Modeling of Measurement-based Quantum Computing on IBM Q Experience Devices

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Introduction

Quantum computation promises to be an exciting field of research. In recent years, advancements of both an experimental and theoretical nature have put the field to forefront of modern science.Much of this excitement stems from a handful of applications i.e. quantum system simulation, prime factorization, quantum search etc. where quantum computers are believed to be far more efficient at such tasks than their classical counterparts [3].

Several framework of quantum computation exist and differ significantly from one another but the advent of measurement-based frameworks have underlined the prominent roles of measurements and entanglement.

Measurement-based quantum computing provides a new conceptual framework in which the role of measurement and entanglement in quantum mechanics can be addressed/understood while also circumventing common practical issues in their experimental realizations [3].

However a few questions are yet to be answered, as it is still unknown whether measurement-based quantum computation performs better than the standard model on faulty qubits. This ongoing study attempts to start a give a glimpse of an answer to some of these questions.

Quantum computation models

There exist several quantum computation models, differing in the ways in which quantum information is processed. Among the popular models this include:

- **Network model** Based on sequences of unitary quantum logic gates that transform the state of qubits.
- **Teleportation-based models** Measurement-based approach which involves transportation of quantum states (or information) through a classical communication channel and a shared entangled state.
- **One way quantum computation** Another measurement-based approach in which a computation proceeds via repeated rounds of single-qubit measurements on highly entangled states i.e. cluster / graph states.

This ongoing study is confined to only one-way and network frameworks of quantum computing. For a specific quantum algorithm, the performances of the two frameworks were compared.

A one way quantum computer

In the framework of one way quantum computing, entanglement and measurements play very prominent roles.

Entanglement produces a resource for the computation in the form of entangled state and a set of local, single-qubit projective measurements encoding logical qubits that drive the computation.

For a sufficiently large entangled state $|\phi\rangle_C$ i.e. a two dimensional lattice, a universal set of quantum logic operations can be realized [3].

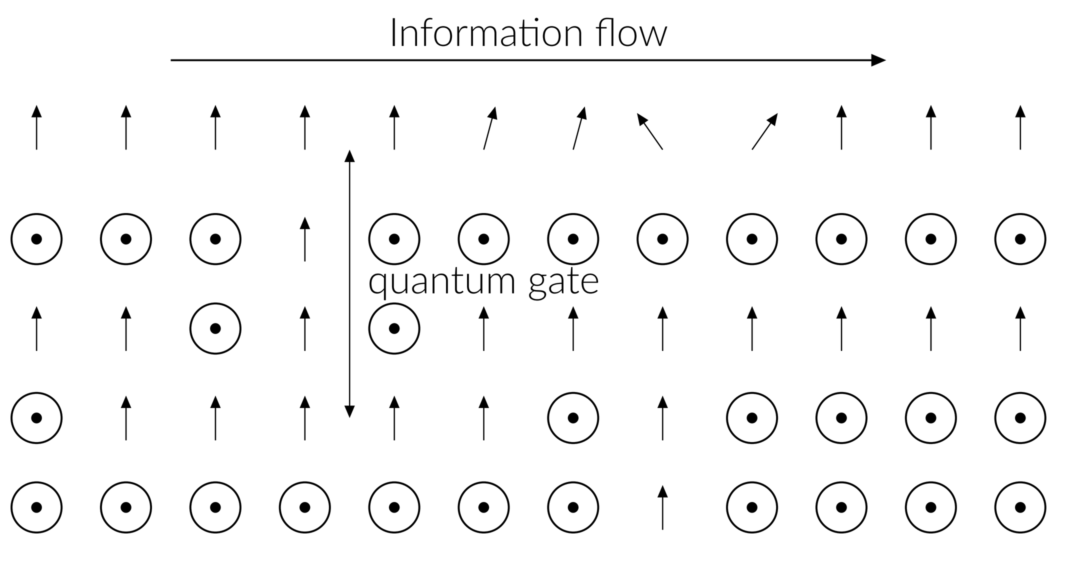


Figure 1. Propagation of information through an entangled state $|\phi\rangle_C$ via measurements. Tilted arrows represented $x-y$ plane measurements, vertical arrows represent measurements of σ_x and the circles denote measurement of σ_z.

