Lista de Exercícios – Métodos Numéricos para Engenharia TC
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2ª QUESTÃO

QUESTÃO 2

(A)

Velocidade Km/h	16	40	64	88	112
Economia Km/L	4,2	9,2	10	10,7	8,6

→ ordem 4:

$$f(x) = \frac{(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)} y_1 + \frac{(x-x_1)(x-x_3)(x-x_4)(x-x_5)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)(x_2-x_5)} y_2 +$$

$$\frac{(x-x_1)(x-x_2)(x-x_4)(x-x_5)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)(x_3-x_5)} y_3 + \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_5)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)(x_4-x_5)} y_4 +$$

$$\frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_5-x_1)(x_5-x_2)(x_5-x_3)(x_5-x_4)} y_5$$

→ substituindo os valores:

$$f(x) = \frac{(x-40)(x-64)(x-88)(x-112)}{(16-40)(16-64)(16-88)(16-112)} 4.2 + \frac{(x-16)(x-64)(x-88)(x-112)}{(40-16)(40-64)(40-88)(40-112)} 9.2 +$$

$$\frac{(x-16)(x-40)(x-88)(x-112)}{(64-16)(64-40)(64-88)(64-112)} 10 + \frac{(x-16)(x-40)(x-64)(x-112)}{(88-16)(88-40)(88-64)(88-112)} 10.7 +$$

$$\frac{(x-16)(x-40)(x-64)(x-88)}{(112-16)(112-40)(112-64)(112-88)} 8.6$$

$$f(105) = \frac{65 \cdot 41 \cdot 17 \cdot (-7)}{(-24)(-48)(-72)(-96)} \cdot 4,2 + \frac{89 \cdot 41 \cdot 17 \cdot (-7)}{24(-24)(-48)(-72)} \cdot 9.2 +$$

$$\frac{89 \cdot 65 \cdot 17 \cdot (-7)}{48 \cdot 24 \cdot (-24)(-48)} \cdot 10 + \frac{89 \cdot 65 \cdot 41 \cdot (-7)}{72 \cdot 48 \cdot 24 \cdot (-24)} 10.7 +$$

$$\frac{89 \cdot 65 \cdot 41 \cdot 17}{96 \cdot 72 \cdot 48 \cdot 24} \cdot 8.6$$

$$f(105) = -0.0398 \cdot 4.2 + 0.2181 \cdot 9.2 - 0.5187 \cdot 10 + 0.8340 \cdot 10.7 + 0.5063 \cdot 8.6$$

$$f(105) = -0.16716 + 2.00652 - 5.187 + 8.9238 + 4.35418$$

$$\boxed{f(105) = 9.93034}$$

Código – Lagrange Ordem 4

lista4questao2a_ordem4.m

```
clc; clear all;

% lagrange

x=[16 40 64 88 112];
y=[4.2 9.2 10 10.7 8.6];
Xint=105;

n = length(x);

for i = 1:n
    L(i) = 1;
    for j = 1:n
        if j ~= i
            L(i) = L(i) * (Xint - x(j)) / (x(i) - x(j));
        endif
    endfor
endfor

Yint = sum(y.*L);

fprintf('Yint = %.4f \n', Yint);
```

Saída

```
Yint = 9.9314
>>
```

→ Ordem 2 :

$$f(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)} y_3$$

→ substituindo :

x	64	88	112
y	10	10.7	8.6

$$f(105) = \frac{17 \cdot (-7)}{(-24)(-48)} \cdot 10 + \frac{41 \cdot (-7)}{24 \, (-24)} \cdot 10.7 + \frac{41 \cdot 17}{48 \cdot 24} \cdot 8.6$$

$$f(105) = -0.10329 \cdot 10 + 0.49826 \cdot 10.7 + 0.60503 \cdot 8.6$$

$$f(105) = -1.0329 + 5.331382 + 5.2033$$

$$\boxed{f(105) = 9.501782}$$

Código – Lagrange Ordem 2

lista4questao2a_ordem2.m

```
clc; clear all;

% lagrange

x=[64 88 112];
y=[10 10.7 8.6];
Xint=105;

n = length(x);

for i = 1:n
    L(i) = 1;
    for j = 1:n
        if j ~= i
            L(i) = L(i) * (Xint - x(j)) / (x(i) - x(j));
        endif
    endfor
endfor

Yint = sum(y.*L);

fprintf('Yint = %.4f \n', Yint);
```

