# CSC 384 Test 4 on Uncertainty

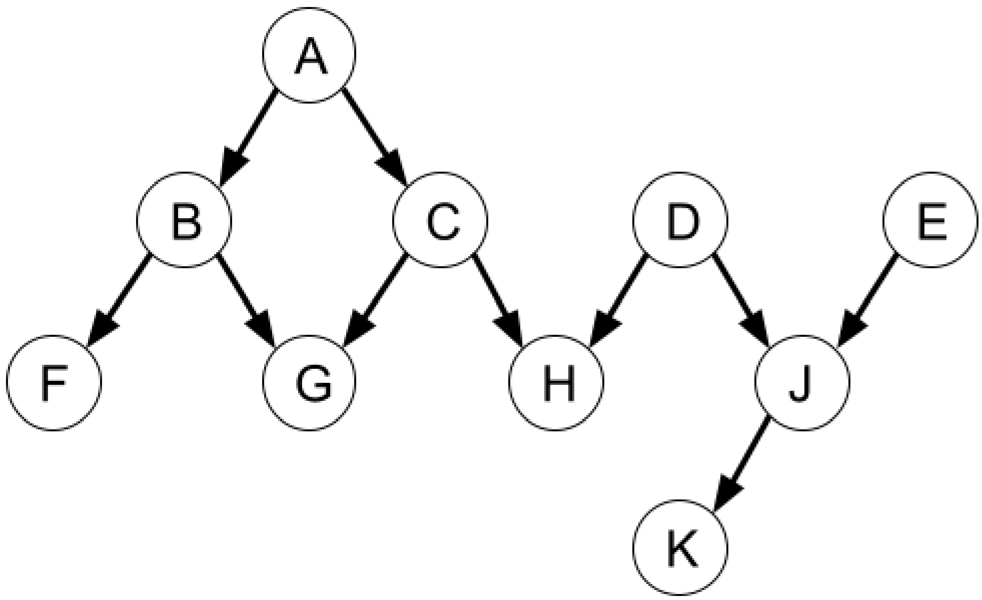
Friday, November 25, 2022

Last Name: ________________________________

First Name: ________________________________

Student Number: ________________________________

**Q1 (6 marks) D-Separation**




Consider the Bayesian network above. Circle the correct answer.

(a) **(1 mark)** C and E are conditionally independent given K.

True or False

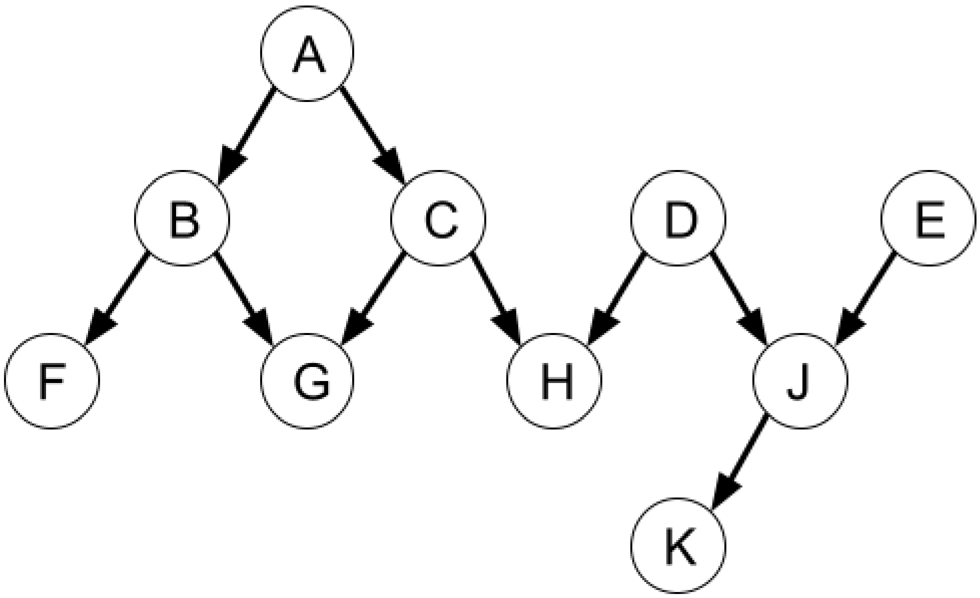
(b) **(1 mark)** C and E are conditionally independent given H, K.

