Alp Eren İnceoğlu– 2019400063
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THEORETICAL ANALYSIS

Basic operation is the comparison marked as (1)

Analyse B(n)

For each iteration of outside for loop it executes exactly 1 time and outside loop iterates n time no matter the input so B(n)=(n-1-0+1)*1=n∈ Θ(n)
Θ

Analyse W(n)

For each iteration of outside for loop it executes exactly 1 time and outside loop iterates n time no matter the input so W(n)=(n-1-0+1)*1=n∈ Θ(n)

Analyse A(n)

For each iteration of outside for loop it executes exactly 1 time and outside loop iterates n time no matter the input so A(n)=(n-1-0+1)*1=n∈Θ(n)

Basic operations are the three assignments marked as (2)

Analyse B(n)

When the arr[i] is 0, first for loops for (n-i) times and the inside while loops for ($\log_2$(n+1)+1)=(k+1) times regardless of the i. Therefore, for an array full of 0's, B(n)= $\sum_{i=0}^{n-1}(n-i)*(k+1)$=(k+1)* $\frac{(n+1)*n}{2}$∈ Θ(n^2logn)

Analyse W(n)

When the arr[i] is 2, outside for loops for n times and the inside one loops for n^2 times so for an array full of 2's, W(n)= $\sum_{i=0}^{n-1}(n^2-1-0+1)*n=\frac{n*n*(n+1)*(2n+1)}{6}$∈ Θ($n^4$)

Analyse A(n)

For an array with equal number of 0's, 1's and 2's,
A(n)= $\sum_{i=0}^{n-1}P(arr[i]=0)*(n-i)*(k+1)+P[arr[i]=1]*n*n^2*(k+1)+$
$P[arr[i]=2]*\frac{n*n*(n+1)*(2n+1)}{6}$
$=\sum_{i=0}^{n-1}\frac{1}{3}*(n-i)*(k+1)+\frac{n}{3}*n^2*(k+1)+\frac{n}{3}*\frac{n*(n+1)*(2n+1)}{6}$
$=\frac{n*(n+1)}{6}*(k+1)+\frac{n}{3}*n^2*(k+1)+\frac{n}{3}*\frac{n*(n+1)*(2n+1)}{6}$∈ Θ($n^4$)

Basic operation is two assignments marked as (3)

Analyse B(n)

Since taking the third assignment as the basic operation is very similar to taking the second ones, except for the fact that there are no operations in the first for loop, for an input array which has 0 for each element B(n)=0∈ Θ(1).

Analyse W(n)

When the arr[i] is 2, outside for loops for n times and the inside one loops for n^2 times so for an array full of 2's, W(n)= $\sum_{i=0}^{n-1}(n^2-1-0+1)*n=\frac{n*n*(n+1)*(2n+1)}{6}$∈ Θ($n^4$)

Analyse A(n)

For an array with equal number of 0's, 1's and 2's,
A(n)= $P(arr[i]=0)*n*0+P[arr[i]=1]*n*n^2*(k+1)+P[arr[i]=2]*n*\frac{n*(n+1)*(2n+1)}{6}$

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$=\frac{n}{3} * (0) + \frac{n}{3} * n^2 * (k+1) + \frac{n}{3} * \frac{n*(n+1)*(2n+1)}{6} \in \Theta(n^4)$

Basic operations are the two loop incrementation marked as (4)

Analyse B(n)

When all elements in the array are equal to 0, for statement never executes so B(n)=0∈ Θ(1)

Analyse W(n)

When all elements in array are equal to 1, W(n)=n*n*n= $n^3 \in \Theta(n^3)$

Analyse A(n)

A(n) = E[X] * n^2+E[Y]*n= $\frac{n}{3}$*n^2+ $\frac{n}{3}$*n= (n^3+ n^2)/3

Basic operation is the assignment marked as (5)

Analyse B(n)

When no element in the array equals to 0, it executes 0 times. B(n)=0∈ Θ(1)

Analyse W(n)

