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EE407-HOMEWORK II

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1) a) $q_o(t) = K w(t) \Rightarrow \frac{dq_o(t)}{dt} = K\frac{dw(t)}{dt}$

$$\frac{dw(t)}{dt} = q_i(t) - q_o(t) \Longrightarrow \frac{1}{K}\frac{dq_o(t)}{dt} = q_i(t) - q_o(t)$$

$$\boxed{\frac{dq_o(t)}{dt} = Kq_i(t) - Kq_o(t)}$$

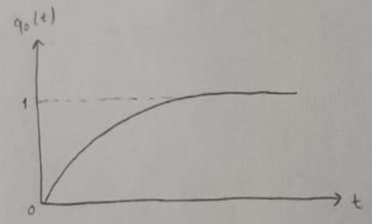
b) $\frac{dq_o(t)}{dt} = Kq_i(t) - Kq_o(t) \xrightarrow[\text{and } q_i(t)=u(t)]{\text{in s domain}} sQ_o(s) = \frac{K}{s} - KQ_o(s)$

$$Q_o(s)\left[s+K\right] = \frac{K}{s}$$

$$Q_o(s) = \frac{K}{s(s+K)} = \frac{1}{s} - \frac{1}{(s+K)}$$

convert it to time domain

$$\boxed{q_o(t) = \left(1 - e^{-Kt}\right)u(t)}$$



c) $\frac{dw(t)}{dt} = q_i(t) - q_o(t)$ and $q_i(t) = BKw_{max}\,u(t)$

s domain

$$sW(s) - w_{min} = \frac{BKw_{max}}{s} - KW(s) \Rightarrow W(s) = \frac{BKw_{max}}{s(s+K)} + \frac{w_{min}}{(s+K)} = \frac{Bw_{max}}{s} - \frac{(Bw_{max} - w_{min})}{(s+K)}$$

time domain

$$w(t) = \left[Bw_{max} - (Bw_{max} - w_{min})e^{-Kt}\right]u(t)$$

5 days = 120 hours

at $t = 120 \Longrightarrow \boxed{w(120) = w_{max}}$

$$w(120) = w_{max} = Bw_{max} - (Bw_{max} - w_{min})e^{-120K}$$

$$Bw_{max}e^{-120K} - w_{min}e^{-120K} = (B-1)w_{max} \Longrightarrow w_{max}\left(Be^{-120K} - B + 1\right) = w_{min}e^{-120K}$$

$$w_{max} \left(\frac{e^{-120k}}{Be^{-120K} - B + 1}\right) w_{min}$$

$w_\gamma = w_{max} - w_{min}$ and $w_{min} = w_{initial}$

$$\boxed{w_\gamma = \left(\frac{e^{-120K}}{Be^{-120K} - B + 1} - 1\right) w_{initial}}$$

2.a)

