Assignment4

EE17B047

February 27, 2019

1 Introduction

This assignment aims to obtain the fourier series of various functions and see how well the model we obtain fit to the actual function. In particular, it involves finding the fourier coefficients of the functions **exp(x)** and **cos(cos(x))**.

Fourier series is primarily used to express periodic functions in the form of sum of sinusoids. Any function f(x) can be represented using the fourier series as :

$$f(x) = a_0 + \sum_{1}^{\infty} \{a_k sin(kx) + b_k cos(kx)\} \tag{1}$$

Now, from the above equation we can determine the coefficients of the series a_k and b_k from the equations :

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(x)dx \tag{2}$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x)cos(nx)dx \tag{3}$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x)sin(nx)dx \tag{4}$$

2 Question1

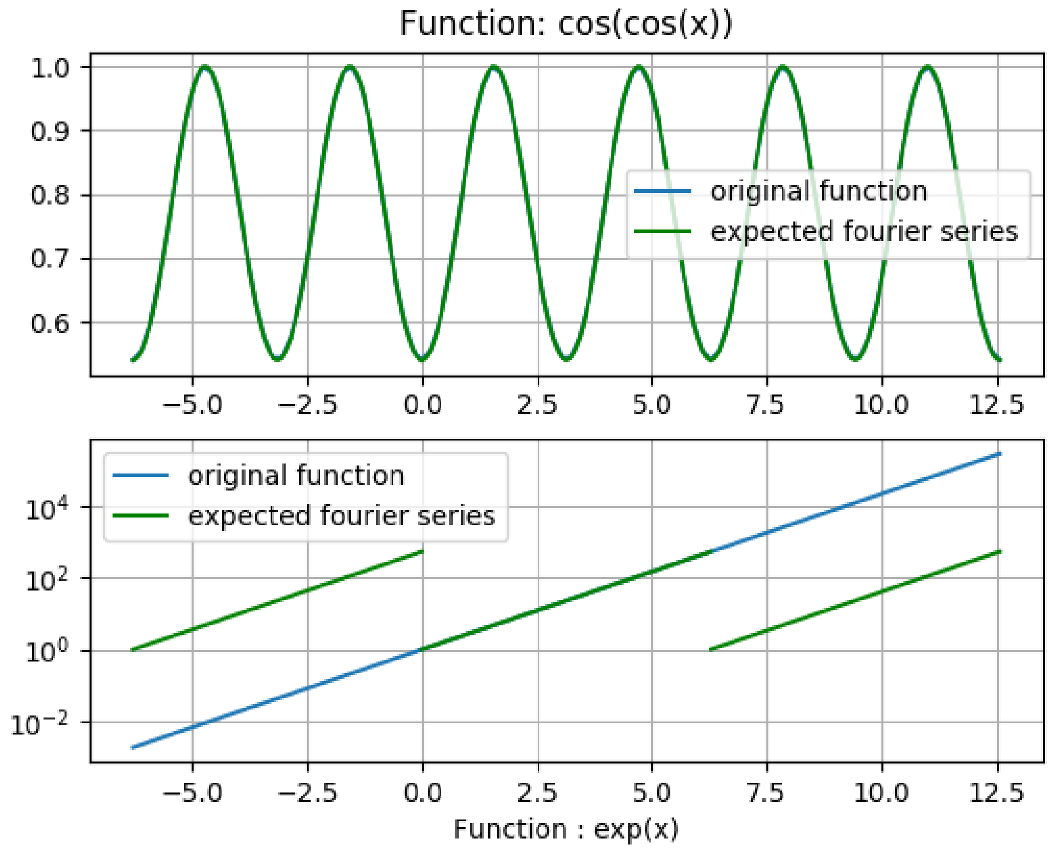

Figure 1: Functions cos(cos(x) and exp(x)

From the above figure, we can conclude that the function exp(x) is non periodic whereas the function cos(cos(x)) is 2π periodic. Since exp(x) is not periodic but, the fourier series is always periodic. Hence, we expect the fourier series to emulate the function in the range [0,2π].