When all elements are equal to 0, outside loop executes n times where the inside executes n-1 times.$\sum_{i=0}^{n-1} n - i \quad = \frac{(n+1)*n}{2} \in \Theta(n^2)$

Analyse A(n)

When one-third of the elements in the array are 0, A(n)= . $\sum_{i=0}^{n-1} P(arr[i] = 0) * (n-i)$= $\frac{1}{3} * \frac{(n+1)*n}{2} = \frac{(n+1)*n}{6} \in \Theta(n^2)$

IDENTIFICATION OF BASIC OPERATION(S)

We have chosen operation 2 as the basic operation, since that operation is the one that is consistently inside the inmost loops and contributes most to the complexity due to it being the operations with most executions in an average run of the code and also most similar to the expected run length's of the code.

REAL EXECUTION

Best Case

N Size	Time Elapsed
1	4.76837158203125e-06
5	1.0728836059570312e-05
10	6.127357482910156e-05
25	0.00031304359436035156
50	0.0015811920166015625
75	0.003261566162109375
100	0.005693674087524414
150	0.014956235885620117
200	0.026244401931762695
250	0.04085350036621094

Worst Case

N Size	Time Elapsed
1	2.6226043701171875e-06
5	2.9325485229492188e-05
10	0.0004661083221435547

25	0.011408329010009766
50	0.17282986640930176
75	0.8671586513519287
100	2.729388475418091
150	13.875182151794434
200	43.729897260665894
250	106.13601636886597

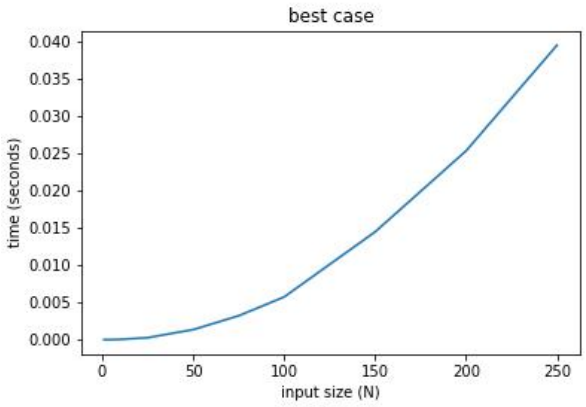
Average Case

N Size	Time Elapsed
1	1.5894571940104167e-06
5	5.014737447102865e-05
10	0.0004661083221435547
25	0.011408329010009766
50	0.10332155227661133
75	0.44780723253885907
100	1.299940824508667
150	6.046857595443726
200	19.307046095530193
250	44.22262501716614

COMPARISON

Best Case

Graph of the real execution time of the algorithm



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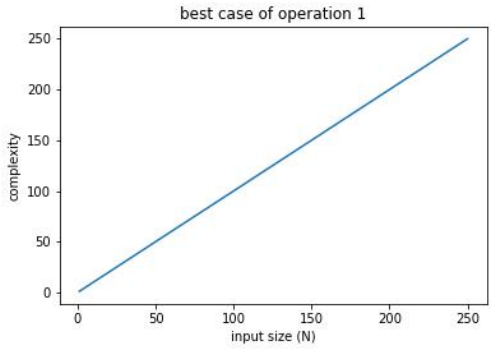
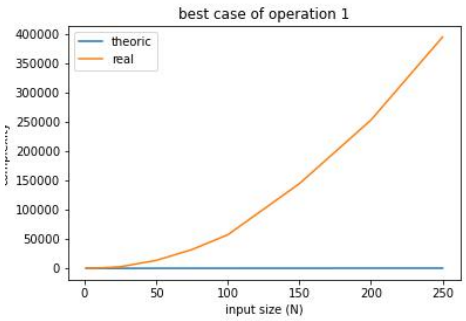
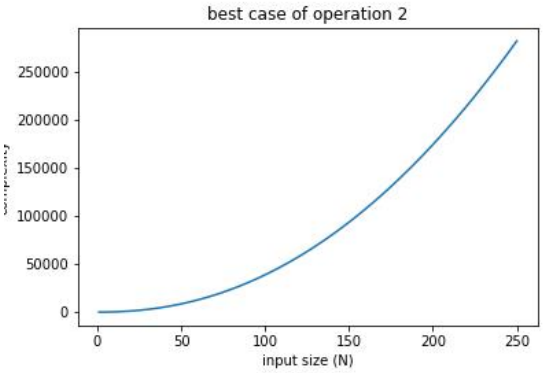
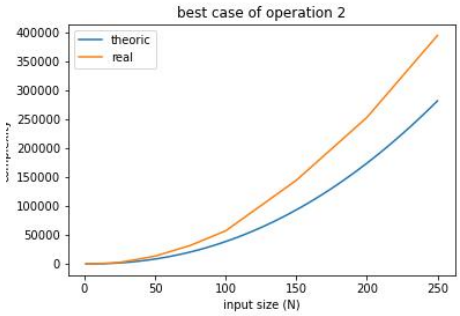
Graph of the theoretical analysis when basic operation is the operation marked as (1)





