

End-to-end Integration of Hyperparameter Tuning into Variational Quantum Algorithms

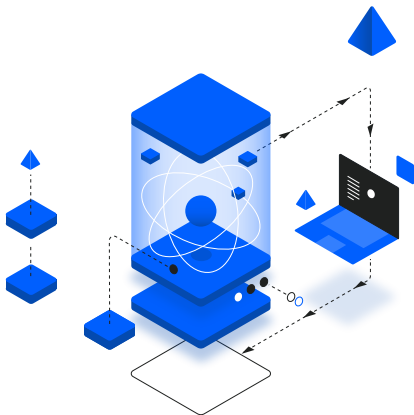
Unathi Skosana and Mark Tame

Stellenbosch University

SAIP2024

Quantum Computing as a New Paradigm for Computation

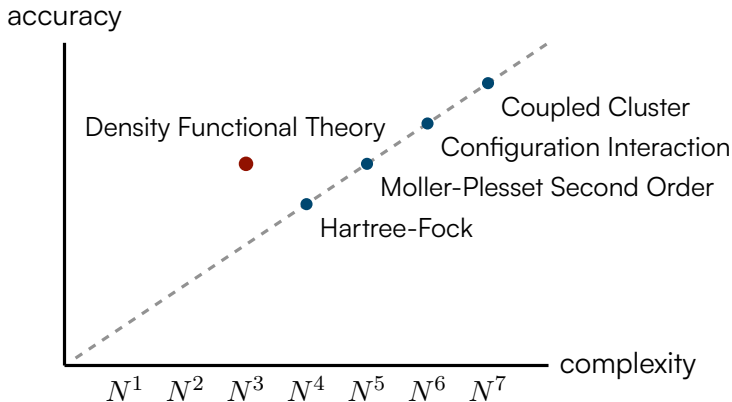
Quantum computing uses specialized **quantum technology** to solve **complex problems** that classical computers cannot solve **quickly enough**.



Credit: PSNC

Examples of Complex Problems

Quantum Chemistry, Electronic Structure of Molecular Systems.



Credit: Janosh Riebesell

Examples of Complex Problems

Discrete logarithm, Prime factorization in cryptography.

Easy to compute forward

$$x = g^a$$

Given a and g

$$G = \{1, g, g^2, \dots, g^{N-1}\}, \quad g^N = 1$$

Given p, x and g

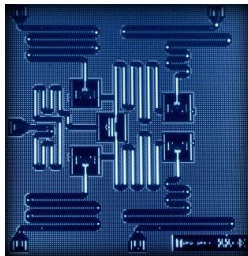
$$a = \log_g(x \bmod p)$$

Hard to compute backward

Credit: Anthropic's Claude Sonnet 3.5 LLM

Quantum Computing Hardware Since 2016

- The first generally available cloud-based quantum processor had 5 quantum bits, or qubits:



Credit: IBM

- Use cases limited to proof-of-concept demonstrations:

arXiv > quant-ph > arXiv:2103.13855

Quantum Physics



[Submitted on 25 Mar 2021 (v1), last revised 19 Sep 2022 (this version, v3)]

Demonstration of Shor's factoring algorithm for $N=21$ on IBM quantum processors

Unathi Skosana, Mark Tame

Scaling Up Quantum Computing Hardware

Generally available state-of-the-art devices can have as many as 433 noisy qubits.

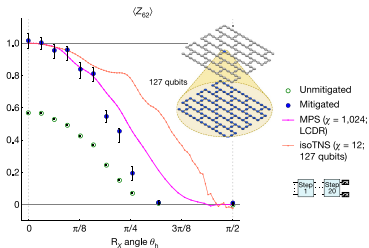
	2022	2023	2024	2025
	433 Qubit	1121 Qubit	1386 Qubit	4158+ Qubit
	25 Qubit	29 Qubit	35 Qubit	64 Qubit
	0 Qubit	0 Qubit	0 Qubit	1000000+ Qubit

Credit: Tobias Osborne

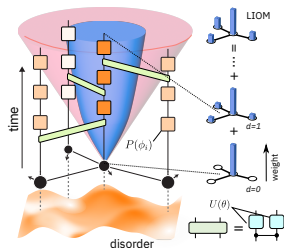
Despite increasing qubit numbers, Quantum Advantage is yet to be realized.

