

## **EJERCICIO 1**

1. Una deuda se expresa en sentido negativo. Luego inicialmente el estado económico de Pedro es - 60 bs.

Al recibir 320 bs. Aumenta su capital tantos bs. como ha recibido; entonces la operación queda expresada como se indica

$$-60+320=+260 \text{ bs.}$$

2.  $1.170-1.515=-345 \text{ sucres}$

3.  $200+56-189=+\$67$

4.  $-665-1.178+2.280=+437 \text{ soles}$

5.  $20-15+40-75=-\$30$

6.  $-67+72-16+2=-\$9$

7.  $200-78-81-93+41-59=-70 \text{ colones}$

8.  $-45-66-79+200-10=0$

## **EJERCICIO 2**

1.  $12-15=-3^\circ$

2.  $-3+8-6=-1^\circ$

3.  $15-(-3)=15+3=18^\circ$

4.  $-(-8)+5=8+5=13^\circ$

5.  $-4+7+2-11=-6^\circ$

6.  $-8+4 \cdot 1=-8+4=-4^\circ \rightarrow 7 \text{ am}$

$$-8+4 \cdot 2=-8+8=0^\circ \rightarrow 8 \text{ am}$$

$$-8+4 \cdot 5=-8+20=+12^\circ \rightarrow 11 \text{ am}$$

7.  $-1-2 \cdot 2=-1-4=-5^\circ \rightarrow 10 \text{ am}$

$$-1-2 \cdot 3=-1-6=-7^\circ \rightarrow 11 \text{ am}$$

$$-1-2 \cdot 3+3 \cdot 1=-1-6+3=-4^\circ \rightarrow 12 \text{ am}$$

$$-1-2 \cdot 3+3 \cdot 3=-1-6+9=+2^\circ \rightarrow 2 \text{ pm}$$

8.  $-56+7=-49^\circ$

9.  $-71+5=-66^\circ \text{ long}$

$$-15+(-5)=-20^\circ \text{ lat.}$$

10.  $18+3=+21^\circ \text{ long}$

$$65-4=+61^\circ \text{ lat.}$$

11.  $-75+135=+60 \text{ años}$

## **EJERCICIO 3**

1.  $+32m; -16m$

2.  $+10m; -4m$

3.  $50-85=-35m$

4.  $-6 \cdot 11=-66m$

5.  $-8 \times 6=-48m$

$$9 \times 6=+54m$$

6.  $400 \cdot 2=+800m \rightarrow \text{corredor}$

$$-400 \cdot 3=-1.200m \rightarrow \text{yo}$$

7. Los 40 pies de longitud del poste se reparten así: 15 pies que sobresalen

- se asumen en sentido positivo - .

25 pies que se encuentran enterrados.

- se asumen en sentido negativo - .

Al introducir 3 pies más, se adicionan a los que están bajo el suelo y se deben descontar de los que están por encima.

Aritméticamente significa:

$$40=15+25$$

$$-25-3=-28 \text{ pies}$$

$$+15-3=+12 \text{ pies}$$

8.  $55-52=+3m$

9.  $-32+15=-17m$

10.  $35-47=-12m$

11.  $-39+56=+17m$

12.  $90-58-36=-4m$

13.  $72-30 \cdot 1=72-30=+42m$

$$72-30 \cdot 2=72-60=+12m$$

$$72-30 \cdot 3=72-90=-18m$$

$$72-30 \cdot 4=72-120=-48m$$

14.  $-120-(-60) \cdot 1=-120+60=-60 \text{ Km}$

$$-120-(-60) \cdot 2=-120+120=0$$

$$-120-(-60) \cdot 3=-120+180=+60 \text{ Km}$$

$$-120-(-60) \cdot 4=-120+240=+120 \text{ Km}$$

## EJERCICIO 7.

1.  $x+2x=3x$
2.  $8a+9a=17a$
3.  $11b+9b=20b$
4.  $-b-5b=-6b$
5.  $-8m-m=-9m$
6.  $-9m-7m=-16m$
7.  $4a^x+5a^x=9a^x$
8.  $6a^{x+1}+8a^{x+1}=14a^{x+1}$
9.  $-m^{x+1}-5m^{x+1}=-6m^{x+1}$
10.  $-3a^{x-2}-a^{x-2}=-4a^{x-2}$
11.  $\frac{1}{2}a+\frac{1}{2}a=\frac{1+1}{2}a=\frac{2}{2}a=a$
12.  $\frac{3}{5}ab+\frac{1}{10}ab=\frac{6ab+ab}{10}=\frac{7}{10}ab$
13.  $\frac{1}{3}xy+\frac{1}{6}xy=\frac{2+1}{6}xy=\frac{3}{6}xy=\frac{1}{2}xy$
14.  $-\frac{1}{5}xy-\frac{4}{5}xy=\frac{-1-4}{5}xy=-\frac{5}{5}xy=-xy$
15.  $-\frac{5}{6}a^2b-\frac{1}{8}a^2b=\frac{-20-3}{24}a^2b=-\frac{23}{24}a^2b$
16.  $-a-\frac{7}{8}a=\frac{-8-7}{8}a=-\frac{15}{8}a$
17.  $8a+9a+6a=23a$
18.  $15x+20x+x=36x$
19.  $-7m-8m-9m=-24m$
20.  $-a^2b-a^2b-3a^2b=-5a^2b$
21.  $a^x+3a^x+8a^x=12a^x$
22.  $-5a^{x+1}-3a^{x+1}-5a^{x+1}=-13a^{x+1}$
23.  $a+\frac{1}{2}a+\frac{2}{3}a=\frac{6+3+4}{6}a=\frac{13}{6}a$
24.  $-x-\frac{2}{3}x-\frac{1}{6}x=\frac{-6-4-1}{6}x=-\frac{11}{6}x$
25.  $\frac{1}{5}ax+\frac{3}{10}ax+ax=\frac{2+3+10}{10}ax=\frac{15}{10}ax=\frac{3}{2}ax$
26.  $-\frac{3}{4}a^2x-\frac{5}{6}a^2x-a^2x=\frac{-9-10-12}{12}a^2x=-\frac{31}{12}a^2x$

27.  $11a+8a+9a+11a=39a$
28.  $m^{x+1}+3m^{x+1}+4m^{x+1}+6m^{x+1}=14m^{x+1}$
29.  $-x^2y-8x^2y-9x^2y-20x^2y=-38x^2y$
30.  $-3a^m-5a^m-6a^m-9a^m=-23a^m$
31.  $\frac{1}{2}a+\frac{1}{4}a+\frac{1}{8}a+a=\frac{4+2+1+8}{8}a=\frac{15}{8}a$
32.  $\frac{2}{5}ax+\frac{1}{2}ax+\frac{1}{10}ax+\frac{1}{20}ax=\frac{8+10+2+1}{20}ax=\frac{21}{20}ax$
33.  $0,5m+0,6m+0,7m+0,8m=2,6m$
34.  $-\frac{1}{7}ab-\frac{1}{14}ab-\frac{1}{28}ab-ab$   
 $=\frac{-4-2-1-28}{28}ab=-\frac{35}{28}ab=-\frac{5}{4}ab$
35.  $-\frac{2}{3}x^3y-\frac{1}{6}x^3y-\frac{1}{9}x^3y-\frac{1}{12}x^3y$   
 $=\frac{-24-6-4-3}{36}x^3y=-\frac{37}{36}x^3y$
36.  $ab^2+ab^2+7ab^2+9ab^2+21ab^2=39ab^2$
37.  $-m-m-8m-7m-3m=-20m$
38.  $-x^{a+1}-8x^{a+1}-4x^{a+1}-5x^{a+1}-x^{a+1}=-19x^{a+1}$
39.  $\frac{1}{2}a+\frac{1}{3}a+\frac{1}{4}a+\frac{1}{5}a+\frac{1}{6}a$   
 $=\frac{30+20+15+12+10}{60}a=\frac{87}{60}a=\frac{29}{20}a$
40.  $-\frac{1}{3}ab-\frac{1}{6}ab-\frac{1}{2}ab-\frac{1}{12}ab-\frac{1}{9}ab$   
 $=\frac{-12-6-18-3-4}{36}ab=-\frac{43}{36}ab$

## EJERCICIO 8

1.  $8a-6a=2a$
2.  $6a-8a=-2a$
3.  $9ab-15ab=-6ab$
4.  $15ab-9ab=6ab$
5.  $2a-2a=0$
6.  $-7b+7b=0$
7.  $-14xy+32xy=18xy$

8.  $-25x^2y + 32x^2y = 7x^2y$
9.  $40x^3y - 51x^3y = -11x^3y$
10.  $-n^2n + 6m^2n = 5m^2n$
11.  $-15xy + 40xy = 25xy$
12.  $55a^3b^2 - 81a^3b^2 = -26a^3b^2$
13.  $-x^2y + x^2y = 0$
14.  $-9ab^2 + 9ab^2 = 0$
15.  $7x^2y - 7x^2y = 0$
16.  $-101mn + 118mn = 17mn$
17.  $502ab - 405ab = 97ab$
18.  $-1024x + 1018x = -6x$
19.  $-15ab + 15ab = 0$
20.  $\frac{1}{2}a - \frac{3}{4}a = \frac{2-3}{4}a = -\frac{1}{4}a$
21.  $\frac{3}{4}a - \frac{1}{2}a = \frac{3-2}{4}a = \frac{1}{4}a$
22.  $\frac{5}{6}a^2b - \frac{5}{12}a^2b = \frac{10-5}{12}a^2b = \frac{5}{12}a^2b$
23.  $-\frac{4}{7}x^2y + \frac{9}{14}x^2y = \frac{-8+9}{14}x^2y = \frac{1}{14}x^2y$
24.  $\frac{3}{8}am - \frac{5}{4}am = \frac{3-10}{8}am = -\frac{7}{8}am$
25.  $-am + \frac{3}{5}am = \frac{-5+3}{5}am = -\frac{2}{5}am$
26.  $\frac{5}{6}mn - \frac{7}{8}mn = \frac{20-21}{24}mn = -\frac{1}{24}mn$
27.  $-a^2b + \frac{3}{11}a^2b = \frac{-11+3}{11}a^2b = -\frac{8}{11}a^2b$
28.  $3,4a^4b^3 - 5,6a^4b^3 = -2,2a^4b^3$
29.  $-1,2yz + 3,4yz = 2,2yz$
30.  $4a^x - 2a^x = 2a^x$
31.  $-8a^{x+1} + 8a^{x+1} = 0$
32.  $25m^{a-1} - 32m^{a-1} = -7m^{a-1}$
33.  $-x^{a+1} + x^{a+1} = 0$
34.  $-\frac{1}{4}a^{m-2} + \frac{1}{2}a^{m-2} = \frac{-1+2}{4}a^{m-2} = \frac{1}{4}a^{m-2}$

35.  $\frac{5}{6}a^{m+1} - \frac{7}{12}a^{m+1} = \frac{10-7}{12}a^{m+1} = \frac{3}{12}a^{m+1} = \frac{1}{4}a^{m+1}$
36.  $4a^2 - \frac{1}{3}a^2 = \frac{12-1}{3}a^2 = \frac{11}{3}a^2$
37.  $-5mn + \frac{3}{4}mn = \frac{-20+3}{4}mn = -\frac{17}{4}mn$
38.  $8a^{x+2}b^{x+3} - 25a^{x+2}b^{x+3} = -17a^{x+2}b^{x+3}$
39.  $-\frac{7}{8}a^mb^n + a^mb^n = \frac{-7+8}{8}a^mb^n = \frac{1}{8}a^mb^n$
40.  $0,85mxy - 0,5mxy = 0,35mxy$

## EJERCICIO 9

1.  $9a - 3a + 5a = 11a$
2.  $-8x + 9x - x = 0$
3.  $12mn - 23mn - 5mn = -16mn$
4.  $-x + 19x - 18x = 0$
5.  $19m - 10m + 6m = 15m$
6.  $-11ab - 15ab + 26ab = 0$
7.  $-5a^x + 9a^x - 35a^x = -31a^x$
8.  $-24a^{x+2} - 15a^{x+2} + 39a^{x+2} = 0$
9.  $\frac{2}{3}y + \frac{1}{3}y - y = \frac{2+1-3}{3}y = \frac{0}{3}y = 0$
10.  $-\frac{3}{5}m + \frac{1}{4}m - \frac{1}{2}m = \frac{-12+5-10}{20}m = -\frac{17}{20}m$
11.  $\frac{3}{8}a^2b + \frac{1}{4}a^2b - a^2b = \frac{3+2-8}{8}a^2b = -\frac{3}{8}a^2b$
12.  $-a + 8a + 9a - 15a = a$
13.  $7ab - 11ab + 20ab - 31ab = -15ab$
14.  $25x^2 - 50x^2 + 11x^2 + 14x^2 = 0$
15.  $-xy - 8xy - 19xy + 40xy = 12xy$
16.  $7ab + 21ab - ab - 80ab = -53ab$
17.  $-25xy^2 + 11xy^2 + 60xy^2 - 82xy^2 = -36xy^2$
18.  $-72ax + 87ax - 101ax + 243ax = 157ax$
19.  $-82bx - 71bx - 53bx + 206bx = 0$
20.  $105a^3 - 464a^3 + 58a^3 + 301a^3 = 0$

$$21. \frac{1}{2}x - \frac{1}{3}x + \frac{1}{4}x - \frac{1}{5}x = \frac{30-20+15-12}{60}x = \frac{13}{60}x$$

$$22. \frac{1}{3}y - \frac{1}{3}y + \frac{1}{6}y - \frac{1}{12}y = \frac{4-4+2-1}{12}y = \frac{1}{12}y$$

$$23. \frac{3}{5}a^2b - \frac{1}{6}a^2b + \frac{1}{3}a^2b - a^2b = \frac{18-5+10-30}{30}a^2b = -\frac{7}{30}a^2b$$

$$24. -\frac{5}{6}ab^2 - \frac{1}{6}ab^2 + ab^2 - \frac{3}{8}ab^2 = \frac{-20-4+24-9}{24}ab^2 = -\frac{9}{24}ab^2 = -\frac{3}{8}ab^2$$

$$25. -a+8a-11a+15a-75a=-64a$$

$$26. -7c+21c+14c-30c+82c=80c$$

$$27. -mn+14mn-31mn-mn+20mn=mn$$

$$28. a^2y-7a^2y-93a^2y+51a^2y+48a^2y=0$$

$$29. -a+a-a+a-3a+6a=3a$$

$$30. \frac{1}{2}x + \frac{2}{3}x - \frac{7}{6}x + \frac{1}{2}x - x = \frac{3+4-7+3-6}{6}x = -\frac{3}{6}x = -\frac{1}{2}x$$

$$31. -2x + \frac{3}{4}x + \frac{1}{4}x + x - \frac{5}{6}x = \frac{-48+18+6+24-20}{24}x = -\frac{20}{24}x = -\frac{5}{6}x$$

$$32. 7a^x - 30a^x - 41a^x - 9a^x + 73a^x = 0$$

$$33. -a^{x+1} + 7a^{x+1} - 11a^{x+1} - 20a^{x+1} + 26a^{x+1} = a^{x+1}$$

$$34. a+6a-20a+150a-80a+31a=88a$$

$$35. -9b-11b-17b-81b-b+110b=-9b$$

$$36. -a^2b+15a^2b+a^2b-85a^2b-131a^2b+39a^2b=-162a^2b$$

$$37. 84m^2x-501m^2x-604m^2x-715m^2x+231m^2x+165m^2x = -1340m^2x$$

$$38. \frac{5}{6}a^3b^2 + \frac{2}{3}a^3b^2 - \frac{1}{4}a^3b^2 - \frac{5}{8}a^3b^2 + 4a^3b^2 = \frac{20+16-6-15+96}{24}a^3b^2 = \frac{111}{24}a^3b^2 = \frac{37}{8}a^3b^2 = 4\frac{5}{8}a^3b^2$$

$$39. 40a-81a+130a+41a-83a-91a+16a=-28a$$

$$40. -21ab+52ab-60ab+84ab-31ab-ab-23ab=0$$

## EJERCICIO 10

$$1. 7a+6a-9b-4b \\ 7a+6a=13a \quad -9b-4b=-13b \\ =13a-13b$$

$$2. a+b-c-b-c+2c-a \\ a-a=0 \quad b-b=0 \quad -c-c+2c=0 \\ =0$$

$$3. 5x-11y-9+20x-1-y \\ 5x+20x=25x \quad -11y-y=-12y \quad -9-1=-10 \\ =25x-12y-10$$

$$4. -6m+8n+5-m-n-6m-11 \\ -6m-m-6m=-13m \quad 8n-n=7n \quad 5-11=-6 \\ =-13m+7n-6$$

$$5. -a+b+2b-2c+3a+2c-3b \\ -a+3a=2a \quad b+2b-3b=0 \quad -2c+2c=0 \\ =2a$$

$$6. -81x+19y-30z+6y+80x+x-25y \\ -81x+80x+x=0 \quad 19y+6y-25y=0 \quad -30z \\ =-30z$$

$$7. 15a^2-6ab-8a^2+20-5ab-31+a^2-ab \\ 15a^2-8a^2+a^2=8a^2 \\ -6ab-5ab-ab=-12ab \quad 20-31=-11 \\ =8a^2-12ab-11$$

$$8. -3a+4b-6a+81b-114b+31a-a-b \\ -3a-6a+31a-a=21a \\ 4b+81b-114b-b=-30b \\ =21a-30b$$

$$9. -71a^3b-84a^4b^2+50a^3b+84a^4b^2-45a^3b+18a^3b \\ -71a^3b+50a^3b-45a^3b+18a^3b=-48a^3b \\ -84a^4b^2+84a^4b^2=0 \\ =-48a^3b$$

$$10. -a+b-c+8+2a+2b-19-2c-3a-3-3b+3c \\ -a+2a-3a=-2a \quad b+2b-3b=0 \\ -c-2c+3c=0 \quad 8-19-3=-14 \\ =-2a-14$$

$$\begin{aligned}
 11. \quad & m^2 + 71mn - 14m^2 - 65mn + m^3 - m^2 - 115m^2 + 6m^3 \\
 & m^3 + 6m^3 = 7m^3 \qquad m^2 - 14m^2 - m^2 - 115m^2 = -129m^2 \qquad 71mn - 65mn = 6mn \\
 & = 7m^3 - 129m^2 + 6mn
 \end{aligned}$$

$$\begin{aligned}
 12. \quad & x^4y - x^3y^2 + x^2y - 8x^4y - x^2y - 10 + x^3y^2 - 7x^3y^2 - 9 + 21x^4y - y^3 + 50 \\
 & x^4y - 8x^4y + 21x^4y = 14x^4y \qquad -x^3y^2 + x^3y^2 - 7x^3y^2 = -7x^3y^2 \\
 & x^2y - x^2y = 0 \qquad -y^3 \qquad -10 - 9 + 50 = 31 \\
 & = 14x^4y - 7x^3y^2 - y^3 + 31
 \end{aligned}$$

$$\begin{aligned}
 13. \quad & 5a^{x+1} - 3b^{x+2} - 8c^{x+3} - 5a^{x+1} - 50 + 4b^{x+2} - 65 - b^{x+2} + 90 + c^{x+3} + 7c^{x+3} \\
 & -8c^{x+3} + c^{x+3} + 7c^{x+3} = 0 \qquad -3b^{x+2} + 4b^{x+2} - b^{x+2} = 0 \qquad 5a^{x+1} - 5a^{x+1} = 0 \qquad -50 - 65 + 90 = -25 \\
 & = -25
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & a^{m+2} - x^{m+3} - 5 + 8 - 3a^{m+2} + 5x^{m+3} - 6 + a^{m+2} - 5x^{m+3} \\
 & -x^{m+3} + 5x^{m+3} - 5x^{m+3} = -x^{m+3} \qquad a^{m+2} - 3a^{m+2} + a^{m+2} = -a^{m+2} \qquad -5 + 8 - 6 = -3 \\
 & = -x^{m+3} - a^{m+2} - 3
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & 0,3a + 0,4b + 0,5c - 0,6a - 0,7b - 0,9c + 3a - 3b - 3c \\
 & 0,3a - 0,6a + 3a = 2,7a \qquad 0,4b - 0,7b - 3b = -3,3b \qquad 0,5c - 0,9c - 3c = -3,4c \\
 & = 2,7a - 3,3b - 3,4c
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \frac{1}{2}a + \frac{1}{3}b + 2a - 3b - \frac{3}{4}a - \frac{1}{6}b + \frac{3}{4} - \frac{1}{2} \\
 & \frac{1}{2}a + 2a - \frac{3}{4}a = \frac{2+8-3}{4}a = \frac{7}{4}a \qquad \frac{1}{3}b - 3b - \frac{1}{6}b = \frac{2-18-1}{6}b = -\frac{17}{6}b \qquad \frac{3}{4} - \frac{1}{2} = \frac{3-2}{4} = \frac{1}{4} \\
 & = \frac{7}{4}a - \frac{17}{6}b + \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & \frac{3}{5}m^2 - 2mn + \frac{1}{10}m^2 - \frac{1}{3}mn + 2mn - 2m^2 \\
 & \frac{3}{5}m^2 + \frac{1}{10}m^2 - 2m^2 = \frac{6+1-20}{10}m^2 = -\frac{13}{10}m^2 \qquad -2mn - \frac{1}{3}mn + 2mn = \frac{-6-1+6}{3}mn = -\frac{1}{3}mn \\
 & = -\frac{13}{10}m^2 - \frac{1}{3}mn
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & -\frac{3}{4}a^2 + \frac{1}{2}ab - \frac{5}{6}b^2 + 2\frac{1}{3}a^2 - \frac{3}{4}ab + \frac{1}{6}b^2 - \frac{1}{3}b^2 - 2ab \\
 & -\frac{3}{4}a^2 + \frac{7}{3}a^2 = \frac{-9+28}{12}a^2 = \frac{19}{12}a^2 \qquad \frac{1}{2}ab - \frac{3}{4}ab - 2ab = \frac{2-3-8}{4}ab = -\frac{9}{4}ab \\
 & -\frac{5}{6}b^2 + \frac{1}{6}b^2 - \frac{1}{3}b^2 = \frac{-5+1-2}{6}b^2 = -\frac{6}{6}b^2 = -b^2 \\
 & = \frac{19}{12}a^2 - \frac{9}{4}ab - b^2
 \end{aligned}$$

$$19. 0,4x^2y + 31 + \frac{3}{8}xy^2 - 0,6y^3 - \frac{2}{5}x^2y - 0,2xy^2 + \frac{1}{4}y^3 - 6$$

$$0,4x^2y - \frac{2}{5}x^2y = \frac{2-2}{5}x^2y = 0$$

$$\frac{3}{8}xy^2 - 0,2xy^2 = \frac{3-1,6}{8}xy^2 = \frac{1,4}{8}xy^2 = 0,175xy^2$$

$$-0,6y^3 + \frac{1}{4}y^3 = \frac{-2,4+1}{4}y^3 = -\frac{1,4}{4}y^3 = -0,35y^3$$

$$31-6=25$$

$$=0,175xy^2 - 0,35y^3 + 25$$

$$20. \frac{3}{25}a^{m-1} - \frac{7}{50}b^{m-2} + \frac{3}{5}a^{m-1} - \frac{1}{25}b^{m-2} - 0,2a^{m-1} + \frac{1}{5}b^{m-2}$$

$$\frac{3}{25}a^{m-1} + \frac{3}{5}a^{m-1} - 0,2a^{m-1} = \frac{3+15-5}{25}a^{m-1} = \frac{13}{25}a^{m-1}$$

$$-\frac{7}{50}b^{m-2} - \frac{1}{25}b^{m-2} + \frac{1}{5}b^{m-2} = \frac{-7-2+10}{50}b^{m-2} = \frac{1}{50}b^{m-2}$$

$$= \frac{13}{25}a^{m-1} + \frac{1}{50}b^{m-2}$$

## EJERCICIO 11

Para resolver los problemas del 1 al 18 las literales toman los siguientes valores:

$$a = 1 \quad b = 2 \quad c = 3 \quad m = 1/2 \quad n = 1/3 \quad p = 1/4$$

$$1. 3ab = 3 \cdot 1 \cdot 2 = 6$$

$$2. 5a^2b^3c = 5 \cdot 1^2 \cdot 2^3 \cdot 3 = 5 \cdot 8 \cdot 3 = 120$$

$$3. b^2mn = 2^2 \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{4}{6} = \frac{2}{3}$$

$$4. 24m^2n^3p = 24 \left(\frac{1}{2}\right)^2 \left(\frac{1}{3}\right)^3 \cdot \frac{1}{4} = 6 \cdot \frac{1}{27} \cdot \frac{1}{4} = \frac{6}{108} = \frac{1}{18}$$

$$5. \frac{2}{3}a^4b^2m^3 = \frac{2}{3} \cdot 1^4 \cdot 2^2 \cdot \left(\frac{1}{2}\right)^3 = \frac{2}{3} \cdot 4 \cdot \frac{1}{8} = \frac{8}{24} = \frac{1}{3}$$

$$6. \frac{7}{12}c^3p^2m = \frac{7}{12} \cdot 3^3 \cdot \left(\frac{1}{4}\right)^2 \cdot \frac{1}{2} = \frac{189}{12} \cdot \frac{1}{16} \cdot \frac{1}{2} = \frac{189}{384} = \frac{63}{128}$$

$$7. m^bn^cp^a = \left(\frac{1}{2}\right)^2 \left(\frac{1}{3}\right)^3 \cdot \frac{1}{4} = \frac{1}{4} \cdot \frac{1}{27} \cdot \frac{1}{4} = \frac{1}{432}$$

$$8. \frac{5}{6}a^{b-1} \cdot m^{c-2} = \frac{5}{6} \cdot 1^{2-1} \cdot \left(\frac{1}{2}\right)^{3-2} = \frac{5}{6} \cdot \frac{1}{2} = \frac{5}{12}$$

$$9. \sqrt{2bc^2} = \sqrt{2 \cdot 2 \cdot 3^2} = \sqrt{4 \cdot 9} = \sqrt{36} = 6$$

$$10. 4m \cdot \sqrt[3]{12bc^2} = 4 \cdot \frac{1}{2} \cdot \sqrt[3]{12 \cdot 2 \cdot 3^2} = 2 \cdot \sqrt[3]{216} = 2 \cdot 6 = 12$$

$$11. mn \cdot \sqrt{8a^4b^3} = \frac{1}{2} \cdot \frac{1}{3} \cdot \sqrt{8 \cdot 1^4 \cdot 2^3} = \frac{1}{6} \cdot \sqrt{64} = \frac{8}{6} = \frac{4}{3}$$

$$12. \frac{4a}{3bc} = \frac{4 \cdot 1}{3 \cdot 2 \cdot 3} = \frac{4}{18} = \frac{2}{9}$$

$$13. \frac{5b^2m^2}{np} = \frac{5 \cdot 2^2 \cdot \left(\frac{1}{2}\right)^2}{\frac{1}{3} \cdot \frac{1}{4}} = \frac{20 \cdot \frac{1}{4}}{\frac{1}{12}} = \frac{5}{\frac{1}{12}} = 60$$

$$14. \frac{\frac{3}{4}b^3}{\frac{2}{3}c^2} = \frac{\frac{3}{4} \cdot 2^3}{\frac{2}{3} \cdot 3^2} = \frac{\frac{3 \cdot 8}{4}}{\frac{2 \cdot 9}{3}} = \frac{\frac{24}{4}}{\frac{18}{3}} = \frac{6}{6} = 1$$

$$15. \frac{2m}{\sqrt{n^2}} = \frac{2 \cdot \frac{1}{2}}{\sqrt{\left(\frac{1}{3}\right)^2}} = \frac{1}{\sqrt{\frac{1}{9}}} = \frac{1}{\frac{1}{3}} = 3$$

$$16. \frac{24mn}{2 \cdot \sqrt{n^2p^2}} = \frac{24 \cdot \frac{1}{2} \cdot \frac{1}{3}}{2 \cdot \sqrt{\left(\frac{1}{3}\right)^2 \left(\frac{1}{4}\right)^2}} = \frac{\frac{24}{6}}{2 \cdot \sqrt{\frac{1}{9} \cdot \frac{1}{16}}} = \frac{4}{2 \cdot \sqrt{\frac{1}{144}}} = \frac{4}{2 \cdot \frac{1}{12}} = \frac{4}{\frac{1}{6}} = 24$$

$$17. \frac{3 \cdot \sqrt[3]{64b^3c^6}}{2m} = \frac{3 \cdot \sqrt[3]{64 \cdot 2^3 \cdot 3^6}}{2 \cdot \frac{1}{2}} = 3 \cdot \sqrt[3]{64 \cdot 8 \cdot 729} = 3 \cdot \sqrt[3]{373.248} = 3 \cdot 72 = 216$$

$$18. \frac{\frac{3}{5} \cdot \sqrt{apb^2}}{\frac{3}{2} \cdot \sqrt[3]{125bm}} = \frac{\frac{3}{5} \cdot \sqrt{1 \cdot \frac{1}{4} \cdot 2^2}}{\frac{3}{2} \cdot \sqrt[3]{125 \cdot 2 \cdot \frac{1}{2}}} = \frac{\frac{3}{5} \cdot \sqrt{\frac{4}{4}}}{\frac{3}{2} \cdot \sqrt[3]{\frac{250}{2}}} = \frac{\frac{3}{5} \cdot \sqrt{1}}{\frac{3}{2} \cdot \sqrt[3]{125}} = \frac{\frac{3}{5}}{\frac{3}{2} \cdot 5} = \frac{\frac{3}{5}}{\frac{15}{2}} = \frac{6}{75} = \frac{2}{25}$$

## EJERCICIO 12

Para resolver los problemas del 1 al 18 las literales toman los siguientes valores:

$$a = 3 \quad b = 4 \quad c = 1/3 \quad d = 1/2 \quad m = 6 \quad n = 1/4$$

$$1. a^2 - 2ab + b^2 = 3^2 - 2 \cdot 3 \cdot 4 + 4^2 = 9 - 24 + 16 = 1$$

$$2. c^2 + 2cd + d^2 = \left(\frac{1}{3}\right)^2 + 2 \cdot \frac{1}{3} \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^2 = \frac{1}{9} + \frac{1}{3} + \frac{1}{4} = \frac{4+12+9}{36} = \frac{25}{36}$$

$$3. \frac{a}{c} + \frac{b}{d} = \frac{3}{\frac{1}{3}} + \frac{4}{\frac{1}{2}} = 9 + 8 = 17$$

$$4. \frac{c}{d} - \frac{m}{n} + 2 = \frac{\frac{1}{3}}{\frac{1}{2}} - \frac{6}{\frac{1}{4}} + 2 = \frac{2}{3} - 24 + 2 = \frac{2-72+6}{3} = -\frac{64}{3}$$

$$5. \frac{a^2}{3} - \frac{b^2}{2} + \frac{m^2}{6} = \frac{3^2}{3} - \frac{4^2}{2} + \frac{6^2}{6} = \frac{9}{3} - \frac{16}{2} + \frac{36}{6} = 3 - 8 + 6 = 1$$

$$6. \frac{3}{5}c - \frac{1}{2}b + 2d = \frac{3}{5} \cdot \frac{1}{3} - \frac{1}{2} \cdot 4 + 2 \cdot \frac{1}{2} = \frac{1}{5} - 2 + 1 = \frac{1-10+5}{5} = -\frac{4}{5}$$

$$7. \frac{ab}{n} + \frac{ac}{d} - \frac{bd}{m} = \frac{3 \cdot 4}{\frac{1}{4}} + \frac{3 \cdot \frac{1}{3}}{\frac{1}{2}} - \frac{4 \cdot \frac{1}{2}}{\frac{1}{6}} = \frac{12}{\frac{1}{4}} + \frac{1}{\frac{1}{2}} - \frac{2}{\frac{1}{6}} = 48 + 2 - \frac{1}{3} = \frac{144+6-1}{3} = \frac{149}{3} = 49\frac{2}{3}$$

$$8. \sqrt{b} + \sqrt{n} + \sqrt{6m} = \sqrt{4} + \sqrt{\frac{1}{4}} + \sqrt{6 \cdot 6} = 2 + \frac{1}{2} + 6 = \frac{4+1+12}{2} = \frac{17}{2} = 8\frac{1}{2}$$

$$9. c\sqrt{3a} - d\sqrt{16b^2} + n\sqrt{8d} = \frac{1}{3} \cdot \sqrt{3 \cdot 3} - \frac{1}{2} \cdot \sqrt{16 \cdot 4^2} + \frac{1}{4} \cdot \sqrt{8 \cdot \frac{1}{2}} = \frac{1}{3} \cdot 3 - \frac{1}{2} \cdot 16 + \frac{1}{4} \cdot 2 = 1 - 8 + \frac{1}{2} = \frac{2-16+1}{2} = -\frac{13}{2} = -6\frac{1}{2}$$

$$10. \frac{m^a}{d^b} = \frac{6^3}{\left(\frac{1}{2}\right)^4} = \frac{216}{\frac{1}{16}} = 216 \cdot 16 = 3.456$$

$$11. \frac{3}{4}c^2 + \frac{4n^2}{m} = \frac{3\left(\frac{1}{3}\right)^2}{4} + \frac{4\left(\frac{1}{4}\right)^2}{6} = \frac{\frac{1}{3}}{4} + \frac{\frac{1}{4}}{6} = \frac{1}{12} + \frac{1}{24} = \frac{2+1}{24} = \frac{3}{24} = \frac{1}{8}$$

$$12. \frac{4d^2}{2} + \frac{16n^2}{2} - 1 = 2 \cdot \left(\frac{1}{2}\right)^2 + 8 \cdot \left(\frac{1}{4}\right)^2 - 1 = \frac{2}{4} + \frac{8}{16} - 1 = \frac{1}{2} + \frac{1}{2} - 1 = 1 - 1 = 0$$

$$13. \frac{a+b}{c} - \frac{b+m}{d} = \frac{3+4}{\frac{1}{3}} - \frac{4+6}{\frac{1}{2}} = \frac{7}{\frac{1}{3}} - \frac{10}{\frac{1}{2}} = 21 - 20 = 1$$

$$14. \frac{b-a}{n} + \frac{m-b}{d} + 5a = \frac{4-3}{\frac{1}{4}} + \frac{6-4}{\frac{1}{2}} + 5 \cdot 3 = \frac{1}{\frac{1}{4}} + \frac{2}{\frac{1}{2}} + 15 = 4 + 4 + 15 = 23$$

$$15. \frac{12c-a}{2b} - \frac{16n-a}{m} + \frac{1}{d} = \frac{12 \cdot \frac{1}{3} - 3}{2 \cdot 4} - \frac{16 \cdot \frac{1}{4} - 3}{6} + \frac{1}{\frac{1}{2}} = \frac{4-3}{8} - \frac{4-3}{6} + 2 = \frac{1}{8} - \frac{1}{6} + 2 = \frac{3-4+48}{24} = \frac{47}{24} = 1\frac{23}{24}$$

$$16. \sqrt{4b} + \frac{\sqrt{3a}}{3} - \frac{\sqrt{6m}}{6} = \sqrt{4 \cdot 4} + \frac{\sqrt{3 \cdot 3}}{3} - \frac{\sqrt{6 \cdot 6}}{6} = \sqrt{16} + \frac{\sqrt{9}}{3} - \frac{\sqrt{36}}{6} = 4 + \frac{3}{3} - \frac{6}{6} = 4 + 1 - 1 = 4$$

$$17. \frac{\sqrt{b} + \sqrt{2d}}{2} - \frac{\sqrt{3c} + \sqrt{8d}}{4} = \frac{\sqrt{4} + \sqrt{2 \cdot \frac{1}{2}}}{2} - \frac{\sqrt{3 \cdot \frac{1}{3}} + \sqrt{8 \cdot \frac{1}{2}}}{4} = \frac{2+1}{2} - \frac{1+2}{4} = \frac{3}{2} - \frac{3}{4} = \frac{6-3}{4} = \frac{3}{4}$$

$$18. \frac{2 \cdot \sqrt{a^2 b^2}}{3} + \frac{3 \cdot \sqrt{2+d^2}}{4} - a \cdot \sqrt{n} = \frac{2 \cdot \sqrt{3^2 \cdot 4^2}}{3} + \frac{3 \cdot \sqrt{2 + \left(\frac{1}{2}\right)^2}}{4} - 3 \cdot \sqrt{\frac{1}{4}} = \frac{2 \cdot \sqrt{144}}{3} + \frac{3 \cdot \sqrt{\frac{9}{4}}}{4} - 3 \cdot \frac{1}{2} \\ = \frac{2 \cdot 12}{3} + \frac{3 \cdot \frac{3}{2}}{4} - \frac{3}{2} = \frac{24}{3} + \frac{\frac{9}{2}}{4} - \frac{3}{2} = 8 + \frac{9}{8} - \frac{3}{2} = \frac{64+9-12}{8} = \frac{61}{8} = 7\frac{5}{8}$$

### ***EJERCICIO 13***

***Para los problemas 1 al 24 las literales toman los siguientes valores:***

***a = 1   b = 2   c = 3   d = 4   m = 1/2   n = 2/3   p = 1/4   x = 0***

$$1. (a+b) \cdot c - d = (1+2) \cdot 3 - 4 = 3 \cdot 3 - 4 = 9 - 4 = 5$$

$$2. (a+b)(b-a) = (1+2)(2-1) = 3 \cdot 1 = 3$$

$$3. (b-m)(c-n) + 4a^2 = \left(2 - \frac{1}{2}\right) \left(3 - \frac{2}{3}\right) + 4 \cdot 1^2 = \left(\frac{4-1}{2}\right) \left(\frac{9-2}{3}\right) + 4 = \frac{3}{2} \cdot \frac{7}{3} + 4 = \frac{7}{2} + 4 = \frac{7+8}{2} = \frac{15}{2} = 7\frac{1}{2}$$

$$4. (2m+3n)(4p+b^2) = \left(2 \cdot \frac{1}{2} + 3 \cdot \frac{2}{3}\right) \left(4 \cdot \frac{1}{4} + 2^2\right) = (1+2)(1+4) = 3 \cdot 5 = 15$$

$$5. (4m+8p)(a^2+b^2)(6n-d) = \left(4 \cdot \frac{1}{2} + 8 \cdot \frac{1}{4}\right) (1^2 + 2^2) \left(6 \cdot \frac{2}{3} - 4\right) = (2+2) \cdot 5 \cdot 0 = 0$$



$$6. (c-b)(d-c)(b-a)(m-p) = (3-2)(4-3)(2-1)\left(\frac{1}{2}-\frac{1}{4}\right) = 1 \cdot 1 \cdot 1 \cdot \left(\frac{2-1}{4}\right) = \frac{1}{4}$$

$$7. b^2(c+d) - a^2(m+n) + 2x = 2^2(3+4) - 1^2\left(\frac{1}{2} + \frac{2}{3}\right) + 2 \cdot 0 = 4 \cdot 7 - \left(\frac{3+4}{6}\right) = 28 - \frac{7}{6} = \frac{168-7}{6} = \frac{161}{6} = 26\frac{5}{6}$$

$$8. 2mx + 6(b^2 + c^2) - 4d^2 = 2 \cdot \frac{1}{2} \cdot 0 + 6(2^2 + 3^2) - 4 \cdot 4^2 = 6(4+9) - 4 \cdot 16 = 6 \cdot 13 - 64 = 78 - 64 = 14$$

$$9. \left(\frac{8m}{9n} + \frac{16p}{b}\right)a = \left(\frac{8 \cdot \frac{1}{2}}{9 \cdot \frac{2}{3}} + \frac{16 \cdot \frac{1}{4}}{2}\right) \cdot 1 = \frac{4}{18} + \frac{4}{2} = \frac{4}{6} + 2 = \frac{2}{3} + 2 = \frac{2+6}{3} = \frac{8}{3} = 2\frac{2}{3}$$

$$10. x + m(a^b + d^c - c^a) = 0 + \frac{1}{2}(1^2 + 4^3 - 3^1) = \frac{1}{2}(1 + 64 - 3) = \frac{1}{2} \cdot 62 = 31$$

$$11. \frac{4(m+p)}{a} \div \frac{a^2+b^2}{c^2} = \frac{4\left(\frac{1}{2} + \frac{1}{4}\right)}{1} \div \frac{1^2+2^2}{3^2} = 4\left(\frac{2+1}{4}\right) \div \frac{1+4}{9} = 4 \cdot \frac{3}{4} \div \frac{5}{9} = \frac{3}{5} = \frac{27}{5} = 5\frac{2}{5}$$

$$12. (2m+3n+4p)(8p+6n-4m)(9n+20p) \\ = \left(2 \cdot \frac{1}{2} + 3 \cdot \frac{2}{3} + 4 \cdot \frac{1}{4}\right)\left(8 \cdot \frac{1}{4} + 6 \cdot \frac{2}{3} - 4 \cdot \frac{1}{2}\right)\left(9 \cdot \frac{2}{3} + 20 \cdot \frac{1}{4}\right) = (1+2+1)(2+4-2)(6+5) = 4 \cdot 4 \cdot 11 = 176$$

$$13. c^2(m+n) - d^2(m+p) + b^2(n+p) \\ = 3^2\left(\frac{1}{2} + \frac{2}{3}\right) - 4^2\left(\frac{1}{2} + \frac{1}{4}\right) + 2^2\left(\frac{2}{3} + \frac{1}{4}\right) = 9 \cdot \frac{7}{6} - 16 \cdot \frac{3}{4} + 4 \cdot \frac{11}{12} = \frac{21}{2} - 12 + \frac{44}{12} = \frac{126-144+44}{12} = \frac{26}{12} = 2\frac{2}{12} = 2\frac{1}{6}$$

$$14. \left(\frac{\sqrt{c^2+d^2}}{a} \div \frac{2}{\sqrt{d}}\right)m = \left(\frac{\sqrt{3^2+4^2}}{1} \div \frac{2}{\sqrt{4}}\right) \cdot \frac{1}{2} = \left(\frac{\sqrt{9+16}}{1} \div \frac{2}{2}\right) \cdot \frac{1}{2} = \left(\frac{\sqrt{25}}{1} \div \frac{2}{2}\right) \cdot \frac{1}{2} = 5 \cdot \frac{1}{2} = \frac{5}{2} = 2\frac{1}{2}$$

$$15. (4p+2b)(18n-24p) + 2(8m+2)(40p+a) \\ = \left(4 \cdot \frac{1}{4} + 2 \cdot 2\right)\left(18 \cdot \frac{2}{3} - 24 \cdot \frac{1}{4}\right) + 2\left(8 \cdot \frac{1}{2} + 2\right)\left(40 \cdot \frac{1}{4} + 1\right) = (1+4)(12-6) + 2(4+2)(11) = 5 \cdot 6 + 2 \cdot 66 = 30 + 132 = 162$$

$$16. \frac{a+\frac{d}{b}}{d-b} \cdot \frac{5+\frac{2}{m^2}}{p^2} = \frac{1+\frac{4}{2}}{4-2} \cdot \frac{5+\frac{\left(\frac{1}{2}\right)^2}{\left(\frac{1}{4}\right)^2}}{\left(\frac{1}{4}\right)^2} = \frac{2+4}{2} \cdot \frac{5+\frac{1}{4}}{\frac{1}{16}} = \frac{6}{2} \cdot \frac{5+8}{\frac{1}{16}} = \frac{3}{2} \cdot \frac{5+8}{\frac{1}{16}} = \frac{3}{2} \cdot \frac{13}{\frac{1}{16}} = \frac{3}{2} \cdot 208 = \frac{624}{2} = 312$$

$$17. (a+b) \cdot \sqrt{c^2+8b} - m \cdot \sqrt{n^2} = (1+2) \cdot \sqrt{3^2+8 \cdot 2} - \frac{1}{2} \cdot \sqrt{\left(\frac{2}{3}\right)^2} = 3 \cdot \sqrt{25} - \frac{1}{2} \cdot \frac{2}{3} = 3 \cdot 5 - \frac{2}{6} = 15 - \frac{2}{6} = \frac{90-2}{6} = \frac{88}{6} = 14\frac{4}{6} = 14\frac{2}{3}$$

$$18. \left(\frac{\sqrt{a+c}}{2} + \frac{\sqrt{6n}}{b}\right) \div (c+d) \cdot \sqrt{p} = \left(\frac{\sqrt{1+3}}{2} + \frac{\sqrt{6 \cdot \frac{2}{3}}}{2}\right) \div (3+4) \cdot \sqrt{\frac{1}{4}} = \left(\frac{\sqrt{4}}{2} + \frac{\sqrt{4}}{2}\right) \div 7 \cdot \frac{1}{2} = \frac{2}{7} = \frac{4}{7}$$

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19.  $3(c-b) \cdot \sqrt{32m} - 2(d-a) \cdot \sqrt{16p} - \frac{2}{n}$   
 $= 3(3-2) \cdot \sqrt{32 \cdot \frac{1}{2}} - 2(4-1) \cdot \sqrt{16 \cdot \frac{1}{4}} - \frac{2}{\frac{2}{3}} = 3 \cdot \sqrt{16} - 6 \cdot \sqrt{4} - \frac{6}{2} = 3 \cdot 4 - 6 \cdot 2 - 3 = 12 - 12 - 3 = -3$

20.  $\frac{\sqrt{6abc}}{2 \cdot \sqrt{8b}} + \frac{\sqrt{3mn}}{2(b-a)} - \frac{cdnp}{abc}$   
 $= \frac{\sqrt{6 \cdot 1 \cdot 2 \cdot 3}}{2 \cdot \sqrt{8 \cdot 2}} + \frac{\sqrt{3 \cdot \frac{1}{2} \cdot \frac{2}{3}}}{2(2-1)} - \frac{3 \cdot 4 \cdot \frac{2}{3} \cdot \frac{1}{4}}{1 \cdot 2 \cdot 3} = \frac{\sqrt{36}}{2 \cdot \sqrt{16}} + \frac{\sqrt{1}}{2} - \frac{2}{6} = \frac{6}{8} + \frac{1}{2} - \frac{1}{3} = \frac{18+12-8}{24} = \frac{22}{24} = \frac{11}{12}$

21.  $\frac{a^2+b^2}{b^2-a^2} + 3(a+b)(2a+3b) = \frac{1^2+2^2}{2^2-1^2} + 3(1+2)(2 \cdot 1 + 3 \cdot 2) = \frac{5}{3} + 3 \cdot 3 \cdot 8 = \frac{5}{3} + 72 = \frac{5+216}{3} = \frac{221}{3} = 73\frac{2}{3}$

22.  $b^2 + \left(\frac{1}{a} + \frac{1}{b}\right)\left(\frac{1}{b} + \frac{1}{c}\right) + \left(\frac{1}{n} + \frac{1}{m}\right)^2$   
 $= 2^2 + \left(\frac{1}{1} + \frac{1}{2}\right)\left(\frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{\frac{2}{3}} + \frac{1}{\frac{1}{2}}\right)^2 = 4 + \left(\frac{3}{2} + \frac{5}{6}\right) + \left(\frac{3}{2} + 2\right)^2 = 4 + \frac{5}{4} + \left(\frac{7}{2}\right)^2 = \frac{21}{4} + \frac{49}{4} = \frac{70}{4} = \frac{35}{2} = 17\frac{1}{2}$

$(2m+3n)(4p+2c) - 4m^2n^2$

23.  $= \left(2 \cdot \frac{1}{2} + 3 \cdot \frac{2}{3}\right)\left(4 \cdot \frac{1}{4} + 2 \cdot 3\right) - 4\left(\frac{1}{2}\right)^2 \left(\frac{2}{3}\right)^2 = (1+2)(1+6) - 4 \cdot \frac{1}{4} \cdot \frac{4}{9} = 3 \cdot 7 - \frac{4}{9} = 21 - \frac{4}{9} = \frac{189-4}{9} = \frac{185}{9} = 20\frac{5}{9}$

24.  $\frac{b^2 - \frac{c}{3}}{2ab-m} - \frac{n}{b-m} = \frac{2^2 - \frac{3}{3}}{2 \cdot 1 \cdot 2 - \frac{1}{2}} - \frac{\frac{2}{3}}{2 - \frac{1}{2}} = \frac{3}{4 - \frac{1}{2}} - \frac{\frac{2}{3}}{\frac{3}{2}} = \frac{3}{\frac{7}{2}} - \frac{4}{9} = \frac{6}{7} - \frac{4}{9} = \frac{54-28}{63} = \frac{26}{63}$

## EJERCICIO 14

- |                          |                               |                               |
|--------------------------|-------------------------------|-------------------------------|
| 1. $a+b+m$               | 10. $\$(x+a-m)$               | 18. $x^2 m^2$                 |
| 2. $m^2 + b^3 + x^4$     | 11. $(m-a-b-c)Km$             | 19. $\$(3a+6b)$ ; $\$(ax+bm)$ |
| 3. $a+1$ ; $a+2$         | 12. $\$(n-300)$               | 20. $(a+b)(x+y)$              |
| 4. $x-1$ ; $x-2$         | 13. $(365-x)$ días            | 21. $\$(8x+48)$               |
| 5. $y+2$ ; $y+4$ ; $y+6$ | 14. $\$8a$ ; $\$15a$ ; $\$ma$ | 22. $bs.(a-8)(x+4)$           |
| 6. $\$(a+x+m)$           | 15. $2a+3b+\frac{c}{2}$       | 23. $\frac{75}{x}$ sucres     |
| 7. $m-n$                 | 16. $a \cdot b m^2$           | 24. $\$\frac{a}{m}$           |
| 8. $bs.(x-6)$            | 17. $23n m^2$                 |                               |
| 9. $(x-m)Km$             |                               |                               |

$$25. \frac{3.000}{n-1} \text{ colones}$$

$$26. \frac{x}{a-3} \text{ soles}$$

$$27. \frac{m}{14} m$$

$$28. \frac{x+1}{a} \text{ km/h}$$

$$29. \$ \frac{a+b}{m-2}$$

$$30. \left( x + 2x + \frac{x}{2} \right) \text{ hab.}$$

$$31. \left[ 1.000 - \left( a + \frac{a}{3} + \frac{2a}{4} \right) \right] \text{ sucres}$$

