



# CSC 384

## Introduction to Artificial Intelligence

### Uncertainty 3

### D-Separation

Alice Gao and Randy Hickey

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# Learning Goals

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By the end of the lecture, you should be able to

1. Explain the independence relationships in the three key structures.
2. Determine whether two random variables  $X$  and  $Y$  are independent given a Bayesian network.
3. Determine whether two random variables  $X$  and  $Y$  are conditionally independent given a third variable  $Z$  given a Bayesian network.

# Outline

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- Independence in Three Key Structures
- D-Separation

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# **INDEPENDENCE IN THREE KEY STRUCTURES**

# Structure 1: Chain

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# Chain: Independence

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Q1: Are ExoticTrip and Fever **independent**?



(A) YES

(B) NO

# Chain: Independence

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A1: Are ExoticTrip and Fever **independent**?



(A) YES

**(B) NO**

- ExoticTrip and Fever affect each other through Malaria.

$$P(f|et) > P(f|\neg et)$$

- ExoticTrip -> More likely to get Malaria -> More likely to have a Fever
- No ExoticTrip -> Less likely to get Malaria -> Less likely to have a Fever

# Chain: Conditional Independence

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Q1: Are ExoticTrip and Fever **conditionally independent** given Malaria?



(A) YES

(B) NO



# Chain: Conditional Independence

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A1: Are ExoticTrip and Fever **conditionally independent** given Malaria?



**(A) YES**

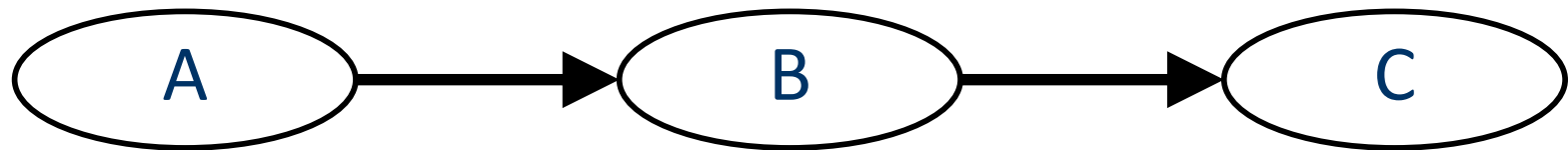
**(B) NO**

Malaria is the only way through which ExoticTrip and Fever influence each other.

$$P(f|\neg et \wedge m) = P(f|et \wedge m)$$

# Chain: Summary

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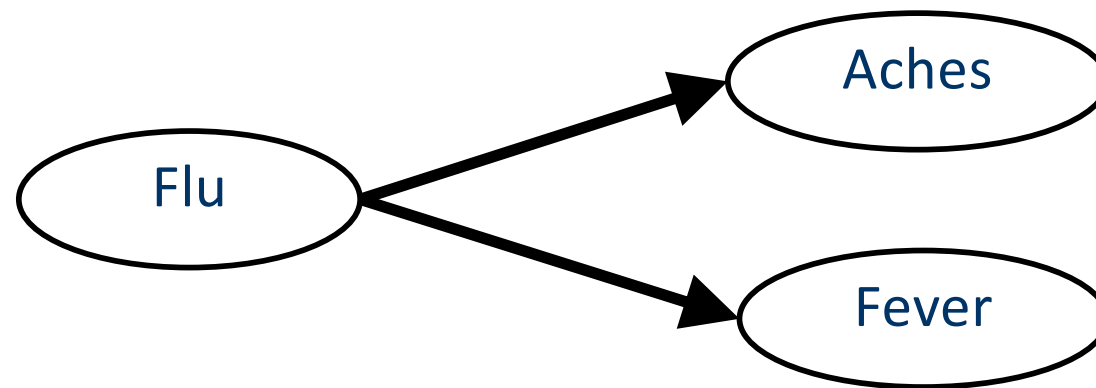


**A** and **C** are NOT independent.

**A** and **C** are conditionally independent given **B**.

## Structure 2: Common Parent *or Common Cause*

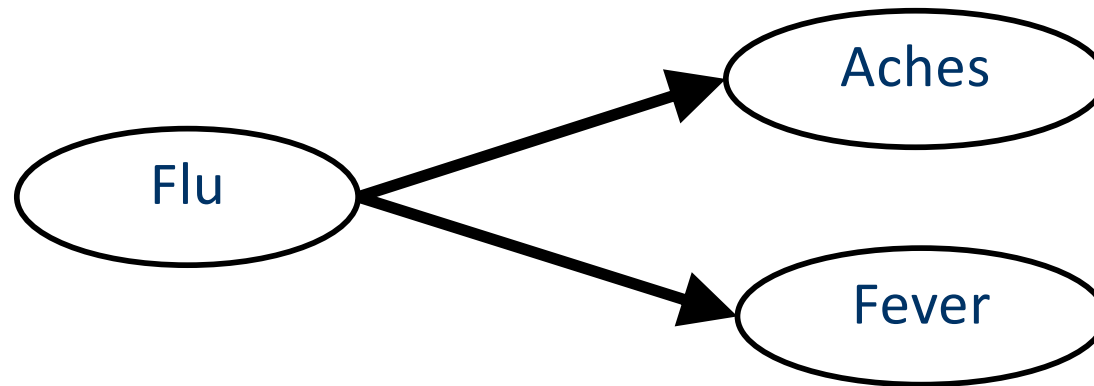
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# Common Parent: Independence

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Q1: Are Aches and Fever **independent**?



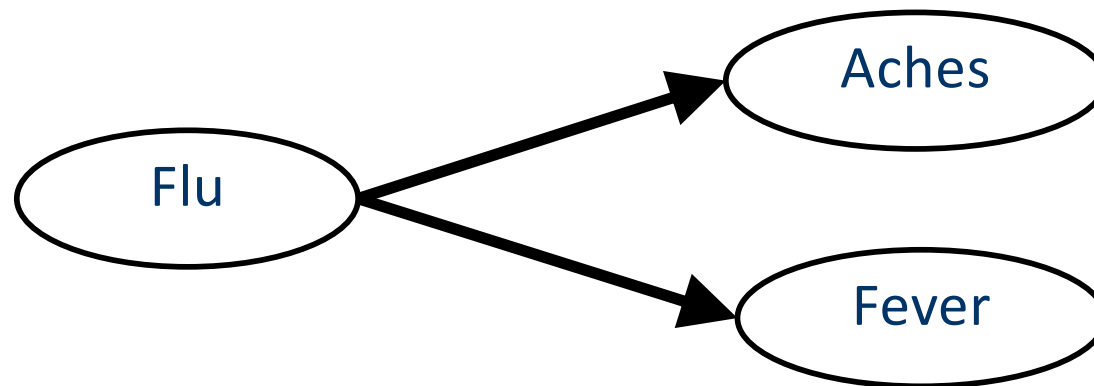
(A) YES

(B) NO

# Common Parent: Independence

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A1: Are Aches and Fever **independent**?



(A) YES

**(B) NO**

Flu explains Aches and Fever.

$$P(f|a) > P(f|\neg a)$$

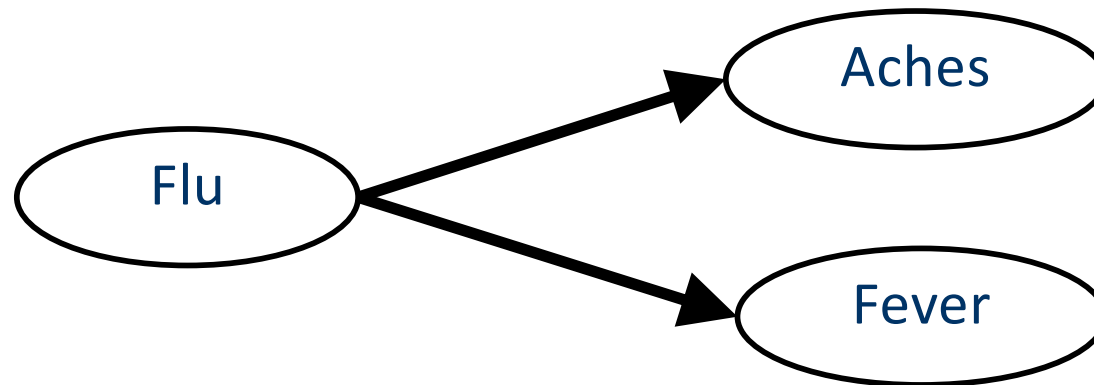
Have aches -> More likely to have flu → More likely to have fever

No aches -> Less likely to have flu → Less likely to have fever

# Common Parent: Conditional Independence

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Q2: Are Aches and Fever **conditionally independent** given Flu?



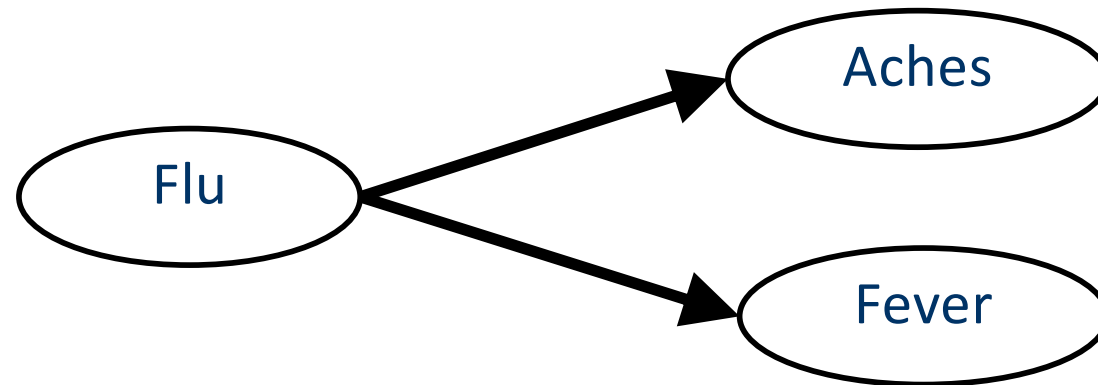
(A) YES

(B) NO

# Common Parent: Conditional Independence

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A2: Are Aches and Fever **conditionally independent** given Flu?



**(A) YES**

**(B) NO**

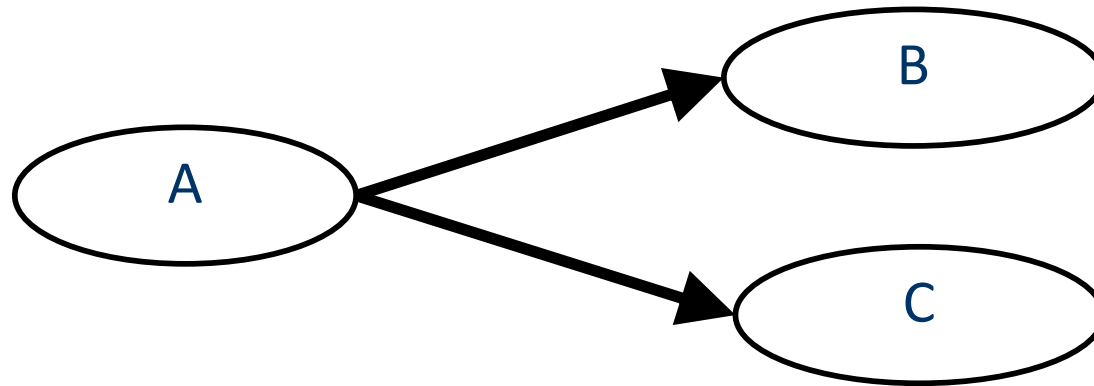
Once Flu is known, the prob of Aches and Fever are fixed.

$$P(fe|\neg a \wedge fl) = P(fe|a \wedge fl)$$

# Common Parent: Summary

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*or Common Cause*



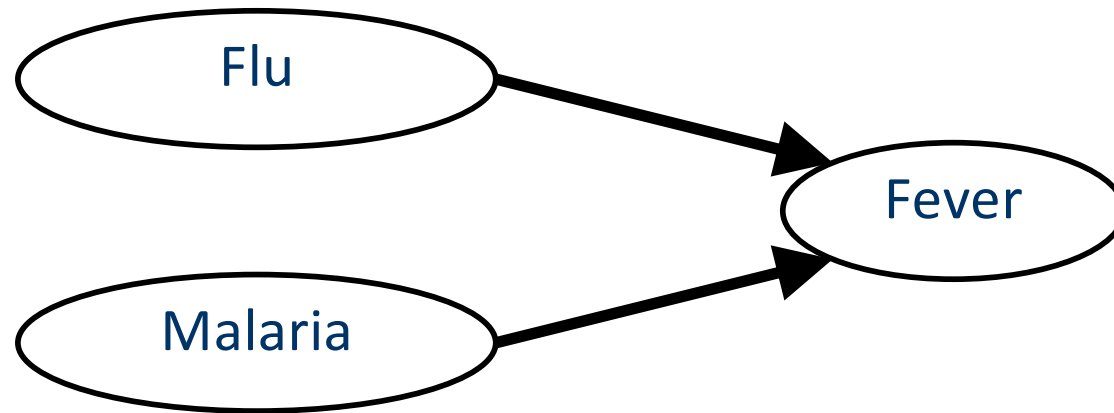
**B** and **C** are NOT independent.

