ASSIGNMENT 5 - EE2703

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1 Introduction

The assignment primarily deals with solving the Laplace equation employing the use of an iterated loop until the error drops below a given threshold.

$$\frac{\partial^2 \phi}{\partial t^2} = 0 \tag{1}$$

Where ϕ is the voltage as a function of x and y coordinates of the plate.

The model consists of a plate of 1cm*1cm dimensions. A wire of certain radius is attached to the plate at the center and a voltage of 1 V is passed through it. Hence, the eqn(1) simplifies into :

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \tag{2}$$

Now, in order to solve the above equation, we need to simply the differential terms. For this, we use the fundamental definition of the derivative i.e.

$$\frac{\partial \phi}{\partial x}|_{(x_i, y_j)} = \frac{\phi(x_{i+1/2}, y_j) - \phi(x_{i-1/2}, y_j)}{\Delta x} \tag{3}$$

From the above definition, we can write :

$$\frac{\partial^2 \phi}{\partial x^2}|_{(x_i, y_j)} = \frac{\phi(x_{i+1}, y_j) + \phi(x_{i-1}, y_j) - 2\phi(x_i, y_j)}{(\Delta x)^2} \tag{4}$$

Combining the above equation with that of the corresponding derivatives of y and then rearranging the equation provides us with the following equation to work with :

$$\phi(x_i, y_j) = \frac{\phi(x_{i+1}, y_j) + \phi(x_{i-1}, y_j) + \phi(x_i, y_{j+1}) + \phi(x_i, y_{j-1})}{4} \tag{5}$$

The above equation holds good for the point in the interior which are surrounded on all the four sides by points. However, in the case of the boundary points, we cannot use the above equation as one or more points in the neighbourhood don't exist.

Hence, to account for such points, we must also enforce boundary conditions :

- At the edges of the plate, the potential gradient is solely in the tangential direction.
- In this case, the bottom edge of the plate is connected to the plate is connected to the ground making the potential zero.

2 Calculating the Potential Function

Now, since we know the potential function and the various boundary conditions, we go forward and implement the python program.

We initialize the various necessary variables and then declare the potential array. Now, in order to find the coordinates of the points for which the potential is 1 at all times, we use :

```
c = np .where(Y*Y + X*X <= 0.35*0.35)
potential[c] = 1.0
```

The second line sets the potential of all the points in c to 1.

Now, we start the implementation of the loop as follows :

```
for  i in range(Niter) :
oldpot = potential.copy()
potential[1:-1,1:-1] = 0.25*(potential[1:-1,0:-2]+potential[1:-1,2:]+
                       potential[0:-2,1:-1]+potential[2:,1:-1])
potential[1:-1,0]=potential[1:-1,1] #1
potential[1:-1,Nx-1]=potential[1:-1,Nx-2] #1
potential[0,1:-1]=potential[1,1:-1] #1
potential[0,0]=potential[0,1] #1
potential[0,Nx-1]=potential[0,Nx-2] #1
potential[Ny-1,1:-1]=0.0 #2
potential[c]=1.0 #2
error.append((abs(oldpot-potential)).max())
```

In the above code, the lines which have been commented with #1 represent the part of the loop which implements the $\frac{\partial \phi}{\partial x} = 0$. The lines of the code commented as #2 implement the initial conditions i.e. the ground and the wire.

In the above loop, to make sure that the system is reaching a steady state as we increase the no of iterations, we use the error list to measure the maximum variation in a given points potential over one iteration. Then, we plot this as a function of no of iterations. If the error decreases as a function of time, it implies the system reaches steady state.

Moreover, we try to fit the error plot to the function Ae^{Bt}. We find the coefficients A,B by making use of the lstsq function.

Here are the graphs obtained from the above procedure.