## EJERCICIO 15

$$1. m, n \rightarrow m+n$$

$$2. m, -n \rightarrow m+(-n)=m-n$$

$$3. -3a, 4b \rightarrow -3a+4b$$

$$4. 5b, 6a \rightarrow -6a+5b$$

$$5. 7, -6 \rightarrow 7+(-6)=7-6=1$$

$$6. -6, 9 \rightarrow -6+9=3$$

$$7. -2x, 3y \rightarrow -2x+3y$$

$$8. 5mn, -m \rightarrow 5mn+(-m)=5mn-m$$

$$9. 5a, 7a \rightarrow 5a+7a=12a$$

$$10. -8x, -5x \rightarrow -8x+(-5x)=-8x-5x=-13x$$

$$11. -11m, 8m \rightarrow -11m+8m=-3m$$

$$12. 9ab, -15ab \rightarrow 9ab-15ab=-6ab$$

$$13. -xy, -9xy \rightarrow -9xy-xy=-10xy$$

$$14. mn, -11mn \rightarrow mn+(-11mn)=mn-11mn=-10mn$$

$$15. \frac{1}{2}a, -\frac{2}{3}b \rightarrow \frac{1}{2}a-\frac{2}{3}b$$

$$16. \frac{3}{5}b, \frac{3}{4}c \rightarrow \frac{3}{5}b+\frac{3}{4}c$$

$$17. \frac{1}{3}b, \frac{2}{3}b \rightarrow \frac{1}{3}b+\frac{2}{3}b=\frac{3}{3}b=b$$

$$18. -\frac{1}{2}xy, -\frac{1}{2}xy \rightarrow -\frac{1}{2}xy-\frac{1}{2}xy=-\frac{2}{2}xy=-xy$$

$$19. -\frac{3}{5}abc, -\frac{2}{5}abc$$

$$-\frac{3}{5}abc-\frac{2}{5}abc=-\frac{5}{5}abc=-abc$$

$$20. -4x^2y, \frac{3}{8}x^2y$$

$$-4x^2y+\frac{3}{8}x^2y=\frac{-32+3}{8}x^2y=-\frac{29}{8}x^2y$$

$$21. \frac{3}{8}mn, -\frac{3}{4}mn$$

$$\frac{3}{8}mn-\frac{3}{4}mn=\frac{3-6}{8}mn=-\frac{3}{8}mn$$

$$22. a, b, c \rightarrow a+b+c$$

$$23. a, -b, c \rightarrow a-b+c$$

$$24. a, -b, 2c \rightarrow a-b+2c$$

$$25. 3m, -2n, 4p \rightarrow 3m-2n+4p$$

$$26. a^2, -7ab, -5b^2 \rightarrow a^2-7ab-5b^2$$

$$27. x^2, -3xy, -4y^2 \rightarrow x^2-3xy-4y^2$$

$$28. x^3, -x^2y, 6 \rightarrow x^3-x^2y+6$$

$$29. 2a, -b, 3a \rightarrow 2a+3a-b=5a-b$$

$$30. -m, -8n, 4n \rightarrow -m-8n+4n=-m-4n$$

$$31. -7a, 8a, -b \rightarrow -7a+8a-b=a-b$$

$$32. \frac{1}{2}x, \frac{2}{3}y, -\frac{3}{4}x$$

$$\frac{1}{2}x-\frac{3}{4}x+\frac{2}{3}y=\frac{4-6}{8}x+\frac{2}{3}y=-\frac{1}{4}x+\frac{2}{3}y$$

$$33. -\frac{3}{5}m, -m, -\frac{2}{3}mn$$

$$-\frac{3}{5}m-m-\frac{2}{3}mn=\frac{-3-5}{5}m-\frac{2}{3}mn=-\frac{8}{5}m-\frac{2}{3}mn$$

$$34. -7a^2, 5ab, 3b^2, -a^2$$

$$-7a^2-a^2+5ab+3b^2=-8a^2+5ab+3b^2$$

$$35. -7mr^2 + 17mr^2 - 5m - 4m = 10mr^2 - 9m$$

$$36. x^3 - 7x^3 - 8x^2y + 4x^2y + 5 = -6x^3 - 4x^2y + 5$$

$$37. 5x^2 - x^2 + 9xy - 6xy + 7y^2 = 4x^2 + 3xy + 7y^2$$

$$38. -8a^2b - a^2b + 5ab^2 - 11ab^2 - 7b^3 \\ = -9a^2b - 6ab^2 - 7b^3$$

$$39. m^3 - 8m^2n + 7m^2n + 7mn^2 - n^3 \\ = m^3 - m^2n + 7mn^2 - n^3$$

$$40. \frac{1}{2}a - \frac{1}{4}a + \frac{2}{3}b + \frac{1}{5}b - 6 \\ = \frac{2-1}{4}a + \frac{10+3}{15}b - 6 = \frac{1}{4}a + \frac{13}{15}b - 6$$

$$41. a - a - 3b + 4b - 8c + 8c = b$$

$$42. m^3 + 5m^3 - 5m^3 - 4m^2n - 4m^2n - 7mn^2 \\ = m^3 - 8m^2n - 7mn^2$$

$$43. 9x - x - 11y - 6y + 4z - 6z = 8x - 17y - 2z$$

$$44. 6a^2 + 9a^2 - 5ab - 7b^2 - 8b^2 - 11 \\ = 15a^2 - 5ab - 15b^2 - 11$$

$$45. -x^2y^2 + x^2y^2 - 5xy^3 + 7xy^3 - 4y^4 - 8 \\ = 2xy^3 - 4y^4 - 8$$

$$46. 3a - \frac{1}{2}a + \frac{1}{2}b - b - 4 + 6 \\ = \frac{6-1}{2}a + \frac{1-2}{2}b + 2 = \frac{5}{2}a - \frac{1}{2}b + 2$$

$$47. \frac{1}{2}x^2 + \frac{3}{4}x^2 + \frac{2}{3}xy - \frac{1}{3}xy + \frac{5}{6}y^2 - \frac{5}{6}y^2 \\ = \frac{2+3}{4}x^2 + \frac{2-1}{3}xy = \frac{5}{4}x^2 + \frac{1}{3}xy$$

$$48. 5ax - 5ax - 6a^{x+1} + a^{x+1} + 5a^{x+1} + 8a^{x+2} = 8a^{x+2}$$

$$49. \frac{3}{4}x^2 + x^2 - \frac{2}{3}xy - \frac{1}{3}xy + \frac{1}{3}y^2 + 5y^2 \\ = \frac{3+4}{4}x^2 - \frac{2}{3}xy + \frac{1+5}{3}y^2 = \frac{7}{4}x^2 - xy + \frac{16}{3}y^2$$

$$50. \frac{3}{4}a^2b - \frac{1}{4}a^2b + a^2b + \frac{1}{2}ab^2 + \frac{1}{2}ab^2 - \frac{5}{6}ab^2 \\ = \frac{3-1+4}{4}a^2b + \frac{3+3-5}{6}ab^2 = \frac{3}{2}a^2b + \frac{1}{6}ab^2$$

## EJERCICIO 16

$$1. \frac{3a+2b-c}{2a+3b+c} \\ \frac{5a+5b}{5a+5b}$$

$$2. \frac{7a-4b+5c}{-7a+4b-6c} \\ -c$$

$$3. \frac{m+n-p}{-m-n+p} \\ 0$$

$$4. \frac{9x-3y+5}{-x-y+4} \\ \frac{-5x+4y-9}{3x}$$

$$5. \frac{a+b-c}{2a+2b-2c} \\ \frac{-3a-b+3c}{2b}$$

$$6. \frac{p+q+r}{-2p-6q+3r} \\ \frac{p+5q-8r}{-4r}$$

$$7. \frac{-7x-4y+6z}{10x-20y-8z} \\ \frac{-5x+24y+2z}{-2x}$$

$$8. \frac{-2m+3n-6}{3m-8n+8} \\ \frac{-5m+n-10}{-4m-4n-8}$$

$$9. \frac{-5a-2b-3c}{7a-3b+5c} \\ \frac{-8a+5b-3c}{-6a-c}$$

$$10. \frac{ab+bc+cd}{-8ab-3bc-3cd} \\ \frac{5ab+2bc+2cd}{-2ab}$$

$$11. \frac{ax-ay-az}{-5ax-7ay-6az} \\ \frac{4ax+9ay+8az}{ay+az}$$

$$12. \frac{5x-7y+8}{-4x-y+6} \\ \frac{-3x+8y+9}{-2x+23}$$

$$13. \frac{-am+6mn-4s}{-am-5mn+6s} \\ \frac{3am-5mn-2s}{am-4mn}$$

$$14. \frac{2a+3b}{6b-4c} \\ \frac{-a+8c}{a+9b+4c}$$

$$15. \frac{6m-3n}{-4n+5p} \\ \frac{-m-5p}{5m-7n}$$

$$16. \frac{2a+3b}{+5c-4} \\ \frac{8a+6}{+7c-9} \\ \frac{10a+3b+12c-7}{10a+3b+12c-7}$$

$$17. \frac{2x-3y}{+5z+9} \\ \frac{6x-4}{3y-5} \\ \frac{8x+5z}{8x+5z}$$

$$\begin{array}{r}
 18. \quad 8a+3b-c \\
 \quad 5a-b+c \\
 \quad -a-b-c \\
 \quad 7a-b+4c \\
 \hline
 19a \quad +3c
 \end{array}$$

$$\begin{array}{r}
 19. \quad 7x+2y \quad -4 \\
 \quad +9y-6z+5 \\
 \quad -y+3z-6 \\
 \quad 8x-3y \quad -5 \\
 \hline
 15x+7y-3z-10
 \end{array}$$

$$\begin{array}{r}
 20. \quad -m-n-p \\
 \quad m+2n \quad -5 \\
 \quad -6m \quad +3p+4 \\
 \quad 5m+2n \quad -8 \\
 \hline
 -m+3n+2p-9
 \end{array}$$

$$\begin{array}{r}
 21. \quad 5a^x-3a^m-7a^n \\
 \quad -8a^x+5a^m-9a^n \\
 \hline
 -11a^x+5a^m+16a^n \\
 \quad -14a^x+7a^m
 \end{array}$$

$$\begin{array}{r}
 22. \quad 6m^{a+1}-7m^{a+2}-5m^{a+3} \\
 \quad 4m^{a+1}-7m^{a+2}-m^{a+3} \\
 \hline
 -5m^{a+1}+3m^{a+2}+12m^{a+3} \\
 \quad 5m^{a+1}-11m^{a+2}+6m^{a+3}
 \end{array}$$

$$\begin{array}{r}
 23. \quad 8x+y+z+u \\
 \quad -3x-4y-2z+3u \\
 \quad 4x+5y+3z-4u \\
 \quad -9x-y+z+2u \\
 \hline
 \quad y+3z+2u
 \end{array}$$

$$\begin{array}{r}
 24. \quad a+b-c+d \\
 \quad a-b+c-d \\
 \quad -2a+3b-2c+d \\
 \quad -3a-3b+4c-d \\
 \hline
 -3a \quad +2c
 \end{array}$$

$$\begin{array}{r}
 25. \quad 5ab-3bc+4cd \\
 \quad +2bc+2cd-3de \\
 \quad -2ab+4bc \quad +3de \\
 \quad -ab-3bc-6cd \\
 \hline
 2ab
 \end{array}$$

$$\begin{array}{r}
 26. \quad a-b \\
 \quad \quad b-c \\
 \quad \quad \quad c+d \\
 \quad a \quad -c \\
 \quad \quad c-d \\
 \quad -a \quad +d \\
 \quad a \quad -d \\
 \hline
 2a
 \end{array}$$

## EJERCICIO 17

$$\begin{array}{r}
 1. \quad x^2+4x \\
 \quad x^2-5x \\
 \hline
 2x^2-x
 \end{array}$$

$$\begin{array}{r}
 2. \quad a^2+ab \\
 \quad -2ab+b^2 \\
 \hline
 a^2-ab+b^2
 \end{array}$$

$$\begin{array}{r}
 3. \quad x^3 \quad +2x \\
 \quad -x^2 \quad +4 \\
 \hline
 x^3-x^2+2x+4
 \end{array}$$

$$\begin{array}{r}
 4. \quad a^4 \quad -3a^2 \\
 \quad +a^3 \quad +4a \\
 \hline
 a^4+a^3-3a^2+4a
 \end{array}$$

$$\begin{array}{r}
 5. \quad -x^2+3x \\
 \quad x^3 \quad +6 \\
 \hline
 x^3-x^2+3x+6
 \end{array}$$

$$\begin{array}{r}
 6. \quad x^2-4x \\
 \quad -7x+6 \\
 \quad 3x^2 \quad -5 \\
 \hline
 4x^2-11x+1
 \end{array}$$

$$\begin{array}{r}
 7. \quad m^2+n^2 \\
 \quad +4n^2-3mn \\
 \hline
 -5m^2-5n^2 \\
 \quad -4m^2 \quad -3mn
 \end{array}$$

$$\begin{array}{r}
 8. \quad 3x+x^3 \\
 \quad -4x^2+5 \\
 \quad -x^3+4x^2-6 \\
 \hline
 3x \quad -1
 \end{array}$$

$$\begin{array}{r}
 9. \quad x^2-3xy+y^2 \\
 \quad -x^2+3xy-2y^2 \\
 \quad x^2+3xy-y^2 \\
 \hline
 x^2+3xy-2y^2
 \end{array}$$

$$\begin{array}{r}
 10. \quad a^2-3ab+b^2 \\
 \quad +a^2-5ab-b^2 \\
 \quad -2a^2+8ab-b^2 \\
 \hline
 -b^2
 \end{array}$$

$$\begin{array}{r}
 11. \quad -7x^2+5x-6 \\
 \quad 4x^2+8x-9 \\
 \quad -x^2-7x+14 \\
 \hline
 -4x^2+6x-1
 \end{array}$$

$$\begin{array}{r}
 12. \quad a^3 \quad -4a+5 \\
 \quad a^3-2a^2 \quad +6 \\
 \quad +a^2-7a+4 \\
 \hline
 2a^3-a^2-11a+15
 \end{array}$$

$$\begin{array}{r}
 13. \quad -x^2+x-6 \\
 \quad x^3-7x^2 \quad +5 \\
 \quad -x^3 \quad +8x-5 \\
 \hline
 -8x^2+9x-6
 \end{array}$$

$$\begin{array}{r}
 14. \quad a^3 \quad -b^3 \\
 \quad +5a^2b-4ab^2 \\
 \quad a^3 \quad -7ab^2-b^3 \\
 \hline
 2a^3+5a^2b-11ab^2-2b^3
 \end{array}$$

$$\begin{array}{r}
 15. \quad x^3 \quad +xy^2+y^3 \\
 \quad x^3-5x^2y \quad -y^3 \\
 \quad 2x^3 \quad -4xy^2-5y^3 \\
 \hline
 4x^3-5x^2y-3xy^2-5y^3
 \end{array}$$

$$\begin{array}{r}
 16. \quad -7m^2n \quad +4n^3 \\
 \quad +m^3 \quad +6mn^2-n^3 \\
 \quad -m^3+7m^2n \quad +5n^3 \\
 \hline
 +6mn^2+8n^3
 \end{array}$$

$$\begin{array}{r}
 17. \quad x^4 \quad \quad - x^2 + x \\
 \quad \quad + x^3 - 4x^2 \quad \quad + 5 \\
 \quad \quad \quad + 7x^2 - 4x + 6 \\
 \hline
 x^4 + x^3 + 2x^2 - 3x + 11
 \end{array}$$

$$\begin{array}{r}
 18. \quad +a^6 \quad \quad + a^4 \quad \quad + 6 \\
 \quad \quad + a^5 \quad \quad - 3a^3 \quad \quad + 8 \\
 \quad \quad \quad + a^3 - a^2 - 14 \\
 \hline
 +a^6 + a^5 + a^4 - 2a^3 - a^2
 \end{array}$$

$$\begin{array}{r}
 19. \quad x^5 \quad \quad \quad + x - 9 \\
 \quad \quad + 3x^4 \quad \quad - 7x^2 \quad \quad + 6 \\
 \quad \quad \quad - 3x^3 \quad \quad - 4x + 5 \\
 \hline
 x^5 + 3x^4 - 3x^3 - 7x^2 - 3x + 2
 \end{array}$$

$$\begin{array}{r}
 20. \quad a^3 + a \\
 \quad \quad + a^2 + 5 \\
 \quad \quad + 4a + 7a^2 \\
 \quad \quad \quad - 8a^2 - 6 \\
 \hline
 a^3 + 5a \quad \quad - 1
 \end{array}$$

$$\begin{array}{r}
 21. \quad x^4 \quad \quad - x^2 y^2 \\
 \quad \quad - 5x^3 y \quad \quad + 6xy^3 \\
 \quad \quad \quad \quad \quad - 4xy^3 + y^4 \\
 \quad \quad \quad \quad \quad - 4x^2 y^2 \quad \quad - 6 \\
 \hline
 x^4 - 5x^3 y - 5x^2 y^2 + 2xy^3 + y^4 - 6
 \end{array}$$

$$\begin{array}{r}
 22. \quad + x^2 + xy \\
 \quad \quad - x^2 + 4xy - 7y^2 \\
 \quad \quad - x^2 + 6xy + 5y^2 \\
 \quad \quad \quad - 6x^2 - 4xy + y^2 \\
 \hline
 -7x^2 + 7xy - y^2
 \end{array}$$

$$\begin{array}{r}
 23. \quad a^3 - 8ax^2 + x^3 \\
 \quad \quad - 6ax^2 - x^3 + 5a^2 x \\
 \quad \quad 3a^3 \quad \quad - x^3 - 5a^2 x \\
 \quad \quad a^3 + 14ax^2 - x^3 \\
 \hline
 5a^3 \quad \quad - 2x^3
 \end{array}$$

$$\begin{array}{r}
 24. \quad \quad - 8a^2 m + 6am^2 - m^3 \\
 \quad \quad + a^3 \quad \quad - 5am^2 + m^3 \\
 \quad \quad - 4a^3 + 4a^2 m - 3am^2 \\
 \quad \quad \quad + 7a^2 m - 4am^2 \quad \quad - 6 \\
 \hline
 -3a^3 + 3a^2 m - 6am^2 \quad \quad - 6
 \end{array}$$

$$\begin{array}{r}
 25. \quad x^5 \quad \quad - x^3 y^2 \quad \quad - xy^4 \\
 \quad \quad + 2x^4 y \quad \quad + 3x^2 y^3 \quad \quad - y^5 \\
 \quad \quad \quad + 3x^3 y^2 \quad \quad - 4xy^4 - y^5 \\
 \hline
 x^5 \quad \quad \quad + 5xy^4 + 2y^5 \\
 2x^5 + 2x^4 y + 2x^3 y^2 + 3x^2 y^3
 \end{array}$$

$$\begin{array}{r}
 26. \quad a^5 + a^6 + a^2 \\
 \quad \quad \quad + a^4 + a^3 + 6 \\
 \quad \quad \quad + 3a^2 \quad \quad - 8 + 5a \\
 \hline
 -a^5 \quad -4a^2 \quad \quad + 6 - 5a \\
 \hline
 + a^6 \quad \quad + a^4 + a^3 + 4
 \end{array}$$

$$\begin{array}{r}
 27. \quad a^4 \quad \quad \quad - b^4 \\
 \quad \quad - a^3 b + a^2 b^2 - ab^3 \\
 \quad \quad - 3a^4 + 5a^3 b - 4a^2 b^2 \\
 \quad \quad \quad - 4a^3 b + 3a^2 b^2 \quad \quad - 3b^4 \\
 \hline
 -2a^4 \quad \quad \quad - ab^3 - 4b^4
 \end{array}$$

$$\begin{array}{r}
 28. \quad m^3 - n^3 + 6m^2 n \\
 \quad \quad + n^3 - 4m^2 n + 5mn^2 \\
 \quad \quad m^3 - n^3 \quad \quad + 6mn^2 \\
 \hline
 -2m^3 + n^3 - 2m^2 n \\
 \hline
 11mn^2
 \end{array}$$

$$\begin{array}{r}
 29. \quad a^x \quad \quad - 3a^{x-2} \\
 \quad \quad + 5a^{x-1} \quad \quad + 6a^{x-3} \\
 \quad \quad \quad \quad \quad + 7a^{x-3} + a^{x-4} \\
 \quad \quad \quad \quad \quad + a^{x-1} \quad \quad - 13a^{x-3} \\
 \hline
 a^x + 6a^{x-1} - 3a^{x-2} \quad \quad + a^{x-4}
 \end{array}$$

$$\begin{array}{r}
 30. \quad -a^x + a^{x+1} \quad \quad + a^{x+2} \\
 \quad \quad \quad - 3a^{x+3} - a^{x-1} + a^{x-2} \\
 \quad \quad - a^x \quad \quad + 4a^{x+3} \quad \quad - 5a^{x+2} \\
 \quad \quad \quad \quad \quad + a^{x-1} - a^{x-2} + a^{x+2} \\
 \hline
 -2a^x + a^{x+1} + a^{x+3} \quad \quad - 3a^{x+2} \\
 \Rightarrow a^{x+3} - 3a^{x+2} + a^{x+1} - 2a^x
 \end{array}$$

## EJERCICIO 18

$$1. \frac{1}{2}x^2 + \frac{1}{3}xy + \frac{1}{2}xy + \frac{1}{4}y^2$$

$$\frac{1}{2}x^2 + \frac{2+3}{6}xy + \frac{1}{4}y^2 = \frac{1}{2}x^2 + \frac{5}{6}xy + \frac{1}{4}y^2$$

$$2. a^2 + \frac{1}{2}ab - \frac{1}{4}ab + \frac{1}{2}b^2 - \frac{1}{4}ab - \frac{1}{5}b^2$$

$$a^2 + \frac{2-1-1}{4}ab + \frac{5-2}{10}b^2 = a^2 + \frac{3}{10}b^2$$

$$3. x^2 + \frac{2}{3}xy - \frac{1}{6}xy + y^2 - \frac{5}{6}xy + \frac{2}{3}y^2$$

$$x^2 + \frac{4-1-5}{6}xy + \frac{3+2}{3}y^2 = x^2 - \frac{2}{6}xy + \frac{5}{3}y^2 = x^2 - \frac{1}{3}xy + \frac{5}{3}y^2$$

$$4. \frac{3}{4}x^2 - \frac{1}{2}y^2 - \frac{2}{5}xy + \frac{1}{6}y^2 + \frac{1}{10}xy + \frac{1}{3}y^2$$

$$\frac{3}{4}x^2 + \frac{-4+1}{10}xy + \frac{-3+1+2}{6}y^2 = \frac{3}{4}x^2 - \frac{3}{10}xy$$

$$5. \frac{2}{3}a^2 + \frac{1}{5}ab - \frac{1}{2}b^2 - \frac{5}{6}a^2 - \frac{1}{10}ab + \frac{1}{6}b^2 - \frac{1}{12}a^2 + \frac{1}{20}ab - \frac{1}{3}b^2$$

$$\frac{8+10-1}{12}a^2 + \frac{4-2+1}{20}ab + \frac{-3+1-2}{6}b^2 = \frac{17}{12}a^2 + \frac{3}{20}ab - \frac{4}{6}b^2 = \frac{17}{12}a^2 + \frac{3}{20}ab - \frac{2}{3}b^2$$

$$6. \frac{5}{6}x^2 - \frac{2}{3}y^2 + \frac{3}{4}xy - \frac{1}{6}x^2 + \frac{1}{8}y^2 - \frac{1}{2}xy - \frac{1}{3}x^2 + \frac{1}{4}y^2 + \frac{5}{6}xy$$

$$\frac{5-1-2}{6}x^2 + \frac{-16+3+6}{24}y^2 + \frac{9-6+10}{12}xy = \frac{2}{6}x^2 - \frac{7}{24}y^2 + \frac{13}{12}xy \Rightarrow \frac{1}{3}x^2 + \frac{13}{12}xy - \frac{7}{24}y^2$$

$$7. a^3 - \frac{1}{2}ab^2 + b^3 - \frac{3}{8}ab^2 - 2b^3 + \frac{5}{6}a^2b - \frac{1}{4}a^3 - \frac{3}{5}b^3 - \frac{1}{2}a^2b$$

$$\frac{4+1}{4}a^3 + \frac{-4-3}{8}ab^2 + \frac{5-10-3}{5}b^3 + \frac{5-3}{6}a^2b = \frac{5}{4}a^3 - \frac{7}{8}ab^2 - \frac{8}{5}b^3 + \frac{2}{6}a^2b \Rightarrow \frac{5}{4}a^3 + \frac{1}{3}a^2b - \frac{7}{8}ab^2 - \frac{8}{5}b^3$$

$$8. x^4 - x^2 + 5 - 3 + \frac{2}{3}x^3 - \frac{3}{8}x - \frac{3}{5}x^4 + \frac{5}{6}x^3 - \frac{3}{4}x$$

$$\frac{5-3}{5}x^4 - x^2 + 2 + \frac{4+5}{6}x^3 + \frac{-3-6}{8}x = \frac{2}{5}x^4 - x^2 + 2 + \frac{9}{6}x^3 - \frac{9}{8}x \Rightarrow \frac{2}{5}x^4 + \frac{3}{2}x^3 - x^2 - \frac{9}{8}x + 2$$

$$9. \frac{2}{3}m^3 - \frac{1}{4}mn^2 + \frac{2}{5}n^3 + \frac{1}{8}mn^2 - \frac{3}{5}n^3 + \frac{1}{6}m^2n - m^3 - n^3 - \frac{1}{2}m^2n$$

$$\frac{2+3}{3}m^3 + \frac{-2+1}{8}mn^2 + \frac{2-3-5}{5}n^3 + \frac{1-3}{6}m^2n = \frac{5}{3}m^3 - \frac{1}{8}mn^2 - \frac{6}{5}n^3 - \frac{2}{6}m^2n \Rightarrow \frac{5}{3}m^3 - \frac{1}{3}m^2n - \frac{1}{8}mn^2 - \frac{6}{5}n^3$$

$$10. x^4 + 2x^2y^2 + \frac{2}{7}y^4 - \frac{5}{6}x^4 + \frac{3}{8}x^2y^2 - \frac{1}{14}y^4 - \frac{1}{6}xy^3 - \frac{1}{4}x^2y^2 + \frac{1}{7}y^4 - \frac{5}{6}x^3y$$

$$\frac{6-5}{6}x^4 + \frac{16+3-2}{8}x^2y^2 + \frac{4-1+2}{14}y^4 - \frac{1}{6}xy^3 - \frac{5}{6}x^3y = \frac{1}{6}x^4 + \frac{17}{8}x^2y^2 + \frac{5}{14}y^4 - \frac{1}{6}xy^3 - \frac{5}{6}x^3y \Rightarrow \frac{1}{6}x^4 - \frac{5}{6}x^3y + \frac{17}{8}x^2y^2 - \frac{1}{6}xy^3 + \frac{5}{14}y^4$$

$$\begin{array}{r}
 11. \quad x^5 - \frac{2}{3}x^3 + \frac{4}{5}x \\
 -3x^5 \qquad \qquad -\frac{1}{10}x + \frac{3}{8}x^2 \\
 + \frac{1}{6}x^3 \qquad \qquad -\frac{1}{4}x^2 - \frac{2}{3}x^4 \\
 -\frac{1}{12}x^3 + \frac{3}{5}x \qquad \qquad -4 \\
 \hline
 -2x^5 + \frac{-8+2-1}{12}x^3 + \frac{8-1+6}{10}x + \frac{3-2}{8}x^2 - \frac{2}{3}x^4 - 4 \\
 \hline
 = -2x^5 - \frac{7}{12}x^3 + \frac{13}{10}x + \frac{1}{8}x^2 - \frac{2}{3}x^4 - 4 \\
 \Rightarrow -2x^5 - \frac{2}{3}x^4 - \frac{7}{12}x^3 + \frac{1}{8}x^2 + \frac{13}{10}x - 4
 \end{array}$$

$$\begin{array}{r}
 12. \quad \frac{2}{9}a^3 + \frac{5}{6}ax^2 - \frac{1}{3}x^3 \\
 -\frac{3}{7}a^2x - \frac{7}{8}ax^2 - \frac{1}{9}x^3 \\
 -\frac{2}{3}a^3 + \frac{1}{2}a^2x - \frac{1}{4}ax^2 \\
 \hline
 \frac{2-6}{9}a^3 + \frac{-6+7}{14}a^2x + \frac{20-21-6}{24}ax^2 + \frac{-3-1}{9}x^3 \\
 \hline
 = -\frac{4}{9}a^3 + \frac{1}{14}a^2x - \frac{7}{24}ax^2 - \frac{4}{9}x^3
 \end{array}$$

$$\begin{array}{r}
 13. \quad a^6 - a^4 + a^2 \\
 + \frac{3}{5}a^5 - \frac{3}{8}a^3 - \frac{1}{2}a \\
 - \frac{3}{7}a^4 - \frac{5}{8}a^2 \qquad \qquad +6 \\
 - \frac{3}{8}a - 6 \\
 \hline
 a^6 + \frac{-7-3}{7}a^4 + \frac{8-5}{8}a^2 + \frac{3}{5}a^5 - \frac{3}{8}a^3 + \frac{-4-3}{8}a \\
 \hline
 = a^6 - \frac{10}{7}a^4 + \frac{3}{8}a^2 + \frac{3}{5}a^5 - \frac{3}{8}a^3 - \frac{7}{8}a \\
 \Rightarrow a^6 + \frac{3}{5}a^5 - \frac{10}{7}a^4 - \frac{3}{8}a^3 + \frac{3}{8}a^2 - \frac{7}{8}a
 \end{array}$$

$$\begin{array}{r}
 14. \quad x^5 - y^5 \\
 + \frac{1}{10}x^3y^2 - \frac{3}{4}xy^4 - \frac{1}{6}y^5 \\
 - \frac{1}{9}y^5 + \frac{3}{5}x^4y - \frac{5}{6}x^2y^3 \\
 - \frac{2}{5}x^3y^2 - \frac{1}{3}y^5 + 2x^4y \\
 \hline
 x^5 + \frac{1-4}{10}x^3y^2 - \frac{3}{4}xy^4 + \frac{-18-3-2-6}{18}y^5 + \frac{3+10}{5}x^4y - \frac{5}{6}x^2y^3 \\
 \hline
 = x^5 - \frac{3}{10}x^3y^2 - \frac{3}{4}xy^4 - \frac{29}{18}y^5 + \frac{13}{5}x^4y - \frac{5}{6}x^2y^3 \\
 \Rightarrow x^5 + \frac{13}{5}x^4y - \frac{3}{10}x^3y^2 - \frac{5}{6}x^2y^3 - \frac{3}{4}xy^4 - \frac{29}{18}y^5
 \end{array}$$

## EJERCICIO 19

Para los problemas 1 al 14 las literales toman los siguientes valores:

$$a = 2 \quad b = 3 \quad c = 10 \quad x = 5 \quad y = 4 \quad m = 2/3 \quad n = 1/5$$

$$\begin{array}{r}
 1. \quad 4x - 5y \\
 -3x + 6y - 8 \\
 -x + y \\
 \hline
 2y - 8 \\
 \Rightarrow 2 \cdot 4 - 8 = 8 - 8 = 0
 \end{array}$$

$$\begin{array}{r}
 2. \quad x^2 - 5x + 8 \\
 -x^2 + 10x - 30 \\
 -6x^2 + 5x - 50 \\
 \hline
 -6x^2 + 10x - 72 \\
 \Rightarrow -6 \cdot 5^2 + 10 \cdot 5 - 72 = -6 \cdot 25 + 50 - 72 \\
 = -150 + 50 - 72 = -172
 \end{array}$$

$$\begin{array}{r}
 3. \quad x^4 - y^4 \\
 2x^4 - 5x^2y^2 - 8 \\
 -4x^4 \qquad \qquad + 7x^3y + 10xy^3 \\
 \hline
 -x^4 - y^4 - 5x^2y^2 - 8 + 7x^3y + 10xy^3 \\
 \Rightarrow -x^4 + 7x^3y - 5x^2y^2 + 10xy^3 - y^4 - 8 \\
 = -5^4 + 7 \cdot 5^3 \cdot 4 - 5 \cdot 5^2 \cdot 4^2 + 10 \cdot 5 \cdot 4^3 - 4^4 - 8 \\
 = -625 + 28 \cdot 125 - 5 \cdot 25 \cdot 16 + 50 \cdot 64 - 256 - 8 \\
 = -625 + 3.500 - 2.000 + 3.200 - 256 - 8 = 3.811
 \end{array}$$

$$\begin{array}{r}
 4. \quad 3m - 5n + 6 \\
 -6m - 20n + 8 \\
 12m - 20n - 12 \\
 \hline
 9m - 45n + 2 \\
 \Rightarrow 9 \cdot \frac{2}{3} - 45 \cdot \frac{1}{5} + 2 = \frac{18}{3} - \frac{45}{5} + 2 = 6 - 9 + 2 = -1
 \end{array}$$



5.  $nx + cn - ab$

$$8nx - 2cn - ab$$

$$\frac{nx}{10nx - cn - 3ab - 5}$$

$$\Rightarrow 10 \cdot \frac{1}{5} \cdot 5 - 10 \cdot \frac{1}{5} \cdot 3 - 2 \cdot 3 - 5$$

$$= \frac{50}{5} - \frac{10}{5} - 6 \cdot 3 - 5$$

$$= 10 - 2 - 18 - 5 = -15$$

6.  $a^3 + b^3$

$$-3a^2b + 8ab^2 - b^3$$

$$-5a^3 - 6ab^2 + 8$$

$$+3a^2b - 2b^3$$

$$\frac{-4a^3 + 2ab^2 - 2b^3 + 8}{-4 \cdot 2^3 + 2 \cdot 2 \cdot 3^2 - 2 \cdot 3^3 + 8}$$

$$\Rightarrow -4 \cdot 2^3 + 2 \cdot 2 \cdot 3^2 - 2 \cdot 3^3 + 8$$

$$= -4 \cdot 8 + 4 \cdot 9 - 2 \cdot 27 + 8$$

$$= -32 + 36 - 54 + 8 = -42$$

7.  $27m^3 + 125n^3$

$$-9m^2n + 25mn^2$$

$$-14mn^2 - 8$$

$$+10m^2n + 11mn^2$$

$$\frac{27m^3 + m^2n + 22mn^2 + 125n^3 - 8}{27 \cdot \left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^2 \cdot \frac{1}{5} + 22 \cdot \frac{2}{3} \cdot \left(\frac{1}{5}\right)^2 + 125 \cdot \left(\frac{1}{5}\right)^3 - 8}$$

$$\Rightarrow 27 \cdot \left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^2 \cdot \frac{1}{5} + 22 \cdot \frac{2}{3} \cdot \left(\frac{1}{5}\right)^2 + 125 \cdot \left(\frac{1}{5}\right)^3 - 8$$

$$= 27 \cdot \frac{8}{27} + \frac{4}{9} \cdot \frac{1}{5} + \frac{44}{3} \cdot \frac{1}{25} + 125 \cdot \frac{1}{125} - 8$$

$$= 8 + \frac{4}{45} + \frac{44}{75} + 1 - 8 = \frac{4}{45} + \frac{44}{75} + 1$$

$$= \frac{20 + 132 + 225}{225} = \frac{377}{225} = 1 \frac{152}{225}$$

8.  $x^{a-1} + y^{b-2} + m^{x-4}$

$$2x^{a-1} - 2y^{b-2} - 2m^{x-4}$$

$$+ 3y^{b-2} - 2m^{x-4}$$

$$\frac{3x^{a-1} + 2y^{b-2} - 3m^{x-4}}{\Rightarrow 3 \cdot 5^{2-1} + 2 \cdot 4^{3-2} - 3 \left(\frac{2}{3}\right)^{5-4}} = 15 + 8 - 2 = 21$$

$$\Rightarrow 3 \cdot 5^{2-1} + 2 \cdot 4^{3-2} - 3 \left(\frac{2}{3}\right)^{5-4} = 15 + 8 - 2 = 21$$

9.  $n^{b-1} - m^{x-3} + 8$

$$-5n^{b-1} - 3m^{x-3} + 10$$

$$4n^{b-1} + 5m^{x-3} - 18$$

$$\frac{m^{x-3}}{\Rightarrow \left(\frac{2}{3}\right)^{5-3} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}}$$

$$\Rightarrow \left(\frac{2}{3}\right)^{5-3} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

10.  $x^3y - xy^3 + 5$

$$5x^3y - 6 + x^4 - x^2y^2$$

$$-6xy^3 + 2 + x^2y^2$$

$$+ 3xy^3 + 1 - y^4$$

$$\frac{6x^3y - 4xy^3 + 2 + x^4 - y^4}{\Rightarrow x^4 + 6x^3y - 4xy^3 - y^4 + 2}$$

$$= 5^4 + 6 \cdot 5^3 \cdot 4 - 4 \cdot 5 \cdot 4^3 - 4^4 + 2$$

$$= 625 + 24 \cdot 125 - 20 \cdot 64 - 256 + 2$$

$$= 625 + 3.000 - 1.280 - 256 + 2 = 2.091$$

11.  $\frac{3}{4}a^2 + \frac{2}{3}b^2$

$$- \frac{1}{3}ab + \frac{1}{9}b^2$$

$$+ \frac{1}{6}ab - \frac{1}{3}b^2$$

$$\frac{\frac{3}{4}a^2 - \frac{-2+1}{6}ab + \frac{6+1-3}{9}b^2}{\frac{3}{4}a^2 - \frac{1}{6}ab + \frac{4}{9}b^2}$$

$$\Rightarrow \frac{3}{4} \cdot 2^2 - \frac{1}{6} \cdot 2 \cdot 3 + \frac{4}{9} \cdot 3^2$$

$$= \frac{3}{4} \cdot 4 - \frac{1}{6} \cdot 6 + \frac{4}{9} \cdot 9 = 3 - 1 + 4 = 6$$

12.  $\frac{9}{17}m^2 + \frac{25}{34}n^2 - \frac{1}{4}$

$$- \frac{7}{34}m^2 + \frac{5}{17}n^2 + \frac{1}{2} - 15mn$$

$$- \frac{7}{34}m^2 - \frac{1}{4} - 30mn$$

$$\frac{18+7-7}{34}m^2 + \frac{25+10}{34}n^2 + \frac{-1+2-1+12}{4} - 45mn$$

$$= \frac{18}{34}m^2 + \frac{35}{34}n^2 + \frac{12}{4} - 45mn$$

$$\Rightarrow \frac{9}{17}m^2 - 45mn + \frac{35}{34}n^2 + 3$$

$$= \frac{9}{17} \left(\frac{2}{3}\right)^2 - 45 \cdot \frac{2}{3} \cdot \frac{1}{5} + \frac{35}{34} \left(\frac{1}{5}\right)^2 + 3$$

$$= \frac{9}{17} \cdot \frac{4}{9} - \frac{90}{15} + \frac{35}{34} \cdot \frac{1}{25} + 3$$

$$= \frac{4}{17} - 6 + \frac{35}{850} + 3 = \frac{4}{17} + \frac{7}{170} - 3$$

$$= \frac{40+7-510}{170} = -\frac{463}{170} = -2 \frac{123}{170}$$

$$\begin{array}{rclcl}
 13. & \frac{1}{2}b^2m & - & \frac{3}{5}cn & - & 2 \\
 & \frac{3}{4}b^2m & - & \frac{1}{10}cn & + & 6 \\
 & -\frac{1}{4}b^2m & + & \frac{1}{25}cn & + & 4 \\
 & -\frac{1}{8}b^2m & + & 2cn & + & \frac{3}{5} \\
 \hline
 & \frac{4+6-2-1}{8}b^2m + \frac{-30-5+2+100}{50}cn + \frac{-10+30+20+3}{5} \\
 \Rightarrow & \frac{7}{8}b^2m + \frac{67}{50}cn + \frac{43}{5} = \frac{7}{8} \cdot 3^2 \cdot \frac{2}{3} + \frac{67}{50} \cdot 10 \cdot \frac{1}{5} + \frac{43}{5} \\
 = & \frac{7 \cdot 9 \cdot 2}{24} + \frac{67}{50} + \frac{43}{5} = \frac{126}{24} + \frac{670}{250} + \frac{43}{5} = \frac{21}{4} + \frac{67}{25} + \frac{43}{5} \\
 = & \frac{525+268+860}{100} = \frac{1.653}{100} = 16 \frac{53}{100}
 \end{array}$$

$$\begin{array}{rclcl}
 14. & 0,2a^3 + 0,4ab^2 - 0,5a^2b & & & \\
 & + 0,6ab^2 - 0,3a^2b - 0,8b^3 & & & \\
 & - 0,4a^3 & - 0,8a^2b & + 6 & \\
 & 0,2a^3 & + 1,5a^2b + 0,9b^3 & & \\
 \hline
 & ab^2 & - 0,1a^2b + 0,1b^3 + 6 & & \\
 \Rightarrow & -0,1a^2b + ab^2 + 0,1b^3 + 6 & & & \\
 = & -0,1 \cdot 2^2 \cdot 3 + 2 \cdot 3^2 + 0,1 \cdot 3^3 + 6 & & & \\
 = & -0,3 \cdot 4 + 2 \cdot 9 + 0,1 \cdot 27 + 6 & & & \\
 = & -1,2 + 18 + 2,7 + 6 & & & \\
 = & 25,5 & & & 
 \end{array}$$

## EJERCICIO 20

1.  $-8-5=-13$

2.  $-7-4=-11$

3.  $8-11=-3$

4.  $-8-(-11)=-8+11=3$

5.  $-1-(-9)=-1+9=8$

6.  $2a-3b=2a-3b$

7.  $3b-2=3b-2$

8.  $4x-6b=4x-6b$

9.  $-5a-6b=-5a-6b$

10.  $-8x-(-3)=-8x+3$

11.  $-9a^2-5b^2=-9a^2-5b^2$

12.  $-7xy-(-5yz)=-7xy+5yz$

13.  $3a-4a=-a$

14.  $11m^2-25m^2=-14m^2$

15.  $-6x^2y-(-x^2y)=-6x^2y+x^2y=-5x^2y$

16.  $11a^3m^2-(-7a^3m^2)=11a^3m^2+7a^3m^2=18a^3m^2$

17.  $-8ab^2-(-8ab^2)=-8ab^2+8ab^2=0$

18.  $31x^2y-(-46x^2y)=31x^2y+46x^2y=77x^2y$

19.  $-84a^2b-(-84a^2b)=-84a^2b+84a^2b=0$

20.  $3a^{x+1}-5b^{x+2}=3a^{x+1}-5b^{x+2}$

21.  $-8x^{a+2}-11=-8x^{a+2}-11$

22.  $6a^n-(-5a^n)=6a^n+5a^n=11a^n$

23.  $45a^{x-1}-(-60a^{x-1})=45a^{x-1}+60a^{x-1}=105a^{x-1}$

24.  $54b^{n-1}-(-86b^{n-1})=54b^{n-1}+86b^{n-1}=140b^{n-1}$

25.  $-35m^a-(-60m^a)=-35m^a+60m^a=25m^a$

26.  $5-\left(-\frac{1}{2}\right)=5+\frac{1}{2}=\frac{10+1}{2}=\frac{11}{2}=5\frac{1}{2}$

27.  $-\frac{2}{3}-\frac{3}{4}=\frac{-8-9}{12}=-\frac{17}{12}$

28.  $\frac{1}{3}x^2-\left(-\frac{2}{3}x^2\right)=\frac{1}{3}x^2+\frac{2}{3}x^2=\frac{3}{3}x^2=x^2$

29.  $\frac{4}{5}x^3y-\left(-\frac{5}{6}x^3y\right)=\frac{24+25}{30}x^3y=\frac{49}{30}x^3y$

30.  $-\frac{1}{8}ab^2-\left(-\frac{3}{4}ab^2\right)=\frac{-1+6}{8}ab^2=\frac{5}{8}ab^2$

31.  $-2-3=-5$

32.  $7-(-1)=7+1=8$

33.  $-8-(-5)=-8+5=-3$

34.  $5-(-4)=5+4=9$

35.  $-7-(-7)=-7+7=0$

36.  $2a-(-5)=2a+5$

$$37. -3x - b = -3x - b$$

$$38. -2n - 5m = -2n - 5m$$

$$39. 3b - (-6a) = 6a + 3b$$

$$40. 8b - (-5a^3) = 5a^3 + 8b$$

$$41. -7a - (-9) = -7a + 9$$

$$42. 25ab - (-25) = 25ab + 25$$

$$43. 3a - (-a) = 3a + a = 4a$$

$$44. -4b - (-3b) = -4b + 3b = -b$$

$$45. 54x^3 - (-11x^3) = 54x^3 + 11x^3 = 65x^3$$

$$46. 78a^2b - 14a^2b = 64a^2b$$

$$47. -54a^2y - (-43a^2y) = -54a^2y + 43a^2y = -11a^2y$$

$$48. -ab - 9ab = -10ab$$

$$49. -31x^2y - (-31x^2y) = -31x^2y + 31x^2y = 0$$

$$50. -3a^x - a^x = -4a^x$$

$$51. 311a^{x+1} - (-7a^{x+1}) = 311a^{x+1} + 7a^{x+1} = 318a^{x+1}$$

$$52. 105m^x - 9m^x = 96m^x$$

$$53. -31a^{x-1} - 18a^{x-1} = -49a^{x-1}$$

$$54. -236m^a - (-19m^a) = -236m^a + 19m^a = -217m^a$$

$$55. -85a^{x+2} - 54a^{x+2} = -139a^{x+2}$$

$$56. \frac{1}{4} - (-6a) = \frac{1}{4} + 6a$$

$$57. -\frac{2}{3} - (-5) = \frac{-2+15}{3} = \frac{13}{3}$$

$$58. -\frac{7}{10}m^3 - \frac{3}{8}m^3 = \frac{-56-30}{80}m^3 = -\frac{86}{80}m^3 = -\frac{43}{40}m^3$$

$$59. \frac{5}{6}a^2b^2 - \left(-\frac{11}{12}a^2b^2\right) = \frac{10+11}{12}a^2b^2 = \frac{21}{12}a^2b^2 = \frac{7}{4}a^2b^2$$

$$60. -\frac{1}{9}a^3b^2 - 45a^3b^2 = \frac{-1-405}{9}a^3b^2 = \frac{-406}{9}a^3b^2 = -45\frac{1}{9}a^3b^2$$

