TW364 Assignment 1 : Solutions

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Solutions

(a)
For even reflection case

$$f(x) = x^2 \qquad -\pi \leq x \leq \pi$$

For odd reflection case

$$f(x) = \begin{cases} x^2 & 0 \leq x \leq \pi \\ -x^2 & -\pi \leq x < 0 \end{cases}$$

For identify shift case

$$f(x) = \begin{cases} x^2 & 0 \leq x \leq \pi \\ \\ (x+\pi)^2 & -\pi \leq x < 0 \end{cases}$$

(b)

Even Reflection Case:

```
a0 = 2/π Integrate[x^2, {x, 0, π}]
```

$$\frac{2\,\pi^2}{3}$$

```
an = 2/π Refine[2/π Integrate[x^2 * Cos[n * x], {x, 0, π}], Element[n, Integers]]
```

$$\frac{8\,(-1)^n}{n^2\,\pi}$$

Odd Reflection Case:

```
bn = Refine[2/π Integrate[x^2 * Sin[n * x], {x, 0, π}], Element[n, Integers]]
```

$$\frac{2\left(-2+(-1)^n\left(2-n^2\,\pi^2\right)\right)}{n^3\,\pi}$$

Identity Shift Case:

```
a0 = 1/π Integrate[x^2, {x, 0, π}] + 1/π Integrate[(x + π) ^2, {x, -π, 0}]
```

$$\frac{2\,\pi^2}{3}$$

```
an = Simplify[1/π Refine[Integrate[x^2 * Cos[n * x], {x, 0, π}] +
      Integrate[(x + π) ^2 * Cos[n * x], {x, -π, 0}], Element[n, Integers]]]
```

$$\frac{2\left(1+(-1)^n\right)}{n^2}$$

```
bn = Simplify[1/π Refine[Integrate[x^2 * Sin[n * x], {x, 0, π}] +
      Integrate[(x + π) ^2 * Sin[n * x], {x, -π, 0}], Element[n, Integers]]]
```

$$-\frac{\left(1+(-1)^n\right)\pi}{n}$$

(c)

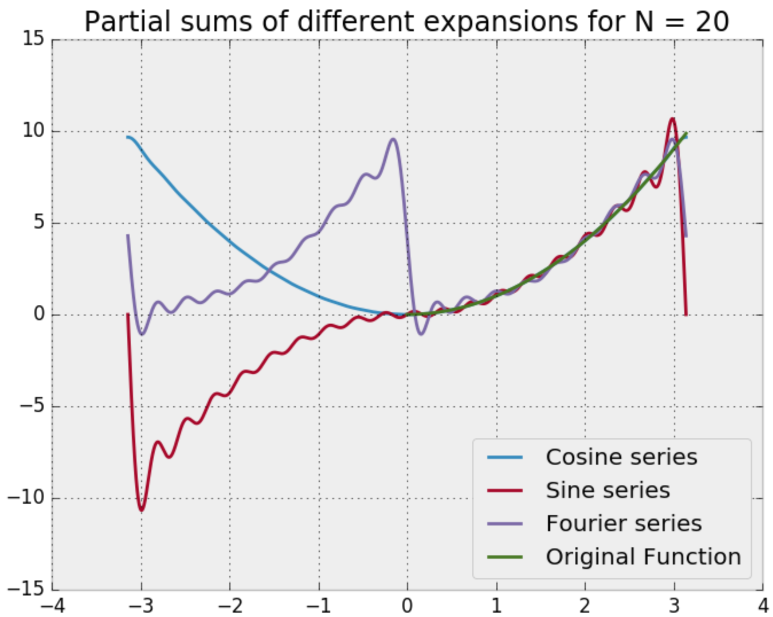

Figure 1: $S_{10}(x)$ for different extensions of x^2 on the interval $x \in [-\pi, \pi]$

