n) - s(n-1)] * s(n)$

$= s(n) * x(n) - s(n-1) * s(n)$

24) The discrete time s/m $y(n) = ny(n-1) + x(n)$, $n \geq 0$ is at [ie, $y(-1) = 0$]. Check if the s/m is linear time invarent and stable

Ⓐ $y(n) = ny(n-1) + x(n)$, $n \geq 0$

$y_1(n) = ny_1(n-1) + x_1(n)$
$y_2(n) = ny_2(n-1) + x_2(n)$ $\Big\}$ ⊕ $\Rightarrow y(n) = ny_1(n-1) + x_1(n) + ny_2(n-1) + x_2(n)$

$y(n) = ny(n-1) + x(n)$

$x(n) = ax_1(n) + bx_2(n)$

$y(n) = ay_1(n) + by_2(n)$

Hence the s/m is Linear

$\Rightarrow y(n-1) = (n-1)\, y(n-2) + x(n-1)$

delayed $\Rightarrow y(n-1) = n\, y(n-2) + x(n-1)$

so, the s/m is time Varient

$\Rightarrow$ If $x(n) = u(n)$ then $|x(n)| \le 1$, for this bounded input, output is $y(0) = 0$, $y(1) = 2$, $y(2) = 5 \ldots$ unbounded so s/m is unstable

25) Consider the signal s/m: $a^n u(n)$; $0 < a < 1$

(a) show that any sequence $x(n)$ can be decomposed as

$$x(n) = \sum_{n=-\infty}^{\infty} k\, \delta(n-k)$$ and Express C_k in terms of $x(n)$

(A) $\delta(n) = s(n) - a\, r(n-1)$

$s(n-k) = \delta(n-k) - a\,\delta(n-k-1)$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k)$$

$$= \sum_{k=-\infty}^{\infty} x(k) \left[\delta(n-k) - a\delta(n-k-1) \right]$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k) - a \cdot \sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k-1)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k) - a \sum_{k=-\infty}^{\infty} x(k-1)\, \delta(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \left[x(k) - a x(k-1) \right] \delta(n-k)$$

Thus $k = x(k) - a x(k-1)$

(b) $g(n) = T[r(n)]$ where $T[\,]$ is an LTI s/m.

(A) $y(n) = T[x(n)]$

$$= T\left[\sum_{k=-\infty}^{\infty} C_k\, \delta(n-k) \right]$$

$$= \sum_{k=-\infty}^{\infty} C_k\, T(r(n-k))$$

$$= \sum_{k=-\infty}^{\infty} C_k\, g(n-k)$$

(c) Express the impulse response $h(n) = T[\delta(n)]$ in terms of $g(n)$

(A) $h(n) = T[\delta(n)]$

$h(n) = T[\delta(n) - a\,\delta(n-1)]$

$= g(n) - a g(n-1)$

26) Determine the zero input response of the s/m described by the second order difference Eq

$$x(n) - 3y(n-1) - 4y(n-2) = 0$$

with $x(n) = 0$

$-3y(n-1) - 4y(n-2) = 0 \quad (\div(-3))$

$y(n-1) + \frac{4}{3} y(n-2) = 0$

at $n = 0$

$y(-1) = -\frac{4}{3} y(-2)$

at $n = 1$

$y(0) = -\frac{4}{3} y(-1) = \left(-\frac{4}{3}\right)^2 y(-2)$

$y(1) = \left(-\frac{4}{3}\right)^3 y(-2)$

$y(k) = \left(-\frac{4}{3}\right)^{k+2} y(-2)$

zero-i/p response

27) Determine the particular solution of the difference Eq^n $y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n)$ when the forcing function is $x(n) = 2^n u(n)$

(A) $y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n)$

$x(n) = y(n) - \frac{5}{6} y(n-1) + \frac{1}{6} y(n-2)$

Characteristic Eq n is

$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0, \quad \lambda = \frac{1}{2}, \frac{1}{3}$

so $y_h(n) = C_1 (1/2)^n + C_2 \left(\frac{1}{3}\right)^n$

$x(n) = 2^n u(n)$

$y_p(n) = K(2^n)\, u(n)$

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so $k(2^n)u(n) - k(5/6)(2^{n-1})u(n-1) + k(1/6)(2^{n-2})u(n-2) = 2^n u(n)$

for $n \geq 2$

$$4k - \frac{5k}{3} + \frac{k}{6} = 4$$

$$k = 8/5$$

Total solution is

$$y_p(n) + y_h(n) = y(n)$$

$$y(n) = \frac{8}{5}(2^n)u(n) + C_1(1/2)^n u(n) + C_2(1/3)^n u(n)$$

Assume $y(-2) = y(-1) = 0$ so $y(0) = 1$

then $y(1) = \frac{5}{6}y(0) + 2 = 17/6$

so, $\frac{8}{5} + C_1 + C_2 = 1$

$C_1 + C_2 = 1 \therefore C_1 + C_2 = 3/5$ —— ①

$$\frac{16}{5} + \frac{1}{2} + C_1 + \frac{1}{3}C_2 = 17/6$$

$34 + 2C_2 = -11/5$ —— ②

by solving ① & ②

$$C_1 = -1, \quad C_2 = 2/5$$

total solution is

$$y(n) = \left[\frac{8}{5}(2)^n - \left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{1}{3}\right)^n\right]u(n)$$

48).

given $y(n) = (-a_1)^{n+1}y(-1) + \frac{\left(1-(-a_1)^{n+1}\right)}{1+a_1}$ for $n \geq 0$