## EJERCICIO 21

$$1. \frac{a+b}{-a+b}$$

$$\frac{-a+b}{2b}$$

$$2. \frac{2x-3y}{x-2y}$$

$$\frac{x-2y}{3x-5y}$$

$$3. \frac{8a+b}{3a}$$

$$\frac{-4}{11a+b-4}$$

$$4. \frac{x^2-3x}{5x-6}$$

$$\frac{x^2+2x-6}{x^2+2x-6}$$

$$5. \frac{a^3-a^2b}{-7a^2b-9ab^2}$$

$$\frac{-7a^2b-9ab^2}{a^3-8a^2b-9ab^2}$$

$$6. \frac{x-y+z}{-x+y-z}$$

$$\frac{-x+y-z}{0}$$

$$7. \frac{x+y-z}{x+y-z}$$

$$\frac{2x+2y-2z}{2x+2y-2z}$$

$$8. \frac{x^2+y^2-3xy}{-3x^2+y^2+4xy}$$

$$\frac{-2x^2+2y^2+xy}{-2x^2+2y^2+xy}$$

$$\Rightarrow -2x^2 + xy + 2y^2$$

$$9. \frac{x^3-x^2+6}{-5x^2-6+4x}$$

$$\frac{-5x^2-6+4x}{x^3-6x^2+4x}$$

$$10. \frac{y^2+6y^3-8}{3y^2-2y^4-6y}$$

$$\frac{4y^2-2y^4+6y^3-6y-8}{4y^2-2y^4+6y^3-6y-8}$$

$$\Rightarrow -2y^4 + 6y^3 + 4y^2 - 6y - 8$$

$$11. \frac{a^3-6ab^2+9a}{+8a-15a^2b-5}$$

$$\frac{a^3-6ab^2+17a-15a^2b-5}{a^3-6ab^2+17a-15a^2b-5}$$

$$\Rightarrow a^3 - 15a^2b - 6ab^2 + 17a - 5$$

$$12. \frac{x^4+9xy^3-11y^4}{-20y^4+8x^3y+6x^2y^2}$$

$$\frac{x^4+9xy^3-31y^4+8x^3y+6x^2y^2}{x^4+9xy^3-31y^4+8x^3y+6x^2y^2}$$

$$\Rightarrow x^4 + 8x^3y + 6x^2y^2 + 9xy^3 - 31y^4$$

$$13. \frac{a+b+c-d}{a+b-c+d}$$

$$\frac{2a+2b}{2a+2b}$$

$$14. \frac{ab+2ac-3cd-5de}{-8ab+4ac+5cd-5de}$$

$$\frac{-7ab+6ac+2cd-10de}{-7ab+6ac+2cd-10de}$$

$$15. \frac{x^3+6x^2-9x-19}{-6x^3+11x^2-21x+43}$$

$$\frac{-6x^3+11x^2-21x+43}{-6x^3+11x^2-21x+43}$$

$$\Rightarrow -5x^3 + 17x^2 - 30x + 24$$

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$$\begin{array}{r}
 16. \quad y^5 - 9y^3 + 6y^2 \quad -31 \\
 \quad -31y^3 + 8y^2 + 11y^4 + 19y \\
 \hline
 y^5 - 40y^3 + 14y^2 + 11y^4 + 19y - 31 \Rightarrow y^5 + 11y^4 - 40y^3 + 14y^2 + 19y - 31
 \end{array}$$

$$\begin{array}{r}
 17. \quad 5m^3 - 9n^3 + 6m^2n - 8mn^2 \\
 \quad -5m^3 \quad + 21m^2n - 14mn^2 + 18 \\
 \hline
 -9n^3 + 27m^2n - 22mn^2 + 18 \Rightarrow 27m^2n - 22mn^2 - 9n^3 + 18
 \end{array}$$

$$\begin{array}{r}
 18. \quad 4x^3y - 19xy^3 + y^4 - 6x^2y^2 \\
 \quad 25x^3y + 51xy^3 \quad - 32x^2y^2 + x^4 \\
 \hline
 29x^3y + 32xy^3 + y^4 - 38x^2y^2 + x^4 \Rightarrow x^4 + 29x^3y - 38x^2y^2 + 32xy^3 + y^4
 \end{array}$$

$$\begin{array}{r}
 19. \quad m^6 + m^4n^2 - 9m^2n^4 + 19 \\
 \quad + 30m^2n^4 + 61 + 13m^3n^3 - 16mn^5 \\
 \hline
 m^6 + m^4n^2 + 21m^2n^4 + 80 + 13m^3n^3 - 16mn^5 \Rightarrow m^6 + m^4n^2 + 13m^3n^3 + 21m^2n^4 - 16mn^5 + 80
 \end{array}$$

$$\begin{array}{r}
 20. \quad -a^5b \quad + 6a^3b^3 \quad - 18ab^5 \quad + 42 \\
 \quad 8a^6 \quad + 11a^4b^2 \quad + 11a^2b^4 \quad - 9b^6 \\
 \hline
 8a^6 - a^5b + 11a^4b^2 + 6a^3b^3 + 11a^2b^4 - 18ab^5 - 9b^6 + 42
 \end{array}$$

$$\begin{array}{r}
 21. \quad 1 - x^2 + x^4 - x^3 + 3x - 6x^5 \\
 \quad 24 + 30x^2 - 8x^4 \quad - 15x \quad + x^6 \\
 \hline
 25 + 29x^2 - 7x^4 - x^3 - 12x - 6x^5 + x^6 \Rightarrow x^6 - 6x^5 - 7x^4 - x^3 + 29x^2 - 12x + 25
 \end{array}$$

$$\begin{array}{r}
 22. \quad -6x^2y^3 + 8x^5 - 23x^4y + 80x^3y^2 - 18 \\
 \quad + 51x^4y + 21x^3y^2 - 80 + y^5 - 9xy^4 \\
 \hline
 -6x^2y^3 + 8x^5 + 28x^4y + 101x^3y^2 - 98 + y^5 - 9xy^4 \Rightarrow 8x^5 + 28x^4y + 101x^3y^2 - 6x^2y^3 - 9xy^4 + y^5 - 98
 \end{array}$$

$$\begin{array}{r}
 23. \quad m^6 \quad - 8m^4n^2 \quad + 21m^2n^4 - 6mn^5 \quad + 8 \\
 \quad 23m^5n \quad - 14m^3n^3 \quad + 24mn^5 - 8n^6 + 14 \\
 \hline
 m^6 + 23m^5n - 8m^4n^2 - 14m^3n^3 + 21m^2n^4 + 18mn^5 - 8n^6 + 22
 \end{array}$$

$$\begin{array}{r}
 24. \quad x^7 - 8x + 16x^5 - 23x^2 - 15 \\
 \quad - 51x \quad + 18 + 8x^6 - 25x^4 + 30x^3 \\
 \hline
 x^7 - 59x + 16x^5 - 23x^2 + 3 + 8x^6 - 25x^4 + 30x^3 \Rightarrow x^7 + 8x^6 + 16x^5 - 25x^4 + 30x^3 - 23x^2 - 59x + 3
 \end{array}$$

$$\begin{array}{r}
 25. \quad 9a^6 - 15a^4b^2 + 31a^2b^4 - b^6 \quad + 14 \\
 \quad + 15a^4b^2 \quad - 3b^6 - 25a^5b - 53a^3b^3 + 9ab^5 \\
 \hline
 9a^6 \quad + 31a^2b^4 - 4b^6 - 25a^5b - 53a^3b^3 + 9ab^5 + 14 \Rightarrow 9a^6 - 25a^5b - 53a^3b^3 + 31a^2b^4 + 9ab^5 - 4b^6 + 14
 \end{array}$$

$$\begin{array}{r}
 a^x + a^{x+1} - a^{x+2} \\
 26. \quad -5a^x + 6a^{x+1} + a^{x+2} \\
 \hline
 -4a^x + 7a^{x+1}
 \end{array}$$

27. 
$$\begin{array}{r} m^a - m^{a-1} + 3m^{a-2} \\ 4m^a \quad -5m^{a-2} - 3m^{a+1} - 8m^{a-3} \\ \hline 5m^a - m^{a-1} - 2m^{a-2} - 3m^{a+1} - 8m^{a-3} \Rightarrow -3m^{a+1} + 5m^a - m^{a-1} - 2m^{a-2} - 8m^{a-3} \end{array}$$
28. 
$$\begin{array}{r} a^{m+4} - 7a^{m+2} - 8a^m + 6a^{m-1} \\ +14a^{m+2} \quad + 8a^{m-1} + 5a^{m+3} + 11a^{m+1} \\ \hline a^{m+4} + 7a^{m+2} - 8a^m + 14a^{m-1} + 5a^{m+3} + 11a^{m+1} \Rightarrow a^{m+4} + 5a^{m+3} + 7a^{m+2} + 11a^{m+1} - 8a^m + 14a^{m-1} \end{array}$$
29. 
$$\begin{array}{r} x^{a+2} - 7x^a + 9x^{a-1} + 25x^{a-2} \\ -19x^a - 45x^{a-1} \quad + 11x^{a+1} - 60x^{a-3} \\ \hline x^{a+2} - 26x^a - 36x^{a-1} + 25x^{a-2} + 11x^{a+1} - 60x^{a-3} \Rightarrow x^{a+2} + 11x^{a+1} - 26x^a - 36x^{a-1} + 25x^{a-2} - 60x^{a-3} \end{array}$$
30. 
$$\begin{array}{r} m^{n+1} - 6m^{n-2} + 8m^{n-3} - 19m^{n-5} \\ - 5m^{n-2} - 6m^{n-3} - 9m^{n-5} - 8m^n - m^{n-4} \\ \hline m^{n+1} - 11m^{n-2} + 2m^{n-3} - 28m^{n-5} - 8m^n - m^{n-4} \Rightarrow m^{n+1} - 8m^n - 11m^{n-2} + 2m^{n-3} - m^{n-4} - 28m^{n-5} \end{array}$$

## EJERCICIO 22

1. 
$$\begin{array}{r} b - a \\ b - a \\ \hline 2b - 2a \Rightarrow -2a + 2b \end{array}$$
2. 
$$\begin{array}{r} 2x + 3y \\ -x + y \\ \hline x + 4y \end{array}$$
3. 
$$\begin{array}{r} -7a + 5 \\ 5a \quad -b \\ \hline -2a + 5 - b \Rightarrow -2a - b + 5 \end{array}$$
4. 
$$\begin{array}{r} -x^2 + 6 \\ -x^2 \quad + 5x \\ \hline -2x^2 + 6 + 5x \Rightarrow -2x^2 + 5x + 6 \end{array}$$
5. 
$$\begin{array}{r} x^2y + 5xy^2 \\ + xy^2 - x^3 \\ \hline x^2y + 6xy^2 - x^3 \Rightarrow -x^3 + x^2y + 6xy^2 \end{array}$$
6. 
$$\begin{array}{r} 7a^2b + 5ab^2 \\ -6a^2b \quad + 8a^3 \\ \hline a^2b + 5ab^2 + 8a^3 \Rightarrow 8a^3 + a^2b + 5ab^2 \end{array}$$
7. 
$$\begin{array}{r} -a + 2b - 3c \\ -a + b - 2c \\ \hline -2a + 3b - 5c \end{array}$$
8. 
$$\begin{array}{r} -3n + 4m + 5p \\ + n - m - p \\ \hline -2n + 3m + 4p \Rightarrow 3m - 2n + 4p \end{array}$$
9. 
$$\begin{array}{r} x + 3y - 6z \\ x - y + z \\ \hline 2x + 2y - 5z \end{array}$$
10. 
$$\begin{array}{r} -5b^2 + 8ab + a^2 \\ 6b^2 - ab - 3a^2 \\ \hline b^2 + 7ab - 2a^2 \Rightarrow -2a^2 + 7ab + b^2 \end{array}$$
11. 
$$\begin{array}{r} -5m^2 - n^2 + 6mn \\ -m^2 + n^2 + 3mn \\ \hline -6m^2 \quad + 9mn \end{array}$$
12. 
$$\begin{array}{r} -8x^2 + 5x - 4 \\ + x - 6 + x^3 \\ \hline -8x^2 + 6x - 10 + x^3 \Rightarrow x^3 - 8x^2 + 6x - 10 \end{array}$$

$$\begin{array}{r}
 13. \quad 14m^2 - 8n + 16 \\
 -14m^2 \quad - 9 - m^3 \\
 \hline
 -8n + 7 - m^3 \Rightarrow -m^3 - 8n + 7
 \end{array}$$

$$\begin{array}{r}
 14. \quad 8ab + 5bc + 6cd \\
 -ab + bc - 6cd \\
 \hline
 7ab + 6bc
 \end{array}$$

$$\begin{array}{r}
 15. \quad a^3 - 9a^2b - b^3 \\
 -25a^2b + b^3 + 8ab^2 \\
 \hline
 a^3 - 34a^2b + 8ab^2
 \end{array}$$

$$\begin{array}{r}
 16. \quad 6x^3 - 8x^2y - 6xy^2 \\
 - xy^2 + 6y^3 - 4 \\
 \hline
 6x^3 - 8x^2y - 7xy^2 + 6y^3 - 4
 \end{array}$$

$$\begin{array}{r}
 17. \quad m^2 - 9n + 11c + 14 \\
 -m^2 - 7n + 8c \quad -d \\
 \hline
 -16n + 19c + 14 - d \Rightarrow 19c - d - 16n + 14
 \end{array}$$

$$\begin{array}{r}
 18. \quad 5a^4 + 9a^3b - 40ab^3 + 6b^4 \\
 -7a^3b + 8a^2b^2 - 5ab^3 - b^4 \\
 \hline
 5a^4 + 2a^3b + 8a^2b^2 - 45ab^3 + 5b^4
 \end{array}$$

$$\begin{array}{r}
 19. \quad x^5 - 8x^4 + 25x^2 + 15 \\
 - 6x^2 + 7 - 6x^3 + 9x \\
 \hline
 x^5 - 8x^4 + 19x^2 + 22 - 6x^3 + 9x \\
 \Rightarrow x^5 - 8x^4 - 6x^3 + 19x^2 + 9x + 22
 \end{array}$$

$$\begin{array}{r}
 20. \quad -3xy^4 - 8x^3y^2 - 19y^5 + 18 \\
 -6xy^4 \quad -25y^5 \quad -x^5 + x^2y^3 \\
 \hline
 -9xy^4 - 8x^3y^2 - 44y^5 + 18 - x^5 + x^2y^3 \\
 \Rightarrow -x^5 - 8x^3y^2 + x^2y^3 - 9xy^4 - 44y^5 + 18
 \end{array}$$

$$\begin{array}{r}
 21. \quad x^3 - 6x^4 + 8x^2 - 9 + 15x \\
 -25x^3 \quad +18x^2 + 46 - 25x + 11x^5 \\
 \hline
 -24x^3 - 6x^4 + 26x^2 + 37 - 10x + 11x^5 \\
 \Rightarrow 11x^5 - 6x^4 - 24x^3 + 26x^2 - 10x + 37
 \end{array}$$

$$\begin{array}{r}
 22. \quad a^5 - 26a^3b^2 + 8ab^4 - b^5 + 6 \\
 - 8a^4b - a^3b^2 + 15a^2b^3 + 45ab^4 + 8 \\
 \hline
 a^5 - 8a^4b - 27a^3b^2 + 15a^2b^3 + 53ab^4 - b^5 + 14
 \end{array}$$

$$\begin{array}{r}
 23. \quad y^6 + y^3 + y^2 + 9 \\
 -23y^3 \quad + 5 - 8y^4 + 15y^5 + 8y \\
 \hline
 y^6 - 22y^3 + y^2 + 14 - 8y^4 + 15y^5 + 8y \\
 \Rightarrow y^6 + 15y^5 - 8y^4 - 22y^3 + y^2 + 8y + 14
 \end{array}$$

$$\begin{array}{r}
 24. \quad x^3 - x^6 + 3x^4 - 5x^2 - 9 \\
 -36 - 7x^7 - 5x^5 + 23x^3 - 51x \\
 \hline
 x^3 - x^6 + 3x^4 - 5x^2 - 45 - 7x^7 - 5x^5 + 23x^3 - 51x \\
 \Rightarrow x^8 - 7x^7 - x^6 - 5x^5 + 3x^4 + 23x^3 - 5x^2 - 51x - 45
 \end{array}$$

$$\begin{array}{r}
 25. \quad x^7 - 3x^5y^2 + 35x^4y^3 - 8x^2y^5 + 60 \\
 + 60x^4y^3 + x^2y^5 \quad - y^7 - 90x^3y^4 + 50xy^6 \\
 \hline
 x^7 - 3x^5y^2 + 95x^4y^3 - 7x^2y^5 + 60 - y^7 - 90x^3y^4 + 50xy^6 \\
 \Rightarrow x^7 - 3x^5y^2 + 95x^4y^3 - 90x^3y^4 - 7x^2y^5 + 50xy^6 - y^7 + 60
 \end{array}$$

$$\begin{array}{r}
 26. \quad a^{x+3} - 8a^{x+1} - 5 \\
 + 5a^{x+1} \quad - a^{x+2} + 6a^x \\
 \hline
 a^{x+3} - 3a^{x+1} - 5 - a^{x+2} + 6a^x \\
 \Rightarrow a^{x+3} - a^{x+2} - 3a^{x+1} + 6a^x - 5
 \end{array}$$

$$\begin{array}{r}
 27. \quad -8a^n + 16a^{n-4} + 15a^{n-2} + a^{n-3} \\
 - 7a^n \quad - 5a^{n-2} - a^{n-3} - 8a^{n-1} \\
 \hline
 -15a^n + 16a^{n-4} + 10a^{n-2} - 8a^{n-1} \\
 \Rightarrow -15a^n - 8a^{n-1} + 10a^{n-2} + 16a^{n-4}
 \end{array}$$

$$\begin{array}{r}
 28. \quad 15x^{a+3} + 5x^{a+2} - 6x^a + 41x^{a-1} \\
 + 9x^{a+2} \quad + 18x^{a-1} - 31x^{a+1} + x^{a+4} \\
 \hline
 15x^{a+3} + 14x^{a+2} - 6x^a + 59x^{a-1} - 31x^{a+1} + x^{a+4} \\
 \Rightarrow x^{a+4} + 15x^{a+3} + 14x^{a+2} - 31x^{a+1} - 6x^a + 59x^{a-1}
 \end{array}$$

$$\begin{array}{r}
 29. \quad 9a^{m-1} - 21a^{m-2} + 26^{m-3} + 14a^{m-5} \\
 5a^{m-1} - 12a^{m-2} \quad + a^m + 8a^{m-4} \\
 \hline
 14a^{m-1} - 33a^{m-2} + 26^{m-3} + 14a^{m-5} + a^m + 8a^{m-4} \\
 \Rightarrow a^m + 14a^{m-1} - 33a^{m-2} + 26^{m-3} + 8a^{m-4} + 14a^{m-5}
 \end{array}$$

$$\begin{array}{r}
 30. \quad -15m^{x+3} \quad + 50m^{x+1} - 14m^x - 6m^{x-1} + 8m^{x-2} \\
 m^{x+4} \quad + 23m^{x+2} + 6m^{x+1} \quad + m^{x-1} \\
 \hline
 m^{x+4} - 15m^{x+3} + 23m^{x+2} + 56m^{x+1} - 14m^x - 5m^{x-1} + 8m^{x-2}
 \end{array}$$

## EJERCICIO 23

$$\begin{array}{r} 1 \\ -a+1 \\ \hline -a+2 \end{array}$$

$$\begin{array}{r} 0 \\ -a+8 \\ \hline -a+8 \end{array}$$

$$\begin{array}{r} -9 \\ -3a-a^2+5 \\ \hline -3a-a^2-4 \Rightarrow -a^2-3a-4 \end{array}$$

$$\begin{array}{r} +16 \\ -5xy+x^2-16 \\ \hline -5xy+x^2 \Rightarrow x^2-5xy \end{array}$$

$$\begin{array}{r} 1 \\ -a^3+a^2b-ab^2 \\ \hline 1-a^3+a^2b-ab^2 \Rightarrow -a^3+a^2b-ab^2+1 \end{array}$$

$$\begin{array}{r} x^3 \\ x^3+8x^2y+6xy^2 \\ \hline 2x^3+8x^2y+6xy^2 \end{array}$$

$$\begin{array}{r} a^3 \\ +8a^2b-6ab^2+b^3 \\ \hline a^3+8a^2b-6ab^2+b^3 \end{array}$$

$$\begin{array}{r} +y^4 \\ 5x^3y-7x^2y^2+8xy^3 \\ \hline 5x^3y-7x^2y^2+8xy^3+y^4 \end{array}$$

$$\begin{array}{r} m^4 \\ -5m^4+18am^3-7a^2m^2-a^3m+a^4 \\ \hline -4m^4+18am^3-7a^2m^2-a^3m+a^4 \end{array}$$

$$\begin{array}{r} 16 \\ 14-b+a-c-d \\ \hline 30-b+a-c-d \Rightarrow a-b-c-d+30 \end{array}$$

$$\begin{array}{r} x^2-1 \\ -xy-y^2 \\ \hline x^2-1-xy-y^2 \Rightarrow x^2-xy-y^2-1 \end{array}$$

$$\begin{array}{r} a^3 \\ -5a^2b+8ab^2-b^3 \\ \hline a^3-5a^2b+8ab^2-b^3+6 \end{array}$$

$$\begin{array}{r} x^3 \\ +5x^2y-17xy^2 \\ \hline x^3+5x^2y-17xy^2+y^3+5 \end{array}$$

$$\begin{array}{r} x^4 \\ -9x^3y+8x^2y^2+15xy^3 \\ \hline x^4-9x^3y+8x^2y^2+15xy^3-1 \end{array}$$

$$\begin{array}{r} a^5 \\ +11a^4b-2a^2b^3-8a^3b^2+4ab^4 \\ \hline a^5+11a^4b-2a^2b^3-8a^3b^2+4ab^4+b^5 \\ \Rightarrow a^5+11a^4b-8a^3b^2-2a^2b^3+4ab^4+b^5 \end{array}$$

$$\begin{array}{r} x^4 \\ -5x^3 \\ \hline x^4-5x^3+x^2+25x+50 \end{array}$$

$$\begin{array}{r} y^6+y-41 \\ -9y^5-17y^4+y^3-18y^2 \\ \hline y^6+y-41-9y^5-17y^4+y^3-18y^2 \\ \Rightarrow y^6-9y^5-17y^4+y^3-18y^2+y-41 \end{array}$$

$$\begin{array}{r} a^6 \\ +15a^5b \\ \hline a^6+15a^5b+9a^4b^2-17a^3b^3+a^2b^4+14ab^5+b^6 \end{array}$$

$$\begin{array}{r} x^4+x^3-11x \\ -5x+x^2+34 \\ \hline x^4+x^3-16x+x^2+34 \Rightarrow x^4+x^3+x^2-16x+34 \end{array}$$

$$\begin{array}{r} m^3 \\ -m^2n-7mn^2+3n^3 \\ \hline m^3-m^2n-7mn^2+3n^3-1 \end{array}$$

## EJERCICIO 24

$$\begin{aligned}
 1. \quad & \frac{1}{2}a^2 \\
 & \frac{\frac{1}{4}a^2 + \frac{1}{3}ab - \frac{2}{5}b^2}{\frac{2+1}{4}a^2 + \frac{1}{3}ab - \frac{2}{5}b^2} \\
 & = \frac{\frac{1}{4}a^2 + \frac{1}{3}ab - \frac{2}{5}b^2}{\frac{3}{4}a^2 + \frac{1}{3}ab - \frac{2}{5}b^2}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \frac{-\frac{4}{5}xy - \frac{2}{3}yz + \frac{5}{9}}{-\frac{4}{5}xy - \frac{2}{3}yz + \frac{135+5}{9}} \\
 & = \frac{-\frac{4}{5}xy - \frac{2}{3}yz + \frac{140}{9}}{-\frac{4}{5}xy - \frac{2}{3}yz + 15\frac{5}{9}}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \frac{\frac{3}{5}bc}{-\frac{1}{6}bc + \frac{3}{4}ab + \frac{2}{9}cd} \\
 & = \frac{\frac{18-5}{30}bc + \frac{3}{4}ab + \frac{2}{9}cd}{\frac{3}{4}ab + \frac{13}{30}bc + \frac{2}{9}cd}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \frac{\frac{1}{2}a - \frac{2}{3}b}{-\frac{4}{5}a - \frac{2}{9}b + \frac{1}{2}} \\
 & = \frac{\frac{5-8}{10}a - \frac{6+2}{9}b + \frac{1}{2}}{-\frac{3}{10}a - \frac{8}{9}b + \frac{1}{2}}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \frac{\frac{5}{9}x^2 - \frac{3}{8}y^2}{-\frac{1}{10}y^2 - \frac{5}{7}xy + \frac{3}{11}} \\
 & = \frac{\frac{5}{9}x^2 - \frac{30+8}{80}y^2 - \frac{5}{7}xy + \frac{3}{11}}{\frac{5}{9}x^2 - \frac{38}{80}y^2 - \frac{5}{7}xy + \frac{3}{11}} \\
 & \Rightarrow \frac{\frac{5}{9}x^2 - \frac{5}{7}xy - \frac{19}{40}y^2 + \frac{3}{11}}{\frac{5}{9}x^2 - \frac{5}{7}xy - \frac{19}{40}y^2 + \frac{3}{11}}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \frac{\frac{5}{6}m^3 + \frac{2}{9}n^3}{\frac{1}{5}n^3 + \frac{1}{2}m^2n - \frac{3}{8}mn^2} \\
 & = \frac{\frac{5}{6}m^3 + \frac{10+9}{45}n^3 + \frac{1}{2}m^2n - \frac{3}{8}mn^2}{\frac{5}{6}m^3 + \frac{1}{2}m^2n - \frac{3}{8}mn^2 + \frac{19}{45}n^3}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & \frac{\frac{3}{7}a^2 + \frac{1}{3}ab - \frac{3}{5}b^2}{-\frac{5}{14}a^2 - \frac{1}{2}ab + \frac{1}{8}} \\
 & = \frac{\frac{6-5}{14}a^2 + \frac{2-3}{6}ab}{\frac{1}{14}a^2 - \frac{1}{6}ab - \frac{3}{5}b^2 + \frac{1}{8}}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & \frac{\frac{3}{8}x^2 + \frac{5}{6}xy - \frac{1}{10}y^2}{\frac{3}{5}x^2 + \frac{3}{10}xy - 2y^2} \\
 & = \frac{\frac{15+24}{40}x^2 + \frac{50+18}{60}xy - \frac{1+20}{10}y^2}{\frac{39}{40}x^2 + \frac{68}{60}xy - \frac{21}{10}y^2} = \frac{39}{40}x^2 + \frac{17}{15}xy - \frac{21}{10}y^2
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & \frac{a^3 + a^2 - a + \frac{5}{6}}{\frac{7}{8}a^2 - \frac{9}{10}a - \frac{7}{8}} \\
 & = \frac{a^3 + \frac{8+7}{8}a^2 - \frac{10+9}{10}a + \frac{20-21}{24}}{a^3 + \frac{15}{8}a^2 - \frac{19}{10}a - \frac{1}{24}}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & \frac{m^3 + \frac{7}{12}mn^2 - \frac{4}{7}n^3}{-\frac{5}{9}mn^2 - n^3 + \frac{5}{21}m^2n + \frac{1}{8}} \\
 & = \frac{m^3 + \frac{21-20}{36}mn^2 - \frac{4+7}{7}n^3 + \frac{5}{21}m^2n + \frac{1}{8}}{\Rightarrow m^3 + \frac{5}{21}m^2n + \frac{1}{36}mn^2 - \frac{11}{7}n^3 + \frac{1}{8}}
 \end{aligned}$$



$$\begin{array}{r}
 11. \quad \frac{3}{5}x^4 + \frac{3}{4}x^3y - \frac{5}{7}xy^3 + \frac{2}{3}y^4 \\
 -x^4 - \frac{5}{8}x^2y^2 + \frac{1}{3}xy^3 - \frac{5}{6}y^4 \\
 \hline
 \frac{3-5}{5}x^4 + \frac{3}{4}x^3y - \frac{5}{8}x^2y^2 - \frac{15-7}{21}xy^3 + \frac{4-5}{6}y^4 \\
 = -\frac{2}{5}x^4 + \frac{3}{4}x^3y - \frac{5}{8}x^2y^2 - \frac{8}{21}xy^3 - \frac{1}{6}y^4
 \end{array}$$

$$\begin{array}{r}
 12. \quad \frac{1}{2}a + \frac{3}{5}b - \frac{7}{8}c + \frac{8}{9}d \\
 + \frac{7}{20}b - \frac{1}{8}c + \frac{1}{9}d - \frac{7}{8} \\
 \hline
 \frac{1}{2}a + \frac{12+7}{20}b - \frac{7+1}{8}c + \frac{8+1}{9}d - \frac{7}{8} \\
 = \frac{1}{2}a + \frac{19}{20}b - \frac{8}{8}c + \frac{9}{9}d - \frac{7}{8} \\
 = \frac{1}{2}a + \frac{19}{20}b - c + d - \frac{7}{8}
 \end{array}$$

## EJERCICIO 25

$$\begin{array}{r}
 1. \quad \frac{3}{8}a^2 - \frac{5}{6}a \\
 -\frac{5}{6}a^2 \\
 \hline
 \frac{9-20}{24}a^2 - \frac{5}{6}a = -\frac{11}{24}a^2 - \frac{5}{6}a
 \end{array}$$

$$\begin{array}{r}
 2. \quad 8a + 6b - 5 \\
 -\frac{1}{2}a + \frac{3}{5}b \\
 \hline
 \frac{16-1}{2}a + \frac{30+3}{5}b - 5 = \frac{15}{2}a + \frac{33}{5}b - 5
 \end{array}$$

$$\begin{array}{r}
 3. \quad x^3 + \frac{2}{3}x^2y - 6 \\
 -\frac{7}{9}x^2y \\
 \hline
 x^3 + \frac{6-7}{9}x^2y - 6 = x^3 - \frac{1}{9}x^2y - 6
 \end{array}$$

$$\begin{array}{r}
 4. \quad a + b - c \\
 -\frac{1}{2}a + \frac{3}{4}b - \frac{2}{3}c \\
 \hline
 \frac{2-1}{2}a + \frac{4+3}{4}b - \frac{3+2}{3}c = \frac{1}{2}a + \frac{7}{4}b - \frac{5}{3}c
 \end{array}$$

$$\begin{array}{r}
 5. \quad \frac{2}{3}m + \frac{5}{6}n + \frac{1}{2}p \\
 -m - n + p \\
 \hline
 \frac{2-3}{3}m + \frac{5-6}{6}n + \frac{1+2}{2}p = -\frac{1}{3}m - \frac{1}{6}n + \frac{3}{2}p
 \end{array}$$

$$\begin{array}{r}
 6. \quad \frac{5}{8}a^2b + \frac{1}{4}ab^2 - \frac{1}{3} \\
 + \frac{7}{8}ab^2 - 6 - \frac{5}{6}a^3 \\
 \hline
 \frac{5}{8}a^2b + \frac{2+7}{8}ab^2 - \frac{1+18}{3} - \frac{5}{6}a^3 \\
 = \frac{5}{8}a^2b + \frac{9}{8}ab^2 - \frac{19}{3} - \frac{5}{6}a^3 \\
 \Rightarrow -\frac{5}{6}a^3 + \frac{5}{8}a^2b + \frac{9}{8}ab^2 - \frac{19}{3}
 \end{array}$$

$$\begin{array}{r}
 7. \quad \frac{2}{11}m^3n + \frac{5}{14}m^2n^2 + \frac{1}{3}mn^3 - 6 \\
 m^4 - \frac{7}{8}m^2n^2 + \frac{2}{9}mn^3 \\
 \hline
 m^4 + \frac{2}{11}m^3n + \frac{20-49}{56}m^2n^2 + \frac{3+2}{9}mn^3 - 6 \\
 = m^4 + \frac{2}{11}m^3n - \frac{29}{56}m^2n^2 + \frac{5}{9}mn^3 - 6
 \end{array}$$

$$\begin{array}{r}
 8. \quad -\frac{7}{8}x^4y + \frac{1}{14}x^3y^2 + \frac{2}{3}x^2y^3 + \frac{1}{3}xy^4 - 7 \\
 \frac{1}{2}x^5 - \frac{3}{7}x^3y^2 + \frac{1}{8}xy^4 - \frac{2}{9} \\
 \hline
 \frac{1}{2}x^5 - \frac{7}{8}x^4y + \frac{1-6}{14}x^3y^2 + \frac{2}{3}x^2y^3 + \frac{8+3}{24}xy^4 - \frac{63+2}{9} \\
 = \frac{1}{2}x^5 - \frac{7}{8}x^4y - \frac{5}{14}x^3y^2 + \frac{2}{3}x^2y^3 + \frac{11}{24}xy^4 - \frac{65}{9} \\
 = \frac{1}{2}x^5 - \frac{7}{8}x^4y - \frac{5}{14}x^3y^2 + \frac{2}{3}x^2y^3 + \frac{11}{24}xy^4 - 7\frac{2}{9}
 \end{array}$$

$$\begin{array}{r}
 9. \quad \frac{7}{9}x^5y + \frac{2}{3}x^4y^2 - \frac{1}{8}x^3y^3 - x^2y^4 + xy^5 + \frac{2}{13}y^6 \\
 -x^6 + \frac{7}{9}x^4y^2 - \frac{1}{11}x^2y^4 - xy^5 + y^6 \\
 \hline
 -x^6 + \frac{7}{9}x^5y + \frac{6+7}{9}x^4y^2 - \frac{1}{8}x^3y^3 - \frac{11+1}{11}x^2y^4 + \frac{2+13}{13}y^6 = -x^6 + \frac{7}{9}x^5y + \frac{13}{9}x^4y^2 - \frac{1}{8}x^3y^3 - \frac{12}{11}x^2y^4 + \frac{15}{13}y^6
 \end{array}$$

$$\begin{array}{r}
 10. \quad \frac{1}{3}x^3 - \frac{7}{9}x^2y + \frac{5}{8}xy^2 - \frac{7}{11}y^3 - \frac{2}{5} \\
 \frac{2}{3}x^3 + \frac{1}{6}x^2y - \frac{3}{4}xy^2 - 6 \\
 \hline
 \frac{1+2}{3}x^3 - \frac{14-3}{18}x^2y + \frac{5-6}{8}xy^2 - \frac{7}{11}y^3 - \frac{2+30}{5} = x^3 - \frac{11}{18}x^2y - \frac{1}{8}xy^2 - \frac{7}{11}y^3 - \frac{32}{5}
 \end{array}$$

$$\begin{array}{r}
 11. \quad \frac{3}{10}m^4n^2 - \frac{3}{7}m^2n^4 + \frac{5}{9}n^6 \\
 \frac{2}{13}m^6 + \frac{7}{20}m^4n^2 - \frac{5}{14}m^2n^4 - \frac{1}{3}n^6 + \frac{3}{5} \\
 \hline
 \frac{2}{13}m^6 + \frac{6+7}{20}m^4n^2 - \frac{6+5}{14}m^2n^4 + \frac{5-3}{9}n^6 + \frac{3}{5} = \frac{2}{13}m^6 + \frac{13}{20}m^4n^2 - \frac{11}{14}m^2n^4 + \frac{2}{9}n^6 + \frac{3}{5}
 \end{array}$$

$$\begin{array}{r}
 12. \quad \frac{3}{8}c^5 + \frac{7}{22}c^4d + \frac{7}{12}c^3d^2 + \frac{1}{2}c^2d^3 - \frac{1}{3}d^5 - 35 \\
 + \frac{5}{11}c^4d + \frac{5}{6}c^3d^2 - \frac{3}{4}cd^4 - \frac{3}{13}d^5 \\
 \hline
 \frac{3}{8}c^5 + \frac{7+10}{22}c^4d + \frac{7+10}{12}c^3d^2 + \frac{1}{2}c^2d^3 - \frac{3}{4}cd^4 - \frac{13+9}{39}d^5 - 35 \\
 = \frac{3}{8}c^5 + \frac{17}{22}c^4d + \frac{17}{12}c^3d^2 + \frac{1}{2}c^2d^3 - \frac{3}{4}cd^4 - \frac{22}{39}d^5 - 35
 \end{array}$$

## EJERCICIO 26

Para los problemas 1 al 14 las literales toman los siguientes valores:

$$a = 1 \quad b = 2 \quad c = 3 \quad x = 4 \quad y = 5 \quad m = 3/2 \quad n = 2/5$$

$$\begin{array}{r}
 1. \quad a^2 - ab \\
 - 3ab - b^2 \\
 \hline
 a^2 - 4ab - b^2 \\
 = 1^2 - 4 \cdot 1 \cdot 2 - 2^2 = 1 - 8 - 4 = -11
 \end{array}$$

$$\begin{array}{r}
 2. \quad a^3 + b^3 \\
 + 2b^3 + 5a^2b - 6ab^2 \\
 \hline
 a^3 + 3b^3 + 5a^2b - 6ab^2 \Rightarrow a^3 + 5a^2b - 6ab^2 + 3b^3 \\
 = 1^3 + 5 \cdot 1^2 \cdot 2 - 6 \cdot 1 \cdot 2^2 + 3 \cdot 2^3 = 1 + 5 \cdot 2 - 6 \cdot 4 + 3 \cdot 8 \\
 = 1 + 10 - 24 + 24 = 11
 \end{array}$$

$$3. \frac{1}{2}a$$

$$-a - \frac{1}{2}b + \frac{5}{3}c$$

$$\frac{1-2}{2}a - \frac{1}{2}b + \frac{5}{3}c = -\frac{1}{2}a - \frac{1}{2}b + \frac{5}{3}c$$

$$\Rightarrow -\frac{1}{2} \cdot 1 - \frac{1}{2} \cdot 2 + \frac{5}{3} \cdot 3 = -\frac{1}{2} - 1 + 5$$

$$= \frac{-1-2+10}{2} = \frac{7}{2} = 3\frac{1}{2}$$

$$4. 3m^2 - 5n^2$$

$$-m^2 - 10n^2 - 8mn$$

$$2m^2 - 15n^2 - 8mn \Rightarrow 2m^2 - 8mn - 15n^2$$

$$= 2\left(\frac{3}{2}\right)^2 - 8 \cdot \frac{3}{2} \cdot \frac{2}{5} - 15\left(\frac{2}{5}\right)^2 = 2 \cdot \frac{9}{4} - \frac{48}{10} - 3 \cdot \frac{4}{5}$$

$$= \frac{9}{2} - \frac{24}{5} - \frac{12}{5} = \frac{45-48-24}{10} = -\frac{27}{10}$$

$$5. x^4 - 18x^2y^2 + 15y^4$$

$$16x^3y + 6xy^3 - 9y^4$$

$$x^4 + 16x^3y - 18x^2y^2 + 6xy^3 + 6y^4$$

$$= 4^4 + 16 \cdot 4^3 \cdot 5 - 18 \cdot 4^2 \cdot 5^2 + 6 \cdot 4 \cdot 5^3 + 6 \cdot 5^4$$

$$= 256 + 80 \cdot 64 - 18 \cdot 16 \cdot 25 + 24 \cdot 125 + 6 \cdot 625$$

$$= 256 + 5120 - 7200 + 3000 + 3750 = 4.926$$

$$6. a^3 - 7am^2 + m^3$$

$$+ 5am^2 + 5m^3 - 8a^2m$$

$$a^3 - 2am^2 + 6m^3 - 8a^2m \Rightarrow a^3 - 8a^2m - 2am^2 + 6m^3$$

$$= 1^3 - 8 \cdot 1^2 \cdot \frac{3}{2} - 2 \cdot 1 \cdot \left(\frac{3}{2}\right)^2 + 6\left(\frac{3}{2}\right)^3 = 1 - \frac{24}{2} - 2 \cdot \frac{9}{4} + 6 \cdot \frac{27}{8}$$

$$= 1 - 12 - \frac{9}{2} + \frac{81}{4} = \frac{4-48-18+81}{4} = \frac{19}{4}$$

$$7. \frac{2}{3}a^2 + \frac{7}{8}ab - \frac{1}{5}b^2$$

$$-\frac{1}{6}a^2 - ab + \frac{1}{10}b^2$$

$$\frac{4-1}{6}a^2 + \frac{7-8}{8}ab - \frac{2-1}{10}b^2 = \frac{1}{2}a^2 - \frac{1}{8}ab - \frac{1}{10}b^2$$

$$\Rightarrow \frac{1}{2} \cdot 1^2 - \frac{1}{8} \cdot 1 \cdot 2 - \frac{1}{10} \cdot 2^2 = \frac{1}{2} - \frac{2}{8} - \frac{4}{10}$$

$$= \frac{1}{2} - \frac{1}{4} - \frac{2}{5} = \frac{10-5-8}{20} = -\frac{3}{20}$$

$$8. \frac{2}{3}m^2n + \frac{3}{4}mn^2 - \frac{1}{2}n^3$$

$$\frac{1}{6}m^2n + \frac{1}{4}mn^2 + \frac{1}{2}n^3 + m^3$$

$$\frac{4+1}{6}m^2n + \frac{3+1}{4}mn^2 + m^3 = m^3 + \frac{5}{6}m^2n + mn^2$$

$$= \left(\frac{3}{2}\right)^3 + \frac{5}{6}\left(\frac{3}{2}\right)^2 \cdot \frac{2}{5} + \frac{3}{2}\left(\frac{2}{5}\right)^2 = \frac{27}{8} + \frac{5}{6} \cdot \frac{18}{20} + \frac{3}{2} \cdot \frac{4}{25}$$

$$= \frac{27}{8} + \frac{3}{4} + \frac{12}{50} = \frac{675+150+48}{200} = \frac{873}{200}$$

$$9. a^5 - 3a^2b^4 + b^5$$

$$-a^4b^2 + 5a^3b^3$$

$$a^5 - a^4b^2 + 5a^3b^3 - 3a^2b^4 + b^5$$

$$= 1^5 - 1^4 \cdot 2^2 + 5 \cdot 1^3 \cdot 2^3 - 3 \cdot 1^2 \cdot 2^4 + 2^5$$

$$= 1 - 4 + 5 \cdot 8 - 3 \cdot 16 + 32 = 1 - 4 + 40 - 48 + 32 = 21$$

$$10. -ab + 10mn - 8mx$$

$$-15ab$$

$$-16ab + 10mn - 8mx$$

$$= -16 \cdot 1 \cdot 2 + 10 \cdot \frac{3}{2} \cdot \frac{2}{5} - 8 \cdot \frac{3}{2} \cdot 4$$

$$= -32 + 6 - 48 = -74$$

$$11. a^3$$

$$-11a^2b + 9ab^2 - b^3$$

$$a^3 - 11a^2b + 9ab^2 - b^3$$

$$= 1^3 - 11 \cdot 1^2 \cdot 2 + 9 \cdot 1 \cdot 2^2 - 2^3$$

$$= 1 - 22 + 36 - 8 = 7$$

$$12. \frac{1}{64}x^4$$

$$-\frac{2}{3}x^2 - \frac{5}{6}x + \frac{3}{8}$$

$$\frac{1}{64}x^4 - \frac{2}{3}x^2 - \frac{5}{6}x + \frac{3}{8}$$

$$= \frac{1}{64} \cdot 4^4 - \frac{2}{3} \cdot 4^2 - \frac{5}{6} \cdot 4 + \frac{3}{8}$$

$$= 4 - \frac{32}{3} - \frac{20}{6} + \frac{3}{8} = \frac{96-256-80+9}{24}$$

$$= -\frac{231}{24} = -\frac{77}{8} = -9\frac{5}{8}$$

$$\begin{aligned}
 13. \quad & x^3 + \frac{3}{16}x^2y - \frac{2}{5}xy^2 \\
 & - \frac{3}{4}x^3 + \frac{3}{5}xy^2 + \frac{1}{25}y^3 \\
 & \hline
 & \frac{4-3}{4}x^3 + \frac{3}{16}x^2y - \frac{2-3}{5}xy^2 + \frac{1}{25}y^3 \\
 & \Rightarrow \frac{1}{4}x^3 + \frac{3}{16}x^2y + \frac{1}{5}xy^2 + \frac{1}{25}y^3 \\
 & = \frac{1}{4} \cdot 4^3 + \frac{3}{16} \cdot 4^2 \cdot 5 + \frac{1}{5} \cdot 4 \cdot 5^2 + \frac{1}{25} \cdot 5^3 \\
 & = \frac{1}{4} \cdot 64 + \frac{15}{16} \cdot 16 + \frac{4}{5} \cdot 25 + \frac{125}{25} \\
 & = 16 + 15 + 20 + 5 = 56
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & \frac{2}{5}a^{x-1} + a^x - \frac{5}{6}a^{x-3} + a^{x-2} \\
 & - a^{x-1} + 9a^{x-3} - a^{x-2} \\
 & \hline
 & \frac{2-5}{5}a^{x-1} + a^x - \frac{5-54}{6}a^{x-3} \\
 & \Rightarrow a^x - \frac{3}{5}a^{x-1} + \frac{49}{6}a^{x-3} \\
 & = 1^4 - \frac{3}{5} \cdot 1^{4-1} + \frac{49}{6} \cdot 1^{4-3} \\
 & = 1 - \frac{3}{5} + \frac{49}{6} = \frac{+30-18+245}{30} = \frac{257}{30} = 8\frac{17}{30}
 \end{aligned}$$

## EJERCICIO 27

$$\begin{array}{r}
 1. \quad \begin{array}{r} +ab + b^2 \\ a^2 - 5b^2 \\ \hline a^2 + ab - 4b^2 \end{array} \quad \begin{array}{r} a^2 \\ -a^2 - ab + 4b^2 \\ \hline -ab + 4b^2 \end{array}
 \end{array}$$

$$\begin{array}{r}
 2. \quad \begin{array}{r} a+8 \\ -a+6 \\ \hline 14 \end{array} \quad \begin{array}{r} 1 \\ -14 \\ \hline -13 \end{array}
 \end{array}$$

$$\begin{array}{r}
 3. \quad \begin{array}{r} -x^3 + 4xy^2 \\ +5x^2y + y^3 \\ \hline -x^3 + 5x^2y + 4xy^2 + y^3 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{r} -7x^2y \\ x^3 - 5x^2y - 4xy^2 - y^3 \\ \hline x^3 - 12x^2y - 4xy^2 - y^3 \end{array}
 \end{array}$$

$$\begin{array}{r}
 4. \quad \begin{array}{r} -3m^3n + 4mn^2 - n^3 \\ 3m^3n - 4mn^2 + 5n^3 \\ \hline 4n^3 \end{array} \quad \begin{array}{r} 5m^4 \\ -4n^3 \\ \hline 5m^4 - 4n^3 \end{array}
 \end{array}$$

$$\begin{array}{r}
 5. \quad \begin{array}{r} 8a+9b-3c \\ -7a-9b+3c \\ \hline a \end{array} \quad \begin{array}{r} 6a \\ -a \\ \hline 5a \end{array}
 \end{array}$$

$$\begin{array}{r}
 6. \quad \begin{array}{r} a-b+c \\ -2a+b-c \\ \hline -a \end{array} \quad \begin{array}{r} a+b-c \\ a \\ \hline 2a+b-c \end{array}
 \end{array}$$

$$\begin{array}{r}
 7. \quad \begin{array}{r} -m+n-p \\ 2m-2n+2p \\ \hline m-n+p \end{array} \quad \begin{array}{r} m-n+p \\ -m+n-p \\ \hline 0 \end{array}
 \end{array}$$

$$\begin{array}{r}
 8. \quad \begin{array}{r} 9ax - a^2 \\ -9ax + 7a^2 + 25x^2 \\ \hline 6a^2 + 25x^2 \end{array} \quad \begin{array}{r} x^2 - 5ax + 3a^2 \\ -25x^2 - 6a^2 \\ \hline -24x^2 - 5ax - 3a^2 \end{array}
 \end{array}$$

$$\begin{array}{r}
 9. \quad \begin{array}{r} 5a^2 + 6a - 4 \\ 2a^3 - 8a + 6 \\ \hline 2a^3 + 5a^2 - 2a + 2 \end{array} \quad \begin{array}{r} a^3 - 1 \\ -2a^3 - 5a^2 + 2a - 2 \\ \hline -a^3 - 5a^2 + 2a - 3 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{r} 5x^3 - 9x^2 + 4 \\ -11x^4 - 7x^3 - 6x \\ \hline -11x^4 - 2x^3 - 9x^2 - 6x + 4 \end{array} \quad \begin{array}{r} x^4 - 1 \\ 11x^4 + 2x^3 + 9x^2 + 6x - 4 \\ \hline 12x^4 + 2x^3 + 9x^2 + 6x - 5 \end{array}
 \end{array}$$

$$\begin{array}{r}
 11. \quad \begin{array}{r} 35a^2b - 7ab^2 - 11 \\ -7a^3 - 35a^2b + 8ab^2 + 6 \\ \hline -7a^3 + ab^2 - 5 \end{array} \quad \begin{array}{r} a^3 + b^3 \\ 7a^3 - ab^2 + 5 \\ \hline 8a^3 - ab^2 + b^3 + 5 \end{array}
 \end{array}$$

$$\begin{array}{r}
 12. \quad \begin{array}{r} -11n^4 + 14n^2 - 25n + 8 \\ 19n^3 - 6n^2 + 9n - 4 \\ \hline -11n^4 + 19n^3 + 8n^2 - 16n + 4 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{r} n^5 - 7n^3 + 4n \\ 11n^4 - 19n^3 - 8n^2 + 16n - 4 \\ \hline n^5 + 11n^4 - 26n^3 - 8n^2 + 20n - 4 \end{array}
 \end{array}$$

$$13. \begin{array}{r} -6a^3m \quad +5am^3 \quad -6 \\ 7a^4 - 5a^3m - 11a^2m^2 \quad -6m^4 \\ \hline 7a^4 - 11a^3m - 11a^2m^2 + 5am^3 - 6m^4 - 6 \end{array}$$

$$\begin{array}{r} a^4 \quad -8a^2m^2 \quad +m^4 \\ -7a^4 + 11a^3m + 11a^2m^2 - 5am^3 + 6m^4 + 6 \\ \hline -6a^4 + 11a^3m + 3a^2m^2 - 5am^3 + 7m^4 + 6 \end{array}$$

$$14. \begin{array}{r} -4x^4y \quad +13x^2y^3 - 9xy^4 \\ -6x^5 \quad +8x^3y^2 \quad +xy^4 - 2y^5 \\ \hline -6x^5 - 4x^4y + 8x^3y^2 + 13x^2y^3 - 8xy^4 - 2y^5 \end{array}$$

$$\begin{array}{r} x^5 \quad -30x^3y^2 \quad +40xy^4 + y^5 \\ 6x^5 + 4x^4y - 8x^3y^2 - 13x^2y^3 + 8xy^4 + 2y^5 \\ \hline 7x^5 + 4x^4y - 38x^3y^2 - 13x^2y^3 + 48xy^4 + 3y^5 \end{array}$$

$$15. \begin{array}{r} a+b \quad 2a \\ a-b \quad -2a+b \\ \hline 2a \quad b \end{array}$$

$$16. \begin{array}{r} 8x \quad +9 \quad 8x+6y+4 \\ 6y-5 \quad 2 \\ \hline 8x+6y+4 \quad 8x+6y+6 \end{array}$$

$$17. \begin{array}{r} x^2 \quad -6y^2 \quad x^2 - 7xy + 34y^2 \\ -7xy + 40y^2 \quad 9y^2 - 16 \\ \hline x^2 - 7xy + 34y^2 \quad x^2 - 7xy + 43y^2 - 16 \end{array}$$

$$18. \begin{array}{r} 4a^2 + 8ab - 5b^2 \quad 5a^2 + ab + b^2 \\ a^2 - 7ab + 6b^2 \quad -4a^2 - ab + b^2 \\ \hline 5a^2 + ab + b^2 \quad a^2 + 2b^2 \end{array}$$

$$19. \begin{array}{r} x^3 \quad -y^3 \\ -14x^2y + 5xy^2 \\ \hline x^3 - 14x^2y + 5xy^2 - y^3 \end{array}$$

$$\begin{array}{r} x^3 - 14x^2y + 5xy^2 - y^3 \\ 3x^3 \quad -19y^3 \\ \hline 4x^3 - 14x^2y + 5xy^2 - 20y^3 \end{array}$$

$$20. \begin{array}{r} x^4 - 6x^2y^2 + y^4 \quad x^4 - 2x^2y^2 + 32y^4 \\ + 8x^2y^2 + 31y^4 \quad -x^4 - 2x^2y^2 - 32y^4 \\ \hline x^4 + 2x^2y^2 + 32y^4 \quad 0 \end{array}$$

$$21. \begin{array}{r} -6n^5 + n^4 \quad +n^2 \\ +7n^3 - n^2 - 8n - 6 \\ \hline -6n^5 + n^4 + 7n^3 \quad -8n - 6 \end{array}$$

$$\begin{array}{r} -6n^5 + n^4 + 7n^3 - 8n - 6 \\ n^6 \quad +3n^4 + 8n^3 \quad -19 \\ \hline n^6 - 6n^5 + 4n^4 + 15n^3 - 8n - 25 \end{array}$$

$$22. \begin{array}{r} a^5 \quad -3a^3b^2 + 6ab^4 \\ +22a^4b + 10a^3b^2 - 11ab^4 - b^5 \\ \hline a^5 + 22a^4b + 7a^3b^2 - 5ab^4 - b^5 \end{array}$$

$$\begin{array}{r} a^5 + 22a^4b + 7a^3b^2 \quad -5ab^4 - b^5 \\ -5a^4b \quad +7a^2b^3 \quad -b^5 \\ \hline a^5 + 17a^4b + 7a^3b^2 + 7a^2b^3 - 5ab^4 - 2b^5 \end{array}$$

$$23. \begin{array}{r} 4m^3 - 5m^2 - 2m \quad -3m^3 - 5m^2 + 6m + 4 \\ -7m^3 \quad +8m + 4 \quad m^4 \quad -5 \\ \hline -3m^3 - 5m^2 + 6m + 4 \quad m^4 - 3m^3 - 5m^2 + 6m - 1 \end{array}$$

$$24. \begin{array}{r} 7a^2 - 11ab + b^2 \quad 2b^2 - 8 \\ -7a^2 + 11ab + b^2 - 8 \quad +4 \\ \hline 2b^2 - 8 \quad 2b^2 - 4 \end{array}$$

$$25. \begin{array}{r} 3a - 4b + 5c \\ -7a + 8b \quad -11 \quad -5a + 6b - 2c - 11 \\ -a + 2b - 7c \quad -a + b + 2c \\ \hline -5a + 6b - 2c - 11 \quad -6a + 7b \quad -11 \end{array}$$

$$26. \begin{array}{r} 5a^3 + 14a^2 - 19a + 8 \\ a^5 \quad +9a - 1 \\ -a^4 \quad +3a^2 \quad -1 \\ \hline a^5 - a^4 + 5a^3 + 17a^2 - 10a + 6 \\ -a^4 + 3a^3 \quad -5 \\ \hline a^5 - 2a^4 + 8a^3 + 17a^2 - 10a + 1 \end{array}$$

$$27. \begin{array}{r} m^4 \quad \quad + 10m^2n^2 \quad \quad + 15n^4 \\ -11m^3n - 14m^2n^2 - 3mn^3 + n^4 \\ \hline m^4 - 11m^3n - 4m^2n^2 - 3mn^3 + 16n^4 \end{array}$$

$$\begin{array}{r} 6m^4 \quad \quad + 7m^2n^2 + 8mn^3 - n^4 \\ -m^4 + 11m^3n + 4m^2n^2 + 3mn^3 - 16n^4 \\ \hline 5m^4 + 11m^3n + 11m^2n^2 + 11mn^3 - 17n^4 \end{array}$$

$$28. \begin{array}{r} a^5 \quad \quad + 4a^3b^2 \quad \quad + 8ab^4 - b^5 \\ -7a^4b \quad \quad + 15a^2b^3 - 25ab^4 + 3b^5 \\ -a^3b^2 + 3a^2b^3 - 5ab^4 \\ \hline a^5 - 7a^4b + 3a^3b^2 + 18a^2b^3 - 22ab^4 + 2b^5 \end{array}$$

$$\begin{array}{r} 3a^5 \quad \quad - 6a^2b^3 - 21ab^4 \quad \quad - 6 \\ -a^5 + 7a^4b - 3a^3b^2 - 18a^2b^3 + 22ab^4 - 2b^5 \\ \hline 2a^5 + 7a^4b - 3a^3b^2 - 24a^2b^3 + ab^4 - 2b^5 - 6 \end{array}$$

$$29. \begin{array}{r} x^5 \quad \quad \quad \quad \quad + y^5 \\ 3x^4y + 21x^3y^2 + 18x^2y^3 - y^5 \\ \hline x^5 + 3x^4y + 21x^3y^2 + 18x^2y^3 \end{array}$$

$$\begin{array}{r} x^5 + 32x^4y - 26x^3y^2 + 18x^2y^3 - 2xy^4 + y^5 \\ -x^5 - 3x^4y - 21x^3y^2 - 18x^2y^3 \\ \hline 29x^4y - 47x^3y^2 - 2xy^4 + y^5 \end{array}$$

$$30. \begin{array}{r} 3a^x + 6a^{x-1} \\ a^x - 7a^{x-1} + a^{x-2} \\ \hline 4a^x - a^{x-1} + a^{x-2} \end{array}$$

$$\begin{array}{r} 8a^{x+2} - 7a^{x+1} - a^x + 12a^{x-1} \\ -4a^x + a^{x-1} - a^{x-2} \\ \hline 8a^{x+2} - 7a^{x+1} - 5a^x + 13a^{x-1} - a^{x-2} \end{array}$$