Saída

```
Yint = 9.5017
>>
```

(B) → ordem 4

x	y					
16	4.2	a_1				
		$f[x_2 x_1]$	a_2			
40	9.2		$f[x_3 x_2 x_1]$	a_3		
		$f[x_3 x_2]$		$f[x_4 x_3 x_2 x_1]$	a_4	
64	10		$f[x_4 x_3 x_2]$			$f[x_5 x_4 x_3 x_2 x_1]$ a_5
		$f[x_4 x_3]$		$f[x_5 x_4 x_3 x_2]$		
88	10.7		$f[x_3 x_4 x_3]$			
		$f[x_5 x_4]$				
112	8.6					

$a_1 = y_1 = \boxed{4.2}$

$a_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5}{24} = \boxed{0.2083}$

$a_3 = \frac{f[x_3 x_2] - f[x_2 x_1]}{x_3 - x_1} = \frac{\frac{y_3 - y_2}{x_3 - x_2} - 0.2083}{x_3 - x_1} = \frac{\frac{0.8}{24} - 0.2083}{48} = \boxed{-0.0036}$

$a_4 = [\ldots] = \frac{\left(\frac{y_4 - y_3}{x_4 - x_3} - \frac{y_3 - y_2}{x_3 - x_2}\right) - \left(\frac{y_3 - y_2}{x_3 - x_2} - \frac{y_2 - y_1}{x_2 - x_1}\right)}{(x_4 - x_2)} = \frac{\left(\frac{0.7}{24} - \frac{0.8}{24}\right)}{48} + 0.00365$

$\frac{}{x_4 - x_1} \qquad \frac{}{72}$

$= \frac{-0.000087 + 0.00365}{72} = \boxed{0.00005}$

$a_5 = \frac{f[x_5 x_4 x_3 x_2] - a_4}{x_5 - x_1} = \frac{\left(\frac{\left(\frac{y_5 - y_4}{x_5 - x_4} - \frac{y_4 - y_3}{x_4 - x_3}\right) - \left(\frac{y_4 - y_3}{x_4 - x_3} - \frac{y_3 - y_2}{x_3 - x_2}\right)}{x_5 - x_3}\right) - a_4}{x_5 - x_1}$

$= \frac{\left(\left(\frac{-0.0875 - 0.0291}{48}\right) + 0.000087\right) - 0.00005}{96} = \frac{-0.00243 + 0.000087 - 0.00005}{96}$

$= \boxed{-0.000025}$

→ polinômio de ordem 4:

$f(x) = a_1 + a_2(x - x_1) + a_3(x - x_1)(x - x_2) + a_4(x - x_1)(x - x_2)(x - x_3) + a_5(x - x_1)(x - x_2)(x - x_3)(x - x_4)$

$f(48) = 4.2 + (0.2083 \cdot 32) - (0.0036 \cdot 32 \cdot 8) + (0.00005 \cdot 32 \cdot 8 \cdot (-16)) - (0.000025 \cdot 32 \cdot 8 \cdot (-16)(-40))$

$f(48) = 4.2 + 6.6656 - 0.9216 - 0.2048 - 4.096$

$\boxed{f(48) = 9.4532}$

Código – Newton Ordem 4

lista4questao2b_ordem4.m

```
clc; clear all;
% newton

x=[16 40 64 88 112];
y=[4.2 9.2 10 10.7 8.6];
Xint=48;

n = length(x);
a(1) = y(1);


for i = 1:n-1
    divDIF(i, 1) = (y(i+1) - y(i))/(x(i+1) - x(i));
endfor


for j = 2:n-1
    for i = 1:n-j
        divDIF(i,j) = (divDIF(i+1, j-1) - divDIF(i, j-1)) / (x(j+i) - x(i));
    endfor
endfor

for j = 2:n
    a(j) = divDIF(1, j-1);
endfor

Yint = a(1);
xn = 1;

for k = 2:n
    xn = xn * (Xint - x(k-1));
    Yint = Yint + a(k) * xn;
endfor

fprintf('Yint = %.4f \n', Yint);
```

Saída

```
Yint = 9.5909
>>
```

→ ordem 2

$a_1 = 4.2$

$a_2 = 0.2083$

$a_3 = 0.0036$

→ polinômio de ordem 2:

