Theory:

1)

Assignment - 2

1) (a) $A = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$, $A_H = \begin{bmatrix} 2 \\ 5 \\ 1 \\ 1 \end{bmatrix}$ (homogeneous co-ordinate)

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T(t_x, t_y, t_z) = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let M be the coordinate transformation matrix, then

$$M = T(-1, 3, 2) . R_x(-\pi/2) . R_y(\pi/2)$$

$$= \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} \cos\pi/2 & 0 & \sin\pi/2 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\pi/2 & 0 & \cos\pi/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\pi/2 & -\sin\pi/2 & 0 \\ 0 & \sin\pi/2 & \cos\pi/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M = T(-1,3,2) . R_x(-\pi/2) . R_y(\pi/2)$$

$$= \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos -\pi/2 & -\sin -\pi/2 & 0 \\ 0 & \sin -\pi/2 & \cos -\pi/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} .$$

$$\begin{bmatrix} \cos \pi/2 & 0 & \sin \pi/2 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \pi/2 & 0 & \cos \pi/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Let A_{com_H} be the new homogeneous coordinates of A_H,

then, $A_{com_H} = M \cdot A_H$

$$= \begin{bmatrix} 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 \\ -2+3 \\ -5+2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ -3 \\ 1 \end{bmatrix}$$

To get the ~~a~~ non homogeneous coords from A_{com_H},

$$A_{com} = \begin{bmatrix} 0/1 \\ 1/1 \\ -3/1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$$

(c) $O_{world} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$O_{camH} = M_1 O_{world}$

$$= \begin{bmatrix} 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

$$O_{cam} = \begin{bmatrix} -1/1 \\ 3/1 \\ 2/1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$$

(ie)

(ie) To get the axis of rotation,

$R = R_x(-\pi/2)\, R_x(\pi/2)$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (Shown Earlier)

Let's find its eigenvectors

Let $\vec{v}$ be $(v_1, v_2, v_3, v_4)^T$ s.t.

$$R\vec{v} = \lambda v,$$

then $|R - \lambda I| = 0$

$$\begin{vmatrix} -\lambda & 0 & 1 & 0 \\ -1 & -\lambda & 0 & 0 \\ 0 & -1 & -\lambda & 0 \\ 0 & 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$$\cancel{-\lambda\left(\lambda^2(1-\lambda)\right) + 1\left(1(1-\lambda)\right) = 0}$$

$$\cancel{(1+\lambda^3)(1-\lambda) = 0}$$

$$\lambda = 1$$

$$\therefore \cancel{\begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = 1 \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}}$$

$$(-\lambda)\big((-\lambda)(-\lambda)(1-\lambda)\big) + 1\big(1(1-\lambda)\big) = 0$$

$$\lambda^4 - \lambda^3 - \lambda + 1 = 0$$

$$\left(\lambda + \frac{i\sqrt{3}+1}{2}\right)\cdot\left(\lambda - \frac{i\sqrt{3}-1}{2}\right)\cdot(\lambda-1)^2 = 0$$

We need to look at real eigen values,

for $\lambda = 1$,

we have, $\begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ as the possible sol^{n}s,

(d)

$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ is trivial since we are using homogeneous coordinates,

$\therefore$ Axis of rotation, $N_H = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$

If we move back to cartesian coords

$$\boxed{\vec{N} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}}, \quad \hat{n} = \begin{bmatrix} -1/\sqrt{3} \\ 1/\sqrt{3} \\ -1/\sqrt{3} \end{bmatrix}$$

$$\hat{N} = \begin{bmatrix} -1/\sqrt{3} \\ 1/\sqrt{3} \\ -1/\sqrt{3} \end{bmatrix}$$

$\cancel{\theta = \cos^{-1}(1)}$

$\boxed{\cancel{\theta = \frac{\pi}{4}}}$ $\qquad \theta = \cos^{-1}\left(\frac{\text{trace}(R) - 1}{2}\right) = \cancel{\cos^{-1}(0.5)}$

$= \cos^{-1}(-0.5) = (2\pi/3) \qquad \cancel{= \pi/3}$

(d) Since, we are dealing with rotation, we can talk in terms of cartesian coordinates.

$\therefore \quad \cancel{\vec{x} = \hat{n}(\hat{n} \cdot \vec{x}) \quad N\vec{x} = \hat{n} \times \vec{x}}$

$$R = I + (\sin\theta) N + (1 - \cos\theta) N^2$$

$$N = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$N^2 = \begin{bmatrix} -2 & -1 & 1 \\ -1 & -2 & -1 \\ 1 & -1 & -2 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \left(\sin\frac{\pi}{3}\right) \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$+ \left(1 - \cos\frac{\pi}{3}\right) \begin{bmatrix} -2 & -1 & 1 \\ -1 & -2 & -1 \\ 1 & -1 & -2 \end{bmatrix}$$

(d) $$R = I + (\sin\theta)\, N + (1-\cos\theta)\, N^2$$

$$N = \begin{bmatrix} 0 & 1/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{3} & 0 & 1/\sqrt{3} \\ -1/\sqrt{3} & -1/\sqrt{3} & 0 \end{bmatrix}$$

$$N^2 = \begin{bmatrix} -2/3 & -1/3 & 1/3 \\ -1/3 & -2/3 & -1/3 \\ 1/3 & -1/3 & -2/3 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cancel{+ \frac{\sqrt{3}}{2} \begin{bmatrix} -2/3 & -1/3 & 1/3 \\ -1/3 & -2/3 & -1/3 \\ 1/3 & -1/3 & -2/3 \end{bmatrix}}$$

$$\cancel{+ \frac{3}{2} \begin{bmatrix} -2/3 & -1/3 & 1/3 \\ -1/3 & -2/3 & -1/3 \\ 1/3 & -1/3 & -2/ \end{bmatrix}}$$

$$+ \frac{\sqrt{3}}{2} \begin{bmatrix} 0 & 1/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{3} & 0 & 1/\sqrt{3} \\ -1/\sqrt{3} & 1/\sqrt{3} & 0 \end{bmatrix}$$

$$+ \frac{3}{2} \begin{bmatrix} -2/3 & -1/3 & 1/3 \\ -1/3 & -2/3 & -1/3 \\ 1/3 & -1/3 & -2/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1/2 & 1/2 \\ -1/2 & 0 & 1/2 \\ -1/2 & -1/2 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} -1 & -1/2 & 1/2 \\ -1/2 & -1 & -1/2 \\ 1/2 & -1/2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

This is same as what we got in the previous step

Hence, Proved.

2)

A2) Let $\vec{x}$ be rotated about $\hat{u}$ (unit vector) for angle θ, then

$$\vec{x} = \underbrace{\hat{u}(\hat{u}\cdot\vec{x})}_{\substack{\text{Parallel}\\ \text{Component}}} - \underbrace{\hat{u}\times(\hat{u}\times\vec{x})}_{\substack{\text{Perpendicular}\\ \text{Component}}}$$

Let, $R\vec{x}$ be the rotated vector, then

$$\cancel{R\vec{x} = \hat{n}.(\hat{n}.\vec{x}) + \sin\theta(\hat{n}}$$

$$R\vec{x} = \hat{u}_*(\hat{u}\cdot\vec{x}) + \sin\theta(\hat{u}\times\vec{x}) - \cos\theta(\hat{u}\times(\hat{u}\times\vec{x}))$$

$$\hat{u}\times\hat{u}\times\vec{x} = (\hat{u}\cdot\vec{x})\,\hat{u} - (\hat{u}\cdot\hat{u})\,\vec{x}$$

$$= (\hat{u}\cdot\vec{x})\,\hat{u} - \vec{x}$$

$$\therefore\ R\vec{x} = \hat{u}(\hat{u}\cdot\vec{x}) + \sin\theta(\hat{u}\times\vec{x})$$

$$-\cos\theta((\hat{u}\cdot\vec{x})\,\hat{u} - \vec{x})$$

$$= \cos\theta\,\vec{x} + \sin\theta(\hat{u}\times\vec{x}) + (1-\cos\theta)(\hat{u}\cdot\vec{x})\hat{u}$$

$$= \cos\theta\,\vec{x} + \sin\theta(\hat{u}\times\vec{x}) + (1-\cos\theta)(\vec{u}^{T}\vec{x})\vec{u}$$

Hence Proved.

A3)

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A3) Let K_1 and K_2 be the ~~~~ intrinsic parameter matrix for C_1 & C_2, then

$$x_1 = K_1[I \mid 0]X \quad -(1)$$

and $$x_2 = K_2[R \mid 0]X \quad -(2)$$

~~We know that~~ $X = X$

$$\therefore \left(K_1[I \mid 0]\right)^{-1} x_1 = \left(K_2[R \mid 0]\right)^{-1} x_2$$

$$x_1 = \left(K_1[I \mid 0]\right)\left([R \mid 0]^{-1} K_2^{-1}\right) x_2$$

$$\therefore \boxed{x_1 = H x_2}$$

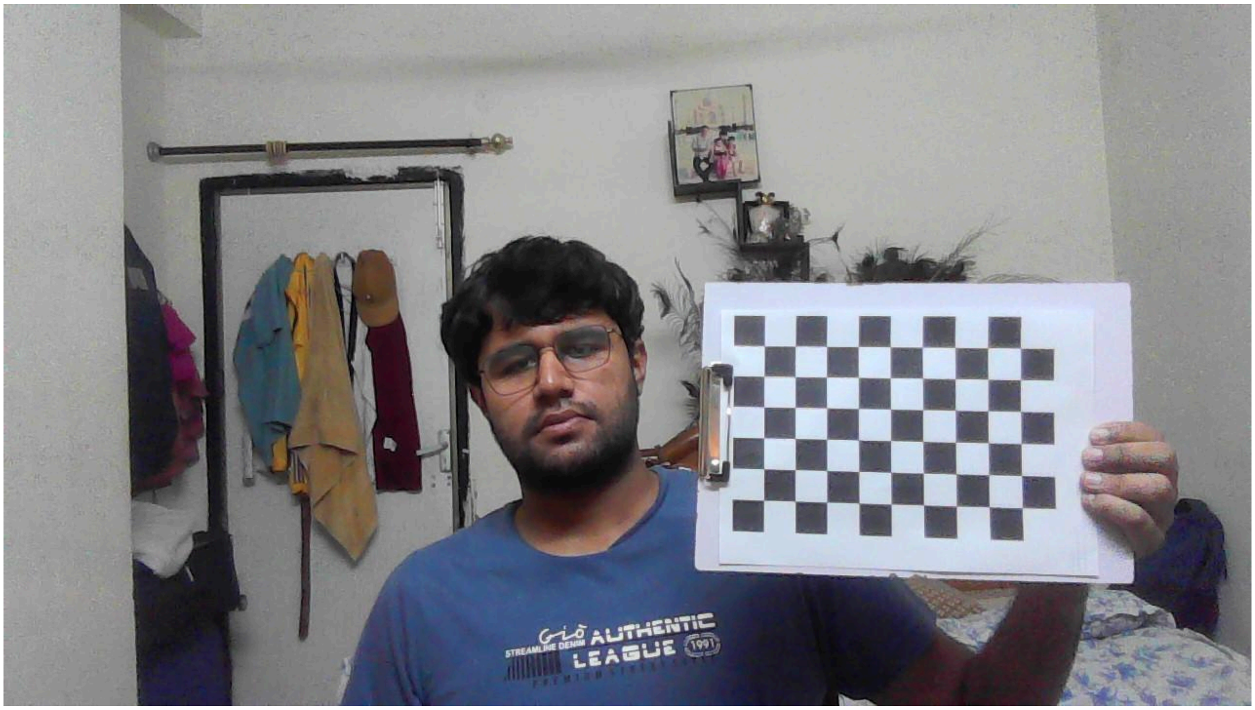
where $H = K_1[I \mid 0][R \mid 0]K_2^{-1}$

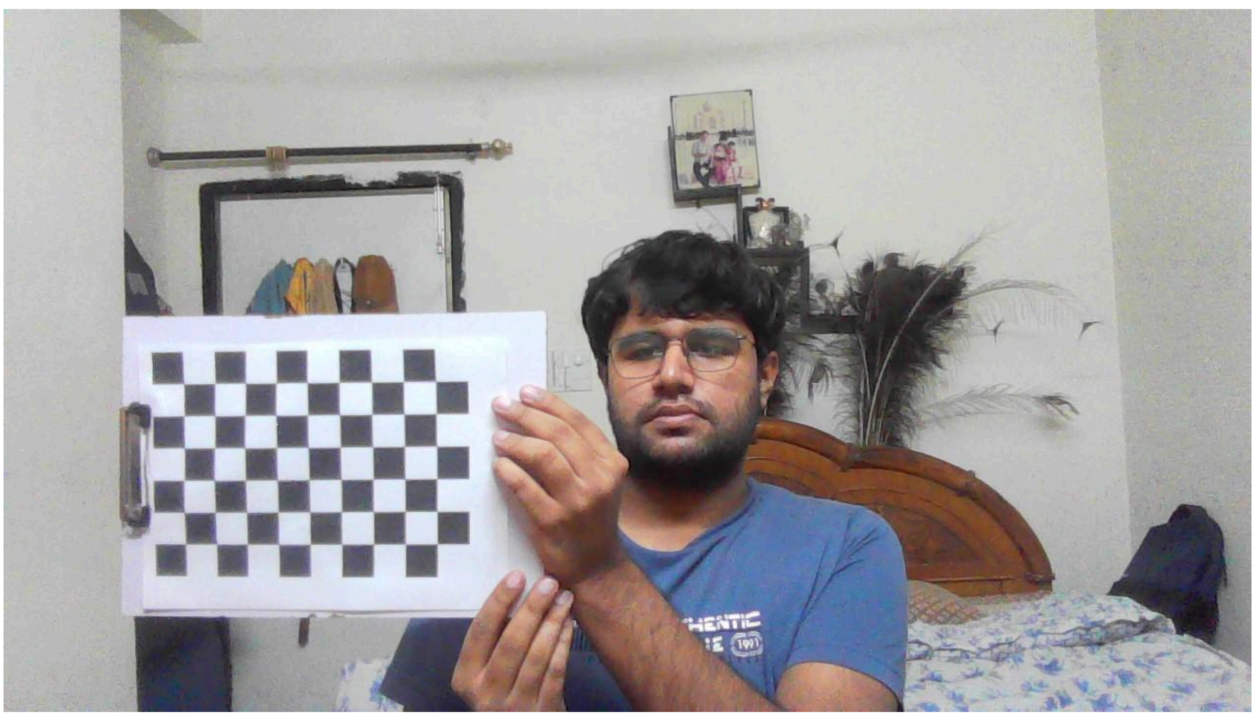
We can clearly observe that H is a 3x3 matrix.

