Modeling of Measurement-based Quantum Computing on IBM Q Experience Devices

Unathi K. Skosana [1] Supervisor: Prof. Mark Tame [2]

Department of Physics, Stellenbosch University

UNIVERSITEIT·STELLENBOSCH·UNIVERSITY
jou kennisvennoot • your knowledge partner

One-way quantum computing

One-way quantum computing is a one of many frameworks of quantum measurement-based quantum computing (MBQC).

In such framework, a quantum computation is not realized through explicit unitary evolution of qubits mediated by unitary quantum gates as in the quantum circuit model.

Rather, a computation is realized through sequential single-qubit measurements on an initial resource state [2].

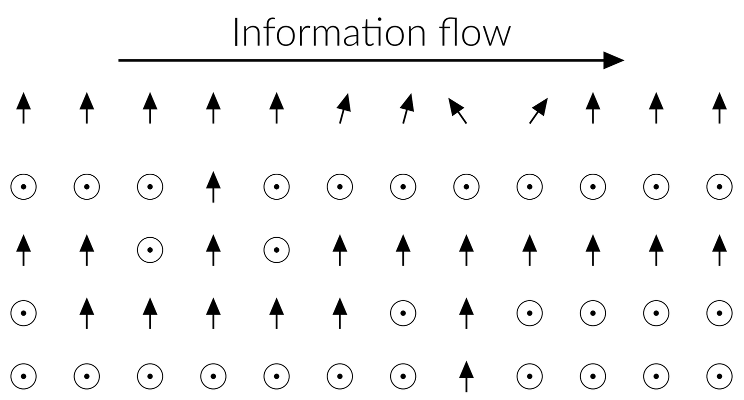


Figure 1. Propagation of information through an entangled state $|\phi\rangle_{\mathcal{C}}$ via measurements. Tilted arrows represented $x-y$ plane measurements, vertical arrows represent measurements of $\hat{\sigma}_x$ and the circles denote measurement of $\hat{\sigma}_z$

Graph states for quantum computation

The initial resource states are provided in the form of highly entangled multi-qubit states called **graph states**

$$|G\rangle = \prod_{\{k,l\}\in E} \mathrm{CZ}^{kl} \, |+\rangle^{\otimes |V|} \tag{1}$$

The two-qubit gate CZ^{kl} (controlled $\hat{\sigma}_z$) is applied between pairs of vertices according to the edge set E of the underlying graph $G = (V, E)$, describing the topology of the state.

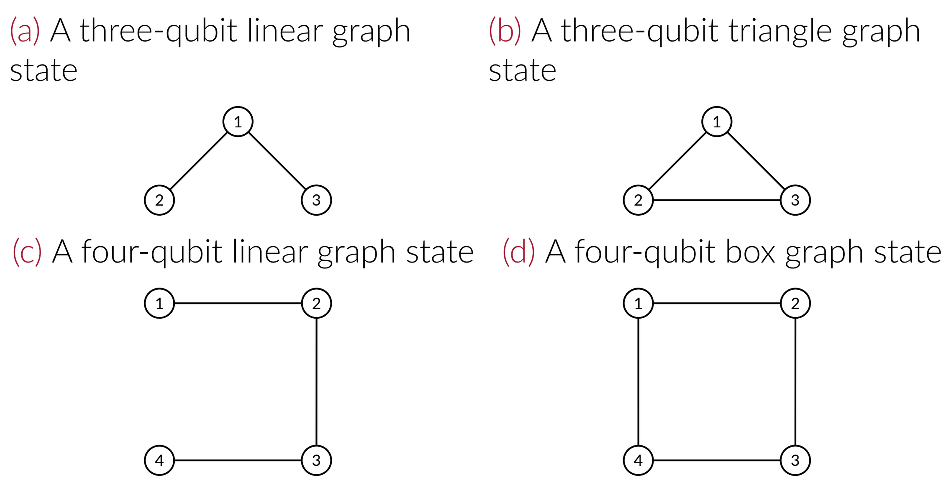


Figure 2. A few representives graph states.

Local unitaries

Two graph state vectors $|G\rangle$ and $|G'\rangle$ and their underlying graphs G and G' are said to equivalent to another under a local unitary $\hat{U}$

$$|G'\rangle = \hat{U}\,|G\rangle \tag{2}$$

The graph states $|G\rangle$ and $|G'\rangle$ are said to belong some equivalence class and within such a class, all graph states are equivalent modulo local unitaries (LU).

