

## **Introduction**

This report presents a detailed quantitative analysis of a selected portfolio of eight funds: TMSRX, CRAZX, NBRVX, VDIGX, FBGRX, PRDGX, BPTRX, and ACMVX. The objective is to analyze their performance, conduct exploratory data analysis (EDA), optimize portfolio allocation, and derive actionable insights for optimal investment strategies.

### Tickers Chosen

The selected tickers represent a diverse range of investment strategies and sectors, providing a comprehensive basis for portfolio optimization. Here's a detailed analysis of why each ticker was chosen:

- **TMSRX**: A multi-strategy alternative fund known for its low correlation with traditional asset classes, providing diversification benefits.
- **CRAZZ**: A mid-cap growth fund focusing on companies with high growth potential, offering opportunities for capital appreciation.
- **NBRVX**: A value-oriented fund investing in large-cap companies, balancing growth with stability.
- **VDIGX**: A dividend growth fund focusing on companies with strong dividend growth, ideal for income-seeking investors.
- **FBGRX**: A large-cap growth fund investing in high-growth companies, aiming for long-term capital appreciation.
- **PRDGX**: A blend fund combining growth and value strategies, providing balanced exposure.
- **BPTRX**: A technology-focused fund investing in innovative companies, offering high growth potential.
- **ACMVX**: A small-cap value fund targeting undervalued companies with strong fundamentals, adding diversification.

Ticker	Fund Type	Strategy
TMSRX	Multi-Strategy	Diversification
CRAZZ	Mid-Cap Growth	Capital Appreciation
NBRVX	Large-Cap Value	Stability and Growth
VDIGX	Dividend Growth	Income and Growth
FBGRX	Large-Cap Growth	Long-term Appreciation
PRDGX	Blend	Balanced Exposure
BPTRX	Technology Focused	High Growth Potential
ACMVX	Small-Cap Value	Diversification and Value

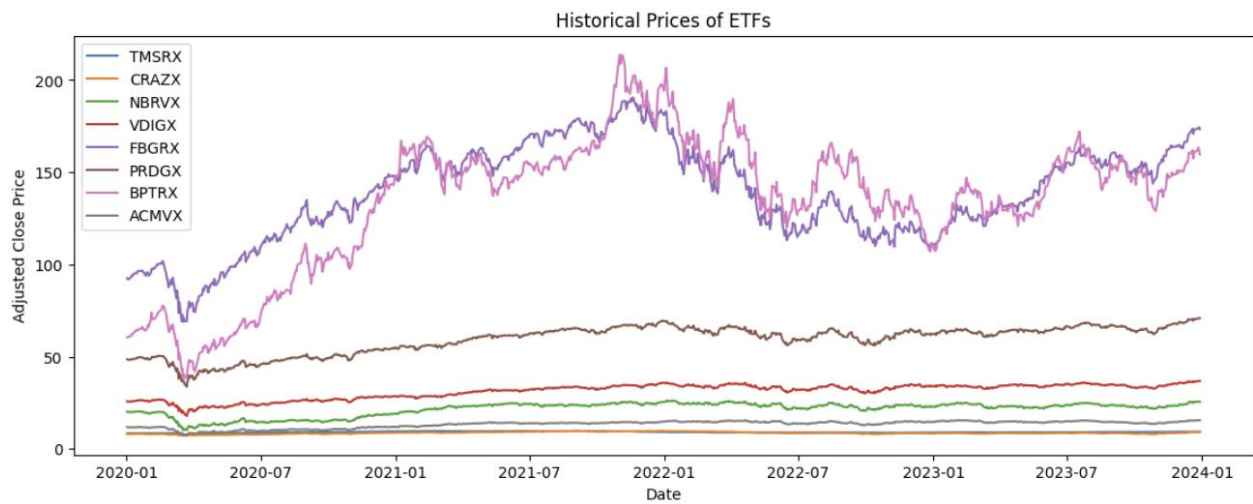
## Exploratory Data Analysis (EDA)

EDA is a crucial step in understanding the historical performance and statistical properties of the selected tickers. This section includes time series analysis, summary statistics, and visualizations to uncover patterns and insights.