for $a_1 = 0.9$

(a) at $y(-1) = 1$

The given Eq'n $y(n) = (-a)^{n+1} + \frac{\left(1-(-a)^{n+1}\right)}{1+a}$

$y(n) = y_{zi}(n) + y_{zs}(n)$

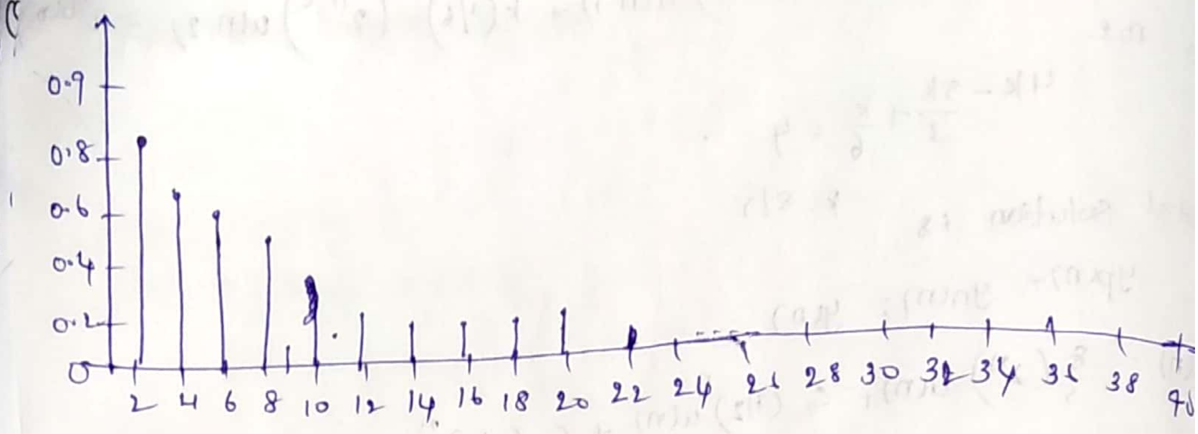
$= \text{Transient} \quad \text{State}$

$y(0) = 0.7$

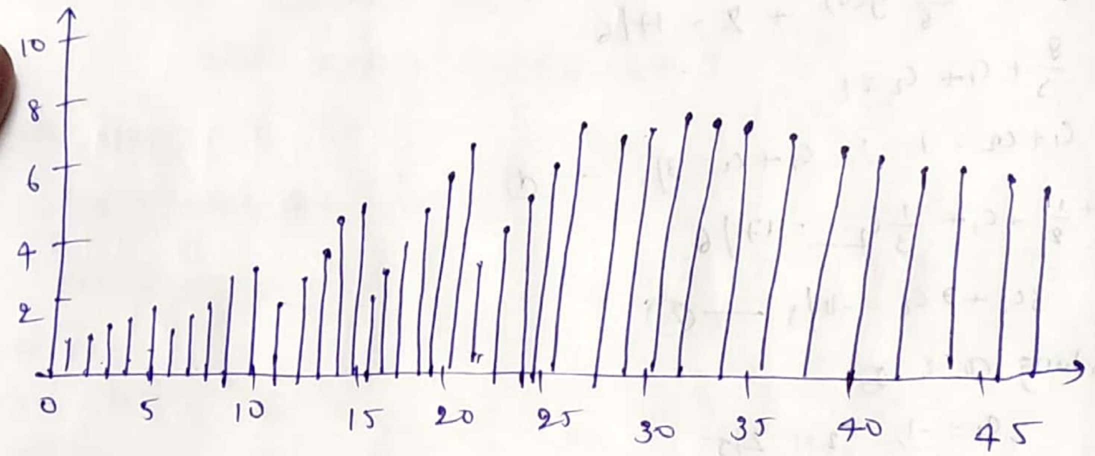
$y(1) = 0.87$

$y(2) = 0.72$

⋮



← Transient Reponse →



← steady state Response →

29) Determine the impulse response for the Cascade of two linear time invarient s/m having impulse response

$h_1(n) = a^n \left[u(n) - u(n-N)\right]$ and $h_2(n) = \left[u(n) - u(n-m)\right]$

$h(n) = h_1(n) * h_2(n)$

$= \sum_{k=-\infty}^{\infty} a^k \left[u(k) - u(k-N)\right]\left[u(n-k) - u(n-k-m)\right]$

$= \sum_{k=-\infty}^{\infty} a^k u(k)\, u(n-k) - \sum_{k=-\infty}^{\infty} a^k u(k)\, u(n-k-m) - \sum_{k=-\infty}^{\infty} a^k u(k-N)\, u(n-k) + \sum_{k=-\infty}^{\infty} a^k u(k-N)\, u(n-k-m)$

$= \left(\sum_{k=0}^{n} a^k - \sum_{k=0}^{n-m} a^k\right) - \left(\sum_{k=N}^{n} a^k - \sum_{k=N}^{n-m} a^k\right)$

$h(n) = 0$

30) Determine the response $y(n)$, $n \geq 0$ of the s/m described by the second order differential eq

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$ to the i/p $x(n) = 4^n u(n)$

Ⓐ $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

Characteristic Eq is

$$\lambda^2 - 3\lambda - 4 = 0$$

$$\lambda = 4, -1$$

so $y_h(n) = c_1 4^n + c_2 (-1)^n$

$x(n) = 4^n u(n)$

$y_p(n) = k n 4^n u(n)$

$k \cdot n 4^n u(n) - 3k(n-1) 4^{n-1} u(n-1) - 4k(n-2) 4^{n-2} u(n-2) = 4^n u(n) + 2(4)^{n-1} u(n-1)$

for $n=2$ $k(32-12) = 4^2 + 8 = 24 \rightarrow k = 6/5$

The total solution is

$y(n) = y_p(n) + y_h(n)$

$= \left[\frac{6}{5} n 4^n + c_1 4^n + c_2 (-1)^n\right] u(n)$

to find c_1 and c_2, let $y(-2)$ & $y(-1) = 0$ then $y(0) = 1$

$y(1) = 3y(0) + 4 + 2 = 9$

$c_2 + c_1 = 1$ — ①

$\frac{24}{5} + 4c_1 - c_2 = 9 \Rightarrow 4c_1 - c_2 = \frac{21}{5}$ — ②

from ① & ②

$c_1 = \frac{26}{25}$ & $c_2 = -1/25$

so $y(n) = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25} (-1)^n\right] u(n)$

31) Determine the impulse response of the following causal s/m

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$.

31) Characteristic Eq $\lambda^2 - 3\lambda - 4 = 0$

$\lambda = -4, 1$

$y_h(n) = c_1 4^n + c_2(-1)^n$

$x(n) = \delta(n)$

$y(0) = 1$, and $y(1) - 3y(0) = 2$

$y(1) = 5$

so $c_1 + c_2 = 1$ —— ①

$4c_1 - c_2 = 5$ —— ②

from ① & ② $c_1 = \frac{6}{5}$ & $c_2 = -1/5$

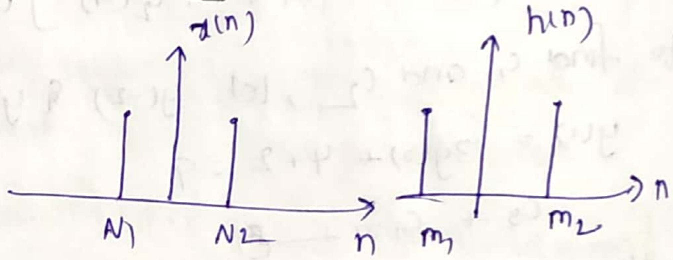
$\therefore h(n) = \left[\frac{6}{5} 4^n - \frac{1}{5}(-1)^n\right] u(n)$

32) Let $x(n)$, $N_1 \le n \le N_2$ and $h(n)$, $m_1 \le n \le m_2$ be two finite duration sig

a) Determine the range $L_1 \le n \le L_2$ of their Convolution in terms of N_1, N_2 and m_1 & m_2

(a) $L_1 = N_1 + m_1$

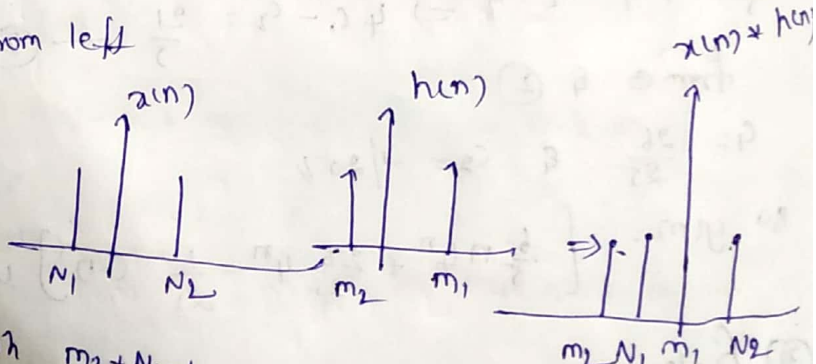
$L_2 = N_2 + m_2$



(b)

partial over lap from left

$\rightarrow x(n) * h(n) \Rightarrow$



Low $N_1 + m_1$ & high $m_2 + N_1 - 1$

If fully overlap then $N_1 + m_2$ (Low) & high $N_2 + m_1$

partial overlap from right

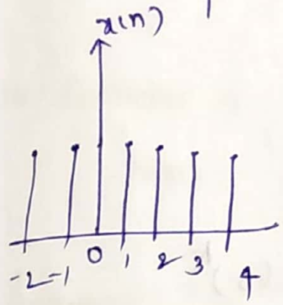
Low $\Rightarrow N_2+m_1+1$

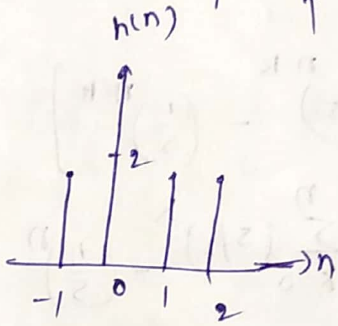
high $\Rightarrow N_2+m_2$

if fully overlapped high N_2+m_1 ; Low $= N_1+m_2$

(c) $x(n) = \begin{cases} 1 & -2 \le n \le 4 \\ 0 & \text{else where} \end{cases}$ $h(n) = \begin{cases} 2 & -1 \le n \le 2 \\ 0, & \text{else where} \end{cases}$

