

math notes

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Chapter 1

Chapter 2

2.1

- A set S is a collection of items $\{2, chair, immortality, \{1\}\}$
- $2 \in S$ denotes that 2 is a member of set S . " $x \in S$ "
" $x \in D$ "
- Important sets
 \mathbb{R} = set of all real numbers
 \mathbb{Z} = set of ints
 \mathbb{Q} = set of rational numbers
 \mathbb{N} = set of natural numbers
- Def y $A \subseteq B$ are sets, and every element of A is inside B , then A is a subset of B $A \subseteq B$
- $\mathbb{N} \subseteq \mathbb{Q}$
 $\mathbb{N} \subseteq \mathbb{R}$
 $\mathbb{N} \subseteq \mathbb{Z}$
 $\mathbb{Z} \subseteq \mathbb{Q}$
 $\mathbb{Z} \subseteq \mathbb{R}$
 $\mathbb{Z} \not\subseteq \mathbb{N}$
 $\mathbb{Q} \subseteq \mathbb{R}$

Chapter 3

Set Theory

3.1 Predicate and quantified statements

- Def a predicate is a sentence that contains a finite number of variables and that becomes a statement when specific values are substituted for the variable
 $P(x) = "x \geq 2"$
 $P(1) = 1 > 2$ *true*
 $P(1) = "1 > 2"$ *false*
- The domain of the predicate is the set of all values that may be substituted for a variable.
 $D = R$
 $D = \{1, 2, 3\}$
 $D = Z^+$ (set of positive ints)
- The truth set of $P(x)$ is the set of elements in D such that $P(x)$ is true.
 $\{x \in D : P(x)\}$
 $\{x = 1, 2, 3 : x > 2\} = \{3\}$
- Ex finding the truth set
 $Q(n) = "n \text{ is a factor of } 8"$
a) $D = Z^+$
 $\{n \in Z^+ : n \text{ is a factor of } 8\}$
 $= \{1, 2, 4, 8\}$
b) $D = Z$
 $\{n \in Z : n \text{ divides } 8 \text{ evenly}\}$
 $= \{1, 2, 4, 8, -1, -2, -4, -8\}$
- Def The symbol \forall denotes "for all" (for every, for any) and is called the universal quantifier.
'all humans are mortal'
" \forall humans h , h is mortal"
define $H = \text{all humans}$ " *forall* $h \in H$, h is mortal "
- Def A universal statement has the form " $\forall x \in D, Q(x)$ " where D is a domain and $Q(x)$ a predicate.
The statement is true if and only if $Q(x)$ is true for each $x \in D$. The statement is false if there is at least one $x \in D$ such that $Q(x)$ is false.
- $\forall x \in D, x^2 \geq x$
a) $D = \{1, 2, 3, 4\}$
 $1 \geq 1, 4 \geq 2, 9 \geq 3, 16 \geq 4$
therefore the statement is true.
b) $D = R$
let $x = .5$ counter example
then $x^2 = .25$
 $.25 < .5$

