SOLUTIONS

CSC 384 Winter 2023 Test 4 Version A

March 27 and 28, 2023

Last Name: ______________________________

First Name: ______________________________

Email: ______________________________

There are 3 questions with a total of 26 marks.

- Q1 (8 marks)
- Q2 (12 marks)
- Q3 (6 marks)

Q1 D-Separation (8 marks)

Consider Figure 1 below. For each question below, circle the best answer and provide an explanation. Use the following format for your explanation (where X, A, B, C, and D are variables).

(Observing/Not observing) X (blocks/doesn't block) the path A-B-C-D by rule 1/2/3.

Q1.1 (2 marks) **C** and **E** are unconditionally independent.

True or False

Explain:

Q1.2 (2 marks) **F** and **E** are conditionally independent given **B**.

True or False

Explain:

Q1.3 (2 marks) **A** and **I** are unconditionally independent.

True or False

Explain:

Q1.4 (2 marks) **C** and **E** are conditionally independent given **I.**

True or False

Explain:

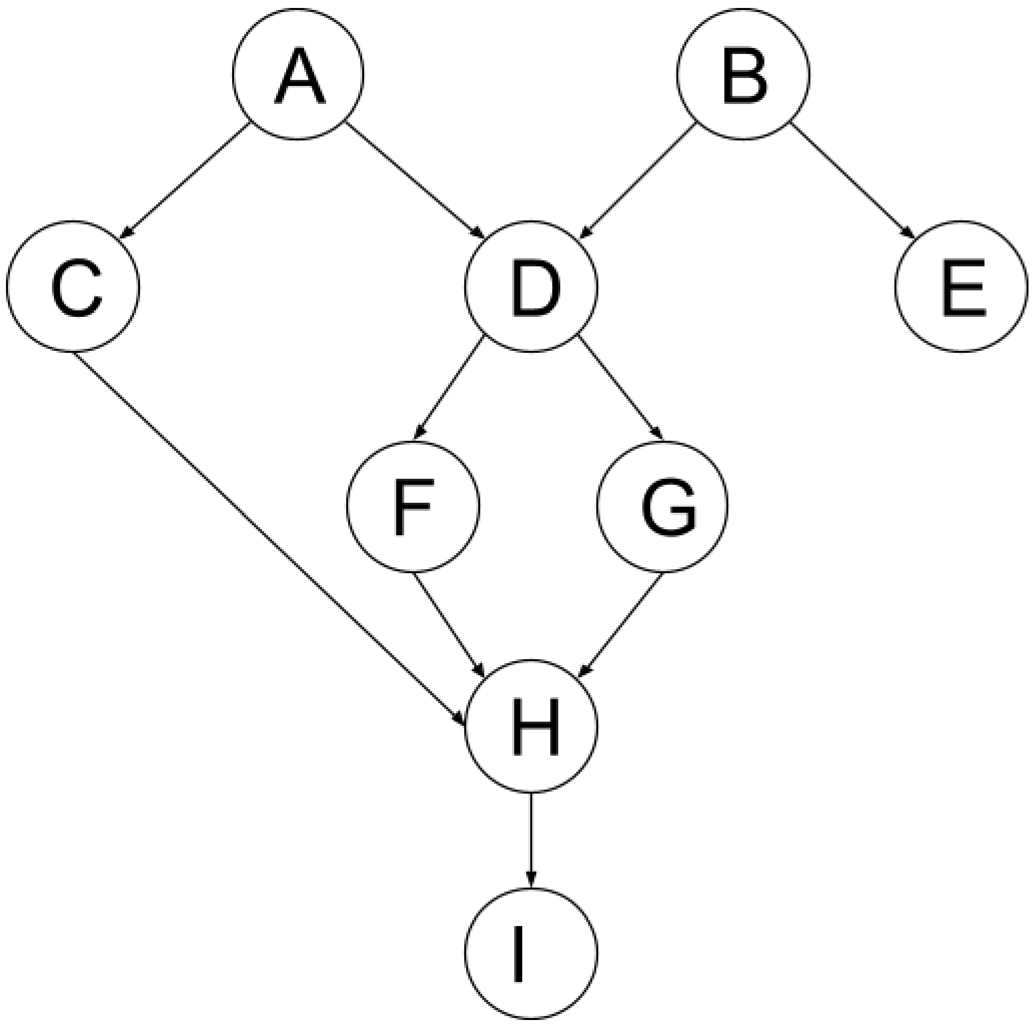


Figure 1 Above

Q1 Solutions

Q1.1 (2 marks) **C** and **E** are unconditionally independent.

True

Not observing D nor D's descendants blocks path CADBE by rule 3. Not observing H nor H's descendants blocks the path CHGDBE and the path CHFDBE by rule 3.

Q1.2 (2 marks) **F** and **E** are conditionally independent given **B**.

