Notes on Charles Pinter's 'A Book Of Abstract Algebra'

Unathi Skosana
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These are notes taken while reading Charles Pinter's 'A Book Of Abstract Algebra'

Contents

Operations

Question 1. *What is an **operation** on a set A?*

Definitions 2 (Informal definition)**.** An operation is any rule which assigns to each ordered pair of elements of A a unique element in A.

Definitions 3 (Formal definition)**.** Let A be any set:
An operation $*$ on A is a rule which assigns to each ordered pairs (a, b) of elements of A exactly one $a * b$ in A, such that:

- $a * b$ is defined for *every* ordered pair (a, b) of elements of A. [1]
- $a * b$ must be *uniquely* defined. [2]
- If $a, b \in A$, then $a * b \in A$. [3]

[1] In $\mathbb{R}$, division does not qualify as operation since it does not satisfy this condition. i.e. the ordered pair $(a, 0)$ has undefined quotient $a/0$.
[2] If $\circ$ is defined on (a, b) to be the number whose square is ab. In $\mathbb{R}$, $\circ$ does not qualify as an operation since $2 \circ 2$ could be either 2, or +2
[3] A is closed under the operation $*$

Definitions 4 (Commutativity)**.** An operation $*$ is said to be *commutative* if it satisfies

$$a * b = b * a \tag{1}$$

for any two elements a and b in A.

Definitions 5 (Associativity)**.** An operation $*$ is said to be *associative* if it satisfies

$$(a * b) * c = a * (b * c) \tag{2}$$

for any three elements a, b and c in A.

Definitions 6 (Identity element)**.** The *identity* element e with respect to the operation $*$ has the property that:

$$e * a = a \quad \text{and} \quad a * e = a \tag{3}$$

is true for every element a in A.

Definitions 7 (Inverses)**.** The inverse of any element a, item denoted by a^{-1} has the property that:

$$a * a^{-1} = e \quad \text{and} \quad a^{-1} * a = e \tag{4}$$

is true for all element a in A.