- Def the symbol " \exists " denotes "there exist" (there is one ... , there are some..) and is called the existential quantifier.
 "there is a cat on the fridge"
 $C = \{all\ cats\}$
 $\exists x \in C, c$ is on the fridge.
- def an existential " $\exists x \in D$ such that $Q(x)$ "
 " $\exists x \in D$ such that $Q(x)$ "
 is true when at least one x in D makes $Q(x)$ true
 is false when every $x \in D$ makes $Q(x)$ false.
 Consider $\exists m \in \mathbb{Z}^+$ such that $m^2 = m$
 $\frac{m^2}{m} = \frac{m}{m} \Rightarrow m = 1$
 which in words is "there exists a positive int such that it is equal to its square."
 "some positive integer equals its square"
- "no dogs have wings"
 All dogs don't have wings
 D = a set of dogs
 $\forall d \in D, d$ doesn't have wings.
 all, every, each, none, no ; are universal words
 at least one, some, there is / exists; are existential words
- Universal conditional statements
 $\forall x, if P(x) then Q(x)$
 $\forall x \in \mathbb{R}, if x > 2, then x^2 > 4$
 Ex : No sloths are fast.
 $\forall s \in U$, if s is a sloth, then s is not fast. $\equiv \forall s \in S, s$ is not fast
 $P(x) = s$ is a sloth"
 $\{x \in U, if P(x), then Q(x)\}$
 $\{x \in U: P(x)\} : D = \{sloths\} / equiv \forall x \in D, Q(x).$
- $Ex \forall x \in R, if x \in Z, then x \in Q \equiv \forall x \in Z, x \in Q.$
 N : check 1,2,3
 Z : check negative
 Q : check $\frac{1}{2}, \frac{3}{4}$
 R : check $\pi, e, \sqrt{2}$
- Ex there exists a number that is both even and prime
 $\exists n$ such that $P(n) \wedge E(n).$
 \forall a prime number such that $E(n)$
 \forall an even number such that $P(n).$
 "implicit quantification"
 y a number is an int, then ...
 $\forall n \in \mathbb{Z}$
 indefinite article "a"
 "24 can be written as the sum of two ints"
 $\forall m, n \in \mathbb{Z}$ such that $m + n = 24$
 $P(x) \Leftrightarrow Q(x).$
 $\{x: P(x)\} = \{x: Q(x)\}$
- homework 3.1 2,3,9,11,13,19

3.2 Negations of quantified statements

- $(\forall x \in D, Q(x)) \equiv \exists x \in D, such\ that\ Q(x)$
 $(\exists x \in D such\ that\ Q(x)) \equiv \forall x \in D, \neg Q(x).$
 Negate "some dogs are bad dogs"

” all dogs are god dogs”

$(\forall x \text{ if } P(x) \text{ then } Q(x)) \equiv \exists x \text{ such that } P(x) \text{ and } \neg Q(x)$

negating \forall

gives \exists

negating \exists

gives \forall

- Universal statements that are vacuously true
Consider an empty jar
"all the grapes in the jar are red"
 $\forall g \in J$
negating that =
"some grapes in the jar are not red"
 $\exists g \in J$ Such that g is not red.
- Universal Conditional
 $\forall x \in D, \text{ if } P(x) \text{ then } Q(x)$

3.3 Statments sith multiple quantifiers

from quiz

any interger equals twice some interger

which is " $\forall n \in \mathbb{Z}$

- "everyone has someone to love"
"for every person, there is another person that they love "
 $\forall p \in H, \exists r \in H$ such that p loves r .
- $(\forall x \in D, \exists y \in E \text{ such that } P(x,y))$
 $\equiv \exists x \in D \text{ such that } \forall y \in E, P(x,y)$
 $(\exists x \in D ; \text{such that } \forall y \text{ in } E, P(x,y))$
 $\equiv \forall x \in D, \exists y \in E \text{ such that } P(x,y).$

3.4 arguments with quantified statments

- Rule of universal instantiation if a property is true of everything in a set, it is true for any particular member of the set.
- hello