True

Observing B blocks the path FDBE by rule 2.

Q1.3 (2 marks) **A** and **I** are unconditionally independent.

False

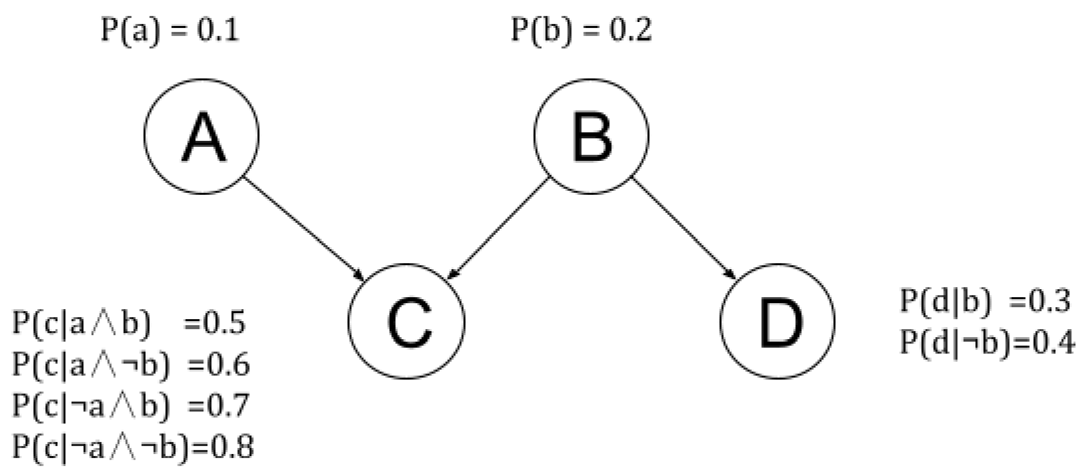
All the nodes on the three paths between A and I follow the chain structure (rule 1). Since none of the nodes are observed, they do not block any path by rule 1.

Q1.4 (2 marks) **C** and **E** are conditionally independent given **I.**

False

Observing I does not block the path CHGDBE by rule 3.

Q2 Variable Elimination Algorithm (12 marks)



Consider the Bayesian network above. A, B, C, and D are binary variables. We use the lower-case letters to denote the values of the variables, e.g. a denotes A = true and $\neg a$ denotes A=false.

Calculate $P(A \mid \neg c)$ by using the Variable Elimination Algorithm.

Eliminate the hidden variables in **alphabetical** order.

For each step, indicate the following.

- Indicate the **operation** (e.g. Restrict, Multiply, Sum out, or Normalize).
- Indicate the **factors** on which you are applying the operations.
- Each operation should **produce a new factor.** Give this factor a unique name and draw a table containing its contents. The table should indicate the variables in the factor and the value for each combination of the variables' values.

Show all your work on pages 6 and 7.

We have created the initial factors for you below.

Factor f1

a	0.1
$\neg a$	0.9

Factor f2

b	0.2
$\neg b$	0.8

Factor f3

d	b	0.3
$\neg d$	b	0.7
d	$\neg b$	0.4
$\neg d$	$\neg b$	0.6

Factor f4

c	a	b	0.5
$\neg c$	a	b	0.5
c	a	$\neg b$	0.6
$\neg c$	a	$\neg b$	0.4
c	$\neg a$	b	0.7
$\neg c$	$\neg a$	b	0.3
c	$\neg a$	$\neg b$	0.8
$\neg c$	$\neg a$	$\neg b$	0.2

Your Q2 final answers:

$P(a \mid \neg c) =$	$P(\neg a \mid \neg c) =$

Your Q2 work starts here.

Your Q2 work continues.

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Q2 Version A Solutions

The final answers:

$P(a \mid \neg c) = 0.175$	$P(\neg a \mid \neg c) = 0.825$

1 Restrict factor f4 to C = false to produce factor f5

f5

a	b	**0.5**
a	$\neg b$	**0.4**
$\neg a$	b	**0.3**
$\neg a$	$\neg b$	**0.2**

The remaining factors are f1, f2, f3, and f5.

There are two hidden variables, B and D. We will eliminate B first.

2. Multiply factors f2, f3, and f5 to produce factor f6.

f6

a	b	d	**0.03**
a	b	$\neg d$	**0.07**
a	$\neg b$	d	**0.128**
a	$\neg b$	$\neg d$	**0.192**
$\neg a$	b	d	**0.018**
$\neg a$	b	$\neg d$	**0.042**
$\neg a$	$\neg b$	d	**0.064**
$\neg a$	$\neg b$	$\neg d$	**0.096**

The remaining factors are f1 and f6.

3. Sum out B from factor f6 to produce factor f7.

f7

a	d	**0.158**
a	$\neg d$	**0.262**
$\neg a$	d	**0.082**
$\neg a$	$\neg d$	**0.138**

The remaining factors are f1 and f7.