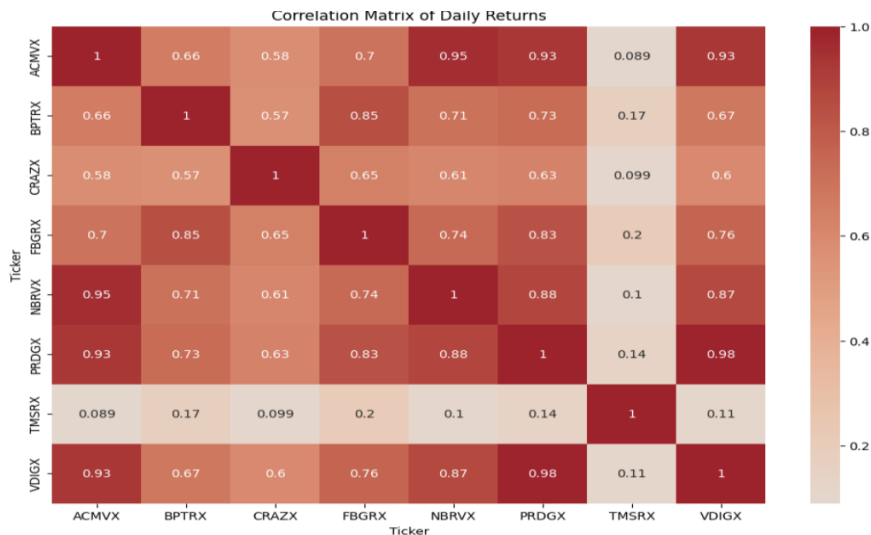
Date	BPTRX	CRAZX	FBGRX	NBRVX	PRDGX	TMSRX	VDIGX
02/01/2020	60.4226	8.1171	92.5334	20.1108	48.5939	8.4103	25.7716
03/01/2020	60.8011	8.0872	91.9003	20.0238	48.3583	8.4103	25.6794
06/01/2020	61.0592	8.1021	92.6009	19.9466	48.4127	8.4277	25.7213
07/01/2020	61.2829	8.0723	92.9556	19.9562	48.2677	8.4537	25.5788
08/01/2020	62.1861	8.1021	93.7069	20.0239	48.4942	8.4537	25.7381

## Visualizations:

Close prices:



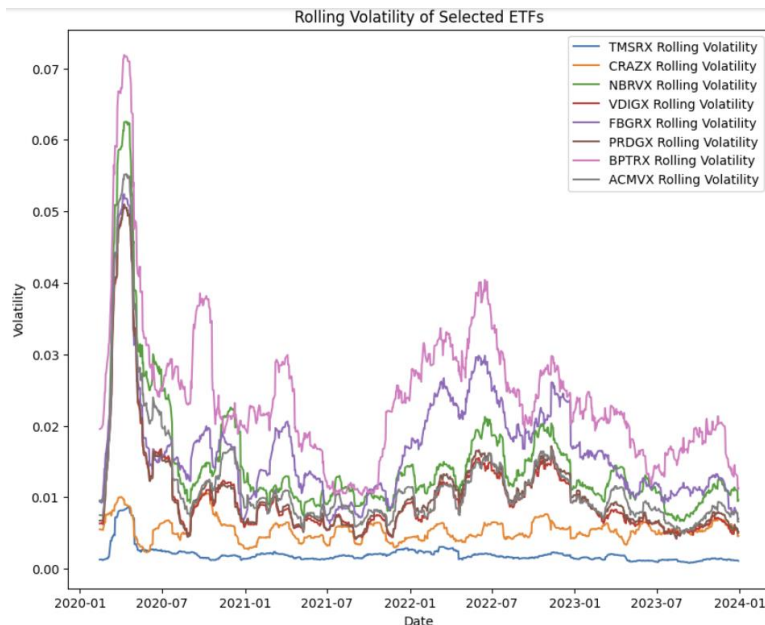
## Co-relation analysis:



- **High Correlations:**

- **ACMVX and NBRVX (0.95)**: This indicates a very strong positive correlation between these two funds, suggesting that they tend to move in the same direction. This high correlation is typical of funds within the same asset class or sector.
- **PRDGX and VDIGX (0.98)**: Another very high correlation, indicating these funds exhibit very similar return patterns.
- **BPTRX and FBGRX (0.85)**: Indicates a strong positive correlation, which could be due to overlapping holdings in technology stocks.
- **Moderate Correlations:**
  - **CRAZX and FBGRX (0.65)**: Indicates a moderate positive correlation, suggesting some level of co-movement but also some diversification benefits.
  - **VDIGX and CRAZX (0.60)**: Reflects a moderate relationship, possibly due to some overlapping sectors or market conditions influencing both funds.
- **Low Correlations:**
  - **TMSRX and Other Funds (0.089 to 0.2)**: TMSRX shows very low correlation with the other funds, underscoring its role as a diversifier in the portfolio due to its multi-strategy approach.
- Funds with high correlations, such as ACMVX and NBRVX or PRDGX and VDIGX, are likely to provide less diversification benefit if held together.
- Including low-correlation assets like TMSRX can reduce overall portfolio volatility.

### Rolling Volatility:



### Initial Volatility Surge:

All funds experienced a surge in volatility at the beginning of 2020, with BPTRX showing the most significant spike due to its tech exposure during the pandemic market upheaval.

#### Volatility Trends:

- **BPTRX:** Maintains the highest volatility throughout the period, reflecting the high-risk, high-reward nature of technology investments.
- **TMSRX:** Consistently exhibits the lowest volatility, showcasing its role as a stabilizing force within the portfolio.