Variational Quantum Algorithms (VQAs) in the Wild

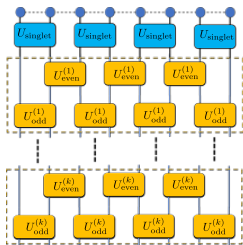
Utility Before Fault Tolerance (Nature 618, 500—505)



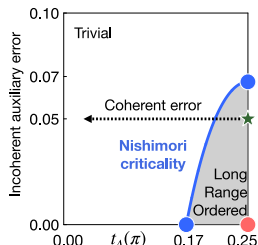
Quantum Many-body Dynamics (arXiv:2307.07552)



Quantum Spin Chains (arXiv:2207.0999)



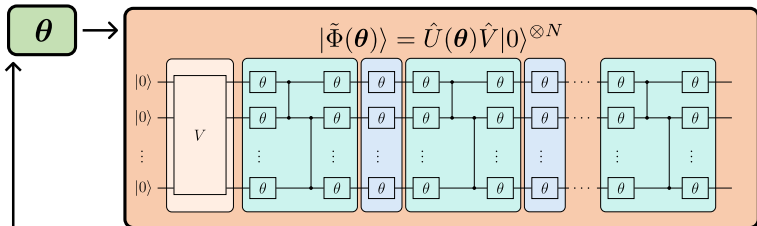
Nishimori transition (arXiv:2309.02863)



Variational Quantum Algorithms (VQAs)

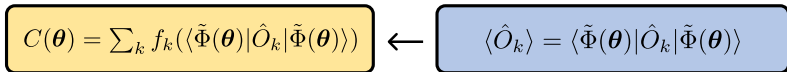
Parameter initialization

State preparation



Cost function evaluation

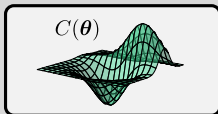
Measurement of observables



Classical optimization

Navigate cost landscape

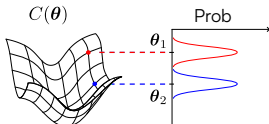
Update parameters



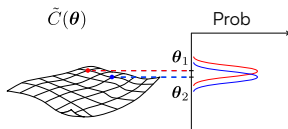
$$\theta'_j = \theta_j - \alpha \frac{\partial C(\theta)}{\partial \theta_j}$$

Cost Landscapes in Variational Quantum Algorithms

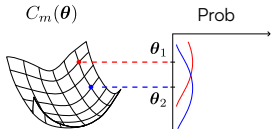
(a) Noiseless cost landscape



(b) Noisy cost landscape



(c) Error-mitigated cost landscape



(a) Clear separation of cost function values.

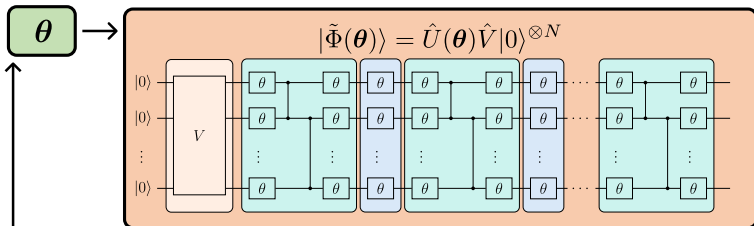
(b) Concentration of cost function values.

(c) Recovery of features keys of noiseless cost function values.

Hyperparameter Tuning

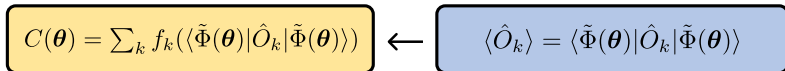
Parameter initialization

State preparation

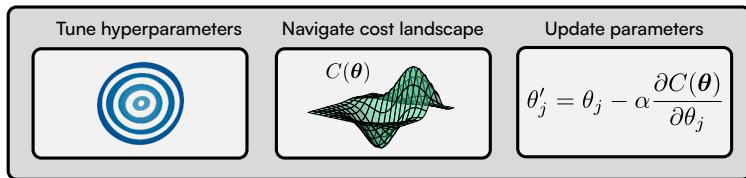


Cost function evaluation

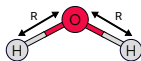
Measurement of observables



Classical optimization



Case study: Estimating Ground State Energies

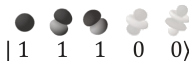


Expand Hamiltonian
in STO-6G basis set

$$\hat{H} = \sum_{ij}^{N_b} \langle i | \hat{T}_e + \hat{V}_{en} | j \rangle \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2} \sum_{ijkl}^{N_b} \langle ik | \hat{V}_{ee} | jl \rangle \hat{a}_i^\dagger \hat{a}_k^\dagger \hat{a}_l \hat{a}_j$$