Graph of the theoretical analysis when basic operation is the operation marked as (2)





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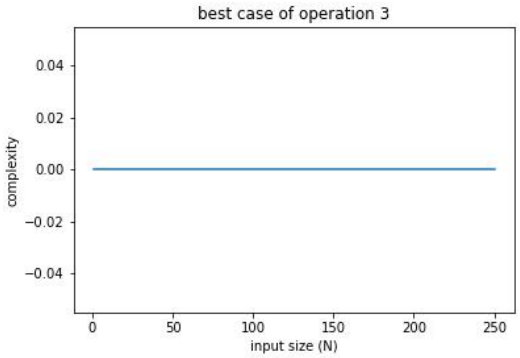
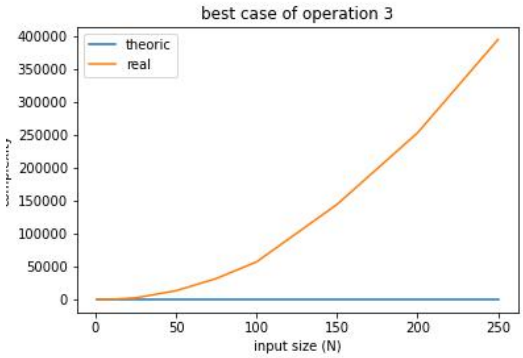
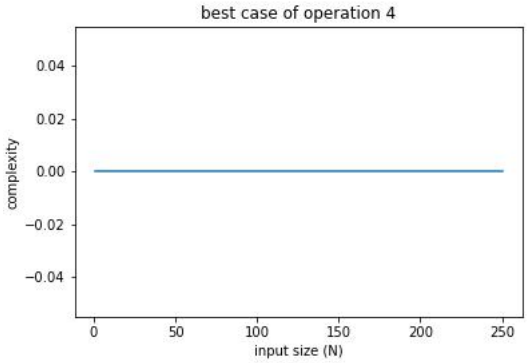
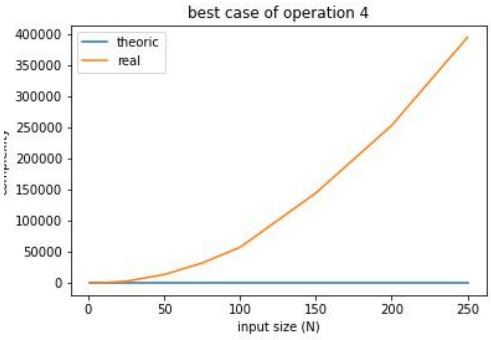
Graph of the theoretical analysis when basic operation is the operation marked as (3)



