Advanced Fixed Income and Credit

Group 13 Submission

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Lecturer: Thibault Lair

Group Members	Student ID
Solal Fuster	72950
Grigory Sheleg	80985
Maël Thuillier	79372
Felix Tremblot de La Croix	87510
Elyazid Benkhadra	80574

Attached: Fixed_Income.zip (Jupyter Notebook with the code used to compute our results, our generated library 'graph.py' for graphs, and requirements.txt file)

Before answering the questions of the assignment, it is important for us to specify that the .zip file should be unzipped entirely.

graph.py and XLS698-XLS-ENG.xls must be in the same directory as the Jupyter Notebook.

graph.py makes the Jupyter Notebook cleaner to follow and read, as more than 100s of lines of codes have been put to generate better looking graphs.

You can also find attached a requirements file with all the libraries used for this project.

Moreover, we decided to use QuantLib, as it is a fast and reliable library for finance computations.

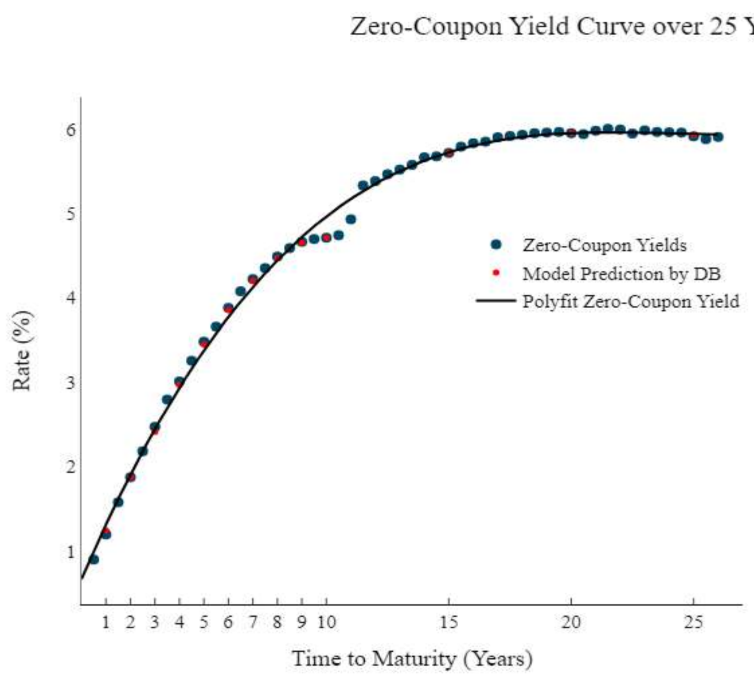
Question 1:
Bootstrapping

To address these questions, we first import and clean the Exhibit 1 and Exhibit 4 datasets in Python. Then, we generate the time to maturity of each bond. We make the assumption that all bonds have a face value of \$100.00, and have been emitted on 15th August 2003.

1.1 Zero-Coupon yields identification

To realize all the computations needed, we use the library QuantLib where we set the evaluation date (15th August 2023), use the United States' Government Bond calendar and the ISDA convention. We compute yields on a compounded basis on the same frequency as the coupon frequency for all periods.

Here is the plot we get from our code with all the yields shown, with a Polyfit curve to have a view on the tendency :



Maturity	Model DB	Yield ZC
1	1.2443	1.1939
2	1.8727	1.8727
3	2.411	2.4711
4	2.9665	3.0083
5	3.4454	3.4798
6	3.8557	3.8817
7	4.1996	4.2203
8	4.4677	4.4865
9	4.6528	4.664
10	4.7107	4.7107
15	5.716	5.7166
20	5.9517	5.9512
25	5.9315	5.9127

It is of interest to note the drop of yield after maturity nine to 12, and the rise of coupon rate for maturity 10.5 and after compared to maturity 10. It will have consequences for KRD computation.

You can find in the appendix the numerical results for every maturity.

1.2 Model Comparison with BEY Predictions

We decided to round the figures we got up to 10^{-4} to match what we have with the Model Prediction (BEY). Due to that rounding, we sometimes get results that are exactly matching. It would not be worth it to short or long these bonds because they are accurately priced with our rounding, hence why we introduced a 'Hold' category.

Here are the results we get when we compare our zero-coupon yields against the BEY model's predictions from Deutsche Bank:

Maturity (years)	Zero-Coupon Yield	Model Prediction (BEY)	Buy or Sell ?
1	1.1939	1.2443	**Sell**
2	1.8727	1.8727	Hold
3	2.4711	2.4110	**Buy**
4	3.0083	2.9665	**Buy**
5	3.4798	3.4454	**Buy**
6	3.8817	3.8557	**Buy**
7	4.2203	4.1996	**Buy**
8	4.4865	4.4677	**Buy**
9	4.6640	4.6528	**Buy**
10	4.7107	4.7107	Hold
15	5.7166	5.7160	**Buy**
20	5.9512	5.9517	**Sell**
25	5.9127	5.9315	**Sell**

We need to buy the 3, 4, 5, 6, 7, 8, 9 and 15 year maturities, sell the 1, 20 and 25 year maturities and Hold/neither buy nor sell the 2 and 10 year maturities among these U.S. Treasury Bonds.

1.3 Strategy Analysis

This strategy is not a risk-free arbitrage strategy. These bonds are all different and we don't know if risks are offset by buying and selling some of them, especially since our results indicate that we need to buy more bonds than we sell. Moreover, there are a lot more concerns we need to consider: market timing, exogenous risk, interest rates risk.

Risk-free arbitrage involves buying and selling securities to profit from price discrepancies without taking on any risk. Thus, in an ideal arbitrage situation, an investor can lock in a profit regardless of future market movements, eliminating exposure to risk.

Nevertheless, if bonds appear mispriced according to our results, holding these bonds still exposes the investor to **changes in interest rates.**

Moreover, even if U.S. Treasury bonds are generally highly liquid, **liquidity** can vary across maturities, which affect the execution of such a strategy.

Finally, some external **economic factors**, such as policy changes, macroeconomic shifts, or sudden market volatility, could affect bond prices, impacting the strategy's outcomes.

Therefore, buying and selling Treasury bonds based on perceived mispricing from Deutsche Bank's model is **not a risk-free arbitrage strategy**.