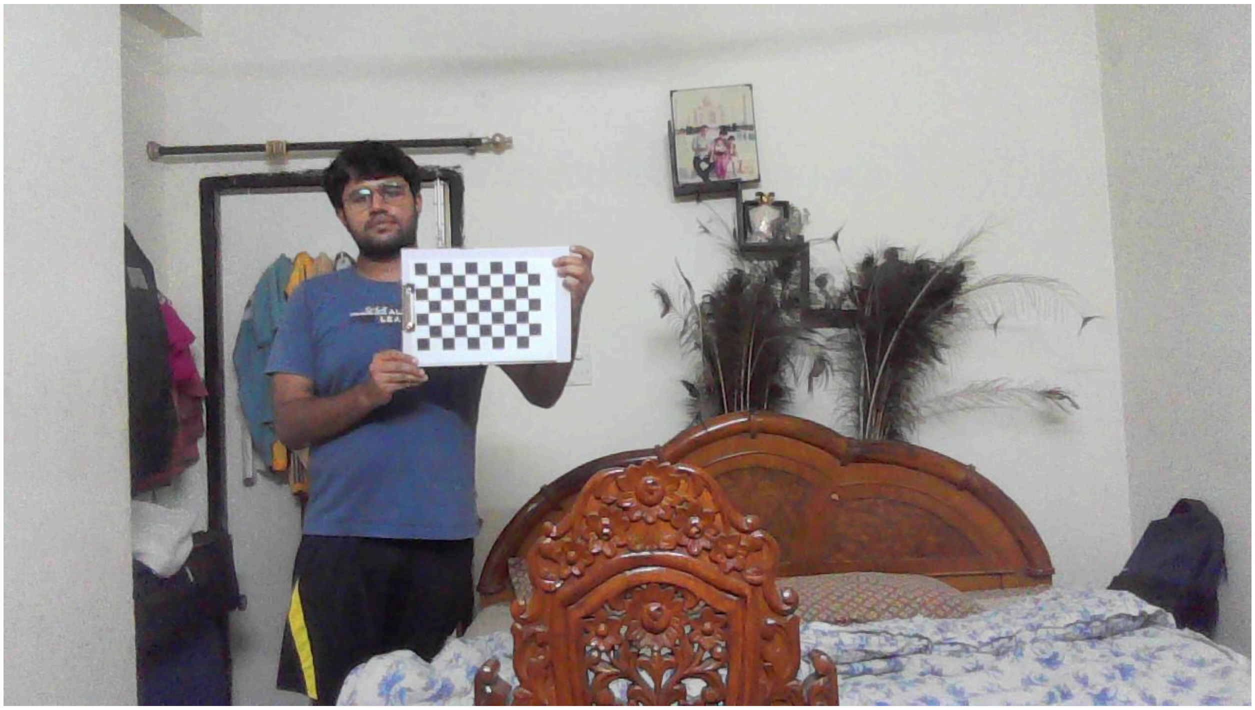
Hence Proved

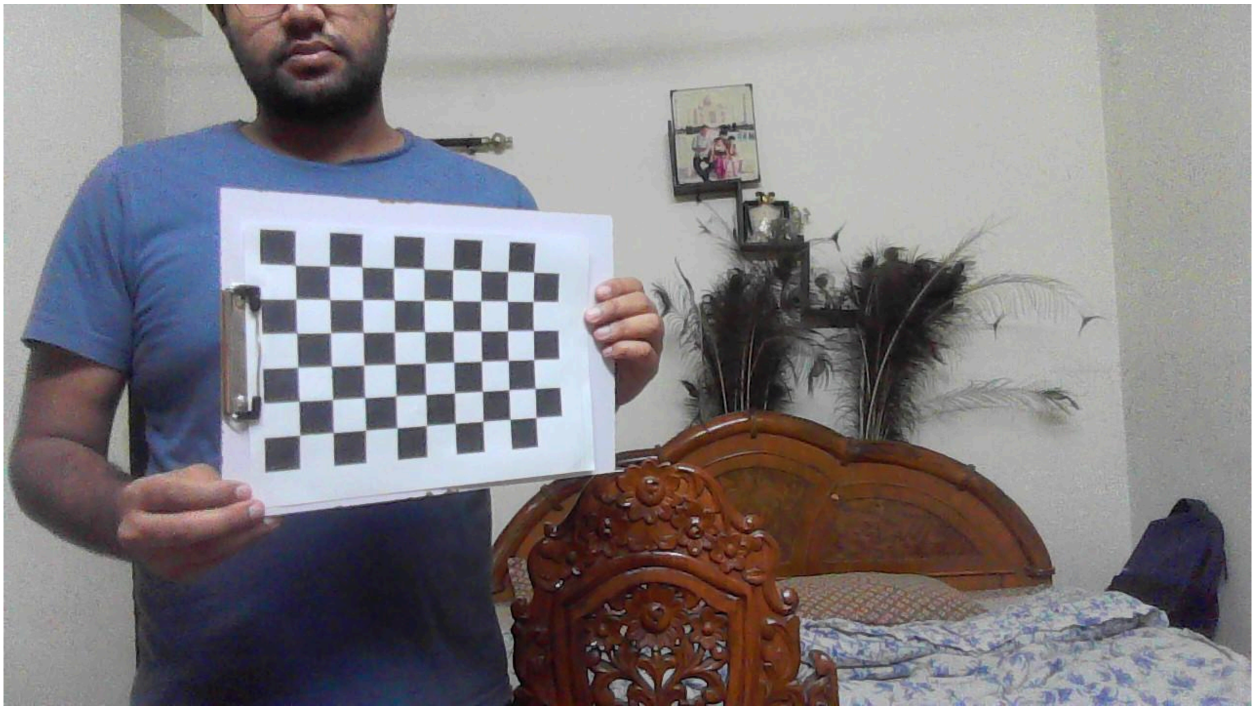
Camera Calibration:

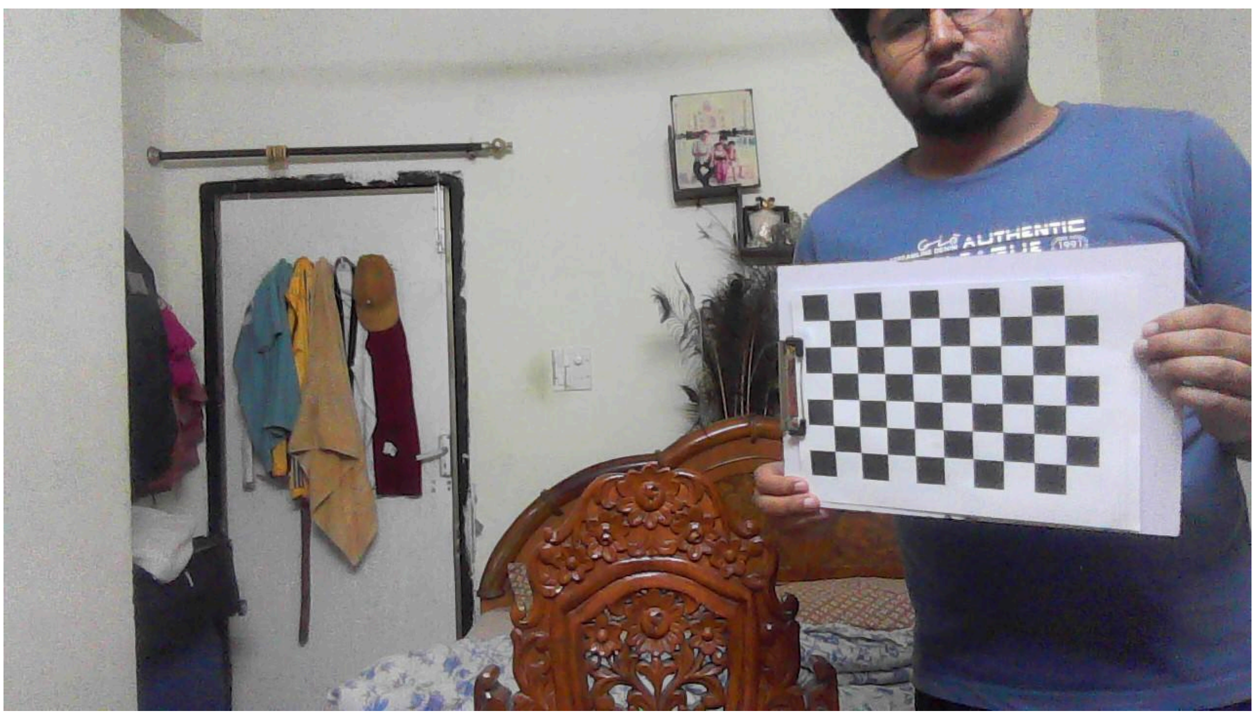
Final set of images used in this question:







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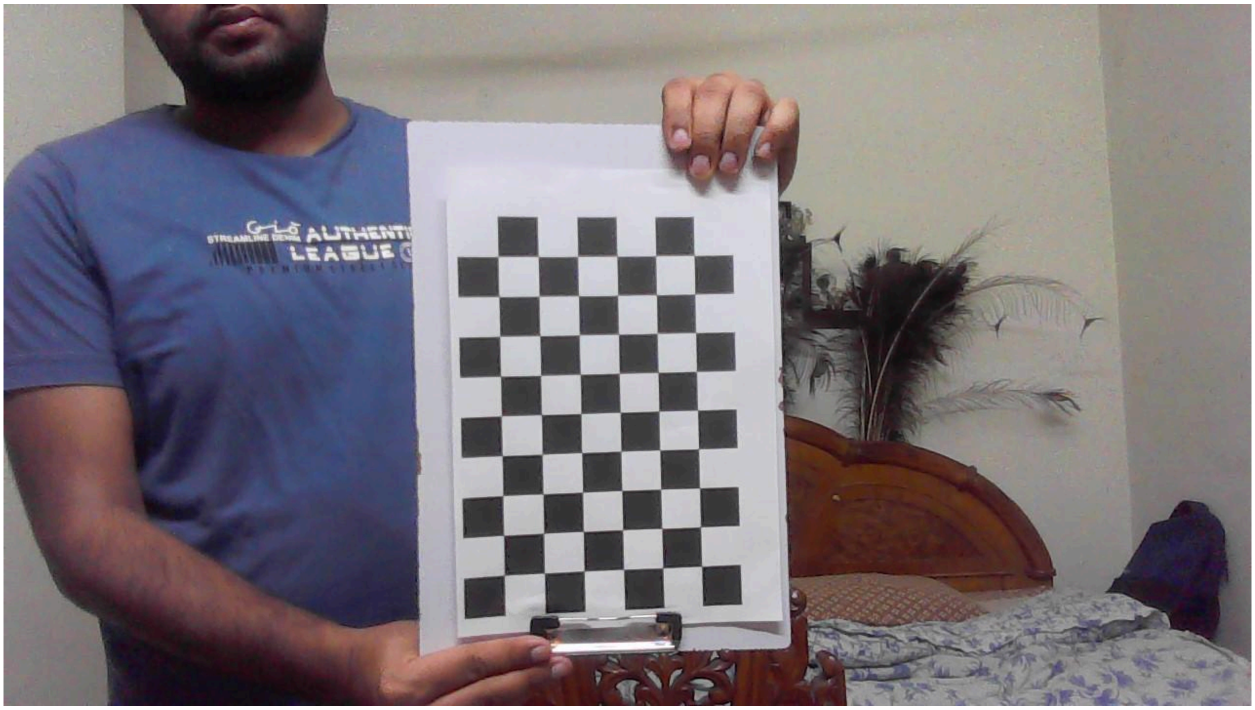

