ASSIGNMENT 6 - EE2703

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1 Introduction

This assignment deals with introduction of the **scipy.signal** module. The module is composed of various functions which can represent and perform operations frequency domain functions such as scipy.signal.lti, scipy.signal.impulse, scipy.signal.lsim

The functions which have been used for the assignment are as follows :

- scipy.signal.lti : This function defines the transfer function of a linear time invariant system wherein one can define the numerator and denominator polynomials as the parameters to the function.

  The below line defines the function $H(s) = \frac{1+s}{1-s}$

  ```
  H = sp.lti([-1,1],[1,1])
  ```

- scipy.signal.impulse : This function takes the transfer function, initial conditions and the time interval as the inputs and provides the impulse response of the system as the output.

  The below line of code will evaluate the impulse response of H over a time interval of 10 seconds.

  ```
  H_imp = sp.impulse(H,None,np.linspace(0,10,101))
  ```

- scipy.signal.lsim : This function computes the convolution of two time domain functions by taking the imput of one function in time domain and the other in frequency domain.

  The line below computes the convolution of H and cos(t) in the time interval of 20 seconds.

  ```
  conv = sp.lsim(H,np.cos(t),np.linspace(0,20,201)
  ```

Now, the various questions in the assignment require us to calculate the transfer functions in various electrical and physical systems and then solve for the function and plot them in time domain with the necessary initial conditions.

2 Question 1

From the given data, we know that the input to the system is :

$$f(t) = cos(1.5t)e^{-0.5t}u(t) \quad F(s) = \frac{s+0.5}{(s+0.5)^2+2.25}$$

The equation of the differential equation is :

$$\frac{d^2x(t)}{dt^2} + 2.25x(t) = f(t) \quad X(s) = \frac{F(s)}{s^2+2.25}$$

Hence, since we now know the form of X(s), we define an LTI function for it and then compute the impulse response which gives us the x(t). The lines of code for the above process are as follows

```
X = sp.lti([-0.5],[-0.5+1.5j,-0.5-1.5j,-1.5j,1.5j],1)
t2,x2=sp.impulse(X,None,np.linspace(0,50,1001))
```

Now, we have obtained x(t) and we proceed to plot it. The obtained plot is :

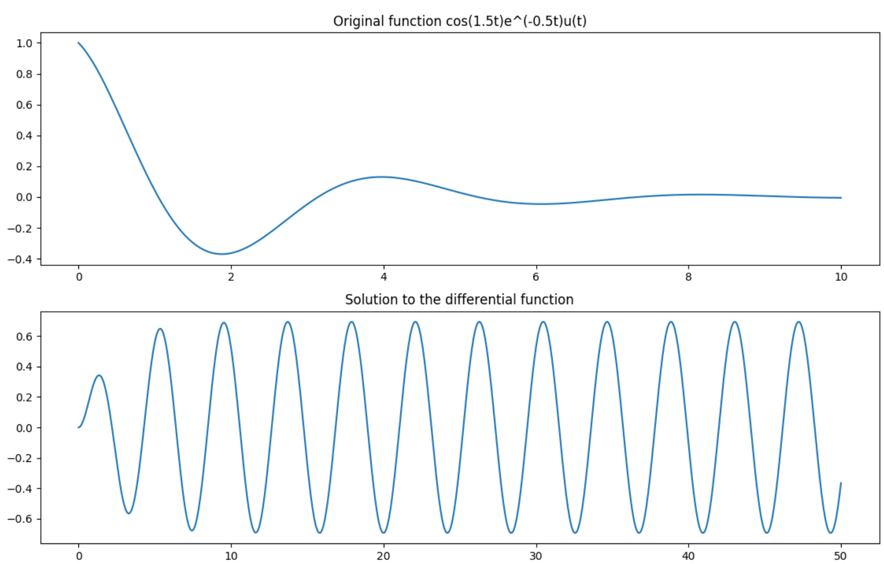

Figure 1: Function and the corresponding Output

3 Question 2

This question involves changing the decay factor of the input from 0.5 to 0.05. Therefore, we obtain the following plot :