Question 2:
Cubic Splines

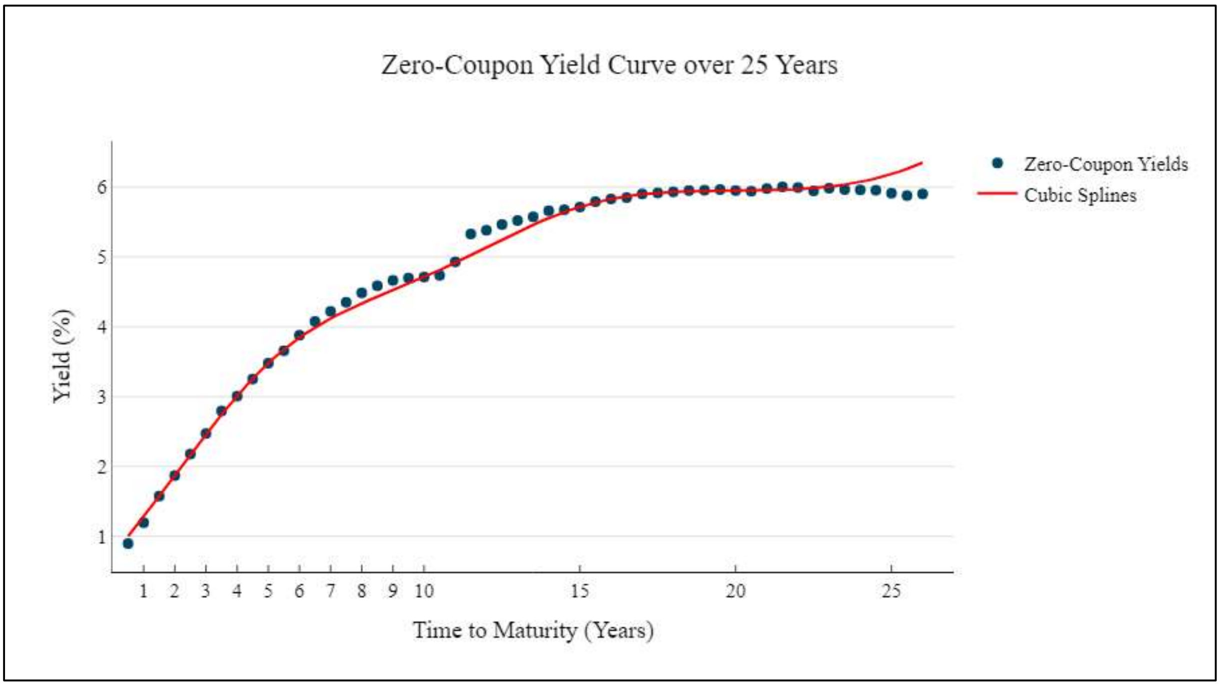
2.1 Zero-Coupon Yields and Discount Function with 5 knots

We defined a range of zero-coupon yields (2, 5, 10, 15 and 20-year maturities) that will be used as Knot-points to interpolate the Zero-Coupon Yields using Cubic Splines method (Graph 1).

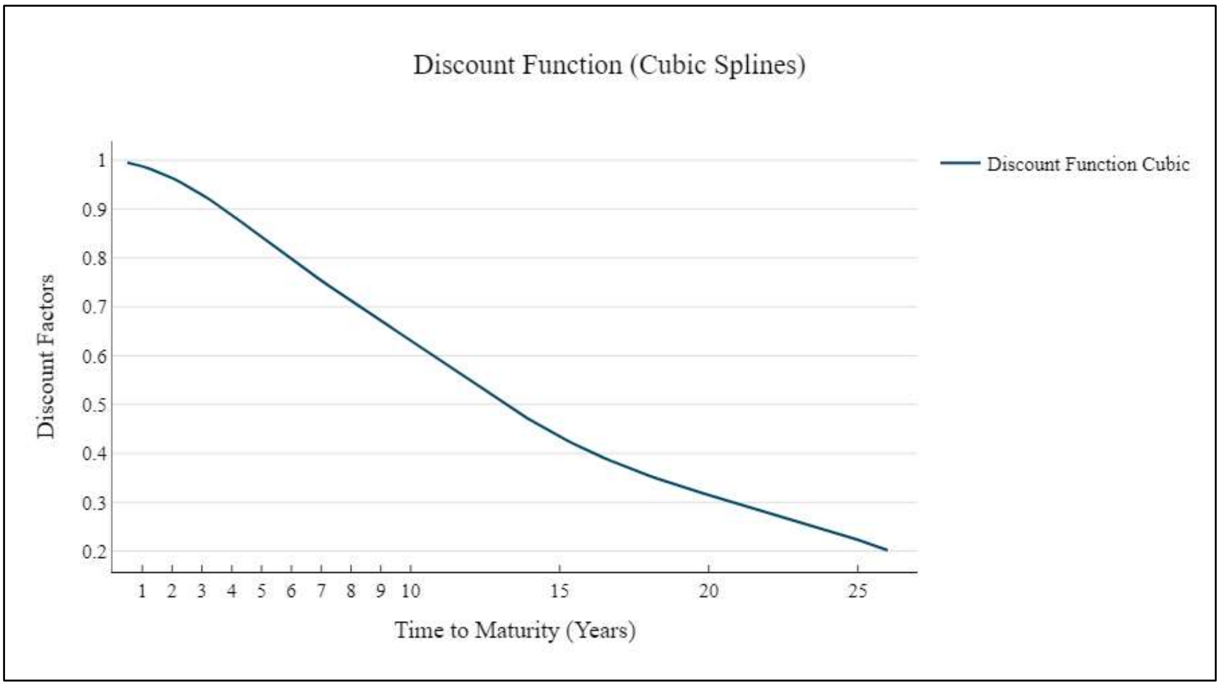
We can then calculate the discount function using these interpolated zero-coupon yields (Graph 2).

We also make a boundary assumption; the second derivative at curve ends are zero (hence why we have bc_type ='natural' in the CubicSpline function).

Graph 1:



Graph 2:



2.2 Discussion about new splined-based zero-coupon

These ‘spline-based’ zero-coupon yields seem to be interesting to use because we might get noisy and outlier data points, which splines can mitigate by smoothing the yield curve.

Also, we ensure that the curve is continuous which is useful for risk management (gamma computation which uses derivation).

First, to determine whether this affects our conclusion let’s compare the model prediction of Deutsche Bank and the spline-based Zero-Coupon Yield (Tab1)

Tab1:

Maturity (years)	Spline Zero-Coupon Yield	Model Prediction (BEY)	Recommendation	Previous Recommendation
1	1.2692	1.2443	Buy	Sell
2	1.8727	1.8727	**Hold**	**Hold**
3	2.4760	2.4110	**Buy**	**Buy**
4	3.0300	2.9665	**Buy**	**Buy**
5	3.4794	3.4454	**Buy**	**Buy**
6	3.8205	3.8557	Sell	Buy
7	4.1316	4.1996	Sell	Buy
8	4.3980	4.4677	Sell	Buy
9	4.5982	4.6528	Sell	Buy
10	4.7107	4.7107	**Hold**	**Hold**
15	5.7166	5.7160	**Buy**	**Buy**
20	5.9512	5.9517	**Sell**	**Sell**
25	6.1208	5.9315	Buy	Sell

For the 2, 5, 10, 15, 20-year maturities, as we used these maturities as knots for the construction of our curve we get the same yield, and thus the same recommendations.

However, due to the methodology we used to interpolate the yields we got, and the choice of knots made for the assignment, we get different yields for every other maturity, which is changing the recommendation results for some of them (i.e. 1, 6, 7, 8, 9 and 25-year maturities).

Since we used knots up to the 20-year maturity, we had to extrapolate the 25-year maturity using 'extrapolate = True' option of Scipy's Cubic Spline function. The difference in yield between our original zero coupon yields and our Cubic Spline zero coupon yields seems to be too high. We think the 25-year maturity is a bit off mark and should not be used.

Indeed, when we look at the Cubic Spline curve we generated earlier, yields start to get back up instead of being stable soon after the last knot we used, hence why the very last point of data we get at the 25 year maturity should not be relied on.

