SOLUTIONS

CSC 384 Test 4 on Uncertainty

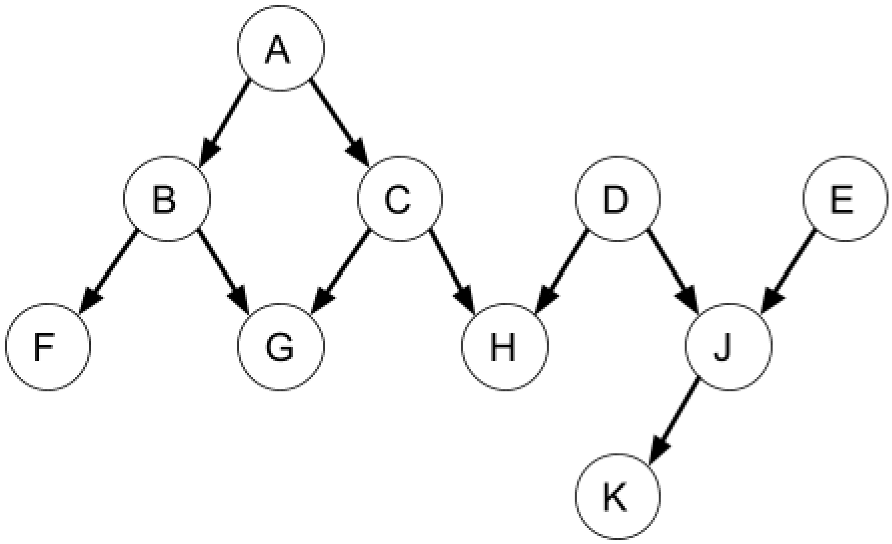
Friday, November 25, 2022

Last Name: ________________________________

First Name: ________________________________

Student Number: ________________________________

Q1 (6 marks) D-Separation



Consider the Bayesian network above. Circle the correct answer.

(a) **(1 mark)** C and E are conditionally independent given K.

True or False

Solutions: Not observing H blocks the path CHDJE by rule 3.

(b) **(1 mark)** C and E are conditionally independent given H, K.

True or False

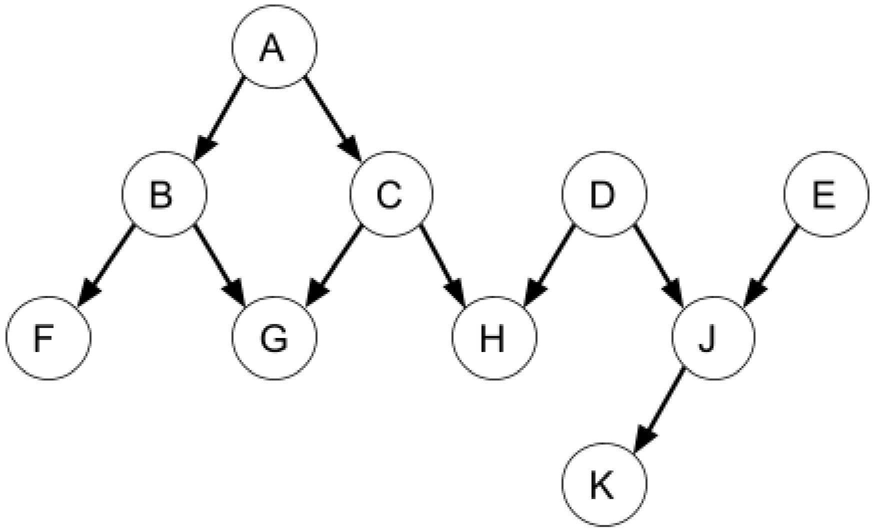
Solutions: Observing H doesn't block the path CHDJE by rule 3. Not observing D doesn't block the path by rule 2. Observing K doesn't block the path by rule 3.

(c) **(1 mark)** H and K are unconditionally independent.

True or False

Solutions: Not observing D doesn't block the path HDJK by rule 2. Not observing J doesn't block the path by rule 1.

Q1 (6 marks) D-Separation (Continued)



(*This diagram is the same as the one on the previous page*.)

(d) **(1 mark)** C and K are conditionally independent given H and J.

True or False

Solutions: Observing J blocks the path by rule 1.

(e) **(1 mark)** F and H are conditionally independent given A and G.

True or False

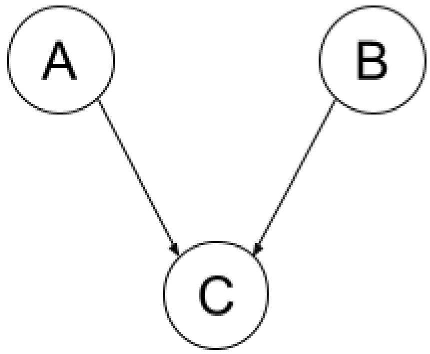
Solutions: The bottom path FBGCH is connected. Not observing B doesn't block the path FBGCH by rule 2. Observing G doesn't block the path FBGCH by rule 3. Not observing C doesn't block the apth FBGCH by rule 2.