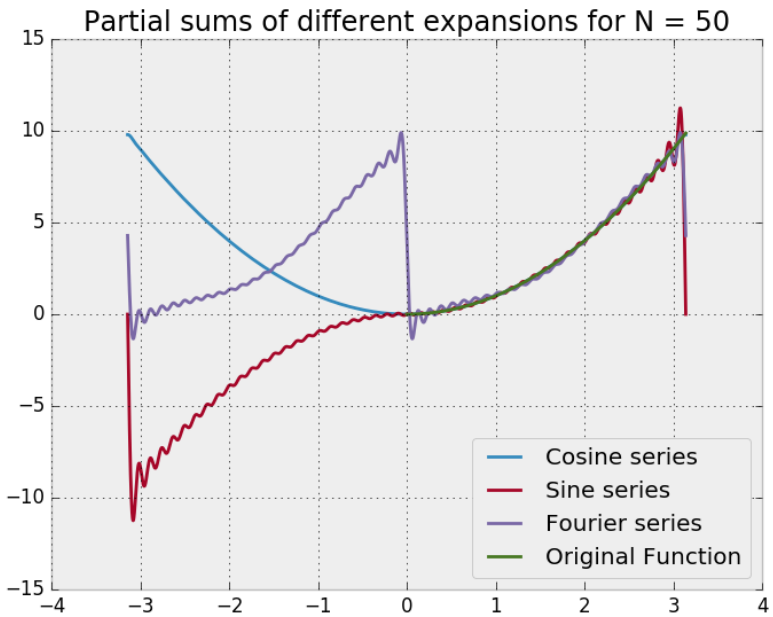

Figure 2: $S_{20}(x)$ for different extensions of x^2 on the interval $x \in [-\pi, \pi]$

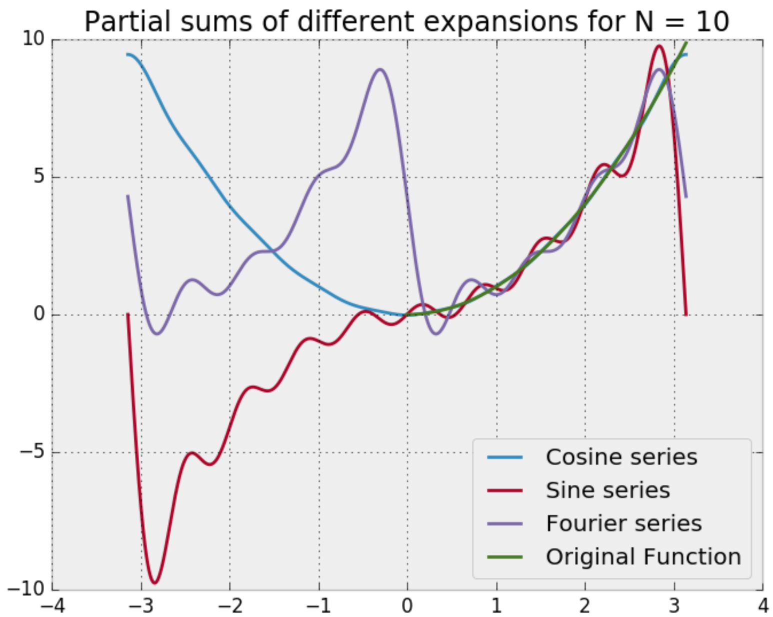

Figure 3: $S_{100}(x)$ for different extensions of x^2 on the interval $x \in [-\pi, \pi]$

(d)