Edge local complementaion and equivalence classes

From an underling graph $G = (V, E)$ of a graph state $|G\rangle$, a new graph $G' = (V, E')$ and an associated graph state $|G'\rangle$ can be realized via the complement of a subgraph induced by the neighborhood of some vertex $k \in V$ of G. Then the two graph states $|G\rangle$ and $|G'\rangle$ are said belong to the same **LU-equivalence** class [1].

Equivalence classes of graph states

Unitary for edge complementaion

The mapping $\tau^k : G \mapsto G'$ under edge local complementaion is mediated by a unitary operation $\hat{U}^k$ [1].

$$\left|\tau^k(G)\right\rangle = \hat{U}^k(G)\,|G\rangle \tag{3}$$

where $\hat{U}^k$ is of the form

$$\hat{U}^k(G) = (-i\hat{\sigma}_x^k)^{1/2} \prod_{l\in\eta_k} (i\hat{\sigma}_z^l)^{1/2} \tag{4}$$

Four-qubit linear graph state $\overset{\text{LU}}{\equiv}$ Four-qubit box graph state

Starting from the four-qubit linear graph state as in Fig. 2c, a sequential application of edge local complementaion on the vertices $3, 2$ and 3 produces a four-qubit box graph state similar to the one in Fig. 2d

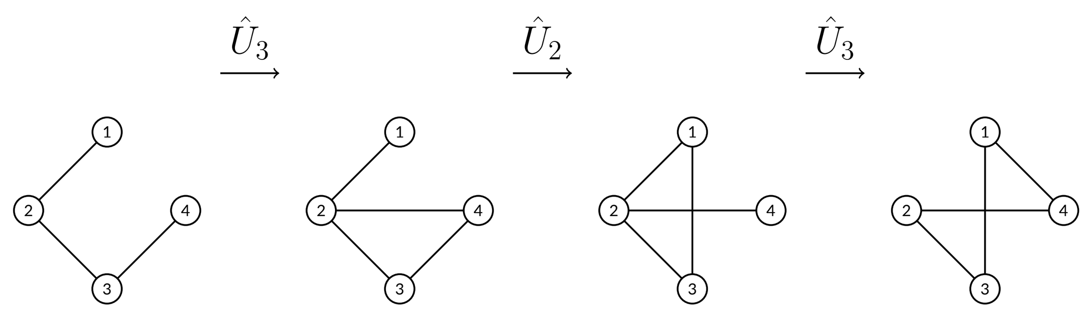


Figure 3. Sequentially applying the edge local complementaion rule on a four-qubit linear graph state showing that such a state is LU-equivalent to four-qubit box graph state under unitary transformation $\hat{U}^3\hat{U}^2\hat{U}^3 = \hat{\sigma}_z^1 \otimes \hat{H}^2 \otimes \hat{H}^3 \otimes \hat{\sigma}_z^4$

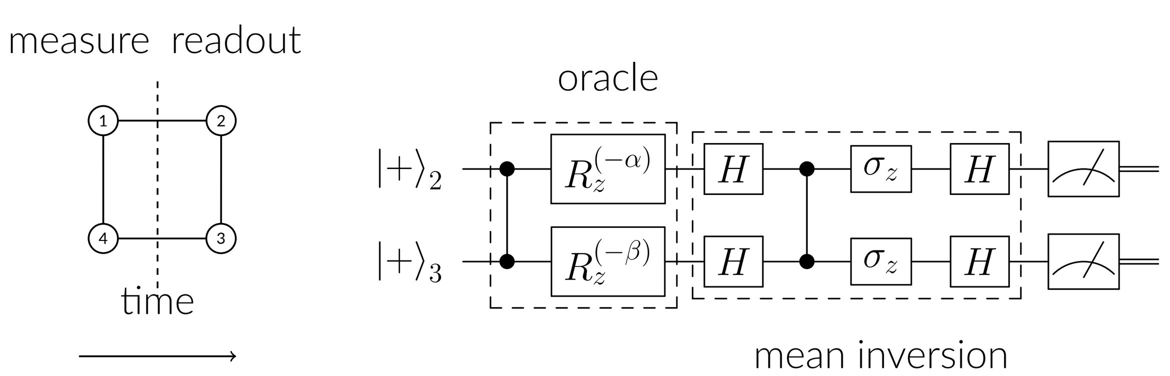
Under the vertex-mapping bijection that swaps vertices 1 and 4, the graphs in the last step of Fig. 3 and Fig. 2d are isomorphic to one another.

Case Study 00 : Grover's quantum search algorithm

Two-qubit quantum search on four-qubit box graph state

A two-qubit measurement-based version of Grover's quantum search algorithm can be realized on the four-qubit box graph state [4].