4. Sum out D from factor f7 to produce factor f8.

f8

a	**0.42**
$\neg a$	**0.2**

The remaining factors are f1 and f8.

5. Multiply factors f1 and f8 to produce factor f9.

f9

a	**0.042**
$\neg a$	**0.198**

The remaining factor is f9.

6. Normalize factor f9 to produce factor f10.

f10

a	**0.175**
$\neg a$	**0.825**

Q3 Filtering (6 marks)

Consider the hidden Markov model on the next page.

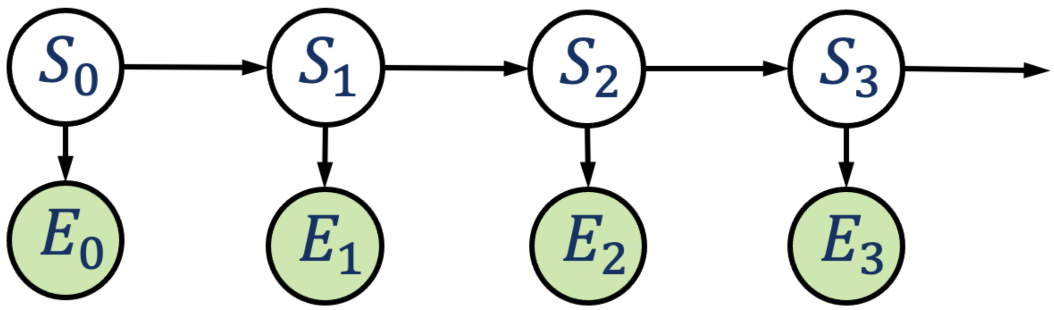
- S_t denotes the hidden state at time t. $S_t = true$ means it rained on day t ($S_t = false$ otherwise).
- E_t denotes the observation at time t. $E_t = true$ means the director brought an umbrella on day t and $E_t = false$ otherwise.
- α is the normalization constant.

Assume that **the first three observations are $\neg e_0$, e_1, and $\neg e_2$.** That is, the director **brought an umbrella on day 1 and didn't bring an umbrella on days 0 and 2.**

Calculate the filtering probabilities for **day 2**. We have provided the filtering formulas on the next page. **For full marks**, **show ALL your work** and present your solutions to **3 decimal places**.

$P(s_0) = 0.45$

$P(s_t|s_{t-1}) \quad = 0.75$
$P(s_t|\neg s_{t-1}) = 0.35$



$P(e_t|s_t) \quad = 0.85$
$P(e_t|\neg s_t) = 0.10$

The Filtering Formulas:

- Base case: $P(S_0|E_0) \; = \; \alpha \, P(S_0) \, P(E_0|S_0)$
- Recursive case:
    - $P(S_k|E_0 \wedge ... \wedge E_{k-1}) \; = \; \sum_{S_{k-1}} P(S_{k-1}|E_0 \wedge ... \wedge E_{k-1}) \; * \; P(S_k|S_{k-1})$
    - $P(S_k|E_0 \wedge ... \wedge E_k) \; = \; \alpha \, P(E_k|S_k) \, P(S_k|E_0 \wedge ... \wedge E_{k-1})$

Assume that

$P(s_1 | \neg e_0 \wedge e_1) = 0.849$ and $P(\neg s_1 | \neg e_0 \wedge e_1) = 0.151$

Your final answers::

$P(s_2 | \neg e_0 \wedge e_1 \wedge \neg e_2) =$	$P(\neg s_2 | \neg e_0 \wedge e_1 \wedge \neg e_2) =$

Your calculations:

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Q3 Version A Solutions

Step 1: (2 marks)

$P(s_2|\neg e_0 \wedge e_1)$
$= P(s_1|\neg e_0 \wedge e_1)P(s_2|s_1) + P(\neg s_1|\neg e_0 \wedge e_1)P(s_2|\neg s_1)$
$= 0.849 * 0.75 + 0.151 * 0.35 = 0.690$

$P(\neg s_2|\neg e_0 \wedge e_1) = 1 - 0.690 = 0.310$

Step 2: (4 marks)

(2 marks)
$P(\neg e_2|s_2)P(s_2|\neg e_0 \wedge e_1) = 0.15 * 0.690 = 0.103$
$P(\neg e_2|\neg s_2)P(\neg s_2|\neg e_0 \wedge e_1) = 0.90 * 0.310 = 0.279$

(2 marks)
$P(s_2|\neg e_0 \wedge e_1 \wedge \neg e_2) = 0.103/(0.103 + 0.279) = 0.270$
$P(\neg s_2|\neg e_0 \wedge e_1 \wedge \neg e_2) = 1 - 0.270 = 0.730$