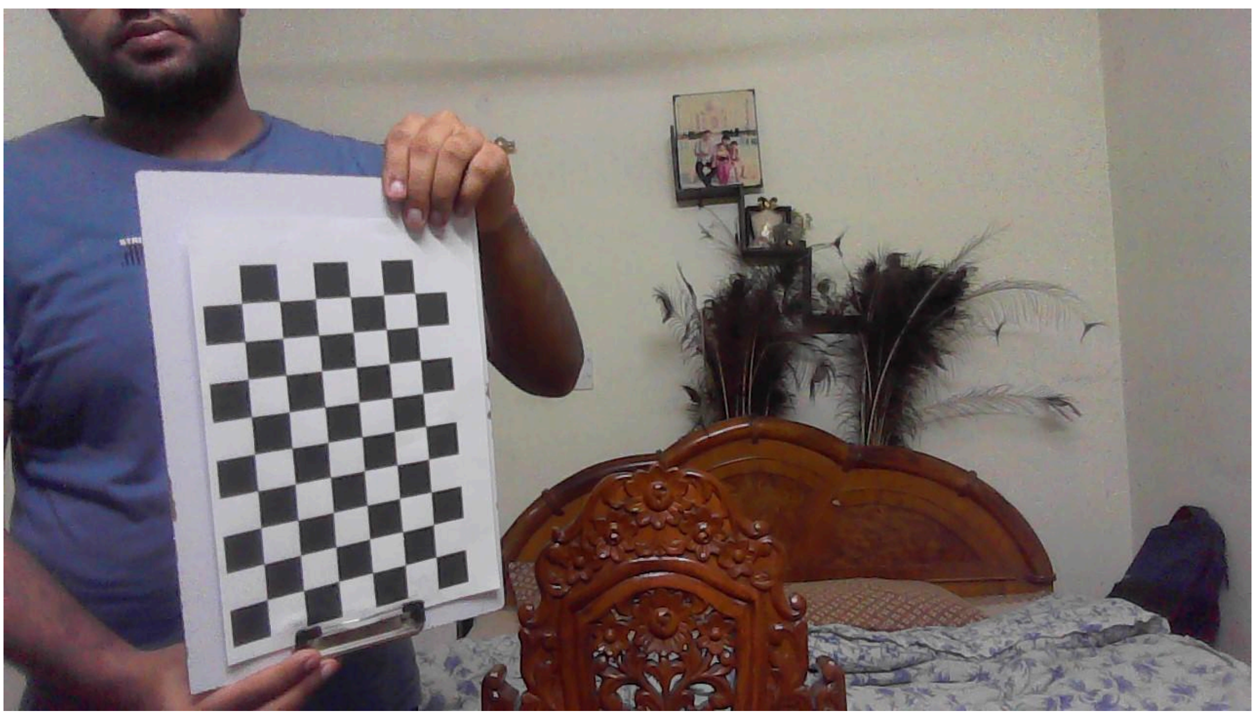

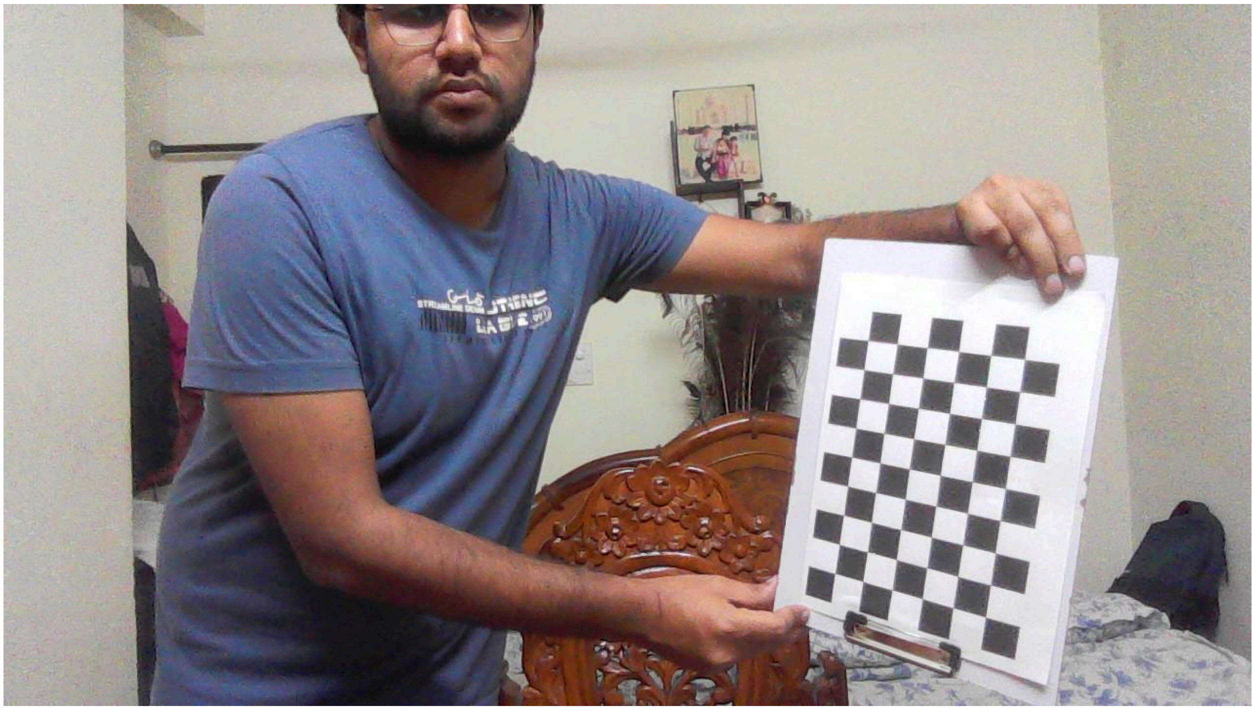

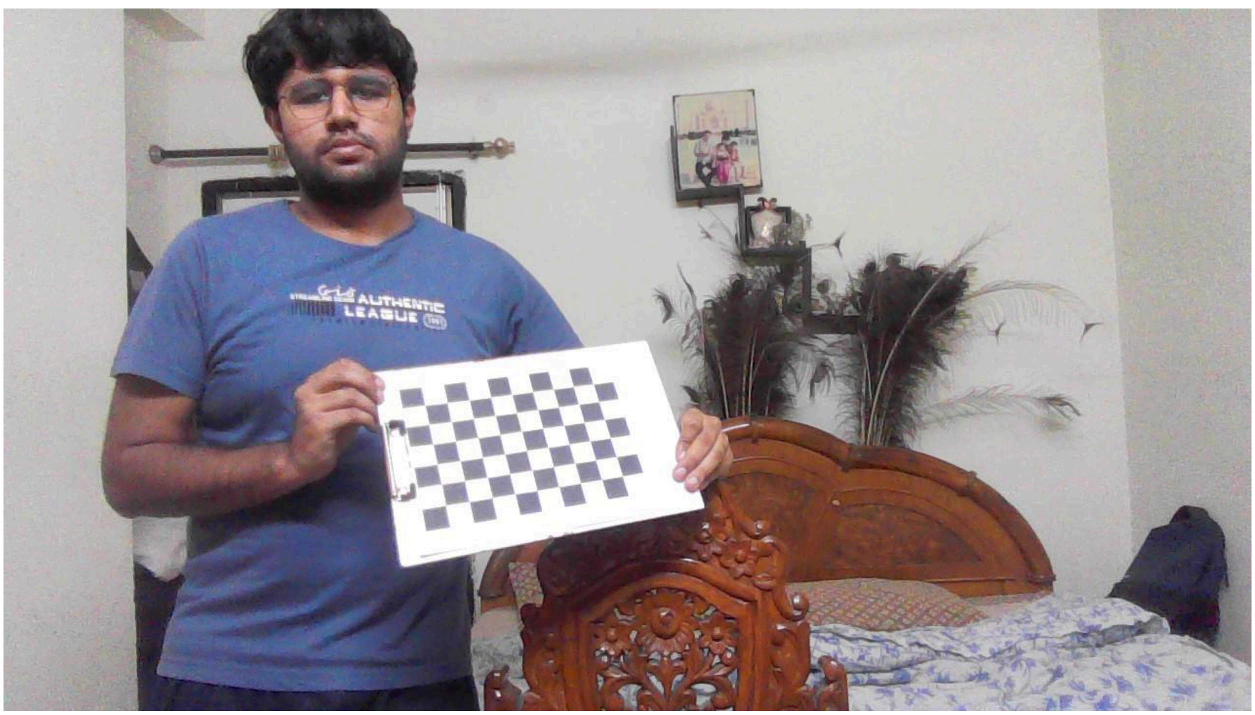
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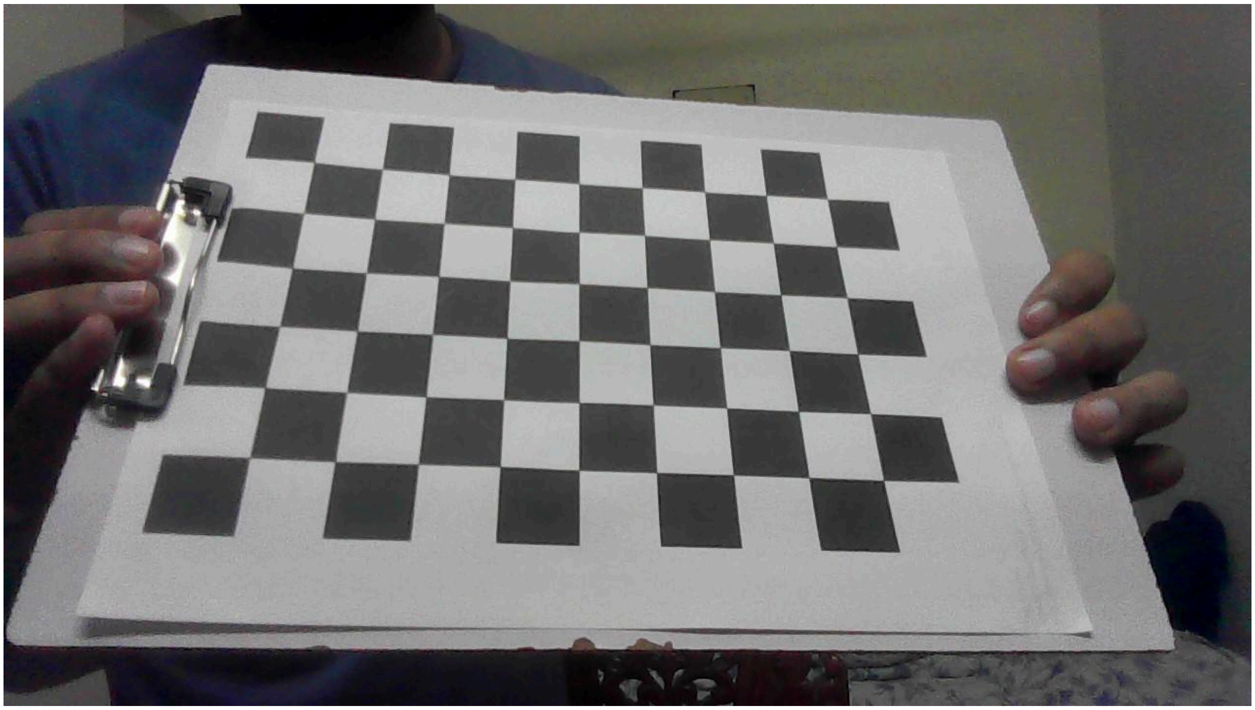