$x(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1\}$; $h(n) = \{2, \underset{\uparrow}{2}, 2, 2\}$



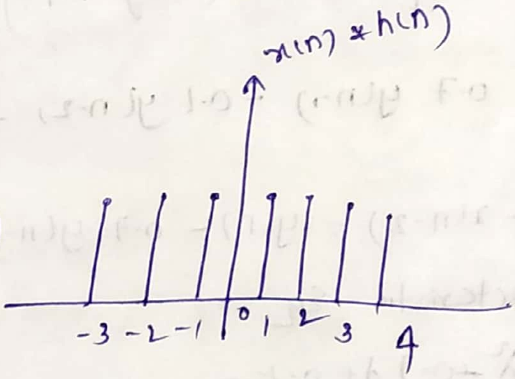


$N_1 = -2$, $N_2 = 4$ $m_1 = -1$, $m_2 = 2$

partial overlap from left

Low $N_1+m_1 = -3$

high $m_2+N_1-1 = 2-2-1 = -1$



full overlap $n=0$ $n=3$

partial right: $n=4$, $n=6$, $L_2 = 6$

33) Determine the impulse response and the unit step response the s/m described by the difference Eqn

(a) $y(n) = 0.6\, y(n-1) - 0.08\, y(n-2) + x(n)$

(i) $x(n) = y(n) - 0.6\, y(n-1) - 0.08\, y(n-2)$

Characteristic Eqn

$\lambda^2 - 0.6\lambda + 0.08 = 0$

$\lambda = 1/2,\ 2/5$

$$y_h(n) = c_1(1/5)^n + c_2(2/5)^n$$

Impulse response $x(n) = \delta(n)$ with $y(0) = 1$

$y(1) - 0.6\,y(0) = 0 \Rightarrow y(1) = 0.6$

so $c_1 + c_2 = 1$ —①

$\frac{1}{5}c_1 + \frac{2}{5}c_2 = 0.6$ —②

from ① & ② $c_1 = -1,\ c_2 = 3$

$$\therefore h(n) = \left[-\left(\tfrac{1}{5}\right)^n + 2\left(\tfrac{2}{5}\right)^n\right]u(n)$$

Step response $x(n) = u(n)$

$$s(n) = \sum_{k=0}^{n} h(n-k) \ ;\ n \geq 0$$

$$= \sum_{k=0}^{n}\left[2(2/5)^{n-k} - \left(\tfrac{1}{5}\right)^{n-k}\right]$$

$$= 2\left(\tfrac{2}{5}\right)^n \sum_{k=0}^{n}(5/2)^k - \left(\tfrac{1}{5}\right)^n \sum_{k=0}^{n}(5)^k$$

$$= \left[2(2/5)^n\left((5/2)^{n+1} - 1\right)\right] - (1/5)^k\left(5^{n+1} - 1\right)\Big]u(n)$$

(b) $y(n) = 0.7\,y(n-1) - 0.1\,y(n-2) + 2x(n) - x(n-2)$

$2x(n) - x(n-2) = y(n) - 0.7\,y(n-1) + 0.1\,y(n-2)$

Characteristic Eq

$\lambda^2 + 0.7\lambda + 0.1 = 0$

$\lambda = 1/2,\ 1/5$

$$y_h(n) = c_1(1/2)^n + c_2(1/5)^n$$

Impulse response $x(n) = \delta(n)\ y(0) = 2$

$y(1) - 0.7\,y(0) = 0 \Rightarrow y(1) = 1.4$

$c_1 + c_2 = 2$

$\frac{1}{2}c_1 + \frac{1}{5}c_2 = 7/5$ —①

$c_1 + \frac{2}{5}c_2 = 14/5$ —②

Solving 1 & 2

$c_1 = \frac{10}{3}, \quad c_2 = -4/3$

So $h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(1/5\right)^n\right] u(n)$

Step Response $s(n) = \sum_{k=0}^{n} h(n-k)$

$$= \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}\left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$= \frac{10}{3}\left[\frac{1}{2}^n\left(2^{n+1}-1\right)u(n)\right] - \frac{4}{3}\left[\frac{1}{5}^n\left[5^{n+1}-1\right]\right]u(n)$$

34) Consider a s/m with Impulse response

$$h(n) = \begin{cases}\left(\frac{1}{2}\right)^n, & 0 \le n \le 4 \\ 0, & \text{else where}\end{cases}$$

Determine the i/p $x(n)$ for $0 \le n \le 8$ that will generate o/p seq $y(n) = \{1, 2, 2.5, 3, 3, 3, 2, 1, 0\}$ (↑ at 1)

A) $h(n) = \{1, 1/2, 1/4, 1/8, 1/16 \ldots\}$ (↑ at 1)

$y(n) = \{1, 2, 2.5, 3, 3, 3, 2, 1, 0, \ldots\}$ (↑ at 1)

$y(0) = x(0)\,h(0)$

$y(0) = x(0) \cdot 1 \Rightarrow x(0) = 1$

$y(1) = x(1) + h(1)\,x(0)$

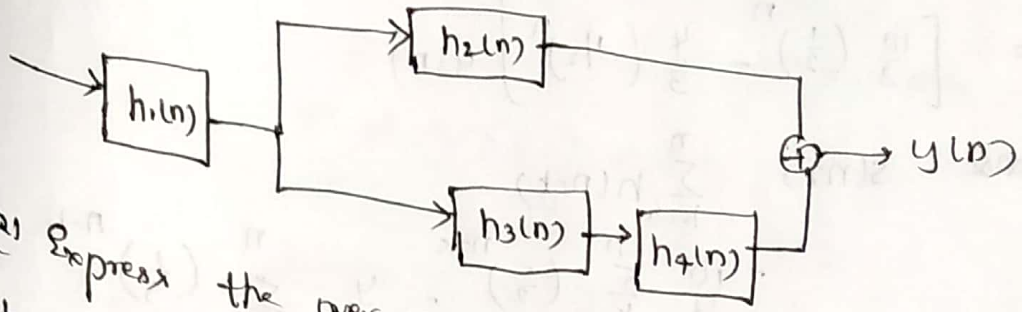
$2 = x(1) + \frac{1}{2}(1) \Rightarrow x(1) = 3/2$

$y(2) = x(2) + h(2)\,x(1) + h_1(0)$

$2.5 = x(2) + \frac{1}{4}(3/2) + 1/2(1)$

So $x[n] = \{1, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, \frac{3}{2} \ldots\}$

Consider the interconnection of LTI s/m as shown



(a) Express the overall impulse response interms of $h_1(n)$, $h_2(n)$, $h_3(n)$, $h_4(n)$

(a) $h(n) = h_1(n) * [h_2(n) - \{h_3(n) * h_4(n)\}]$

(b) Determine $h(n)$ when $h_1(n) = \{1/2, 1/4, 1/2\}$

$h_2(n) = h_3 = (n+1)u(n)$

$h_4(n) = \delta(n-2)$

$h_3(n) * h_4(n) = (n+1)u(n) * \delta(n-2)$

$= (n+1)u(n-2) = (n+1)u(n-2)$

$h_2(n) - [h_3(n) * h_4(n)] = (n+1)u(n) - (n+1)u(n-2)$

$= 2u(n) - \delta(n)$

$h_1(n) = \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)$

$h(n) = \left[\frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)\right] * [2u(n) - \delta(n)]$

