CSC 384
Introduction to Artificial Intelligence

Knowledge Representation 1

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Learning Goals

By the end of this lecture, you should be able to

- Define terms recursively.
- Define formulas recursively.
- Define a model for a given syntax.
- Interpret a formula given a model.

Outline

1. Introducing First Order Logic
2. Syntax
3. Semantics

FIRST ORDER LOGIC

Knowledge-Based Agents

- Humans know things.
  - An internal representation of knowledge.

- What they know helps them do things.
  - Can perform reasoning using the knowledge representation.

First-Order Logic (FOL)

- A.k.a. Predicate Logic.
- A Knowledge Representation Language.

Representation:

- Syntax: grammar or rules for forming proper sentences.
- Semantics: meanings of the sentences.
- Translating English into FOL

Reasoning: Resolution proofs

FIRST-ORDER LOGIC SYNTAX

First-Order Logic (FOL)

- Two components: Syntax and Semantics.
- Syntax: grammar or rules for forming proper sentences.
- Semantics: the meaning of the sentences.

The Language of FOL

- Domain: a non-empty set of objects.

- Objects may have properties.
  - Modeled using predicates.

- There are relationships between objects.
  - Modeled using predicates and functions.

Syntax of FOL

- Constant and variable symbols represent objects in the domain.
- Predicate symbols represent properties or relationships.
- Function symbols represent functions.

Constant and Variable Symbols

- Map to objects in the domain.
- A constant maps to a particular object.
- A variable maps to an arbitrary object.

Example:

- Domain is {Avery, Parker, Hayden, Coffee, Tea, ...}.
- A constant symbol A can map to Avery.
- A variable symbol X can map to any of the three people.

Predicate Symbols

- Describe properties of objects or relationships among objects.
- Takes objects in the domain as arguments, and
- Returns true or false.
- Arity: the number of arguments in the predicate.

- A unary predicate describes properties of the objects.
  - pianist(Avery): Avery is a pianist.
  - likescoffee(Hayden): Hayden likes coffee.

- Higher-order predicate describe relationships among the objects.
  - friends(Hayden, Parker): Hayden and Parker are friends.
  - =(Avery, Avery): Equality is a commonly used predicate.

Function Symbols

- Stand for functions 😂
- Takes objects in the domain as arguments, and
- Returns an object of the domain.

- Examples:
  - sibling(Parker)
  - bothlike(Avery, Parker)

A Summary of Symbols

Syntax	Semantics
Constant symbol A, B, C	A particular object of the domain
Variable symbol X, Y, Z	An arbitrary object of the domain
Function symbols f, g, h	A relationship Returns an object of the domain
Predicate symbol p, q, r A special example: = (equality)	A property or a relationship Returns true or false
Logical connectives $\wedge$, $\vee$, $\neg$, $\rightarrow$	
Quantifiers $\forall$, $\exists$	

The Grammar of FOL

- We have basic syntactic symbols:
  - constants, variables, predicates, and functions.
- Use symbols to build up terms and formulas.

- Terms represent objects of the domain.
- Formulas (sentences) represent true/false assertions.

Terms

Terms represent objects of the domain.

A Recursive Definition of Terms:

1. A constant symbol A is a term.
2. A variable symbol X is a term.
3. If $t_1, t_2, \ldots, t_n$ are terms and f is an n-ary function symbol, then $f(t_1, t_2, \ldots, t_n)$ is a term.
4. Nothing else is a term.

Which Expressions are Terms?

Which of the following expressions is a term?

A. Z

B. $\mathrm{h(A, X)}$

C. $\mathrm{p(g(X, Y), A)}$

D. $\mathrm{g(X, h(Y, Z), A)}$

E. $\mathrm{h(X, g(X, Y), A)}$

Constant symbols: A.

Variable symbols: X, Y, Z.

Predicate symbols: p is a binary predicate.

Function symbols: g is a binary function, and h is a 3-ary function.

Atomic Formulas (Sentences)

A formula/sentence is a true/false assertion.

If $t_1, \ldots, t_n$ $(n \geq 1)$ are terms and
p is an n-ary predicate symbol,
then $p(t_1, \ldots, t_n)$ is an atomic formula.