Chapter 4

4

4.1 4.1

4.2 4.2

- hello

4.3 4.3

from quiz

1 for all ints m if m is even then $3m + 5$ is odd

$m = 2k$ for some $k \in \mathbb{Z}$ $\therefore 3m + 5 = 3(2k) + 5 = 6k + 5$

want $6k + 5 = 2j + 1$ for some $j \in \mathbb{Z}$ find the j

$6k + 5 = 2j + 1$ $6k + 4 = 2j$ $3k + 2 = j$

set $j = 3k + 2$ since $k \in \mathbb{Z}, j \in \mathbb{Z}$

we have $3m + 5 = 2j + 1$ for $j \in \mathbb{Z}$

2 for all ints a,b, and c if $a \mid b$ and $a \mid c$ then $a \mid (b+c)$

$a \mid b \Leftrightarrow b = ar$ for some $r \in \mathbb{Z}$

$a \mid c \Leftrightarrow c = as$ for some $s \in \mathbb{Z}$

$b + c = ar + as = a(r + s)$

since $r, s \in \mathbb{Z}, r + s \in \mathbb{Z}$

$a \mid (b + c)$

4.4 Division into cases and the quotient-remainder theorem

- (quotient-remainder)
given any integer n and integer $d > 0$, there exist unique integers q, r such that $n = dq + r$ where $0 \leq r < d$
- given $n = dq + r$ we say "n div d" is the integer quotient q obtained when n is divided by d, and "n mod d" is the integer remainder obtained when n is divided by d.
- representations of integers
we can prove all integers are even or odd using the QR
given any n, $2 \mid d$ then $n = 2q + r$ where $0 \leq r < 2$
so $r = 0$ or $r = 1$
two cases
 $n = 2q + 0$ or $n = 2q + 1$
but $q \in \mathbb{Z}$
so n is even or n is odd
- prove that any two consecutive integers have opposite parity (that is one is even and one is odd).
let m, $m + 1$ be two consecutive integers
case 1

m is even therefore $m = 2k = 2k + 1$ for the same $k \in \mathbb{Z}$ therefore $m + 1$ is odd

case 2

therefore $m = 2k + 1$ for some $k \in \mathbb{Z}$ then $m + 1 = 2k + 1 + 1$

$$= 2k + 2$$

$$2(k + 1)$$

therefore $(k + 1) \in \mathbb{Z}$ therefore $m + 1$ is even

- can have more than 2 cases
- show that any integer n can be written in one of the four forms

$$n = 4q$$

$$n = 4q + 1$$

$$n = 4q + 2$$

$$n = 4q + 3$$

for some $q \in \mathbb{Z}$

proof

QR - ther $6 + d = 4$ then for all integers , n , $n = 4q + r$ where $0 \leq r < 4$

four cases $r = 0, 1, 2, 3$

$$n = 4q$$

$$n = 4q + 1$$

$$n = 4q + 2$$

$$n = 4q + 3$$

4.5

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Chapter 5

Sequences and math induction

5.1 sequences

- def a sequence ins a function whose domain is either all the integers or all the integers greater than or equal to an integer.
- a_0, a_1, a_2, a_n
an element a_k (" a sub k ") is called a term of the sequence and k is its index
- when the sequence doesn't have a final term it is an infinite sequence

- Explicit formulae for sequences

$$a_k = \frac{k}{k+1} \text{ for all integers } k \geq 1$$

$$a_1 = \frac{1}{2}$$

$$b_i = \frac{i-1}{i} \text{ for all integers } i \geq 2$$

$$b_2 = 1/2$$

$$b_3 = 2/3$$

$$b_4 = 3/4 \text{ etc}$$

$$a_k = b_i \text{ where } k = i - 1$$

- alternating sequences

$$c_j = (-1)^j \text{ for } j \geq 0$$

$$c_0 = (-1)^0 = 1$$

$$c_1 = (-1)^1 = -1$$

$$c_2 = (-1)^2 = 1$$

$$c_3 = (-1)^3 = -1$$

example

$$1, -1/4, 1/9, -1/16, 1/25, -1/36$$

1 in numerator

signs alternate

perfect squares in denominator

list $k = 1, 2, 3, \text{etc}$ for each term

$$a_k = \frac{(-1)^{k+1}}{k^2}$$

positive a_k when k is odd

- summation notation

want to take the sum of all the elements in a sequence $(a_1 + a_2 + a_3 + \dots)$

$$\sum_{k=1}^n a_k$$

let $a_1 = -2, a_2 = -1, a_3 = 0, a_4 = 1, a_5 = 2$

compute

$$\sum_{k=1}^5 ak = a_1 + a_2 + a_3 + a_4 + a_5 = -2, + -1 + 0_1 + 2 = 0$$

$$\sum_{k=2}^2 a_k = a_2 = -1$$

$$\sum_{k=1}^2 a_2 k = a_2(1) + a_2(2) = a_2 + a_4 = -1 + 1 = 0$$

$$\sum_{k=1}^5 k^2 \text{ or } \sum_{i=0}^8 \frac{(-1)^i}{i+1}$$

$$\sum_{i=0}^n \frac{(-1)^i}{i+1} = \frac{(-1)^0}{0+1} + \frac{(-1)^1}{1+1} + \frac{(-1)^2}{2+1} + \frac{(-1)^3}{3+1} + \dots + \frac{(-1)^n}{n+1}$$