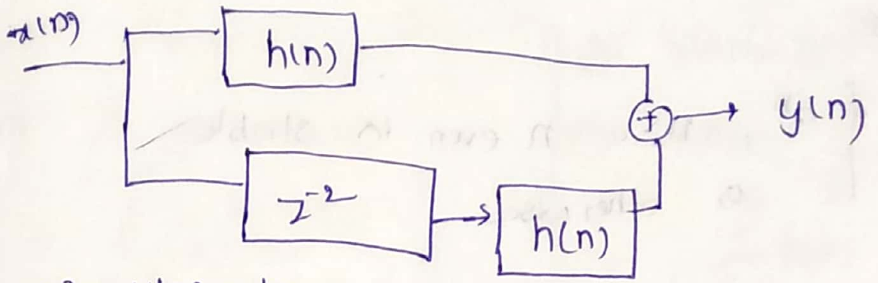
$= \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)$

(c) Determine the response of the s/m in part (b) if

$x(n) = \delta(n+2) + 3\delta(n-1) - 4\delta(n-3)$

$x(n) = \{1, 0, \underset{\uparrow}{0}, 3, 0, -4\}$

(36) $h(n) = a^n u(n)$, $-1 < a < 1$ Determine the response $y(n)$ of s/m of Excitation. $x(n) = u(n+5) - u(n-10)$



(A) $s(n) = u(n) * h(n)$

$$s(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{\infty} h(n-k)$$

$$= \sum_{k=0}^{\infty} a^{n-k} = \frac{a^{n+1} - 1}{a-1} \; ; \; n \geq 0$$

for $x(n) = u(n+5) - u(n-10)$ then

$$s(n+5) - s(n-10) = \frac{a^{n+6} - 1}{a-1} u(n+5) - \frac{a^{n-9} - 1}{a-1} u(n-10)$$

from given fig $y(n) = x(n) * h(n) - x(n) * h(n-2)$

$$y(n) = \frac{a^{n+6} - 1}{a-1} u(n+5) - \frac{a^{n-9} - 1}{a-1} u(n-10) -$$

$$= \frac{a^{n+4} - 1}{a-1} u(n+3) + \frac{a^{n-11} - 1}{a-1} u(n-11)$$

(37) Compute and sketch step response of the s/m

$$y(n) = \frac{1}{M} \sum_{k=0}^{m-1} x(n-k)$$

$$h(n) = \left[\frac{u(n) - u(n-m)}{m} \right]$$

$$s(n) = \sum_{k=-\infty}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} h(n-k) = \begin{cases} \frac{n+1}{m}, & n < m \\ 1, & n \geq m \end{cases}$$

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(38) Determine the range of values of the parameter a for which the linear time-invariant s/m with impulse response

$$h(n) = \begin{cases} a^n, & n \geq 0 \text{ n even} \\ 0, & \text{otherwise} \end{cases}$$ is stable

(A) $$\sum_{k=-\infty}^{\infty} |h(n)| = \sum_{n=0,\ n=\text{even}}^{\infty} |a|^n$$

$$= \sum_{n=0}^{\infty} |a|^{2n} = \frac{1}{1-|a|^2} \quad \text{stable if } |a|<1$$

(39) Determine the response of the s/m. with impulse response $h(n) = a^n u(n)$ to i/p s/g $x(n) = u(n) - u(n-10)$

(A) $h(n) = a^n u(n)$

$$y_1(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} a^{n-k} = a^n \sum_{k=0}^{n} a^{-k} = \frac{1-a^{n+1}}{1-a} u(n)$$

$$y(n) = y_1(n) - y_1(n-10)$$

$$= \frac{1}{1-a}\left[\left(1-a^{n+1}\right)u(n) - \left(1-a^{n-9}\right)u(n-10)\right]$$

40) $h(n) = \left(\frac{1}{2}\right)^n u(n)$ to i/p s/g $x(n) = \begin{cases} 1, & 0 \leq n \leq 10 \\ 0, & \text{otherwise} \end{cases}$

Ans from 36 problem with $a = 1/2$

$$y(n) = 2\left[1-\left(\tfrac{1}{2}\right)^{n+1}\right]u(n) - 2\left[1-\left(\tfrac{1}{2}\right)^{n-9}\right]u(n-10)$$

41) Determine the response of s/m characterised by the impulse response $h(n) = \left(\frac{1}{2}\right)^n u(n)$ to i/p signals

(a) $x(n) = 2^n u(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k 2^{n-k}$$

$$= 2^n \sum_{k=0}^{n} \left(\frac{1}{4}\right)^k$$

$$= 2^n \left[1 - \left(\frac{1}{4}\right)^{n+1}\right] \left(\frac{4}{3}\right)$$

$$= \frac{2}{3}\left[2^{n+1} - \left(\frac{1}{2}\right)^{n+1}\right] u(n)$$

(b) $x(n) = u(-n)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=0}^{\infty} h(k)$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k = 2, \quad n < 0$$

$$y(n) = \sum_{k=n}^{\infty} h(k)$$

$$= \sum_{k=n}^{\infty} \left(\frac{1}{2}\right)^k$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k - \sum_{k=0}^{n-1} \left(\frac{1}{2}\right)^k$$

$$= 2 - \left(\frac{1 - \left(\frac{1}{2}\right)^n}{1/2}\right)$$

$$= 2\left(1/2\right)^n, \quad n \geq 0.$$

42)

Three s/m's with impulse responses $h(n) = \delta(n) - \delta(n-1)$, $h_2(n) = h(n)$. and $h_3(n) = u(n)$, are connected in cascade

(a) What is the impulse response of $h_c(n)$ of the overall s/m

$$h_c(n) = h_1(n) * h_2(n) * h_3(n)$$

$$= [\delta(n) - \delta(n-1)] * u(n) * h(n)$$

$$= [u(n) - u(n-1)] * h(n)$$

$$= \delta(n) * h(n) = h(n)$$

(b) No

(43) (a) prove and Explain graphically the difference b/w relations $x(n)\,\delta(n-n_0) = x(n_0)\,\delta(n-n_0)$ &

$$x(n) * \delta(n-n_0) = x(n-n_0)$$

$x(n)\,\delta(n-n_0) = x(n_0)$. Thus only the value of $x(n)$ at $n = n_0$ is of interest.

$x(n) * \delta(n-n_0) = x(n-n_0)$. Thus we obtain shifted version of $x(n)$ sequence.

(b) Show that discrete time s/m, which is described by convolution summation is LTI are relaxed.

A) $$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= h(n) * x(n)$$

Linearity:

$$x_1(n) \longrightarrow y_1(n) = h(n) * x_1(n)$$

$$x_2(n) \longrightarrow y_2(n) = h(n) * x_2(n)$$

$$= \alpha h(n) * x_1(n) + \beta h(n) * x_2(n)$$

$$= \alpha y_1(n) + \beta y_2(n)$$

Time Invariance:

$$x(n) \longrightarrow y_1(n) = h(n) * x(n)$$

$$x(n-n_0) \longrightarrow y_1(n) = h(n) * x(n-n_0)$$

$$= \sum_k h(k)\, x(n-n_0-k)$$

$$= y(n-n_0)$$

c) Impulse response of $y(n) = x(n-n_0)$

A) $h(n) = \delta(n-n_0)$

(45) Compute the zero state response of the s/m described by the difference equation $y(n) + \frac{1}{2} y(n-1) = x(n) + 2x(n-2)$ to the i/p $x(n) = \{1, 2, 3, 4, 2, 1\}$ by solving the difference equation recursively.