Cluster state as computational resource

The computational resource is provided in the form of highly entangled multi-particle states called **graph states** [3], which can be parameterized in terms of mathematical graphs .i.e. a set of vertices and edges connecting them $G = (V, E)$. **Cluster states** belong to a subclass of the graph states.

These states are prepared beforehand and independent of the computation to be performed, which means numerous computations can be done on the same cluster state if sufficiently large.

Cluster states

For such a graph G, a graph state $|\phi\rangle_G$ is associated. The graph state $|\phi\rangle_G$ is completely determined by eigenvalue equation [1, 4].

$$K_j\,|\phi\rangle_G = \sigma_x^{(j)} \otimes_{i\in \mathrm{nghb}(a)} \sigma_z^{(i)}\,|\phi\rangle_G = |\phi\rangle_G \tag{1}$$

The cluster states of size $N = 2, 3, 4$

$$|\phi\rangle_{\mathcal{C}_2} = \frac{1}{\sqrt{2}}(|0\rangle_1 |+\rangle_2 + |1\rangle_1 |-\rangle_2) \tag{2}$$

$$|\phi\rangle_{\mathcal{C}_3} = \frac{1}{\sqrt{2}}(|+\rangle_1 |0\rangle_2 |+\rangle_3 + |-\rangle_1 |1\rangle_2 |-\rangle_2) \tag{3}$$

$$|\phi\rangle_{\mathcal{C}_4} = \frac{1}{2}\big(|+\rangle_1 |0\rangle_2 |+\rangle_3 |0\rangle_4 + |+\rangle_1 |0\rangle_2 |-\rangle_3 |1\rangle_4 + |-\rangle_1 |1\rangle_2 |-\rangle_3 |0\rangle_4 + |-\rangle_1 |1\rangle_2 |+\rangle_3 |1\rangle_4\big) \tag{4}$$

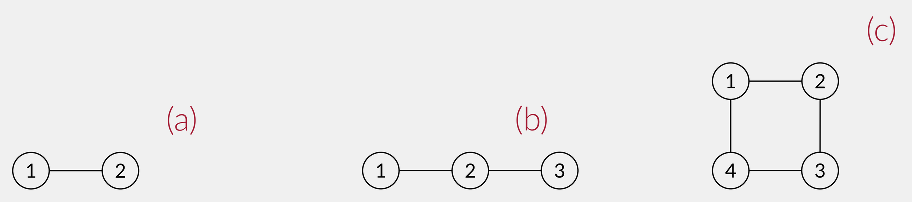


Figure 2. **a.** Bell state $|\phi\rangle_{C_2}$ **b.** Greenberger-Horne-Zeilinger state $|\phi\rangle_{C_3}$ **c.** Multi-party entanglement $|\phi\rangle_{C_4}$

Case study : Grover's search algorithm

General algorithm

For a search space of size $N = 2^n$ indexed (by some n-bit integer) elements, we wish to find a specific item x^* identified by some unique index $i^* \in \{0, N-1\}$. A function $f(x)$ implemented by an **oracle** O defines our a particular search problem [2].

$$f(x) = \begin{cases} 1 & \text{if } x = x^* \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

For a classical oracle one would have to consult the oracle at most $N-1$ times.

A quantum oracle $\hat{O} : |x\rangle \xrightarrow{\hat{O}} (-1)^{f(x)}\,|x\rangle$ can be constructed such that at most, the oracle is only consulted $\mathcal{O}(\sqrt{N})$ times.