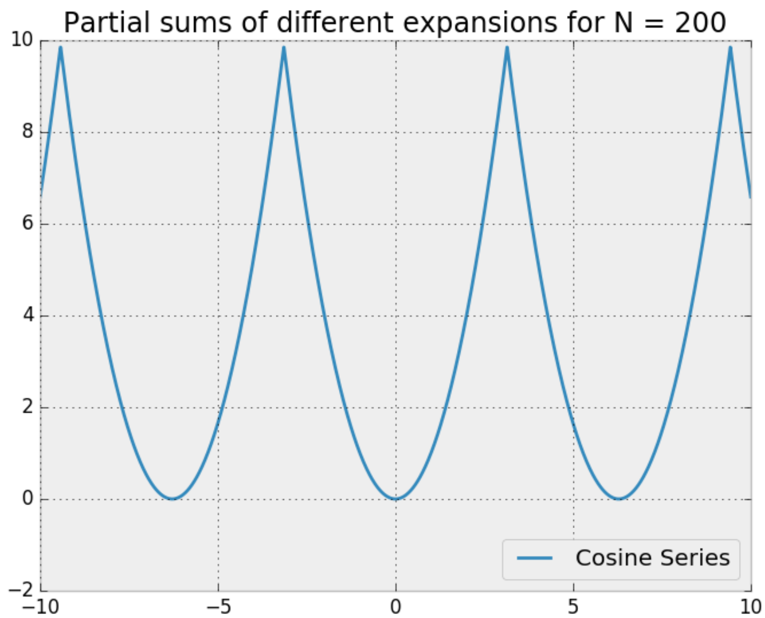

Figure 4: Even periodic extension of x^2 on the interval $x \in [-100, 100]$ for $N = 200$

We observe no Gibbs oscillations for this particular periodic extension because the even periodic extensions of x^2 on the real line do suffer from any jump discontinuities.

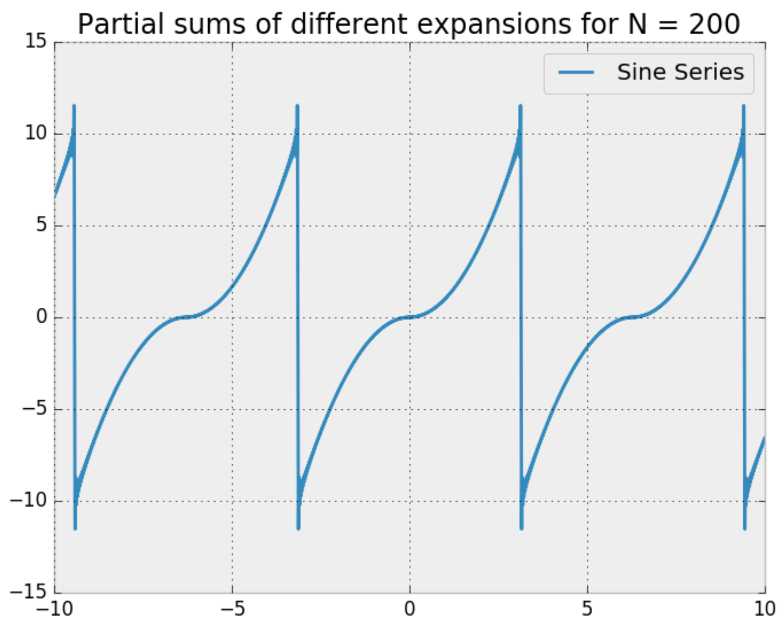

Figure 5: Odd periodic extension of x^2 on the interval $x \in [-10, 10]$ for $N = 200$

The size of the overshoot should be approximately8.95% of the size of the jump, the size of the jump in this case being $2\pi * 2$. Thus