$f(x) = a_1 + a_2(x - x_1) + a_3(x - x_1)(x - x_2)$

$f(48) = 9.944$

Código – Newton Ordem 2

lista4questao2b_ordem2.m

```
clc; clear all;
% newton

x=[16 40 64];
y=[4.2 9.2 10];
Xint=48;

n = length(x);
a(1) = y(1);


for i = 1:n-1
    divDIF(i, 1) = (y(i+1) - y(i))/(x(i+1) - x(i));
endfor


for j = 2:n-1
    for i = 1:n-j
        divDIF(i,j) = (divDIF(i+1, j-1) - divDIF(i, j-1)) / (x(j+i) - x(i));
    endfor
endfor

for j = 2:n
    a(j) = divDIF(1, j-1);
endfor

Yint = a(1);
xn = 1;

for k = 2:n
    xn = xn * (Xint - x(k-1));
    Yint = Yint + a(k) * xn;
endfor

fprintf('Yint = %.4f \n', Yint);
```

Saída

```
Yint = 9.9333
>>
```

3ª QUESTÃO

QUESTÃO 3

x	1.6	2	2,5	3.2	4	4.5
f(x)	2	8	14	15	8	2

(A) → primeiro grau

$$f(x) = \frac{(x-x_2)}{(x_1-x_2)} y_1 + \frac{(x-x_1)}{(x_2-x_1)} y_2$$

→ segundo grau

$$f(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)} y_3$$

→ terceiro grau

$$f(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1 + \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2 + \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3 + \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)} y_4$$

(B) há 6 pontos (n=6) ∴ 5 splines ($i = 1, 2, 3, 4, 5$) → $f_i(x) = a_i x^2 + b_i x + c_i$

$i = 1$ $\quad f_1(x) = a_1 x_1^2 + b_1 x_1 + c_1 = b_1\, 1.6 + c_1 = 2$

$\quad f_1(x) = a_1 x_2^2 + b_1 x_2 + c_1 = b_1\, 2 + c_1 = 8$

$i = 2$ $\quad f_2(x) = a_2\, 2^2 + b_2\, 2 + c_2 = 8$

$\quad f_2(x) = a_2\, 2.5^2 + b_2\, 2.5 + c_2 = 14$

$i = 3$ $\quad f_3(x) = a_3\, 2.5^2 + b_3\, 2.5 + c_3 = 14$

$\quad f_3(x) = a_3\, 3.2^2 + b_3\, 3.2 + c_3 = 15$

$i = 4$ $\quad f_4(x) = a_4\, 3.2^2 + b_4\, 3.2 + c_4 = 15$

$\quad f_4(x) = a_4\, 4^2 + b_4\, 4^2 + c_4 = 8$

$i = 5$ $\quad f_5(x) = a_5\, 4^2 + b_5\, 4^2 + c_5 = 8$

$\quad f_5(x) = a_5\, 4.5^2 + b_5\, 4.5^2 + c_5 = 2$

2 (n-1) equações

10 equações

* equações obtidas a partir da condição que diz que:
> em cada intervalo, o polinômio deve passar pelos pontos finais

---

$i = 2$ $\quad 2a_1 x_2 + b_1 = 2a_2 x_2 + b_2$

$\quad b_1 = 2a_2\, 2 + b_2$

$\quad b_1 - 4a_2 - b_2 = 0$

$i = 3$ $\quad 2a_2 x_3 + b_2 = 2a_3 x_3 + b_3$

$\quad 2a_2\, 2.5 + b_2 = 2a_3\, 2.5 + b_3$

$\quad 5a_2 + b_2 - 5a_3 - b_3 = 0$

$i = 4$ $\quad 2a_3 x_4 + b_3 = 2a_4 x_4 + b_4$

$\quad 2a_3\, 3.2 + b_4 = 2a_4\, 3.2 + b_4$

$\quad 6.4a_3 + b_4 - 6.4a_4 - b_4 = 0$

$i = 5$ $\quad 2a_4 x_5 + b_4 = 2a_5 x_5 + b_5$

$\quad 2a_4\, 4 + b_4 = 2a_5\, 4 + b_5$

$\quad 8a_4 + b_4 - 8a_4 - b_5 = 0$

* equações obtidas a partir da condição que diz que:
> nos nós interiores, as inclinações (derivadas primeiras) dos polinômios de intervalos adjacentes são iguais