2.3 Knots analysis

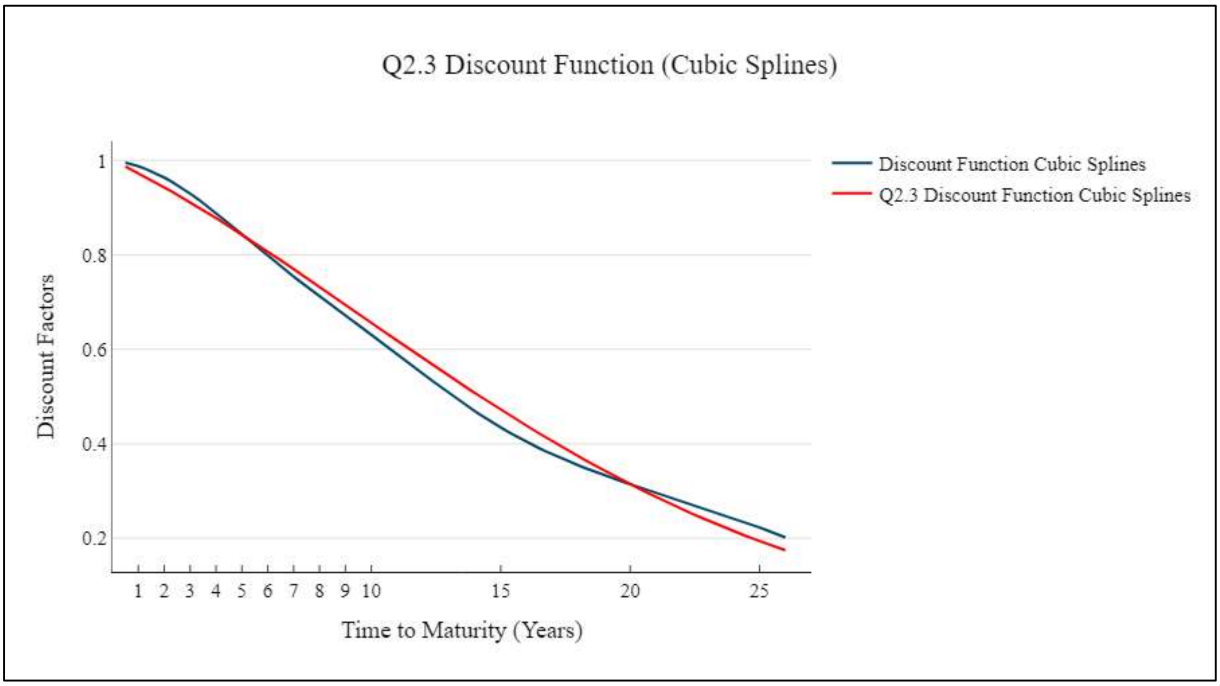
When doing a Cubic Spline interpolation, in the case of two knots, you only have one interval between them, which prevents creating the curvature associated with cubic splines.

Indeed, there is only one segment, thus we lose the cubic characteristics, with just two points, the entire interval is forced into a single linear equation to fit both points (Graph 3).

Graph 3:

Changing the number and placement of knots in the spline affects the estimated discount function's flexibility, stability, and accuracy. Indeed, more knots allow for a closer fit to market data, capturing detailed variations in the yield curve, whereas fewer knots produce a smoother function but less sensible to local variations (Graph 4).

Graph 4:



The goal is to strike a balance, optimizing knot placement to capture key yield curve behaviors without overfitting, ensuring a reliable discount function across maturities.

Question 3:
Nelson-Siegel-Svensson (NSS) Model

3.1 Discount function of NSS

We use the 'Nelson-Siegel-Svensson' package in Python to estimate our NSS model values.

First, we calibrate the model using ordinary least squares (OLS) for betas and nonlinear optimization for taus.

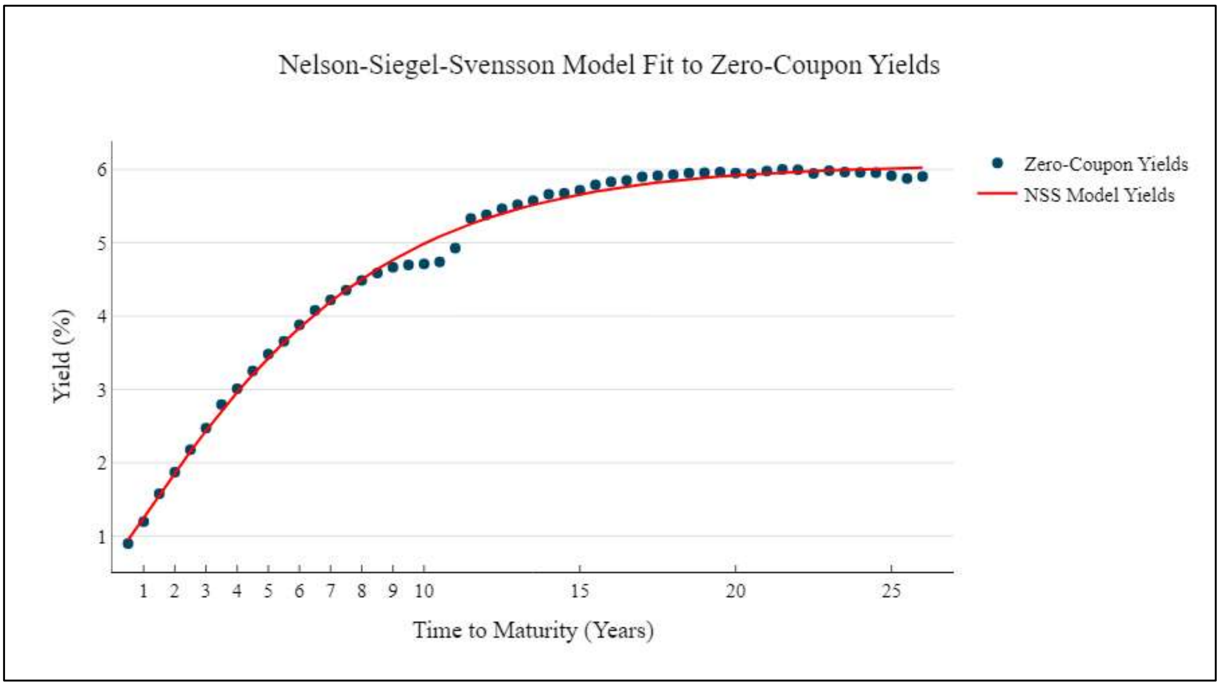
Moreover, we use the zero-coupon yields we computed in Q1.1 directly with the NSS model when calibrating it.

The calibration gives us these results as parameters:

- $\beta_0 = 0.06033$
- $\beta_1 = -0.05391$
- $\beta_2 = -0.06016$
- $\beta_3 = 0.07206$
- $\tau_1 = 3.14798$
- $\tau_2 = 5.15351$

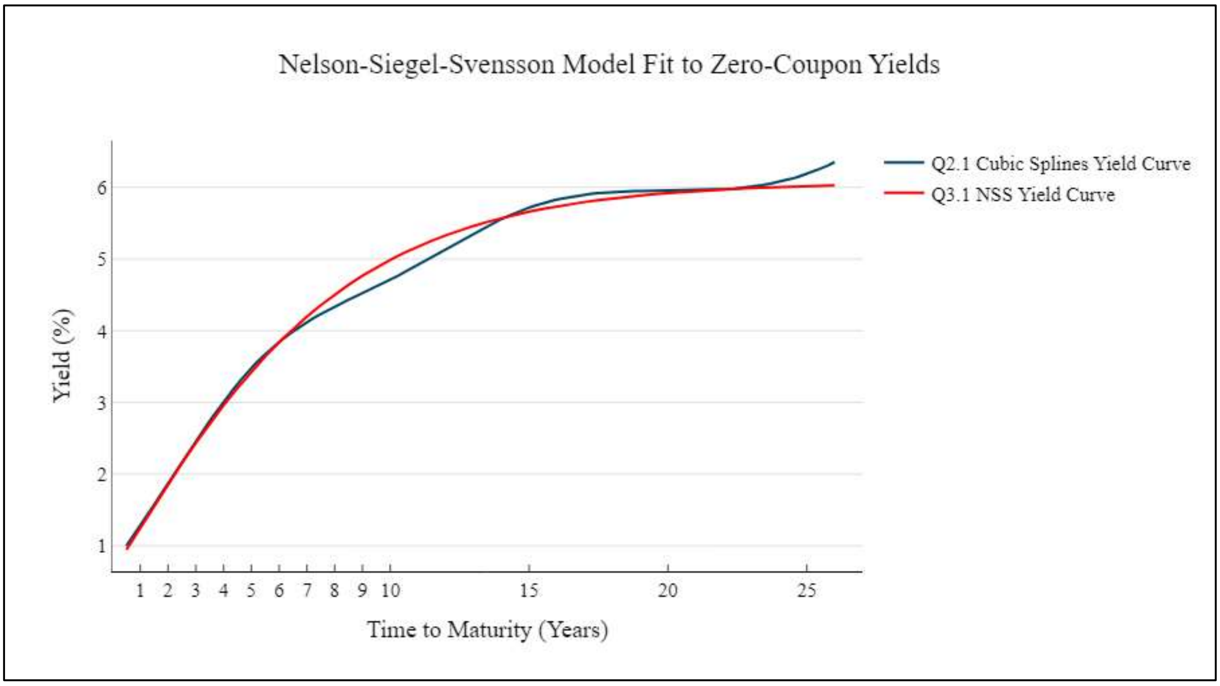
We get the following curve (Graph 5) with these parameters.