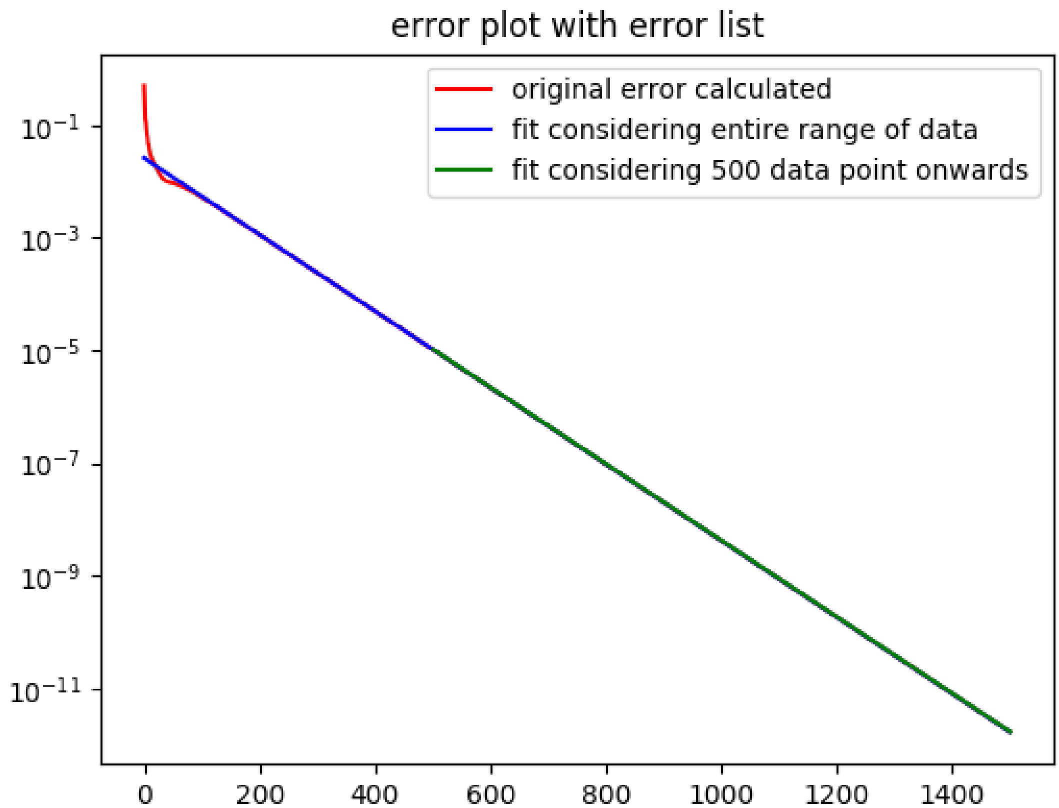

Figure 1: Error Plots

From the above graph we see that the error function decreases as the no of the iterations increases. The fit of the curve is exact only after a certain number of iterations. Initially, the curve doesn't fit the data as the data doesn't vary exponentially in the beginning.