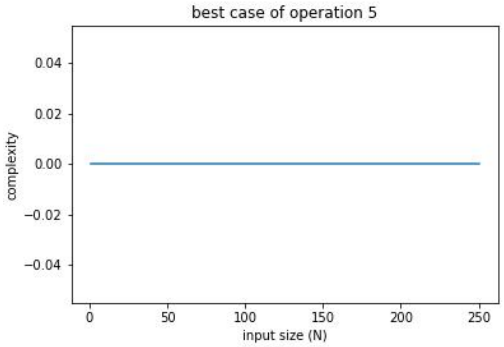


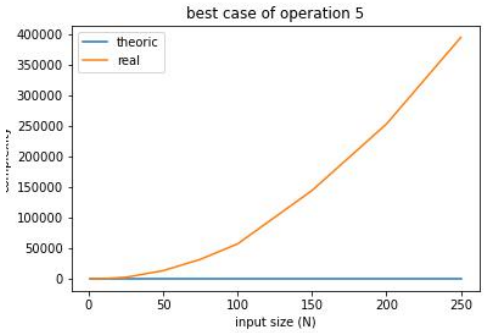
Graph of the theoretical analysis when basic operation is the operation marked as (4)





Graph of the theoretical analysis when basic operation is the operation marked as (5)





Comments

After graphing all the graphs, we figured out that putting the actual graphs, with the modification of multiplying the time with 10^7 in order to convert the seconds into complexities, behind the theoretical ones would give us the easiest way of figuring out how incorrect some of the options are. For example, 3, 4 and 5 ones result in 0, so those are obviously not the case. While the 1 one is closer in shape to the actual graph, it's actual values are quite far from the real times which makes it an obvious wrong choice. 2'nd one however is quite close in shape to the real one, and while it's values are remarkably lower than the actual one, this is mostly because of how little time it takes in the best cases so that small differences look really bigger.

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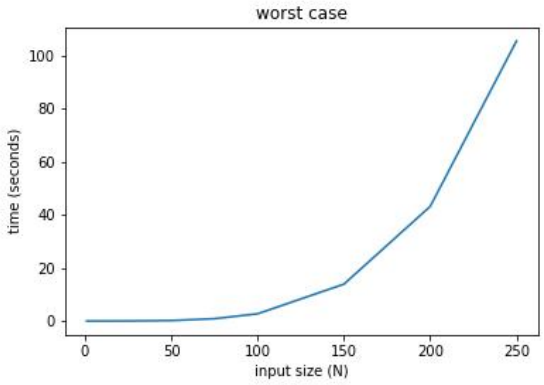
Worst Case

Graph of the real execution time of the algorithm



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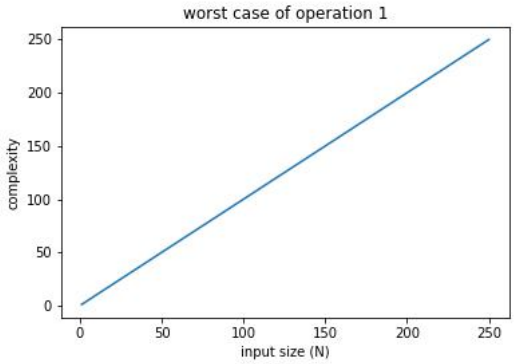
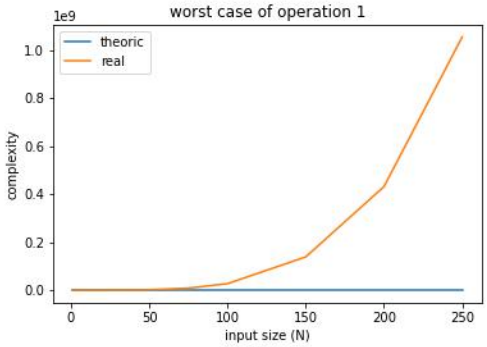
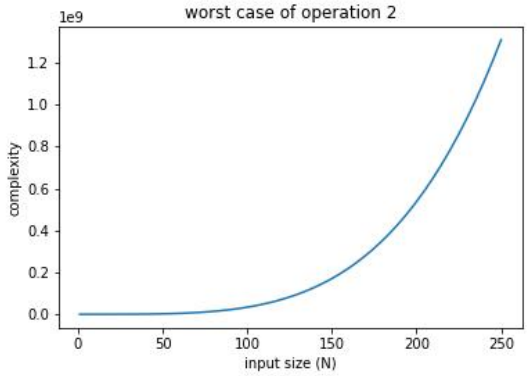
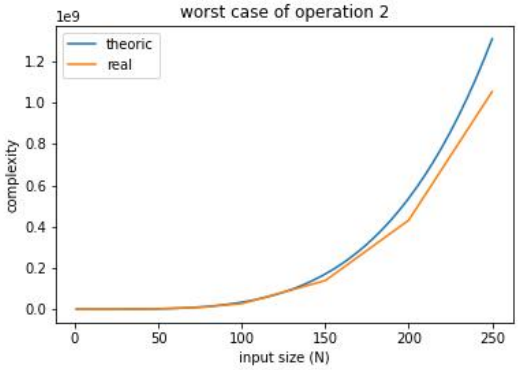
Graph of the theoretical analysis when basic operation is the operation marked as (1)