Graph 5:



3.2 Comparison of NSS and Cubic Splines

Graph 6:



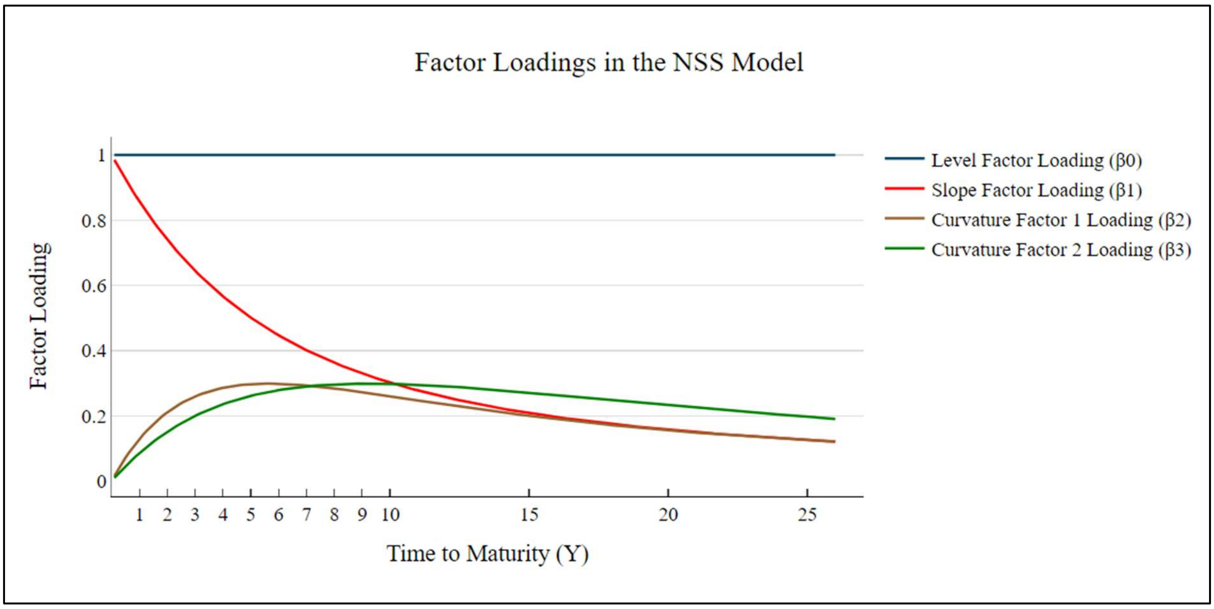
What we are seeing is that the NSS yield curve we get in Q3.1 is smoother than the Q2.1 cubic splines yield curve. We see bumps on the cubic splines curve that seem to be starting around 7 years, up until a maturity of 14 years.

After 14 years, we can see the Q2.1 curve going higher than the Q3.1 curve. It is mainly because the Q3.1 curve shows yields smoothened by the NSS parameters, whereas the Q2.1 curve shows the "gap" in yields that was computed in 1.1 as well, but a bit smoothened by the cubic splines model due to the knots we had taken earlier.

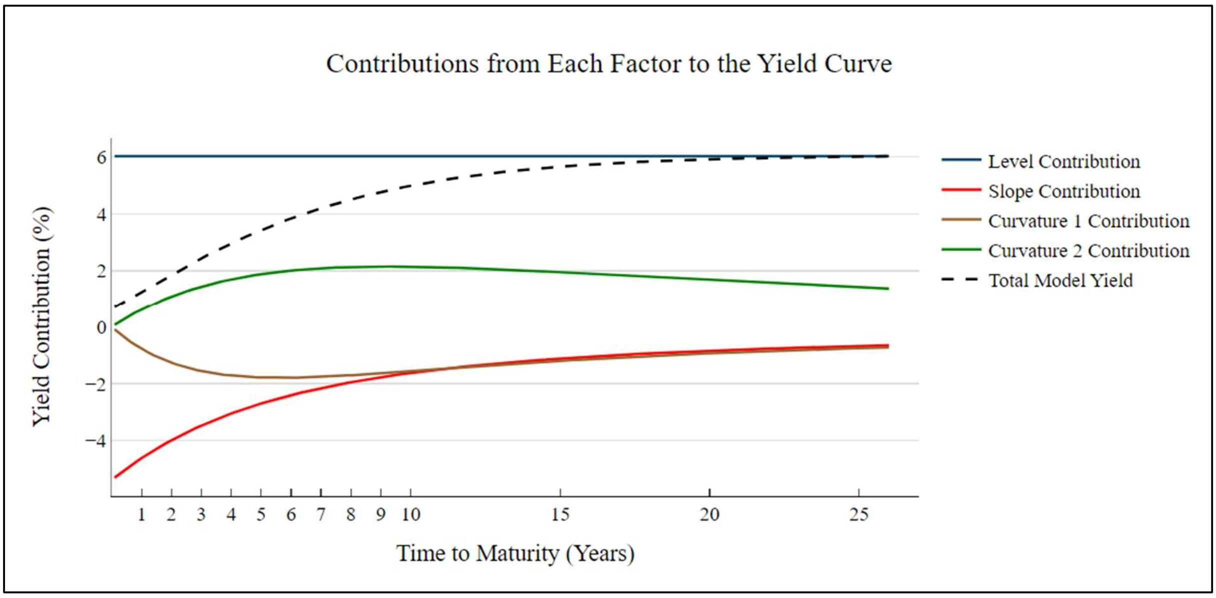
The extreme end is also smoother and seems more closely related to Zero-Coupon Yield computed in Q1.1.

3.3 Analysis of factor loadings and contributions

Graph 7 :



Graph 8:



Within the NSS model, we have 4 different parameters:

- β_0 adjusts the level of the entire curve (long-term rates). It represents the long-term level of the interest rate of the yield curve. For our curves, it is coherent to see its factor loading always constant across all maturities and to see it always around 6% as it seems to be the close to the very last zero-coupon yields. It seems to be driven by long-term monetary policies and what the market thinks will be the long-term central bank interest rate.

- β_1 however controls the slope at the short end (short-term rates). On the factor loading level, we find that it has a very high loading at first that decreases when maturities are going up, which is logical considering the fact that it adjusts the curves for short-term rates differences. It is also logical to see that it contributes very negatively to rates at first to compensate for the very high and stable level of β_0. The key drivers for this parameter seems to be driven by short term monetary policies and liquidity preferences from investors (the demand for short-term maturities vs long-term maturities).