## EJERCICIO 28

$$1. \begin{array}{r} x^2 \quad + 5 \quad \quad x-4 \quad \quad x^2+2x-1 \\ 2x-6 \quad \quad -x+6 \quad \quad -2 \\ \hline x^2+2x-1 \quad \quad 2 \quad \quad x^2+2x-3 \end{array}$$

$$2. \begin{array}{r} 3a-5b+c \quad \quad 7a+b \quad \quad 4a-6b-2c \\ a-b-3c \quad \quad -8b-3c \quad \quad -7a+7b+3c \\ \hline 4a-6b-2c \quad \quad 7a-7b-3c \quad \quad -3a+b+c \end{array}$$

$$3. \begin{array}{r} x^3 \quad + 1 \quad \quad 9x+4 \quad \quad 6x^3-x^2 \quad \quad + 8 \\ 5x^3-x^2+7 \quad \quad -3x^2-x+1 \quad \quad 3x^2-8x-5 \\ \hline 6x^3-x^2+8 \quad \quad -3x^2+8x+5 \quad \quad 6x^3+2x^2-8x+3 \end{array}$$

$$4. \begin{array}{r} a^2+1 \quad \quad a^4 \quad + 2 \quad \quad a^3+a^2 \\ a^3 \quad - 1 \quad \quad a-2 \quad \quad -a^4 \quad \quad -a \\ \hline a^3+a^2 \quad \quad a^4+a \quad \quad -a^4+a^3+a^2-a \end{array}$$

$$5. \begin{array}{r} ab+bc+ac \quad \quad 5ab-3bc+4ac \quad \quad ab-6bc+9ac-9 \\ -7bc+8ac-9 \quad \quad -ab+3bc+5ac \quad \quad -4ab \quad \quad -9ac \\ \hline ab-6bc+9ac-9 \quad \quad 4ab \quad \quad +9ac \quad \quad -3ab-6bc \quad \quad -9 \end{array}$$

$$6. \begin{array}{r} a^2x \quad \quad -3x^3 \quad \quad -5a^2x+11ax^2-11x^3 \\ a^3 \quad \quad +3ax^2 \quad \quad a^3-4a^2x+6ax^2+8x^3 \\ \hline a^3+a^2x+3ax^2-3x^3 \quad \quad a^3-9a^2x+17ax^2-3x^3 \end{array}$$

$$\begin{array}{r} a^3 + a^2x + 3ax^2 - 3x^3 \\ -a^3 + 9a^2x - 17ax^2 + 3x^3 \\ \hline 10a^2x - 14ax^2 \end{array}$$

$$7. \begin{array}{r} x^4 \quad \quad + x^2 \quad \quad - 3 \\ -x^3 \quad \quad -3x+5 \quad \quad -7x^3+8x^2-3x+4 \\ x^4 \quad \quad -5x^2+4x \quad \quad x^4 \quad \quad - 3 \\ \hline 2x^4-x^3-4x^2+x+2 \quad \quad x^4-7x^3+8x^2-3x+1 \end{array}$$

$$\begin{array}{r} 2x^4 - x^3 - 4x^2 + x + 2 \\ -x^4 + 7x^3 - 8x^2 + 3x - 1 \\ \hline x^4 + 6x^3 - 12x^2 + 4x + 1 \end{array}$$

$$8. \begin{array}{r} m^4 \qquad \qquad \qquad - n^4 \\ 17m^3n - 4m^2n^2 - 7mn^3 \qquad \qquad - m^4 \qquad \qquad + 6 \qquad \qquad 17m^3n + 2m^2n^2 - 7mn^3 - 81n^4 \\ - m^4 \qquad \qquad + 6m^2n^2 \qquad \qquad - 80n^4 \qquad \qquad - m^2n^2 + mn^3 - 4 \qquad \qquad m^4 \qquad \qquad + m^2n^2 \qquad \qquad - mn^3 \qquad \qquad - 2 \\ \hline 17m^3n + 2m^2n^2 - 7mn^3 - 81n^4 \qquad \qquad - m^4 - m^2n^2 + mn^3 + 2 \qquad \qquad m^4 + 17m^3n + 3m^2n^2 - 8mn^3 - 81n^4 - 2 \end{array}$$

$$9. \begin{array}{r} + a^3 \qquad \qquad + a \qquad - 7 \\ a^5 - a^4 \qquad - 6a^2 \qquad \qquad + 8 \qquad \qquad - a^4 - 4a^3 + a^2 \qquad \qquad a^5 - a^4 + a^3 - 11a^2 - 10a + 27 \\ \qquad \qquad - 5a^2 - 11a + 26 \qquad \qquad 16a^3 - 8a^2 - 7a - 15 \qquad \qquad + a^4 - 12a^3 + 7a^2 + 7a + 15 \\ \hline a^5 - a^4 + a^3 - 11a^2 - 10a + 27 \qquad \qquad - a^4 + 12a^3 - 7a^2 - 7a - 15 \qquad \qquad a^5 \qquad \qquad - 11a^3 - 4a^2 - 3a + 42 \end{array}$$

$$10. \begin{array}{r} 3x^2 \qquad \qquad - y^2 \qquad \qquad x^2 - 3xy - y^2 \qquad \qquad 20x^2 - 11xy + 8y^2 \\ - 11xy + 9y^2 - 14 \qquad \qquad 19x^2 - 8xy + 9y^2 \qquad \qquad - 3x^2 + 11xy - 8y^2 + 14 \\ \hline 3x^2 - 11xy + 8y^2 - 14 \qquad \qquad 20x^2 - 11xy + 8y^2 \qquad \qquad 17x^2 \qquad \qquad + 14 \end{array}$$

$$11. \begin{array}{r} a-1 \qquad \qquad a^2 \qquad - 3 \\ - a + 1 \qquad \qquad a - 4 \\ \hline 0 \qquad \qquad - 3a + 8 \\ \hline a^2 - 2a + 1 \end{array} \quad 12. \begin{array}{r} a^2 - ab + b^2 \\ 3a^2 - 8ab + 7b^2 \qquad \qquad - a^2 + 9ab + 3b^2 \qquad \qquad - a^2 + ab - 4b^2 \\ - 5a^2 + 11ab - 17b^2 \qquad \qquad - 8ab - 7b^2 \qquad \qquad a^2 - 2ab + 9b^2 \\ \hline - a^2 + 2ab - 9b^2 \qquad \qquad - a^2 + ab - 4b^2 \qquad \qquad - ab + 5b^2 \end{array}$$

$$13. \begin{array}{r} m^4 \qquad \qquad \qquad - 1 \\ - m^3 + 8m^2 - 6m + 5 \qquad \qquad m^5 \qquad \qquad - 16 \qquad \qquad m^5 - 16m^4 \qquad \qquad + 7m^2 \qquad \qquad - 19 \\ \qquad \qquad - m^2 - 7m + 1 \qquad \qquad - 16m^4 + 7m^2 - 3 \qquad \qquad - m^4 + m^3 - 7m^2 + 13m - 5 \\ \hline m^4 - m^3 + 7m^2 - 13m + 5 \qquad \qquad m^5 - 16m^4 + 7m^2 - 19 \qquad \qquad m^5 - 17m^4 + m^3 \qquad \qquad + 13m - 24 \end{array}$$

$$14. \begin{array}{r} x^5 \qquad \qquad \qquad - y^5 \\ - 2x^4y + 5x^3y^2 - 7x^2y^3 \qquad \qquad - 3y^5 \qquad \qquad 7x^4y - x^3y^2 + 11xy^4 \qquad \qquad - x^5 + 2x^4y + 2x^3y^2 + 7x^2y^3 - 6xy^4 + 4y^5 + 8 \\ \qquad \qquad - 7x^3y^2 \qquad \qquad + 6xy^4 \qquad \qquad - 8 \qquad \qquad - xy^4 - 1 \qquad \qquad 7x^4y - x^3y^2 \qquad \qquad + 10xy^4 \qquad \qquad - 1 \\ \hline x^5 - 2x^4y - 2x^3y^2 - 7x^2y^3 + 6xy^4 - 4y^5 - 8 \qquad \qquad 7x^4y - x^3y^2 + 10xy^4 - 1 \qquad \qquad - x^5 + 9x^4y + x^3y^2 + 7x^2y^3 + 4xy^4 + 4y^5 + 7 \end{array}$$

$$15. \begin{array}{r} - a^6 \qquad \qquad + 7a^4 \qquad \qquad - 8a \\ - 3a^5 \qquad \qquad + 11a^3 - a^2 \qquad \qquad + 4 \\ \qquad \qquad - 6a^4 - 11a^3 \qquad \qquad - 2a + 8 \qquad \qquad - 3a^4 \qquad \qquad + 7a^2 - 8a + 5 \qquad \qquad 5a^5 - 3a^4 - 7a^3 + 48a^2 - 58a + 13 \\ \qquad \qquad - 5a^3 + 5a^2 - 4a + 1 \qquad \qquad 5a^5 \qquad \qquad - 7a^3 + 41a^2 - 50a + 8 \qquad \qquad a^6 + 3a^5 - a^4 + 5a^3 - 4a^2 + 14a - 13 \\ \hline - a^6 - 3a^5 + a^4 - 5a^3 + 4a^2 - 14a + 13 \qquad \qquad 5a^5 - 3a^4 - 7a^3 + 48a^2 - 58a + 13 \qquad \qquad a^6 + 8a^5 - 4a^4 - 2a^3 + 44a^2 - 44a \end{array}$$

$$16. \begin{array}{r} a^5 \qquad \qquad - 7a^3x^2 \qquad \qquad + 9 \qquad \qquad + 18a^3x^2 \qquad \qquad - 4x^5 - 8 \\ - 20a^4x \qquad \qquad + 21a^2x^3 - 19ax^4 \qquad \qquad - 9a^4x - 17a^3x^2 + 11a^2x^3 \\ \qquad \qquad + 9a^3x^2 \qquad \qquad - 7ax^4 + x^5 - 80 \qquad \qquad a^5 \qquad \qquad \qquad \qquad + 36 \\ \hline a^5 - 20a^4x + 2a^3x^2 + 21a^2x^3 - 26ax^4 + x^5 - 71 \qquad \qquad a^5 - 9a^4x + a^3x^2 + 11a^2x^3 - 4x^5 + 28 \end{array}$$

$$\begin{array}{r} a^5 - 9a^4x + a^3x^2 + 11a^2x^3 \qquad \qquad - 4x^5 + 28 \\ - a^5 + 20a^4x - 2a^3x^2 - 21a^2x^3 + 26ax^4 - x^5 + 71 \\ \hline 11a^4x - a^3x^2 - 10a^2x^3 + 26ax^4 - 5x^5 + 99 \end{array}$$

## EJERCICIO 29

$$\begin{array}{r}
 1. \quad a + \frac{1}{2}b \quad \frac{3}{4}a \\
 -\frac{2}{3}a + \frac{3}{4}b \quad -\frac{1}{3}a - \frac{5}{4}b \\
 \hline
 \frac{3-2}{3}a + \frac{2+3}{4}b \quad \frac{9-4}{12}a - \frac{5}{4}b \\
 = \frac{1}{3}a + \frac{5}{4}b \quad = \frac{5}{12}a - \frac{5}{4}b
 \end{array}$$

$$\begin{array}{r}
 2. \quad \frac{3}{8}a-6 \quad \frac{1}{2}a^3 + \frac{3}{5}a^2 \\
 -\frac{5}{6}a^3 + \frac{3}{5}a^2 \quad \frac{5}{6}a^3 - \frac{3}{5}a^2 - \frac{3}{8}a+6 \\
 \hline
 -\frac{5}{6}a^3 + \frac{3}{5}a^2 + \frac{3}{8}a-6 \quad \frac{3+5}{6}a^3 - \frac{3}{8}a+6 \\
 = \frac{4}{3}a^3 - \frac{3}{8}a+6
 \end{array}$$

$$\begin{array}{r}
 3. \quad a + 3b \quad \frac{3}{5}a + \frac{7}{3}b + 6 \\
 -\frac{2}{5}a - \frac{2}{3}b + 6 \quad -\frac{1}{5}a + \frac{1}{6}b \\
 \hline
 \frac{5-2}{5}a + \frac{9-2}{3}b+6 \quad \frac{3-1}{5}a + \frac{14+1}{6}b+6 \\
 = \frac{3}{5}a + \frac{7}{3}b+6 \quad = \frac{2}{5}a + \frac{5}{2}b+6
 \end{array}$$

$$\begin{array}{r}
 4. \quad \frac{1}{3}x^3 - \frac{3}{7}x^2 + \frac{1}{5} \\
 \quad \frac{1}{14}x^2 - \frac{2}{9}x+6 \\
 \hline
 \frac{1}{3}x^3 - \frac{6-1}{14}x^2 - \frac{2}{9}x + \frac{1+30}{5} \\
 = \frac{1}{3}x^3 - \frac{5}{14}x^2 - \frac{2}{9}x + \frac{31}{5} \\
 -\frac{5}{6}x^3 \\
 -\frac{1}{3}x^3 + \frac{5}{14}x^2 + \frac{2}{9}x - \frac{31}{5} \\
 \hline
 -\frac{5+2}{6}x^3 + \frac{5}{14}x^2 + \frac{2}{9}x - \frac{31}{5} \\
 = -\frac{7}{6}x^3 + \frac{5}{14}x^2 + \frac{2}{9}x - \frac{61}{5}
 \end{array}$$

$$\begin{array}{r}
 5. \quad \frac{7}{12}a^4 \quad \frac{7}{12}a^4 - \frac{3}{7}a^3 + \frac{2}{5}a^2 - 6 \\
 -\frac{3}{7}a^3 + \frac{2}{5}a^2 - 6 \quad \frac{3}{4}a^4 - \frac{1}{5}a + \frac{1}{3} \\
 \hline
 \frac{7}{12}a^4 - \frac{3}{7}a^3 + \frac{2}{5}a^2 - 6 \quad \frac{7+9}{12}a^4 - \frac{3}{7}a^3 + \frac{2}{5}a^2 - \frac{1}{5}a - \frac{18-1}{3} \\
 = \frac{4}{3}a^4 - \frac{3}{7}a^3 + \frac{2}{5}a^2 - \frac{1}{5}a - \frac{17}{3}
 \end{array}$$

$$\begin{array}{r}
 6. \quad -\frac{1}{2}x + \frac{2}{3}y - \frac{1}{4}z \quad \frac{5}{9}y - \frac{2}{5} \\
 -\frac{1}{9}y - \frac{2}{5}z + 3 \quad \frac{1}{2}x - \frac{5}{9}y + \frac{13}{20}z - 3 \\
 \hline
 -\frac{1}{2}x + \frac{6-1}{9}y - \frac{5+8}{20}z + 3 \quad \frac{1}{2}x + \frac{13}{20}z - \frac{2+15}{5} \\
 = -\frac{1}{2}x + \frac{5}{9}y - \frac{13}{20}z + 3 \quad = \frac{1}{2}x + \frac{13}{20}z - \frac{17}{5}
 \end{array}$$

$$\begin{array}{r}
 7. \quad -\frac{3}{2}a^2b + \frac{3}{4}ab^2 - b^3 \\
 -\frac{1}{8}a^2b - \frac{5}{6}ab^2 + \frac{2}{3}b^3 \\
 \hline
 -\frac{12-1}{8}a^2b + \frac{9-10}{12}ab^2 - \frac{3-2}{3}b^3 \\
 = -\frac{11}{8}a^2b - \frac{1}{12}ab^2 - \frac{1}{3}b^3
 \end{array}$$

$$\begin{array}{r}
 \frac{1}{2}a^3 \quad -\frac{1}{3}b^3 \\
 \frac{11}{8}a^2b + \frac{1}{12}ab^2 + \frac{1}{3}b^3 \\
 \hline
 \frac{1}{2}a^3 + \frac{11}{8}a^2b + \frac{1}{12}ab^2
 \end{array}$$

$$\begin{array}{r}
 8. \quad \frac{1}{2}a - \frac{2}{9}b \quad \frac{2}{3}b + \frac{1}{5}c \quad \frac{1}{2}a + \frac{1}{9}b - \frac{3}{5}c \\
 \frac{1}{3}b - \frac{3}{5}c \quad -\frac{5}{9}b - \frac{1}{10}c \quad -\frac{1}{9}b - \frac{1}{10}c \\
 \hline
 \frac{1}{2}a - \frac{2-3}{9}b - \frac{3}{5}c \quad \frac{6-5}{9}b + \frac{2-1}{10}c \quad \frac{1}{2}a - \frac{6+1}{10}c \\
 = \frac{1}{2}a + \frac{1}{9}b - \frac{3}{5}c \quad = \frac{1}{9}b + \frac{1}{10}c \quad = \frac{1}{2}a - \frac{7}{10}c
 \end{array}$$



$$\begin{array}{l}
 9. \quad \frac{1}{3}a^3 + \frac{1}{8}a^2 + \frac{1}{5} \\
 \quad \quad \quad -\frac{3}{5}a^2 - \frac{3}{4}a - \frac{1}{10} \\
 \hline
 \frac{1}{3}a^3 + \frac{5-24}{40}a^2 - \frac{3}{4}a + \frac{2-1}{10} \\
 = \frac{1}{3}a^3 - \frac{19}{40}a^2 - \frac{3}{4}a + \frac{1}{10} \\
 \end{array}
 \quad
 \begin{array}{l}
 \frac{1}{4}a^2 - \frac{2}{3}a + \frac{1}{4} \\
 \frac{1}{3}a^3 - \frac{29}{40}a^2 - \frac{1}{8} \\
 \hline
 \frac{1}{3}a^3 + \frac{10-29}{40}a^2 - \frac{2}{3}a + \frac{2-1}{8} \\
 = \frac{1}{3}a^3 - \frac{19}{40}a^2 - \frac{2}{3}a + \frac{1}{8} \\
 \end{array}
 \quad
 \begin{array}{l}
 \frac{1}{3}a^3 - \frac{19}{40}a^2 - \frac{2}{3}a + \frac{1}{8} \\
 -\frac{1}{3}a^3 + \frac{19}{40}a^2 + \frac{3}{4}a - \frac{1}{10} \\
 \hline
 -\frac{8-9}{12}a + \frac{10-8}{80} \\
 = \frac{1}{12}a + \frac{2}{80} = \frac{1}{12}a + \frac{1}{40} \\
 \end{array}$$

$$\begin{array}{l}
 10. \quad \frac{3}{5}x^2 - \frac{5}{6}xy + \frac{2}{9}y^2 \\
 \quad \quad \quad -\frac{3}{2}xy - \frac{1}{3}y^2 + \frac{1}{4} \\
 \hline
 \frac{3}{5}x^2 - \frac{10+18}{12}xy + \frac{2-3}{9}y^2 + \frac{1}{4} \\
 = \frac{3}{5}x^2 - \frac{28}{12}xy - \frac{1}{9}y^2 + \frac{1}{4} \\
 = \frac{3}{5}x^2 - \frac{7}{3}xy - \frac{1}{9}y^2 + \frac{1}{4} \\
 \end{array}
 \quad
 \begin{array}{l}
 \frac{2}{9}x^2 + \frac{1}{9}xy - \frac{2}{3}y^2 \\
 \frac{17}{45}x^2 - \frac{22}{9}xy - \frac{3}{2}y^2 - \frac{1}{2} \\
 \hline
 \frac{10+17}{45}x^2 + \frac{1-22}{9}xy - \frac{4+9}{6}y^2 - \frac{1}{2} \\
 = \frac{27}{45}x^2 - \frac{21}{9}xy - \frac{13}{6}y^2 - \frac{1}{2} \\
 = \frac{3}{5}x^2 - \frac{7}{3}xy - \frac{13}{6}y^2 - \frac{1}{2} \\
 \end{array}
 \quad
 \begin{array}{l}
 -\frac{3}{5}x^2 + \frac{7}{3}xy + \frac{13}{6}y^2 + \frac{1}{2} \\
 \frac{3}{5}x^2 - \frac{7}{3}xy - \frac{1}{9}y^2 + \frac{1}{4} \\
 \hline
 \frac{39-2}{18}y^2 + \frac{2+1}{4} \\
 = \frac{37}{18}y^2 + \frac{3}{4} \\
 \end{array}$$

$$\begin{array}{l}
 11. \quad \frac{2}{7}a^3 - \frac{1}{5}b^3 \\
 \quad \quad \quad -\frac{3}{4}a^2b + \frac{3}{8}ab^2 + \frac{1}{10}b^3 \\
 \hline
 \frac{2}{7}a^3 - \frac{3}{4}a^2b + \frac{3}{8}ab^2 - \frac{2-1}{10}b^3 \\
 = \frac{2}{7}a^3 - \frac{3}{4}a^2b + \frac{3}{8}ab^2 - \frac{1}{10}b^3 \\
 \end{array}
 \quad
 \begin{array}{l}
 \frac{1}{2}a^2b + \frac{1}{4}ab^2 - \frac{1}{5} \\
 -\frac{5}{4}a^2b + \frac{1}{8}ab^2 - \frac{3}{2}b^3 - \frac{1}{2} \\
 \hline
 \frac{2-5}{4}a^2b + \frac{2+1}{8}ab^2 - \frac{3}{2}b^3 - \frac{2+5}{10} \\
 = -\frac{3}{4}a^2b + \frac{3}{8}ab^2 - \frac{3}{2}b^3 - \frac{7}{10} \\
 \end{array}
 \quad
 \begin{array}{l}
 -\frac{3}{4}a^2b + \frac{3}{8}ab^2 - \frac{3}{2}b^3 - \frac{7}{10} \\
 -\frac{2}{7}a^3 + \frac{3}{4}a^2b - \frac{3}{8}ab^2 + \frac{1}{10}b^3 \\
 \hline
 -\frac{2}{7}a^3 - \frac{15-1}{10}b^3 - \frac{7}{10} \\
 = -\frac{2}{7}a^3 - \frac{14}{10}b^3 - \frac{7}{10} = -\frac{2}{7}a^3 - \frac{7}{5}b^3 - \frac{7}{10} \\
 \end{array}$$

$$\begin{array}{l}
 12. \quad \frac{1}{3}m^2n^2 - \frac{1}{4}mn^3 - n^4 \\
 \frac{2}{7}m^4 + \frac{3}{5}m^3n - \frac{2}{5}m^2n^2 + \frac{5}{3}n^4 \\
 \frac{1}{14}m^4 - \frac{7}{20}m^3n + \frac{1}{4}m^2n^2 - \frac{2}{3}n^4 \\
 \hline
 \frac{4+1}{14}m^4 + \frac{12-7}{20}m^3n + \frac{20-24+15}{60}m^2n^2 - \frac{1}{4}mn^3 - \frac{3-5+2}{3}n^4 \\
 = \frac{5}{14}m^4 + \frac{5}{20}m^3n + \frac{11}{60}m^2n^2 - \frac{1}{4}mn^3 - \frac{0}{3}n^4 \\
 = \frac{5}{14}m^4 + \frac{1}{4}m^3n + \frac{11}{60}m^2n^2 - \frac{1}{4}mn^3 \\
 \end{array}
 \quad
 \begin{array}{l}
 \frac{5}{14}m^4 - \frac{2}{5}n^4 \\
 -\frac{5}{14}m^4 - \frac{1}{4}m^3n - \frac{11}{60}m^2n^2 + \frac{1}{4}mn^3 \\
 \hline
 -\frac{1}{4}m^3n - \frac{11}{60}m^2n^2 + \frac{1}{4}mn^3 - \frac{2}{5}n^4 \\
 \end{array}$$

$$\begin{array}{r}
 13. \quad \frac{1}{2}x + \frac{1}{3}y \\
 \quad \quad \frac{3}{4}y - \frac{1}{6}z \\
 \quad \quad \quad \frac{2}{5}z + \frac{1}{4}m \\
 \quad \quad \quad - \frac{1}{2}m + \frac{1}{3}n + \frac{3}{8} \\
 \hline
 \frac{1}{2}x + \frac{4+9}{12}y - \frac{5-12}{30}z + \frac{1-2}{4}m + \frac{1}{3}n + \frac{3}{8} \\
 = \frac{1}{2}x + \frac{13}{12}y + \frac{7}{30}z - \frac{1}{4}m + \frac{1}{3}n + \frac{3}{8}
 \end{array}
 \qquad
 \begin{array}{r}
 5 \\
 -\frac{1}{2}x - \frac{13}{12}y - \frac{7}{30}z + \frac{1}{4}m - \frac{1}{3}n - \frac{3}{8} \\
 \hline
 -\frac{1}{2}x - \frac{13}{12}y - \frac{7}{30}z + \frac{1}{4}m - \frac{1}{3}n + \frac{40-3}{8} \\
 = -\frac{1}{2}x - \frac{13}{12}y - \frac{7}{30}z + \frac{1}{4}m - \frac{1}{3}n + \frac{37}{8}
 \end{array}$$

$$\begin{array}{r}
 14. \quad \frac{5}{6}a^4 + \frac{1}{2}a^3 - \frac{3}{5}a \\
 \quad \quad - \frac{2}{3}a^2 - \frac{3}{8}a + 5 \\
 \quad \quad - \frac{3}{8}a^4 - \frac{3}{4}a^3 + \frac{1}{6}a^2 - \frac{2}{3} \\
 \quad \quad + \frac{1}{6}a^3 + \frac{39}{40}a + \frac{3}{11} \\
 \hline
 \frac{20-9}{24}a^4 + \frac{6-9+2}{12}a^3 - \frac{4-1}{6}a^2 - \frac{24+15-39}{40}a + \frac{165-22+9}{33} \\
 = \frac{11}{24}a^4 - \frac{1}{12}a^3 - \frac{3}{6}a^2 - \frac{0}{40}a + \frac{152}{33} = \frac{11}{24}a^4 - \frac{1}{12}a^3 - \frac{1}{2}a^2 + \frac{152}{33}
 \end{array}
 \qquad
 \begin{array}{r}
 \frac{11}{24}a^4 - \frac{1}{12}a^3 - \frac{1}{2}a^2 + \frac{152}{33} \\
 -a^4 + \frac{1}{12}a^3 - \frac{3}{8} \\
 \hline
 \frac{11-24}{24}a^4 - \frac{1}{2}a^2 + \frac{1216-99}{264} \\
 = -\frac{13}{24}a^4 - \frac{1}{2}a^2 + \frac{1117}{264}
 \end{array}$$

### EJERCICIO 30

$  \begin{array}{r}  1. \quad x^3 - x^2 + 5 \\  -x^3 + x^2 + 3x - 5 - 6 \\  \hline  3x - 6 \\  \text{Rta. } -x^3 + x^2 + 3x - 11  \end{array}  $	$  \begin{array}{r}  4. \quad x^3 - 4x^2 + 8 \\  -x^3 + 4x^2 + x - 13 \\  \hline  x - 5 \\  \text{Rta. } x^3 - 4x^2 - x + 13  \end{array}  $	$  \begin{array}{r}  7. \quad 5a^3 + 8ab^2 - b^3 - 11 \\  -a^3 + b^3 \\  \hline  4a^3 + 8ab^2 - 11 \\  \text{Rta. } 5a^3 + 8ab^2 - b^3 - 11  \end{array}  $
$  \begin{array}{r}  2. \quad -5a + 9b - 6c \\  5a - 9b + 6c + 8x + 9 \\  \hline  8x + 9 \\  \text{Rta. } 5a - 9b + 6c + 8x + 9  \end{array}  $	$  \begin{array}{r}  5. \quad m^4 - 3mn^3 + 6n^4 \\  -m^4 + 4m^2n^2 + 3mn^3 - 6n^4 - 8 \\  \hline  4m^2n^2 - 8 \\  \text{Rta. } m^4 - 4m^2n^2 - 3mn^3 + 6n^4 + 8  \end{array}  $	$  \begin{array}{r}  8. \quad \frac{1}{2}x - \frac{1}{3}y - 4 \\  -\frac{1}{2}x + \frac{1}{3}y \\  \hline  -4 \\  \text{Rta. } \frac{1}{2}x - \frac{1}{3}y - 4  \end{array}  $
$  \begin{array}{r}  3. \quad a^3 - b^3 \\  -a^3 - 8a^2b + 5ab^2 - 3b^3 \\  \hline  -8a^2b + 5ab^2 - 4b^3 \\  \text{Rta. } -a^3 - 8a^2b + 5ab^2 - 3b^3  \end{array}  $	$  \begin{array}{r}  6. \quad 4x^3 + 5x^2 - 5x - 2 \\  -5x^2 - 4x + 8 \\  \hline  4x^3 - 9x + 6 \\  \text{Rta. } 4x^3 + 5x^2 - 5x - 2  \end{array}  $	

$$\begin{array}{r}
 9. \quad 5x^2 - 7xy - 8y^2 \\
 -5x^2 + 7xy + 8y^2 + 1 \\
 \hline
 1 \\
 \text{Rta. } -5x^2 + 7xy + 8y^2 + 1
 \end{array}$$

$$\begin{array}{r}
 10. \quad \frac{n^3}{-9m^3 + 8m^2n - 5mn^2 + n^3} \\
 \frac{-9m^3 + 8m^2n - 5mn^2 + 2n^3}{10m^3 - 8m^2n + 5mn^2 - 2n^3} \\
 \hline
 m^3 \\
 \text{Rta. } 10m^3 - 8m^2n + 5mn^2 - 2n^3
 \end{array}$$

$$\begin{array}{r}
 11. \quad \frac{0}{-a^3 + 5a - 8} \\
 \frac{-a^3 + 5a - 8}{-a^3 + 5a - 8} \\
 \hline
 \text{Rta. } 0
 \end{array}$$

## EJERCICIO 31

$$\begin{array}{l}
 1. \quad x - (x - y) \\
 = x - x + y \\
 = y
 \end{array}$$

$$\begin{array}{l}
 6. \quad a + (a - b) + (-a + b) \\
 = a + a - b - a + b \\
 = 2a - a = a
 \end{array}$$

$$\begin{array}{l}
 11. \quad x + y + (x - y + z) - (x + y - z) \\
 = x + y + x - y + z - x - y + z \\
 = x - y + 2z
 \end{array}$$

$$\begin{array}{l}
 2. \quad x^2 + (-3x - x^2 + 5) \\
 = x^2 - 3x - x^2 + 5 \\
 = -3x + 5
 \end{array}$$

$$\begin{array}{l}
 7. \quad a^2 + [-b^2 + 2a^2] - [a^2 - b^2] \\
 = a^2 - b^2 + 2a^2 - a^2 + b^2 \\
 = 3a^2 - a^2 = 2a^2
 \end{array}$$

$$\begin{array}{l}
 12. \quad a - (b + a) + (-a + b) - (-a + 2b) \\
 = a - b - a - a + b + a - 2b \\
 = -2b
 \end{array}$$

$$\begin{array}{l}
 3. \quad a + b - (-2a + 3) \\
 = a + b + 2a - 3 \\
 = 3a + b - 3
 \end{array}$$

$$\begin{array}{l}
 8. \quad 2a - \{-x + a - 1\} - \{a + x - 3\} \\
 = 2a + x - a + 1 - a - x + 3 \\
 = 1 + 3 = 4
 \end{array}$$

$$\begin{array}{l}
 13. \quad -(x^2 - y^2) + xy + (-2x^2 + 3xy) - [-y^2 + xy] \\
 = -x^2 + y^2 + xy - 2x^2 + 3xy + y^2 - xy \\
 = -3x^2 + 3xy + 2y^2
 \end{array}$$

$$\begin{array}{l}
 4. \quad 4m - (-2m - n) \\
 = 4m + 2m + n \\
 = 6m + n
 \end{array}$$

$$\begin{array}{l}
 9. \quad x^2 + y^2 - (x^2 + 2xy + y^2) + [-x^2 + y^2] \\
 = x^2 + y^2 - x^2 - 2xy - y^2 - x^2 + y^2 \\
 = -x^2 - 2xy + y^2
 \end{array}$$

$$\begin{array}{l}
 14. \quad 8x^2 + [-2xy + y^2] - \{-x^2 + xy - 3y^2\} - (x^2 - 3xy) \\
 = 8x^2 - 2xy + y^2 + x^2 - xy + 3y^2 - x^2 + 3xy \\
 = 8x^2 + 4y^2
 \end{array}$$

$$\begin{array}{l}
 5. \quad 2x + 3y - (4x + 3y) \\
 = 2x + 3y - 4x - 3y \\
 = -2x
 \end{array}$$

$$\begin{array}{l}
 10. \quad (-5m + 6) + (-m + 5) - 6 \\
 = -5m + 6 - m + 5 - 6 \\
 = -6m + 5
 \end{array}$$

$$\begin{array}{l}
 15. \quad -(a + b) + (-a - b) - (-b + a) + (3a + b) \\
 = -a - b - a - b + b - a + 3a + b \\
 = 0
 \end{array}$$

## EJERCICIO 32

$$\begin{array}{l}
 1. \quad 2a + [a - (a + b)] \\
 = 2a + [a - a - b] \\
 = 2a + a - a - b \\
 = 2a - b
 \end{array}$$

$$\begin{array}{l}
 2. \quad 3x - [x + y - (2x + y)] \\
 = 3x - [x + y - 2x - y] \\
 = 3x - x - y + 2x + y \\
 = 4x
 \end{array}$$

$$\begin{array}{l}
 3. \quad 2m - [(m - n) - (m + n)] \\
 = 2m - [m - n - m - n] \\
 = 2m - m + n + m + n \\
 = 2m + 2n
 \end{array}$$

$$\begin{aligned}
4. & 4x^2 + \left[ - (x^2 - xy) + (-3y^2 + 2xy) - (-3x^2 + y^2) \right] \\
&= 4x^2 + \left[ -x^2 + xy - 3y^2 + 2xy + 3x^2 - y^2 \right] \\
&= 4x^2 - x^2 + xy - 3y^2 + 2xy + 3x^2 - y^2 \\
&= 6x^2 + 3xy - 4y^2
\end{aligned}$$

$$\begin{aligned}
5. & a + \{ (-2a + b) - (-a + b - c) + a \} \\
&= a + \{ -2a + b + a - b + c + a \} \\
&= a - 2a + b + a - b + c + a \\
&= a + c
\end{aligned}$$

$$\begin{aligned}
6. & 4m - [2m + (n - 3)] + [-4n - (2m + 1)] \\
&= 4m - [2m + n - 3] + [-4n - 2m - 1] \\
&= 4m - 2m - n + 3 - 4n - 2m - 1 \\
&= -5n + 2
\end{aligned}$$

$$\begin{aligned}
7. & 2x + [-5x - (-2y + \{-x + y\})] \\
&= 2x + [-5x - (-2y - x + y)] \\
&= 2x + [-5x + 2y + x - y] \\
&= 2x - 5x + 2y + x - y \\
&= -2x + y
\end{aligned}$$

$$\begin{aligned}
8. & x^2 - \{ -7xy + [-y^2 + (-x^2 + 3xy - 2y^2)] \} \\
&= x^2 - \{ -7xy + [-y^2 - x^2 + 3xy - 2y^2] \} \\
&= x^2 - \{ -7xy - y^2 - x^2 + 3xy - 2y^2 \} \\
&= x^2 + 7xy + y^2 + x^2 - 3xy + 2y^2 \\
&= 2x^2 + 4xy + 3y^2
\end{aligned}$$

$$\begin{aligned}
9. & -(a + b) + [-3a + b - \{-2a + b - (a - b)\} + 2a] \\
&= -a - b + [-a + b - \{-2a + b - a + b\}] \\
&= -a - b + [-a + b + 2a - b + a - b] \\
&= -a - b - a + b + 2a - b + a - b \\
&= a - 2b
\end{aligned}$$

$$\begin{aligned}
10. & (-x + y) - \{ 4x + 2y + [-x - y - (x + y)] \} \\
&= -x + y - \{ 4x + 2y + [-x - y - x - y] \} \\
&= -x + y - \{ 4x + 2y - x - y - x - y \} \\
&= -x + y - 4x - 2y + x + y + x + y \\
&= -3x + y
\end{aligned}$$

$$\begin{aligned}
11. & -(-a + b) + [- (a + b) - (-2a + 3b) + (-b + a - b)] \\
&= a - b + [-a - b + 2a - 3b - b + a - b] \\
&= a - b - a - b + 2a - 3b - b + a - b \\
&= 3a - 7b
\end{aligned}$$

$$\begin{aligned}
12. & 7m^2 - \{ -[m^2 + 3n - (5 - n) - (-3 + m^2)] \} - (2n + 3) \\
&= 7m^2 - \{ -[m^2 + 3n - 5 + n + 3 - m^2] \} - 2n - 3 \\
&= 7m^2 - \{ -m^2 - 3n + 5 - n + 3 + m^2 \} - 2n - 3 \\
&= 7m^2 + m^2 + 3n - 5 + n + 3 - m^2 - 2n - 3 \\
&= 7m^2 + 2n - 5
\end{aligned}$$

$$\begin{aligned}
13. & 2a - (-4a + b) - \{ -[-4a + (b - a) - (-b + a)] \} \\
&= 2a + 4a - b - \{ -[-4a + b - a + b - a] \} \\
&= 6a - b - \{ 4a - b + a - b + a \} \\
&= 6a - b - 4a + b - a + b - a \\
&= b
\end{aligned}$$

$$\begin{aligned}
14. & 3x - \{ 5y + [-2x + \{y - (6 + x)\} - (-x + y)] \} \\
&= 3x - \{ 5y + [-2x + \{y - 6 - x\} + x - y] \} \\
&= 3x - \{ 5y + [-2x + y - 6 - x + x - y] \} \\
&= 3x - \{ 5y - 2x + y - 6 - x + x - y \} \\
&= 3x - 5y + 2x - y + 6 + x - x + y \\
&= 5x - 5y + 6
\end{aligned}$$

$$\begin{aligned}
15. & 6c - \{ -(2a + c) + \{ -(a + c) - 2a - (a + c) \} + 2c \} \\
&= 6c - \{ -2a - c + \{ -a - c - 2a - a - c \} + 2c \} \\
&= 6c - \{ -2a - c - a - c - 2a - a - c + 2c \} \\
&= 6c + 2a + c + a + c + 2a + a + c - 2c \\
&= 6a + 7c
\end{aligned}$$

$$\begin{aligned}
16. & -(3m + n) - \{ 2m + \{ -m + (2m - (2n - 5)) \} \} - (n + 6) \\
&= -3m - n - \{ 2m + \{ -m + (2m - 2n + 5) \} \} - n - 6 \\
&= -3m - n - \{ 2m + \{ -m + 2m - 2n + 5 \} \} - n - 6 \\
&= -3m - n - \{ 2m - m + 2m - 2n + 5 - n - 6 \} \\
&= -3m - n - 2m + m - 2m + 2n - 5 + n + 6 \\
&= -6m + 2n + 1
\end{aligned}$$

$$\begin{aligned}
17. & 2a + \left\{ - \left[ 5b + (3a - c) + 2 - (-a + b - (c + 4)) \right] - (-a + b) \right\} \\
&= 2a + \left\{ - \left[ 5b + 3a - c + 2 - (-a + b - c - 4) \right] + a - b \right\} \\
&= 2a + \left\{ - \left[ 5b + 3a - c + 2 + a - b + c + 4 \right] + a - b \right\} \\
&= 2a + \left\{ - 5b - 3a + c - 2 - a + b - c - 4 + a - b \right\} \\
&= 2a - 5b - 3a + c - 2 - a + b - c - 4 + a - b \\
&= -a - 5b - 6
\end{aligned}$$

$$\begin{aligned}
18. & - \left[ -3x + (-x - (2y - 3)) \right] + \left\{ - (2x + y) + (-x - 3) + 2 - (x + y) \right\} \\
&= - \left[ -3x + (-x - 2y + 3) \right] + \left\{ -2x - y - x - 3 + 2 - x - y \right\} \\
&= - \left[ -3x - x - 2y + 3 \right] - 2x - y - x - 3 + 2 - x - y \\
&= 3x + x + 2y - 3 - 4x - 2y - 1 \\
&= -4
\end{aligned}$$

$$\begin{aligned}
19. & - \left[ -(-a) \right] - \left[ +(-a) \right] + \left\{ -[-b + c] - \left[ +(-c) \right] \right\} \\
&= -[a] - [-a] + \{b - c - [-c]\} \\
&= -a + a + \{b - c + c\} \\
&= b - c + c \\
&= b
\end{aligned}$$

$$\begin{aligned}
20. & - \left\{ - \left[ -(a + b) \right] \right\} - \left\{ + \left[ -(-b - a) \right] \right\} - (a + b) \\
&= - \left\{ - \left[ -a - b \right] \right\} - \left\{ + \left[ b + a \right] \right\} - a - b \\
&= - \{a + b\} - \{b + a\} - a - b \\
&= -a - b - b - a - a - b \\
&= -3a - 3b
\end{aligned}$$

$$\begin{aligned}
21. & - \left\{ - \left[ -(a + b - c) \right] \right\} - \left\{ + \left[ -(c - a + b) \right] \right\} + \left[ - \{ -a + (-b) \} \right] \\
&= - \left\{ - \left[ -a - b + c \right] \right\} - \left\{ + \left[ -c + a - b \right] \right\} + \left[ - \{ -a - b \} \right] \\
&= - \{a + b - c\} - \{-c + a - b\} + [a + b] \\
&= -a - b + c + c - a + b + a + b \\
&= -a + b + 2c
\end{aligned}$$

$$\begin{aligned}
22. & - \left[ 3m + \left\{ -m - (n - (m + 4)) \right\} \right] + \left\{ - (m + n) + (-2n + 3) \right\} \\
&= - \left[ 3m + \left\{ -m - (n - m - 4) \right\} \right] + \left\{ -m - n - 2n + 3 \right\} \\
&= - \left[ 3m + \left\{ -m - n + m + 4 \right\} - m - n - 2n + 3 \right] \\
&= - \left[ 3m - m - n + m + 4 - m - n - 2n + 3 \right] \\
&= -3m + m + n - m - 4 + m + n + 2n - 3 \\
&= -2m + 4n - 7
\end{aligned}$$

$$\begin{aligned}
23. & - \left[ x + \left\{ - (x + y) - \left[ -x + (y - z) - (-x + y) \right] - y \right\} \right] \\
&= - \left[ x + \left\{ -x - y - \left[ -x + y - z + x - y \right] - y \right\} \right] \\
&= - \left[ x + \left\{ -x - y + x - y + z - x + y - y \right\} \right] \\
&= - \left[ x - x - 2y + z \right] \\
&= -x + x + 2y - z = 2y - z
\end{aligned}$$

$$\begin{aligned}
24. & - \left[ -a + \left\{ -a + (a - b) - (a - b + c) - \left[ -(-a) + b \right] \right\} \right] \\
&= - \left[ -a + \left\{ -a + a - b - a + b - c - [a + b] \right\} \right] \\
&= - \left[ -a + \left\{ -a - c - a - b \right\} \right] \\
&= - \left[ -a - a - c - a - b \right] = 3a + b + c
\end{aligned}$$

## EJERCICIO 33

$$\begin{aligned}
1. & a - b + c - d \\
&= a + (-b + c - d)
\end{aligned}$$

$$\begin{aligned}
2. & x^2 - 3xy - y^2 + 6 \\
&= x^2 + (-3xy - y^2 + 6)
\end{aligned}$$

$$\begin{aligned}
3. & x^3 + 4x^2 - 3x + 1 \\
&= x^3 + (4x^2 - 3x + 1)
\end{aligned}$$

$$\begin{aligned}
4. & a^3 - 5a^2b + 3ab^2 - b^3 \\
&= a^3 + (-5a^2b + 3ab^2 - b^3)
\end{aligned}$$

$$\begin{aligned}
5. & x^4 - x^3 + 2x^2 - 2x + 1 \\
&= x^4 - x^3 + (2x^2 - 2x + 1)
\end{aligned}$$

$$\begin{aligned}
6. & 2a + b - c + d \\
&= 2a - (-b + c - d)
\end{aligned}$$

$$\begin{aligned}
7. & x^3 + x^2 + 3x - 4 \\
&= x^3 - (-x^2 - 3x + 4)
\end{aligned}$$

$$\begin{aligned}
8. & x^3 - 5x^2y + 3xy^2 - y^3 \\
&= x^3 - (5x^2y - 3xy^2 + y^3)
\end{aligned}$$

$$\begin{aligned}
9. & a^2 - x^2 - 2xy - y^2 \\
&= a^2 - (x^2 + 2xy + y^2)
\end{aligned}$$

$$\begin{aligned}
10. & a^2 + b^2 - 2bc - c^2 \\
&= a^2 - (-b^2 + 2bc + c^2)
\end{aligned}$$