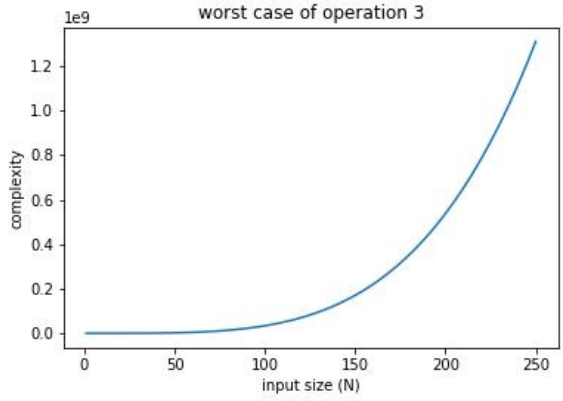


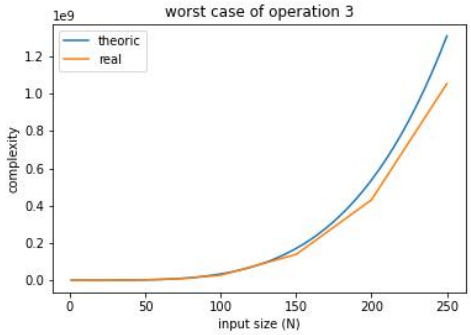
Graph of the theoretical analysis when basic operation is the operation marked as (2)



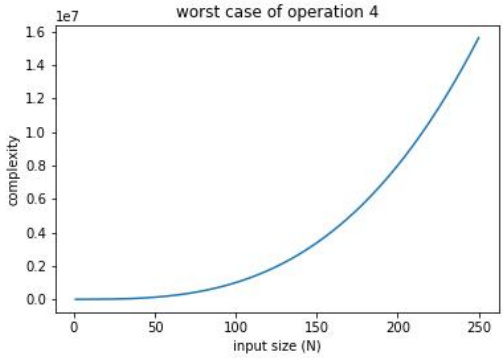


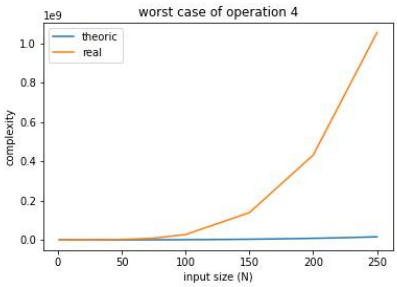
Graph of the theoretical analysis when basic operation is the operation marked as (3)





Graph of the theoretical analysis when basic operation is the operation marked as (4)





Graph of the theoretical analysis when basic operation is the operation marked as (5)

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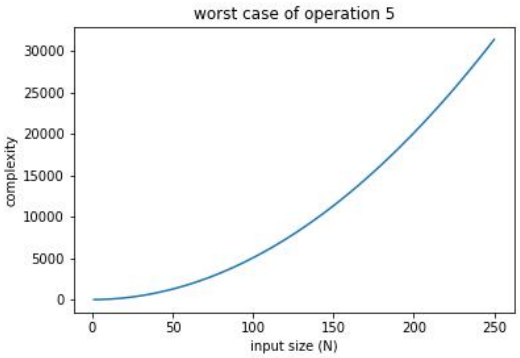