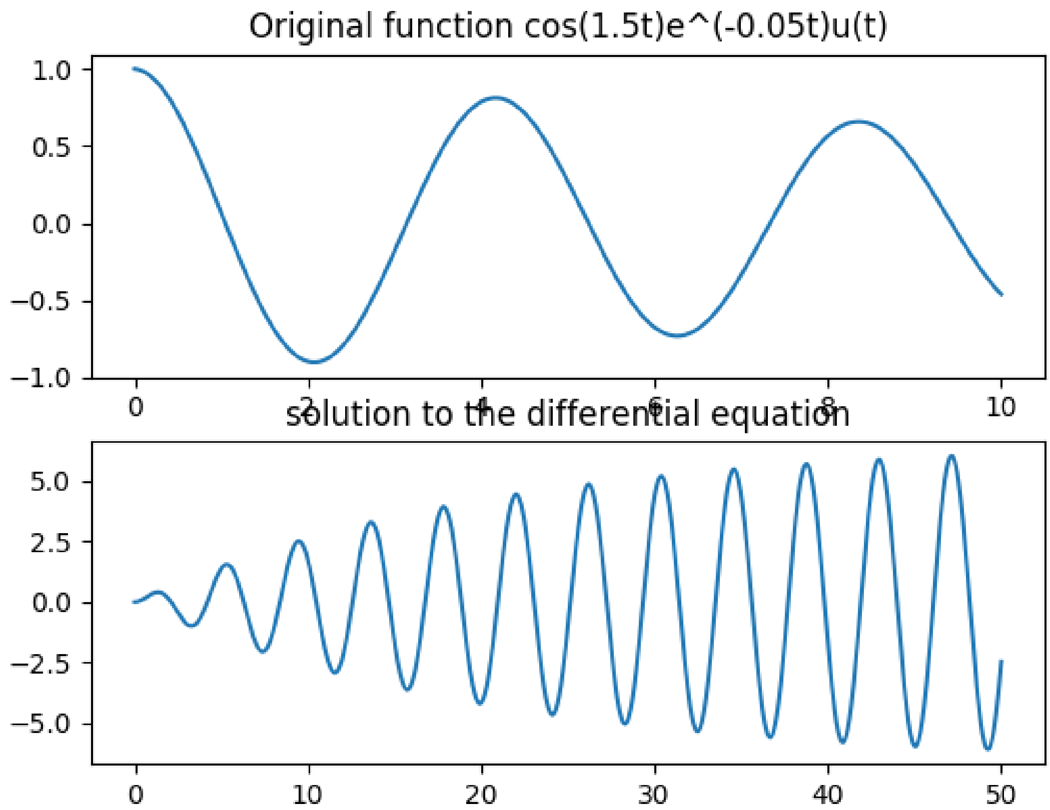

Figure 2: Decay factor of 0.05

In the above plot, there is a change in input transfer function :

$$F(s) = \frac{s + 0.05}{(s + 0.05)^2 + 2.25}$$

4 Question 3

Here, we are required to generate the output continuous time function for a range of frequencies from 1.4 to 1.6. The code used to accomplish the task is :

```
t1,tran_func_cont=sp.impulse(tran_func,None,np.linspace(0,50,501))
input_func = sp.lti([-0.05],[-a-0.05,+a-0.05],1)
t,y,svec = sp.lsim(input_func,tran_func_cont,np.linspace(0,50,501))
```

The above lines with intitial value of a as 1.4j and a loop gives the graph as shown below :