Procedure

1. Prepare input register in the state $|0\rangle^{\otimes n}$
2. Apply Hadamard transform $H^{\otimes n}$ to put input register in $|+\rangle^{\otimes n}$ to prepare $|\psi\rangle$
3. Apply oracle O
4. Apply mean inversion operator : $H^{\otimes n}(2\,|0\rangle\,\langle 0| - I)H^{\otimes n} = 2\,|\psi\rangle\,\langle\psi| - I$
5. Repeat step 2 and 3 $\mathcal{O}(\sqrt{N})$ times
6. Measure

Case study : Grover's search algorithm

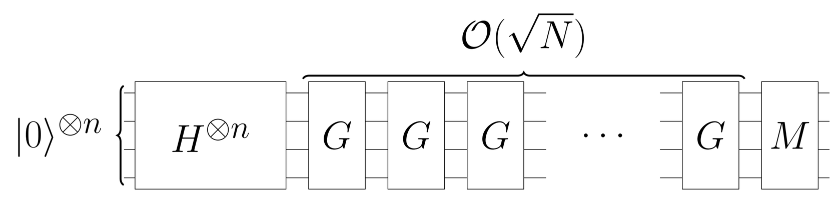


Figure 3. A schematic for Grover's search algorithm

Quantum search on the network model

A different but equivalent circuit was considered for the 2-qubit case.

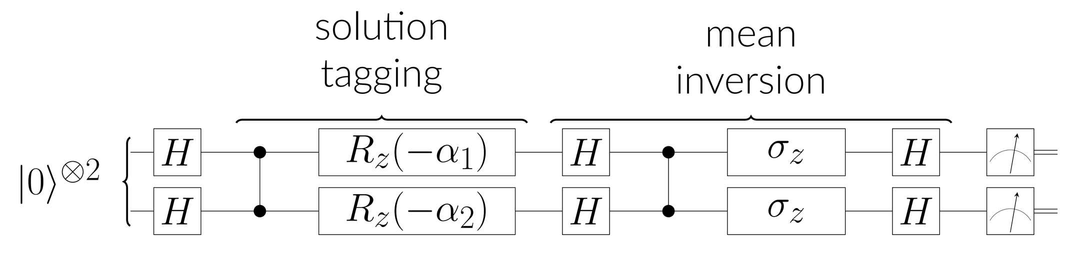


Figure 4. Schematic for a 2-qubit Grover's search algorithm

The specific values of $\alpha_1\alpha_2$ tag a specific item x^* for which our oracle identifies as the solution to our search problem. 00, 0π $\pi 0$ and $\pi\pi$ tag 11, 10, 01 and 11 respectively.

Quantum search on cluster states

Measurements of qubit 1 and qubit 4 on the cluster state in figure 2c in the basis $B(\alpha)_j = \{|+\alpha\rangle_j, |-\alpha\rangle_j\}$ where $|\pm\alpha_j\rangle = (|0\rangle_j \pm e^{i\alpha}\,|1\rangle_j)$ has an equivalent effect to the circuit in figure 4. After these measurements the result of the computation will reside on qubit 2 and qubit 3.

This computation will be equivalent to Grover if the final measurements (readout) of qubit 2 and qubit 3 are made in the $B(\pi)$ basis. The outcome should be **fedforward** via $\{o_2, o_3\} \to \{o_2 \oplus o_4, o_3 \oplus o_1\}$ where o_i is 1 if the outcome on qubit i is $|-\alpha\rangle_i$ and 0 if $|\alpha\rangle_i$.