→ Sistema com as 14 equações

$$\begin{bmatrix}
1,6 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2^2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2,5^2 & 2,5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2,5^2 & 2,5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 3,2^2 & 3,2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3,2^2 & 3,2 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4^2 & 4 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4^2 & 4 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4,5^2 & 4,5 & 1 \\
1 & 0 & -4 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 8 & 1 & 0 & -5 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 6,4 & 1 & 0 & -6,4 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & 1 & 0 & -8 & -1 & 0
\end{bmatrix}
\begin{bmatrix} b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \\ a_3 \\ b_3 \\ c_3 \\ a_4 \\ b_4 \\ c_4 \\ a_5 \\ b_5 \\ c_5 \end{bmatrix}
=
\begin{bmatrix} 2 \\ 8 \\ 8 \\ 14 \\ 14 \\ 15 \\ 15 \\ 8 \\ 8 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

→ efetuando o cálculo $A \cdot X = B$ no Octave:

$$x = \begin{bmatrix}
15 \\ -22 \\ -6 \\ 39 \\ -46 \\ -10.8163 \\ 63.0816 \\ -76.1020 \\ -3.2589 \\ 14.7143 \\ 1.2857 \\ -1.2857 \\ -1.0714 \\ 32.8571
\end{bmatrix}$$

→ as equações finais são:

$f_1(x) = 15x - 22$ para $1.6 < x < 2$

$f_2(x) = -6x^2 + 39x - 46$ para $2 < x < 2.5$

$f_3(x) = -10.8163x^2 + 63.0816x - 76.1020$ para $2.5 < x < 3.2$

$f_4(x) = -3.2589x^2 + 14.7143x + 1.2857$ para $3.2 < x < 4$

$f_5(x) = -1.2857x^2 - 1.0714x + 32.8571$ para $4 < x < 4.5$

Código – Matriz Spline Quadrática

lista4questao3b_matriz.m

```
clc; clear all;

A = [1.6, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0; ...
     2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0; ...
     0, 0, 4, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0; ...
     0, 0, 6.25, 2.5, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0; ...
     0, 0, 0, 0, 0, 6.25, 2.5, 1, 0, 0, 0, 0, 0, 0; ...
     0, 0, 0, 0, 0, 10.24, 3.2, 1, 0, 0, 0, 0, 0, 0; ...
     0, 0, 0, 0, 0, 0, 0, 0, 10.24, 3.2, 1, 0, 0, 0; ...
     0, 0, 0, 0, 0, 0, 0, 0, 16, 4, 1, 0, 0, 0; ...
     0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 16, 4, 1; ...
     0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 20.25, 4.5, 1; ...
     1, 0, -4, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0; ...
     0, 0, 5, 1, 0, -5, -1, 0, 0, 0, 0, 0, 0, 0; ...
     0, 0, 0, 0, 0, 6.4, 1, 0, -6.4, -1, 0, 0, 0, 0; ...
     0, 0, 0, 0, 0, 0, 0, 0, 8, 1, 0, -8, -1, 0];

B = [2;8;8;14;14;15;15;8;8;2;0;0;0;0];


X = inv(A)*B
```

Saída

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```
X =

   15.0000
  -22.0000
   -6.0000
   39.0000
  -46.0000
  -10.8163
   63.0816
  -76.1020
   -3.2589
   14.7143
    1.2857
   -1.2857
   -1.0714
   32.8571