**B** and **C** are independent given **A**.



## Structure 3: Common Child *or Common Effect*

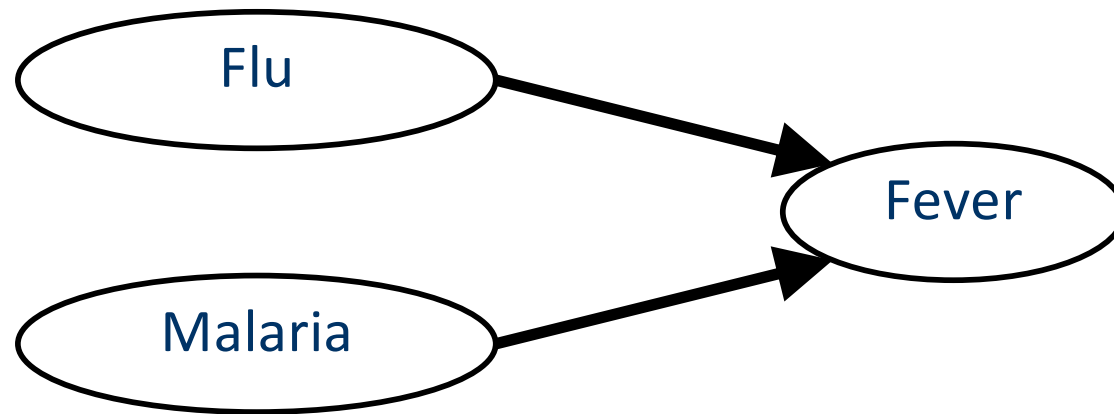
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# Common Child: Independence

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Q1: Are Flu and Malaria **independent**?



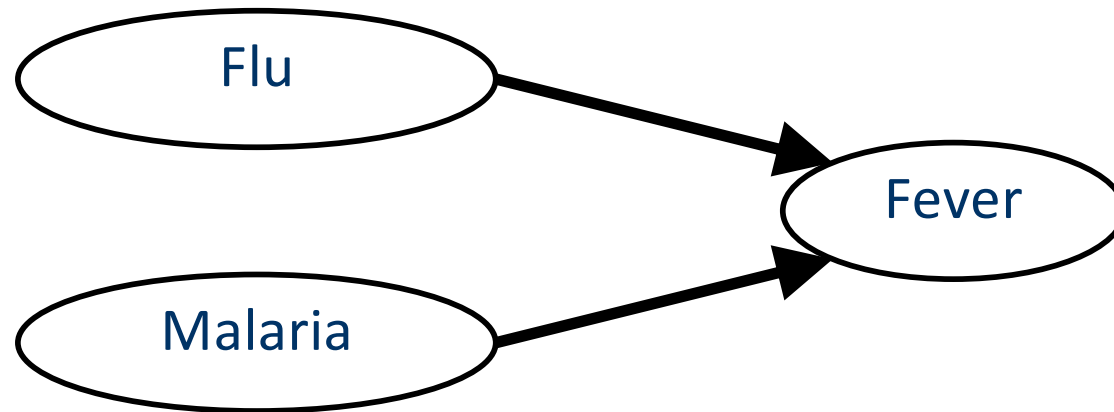
(A) YES

(B) NO

# Common Child: Independence

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A1: Are Flu and Malaria **independent**?



**(A) YES**

**(B) NO**

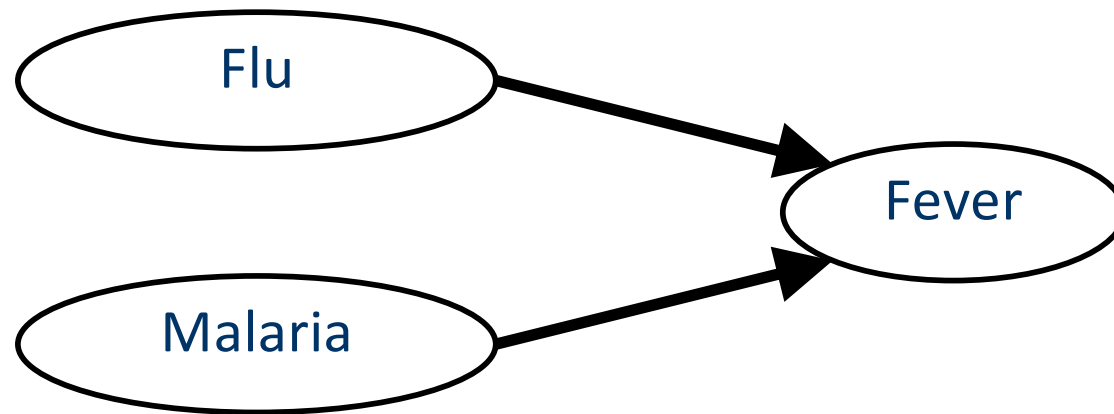
Flu and Malaria are two independent causes of Fever.

The absence of an arrow between Flu and Malaria means that they are required to be independent.

# Common Child: Conditional Independence

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Q2: Are Flu and Malaria **conditionally independent** given Fever?

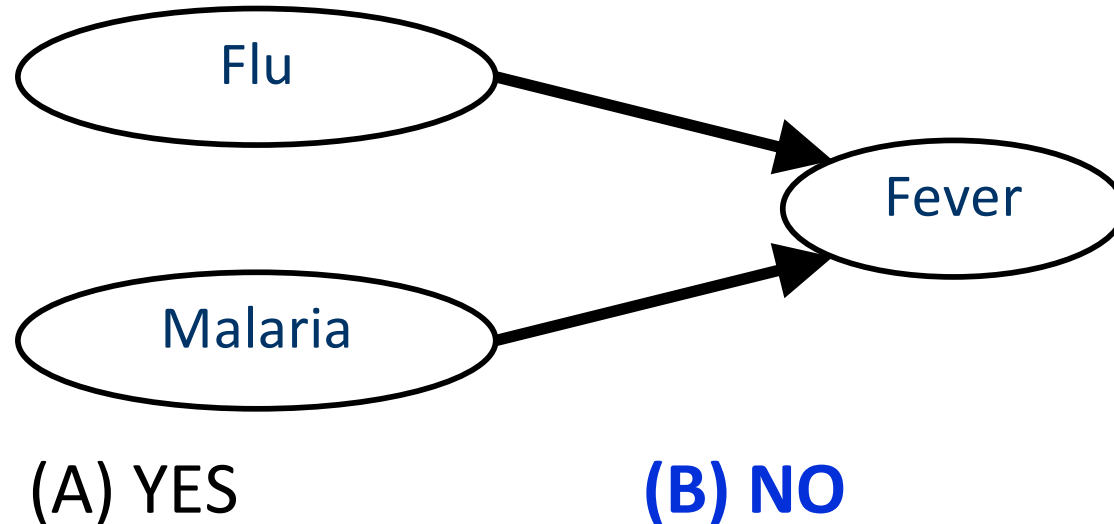


(A) YES

(B) NO

# Common Child: Conditional Independence

A2: Are Flu and Malaria conditionally independent given Fever?



$$P(fl|\neg m \wedge fe) > P(fl|m \wedge fe)$$

Assume that I have a Fever.

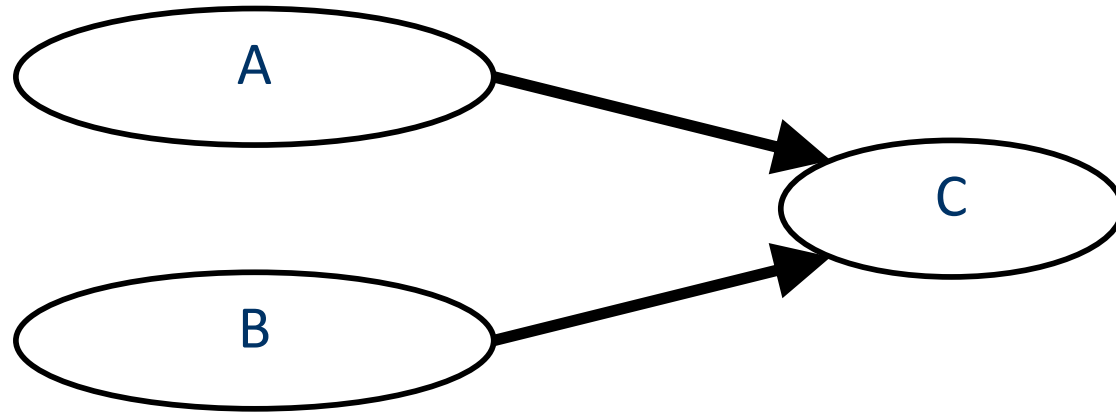
If I do not have Malaria, it is more likely that I have the Flu.

If I have Malaria, it is less likely that I have the Flu.

# Common Child: Summary

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*or Common Effect*



**A** and **B** are independent.

**A** and **B** are NOT independent given **C**.

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# D-SEPARATION

# D-Separation

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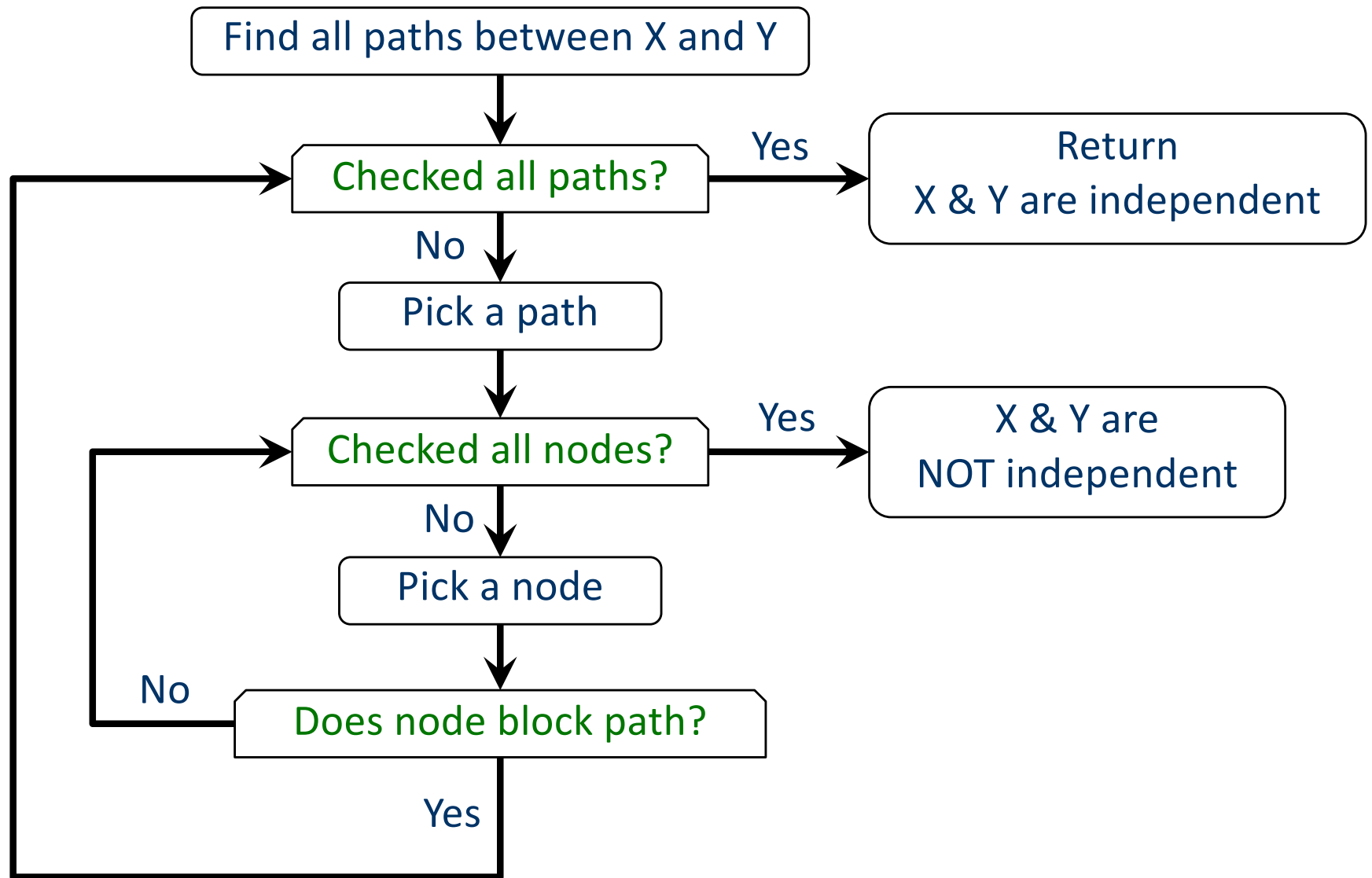
Are two variables  $X$  and  $Y$  independent given the set of observed variables  $E$ ?