Ans $y(n) = -\frac{1}{2}y(n-1) + x(n) + 2x(n-2)$

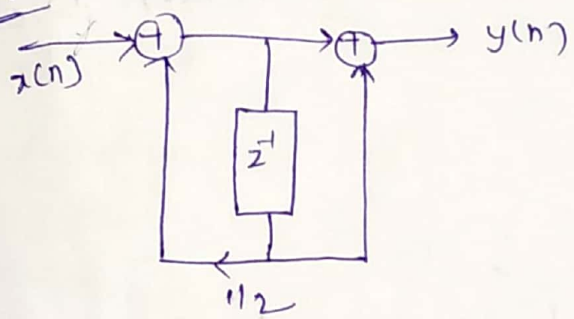
at $y(-2) = -\frac{1}{2}y(-3) + x(-2) + 2x(-4) = 1$

$y(-1) = -\frac{1}{2}y(-2) + x(-1) + 2x(-3) = 3/2$

$y(0) = -1/2\, y(-1) + 2x(-2) + x(0) = 17/4$

$y(1) = -\frac{1}{2}y(0) + x(1) + 2x(-1) = 47/8$

2.47



(a) $x(n) = \{\underset{\uparrow}{1}, 0, 0, \ldots\}$

$y(n) = \frac{1}{2}y(n-1) + x(n) + x(n-1)$

$y(0) = x(0) = 1$

$y(1) = \frac{1}{2}y(0) + x(1) + x(0) = 3/2$

$y(2) = \frac{1}{2}y(1) + x(2) + x(1) = 3/4$, thus we obtain

$y(n) = \{1, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{3}{32} \ldots\}$

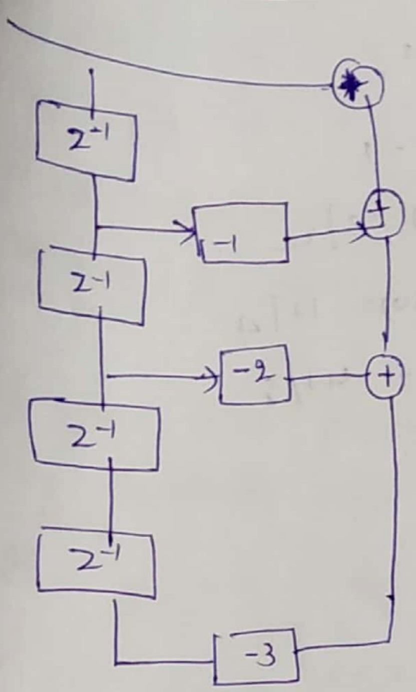
(b) $y(n) = \frac{1}{2}y(n-1) + x(n) + x(n-1)$

(c) as in part (a) we obtain

$y(n) = \{1, 5/2, 13/4, \frac{29}{8}, \frac{61}{16} \ldots\}$

(d) $y(n) = u(n) * h(n)$

$= \sum_k u(k)\, h(n-k)$



$$= \sum_{k=0}^{n} h(n-k)$$

$y(0) = h(0) = 1$

$y(1) = h(0) + h(1) = 5/2$

$y(2) = h(0) + h(1) + h(2) = \frac{13}{4}$ etc

(e) from part (a), $h(n) = 0$ for $n < 0 \Rightarrow$ the s/m is causal

$\sum_{n=0}^{\infty} |h(n)| = 1 + 3/2\left(1 + 1/2 + 1/4 + \dots\right) = 4 \Rightarrow$ s/m is stable

(48) Consider the s/m described by the differential eq

$y(n) = a\,y(n-1) + b\,x(n)$

(a) Determine b in terms of a so that $\sum_{n=-\infty}^{\infty} h(n) = 1$

Ans $y(n) = a\,y(n-1) + b\,x(n)$

$h(n) = b\,a^n u(n)$

$\sum_{n=0}^{\infty} h(n) = \frac{b}{1-a} = 1$

$b = 1-a$

(b) Compute the zero-state step response $s(n)$ of the s/m & choose b so that $s(\infty) = 1$

(c) Compare the values of b obtained in parts (a) & (b). What did you notice

(b) $S(n) = \sum_{k=0}^{n} h(n-k)$

$$= b\left[\frac{1-a^{n+1}}{1-a}\right] u(n)$$

$S(\infty) = \frac{b}{1-a} = 1 \qquad b = 1-a$

(c) $b = 1-a$ in both the cases

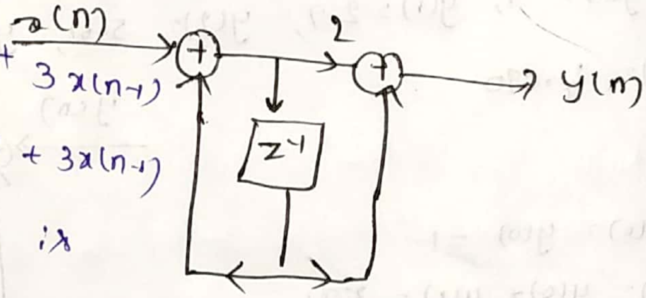
49) A discrete time s/m is relized by the structure shown in fig.

(a) Determine the impulse response

(b) Determine a realization for its inverse s/m, that is the s/m which produces $x(n)$ as an o/p when $y(n)$ is used as an i/p.

(a) $y(n) = 0.8y(n-1) + 2x(n) + 3x(n-1)$

$y(n) - 0.8y(n-1) = 2x(n) + 3x(n-1)$



The characteristic Eq n is

$\lambda - 0.8 = 0$

$\lambda = 0.8$

$y_h(n) = c(0.8)^n$

let us first consider the response of the s/m

$y(n) - 0.8y(n-1) = x(n)$

to $x(n) = \delta(n)$. Since $y(0) = 1$ it follows that $c = 1$, then the impulse response of original s/m is

$h(n) = 2(0.8)^n u(n) + 3(0.8)^{n-1} u(n-1)$

$= 2\delta(n) + 4.6(0.8)^{n-1} u(n-1)$

(b) The inverse s/m is characterized by the difference Eq n

$x(n) = -1.5x(n-1) + \frac{1}{2}y(n) - 0.4y(n-1)$

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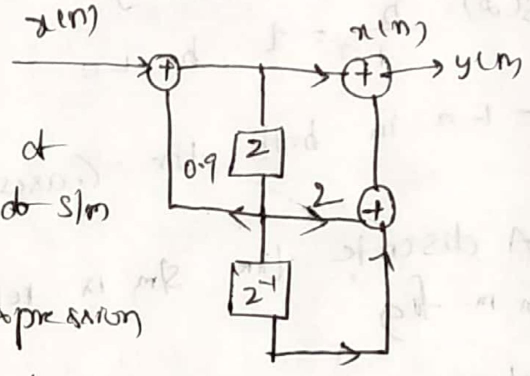
2.50)

(a) Compute 1st six values of the x(n) impulse response of S/m



(b) Compute the 1st six values of Zero state step response of S/m

(c) Determine an analytical expression for the impulse response of the S/m.