>>
```

(D) → para $f(2.2)$

• lagrange ordem 1

$$f(2.2) = \frac{2.2 - 2.5}{2 - 2.5} \cdot 8 + \frac{2.2 - 2}{2.5 - 2} \cdot 14 = 4.8 + 5.6 = \boxed{10.4}$$

• lagrange ordem 2

$$f(2.2) = \frac{(2.2 - 2)(2.2 - 2.5)}{(1.6 - 2)(1.6 - 2.5)} 2 + \frac{(2.2 - 1.6)(2.2 - 2.5)}{(2 - 1.6)(2 - 2.5)} 8 + \frac{(2.2 - 1.6)(2.2 - 2)}{(2.5 - 1.6)(2.5 - 2)} 14$$

$$f(2.2) = -0.333 + 7.2 + 3.73$$

$$\boxed{f(2.2) = 10.6}$$

• lagrange ordem 3

$\boxed{f(2.2) = 10.723}$ ⇝ calculado no octave

• spline quadrática

$$f(2.2) = -6(2.2)^2 + 39 \cdot 2.2 - 46 = \boxed{10.76}$$

———— // ————

→ para $f(3.4)$

• lagrange ordem 1

$$f(3.4) = \frac{3.4 - 4}{3.2 - 4} 15 + \frac{3.4 - 3.2}{4 - 3.2} 8 = 11.25 + 2 = \boxed{13.25}$$

• lagrange ordem 2

$$f(3.4) = \frac{(3.4 - 3.2)(3.4 - 4)}{(2.5 - 3.2)(2.5 - 4)} 14 + \frac{(3.4 - 2.5)(3.4 - 4)}{(3.2 - 2.5)(3.2 - 4)} 15 + \frac{(3.4 - 2.5)(3.4 - 3.2)}{(4 - 2.5)(4 - 3.2)} 8$$

$$f(3.4) = -1.6 + 14.46 + 1.2$$

$$f(3.4) = 14.06$$

• lagrange ordem 3

$\boxed{f(3.4) = 13.955}$ ⇝ calculado no octave

• spline quadrática

$$f(3.4) = -3.2589 \cdot (3,4)^2 + 14.7143 \cdot 3.4 + 1.2857 = \boxed{13.64}$$

———— // ————

→ para $f(4.2)$

• lagrange ordem 1

$$f(4.2) = \frac{(4.2 - 4.5)}{(4 - 4.5)} 8 + \frac{(4.2 - 4)}{(4.5 - 4)} \cdot 2 = 4.8 + 0.8 = \boxed{5.6}$$

• lagrange ordem 2

$$f(4.2) = \frac{(4.2 - 4)(4.2 - 4.5)}{(3.2 - 4)(3.2 - 4.5)} 15 + \frac{(4.2 - 3.2)(4.2 - 4.5)}{(4 - 3.2)(4 - 4.5)} 8 + \frac{(4.2 - 3.2)(4.2 - 4)}{(4.5 - 3.2)(4.5 - 4)} 2$$

$f(4.2) = -0.365 + 5 + 0.565$

$f(4.2) = 5.25$

- lagrange ordem 3

$f(4.2) = 5.3214$ ⇝ calculado no octave

- spline quadrática

$f(4.2) = -1.2857 \cdot (4.2)^2 - 1.0714 \cdot 4.2 + 32.8571$

$f(4.2) = 5.677$

Código – Lagrange

lista4questao3d_lagrange.m

```
function Yint = lista4questao3d_lagrange(x, y, Xint)

n = length(x);

for i = 1:n
    L(i) = 1;
    for j = 1:n
        if j ~= i
            L(i) = L(i) * (Xint - x(j)) / (x(i) - x(j));
        endif
    endfor
endfor

Yint = sum(y.*L);
fprintf('\n\n')
```

Saída f(2.2)

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```
>> x=[2 2.5];
>> y=[8 14];
>> Yint = lista4questao3d_lagrange(x, y, 2.2)
Yint =  10.400
>>
>> x=[1.6 2 2.5];
>> y=[2 8 14];
>> Yint = lista4questao3d_lagrange(x, y, 2.2)
Yint =  10.600
>>
>> x=[1.6 2 2.5 3.2];
>> y=[2 8 14 15];
>> Yint = lista4questao3d_lagrange(x, y, 2.2)
Yint =  10.723
>>
```

Saída f(3.4)

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```
>> x=[3.2 4];
>> y=[15 8];
>> Yint = lista4questao3d_lagrange(x, y, 3.4)
Yint =  13.250
>>
>> x=[2.5 3.2 4];
>> y=[14 15 8];
>> Yint = lista4questao3d_lagrange(x, y, 3.4)
Yint =  14.064
>>
>> x=[2 2.5 3.2 4];
>> y=[8 14 15 8];
>> Yint = lista4questao3d_lagrange(x, y, 3.4)
Yint =  13.955
>>
```

Saída f(4.2)

Janela de Comandos

```
>> x=[4 4.5];
>> y=[8 2];
>> Yint = lista4questao3d_lagrange(x, y, 4.2)
Yint =  5.6000
>>
>> x=[3.2 4 4.5];
>> y=[15 8 2];
>> Yint = lista4questao3d_lagrange(x, y, 4.2)
Yint =  5.7500
>>
>> x=[2.5 3.2 4 4.5];
>> y=[14 15 8 2];
>> Yint = lista4questao3d_lagrange(x, y, 4.2)
Yint =  5.6214
>>
```

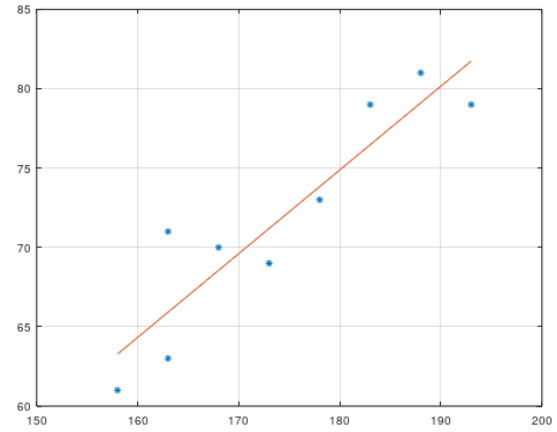
4ª QUESTÃO

Código Letra A e B

lista4questao4ab_dispersao_dados.m