$\hat{a}_j \rightarrow \frac{\hat{\sigma}_x + i\hat{\sigma}_y}{2} \otimes \hat{\sigma}_z \otimes \dots \otimes \hat{\sigma}_z$ Qubit-mapping
Electronic Hamiltonian

$$\hat{H}_q = \sum_i c_i \prod_k^{N_b} \hat{\sigma}_k^{(i)}$$



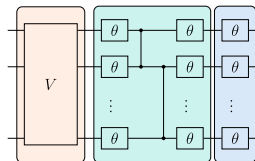
Qubit-mapping
Electronic States

$$C(\theta) = \sum_i c_i \langle \Phi_0 | \hat{U}^\dagger(\theta) \prod_k^{N_b} \hat{\sigma}_k^{(i)} \hat{U}(\theta) | \Phi_0 \rangle$$

VQA

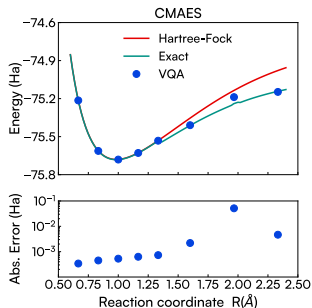
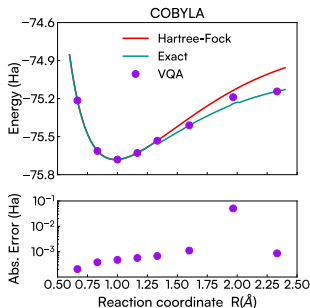
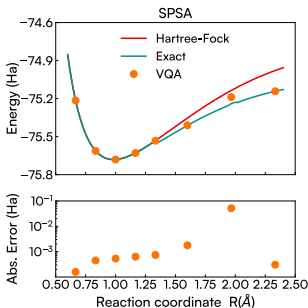
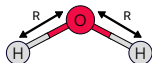
$$\theta^* = \underset{\theta}{\operatorname{argmin}} C(\theta)$$

Layers of rotation
and entangling gates

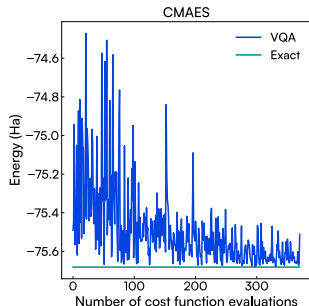
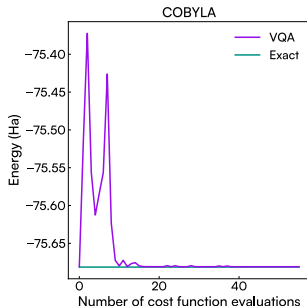
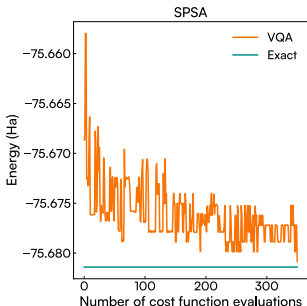
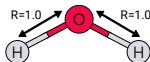


$$U(\theta) =$$

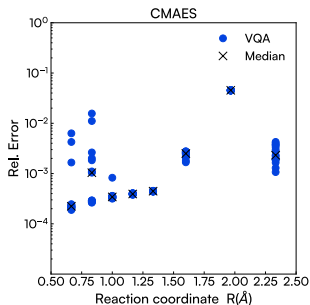
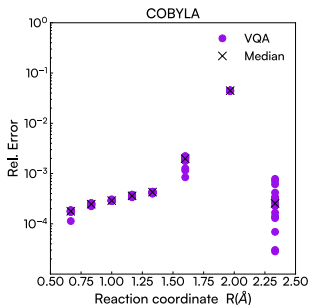
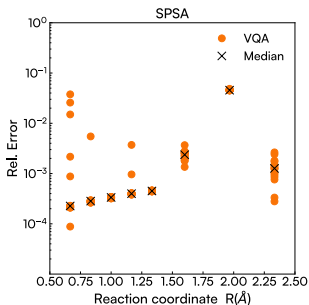
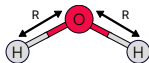
Noiseless Simulation: Potential Energy Surface