$$= 1 - 1/2 + 1/3 + \dots + \frac{(-1)^n}{n+1}$$

$$1/n + 2/(n+1) + 3/(n+3) + \dots + (n+1)/(2n)$$

$$\text{the general term } a_k = (k+1)/(n+k) \quad 2n = n+n$$

$$\text{the ints range from } (k=0) \text{ to } (k=n)$$

$$\sum_{k=0}^n \frac{k+1}{n+k}$$

- telescoping sum

$$\sum_{k=1}^n \frac{1}{k(k+1)}$$

$$1/k - 1/(k+1) = (k+1)/(k(k+1)) = (k)/(k(k+1)) = 1/(k(k+1))$$

$$= \sum_{k=1}^n \frac{1/k}{1/(k+1)} = ((1/1)-(1/2))+((1/2)+(1/3))+((1/3)-(1/4))+\dots+((1/(n-1))-(1/n))+((1/n)-(1/(n+1)))$$

- product notation

$$\prod_{k=1}^n a_k = (a_1)(a_2)(a_3)\dots(a_n)$$

$$\prod_{k=1}^5 k = 1 * 2 * 3 * 4 * 5 = 120$$

$$\sum_{k=1}^n a_k + \sum_{k=1}^n b_k = \sum_{k=1}^n (a_k + b_k)$$

$$\sum_{k=0}^n a_k = a_0 + \sum_{k=1}^n a_k$$

$$2 \ c * \sum_{k=1}^n a_k = \sum_{k=1}^n a_k * c$$

$$(\prod_{k=1}^n a_k)(\prod_{k=1}^n b_k) = \prod_{k=1}^n (a_k * b_k)$$

- factorials

$$n! = n(n-1)(n-2)\dots 3 * 2 * 1 = n(n-1)!$$

$$n! = \prod_{k=1}^n k$$

$$\text{we define } 0! = 1 \quad (8!)/(7!) = (8 * 7!)/(7!) = 8$$

$$n, r \in \mathbb{Z} \quad 0 \leq r \leq n$$

homework

section 5.1

3,12,19,31,40,49,55,65,75

test october 9th chapters 2-5

defs, proofs 4 questions 3 to do and bonus

5.2 Math induction

- principle of math induction

let $P(n)$ be a property defined for integers n and let a be a fixed int

suppose the following to be true

$P(a)$ is true

For all ints $k \geq a$, if $P(k)$ is true, then $P(k+1)$ is true.

then the statement $\forall \text{ints } n \geq a, P(n)$ is true

- method of proof by math induction

show that $P(a)$ is true

suppose $P(k)$ is true for $k \geq a$, use this to show $P(k+1)$ true

- example

$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$ for all ints $n \geq 1$ which is $P(n)$

1 show $P(1)$ is true $n = 1$

$$1 = \frac{1(1+1)}{2} = \frac{2}{2} = 1$$

2 assume $P(k)$ is true

$$\sum_{j=1}^k j = \frac{k(k+1)}{2}$$

$$\text{show } P(k+1) \quad \sum_{j=1}^{k+1} (k+1)j = \sum_{j=1}^k kj + (k+1) = \frac{k(K+1)}{2} + k+1 = \frac{k^2 + k + 2k + 2}{2} = \frac{k^2 + 3k + 2}{2} = \frac{(k+1)(k+2)}{2} \text{ so } P(k+1) \text{ is true}$$