(f) **(1 mark)** B and H are conditionally independent given C and G.

True or False

Solutions: Observing C blocks both paths between B and H by rule 1 and rule 2.

Q2 (5 marks) Variable Elimination Algorithm



Consider the Bayesian network above. The conditional probability tables are given below. Recall that the small case letters denote observed values. For example, a and $\neg a$ denote A is true and A is false respectively.

A	$P(a) = 0.4$	
B	$P(b) = 0.7$	
C	$P(c \mid \neg a \wedge \neg b) = 0.8$ $P(c \mid \neg a \wedge b) = 0.5$	$P(c \mid a \wedge \neg b) = 0.6$ $P(c \mid a \wedge b) = 0.1$

Suppose that we want to calculate the probability below using the variable elimination algorithm.

$$P(B \mid \neg c)$$

We have written down the steps of VEA on the following pages. **Complete "Step 4 Eliminate the Hidden Variables."**

1. **Categorize the variables.**

- Query variables: B
- Evidence variables: C
- Hidden variables: A

2. **Define factors.**

Define factor f1(A):

a	0.4
$\neg a$	0.6

Define factor f2(B):

b	0.7
$\neg b$	0.3

Define factor f3(A,B,C):

a	b	c	0.1
a	b	$\neg c$	0.9
a	$\neg b$	c	0.6
a	$\neg b$	$\neg c$	0.4
$\neg a$	b	c	0.5
$\neg a$	b	$\neg c$	0.5
$\neg a$	$\neg b$	c	0.8
$\neg a$	$\neg b$	$\neg c$	0.2

3. **Restrict factors.**

Restrict factor f3(A,B,C) to produce factor f4(A,B).

a	b	0.9
a	$\neg b$	0.4
$\neg a$	b	0.5
$\neg a$	$\neg b$	0.2

4. **(5 marks) Eliminate the hidden variables.**

(1 mark)
Multiply factors ____________________ to produce factor f5(________)

factor f5 (2 marks):

(1 mark)
Sum out ____ from factor __________ to produce factor f6(________)

factor f6 (1 mark):

5. **Multiply the remaining factors. …**
6. **Normalize the remaining factor. …**

Q2 SOLUTIONS:

1. **Categorize the variables.**

- Query variables: B
- Evidence variables: C
- Hidden variables: A

2. **Define factors.**

Define factor f1(A):

a	0.4
$\neg a$	0.6

Define factor f2(B):

b	0.7
$\neg b$	0.3

Define factor f3(A,B,C):

a	b	c	0.1
a	b	$\neg c$	0.9
a	$\neg b$	c	0.6
a	$\neg b$	$\neg c$	0.4
$\neg a$	b	c	0.5
$\neg a$	b	$\neg c$	0.5
$\neg a$	$\neg b$	c	0.8
$\neg a$	$\neg b$	$\neg c$	0.2

3. **Restrict factors.**

Restrict factor f3(A,B,C) to produce factor f4(A,B)

a	b	0.9
a	$\neg b$	0.4
$\neg a$	b	0.5
$\neg a$	$\neg b$	0.2

4. **(5 marks) Eliminate the hidden variables.**

(1 mark)
Multiply factors f1(A) and f4(A,B) to produce factor f5(A,B)

f5(A,B) (2 marks):

a	b	0.36
a	$\neg b$	0.16
$\neg a$	b	0.30
$\neg a$	$\neg b$	0.12

(1 mark)
Sum out A from f5(A,B) to produce factor f6(B)

f6(B) (1 mark):

b	0.66
$\neg b$	0.28

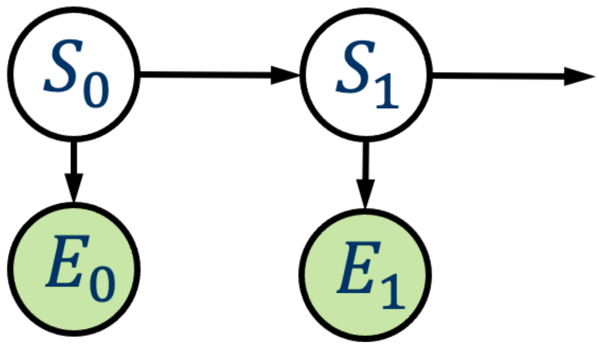
5. **Multiply the remaining factors.** …
6. **Normalize the remaining factor.** …

Q3 (6 marks) Filtering

$P(s_0) = 0.5$

$P(s_t|s_{t-1}) = 0.7$
$P(s_t|\neg s_{t-1}) = 0.4$

$P(e_t|s_t) = 0.9$
$P(e_t|\neg s_t) = 0.2$



Consider the Hidden Markov Model discussed in class.