## EJERCICIO 34

1.  $x + 2y + (x - y)$

$$= x - [-2y - (x - y)]$$

2.  $4m - 2n + 3 - (-m + n) + (2m - n)$

$$= 4m - [2n - 3 + (-m + n) - (2m - n)]$$

3.  $x^2 - 3xy + [(x^2 - xy) + y^2]$

$$= x^2 - \{3xy - [(x^2 - xy) + y^2]\}$$

4.  $x^3 - 3x^2 + [-4x + 2] - 3x - (2x + 3)$

$$= x^3 - [3x^2 - [-4x + 2] + 3x + (2x + 3)]$$

5.  $2a + 3b - \{-2a + [a + (b - a)]\}$

$$= 2a - [-3b + \{-2a + [a + (b - a)]\}]$$

6.  $-2a + (-3a + b)$

$$= -[2a - (-3a + b)]$$

7.  $2x^2 + 3xy - (y^2 + xy) + (-x^2 + y^2)$

$$= -[-2x^2 - 3xy + (y^2 + xy) - (-x^2 + y^2)]$$

8.  $x^3 - [-3x^2 + 4x - 2]$

$$= -\{x^3 + [-3x^2 + 4x - 2]\}$$

9.  $[m^4 - (3m^2 + 2m + 3)] + (-2m + 3)$

$$= -\{-[m^4 - (3m^2 + 2m + 3)] - (-2m + 3)\}$$

## EJERCICIO 35

1.  $2 \cdot -3 = -6$

2.  $-4 \cdot -8 = 32$

3.  $-15 \cdot 16 = -240$

4.  $ab \cdot -ab = -a^{1+1}b^{1+1} = -a^2b^2$

5.  $2x^2 \cdot -3x = -6x^{2+1} = -6x^3$

6.  $-4a^2b \cdot -ab^2 = 4a^{2+1}b^{1+2} = 4a^3b^3$

7.  $-5x^3y \cdot xy^2 = -5x^{3+1}y^{1+2} = -5x^4y^3$

8.  $a^2b^3 \cdot 3a^2x = 3a^{2+2}b^3x = 3a^4b^3x$

9.  $-4m^2 \cdot -5mn^2p = 20m^{2+1}n^2p = 20m^3n^2p$

10.  $5a^2y \cdot -6x^2 = -30a^2x^2y$

11.  $-x^2y^3 \cdot -4y^3z^4 = 4x^2y^{3+3}z^4 = 4x^2y^6z^4$

12.  $abc \cdot cd = abc^{1+1}d = abc^2d$

13.  $-15x^4y^3 \cdot -16a^2x^3 = 240a^2x^{4+3}y^3 = 240a^2x^7y^3$

14.  $3a^2b^3 \cdot -4x^2y = -12a^2b^3x^2y$

15.  $3a^2bx \cdot 7b^3x^5 = 21a^2b^{1+3}x^{1+5} = 21a^2b^4x^6$

16.  $-8m^2n^3 \cdot -9a^2mx^4 = 72a^2m^{2+1}n^3x^4 = 72a^2m^3n^3x^4$

17.  $a^mb^n \cdot -ab = -a^{m+1}b^{n+1}$

18.  $-5a^mb^n \cdot -6a^2b^3x = 30a^{m+2}b^{n+3}x$

19.  $x^my^n c \cdot -x^m y^n c^x = -x^{m+m} y^{n+n} c^{1+x} = -x^{2m} y^{2n} c^{1+x}$

20.  $-m^x n^a \cdot -6m^2 n = 6m^{x+2} n^{a+1}$

## EJERCICIO 36

1.  $a^m \cdot a^{m+1} = a^{m+m+1} = a^{2m+1}$

2.  $-x^a \cdot -x^{a+2} = x^{a+a+2} = x^{2a+2}$

3.  $4a^nb^x \cdot -ab^{x+1} = -4a^{n+1}b^{x+x+1} = -4a^{n+1}b^{2x+1}$

4.  $-a^{n+1}b^{n+2} \cdot a^{n+2}b^n = -a^{n+1+n+2}b^{n+2+n} = -a^{2n+3}b^{2n+2}$

5.  $-3a^{n+4}b^{n+1} \cdot -4a^{n+2}b^{n+3} = 12a^{2n+6}b^{2n+4}$

6.  $3x^2y^3 \cdot 4x^{m+1}y^{m+2} = 12x^{2+m+1}y^{3+m+2} = 12x^{m+3}y^{m+5}$

7.  $4x^{a+2}b^{a+4} \cdot -5x^{a+5}b^{a+1} = -20x^{2a+7}b^{2a+5}$

8.  $a^mb^nc \cdot -a^mb^{2n} = -a^{m+m}b^{n+2n}c = -a^{2m}b^{3n}c$

9.  $-x^{m+1}y^{a+2} \cdot -4x^{m-3}y^{a-5}c^2 = 4x^{2m-2}y^{2a-3}c^2$

10.  $-5m^an^{b-1}c \cdot -7m^{2a-3}n^{b-4} = 35m^{3a-3}n^{2b-5}c$

## EJERCICIO 37

$$1. \frac{1}{2}a^2 \cdot \frac{4}{5}a^3b = \frac{1}{2} \cdot \frac{4}{5}a^5b = \frac{4}{10}a^5b = \frac{2}{5}a^5b$$

$$2. -\frac{3}{7}m^2n \cdot -\frac{7}{14}a^2m^3 \\ = -\frac{3}{7} \cdot -\frac{7}{14}a^2m^5n = \frac{21}{98}a^2m^5n = \frac{3}{14}a^2m^5n$$

$$3. \frac{2}{3}x^2y^3 \cdot -\frac{3}{5}a^2x^4y = -\frac{6}{15}a^2x^6y^4 = -\frac{2}{5}a^2x^6y^4$$

$$4. -\frac{1}{8}m^3n^4 \cdot -\frac{4}{5}a^3m^2n = \frac{4}{40}a^3m^5n^5 = \frac{1}{10}a^3m^5n^5$$

$$5. -\frac{7}{8}abc \cdot \frac{2}{7}a^3 = -\frac{14}{56}a^4bc = -\frac{1}{4}a^4bc$$

$$6. -\frac{3}{5}x^4y^4 \cdot -\frac{5}{6}a^2by^5 = \frac{3}{6}a^2bx^4y^9 = \frac{1}{2}a^2bx^4y^9$$

$$7. \frac{1}{3}a \cdot \frac{3}{5}a^m = \frac{3}{15}a^{m+1} = \frac{1}{5}a^{m+1}$$

$$8. -\frac{3}{4}a^m \cdot -\frac{2}{5}ab^3 = \frac{3}{2} \cdot \frac{1}{5}a^{m+1}b^3 = \frac{3}{10}a^{m+1}b^3$$

$$9. \frac{5}{6}a^mb^n \cdot -\frac{3}{10}ab^2c \\ = -\frac{1}{2} \cdot \frac{1}{2}a^{m+1}b^{n+2}c = -\frac{1}{4}a^{m+1}b^{n+2}c$$

$$10. -\frac{2}{9}a^xb^{m+1} \cdot -\frac{3}{5}a^{x-1}b^m \\ = \frac{2}{3} \cdot \frac{1}{5}a^{2x-1}b^{2m+1} = \frac{2}{15}a^{2x-1}b^{2m+1}$$

$$11. \frac{3}{8}a^mb^n \cdot -\frac{4}{5}a^{2m}b^n = -\frac{12}{40}a^{3m}b^{2n} = -\frac{3}{10}a^{3m}b^{2n}$$

$$12. -\frac{2}{11}a^{x+1}b^{x-3}c^2 \cdot -\frac{44}{7}a^{x-3}b^2 \\ = 2 \cdot \frac{4}{7}a^{2x-2}b^{x-1}c^2 = \frac{8}{7}a^{2x-2}b^{x-1}c^2$$

## EJERCICIO 38

$$1. a \cdot -3a \cdot a^2 = -3a^{1+1+2} = -3a^4$$

$$2. 3x^2 \cdot -x^3y \cdot -a^2x = 3a^2x^{2+3+1}y = 3a^2x^6y$$

$$3. -m^2n \cdot -3m^2 \cdot -5mn^3 = -15m^{2+2+1}n^{1+3} = -15m^5n^4$$

$$4. 4a^2 \cdot -5a^3x^2 \cdot -ay^2 = 20a^{2+3+1}x^2y^2 = 20a^6x^2y^2$$

$$5. -a^m \cdot -2ab \cdot -3a^2b^x = -6a^{m+1+2}b^{1+x} = -6a^{m+3}b^{x+1}$$

$$6. \frac{1}{2}x^3 \cdot -\frac{2}{3}a^2x \cdot -\frac{3}{5}a^4m = \frac{6}{30}a^{2+4}x^{3+1}m = \frac{1}{5}a^6x^4m$$

$$7. \frac{2}{3}a^m \cdot \frac{3}{4}a^2b^4 \cdot -3a^4b^{x+1} \\ = -\frac{18}{12}a^{m+2+4}b^{4+x+1} = -\frac{3}{2}a^{m+6}b^{x+5}$$

$$8. -\frac{3}{5}m^3 \cdot -5a^2m \cdot -\frac{1}{10}a^xm^a \\ = -\frac{15}{50}a^{2+x}m^{3+1+a} = -\frac{3}{10}a^{x+2}m^{a+4}$$

$$9. 2a \cdot -a^2 \cdot -3a^3 \cdot 4a = 24a^{1+2+3+1} = 24a^7$$

$$10. -3b^2 \cdot -4a^3b \cdot ab \cdot -5a^2x \\ = -60a^{3+1+2}b^{2+1+1}x = -60a^6b^4x$$

$$11. a^mb^x \cdot -a^2 \cdot -2ab \cdot -3a^2x \\ = -6a^{m+2+1+2}b^{x+1}x = -6a^{m+5}b^{x+1}x$$

$$12. -\frac{1}{2}x^2y \cdot -\frac{3}{5}xy^2 \cdot -\frac{10}{3}x^3 \cdot -\frac{3}{4}x^2y \\ = \frac{6}{8}x^{2+1+3+2}y^{1+2+1} = \frac{3}{4}x^8y^4$$

## EJERCICIO 39

$$1. \begin{array}{r} 3x^3 - x^2 \\ -2x \\ \hline -6x^4 + 2x^3 \end{array}$$

$$2. \begin{array}{r} 8x^2y - 3y^2 \\ 2ax^3 \\ \hline 16ax^5y - 6ax^3y^2 \end{array}$$

$$3. \begin{array}{r} x^2 - 4x + 3 \\ -2x \\ \hline -2x^3 + 8x^2 - 6x \end{array}$$

$$4. \begin{array}{r} a^3 - 4a^2 + 6a \\ 3ab \\ \hline 3a^4b - 12a^3b + 18a^2b \end{array}$$

$$\begin{array}{r} 5. \quad a^2 - 2ab + b^2 \\ - ab \\ \hline - a^3b + 2a^2b^2 - ab^3 \end{array}$$

$$\begin{array}{r} 6. \quad x^5 - 6x^3 - 8x \\ 3a^2x^2 \\ \hline 3a^2x^7 - 18a^2x^5 - 24a^2x^3 \end{array}$$

$$\begin{array}{r} 7. \quad m^4 - 3m^2n^2 + 7n^4 \\ - 4m^3x \\ \hline - 4m^7x + 12m^5n^2x - 28m^3n^4x \end{array}$$

$$\begin{array}{r} 8. \quad x^3 - 4x^2y + 6xy^2 \\ ax^3y \\ \hline ax^6y - 4ax^5y^2 + 6ax^4y^3 \end{array}$$

$$\begin{array}{r} 9. \quad a^3 - 5a^2b - 8ab^2 \\ - 4a^4m^2 \\ \hline - 4a^7m^2 + 20a^6bm^2 + 32a^5b^2m^2 \end{array}$$

$$\begin{array}{r} 10. \quad a^m - a^{m-1} + a^{m-2} \\ - 2a \\ \hline - 2a^{m+1} + 2a^{m-1+1} - 2a^{m-2+1} \\ = - 2a^{m+1} + 2a^m - 2a^{m-1} \end{array}$$

$$\begin{array}{r} 11. \quad x^{m+1} + 3x^m - x^{m-1} \\ 3x^{2m} \\ \hline 3x^{3m+1} + 9x^{3m} - 3x^{3m-1} \end{array}$$

$$\begin{array}{r} 12. \quad a^mb^n + a^{m-1}b^{n+1} - a^{m-2}b^{n+2} \\ 3a^2b \\ \hline 3a^{m+2}b^{n+1} + 3a^{m+1}b^{n+2} - 3a^mb^{n+3} \end{array}$$

$$\begin{array}{r} 13. \quad x^3 - 3x^2 + 5x - 6 \\ - 4x^2 \\ \hline - 4x^5 + 12x^4 - 20x^3 + 24x^2 \end{array}$$

$$\begin{array}{r} 14. \quad a^4 - 6a^3x + 9a^2x^2 - 8 \\ 3bx^3 \\ \hline 3a^4bx^3 - 18a^3bx^4 + 27a^2bx^5 - 24bx^3 \end{array}$$

$$\begin{array}{r} 15. \quad a^{n+3} - 3a^{n+2} - 4a^{n+1} - a^n \\ - a^n x^2 \\ \hline - a^{2n+3}x^2 + 3a^{2n+2}x^2 + 4a^{2n+1}x^2 + a^{2n}x^2 \end{array}$$

$$\begin{array}{r} 16. \quad x^4 - 6x^3 + 8x^2 - 7x + 5 \\ - 3a^2x^3 \\ \hline - 3a^2x^7 + 18a^2x^6 - 24a^2x^5 + 21a^2x^4 - 15a^2x^3 \end{array}$$

$$\begin{array}{r} 17. \quad -3x^3 + 5x^2y - 7xy^2 - 4y^3 \\ 5a^2xy^2 \\ \hline - 15a^2x^4y^2 + 25a^2x^3y^3 - 35a^2x^2y^4 - 20a^2xy^5 \end{array}$$

$$\begin{array}{r} 18. \quad x^{a+5} - 3x^{a+4} + x^{a+3} - 5x^{a+1} \\ - 2x^2 \\ \hline - 2x^{a+7} + 6x^{a+6} - 2x^{a+5} + 10x^{a+3} \end{array}$$

$$\begin{array}{r} 19. \quad a^8 - 3a^6b^2 + a^4b^4 - 3a^2b^6 + b^8 \\ - 5a^3y^2 \\ \hline - 5a^{11}y^2 + 15a^9b^2y^2 - 5a^7b^4y^2 + 15a^5b^6y^2 - 5a^3b^8y^2 \end{array}$$

$$\begin{array}{r} 20. \quad a^mb^n + 3a^{m-1}b^{n+2} - a^{m-2}b^{n+4} + a^{m-3}b^{n+6} \\ 4a^mb^3 \\ \hline 4a^{2m}b^{n+3} + 12a^{2m-1}b^{n+5} - 4a^{2m-2}b^{n+7} + 4a^{2m-3}b^{n+9} \end{array}$$

## EJERCICIO 40

$$\begin{array}{r} 1. \quad \frac{1}{2}a - \frac{2}{3}b \\ \frac{2}{5}a^2 \\ \hline \frac{2}{10}a^{1+2} - \frac{4}{15}a^2b \\ = \frac{1}{5}a^3 - \frac{4}{15}a^2b \end{array}$$

$$\begin{array}{r} 2. \quad \frac{2}{3}a - \frac{3}{4}b \\ - \frac{2}{3}a^3b \\ \hline - \frac{4}{9}a^{3+1}b + \frac{6}{12}a^3b^{1+1} \\ = - \frac{4}{9}a^4b + \frac{1}{2}a^3b^2 \end{array}$$

$$\begin{array}{r} 3. \quad \frac{3}{5}a - \frac{1}{6}b + \frac{2}{5}c \\ - \frac{5}{3}ac^2 \\ \hline - \frac{15}{15}a^{1+1}c^2 + \frac{5}{18}abc^2 - \frac{10}{15}ac^{2+1} \\ = - a^2c^2 + \frac{5}{18}abc^2 - \frac{2}{3}ac^3 \end{array}$$

$$\begin{array}{r} 4. \quad \frac{2}{5}a^2 + \frac{1}{3}ab - \frac{2}{9}b^2 \\ 3a^2x \\ \hline \frac{6}{5}a^{2+2}x + \frac{3}{3}a^{2+1}bx - \frac{6}{9}a^2b^2x \\ = \frac{6}{5}a^4x + a^3bx - \frac{2}{3}a^2b^2x \end{array}$$



$$5. \frac{1}{3}x^2 - \frac{2}{5}xy - \frac{1}{4}y^2$$

$$\frac{3}{2}y^3$$

$$\frac{3}{6}x^2y^3 - \frac{6}{10}xy^{3+1} - \frac{3}{8}y^{2+3}$$

$$= \frac{1}{2}x^2y^3 - \frac{3}{5}xy^4 - \frac{3}{8}y^5$$

$$6. 3a - 5b + 6c$$

$$-\frac{3}{10}a^2x^3$$

$$-\frac{9}{10}a^{2+1}x^3 + \frac{15}{10}a^2bx^3 - \frac{18}{10}a^2cx^3$$

$$= -\frac{9}{10}a^3x^3 + \frac{3}{2}a^2bx^3 - \frac{9}{5}a^2cx^3$$

$$7. \frac{2}{9}x^4 - x^2y^2 + \frac{1}{3}y^4$$

$$\frac{3}{7}x^3y^4$$

$$\frac{6}{63}x^{4+3}y^4 - \frac{3}{7}x^{2+3}y^{2+4} + \frac{3}{21}x^3y^{4+4}$$

$$= \frac{2}{21}x^7y^4 - \frac{3}{7}x^5y^6 + \frac{1}{7}x^3y^8$$

$$\frac{1}{2}a^2 - \frac{1}{3}b^2 + \frac{1}{4}x^2 - \frac{1}{5}y^2$$

$$-\frac{5}{8}a^2m$$

$$8. -\frac{5}{16}a^{2+2}m + \frac{5}{24}a^2b^2m - \frac{5}{32}a^2mx^2 + \frac{5}{40}a^2my^2$$

$$= -\frac{5}{16}a^4m + \frac{5}{24}a^2b^2m - \frac{5}{32}a^2mx^2 + \frac{1}{8}a^2my^2$$

$$9. \frac{2}{3}m^3 + \frac{1}{2}m^2n - \frac{5}{6}mn^2 - \frac{1}{9}n^3$$

$$\frac{3}{4}m^2n^3$$

$$\frac{6}{12}m^{3+2}n^3 + \frac{3}{8}m^{2+2}n^{1+3} - \frac{15}{24}m^{1+2}n^{2+3} - \frac{3}{36}m^2n^{3+3}$$

$$= \frac{1}{2}m^5n^3 + \frac{3}{8}m^4n^4 - \frac{5}{8}m^3n^5 - \frac{1}{12}m^2n^6$$

$$\frac{2}{5}x^6 - \frac{1}{3}x^4y^2 + \frac{3}{5}x^2y^4 - \frac{1}{10}y^6$$

$$-\frac{5}{7}a^3x^4y^3$$

$$10. -\frac{10}{35}a^3x^{6+4}y^3 + \frac{5}{21}a^3x^{4+4}y^{2+3} - \frac{15}{35}a^3x^{2+4}y^{4+3} + \frac{5}{70}a^3x^4y^{6+3}$$

$$= -\frac{2}{7}a^3x^{10}y^3 + \frac{5}{21}a^3x^8y^5 - \frac{3}{7}a^3x^6y^7 + \frac{1}{14}a^3x^4y^9$$

## EJERCICIO 41

$$1. a+3$$

$$a-1$$

$$\frac{a^2+3a}{a-3}$$

$$\frac{a^2+2a-3}{a^2+2a-3}$$

$$3. x+5$$

$$x-4$$

$$\frac{x^2+5x}{-4x-20}$$

$$\frac{x^2+x-20}{x^2+x-20}$$

$$5. -x+3$$

$$-x+5$$

$$\frac{x^2-3x}{-5x+15}$$

$$\frac{x^2-8x+15}{x^2-8x+15}$$

$$7. 3x-2y$$

$$2x+y$$

$$\frac{6x^2-4xy}{+3xy-2y^2}$$

$$\frac{6x^2-xy-2y^2}{6x^2-xy-2y^2}$$

$$2. a-3$$

$$a+1$$

$$\frac{a^2-3a}{a-3}$$

$$\frac{a^2-2a-3}{a^2-2a-3}$$

$$4. m-6$$

$$m-5$$

$$\frac{m^2-6m}{-5m+30}$$

$$\frac{m^2-11m+30}{m^2-11m+30}$$

$$6. -a-2$$

$$-a-3$$

$$\frac{a^2+2a}{+3a+6}$$

$$\frac{a^2+5a+6}{a^2+5a+6}$$

$$8. 5x-4y$$

$$-3x+2y$$

$$\frac{-15x^2+12xy}{+10xy-8y^2}$$

$$\frac{-15x^2+22xy-8y^2}{-15x^2+22xy-8y^2}$$

$$\begin{array}{r}
 9. \quad 5a-7b \\
 \quad a+3b \\
 \hline
 5a^2-7ab \\
 \quad +15ab-21b^2 \\
 \hline
 5a^2+8ab-21b^2
 \end{array}$$

$$\begin{array}{r}
 10. \quad 7x-3 \\
 \quad 2x+4 \\
 \hline
 14x^2-6x \\
 \quad +28x-12 \\
 \hline
 14x^2+22x-12
 \end{array}$$

$$\begin{array}{r}
 11. \quad -a+b \\
 \quad 8a-4b \\
 \hline
 -8a^2+8ab \\
 \quad +4ab-4b^2 \\
 \hline
 -8a^2+12ab-4b^2
 \end{array}$$

$$\begin{array}{r}
 12. \quad 6m-5n \\
 \quad m-n \\
 \hline
 6m^2-5mn \\
 \quad -6mn+5n^2 \\
 \hline
 6m^2-11mn+5n^2
 \end{array}$$

$$\begin{array}{r}
 13. \quad -9m+8n \\
 \quad 6m+4n \\
 \hline
 -54m^2+48mn \\
 \quad -36mn+32n^2 \\
 \hline
 -54m^2+12mn+32n^2
 \end{array}$$

$$\begin{array}{r}
 14. \quad -7y-3 \\
 \quad 2y-11 \\
 \hline
 -14y^2-6y \\
 \quad +77y+33 \\
 \hline
 -14y^2+71y+33
 \end{array}$$

## EJERCICIO 42

$$\begin{array}{r}
 1. \quad x^2+xy+y^2 \\
 \quad x-y \\
 \hline
 x^3+x^2y+xy^2 \\
 \quad -x^2y-xy^2-y^3 \\
 \hline
 x^3 \qquad \qquad -y^3
 \end{array}$$

$$\begin{array}{r}
 2. \quad a^2-2ab+b^2 \\
 \quad a-b \\
 \hline
 a^3-2a^2b+ab^2 \\
 \quad -a^2b+2ab^2-b^3 \\
 \hline
 a^3-3a^2b+3ab^2-b^3
 \end{array}$$

$$\begin{array}{r}
 3. \quad a^2+2ab+b^2 \\
 \quad a+b \\
 \hline
 a^3+2a^2b+ab^2 \\
 \quad +a^2b+2ab^2+b^3 \\
 \hline
 a^3+3a^2b+3ab^2+b^3
 \end{array}$$

$$\begin{array}{r}
 4. \quad x^3-3x^2+1 \\
 \quad x+3 \\
 \hline
 x^4-3x^3 \quad +x \\
 \quad +3x^3-9x^2 \quad +3 \\
 \hline
 x^4 \quad \quad -9x^2+x+3
 \end{array}$$

$$\begin{array}{r}
 5. \quad a^3+a^2-a \\
 \quad a-1 \\
 \hline
 a^4+a^3-a^2 \\
 \quad -a^3-a^2+a \\
 \hline
 a^4 \quad \quad -2a^2+a
 \end{array}$$

$$\begin{array}{r}
 6. \quad m^4+m^2n^2+n^4 \\
 \quad m^2-n^2 \\
 \hline
 m^6+m^4n^2+n^2n^4 \\
 \quad -m^4n^2-m^2n^4-n^6 \\
 \hline
 m^6 \qquad \qquad -n^6
 \end{array}$$

$$\begin{array}{r}
 7. \quad x^3-2x^2+3x-1 \\
 \quad 2x+3 \\
 \hline
 2x^4-4x^3+6x^2-2x \\
 \quad +3x^3-6x^2+9x-3 \\
 \hline
 2x^4-x^3 \quad \quad +7x-3
 \end{array}$$

$$\begin{array}{r}
 9. \quad 3y^3-6y+5 \\
 \quad y^2+2 \\
 \hline
 3y^5-6y^3+5y^2 \\
 \quad +6y^3 \quad \quad -12y+10 \\
 \hline
 3y^5 \quad \quad +5y^2-12y+10
 \end{array}$$

$$\begin{array}{r}
 9. \quad m^3-m^2+m-2 \\
 \quad am+a \\
 \hline
 am^4-am^3+am^2-2am \\
 \quad +am^3-am^2+am-2a \\
 \hline
 am^4 \qquad \qquad -am-2a
 \end{array}$$

$$\begin{array}{r}
 10. \quad 3a^2-5ab+2b^2 \\
 \quad 4a-5b \\
 \hline
 12a^3-20a^2b+8ab^2 \\
 \quad -15a^2b+25ab^2-10b^3 \\
 \hline
 12a^3-35a^2b+33ab^2-10b^3
 \end{array}$$

$$\begin{array}{r}
 11. \quad 5m^4-3m^2n^2+n^4 \\
 \quad 3m-n \\
 \hline
 15m^5 \quad \quad -9m^3n^2 \quad \quad +3mn^4 \\
 \quad -5m^4n \quad \quad +3m^2n^3 \quad \quad -n^5 \\
 \hline
 15m^5-5m^4n-9m^3n^2+3m^2n^3+3mn^4-n^5
 \end{array}$$

$$\begin{array}{r}
 12. \quad a^2+a+1 \\
 \quad a^2-a-1 \\
 \hline
 a^4+a^3+a^2 \\
 \quad -a^3-a^2-a \\
 \hline
 \quad \quad -a^2-a-1 \\
 \hline
 a^4 \quad \quad -a^2-2a-1
 \end{array}$$

$$13. \begin{array}{r} x^3 + 2x^2 - x \\ x^2 - 2x + 5 \\ \hline x^5 + 2x^4 - x^3 \\ - 2x^4 - 4x^3 + 2x^2 \\ + 5x^3 + 10x^2 - 5x \\ \hline x^5 \qquad \qquad + 12x^2 - 5x \end{array}$$

$$14. \begin{array}{r} m^3 - 3m^2n + 2mn^2 \\ m^2 - 2mn - 8n^2 \\ \hline m^5 - 3m^4n + 2m^3n^2 \\ - 2m^4n + 6m^3n^2 - 4m^2n^3 \\ - 8m^3n^2 + 24m^2n^3 - 16mn^4 \\ \hline m^5 - 5m^4n \qquad \qquad + 20m^2n^3 - 16mn^4 \end{array}$$

$$15. \begin{array}{r} x^2 + x + 1 \\ x^2 - x - 1 \\ \hline x^4 + x^3 + x^2 \\ - x^3 - x^2 - x \\ - x^2 - x - 1 \\ \hline x^4 \qquad - x^2 - 2x - 1 \end{array}$$

$$16. \begin{array}{r} x^4 - 3x^2 + 2 \\ x^2 - 2x + 3 \\ \hline x^6 \qquad - 3x^4 \qquad + 2x^2 \\ - 2x^5 \qquad + 6x^3 \qquad - 4x \\ + 3x^4 \qquad - 9x^2 \qquad + 6 \\ \hline x^6 - 2x^5 \qquad + 6x^3 - 7x^2 - 4x + 6 \end{array}$$

$$17. \begin{array}{r} m^3 + m^2 - 4m - 1 \\ m^3 + 1 \\ \hline m^6 + m^5 - 4m^4 - m^3 \\ + m^3 + m^2 - 4m - 1 \\ \hline m^6 + m^5 - 4m^4 \qquad + m^2 - 4m - 1 \end{array}$$

$$18. \begin{array}{r} a^3 - 5a + 2 \\ a^2 - a + 5 \\ \hline a^5 \qquad - 5a^3 + 2a^2 \\ - a^4 \qquad + 5a^2 - 2a \\ + 5a^3 \qquad - 25a + 10 \\ \hline a^5 - a^4 \qquad + 7a^2 - 27a + 10 \end{array}$$

$$19. \begin{array}{r} x^2 - 2xy + y^2 \\ - x^2 + xy + 3y^2 \\ \hline - x^4 + 2x^3y - x^2y^2 \\ + x^3y - 2x^2y^2 + xy^3 \\ + 3x^2y^2 - 6xy^3 + 3y^4 \\ \hline - x^4 + 3x^3y \qquad - 5xy^3 + 3y^4 \end{array}$$

$$20. \begin{array}{r} n^2 - 2n + 1 \\ n^2 - 1 \\ \hline n^4 - 2n^3 + n^2 \\ - n^2 + 2n - 1 \\ \hline n^4 - 2n^3 \qquad + 2n - 1 \end{array}$$

$$21. \begin{array}{r} a^3 - 3a^2b + 4ab^2 \\ a^2b - 2ab^2 - 10b^3 \\ \hline a^5b - 3a^4b^2 + 4a^3b^3 \\ - 2a^4b^2 + 6a^3b^3 - 8a^2b^4 \\ - 10a^3b^3 + 30a^2b^4 - 40ab^5 \\ \hline a^5b - 5a^4b^2 \qquad + 22a^2b^4 - 40ab^5 \end{array}$$

$$22. \begin{array}{r} 8x^3 - 12x^2y + 6xy^2 - 9y^3 \\ 2x + 3y \\ \hline 16x^4 - 24x^3y + 12x^2y^2 - 18xy^3 \\ + 24x^3y - 36x^2y^2 + 18xy^3 - 27y^4 \\ \hline 16x^4 \qquad - 24x^2y^2 \qquad - 27y^4 \end{array}$$

$$23. \begin{array}{r} 2y^3 - 3y^2 + y - 4 \\ 2y + 5 \\ \hline 4y^4 - 6y^3 + 2y^2 - 8y \\ + 10y^3 - 15y^2 + 5y - 20 \\ \hline 4y^4 + 4y^3 - 13y^2 - 3y - 20 \end{array}$$

$$24. \begin{array}{r} -a^3 + 2ax^2 + 3x^3 \\ 2a^2 - 3ax - x^2 \\ \hline -2a^5 \qquad + 4a^3x^2 + 6a^2x^3 \\ + 3a^4x \qquad - 6a^2x^3 - 9ax^4 \\ + a^3x^2 \qquad - 2ax^4 - 3x^5 \\ \hline -2a^5 + 3a^4x + 5a^3x^2 \qquad - 11ax^4 - 3x^5 \end{array}$$

$$\begin{array}{r}
 25. \quad x^4 - 3x^3y + 2x^2y^2 + xy^3 \\
 \underline{-x^2 - xy - y^2} \\
 -x^6 + 3x^5y - 2x^4y^2 - x^2y^3 \\
 \quad -x^5y + 3x^4y^2 - 2x^3y^3 - x^2y^4 \\
 \quad \quad -x^4y^2 + 3x^3y^3 - 2x^2y^4 - xy^5 \\
 \hline
 -x^6 + 2x^5y \qquad \qquad -3x^2y^4 - xy^5
 \end{array}$$

$$\begin{array}{r}
 26. \quad a^3 - 5a^2 + 2a - 3 \\
 \underline{a^3 - 2a - 7} \\
 a^6 - 5a^5 + 2a^4 - 3a^3 \\
 \quad -2a^4 + 10a^3 - 4a^2 + 6a \\
 \quad \quad -7a^3 + 35a^2 - 14a + 21 \\
 \hline
 a^6 - 5a^5 \qquad \qquad + 31a^2 - 8a + 21
 \end{array}$$

$$\begin{array}{r}
 27. \quad m^4 + m^3 - m^2 + 3 \\
 \underline{m^2 - 2m + 3} \\
 m^6 + m^5 - m^4 \qquad \quad + 3m^2 \\
 \quad -2m^5 - 2m^4 + 2m^3 \qquad -6m \\
 \quad \quad + 3m^4 + 3m^3 - 3m^2 \qquad + 9 \\
 \hline
 m^6 - m^5 \qquad + 5m^3 \qquad -6m + 9
 \end{array}$$

$$\begin{array}{r}
 28. \quad a^4 + a^3b - 3a^2b^2 - ab^3 + b^4 \\
 \underline{a^2 - 2ab + b^2} \\
 a^6 + a^5b - 3a^4b^2 - a^3b^3 + a^2b^4 \\
 \quad -2a^5b - 2a^4b^2 + 6a^3b^3 + 2a^2b^4 - 2ab^5 \\
 \quad \quad + a^4b^2 + a^3b^3 - 3a^2b^4 - ab^5 + b^6 \\
 \hline
 a^6 - a^5b - 4a^4b^2 + 6a^3b^3 \qquad -3ab^5 + b^6
 \end{array}$$

$$\begin{array}{r}
 29. \quad x^4 - x^3y + x^2y^2 - xy^3 + y^4 \\
 \underline{x^2 + xy - 2y^2} \\
 x^6 - x^5y + x^4y^2 - x^3y^3 + x^2y^4 \\
 \quad + x^5y - x^4y^2 + x^3y^3 - x^2y^4 + xy^5 \\
 \quad \quad - 2x^4y^2 + 2x^3y^3 - 2x^2y^4 + 2xy^5 - 2y^6 \\
 \hline
 x^6 \qquad - 2x^4y^2 + 2x^3y^3 - 2x^2y^4 + 3xy^5 - 2y^6
 \end{array}$$

$$\begin{array}{r}
 30. \quad y^2 - 2y + 1 \\
 \underline{y^4 - 2y^2 + 2} \\
 y^6 - 2y^5 + y^4 \\
 \quad -2y^4 + 4y^3 - 2y^2 \\
 \quad \quad + 2y^2 - 4y + 2 \\
 \hline
 y^6 - 2y^5 - y^4 + 4y^3 \qquad -4y + 2
 \end{array}$$

$$\begin{array}{r}
 31. \quad m^4 - 3m^2 + 4 \\
 \underline{3m^3 - 2m + 1} \\
 3m^7 - 9m^5 \qquad \quad + 12m^3 \\
 \quad -2m^5 \qquad \quad + 6m^3 \qquad \quad -8m \\
 \quad \quad + m^4 \qquad \quad -3m^2 \qquad \quad + 4 \\
 \hline
 3m^7 - 11m^5 + m^4 + 18m^3 - 3m^2 - 8m + 4
 \end{array}$$

$$\begin{array}{r}
 32. \quad a^3 + a^2 - a + 1 \\
 \underline{a^3 + a^2 - 2a - 1} \\
 a^6 + a^5 - a^4 \qquad + a^3 \\
 \quad + a^5 + a^4 - a^3 + a^2 \\
 \quad \quad - 2a^4 - 2a^3 + 2a^2 - 2a \\
 \quad \quad \quad - a^3 - a^2 + a - 1 \\
 \hline
 a^6 + 2a^5 - 2a^4 - 3a^3 + 2a^2 - a - 1
 \end{array}$$

$$\begin{array}{r}
 33. \quad 8x^3 - 12x^2y - 6xy^2 + y^3 \\
 \underline{3x^2 - 2xy + 4y^2} \\
 24x^5 - 36x^4y - 18x^3y^2 + 3x^2y^3 \\
 \quad -16x^4y + 24x^3y^2 + 12x^2y^3 - 2xy^4 \\
 \quad \quad + 32x^3y^2 - 48x^2y^3 - 24xy^4 + 4y^5 \\
 \hline
 24x^5 - 52x^4y + 38x^3y^2 - 33x^2y^3 - 26xy^4 + 4y^5
 \end{array}$$

$$\begin{array}{r}
 34. \quad 5a^4 - 4a^3 + 2a^2 - 3a - 1 \\
 \underline{a^4 - 2a^2 + 2} \\
 5a^8 - 4a^7 + 2a^6 - 3a^5 - a^4 \\
 \quad -10a^6 + 8a^5 - 4a^4 + 6a^3 + 2a^2 \\
 \quad \quad + 10a^4 - 8a^3 + 4a^2 - 6a - 2 \\
 \hline
 5a^8 - 4a^7 - 8a^6 + 5a^5 + 5a^4 - 2a^3 + 6a^2 - 6a - 2
 \end{array}$$

$$35. \begin{array}{r} x^4 - x^3 + x^2 - x + 1 \\ x^3 - 2x^2 + 3x + 6 \\ \hline x^7 - x^6 + x^5 - x^4 + x^3 \\ - 2x^6 + 2x^5 - 2x^4 + 2x^3 - 2x^2 \\ + 3x^5 - 3x^4 + 3x^3 - 3x^2 + 3x \\ + 6x^4 - 6x^3 + 6x^2 - 6x + 6 \\ \hline x^7 - 3x^6 + 6x^5 \phantom{+ 6x^4 - 6x^3 + 6x^2 - 6x + 6} + x^2 - 3x + 6 \end{array}$$

$$36. \begin{array}{r} 3a^3 + 2a^2 - 5a - 4 \\ a^3 + a^2 - 2a + 1 \\ \hline 3a^6 + 2a^5 - 5a^4 - 4a^3 \\ + 3a^5 + 2a^4 - 5a^3 - 4a^2 \\ - 6a^4 - 4a^3 + 10a^2 + 8a \\ + 3a^3 + 2a^2 - 5a - 4 \\ \hline 3a^6 + 5a^5 - 9a^4 - 10a^3 + 8a^2 + 3a - 4 \end{array}$$

$$37. \begin{array}{r} 5y^4 - 3y^3 + 4y^2 + 2y \\ y^4 - 3y^2 - 1 \\ \hline 5y^8 - 3y^7 + 4y^6 + 2y^5 \\ - 15y^6 + 9y^5 - 12y^4 - 6y^3 \\ - 5y^4 + 3y^3 - 4y^2 - 2y \\ \hline 5y^8 - 3y^7 - 11y^6 + 11y^5 - 17y^4 - 3y^3 - 4y^2 - 2y \end{array}$$

$$38. \begin{array}{r} m^4 - 2m^3n + 3m^2n^2 - 4n^4 \\ - m^3 + 3m^2n - 5mn^2 + n^3 \\ \hline - m^7 + 2m^6n - 3m^5n^2 + 4m^3n^4 \\ + 3m^6n - 6m^5n^2 + 9m^4n^3 - 12m^2n^5 \\ - 5m^5n^2 + 10m^4n^3 - 15m^3n^4 + 20mn^6 \\ + m^4n^3 - 2m^3n^4 + 3m^2n^5 - 4n^7 \\ \hline - m^7 + 5m^6n - 14m^5n^2 + 20m^4n^3 - 13m^3n^4 - 9m^2n^5 + 20mn^6 - 4n^7 \end{array}$$

$$39. \begin{array}{r} x^6 - 3x^4y^2 - x^2y^4 + y^6 \\ x^5 - 2x^3y^2 + 3xy^4 \\ \hline x^{11} - 3x^9y^2 - x^7y^4 + x^5y^6 \\ - 2x^9y^2 + 6x^7y^4 + 2x^5y^6 - 2x^3y^8 \\ + 3x^7y^4 - 9x^5y^6 - 3x^3y^8 + 3xy^{10} \\ \hline x^{11} - 5x^9y^2 + 8x^7y^4 - 6x^5y^6 - 5x^3y^8 + 3xy^{10} \end{array}$$

$$40. \begin{array}{r} 3a^5 - 6a^3 + 2a^2 - 3a + 2 \\ a^4 - 3a^2 + 4a - 5 \\ \hline 3a^9 - 6a^7 + 2a^6 - 3a^5 + 2a^4 \\ - 9a^7 + 18a^5 - 6a^4 + 9a^3 - 6a^2 \\ + 12a^6 - 24a^4 + 8a^3 - 12a^2 + 8a \\ - 15a^5 + 30a^3 - 10a^2 + 15a - 10 \\ \hline 3a^9 - 15a^7 + 14a^6 - 28a^4 + 47a^3 - 28a^2 + 23a - 10 \end{array}$$

$$41. \begin{array}{r} a + b - c \\ a - b + c \\ \hline a^2 + ab - ac \\ - ab - b^2 + bc \\ + ac + bc - c^2 \\ \hline a^2 - b^2 + 2bc - c^2 \end{array}$$

$$42. \begin{array}{r} x + 2y - z \\ x - y + z \\ \hline x^2 + 2xy - xz \\ - xy - 2y^2 + yz \\ + xz + 2yz - z^2 \\ \hline x^2 + xy - 2y^2 + 3yz - z^2 \end{array}$$

$$43. \begin{array}{r} 2x - 3y + 5z \\ - x + y + 2z \\ \hline - 2x^2 + 3xy - 5xz \\ + 2xy - 3y^2 + 5yz \\ + 4xz - 6yz + 10z^2 \\ \hline - 2x^2 + 5xy - xz - 3y^2 - yz + 10z^2 \end{array}$$

$$44. \begin{array}{r} x^2 - xy - xz + y^2 - yz + z^2 \\ x + y + z \\ \hline x^3 - x^2y - x^2z + xy^2 - xyz + xz^2 \\ + x^2y - xy^2 - xyz + y^3 - y^2z + yz^2 \\ + x^2z - xyz - xz^2 + y^2z - yz^2 + z^3 \\ \hline x^3 - 3xyz + y^3 + z^3 \end{array}$$

## EJERCICIO 43

1.  $a^{x+2} - a^{x+1} + a^x$

$$\begin{array}{r} a+1 \\ a^{x+3} - a^{x+2} + a^{x+1} \\ + a^{x+2} - a^{x+1} + a^x \\ \hline a^{x+3} \qquad \qquad + a^x \end{array}$$

2.  $-x^{n+3} + 2x^{n+2} + x^{n+1}$

$$\begin{array}{r} x^2 + x \\ -x^{n+5} + 2x^{n+4} + x^{n+3} \\ - x^{n+4} + 2x^{n+3} + x^{n+2} \\ \hline -x^{n+5} + x^{n+4} + 3x^{n+3} + x^{n+2} \end{array}$$

3.  $m^{a+2} + m^{a+1} - m^a + m^{a-1}$

$$\begin{array}{r} m^2 - 2m + 3 \\ m^{a+4} + m^{a+3} - m^{a+2} + m^{a+1} \\ - 2m^{a+3} - 2m^{a+2} + 2m^{a+1} - 2m^a \\ + 3m^{a+2} + 3m^{a+1} - 3m^a + 3m^{a-1} \\ \hline m^{a+4} - m^{a+3} \qquad + 6m^{a+1} - 5m^a + 3m^{a-1} \end{array}$$

4.  $a^{n+2} + 3a^{n+1} - 2a^n$

$$\begin{array}{r} a^{n+1} + a^n \\ a^{2n+3} + 3a^{2n+2} - 2a^{2n+1} \\ + a^{2n+2} + 3a^{2n+1} - 2a^{2n} \\ \hline a^{2n+3} + 4a^{2n+2} + a^{2n+1} - 2a^{2n} \end{array}$$

5.  $x^{a+2} + 2x^{a+1} - x^a$

$$\begin{array}{r} x^{a+3} - 2x^{a+1} \\ x^{2a+5} + 2x^{2a+4} - x^{2a+3} \\ - 2x^{2a+3} - 4x^{2a+2} + 2x^{2a+1} \\ \hline x^{2a+5} + 2x^{2a+4} - 3x^{2a+3} - 4x^{2a+2} + 2x^{2a+1} \end{array}$$

6.  $a^x - 2a^{x-1} + 3a^{x-2}$

$$\begin{array}{r} a^2 + 2a - 1 \\ a^{x+2} - 2a^{x+1} + 3a^x \\ + 2a^{x+1} - 4a^x + 6a^{x-1} \\ - a^x + 2a^{x-1} - 3a^{x-2} \\ \hline a^{x+2} \qquad \qquad - 2a^x + 8a^{x-1} - 3a^{x-2} \end{array}$$

7.  $a^x + 3a^{x-1} - 2a^{x-2}$

$$\begin{array}{r} a^x - a^{x-1} + a^{x-2} \\ a^{2x} + 3a^{2x-1} - 2a^{2x-2} \\ - a^{2x-1} - 3a^{2x-2} + 2a^{2x-3} \\ + a^{2x-2} + 3a^{2x-3} - 2a^{2x-4} \\ \hline a^{2x} + 2a^{2x-1} - 4a^{2x-2} + 5a^{2x-3} - 2a^{2x-4} \end{array}$$

8.  $m^{a+4} - m^{a+3} - 2m^{a+2} + m^{a+1}$

$$\begin{array}{r} -m^{a+1} + m^{a+2} + m^{a+3} \\ -m^{2a+3} + m^{2a+2} + 2m^{2a+1} - m^{2a} \\ + m^{2a+2} - m^{2a+1} - 2m^{2a} + m^{2a-1} \\ + m^{2a+1} - m^{2a} - 2m^{2a-1} + m^{2a-2} \\ \hline -m^{2a+3} + 2m^{2a+2} + 2m^{2a+1} - 4m^{2a} - m^{2a-1} + m^{2a-2} \end{array}$$

9.  $x^{a-1} + 2x^{a-2} - x^{a-3} + x^{a-4}$

$$\begin{array}{r} x^{a-1} - x^{a-2} - x^{a-3} \\ x^{2a-2} + 2x^{2a-3} - x^{2a-4} + x^{2a-5} \\ - x^{2a-3} - 2x^{2a-4} + x^{2a-5} - x^{2a-6} \\ - x^{2a-4} - 2x^{2a-5} + x^{2a-6} - x^{2a-7} \\ \hline x^{2a-2} + x^{2a-3} - 4x^{2a-4} \qquad \qquad - x^{2a-7} \end{array}$$

10.  $a^n b - a^{n-1} b^2 + 2a^{n-2} b^3 - a^{n-3} b^4$

$$\begin{array}{r} a^n b^2 - a^{n-2} b^4 \\ a^{2n} b^3 - a^{2n-1} b^4 + 2a^{2n-2} b^5 - a^{2n-3} b^6 \\ - a^{2n-2} b^5 + a^{2n-3} b^6 - 2a^{2n-4} b^7 + a^{2n-5} b^8 \\ \hline a^{2n} b^3 - a^{2n-1} b^4 + a^{2n-2} b^5 \qquad \qquad - 2a^{2n-4} b^7 + a^{2n-5} b^8 \end{array}$$

11.  $a^x + b^x$

$$\begin{array}{r} a^m + b^m \\ a^{m+x} + a^m b^x \\ + a^x b^m + b^{m+x} \\ \hline a^{m+x} + a^m b^x + a^x b^m + b^{m+x} \end{array}$$

12.  $a^{x-1} - b^{n-1}$

$$\begin{array}{r} a - b \\ a^x - ab^{n-1} \\ - a^{x-1} b + b^n \\ \hline a^x - ab^{n-1} - a^{x-1} b + b^n \end{array}$$

$$\begin{array}{r}
 13. \quad -5a^{2m+2} + a^{2m+1} + 3a^{2m} \\
 6a^{3m-1} - 8a^{3m-2} + a^{3m-3} \\
 \hline
 -30a^{5m+1} + 6a^{5m} + 18a^{5m-1} \\
 + 40a^{5m} - 8a^{5m-1} - 24a^{5m-2} \\
 - 5a^{5m-1} + a^{5m-2} + 3a^{5m-3} \\
 \hline
 -30a^{5m+1} + 46a^{5m} + 5a^{5m-1} - 23a^{5m-2} + 3a^{5m-3}
 \end{array}$$

$$\begin{array}{r}
 14. \quad x^{a+2}y^{x-1} - 4x^{a+1}y^x + 3x^ay^{x+1} \\
 - 2x^{2a-1}y^{x-2} - 4x^{2a-2}y^{x-1} - 10x^{2a-3}y^x \\
 \hline
 - 2x^{3a+1}y^{2x-3} + 8x^{3a}y^{2x-2} - 6x^{3a-1}y^{2x-1} \\
 - 4x^{3a}y^{2x-2} + 16x^{3a-1}y^{2x-1} - 12x^{3a-2}y^{2x} \\
 - 10x^{3a-1}y^{2x-1} + 40x^{3a-2}y^{2x} - 30x^{3a-3}y^{2x+1} \\
 \hline
 - 2x^{3a+1}y^{2x-3} + 4x^{3a}y^{2x-2} + 28x^{3a-2}y^{2x} - 30x^{3a-3}y^{2x+1}
 \end{array}$$