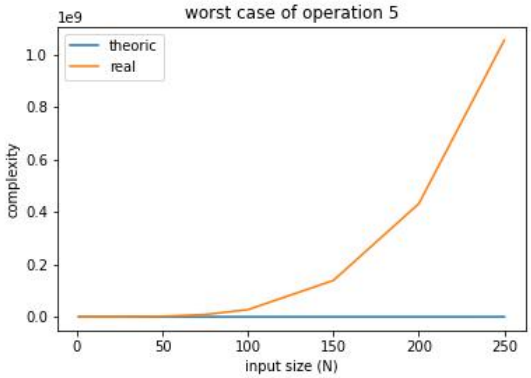

Comments

1’st one is not in a similar shape to the actual results due to it’s linear nature, so it’s clearly out of question. Other than that, while the rest are similar enough in shape, 4 and 5’th ones numbers are so low that they appear 0 when put with the real one, however the 2’nd and 3rd ones are so similar to the actual case that it actually feels like they came out of the same algorithm. While this might feel like it would be difficult to chose the true case, due to the prior knowledge of best cases and how much the 3rd ones differs from the actual outputs 2’nd one remains the more true option.

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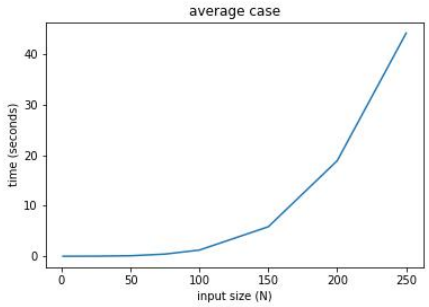
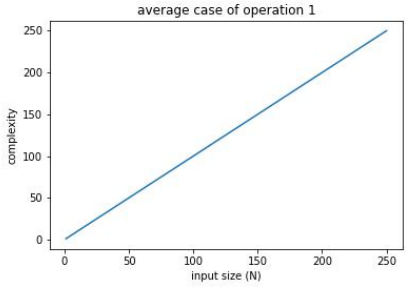
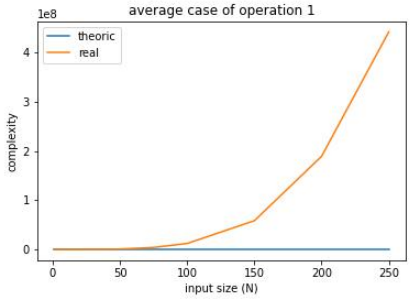
Average Case

Graph of the real execution time of the algorithm

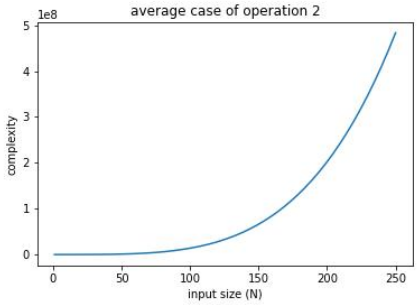


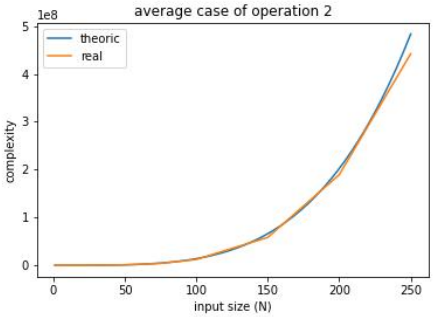
Graph of the theoretical analysis when basic operation is the operation marked as (1)