If  $E$  d-separates  $X$  and  $Y$ ,  
then  $X$  and  $Y$  are independent given  $E$ .

$E$  d-separates  $X$  and  $Y \iff$   
 $E$  blocks every undirected path between  $X$  and  $Y$ .



# Does E D-Separate X and Y?



# Does E D-Separate X and Y?

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One path connects X and Y

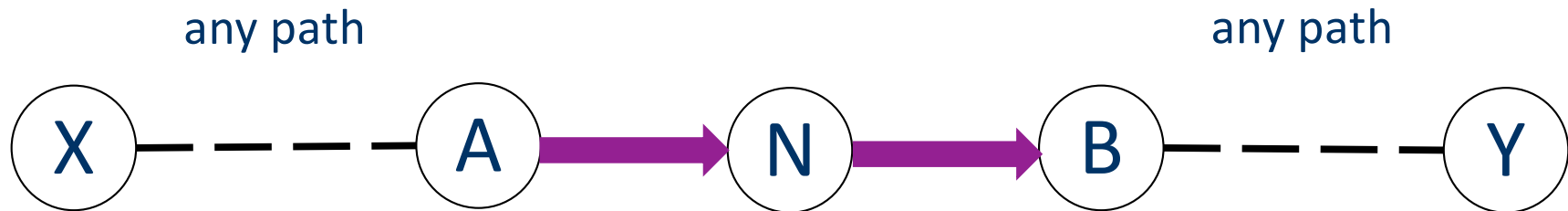
→ X and Y are NOT independent

Every path between X and Y is blocked

→ X and Y are independent

# Blocked Path – Scenario 1/3 (Chain)

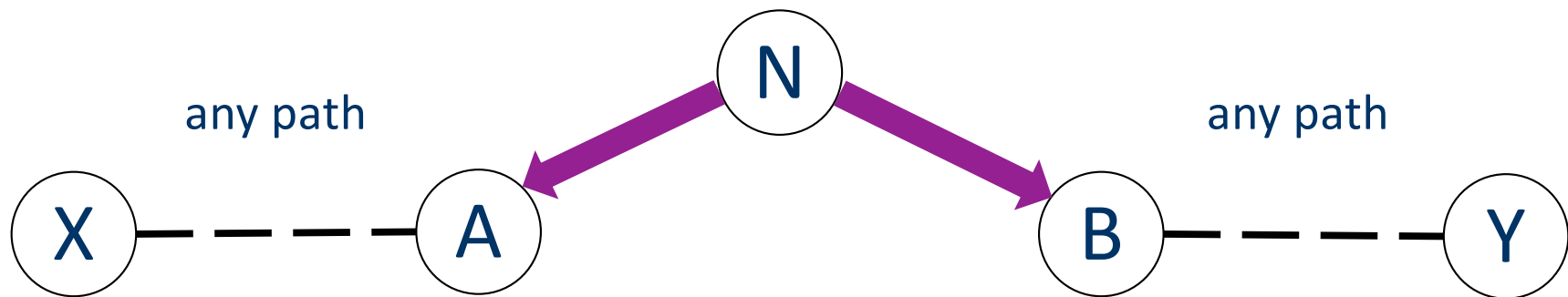
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If N is **observed**,  
N blocks the path between X and Y.

## Blocked Path – Scenario 2/3 (Common Parent)

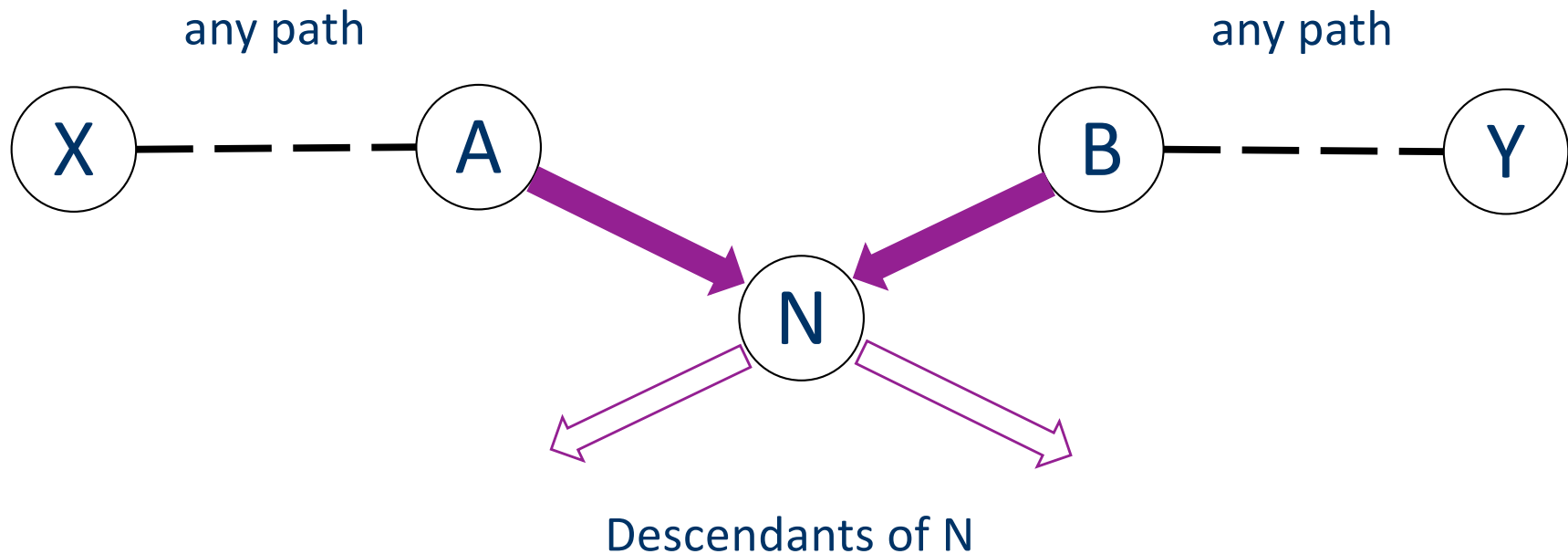
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If N is observed,  
N blocks the path between X and Y.

## Blocked Path – Scenario 3/3 (Common Descendants)

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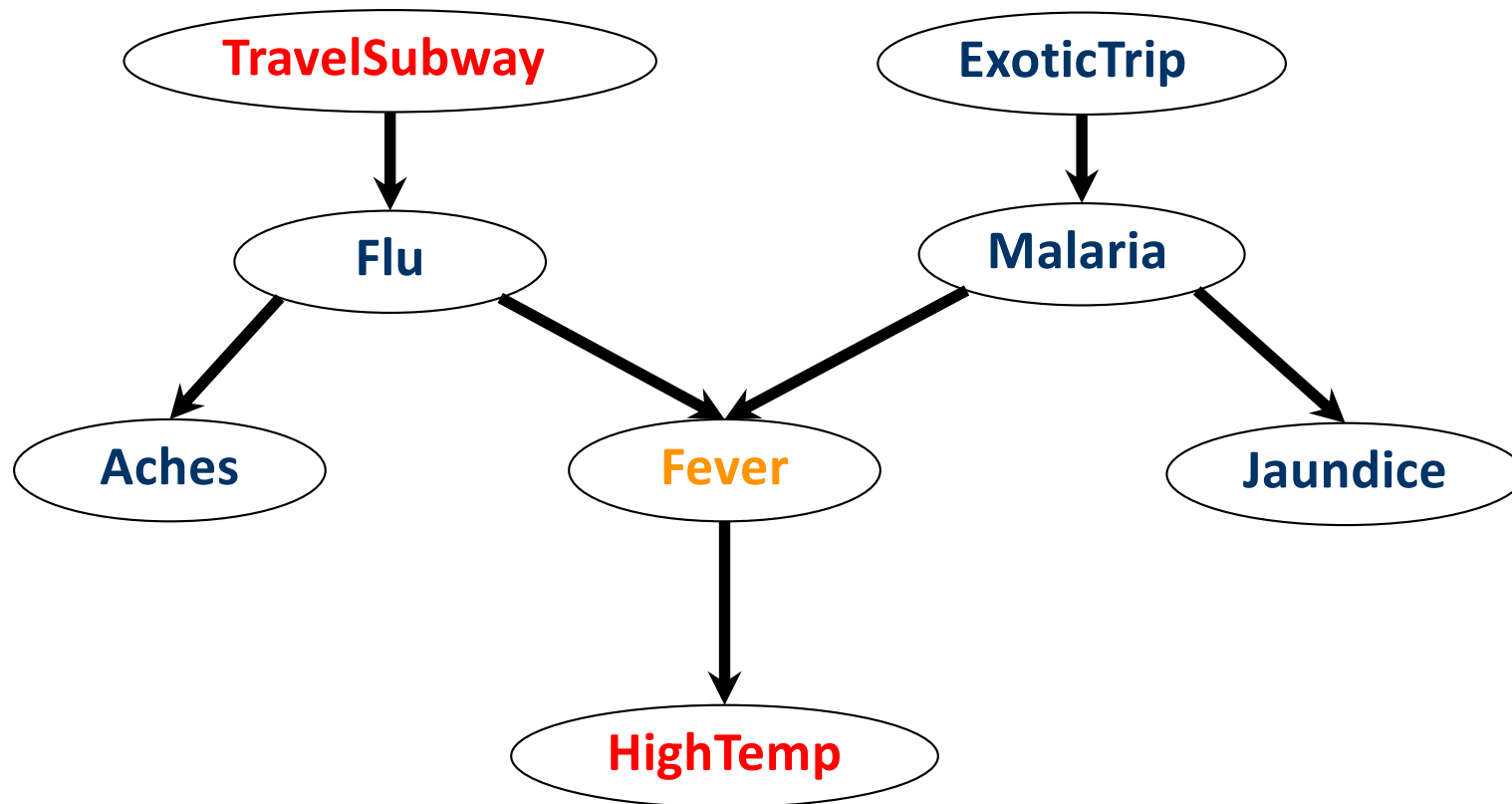


If N and N's descendants are **NOT** observed,  
they block the path between X and Y.

