CSC 384 Winter 2023 Test 3 Version A

March 6 and 7, 2023

Last Name: ______________________________

First Name: ______________________________

Email: ______________________________

Q1 Conceptual Questions

Let X and Y be two random variables.

Let c(X, Y) denote a binary constraint between X and Y.

Q1.1 (1 mark)

If <X, c(X, Y)> IS arc-consistent, then <Y, c(X, Y)> IS also arc-consistent.

Circle the correct answer: True or False

- If your answer is true, explain why in a few sentences.
- If your answer is false, give a counterexample below.

Q1.2 (1 mark)

If <X, c(X, Y)> IS arc-consistent,

then <X, c(X, Y)> IS arc-consistent after removing one value v from X's domain.

Circle the correct answer: True or False

- If your answer is true, explain why in a few sentences.
- If your answer is false, give a counterexample below.

Q1.3 (1 mark)

If <X, c(X, Y)> IS arc-consistent,

then <X, c(X, Y)> IS arc-consistent after removing one value w from Y’s domain.

Circle the correct answer: True or False

- If your answer is true, explain why in a few sentences.
- If your answer is false, give a counterexample below.

Q1.4 (1 mark)

While executing the AC-3 algorithm, as soon as every variable’s domain has one value left, we can conclude that the CSP has a unique solution.

Circle the correct answer: True or False

- If your answer is true, explain why in a few sentences.
- If your answer is false, give a counterexample below.

Q1.5 (1 mark)

Assume that we combine Backtracking Search with constraint propagation (e.g. Forward Checking or the AC-3 algorithm). Compare the two approaches below.

(1) Perform Forward Checking after executing the AC-3 algorithm.

(2) Perform the AC-3 algorithm only.

There exists one CSP such that using the first approach leads to removing more values from the variables' domains than using the second approach.

Circle the best answer: True or False

Q2 (2 marks)

Suppose you are doing Backtracking Search. The last variable assigned was x, and it was assigned a value of v. A variable's domain became empty. You are backtracking to before x was assigned v.

Q2.1 (1 mark)

If **no constraint propagation** (e.g. forward checking or AC-3) was used, we must unassign x and restore the domains of all the other variables to their states before x was assigned v.

Circle the best answer: True or False

Q2.2 (1 mark)

If **only the AC-3 algorithm** were used for constraint propagation, we must unassign x and restore the domains of all the other variables to their states before x was assigned v.

Circle the best answer: True or False

Q3 Backtracking Search and Forward Checking

We want to schedule five activities in four time slots. Let the variables A, B, C, D, and E represent the activities. Each variable's domain is {1, 2, 3, 4}. The constraints are below.

$A \leq B$	$A > D$	$C \neq D$	$C > E$
$A \neq C$	$B \neq C$	$C \neq D + 1$	$D > E$

Complete the table below. **Please follow the rules below very carefully!**

- Choose the next variable using the **minimum-remaining-value** heuristic. If there is a tie, choose the variable that comes first in **alphabetical** order.
- For each variable, choose the next value using the **least-constraining-value** heuristic. If there is a tie, choose the **smallest** value.
- We have completed the first step for you as an example. Note that we chose to assign variable **C** in the first step.
-

For each step, fill in the following information.

- Choose a variable and a value to assign. Write the assignment in the variable's column..
- For other variables, write the updated domains resulting from Forward Checking. If a domain is empty, write "empty."
- If a variable was assigned in a previous step and the assignment has not changed, there is no need to write the assignment again.
- In the "What Next?" column, write "Continue," "Backtrack," or "Solution Found!"

Use as many rows in the table as necessary.

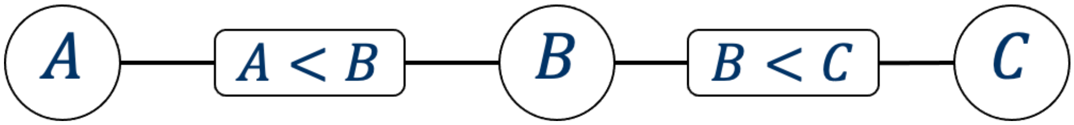
The CSP from Q3 is repeated here for your convenience.

We want to schedule five activities in four time slots. Let the variables A, B, C, D, and E represent the activities. Each variable's domain is {1, 2, 3, 4}. The constraints are below.

$A \leq B$	$A > D$	$C \neq D$	$C > E$
$A \neq C$	$B \neq C$	$C \neq D + 1$	$D > E$

<table>
<tr><th>Step</th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th><th>What Next?</th></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="7"></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="7"></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="7"></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="7"></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="7"></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Q4 The AC-3 Algorithm



Consider the CSP above. There are three variables (A, B, C) and two binary constraints (A<B and B<C). The initial domains of the variables are below.

$$D_A = \{1, 2, 3, 4\}, D_B = \{1, 2, 3, 4\}, D_C = \{1, 2, 3, 4\}$$

Q4.1 (10 marks)

Execute the AC-3 algorithm on this CSP until the algorithm terminates. To **ensure a unique solution**, follow the directions below.

- Start with the initial queue on the next page.
- At each step, remove the first (leftmost) arc from the queue.
- Always add arc(s) to the queue's end (right side).
- When adding multiple arcs to the queue, add them in alphabetical order.

Complete the table on the next page. For each step, indicate

- The arc removed
- The value(s) deleted from any domains
- The updated domains of the variables
- Any arc(s) added back to the queue

We completed the first step for you as an example.

Initial Queue: <C, B<C >, <A, A<B >, <B, B<C >, <B, A<B >

Step	**Arc removed**	**Value(s) deleted**	**Updated domains**	**Arc(s) added**

Q4.2 (1 mark)

Consider the CSP defined at the beginning of Q4.

Once the AC-3 algorithm terminates, what can we conclude about this CSP **based on the result of the AC-3 algorithm execution only**?

Circle the best answer.

(A) The CSP has no solution.

(B) The CSP has a unique solution.

(C) The CSP has at least one solution.

(D) The CSP has at least two solutions.

(E) The CSP may or may not have a solution.