

# Notes on Charles Pinter's book of abstract algebra

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These are notes taken while reading Charles Pinter's 'A Book Of Abstract Algebra'

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Operations 1

## Operations

**Question 1.** What is an *operation* on a set  $A$ ?

**Definitions 2** (Informal definition). An operation is any rule which assigns to each ordered pair of elements of  $A$  a unique element in  $A$ .

**Definitions 3** (Formal definition). Let  $A$  be any set:

An operation  $*$  on  $A$  is a rule which assigns to each ordered pairs  $(a, b)$  of elements of  $A$  exactly one  $a * b$  in  $A$

- $a * b$  is defined for *every* ordered pair  $(a, b)$  of elements of  $A$ .<sup>1</sup>
- $a * b$  must be *uniquely* defined.<sup>2</sup>
- If  $a, b \in A$ , then  $a * b \in A$ .<sup>3</sup>

**Definitions 4** (Superfluous properties). • An operation  $*$  is said to be *commutative* if it satisfies

$$a * b = b * a \quad (1)$$

for any two elements  $a$  and  $b$  in  $A$ .

- An operation  $*$  is said to be *associative* if it satisfies

$$(a * b) * c = a * (b * c) \quad (2)$$

for any three elements  $a, b$  and  $c$  in  $A$ .

- The *identity* element  $e$  with respect to the operation  $*$  has the property that:

$$e * a = a \quad \text{and} \quad a * e = a \quad (3)$$

for every element in  $a$  in  $A$ .

- The inverse of any element  $a$ , denoted by  $a^{-1}$  has the property that;

$$a * a^{-1} = e \quad \text{and} \quad a^{-1} * a = e \quad (4)$$

<sup>1</sup> In  $\mathbb{R}$ , division does not qualify as operation since it does not satisfy this condition. i.e. the ordered pair  $(a, 0)$  has undefined quotient  $a/0$ .

<sup>2</sup> If  $\diamond$  is defined on  $(a, b)$  to be the number whose square is  $ab$ . In  $\mathbb{R}$ ,  $\diamond$  does not qualify as an operation since  $2 \diamond 2$  could be either 2, or  $+2$

<sup>3</sup>  $A$  is closed under the operation  $*$