3 Question2

Now, by employing the equations (2),(3),(4) we can obtain the fourier series of the functions by using the quad() function for the integration. In order to facilitate the use of quad(), we define two more functions namely :

$$u(x) = f(x)cos(nx) \quad v(x) = f(x)sin(nx) \tag{5}$$

4 Question 3

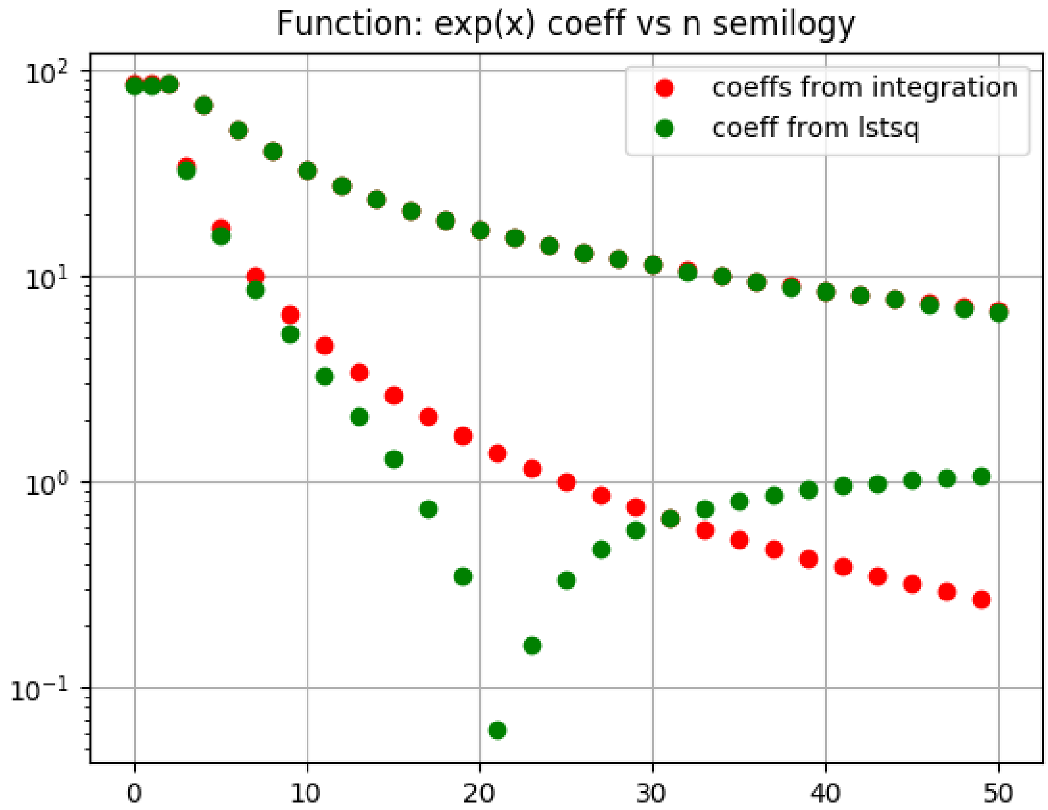

Figure 2: semilogy plot for coefficients of exp(x)

The above graphs are composed of the plot of the coefficients obtained from fourier series and lstsq as well.

a) The b_n coefficients in the cos(cos(x)) case go to zero because it is an even function and all the b_n fourier coefficients of even functions are zero.

The cos(cos(x)) being a periodic function, has contributions from fixed no of sinusoids. Thereby, its coefficients quickly decay to zero after a given frequency. Whereas, in the case of exp(x) the function being non periodic, has contributions from higher frequency components as well. Thereby, its components don't decay as fast as that of cos(cos(x)). Moreover, exp(x) is always greater than cos(cos(x)).

c) Hence, from the above expressions we can conclude that log(a_n) is propotional to log(n) thereby proving that the loglog plot for coefficients of exp(x) is linear.

$$\int_0^{2\pi} e^x cos(nx) = \frac{e^{2\pi}-1}{n^2+1} \quad \int_0^{2\pi} e^x sin(nx) = \frac{n(1-e^{2\pi})}{n^2+1} \tag{6}$$

In case of cos(cos(x)), the coefficients a_n and b_n are proporional to e^{-n} thereby making the semilogy graph to look linear.