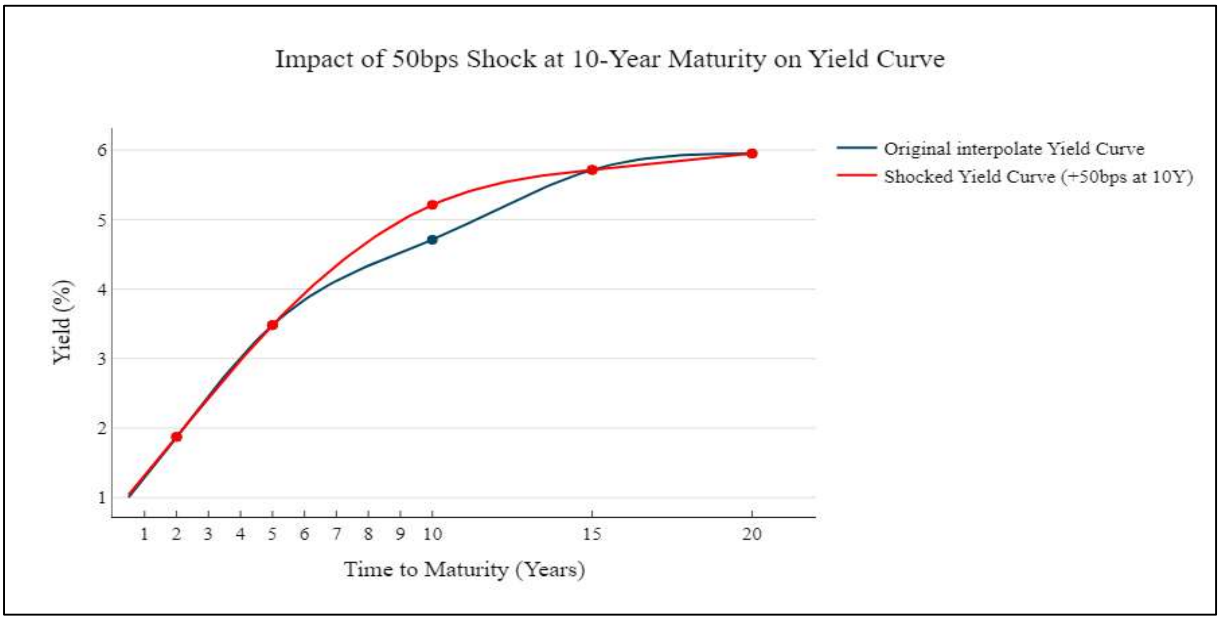
- β_2 and β_3 should both be analysed together as they are curve parameters. β_2 is the first curvature parameter that controls for humps and dips around medium maturities. It is driven mainly by market sentiment and changes in medium-term economic outlook. β_3 is a continuity of β_2 in the sense that it is the second curvature parameter and allow for extra flexibility of curvatures (second humps/dips, complex curvature patterns). It also aims at reducing term structure anomalies. What we see here confirms our analysis: the first curvature parameter is more present in terms of loading short/medium term maturities, whereas the second curvature parameter kicks in on longer-term maturities. What we see on the contributions though is that the two curvature parameters seems to offset each other most of the time, with β_3 having a slightly higher contribution around 5 years, and stabilizing at a higher level for a longer time compared to β_2.

Question 4 :
Hedging

4.1 Impact of a 10-year rate change of ±50bps

To answer this question, we simply plot the same interpolated Yield Curve from Q2.1 with a rise of 50bps for 10-Year zero coupon bond.

Graph 9:

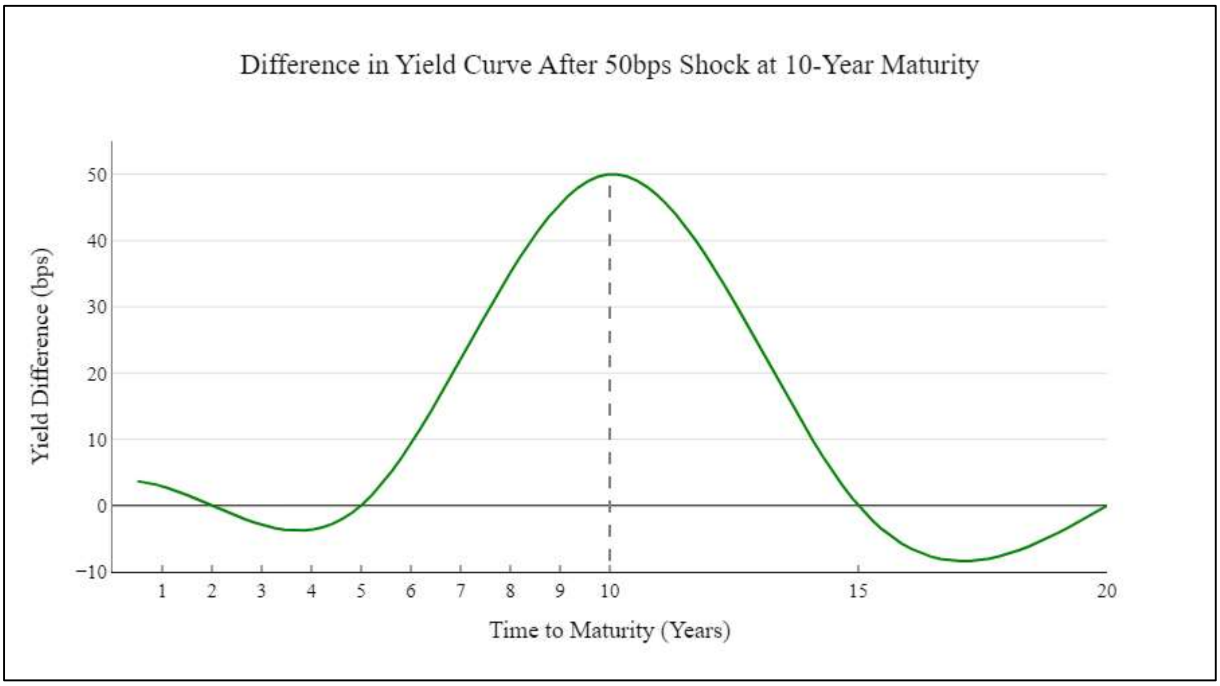


The impact is quite important, while the “speed” of curve 2.1 is falling between maturity from 5 to 10 year, because of the slowing increase of zero-coupon yields in this interval and get higher after.

The new interpolated curve is not affected by this decrease and look smoother. The curve is close to the one implied by the NSS model.

Thus, the new curve is higher than the previous one and increase faster, until being caught up by old curve, as seen in graph 10.

Graph 10:



4.2 KRD of portfolio with a 10-year rate change

Firstly, we created a function that allowed us to compute all the bonds present values.

We created a new variable for our bonds, which is the number of bonds in our possession for every maturity for an equally invested portfolio of 1 million:

$\frac{\frac{100000}{52}}{price\ of\ bond\ at\ maturity\ i}$, is the number of bonds of maturity I, that we possess in the portfolio.

