CSC 384 Winter 2023 Test 4 Version A

March 27 and 28, 2023

Last Name: ________________________________

First Name: ________________________________

Email: ________________________________

There are 3 questions with a total of 26 marks.

- Q1 (8 marks)
- Q2 (12 marks)
- Q3 (6 marks)

Q1 D-Separation (8 marks)

Consider Figure 1 below. For each question below, circle the best answer and provide an explanation. Use the following format for your explanation (where X, A, B, C, and D are variables).

(Observing/Not observing) X (blocks/doesn't block) the path A-B-C-D by rule 1/2/3.

Q1.1 (2 marks) **C** and **E** are unconditionally independent.

True or False

Explain:

Q1.2 (2 marks) **F** and **E** are conditionally independent given **B**.

True or False

Explain:

Q1.3 (2 marks) **A** and **I** are unconditionally independent.

True or False

Explain:

Q1.4 (2 marks) **C** and **E** are conditionally independent given **I.**

True or False

Explain:

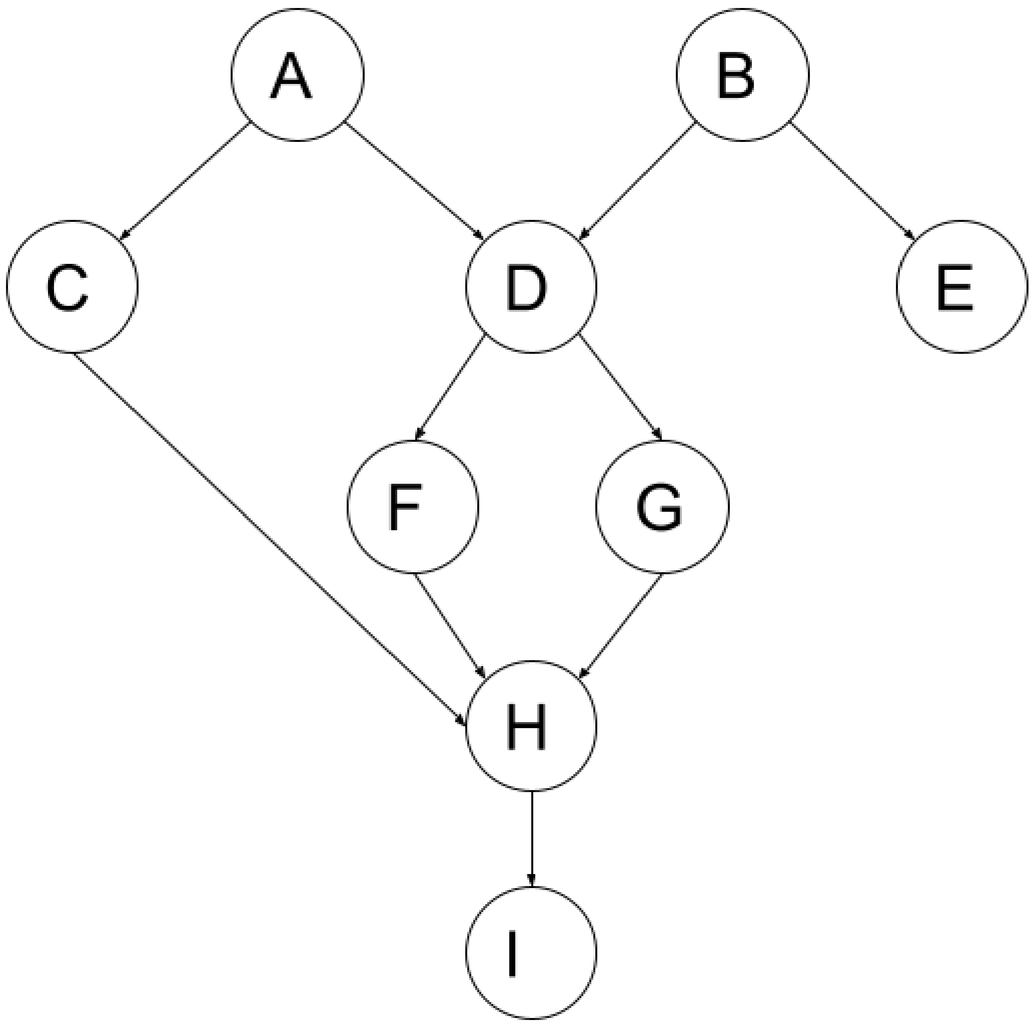
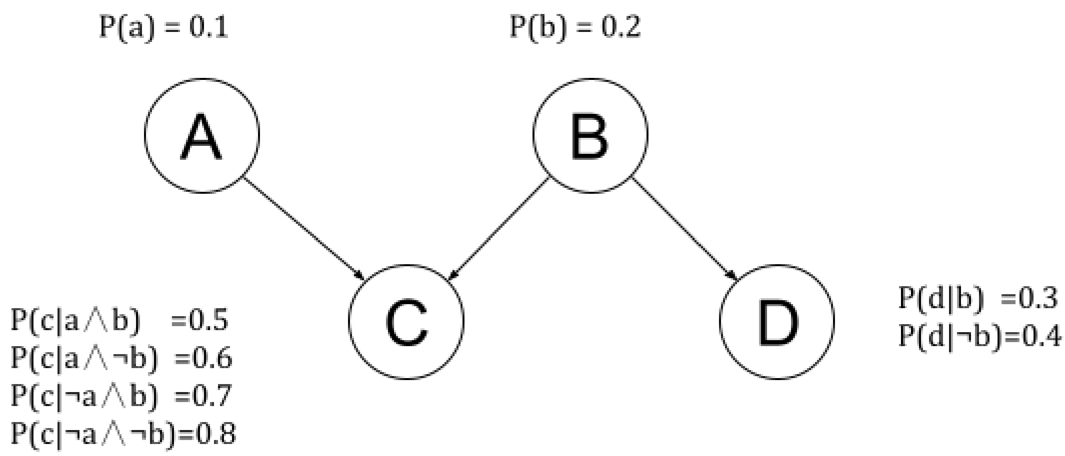


Figure 1 Above

Q2 Variable Elimination Algorithm (12 marks)



Consider the Bayesian network above. A, B, C, and D are binary variables. We use the lower-case letters to denote the values of the variables, e.g. a denotes A = true and $\neg a$ denotes A=false.