# Let's Apply D-Separation

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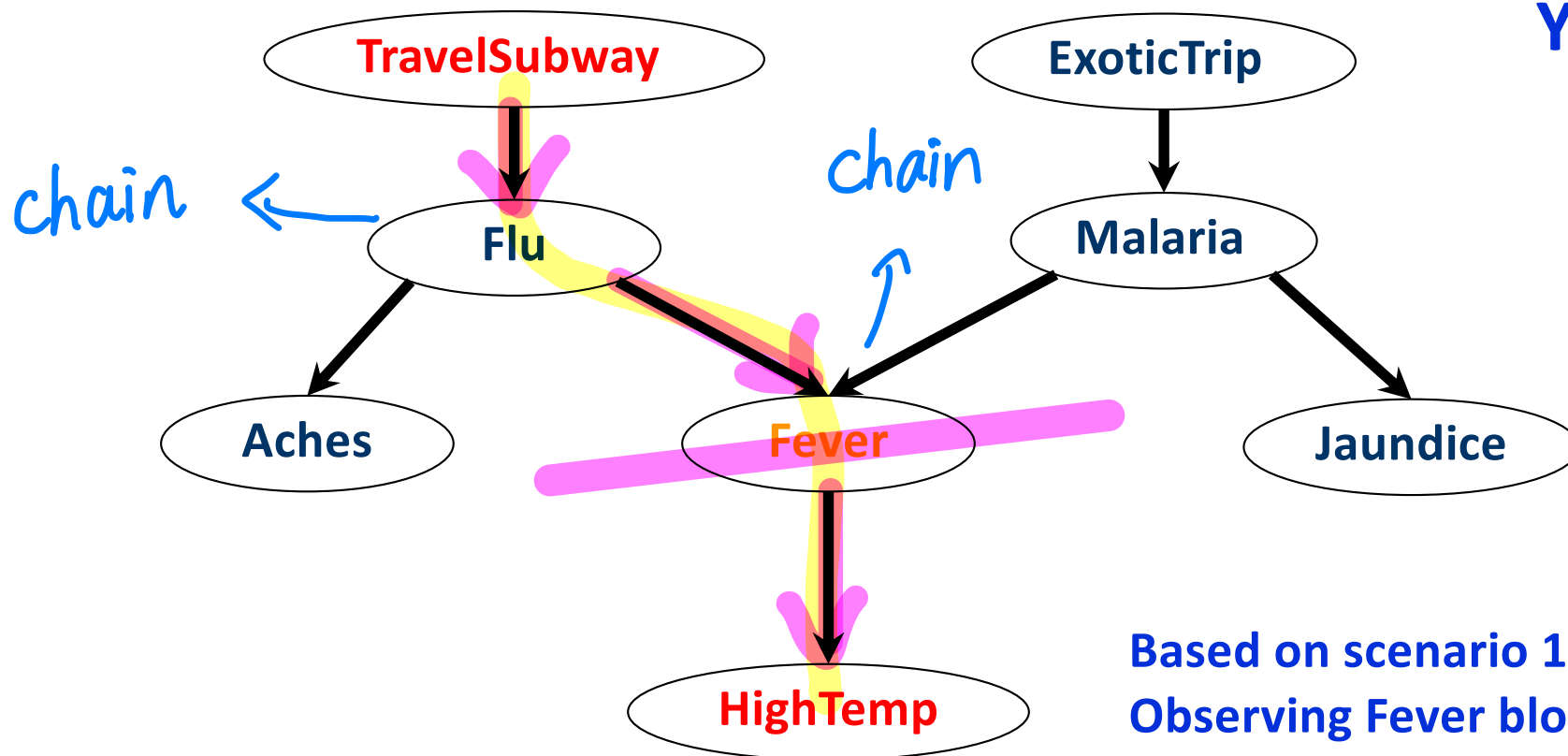
Q1a: Are **TravelSubway** and **HighTemp** conditionally independent given **Fever**?



# Let's Apply D-Separation

Q1a: Are **TravelSubway** and **HighTemp** conditionally independent given **Fever**?

**Answer:**  
**YES**

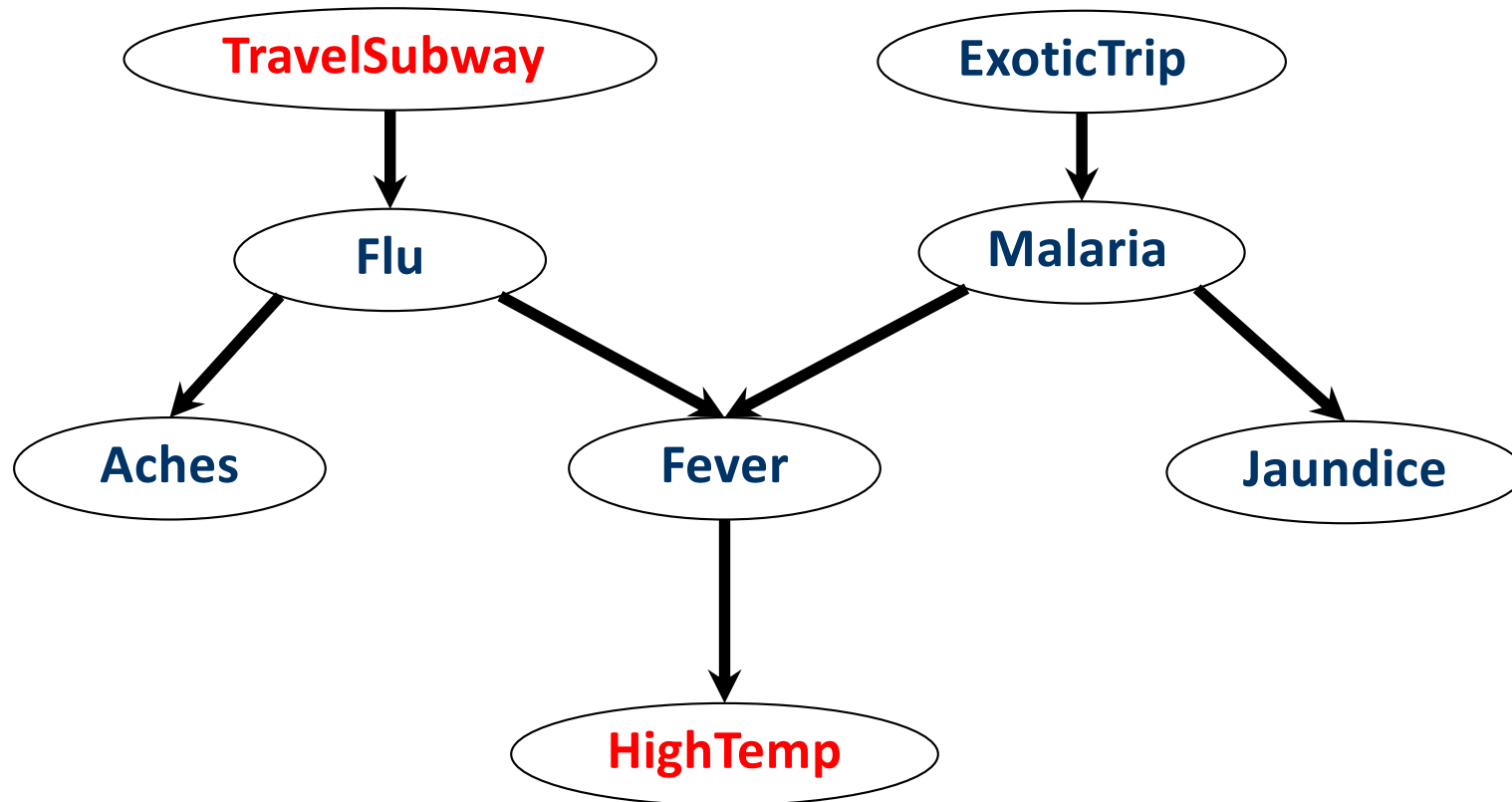


Based on scenario 1,  
Observing Fever blocks the  
path TS → F → F → HT.

# Let's Apply D-Separation

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Q1b: Are **TravelSubway** and **HighTemp** independent?

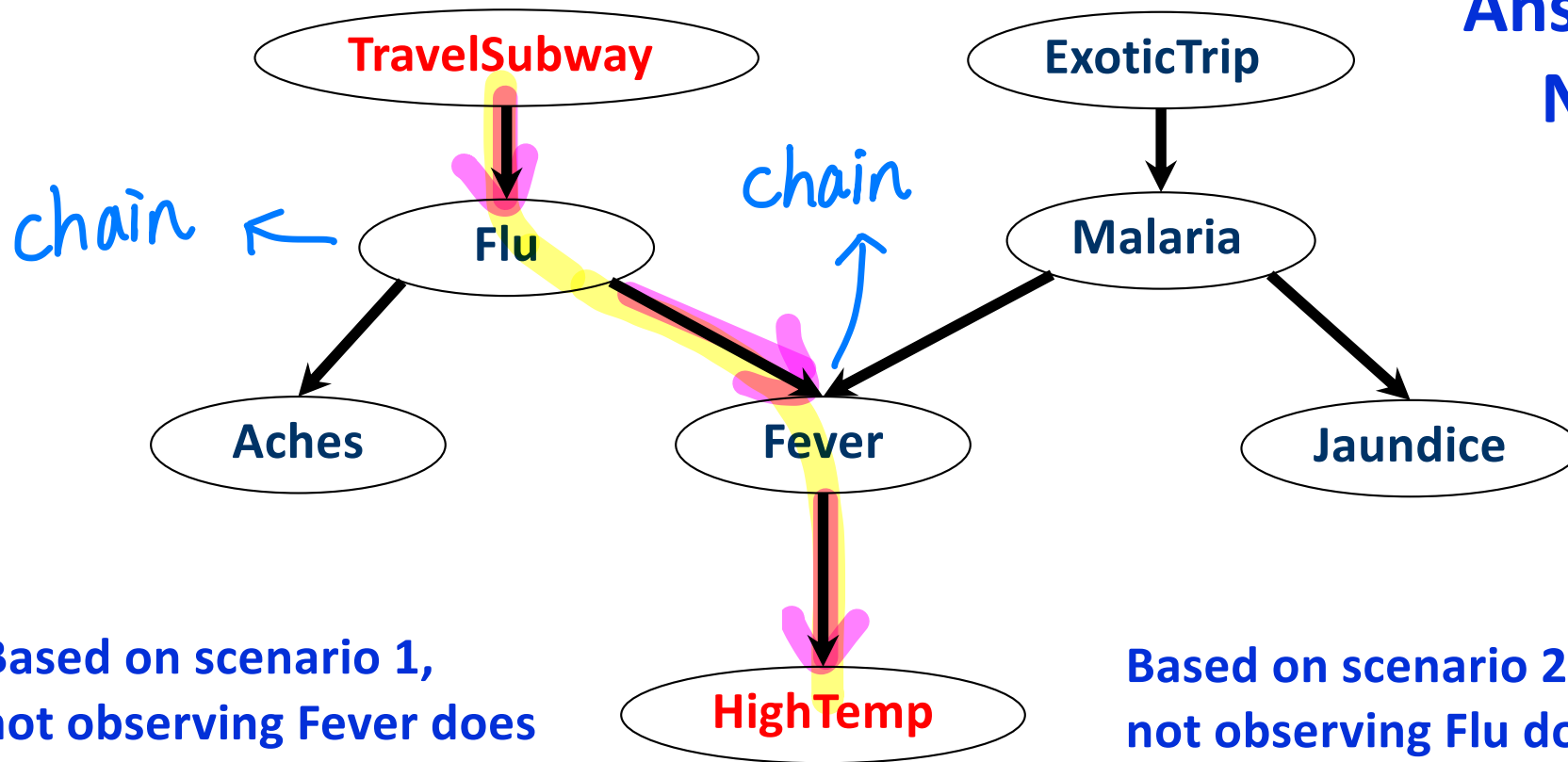




# Let's Apply D-Separation

Q1b: Are **TravelSubway** and **HighTemp** independent?

**Answer:**  
**NO**



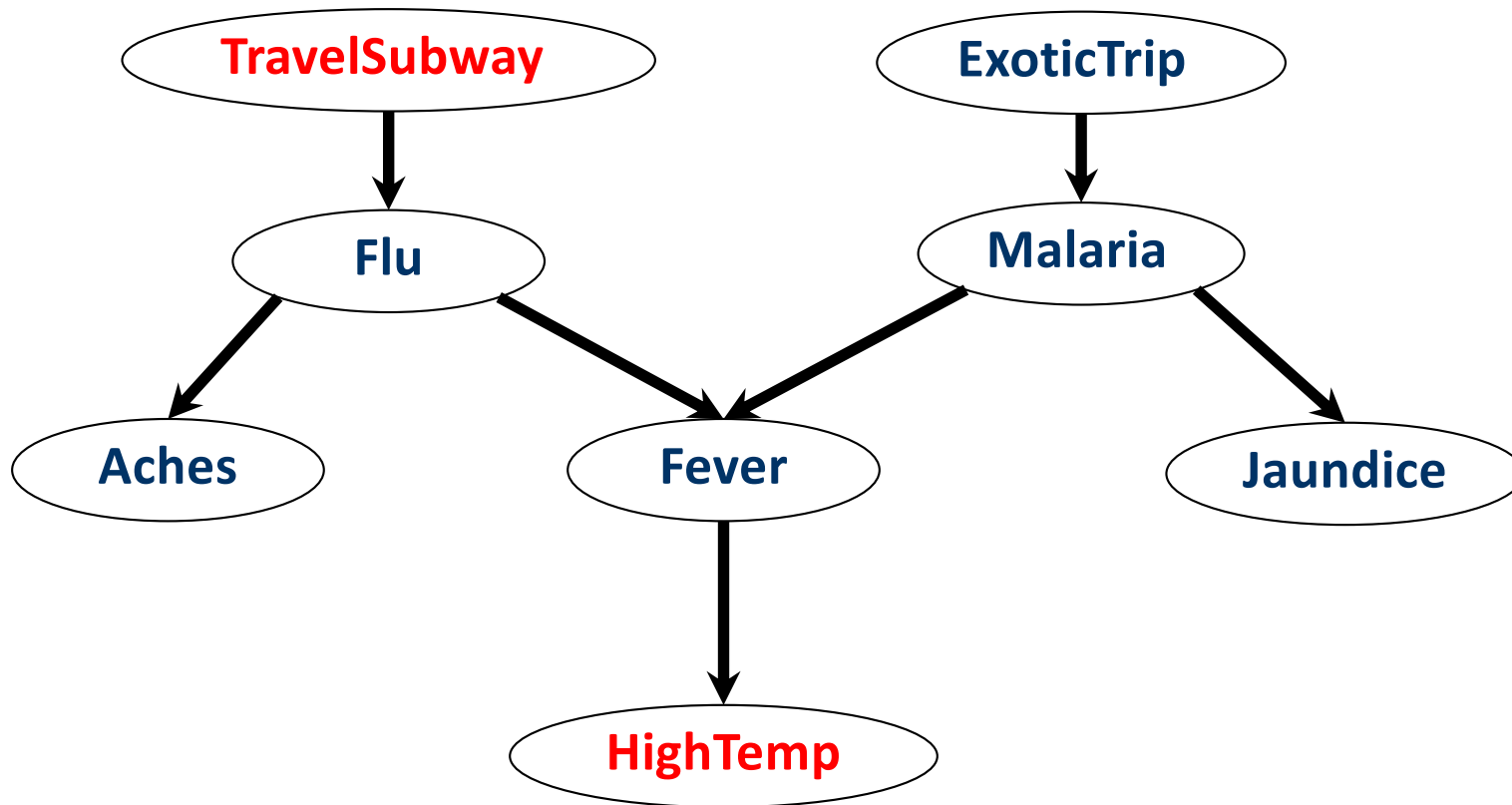
Based on scenario 1,  
not observing Fever does  
not block the path TS → F →  
F → HT.