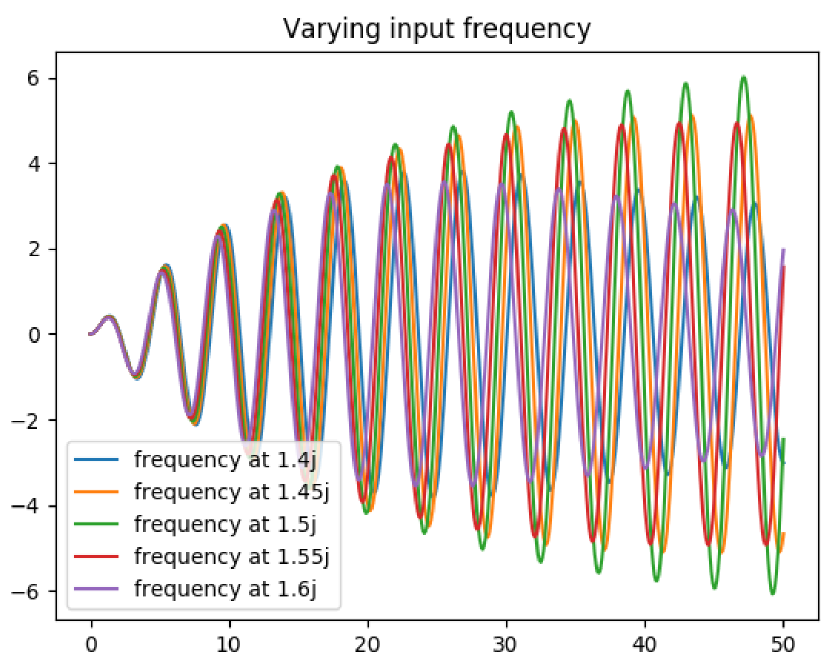

Figure 3: Variation of Amplitude with change in frequency

The plot shows that the amplitude of the output is maximum when the frequency of the forced and natural response are equal. Moreover, any deviation from the natural frequency results in a lower amplitude.

5 Question 4

Firstly, we must convert the two differential equations into their frequency counterparts for the ease of solving :

$$Y(s) = X(s)(s^2 + 1) - s$$

$$Y(s)(s^2 + 2) = 2X(s)$$

From solving the two equations, we obtain the expressions :

$$X(s) = \frac{s^2 + 2}{s^3 + 3s} \qquad Y(s) = \frac{2}{s^3 + 3s}$$

Thereby, we can obtain the time domain functions using the sp.impulse function.

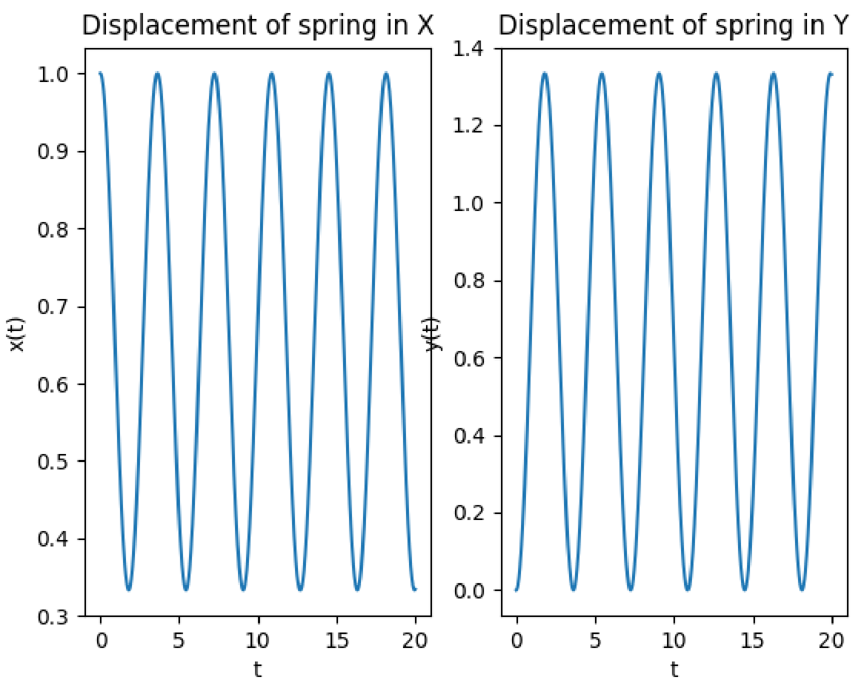

Figure 4: The displacement of spring in both the directions

6 Question 5

Using the circuit analysis, the tranfer function we obtain is :

$$H(s) = \frac{10^{12}}{s^2 + 10^8 s + 10^{12}}$$

The bode plot of the above obtained transfer function can be calculated using the given code :

```
H_5 = sp.lti([10**12],[1,10**8,10**12])
w,mod,phi = H_5.bode()
```