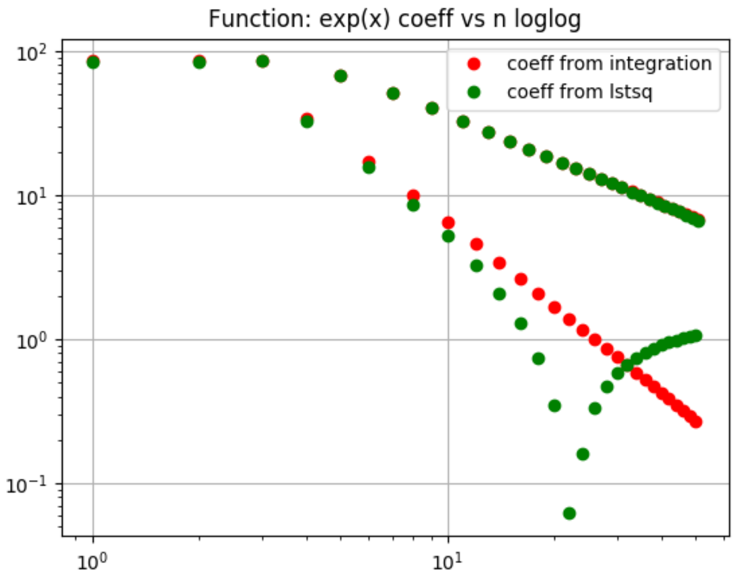

Figure 3: loglog plot of coefficients of exp(x)

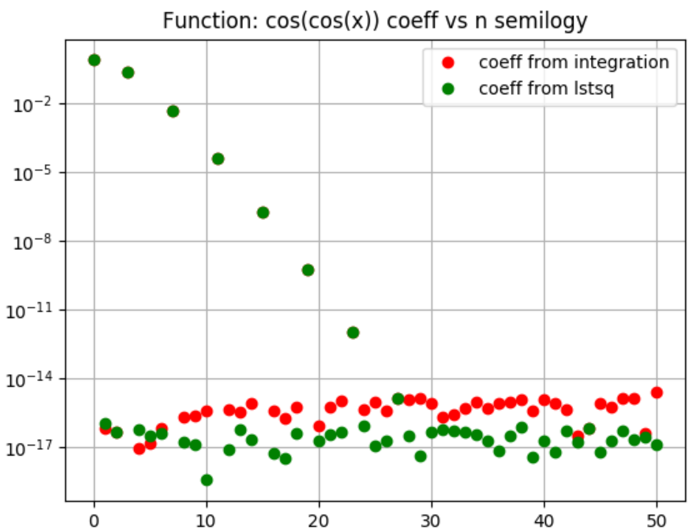

Figure 4: semilogy plot of coefficients of cos(cos(x))

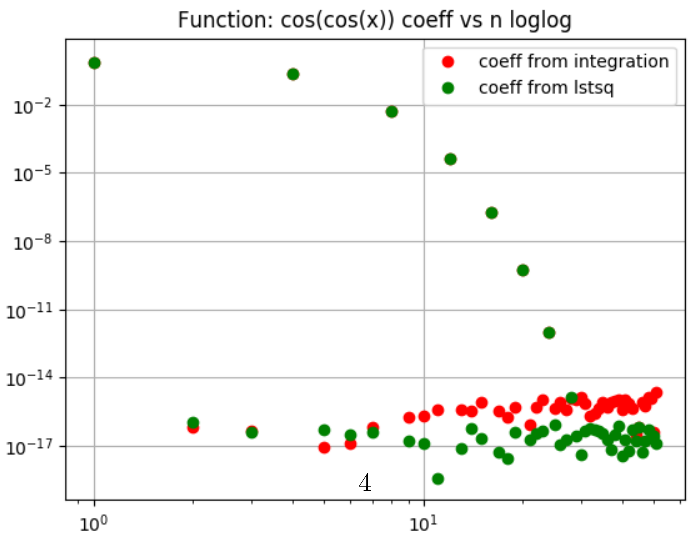

Figure 5: loglog plot of coefficients of cos(cos(x))

5 Question 4

Now, we make use of the method lstsq by assuming the function f(x) to be:

$$f(x_i) \approx a_0 + \sum_{1}^{25}\{a_i cos(nx_i) + b_i sin(nx_i)\} \tag{7}$$

In order to proceed with lstsq, we have to from the matrices A,b in the equation :

$$Ac = b \tag{8}$$

Where c is the matrix composed of the coefficients of the sinusoids.