Based on scenario 2,  
not observing Flu does not  
block the path TS → F → F →  
HT.

# Let's Apply D-Separation

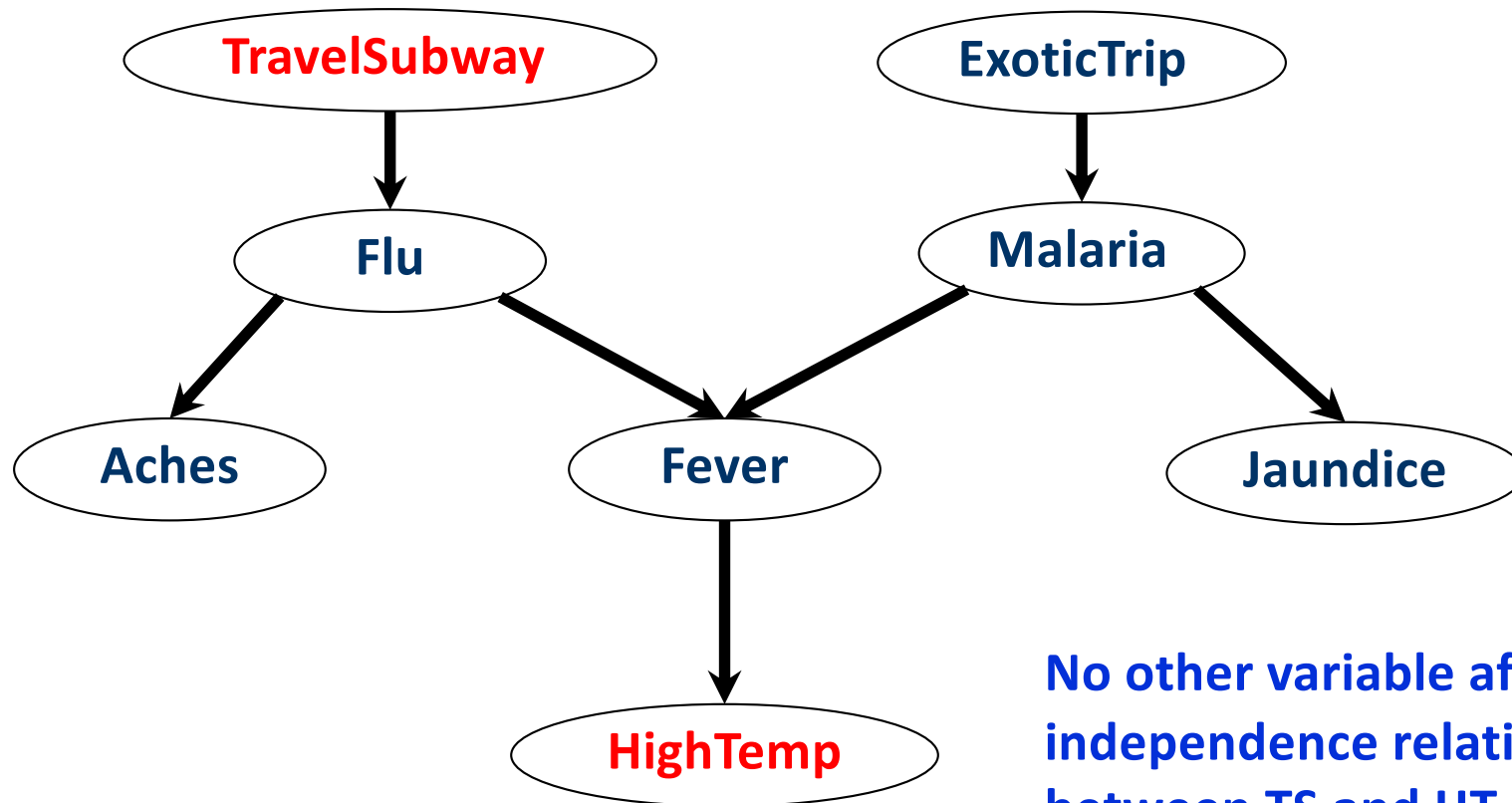
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Q1c: How do other variables affect the **independence** relationship between **TravelSubway** and **HighTemp**?



# Let's Apply D-Separation

Q1c: How do other variables affect the **independence** relationship between **TravelSubway** and **HighTemp**?

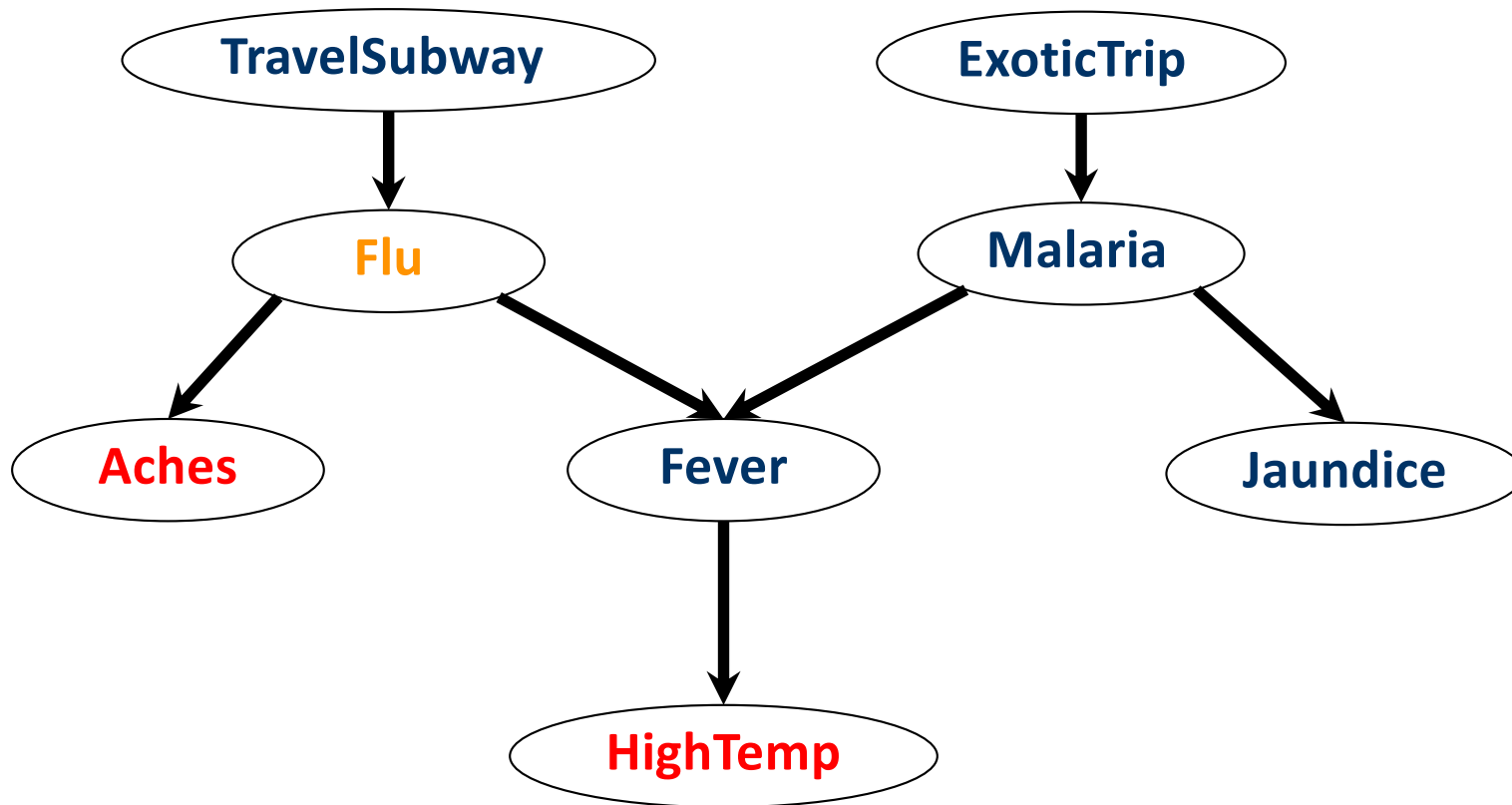


No other variable affects the independence relationship between TS and HT.

# Let's Apply D-Separation

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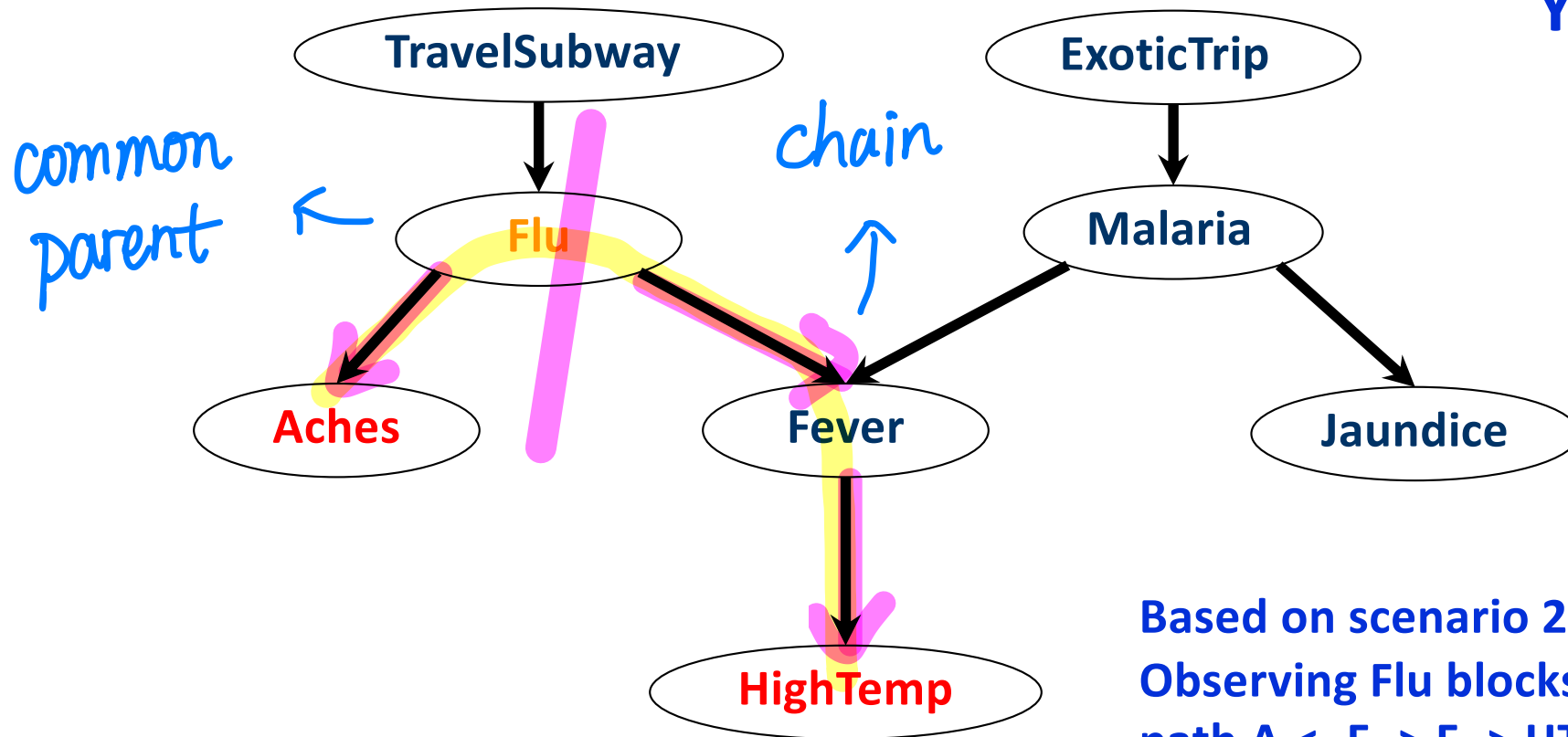
Q2a: Are **Aches** and **HighTemp** conditionally independent given **Flu**?



