CSC 384
Introduction to Artificial Intelligence

Knowledge Representation 3

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Learning Goals

By the end of this lecture, you should be able to

- Construct a resolution proof using forward chaining
- Construct a resolution proof as a refutation proof.

Outline

1. Resolution Proof Procedure
2. Resolution Proof Example

RESOLUTION PROOF PROCEDURE

Clausal Form

- To construct a resolution proof, we must convert each formula to clausal form.

- A literal is an atomic formula or the negation of an atomic formula.
  - e.g. $p(X)$ or $\neg p(X)$
- A clause is a disjunction (OR) of literals.
  - e.g. $p(X) \vee q(Y) \vee \neg r(Y, Z)$
  - We rewrite it: $(p(X), q(Y), \neg r(Y, Z))$

Resolution Rule

Resolution proof has one rule:

From the two clauses

$$(P, Q_1, \dots, Q_k)$$
$$(\neg P, R_1, \dots, R_n)$$

We infer the new clause

$$(Q_1, \dots, Q_k, R_1, \dots, R_n)$$

Two Proof Approaches

Suppose we want to prove f from KB.

- Forward Chaining (or Direct Proof)
  - Start with the clauses from the KB.
  - Generate a new clause using the resolution rule.
  - Stop when we generate f.

- Refutation Proof (or Proof by Contradiction)
  - Start with the clauses from KB and a new clause $\neg f$.
  - Generate a new clause using the resolution rule.
  - Stop when we generate the empty clause.

Forward Chaining Example

Want to prove likes(C,peanuts) from:

1. $(\text{elephant}(C), \text{giraffe}(C))$
2. $(\neg\text{elephant}(C), \text{likes}(C, \text{peanuts}))$
3. $(\neg\text{giraffe}(C), \text{likes}(C, \text{leaves}))$
4. $\neg\text{likes}(C, \text{leaves})$

Forward Chaining Proof:

Step	**Clauses to combine**	**Resulting clause**
5		
6		
7		

Forward Chaining Example

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1. $(\text{elephant}(C), \text{giraffe}(C))$
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3. $(\neg\text{giraffe}(C), \text{likes}(C, \text{leaves}))$
4. $\neg\text{likes}(C, \text{leaves})$

Forward Chaining Proof:

Step	**Clauses to combine**	**Resulting clause**
5	3 and 4	¬giraffe(C)
6	1 and 5	elephant(C)
7	2 and 6	likes(C,peanuts)

Refutation Proof Example

Want to prove likes(C,peanuts) from:

1. $(\text{elephant}(C), \text{giraffe}(C))$
2. $(\neg\text{elephant}(C), \text{likes}(C, \text{peanuts}))$
3. $(\neg\text{giraffe}(C), \text{likes}(C, \text{leaves}))$
4. $\neg\text{likes}(C, \text{leaves})$

Refutation Proof:

Step	**Clauses to combine**	**Resulting clause**
5	Refutation clause	¬likes(C,peanuts)
6	2 and 5	¬elephant(C)
7	1 and 6	giraffe(C)
8	3 and 7	likes(C,leaves)
9	4 and 8	Empty clause

RESOLUTION PROOF EXAMPLE

Resolution Proof Example

The assertions:

- Some patients like every doctor.
- No patient likes any quack.

Prove that no doctor is a quack.

Step 1: Define symbols.

Step 1: Define predicates.

The assertions:

- Some patients like every doctor.
- No patient likes any quack.
- Prove that no doctor is a quack.

Step 1: Define symbols.

Step 1: Define predicates.

- $p(X)$: X is a patient.
- $d(X)$: X is a doctor.
- $q(X)$: X is a quack.
- $likes(X, Y)$: X likes Y.

The assertions:

- Some patients like every doctor.
- No patient likes any quack.
- Prove that no doctor is a quack.

Step 2: Convert each assertion to a formula.

Some patients like every doctor.

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Some patients like every doctor.

$$\exists X \left(p(X) \wedge \left(\forall Y \big(d(Y) \to likes(X,Y) \big) \right) \right)$$

Step 2: Convert each assertion to a formula.

No patient likes any quack.