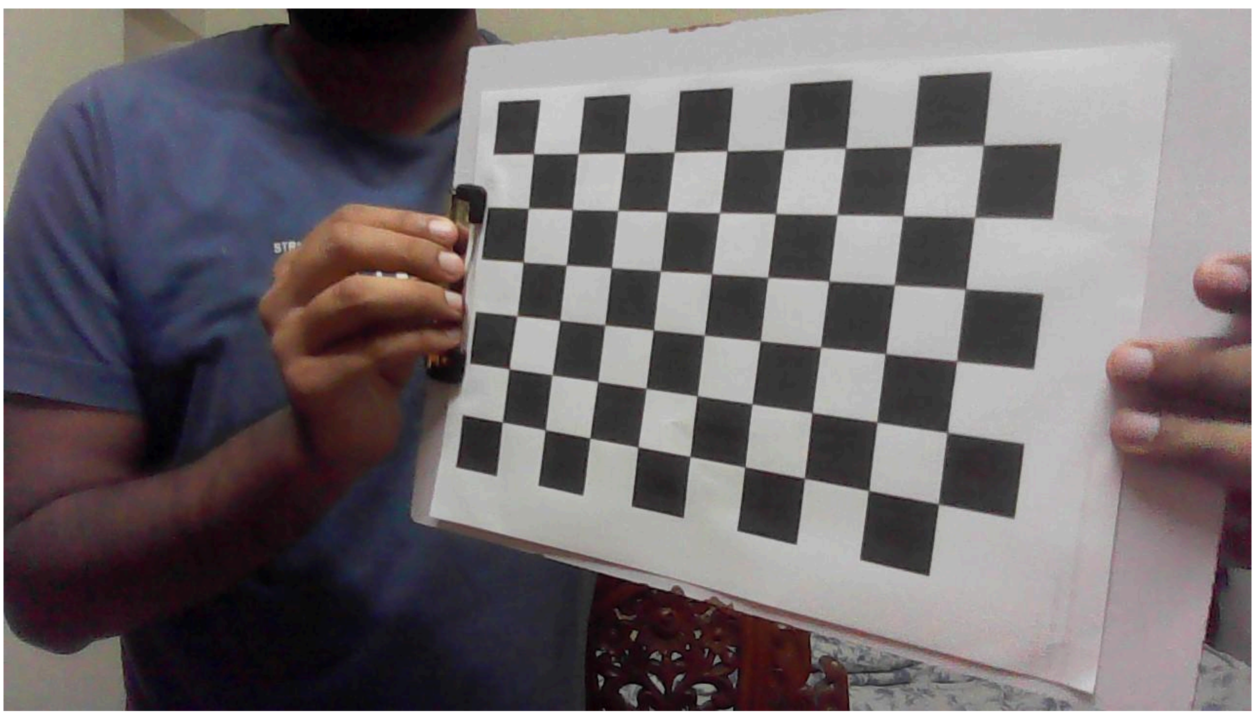

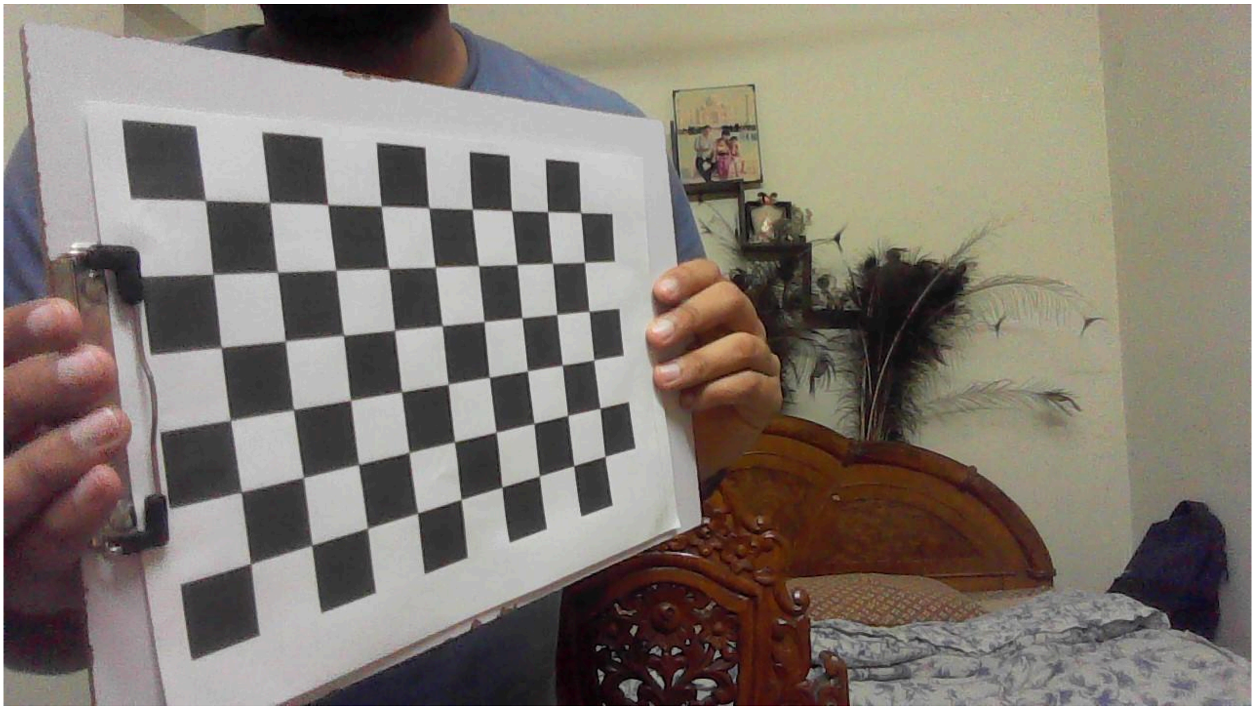

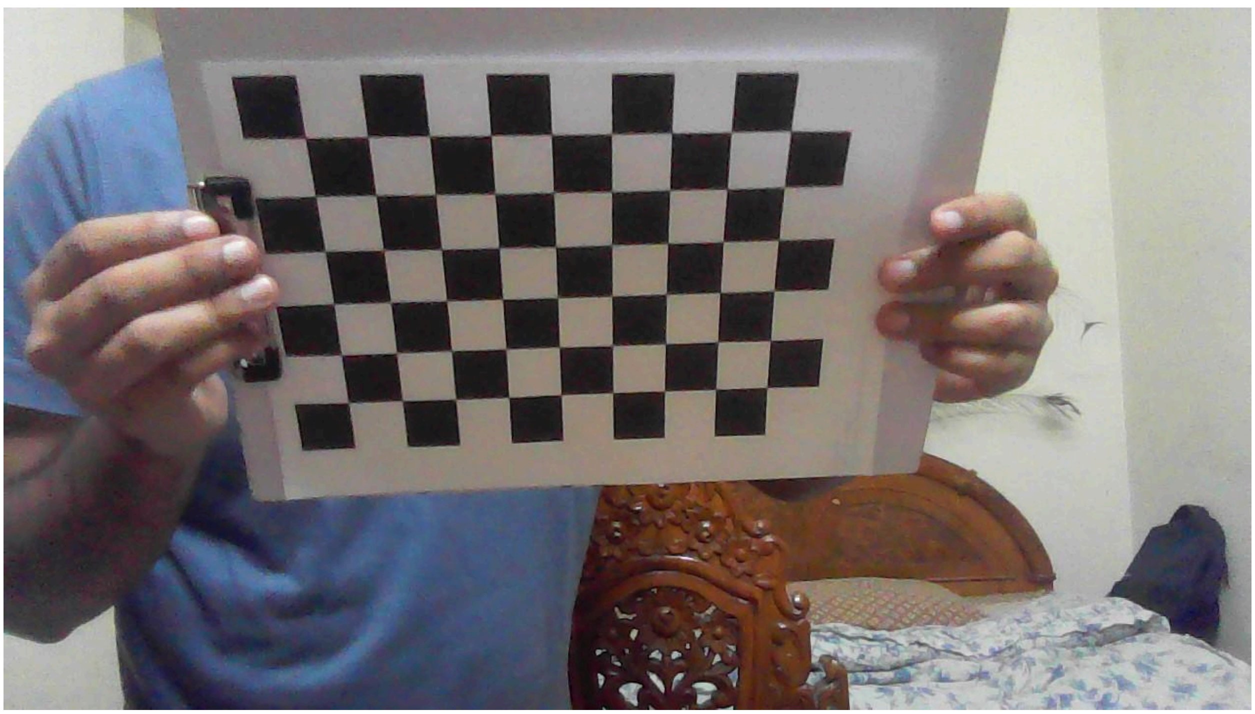

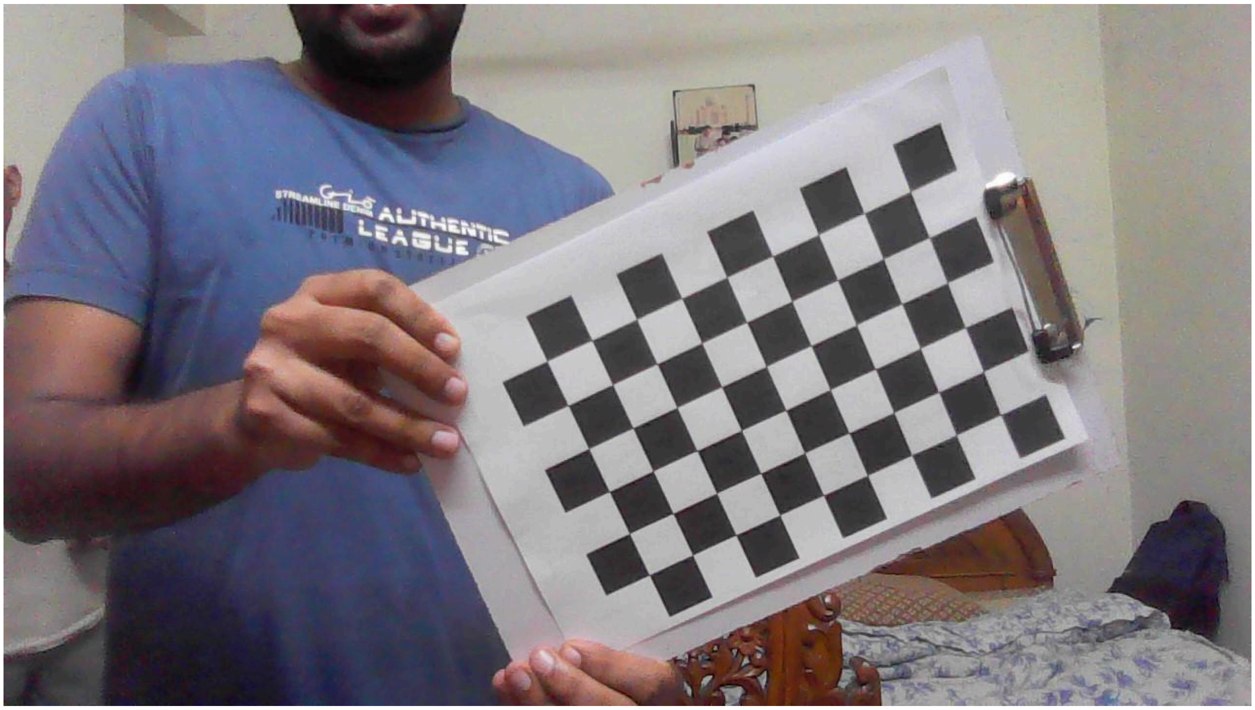
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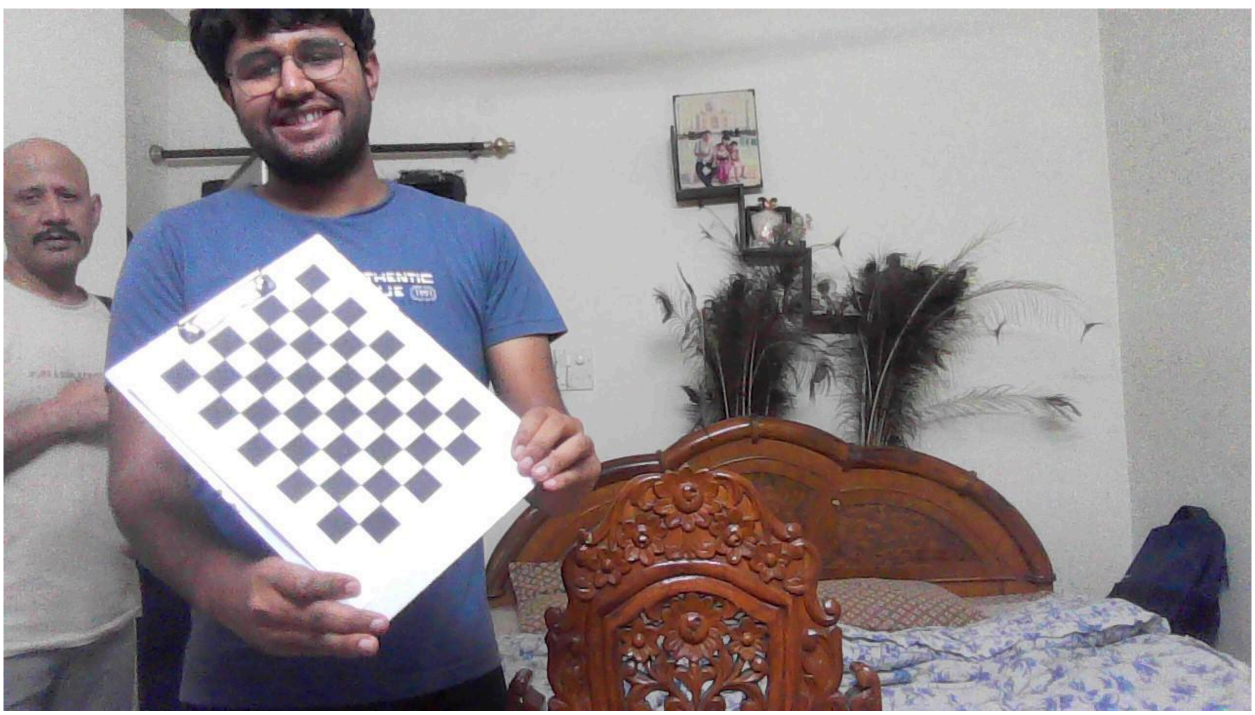