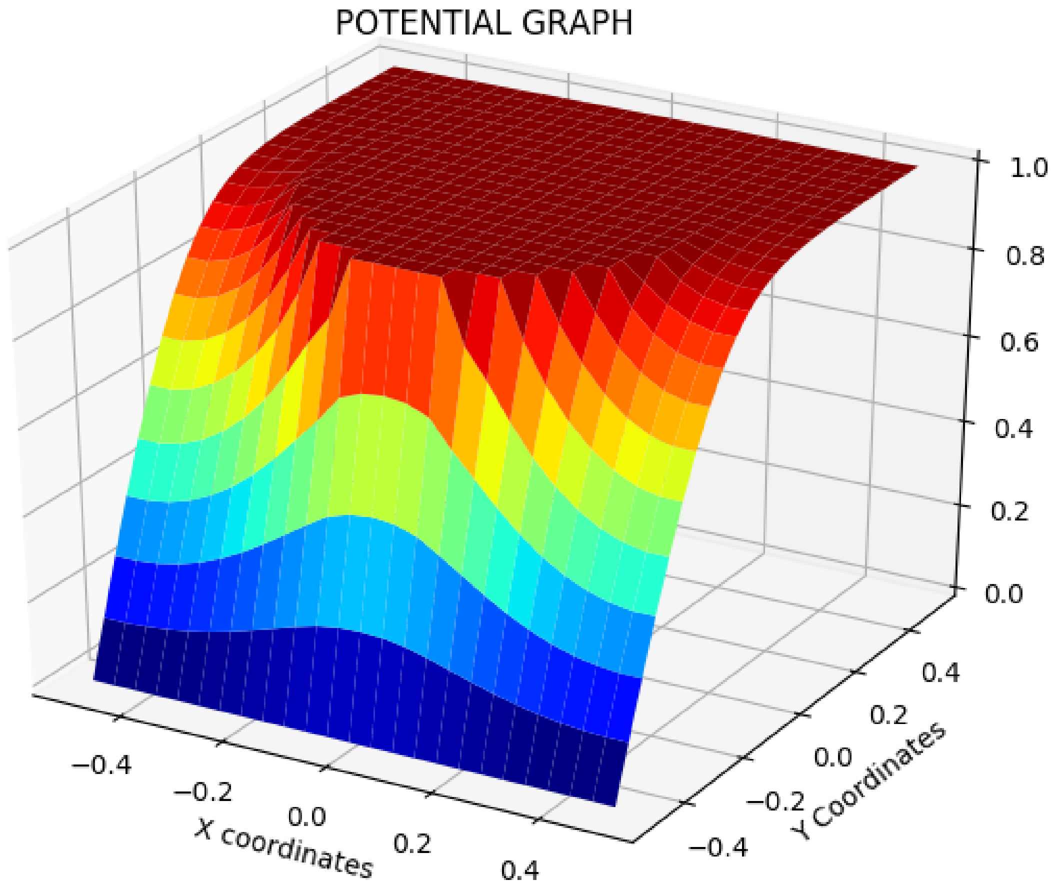

Figure 2: Potential vs Coordinates

From the above graph, we can conclude that the potential at the center and the top half of the conductor remains mostly at 1V. Since the bottom of the plate is set to 0V, in potential varies from 1V to 0V as we go from middle to the bottom of the plate. Thereby, all of our boundary conditions are met in the above graph.

Now, the obtained potential graph is what we expect because, current can't flow across any of the other three edges as they don't form a closed loop for the current to flow. Therefore, current can flow only along the bottom thereby potential can drop only in the specified direction.

3 Contour Plot of Potential

The plot below represents the contour plot of the potential. The red dots on the plot represent the points where the wire is in contact with the plate and the potential is 1V.

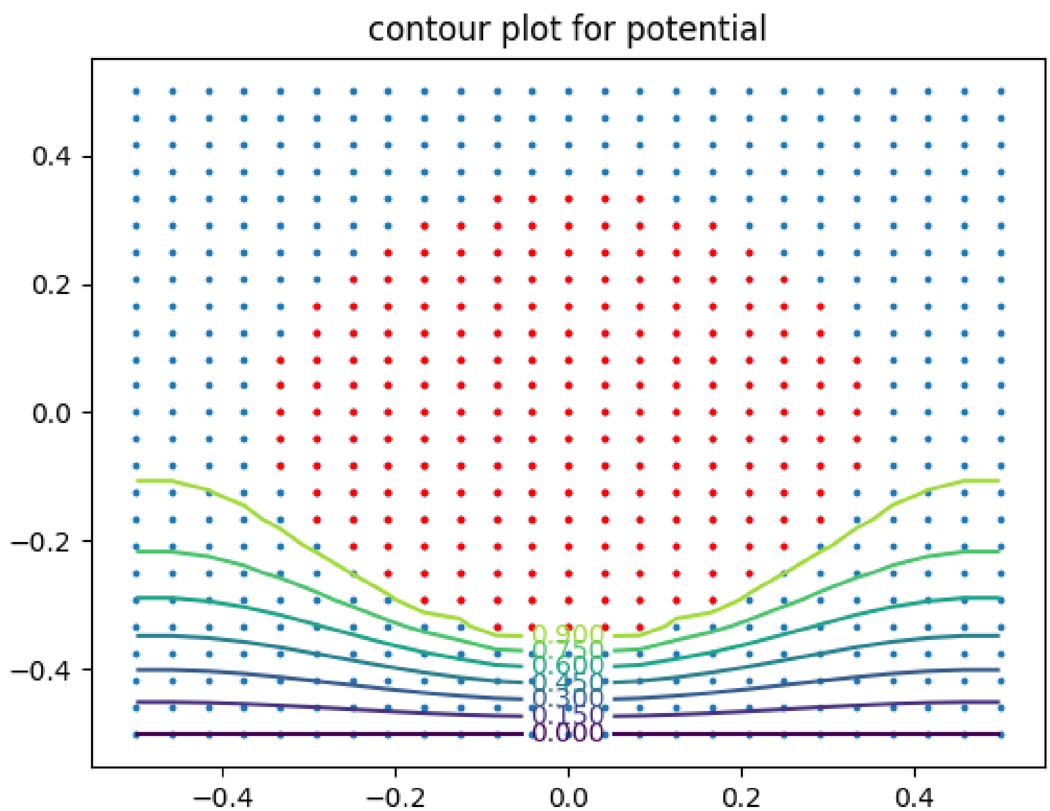

Figure 3: Contour plot of the potential

4 Current Densities

Now, since we have the potential map of the plate, we try to find the current densities of the plate as function of the cartesian coordinates. We know the following equations :

$$j_x = -\frac{\partial phi}{\partial x} \quad j_y = -\frac{\partial phi}{\partial y} \tag{6}$$