Formulas (Sentences)

A recursive definition of formulas/sentences

1. An atomic formula is a formula.
2. If p is a formula,
   then $\neg p$ is a formula.
3. If p and q are formulas, and $*$ is one of $\wedge$, $\vee$, $\rightarrow$,
   then $(p * q)$ is a formula.
4. If f is a formula and X is a variable,
   then $\forall X. f$ and $\exists X. f$ are formulas.

Which of the following is a formula?

Which of the following is a formula?

A. $\mathrm{f(Y) \rightarrow p(Y, Z)}$

B. $\mathrm{\forall X. p(A, f(X))}$

C. $\mathrm{p(Y, \mathit{Z}) \rightarrow q(q(Y))}$

D. $\mathrm{q(A, f(A))}$

E. $\mathrm{p(A, f(q(Y, Z)))}$

Constant symbols: A. Variable symbols: $\mathrm{X, Y, Z}$.

Predicate symbols: p and q are binary predicates.

Function symbols: f is a unary function.

FIRST ORDER LOGIC SEMANTICS

Semantics

- An interpretation (model) is a tuple $< D, \Phi, \Psi, V >$ mapping the symbols to semantic entities.

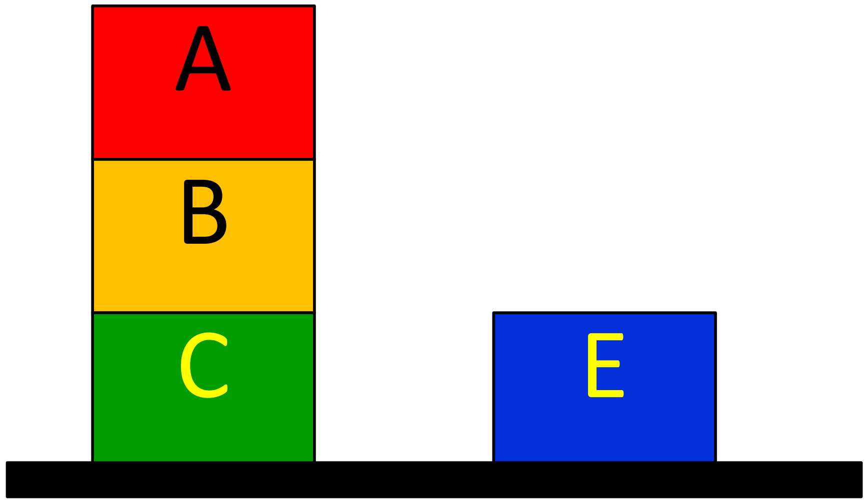
- D is a non-empty set of objects.
- Φ specifies the meaning of each constant and function symbol.
- Ψ specifies the meaning of each predicate symbol.
- V specifies the meaning of each variable.

An Example

Syntax of our language

- Constants: a, b, c, e
- Predicates:
  - on: binary
  - above: binary
  - clear: unary
  - ontable: unary

Our Environment



Syntax versus Semantics

Syntax of our language

- Constants: a, b, c, e
- Predicates:
  - on: binary
  - above: binary
  - clear: unary
  - ontable: unary

Model 1

- $D = \{A, B, C, E\}$
- $\Phi(a) = A, \Phi(b) = B,$
- $\Phi(c) = C, \Phi(e) = E.$
- $\Psi(on) = \{(A, B), (B, C)\}$
- $\Psi(above) = \{(A, B), (B, C), (A, C)\}$
- $\Psi(clear) = \{A, E\}$
- $\Psi(ontable) = \{C, E\}$