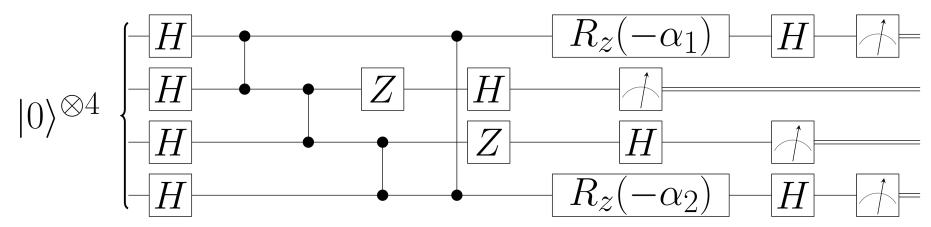
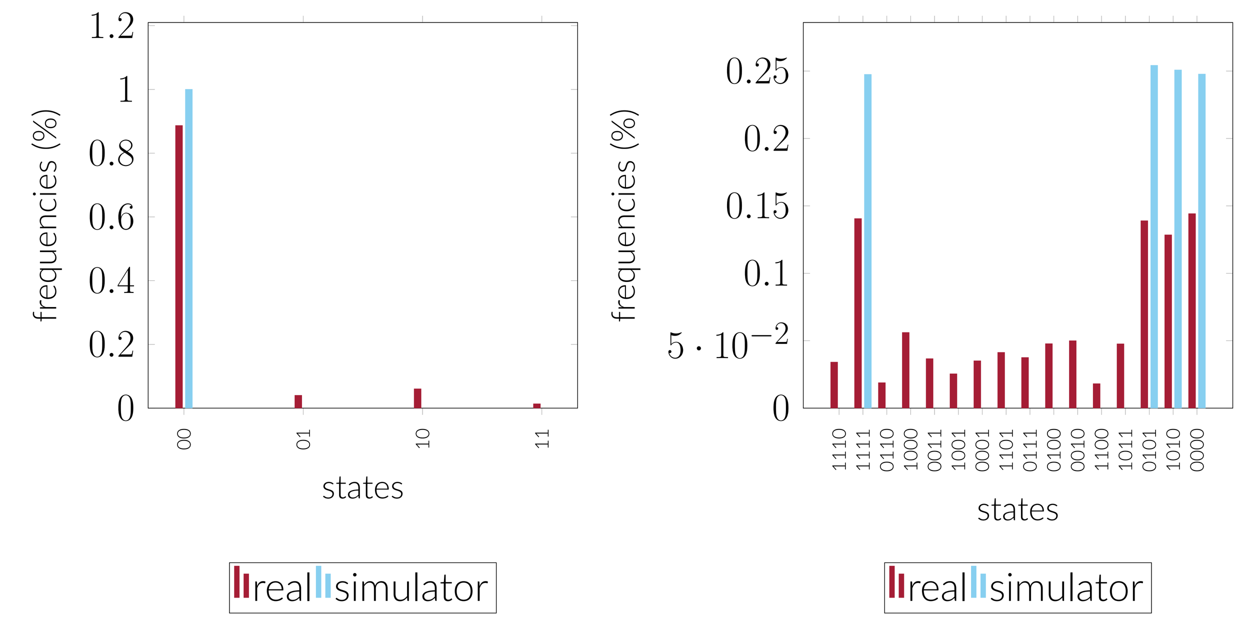


Figure 5. Circuit schematic for a 2-qubit Grover's search algorithm on a cluster state

Results

The two models were simulated on subgraphs (choosing qubits with lowest error rates) of the 14-qubit IBM Q Experience architecture ibmq_16_melbourne and also on the ibmq_qasm_simulator for comparison with the nearly ideal case 8192 iterations of the experiments were ran.