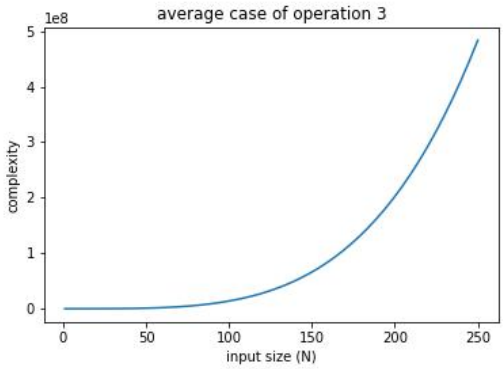


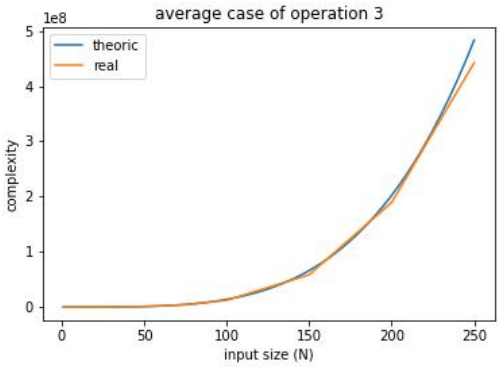
Graph of the theoretical analysis when basic operation is the operation marked as (2)





Graph of the theoretical analysis when basic operation is the operation marked as (3)





Graph of the theoretical analysis when basic operation is the operation marked as (4)

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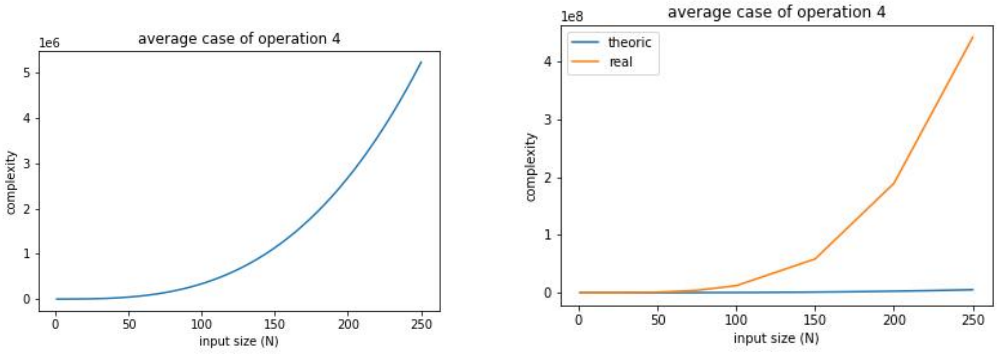
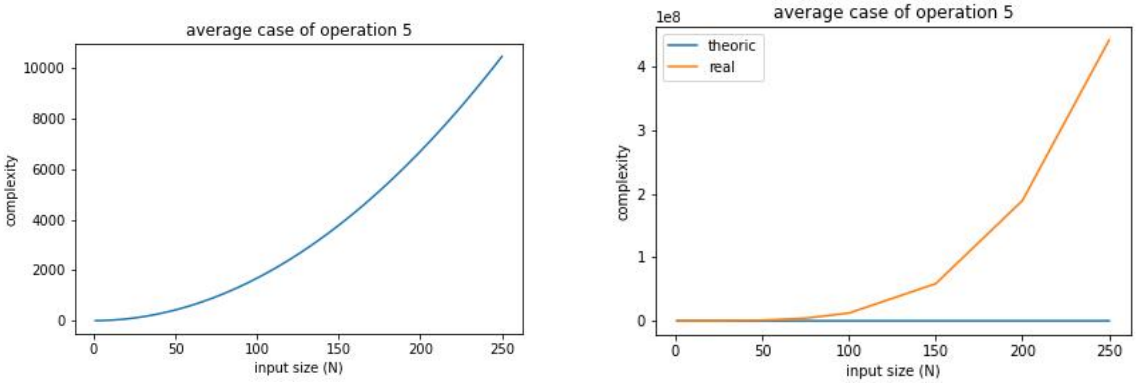


Graph of the theoretical analysis when basic operation is the operation marked as (5)



Comments

As with the best cases, 1st one is very different in shape and 4th and 5th ones are seriously low in values, while the 2nd and 3rd are almost the same with the actual results. Just like worst cases, those are hard to distinguish due to the fact that 2nd and 3rd operations were chosen in a quite similar manner where the only difference was that the 3rd operation excluded a relatively low complex part of the actual operation, which made zero difference in the worst cases and indistinguishably low difference in the average cases however created a huge difference in the best cases, which makes the 2nd operation clearly the correct one.