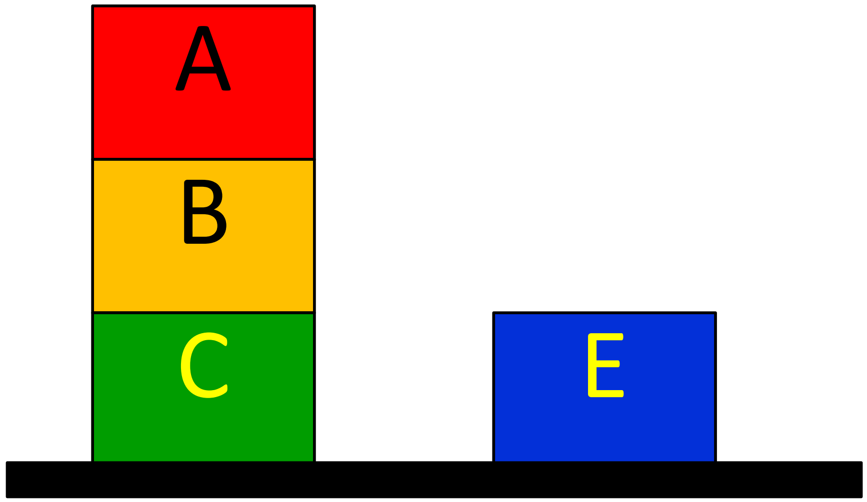
Model 1 Matches Our Environment

Model 1

- $D = \{A, B, C, E\}$
- $\Phi(a) = A, \Phi(b) = B,$
- $\Phi(c) = C, \Phi(e) = E.$

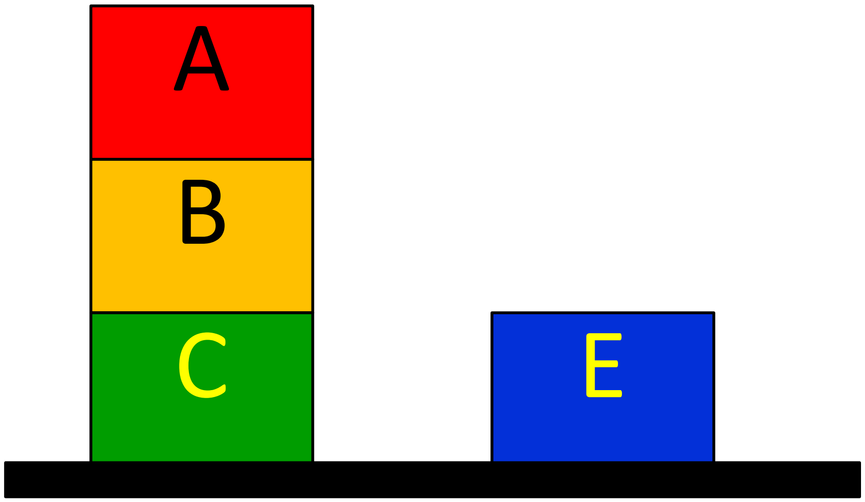
- $\Psi(on) = \{(A, B), (B, C)\}$
- $\Psi(above) =$
  $\{(A, B), (B, C), (A, C)\}$
- $\Psi(clear) = \{A, E\}$
- $\Psi(ontable) = \{C, E\}$

Our Environment



Evaluating Formulas in Model 1 (Part 1)

Our Environment



$\forall X \forall Y. (on(X, Y) \rightarrow above(X, Y))$

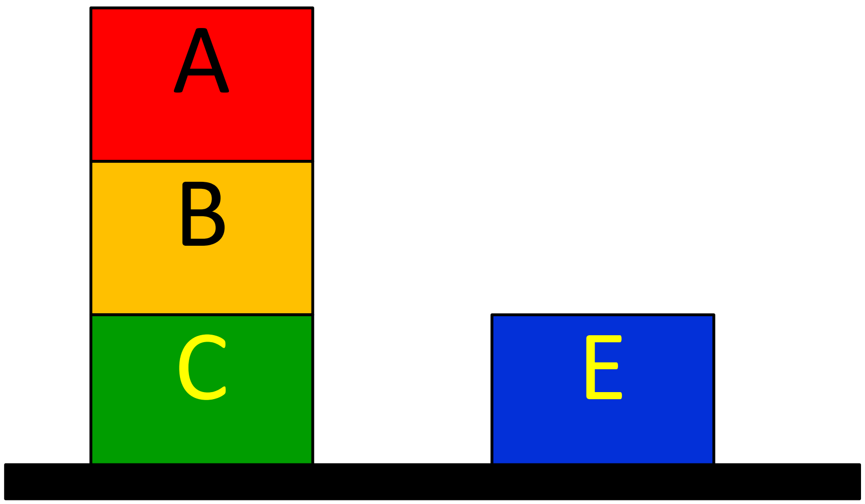
True or False

$\forall X \forall Y. (above(X, Y) \rightarrow on(X, Y))$

True or False

Evaluating Formulas in Model 1 (Part 1)