We then compute the present value of the portfolio, without any shock :

$$\sum_{i \in Maturity} PV_{(z_{0.5},\dots,z_{26})}(Bonds\ i) * Number\ of\ Bonds\ i = \$999\ 965.67845$$

And the present value of the portfolio, with a shock for the 10-Year zero-coupon yield.

$$\sum_{i \in Maturity} PV_{(z_{0.5},\dots z_{10}+0.5,\dots,z_{26})}(Bonds\ i) * Number\ of\ Bonds\ i = \$998\ 796.3410$$

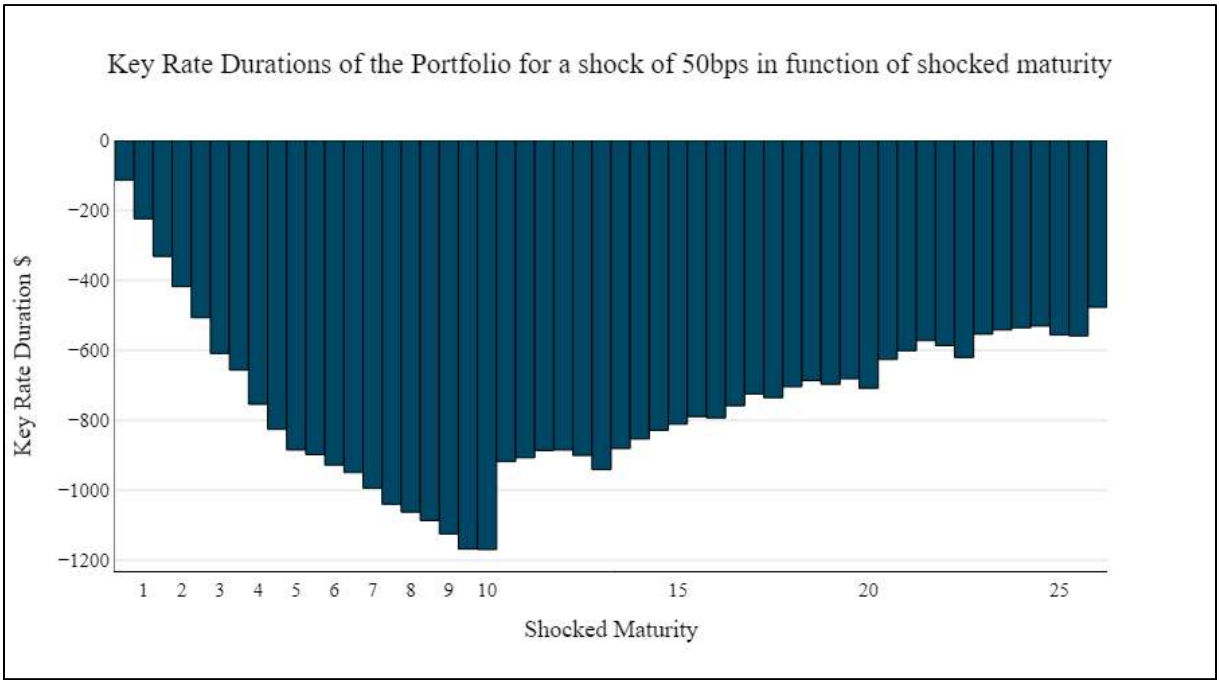
Thus, the key rate duration in dollar for a shock of the 10-year zero coupon rate is approximately:

KRD_{10} = $-1169.34

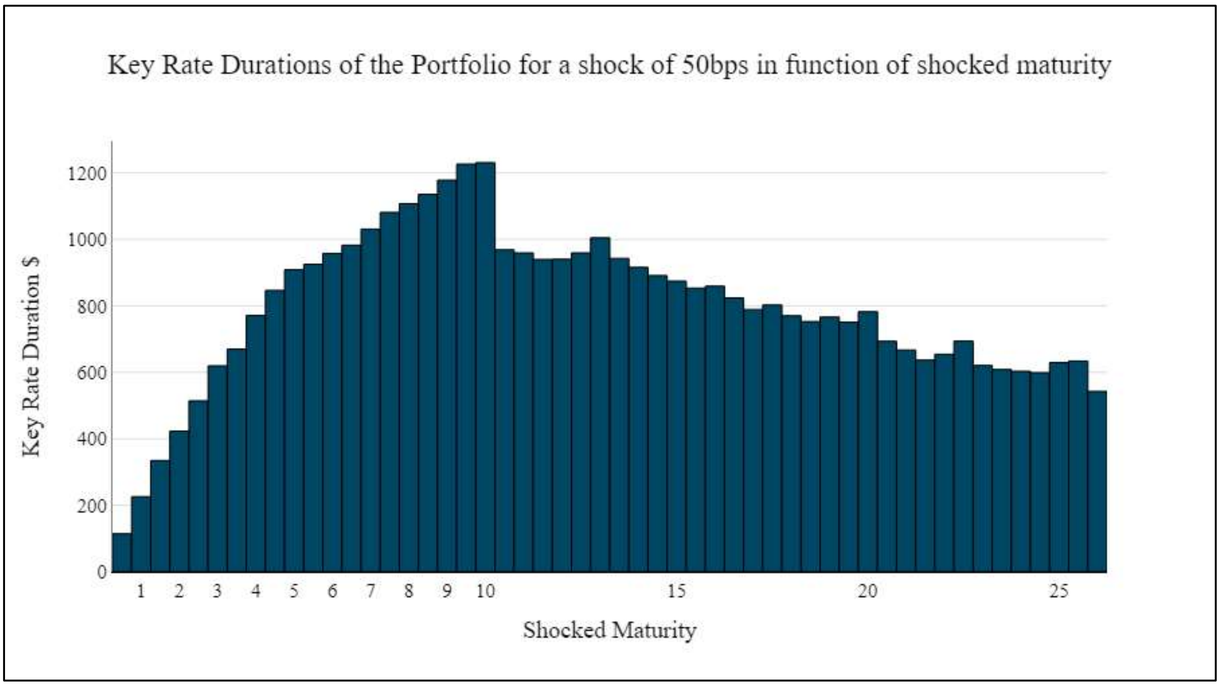
4.3 KRD of portfolio with a change for every maturity

We apply the same methodology as in Question 4.2, but for every maturity from 0.5 to 26 years. We have plotted what we obtain for a positive shock of 50bps on the graph 11, and for a negative shock of 50bps on the graph 12.

Graph 11:



Graph 12 :



We see the curve is steady until the 10 year maturity, which is the maturity for which the key rate duration is maximum, for negative shock, and minimum for positive shock.

The portfolio is particularly sensitive to interest rate changes around this 10-Year maturity. It indicates a concentration of bonds affected by the shock around this maturity. This high sensitivity near the 10-year maturity range suggests that risk management strategies should focus on this segment of the yield curve.

An important discontinuity follows, consequences of the coupon rate huge increase after 10-Year maturity (4.25 to 13.25). The curve stays constant for a semi dozen maturities, and goes down (positive shock), or up (negative shock) to lower levels at a slow speed afterwards.

Finally, due to the portfolio's high interest rate sensitivity, shocks could result in either huge gains or losses, depending on the direction of rate moves, especially around the 10-year maturity level. It is important for making informed hedging or rebalancing decisions.