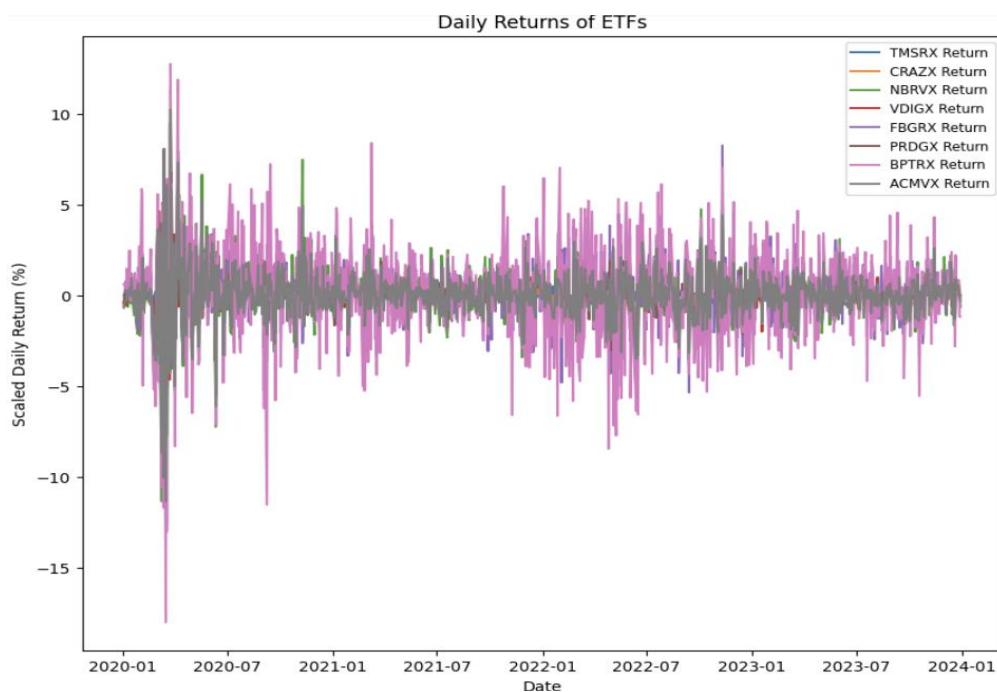
#### Volatility Patterns:

- **Mid-2020 to 2021:** A general decline in volatility can be observed across all funds as markets began to stabilize post-pandemic.
- **2022 onwards:** Slight upticks in volatility are observed, potentially due to geopolitical events, inflation concerns, and changes in monetary policy.

#### Implications:

- High volatility funds like BPTRX can significantly impact the portfolio's risk profile. Allocations to such funds should be carefully managed to align with the investor's risk tolerance.
- Low-volatility funds like TMSRX provide stability and are essential for maintaining a balanced risk profile, especially during periods of market uncertainty.

#### Daily Returns:



#### Daily Returns of ETFs

#### Volatility Spikes:

- Early 2020: The significant spikes in returns, both positive and negative, around the beginning of 2020 correspond to market reactions to the COVID-19 pandemic. This period shows high market volatility across all funds.

**Normalization:**

- Post-2020: A noticeable stabilization in daily returns can be observed after the initial shock of the pandemic, with returns becoming less volatile.

**Return Patterns:**

- **BPTRX:** Exhibits the highest volatility, consistent with its tech-focused investment strategy which is more sensitive to market swings.
- **TMSRX:** Shows the lowest volatility, reflecting its strategy of reducing correlation with traditional asset classes and providing steady returns.

**Implications:**

- Investors seeking high returns must be willing to tolerate significant short-term volatility, as seen with BPTRX.
- For those seeking stability, TMSRX is a preferable choice due to its low volatility.

**Statistical Analysis:**

TF	Initial ADF Statistic	Initial p-value	Post-Differencing ADF Statistic	Post-Differencing p-value
ACMVX	-1.4297	0.5679	-9.1963	2.06E-15
BPTRX	-2.1013	0.244	-9.3615	7.80E-16
PRDGX	-1.2042	0.6718	-9.5721	2.27E-16
FBGRX	-1.5792	0.494	-10.1775	6.83E-18
VDIGX	-1.1841	0.6804	-9.7439	8.36E-17
NBRVX	-1.3933	0.5855	-9.4643	4.27E-16
CRAZX	-1.544	0.5117	-29.7797	0

#### Findings:

1. **Initial Non-Stationarity:** All ETFs exhibited non-stationarity initially, indicated by p-values greater than 0.05.
2. **Post-Differencing Stationarity:** Upon applying differencing, all ETFs achieved stationarity, with significantly low p-values ( $\ll 0.05$ ).
3. **Strong Stationarity for CRAZX:** CRAZX showed the most robust transformation, achieving strong stationarity post-differencing.