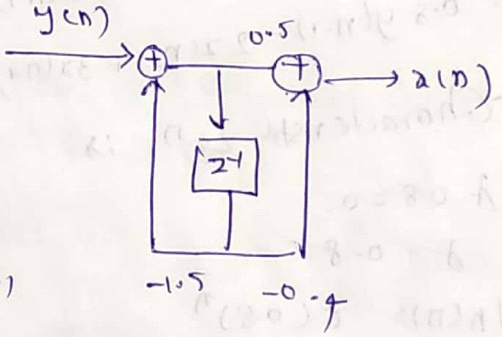
(a) $y(n) = 0.9\, y(n-1) + x(n) + 2x(n-1) + 3x(n-2)$

$y(n) - 0.9\, y(n-1) = x(n) + 2x(n-1) + 3x(n-2)$

(a) for $x(n) = \delta(n)$ we have

$y(0) = 1,\ y(1) = 2.9,\ y(2) = 5.61,\ y(3) = 5.049,\ y(4) = 4.54$

$y(5) = 4.090$



(b) $s(0) = y(0) = 1$

$s(1) = y(0) + y(1) = 3.9,$

$s(2) = y(0) + y(1) + y(2) = 9.51$

$s(3) = y(0) + y(1) + y(2) + y(3) = 14.56$

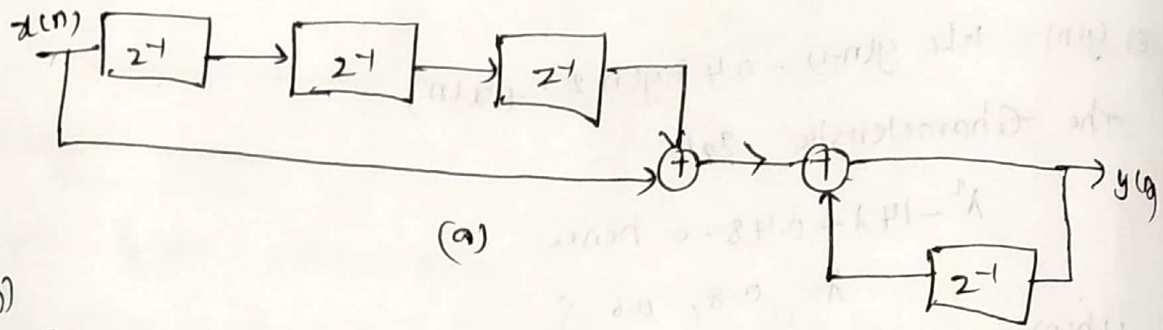
$s(4) = \sum_{0}^{4} y(n) = 19.10$

$s(5) = \sum_{0}^{5} y(n) = 23.19$

(c) $h(n) = (0.9)^n u(n) + 2(0.9)^{n-1} u(n-1) + 3(0.9)^{n-2} u(n-2)$

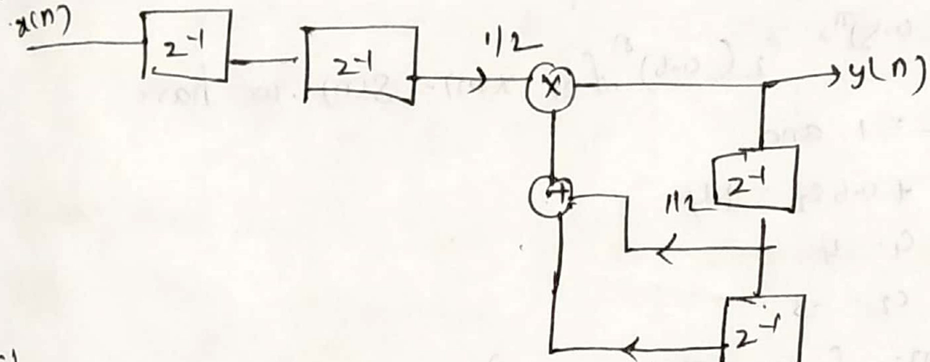
$= \delta(n) + 2.9\,\delta(n-1) + 5.16\,(0.9)^{n-2} u(n-2)$

5) Determine and sketch the impulse response of the following s/m for n = 0, 1, .. 9)

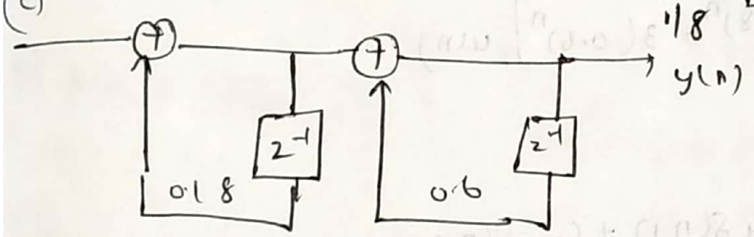


(a)

(b)



(c)



(a) $y(n) = \frac{1}{3} x(n) + \frac{1}{3} x(n-3) + y(n-1)$

for $x(n) = \delta(n)$, we have

$h(n) = \left\{ \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3} \ldots \right\}$

(b) $y(n) = \frac{1}{2} y(n-1) + \frac{1}{8} y(n-2) + \frac{1}{2} x(n-2)$

With $x(n) = \delta(n)$ and

$y(-1) = y(-2) = 0$, we obtain

$h(n) = \left\{ 0, 0, 1/2, 1/4, 3/16, 1/8, 11/128, \frac{15}{256}, \frac{41}{1024}, \ldots \right\}$

(c) $y(n) = 1.4\, y(n-1) - 0.48\, y(n-2) + x(n)$

with $x(n) = \delta(n)$ &

$y(-1) = y(-2) = 0$ we obtain

$h(n) = \{1, 1.4, 1.48, 1.4, 1.2496, 1.0774, 0.9086 \ldots\}$

d) All three S/m's are IIR

(e) $y(n) = 1.4\, y(n-1) - 0.48\, y(n-2) + x(n)$

The Characteristic Eqn

$$\lambda^2 - 1.4\lambda + 0.48 = 0 \text{ hence}$$

$$\lambda = 0.8,\ 0.6 \ \{$$

$y_h(n) = C_1(0.8)^n + C_2(0.6)^n$ for $x(n) = \delta(n)$, we have

$C_1 + C_2 = 1$ and

$0.8C_1 + 0.6C_2 = 1.4$

$C_1 = 4$

$C_2 = -3$

$h(n) = \left[4(0.8)^n - 3(0.6)^n\right] u(n)$

2.52

(a) $h_1(n) = c_0\,\delta(n) + c_1\,\delta(n-1) + c_2\,\delta(n-2)$

$h_2(n) = b_2\,\delta(n) + b_1\,\delta(n-1) + b_0\,\delta(n-2)$

$h_3(n) = a_0\,\delta(n) + (a_1 + a_0 a_2)\,\delta(n-1) + a_1 a_2\,\delta(n-2)$

(b) The only question is whether

$h_3(n) \not= h_2(n) = h_1(n)$

let $a_0 = c_0$

$a_1 + a_2 c_0 = c_1, \Rightarrow a_1 + a_2 c_0 - c_1 = 0$

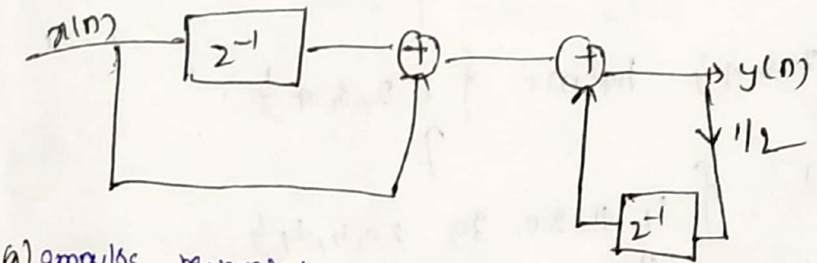
$a_2 a_1 = c_2 \Rightarrow \frac{c_2}{a_2} = a_1$

$\Rightarrow \frac{c_2}{a_2} + a_2 c_0 - c_1 = 0$

$\Rightarrow c_0 a_2^2 - c_1 a_2 + c_2 = 0$

for $c_0 \neq 0$, the quadratic has a real solution if and only if $c_1^2 - 4c_0 c_2 \geq 0$

(53)



(a) Impulse response.

