

Notes on Charles Pinter's 'A Book Of Abstract Algebra'

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These are notes taken while reading Charles Pinter's 'A Book Of Abstract Algebra'

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Operations 1

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Question 1. What is an *operation* on a set A ?

Definitions 2 (Informal definition). An operation is any rule which assigns to each ordered pair of elements of A a unique element in A .

Definitions 3 (Formal definition). Let A be any set:

An operation $*$ on A is a rule which assigns to each ordered pairs (a, b) of elements of A exactly one $a * b$ in A

- $a * b$ is defined for *every* ordered pair (a, b) of elements of A .¹
- $a * b$ must be *uniquely* defined.²
- If $a, b \in A$, then $a * b \in A$.³

¹ In \mathbb{R} , division does not qualify as operation since it does not satisfy this condition. i.e. the ordered pair $(a, 0)$ has undefined quotient $a/0$.

² If \diamond is defined on (a, b) to be the number whose square is ab . In \mathbb{R} , \diamond does not qualify as an operation since $2 \diamond 2$ could be either 2, or ± 2

³ A is closed under the operation $*$

Definitions 4 (Superfluous properties). • An operation $*$ is said to be *commutative* if it satisfies

$$a * b = b * a \quad (1)$$

for any two elements a and b in A .

- An operation $*$ is said to be *associative* if it satisfies

$$(a * b) * c = a * (b * c) \quad (2)$$

for any three elements a, b and c in A .

- The *identity* element e with respect to the operation $*$ has the property that:

$$e * a = a \quad \text{and} \quad a * e = a \quad (3)$$

for every element in a in A .

- The inverse of any element a , denoted by a^{-1} has the property that;

$$a * a^{-1} = e \quad \text{and} \quad a^{-1} * a = e \quad (4)$$