



Lista de Exercícios – Métodos Numéricos para Engenharia TC
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1ª QUESTÃO

$$(a) \begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 & (1) \\ a_{21}x_1 + a_{22}x_2 = b_2 & (2) \end{cases} \quad \left| \quad \begin{bmatrix} 0,0003 & 1,566 \\ 0,3454 & -2,436 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1,569 \\ 1,018 \end{bmatrix} \right.$$

$$\begin{cases} x_2 = -\left(\frac{a_{11}}{a_{12}}\right)x_1 + \frac{b_1}{a_{12}} & (1) \\ x_2 = -\left(\frac{a_{21}}{a_{22}}\right)x_1 + \frac{b_2}{a_{22}} & (2) \end{cases} \quad \left| \quad \begin{cases} x_2 = -\left(\frac{0,0003}{1,566}\right)x_1 + \frac{1,569}{1,566} & a(1) \\ x_2 = +\frac{0,3454}{2,436}x_1 - \left(\frac{1,018}{2,436}\right) & a(2) \end{cases} \right.$$

inclinação intersecção

$$-\left(\frac{0,0003}{1,566}\right)x_1 + \frac{1,569}{1,566} = \frac{0,3454}{2,436}x_1 - \left(\frac{1,018}{2,436}\right)$$

$$-\left(\frac{0,0003}{1,566}\right)x_1 - \left(\frac{0,3454}{2,436}\right)x_1 = -\left(\frac{1,018}{2,436}\right) - \left(\frac{1,569}{1,566}\right) \quad \times(-1)$$

$$\left[\left(\frac{0,0003}{1,566}\right) + \left(\frac{0,3454}{2,436}\right)\right]x_1 = \left(\frac{1,018}{2,436}\right) + \left(\frac{1,569}{1,566}\right)$$

$$0,14198139 x_1 = 1,41981390$$

$$\boxed{x_1 = 10,00}$$

substituindo x_1 em (2):

$$x_2 = \left(\frac{0,3454}{2,436}\right)10 - \left(\frac{1,018}{2,436}\right)$$

$$x_2 = 1,41789819 - 0,41789819$$

$$\boxed{x_2 = 1,000}$$

$$(b) \begin{bmatrix} 0,3454 & -2,436 \\ 0,0003 & 1,566 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1,018 \\ 1,569 \end{bmatrix}$$

$$\begin{cases} x_2 = +\left(\frac{0,3454}{2,436}\right)x_1 - \left(\frac{1,018}{2,436}\right) & b(1) \\ x_2 = -\left(\frac{0,0003}{1,566}\right)x_1 + \left(\frac{1,569}{1,566}\right) & b(2) \end{cases}$$

como as equações são iguais às obtidas no item a, trocando apenas a ordem:

$$b(1) = a(2)$$

$$b(2) = a(1)$$

logo, $\boxed{x_1 = 10,00}$ e $\boxed{x_2 = 1,000}$

prova real:

$$0,0003 \cdot 10 + 1,566 \cdot 1 = 1,569 ?$$

$$0,003 + 1,566 = 1,569$$

$$1,569 = 1,569 \quad \checkmark$$

$$0,3454 \cdot 10 - 2,436 \cdot 1 = 1,018 ?$$

$$3,454 - 2,436 = 1,018$$

$$1,018 = 1,018 \quad \checkmark$$

A ordem das equações não interfere no resultado!



