SOLUTIONS

CSC 384 Winter 2023 Test 3 Version A

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Last Name: ______________________________

First Name: ______________________________

Email: ______________________________

Q1 Conceptual Questions

Let X and Y be two random variables.

Let c(X, Y) denote a binary constraint between X and Y.

Q1.1 (1 mark)

If <X, c(X, Y)> IS arc-consistent, then <Y, c(X, Y)> IS also arc-consistent.

Circle the correct answer: True or False

- If your answer is true, explain why in a few sentences.
- If your answer is false, give a counterexample below.

Q1.1 Answer: False.

Consider the arc <X, X<Y > where X's domain is {1} and Y's domain is {1, 2}.

<X, X<Y> is arc-consistent but <Y, X<Y> is not arc-consistent.

Q1.2 (1 mark)

If <X, c(X, Y)> IS arc-consistent,

then <X, c(X, Y)> IS arc-consistent after removing one value v from X's domain.

Circle the correct answer: True or False

- If your answer is true, explain why in a few sentences.
- If your answer is false, give a counterexample below.

Q1.2 Answer: True.

If <X, c(X, Y)> is arc-consistent before the change. Then, every value of X has a corresponding value of Y that satisfies the constraint. If we remove one value from X's domain, every remaining value in X still has a corresponding value of Y that satisfies the constraint.

Q1.3 (1 mark)

If <X, c(X, Y)> IS arc-consistent,

then <X, c(X, Y)> IS arc-consistent after removing one value w from Y's domain.

Circle the correct answer: True or False

- If your answer is true, explain why in a few sentences.
- If your answer is false, give a counterexample below.

Q1.3 Answer: False.

Consider the arc <X, X < Y>. X's domain is {1, 2}, and Y's domain is {2, 3}.

After removing 3 from Y's domain, <X, X < Y> is no longer arc-consistent.

Q1.4 (1 mark)

While executing the AC-3 algorithm, as soon as every variable's domain has one value left, we can conclude that the CSP has a unique solution.

Circle the correct answer: True or False

- If your answer is true, explain why in a few sentences.
- If your answer is false, give a counterexample below.

Q1.4 Answer: False.

We cannot reach a conclusion regarding the solution until the AC-3 algorithm terminates. The AC-3 algorithm does not terminate until the queue is empty.

Consider a problem with three variables: A, B, C. D_A = {1}, D_B = {2, 3}, D_C = {3}. The constraints are A < B, B < C, and C < A. Suppose we first remove the arc <B, B<C > from the queue and remove 3 from B's domain. At this point, each domain has 1 value left, but this problem has no solution.

Q1.5 (1 mark)

Assume that we combine Backtracking Search with constraint propagation (e.g. Forward Checking or the AC-3 algorithm). Compare the two approaches below.

(1) Perform Forward Checking after executing the AC-3 algorithm.

(2) Perform the AC-3 algorithm only.

There exists one CSP such that using the first approach leads to removing more values from the variables' domains than using the second approach.

Circle the best answer: True or False

Q1.5 Answer: False

The AC-3 algorithm prunes strictly more domain values than Forward Checking. Therefore, it is redundant to perform Forward Checking after AC-3.

Q2 (2 marks)

Suppose you are doing Backtracking Search. The last variable assigned was x, and it was assigned a value of v. A variable's domain became empty. You are backtracking to before x was assigned v.

Q2.1 (1 mark)

If **no constraint propagation** (e.g. forward checking or AC-3) was used, we must unassign x and restore the domains of all the other variables to their states before x was assigned v.

Circle the best answer: True or False

Q2.1 Answer: False

If we do not perform constraint propagation, we will not remove values from the variables' domains. Then, there is no need to restore these values when backtracking.

Q2.2 (1 mark)

If **only the AC-3 algorithm** were used for constraint propagation, we must unassign x and restore the domains of all the other variables to their states before x was assigned v.