5.3 more math induction

1 show $P(a)$ true

2: Assume $P(k)$ is true for some $k \geq a$ use this to prove $P(k+1)$ is true

For all integers $n \geq 0, 2^{(2n)} - 1$ is divisible by 3

pf: show that $P(0)$ is true

$n = 0 : 2 - 1$ is divisible by 3

$$2^0 - 1 = 1 - 1 = 0. 0^n = 3^d * 0^k$$

so 0 is divisible by 3

2. Assume $P(k)$ is true for $k \geq 0$

know : $2^{(2k)} - 1$ is divisible by 3 for $k \geq 0$

So $2^{(2k)} - 1 = 3r$ for some $r \in \mathbb{Z}$

$$2^{(2(k+1))} - 1 = 2^{(2k+2)} - 1 = 2^{2k} * 2^2 - 1 = 4 * 2^{2k} - 1 = 3 * 2^{2k} + 2^{2k} - 1 = 3 * 2^{2k} + 3r = 3(2^{2k} + r)$$

let $s = 2^{2k} + r$ since $k, r \in \mathbb{Z}, s \in \mathbb{Z}$ Then $2^{(2(k+1))} - 1 = 3s, s \in \mathbb{Z}$ So $2^{(2(k+1))}$ is divisible by 3 Thus $P(k+1)$ is true So $P(n)$ is true for all integers $n \geq 0$

Prove an inequality

For all integers $n \geq 3, 2n + 1 < 2^n$

show that $P(3)$ is true: $n = 3 : 2(3) + 1 = 7 < 2^3 = 8$

assume $2k + 1 < 2^k$ for $k \geq 3$ then $2(k+1) + 1 = 2k + 2 + 1 = (2k + 1) + 2 < 2^k + 2^1$ we know $2 < 2^n \forall n > 1$

remind: if $a < b$ then $a + c < b + c$

In particular, for $k \geq 3, 2 < 2^k$

then $2(k+1) + 1 < 2^k + 2 < 2^k + 2^k$ Now $2^k + 2^k = 2 * 2^k = 2^{(k+1)}$

thus $2(k+1) + 1 < 2^{(k+1)}$

define a sequence a_1, a_2, a_3, \dots as follows : $a_1 = 2a_k = 5 * a_k - 1, \forall k \geq 2$

($a_2 = 5a_1 = 5 * 2 = 10$) prove that for $n \geq 1, a_n = 2 * 5^{(n-1)}$

pf: $P(1) : a_1 = 2 * 5^{(1-1)}$ (check) $a_1 = 2 * 5^0 = 2$ so $P(1)$ is true

Assume $P(k)$ is true. then $a_k = 2 * 5^k - 1$ for $k \geq 1. a_{k+1} = 5a_k = 5 * 2 * 5^k - 1 = 2 * (5 * 5^k - 1) = 2 * 5^{k+1} - 1 + 1 = 2 * 5^{k+1}$ So $a_{k+1} = 2 * 5^{k+1}$ So $P(k+1)$ is true

MidTerm

write the definitions for even, odd, prime, composite, divisibility

ex: even $h = 2k$ for some $k \in \mathbb{Z}$

truth tables for $p \Rightarrow q, p^q, p \vee q$

determine whether an argument is valid

identify the major and minor premises, the conclusion and any critical rows

T/F ch 2 thru 3 : negations of logical statements ; contrapositives of logical statements; valid and invalid argument forms ; translating between formal and informal language pg 121

four proofs pick three 2 are direct proofs pg 180

1 is by contradiction or contraposition pg 198

1 by math induction pg 244

memorize

all

the

things

5.3 homework

6,8,19,24

5.4 strong math induction

Let $P(n)$ be a property defined for integration, and let a and b be fixed ints with $a \leq b$

Suppose the following statements are true

1 $P(a), P(a+1), \dots, P(b)$ are all true

2 For any $k \geq b$ if $P(i)$ is true for all $i, a \leq i \leq k$, then $P(k+1)$ is true

then $P(n)$ is true for $n \geq a$

Method

1 Prove $P(a), P(a+1), \dots, P(b)$ are true

2 Suppose for all i such that $a \leq i \leq k$ $P(i)$ is true and use this to show $P(k+1)$ is true

example :

Any int $n > 1$ is divisible by a prime number.