# Let's Apply D-Separation

Q2a: Are **Aches** and **HighTemp** conditionally independent given **Flu**?

**Answer:**  
**YES**

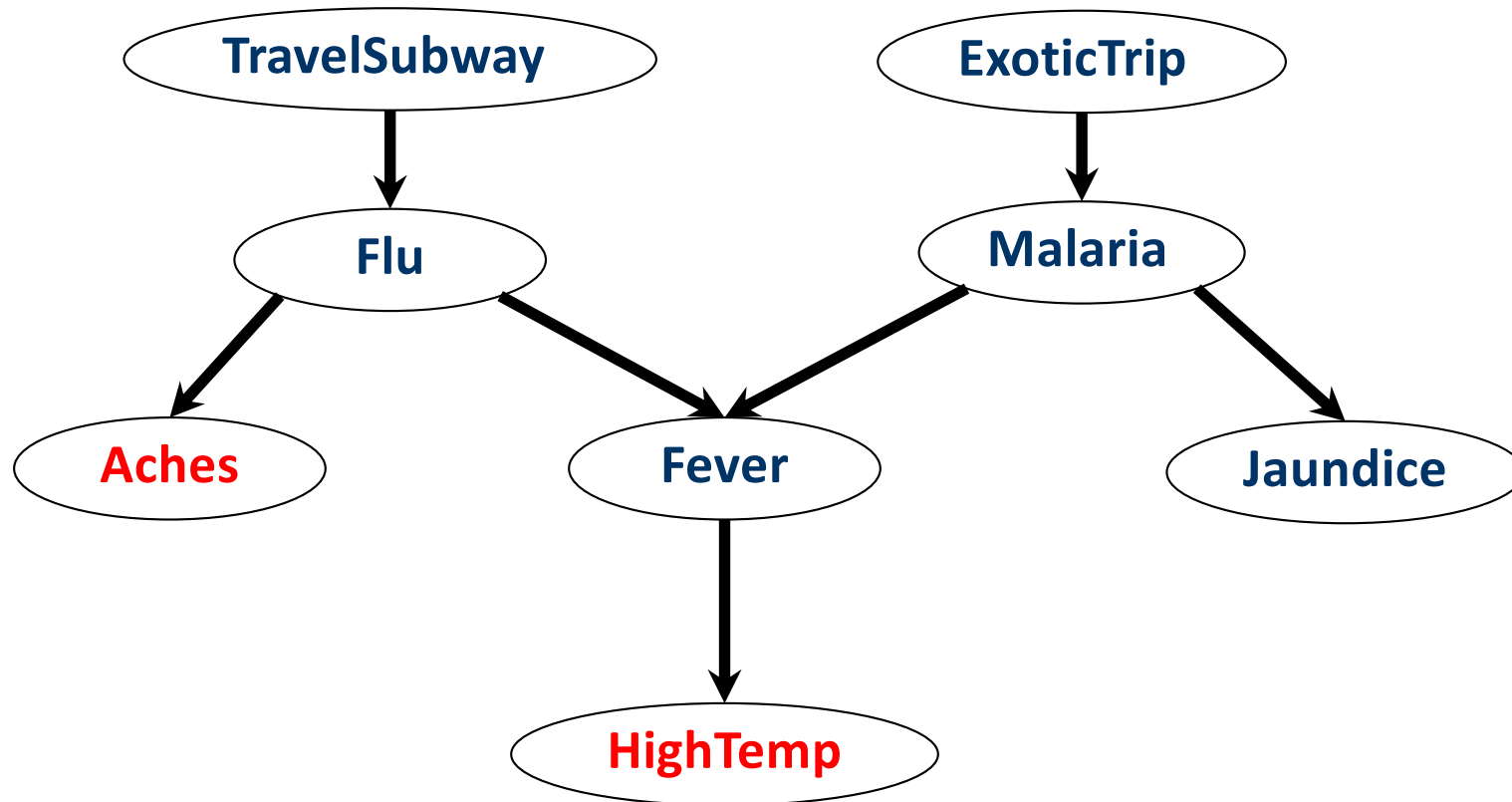


Based on scenario 2,  
Observing Flu blocks the  
path  $A \leftarrow F \rightarrow F \rightarrow HT$ .

# Let's Apply D-Separation

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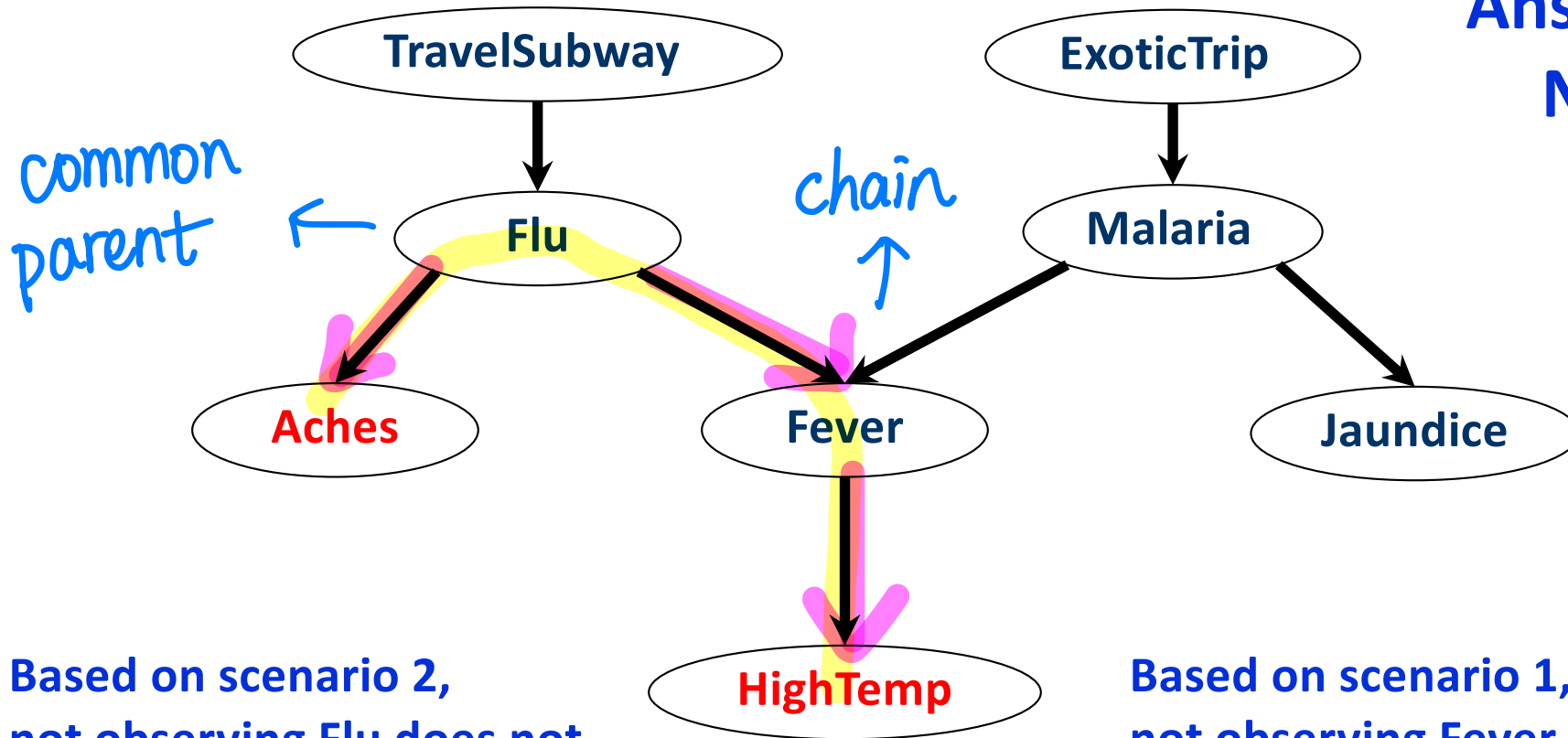
Q2b: Are **Aches** and **HighTemp** independent?



# Let's Apply D-Separation

Q2b: Are **Aches** and **HighTemp** independent?

**Answer:  
NO**



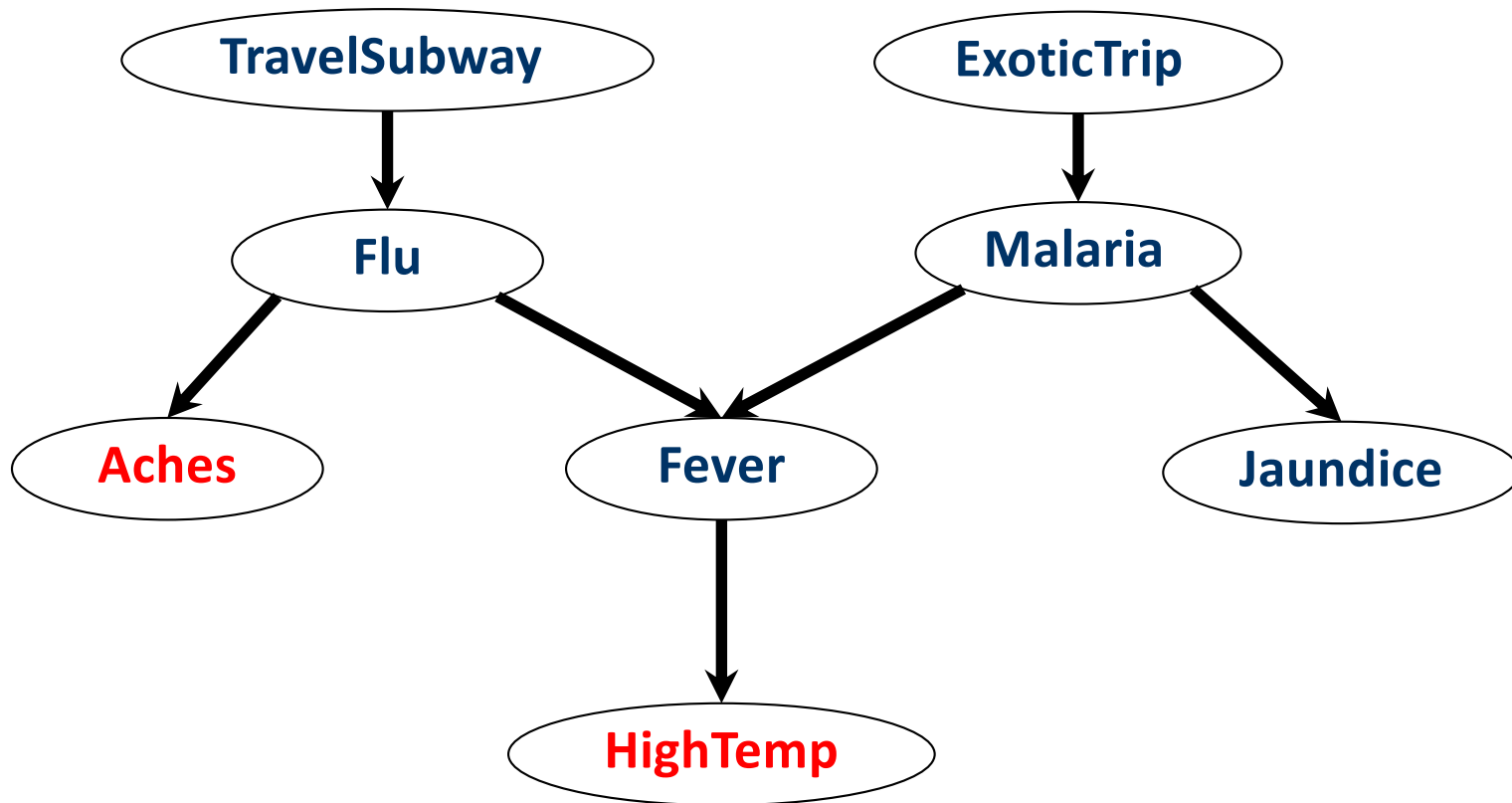
Based on scenario 2,  
not observing Flu does not  
block the path  $A \leftarrow F \rightarrow F \rightarrow$   
HT.