Through measurements of qubits 1 and 4 of Fig. 2d in the basis $B_j(\alpha) = \{|+\alpha\rangle_j\,, |-\alpha\rangle_j\}$ where $|\pm\alpha\rangle = \frac{1}{\sqrt{2}}(|0\rangle_j \pm e^{i\alpha}\,|1\rangle_j) \quad (\alpha \in \mathbb{R})$. Reading out qubits 2 and 3 in the basis $B(\pi) = \frac{1}{\sqrt{2}}(|0\rangle_j \mp |1\rangle_j)$.



Each of the four cases, where the oracle tags the elements 00, 01, 10 and 11 are realized by the measurement settings $\alpha\beta = \pi\pi$, $\pi 0$, 0π and 00 respectively.

Caveats

The outcome of a measurement in the basis $B_i(\alpha)$ on qubit i is o_i, which is 0(1) for a measurement of $|+\alpha\rangle$ ($|-\alpha\rangle$).

If the measurement on qubits 1 and 4 are anything either than $o_1 = 0$ and $o_4 = 0$, a Pauli errors are introduced and there is a need to reinterpret the outcomes on qubits 2 and 3. In such a scenario, a feedforward of the outcomes must be applied $\{o_2 \oplus o_4, o_3 \oplus o_1\}$.

Case study 01 : Improved quantum search

The LU-equivalence and isomorphism of the four-qubit linear and box graph state offers a way to improve the performance of the former implementation.

The resultant LU-equivalent four-qubit linear graph state has one less CZ gate than the four-qubit graph state, which significantly improves upon the performance of the former implementation since two-qubit gates have error rates that are much higher than single-qubit gates.

The measurement procedure is the same as in the former implementation, however the feedforward relation has modified to $\{o_2 \oplus o_1, o_3 \oplus o_4\}$ since qubits 2 and 3 have to swapped. Similarly measurement outcomes have to swapped.

Qiskit experiments

To model the algorithms described in two case studies, the quantum circuits in Fig. 4 and Fig. 5 were executed on the IBM Q 5 vigo machine, with trials of 8100 shots.

Since the IBM Q 5 vigo machine doesn't have the necessary topology to realize a four-qubit box graph state, the mapping of logical qubits to physical qubits was chosen as to minimize the number of swaps gate in the resulting transpiled circuit.

To further limit the number of gates, the feedforward operations which would be CNOT gates on the relevant qubits were done post-experiment. Similarly, the swap of qubits 2 and 3 in the algorithm in case study 01 was also done post-experiment.

The four-qubit linear graph state passed an multi-partite stabilizer-based entanglement witness test [3], giving an expectation value of $\langle \mathcal{W} \rangle = -0.45 \pm 0.261$ while the four-qubit box graph state narrowly failed the entanglement witness test, giving $\langle \mathcal{W} \rangle = 0.003 \pm 0.260$.

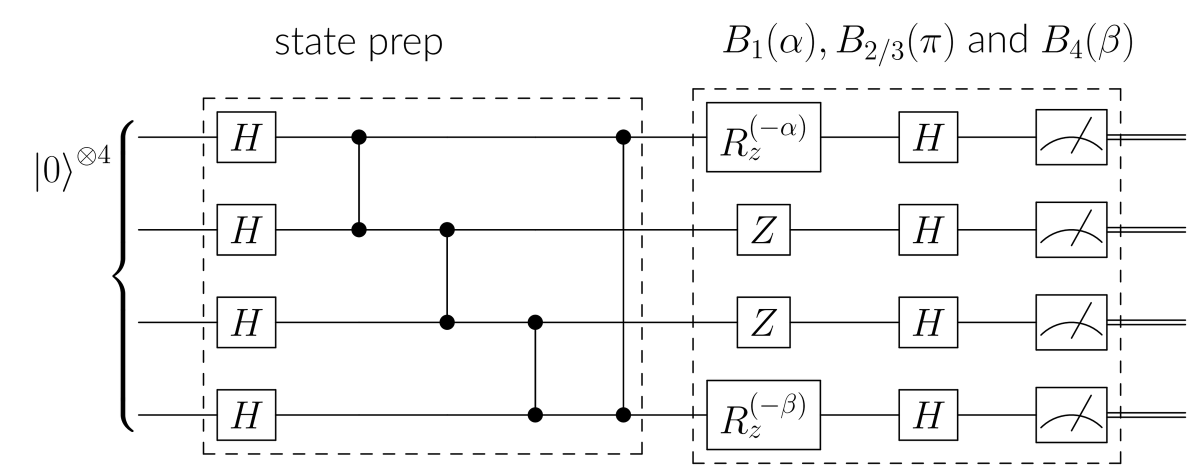


Figure 4. Measurement-based two-qubit Grover's algorithm on a four-qubit box graph state