$P(n)$: " n is divisible by a prime number " " \exists prime p such that $p|n$."

Show $P(2)$ is true

2 is a prime number and $2 \leq 2$

Suppose that for some $k \geq 2, P(i)$ is true for $2 \leq i \leq k$

Want to show : $k+1$ is divisible by a prime

Case 1 $k+1$ is prime. Then $k+1|k+1$ so $\exists p$ such that $p|k+1$

case 2 ; $k+1$ is composite then $\exists a, b \in \mathbb{N}$ such that $k+1 = ab$. $a < k+1$ and $1 < b < k+1$ then $2 \leq a \leq k$ so $P(a)$ is true

Chapter 6

Set Theory

6.1 definitions and the elements method of proof if S is a set and $P(x)$ is a property that elements of S may or may not satisfy, we define $A = \{x \in S | P(x)\}$ "the set of all x in S such that $P(x)$ is true" $N = \{n \in \mathbb{Z} : n > 0\}$
 $N \leq \mathbb{Z}$

Subsets : Def : $A \leq B$ is $x \in A$ then $x \in B$ Negation : $A \not\leq B$ if $\exists x$ such that $x \in A$ and $x \notin B$

element method of proof To prove a set is a subset of another let x, y be sets Prove $x \leq y$:

- 1) suppose $x \in X$ is a particular but arbitrary element of X
- 2) show that $x \in y$

example $A = \{m \in \mathbb{Z} | m = 6r + 12 \text{ for some } r \in \mathbb{Z}\}$

$B = \{n \in \mathbb{Z} | n = 3s \text{ for some } s \in \mathbb{Z}\}$

a) Prove $A \leq B$ Let k be a particular but arbitrary member of A . then $k \in \mathbb{Z}$ and there exists $r \in \mathbb{Z}$ such that $k = 6r + 12$

Want : $k \in B \Leftrightarrow \exists s \in \mathbb{Z}$ such that $k = 3s$

$k = 6r + 12 = 3(2r + 4)$

Let $s = 2r + 4$ Since $r \in \mathbb{Z}, s \in \mathbb{Z}$ So $k = 3s, s \in \mathbb{Z}$ and $k \in B$

$\therefore k \in B$

$\therefore A \leq B$

b) disprove $B \leq A$ (prove $B \not\leq A$)

$\exists x \in B$ such that $x \notin A$ $x \in B$ so $x = 3s$ for some integer s

Let $s = 1$ then $x = 3$ if $3 = 6r + 12$ then $6r = -9$ $r = -3/2 \notin \mathbb{Z}$

$\therefore x \notin A$

$\therefore B \not\leq A$ ■

Set equality $A = B$ iff $A \leq B$ and $B \leq A$

$A = \{m \in \mathbb{Z} | m = 3a \text{ for some } a \in \mathbb{Z}\}$

$B = \{n \in \mathbb{Z} | n = 3b - 3 \text{ for some } b \in \mathbb{Z}\}$

1) Prove $A \leq B$ Let $x \in A$ then $x \in \mathbb{Z}$ and $\exists a \in \mathbb{Z}$ such that $x = 3a$ Since $a \in \mathbb{Z}, (b - 1) \in \mathbb{Z}$ Let $b - 1 = a$ Then $x = 3a = 3(b - 1) = 3b - 3, b \in \mathbb{Z}$ So $x \in B$

$\therefore A \leq B$

Prove $B \leq A$: Let $y \in B$

homework 6.1

1, (a, c, e), 2, 3(a), 5

home work 6.1 number 5 $C = \{n \in \mathbb{Z} | n = 6r - 5 \text{ for some } r \in \mathbb{Z}\}$

$D = \{m \in \mathbb{Z} | m = 3s + 1 \text{ for some } s \in \mathbb{Z}\}$

a: $C \leq D$ Let $n \in C$ Then $\exists r \in \mathbb{Z}$ such that $n = 6r - 5$