```
clc; clear all;  clc;

% letra a - dispersao de dados
x = [183 173 168 188 158 163 193 163 178];
y = [79 69 70 81 61 63 79 71 73];
plot(x,y,'*')
grid on
hold on

% letra b - ajuste de reta
p=polyfit(x,y,1);

xl=linspace(158,193,9)
yl=polyval(p, xl)

plot(xl,yl)
```

Saída



Janela de Comandos

```
xl =

   158.00   162.38   166.75   171.12   175.50   179.88   184.25   188.62   193.00

yl =

   63.278   65.586   67.894   70.202   72.511   74.819   77.127   79.435   81.743

>>
```

Código Letra C – altura=175cm (Lagrange)

lista4questao4c_lagrange.m

```
clc; clear all;

% x e y gerados a partir do ajuste de reta

x = [158.00 162.38 166.75 171.12 175.50 179.88 184.25 188.62 193.00];
y = [63.278 65.586 67.894 70.202 72.511 74.819 77.127 79.435 81.743];
Xint=175;

n = length(x);

for i = 1:n
    L(i) = 1;
    for j = 1:n
        if j ~= i
            L(i) = L(i) * (Xint - x(j)) / (x(i) - x(j));
        endif
    endfor
endfor

Yint = sum(y.*L);

fprintf('Yint = %.4f \n', Yint);
```

Saída

Janela de Comandos

```
Yint = 72.2476
>>
```

Código Letra C – peso=80kg (Lagrange)

lista4questao4c_lagrange.m

```
clc; clear all;

% x e y gerados a partir do ajuste de reta

% para altura=175:
% x = [158.00 162.38 166.75 171.12 175.50 179.88 184.25 188.62 193.00];
% y = [63.278 65.586 67.894 70.202 72.511 74.819 77.127 79.435 81.743];
% Xint=175;

% para peso=80:
x = [63.278 65.586 67.894 70.202 72.511 74.819 77.127 79.435 81.743];
y = [158.00 162.38 166.75 171.12 175.50 179.88 184.25 188.62 193.00];
Xint=80;

n = length(x);

for i = 1:n
    L(i) = 1;
    for j = 1:n
        if j ~= i
            L(i) = L(i) * (Xint - x(j)) / (x(i) - x(j));
        endif
    endfor
endfor

Yint = sum(y.*L);

fprintf('Yint = %.4f \n', Yint);
```

Saída

Janela de Comandos

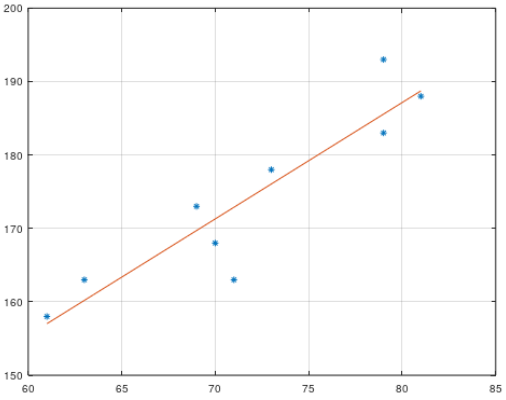
```
Yint = 189.6926
>>
```

Código Letra D

lista4questao4d_dispersao_dados.m

```
clc; clear all;  clc;

% dispersao de dados
x = [79 69 70 81 61 63 79 71 73];
y = [183 173 168 188 158 163 193 163 178];
plot(x,y,'*')
grid on
hold on

% ajuste de reta
p=polyfit(x,y,1);

xl=linspace(61,81,9)
yl=polyval(p, xl)

plot(xl,yl)
```

Saída



Janela de Comandos

```
xl =

   61.000   63.500   66.000   68.500   71.000   73.500   76.000   78.500   81.000

yl =

   157.02   160.99   164.95   168.91   172.88   176.84   180.81   184.77   188.73

>>
```

Código Letra E – peso=80kg (Lagrange)

lista4questao4e_lagrange.m

```
clc; clear all;