$$y(n) = \frac{1}{2}y(n-1) + x(n) + x(n-1)$$

for $y(n) - \frac{1}{2}y(n-1) = x(n) + x(n-1)$ $\quad x(n) = \delta(n)$

$$h(n) = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1)$$

b) show that $h(n)$ is Equal to the convolution of the following sig's: $h_1(n) = \delta(n) + \delta(n-1)$

$h_2(n) = (1/2)^n u(n)$

$$h_1(n) * [\delta(n) + \delta(n-1)] = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1)$$

(54)

(a) $x_1(n) = \{\underset{\uparrow}{1}, 2, 4\}$; $h(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

Convolution: $y_1(n) = \{\underset{\uparrow}{1}, 3, 7, 7, 7, 6, 4\}$

Correlation ; $r_1(n) = \{1, 3, 7, 7, \underset{\uparrow}{7}, 6, 4\}$

(b) $x_2(n) = \{\underset{\uparrow}{0}, 1, -2, 3, -4\}$; $h_2(n) = \{1/2, 1, \underset{\uparrow}{2}, 1, 1/2\}$

Convolution : $y_2(n) = \{1/2, \underset{\uparrow}{0}, 3/2, -2, 1/2, -6, -5/2, -2\}$

Correlation $r_1(n) = \{\frac{1}{2}, \underset{\uparrow}{0}, 3/2, -2, 1/2, -6, -5/2, -2\}$

(c) $x_3(n) = \{1, 2, 3, 4\}$, $h_3(n) = \{4, 3, 2, 1\}$

(d) $x_4(n) = \{1, 2, 3, 4\}$ $h_4(n) = \{1, 2, 3, 4\}$

(c) Convolution $y_3(n) = \{4, 11, 20, 30, 20, 11, 4\}$

R_{xx} $r_1(n) = \{1, 4, 10, 20, 25, 24, 16\}$

Convolution : $y_4(n) = \{1, 4, 10, 20, 25, 24, 16\}$

Correlation : $r_4(n) = \{4, 11, 20, 30, 20, 11, 4\}$

Note that $h_3(-n) = h_4(n+3)$

hence $r_3(n) = y_4(n+3)$

$h_4(-n) = h_3(n+3)$

$r_4(n) = y_3(n+3)$

(55)

$x(n) = \{1, 3, 3, 1\}$; $y(n) = \{1, 4, 6, 4, 1\}$

$x(n) * y(n) = h(n)$

length of $h(n) = 2$ $h(n) = \{h_0, h_1\}$

$h_0 = 1$

$3h_0 + h_1 = 4$

$\Rightarrow h_0 = 1, h_1 = 1$

(56)

(2.5.6) $y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$

(2.5.9) $w(n) = -\sum_{k=1}^{N} a_k w(n-k) + x(n)$

(2.5.10) $y(n) = \sum_{k=0}^{M} b_k w(n-k)$

from 2.5.9 we obtain $x(n) = w(n) + \sum_{k=1}^{N} a_k w(n-k)$

by substituting (2.5.10) for $y(n)$ & (A) into (2.5.6) we obtain L.H.S = R.H.S

(5.7)

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

i/p $x(n) = (-1)^n u(n)$ and initial conditions are

$y(-1) = y(-2) = 0$

$y(n) - 4y(n-1) + 4y(n-2) \Rightarrow x(n) - x(n-1)$

The characteristic Eq is

$$\lambda^2 - 4\lambda + 4 = 0$$

$\lambda = 2, 2.$ Hence

$$y_h(n) = C_1 2^n + C_2 n 2^n$$

the particular solution is

$$y_p(n) = k(-1)^n u(n)$$

Substituting this sol^n into the difference Eq^n, we obtain

$k(-1)^n u(n) - 4k(-1)^n u(n-1) + 4k(-1)^{n-2} u(n-2) = (-1)^n u(n) - (-1)^{n-1}$.

for $n=2$, $k(1+4+4) = 2 \Rightarrow k = 2/9$. The total solution is

$$y(n) = \left[C_1 2^n + C_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n)$$

from initial condition we obtain $y(0) = 1$, $y(1) = 2$

$C_1 + \frac{2}{9} = 1$

$\Rightarrow C_1 = 7/9$

$2C_1 + 2C_2 - 2/9 = 2$

$\Rightarrow C_2 = 1/3$

(58) $y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

from prob (57)

$$h(n) = [c_1 2^n + c_1 n 2^n] u(n)$$

With

$y(0) = 1$, $y(1) = 3$, we have

$c_1 = 1$ $\quad 2c_1 + 3c_1$

$c_2 = 1/2$

Thus $h(n) = \left[2^n + \frac{1}{2} n 2^n\right] u(n)$

(59)

$$x(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]\, u(n-k)$$ where $u(n-k)$

is delayed by k unit.

$$u(n-k) = \begin{cases} 1 & n \geq k \\ 0 & \text{otherwise} \end{cases}$$

(a) $x(n) = x(n) * \delta(n)$

$= x(n) * [u(n) - u(n-1)]$

$= [x(n) - x(n-1)] * u(n)$

$$x(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]\, u(n-k)$$

(b) let $h(n)$ be the impulse response of s/m

$$s(k) = \sum_{m=-\infty}^{k} h(m)$$

$h(k) = s(k) - s(k-1)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)]\, x(n-k)$$

$\therefore\ r_{xx}(l) = \{1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1\}$

(b) $r_{yy}(l) = \sum_{n=-\infty}^{\infty} y(n)\, y(n-l)$

We obtain $r_{yy}(l) = \{1, 3, 5, 7, 5, 3, 1\}$

We obtain $y(n) = x(-n+3)$ which is Equivalent to receiving the Sequence $x(n)$. This has not changed the

(63) $x(n) = \begin{cases} 1, & -N \le n \le N \\ 0, & \text{otherwise} \end{cases}$

$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$

$= \begin{cases} 2N+1-|l|, & -2N \le l \le 2N \\ 0, & \text{else} \end{cases}$

$r_{xx}(0) = 2N+1$

$\therefore$ the normalized auto Correlation is

$\rho_{xx}(l) = \begin{cases} \frac{1}{2N+1}(2N+1-|l|), & -2N \le l \le 2N \\ 0, & \text{else} \end{cases}$

(64)

(a) $r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$

$= \sum_{n=-\infty}^{\infty} [\delta(n) + r_1\,\delta(n-k_1) + r_2\,\delta(n-k_2)] *$
$[\delta(n-l) + r_1\,\delta(n-l-k_1) + r_2\,\delta(n-l-k_2)$

$= (1 + r_1^2 + r_2^2)\, r_{\delta\delta}(l) + r_1 [r_{\delta\delta}(l+k_1) + r_{\delta\delta}(l-k_1)$
$+ r_2 [r_{\delta\delta}(l+k_2) + r_{\delta\delta}(l-k_2) + r_1 r_2 [r_{\delta\delta}(l+$

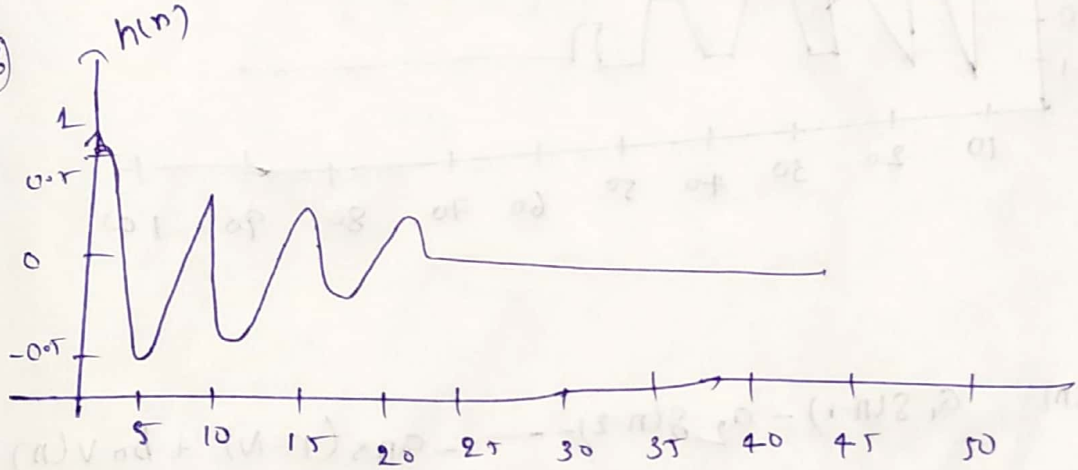
$k_2) + r_{xs}\left[l + k_2 - k_1)\right]$

(b) $r_{xx}(l)$ has peaks at $l = 0$, $\pm k_1$, $\pm k_2$ and $\pm(k_1 + k_2)$. Suppose that $k_1 < k_2$. Then, we can determine r_1 and k_1. The problem is to determine r_2 and k_2 from the other peaks.