#### Skewness and Kurtosis analysis:

##### ACMVX (American Century Mid Cap Value Fund)

- **Kurtosis: 12.50**
- **Skewness: -0.53**

ACMVX exhibits a very high kurtosis, indicating a significant likelihood of extreme return values. This heightened risk of extreme outcomes, whether gains or losses, necessitates careful consideration in portfolio diversification. The negative skewness of -0.53 further suggests that ACMVX is prone to more frequent extreme negative returns. For risk management, this ETF should be balanced with other assets to mitigate potential downturns.

##### BPTRX (Baron Partners Fund)

- **Kurtosis: 4.38**
- **Skewness: -0.54**

BPTRX has a moderately high kurtosis, indicating a reasonable risk of experiencing extreme returns. The negative skewness signifies a tendency towards more frequent significant negative returns. While less extreme than ACMVX, BPTRX still requires strategic risk management and should be included in a diversified portfolio to balance its risk profile.

- **Moderate Risk:** Manageable with proper diversification.
- **Balancing Act:** Including more stable, positively skewed assets.

##### CRAZX (Virtus KAR Small-Cap Growth Fund)

- **Kurtosis: 1.92**
- **Skewness: -0.45**

CRAZX presents low kurtosis, indicating fewer extreme outcomes compared to other ETFs analyzed. However, the negative skewness still implies a risk of frequent negative returns. This fund can act as a stabilizer in a portfolio due to its relatively normal distribution but still requires monitoring for downside risks.

- **Stabilizing Influence:** Useful for balancing higher-risk assets.
- **Downside Risk:** Monitoring and occasional rebalancing needed/

#### **FBGRX (Fidelity Blue Chip Growth Fund)**

- **Kurtosis: 4.75**
- **Skewness: -0.48**

FBGRX shows a moderately high kurtosis, suggesting some risk of extreme return values. The negative skewness indicates a propensity for extreme negative returns. FBGRX's risk profile necessitates careful integration into a diversified portfolio to mitigate its downside potential.

#### **Implications:**

- **Moderate Risk Profile:** Balanced with other low-kurtosis assets.
- **Mitigation Required:** Strategies to offset negative skewness.

#### **NBRVX (Neuberger Berman Real Estate Fund)**

- **Kurtosis: 11.60**
- **Skewness: -0.90**

NBRVX has a very high kurtosis, highlighting a significant probability of extreme return events. With a skewness of -0.90, this ETF is particularly susceptible to extreme negative returns. NBRVX's inclusion in a portfolio demands careful risk mitigation, potentially through pairing with assets having lower risk profiles.

- **High Risk:** Requires strong risk management.
- **Diversification Strategy:** Essential to pair with less risky assets.

#### **PRDGX (T. Rowe Price Dividend Growth Fund)**

- **Kurtosis: 14.04**
- **Skewness: -0.41**

PRDGX exhibits very high kurtosis, suggesting a high likelihood of extreme returns. The negative skewness indicates a greater frequency of significant negative returns. This ETF necessitates a cautious approach in portfolio construction to manage its inherent risks.

- **High Risk and Reward:** Manageable with careful portfolio balancing.
- **Risk Mitigation:** Essential to pair with more stable investments.

#### **TMSRX (Transamerica Managed Risk Strategy ETF)**

- **Kurtosis: 37.06**
- **Skewness: -3.13**

TMSRX presents extremely high kurtosis, indicating a very high risk of extreme returns. Its skewness is also highly negative, suggesting a strong propensity for frequent and severe negative returns. This ETF



demands stringent risk management and should be carefully balanced within a diversified portfolio to mitigate its high-risk profile.

- **Extremely High Risk:** Needs robust risk management strategies.
- **Portfolio Stabilization:** Pair with low-risk, stable assets.