Circle the best answer: True or False

Q2.1 Answer: True

If we perform the AC-3 algorithm, we may remove values from the variables' domains. Then, it is necessary to restore these values when backtracking.

Q3 Backtracking Search and Forward Checking

We want to schedule five activities in four time slots. Let the variables A, B, C, D, and E represent the activities. Each variable's domain is {1, 2, 3, 4}. The constraints are below.

$A \leq B$	$A > D$	$C \neq D$	$C > E$
$A \neq C$	$B \neq C$	$C \neq D + 1$	$D > E$

Complete the table below. **Please follow the rules below very carefully!**

- Choose the next variable using the **minimum-remaining-value** heuristic. If there is a tie, choose the variable that comes first in **alphabetical** order.
- For each variable, choose the next value using the **least-constraining-value** heuristic. If there is a tie, choose the **smallest** value.
- We have completed the first step for you as an example. Note that we chose to assign variable **C** in the first step.
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For each step, fill in the following information.

- Choose a variable and a value to assign. Write the assignment in the variable's column..
- For other variables, write the updated domains resulting from Forward Checking. If a domain is empty, write "empty."
- If a variable was assigned in a previous step and the assignment has not changed, there is no need to write the assignment again.
- In the "What Next?" column, write "Continue," "Backtrack," or "Solution Found!"

Use as many rows in the table as necessary.

The CSP from Q3 is repeated here for your convenience.

We want to schedule five activities in four time slots. Let the variables A, B, C, D, and E represent the activities. Each variable's domain is {1, 2, 3, 4}. The constraints are below.

$A \leq B$ $\quad$ $A > D$ $\quad$ $C \neq D$ $\quad$ $C > E$

$A \neq C$ $\quad$ $B \neq C$ $\quad$ $C \neq D + 1$ $\quad$ $D > E$

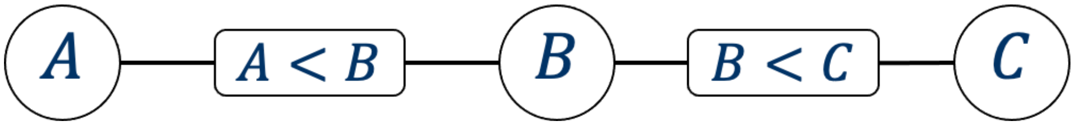
Step	A	B	C	D	E	What Next?
1	2, 3, 4	2, 3, 4	C = 1	2, 3, 4	empty	Backtrack
We need to assign C another value from its domain. **The values in the grey rows are not actual steps. They are provided to show our reasoning.**						
	1, 3, 4	1, 3, 4	C = 2	2, 3, 4	1	
	1, 3, 4	1, 3, 4	C = 3	1, 4	1, 2	
	1, 2, 3	1, 2, 3	C = 4	1, 2	1, 2, 3	
Apply the LCV heuristic. C = 4 leaves the largest number of values in the other variables' domains.						
2	1, 2, 3	1, 2, 3	C = 4	1, 2	1, 2, 3	Continue
	2, 3	1, 2, 3		D = 1	empty	
	3	1, 2, 3		D = 2	1	
Apply the LCV heuristic. The two values of D are equivalent. Choose D = 1 by tie-breaking rule.						
3	2, 3	1, 2, 3		D = 1	empty	Backtrack
4	3	1, 2, 3		D = 2	1	Continue
Apply the MRV heuristic. Both A and E have 1 value left. Choose A by the tie-breaking rule.						
5	A = 3	3			1	Continue
Apply the MRV heuristic. Both B and E have 1 value left. Choose B by the tie-breaking rule.						
6		B = 3			1	Continue
7					E = 1	Solution

						found!