Our Environment



$\forall X \forall Y.\,(on(X,Y) \rightarrow above(X,Y))$

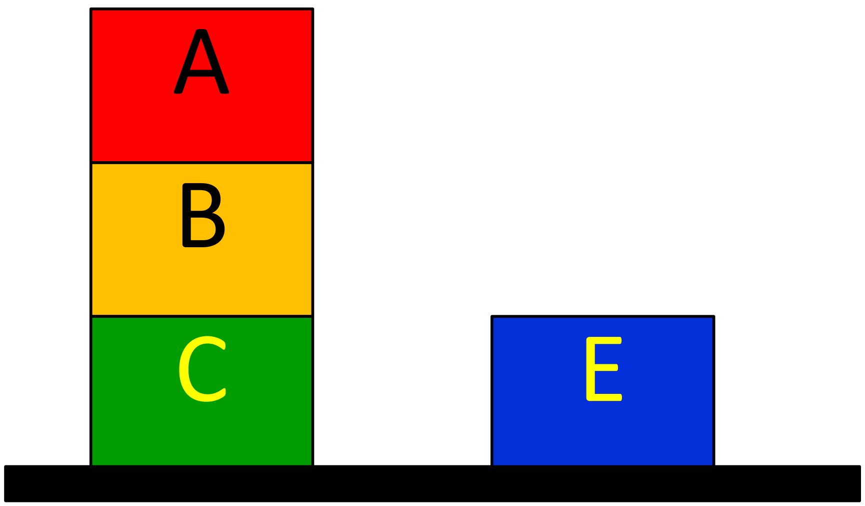
True

$\forall X \forall Y.\,(above(X,Y) \rightarrow on(X,Y))$

False

Evaluating Formulas in Model 1 (Part 2)

Our Environment



$\forall X \exists Y . (clear(X) \lor on(Y, X))$

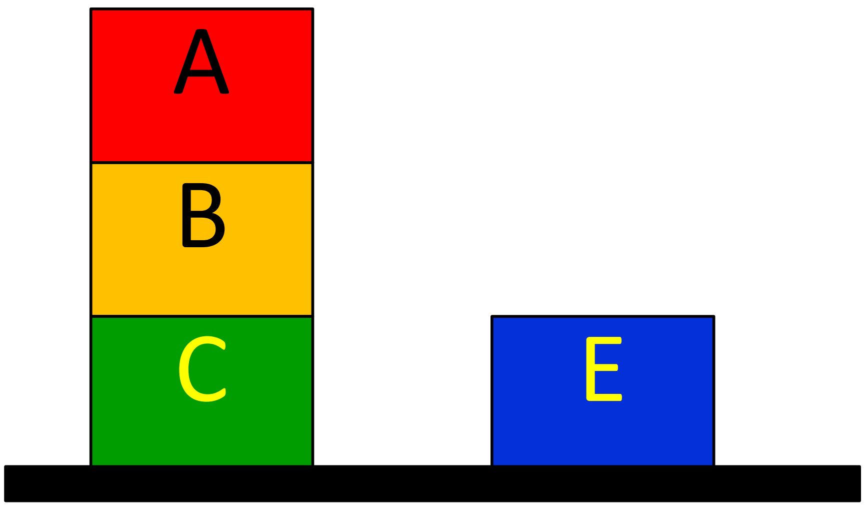
True or False

$\exists Y \forall X . (clear(X) \lor on(Y, X))$

True or False

Evaluating Formulas in Model 1 (Part 2)

Our Environment



$\forall X \exists Y . (clear(X) \vee on(Y, X))$

True

$\exists Y \forall X . (clear(X) \vee on(Y, X))$

False