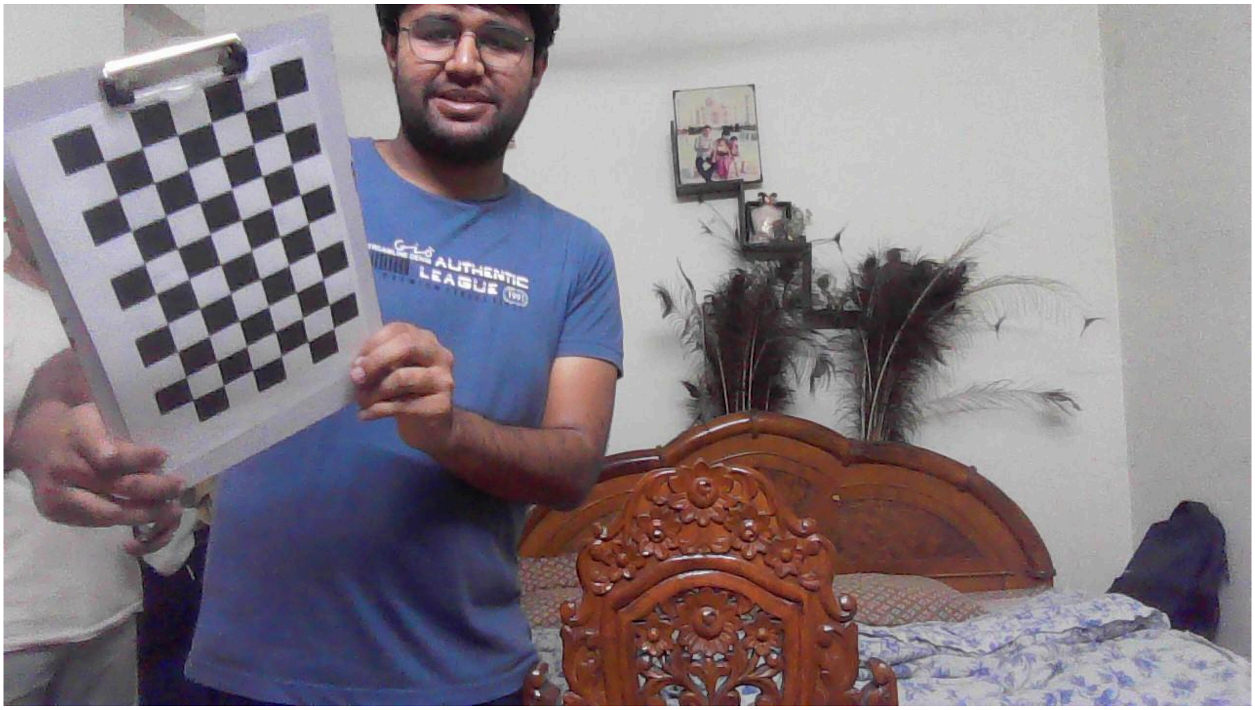
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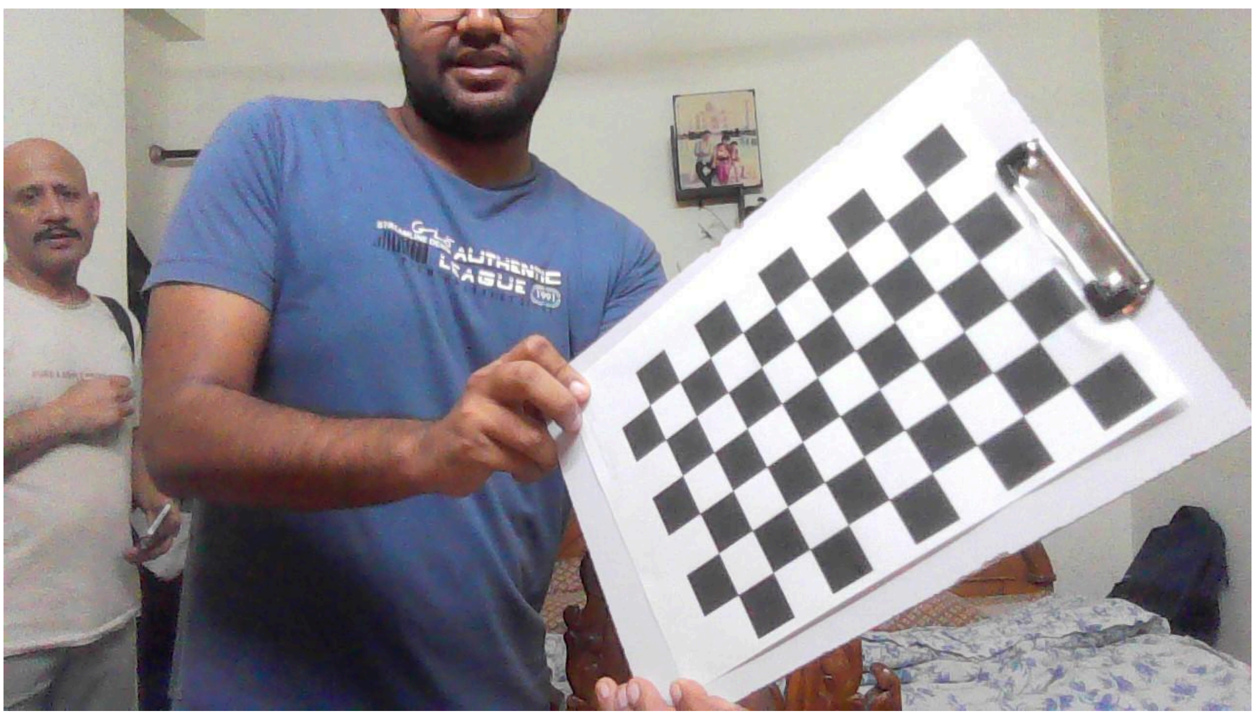
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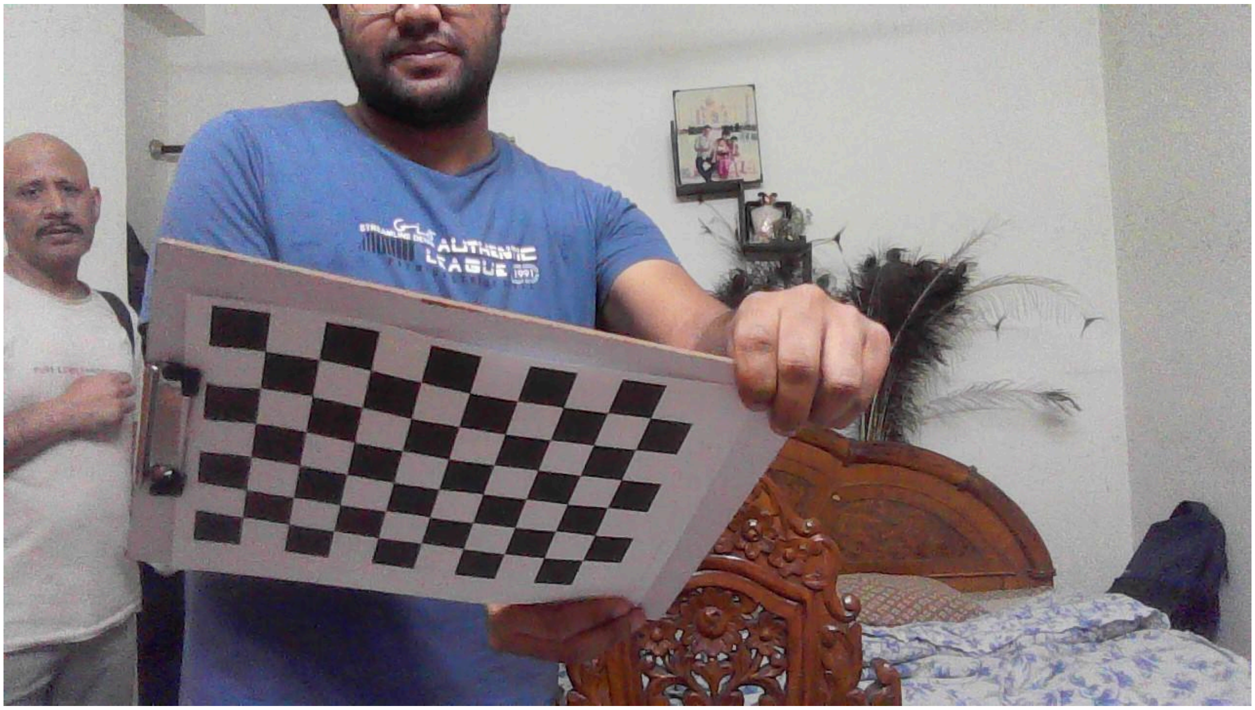
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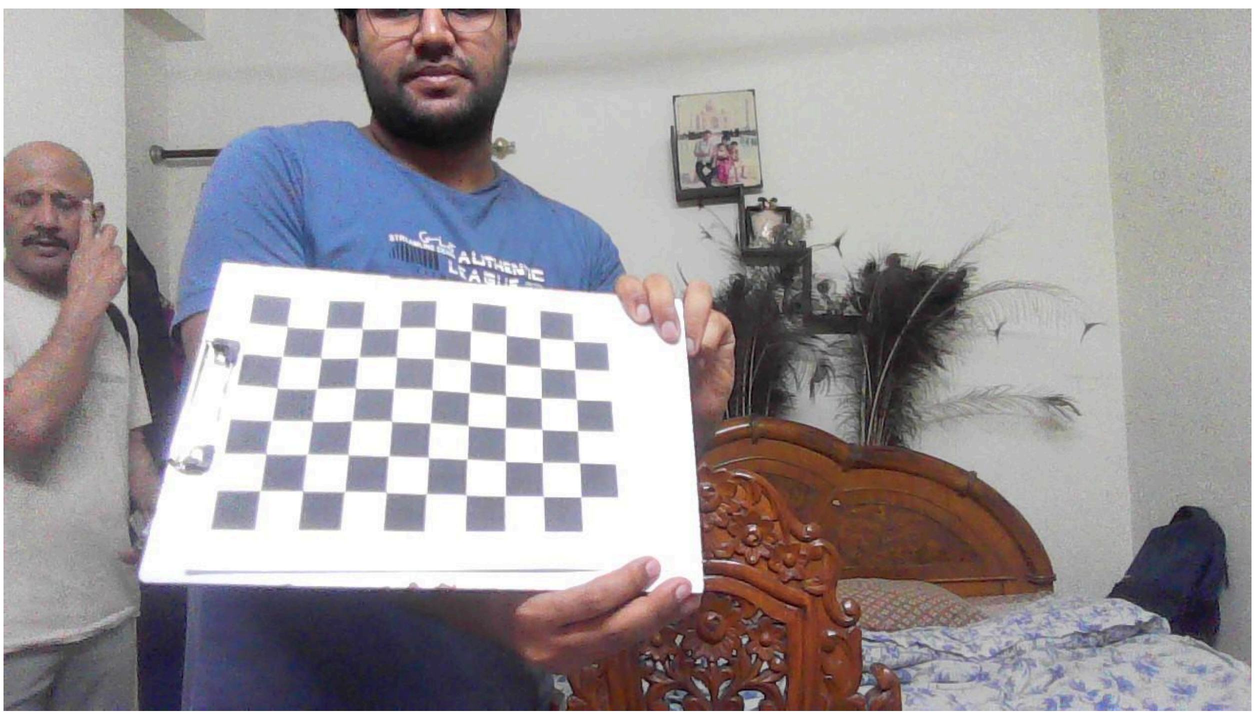
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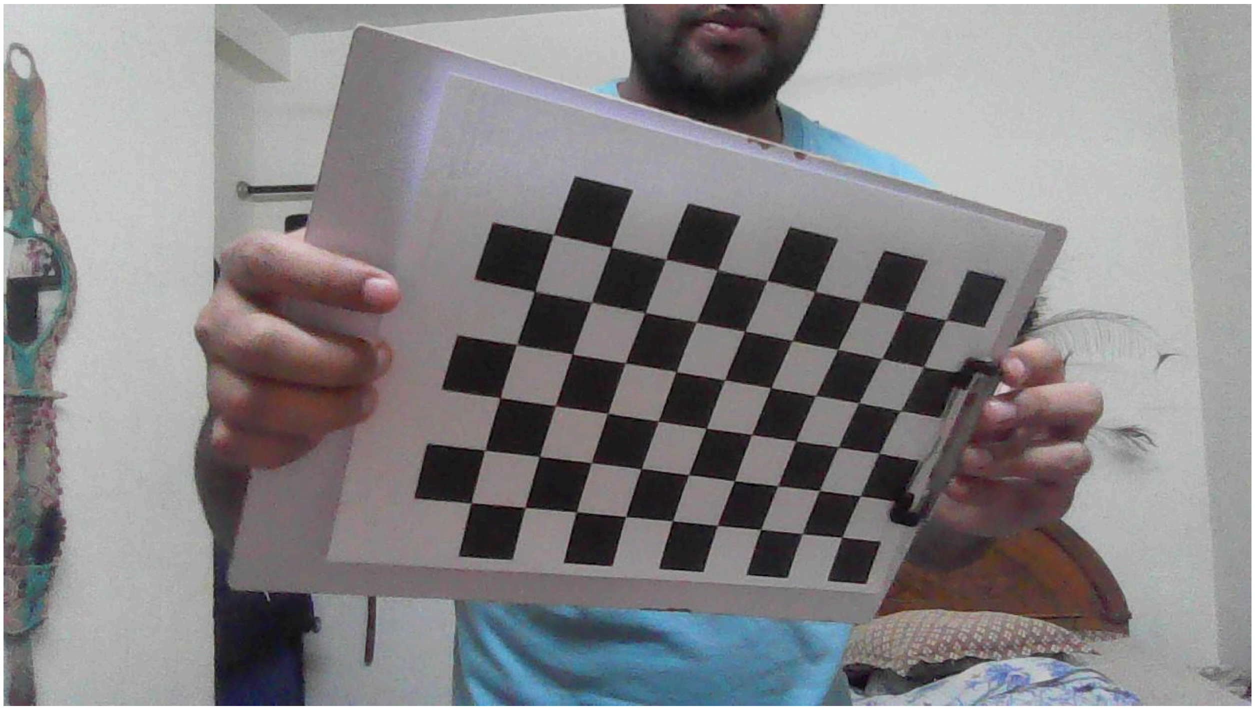
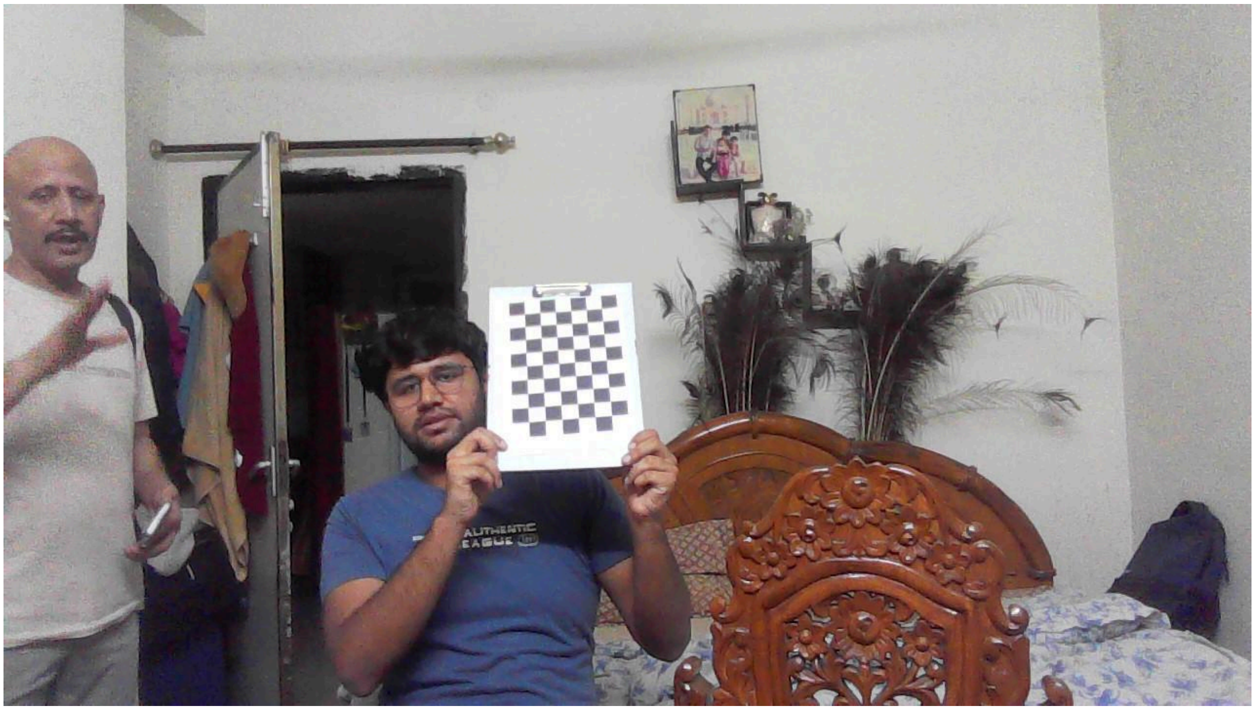
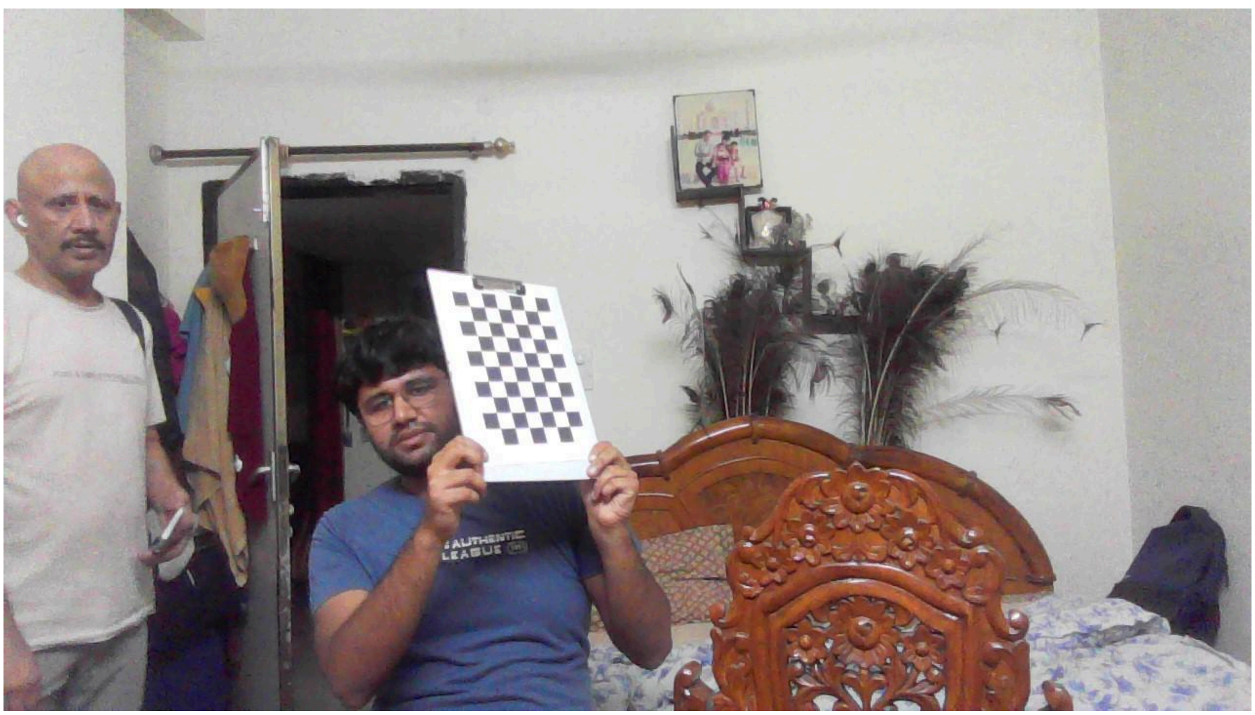