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No patient likes any quack.

$$\neg\left(\exists X\ \left(p(X) \wedge \left(\exists Y\ \left(q(Y) \wedge likes(X,Y)\right)\right)\right)\right)$$

Step 2: Convert each assertion to a formula.

No doctor is a quack.

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No doctor is a quack.

$$\neg \left(\exists X \left(d(X) \wedge q(X) \right) \right)$$

Step 3: Convert to clausal form

$$\exists X \left(p(X) \land \left(\forall Y \left(d(Y) \rightarrow likes(X, Y) \right) \right) \right)$$

Convert to clausal form:

Step 3: Convert to clausal form

$$\exists X \left(p(X) \wedge \left(\forall Y \left(d(Y) \rightarrow likes(X, Y) \right) \right) \right)$$

Convert to clausal form:

1. $\exists X \left(p(X) \wedge \left(\forall Y \left(\neg d(Y) \vee likes(X, Y) \right) \right) \right)$
2. $p(a) \wedge \left(\forall Y \left(\neg d(Y) \vee likes(a, Y) \right) \right)$
3. $\forall Y \; p(a) \wedge \left(\neg d(Y) \vee likes(a, Y) \right)$
4. *Clause 1:* $p(a)$ *Clause 2:* $\left(\neg d(Y), likes(a, Y) \right)$

Step 3: Convert to clausal form

$$\neg\left(\exists X\ \left(p(X) \wedge \left(\exists Y\ \left(q(Y) \wedge likes(X, Y)\right)\right)\right)\right)$$

Convert to clausal form:

Step 3: Convert to clausal form

$$\neg\left(\exists X\ \left(p(X) \wedge \left(\exists Y\ \left(q(Y) \wedge likes(X,Y)\right)\right)\right)\right)$$

Convert to clausal form:

1. $\forall X\left(\neg p(X) \vee \left(\forall Y\left(\neg q(Y) \vee \neg likes(X,Y)\right)\right)\right)$
2. $\forall X \forall Y\left(\neg p(X) \vee \left(\neg q(Y) \vee \neg likes(X,Y)\right)\right)$
3. $\forall X \forall Y\left(\neg p(X) \vee \neg q(Y) \vee \neg likes(X,Y)\right)$
4. Clause: $(\neg p(X), \neg q(Y), \neg likes(X,Y))$

Step 3: Negate the query and convert to clausal form.

$$\neg\neg\left(\exists X\left(d(X) \land q(X)\right)\right)$$

Convert to clausal form:

Step 3: Negate the query and convert to clausal form.

$$\neg\neg\left(\exists X\ (d(X) \land q(X))\right)$$

Convert to clausal form:

1. $\exists X\ \left(d(X) \land q(X)\right)$
2. $d(b) \land q(b)$
3. *Clause 1:* $d(b)$ *Clause 2:* $q(b)$

Step 4: Construct resolution proof from clauses

1. $p(a)$
2. $(\neg d(Y), likes(a, Y))$
3. $(\neg p(X), \neg q(Y), \neg likes(X, Y))$

Step	Clauses to combine	Resulting clause
4	Refutation clause	$d(b)$
5	Refutation clause	$q(b)$
6		
7		
8		
9		

Step 4: Construct resolution proof from clauses

1. $p(a)$
2. $(\neg d(Y), likes(a, Y))$
3. $(\neg p(X), \neg q(Z), \neg likes(X, Z))$

Rename Y to Z to make the variable names unique.

Step	Clauses to combine	Resulting clause
4	Refutation clause	$d(b)$
5	Refutation clause	$q(b)$
6		
7		
8		
9		

Step 4: Construct resolution proof from clauses

1. $p(a)$
2. $\left(\neg d(Y), likes(a, Y)\right)$
3. $\left(\neg p(X), \neg q(Z), \neg likes(X, Z)\right)$

Step	Clauses to combine	Resulting clause
4	Refutation clause	$d(b)$
5	Refutation clause	$q(b)$
6	3 and 5, Z = b	$(\neg p(X), \neg likes(X, b))$
7	1 and 6, X = a	$\neg likes(a, b)$
8	2 and 7, Y = b	$\neg d(b)$
9	4 and 8	Empty clause

THANK YOU FOR TAKING THIS COURSE WITH US!