The bode plots of the magnitude and frequency are as follows :

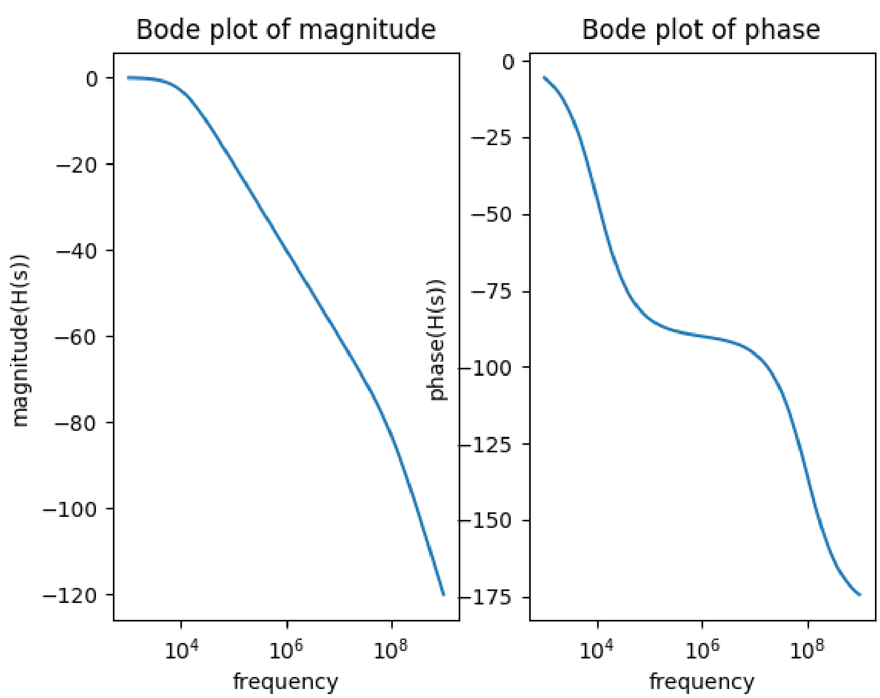

Figure 5: Bode Plots

7 Question 6

: The input to the system given in the question 5 is :

$$v_i(t) = cos(10^3t) - cos(10^6t)$$

Using the sp.lsim function, we find the convolution of the transfer function and input thereby finding the output of the system in continuous time form.