- S_t denotes the hidden state at time t. $S_t = true$ means it rained on day t ($S_t = false$ otherwise).
- E_t denotes the observation at time t. $E_t = true$ means the director brought an umbrella on day t and $E_t = false$ otherwise.
- α is the normalization constant.
- Assume that the first two observations are $\neg e_0$ and $\neg e_1$. That is, the director did not bring an umbrella on days 0 or 1.

On the following pages, calculate the **filtering** probabilities for **Day 1** by completing **parts (a) and (b)**. **For full marks, show ALL your work** and present your solutions to **3 decimal places.**

(a) (2 marks) Calculate $P(s_1|\neg e_0)$ and $P(\neg s_1|\neg e_0)$.
Assume that $P(s_0|\neg e_0) = 0.1$ and $P(\neg s_0|\neg e_0) = 0.9$.
Show all of your work.

Your Answers:

$P(s_1\|\neg e_0) =$	$P(\neg s_1\|\neg e_0) =$

Your calculations:

The Filtering Formulas:

- Base case: $P(S_0|E_0) = \alpha * P(S_0) * P(E_0|S_0)$
- Recursive case:
  - $P(S_k|E_0\wedge ... \wedge E_{k-1}) = \sum_{S_{k-1}} P(S_{k-1}|E_0\wedge ... \wedge E_{k-1}) * P(S_k|S_{k-1})$
  - $P(S_k|E_0\wedge ... \wedge E_k) = \alpha * P(E_k|S_k) * P(S_k|E_0\wedge ... \wedge E_{k-1})$

Q3a Solutions

Calculate $P(s_1|\neg e_0)$ and $P(\neg s_1|\neg e_0)$.
Assume that $P(s_0|\neg e_0) = 0.1$ and $P(\neg s_0|\neg e_0) = 0.9$.
Show all of your work.

Your Answers:

$P(s_1|\neg e_0)$ = 0.430	$P(\neg s_1|\neg e_0)$ = 0.570

Your calculations:

$P(s_1|\neg e_0) = [P(s_0|\neg e_0)P(s_1|s_0) + P(\neg s_0|\neg e_0)P(s_1|\neg s_0)]$
$= (0.1 * 0.7 + 0.9 * 0.4) = 0.43$

$P(\neg s_1|\neg e_0) = [P(s_0|\neg e_0)P(\neg s_1|s_0) + P(\neg s_0|\neg e_0)P(\neg s_1|\neg s_0)]$
$= (0.1 * 0.3 + 0.9 * 0.6) = 0.57$

1 mark for the calculation process.
1 mark for the correct answers.

(b) (4 marks) Calculate $P(s_1|\neg e_0 \wedge \neg e_1)$ and $P(\neg s_1|\neg e_0 \wedge \neg e_1)$.
Assume that $P(s_1|\neg e_0) = 0.4$ and $P(\neg s_1|\neg e_0) = 0.6$.
Show all of your work.

Your Answers:

$P(s_1|\neg e_0 \wedge \neg e_1) =$	$P(\neg s_1|\neg e_0 \wedge \neg e_1) =$

Your calculations:

The Filtering Formulas:

- Base case: $P(S_0|E_0) = \alpha * P(S_0) * P(E_0|S_0)$
- Recursive case:
  - $P(S_k|E_0 \wedge ... \wedge E_{k-1}) = \sum_{S_{k-1}} P(S_{k-1}|E_0 \wedge ... \wedge E_{k-1}) * P(S_k|S_{k-1})$
  - $P(S_k|E_0 \wedge ... \wedge E_k) = \alpha * P(E_k|S_k) * P(S_k|E_0 \wedge ... \wedge E_{k-1})$

Q3b Solutions

Calculate $P(s_1|\neg e_0 \wedge \neg e_1)$ and $P(\neg s_1|\neg e_0 \wedge \neg e_1)$.
Assume that $P(s_1|\neg e_0) = 0.4$ and $P(\neg s_1|\neg e_0) = 0.6$.
Show all of your work.

Your Answers:

$P(s_1|\neg e_0 \wedge \neg e_1)$ = 0.077	$P(\neg s_1|\neg e_0 \wedge \neg e_1)$ = 0.923

Your calculations:

$P(s_1|\neg e_0)P(\neg e_1|s_1) = 0.4 * 0.1 = 0.04$

$P(\neg s_1|\neg e_0)P(\neg e_1|\neg s_1) = 0.6 * 0.8 = 0.48$

$P(s_1|\neg e_0 \wedge \neg e_1) = \alpha P(s_1|\neg e_0)P(\neg e_1|s_1)$

$= 0.04/(0.04 + 0.48) = 0.077$

$P(\neg s_1|\neg e_0 \wedge \neg e_1) = 1 - P(s_1|\neg e_0 \wedge \neg e_1) = 0.923$

2 marks for calculating the unnormalized probabilities. 1 mark for the calculation process and 1 mark for the correct answers.

2 marks for normalizing the probabilities. 1 mark for the calculation process and 1 mark for the correct answers.