% x e y gerados a partir do ajuste de reta

% para peso=80:
x = [61.000 63.500 66.000 68.500 71.000 73.500 76.000 78.500 81.000];
y = [157.02 160.99 164.95 168.91 172.88 176.84 180.81 184.77 188.73];
Xint=80;

% para altura=175:
% x = [157.02 160.99 164.95 168.91 172.88 176.84 180.81 184.77 188.73];
% y = [61.000 63.500 66.000 68.500 71.000 73.500 76.000 78.500 81.000];
% Xint=75;

n = length(x);

for i = 1:n
    L(i) = 1;
    for j = 1:n
        if j ~= i
            L(i) = L(i) * (Xint - x(j)) / (x(i) - x(j));
        endif
    endfor
endfor

Yint = sum(y.*L);

fprintf('Yint = %.4f \n', Yint);
```

Saída

Janela de Comandos

```
Yint = 187.1247
>>
```

Código Letra E – altura=175cm (Lagrange)

lista4questao4e_lagrange.m

```
clc; clear all;

% x e y gerados a partir do ajuste de reta

% para peso=80:
% x = [61.000 63.500 66.000 68.500 71.000 73.500 76.000 78.500 81.000];
% y = [157.02 160.99 164.95 168.91 172.88 176.84 180.81 184.77 188.73];
% Xint=80;

% para altura=175:
x = [157.02 160.99 164.95 168.91 172.88 176.84 180.81 184.77 188.73];
y = [61.000 63.500 66.000 68.500 71.000 73.500 76.000 78.500 81.000];
Xint=175;

n = length(x);

for i = 1:n
    L(i) = 1;
    for j = 1:n
        if j ~= i
            L(i) = L(i) * (Xint - x(j)) / (x(i) - x(j));
        endif
    endfor
endfor

Yint = sum(y.*L);

fprintf('Yint = %.4f \n', Yint);
```

Saída

Janela de Comandos

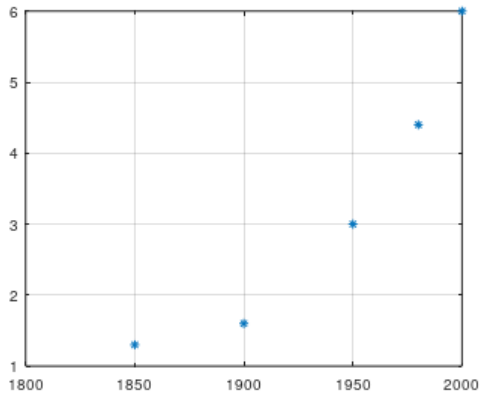
```
Yint = 72.3380
>>
```

5ª QUESTÃO

Entrada

```
Janela de Comandos
>> x = [1850 1900 1950 1980 2000];
>> y = [1.3 1.6 3 4.4 6];
>> plot(x,y,'*')
>> grid on
>>
```

Saída



QUESTÃO 5

x	1850	1900	1950	1980	2000
y	1,3	1,6	3	4,4	6
$z = \ln(y)$	0,26236	0,47	1,09861	1,4816	1,79175

função exponencial → $p = be^{mx}$

$n = 5$

$S_x = 1850 + 1900 + 1950 + 1980 + 2000 = 9680$

$S_{xy} = 1850 \cdot 0,26236 + 1900 \cdot 0,47 + 1950 \cdot 1,09861 + 1980 \cdot 1,4816 + 2000 \cdot 1,79175 = 10037,7235$

$S_{xx} = 1850^2 + 1900^2 + 1950^2 + 1980^2 + 2000^2 = 18.755.400$

$S_y = 0,26236 + 0,47 + 1,09861 + 1,4816 + 1,79175 = 5,10432$

$y = be^{mx}$

$\ln(y) = \ln(be^{mx})$

$\ln(y) = \ln(b) + mx$

$z = a_0 + a_1 x$

$$a_1 = \frac{n S_{xy} - S_x S_y}{n S_{xx} - (S_x)^2} = \frac{5 \cdot 10037,7235 - 9680 \cdot 5,10432}{5 \cdot 18755400 - (9680)^2} = \frac{778,8}{74600} = 0,01044$$

$$a_0 = \frac{S_{xx} S_y - S_{xy} S_x}{n S_{xx} - (S_x)^2} = \frac{18755400 \cdot 5,10432 - 10037,7235 \cdot 9680}{74600} = -19,19$$

→ substituindo:

$z = -19,19 + 0,01044\,(1270)$

$z = 1,3768$

Código

lista4questao5_regressao_linear.m