$$\text{overshoot} = \frac{8.95}{100}(2\pi^2) \approx 1.77$$

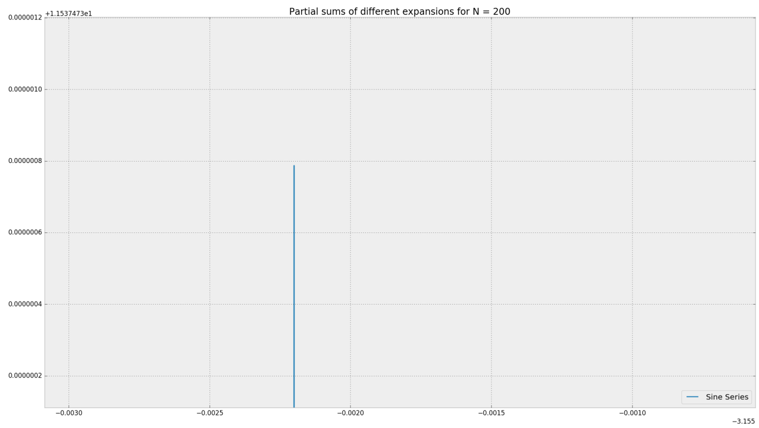

Figure 6: Zoom in on the peak at $x_0 = 0$

The picture above shows hat the value of the partial sums around the discontinuity at $x = 0$ goes to 11.54, which approximately $1.77 + 2 * \pi^2$

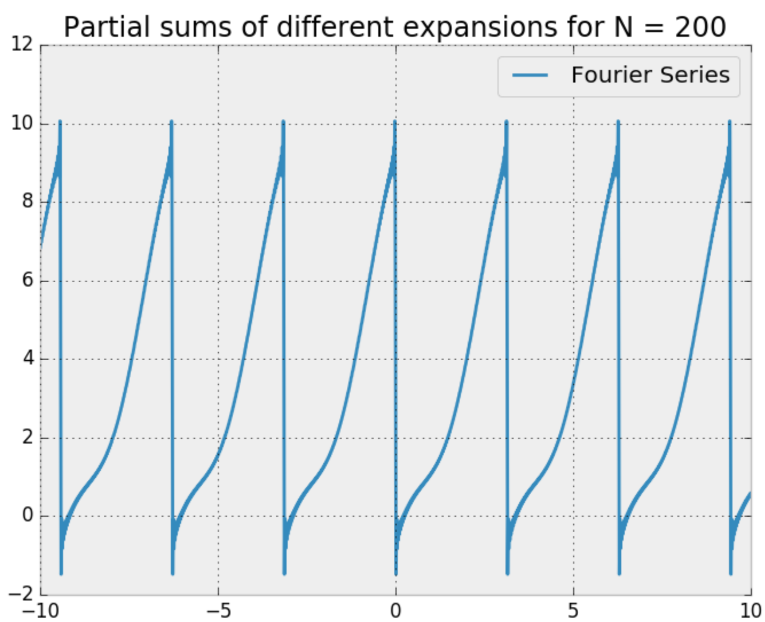

Figure 7: Identity shift periodic extension of x^2 on the interval $x \in [-10, 10]$ for $N = 200$

The size of the jump in this case is π^2, thus

$$\text{overshoot} = \frac{8.95}{100}(\pi^2) \approx 0.89$$

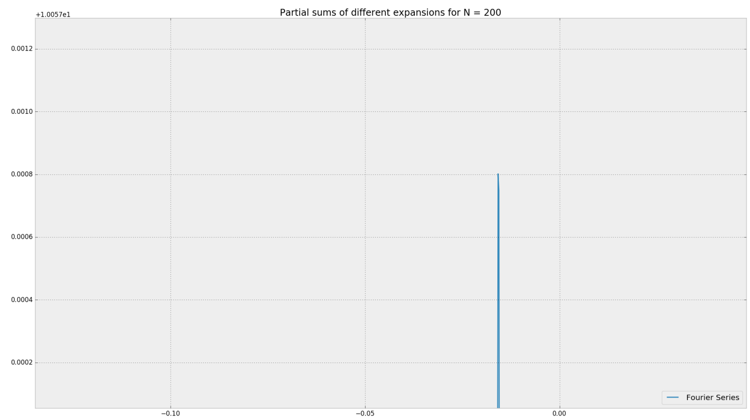

Figure 8: Zoom in on the peak at $x_0 = 0$

The picture above shows hat the value of the partial sums around the discontinuity at $x = 0$ goes to 10.057, which approximately $0.89 + \pi^2$. In the limit to infinity the overshoot should exactly match the theoretical value.