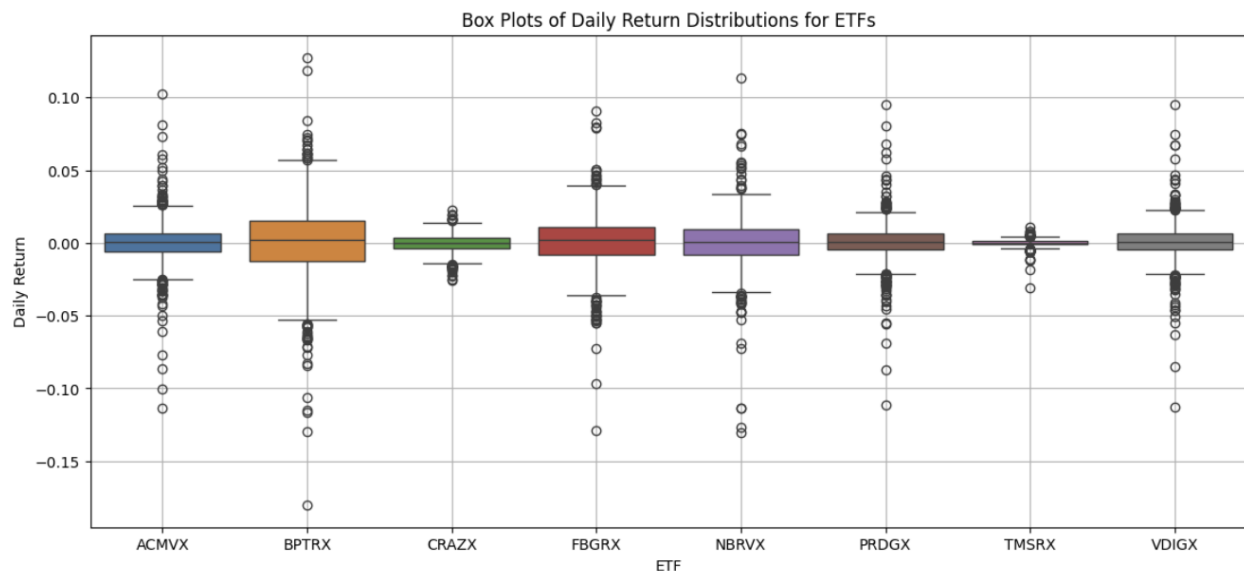
### VDIGX (Vanguard Dividend Growth Fund)

- **Kurtosis: 15.26**
- **Skewness: -0.40**

VDIGX has very high kurtosis, indicating a significant risk of extreme return events. The negative skewness suggests a tendency for frequent extreme negative returns. VDIGX should be managed with a focus on risk mitigation and balanced with less volatile assets in a diversified portfolio.

- **Significant Risk:** Requires comprehensive risk management.
- **Diversification Essential:** Balance with stable, low-risk assets.

### Box Plot



### Detailed Analysis by ETF:

#### ACMVX (American Century Mid Cap Value Fund)

- **Median Return:** Slightly above zero, indicating modest positive daily returns.
- **IQR:** Shows moderate variability in returns.
- **Outliers:** Significant number of negative outliers, suggesting occasional extreme negative returns.

Moderate risk profile with potential for sudden losses. Suitable for a balanced portfolio but requires risk management strategies to mitigate extreme negative events.

#### BPTRX (Baron Partners Fund)

- **Median Return:** Positive, indicating consistent positive daily returns.
- **IQR:** Larger than ACMVX, indicating higher variability.

- **Outliers:** Both positive and negative outliers, but more concentrated on the negative side.

Higher variability and risk compared to ACMVX. The presence of numerous outliers suggests potential for substantial gains but also significant risks. Suitable for aggressive growth portfolios with risk mitigation measures.

#### **CRAZX (Virtus KAR Small-Cap Growth Fund)**

- **Median Return:** Slightly positive, suggesting small daily gains.
- **IQR:** Relatively narrow, indicating lower variability in returns.
- **Outliers:** Some outliers, mostly on the negative side.

Lower risk profile with stable returns. The ETF can act as a stabilizer within a diversified portfolio, balancing out higher-risk assets.

#### **FBGRX (Fidelity Blue Chip Growth Fund)**

- **Median Return:** Positive, reflecting consistent daily gains.
- **IQR:** Moderate, indicating balanced variability.
- **Outliers:** Significant number of negative outliers.

Balanced risk and return profile, but the presence of negative outliers indicates potential for sudden losses. Suitable for growth-oriented portfolios with adequate risk management.

#### **NBRVX (Neuberger Berman Real Estate Fund)**

- **Median Return:** Slightly above zero, suggesting small positive returns.
- **IQR:** Larger, indicating higher variability.
- **Outliers:** Numerous, particularly on the negative side.

High-risk profile with significant variability. Suitable for portfolios looking to capitalize on real estate sector growth but requires strong risk management to handle extreme negative events.

#### **PRDGX (T. Rowe Price Dividend Growth Fund)**

- **Median Return:** Positive, indicating consistent returns.
- **IQR:** Moderate, suggesting balanced variability.
- **Outliers:** Presence of negative outliers.

Balanced ETF suitable for income-focused portfolios. The negative outliers highlight the need for diversification to mitigate risks associated with extreme events.

#### **TMSRX (Transamerica Managed Risk Strategy ETF)**

- **Median Return:** Close to zero, indicating very modest returns.
- **IQR:** Wide, indicating high variability.
- **Outliers:** Numerous extreme negative outliers.

High-risk profile with significant potential for extreme negative returns. Suitable for high-risk, high-reward strategies but necessitates robust risk management frameworks.

#### **VDIGX (Vanguard Dividend Growth Fund)**

- **Median Return:** Positive, reflecting consistent gains.
- **IQR:** Moderate, indicating balanced variability.

- **Outliers:** Negative outliers are present.

Suitable for growth and income-focused portfolios. Requires risk management to handle the potential for significant negative returns.