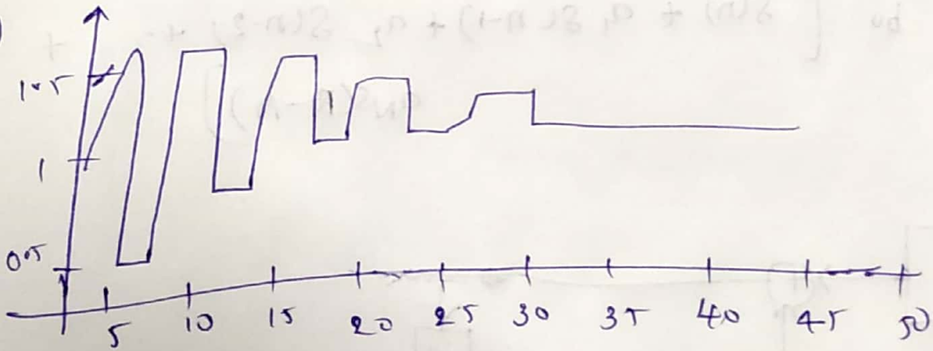
(c) If $r_2 = 0$ the peaks occur at $l = 0$ and $l = \pm k_1$ then it is easy to obtain r_1 and k_1

(66)

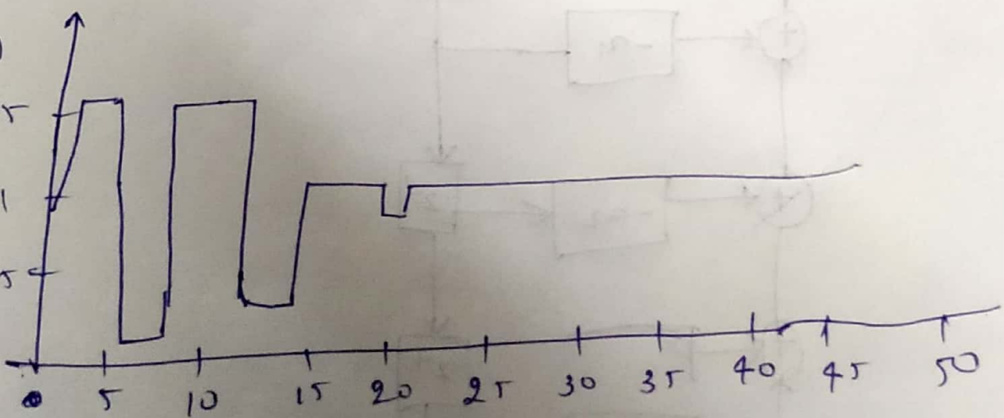
(a)



(b)

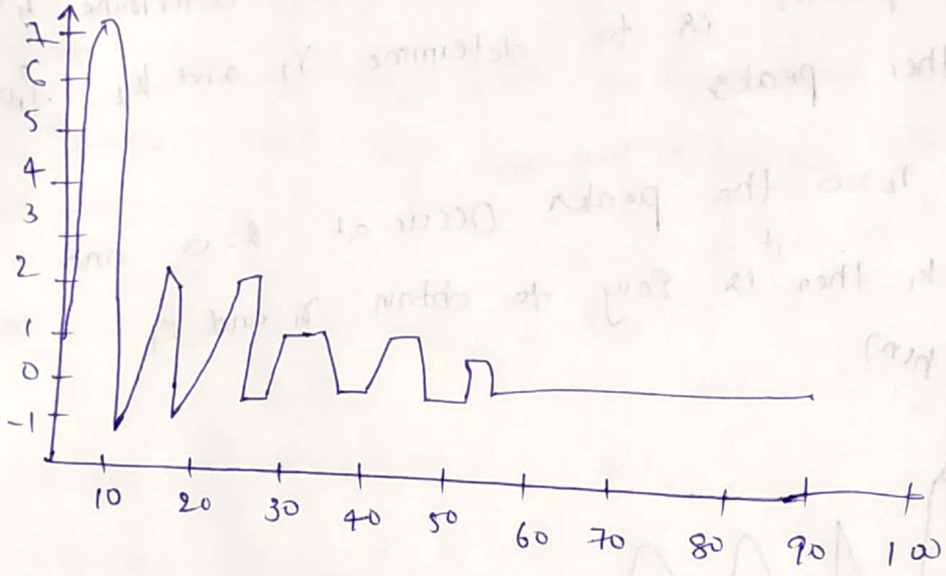


(c)



(d) c, b are similar except c have steady state at $n=20$ where b have nearly at zero ($n=0$)

(67)

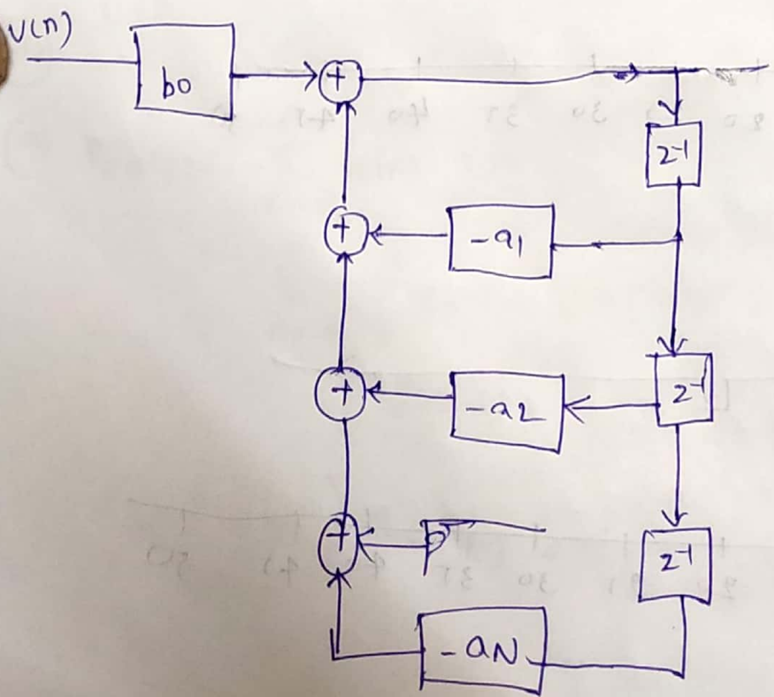


(44)

(a) $s(n) = a_1 s(n-1) - a_2 s(n-2) - \cdots - a_N s(n-N) + b_0 v(n)$

(b) $v(n) = \frac{1}{b_0}\left[s(n) + a_1 s(n-1) + a_2 s(n-2) + \cdots + a_N s(n-N) \right]$

(c)



(b)