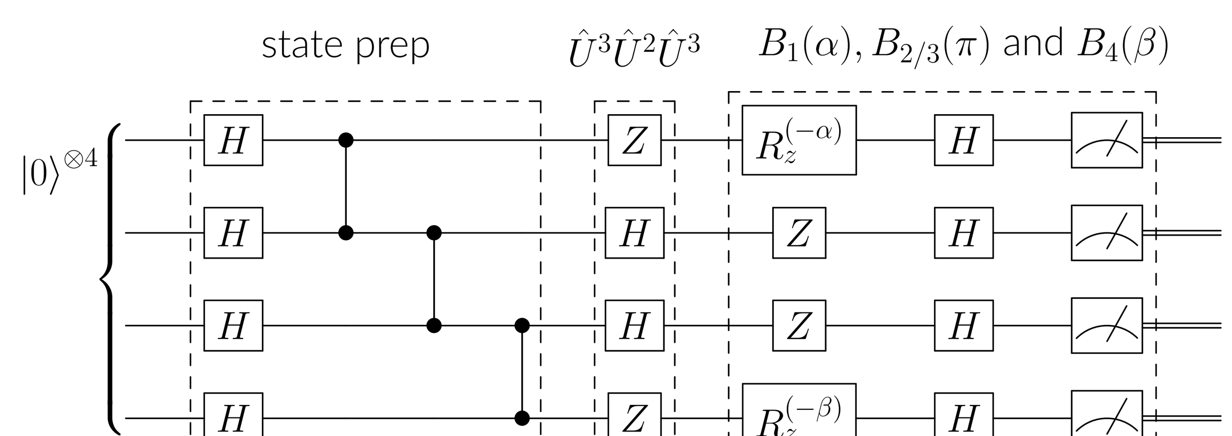


Figure 5. Measurement-based two-qubit Grover's algorithm on a four-qubit linear graph state

Results

Case study 00

The results below were obtained for the algorithm described in case study 00, showing the feedforward and no feedforward results. In all cases,the success probabilities are approximately 75%.

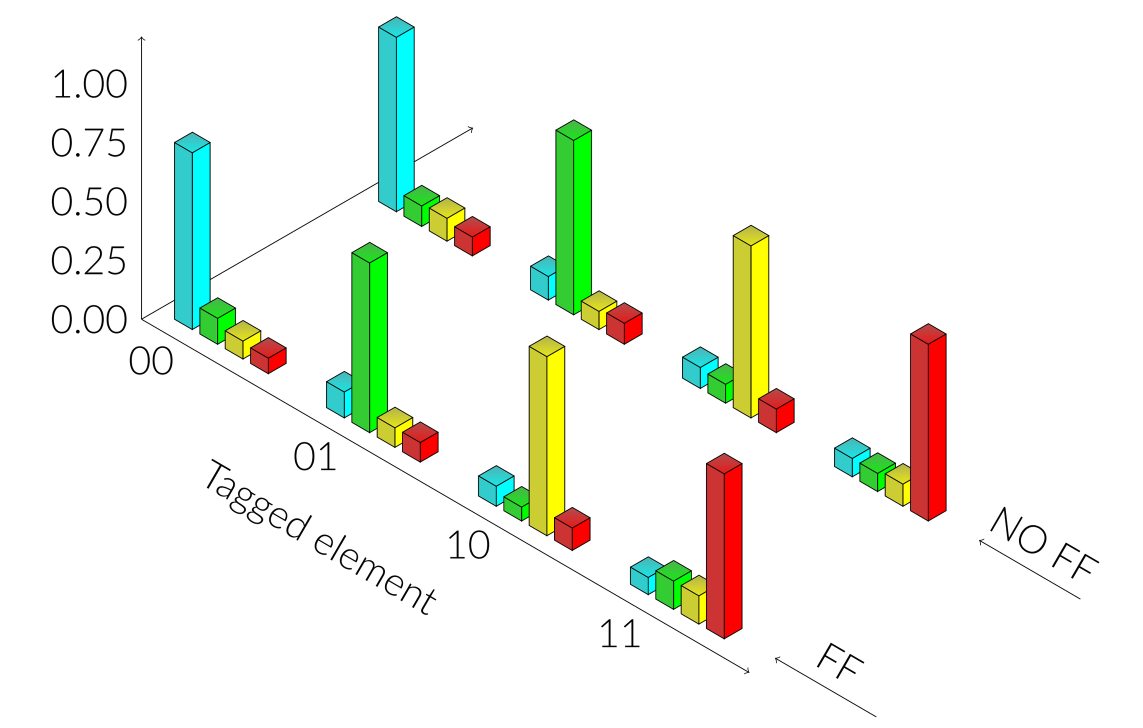


Figure 6. The measured outputs of Grover's algorithm on ibmq_vigo. The data labeled as 'NO FF' show the outputs in those cases the outputs $\{o_1, o_4\}$ were both zero. The data labeled 'FF' show the outputs to which the feedfoward relation was applied