## EJERCICIO 44

$$\begin{array}{r}
 1. \quad \frac{1}{2}a - \frac{1}{3}b \\
 \frac{1}{3}a + \frac{1}{2}b \\
 \hline
 \frac{1}{6}a^2 - \frac{1}{9}ab \\
 + \frac{1}{4}ab - \frac{1}{6}b^2 \\
 \hline
 \frac{1}{6}a^2 - \frac{4-9}{36}ab - \frac{1}{6}b^2 \\
 = \frac{1}{6}a^2 + \frac{5}{36}ab - \frac{1}{6}b^2
 \end{array}$$

$$\begin{array}{r}
 2. \quad x - \frac{2}{5}y \\
 \frac{1}{3}x + \frac{5}{6}y \\
 \hline
 \frac{1}{3}x^2 - \frac{2}{15}xy \\
 + \frac{5}{6}xy - \frac{10}{30}y^2 \\
 \hline
 \frac{1}{3}x^2 - \frac{4-25}{30}xy - \frac{10}{30}y^2 \\
 = \frac{1}{3}x^2 + \frac{21}{30}xy - \frac{1}{3}y^2 \\
 = \frac{1}{3}x^2 + \frac{7}{10}xy - \frac{1}{3}y^2
 \end{array}$$

$$\begin{array}{r}
 3. \quad \frac{1}{2}x^2 - \frac{1}{3}xy + \frac{1}{4}y^2 \\
 \frac{2}{3}x - \frac{3}{2}y \\
 \hline
 \frac{2}{6}x^3 - \frac{2}{9}x^2y + \frac{2}{12}xy^2 \\
 - \frac{3}{4}x^2y + \frac{3}{6}xy^2 - \frac{3}{8}y^3 \\
 \hline
 \frac{2}{6}x^3 - \frac{8+27}{36}x^2y + \frac{2+6}{12}xy^2 - \frac{3}{8}y^3 \\
 = \frac{1}{3}x^3 - \frac{35}{36}x^2y + \frac{8}{12}xy^2 - \frac{3}{8}y^3 \\
 = \frac{1}{3}x^3 - \frac{35}{36}x^2y + \frac{2}{3}xy^2 - \frac{3}{8}y^3
 \end{array}$$

$$\begin{array}{r}
 4. \quad \frac{1}{4}a^2 - ab + \frac{2}{3}b^2 \\
 \frac{1}{4}a - \frac{3}{2}b \\
 \hline
 \frac{1}{16}a^3 - \frac{1}{4}a^2b + \frac{2}{12}ab^2 \\
 - \frac{3}{8}a^2b + \frac{3}{2}ab^2 - \frac{6}{6}b^3 \\
 \hline
 \frac{1}{16}a^3 - \frac{2+3}{8}a^2b + \frac{2+18}{12}ab^2 - b^3 \\
 = \frac{1}{16}a^3 - \frac{5}{8}a^2b + \frac{20}{12}ab^2 - b^3 \\
 = \frac{1}{16}a^3 - \frac{5}{8}a^2b + \frac{5}{3}ab^2 - b^3
 \end{array}$$

$$5. \frac{2}{5}m^2 + \frac{1}{3}mn - \frac{1}{2}n^2$$

$$\frac{3}{2}m^2 - mn + 2n^2$$

$$\frac{\frac{6}{10}m^4 + \frac{3}{6}m^3n - \frac{3}{4}m^2n^2}{- \frac{2}{5}m^3n - \frac{1}{3}m^2n^2 + \frac{4}{5}m^2n^2}$$

$$+ \frac{1}{2}mn^3 + \frac{2}{3}mn^3 - \frac{2}{2}n^4$$

$$\frac{\frac{6}{10}m^4 + \frac{15-12}{30}m^3n - \frac{45+20-48}{60}m^2n^2 + \frac{3+4}{6}mn^3 - \frac{2}{2}n^4}{= \frac{6}{10}m^4 + \frac{3}{30}m^3n - \frac{17}{60}m^2n^2 + \frac{7}{6}mn^3 - n^4}$$

$$= \frac{3}{5}m^4 + \frac{1}{10}m^3n - \frac{17}{60}m^2n^2 + \frac{7}{6}mn^3 - n^4$$

$$= \frac{3}{5}m^4 + \frac{1}{10}m^3n - \frac{17}{60}m^2n^2 + \frac{7}{6}mn^3 - n^4$$

$$6. \frac{3}{8}x^2 + \frac{1}{4}x - \frac{2}{5}$$

$$2x^3 - \frac{1}{3}x + 2$$

$$\frac{\frac{6}{8}x^5 + \frac{2}{4}x^4 - \frac{4}{5}x^3}{- \frac{3}{24}x^3 - \frac{1}{12}x^2 + \frac{2}{15}x}$$

$$+ \frac{6}{8}x^2 + \frac{2}{4}x - \frac{4}{5}$$

$$\frac{\frac{6}{8}x^5 + \frac{2}{4}x^4 - \frac{96+15}{120}x^3 - \frac{2-18}{24}x^2 + \frac{8+30}{60}x - \frac{4}{5}}{= \frac{3}{4}x^5 + \frac{1}{2}x^4 - \frac{111}{120}x^3 + \frac{16}{24}x^2 + \frac{38}{60}x - \frac{4}{5}}$$

$$= \frac{3}{4}x^5 + \frac{1}{2}x^4 - \frac{37}{40}x^3 + \frac{2}{3}x^2 + \frac{19}{30}x - \frac{4}{5}$$

$$= \frac{3}{4}x^5 + \frac{1}{2}x^4 - \frac{37}{40}x^3 + \frac{2}{3}x^2 + \frac{19}{30}x - \frac{4}{5}$$

$$7. -\frac{1}{2}x^2 + \frac{1}{3}ax + \frac{3}{2}a^2$$

$$\frac{3}{2}x^2 - ax + \frac{2}{3}a^2$$

$$- \frac{3}{4}x^4 + \frac{3}{6}ax^3 + \frac{9}{4}a^2x^2$$

$$+ \frac{1}{2}ax^3 - \frac{1}{3}a^2x^2 - \frac{3}{2}a^3x - \frac{2}{6}a^2x^2 + \frac{2}{9}a^3x + \frac{6}{6}a^4$$

$$- \frac{3}{4}x^4 + \frac{3+3}{6}ax^3 + \frac{27-4-4}{12}a^2x^2 - \frac{27-4}{18}a^3x + \frac{6}{6}a^4$$

$$= -\frac{3}{4}x^4 + \frac{6}{6}ax^3 + \frac{19}{12}a^2x^2 - \frac{23}{18}a^3x + \frac{6}{6}a^4$$

$$= -\frac{3}{4}x^4 + ax^3 + \frac{19}{12}a^2x^2 - \frac{23}{18}a^3x + a^4$$

$$8. \frac{2}{7}x^3 - \frac{1}{5}x^2y + \frac{1}{2}xy^2$$

$$\frac{1}{4}x^2 - \frac{2}{3}xy + \frac{5}{6}y^2$$

$$\frac{\frac{2}{28}x^5 - \frac{1}{20}x^4y + \frac{1}{8}x^3y^2}{- \frac{4}{21}x^4y + \frac{2}{15}x^3y^2 - \frac{2}{6}x^2y^3}$$

$$+ \frac{10}{42}x^3y^2 - \frac{5}{30}x^2y^3 + \frac{5}{12}xy^4$$

$$\frac{\frac{2}{28}x^5 - \frac{21+80}{420}x^4y + \frac{105+112+200}{840}x^3y^2 - \frac{10+5}{30}x^2y^3 + \frac{5}{12}xy^4}{= \frac{2}{28}x^5 - \frac{101}{420}x^4y + \frac{417}{840}x^3y^2 - \frac{15}{30}x^2y^3 + \frac{5}{12}xy^4}$$

$$= \frac{1}{14}x^5 - \frac{101}{420}x^4y + \frac{139}{280}x^3y^2 - \frac{1}{2}x^2y^3 + \frac{5}{12}xy^4$$

$$= \frac{1}{14}x^5 - \frac{101}{420}x^4y + \frac{139}{280}x^3y^2 - \frac{1}{2}x^2y^3 + \frac{5}{12}xy^4$$

$$9. \frac{1}{4}x^3 + \frac{1}{3}x^2 - \frac{1}{4}x + \frac{1}{2}$$

$$\frac{3}{2}x^2 + \frac{1}{10}x - \frac{1}{5}$$

$$\frac{\frac{3}{8}x^5 + \frac{3}{6}x^4 - \frac{3}{8}x^3 + \frac{3}{4}x^2}{+ \frac{1}{40}x^4 + \frac{1}{30}x^3 - \frac{1}{40}x^2 + \frac{1}{20}x}$$

$$- \frac{1}{20}x^3 - \frac{1}{15}x^2 + \frac{1}{20}x - \frac{1}{10}$$

$$\frac{\frac{3}{8}x^5 + \frac{60+3}{120}x^4 - \frac{45-4+6}{120}x^3 + \frac{90-3-8}{120}x^2 + \frac{1+1}{20}x - \frac{1}{10}}{= \frac{3}{8}x^5 + \frac{63}{120}x^4 - \frac{47}{120}x^3 + \frac{79}{120}x^2 + \frac{2}{20}x - \frac{1}{10} = \frac{3}{8}x^5 + \frac{21}{40}x^4 - \frac{47}{120}x^3 + \frac{79}{120}x^2 + \frac{1}{10}x - \frac{1}{10}}$$

$$= \frac{3}{8}x^5 + \frac{63}{120}x^4 - \frac{47}{120}x^3 + \frac{79}{120}x^2 + \frac{2}{20}x - \frac{1}{10} = \frac{3}{8}x^5 + \frac{21}{40}x^4 - \frac{47}{120}x^3 + \frac{79}{120}x^2 + \frac{1}{10}x - \frac{1}{10}$$



$$\begin{array}{r}
 10. \quad \frac{3}{4}m^3 - \frac{1}{2}m^2n + \frac{2}{5}mn^2 - \frac{1}{4}n^3 \\
 \frac{2}{3}m^2 - \frac{2}{3}mn + \frac{5}{2}n^2 \\
 \hline
 \frac{6}{12}m^5 - \frac{2}{6}m^4n + \frac{4}{15}m^3n^2 - \frac{2}{12}m^2n^3 \\
 - \frac{6}{12}m^4n + \frac{2}{6}m^3n^2 - \frac{4}{15}m^2n^3 + \frac{2}{12}mn^4 \\
 + \frac{15}{8}m^3n^2 - \frac{5}{4}m^2n^3 + \frac{10}{10}mn^4 - \frac{5}{8}n^5 \\
 \hline
 \frac{6}{12}m^5 - \frac{4+6}{12}m^4n + \frac{32+40+225}{120}m^3n^2 - \frac{10+16+75}{60}m^2n^3 + \frac{20+120}{120}mn^4 - \frac{5}{8}n^5 \\
 = \frac{1}{2}m^5 - \frac{10}{12}m^4n + \frac{297}{120}m^3n^2 - \frac{101}{60}m^2n^3 + \frac{140}{120}mn^4 - \frac{5}{8}n^5 = \frac{1}{2}m^5 - \frac{5}{6}m^4n + \frac{99}{40}m^3n^2 - \frac{101}{60}m^2n^3 + \frac{7}{6}mn^4 - \frac{5}{8}n^5
 \end{array}$$

## **EJERCICIO 45**

1.  $x^3 - x^2 + x$  por  $x^2 - 1$

$$\begin{array}{r}
 1-1+1 \\
 1+0-1 \\
 \hline
 1-1+1 \\
 -1+1-1 \\
 \hline
 1-1+0+1-1 \\
 =x^5 - x^4 + x^2 - x
 \end{array}$$

2.  $x^4 + 3x^3 - 5x^2 + 8$  por  $x^3 - 2x^2 - 7$

$$\begin{array}{r}
 1+3-5+0+8 \\
 1-2+0-7 \\
 \hline
 1+3-5+0+8 \\
 -2-6+10+0-16 \\
 -7-21+35+0-56 \\
 \hline
 1+1-11+3-13+19+0-56 \\
 =x^7 + x^6 - 11x^5 + 3x^4 - 13x^3 + 19x^2 - 56
 \end{array}$$

3.  $a^4 + 3a^3b - 2a^2b^2 + 5ab^3 - b^4$  por  $a^2 - 2ab + b^2$

$$\begin{array}{r}
 1+3-2+5-1 \\
 1-2+1 \\
 \hline
 1+3-2+5-1 \\
 -2-6+4-10+2 \\
 +1+3-2+5-1 \\
 \hline
 1+1-7+12-13+7-1 \\
 =a^6 + a^5b - 7a^4b^2 + 12a^3b^3 - 13a^2b^4 + 7ab^5 - b^6
 \end{array}$$

4.  $m^3 - 5m^2n + 6mn^2 + n^3$  por  $m^3 - 4mn^2 - n^3$

$$\begin{array}{r}
 1-5+6+1 \\
 1+0-4-1 \\
 \hline
 1-5+6+1 \\
 -4+20-24-4 \\
 -1+5-6-1 \\
 \hline
 1-5+2+20-19-10-1 \\
 =m^6 - 5m^5n + 2m^4n^2 + 20m^3n^3 - 19m^2n^4 - 10mn^5 - n^6
 \end{array}$$

5.  $x^4 - 8x^2 + 3$  por  $x^4 + 6x^2 - 5$

$$\begin{array}{r}
 1+0-8+0+3 \\
 1+0+6+0-5 \\
 \hline
 1+0-8+0+3 \\
 +6+0-48+0+18 \\
 -5+0+40+0-15 \\
 \hline
 1+0-2+0-50+0+58+0-15 \\
 =x^8 - 2x^6 - 50x^4 + 58x^2 - 15
 \end{array}$$

6.  $a^6 - 3a^4 - 6a^2 + 10$  por  $a^8 - 4a^6 + 3a^4 - 2a^2$

$$\begin{array}{r}
 1+0-3+0-6+0+10 \\
 1+0-4+0+3+0-2 \\
 \hline
 1+0-3+0-6+0+10 \\
 -4+0+12+0+24+0-40 \\
 +3+0-9+0-18+0+30 \\
 -2+0+6+0+12+0-20 \\
 \hline
 1+0-7+0+9+0+23+0-52+0+42+0-20 \\
 =a^{14} - 7a^{12} + 9a^{10} + 23a^8 - 52a^6 + 42a^4 - 20a^2
 \end{array}$$

7.  $x^3 - 4x^6 + 3x^3 - 2$  por  $3x^6 - 8x^3 + 10$

$$1+0+0-4+0+0+3+0+0-2$$

$$3+0+0-8+0+0+10$$

$$\hline 3+0+0-12+0+0+9+0+0-6$$

$$-8+0+0+32+0+0-24+0+0+16$$

$$+10+0+0-40+0+0+30+0+0-20$$

$$\hline 3+0+0-20+0+0+51+0+0-70+0+0+46+0+0-20$$

$$= 3x^{15} - 20x^{12} + 51x^9 - 70x^6 + 46x^3 - 20$$

8.  $m^{12} - 7m^8 + 9m^4 - 15$  por  $m^{16} - 5m^{12} + 9m^8 - 4m^4 + 3$

$$1-7+9-15$$

$$1-5+9-4+3$$

$$\hline 1-7+9-15$$

$$-5+35-45+75$$

$$+9-63+81-135$$

$$-4-28-36+60$$

$$+3-21+27-45$$

$$\hline 1-12+53-127+187-192+87-45$$

$$= m^{28} - 12m^{24} + 53m^{20} - 127m^{16} + 187m^{12} - 192m^8 + 87m^4 - 45$$

9.  $x^5 - 3x^4y - 6x^3y^2 - 4x^2y^3 - y^5$  por  $2x^2 + 4y^2$

$$1-3-6-4+0-1$$

$$2+0+4$$

$$\hline 2-6-12-8+0-2$$

$$+4-12-24-16+0-4$$

$$\hline 2-6-8-20-24-18+0-4$$

$$= 2x^7 - 6x^6y - 8x^5y^2 - 20x^4y^3 - 24x^3y^4 - 18x^2y^5 - 4y^7$$

10.  $6a^5 - 4a^2 + 6a - 2$  por  $a^4 - 2a^2 + a - 7$

$$6+0+0-4+6-2$$

$$1+0-2+1-7$$

$$\hline 6+0+0-4+6-2$$

$$-12+0+0+8-12+4$$

$$+6+0+0-4+6-2$$

$$-42+0+0+28-42+14$$

$$\hline 6+0-12+2-36+6-16+38-44+14$$

$$= 6a^9 - 12a^7 + 2a^6 - 36a^5 + 6a^4 - 16a^3 + 38a^2 - 44a + 14$$

11.  $n^6 - 3n^4 + 5n^3 - 8n + 4$  por  $n^4 - 3n^2 + 4$

$$1+0-3+5+0-8+4$$

$$1+0-3+0+4$$

$$\hline 1+0-3+5+0-8+4$$

$$-3+0+9-15+0+24-12$$

$$+4+0-12+20+0-32+16$$

$$\hline 1+0-6+5+13-23-8+44-12-32+16$$

$$= n^{10} - 6n^8 + 5n^7 + 13n^6 - 23n^5 - 8n^4 + 44n^3 - 12n^2 - 32n + 16$$

12.  $3x^4 - 4x^3y - y^4$  por  $x^3 - 5xy^2 + 3y^3$

$$3-4+0+0-1$$

$$1+0-5+3$$

$$\hline 3-4+0+0-1$$

$$-15+20+0+0+5$$

$$+9-12+0+0-3$$

$$\hline 3-4-15+29-13+0+5-3$$

$$= 3x^7 - 4x^6y - 15x^5y^2 + 29x^4y^3 - 13x^3y^4 + 5xy^6 - 3y^7$$

13.  $x^{10} - 5x^6y^4 + 3x^2y^8 - 6y^{10}$  por  $x^6 - 4x^4y^2 - 5x^2y^4 + y^6$

$$1+0+0+0-5+0+0+0+3+0-6$$

$$1+0-4+0-5+0+1$$

$$\hline 1+0+0+0-5+0+0+0+3+0-6$$

$$-4+0+0+0+20+0+0+0-12+0+24$$

$$-5+0+0+0+25+0+0+0-15+0+30$$

$$+1+0+0+0-5+0+0+0+3+0-6$$

$$\hline 1+0-4+0-10+0+21+0+28+0-23+0+9+0+33+0-6$$

$$= x^{16} - 4x^{14}y^2 - 10x^{12}y^4 + 21x^{10}y^6 + 28x^8y^8 - 23x^6y^{10} + 9x^4y^{12} + 33x^2y^{14} - 6y^{16}$$

14.  $a^m - 3a^{m-1} + 5a^{m-3}$  por  $a^2 - 5$

$$1-3+0+5$$

$$1+0-5$$

$$\hline 1-3+0+5$$

$$-5+15+0-25$$

$$\hline 1-3-5+20+0-25$$

$$= a^{m+2} - 3a^{m+1} - 5a^m + 20a^{m-1} - 25a^{m-3}$$

15.  $a^{x+2} - 5a^{x+1} - 7a^{x-1}$  por  $7a^{x+3} + 6a^{x+1} + a^x$

$$1-5+0-7$$

$$7+0+6+1$$

$$\hline 7-35+0-49$$

$$+6-30+0-42$$

$$+1-5+0-7$$

$$\hline 7-35+6-78-5-42-7$$

$$= 7a^{2x+5} - 35a^{2x+4} + 6a^{2x+3}$$

$$- 78a^{2x+2} - 5a^{2x+1} - 42a^{2x} - 7a^{2x-1}$$

$$\begin{array}{r}
 16. \ x^{a+2} - 5x^a - 6x^{a-2} \text{ por } 6x^{a+1} - 4x^a + 2x^{a-1} + x^{a-2} \\
 1+0-5+0-6 \\
 6-4+2+1 \\
 \hline
 6+0-30+0-36 \\
 -4+0+20+0+24 \\
 +2+0-10+0-12 \\
 +1+0-5+0-6 \\
 \hline
 6-4-28+21-46+19-12-6 \\
 = 6x^{2a+3} - 4x^{2a+2} - 28x^{2a+1} + 21x^{2a} \\
 - 46x^{2a-1} + 19x^{2a-2} - 12x^{2a-3} - 6x^{2a-4}
 \end{array}$$

$$\begin{array}{r}
 17. \ a^{2x+2} - 3a^{2x+1} - a^{2x} - 5a^{2x-1} \text{ por } 6a^{3x+1} - 5a^{3x} + 3a^{3x-1} \\
 1-3-1-5 \\
 6-5+3 \\
 \hline
 6-18-6-30 \\
 -5+15+5+25 \\
 +3-9-3-15 \\
 \hline
 6-23+12-34+22-15 \\
 = 6a^{5x+3} - 23a^{5x+2} + 12a^{5x+1} - 34a^{5x} + 22a^{5x-1} - 15a^{5x-2}
 \end{array}$$

## EJERCICIO 46

$$\begin{array}{r}
 1. \ 4(a+5)(a-3) \\
 4a+20 \\
 a-3 \\
 \hline
 4a^2+20a \\
 -12a-60 \\
 \hline
 4a^2+8a-60
 \end{array}$$

$$\begin{array}{r}
 2. \ 3a^2(x+1)(x-1) \\
 3a^2x+3a^2 \\
 x-1 \\
 \hline
 3a^2x^2+3a^2x \\
 -3a^2x-3a^2 \\
 \hline
 3a^2x^2-3a^2
 \end{array}$$

$$\begin{array}{r}
 3. \ 2(a-3)(a-1)(a+4) \\
 2a-6 \\
 a-1 \\
 \hline
 2a^2-6a \\
 -2a+6 \\
 \hline
 2a^2-8a+6 \\
 a+4 \\
 \hline
 2a^3-8a^2+6a \\
 +8a^2-32a+24 \\
 \hline
 2a^3-26a+24
 \end{array}$$

$$\begin{array}{r}
 4. \ (x^2+1)(x^2-1)(x^2+1) \\
 x^2+1 \\
 x^2+1 \\
 \hline
 x^4+x^2 \\
 +x^2+1 \\
 \hline
 x^4+2x^2+1 \\
 x^2-1 \\
 \hline
 x^6+2x^4+x^2 \\
 -x^4-2x^2-1 \\
 \hline
 x^6+x^4-x^2-1
 \end{array}$$

$$\begin{array}{r}
 5. \ m(m-4)(m-6)(3m+2) \\
 m^2-4m \\
 m-6 \\
 \hline
 m^3-4m^2 \\
 -6m^2+24m \\
 \hline
 m^3-10m^2+24m \\
 3m+2 \\
 \hline
 3m^4-30m^3+72m^2 \\
 +2m^3-20m^2+48m \\
 \hline
 3m^4-28m^3+52m^2+48m
 \end{array}$$

$$\begin{array}{r}
 6. \ (a-b)(a^2-2ab+b^2)(a+b) \\
 a^2-2ab+b^2 \\
 a+b \\
 \hline
 a^3-2a^2b+ab^2 \\
 +a^2b-2ab^2+b^3 \\
 \hline
 a^3-a^2b-ab^2+b^3 \\
 a-b \\
 \hline
 a^4-a^3b-a^2b^2+ab^3 \\
 -a^3b+a^2b^2+ab^3-b^4 \\
 \hline
 a^4-2a^3b+2ab^3-b^4
 \end{array}$$

$$\begin{array}{r}
 7. \ 3x(x^2-2x+1)(x-1)(x+1) \\
 3x^3-6x^2+3x \\
 x-1 \\
 \hline
 3x^4-6x^3+3x^2 \\
 -3x^3+6x^2-3x \\
 \hline
 3x^4-9x^3+9x^2-3x \\
 x+1 \\
 \hline
 3x^5-9x^4+9x^3-3x^2 \\
 +3x^4-9x^3+9x^2-3x \\
 \hline
 3x^5-6x^4+6x^2-3x
 \end{array}$$

$$8. (x^2 - x + 1)(x^2 + x - 1)(x - 2)$$

$$\begin{array}{r} x^2 - x + 1 \\ x^2 + x - 1 \\ \hline x^4 - x^3 + x^2 \\ + x^3 - x^2 + x \\ - x^2 + x - 1 \\ \hline x^4 - x^2 + 2x - 1 \\ x - 2 \\ \hline x^5 - x^3 + 2x^2 - x \\ - 2x^4 + 2x^2 - 4x + 2 \\ \hline x^5 - 2x^4 - x^3 + 4x^2 - 5x + 2 \end{array}$$

$$9. (a^m - 3)(a^{m-1} + 2)(a^{m-1} - 1)$$

$$\begin{array}{r} a^m - 3 \\ a^{m-1} + 2 \\ \hline a^{2m-1} - 3a^{m-1} \\ 2a^m - 6 \\ \hline 2a^m + a^{2m-1} - 3a^{m-1} - 6 \\ a^{m-1} - 1 \\ \hline 2a^{2m-1} + a^{3m-2} - 3a^{2m-2} - 6a^{m-1} \\ - a^{2m-1} + 3a^{m-1} - 2a^m + 6 \\ \hline a^{2m-1} + a^{3m-2} - 3a^{2m-2} - 3a^{m-1} - 2a^m + 6 \\ \Rightarrow a^{3m-2} + a^{2m-1} - 3a^{2m-2} - 2a^m - 3a^{m-1} + 6 \end{array}$$

$$10. a(a-1)(a-2)(a-3)$$

$$\begin{array}{r} a^2 - a \\ a - 2 \\ \hline a^3 - a^2 \\ - 2a^2 + 2a \\ \hline a^3 - 3a^2 + 2a \\ a - 3 \\ \hline a^4 - 3a^3 + 2a^2 \\ - 3a^3 + 9a^2 - 6a \\ \hline a^4 - 6a^3 + 11a^2 - 6a \end{array}$$

$$11. (x-3)(x+4)(x-5)(x+1)$$

$$\begin{array}{r} x-3 \quad x-5 \\ x+4 \quad x+1 \\ \hline x^2-3x \quad x^2-5x \\ + 4x-12 \quad + x-5 \\ \hline x^2+x-12 \quad x^2-4x-5 \\ x^2-4x-5 \\ \hline x^4+x^3-12x^2 \\ - 4x^3-4x^2+48x \\ - 5x^2-5x+60 \\ \hline x^4-3x^3-21x^2+43x+60 \end{array}$$

$$12. (x^2-3)(x^2+2x+1)(x-1)(x^2+3)$$

$$\begin{array}{r} x^2+2x+1 \quad x^2-3 \\ x^2+3 \quad x-1 \\ \hline x^4+2x^3+x^2 \quad x^3-x^2-3x+3 \\ + 3x^2+6x+3 \quad - x^2+3 \\ \hline x^4+2x^3+4x^2+6x+3 \quad x^3-x^2-3x+3 \\ x^3-x^2-3x+3 \\ \hline x^7+2x^6+4x^5+6x^4+3x^3 \\ - x^6-2x^5-4x^4-6x^3-3x^2 \\ - 3x^5-6x^4-12x^3-18x^2-9x \\ + 3x^4+6x^3+12x^2+18x+9 \\ \hline x^7+x^6-x^5-x^4-9x^3-9x^2+9x+9 \end{array}$$

$$13. 9a^2(3a-2)(2a+1)(a-1)(2a-1)$$

$$\begin{array}{r} 27a^3-18a^2 \quad 2a-1 \\ 2a+1 \quad a-1 \\ \hline 54a^4-36a^3 \quad 2a^2-a \\ + 27a^3-18a^2 \quad - 2a+1 \\ \hline 54a^4-9a^3-18a^2 \quad 2a^2-3a+1 \\ 2a^2-3a+1 \\ \hline 108a^6-18a^5-36a^4 \\ - 162a^5+27a^4+54a^3 \\ + 54a^4-9a^3-18a^2 \\ \hline 108a^6-180a^5+45a^4-45a^3-18a^2 \end{array}$$

$$14. a^x(a^{x+1}+b^{x+2})(a^{x+1}-b^{x+2})b^x$$

$$\begin{array}{r} a^{2x+1}+a^xb^{x+2} \\ a^{x+1}b^x-b^{2x+2} \\ \hline a^{3x+2}b^x+a^{2x+1}b^{2x+2} \\ - a^{2x+1}b^{2x+2}-a^xb^{3x+4} \\ \hline a^{3x+2}b^x \quad - a^xb^{3x+4} \end{array}$$

## EJERCICIO 47

$$\begin{aligned} 1. & 4(x+3)+5(x+2) \\ &= 4x+12+5x+10 \\ &= 9x+22 \end{aligned}$$

$$\begin{aligned} 2. & 6(x^2+4)-3(x^2+1)+5(x^2+2) \\ &= 6x^2+24-3x^2-3+5x^2+10 \\ &= 8x^2+31 \end{aligned}$$

$$\begin{aligned} 3. & a(a-x)+3a(x+2a)-a(x-3a) \\ &= a^2-ax+3ax+6a^2-ax+3a^2 \\ &= 10a^2+ax \end{aligned}$$

$$\begin{aligned} 4. & x^2(y^2+1)+y^2(x^2+1)-3x^2y^2 \\ &= x^2y^2+x^2+x^2y^2+y^2-3x^2y^2 \\ &= -x^2y^2+x^2+y^2 \end{aligned}$$

$$\begin{aligned} 5. & 4m^3-5mn^2+3m^2(m^2+n^2)-3m(m^2-n^2) \\ &= 4m^3-5mn^2+3m^4+3m^2n^2-3m^3+3mn^2 \\ &= 3m^4+m^3+3m^2n^2-2mn^2 \end{aligned}$$

$$\begin{aligned} 6. & y^2+x^2y^3-y^3(x^2+1)+y^2(x^2+1)-y^2(x^2-1) \\ &= y^2+x^2y^3-x^2y^3-y^3+x^2y^2+y^2-x^2y^2+y^2 \\ &= -y^3+3y^2 \end{aligned}$$

$$\begin{aligned} 7. & 5(x+2)-(x+1)(x+4)-6x \\ &= 5x+10-(x^2+4x+x+4)-6x \\ &= -x+10-x^2-4x-x-4 \\ &= -x^2-6x+6 \end{aligned}$$

$$\begin{aligned} 8. & (a+5)(a-5)-3(a+2)(a-2)+5(a+4) \\ &= a^2-5a+5a-25-3(a^2-2a+2a-4)+5a+20 \\ &= a^2+5a-5-3(a^2-4) \\ &= a^2+5a-5-3a^2+12 \\ &= -2a^2+5a+7 \end{aligned}$$

$$\begin{aligned} 9. & (a+b)(4a-3b)-(5a-2b)(3a+b)-(a+b)(3a-6b) \\ &= 4a^2+ab-3b^2-(15a^2-ab-2b^2)-(3a^2-3ab-6b^2) \\ &= 4a^2+ab-3b^2-15a^2+ab+2b^2-3a^2+3ab+6b^2 \\ &= -14a^2+5ab+5b^2 \end{aligned}$$

$$\begin{aligned} 10. & (a+c)^2-(a-c)^2 \\ &= (a+c)(a+c)-(a-c)(a-c) \\ &= a^2+ac+ac+c^2-(a^2-ac-ac+c^2) \\ &= a^2+2ac+c^2-a^2+ac+ac-c^2 \\ &= 4ac \end{aligned}$$

$$\begin{aligned} 11. & 3(x+y)^2-4(x-y)^2+3x^2-3y^2 \\ &= 3(x+y)(x+y)-4(x-y)(x-y)+3x^2-3y^2 \\ &= 3(x^2+xy+xy+y^2)-4(x^2-xy-xy+y^2)+3x^2-3y^2 \\ &= 3(x^2+2xy+y^2)-4(x^2-2xy+y^2)+3x^2-3y^2 \\ &= 3x^2+6xy+3y^2-4x^2+8xy-4y^2+3x^2-3y^2 \\ &= 2x^2+14xy-4y^2 \end{aligned}$$

$$\begin{aligned} 12. & (m+n)^2-(2m+n)^2+(m-4n)^2 \\ &= (m+n)(m+n)-(2m+n)(2m+n)+(m-4n)(m-4n) \\ &= m^2+2mn+n^2-(4m^2+4mn+n^2)+m^2-8mn+16n^2 \\ &= 2m^2-6mn+17n^2-4m^2-4mn-n^2 \\ &= -2m^2-10mn+16n^2 \end{aligned}$$

$$\begin{aligned} 13. & x(a+x)+3x(a+1)-(x+1)(a+2x)-(a-x)^2 \\ &= ax+x^2+3ax+3x-(ax+2x^2+a+2x)-(a-x)(a-x) \\ &= 4ax+x^2+3x-ax-2x^2-a-2x-(a^2-ax-ax+x^2) \\ &= 3ax-x^2+x-a-a^2+ax+ax-x^2 \\ &= -2x^2+x+5ax-a-a^2 \end{aligned}$$

$$\begin{aligned} 14. & (a+b-c)^2+(a-b+c)^2-(a+b+c)^2 \\ & \begin{array}{r} a+b-c \\ a+b-c \\ \hline a^2+ab \quad -ac \\ \quad +ab \quad \quad +b^2-bc \\ \quad \quad -ac \quad -bc+c^2 \\ \hline a^2+2ab-2ac+b^2-2bc+c^2 \end{array} \quad \begin{array}{r} a-b+c \\ a-b+c \\ \hline a^2-ab \quad +ac \\ \quad -ab \quad \quad +b^2-bc \\ \quad \quad +ac \quad -bc+c^2 \\ \hline a^2-2ab+2ac+b^2-2bc+c^2 \end{array} \\ & \begin{array}{r} a+b+c \\ a+b+c \\ \hline a^2+ab \quad +ac \\ \quad +ab \quad \quad +b^2+bc \\ \quad \quad +ac \quad +bc+c^2 \\ \hline a^2+2ab+2ac+b^2+2bc+c^2 \end{array} \\ & \begin{array}{r} a^2+2ab-2ac+b^2-2bc+c^2 \\ a^2-2ab+2ac+b^2-2bc+c^2 \\ -a^2-2ab-2ac-b^2-2bc-c^2 \\ \hline a^2-2ab-2ac+b^2-6bc+c^2 \\ \Rightarrow a^2+b^2+c^2-2ab-2ac-6bc \end{array} \end{aligned}$$

$$\begin{array}{r}
 15. \quad (x^2+x-3)^2 - (x^2-2+x)^2 + (x^2-x-3)^2 \\
 \begin{array}{r}
 x^2+x-3 \\
 \hline
 x^4+x^3-3x^2 \\
 +x^3+x^2-3x \\
 -3x^2-3x+9 \\
 \hline
 x^4+2x^3-5x^2-6x+9
 \end{array}
 \begin{array}{r}
 x^2+x-2 \\
 \hline
 x^4+x^3-2x^2 \\
 +x^3+x^2-2x \\
 -2x^2-2x+4 \\
 \hline
 x^4+2x^3-3x^2-4x+4
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 x^2-x-3 \\
 \hline
 x^4-x^3-3x^2 \\
 -x^3+x^2+3x \\
 -3x^2+3x+9 \\
 \hline
 x^4-2x^3-5x^2+6x+9
 \end{array}
 \begin{array}{r}
 x^4+2x^3-5x^2-6x+9 \\
 -x^4-2x^3+3x^2+4x-4 \\
 \hline
 x^4-2x^3-5x^2+6x+9 \\
 \hline
 x^4-2x^3-7x^2+4x+14
 \end{array}$$

$$\begin{aligned}
 17. \quad & [x+(2x-3)][3x-(x+1)]+4x-x^2 \\
 & = [x+2x-3][3x-x-1]+4x-x^2 \\
 & = [3x-3][2x-1]+4x-x^2 \\
 & = 6x^2-3x-6x+3+4x-x^2 \\
 & = 5x^2-5x+3
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & [(m+n)(m-n)-(m+n)(m+n)][2(m+n)-3(m-n)] \\
 & = [m^2-n^2-(m^2+2mn+n^2)][2m+2n-3m+3n] \\
 & = [m^2-n^2-m^2-2mn-n^2][-m+5n] \\
 & = [-2mn-2n^2][-m+5n] \\
 & = 2m^2n-10mn^2+2mn^2-10n^3 \\
 & = 2m^2n-8mn^2-10n^3
 \end{aligned}$$

## EJERCICIO 48

$$\begin{aligned}
 1. \quad & x-[3a+2(-x+1)] \\
 & = x-[3a-2x+2] \\
 & = x-3a+2x-2 \\
 & = 3x-3a-2
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & -(a+b)-3[2a+b(-a+2)] \\
 & = -a-b-3[2a-ab+2b] \\
 & = -a-b-6a+3ab-6b \\
 & = -7a-7b+3ab \Rightarrow -7a+3ab-7b
 \end{aligned}$$

$$\begin{array}{r}
 16. \quad (x+y+z)^2 - (x+y)(x-y) + 3(x^2+xy+y^2) \\
 \begin{array}{r}
 x+y+z \\
 \hline
 x^2+xy+xz \\
 +xy \quad +y^2+yz \\
 +xz \quad +yz \quad +z^2 \\
 \hline
 x^2+2xy+2xz+y^2+2yz+z^2
 \end{array}
 \begin{array}{r}
 x+y \\
 \hline
 x-y \\
 \hline
 x^2+xy \\
 -xy-y^2 \\
 \hline
 x^2-y^2
 \end{array}
 \end{array}$$

$$\begin{aligned}
 & x^2+2xy+2xz+y^2+2yz+z^2 \\
 & -x^2 \quad +y^2 \\
 & 3x^2+3xy \quad +3y^2 \\
 & \hline
 & 3x^2+5xy+2xz+5y^2+2yz+z^2 \\
 & \Rightarrow 3x^2+5y^2+z^2+5xy+2xz+2yz
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & [3(x+2)-4(x+1)][3(x+4)-2(x+2)] \\
 & = [3x+6-4x-4][3x+12-2x-4] \\
 & = [-x+2][x+8] \\
 & = -x^2-8x+2x+16 \\
 & = -x^2-6x+16
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & [(x+y)^2-3(x-y)^2][(x+y)(x-y)+x(y-x)] \\
 & = [x^2+2xy+y^2-3(x^2-2xy+y^2)][x^2-y^2+xy-x^2] \\
 & = [x^2+2xy+y^2-3x^2+6xy-3y^2][xy-y^2] \\
 & = [-2x^2+8xy-2y^2][xy-y^2] \\
 & = -2x^3y+2x^2y^2+8x^2y^2-8xy^3-2xy^3+2y^4 \\
 & = -2x^3y+10x^2y^2-10xy^3+2y^4
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & -[3x-2y+(x-2y)-2(x+y)-3(2x+1)] \\
 & = -[3x-2y+x-2y-2x-2y-6x-3] \\
 & = -[-4x-6y-3] \\
 & = 4x+6y+3
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & 4x^2 - \left\{ -3x+5 - [-x+x(2-x)] \right\} \\
 & = 4x^2 - \left\{ -3x+5 - [-x+2x-x^2] \right\} \\
 & = 4x^2 - \left\{ -3x+5+x-2x+x^2 \right\} \\
 & = 4x^2 - \left\{ -4x+5+x^2 \right\} \\
 & = 4x^2+4x-5-x^2 \\
 & = 3x^2+4x-5
 \end{aligned}$$

$$\begin{aligned}
5. & 2a - \left\{ -3x + 2 \left[ -a + 3x - 2(-a + b - (2 + a)) \right] \right\} \\
&= 2a - \left\{ -3x + 2 \left[ -a + 3x - 2(-a + b - 2 - a) \right] \right\} \\
&= 2a - \left\{ -3x + 2 \left[ -a + 3x + 2a - 2b + 4 + 2a \right] \right\} \\
&= 2a - \left\{ -3x + 2 \left[ 3a + 3x - 2b + 4 \right] \right\} \\
&= 2a - \left\{ -3x + 6a + 6x - 4b + 8 \right\} \\
&= 2a + 3x - 6a - 6x + 4b - 8 \\
&= -4a - 3x + 4b - 8 \Rightarrow -4a + 4b - 3x - 8
\end{aligned}$$

$$\begin{aligned}
6. & a - (x + y) - 3(x - y) + 2 \left[ -(x - 2y) - 2(-x - y) \right] \\
&= a - x - y - 3x + 3y + 2 \left[ -x + 2y + 2x + 2y \right] \\
&= a - 4x + 2y + 2 \left[ x + 4y \right] \\
&= a - 4x + 2y + 2x + 8y \\
&= a - 2x + 10y
\end{aligned}$$

$$\begin{aligned}
7. & m - (m + n) - 3 \left\{ -2m + \left[ -2m + n + 2(-1 + n) - (m + n - 1) \right] \right\} \\
&= m - m - n - 3 \left\{ -2m + \left[ -2m + n - 2 + 2n - m - n + 1 \right] \right\} \\
&= -n - 3 \left\{ -2m + \left[ -3m + 2n - 1 \right] \right\} \\
&= -n - 3 \left\{ -2m - 3m + 2n - 1 \right\} \\
&= -n - 3 \left\{ -5m + 2n - 1 \right\} \\
&= -n + 15m - 6n + 3 \\
&= -7n + 15m + 3 \Rightarrow 15m - 7n + 3
\end{aligned}$$

$$\begin{aligned}
8. & -2(a - b) - 3(a + 2b) - 4 \left\{ a - 2b + 2 \left[ -a + b - 1 + 2(a - b) \right] \right\} \\
&= -2a + 2b - 3a - 6b - 4 \left\{ a - 2b + 2 \left[ -a + b - 1 + 2a - 2b \right] \right\} \\
&= -5a - 4b - 4 \left\{ a - 2b + 2 \left[ a - b - 1 \right] \right\} \\
&= -5a - 4b - 4 \left\{ a - 2b + 2a - 2b - 2 \right\} \\
&= -5a - 4b - 4 \left\{ 3a - 4b - 2 \right\} \\
&= -5a - 4b - 12a + 16b + 8 \\
&= -17a + 12b + 8
\end{aligned}$$

$$\begin{aligned}
9. & -5(x + y) - \left[ 2x - y + 2 \left\{ -x + y - 3 - (x - y - 1) \right\} \right] + 2x \\
&= -5x - 5y - \left[ 2x - y + 2 \left\{ -x + y - 3 - x + y + 1 \right\} \right] + 2x \\
&= -3x - 5y - \left[ 2x - y + 2 \left\{ -2x + 2y - 2 \right\} \right] \\
&= -3x - 5y - \left[ 2x - y - 4x + 4y - 4 \right] \\
&= -3x - 5y - \left[ -2x + 3y - 4 \right] \\
&= -3x - 5y + 2x - 3y + 4 \\
&= -x - 8y + 4
\end{aligned}$$

$$\begin{aligned}
10. & m - 3(m + n) + \left\{ - \left\{ -(-2m + n - 2 - 3 \left[ m - n + 1 \right]) + m \right\} \right\} \\
&= m - 3m - 3n + \left\{ - \left\{ -(-2m + n - 2 - 3m + 3n - 3) + m \right\} \right\} \\
&= -2m - 3n + \left\{ - \left\{ -(-5m + 4n - 5) + m \right\} \right\} \\
&= -2m - 3n + \left\{ - \left\{ 5m - 4n + 5 + m \right\} \right\} \\
&= -2m - 3n + \left\{ - \left\{ 6m - 4n + 5 \right\} \right\} \\
&= -2m - 3n + \left\{ -6m + 4n - 5 \right\} \\
&= -2m - 3n - 6m + 4n - 5 \\
&= -8m + n - 5
\end{aligned}$$

$$\begin{aligned}
11. & -3(x - 2y) + 2 \left\{ -4 \left[ -2x - 3(x + y) \right] \right\} - \left\{ - \left[ -(x + y) \right] \right\} \\
&= -3x + 6y + 2 \left\{ -4 \left[ -2x - 3x - 3y \right] \right\} - \left\{ - \left[ -x - y \right] \right\} \\
&= -3x + 6y + 2 \left\{ -4 \left[ -5x - 3y \right] \right\} - \left\{ x + y \right\} \\
&= -3x + 6y + 2 \left\{ 20x + 12y \right\} - x - y \\
&= -4x + 5y + 40x + 24y \\
&= 36x + 29y
\end{aligned}$$

$$\begin{aligned}
12. & 5 \left\{ -(a + b) - 3 \left[ -2a + 3b - (a + b) + (-a - b) + 2(-a + b) \right] - a \right\} \\
&= 5 \left\{ -a - b - 3 \left[ -2a + 3b - a - b - a - b - 2a + 2b \right] - a \right\} \\
&= 5 \left\{ -a - b - 3 \left[ -6a + 3b \right] - a \right\} \\
&= 5 \left\{ -2a - b + 18a - 9b \right\} \\
&= 5 \left\{ 16a - 10b \right\} \\
&= 80a - 50b
\end{aligned}$$

$$\begin{aligned}
13. & -3 \left\{ - \left[ +(-a + b) \right] \right\} - 4 \left\{ - \left[ -(-a - b) \right] \right\} \\
&= -3 \left\{ - \left[ -a + b \right] \right\} - 4 \left\{ - \left[ a + b \right] \right\} \\
&= -3 \left\{ a - b \right\} - 4 \left\{ -a - b \right\} \\
&= -3a + 3b + 4a + 4b \\
&= a + 7b
\end{aligned}$$

$$\begin{aligned}
14. & - \left\{ a + b - 2(a - b) + 3 \left\{ - \left[ 2a + b - 3(a + b - 1) \right] \right\} - 3 \left[ -a + 2(-1 + a) \right] \right\} \\
&= - \left\{ a + b - 2a + 2b + 3 \left\{ - \left[ 2a + b - 3a - 3b + 3 \right] \right\} - 3 \left[ -a - 2 + 2a \right] \right\} \\
&= - \left\{ -a + 3b + 3 \left\{ a + 2b - 3 \right\} - 3a + 6 \right\} \\
&= - \left\{ -4a + 3b + 6 + 3a + 6b - 9 \right\} \\
&= - \left\{ -a + 9b - 3 \right\} \\
&= a - 9b + 3
\end{aligned}$$

## EJERCICIO 49

- $-24 \div 8 = -3$
- $-63 \div -7 = 9$
- $-5a^2 \div -a = 5a^{2-1} = 5a$
- $14a^3b^4 \div -2ab^2 = -7a^{3-1}b^{4-2} = -7a^2b^2$
- $-a^3b^4c \div a^3b^4 = -a^{3-3}b^{4-4}c = -c$
- $-a^2b \div -ab = a^{2-1}b^{1-1} = a$
- $54x^2y^2z^3 \div -6xy^2z^3 = -9x^{2-1}y^{2-2}z^{3-3} = -9x$
- $-5m^2n \div m^2n = -5m^{2-2}n^{1-1} = -5$
- $-8a^2x^3 \div -8a^2x^3 = a^{2-2}x^{3-3} = 1$
- $-xy^2 \div 2y = -\frac{xy^2-1}{2} = -\frac{xy}{2}$
- $5x^4y^5 \div -6x^4y = -\frac{5}{6}x^{4-4}y^{5-1} = -\frac{5}{6}y^4$
- $-a^8b^9c^4 \div 8c^4 = -\frac{1}{8}a^8b^9c^{4-4} = -\frac{1}{8}a^8b^9$
- $16m^6n^4 \div -5n^3 = -\frac{16}{5}m^6n^{4-3} = -\frac{16}{5}m^6n$
- $-108a^7b^6c^8 \div -20b^6c^8 = \frac{108}{20}a^7b^{6-6}c^{8-8} = \frac{27}{5}a^7$
- $-2m^2n^6 \div -3mn^6 = \frac{2}{3}m^{2-1}n^{6-6} = \frac{2}{3}m$
- $a^x \div a^2 = a^{x-2}$
- $-3a^xb^m \div ab^2 = -3a^{x-1}b^{m-2}$
- $5a^mb^nc \div -6a^3b^4c = -\frac{5}{6}a^{m-3}b^{n-4}c^{1-1} = -\frac{5}{6}a^{m-3}b^{n-4}$
- $a^xb^m \div -4a^mb^n = -\frac{1}{4}a^{x-m}b^{m-n}$
- $-3m^an^xx^3 \div -5m^xn^2x^3 = \frac{3}{5}m^{a-x}n^{x-2}x^{3-3} = \frac{3}{5}m^{a-x}n^{x-2}$

## EJERCICIO 50

- $a^{m+3} \div a^{m+2} = a^{m+3-1} = a^{m+2}$
- $-3a^{m-2} \div -5a^{m-5} = \frac{3}{5}a^{m-2-1} = \frac{3}{5}a^{m-3}$
- $-4a^{x-2}b^n \div -5a^3b^2 = \frac{4}{5}a^{x-2-3}b^{n-2} = \frac{4}{5}a^{x-5}b^{n-2}$
- $5a^{2m-1}b^{x-3} \div -6a^{2m-2}b^{x-4} = -\frac{5}{6}a^{2m-1-2m+2}b^{x-3-x+4} = -\frac{5}{6}ab$
- $a^{m+n}b^{x+a} \div a^mb^a = a^{m+n-m}b^{x+a-a} = a^n b^x$
- $2x^{a+4} \div -x^{a+2} = -2x^{a+4-1} = -2x^{a+3}$
- $x^{2n+3} \div -4x^{n+4} = -\frac{1}{4}x^{2n+3-n-4} = -\frac{1}{4}x^{n-1}$
- $-7x^{m+3}y^{m-1} \div -8x^4y^2 = \frac{7}{8}x^{m+3-4}y^{m-1-2} = \frac{7}{8}x^{m-1}y^{m-3}$
- $-4x^{n-1}y^{n+1} \div 5x^{n-1}y^{n+1} = -\frac{4}{5}x^{n-1-n+1}y^{n+1-n-1} = -\frac{4}{5}$
- $-5ab^2c^3 \div 6a^mb^nc^x = -\frac{5}{6}a^{1-m}b^{2-n}c^{3-x}$