6 Question 5

After obtaining the coefficients using lstsq, we plot the coefficients in the figures 2,3,4,5 respectively.The line of code for lstsq is :

```
c = lstsq(A,b)[0]
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7 Question 6

From the obtained values, the coefficients from both the methods are almost the same with minor deviations in values. The maximum deviation in the coefficients of the function exp(x) is at the coefficient number 1 and the value is 1.3327. Correspondingly, the value of the largest deviation in coefficients for the function cos(cos(x)) is at the coefficient 50 and the value is 2.37e-15 which is almost about zero indicating the fact that the coefficients for cos(cos(x)) are almost equal. The maximum of absolute difference can be found using the max function.

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max(abs(c1-c2))
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8 Question 7

From the plot in the figures below, we can see that the function exp(x) doesn't fit properly whereas the function cos(cos(x)) fits perfectly. This can be attributed to the fact that the fourier series can be used to fit periodic functions properly whereas they don't represent non periodic functions perfectly. Hence, since cos(cos(x)) is periodic, the coefficients fit well but they don't fit for exp(x). This is because exp(x) is has a discontinuity at 0 due to which the parameters don't fit propoerly.

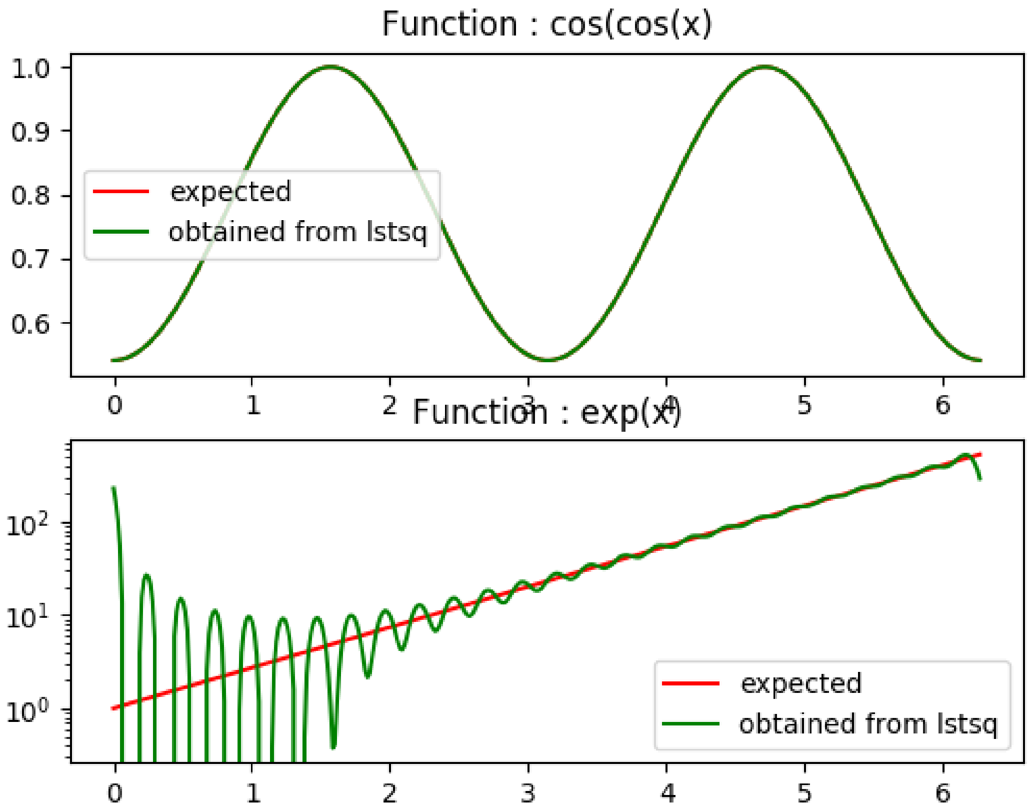

Figure 6: Plot for expected vs fitted curves

9 Conclusion

- Hence, from the above assignment, we can conclude that fourier series can fit periodic functions perfectly whereas they give huge variations when being fitted to non periodic functions.
- The functions which have a discontinuity at the boundary i.e at 0 or 2π have some distortions in the fit.