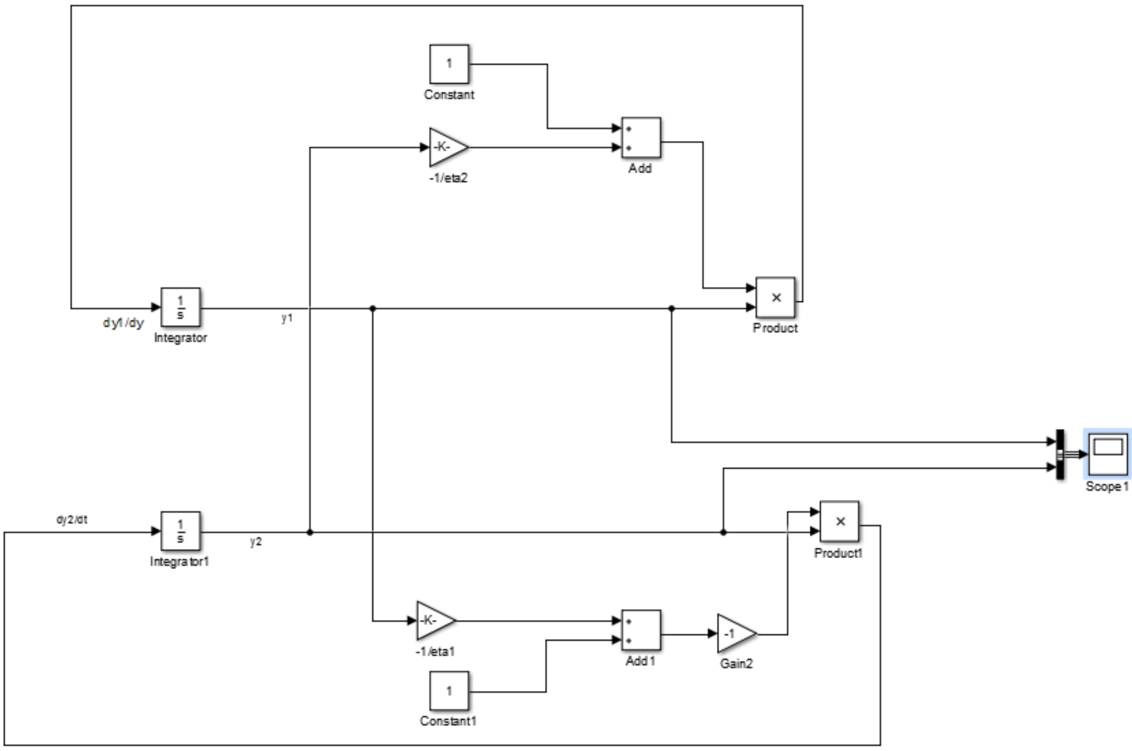


Figure 1: The realization of the population system

2.b)

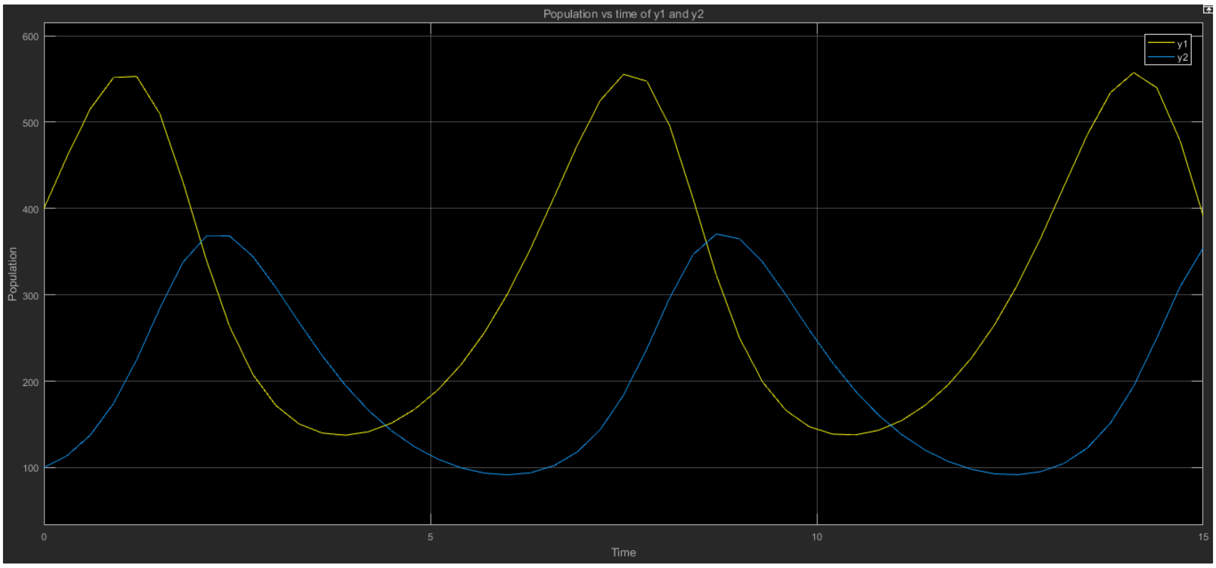


Figure 2: The result of population vs time simulation ($y_1(0)=400, y_2(0)=100$)

2.c) This choice of parameters leads to periodic dynamics in which the prey population initially increases, leading to an abundance of food for the predators. The predators increase in response (lagging the prey population), eventually overwhelming the prey population, which crashes. This in turn causes the predators to crash, and the cycle repeats. The period of these dynamics is about 6 seconds, with the predators lagging the prey by about a second. Disease, competition and parasitism relationships are nonlinear systems with similar properties.