Based on scenario 1,  
not observing Fever does  
not block the path  $A \leftarrow F \rightarrow F \rightarrow$   
HT.

# Let's Apply D-Separation

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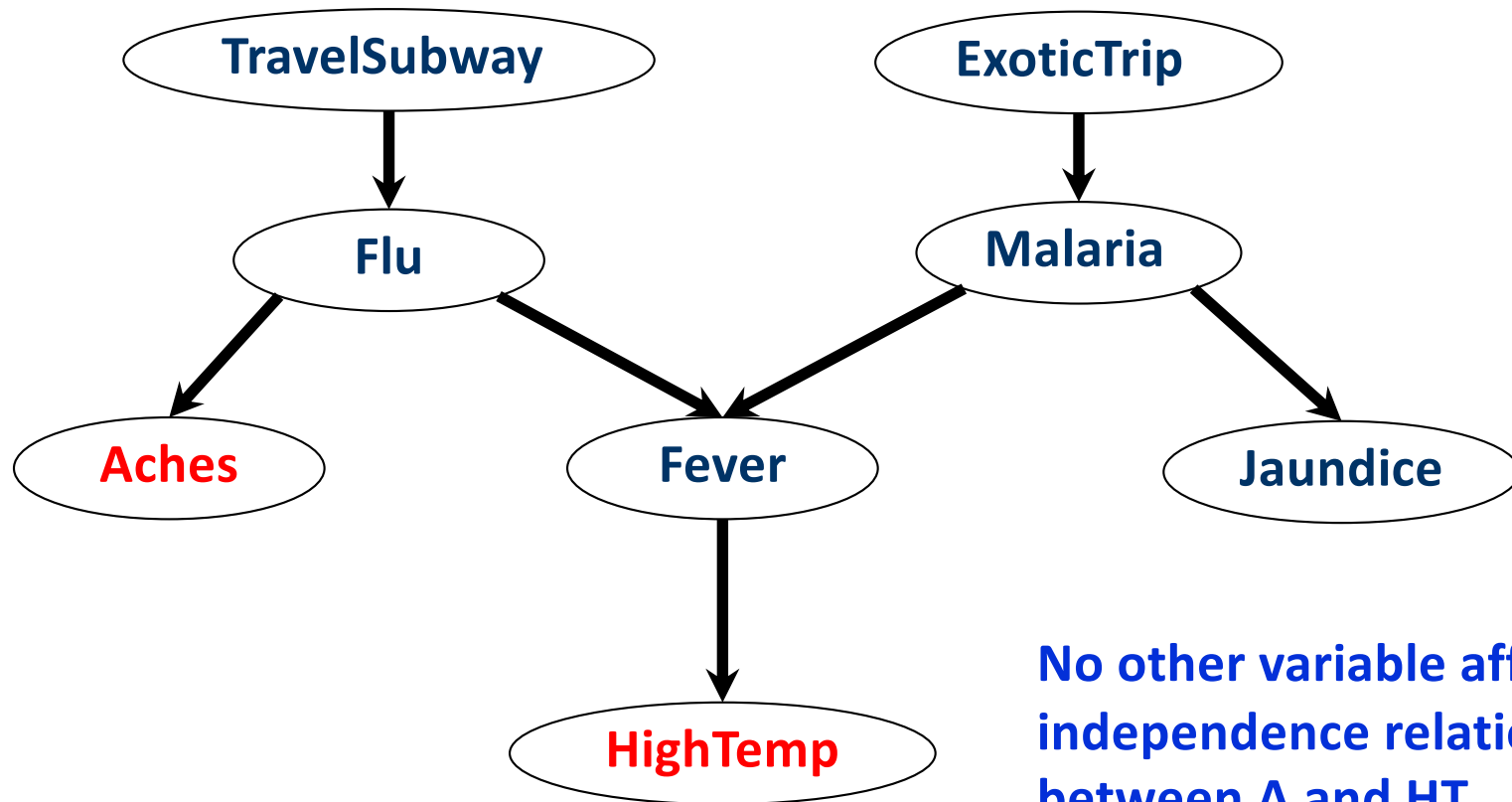
Q2c: How do other variables affect the **independence** relationship between **Aches** and **HighTemp**?





# Let's Apply D-Separation

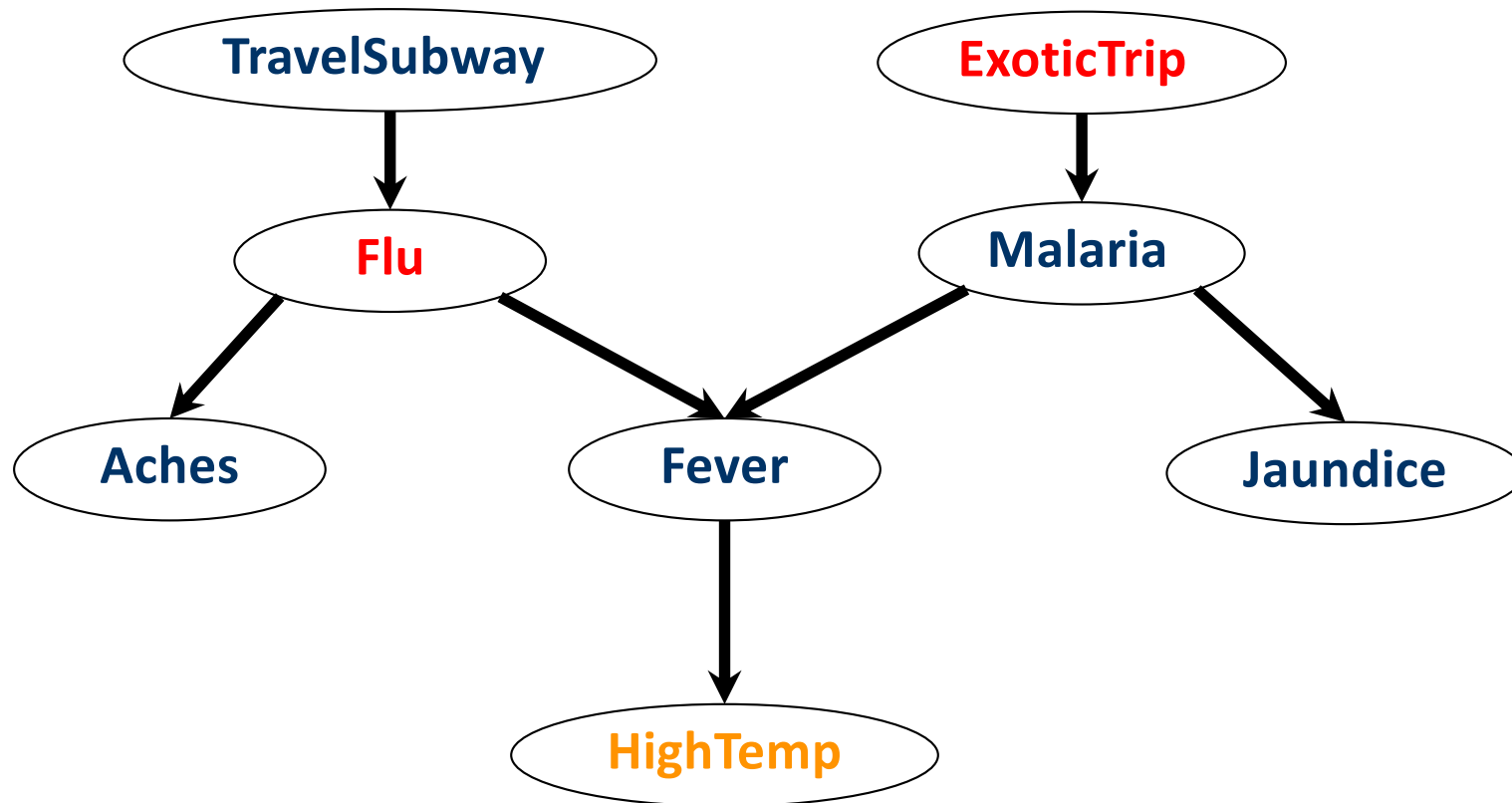
Q2c: How do other variables affect the **independence** relationship between **Aches** and **HighTemp**?



# Let's Apply D-Separation

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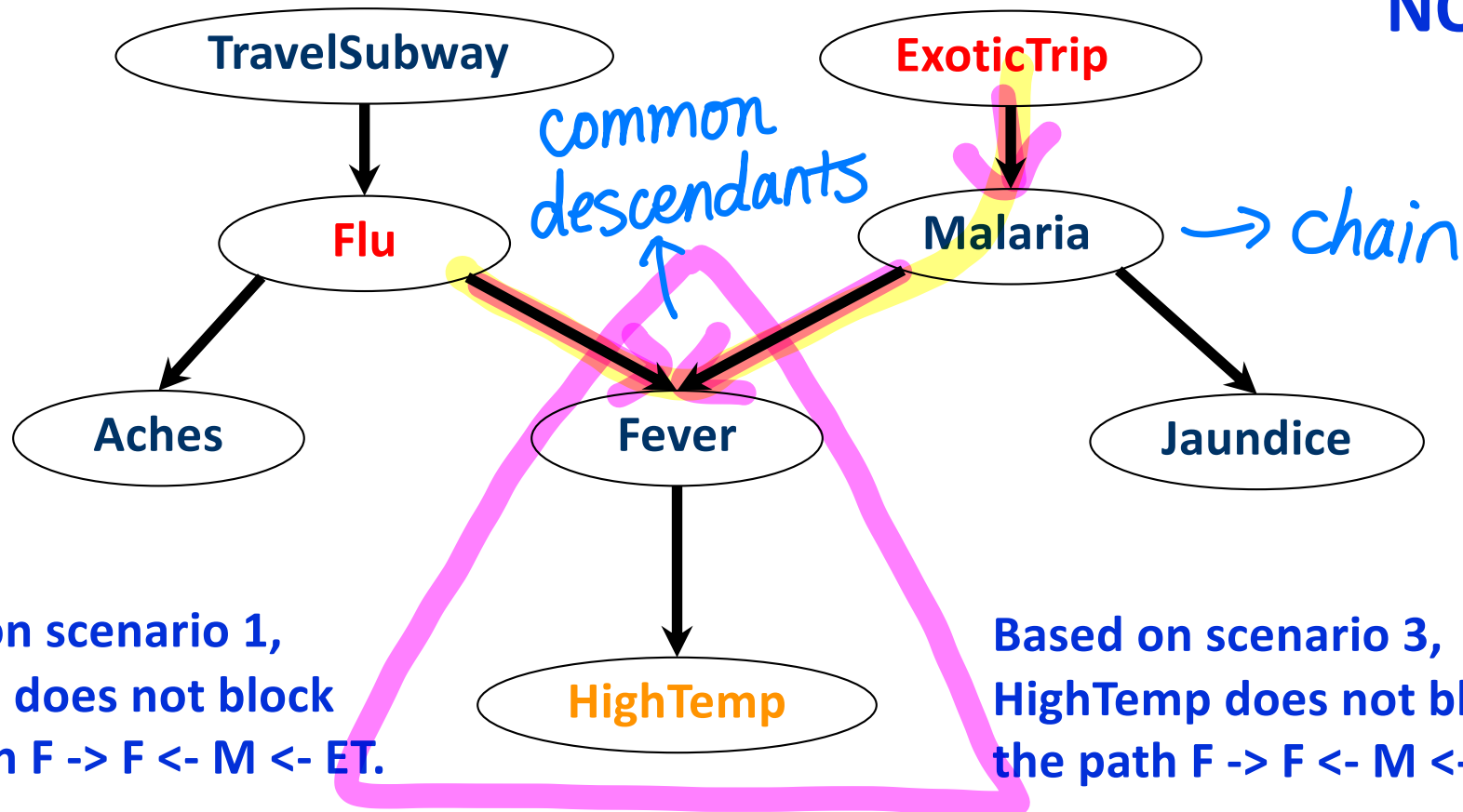
Q3a: Are **Flu** and **ExoticTrip** conditionally independent given **HighTemp**?