True or False

(c) **(1 mark)** H and K are unconditionally independent.

True or False

**Q1 (6 marks) D-Separation (Continued)**




(*This diagram is the same as the one on the previous page*.)

(d) **(1 mark)** C and K are conditionally independent given H and J.

True or False

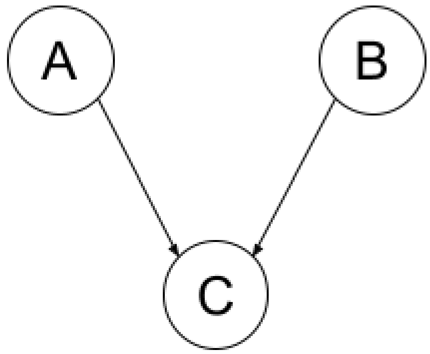
(e) **(1 mark)** F and H are conditionally independent given A and G.

True or False

(f) **(1 mark)** B and H are conditionally independent given C and G.

True or False

**Q2 (5 marks) Variable Elimination Algorithm**



Consider the Bayesian network above. The conditional probability tables are given below. Recall that the small case letters denote observed values. For example, $a$ and $\neg a$ denote A is true and A is false respectively.

| | | |
|---|---|---|
| A | $P(a) = 0.4$ | |
| B | $P(b) = 0.7$ | |
| C | $P(c \vert \neg a \land \neg b) = 0.8$<br>$P(c \vert \neg a \land b) = 0.5$ | $P(c \vert a \land \neg b) = 0.6$<br>$P(c \vert a \land b) = 0.1$ |

Suppose that we want to calculate the probability below using the variable elimination algorithm.

$$P(B|\neg c)$$

We have written down the steps of VEA on the following pages.
**Complete "Step 4 Eliminate the Hidden Variables."**

1. **Categorize the variables.**

- Query variables: B
- Evidence variables: C
- Hidden variables: A

2. **Define factors.**

Define factor f1(A):

| | |
|---|---|
| $a$ | 0.4 |
| $\neg a$ | 0.6 |

Define factor f2(B):

| | |
|---|---|
| $b$ | 0.7 |
| $\neg b$ | 0.3 |

Define factor f3(A,B,C):

| | | | |
|---|---|---|---|
| $a$ | $b$ | $c$ | 0.1 |
| $a$ | $b$ | $\neg c$ | 0.9 |
| $a$ | $\neg b$ | $c$ | 0.6 |
| $a$ | $\neg b$ | $\neg c$ | 0.4 |
| $\neg a$ | $b$ | $c$ | 0.5 |
| $\neg a$ | $b$ | $\neg c$ | 0.5 |
| $\neg a$ | $\neg b$ | $c$ | 0.8 |
| $\neg a$ | $\neg b$ | $\neg c$ | 0.2 |

3. **Restrict factors.**

Restrict factor f3(A,B,C) to produce factor f4(A,B).

| | | |
|---|---|---|
| $a$ | $b$ | 0.9 |
| $a$ | $\neg b$ | 0.4 |
| $\neg a$ | $b$ | 0.5 |
| $\neg a$ | $\neg b$ | 0.2 |

4. **(5 marks) Eliminate the hidden variables.**

(1 mark)

Multiply factors ___________________ to produce factor f5(_______)

factor f5 (2 marks):

| | | |
|---|---|---|
| | | |
| | | |
| | | |
| | | |

(1 mark)