## EJERCICIO 51

- $\frac{1}{2}x^2 \div \frac{2}{3} = \frac{1}{2}x^2 \cdot \frac{3}{2} = \frac{3}{4}x^2$
- $-\frac{3}{5}a^3b \div -\frac{4}{5}a^2b = \frac{3}{5}a^{3-2}b^{1-1} = \frac{3}{5}a$
- $\frac{2}{3}xy^5z^3 \div -\frac{1}{6}z^3 = -\frac{2}{3}xy^5z^{3-3} = -\frac{2}{3}xy^5 = -4xy^5$
- $\frac{7}{8}a^mb^n \div -\frac{3}{4}ab^2 = -\frac{7}{8}a^{m-1}b^{n-2} = -\frac{7}{8}a^{m-1}b^{n-2}$
- $-\frac{2}{9}x^4y^5 \div -2 = \frac{2}{9}x^4y^5 = \frac{2}{18}x^4y^5 = \frac{1}{9}x^4y^5$
- $3m^4n^5p^6 \div -\frac{1}{3}m^4np^5 = -\frac{3}{1}m^{4-4}n^{5-1}p^{6-5} = -9n^4p$
- $-\frac{7}{8}a^2b^5c^6 \div -\frac{5}{2}ab^5c^6 = \frac{7}{8}a^{2-1}b^{5-5}c^{6-6} = \frac{7}{8}a$
- $\frac{2}{3}a^xb^m \div -\frac{3}{5}ab^2 = -\frac{2}{3}a^{x-1}b^{m-2} = -\frac{10}{9}a^{x-1}b^{m-2}$



$$9. -\frac{3}{8}c^3d^5 + \frac{3}{4}d^x = -\frac{\frac{3}{8}c^3d^{5-x}}{\frac{4}{4}} = -\frac{12}{24}c^3d^{5-x} = -\frac{1}{2}c^3d^{5-x}$$

$$10. \frac{3}{4}a^mb^n + -\frac{3}{2}b^3 = -\frac{\frac{3}{4}a^mb^{n-3}}{\frac{2}{2}} = -\frac{6}{12}a^mb^{n-3} = -\frac{1}{2}a^mb^{n-3}$$

$$11. -2a^{x+4}b^{m-3} + -\frac{1}{2}a^4b^3 = \frac{2}{1}a^{x+4-4}b^{m-3-3} = 4a^xb^{m-6}$$

$$12. -\frac{1}{15}a^{x-3}b^{m+5}c^2 + \frac{3}{5}a^{x-4}b^{m-1} = -\frac{\frac{1}{15}a^{x-3-x+4}b^{m+5-m+1}c^2}{\frac{5}{5}} = -\frac{5}{45}ab^6c^2 = -\frac{1}{9}ab^6c^2$$

## EJERCICIO 52

$$1. a^2 - ab \div a = \frac{a^2 - ab}{a} = \frac{a^2}{a} - \frac{ab}{a} = a - b$$

$$2. \frac{3x^2y^3 - 5a^2x^4}{-3x^2} \div -3x^2 = \frac{3x^2y^3 - 5a^2x^4}{-3x^2} \cdot \frac{1}{-3x^2} = \frac{3x^2y^3}{-3x^2} - \frac{5a^2x^4}{-3x^2} = -y^3 + \frac{5}{3}a^2x^2$$

$$3. \frac{3a^3 - 5ab^2 - 6a^2b^3}{-2a} = \frac{3a^3}{-2a} - \frac{5ab^2}{-2a} - \frac{6a^2b^3}{-2a} = -\frac{3}{2}a^2 + \frac{5}{2}b^2 + 3ab^3$$

$$4. \frac{x^3 - 4x^2 + x}{x} = \frac{x^3}{x} - \frac{4x^2}{x} + \frac{x}{x} = x^2 - 4x + 1$$

$$5. \frac{4x^8 - 10x^6 - 5x^4}{2x^3} = \frac{4x^8}{2x^3} - \frac{10x^6}{2x^3} - \frac{5x^4}{2x^3} = 2x^5 - 5x^3 - \frac{5}{2}x$$

$$6. \frac{6m^3 - 8m^2n + 20mn^2}{-2m} = \frac{6m^3}{-2m} - \frac{8m^2n}{-2m} + \frac{20mn^2}{-2m} = -3m^2 + 4mn - 10n^2$$

$$7. \frac{6a^8b^8 - 3a^6b^6 - a^2b^3}{3a^2b^3} = \frac{6a^8b^8}{3a^2b^3} - \frac{3a^6b^6}{3a^2b^3} - \frac{a^2b^3}{3a^2b^3} = 2a^6b^5 - a^4b^3 - \frac{1}{3}$$

$$8. \frac{x^4 - 5x^3 - 10x^2 + 15x}{-5x} = \frac{x^4}{-5x} - \frac{5x^3}{-5x} - \frac{10x^2}{-5x} + \frac{15x}{-5x} = -\frac{1}{5}x^3 + x^2 + 2x - 3$$

$$9. \frac{8m^9n^2 - 10m^7n^4 - 20m^5n^6 + 12m^3n^8}{2m^2} = \frac{8m^9n^2}{2m^2} - \frac{10m^7n^4}{2m^2} - \frac{20m^5n^6}{2m^2} + \frac{12m^3n^8}{2m^2} = 4m^7n^2 - 5m^5n^4 - 10m^3n^6 + 6mn^8$$

$$10. \frac{a^x + a^{m-1}}{a^2} = \frac{a^x}{a^2} + \frac{a^{m-1}}{a^2} = a^{x-2} + a^{m-3}$$

$$11. \frac{2a^m - 3a^{m+2} + 6a^{m+4}}{-3a^3} = \frac{2a^m}{-3a^3} - \frac{3a^{m+2}}{-3a^3} + \frac{6a^{m+4}}{-3a^3} = -\frac{2}{3}a^{m-3} + a^{m-1} - 2a^{m+1}$$

$$12. \frac{a^mb^n + a^{m-1}b^{n+2} - a^{m-2}b^{n+4}}{a^2b^3} = \frac{a^mb^n}{a^2b^3} + \frac{a^{m-1}b^{n+2}}{a^2b^3} - \frac{a^{m-2}b^{n+4}}{a^2b^3} = a^{m-2}b^{n-3} + a^{m-3}b^{n-1} - a^{m-4}b^{n+1}$$

$$13. \frac{x^{m+2} - 5x^m + 6x^{m+1} - x^{m-1}}{x^{m-2}} = \frac{x^{m+2}}{x^{m-2}} - \frac{5x^m}{x^{m-2}} + \frac{6x^{m+1}}{x^{m-2}} - \frac{x^{m-1}}{x^{m-2}} = x^4 - 5x^2 + 6x^3 - x \Rightarrow x^4 + 6x^3 - 5x^2 - x$$

$$14. \frac{4a^{x+4}b^{m-1} - 6a^{x+3}b^{m-2} + 8a^{x+2}b^{m-3}}{-2a^{x+2}b^{m-4}} = \frac{4a^{x+4}b^{m-1}}{-2a^{x+2}b^{m-4}} - \frac{6a^{x+3}b^{m-2}}{-2a^{x+2}b^{m-4}} + \frac{8a^{x+2}b^{m-3}}{-2a^{x+2}b^{m-4}} = -2a^2b^3 + 3ab^2 - 4b$$

## EJERCICIO 53

$$1. \frac{\frac{1}{2}x^2}{\frac{2}{3}x} - \frac{\frac{2}{3}x}{\frac{2}{3}x} = \frac{3}{4}x - \frac{6}{6} = \frac{3}{4}x - 1$$

$$2. \frac{\frac{1}{3}a^3}{-\frac{3}{5}} - \frac{\frac{3}{5}a^2}{-\frac{3}{5}} + \frac{\frac{1}{4}a}{-\frac{3}{5}} = -\frac{5}{9}a^3 + \frac{15}{15}a^2 - \frac{5}{12}a = -\frac{5}{9}a^3 + a^2 - \frac{5}{12}a$$

$$3. \frac{\frac{1}{4}m^4}{\frac{1}{4}m^2} - \frac{\frac{2}{3}m^3n}{\frac{1}{4}m^2} + \frac{\frac{3}{8}m^2n^2}{\frac{1}{4}m^2}$$

$$= \frac{4}{4}m^2 - \frac{8}{3}mn + \frac{12}{8}n^2 = m^2 - \frac{8}{3}mn + \frac{3}{2}n^2$$

$$4. \frac{\frac{2}{3}x^4y^3}{-\frac{1}{5}xy^3} - \frac{\frac{1}{5}x^3y^4}{-\frac{1}{5}xy^3} + \frac{\frac{1}{4}x^2y^5}{-\frac{1}{5}xy^3} - \frac{xy^6}{-\frac{1}{5}xy^3}$$

$$= -\frac{10}{3}x^3 + \frac{5}{5}x^2y - \frac{5}{4}xy^2 + 5y^3$$

$$= -\frac{10}{3}x^3 + x^2y - \frac{5}{4}xy^2 + 5y^3$$

$$5. \frac{\frac{2}{5}a^5}{5a} - \frac{\frac{1}{3}a^3b^3}{5a} - \frac{ab^5}{5a} = \frac{2}{25}a^4 - \frac{1}{15}a^2b^3 - \frac{1}{5}b^5$$

$$6. \frac{\frac{1}{3}a^m}{\frac{1}{2}a} + \frac{\frac{1}{4}a^{m-1}}{\frac{1}{2}a} = \frac{2}{3}a^{m-1} + \frac{2}{4}a^{m-2} = \frac{2}{3}a^{m-1} + \frac{1}{2}a^{m-2}$$

$$7. \frac{\frac{2}{3}a^{x+1}}{\frac{1}{6}a^{x-2}} - \frac{\frac{1}{4}a^{x-1}}{\frac{1}{6}a^{x-2}} - \frac{\frac{2}{5}a^x}{\frac{1}{6}a^{x-2}}$$

$$= \frac{12}{3}a^3 - \frac{6}{4}a - \frac{12}{5}a^2 \Rightarrow 4a^3 - \frac{12}{5}a^2 - \frac{3}{2}a$$

$$8. \frac{-\frac{3}{4}a^{n-1}x^{m+2}}{-\frac{2}{5}a^3x^2} + \frac{\frac{1}{8}a^n x^{m+1}}{-\frac{2}{5}a^3x^2} - \frac{\frac{2}{3}a^{n+1}x^m}{-\frac{2}{5}a^3x^2}$$

$$= \frac{15}{8}a^{n-4}x^m - \frac{5}{16}a^{n-3}x^{m-1} + \frac{10}{6}a^{n-2}x^{m-2}$$

$$= \frac{15}{8}a^{n-4}x^m - \frac{5}{16}a^{n-3}x^{m-1} + \frac{5}{3}a^{n-2}x^{m-2}$$

## EJERCICIO 54

$$1. \begin{array}{r|l} a^2+2a-3 & a+3 \\ -a^2-3a & a-1 \\ \hline -a-3 & \\ +a+3 & \end{array}$$

$$2. \begin{array}{r|l} a^2-2a-3 & a+1 \\ -a^2-a & a-3 \\ \hline -3a-3 & \\ +3a+3 & \end{array}$$

$$3. \begin{array}{r|l} x^2+x-20 & x+5 \\ -x^2-5x & x-4 \\ \hline -4x-20 & \\ +4x+20 & \end{array}$$

$$4. \begin{array}{r|l} m^2-11m+30 & m-6 \\ -m^2+6m & m-5 \\ \hline -5m+30 & \\ +5m-30 & \end{array}$$

$$5. \begin{array}{r|l} x^2-8x+15 & -x+3 \\ -x^2+3x & -x+5 \\ \hline -5x+15 & \\ +5x-15 & \end{array}$$

$$6. \begin{array}{r|l} a^2+5a+6 & a+2 \\ -a^2-2a & a+3 \\ \hline 3a+6 & \\ -3a-6 & \end{array}$$

$$7. \begin{array}{r|l} 6x^2-xy-2y^2 & 2x+y \\ -6x^2-3xy & 3x-2y \\ \hline -4xy-2y^2 & \\ +4xy+2y^2 & \end{array}$$

$$8. \begin{array}{r|l} -15x^2+22xy-8y^2 & -3x+2y \\ +15x^2-10xy & 5x-4y \\ \hline 12xy-8y^2 & \\ -12xy+8y^2 & \end{array}$$

$$9. \begin{array}{r|l} 5a^2+8ab-21b^2 & a+3b \\ -5a^2-15ab & 5a-7b \\ \hline -7ab-21b^2 & \\ +7ab+21b^2 & \end{array}$$

$$10. \begin{array}{r|l} 14x^2+22x-12 & 7x-3 \\ -14x^2+6x & 2x+4 \\ \hline 28x-12 & \\ -28x+12 & \end{array}$$

$$11. \begin{array}{r|l} -8a^2+12ab-4b^2 & -a+b \\ +8a^2-8ab & 8a-4b \\ \hline 4ab-4b^2 & \\ -4ab+4b^2 & \end{array}$$

$$12. \begin{array}{r|l} 6m^2-11mn+5n^2 & m-n \\ -6m^2+6mn & 6m-5n \\ \hline -5mn+5n^2 & \\ +5mn-5n^2 & \end{array}$$

$$13. \begin{array}{r|l} -54m^2+12mn+32n^2 & -9m+8n \\ +54m^2-48mn & 6m+4n \\ \hline -36mn+32n^2 & \\ +36mn-32n^2 & \end{array}$$

$$14. \begin{array}{r|l} -14y^2+71y+33 & -7y-3 \\ +14y^2+6y & 2y-11 \\ \hline +77y+33 & \\ -77y-33 & \end{array}$$

$$\begin{array}{r}
 15. \quad \begin{array}{r} x^3 \\ -x^3 + x^2y \\ \hline x^2y \\ -x^2y + xy^2 \\ \hline xy^2 - y^3 \\ -xy^2 + y^3 \\ \hline \end{array} \quad \begin{array}{r} -y^3 \mid x-y \\ x^2 + xy + y^2 \end{array}
 \end{array}$$

$$\begin{array}{r}
 21. \quad \begin{array}{r} 3y^5 \\ -3y^5 - 6y^3 \\ \hline -6y^3 - 12y \\ +6y^3 + 12y \\ \hline 5y^2 + 10 \\ -5y^2 - 10 \\ \hline \end{array} \quad \begin{array}{r} -12y + 5y^2 + 10 \mid y^2 + 2 \\ 3y^3 - 6y + 5 \end{array}
 \end{array}$$

$$\begin{array}{r}
 16. \quad \begin{array}{r} a^3 - 3a^2b + 3ab^2 - b^3 \\ -a^3 + a^2b \\ \hline -2a^2b + 3ab^2 \\ +2a^2b - 2ab^2 \\ \hline ab^2 - b^3 \\ -ab^2 + b^3 \\ \hline \end{array} \quad \begin{array}{r} a-b \mid a-b \\ a^2 - 2ab + b^2 \end{array}
 \end{array}$$

$$\begin{array}{r}
 22. \quad \begin{array}{r} am^4 \\ -am^4 - am^3 \\ \hline -am^3 \\ +am^3 + am^2 \\ \hline am^2 - am \\ -am^2 - am \\ \hline -2am - 2a \\ + 2am + 2a \\ \hline \end{array} \quad \begin{array}{r} -am - 2a \mid am + a \\ m^3 - m^2 + m - 2 \end{array}
 \end{array}$$

$$\begin{array}{r}
 17. \quad \begin{array}{r} x^4 \\ -x^4 - 3x^3 \\ \hline -3x^3 - 9x^2 \\ +3x^3 + 9x^2 \\ \hline x+3 \\ -x-3 \\ \hline \end{array} \quad \begin{array}{r} -9x^2 + x + 3 \mid x+3 \\ x^3 - 3x^2 + 1 \end{array}
 \end{array}$$

$$\begin{array}{r}
 23. \quad \begin{array}{r} 12a^3 - 35a^2b + 33ab^2 - 10b^3 \\ -12a^3 + 15a^2b \\ \hline -20a^2b + 33ab^2 \\ +20a^2b - 25ab^2 \\ \hline 8ab^2 - 10b^3 \\ -8ab^2 + 10b^3 \\ \hline \end{array} \quad \begin{array}{r} 4a-5b \mid 4a-5b \\ 3a^2 - 5ab + 2b^2 \end{array}
 \end{array}$$

$$\begin{array}{r}
 18. \quad \begin{array}{r} a^4 \\ -a^4 - a^3 \\ \hline -a^3 \\ +a^3 + a^2 \\ \hline a^2 + a \\ -a^2 - a \\ \hline \end{array} \quad \begin{array}{r} +a \mid a+1 \\ a^3 - a^2 + a \end{array}
 \end{array}$$

$$\begin{array}{r}
 24. \quad \begin{array}{r} 15m^5 - 5m^4n - 9m^3n^2 + 3m^2n^3 + 3mn^4 - n^5 \\ -15m^5 + 5m^4n \\ \hline -9m^3n^2 + 3m^2n^3 \\ +9m^3n^2 - 3m^2n^3 \\ \hline 3mn^4 - n^5 \\ -3mn^4 + n^5 \\ \hline \end{array} \quad \begin{array}{r} 3m-n \mid 3m-n \\ 5m^4 - 3m^2n^2 + n^4 \end{array}
 \end{array}$$

$$\begin{array}{r}
 19. \quad \begin{array}{r} m^6 \\ -m^6 + m^4n^2 \\ \hline m^4n^2 \\ -m^4n^2 + m^2n^4 \\ \hline m^2n^4 - n^6 \\ -m^2n^4 + n^6 \\ \hline \end{array} \quad \begin{array}{r} -n^6 \mid m^2 - n^2 \\ m^4 + m^2n^2 + n^4 \end{array}
 \end{array}$$

## EJERCICIO 55

$$\begin{array}{r}
 20. \quad \begin{array}{r} 2x^4 - x^3 \\ -2x^4 - 3x^3 \\ \hline -4x^3 \\ +4x^3 + 6x^2 \\ \hline 6x^2 + 7x \\ -6x^2 - 9x \\ \hline -2x - 3 \\ 2x + 3 \\ \hline \end{array} \quad \begin{array}{r} +7x-3 \mid 2x+3 \\ x^3 - 2x^2 + 3x - 1 \end{array}
 \end{array}$$

$$\begin{array}{r}
 1. \quad \begin{array}{r} a^4 \\ -a^4 - a^3 - a^2 \\ \hline -a^3 - 2a^2 - 2a \\ +a^3 + a^2 + a \\ \hline -a^2 - a - 1 \\ +a^2 + a + 1 \\ \hline \end{array} \quad \begin{array}{r} -a^2 - 2a - 1 \mid a^2 + a + 1 \\ a^2 - a - 1 \end{array}
 \end{array}$$

$$\begin{array}{r}
 2. \quad x^5 \qquad +12x^2-5x \quad \Big| \quad x^2-2x+5 \\
 \underline{-x^5+2x^4-5x^3} \qquad \qquad \qquad x^3+2x^2-x \\
 \qquad \qquad \qquad 2x^4-5x^3+12x^2 \\
 \underline{-2x^4+4x^3-10x^2} \\
 \qquad \qquad \qquad -x^3+2x^2-5x \\
 \underline{+x^3-2x^2+5x} \\
 \qquad \qquad \qquad 0
 \end{array}$$

$$\begin{array}{r}
 8. \quad -x^4+3x^3y \qquad -5xy^3+3y^4 \quad \Big| \quad x^2-2xy+y^2 \\
 \underline{+x^4-2x^3y+x^2y^2} \qquad \qquad \qquad -x^2+xy+3y^2 \\
 \qquad \qquad \qquad x^3y+x^2y^2-5xy^3 \\
 \underline{-x^3y+2x^2y^2-xy^3} \\
 \qquad \qquad \qquad 3x^2y^2-6xy^3+3y^4 \\
 \underline{-3x^2y^2+6xy^3-3y^4} \\
 \qquad \qquad \qquad 0
 \end{array}$$

$$\begin{array}{r}
 3. \quad m^5-5m^4n \qquad +20m^2n^3-16mn^4 \quad \Big| \quad m^2-2mn-8n^2 \\
 \underline{-m^5+2m^4n+8m^3n^2} \qquad \qquad \qquad m^3-3m^2n+2mn^2 \\
 \qquad \qquad \qquad -3m^4n+8m^3n^2+20m^2n^3 \\
 \underline{+3m^4n-6m^3n^2-24m^2n^3} \\
 \qquad \qquad \qquad 2m^3n^2-4m^2n^3-16mn^4 \\
 \underline{-2m^3n^2+4m^2n^3+16mn^4} \\
 \qquad \qquad \qquad 0
 \end{array}$$

$$\begin{array}{r}
 9. \quad n^4-2n^3 \qquad +2n-1 \quad \Big| \quad n^2-2n+1 \\
 \underline{-n^4+2n^3-n^2} \qquad \qquad \qquad n^2-1 \\
 \qquad \qquad \qquad -n^2+2n-1 \\
 \underline{+n^2-2n+1} \\
 \qquad \qquad \qquad 0
 \end{array}$$

$$\begin{array}{r}
 4. \quad x^4 \qquad -x^2-2x-1 \quad \Big| \quad x^2-x-1 \\
 \underline{-x^4+x^3+x^2} \qquad \qquad \qquad x^2+x+1 \\
 \qquad \qquad \qquad +x^3 \qquad -2x \\
 \underline{-x^3+x^2+x} \\
 \qquad \qquad \qquad x^2-x-1 \\
 \underline{-x^2+x+1} \\
 \qquad \qquad \qquad 0
 \end{array}$$

$$\begin{array}{r}
 10. \quad a^5b-5a^4b^2 \qquad +22a^2b^4-40ab^5 \quad \Big| \quad a^2b-2ab^2-10b^3 \\
 \underline{-a^5b+2a^4b^2+10a^3b^3} \qquad \qquad \qquad a^3-3a^2b+4ab^2 \\
 \qquad \qquad \qquad -3a^4b^2+10a^3b^3+22a^2b^4 \\
 \underline{+3a^4b^2-6a^3b^3-30a^2b^4} \\
 \qquad \qquad \qquad 4a^3b^3-8a^2b^4-40ab^5 \\
 \underline{-4a^3b^3+8a^2b^4+40ab^5} \\
 \qquad \qquad \qquad 0
 \end{array}$$

$$\begin{array}{r}
 5. \quad x^6-2x^5 \qquad +6x^3-7x^2-4x+6 \quad \Big| \quad x^4-3x^2+2 \\
 \underline{-x^6+3x^4-2x^2} \qquad \qquad \qquad x^2-2x+3 \\
 \qquad \qquad \qquad -2x^5+3x^4+6x^3-9x^2-4x \\
 \underline{+2x^5-6x^3+4x} \\
 \qquad \qquad \qquad 3x^4-9x^2+6 \\
 \underline{-3x^4+9x^2-6} \\
 \qquad \qquad \qquad 0
 \end{array}$$

$$\begin{array}{r}
 11. \quad 16x^4 \qquad -24x^2y^2 \qquad -27y^4 \quad \Big| \quad 8x^3-12x^2y+6xy^2-9y^3 \\
 \underline{-16x^4+24x^3y-12x^2y^2+18xy^3} \qquad \qquad \qquad 2x+3y \\
 \qquad \qquad \qquad 24x^3y-36x^2y^2+18xy^3-27y^4 \\
 \underline{-24x^3y+36x^2y^2-18xy^3+27y^4} \\
 \qquad \qquad \qquad 0
 \end{array}$$

$$\begin{array}{r}
 6. \quad m^6+m^5-4m^4 \qquad +m^2-4m-1 \quad \Big| \quad m^3+m^2-4m-1 \\
 \underline{-m^6-m^5+4m^4+m^3} \qquad \qquad \qquad m^3+1 \\
 \qquad \qquad \qquad m^3+m^2-4m-1 \\
 \underline{-m^3-m^2+4m+1} \\
 \qquad \qquad \qquad 0
 \end{array}$$

$$\begin{array}{r}
 12. \quad 4y^4+4y^3-13y^2-3y-20 \quad \Big| \quad 2y+5 \\
 \underline{-4y^4-10y^3} \qquad \qquad \qquad 2y^3-3y^2+y-4 \\
 \qquad \qquad \qquad -6y^3-13y^2 \\
 \underline{+6y^3+15y^2} \\
 \qquad \qquad \qquad 2y^2-3y \\
 \underline{-2y^2-5y-20} \\
 \qquad \qquad \qquad -8y-20 \\
 \underline{8y+20} \\
 \qquad \qquad \qquad 0
 \end{array}$$

$$\begin{array}{r}
 7. \quad a^5-a^4 \qquad +7a^2-27a+10 \quad \Big| \quad a^2-a+5 \\
 \underline{-a^5+a^4-5a^3} \qquad \qquad \qquad a^3-5a+2 \\
 \qquad \qquad \qquad -5a^3+7a^2-27a \\
 \underline{+5a^3-5a^2+25a} \\
 \qquad \qquad \qquad 2a^2-2a+10 \\
 \underline{-2a^2+2a-10} \\
 \qquad \qquad \qquad 0
 \end{array}$$

$$\begin{array}{r}
 13. \quad -2a^5-3a^4x+5a^3x^2 \qquad -11ax^4-3x^5 \quad \Big| \quad -a^3+2ax^2+3x^3 \\
 \underline{+2a^5-4a^3x^2-6a^2x^3} \qquad \qquad \qquad 2a^2-3ax-x^2 \\
 \qquad \qquad \qquad 3a^4x+a^3x^2-6a^2x^3-11ax^4 \\
 \underline{-3a^4x+6a^2x^3+9ax^4} \\
 \qquad \qquad \qquad a^3x^2-2ax^4-3x^5 \\
 \underline{-a^3x^2+2ax^4+3x^5} \\
 \qquad \qquad \qquad 0
 \end{array}$$

$$14. \begin{array}{r} -x^6 + 2x^5y \\ + x^6 - 3x^5y + 2x^4y^2 + x^3y^3 \end{array} \quad \begin{array}{r} -3x^2y^4 - xy^5 \\ -x^2 - xy - y^2 \end{array} \quad \begin{array}{r} x^4 - 3x^3y + 2x^2y^2 + xy^3 \end{array}$$

$$\begin{array}{r} -x^5y + 2x^4y^2 + x^3y^3 - 3x^2y^4 \\ + x^5y - 3x^4y^2 + 2x^3y^3 + x^2y^4 \\ - x^4y^2 + 3x^3y^3 - 2x^2y^4 - xy^5 \\ + x^4y^2 - 3x^3y^3 + 2x^2y^4 + xy^5 \end{array}$$

$$19. \begin{array}{r} y^6 - 2y^5 - y^4 + 4y^3 \\ - y^6 + 2y^4 - 2y^2 \end{array} \quad \begin{array}{r} -4y + 2 \\ y^2 - 2y + 1 \end{array}$$

$$\begin{array}{r} -2y^5 + y^4 + 4y^3 - 2y^2 - 4y \\ + 2y^5 - 4y^3 + 4y \end{array}$$

$$15. \begin{array}{r} a^6 - 5a^5 \\ - a^6 + 2a^4 + 7a^3 \end{array} \quad \begin{array}{r} + 31a^2 - 8a + 21 \\ a^3 - 5a^2 + 2a - 3 \end{array} \quad \begin{array}{r} | a^3 - 2a - 7 \end{array}$$

$$\begin{array}{r} -5a^5 + 2a^4 + 7a^3 + 31a^2 \\ + 5a^5 - 10a^3 - 35a^2 \\ 2a^4 - 3a^3 - 4a^2 - 8a \\ - 2a^4 + 4a^2 + 14a \\ - 3a^3 + 6a + 21 \\ + 3a^3 - 6a - 21 \end{array}$$

$$20. \begin{array}{r} 3m^7 - 11m^5 + m^4 + 18m^3 - 3m^2 - 8m + 4 \\ - 3m^7 + 9m^5 - 12m^3 \end{array} \quad \begin{array}{r} | m^4 - 3m^2 + 4 \\ 3m^3 - 2m + 1 \end{array}$$

$$\begin{array}{r} -2m^5 + m^4 + 6m^3 - 8m \\ + 2m^5 - 6m^3 + 8m \\ m^4 - 3m^2 + 4 \\ - m^4 + 3m^2 - 4 \end{array}$$

$$16. \begin{array}{r} m^6 - m^5 + 5m^3 \\ - m^6 - m^5 + m^4 \end{array} \quad \begin{array}{r} -6m + 9 \\ -3m^2 \end{array} \quad \begin{array}{r} | m^4 + m^3 - m^2 + 3 \\ m^2 - 2m + 3 \end{array}$$

$$\begin{array}{r} -2m^5 + m^4 + 5m^3 - 3m^2 - 6m \\ + 2m^5 + 2m^4 - 2m^3 + 6m \\ 3m^4 + 3m^3 - 3m^2 + 9 \\ - 3m^4 - 3m^3 + 3m^2 - 9 \end{array}$$

$$21. \begin{array}{r} a^6 + 2a^5 - 2a^4 - 3a^3 + 2a^2 - a - 1 \\ - a^6 - a^5 + a^4 - a^3 \end{array} \quad \begin{array}{r} | a^3 + a^2 - a + 1 \\ a^3 + a^2 - 2a - 1 \end{array}$$

$$\begin{array}{r} a^5 - a^4 - 4a^3 + 2a^2 \\ - a^5 - a^4 + a^3 - a^2 \\ -2a^4 - 3a^3 + a^2 - a \\ + 2a^4 + 2a^3 - 2a^2 + 2a - 1 \end{array}$$

$$17. \begin{array}{r} a^6 - a^5b - 4a^4b^2 + 6a^3b^3 \\ - a^6 + 2a^5b - a^4b^2 \end{array} \quad \begin{array}{r} -3ab^5 + b^6 \\ a^4 + a^3b - 3a^2b^2 - ab^3 + b^4 \end{array} \quad \begin{array}{r} | a^2 - 2ab + b^2 \\ - a^3 - a^2 + a - 1 \\ + a^3 + a^2 - a + 1 \end{array}$$

$$\begin{array}{r} a^5b - 5a^4b^2 + 6a^3b^3 \\ - a^5b + 2a^4b^2 - a^3b^3 \\ -3a^4b^2 + 5a^3b^3 \\ + 3a^4b^2 - 6a^3b^3 + 3a^2b^4 \\ - a^3b^3 + 3a^2b^4 - 3ab^5 \\ + a^3b^3 - 2a^2b^4 + ab^5 \\ a^2b^4 - 2ab^5 + b^6 \\ - a^2b^4 + 2ab^5 - b^6 \end{array}$$

$$22. \begin{array}{r} 24x^5 - 52x^4y + 38x^3y^2 - 33x^2y^3 - 26xy^4 + 4y^5 \\ - 24x^5 + 36x^4y + 18x^3y^2 - 3x^2y^3 \end{array} \quad \begin{array}{r} | 8x^3 - 12x^2y - 6xy^2 + y^3 \\ 3x^2 - 2xy + 4y^2 \end{array}$$

$$\begin{array}{r} -16x^4y + 56x^3y^2 - 36x^2y^3 - 26xy^4 \\ + 16x^4y - 24x^3y^2 - 12x^2y^3 + 2xy^4 \\ 32x^3y^2 - 48x^2y^3 - 24xy^4 + 4y^5 \\ - 32x^3y^2 - 48x^2y^3 + 24xy^4 - 4y^5 \end{array}$$

$$18. \begin{array}{r} x^6 - 2x^4y^2 + 2x^3y^3 - 2x^2y^4 + 3xy^5 - 2y^6 \\ - x^6 - x^5y + 2x^4y^2 \end{array} \quad \begin{array}{r} | x^2 + xy - 2y^2 \\ x^4 - x^3y + x^2y^2 - xy^3 + y^4 \end{array}$$

$$\begin{array}{r} -x^5y + 2x^3y^3 \\ x^5y + x^4y^2 - 2x^3y^3 \\ + x^4y^2 - 2x^2y^4 \\ - x^4y^2 - x^3y^3 + 2x^2y^4 \\ - x^3y^3 + 3xy^5 \\ + x^3y^3 + x^2y^4 - 2xy^5 \\ x^2y^4 + xy^5 - 2y^6 \\ - x^2y^4 - xy^5 + 2y^6 \end{array}$$

$$23. \begin{array}{r} 5a^8 - 4a^7 - 8a^6 + 5a^5 + 6a^4 - 2a^3 + 4a^2 - 6a \\ - 5a^8 + 10a^6 - 10a^4 \end{array} \quad \begin{array}{r} | a^4 - 2a^2 + 2 \\ 5a^4 - 4a^3 + 2a^2 - 3a \end{array}$$

$$\begin{array}{r} -4a^7 + 2a^6 + 5a^5 - 4a^4 - 2a^3 \\ + 4a^7 - 8a^5 + 8a^3 \\ + 2a^6 - 3a^5 - 4a^4 + 6a^3 + 4a^2 \\ - 2a^6 + 4a^4 - 4a^2 \\ - 3a^5 + 6a^3 - 6a \\ 3a^5 - 6a^3 + 6a \end{array}$$

$$24. \begin{array}{r} x^7 - 3x^6 + 6x^5 \\ -x^7 + 2x^6 - 3x^5 - 6x^4 \\ \hline -x^6 + 3x^5 - 6x^4 \\ +x^6 - 2x^5 + 3x^4 + 6x^3 \\ \hline x^5 - 3x^4 + 6x^3 + x^2 \\ -x^5 + 2x^4 - 3x^3 - 6x^2 - 3x \\ \hline -x^4 + 3x^3 - 5x^2 - 3x \\ +x^4 - 2x^3 + 3x^2 + 6x + 6 \\ \hline x^3 - 2x^2 + 3x + 6 \\ -x^3 + 2x^2 - 3x - 6 \\ \hline \end{array} \quad \begin{array}{r} +x^2 - 3x + 6 \\ \hline x^3 - 2x^2 + 3x + 6 \\ x^4 - x^3 + x^2 - x + 1 \end{array}$$

$$29. \begin{array}{r} 3a^9 - 15a^7 + 14a^6 \\ -3a^9 + 6a^7 - 2a^6 + 3a^5 - 2a^4 \\ \hline -9a^7 + 12a^6 + 3a^5 - 30a^4 + 47a^3 - 28a^2 \\ +9a^7 - 18a^6 + 6a^4 - 9a^3 + 6a^2 \\ \hline +12a^6 - 15a^5 - 24a^4 + 38a^3 - 22a^2 + 23a \\ -12a^6 + 24a^4 - 8a^3 + 12a^2 - 8a \\ \hline -15a^5 + 30a^3 - 10a^2 + 15a - 10 \\ +15a^5 - 30a^3 + 10a^2 - 15a + 10 \\ \hline \end{array} \quad \begin{array}{r} -28a^4 + 47a^3 - 28a^2 + 23a - 10 \\ \hline 3a^5 - 6a^3 + 2a^2 - 3a + 2 \\ a^4 - 3a^2 + 4a - 5 \end{array}$$

$$25. \begin{array}{r} 3a^6 + 5a^5 - 9a^4 - 10a^3 + 8a^2 + 3a - 4 \\ -3a^6 - 2a^5 + 5a^4 + 4a^3 \\ \hline +3a^5 - 4a^4 - 6a^3 + 8a^2 \\ -3a^5 - 2a^4 + 5a^3 + 4a^2 \\ \hline -6a^4 - a^3 + 12a^2 + 3a \\ +6a^4 + 4a^3 - 10a^2 - 8a \\ \hline 3a^3 + 2a^2 - 5a - 4 \\ -3a^3 - 2a^2 + 5a + 4 \\ \hline \end{array} \quad \begin{array}{r} 3a^3 + 2a^2 - 5a - 4 \\ a^3 + a^2 - 2a + 1 \end{array}$$

$$30. \begin{array}{r} a^2 - b^2 + 2bc - c^2 \\ -a^2 - ab + ac \\ \hline -ab - b^2 + ac + 2bc \\ +ab + b^2 - bc \\ \hline ac + bc - c^2 \\ -ac - bc + c^2 \\ \hline \end{array} \quad \begin{array}{r} a + b - c \\ a - b + c \end{array}$$

$$26. \begin{array}{r} 5y^8 - 3y^7 - 11y^6 + 11y^5 - 17y^4 - 3y^3 - 4y^2 - 2y \\ -5y^8 + 3y^7 - 4y^6 - 2y^5 \\ \hline -15y^6 + 9y^5 - 17y^4 - 3y^3 \\ +15y^6 - 9y^5 + 12y^4 + 6y^3 \\ \hline -5y^4 + 3y^3 - 4y^2 - 2y \\ +5y^4 - 3y^3 + 4y^2 + 2y \\ \hline \end{array} \quad \begin{array}{r} 5y^4 - 3y^3 + 4y^2 + 2y \\ y^4 - 3y^2 - 1 \end{array}$$

$$31. \begin{array}{r} -2x^2 + 5xy - xz - 3y^2 - yz + 10z^2 \\ +2x^2 - 3xy + 5xz \\ \hline 2xy + 4xz - 3y^2 - yz \\ -2xy + 3y^2 - 5yz \\ \hline 4xz - 6yz + 10z^2 \\ -4xz + 6yz - 10z^2 \\ \hline \end{array} \quad \begin{array}{r} 2x - 3y + 5z \\ -x + y + 2z \end{array}$$

$$27. \begin{array}{r} -m^7 + 5m^6n - 14m^5n^2 + 20m^4n^3 - 13m^3n^4 - 9m^2n^5 + 20mn^6 - 4n^7 \\ +m^7 - 3m^6n + 5m^5n^2 - m^4n^3 \\ \hline 2m^6n - 9m^5n^2 + 19m^4n^3 - 13m^3n^4 \\ -2m^6n + 6m^5n^2 - 10m^4n^3 + 2m^3n^4 \\ \hline -3m^5n^2 + 9m^4n^3 - 11m^3n^4 - 9m^2n^5 \\ +3m^5n^2 - 9m^4n^3 + 15m^3n^4 - 3m^2n^5 \\ \hline 4m^3n^4 - 12m^2n^5 + 20mn^6 - 4n^7 \\ -4m^3n^4 + 12m^2n^5 - 20mn^6 + 4n^7 \\ \hline \end{array} \quad \begin{array}{r} -m^3 + 3m^2n - 5mn^2 + n^3 \\ m^4 - 2m^3n + 3m^2n^2 - 4n^4 \end{array}$$

$$28. \begin{array}{r} x^{11} - 5x^9y^2 + 8x^7y^4 - 6x^5y^6 - 5x^3y^8 + 3xy^{10} \\ -x^{11} + 2x^9y^2 - 3x^7y^4 \\ \hline -3x^9y^2 + 5x^7y^4 - 6x^5y^6 \\ +3x^9y^2 - 6x^7y^4 + 9x^5y^6 \\ \hline -x^7y^4 + 3x^5y^6 - 5x^3y^8 \\ +x^7y^4 - 2x^5y^6 + 3x^3y^8 \\ \hline +x^5y^6 - 2x^3y^8 + 3xy^{10} \\ -x^5y^6 + 2x^3y^8 - 3xy^{10} \\ \hline \end{array} \quad \begin{array}{r} x^5 - 2x^4y^2 + 3xy^4 \\ x^6 - 3x^4y^2 - x^2y^4 + y^6 \end{array}$$

$$32. \begin{array}{r} x^3 + y^3 - 3xyz + z^3 \\ -x^3 + x^2y + x^2z - xy^2 + xyz - xz^2 \\ \hline x^2y + x^2z + y^3 - xy^2 - 2xyz + z^3 - xz^2 \\ -x^2y - y^3 + xy^2 + xyz + y^2z - yz^2 \\ \hline x^2z - xyz + z^3 - xz^2 + y^2z - yz^2 \\ -x^2z + xyz - z^3 + xz^2 - y^2z + yz^2 \\ \hline \end{array} \quad \begin{array}{r} x^2 - xy - xz + y^2 - yz + z^2 \\ x + y + z \end{array}$$

$$33. \begin{array}{r} a^5 \\ -a^5 - a^4b \\ \hline -a^4b \\ +a^4b + a^3b^2 \\ \hline a^3b^2 \\ -a^3b^2 - a^2b^3 \\ \hline -a^2b^3 \\ +a^2b^3 + ab^4 \\ \hline ab^4 + b^5 \\ -ab^4 - b^5 \\ \hline \end{array} \quad + b^5 \overline{) a+b}$$

$$\begin{array}{r} -a^4b \\ +a^4b + a^3b^2 \\ \hline a^3b^2 \end{array}$$

$$\begin{array}{r} -a^3b^2 - a^2b^3 \\ \hline -a^2b^3 \end{array}$$

$$\begin{array}{r} +a^2b^3 + ab^4 \\ \hline ab^4 + b^5 \end{array}$$

$$\begin{array}{r} -ab^4 - b^5 \\ \hline \end{array}$$

$$37. \begin{array}{r} x^{15} \\ -x^{15} - x^{12}y^3 \\ \hline -x^{12}y^3 \\ x^{12}y^3 + x^9y^6 \\ \hline x^9y^6 \\ -x^9y^6 - x^6y^9 \\ \hline -x^6y^9 \\ x^6y^9 + x^3y^{12} \\ \hline \end{array}$$

$$+ y^{15} \overline{) x^3 + y^3}$$

$$\begin{array}{r} x^{12} - x^9y^3 + x^6y^6 - x^3y^9 + y^{12} \end{array}$$

$$34. \begin{array}{r} 21x^5 \\ -21x^5 + 21x^4y \\ \hline 21x^4y \end{array}$$

$$\begin{array}{r} 21x^4y \\ -21x^4y + 21x^3y^2 \\ \hline 21x^3y^2 \end{array}$$

$$\begin{array}{r} -21x^3y^2 + 21x^2y^3 \\ \hline 21x^2y^3 \end{array}$$

$$\begin{array}{r} -21x^2y^3 + 21xy^4 \\ \hline -21xy^4 + 21y^5 \\ \hline \end{array}$$

$$\begin{array}{r} 21xy^4 - 21y^5 \\ -21xy^4 + 21y^5 \\ \hline \end{array}$$

$$-21y^5 \overline{) 3x-3y}$$

$$7x^4 + 7x^3y + 7x^2y^2 + 7xy^3 + 7y^4$$

$$\begin{array}{r} x^3y^{12} + y^{15} \\ -x^3y^{12} - y^{15} \\ \hline \end{array}$$

$$38. \begin{array}{r} x^3 + 3x^2y + 3xy^2 + y^3 - 1 \\ -x^3 - 2x^2y - xy^2 \\ \hline x^2y + 2xy^2 + y^3 - 1 - x^2 - xy - x \end{array}$$

$$\begin{array}{r} -x^2 - xy - x \\ -x^2y - 2xy^2 - y^3 \\ \hline -xy - y^2 - y \end{array}$$

$$\begin{array}{r} -xy - y^2 - y \\ -1 - x^2 - 2xy - x - y^2 - y \\ \hline +1 + x^2 + 2xy + x + y^2 + y \end{array}$$

$$\begin{array}{r} -1 - x^2 - 2xy - x - y^2 - y \\ +1 + x^2 + 2xy + x + y^2 + y \\ \hline \end{array}$$

$$35. \begin{array}{r} 16x^8 \\ -16x^8 - 16x^6y^2 \\ \hline -16x^6y^2 \end{array}$$

$$\begin{array}{r} -16x^6y^2 \\ +16x^6y^2 + 16x^4y^4 \\ \hline 16x^4y^4 \end{array}$$

$$\begin{array}{r} -16x^4y^4 - 16x^2y^6 \\ \hline -16x^2y^6 - 16y^8 \\ \hline \end{array}$$

$$\begin{array}{r} -16x^2y^6 - 16y^8 \\ +16x^2y^6 + 16y^8 \\ \hline \end{array}$$

$$-16y^8 \overline{) 2x^2 + 2y^2}$$

$$39. \begin{array}{r} x^5 \\ -x^5 + x^4y - x^3y^2 + x^2y^3 - xy^4 \\ \hline x^4y - x^3y^2 + x^2y^3 - xy^4 + y^5 \end{array}$$

$$\begin{array}{r} -x^4y + x^3y^2 - x^2y^3 + xy^4 - y^5 \\ \hline \end{array}$$

$$+ y^5 \overline{) x^4 - x^3y + x^2y^2 - xy^3 + y^4}$$

$$\begin{array}{r} x^4 - x^3y + x^2y^2 - xy^3 + y^4 \\ x + y \end{array}$$

$$36. \begin{array}{r} x^{10} \\ -x^{10} + x^8y^2 \\ \hline x^8y^2 \end{array}$$

$$\begin{array}{r} x^8y^2 \\ -x^8y^2 + x^6y^4 \\ \hline x^6y^4 \end{array}$$

$$\begin{array}{r} -x^6y^4 + x^4y^6 \\ \hline x^4y^6 \end{array}$$

$$\begin{array}{r} -x^4y^6 + x^2y^8 \\ \hline x^2y^8 - y^{10} \\ \hline \end{array}$$

$$\begin{array}{r} x^2y^8 - y^{10} \\ -x^2y^8 + y^{10} \\ \hline \end{array}$$

$$-y^{10} \overline{) x^2 - y^2}$$

$$x^8 + x^6y^2 + x^4y^4 + x^2y^6 + y^8$$

## EJERCICIO 56

$$1. \begin{array}{r} a^{x+3} \\ -a^{x+3} - a^{x+2} \\ \hline -a^{x+2} \\ +a^{x+2} + a^{x+1} \\ \hline a^{x+1} + a^x \\ -a^{x+1} - a^x \\ \hline \end{array} \quad + a^x \overline{) a+1}$$

$$\begin{array}{r} a^{x+2} - a^{x+1} + a^x \end{array}$$

$$\begin{array}{r}
 2. \quad -x^{n+5} + x^{n+4} + 3x^{n+3} + x^{n+2} \quad \Big| \quad x^2 + x \\
 \hline
 + x^{n+5} + x^{n+4} \qquad \qquad \qquad -x^{n+3} + 2x^{n+2} + x^{n+1} \\
 \hline
 + 2x^{n+4} + 3x^{n+3} \\
 \hline
 - 2x^{n+4} - 2x^{n+3} \\
 \hline
 \qquad \qquad \qquad x^{n+3} + x^{n+2} \\
 \hline
 \qquad \qquad \qquad -x^{n+3} - x^{n+2} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 8. \quad -m^{2a+3} + 2m^{2a+2} + 2m^{2a+1} - 4m^{2a} - m^{2a-1} + m^{2a-2} \quad \Big| \quad -m^{a-1} + m^{a-2} + m^{a-3} \\
 \hline
 + m^{2a+3} - m^{2a+2} - m^{2a+1} \qquad \qquad \qquad m^{a+4} - m^{a+3} - 2m^{a+2} + m^{a+1} \\
 \hline
 + m^{2a+2} + m^{2a+1} - 4m^{2a} \\
 \hline
 - m^{2a+2} + m^{2a+1} + m^{2a} \\
 \hline
 \qquad \qquad \qquad 2m^{2a+1} - 3m^{2a} - m^{2a-1} \\
 \hline
 - 2m^{2a+1} + 2m^{2a} + 2m^{2a-1} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 3. \quad m^{a+4} - m^{a+3} + 6m^{a+1} - 5m^a + 3m^{a-1} \quad \Big| \quad m^2 - 2m + 3 \\
 \hline
 - m^{a+4} + 2m^{a+3} - 3m^{a+2} \qquad \qquad \qquad m^{a+2} + m^{a+1} - m^a + m^{a-1} \\
 \hline
 \qquad \qquad \qquad m^{a+3} - 3m^{a+2} + 6m^{a+1} \\
 \hline
 - m^{a+3} + 2m^{a+2} - 3m^{a+1} \\
 \hline
 \qquad \qquad \qquad -m^{a+2} + 3m^{a+1} - 5m^a \\
 \hline
 + m^{a+2} - 2m^{a+1} + 3m^a \\
 \hline
 \qquad \qquad \qquad m^{a+1} - 2m^a + 3m^{a-1} \\
 \hline
 - m^{a+1} + 2m^a - 3m^{a-1} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 9. \quad x^{2a-2} + x^{2a-3} - 4x^{2a-4} \quad \quad \quad -x^{2a-7} \quad \Big| \quad x^{a-1} - x^{a-2} - x^{a-3} \\
 \hline
 -x^{2a-2} + x^{2a-3} + x^{2a-4} \qquad \qquad \qquad x^{a-1} + 2x^{a-2} - x^{a-3} + x^{a-4} \\
 \hline
 + 2x^{2a-3} - 3x^{2a-4} \\
 \hline
 - 2x^{2a-3} + 2x^{2a-4} + 2x^{2a-5} \\
 \hline
 - x^{2a-4} + 2x^{2a-5} \\
 \hline
 + x^{2a-4} - x^{2a-5} - x^{2a-6} \\
 \hline
 + x^{2a-5} - x^{2a-6} - x^{2a-7} \\
 \hline
 - x^{2a-5} + x^{2a-6} + x^{2a-7} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 4. \quad a^{2n+3} + 4a^{2n+2} + a^{2n+1} - 2a^{2n} \quad \Big| \quad a^{n+1} + a^n \\
 \hline
 - a^{2n+3} - a^{2n+2} \qquad \qquad \qquad a^{n+2} + 3a^{n+1} - 2a^n \\
 \hline
 \qquad \qquad \qquad 3a^{2n+2} + a^{2n+1} \\
 \hline
 - 3a^{2n+2} - 3a^{2n+1} \\
 \hline
 \qquad \qquad \qquad -2a^{2n+1} - 2a^{2n} \\
 \hline
 \qquad \qquad \qquad 2a^{2n+1} + 2a^{2n} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 10. \quad a^{2n}b^3 - a^{2n-1}b^4 + a^{2n-2}b^5 \quad \quad \quad -2a^{2n-4}b^7 + a^{2n-5}b^8 \quad \Big| \quad a^nb - a^{n-1}b^2 + 2a^{n-2}b^3 - a^{n-3}b^4 \\
 \hline
 -a^{2n}b^3 + a^{2n-1}b^4 - 2a^{2n-2}b^5 + a^{2n-3}b^6 \qquad \qquad \qquad a^nb^2 - a^{n-2}b^4 \\
 \hline
 -a^{2n-2}b^5 + a^{2n-3}b^6 - 2a^{2n-4}b^7 + a^{2n-5}b^8 \\
 \hline
 + a^{2n-2}b^5 - a^{2n-3}b^6 + 2a^{2n-4}b^7 - a^{2n-5}b^8 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 5. \quad x^{2a+5} + 2x^{2a+4} - 3x^{2a+3} - 4x^{2a+2} + 2x^{2a+1} \quad \Big| \quad x^{a+3} - 2x^{a+1} \\
 \hline
 -x^{2a+5} \qquad \qquad \qquad + 2x^{2a+3} \qquad \qquad \qquad x^{a+2} + 2x^{a+1} - x^a \\
 \hline
 + 2x^{2a+4} - x^{2a+3} \\
 \hline
 - 2x^{2a+4} \qquad \qquad \qquad + 4x^{2a+2} \\
 \hline
 \qquad \qquad \qquad -x^{2a+3} \qquad \qquad \qquad + 2x^{2a+1} \\
 \hline
 + x^{2a+3} \qquad \qquad \qquad - 2x^{2a+1} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 11. \quad a^{m+x} + a^mb^x + a^xb^m + b^{m+x} \quad \Big| \quad a^x + b^x \\
 \hline
 -a^{m+x} - a^mb^x \qquad \qquad \qquad a^xb^m + b^{m+x} \\
 \hline
 \qquad \qquad \qquad -a^xb^m - b^{m+x} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 6. \quad a^{x+2} - 2a^x + 8a^{x-1} - 3a^{x-2} \quad \Big| \quad a^x - 2a^{x-1} + 3a^{x-2} \\
 \hline
 -a^{x+2} + 2a^{x+1} - 3a^x \qquad \qquad \qquad a^2 + 2a - 1 \\
 \hline
 \qquad \qquad \qquad 2a^{x+1} - 5a^x + 8a^{x-1} \\
 \hline
 - 2a^{x+1} + 4a^x - 6a^{x-1} \\
 \hline
 \qquad \qquad \qquad -a^x + 2a^{x-1} - 3a^{x-2} \\
 \hline
 + a^x - 2a^{x-1} + 3a^{x-2} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 12. \quad a^x - a^{x-1}b + b^n - ab^{n-1} \quad \Big| \quad a - b \\
 \hline
 -a^x + a^{x-1}b \qquad \qquad \qquad b^n - ab^{n-1} \\
 \hline
 \qquad \qquad \qquad -b^n + ab^{n-1} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 7. \quad a^{2x} + 2a^{2x-1} - 4a^{2x-2} + 5a^{2x-3} - 2a^{2x-4} \quad \Big| \quad a^x - a^{x-1} + a^{x-2} \\
 \hline
 -a^{2x} + a^{2x-1} - a^{2x-2} \qquad \qquad \qquad a^x + 3a^{x-1} - 2a^{x-2} \\
 \hline
 \qquad \qquad \qquad 3a^{2x-1} - 5a^{2x-2} + 5a^{2x-3} \\
 \hline
 - 3a^{2x-1} + 3a^{2x-2} - 3a^{2x-3} \\
 \hline
 \qquad \qquad \qquad -2a^{2x-2} + 2a^{2x-3} - 2a^{2x-4} \\
 \hline
 + 2a^{2x-2} - 2a^{2x-3} + 2a^{2x-4} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 13. \quad 3a^{5m-3} - 23a^{5m-2} + 5a^{5m-1} + 46a^{5m} - 30a^{5m+1} \quad \Big| \quad a^{3m-3} - 8a^{3m-2} + 6a^{3m-1} \\
 \hline
 - 3a^{5m-3} + 24a^{5m-2} - 18a^{5m-1} \qquad \qquad \qquad 3a^{2m} + a^{2m+1} - 5a^{2m+2} \\
 \hline
 + a^{5m-2} - 13a^{5m-1} + 46a^{5m} \\
 \hline
 - a^{5m-2} + 8a^{5m-1} - 6a^{5m} \\
 \hline
 - 5a^{5m-1} + 40a^{5m} - 30a^{5m+1} \\
 \hline
 5a^{5m-1} - 40a^{5m} + 30a^{5m+1} \\
 \hline
 \end{array}$$



$$\begin{array}{r}
 14. \quad \begin{array}{r} 2x^{3a+1}y^{2x-3} - 4x^{3a}y^{2x-2} \\ -2x^{3a+1}y^{2x-3} + 8x^{3a}y^{2x-2} - 6x^{3a-1}y^{2x-1} \end{array} \quad \begin{array}{r} -28x^{3a-2}y^{2x} + 30x^{3a-3}y^{2x+1} \\ -28x^{3a-2}y^{2x} + 30x^{3a-3}y^{2x+1} \end{array} \quad \begin{array}{r} -x^{a+2}y^{x-1} + 4x^{a+1}y^x - 3x^ay^{x+1} \\ -2x^{2a-1}y^{x-2} - 4x^{2a-2}y^{x-1} - 10x^{2a-3}y^x \end{array} \\
 \hline
 \begin{array}{r} +4x^{3a}y^{2x-2} - 6x^{3a-1}y^{2x-1} - 28x^{3a-2}y^{2x} \\ -4x^{3a}y^{2x-2} + 16x^{3a-1}y^{2x-1} - 12x^{3a-2}y^{2x} \end{array} \\
 \hline
 \begin{array}{r} 10x^{3a-1}y^{2x-1} - 40x^{3a-2}y^{2x} + 30x^{3a-3}y^{2x+1} \\ -10x^{3a-1}y^{2x-1} + 40x^{3a-2}y^{2x} - 30x^{3a-3}y^{2x+1} \end{array} \\
 \hline
 \end{array}$$