2.e)

To find nonzero equilibrium point:

$\frac{dy1}{dt}$ =0 so by solving equation y1(0)=η_1 y2(0)= η_2 We gave the inputs as $y_1(0)$=302 and $y_2(0)$=202.

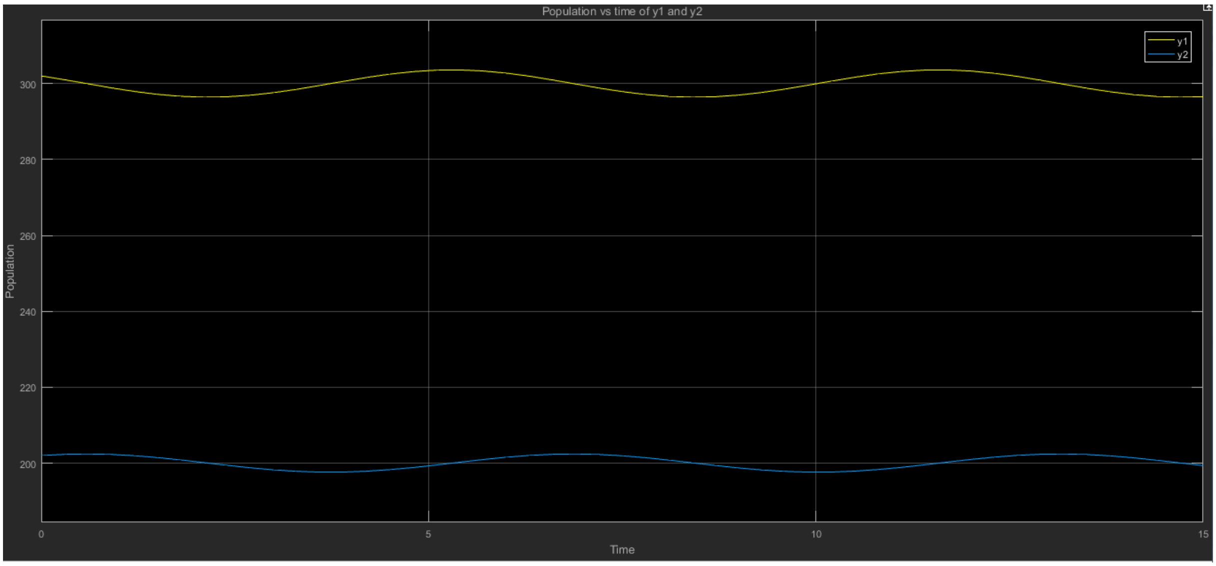


Figure 3: The result of population vs time simulation at nearly equilibrium point (y1(0)=302,y2(0)=202)

As seen on the figure 3 the response of the system changes. Its period did not change, it is again nearly 6 seconds, but this has better sinusoidal characteristic which is not distorted.

2.f)