Sum out ____ from factor __________ to produce factor f6(_______)

factor f6 (1 mark):

| | |
|---|---|
| | |
| | |

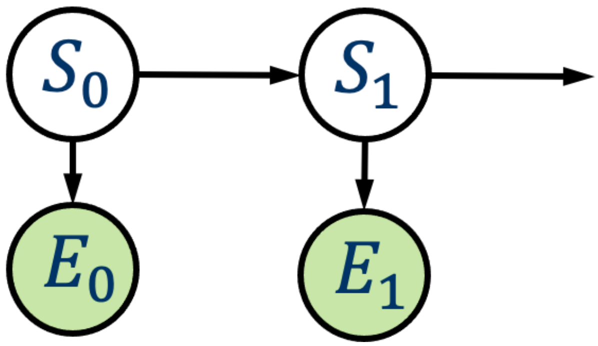
5. **Multiply the remaining factors. …**
6. **Normalize the remaining factor. …**

## Q3 (6 marks) Filtering

$P(s_0) = 0.5$

$P(s_t | s_{t-1}) = 0.7$
$P(s_t | \neg s_{t-1}) = 0.4$

$P(e_t | s_t) = 0.9$
$P(e_t | \neg s_t) = 0.2$




Consider the Hidden Markov Model discussed in class.

- $S_t$ denotes the hidden state at time $t$. $S_t = true$ means it rained on day $t$ ($S_t = false$ otherwise).
- $E_t$ denotes the observation at time $t$. $E_t = true$ means the director brought an umbrella on day $t$ and $E_t = false$ otherwise.
- $\alpha$ is the normalization constant.
- Assume that the first two observations are $\neg e_0$ and $\neg e_1$**.** That is, the director did not bring an umbrella on days 0 or 1.

On the following pages, calculate the **filtering** probabilities for **Day 1** by completing **parts (a) and (b)**. **For full marks, show ALL your work** and present your solutions to **3 decimal places.**

**Q3 (a) (2 marks)** Calculate $P(s_1|\neg e_0)$ and $P(\neg s_1|\neg e_0)$.
Assume that $P(s_0|\neg e_0) = 0.1$ and $P(\neg s_0|\neg e_0) = 0.9$.
Show all of your work.

Your Answers:

| | |
|---|---|
| $P(s_1|\neg e_0) =$ | $P(\neg s_1|\neg e_0) =$ |

Your calculations:

The Filtering Formulas:

- Base case: $P(S_0|E_0) = \alpha * P(S_0) * P(E_0|S_0)$
- Recursive case:
  - $P(S_k|E_0 \wedge ... \wedge E_{k-1}) = \sum_{S_{k-1}} P(S_{k-1}|E_0 \wedge ... \wedge E_{k-1}) * P(S_k|S_{k-1})$
  - $P(S_k|E_0 \wedge ... \wedge E_k) = \alpha * P(E_k|S_k) * P(S_k|E_0 \wedge ... \wedge E_{k-1})$

**Q3 (b) (4 marks)** Calculate $P(s_1|\neg e_0 \wedge \neg e_1)$ and $P(\neg s_1|\neg e_0 \wedge \neg e_1)$.
Assume that $P(s_1|\neg e_0) = 0.4$ and $P(\neg s_1|\neg e_0) = 0.6$.
Show all of your work.

Your Answers:

| | |
|---|---|
| $P(s_1|\neg e_0 \wedge \neg e_1) =$ | $P(\neg s_1|\neg e_0 \wedge \neg e_1) =$ |

Your calculations:

The Filtering Formulas:

- Base case: $P(S_0|E_0) = \alpha * P(S_0) * P(E_0|S_0)$
- Recursive case:
  - $P(S_k|E_0 \wedge ... \wedge E_{k-1}) = \sum_{S_{k-1}} P(S_{k-1}|E_0 \wedge ... \wedge E_{k-1}) * P(S_k|S_{k-1})$
  - $P(S_k|E_0 \wedge ... \wedge E_k) = \alpha * P(E_k|S_k) * P(S_k|E_0 \wedge ... \wedge E_{k-1})$

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