$6r - 5 = 3s + 1$

$6r - 6 = 3s$

$2r - 2 = s$

Let $s = 2r - 2$ since $r \in \mathbb{Z}, s \in \mathbb{Z}$

then $3s = 6r - 6$ and $3s + 1 = 6r - 5$ then $n = 6r - 5 = 3s + 1$ // So $n \in D$ So $C \leq D$

operations on sets Let A, B be subsets of set U

1. The union of A and B , $A \cup B$ is the set of all element that are in A or B
2. the intersection of A and B $A \cap B$ is the set of all elements that are in A and B

the difference $B - A$ is the set of all elements in B that are not in A

The complement of A , A^c , is the set of all elements not in A

$$A \cup B = [x \in U | x \in A \text{ or } x \in B]$$

$$A \cap B = [x \in U | x \in A \text{ and } x \in B]$$

$$B - A = [x \in U : x \notin A \text{ and } x \in B]$$

$$A^c = [x \in U | x \notin A]$$

$$\mathbf{t} \quad (a, b] = [x \in R | a < x \leq b]$$

$$[a, b] = [x \in R | a \leq x \leq b]$$

$$A = (-1, 0] = [x \in R | -1 < x \leq 0]$$

$$B = [0, 1) = [x \in R | 0 \leq x < 1]$$

$$A \cup B = [x \in R | -1 < x < 1]$$

$$A \cap B = [x \in R | x = 0] = [0]$$

$$B - A = [x \in R | 0 < x < 1] = (0, 1)$$

$$A^c = [x \in R | x \leq -1 \text{ or } x > 0] = (-\infty, -1] \cup (0, \infty)$$

$[B_i]_{i \geq 1}$ is an infinite sequence of subsets of U

$$\bigcup_{i=1}^{\infty} B_i = [x \in U | x \in B_i \text{ for some } i \geq 1]$$

$$\bigcap_{i=1}^{\infty} B_i = [x \in U | x \in B_i \text{ for all } i \geq 1]$$

DEF We Define *null*, the empty set (the null set) as the set containd no elements $[0, 1] \cap [2, 4] = \text{null}$

Def A and B are disjoint if $A \cap B = \text{null}$

Def A_1, \dots, A_n are mutually disjoint if $A_i \cap A_j = \text{null}$ whenever $i \neq j$

homework. 6.1 10,11,19,21,24

Chapter 7

7

7.1 1,15,38,40,41

7.2 one to one onto and inverse functions def $F : X \rightarrow Y$ is one to one if for all $x_1, x_2 \in X$ if $F(x_1) = F(x_2)$ then $x_1 = x_2 \iff$ if $x_1 \neq x_2$, then $F(x_1) \neq F(x_2)$

negation $F : X \rightarrow Y$ is not one to one iff $\exists x_1, x_2 \in X$ such that $F(x_1) = F(x_2)$ and $x_1 \neq x_2$

to prove a function is one to one :

1 Suppose $x_1, x_2 \in X$ such that $f(x_1) = f(x_2)$

2 Prove $x_1 = x_2$

onto functions

def $F : X \rightarrow Y$ is surjective or onto iff $\forall y \in Y, \exists x \in X$ such that $F(x) = y$ Negation: $F : X \rightarrow Y$ is not onto iff

$\exists y \in Y$ such that $\forall x \in X F(x) \neq y$

to prove a function is onto

1) Suppose y is any element of Y

2) show $\exists x \in X$ such that $F(x) = y$

Def $F : X \rightarrow Y$ is a one to one correspondence or a bijection iff F is both one to one and onto

Def: if $F : X \rightarrow Y$ is a bijection then there is an inverse function $F^{-1} : Y \rightarrow X$ such that $F^{-1}(F(x)) = x \iff F(F^{-1}(y)) = y$

7.2 homework : 5,10,16,36,40

7.3 composition of functions Def : let $F : X \rightarrow Y$ and $g : Y \rightarrow Z$ such that the range of f is contained in the domain of g

then $g \circ f : X \rightarrow Z$ is defined as $(g \circ f)(x) = g(f(x))$ for all $x \in X$ homework 7.3 1,3,9,16,21