Question 5: Portfolio Strategy

5.1 New model for zero-coupon yield curve

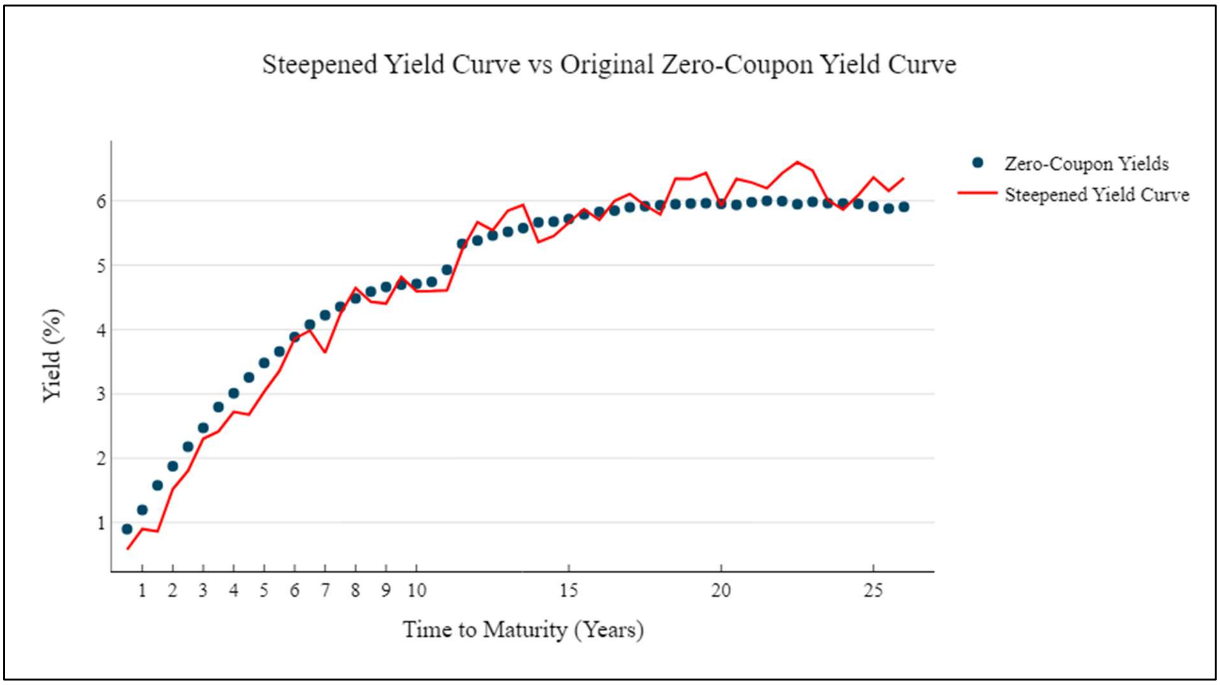
We generate a new zero-coupon yield curve given the formula we have in the assignment:

$$y^T_{steepen} = y^T + [(T - \bar{T})/\bar{T} * 0.0050] + \varepsilon \text{ with } \varepsilon \sim N(0, 0.0025)$$

We have set a seed at the beginning of the assignment (np.random.seed(2**32 - 1)) so that we could get reproducible results.

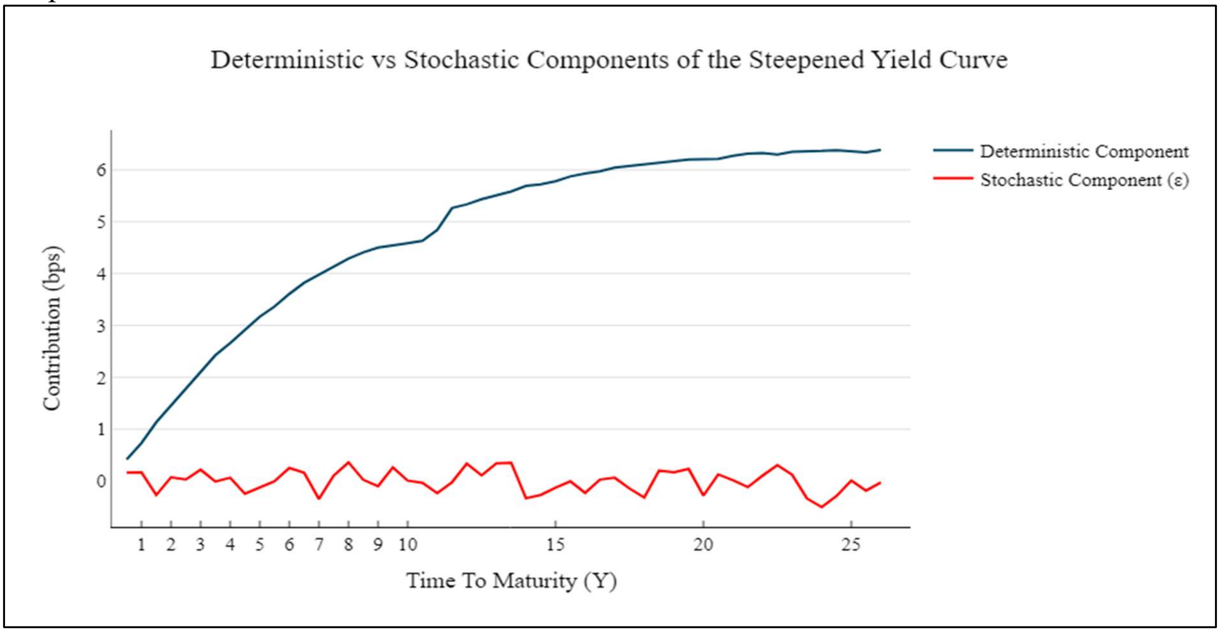
Here is the plot of the steepened yield curve against zero-coupon yields from Question 1.1 (Graph 13):

Graph 13:



And here is the plot with the separation between the stochastic and the deterministic component of the steepened yield curve (Graph 14):

Graph 14:



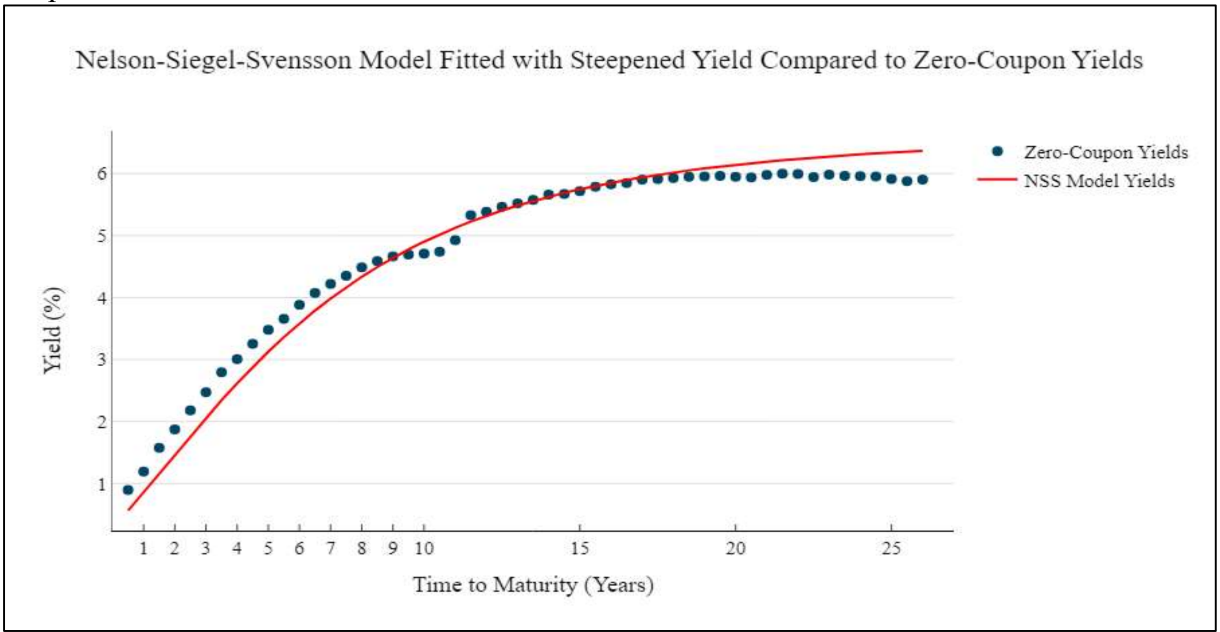
5.2 NSS Curve Optimization

For the NSS curve generation, we use the exact same methodology as in Question 3.1, but we get different results:

- $\beta_0 = 0.06886$
- $\beta_1 = -0.06603$
- $\beta_2 = -0.06592$
- $\beta_3 = 0.05366$
- $\tau_1 = 2.99678$
- $\tau_2 = 5.02365$

It is expected as we use the yields generated in Question 5.1. Here is the plot of the NSS curve against the Question 1.1 zero-coupon yields (Graph 15):

Graph 15:



5.3 Strategy Proposal and Investment Allocation

We suggest a strategy where we buy cheap bonds (bonds where our NSS price is higher than the market-observed price) and sell expensive bonds (bonds where our NSS price is lower than the market-observed price). We invest in each bond according to the weights that we generate with the expected profit per dollar that we get. We make the strategy zero-cost by having exactly $100 invested in short bonds and $100 invested in long bonds, so a total of $200 invested. The higher the expected profit for a bond, the higher the investment we are going to make in it.