1) Intrinsic Camera Parameters:
   Focal Length (fx, fy): 977.637490505218 , 979.841355621112
   Principal Point (cx, cy): 649.3383701208371 , 344.4417845311031
   Skew Parameter: 0.0

2) Extrinsic camera parameters:

```
Image 1 :
Rotation Matrix:
[[ 0.0196569   0.99791545  0.06146836]
 [-0.99573075  0.01399387  0.09123844]
 [ 0.09018806 -0.06299941  0.99393017]]
Translation Vector:
[[ 4.06724911]
 [ 5.16974942]
 [30.89781957]]
Image 2 :
Rotation Matrix:
[[-0.77457542 -0.05169747 -0.6303652 ]
 [ 0.09400191 -0.9949945  -0.03390553]
 [-0.62545708 -0.08551792  0.77555795]]
Translation Vector:
[[-6.13483225]
 [ 6.44318051]
 [28.99330864]]
Image 3 :
Rotation Matrix:
[[-0.79794423  0.13985191  0.58628189]
 [-0.1202107  -0.99009261  0.07256731]
 [ 0.59062204 -0.01257269  0.80685038]]
Translation Vector:
[[10.70810078]
 [ 8.55725219]
 [29.77444243]]
Image 4 :
Rotation Matrix:
[[-0.28802004  0.92884781 -0.23299398]
 [-0.81028451 -0.10671055  0.57623942]
 [ 0.51037581  0.35475992  0.78336577]]
Translation Vector:
[[-8.09143033]
 [ 6.79446195]
 [40.345179  ]]
Image 5 :
Rotation Matrix:
[[-0.03406355  0.99089639 -0.13024599]
 [-0.81674377  0.04750814  0.57504139]
```

```
 [ 0.57599418  0.12596555  0.80769015]]
Translation Vector:
[[-4.70007853]
 [ 1.46116471]
 [11.61987432]]
Image 6 :
Rotation Matrix:
[[ 0.11980496  0.83679589  0.53424658]
 [-0.99276971  0.09695441  0.07076822]
 [ 0.00742099 -0.53886221  0.84236124]]
Translation Vector:
[[-2.48706994]
 [ 1.44401981]
 [17.78130317]]
Image 7 :
Rotation Matrix:
[[-0.10241653  0.5898153  -0.80101733]
 [-0.99414906 -0.08847993  0.06195931]
 [-0.03432941  0.80267628  0.59542613]]
Translation Vector:
[[-5.38363658]
 [ 3.1072592 ]
 [13.2238147 ]]
Image 8 :
Rotation Matrix:
[[ 0.00606969  0.99642547 -0.08425824]
 [-0.88825886 -0.03332911 -0.45813247]
 [-0.45930311  0.07762385  0.88488146]]
Translation Vector:
[[-5.44204059]
 [ 1.0983086 ]
 [18.01273991]]
Image 9 :
Rotation Matrix:
[[ 0.41386626 -0.90767025 -0.06963789]
 [ 0.89781275  0.41962182 -0.13360309]
 [ 0.15048913 -0.00722797  0.98858524]]
Translation Vector:
[[ 5.44414808]
 [-2.92425731]
```

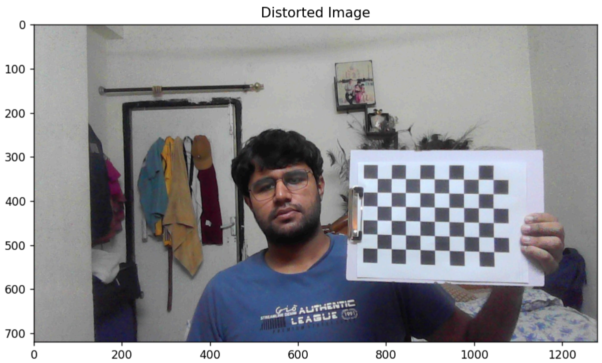
```
 [19.30281298]]
Image 10 :
Rotation Matrix:
[[ 0.61956668  0.75730522 -0.20646049]
 [-0.59964457  0.62637104  0.49808203]
 [ 0.50652099 -0.18479212  0.84219259]]
Translation Vector:
[[-15.61927978]
 [  1.17050128]
 [ 34.31168559]]
Image 11 :
Rotation Matrix:
[[ 0.65797233  0.01342994 -0.75292234]
 [-0.46068573  0.79809153 -0.38835366]
 [ 0.59568538  0.60238654  0.53130913]]
Translation Vector:
[[-13.51991805]
 [ -4.42453763]
 [ 24.27444778]]
Image 12 :
Rotation Matrix:
[[-0.01533827  0.99878408  0.0468519 ]
 [-0.99971429 -0.01617795  0.01759553]
 [ 0.0183321  -0.04656863  0.99874686]]
Translation Vector:
[[-14.15097576]
 [  6.24396602]
 [ 29.83892155]]
Image 13 :
Rotation Matrix:
[[ 0.37606202 -0.70545417  0.60075932]
 [ 0.78980452  0.5830892   0.19030453]
 [-0.4845474   0.40291612  0.7764486 ]]
Translation Vector:
[[ 5.94735939]
 [-3.1970314 ]
 [19.98956364]]
Image 14 :
Rotation Matrix:
[[ 0.33120686  0.93668872 -0.11364969]
```

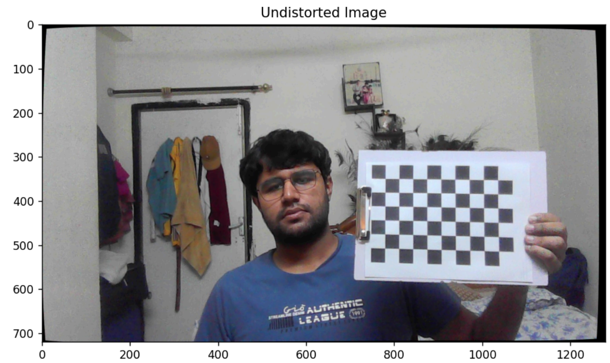
```
 [-0.75082923  0.188689   -0.63297072]
 [-0.57145209  0.29497575  0.76578836]]
Translation Vector:
[[-8.84765609]
 [ 4.03508807]
 [21.37455062]]
Image 15 :
Rotation Matrix:
[[-0.06771307  0.97911182 -0.19171589]
 [-0.83395836  0.0499332   0.54956359]
 [ 0.54765719  0.1970957   0.81315735]]
Translation Vector:
[[-9.32390441]
 [ 3.61285895]
 [23.67382171]]
Image 16 :
Rotation Matrix:
[[-0.31669555 -0.89166783 -0.32346933]
 [ 0.89075771 -0.16238476 -0.42447837]
 [ 0.32596722 -0.42256321  0.84568653]]
Translation Vector:
[[ 5.80776587]
 [-0.3743135 ]
 [18.31816678]]
Image 17 :
Rotation Matrix:
[[ 0.99507581  0.09744751 -0.0181139 ]
 [-0.06938498  0.81536043  0.57478091]
 [ 0.07078032 -0.57069375  0.81810683]]
Translation Vector:
[[-7.27122309]
 [-1.63191461]
 [61.62814962]]
Image 18 :
Rotation Matrix:
[[ 0.70859781  0.28783588 -0.64423571]
 [ 0.0343685   0.89785347  0.43895097]
 [ 0.7047751  -0.33318112  0.62632452]]
Translation Vector:
[[-10.02944463]
```

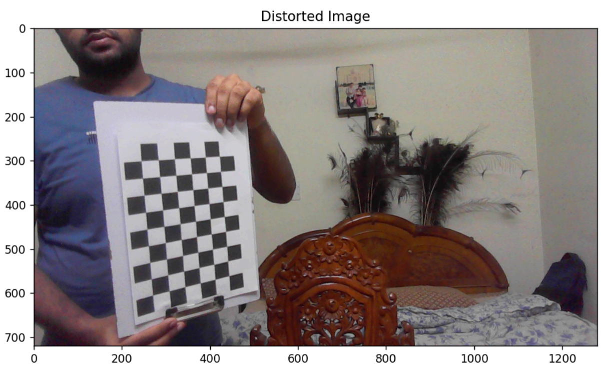
```
 [ -2.31688575]
 [ 58.08680646]]
Image 19 :
Rotation Matrix:
[[ 0.08948559  0.96541521 -0.24487914]
 [-0.88908687 -0.03338625 -0.45651932]
 [-0.4489063   0.25857073  0.8553504 ]]
Translation Vector:
[[-17.01273075]
 [ -0.23084291]
 [ 79.06240874]]
Image 20 :
Rotation Matrix:
[[-0.10162813  0.96464895  0.24315454]
 [-0.96640464 -0.03772765 -0.25424141]
 [-0.23608006 -0.26082375  0.93607541]]
Translation Vector:
[[22.15903281]
 [ 9.3023612 ]
 [89.82016253]]
Image 21 :
Rotation Matrix:
[[-0.01352742  0.8082538  -0.58867886]
 [-0.99981763 -0.01887071 -0.00293431]
 [-0.01348045  0.5885318   0.80836167]]
Translation Vector:
[[-9.98644565]
 [ 2.83663173]
 [28.27074289]]
Image 22 :
Rotation Matrix:
[[-0.06032113  0.84938764  0.52431097]
 [-0.99400546 -0.00313154 -0.1092856 ]
 [-0.09118394 -0.5277602   0.84448485]]
Translation Vector:
[[ 7.55147375]
 [ 4.06596676]
 [36.69812419]]
Image 23 :
Rotation Matrix:
```