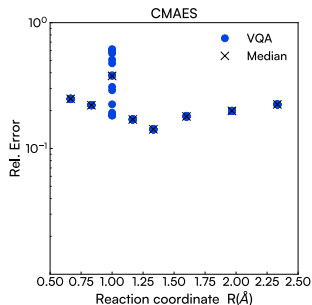
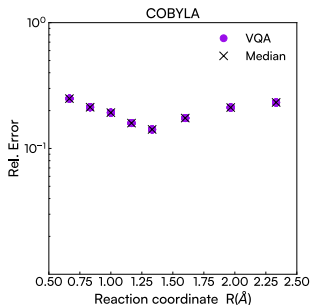
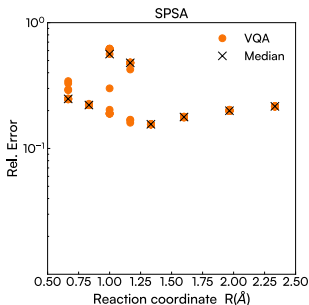
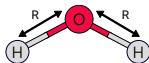
Noiseless Simulation: Cost Function Evaluations



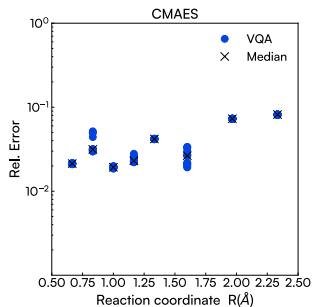
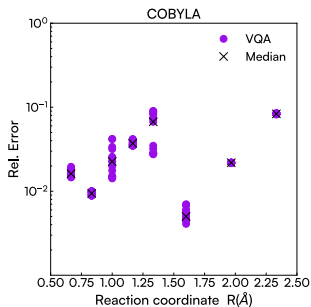
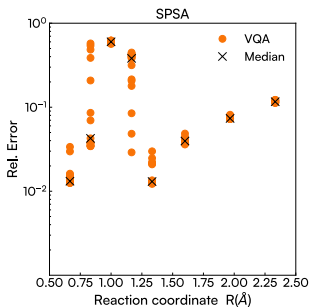
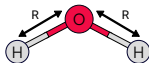
1. Noiseless Simulation: Relative Error



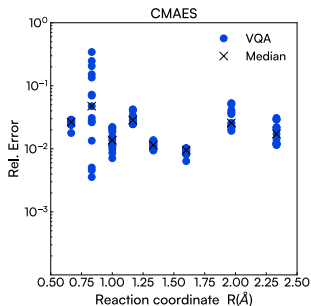
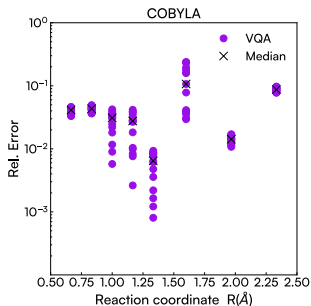
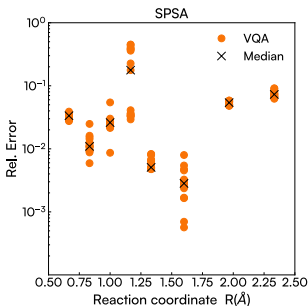
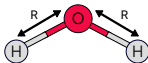
2. Noisy Simulation: Relative Error



3. Noisy + Readout Error Mitigation Simulation: Relative Error



4. Noisy + Readout Error Mitigation + Zero Noise Extrapolation Simulation: Relative Error



Concluding remarks

- Variational Quantum Algorithms as stepping stones towards Quantum Advantage on start-of-the-art quantum hardware.
- Performance and reliability of Variational Quantum Algorithms is significantly influenced by the behavior of the chosen optimization algorithm.
- Hyperparameter tuning as means to get the best out of classical resources on the way to Quantum Advantage.

Thank You for Listening