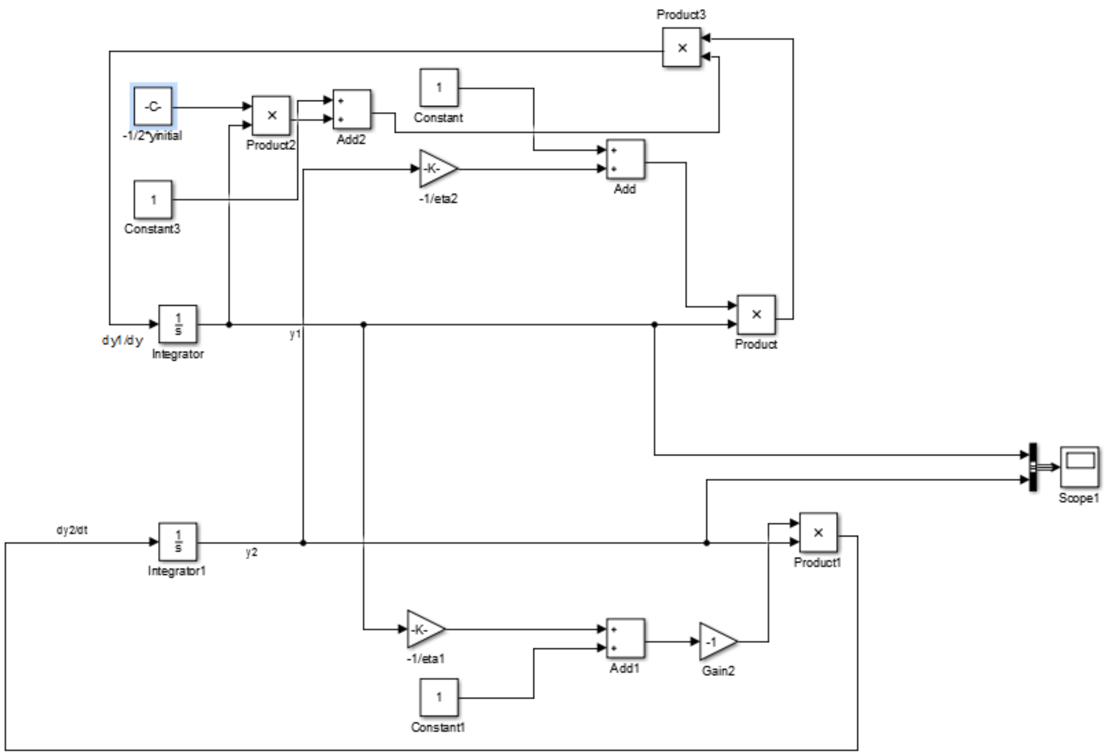


Figure4: The realization of the population system on part 2 f.

2.g)

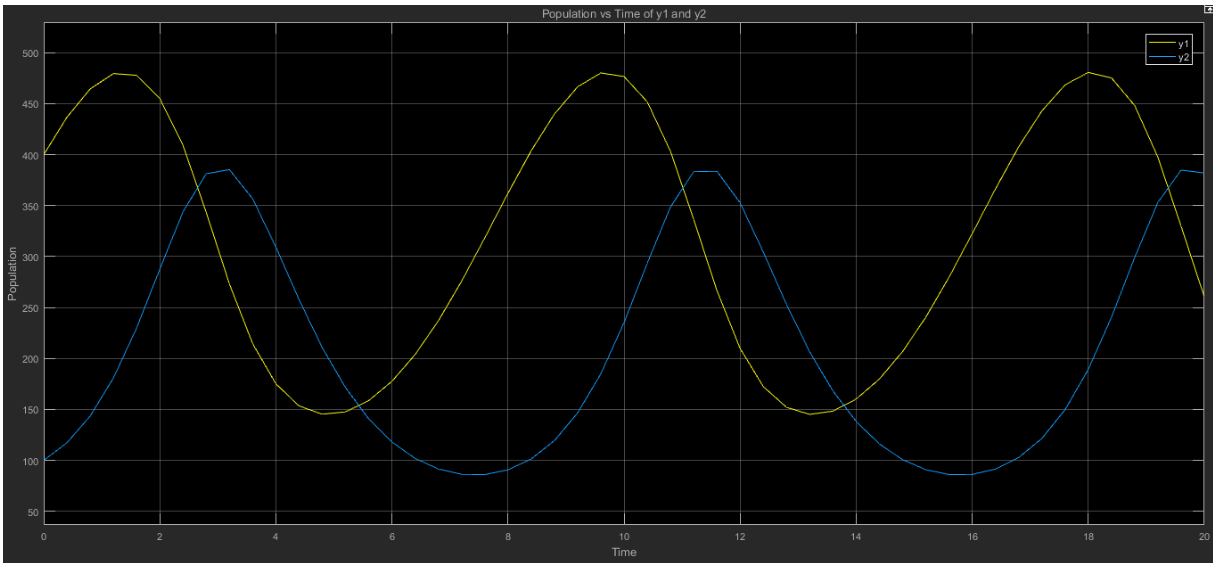


Figure 5: The result of population vs time simulation (y1(0)=400,y2(0)=100)

The system is still periodic, but the period of the new system becomes nearly 8.5 seconds as seen on the figure 5. The maximum number of the prey and predator changed.