```
clc; clear all;
% regressao linear

x = [1850 1900 1950 1980 2000];
y = [0.26236 0.47 1.09861 1.4816 1.79175];

nx = length(x);
ny = length(y);

if nx ~= ny
    disp('ERROR: The number of elements in x must be the same as in y')
    a1 = 'Error';
    a0 = 'Error';

else
    Sx = sum(x);
    Sy = sum(y);
    Sxy = sum(x.*y);
    Sxx = sum(x.^2);
    a1 = (nx*Sxy - Sx*Sy)/(nx*Sxx - Sx^2);
    a0 = (Sxx*Sy - Sxy*Sx)/(nx*Sxx - Sx^2);
end

fprintf('a1 = %.6f\n', a1);
fprintf('a0 = %.6f\n', a0);
```

Saída

```
Janela de Comandos
a1 = 0.010440
a0 = -19.190351
>>
```

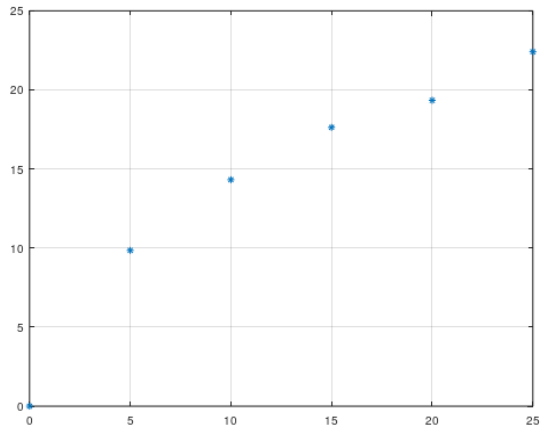
6ª QUESTÃO

Entrada

lista4questao6_grafico.m

```
clc; clear all;  clc;

x = [0 5 10 15 20 25];
y = [0 9.85 14.32 17.63 19.34 22.41];
plot(x,y,'*')
grid on
```

Saída



QUESTÃO 6

x	0	5	10	15	20	25
y	0	9.85	14.32	17.63	19.34	22.41

$v^2 = 2gx \rightsquigarrow y = ax$, onde $y = v^2$ e $x = 2x$

$n = 6$

→ então, a tabela será:

x	0	10	20	30	40	50
y	0	97.0225	205.0624	310.8169	374.0356	502.2081

$S_x = 0 + 10 + 20 + 30 + 40 + 50 = 150$

$S_{xx} = 0^2 + 10^2 + 20^2 + 30^2 + 40^2 + 50^2 = 5500$

$S_{xy} = 0 \cdot 0 + 10 \cdot 97{,}0225 + 20 \cdot 205{,}0624 + 30 \cdot 310{,}8169 + 40 \cdot 374{,}0356 + 50 \cdot 502{,}2081 = 54467{,}809$

$S_y = 0 + 97{,}0225 + 205{,}0624 + 310{,}8169 + 374{,}0356 + 502{,}2081 = 1489{,}1455$

$$a_1 = \frac{6 \cdot 54467{,}809 - 150 \cdot 1489{,}1455}{6 \cdot 5500 - (150)^2} = \frac{103435{,}029}{10500} = 9{,}851$$

$$a_0 = \frac{5500 \cdot 1489{,}1455 - 54467{,}809 \cdot 150}{10500} = 1{,}917$$

$g = a_1 = 9{,}851$

Código

lista4questao6_regressao_linear.m

```
clc; clear all;
% regressao linear

x = [0 10 20 30 40 50];
y = [0 97.0225 205.0624 310.8169 374.0356 502.2081];

nx = length(x);
ny = length(y);

if nx ~= ny
    disp('ERROR: The number of elements in x must be the same as in y')
    a1 = 'Error';
    a0 = 'Error';

else
    Sx = sum(x);
    Sy = sum(y);
    Sxy = sum(x.*y);
    Sxx = sum(x.^2);
    a1 = (nx*Sxy - Sx*Sy)/(nx*Sxx - Sx^2);
    a0 = (Sxx*Sy - Sxy*Sx)/(nx*Sxx - Sx^2);
end

fprintf('a1 = %.6f\n', a1);
fprintf('a0 = %.6f\n', a0);
```

Saída

Janela de Comandos

```
a1 = 9.850955
a0 = 1.917038
>>
```