All the data we show is in dollar, except for the coupon rate, the starting/maturity dates, and the time to maturity.

We find using Python that we expect a total expected profit of $1.16 for cheap bonds and $2.30 for expensive bonds, which gives us a Total Expected Profit of $3.46.

It is also equivalent to an expected return of 1.73% (3.46/2) as we invested a total of $200 in our strategy. If we only consider the fact that we only used $100 from our own wealth, the strategy yields an expected return of 3.46%.

APPENDIX

Question 1.1:

Maturity (years)	Coupon Rate (%)	Current Price ($)	Zero-Coupon Yield (%)
0.5	3	101.0544	0.8975
1.0	2.125	100.9254	1.1939
1.5	1.5	99.8942	1.5750
2.0	6.5	109.0934	1.8727
2.5	5.625	108.4380	2.1796
3.0	2.375	99.7848	2.4711
3.5	6.25	111.7184	2.7943
4.0	3.25	101.0841	3.0083
4.5	3	99.1692	3.2541
5.0	3.25	99.2710	3.4798
5.5	5.5	109.7707	3.6563
6.0	6	112.1450	3.8817
6.5	6.5	114.9084	4.0765
7.0	5.75	110.3894	4.2203
7.5	5	105.2934	4.3520
8.0	5	104.7607	4.4865
8.5	4.875	103.4391	4.5879
9.0	4.375	99.2806	4.6640
9.5	3.875	95.0288	4.6961
10.0	4.25	97.7693	4.7107
10.5	13.25	174.3251	4.7389
11.0	12.5	168.9389	4.9285
11.5	11.25	157.0552	5.3308
12.0	10.625	152.4222	5.3825
12.5	9.25	140.0135	5.4634
13.0	7.5	123.3044	5.5197
13.5	8.75	136.0598	5.5751
14.0	8.875	137.5040	5.6634
14.5	9.125	140.7920	5.6744
15.0	9	139.9079	5.7166
15.5	8.875	138.7431	5.7904
16.0	8.125	130.7162	5.8297
16.5	8.5	135.2938	5.8507
17.0	8.75	138.3466	5.9023
17.5	7.875	128.4995	5.9165
18.0	8.125	131.7341	5.9297
18.5	8	130.4736	5.9495
19.0	7.25	121.5800	5.9562
19.5	7.125	120.1744	5.9661
20.0	6.25	109.4538	5.9512
20.5	7.5	125.4600	5.9398

21.0	7.5	125.4466	5.9788
21.5	7.625	127.1477	6.0017
22.0	6.875	117.5509	5.9961
22.5	6	106.3626	5.9455
23.0	6.75	116.1986	5.9822
23.5	6.625	114.7086	5.9647
24.0	6.375	111.4036	5.9602
24.5	6.125	108.0391	5.9556
25.0	5.5	99.6330	5.9127
25.5	5.25	96.2876	5.8781
26.0	6.125	108.4062	5.9048

Question 5.3:

Here is the investment allocation we generated for cheap bonds only:

Time to Maturity	Current Price ($)	NSS Theoretical Price ($)	Price Difference ($)	EP per Dollar	Investment
0.5	101.0544	101.2263	0.1719	0.001701	0.874933
1.0	100.9254	101.2617	0.3363	0.003332	1.713881
1.5	99.8942	100.5103	0.6161	0.006168	3.172234
2.0	109.0934	109.9289	0.8355	0.007659	3.939148
2.5	108.4380	109.4864	1.0484	0.009668	4.972787
3.0	99.7848	100.9450	1.1602	0.011627	5.980298
3.5	111.7184	113.2761	1.5577	0.013943	7.171557
4.0	101.0841	102.5096	1.4255	0.014102	7.253351
4.5	99.1692	100.6692	1.5000	0.015126	7.779806
5.0	99.2710	100.7962	1.5252	0.015364	7.902395
5.5	109.7707	111.2615	1.4908	0.013581	6.985336
6.0	112.1450	113.7575	1.6125	0.014379	7.395612
6.5	114.9084	116.5836	1.6752	0.014579	7.498411
7.0	110.3894	111.8043	1.4149	0.012817	6.592539
7.5	105.2934	106.4386	1.1452	0.010876	5.594155
8.0	104.7607	105.7194	0.9587	0.009151	4.706940
8.5	103.4391	104.0840	0.6449	0.006235	3.206727
9.0	99.2806	99.4523	0.1717	0.001729	0.889529
11.5	157.0552	157.8307	0.7755	0.004938	2.539708
12.0	152.4222	152.8855	0.4633	0.003040	1.563394
12.5	140.0135	140.3712	0.3577	0.002555	1.314024
13.0	123.3044	123.4033	0.0989	0.000802	0.412546
13.5	136.0598	136.0830	0.0232	0.000171	0.087703
14.0	137.5040	137.6251	0.1211	0.000881	0.452984

And here is the investment allocation we generated for expensive bonds only:

Time to Maturity	Current Price ($)	NSS Theoretical Price ($)	Price Difference ($)	EP per Dollar	Investment (%)
9.5	95.0288	94.5162	-0.5126	0.005394	1.286216
10.0	97.7693	96.5296	-1.2397	0.012680	3.023463
10.5	174.3251	172.9631	-1.3620	0.007813	1.862978
11.0	168.9389	167.8970	-1.0419	0.006167	1.470574
14.5	140.7920	140.5727	-0.2193	0.001558	0.371408
15.0	139.9079	139.5320	-0.3759	0.002687	0.640650
15.5	138.7431	138.4628	-0.2803	0.002020	0.481728
16.0	130.7162	130.3014	-0.4148	0.003173	0.756659
16.5	135.2938	134.7028	-0.5910	0.004368	1.041598
17.0	138.3466	137.7724	-0.5742	0.004150	0.989658
17.5	128.4995	127.7159	-0.7836	0.006098	1.454064
18.0	131.7341	130.7582	-0.9759	0.007408	1.766435
18.5	130.4736	129.3709	-1.1027	0.008452	2.015233
19.0	121.5800	120.2678	-1.3122	0.010793	2.573526
19.5	120.1744	118.7062	-1.4682	0.012217	2.913157
20.0	109.4538	107.6849	-1.7689	0.016161	3.853568
20.5	125.4600	123.3977	-2.0623	0.016438	3.919559
21.0	125.4466	123.4163	-2.0303	0.016185	3.859153
21.5	127.1477	125.0678	-2.0799	0.016358	3.900539
22.0	117.5509	115.2919	-2.2590	0.019217	4.582272
22.5	106.3626	103.6967	-2.6659	0.025064	5.976481
23.0	116.1986	113.5351	-2.6635	0.022922	5.465658
23.5	114.7086	111.8068	-2.9018	0.025297	6.032011
24.0	111.4036	108.3493	-3.0543	0.027417	6.537370
24.5	108.0391	104.8534	-3.1857	0.029487	7.030959
25.0	99.6330	96.1442	-3.4888	0.035017	8.349558
25.5	96.2876	92.5182	-3.7694	0.039147	9.334531
26.0	108.4062	104.5368	-3.8694	0.035694	8.510991

END