From the equation (3) in the report, we can write the above equations as :

$$J_{x_{i,j}} = \frac{1}{2}(\phi_{i,j-1} - \phi_{i,j+1}) \tag{7}$$

$$J_{y_{i,j}} = \frac{1}{2}(\phi_{i-1,j} - \phi_{i+1,j}) \tag{8}$$

Now, we create J_x,J_y with the appropriate dimensions and then run the below code to obtain the current densities as a function of current densities.

```
currentx[:,1:Nx-2] = 0.5*(potential[:,0:Nx-3]-potential[:,2:Nx-1])
currenty[1:Ny-2,:] =-0.5*(potential[0:Ny-3,:]-potential[2:Ny-1,:])
```

Now, in order to plot the above vector quantity, we use the quiver function in python which is a part of pyplot.

```
quiver(x,y,currentx,currenty,scale= 4)
```

The current coming out of the top part of the plate is almost zero because the potential gradient across the top half of the plate is almost zero i.e. most of the top half of the plate have a potential 1V therefore making the current almost zero.

5 Inferences

- Firstly, we understand that by using suitable approximations and boundary conditions, we can solve differential equations and obtain meaningful physical quantities.
- However, the method which has been employed in the assignment is obselete to solve the Laplace equation and various advanced techniques exist.
- The function quiver can be used to plot vectorial quantities thereby providing an effective way to represent the vectorial quantities pictorially.

The plot below represents the current density of the system.

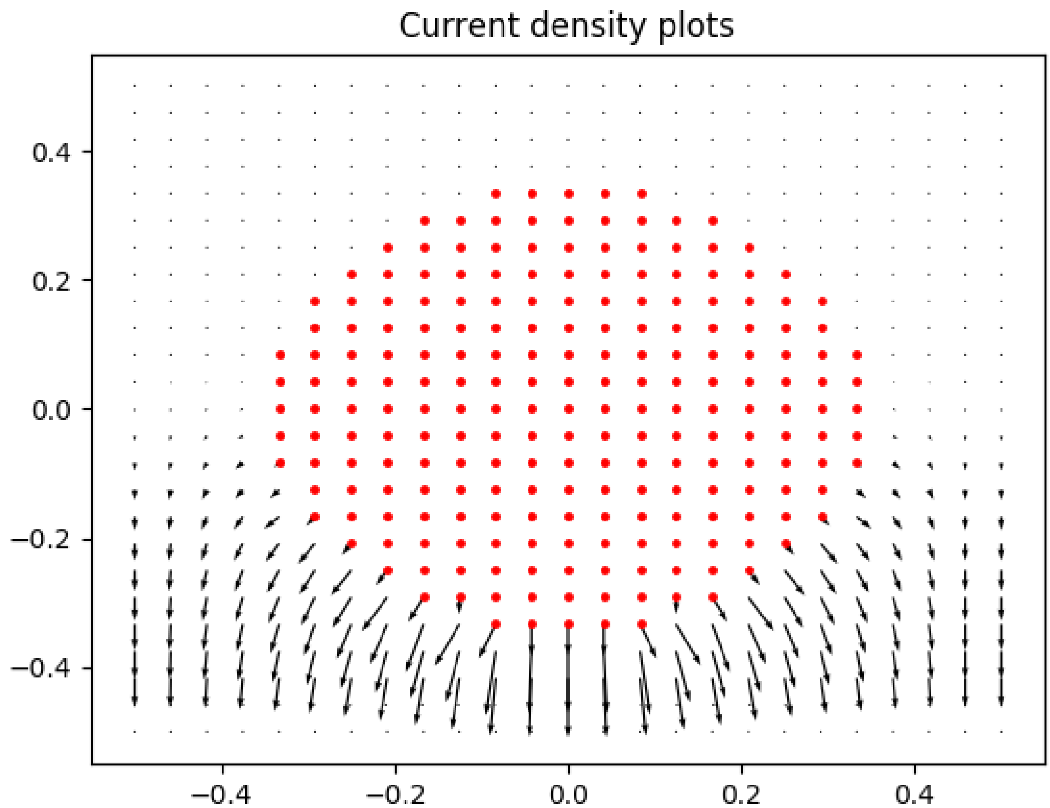

Figure 4: Current Density J = Jx + Jy