Q3 Marking Scheme:

1. Deduct 1 mark for not starting with the variable stated in the question.
2. Deduct 1 mark (up to 4 marks) for incorrectly choosing the next variable using MRV.
3. Deduct 1 mark (up to 2 marks) for incorrectly choosing the next value using LCV.
4. Deduct 1 mark for not following the tie-breaking rule correctly.
5. Deduct 1 mark (up to 5 marks) for pruning values from domains incorrectly (incorrect forward checking). Value pruning happens in steps 2-6.
6. Deduct 1 mark (up to 5 marks) for the incorrect next step (Continue or Backtrack).
7. Deduct 2 marks for each missing step if the answer is unfinished.

Q4 The AC-3 Algorithm



Consider the CSP above. There are three variables (A, B, C) and two binary constraints (A<B and B<C). The initial domains of the variables are below.

$$D_A = \{1, 2, 3, 4\}, D_B = \{1, 2, 3, 4\}, D_C = \{1, 2, 3, 4\}$$

Q4.1 (10 marks)

Execute the AC-3 algorithm on this CSP until the algorithm terminates. To **ensure a unique solution**, follow the directions below.

- Start with the initial queue on the next page.
- At each step, remove the first (leftmost) arc from the queue.
- Always add arc(s) to the queue's end (right side).
- When adding multiple arcs to the queue, add them in alphabetical order.

Complete the table on the next page. For each step, indicate

- The arc removed
- The value(s) deleted from any domains
- The updated domains of the variables
- Any arc(s) added back to the queue

We completed the first step for you as an example.

Initial Queue: <C, B<C >, <A, A<B >, <B, B<C >, <B, A<B >

Step	Arc removed	Value(s) deleted	Updated domains	Arc(s) added
1	**<C, B<C >**	**Remove 1 from D_C**	$D_A = \{1, 2, 3, 4\}$, $D_B = \{1, 2, 3, 4\}$ $D_C = \{2, 3, 4\}$	**None**
Updated Queue: <A, A<B >, <B, B<C >, <B, A<B >				
2	<A, A<B >	Remove 4 from D_A	$D_A = \{1, 2, 3\}$, $D_B = \{1, 2, 3, 4\}$ $D_C = \{2, 3, 4\}$	None
Updated Queue: <B, B<C >, <B, A<B >				
3	<B, B<C >	Remove 4 from D_B	$D_A = \{1, 2, 3\}$, $D_B = \{1, 2, 3\}$ $D_C = \{2, 3, 4\}$	<A, A<B >
Updated Queue: <B, A<B >, <A, A<B >				
4	<B, A<B >	Remove 1 from D_B	$D_A = \{1, 2, 3\}$, $D_B = \{2, 3\}$ $D_C = \{2, 3, 4\}$	<C, B<C>
Updated Queue: <A, A<B >, <C, B<C>				
5	<A, A<B >	Remove 3 from D_A	$D_A = \{1, 2\}$, $D_B = \{2, 3\}$ $D_C = \{2, 3, 4\}$	None
Updated Queue: <C, B<C>				
6	<C, B<C>	Remove 2 from D_C	$D_A = \{1, 2\}$, $D_B = \{2, 3\}$ $D_C = \{3, 4\}$	None

Q4.1 Marking Scheme:

1. **Deduct 1 mark for not removing the first arc from the queue (up to a maximum of 1 mark deduction)**
2. **Deduct 2 marks for not adding the correct arcs to the queue (up to a maximum of 4 mark deduction)**
3. **Deduct 1 mark for not pruning the values correctly (up to a maximum of 4 mark deduction)**
4. **Deduct 1 mark for not adding the new arcs to the end of the queue alphabetically (up to a maximum of 1 mark deduction)**

Q4.2 (1 mark)

Consider the CSP defined at the beginning of Q4.

Once the AC-3 algorithm terminates, what can we conclude about this CSP **based on the result of the AC-3 algorithm execution only**?

Circle the best answer.

(A) The CSP has no solution.

(B) The CSP has a unique solution.

(C) The CSP has at least one solution.

(D) The CSP has at least two solutions.

(E) The CSP may or may not have a solution.

Q4.2 Answer: (E) The CSP may or may not have a solution.