2ª QUESTÃO

$$\left[\begin{array}{ccc|ccc} -4/5 & -3/5 & -2/5 & 1 & 0 & 0 \\ -3/5 & -6/5 & -4/5 & 0 & 1 & 0 \\ -2/5 & -4/5 & -6/5 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 4 & 3 & 2 & -5 & 0 & 0 \\ 3 & 6 & 4 & 0 & -5 & 0 \\ 2 & 4 & 6 & 0 & 0 & -5 \end{array} \right] \begin{array}{l} \rightarrow \frac{L_1}{4} \\ \rightarrow \frac{L_3}{2} \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 3/4 & 1/2 & -5/4 & 0 & 0 \\ 3 & 6 & 4 & 0 & -5 & 0 \\ 1 & 2 & 3 & 0 & 0 & -5/2 \end{array} \right] \begin{array}{l} \\ \rightarrow -3L_1 \\ \rightarrow -L_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 3/4 & 1/2 & -5/4 & 0 & 0 \\ 0 & 15/4 & 5/2 & -15/4 & -5 & 0 \\ 0 & 5/4 & 5/2 & 5/4 & 0 & -5/2 \end{array} \right] \begin{array}{l} \\ \rightarrow 4L_2 \\ \rightarrow 4L_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 3/4 & 1/2 & -5/4 & 0 & 0 \\ 0 & 15 & 10 & -15 & -20 & 0 \\ 0 & 5 & 10 & 5 & 0 & -10 \end{array} \right] \begin{array}{l} \\ \rightarrow L_2/15 \\ \rightarrow L_3/5 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 3/4 & 1/2 & -5/4 & 0 & 0 \\ 0 & 1 & 2/3 & -1 & -4/3 & 0 \\ 0 & 1 & 2 & 1 & 0 & -2 \end{array} \right] \begin{array}{l} \rightarrow L_1 - \frac{3}{4}L_3 \\ \\ \rightarrow L_3 - L_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 0 & 3/2 \\ 0 & 1 & 2/3 & -1 & -4/3 & 0 \\ 0 & 0 & 4/3 & 2 & 4/3 & -2 \end{array} \right] \begin{array}{l} \\ \rightarrow L_2 + L_3 \\ \rightarrow 3/4 L_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 0 & 3/2 \\ 0 & 1 & 2 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3/2 & 1 & -3/2 \end{array} \right] \begin{array}{l} \rightarrow L_1 + L_3 \\ \rightarrow L_2 - 2L_3 \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/2 & 1 & 0 \\ 0 & 1 & 0 & -2 & -2 & 1 \\ 0 & 0 & 1 & 0 & 1 & -3/2 \end{array} \right]$$

$$I = \begin{bmatrix} -1/2 & -1 & 0 \\ -2 & -2 & 1 \\ 0 & 1 & -3/2 \end{bmatrix}$$



3ª QUESTÃO

Handwritten solution for a system of linear equations using the Gauss-Jordan method.

Initial System:

$$\begin{array}{cccc|c} 2 & 3 & 1 & 5 & 14 \\ 1 & 3,75 & 1 & 7,5 & 13 \\ 1,4 & 2,7 & 5,5 & 12 & 21,6 \\ -2 & 1 & 3 & 28 & 30 \end{array}$$

Row Operations:

- $L_1 \leftrightarrow L_2$
- $L_3 - L_1$
- $L_4 + 2L_1$

Intermediate System:

$$\begin{array}{cccc|c} 1 & 1,5 & 0,5 & 2,5 & 5,5 \\ 1 & 3,75 & 1 & 7,5 & 13 \\ 1,4 & 2,7 & 5,5 & 12 & 21,6 \\ -2 & 1 & 3 & 28 & 30 \end{array}$$

Row Operations:

- $L_2 - L_1$
- $L_3 - 1,4L_1$
- $L_4 + 2L_1$

Intermediate System:

$$\begin{array}{cccc|c} 1 & 1,5 & 0,5 & 2,5 & 5,5 \\ 0 & 2 & 0,5 & 5 & 7,5 \\ 0 & 0,6 & 4,8 & 8,5 & 13,9 \\ 0 & 4 & 4 & 3 & 19 \end{array}$$

Row Operations:

- $L_2 \rightarrow \frac{L_2}{2}$
- $L_3 - 1,5L_2$
- $L_4 - 4L_2$

Intermediate System:

$$\begin{array}{cccc|c} 1 & 1,5 & 0,5 & 2,5 & 5,5 \\ 0 & 1 & 0,25 & 2,5 & 3,75 \\ 0 & 0,6 & 4,8 & 8,5 & 13,9 \\ 0 & 4 & 4 & 3 & 19 \end{array}$$

Row Operations:

- $L_3 - 0,6L_2$
- $L_4 - 4L_2$

Intermediate System:

$$\begin{array}{cccc|c} 1 & 0 & 0,125 & -1,25 & -1,25 \\ 0 & 1 & 0,25 & 2,5 & 3,75 \\ 0 & 0,6 & 4,8 & 8,5 & 13,9 \\ 0 & 4 & 4 & 3 & 19 \end{array}$$

