Even Reflection Case:

$$a0 = \frac{2}{\pi}\,\text{Integrate}[x\text{^}2,\ \{x,\ 0,\ \pi\}]$$

$$\frac{2\,\pi^2}{3}$$

$$an = \frac{2}{\pi}\,\text{Refine}\left[\frac{2}{\pi}\,\text{Integrate}[x\text{^}2 * \text{Cos}[n * x],\ \{x,\ 0,\ \pi\}],\ \text{Element}[n,\ \text{Integers}]\right]$$

$$\frac{8\,(-1)^n}{n^2\,\pi}$$

Odd Reflection Case:

$$bn = \text{Refine}\left[\frac{2}{\pi}\,\text{Integrate}[x\text{^}2 * \text{Sin}[n * x],\ \{x,\ 0,\ \pi\}],\ \text{Element}[n,\ \text{Integers}]\right]$$

$$\frac{2\left(-2 + (-1)^n\left(2 - n^2\,\pi^2\right)\right)}{n^3\,\pi}$$

Identity Shift Case:

$$a0 = \frac{1}{\pi}\,\text{Integrate}[x\text{^}2,\ \{x,\ 0,\ \pi\}] + \frac{1}{\pi}\,\text{Integrate}[(x+\pi)\text{^}2,\ \{x,\ -\pi,\ 0\}]$$

$$\frac{2\,\pi^2}{3}$$

$$an = \text{Simplify}\left[\frac{1}{\pi}\,\text{Refine}[\text{Integrate}[x\text{^}2 * \text{Cos}[n * x],\ \{x,\ 0,\ \pi\}] + \text{Integrate}[(x+\pi)\text{^}2 * \text{Cos}[n * x],\ \{x,\ -\pi,\ 0\}],\ \text{Element}[n,\ \text{Integers}]]\right]$$

$$\frac{2\left(1 + (-1)^n\right)}{n^2}$$

$$bn = \text{Simplify}\left[\frac{1}{\pi}\,\text{Refine}[\text{Integrate}[x\text{^}2 * \text{Sin}[n * x],\ \{x,\ 0,\ \pi\}] + \text{Integrate}[(x+\pi)\text{^}2 * \text{Sin}[n * x],\ \{x,\ -\pi,\ 0\}],\ \text{Element}[n,\ \text{Integers}]]\right]$$

$$-\frac{\left(1 + (-1)^n\right)\pi}{n}$$