## **General Insights for Portfolio Construction**

### **Risk Diversification:**

- ETFs like CRAZX, with lower variability, can be used to balance out higher-risk ETFs like TMSRX and NBRVX.
- Diversifying across sectors and risk profiles helps in mitigating the overall portfolio risk.

### **Risk Management:**

- High-kurtosis and negatively skewed ETFs will necessitate active risk management strategies, including stop-loss orders and portfolio insurance.
- Regular rebalancing ensures that the portfolio maintains its desired risk-return profile.

### **Growth vs. Stability:**

- Combining high-growth ETFs (like BPTRX and FBGRX) with more stable ETFs (like CRAZX) can optimize the risk-return trade-off.
- Balancing aggressive and conservative assets ensures potential for growth while maintaining stability.

## Methodology

Portfolio optimization was conducted using the Modern Portfolio Theory (MPT) framework, focusing on three key portfolio strategies: Max Sharpe Ratio Portfolio, Minimum Variance Portfolio, and Efficient Portfolio for Target Return. The results for each portfolio are detailed below.

- **Max Sharpe Ratio Portfolio:** Maximizes the risk-adjusted return.
- **Minimum Variance Portfolio:** Minimizes the portfolio's overall risk.
- **Efficient Portfolio for Target Return:** Achieves a specified return with the lowest risk.

### Max Sharpe Ratio Portfolio

The Max Sharpe Ratio Portfolio aims to maximize the risk-adjusted return. This is achieved by finding the optimal combination of assets that provides the highest Sharpe ratio.

Ticker	Weight
TMSRX	0
CRAZX	0.0914
NBRVX	0
VDIGX	0
FBGRX	0
PRDGX	0
BPTRX	0.9007

- **Expected Annual Return:** 5.48%
- **Annual Volatility:** 5.59%

The portfolio is heavily concentrated in BPTRX (90.07%), reflecting the high return potential of this technology-focused fund. While this concentration provides high expected returns, it also introduces significant risk, as the portfolio's performance is highly dependent on the tech sector's success. CRAZX contributes a modest 9.14%, adding some diversification but not enough to significantly reduce overall risk. TMSRX and other funds are not included, highlighting their lower contribution to risk-adjusted returns for this specific optimization.

### Minimum Variance Portfolio

The Efficient Portfolio for Target Return seeks to achieve a specific return (in this case, 4.50%) with the lowest possible risk.

Ticker	Weight
TMSRX	0
CRAZX	0
NBRVX	0.1189
VDIGX	0
FBGRX	0
PRDGX	0
BPTRX	0.8811

- **Expected Annual Return:** 3.54%
- **Annual Volatility:** 3.64%

This portfolio allocates the majority of its weight to BPTRX (88.11%), with a smaller portion allocated to NBRVX (11.89%). The significant allocation to BPTRX suggests that even within a risk-minimized portfolio, BPTRX's high volatility is somewhat tempered by the stability of NBRVX. The exclusion of other funds indicates that their inclusion would not reduce volatility as effectively. This portfolio is suitable for risk-averse investors seeking to minimize potential losses while accepting lower expected returns.

### Efficient Portfolio for Target Return

The Efficient Portfolio for Target Return seeks to achieve a specific return (in this case, 4.50%) with the lowest possible risk.

Ticker	Weight
TMSRX	0
CRAZX	0.5674
NBRVX	0
VDIGX	0
FBGRX	0
PRDGX	0
BPTRX	0.4326

- **Target Return:** 4.50%
- **Annual Volatility:** 4.85%

This portfolio balances between CRAZX (56.74%) and BPTRX (43.26%) to meet the target return of 4.50%. CRAZX, a mid-cap growth fund, and BPTRX, a tech-focused fund, together achieve the desired return with moderate risk. This balance provides a diversified approach that leverages the growth potential of CRAZX while still capitalizing on the high returns of BPTRX. It is ideal for investors aiming for a specific return objective with controlled risk.