Row Operations:

- $L_3 - 0,125L_2$
- $L_4 - 4L_2$

Intermediate System:

$$\begin{array}{cccc|c} 1 & 0 & 0,125 & -1,25 & -1,25 \\ 0 & 1 & 0,25 & 2,5 & 3,75 \\ 0 & 0 & 4,68 & 7 & 11,65 \\ 0 & 0 & 3 & -3 & 4 \end{array}$$

Row Operations:

- $L_3 - 0,125L_2$
- $L_4 - 3L_3$

Final System:

$$\begin{array}{cccc|c} 1 & 0 & 0 & 1,625 & 0,875 \\ 0 & 1 & 0 & 3,25 & 2,95 \\ 0 & 0 & 1 & 1,51 & 2,51 \\ 0 & 0 & 3 & -3 & 4 \end{array}$$

Row Operations:

- $L_4 \rightarrow \frac{L_4}{3}$
- $L_4 - L_3$

Final System:

$$\begin{array}{cccc|c} 1 & 0 & 0 & 1,625 & 0,875 \\ 0 & 1 & 0 & 3,25 & 2,95 \\ 0 & 0 & 1 & 1,51 & 2,51 \\ 0 & 0 & 1 & -1 & 4/3 \end{array}$$

Row Operations:

- $L_4 - L_3$

Final System:

$$\begin{array}{cccc|c} 1 & 0 & 0 & 1,625 & 0,875 \\ 0 & 1 & 0 & 3,25 & 2,95 \\ 0 & 0 & 1 & 1,51 & 2,51 \\ 0 & 0 & 0 & -2,51 & -1,18 \end{array}$$

Row Operations:

- $L_4 \rightarrow \frac{L_4}{-2,51}$

Final System:

$$\begin{array}{cccc|c} 1 & 0 & 0 & 1,625 & 0,875 \\ 0 & 1 & 0 & 3,25 & 2,95 \\ 0 & 0 & 1 & 1,51 & 2,51 \\ 0 & 0 & 0 & 1 & 0,49 \end{array}$$

Row Operations:

- $L_1 - 1,625L_4$
- $L_2 - 3,25L_4$
- $L_3 - 1,51L_4$

Final System:

$$\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0,125 \\ 0 & 1 & 0 & 0 & 2,16 \\ 0 & 0 & 1 & 0 & 2,185 \\ 0 & 0 & 0 & 1 & 0,49 \end{array}$$

Row Operations:

- $L_1 \rightarrow \frac{L_1}{0,125}$
- $L_2 \rightarrow \frac{L_2}{2,16}$
- $L_3 \rightarrow \frac{L_3}{2,185}$

Final System:

$$\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0,49 \end{array}$$



5ª QUESTÃO

Código

```
1 clc; clear;
2
3 A = [2, 3, 1, 5;
4       1, 3.5, 1, 7.5;
5       1.4, 2.7, 5.5, 12;
6       -2, 1, 3, 28];
7
8 L = zeros(length(A));
9 U = zeros(length(A));
10
11 for j=1: length(A)
12     for i=1:length(A)
13         sum=0;
14         p=j-1;
15         for k=1:p
16             sum=sum+(L(i,k)*U(k,j));
17         end
18
19         if i==j
20             U(i,j) = 1;
21         end
22
23         if i>=j
24             L(i,j) = A(i,j)-sum;
25         else
26             U(i,j) = (1/L(i,i))*(A(i,j)-sum);
27         end
28     end
29 end
30 A
31 L
32 U
```

Saída

A =

2.0000	3.0000	1.0000	5.0000
1.0000	3.5000	1.0000	7.5000
1.4000	2.7000	5.5000	12.0000
-2.0000	1.0000	3.0000	28.0000

L =

2.00000	0.00000	0.00000	0.00000
1.00000	2.00000	0.00000	0.00000
1.40000	0.60000	4.65000	0.00000
-2.00000	4.00000	3.00000	18.48387

U =

1.00000	1.50000	0.50000	2.50000
0.00000	1.00000	0.25000	2.50000
0.00000	0.00000	1.00000	1.50538
0.00000	0.00000	0.00000	1.00000

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