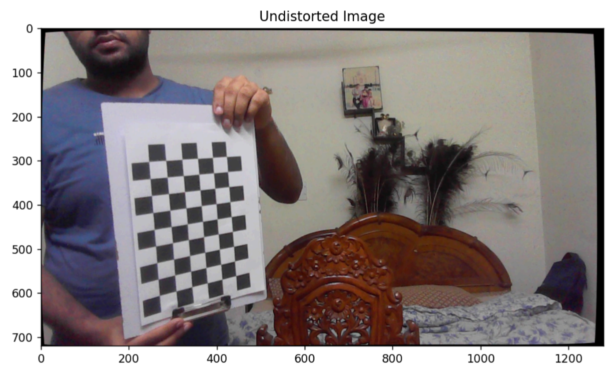
[[2.44604426e-04 9.73951811e-01 2.26754957e-01]
 [-9.47606119e-01 -7.22090946e-02 3.11172765e-01]
 [3.19441048e-01 -2.14950499e-01 9.22905033e-01]]
Translation Vector:
[[5.85585276]
 [1.44751509]
 [41.09969776]]
Image 24 :
Rotation Matrix:
[[-0.14637366 0.96039203 0.2371116]
 [-0.89531199 -0.02667742 -0.44464003]
 [-0.42070322 -0.27737244 0.86375537]]
Translation Vector:
[[1.81556228]
 [-1.20151418]
 [39.3193348]]
Image 25 :
Rotation Matrix:
[[-0.99367801 -0.0208407 -0.11031626]
 [0.02049984 -0.99978094 0.00422328]
 [-0.11038011 0.00193511 0.99388756]]
Translation Vector:
[[1.63749089]
 [5.8202569]
 [24.61631765]]

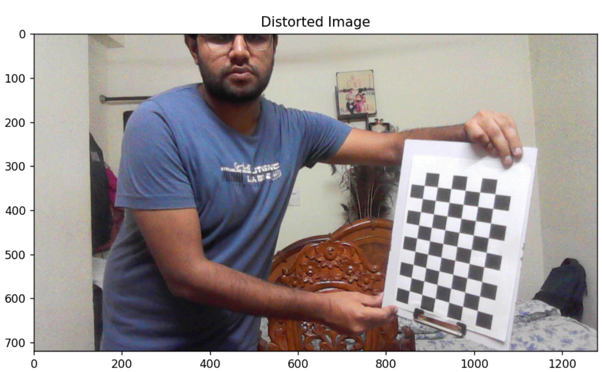
3) Radial Distortion Coefficients:
   Estimated Radial Distortion Coefficients: [[0.15357408 -0.61112962 -0.00688358 0.00728497 0.77132159]]

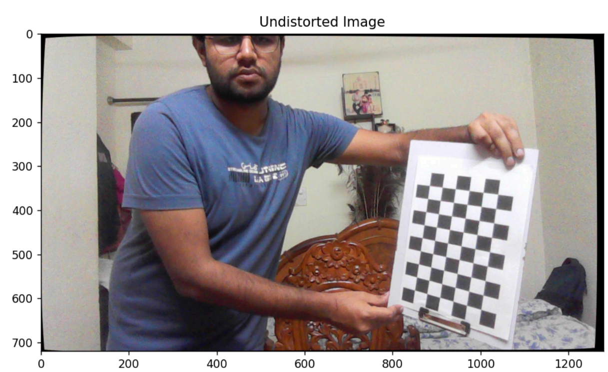

Distorted Image

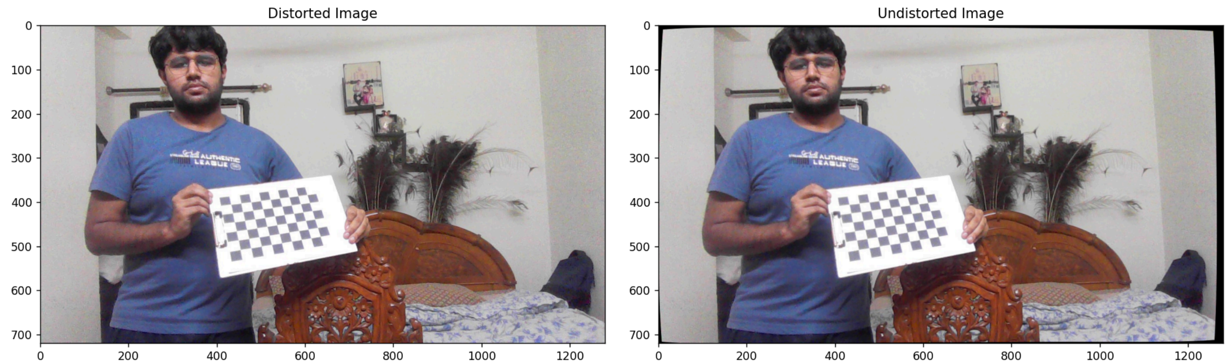

Undistorted Image

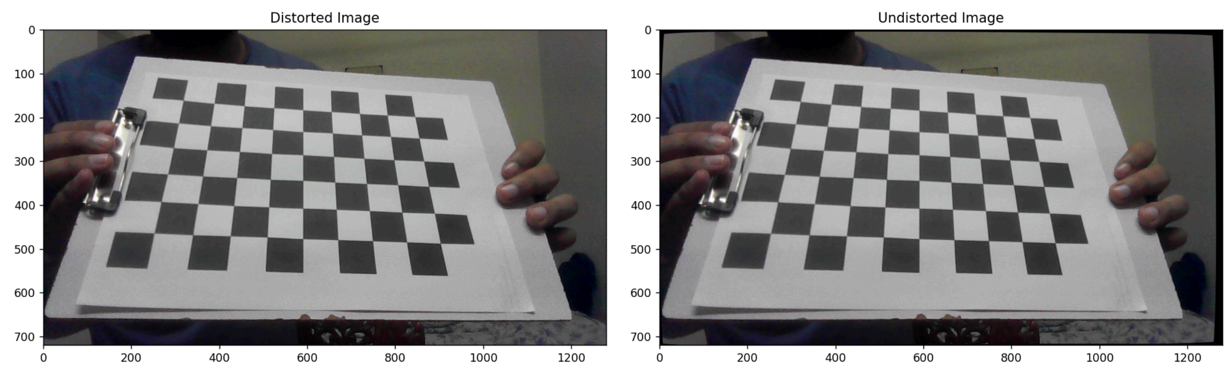

Distorted Image


Undistorted Image


Distorted Image


Undistorted Image





Radial distortion is the highest near the edges and lowest near the center. This is why, when we undistort the images, lines which are slightly curved near the edges appear straighter. A pronounced example of this would be the wall in the images above, in the left image the line of intersection of the two planes of the wall appears slanted while it is straighter in the image on the right.

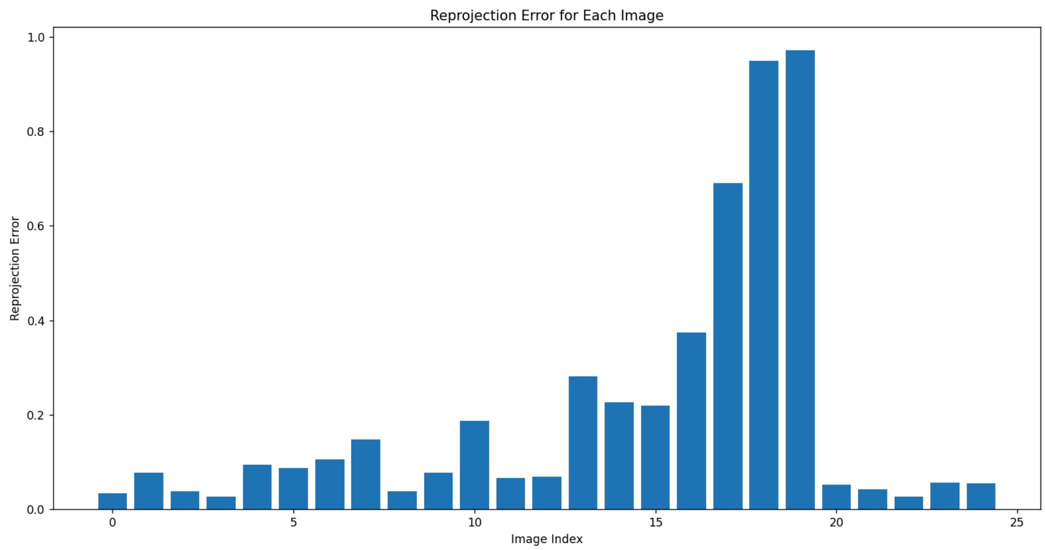
4) Reprojection error

Image 1 :
0.034464626959015456
Image 2 :
0.077024583460803
Image 3 :
0.03841829521369184
Image 4 :

0.027214982276801962
Image 5 :
0.09432761286789638
Image 6 :
0.08764023395019488
Image 7 :
0.10600936957703955
Image 8 :
0.14833361590980998
Image 9 :
0.038962687749832255
Image 10 :
0.07781051484535537
Image 11 :
0.18779751746717843
Image 12 :
0.06684291222225104
Image 13 :
0.06954842390651478
Image 14 :
0.28178498017617093
Image 15 :
0.226372755743481
Image 16 :
0.2190113073561198
Image 17 :
0.37419497002800345
Image 18 :
0.6907029244494277
Image 19 :
0.9493357287281895
Image 20 :
0.971634803043252
Image 21 :
0.05262931985314376
Image 22 :
0.04261771710216406
Image 23 :
0.026942565306696312
Image 24 :

0.05625066188358191
Image 25 :
0.055348618257026645

Mean Error: 0.20004886913334566
Standard Deviation of Errors: 0.26558397198361566



5)

Using camera calibration, both intrinsic and extrinsic parameters are computed for each image. The corners of the chessboard pattern detected in the images are stored in the objpoints array. After camera calibration, the detected corners are re-projected onto the images using the rotation matrix and translation vector obtained during calibration. Subsequently, the distance between the corresponding corners in the images and their re-projected positions is calculated. This distance is then normalized by the total number of corners to obtain the reprojection error for each image