### Over-all results:

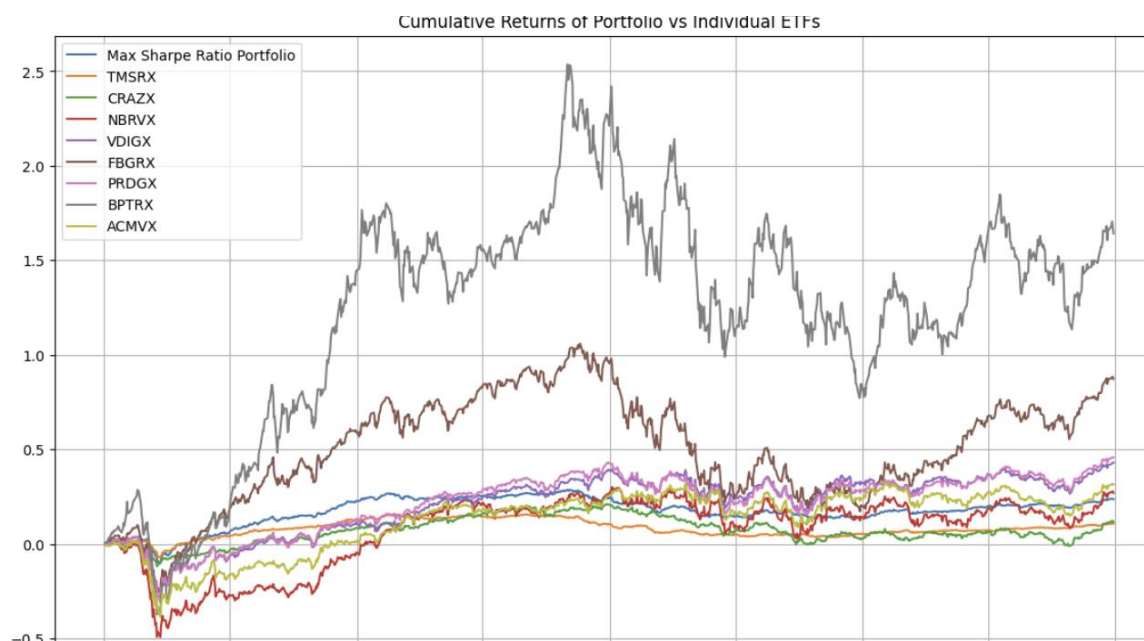
Ticker	Max Sharpe Ratio	Min Variance	Efficient Portfolio
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TMSRX	0	0	0
CRAZX	0.0914	0	0.5674
NBRVX	0	0.1189	0
VDIGX	0	0	0
FBGRX	0	0	0
PRDGX	0	0	0
BPTRX	0.9007	0.8811	0.4326

- **Max Sharpe Ratio Portfolio:** This portfolio provides the highest risk-adjusted return, making it suitable for investors willing to take on significant risk for potentially higher rewards. However, its heavy reliance on BPTRX indicates high vulnerability to sector-specific downturns.
- **Minimum Variance Portfolio:** This portfolio is designed for risk-averse investors, offering the lowest volatility but also lower returns. It balances stability with modest growth, primarily through BPTRX and NBRVX.
- **Efficient Portfolio for Target Return:** This portfolio is ideal for investors with specific return goals, providing a balanced approach to achieving desired returns with moderate risk. The allocation between CRAZX and BPTRX highlights a strategic mix to meet target returns efficiently.

#### For Low-risk Low reward Case:

The chart below shows the cumulative returns of the Max Sharpe Ratio Portfolio compared to the individual ETFs from January 2020 to January 2024.



#### Max Sharpe Ratio Portfolio Performance:

- **Max Sharpe Ratio Portfolio (Blue Line):** This portfolio shows a consistent upward trend, indicating a strong performance over the period. The cumulative return for this portfolio outperforms most individual ETFs, especially during market upswings. However, it also mirrors the significant drawdowns during market downturns, particularly evident during the COVID-19 market crash in early 2020 and other volatility spikes thereafter.

