Chapter 1

1.6 Exercises

1.1

1.3 Products

Semigroups can be combined to create more semi groups

Direct product of S and T

Let S, T be semigroups. Consider the set $S \times T$ with a multiplication defined as follows

$(a, x) * (b, y) = (ab, xy)$

where $a, b \in S$ and $x, y \in T$. The result is a semigroup $(S \times T, *)$.

Lema 1.3.

$(S \times T) \times W$ is isomorphic to $S \times (T \times W)$.

Semidirect Product

Let S, T be a semigroup and $\Theta : T \to \text{End}(S)$ a semigroup homomorphism. Consider the set $S \times T$ with a multiplication defined as follows

$(s, t) \circledast (s', t') = (s\theta(t)(s'), tt')$

where $s, s' \in S, t, t' \in T$, $\Theta(t) : S \to S$ and so $\Theta(t)(s) \in S$

Denoted by $S \times_\Theta T$. We have that $(S \times T, \times_\Theta)$ is a semigroup.

TBD

Let S^T denote the set of al functions from the monoid T to the semigroup S.

We define a multiplication $\circ$ on $S^T \times T$ by $(f, t) \circ (g', t') = (f \circ g', tt')$