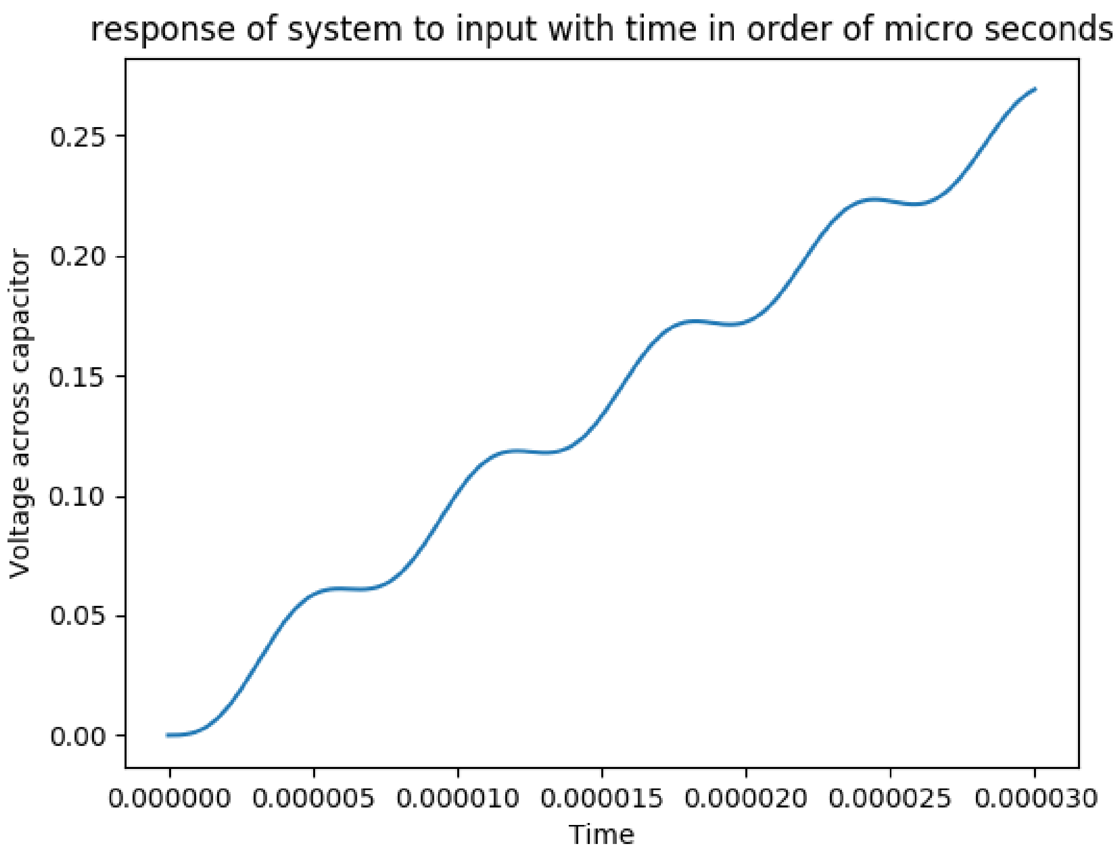

Figure 6: Output Voltage for Micro second range

The output response for small time intervals just after t=0, we can see that the output voltage is increasing almost linearly. This is because the value of the output voltage is non zero at $t = 0^-$ and the voltage at $t = 0^+$ is zero. In order to reach the voltage, an impulsive current will flow thereby resulting in a rapid increase in voltage.

Now, in the case of large time intervals such as milli seconds, we see that the transfer function acts as a low pass filter thereby the $10^6 rad/s$ component of the input gets attenuated and the $10^3 rad/s$ component remains. Thereby, we get a periodic function overall.

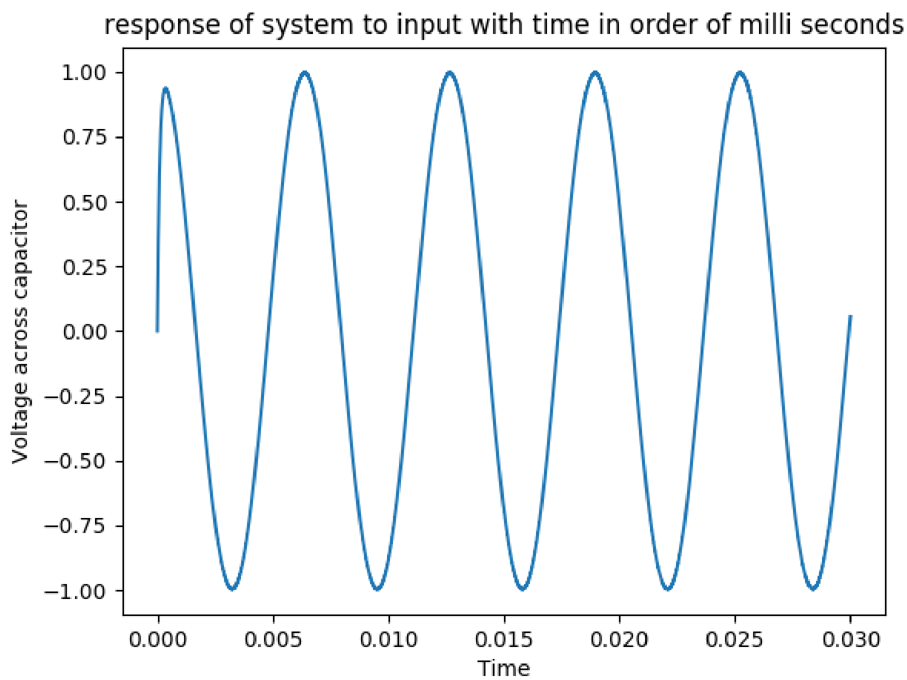

Figure 7: For Large time intervals

8 Inferences

- Firstly, we can perform signal analysis in the frequency domain using python.
- Python makes solving differential equations easier.