# Let's Apply D-Separation

Q3a: Are **Flu** and **ExoticTrip** conditionally independent given **HighTemp**?

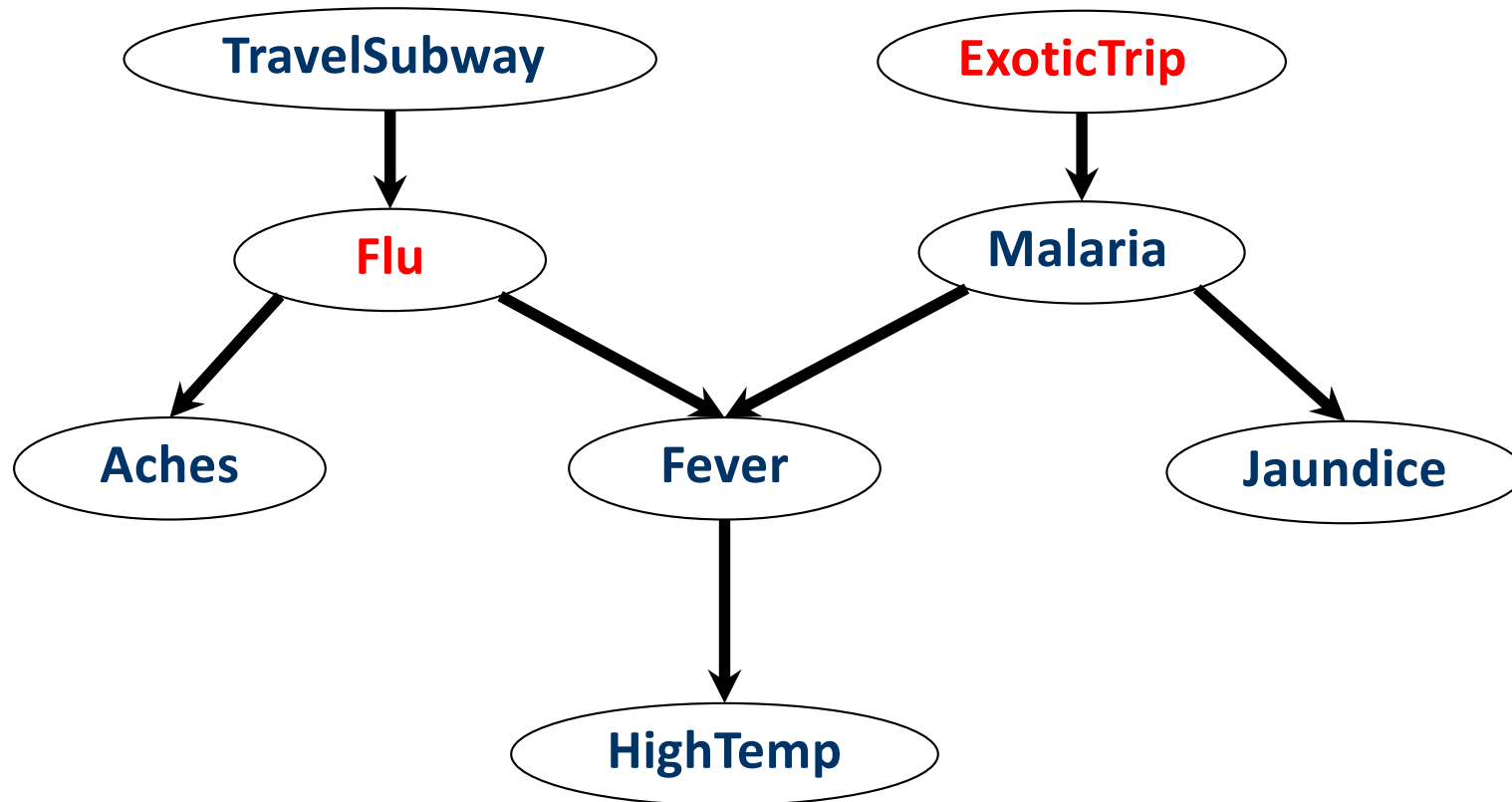
Answer:  
**NO**



# Let's Apply D-Separation

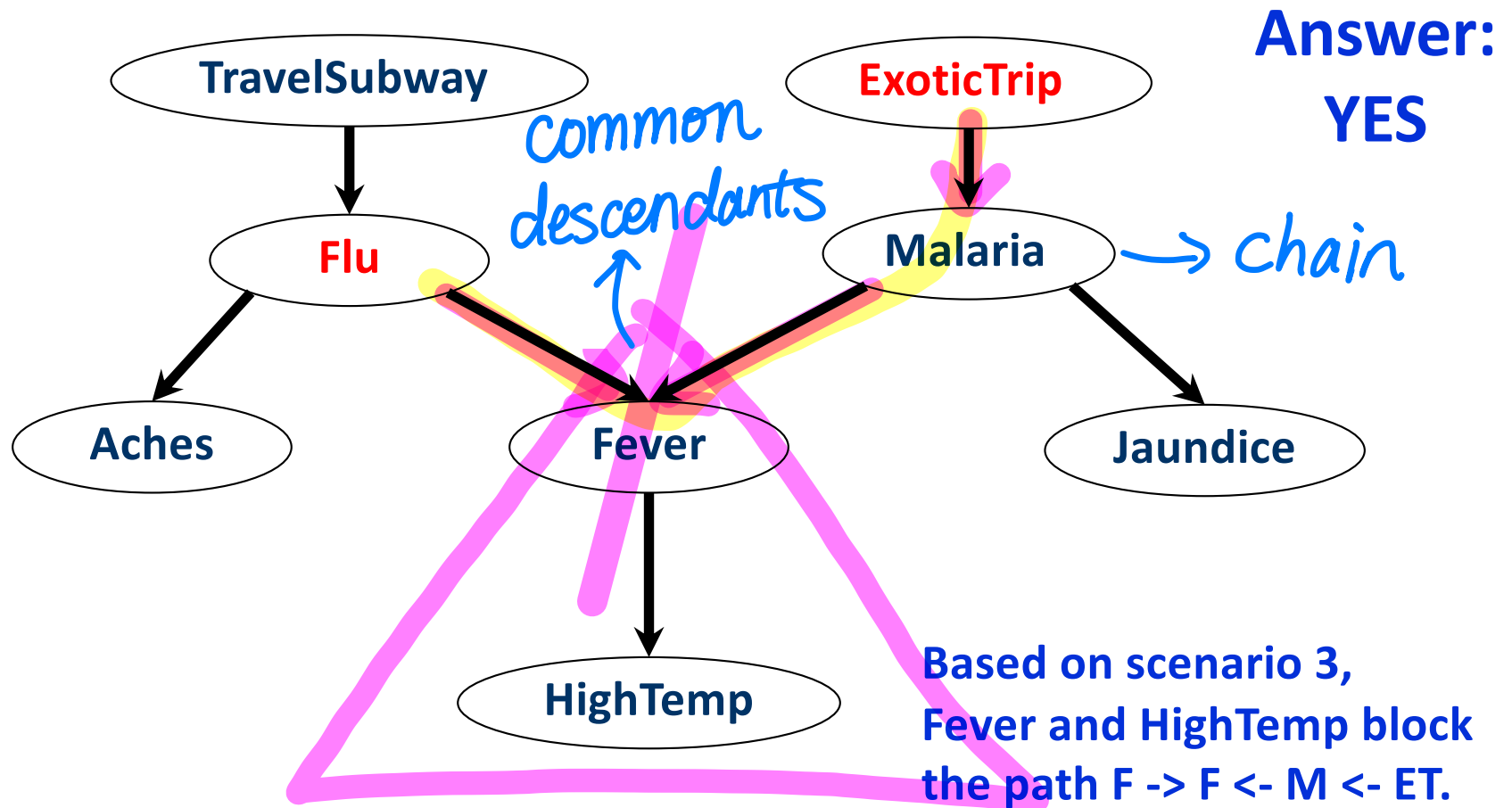
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Q3b: Are **Flu** and **ExoticTrip** independent?



# Let's Apply D-Separation

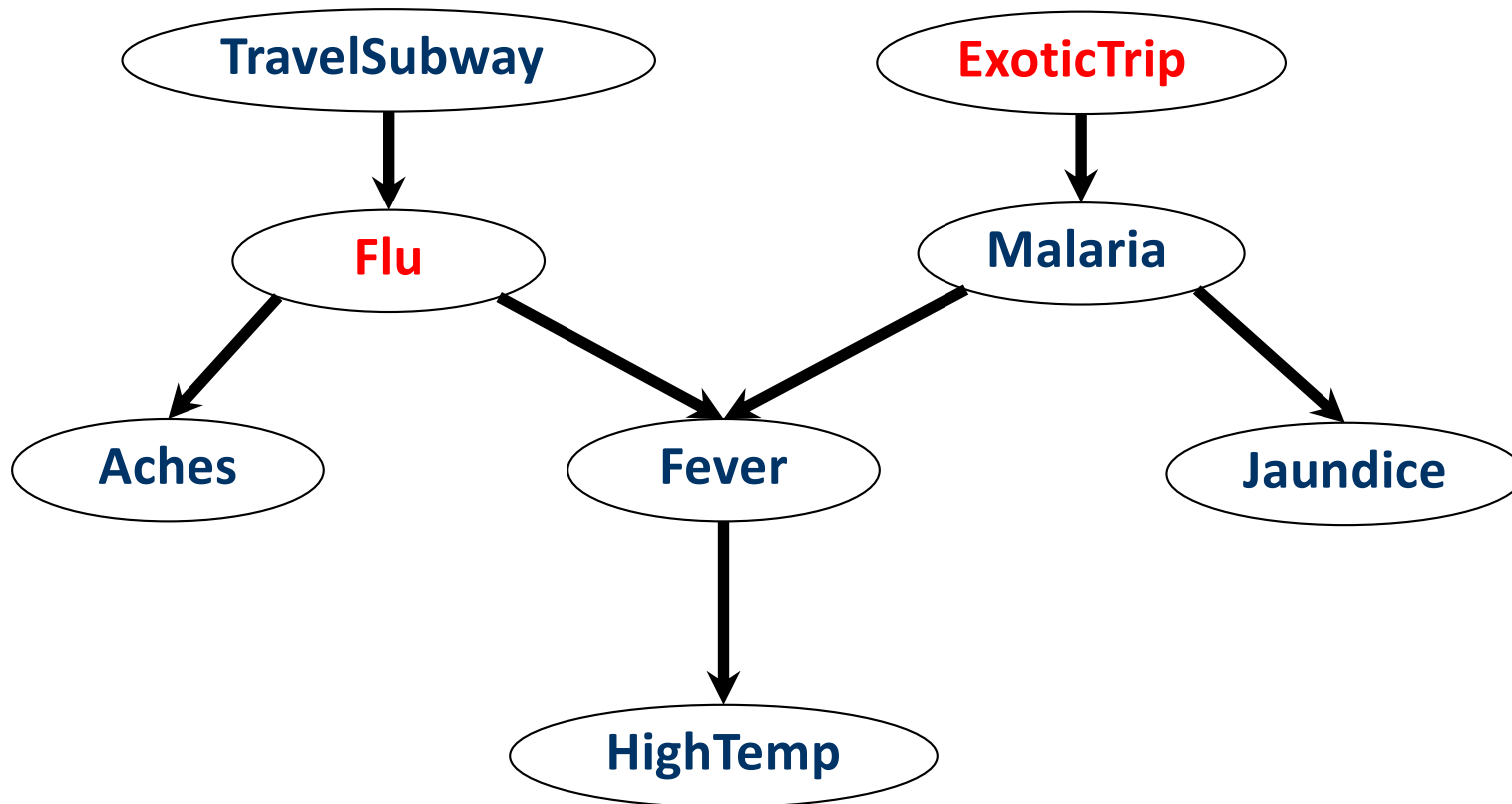
Q3b: Are **Flu** and **ExoticTrip** independent?



# Let's Apply D-Separation

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Q3c: How do other variables affect the **independence** relationship between **Flu** and **ExoticTrip**?



# Let's Apply D-Separation

Q3c: How do other variables affect the **independence** relationship between **Flu** and **ExoticTrip**?