Case study 01

The results below were those obtained for the algorithm described in case study 01. Success probabilities are all well over 80%, making this computation almost as good as the computation in the quantum circuit model computation albeit the greater number of gates.

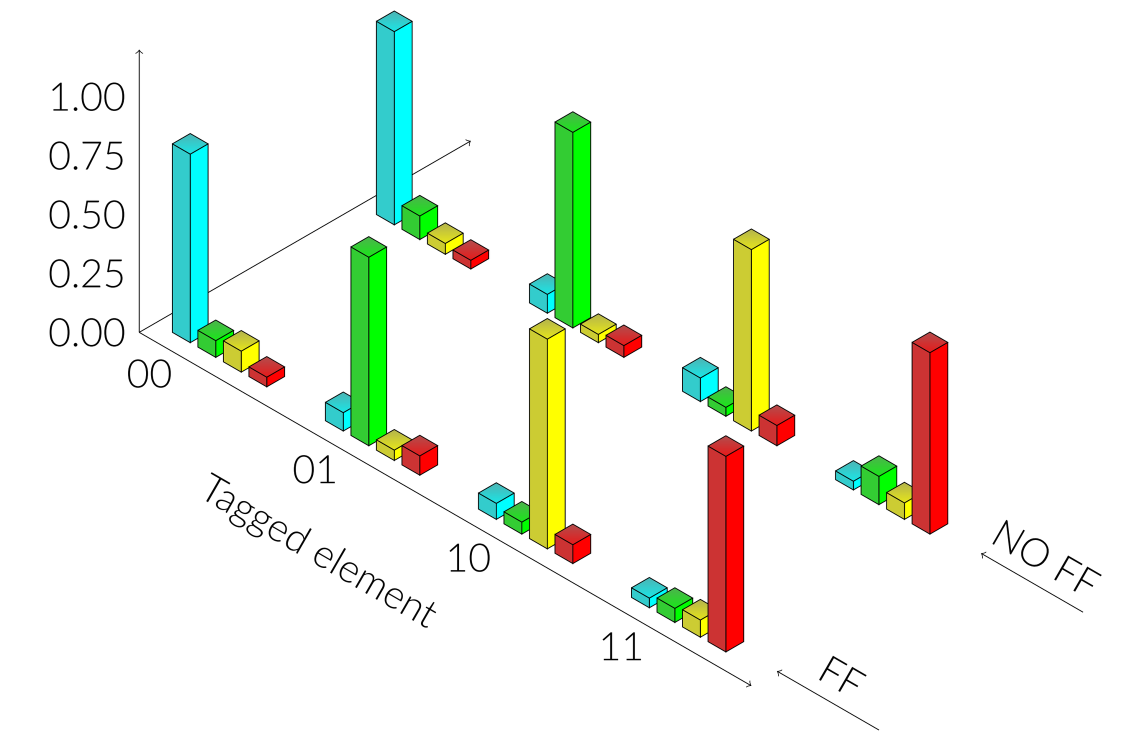


Figure 7. The measured outputs of Grover's algorithm on ibmq_vigo. The data labeled as 'NO FF' show the outputs in those cases the outputs $\{o_1, o_4\}$ were both zero. The data labeled 'FF' show the outputs to which the feedfoward relation was applied

References

[1] M. Hein, J. Eisert, and H. J. Briegel.
Multiparty entanglement in graph states.
Phys. Rev. A, 69:062311, Jun 2004.

[2] Robert Raussendorf, Daniel E. Browne, and Hans J. Briegel.
Measurement-based quantum computation on cluster states.
Phys. Rev. A, 68:022312, Aug 2003.

[3] Géza Tóth and Otfried Gühne.
Entanglement detection in the stabilizer formalism.
Phys. Rev. A, 72:022340, Aug 2005.

[4] P. Walther, K. J. Resch, T. Rudolph, E. Schenck, H. Weinfurter, V. Vedral, M. Aspelmeyer, and A. Zeilinger.
Experimental one-way quantum computing.
Nature, 434(7030):169–176, Mar 2005.