Image 1 - Detected Corners

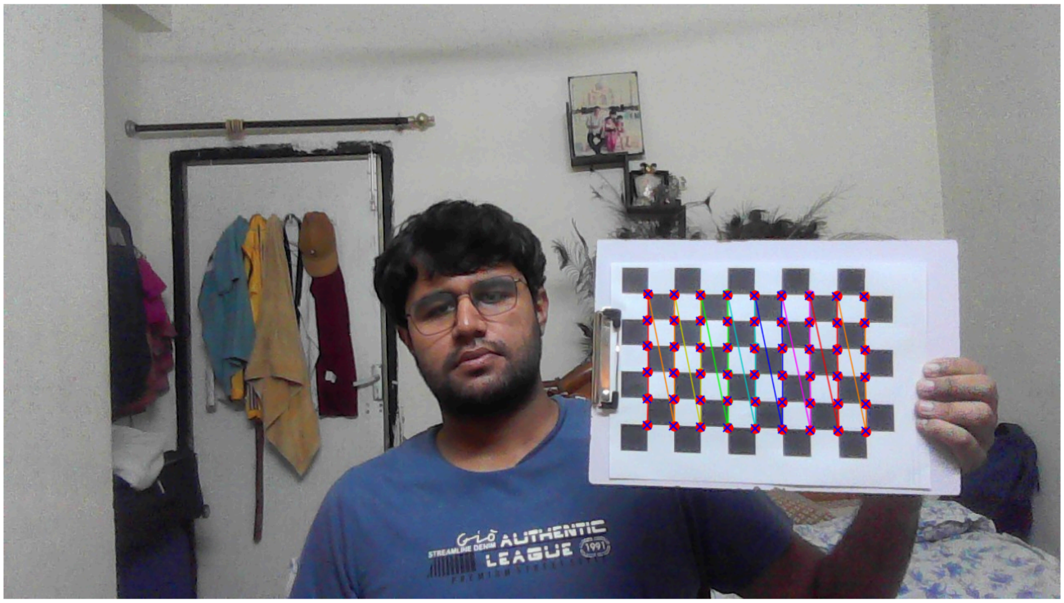


Image 2 - Detected Corners

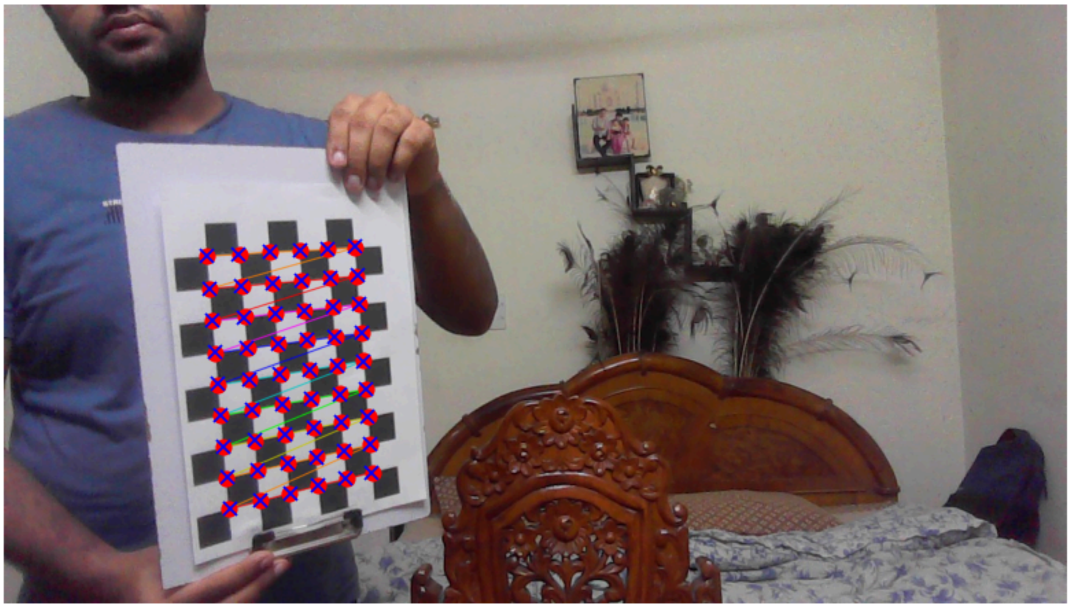


Image 3 - Detected Corners

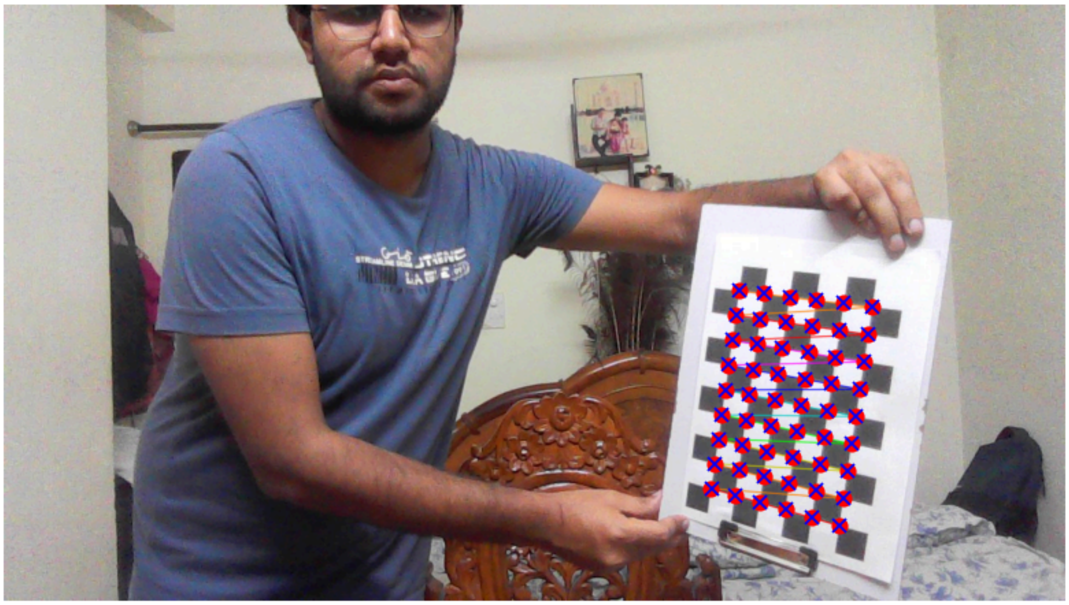


Image 4 - Detected Corners

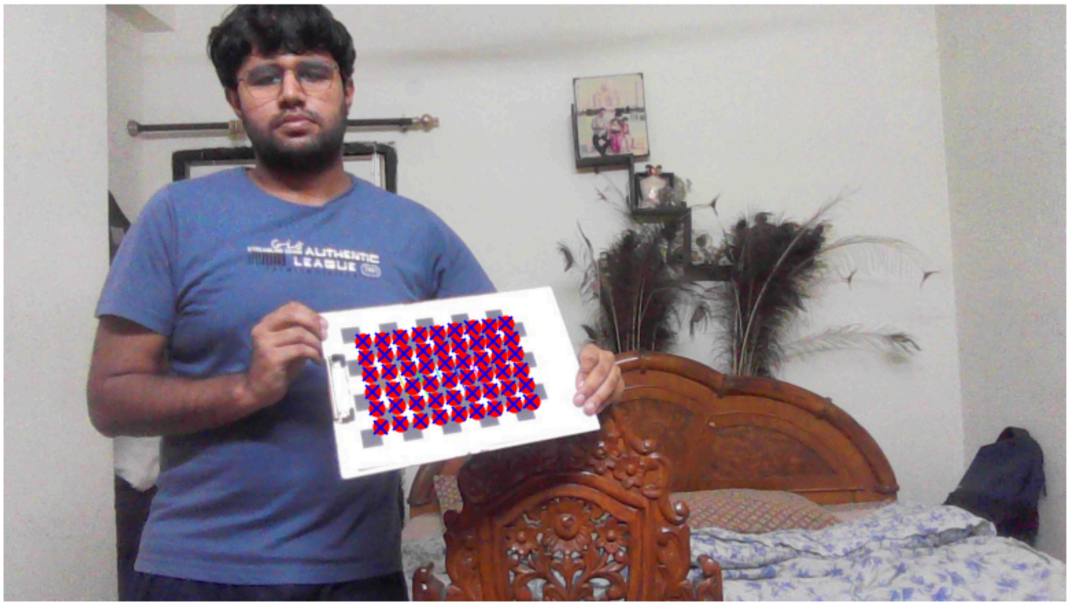
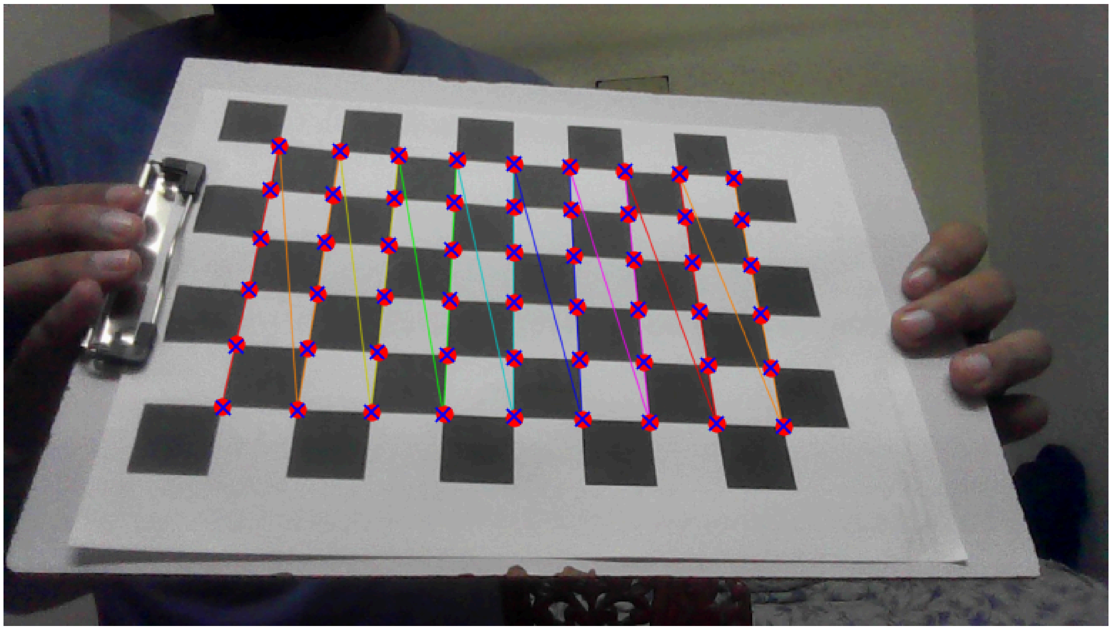


Image 5 - Detected Corners



6)

Image 1: Normal in camera coordinate frame of reference= [0.06146836 0.09123844 0.99393017]

Image 2: Normal in camera coordinate frame of reference= [-0.6303652 -0.03390553 0.77555795]

Image 3: Normal in camera coordinate frame of reference= [0.58628189 0.07256731 0.80685038]

Image 4: Normal in camera coordinate frame of reference= [-0.23299398 0.57623942 0.78336577]

Image 5: Normal in camera coordinate frame of reference= [-0.13024599 0.57504139 0.80769015]

Image 6: Normal in camera coordinate frame of reference= [0.53424658 0.07076822 0.84236124]

Image 7: Normal in camera coordinate frame of reference= [-0.80101733 0.06195931 0.59542613]

Image 8: Normal in camera coordinate frame of reference= [-0.08425824 -0.45813247 0.88488146]

Image 9: Normal in camera coordinate frame of reference= [-0.06963789 -0.13360309 0.98858524]
Image 10: Normal in camera coordinate frame of reference= [-0.20646049 0.49808203 0.84219259]
Image 11: Normal in camera coordinate frame of reference= [-0.75292234 -0.38835366 0.53130913]
Image 12: Normal in camera coordinate frame of reference= [0.0468519 0.01759553 0.99874686]
Image 13: Normal in camera coordinate frame of reference= [0.60075932 0.19030453 0.7764486]
Image 14: Normal in camera coordinate frame of reference= [-0.11364969 -0.63297072 0.76578836]
Image 15: Normal in camera coordinate frame of reference= [-0.19171589 0.54956359 0.81315735]
Image 16: Normal in camera coordinate frame of reference= [-0.32346933 -0.42447837 0.84568653]
Image 17: Normal in camera coordinate frame of reference= [-0.0181139   0.57478091 0.81810683]
Image 18: Normal in camera coordinate frame of reference= [-0.64423571 0.43895097 0.62632452]
Image 19: Normal in camera coordinate frame of reference= [-0.24487914 -0.45651932 0.8553504]
Image 20: Normal in camera coordinate frame of reference= [0.24315454 -0.25424141 0.93607541]
Image 21: Normal in camera coordinate frame of reference= [-0.58867886 -0.00293431 0.80836167]
Image 22: Normal in camera coordinate frame of reference= [0.52431097 -0.1092856 0.84448485]
Image 23: Normal in camera coordinate frame of reference= [0.22675496 0.31117277 0.92290503]
Image 24: Normal in camera coordinate frame of reference= [0.2371116 -0.44464003 0.86375537]
Image 25: Normal in camera coordinate frame of reference= [-0.11031626 0.00422328 0.99388756]

Camera LIDAR Cross Calibration:

1) Chessboard normals and offsets

Plane 1: Normal = [0.63693437 -0.76499839 0.09535237], Offset = -4.993811298096004

Plane 2: Normal = [0.70169742 -0.70118932 0.1263102], Offset = -4.720779659728762
Plane 3: Normal = [0.93809241 -0.22958973 0.25936688], Offset = -5.204305546806318
Plane 4: Normal = [0.72352642 -0.68812782 -0.05467752], Offset = -4.79350162621247
Plane 5: Normal = [0.83333533 -0.55208241 0.02751785], Offset = -5.220803337324185
Plane 6: Normal = [0.91935323 -0.3607026   0.15710911], Offset = -5.70252457528633
Plane 7: Normal = [0.94151039 0.18949708 0.27865578], Offset = -6.323572215248566
Plane 8: Normal = [0.81878735 0.50763718 -0.26812639], Offset = -6.445439719171263
Plane 9: Normal = [0.93288206 -0.2440387   0.26490788], Offset = -5.6742584632412605
Plane 10: Normal = [-0.95207118 0.1539658 -0.26430097], Offset = 5.743694294895789
Plane 11: Normal = [0.60407281 -0.77412903 0.18926246], Offset = -5.062352211986536
Plane 12: Normal = [0.75932657 -0.56254883 0.32705042], Offset = -4.954015451590528
Plane 13: Normal = [-0.47346333 0.7599202   0.44536924], Offset = 5.2549842264287
Plane 14: Normal = [-0.52528321 0.84450165 0.1043768], Offset = 6.103645571922049
Plane 15: Normal = [-0.40409023 0.88184713 -0.24301591], Offset = 5.436720512190618
Plane 16: Normal = [-0.90149712 -0.39908052 0.16744454], Offset = 8.140582366167767
Plane 17: Normal = [-0.89093965 -0.24536379 0.38212975], Offset = 7.618700179547204
Plane 18: Normal = [0.99121488 -0.12226682 -0.05043704], Offset = -6.093678679246951
Plane 19: Normal = [-0.825289   0.10756967 0.55437067], Offset = 5.978131037863134
Plane 20: Normal = [0.93223066 -0.19842704 -0.30260984], Offset = -6.26535811591381
Plane 21: Normal = [0.98936009 -0.13850217 -0.04453952], Offset = -7.325197139472717

Plane 22: Normal = [0.96691202 -0.15794554 -0.2003356], Offset = -8.493221054907726
Plane 23: Normal = [0.99427448 -0.1042191   0.02359328], Offset = -8.846168921169088
Plane 24: Normal = [0.662242   -0.74563698 0.07389876], Offset = -7.749464153135404
Plane 25: Normal = [0.73370584 -0.67940958 0.00885226], Offset = -6.96918888440808

2)