Calculate $P(A \mid \neg c)$ by using the Variable Elimination Algorithm.

Eliminate the hidden variables in **alphabetical** order.

For each step, indicate the following.

- Indicate the **operation** (e.g. Restrict, Multiply, Sum out, or Normalize).
- Indicate the **factors** on which you are applying the operations.
- Each operation should **produce a new factor.** Give this factor a unique name and draw a table containing its contents. The table should indicate the variables in the factor and the value for each combination of the variables' values.

Show all your work on pages 6 and 7.

We have created the initial factors for you below.

Factor f1

a	0.1
$\neg a$	0.9

Factor f2

b	0.2
$\neg b$	0.8

Factor f3

d	b	0.3
$\neg d$	b	0.7
d	$\neg b$	0.4
$\neg d$	$\neg b$	0.6

Factor f4

c	a	b	0.5
$\neg c$	a	b	0.5
c	a	$\neg b$	0.6
$\neg c$	a	$\neg b$	0.4
c	$\neg a$	b	0.7
$\neg c$	$\neg a$	b	0.3
c	$\neg a$	$\neg b$	0.8
$\neg c$	$\neg a$	$\neg b$	0.2

Your Q2 final answers:

$P(a \mid \neg c) =$	$P(\neg a \mid \neg c) =$

Your Q2 work starts here.

Your Q2 work continues.

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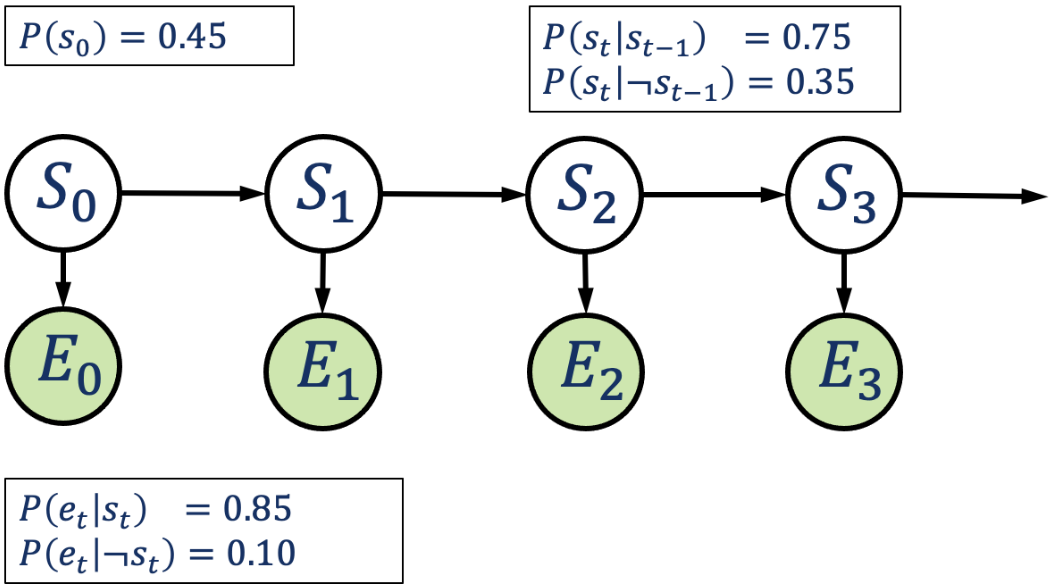
Q3 Filtering (6 marks)

Consider the hidden Markov model on the next page.

- S_t denotes the hidden state at time t. $S_t = true$ means it rained on day t ($S_t = false$ otherwise).
- E_t denotes the observation at time t. $E_t = true$ means the director brought an umbrella on day t and $E_t = false$ otherwise.
- α is the normalization constant.

Assume that **the first three observations are $\neg e_0$, e_1, and $\neg e_2$.** That is, the director **brought an umbrella on day 1 and didn't bring an umbrella on days 0 and 2.**

Calculate the filtering probabilities for **day 2**. We have provided the filtering formulas on the next page. **For full marks**, **show ALL your work** and present your solutions to **3 decimal places**.



The Filtering Formulas:

- Base case: $P(S_0|E_0) = \alpha\, P(S_0)\, P(E_0|S_0)$
- Recursive case:
    - $P(S_k|E_0 \wedge \ldots \wedge E_{k-1}) = \sum_{S_{k-1}} P(S_{k-1}|E_0 \wedge \ldots \wedge E_{k-1}) * P(S_k|S_{k-1})$
    - $P(S_k|E_0 \wedge \ldots \wedge E_k) = \alpha\, P(E_k|S_k)\, P(S_k|E_0 \wedge \ldots \wedge E_{k-1})$

Assume that

$P(s_1|\neg e_0 \wedge e_1) = 0.849$ and $P(\neg s_1|\neg e_0 \wedge e_1) = 0.151$

Your final answers::

$P(s_2|\neg e_0 \wedge e_1 \wedge \neg e_2) =$	$P(\neg s_2|\neg e_0 \wedge e_1 \wedge \neg e_2) =$

Your calculations:

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