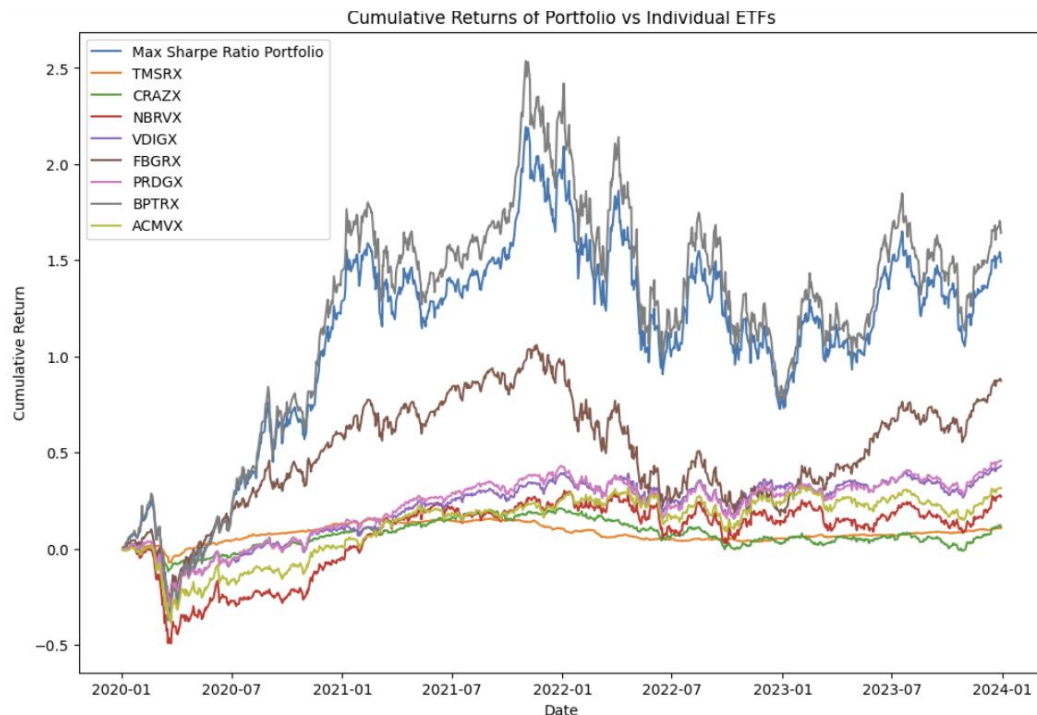
#### **Individual ETFs Performance:**

- **BPTRX (Gray Line):** Exhibits the highest cumulative return among the individual ETFs. Its performance is characterized by high volatility, reflecting its tech-focused investment strategy. The sharp increases and decreases correspond to the tech sector's performance, particularly during the pandemic when tech stocks surged but also faced significant corrections.
- **CRAZX (Green Line):** Shows a steady performance with moderate cumulative returns. This mid-cap growth fund offers a balance between stability and growth, avoiding the extreme volatility seen in tech-focused funds.
- **TMSRX (Orange Line):** Reflects the lowest cumulative return, emphasizing its role as a stabilizer with low volatility. TMSRX's steady but unspectacular performance is ideal for conservative investors seeking minimal risk.
- **NBRVX (Red Line) and VDIGX (Purple Line):** Both show similar moderate performance, consistent with their value-oriented and dividend growth strategies, respectively. They offer lower volatility but also lower returns compared to high-growth funds like BPTRX.
- **FBGRX (Brown Line) and PRDGX (Magenta Line):** These funds show performance trends similar to CRAZX, with balanced exposure to growth and stability.
- **ACMVX (Yellow Line):** Lags behind other ETFs, indicating lower cumulative returns, which is typical for small-cap value funds that may not perform as well in volatile markets.

#### **Analysis of Cumulative Return Patterns:**

- **Early 2020:** The sharp drop across all ETFs and the portfolio in early 2020 corresponds to the market's reaction to the COVID-19 pandemic. This period of high volatility impacted all sectors, with tech and growth stocks experiencing significant fluctuations.
- **Recovery Phase:** Post the initial crash, the tech-heavy BPTRX showed the most substantial recovery, reflecting the market's shift towards technology and digital solutions during the pandemic. The Max Sharpe Ratio Portfolio also benefitted from this recovery due to its high allocation to BPTRX.
- **2021 to 2022:** This period saw varying performance trends. The Max Sharpe Ratio Portfolio continued to outperform, capitalizing on tech stock gains. Value-oriented funds like NBRVX and dividend-focused funds like VDIGX showed more stability with moderate returns.
- **2022 Volatility:** A noticeable dip in mid-2022 aligns with geopolitical tensions, inflation concerns, and changing monetary policies. This affected high-growth funds more significantly, as seen with BPTRX, but the Max Sharpe Ratio Portfolio also experienced drawdowns due to its high exposure to BPTRX.
- **Late 2022 to 2023:** The market began stabilizing, with the Max Sharpe Ratio Portfolio and individual ETFs showing recovery. However, the volatility remained evident, impacting cumulative returns across the board.

### For Low/medium-risk High reward Case:



### Importance of Portfolio Diversification:

The cumulative return analysis underscores the importance of portfolio diversification. The Max Sharpe Ratio Portfolio, while providing high returns, also carries significant risk due to its concentration in BPTRX. In contrast, funds like TMSRX offer stability but lower returns. An optimal investment strategy should balance these aspects, considering an investor's risk tolerance and return objectives.

- **High Returns with High Risk:** Funds like BPTRX can drive substantial portfolio growth but also introduce volatility.
- **Stability with Lower Returns:** Funds like TMSRX and NBRVX provide stability, essential during market downturns.



- **Balanced Approach:** Combining high-growth and stable funds can create a balanced portfolio that mitigates risk while pursuing growth.

### **Future Recommendations: Leveraging ML/DL with Bayesian Optimization, LSTM, and Gradient Boosting**

#### **Combined Approach:**

Integrating machine learning (ML) and deep learning (DL) techniques with Bayesian optimization, Long Short-Term Memory (LSTM) networks, and Gradient Boosting offers significant advancements in quantitative finance and portfolio management. These approaches can enhance predictive accuracy, optimize portfolio performance, and manage risk more effectively.

A combined approach utilizing Bayesian optimization, Bi-LSTM networks, and Gradient Boosting can significantly enhance portfolio optimization. Bayesian optimization can fine-tune LSTM and GBM hyperparameters, ensuring optimal performance. Bi-LSTM networks can provide accurate forecasts of market trends, while GBMs can identify complex relationships in the data. This synergy allows for dynamic and adaptive portfolio management strategies that can respond to changing market conditions with higher precision and efficiency.