Results

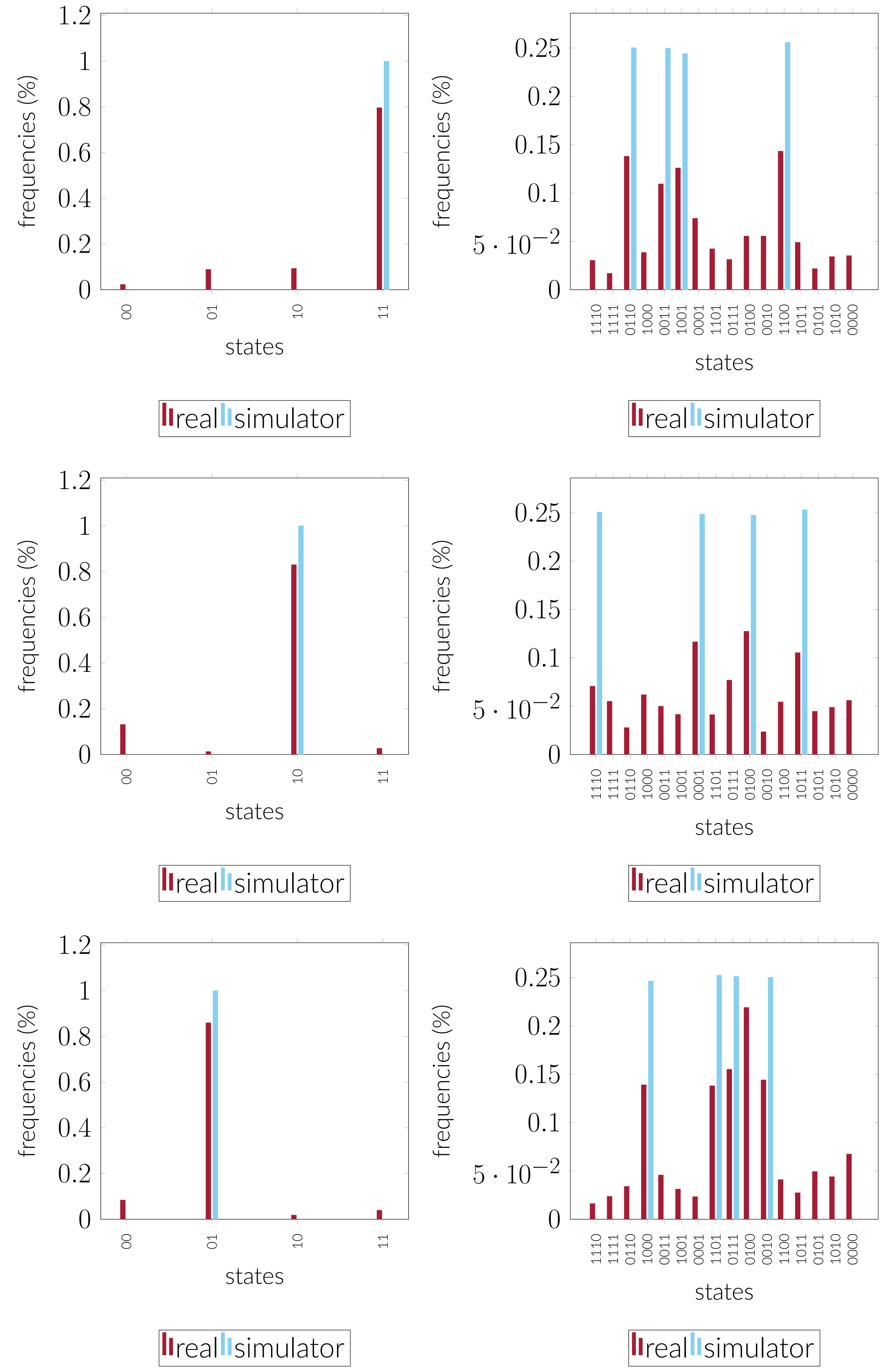


Figure 6. Four different cases of the algorithm

Discussion

The data in the preceding section seems to suggest the network model outperforms the cluster state computation on the 14-qubit machine. This might be due to a few reasons.

- Noise in the circuit scales with number of gates in a circuit.
- Faulty state preparation, fidelity between state prepared and $|\phi\rangle_{C_4}$ small. Can be characterized via entanglement witnesses for cluster states [4]

References

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