## EJERCICIO 57

$$\begin{array}{ll}
 1. \quad \begin{array}{r} \frac{1}{6}a^2 + \frac{5}{36}ab - \frac{1}{6}b^2 \\ -\frac{1}{6}a^2 - \frac{1}{4}ab \\ -\frac{1}{9}ab - \frac{1}{6}b^2 \\ \frac{1}{9}ab + \frac{1}{6}b^2 \end{array} \quad \begin{array}{r} \frac{1}{3}a + \frac{1}{2}b \\ \frac{1}{2}a - \frac{1}{3}b \end{array} & 5. \quad \begin{array}{r} \frac{3}{5}m^4 + \frac{1}{10}m^3n - \frac{17}{60}m^2n^2 + \frac{7}{6}mn^3 - n^4 \\ -\frac{3}{5}m^4 + \frac{2}{5}m^3n - \frac{4}{5}m^2n^2 \\ \frac{1}{2}m^3n - \frac{65}{60}m^2n^2 + \frac{7}{6}mn^3 \\ -\frac{1}{2}m^3n + \frac{1}{3}m^2n^2 - \frac{2}{3}mn^3 \\ -\frac{3}{4}m^2n^2 + \frac{1}{2}mn^3 - n^4 \\ \frac{3}{4}m^2n^2 - \frac{1}{2}mn^3 + n^4 \end{array} \quad \begin{array}{r} \frac{3}{2}m^2 - mn + 2n^2 \\ \frac{2}{5}m^2 + \frac{1}{3}mn - \frac{1}{2}n^2 \end{array} \\
 2. \quad \begin{array}{r} \frac{1}{3}x^2 + \frac{7}{10}xy - \frac{1}{3}y^2 \\ -\frac{1}{3}x^2 + \frac{2}{15}xy \\ \frac{5}{6}xy - \frac{1}{3}y^2 \\ -\frac{5}{6}xy + \frac{1}{3}y^2 \end{array} \quad \begin{array}{r} x - \frac{2}{5}y \\ \frac{1}{3}x + \frac{5}{6}y \end{array} & 6. \quad \begin{array}{r} \frac{3}{4}x^5 + \frac{1}{2}x^4 - \frac{37}{40}x^3 + \frac{2}{3}x^2 + \frac{19}{30}x - \frac{4}{5} \\ -\frac{3}{4}x^5 + \frac{1}{8}x^3 - \frac{3}{4}x^2 \\ \frac{1}{2}x^4 - \frac{4}{5}x^3 - \frac{1}{12}x^2 + \frac{19}{30}x \\ -\frac{1}{2}x^4 + \frac{1}{12}x^2 - \frac{1}{2}x \\ -\frac{4}{5}x^3 + \frac{2}{15}x - \frac{4}{5} \\ \frac{4}{5}x^3 - \frac{2}{15}x + \frac{4}{5} \end{array} \quad \begin{array}{r} 2x^3 - \frac{1}{3}x + 2 \\ \frac{3}{8}x^2 + \frac{1}{4}x - \frac{2}{5} \end{array} \\
 3. \quad \begin{array}{r} \frac{1}{3}x^3 - \frac{35}{36}x^2y + \frac{2}{3}xy^2 - \frac{3}{8}y^3 \\ -\frac{1}{3}x^3 + \frac{2}{9}x^2y - \frac{1}{6}xy^2 \\ -\frac{3}{4}x^2y + \frac{1}{2}xy^2 - \frac{3}{8}y^3 \\ \frac{3}{4}x^2y - \frac{1}{2}xy^2 + \frac{3}{8}y^3 \end{array} \quad \begin{array}{r} \frac{1}{2}x^2 - \frac{1}{3}xy + \frac{1}{4}y^2 \\ \frac{2}{3}x - \frac{3}{2}y \end{array} & 7. \quad \begin{array}{r} \frac{9}{4}a^4 - a^3x - \frac{1}{12}a^2x^2 + \frac{13}{18}ax^3 - \frac{1}{3}x^4 \\ -\frac{9}{4}a^4 + \frac{3}{2}a^3x - a^2x^2 \\ \frac{1}{2}a^3x - \frac{13}{12}a^2x^2 + \frac{13}{18}ax^3 \\ -\frac{1}{2}a^3x + \frac{1}{3}a^2x^2 - \frac{2}{9}ax^3 \\ -\frac{3}{4}a^2x^2 + \frac{1}{2}ax^3 - \frac{1}{3}x^4 \\ \frac{3}{4}a^2x^2 - \frac{1}{2}ax^3 + \frac{1}{3}x^4 \end{array} \quad \begin{array}{r} \frac{3}{2}a^2 - ax + \frac{2}{3}x^2 \\ \frac{3}{2}a^2 + \frac{1}{3}ax - \frac{1}{2}x^2 \end{array} \\
 4. \quad \begin{array}{r} \frac{1}{16}a^3 - \frac{5}{8}a^2b + \frac{5}{3}ab^2 - b^3 \\ -\frac{1}{16}a^3 + \frac{3}{8}a^2b \\ -\frac{1}{4}a^2b + \frac{5}{3}ab^2 \\ \frac{1}{4}a^2b - \frac{3}{2}ab^2 \\ \frac{1}{6}ab^2 - b^3 \\ -\frac{1}{6}ab^2 + b^3 \end{array} \quad \begin{array}{r} \frac{1}{4}a - \frac{3}{2}b \\ \frac{1}{4}a^2 - ab + \frac{2}{3}b^2 \end{array}
 \end{array}$$

$$\begin{array}{l}
 8. \quad \frac{1}{14}x^5 - \frac{101}{420}x^4y + \frac{139}{280}x^3y^2 - \frac{1}{2}x^2y^3 + \frac{5}{12}xy^4 \quad \left| \begin{array}{l} \frac{2}{7}x^3 - \frac{1}{5}x^2y + \frac{1}{2}xy^2 \\ \frac{1}{4}x^2 - \frac{2}{3}xy + \frac{5}{6}y^2 \end{array} \right. \\
 \hline
 -\frac{1}{14}x^5 + \frac{1}{20}x^4y - \frac{1}{8}x^3y^2 \\
 \hline
 -\frac{4}{21}x^4y + \frac{13}{35}x^3y^2 - \frac{1}{2}x^2y^3 \\
 +\frac{4}{21}x^4y - \frac{2}{15}x^3y^2 + \frac{1}{3}x^2y^3 \\
 \hline
 \frac{5}{21}x^3y^2 - \frac{1}{6}x^2y^3 + \frac{5}{12}xy^4 \\
 -\frac{5}{21}x^3y^2 + \frac{1}{6}x^2y^3 - \frac{5}{12}xy^4 \\
 \hline
 \hline
 9. \quad \frac{3}{8}x^5 + \frac{21}{40}x^4 - \frac{47}{120}x^3 + \frac{79}{120}x^2 + \frac{1}{10}x - \frac{1}{10} \quad \left| \begin{array}{l} \frac{1}{4}x^3 + \frac{1}{3}x^2 - \frac{1}{4}x + \frac{1}{2} \\ \frac{3}{2}x^2 + \frac{1}{10}x - \frac{1}{5}x \end{array} \right. \\
 \hline
 -\frac{3}{8}x^5 - \frac{1}{2}x^4 + \frac{3}{8}x^3 - \frac{3}{4}x^2 \\
 \hline
 \frac{1}{40}x^4 - \frac{1}{60}x^3 - \frac{11}{120}x^2 + \frac{1}{10}x \\
 -\frac{1}{40}x^4 - \frac{1}{30}x^3 + \frac{1}{40}x^2 - \frac{1}{20}x \\
 \hline
 -\frac{1}{20}x^3 - \frac{1}{15}x^2 + \frac{1}{20}x - \frac{1}{10} \\
 \frac{1}{20}x^3 + \frac{1}{15}x^2 - \frac{1}{20}x + \frac{1}{10} \\
 \hline
 \hline
 10. \quad \frac{1}{2}m^5 - \frac{5}{6}m^4n + \frac{99}{40}m^3n^2 - \frac{101}{60}m^2n^3 + \frac{7}{6}mn^4 - \frac{5}{8}n^5 \quad \left| \begin{array}{l} \frac{3}{4}m^3 - \frac{1}{2}m^2n + \frac{2}{5}mn^2 - \frac{1}{4}n^3 \\ \frac{2}{3}m^2 - \frac{2}{3}mn + \frac{5}{2}n^2 \end{array} \right. \\
 \hline
 -\frac{1}{2}m^5 + \frac{1}{3}m^4n - \frac{4}{15}m^3n^2 + \frac{1}{6}m^2n^3 \\
 \hline
 -\frac{1}{2}m^4n + \frac{53}{24}m^3n^2 - \frac{91}{60}m^2n^3 + \frac{7}{6}mn^4 \\
 \frac{1}{2}m^4n - \frac{1}{3}m^3n^2 + \frac{4}{15}m^2n^3 - \frac{1}{6}mn^4 \\
 \hline
 \frac{15}{8}m^3n^2 - \frac{5}{4}m^2n^3 + mn^4 - \frac{5}{8}n^5 \\
 -\frac{15}{8}m^3n^2 + \frac{5}{4}m^2n^3 - mn^4 + \frac{5}{8}n^5 \\
 \hline
 \hline
 \end{array}$$

## EJERCICIO 58

$$\begin{array}{l}
 1. \quad x^5 - x^4 + x^2 - x \div x^3 - x^2 + x \\
 \begin{array}{r}
 1-1+0+1-1 \quad | \quad 1-1+1 \\
 -1+1-1 \quad \quad \quad 1+0-1 \\
 \hline
 -1+1-1 \quad \Rightarrow x^2 - 1 \\
 +1-1+1 \\
 \hline
 \end{array} \\
 \\
 2. \quad x^7 + x^6 - 11x^5 + 3x^4 - 13x^3 + 19x^2 - 56 \div x^3 - 2x^2 - 7 \\
 \begin{array}{r}
 1+1-11+3-13+19+0-56 \quad | \quad 1-2+0-7 \\
 -1+2-0+7 \\
 \hline
 3-11+10-13 \quad \Rightarrow x^4 + 3x^3 - 5x^2 + 8 \\
 -3+6-0+21 \\
 \hline
 -5+10+8+19 \\
 +5-10+0-35 \\
 \hline
 8-16+0-56 \\
 -8+16-0+56 \\
 \hline
 \end{array} \\
 \\
 3. \quad a^6 + a^5b - 7a^4b^2 + 12a^3b^2 - 13a^2b^4 + 7ab^5 - b^6 \div a^2 - 2ab + b^2 \\
 \begin{array}{r}
 1+1-7+12-13+7-1 \quad | \quad 1-2+1 \\
 -1+2-1 \quad \quad \quad 1+3-2+5-1 \\
 \hline
 3-8+12 \quad \Rightarrow a^4 + 3a^3b - 2a^2b^2 + 5ab^3 - b^4 \\
 -3+6-3 \\
 \hline
 -2+9-13 \\
 2-4+2 \\
 \hline
 5-11+7 \\
 -5+10-5 \\
 \hline
 -1+2-1 \\
 +1-2+1 \\
 \hline
 \end{array}
 \end{array}$$

$$4. m^6 - 5m^5n + 2m^4n^2 + 20m^3n^3 - 19m^2n^4 - 10mn^5 - n^6 + m^3 - 4mn^2 - n^3$$

$$\begin{array}{r} 1-5+2+20-19-10-1 \quad | \quad 1+0-4-1 \\ -1-0+4+1 \quad \quad \quad 1-5+6+1 \end{array}$$

$$\begin{array}{r} -5+6+21-19 \\ 5+0-20-5 \end{array} \Rightarrow m^3 - 5m^2n + 6mn^2 + n^3$$

$$\begin{array}{r} 6+1-24-10 \\ -6-0+24+6 \end{array}$$

$$\begin{array}{r} 1+0-4-1 \\ -1-0+4+1 \end{array}$$

$$8. m^{28} - 12m^{24} + 53m^{20} - 127m^{16} + 187m^{12} - 192m^8 + 87m^4 - 45 + m^{12} - 7m^8 + 9m^4 - 15$$

$$\begin{array}{r} 1-12+53-127+187-192+87-45 \quad | \quad 1-7+9-15 \\ -1+7-9+15 \quad \quad \quad 1-5+9-4+3 \end{array}$$

$$\begin{array}{r} -5+44-112+187 \\ +5-35+45-75 \end{array} \Rightarrow m^{16} - 5m^{12} + 9m^8 - 4m^4 + 3$$

$$\begin{array}{r} 9-67+112-192 \\ -9+63-81+135 \end{array}$$

$$\begin{array}{r} -4+31-57+87 \\ +4-28+36-60 \end{array}$$

$$\begin{array}{r} 3-21+27-45 \\ -3+21-27+45 \end{array}$$

$$5. x^8 - 2x^6 - 50x^4 + 58x^2 - 15 + x^4 + 6x^2 - 5$$

$$\begin{array}{r} 1-2-50+58-15 \quad | \quad 1+6-5 \\ -1-6+5 \quad \quad \quad 1-8+3 \end{array}$$

$$\begin{array}{r} -8-45+58 \\ +8+48-40 \end{array} \Rightarrow x^4 - 8x^2 + 3$$

$$\begin{array}{r} 3+18-15 \\ -3-18+15 \end{array}$$

$$6. a^{14} - 7a^{12} + 9a^{10} + 23a^8 - 52a^6 + 42a^4 - 20a^2 + a^8 - 4a^6 + 3a^4 - 2a^2$$

$$\begin{array}{r} 1-7+9+23-52+42-20 \quad | \quad 1-4+3-2 \\ -1+4-3+2 \quad \quad \quad 1-3-6+10 \end{array}$$

$$\begin{array}{r} -3+6+25-52 \\ 3-12+9-6 \end{array} \Rightarrow a^6 - 3a^4 - 6a^2 + 10$$

$$\begin{array}{r} -6+34-58+42 \\ +6-24+18-12 \end{array}$$

$$\begin{array}{r} 10-40+30-20 \\ -10+40-30+20 \end{array}$$

$$9. 2x^7 - 6x^6y - 8x^5y^2 - 20x^4y^3 - 24x^3y^4 - 18x^2y^5 - 4y^7 + 2x^2 + 4y^2$$

$$\begin{array}{r} 2-6-8-20-24-18+0-4 \quad | \quad 2+0+4 \\ -2-0-4 \quad \quad \quad 1-3-6-4+0-1 \end{array}$$

$$\begin{array}{r} -6-12-20 \\ 6+0+12 \end{array} \Rightarrow x^5 - 3x^4y - 6x^3y^2 - 4x^2y^3 - y^5$$

$$\begin{array}{r} 6+0+12 \\ -12-8-24 \end{array}$$

$$\begin{array}{r} +12+0+24 \\ -8+0-18 \end{array}$$

$$\begin{array}{r} +8+0+16 \\ -2+0-4 \end{array}$$

$$\begin{array}{r} +12+0+24 \\ -8+0-18 \end{array}$$

$$\begin{array}{r} +8+0+16 \\ -2+0-4 \end{array}$$

$$\begin{array}{r} +12+0+24 \\ -8+0-18 \end{array}$$

$$\begin{array}{r} +8+0+16 \\ -2+0-4 \end{array}$$

$$\begin{array}{r} +12+0+24 \\ -8+0-18 \end{array}$$

$$7. 3x^{15} - 20x^{12} + 51x^9 - 70x^6 + 46x^3 - 20 + 3x^6 - 8x^3 + 10$$

$$\begin{array}{r} 3-20+51-70+46-20 \quad | \quad 3-8+10 \\ -3+8-10 \quad \quad \quad 1-4+3-2 \end{array}$$

$$\begin{array}{r} -12+41-70 \\ 12-32+40 \end{array} \Rightarrow x^9 - 4x^6 + 3x^3 - 2$$

$$\begin{array}{r} 12-32+40 \\ 9-30+46 \end{array}$$

$$\begin{array}{r} -9+24-30 \\ -6+16-20 \end{array}$$

$$\begin{array}{r} -6+16-20 \\ 6-16+20 \end{array}$$

$$10. 6a^9 - 12a^7 + 2a^6 - 36a^5 + 6a^4 - 16a^3 + 38a^2 - 44a + 14 + a^4 - 2a^2 + a - 7$$

$$\begin{array}{r} 6+0-12+2-36+6-16+38-44+14 \quad | \quad 1+0-2+1-7 \\ -6-0+12-6+42 \quad \quad \quad 6+0+0-4+6-2 \end{array}$$

$$\begin{array}{r} -4+6+6-16+38 \\ +4+0-8+4-28 \end{array}$$

$$\begin{array}{r} -4+6+6-16+38 \\ +4+0-8+4-28 \end{array}$$

$$\begin{array}{r} -4+6+6-16+38 \\ +4+0-8+4-28 \end{array}$$

$$\begin{array}{r} -4+6+6-16+38 \\ +4+0-8+4-28 \end{array}$$

$$\begin{array}{r} -4+6+6-16+38 \\ +4+0-8+4-28 \end{array}$$

$$\begin{array}{r} -4+6+6-16+38 \\ +4+0-8+4-28 \end{array}$$

$$11. n^{10} - 6n^8 + 5n^7 + 13n^6 - 23n^5 - 8n^4 + 44n^3 - 12n^2 - 32n + 16 \div n^6 - 3n^4 + 5n^3 - 8n + 4$$

$$\begin{array}{r} 1+0-6+5+13-23-8+44-12-32+16 \quad | \quad 1+0-3+5+0-8+4 \\ -1-0+3-5-0+8-4 \end{array}$$

$$\begin{array}{r} -3+0+13-15-12+44-12 \\ +3+0-9+15+0-24+12 \end{array} \quad \Rightarrow n^4 - 3n^2 + 4$$

$$\begin{array}{r} 4+0-12+20+0-36+16 \\ -4+0+12-20-0+36-16 \end{array}$$

$$12. 3x^7 - 4x^6y - 15x^5y^2 + 29x^4y^3 - 13x^3y^4 + 5xy^6 - 3y^7 \div x^3 - 5xy^2 + 3y^3$$

$$\begin{array}{r} 3-4-15+29-13+0+5-3 \quad | \quad 1+0-5+3 \\ -3-0+15-9 \end{array}$$

$$\begin{array}{r} -4+0+20-13 \\ +4+0-20+12 \end{array} \quad \Rightarrow 3x^4 - 4x^3y - y^4$$

$$-1+0+5-3$$

$$1-0-5+3$$

$$13. x^{16} - 4x^{14}y^2 - 10x^{12}y^4 + 21x^{10}y^6 + 28x^8y^8 - 23x^6y^{10} + 9x^4y^{12} + 33x^2y^{14} - 6y^{16} \div x^6 - 4x^4y^2 - 5x^2y^4 + y^6$$

$$\begin{array}{r} 1-4-10+21+28-23+9+33-6 \quad | \quad 1-4-5+1 \\ -1+4+5-1 \end{array}$$

$$\begin{array}{r} -5+20+28-23 \\ +5-20-25+5 \end{array} \quad \Rightarrow x^{10} - 5x^6y^4 + 3x^2y^8 - 6y^{10}$$

$$3-18+9+33$$

$$-3+12+15-3$$

$$-6+24+30-6$$

$$+6-24-30+6$$

$$14. a^{m+2} - 3a^{m+1} - 5a^m + 20a^{m-1} - 25a^{m-3} + a^2 - 5$$

$$\begin{array}{r} 1-3-5+20+0-25 \quad | \quad 1+0-5 \\ -1-0+5 \end{array}$$

$$-3+0+20 \quad \Rightarrow a^m - 3a^{m-1} + 5a^{m-3}$$

$$+3+0-15$$

$$+5+0-25$$

$$-5-0+25$$

$$15. 7a^{2x+5} - 35a^{2x+4} + 6a^{2x+3} - 78a^{2x+2} - 5a^{2x+1} - 42a^{2x} - 7a^{2x-1} + 7a^{x+3} + 6a^{x+1} + a^x$$

$$\begin{array}{r} 7-35+6-78-5-42-7 \quad | \quad 7+0+6+1 \\ -7-0-6-1 \end{array}$$

$$-35+0-79-5$$

$$+35+0+30+5$$

$$-49+0-42-7$$

$$49-0+42+7$$

$$1-5+0-7 \quad \Rightarrow a^{x+2} - 5a^{x+1} - 7a^{x-1}$$

$$16. 6x^{2a+3} - 4x^{2a+2} - 28x^{2a+1} + 21x^{2a} - 46x^{2a-1} + 19x^{2a-2} - 12x^{2a-3} - 6x^{2a-4} \div 6x^{a+1} - 4x^a + 2x^{a-1} + x^{a-2}$$

$$\begin{array}{r} 6-4-28+21-46+19-12-6 \quad | \quad 6-4+2+1 \\ -6+4-2-1 \end{array}$$

$$-30+20-46+19$$

$$+30-20+10+5$$

$$-36+24-12-6$$

$$36-24+12+6$$

$$1+0-5+0-6 \quad \Rightarrow x^{a+2} - 5x^a - 6x^{a-2}$$

$$17. 6a^{5x+3} - 23a^{5x+2} + 12a^{5x+1} - 34a^{5x} + 22a^{5x-1} - 15a^{5x-2} + a^{2x+2} - 3a^{2x+1} - a^{2x} - 5a^{2x-1}$$

$$\begin{array}{r} 6-23+12-34+22-15 \quad | \quad 1-3-1-5 \\ -6+18+6+30 \end{array}$$

$$-5+18-4+22$$

$$+5-15-5-25$$

$$3-9-3-15$$

$$-3+9+3+15$$

$$6-5+3$$

$$\Rightarrow 6a^{3x+1} - 5a^{3x} + 3a^{3x-1}$$

## EJERCICIO 59

$$\begin{array}{r} 1. \quad a^2 + b^2 \overline{) a^2} \\ \underline{-a^2} \phantom{+} 1 + \frac{b^2}{a^2} \\ b^2 \end{array}$$

$$\begin{array}{r} 2. \quad a^4 + 2 \overline{) a^3} \\ \underline{-a^4} \phantom{+} a + \frac{2}{a^3} \\ +2 \end{array}$$

$$\begin{array}{r} 3. \quad 9x^3 + 6x^2 + 7 \overline{) 3x^2} \\ \underline{-9x^3} \phantom{+} 3x + 2 + \frac{7}{3x^2} \\ +6x^2 \\ \underline{-6x^2} \\ +7 \end{array}$$

$$\begin{array}{r} 4. \quad 16a^4 - 20a^3b + 8a^2b^2 + 7ab^3 \overline{) 4a^2} \\ \underline{-16a^4} \phantom{+} 20a^3b \\ \underline{+20a^3b} \phantom{+} 8a^2b^2 \\ \underline{-8a^2b^2} \phantom{+} 7ab^3 \\ +7ab^3 \end{array}$$

$$\begin{array}{r} 9. \quad x^3 - x^2 + 3x + 2 \overline{) x^2 - x + 1} \\ \underline{-x^3 + x^2 - x} \phantom{+} 2x + 2 \\ +2x + 2 \end{array}$$

$$\begin{array}{r} 10. \quad x^3 \phantom{+} y^3 \overline{) x - y} \\ \underline{-x^3 + x^2y} \phantom{+} x^2y \\ \phantom{-} x^2y \\ \underline{-x^2y + xy^2} \phantom{+} xy^2 + y^3 \\ \phantom{-} xy^2 + y^3 \\ \underline{-xy^2 + y^3} \\ 2y^3 \end{array}$$

$$\begin{array}{r} 5. \quad x^2 + 7x + 10 \overline{) x + 6} \\ \underline{-x^2 - 6x} \phantom{+} x + 10 + \frac{4}{x+6} \\ x + 10 \\ \underline{-x - 6} \\ +4 \end{array}$$

$$\begin{array}{r} 6. \quad x^2 - 5x + 7 \overline{) x - 4} \\ \underline{-x^2 + 4x} \phantom{+} x - 1 + \frac{3}{x-4} \\ -x + 7 \\ \underline{x - 4} \\ +3 \end{array}$$

$$\begin{array}{r} 11. \quad x^5 \phantom{+} y^5 \overline{) x - y} \\ \underline{-x^4 + x^4y} \phantom{+} x^4y \\ \phantom{-} x^4y \\ \underline{-x^4y + x^3y^2} \phantom{+} x^3y^2 \\ \phantom{-} x^3y^2 \\ \underline{-x^3y^2 + x^2y^3} \phantom{+} x^2y^3 \\ \phantom{-} x^2y^3 \\ \underline{-x^2y^3 + xy^4} \phantom{+} xy^4 + y^5 \\ \phantom{-} xy^4 + y^5 \\ \underline{-xy^4 + y^5} \\ 2y^5 \end{array}$$

$$\begin{array}{r} 7. \quad m^4 - 11m^2 + 34 \overline{) m^2 - 3} \\ \underline{-m^4 + 3m^2} \phantom{+} m^2 - 8 + \frac{10}{m^2-3} \\ -8m^2 + 34 \\ \underline{8m^2 - 24} \\ +10 \end{array}$$

$$\begin{array}{r} 8. \quad x^2 - 6xy + y^2 \overline{) x + y} \\ \underline{-x^2 - xy} \phantom{+} x - 7y + \frac{8y^2}{x+y} \\ -7xy + y^2 \\ \underline{+7xy + 7y^2} \\ +8y^2 \end{array}$$

$$\begin{array}{r} 12. \quad x^3 + 4x^2 - 5x + 8 \overline{) x^2 - 2x + 1} \\ \underline{-x^3 + 2x^2 - x} \phantom{+} x + 6 + \frac{6x+2}{x^2-2x+1} \\ 6x^2 - 6x + 8 \\ \underline{-6x^2 + 12x - 6} \\ 6x + 2 \end{array}$$

$$\begin{array}{r}
 13. \quad \begin{array}{r} 8a^3 - 6a^2b + 5ab^2 - 9b^3 \\ -8a^3 + 12a^2b \\ \hline 6a^2b + 5ab^2 \\ -6a^2b + 9ab^2 \\ \hline 14ab^2 - 9b^3 \\ -14ab^2 + 21b^3 \\ \hline 12b^3 \end{array} \quad \begin{array}{r} 2a - 3b \\ 4a^2 + 3ab + 7b^2 + \frac{12b^3}{2a-3b} \end{array} \\
 14. \quad \begin{array}{r} x^5 - 3x^4 \\ -x^5 + 3x^4 - 2x^3 \\ \hline -2x^3 + 9x^2 + 7x \\ + 2x^3 - 6x^2 + 4x \\ \hline 3x^2 + 11x - 4 \\ -3x^2 + 9x - 6 \\ \hline 20x - 10 \end{array} \quad \begin{array}{r} +9x^2 + 7x - 4 \\ x^2 - 3x + 2 \\ x^3 - 2x + 3 + \frac{20x-10}{x^2-3x+2} \end{array}
 \end{array}$$

## EJERCICIO 60

Para los problemas del 1 al 9 las literales toman los siguientes valores:

$$a = -1 \quad b = -2 \quad c = -1/2$$

$$\begin{array}{lll}
 1. a^2 - 2ab + b^2 & 2. 3a^3 - 4a^2b + 3ab^2 - b^3 & 3. a^4 - 3a^3 + 2ac - 3bc \\
 = (-1)^2 - 2(-1) \cdot 2 + 2^2 & = 3(-1)^3 - 4(-1)^2 \cdot 2 + 3(-1)2^2 - 2^3 & = (-1)^4 - 3(-1)^3 + 2(-1)\left(-\frac{1}{2}\right) - 3 \cdot 2\left(-\frac{1}{2}\right) \\
 = 1 + 2 \cdot 2 + 4 = 1 + 4 + 4 = 9 & = -3 - 4 \cdot 2 - 3 \cdot 4 - 8 = -3 - 8 - 12 - 8 = -31 & = 1 + 3 + 2 \cdot \frac{1}{2} + 6 \cdot \frac{1}{2} = 1 + 3 + 1 + 3 = 8
 \end{array}$$

$$\begin{array}{l}
 4. a^5 - 8a^4c + 16a^3c^2 - 20a^2c^3 + 40ac^4 - c^5 \\
 = (-1)^5 - 8(-1)^4\left(-\frac{1}{2}\right) + 16(-1)^3\left(-\frac{1}{2}\right)^2 - 20(-1)^2\left(-\frac{1}{2}\right)^3 + 40(-1)\left(-\frac{1}{2}\right)^4 - \left(-\frac{1}{2}\right)^5 \\
 = -1 + 8 \cdot \frac{1}{2} - 16 \cdot \frac{1}{4} + 20 \cdot \frac{1}{8} - 40 \cdot \frac{1}{16} + \frac{1}{32} = -1 + 4 - 4 + \frac{5}{2} - \frac{5}{2} + \frac{1}{32} = -1 + \frac{1}{32} = -\frac{32+1}{32} = -\frac{31}{32}
 \end{array}$$

$$\begin{array}{ll}
 5. (a-b)^2 + (b-c)^2 - (a-c)^2 & 6. (b+a)^3 - (b-c)^3 - (a-c)^3 \\
 = (-1-2)^2 + \left[2 - \left(-\frac{1}{2}\right)\right]^2 - \left[-1 - \left(-\frac{1}{2}\right)\right]^2 & = [2+(-1)]^3 - \left[2 - \left(-\frac{1}{2}\right)\right]^3 - \left[-1 - \left(-\frac{1}{2}\right)\right]^3 \\
 = (-3)^2 + \left[2 + \frac{1}{2}\right]^2 - \left[-1 + \frac{1}{2}\right]^2 = 9 + \left(\frac{5}{2}\right)^2 - \left(-\frac{1}{2}\right)^2 & = [2-1]^3 - \left[2 + \frac{1}{2}\right]^3 - \left[-1 + \frac{1}{2}\right]^3 = 1^3 - \left[\frac{5}{2}\right]^3 - \left[-\frac{1}{2}\right]^3 \\
 = 9 + \frac{25}{4} - \frac{1}{4} = 9 + \frac{24}{4} = 9 + 6 = 15 & = 1 - \frac{125}{8} + \frac{1}{8} = 1 - \frac{124}{8} = 1 - \frac{31}{2} = \frac{2-31}{2} = -\frac{29}{2} = -14\frac{1}{2}
 \end{array}$$

$$\begin{array}{ll}
 7. \frac{ab}{c} + \frac{ac}{b} - \frac{bc}{a} & 8. (a+b+c)^2 - (a-b-c)^2 + c \\
 = \frac{-1 \cdot 2}{-\frac{1}{2}} + \frac{-1\left(-\frac{1}{2}\right)}{2} - \frac{2\left(-\frac{1}{2}\right)}{-1} = \frac{2}{\frac{1}{2}} + \frac{\frac{1}{2}}{2} - \frac{2}{-2} & = \left[-1+2+\left(-\frac{1}{2}\right)\right]^2 - \left[-1-2-\left(-\frac{1}{2}\right)\right]^2 + \left(-\frac{1}{2}\right) \\
 = 4 + \frac{1}{4} - 1 = 3 + \frac{1}{4} = \frac{12+1}{4} = \frac{13}{4} = 3\frac{1}{4} & = \left[1-\frac{1}{2}\right]^2 - \left[-3+\frac{1}{2}\right]^2 - \frac{1}{2} = \left[\frac{1}{2}\right]^2 - \left[-\frac{5}{2}\right]^2 - \frac{1}{2} \\
 & = \frac{1}{4} - \frac{25}{4} - \frac{1}{2} = \frac{1-25-2}{4} = -\frac{26}{4} = -6\frac{3}{4} = -6\frac{1}{2}
 \end{array}$$

$$\begin{array}{l}
 9. 3(2a+b) - 4a(b+c) - 2c(a-b) \\
 = 3[2(-1)+2] - 4(-1)\left[2 + \left(-\frac{1}{2}\right)\right] - 2\left(-\frac{1}{2}\right)(-1-2) = 3[-2+2] + 4\left[2 - \frac{1}{2}\right] + \frac{2}{2}(-3) = 3 \cdot 0 + 4\left[\frac{3}{2}\right] - \frac{6}{2} = 6 - 3 = 3
 \end{array}$$

**Para los problemas 10 al 16 las literales toman los siguientes valores:**

$$a = 2 \quad b = 1/3 \quad x = -2 \quad y = -1 \quad m = 3 \quad n = 1/2$$

$$\begin{aligned} 10. \quad & \frac{x^4}{8} - \frac{x^2 y}{2} + \frac{3xy^2}{2} - y^3 \\ &= \frac{(-2)^4}{8} - \frac{(-2)^2(-1)}{2} + \frac{3(-2)(-1)^2}{2} - (-1)^3 \\ &= \frac{16}{8} - \frac{-4}{2} + \frac{-6}{2} + 1 = 2 + 2 - 3 + 1 = 2 \end{aligned}$$

$$\begin{aligned} 11. \quad & (a-x)^2 + (x-y)^2 + (x^2-y^2)(m+x-n) \\ &= [2-(-2)]^2 + [-2-(-1)]^2 + [(-2)^2 - (-1)^2] \left[ 3+(-2) - \frac{1}{2} \right] \\ &= [2+2]^2 + [-2+1]^2 + [4-1] \left[ 3-2-\frac{1}{2} \right] = [4]^2 + [-1]^2 + 3 \left[ 1-\frac{1}{2} \right] \\ &= 16 + 1 + 3 \left[ \frac{1}{2} \right] = 17 + \frac{3}{2} = \frac{34+3}{2} = \frac{37}{2} = 18\frac{1}{2} \end{aligned}$$

$$\begin{aligned} 12. \quad & -(x-y) + (x^2+y^2)(x-y-m) + 3b(x+y+n) \\ &= -[-2-(-1)] + [(-2)^2 + (-1)^2] [-2-(-1)-3] + 3 \cdot \frac{1}{3} \left[ -2+(-1) + \frac{1}{2} \right] \\ &= -[-2+1] + [4+1] [-3] + 1 \left[ -2-1 + \frac{1}{2} \right] \\ &= 1 + 5[-4] + \left[ -3 + \frac{1}{2} \right] = 1 - 20 + \left[ -\frac{5}{2} \right] = -19 - \frac{5}{2} = -\frac{38+5}{2} = -\frac{43}{2} = -21\frac{1}{2} \end{aligned}$$

$$\begin{aligned} 13. \quad & (3x-2y)(2n-4m) + 4x^2y^2 - \frac{x-y}{2} \\ &= [3(-2)-2(-1)] \left[ 2 \cdot \frac{1}{2} - 4 \cdot 3 \right] + 4(-2)^2(-1)^2 - \frac{(-2)-(-1)}{2} \\ &= [-6+2] [1-12] + 4 \cdot 4 \cdot 1 - \frac{-2+1}{2} = -4 [-11] + 16 - \frac{-1}{2} \\ &= 44 + 16 + \frac{1}{2} = 60 + \frac{1}{2} = 60\frac{1}{2} \end{aligned}$$

$$\begin{aligned} 14. \quad & \frac{4x}{3y} - \frac{x^3}{2+y^3} + \left( \frac{1}{n} - \frac{1}{b} \right) x + x^4 - m \\ &= \frac{4(-2)}{3(-1)} - \frac{(-2)^3}{2+(-1)^3} + \left( \frac{1}{\frac{1}{2}} - \frac{1}{\frac{1}{3}} \right) (-2) + (-2)^4 - 3 \\ &= \frac{-8}{-3} - \frac{-8}{2-1} + (2-3)(-2) + 16 - 3 = \frac{8}{3} + 8 + (-1)(-2) + 13 \\ &= \frac{8}{3} + 21 + 2 = \frac{8}{3} + 23 = \frac{8+69}{3} = \frac{77}{3} = 25\frac{2}{3} \end{aligned}$$

$$\begin{aligned} 15. \quad & x^2(x-y+m) - (x-y)(x^2+y^2-n) + (x+y)^2(m^2-2n) \\ &= (-2)^2 [-2-(-1)+3] - [-2-(-1)] \left[ (-2)^2 + (-1)^2 - \frac{1}{2} \right] + [-2+(-1)]^2 \left( 3^2 - 2 \cdot \frac{1}{2} \right) \\ &= 4[1+1] - [-2+1] \left[ 4+1 - \frac{1}{2} \right] + [-2-1]^2 (9-1) = 4 \cdot 2 - [-1] \left[ 5 - \frac{1}{2} \right] + [-3]^2 (8) = 8 + \frac{9}{2} + 9 \cdot 8 = 80 + \frac{9}{2} = \frac{160+9}{2} = \frac{169}{2} = 84\frac{1}{2} \end{aligned}$$

$$\begin{aligned} 16. \quad & \frac{3a}{x} + \frac{2y}{m} + \frac{3n}{y} - \frac{m}{n} + 2(x^3 - y^2 + 4) = \frac{3 \cdot 2}{-2} + \frac{2(-1)}{3} + \frac{3 \cdot \frac{1}{2}}{-1} - \frac{\frac{3}{2}}{\frac{1}{2}} + 2[(-2)^3 - (-1)^2 + 4] \\ &= -3 - \frac{2}{3} - \frac{3}{2} - 6 + 2[-8 - 1 + 4] \\ &= -9 - \frac{4+9}{6} + 2[-5] = -9 - \frac{13}{6} - 10 = -19 - \frac{13}{6} = -\frac{114+13}{6} = -\frac{127}{6} = -21\frac{1}{6} \end{aligned}$$

## EJERCICIO 61

$$\begin{array}{cccc} 1. & 7am & 8am & 9am & 10am \\ & \downarrow & \downarrow & \downarrow & \downarrow \\ & +5^\circ & +2^\circ & -1^\circ & -4^\circ \end{array}$$

$$\begin{array}{rcl} 3. & x^2 - 3xy & x^2 \\ & \frac{3xy - y^2}{x^2 - y^2} & -x^2 + y^2 \\ & & y^2 \quad Rta \end{array}$$

$$\begin{array}{rcl} 4. & 3x^2 - 5x + 6 & \\ & \frac{-3x^2 + 8x - 6}{3x} & Rta \end{array}$$

$$\begin{array}{rcl} 5. & 3 & 2a^2 - 3a + 8 \\ & \frac{2a^2 - 3a + 5}{2a^2 - 3a + 8} & \frac{8a + 5}{2a^2 + 5a + 13} \quad Rta \end{array}$$

$$\begin{aligned} 6. & -3x^2 - \left\{ -[4x^2 + 5x - (x^2 - (x+6))] \right\} \\ &= -3x^2 - \left\{ -[4x^2 + 5x - (x^2 - x - 6)] \right\} \\ &= -3x^2 - \left\{ -[4x^2 + 5x - x^2 + x + 6] \right\} \\ &= -3x^2 - \left\{ -3x^2 - 6x - 6 \right\} = -3x^2 + 3x^2 + 6x + 6 = 6x + 6 \end{aligned}$$

$$\begin{aligned}
 7. & (x+y)(x-y) - (x+y)^2 \\
 &= x^2 - xy + xy - y^2 - (x+y)(x+y) \\
 &= x^2 - y^2 - (x^2 + 2xy + y^2) \\
 &= x^2 - y^2 - x^2 - 2xy - y^2 \\
 &= -2xy - 2y^2
 \end{aligned}$$

$$\begin{aligned}
 8. & a=2 \quad b=3 \quad c=1 \\
 & 3(a+b) - 4(c-b) + \sqrt{\frac{c-b}{-a}} \\
 &= 3(2+3) - 4(1-3) + \sqrt{\frac{1-3}{-2}} \\
 &= 3 \cdot 5 - 4(-2) + \sqrt{\frac{-2}{-2}} \\
 &= 15 + 8 + \sqrt{1} = 23 + 1 = 24
 \end{aligned}$$

$$\begin{array}{r}
 9. \quad 3x^2 \quad -5y^2 \quad 2x^2+5xy+6y^2 \\
 \underline{-x^2+3xy-y^2} \quad \underline{-x^2-5xy} \\
 2x^2+3xy-6y^2 \quad x^2 \quad +6y^2
 \end{array}$$

$$\begin{array}{r}
 2x^2+3xy-6y^2 \\
 \underline{x^2 \quad +6y^2} \\
 3x^2+3xy \quad Rta
 \end{array}$$

$$\begin{aligned}
 10. & \frac{2}{3}a^2 - \frac{1}{2}ab + \frac{1}{5}b^2 \\
 & \frac{1}{2}a^2 + \frac{3}{4}ab - 2b^2 \\
 & \frac{1}{3}a^4 - \frac{1}{4}a^3b + \frac{1}{10}a^2b^2 \\
 & + \frac{1}{2}a^3b - \frac{3}{8}a^2b^2 + \frac{3}{20}ab^3 \\
 & - \frac{4}{3}a^2b^2 + ab^3 - \frac{2}{5}b^4 \\
 & \frac{1}{3}a^4 + \frac{1}{4}a^3b - \frac{193}{120}a^2b^2 + \frac{23}{20}ab^3 - \frac{2}{5}b^4
 \end{aligned}$$

$$\begin{array}{r}
 11. \quad x^5 \quad -x^3+5x^2 \\
 \quad -2x^4 \quad +2x^2-10x \\
 \quad \quad +6x^3 \quad -6x+30 \\
 \hline
 x^5-2x^4+5x^3+7x^2-16x+30 \quad | \quad x^2-2x+6 \\
 \underline{-x^5+2x^4-6x^3} \quad \quad \quad x^3-x+5 \\
 \quad -x^3+7x^2-16x \\
 \quad +x^3-2x^2+6x \\
 \quad \quad 5x^2-10x+30 \\
 \quad \quad \underline{-5x^2+10x-30}
 \end{array}$$

$$\begin{array}{r}
 12. \quad \frac{1}{4}a^3 \quad -\frac{1}{90}ab^2 + \frac{1}{15}b^3 \quad \left| \quad \frac{1}{2}a + \frac{1}{3}b \right. \\
 \underline{-\frac{1}{4}a^3 - \frac{1}{6}a^2b} \quad \quad \quad \frac{1}{2}a^2 - \frac{1}{3}ab + \frac{1}{5}b^2 \\
 \quad -\frac{1}{6}a^2b - \frac{1}{90}ab^2 \\
 \quad \quad \underline{\frac{1}{6}a^2b + \frac{1}{9}ab^2} \quad \quad \quad \frac{1}{2}a^2 + ab + \frac{1}{5}b^2 \\
 \quad \quad \quad \frac{1}{10}ab^2 + \frac{1}{15}b^3 \quad \quad \quad \underline{-\frac{1}{2}a^2 + \frac{1}{3}ab - \frac{1}{5}b^2} \\
 \quad \quad \quad \underline{-\frac{1}{10}ab^2 - \frac{1}{15}b^3} \quad \quad \quad \frac{3+1}{3}ab = \frac{4}{3}ab
 \end{array}$$

$$\begin{array}{r}
 13. \quad \frac{-3ab^2 - b^3}{2a^2b + 3ab^2 - b^3} \quad \frac{a^3 - a^2b + b^3}{-2a^2b + 2b^3} \\
 \frac{2a^2b}{2a^2b} \quad \frac{-2b^3}{-2b^3} \quad \frac{a^3 - 3a^2b + 3b^3}{a^3 - 3a^2b + 3b^3}
 \end{array}$$

$$\begin{array}{r}
 a^3 - 3a^2b + 3b^3 \\
 \underline{a^2 - ab + b^2} \\
 a^5 - 3a^4b \quad + 3a^2b^3 \\
 \quad - a^4b + 3a^3b^2 \quad - 3ab^4 \\
 \quad \quad + a^3b^2 - 3a^2b^3 \quad + 3b^5 \\
 \hline
 a^5 - 4a^4b + 4a^3b^2 \quad - 3ab^4 + 3b^5
 \end{array}$$

$$\begin{array}{r}
 14. \quad x^3 - 5x^2 + 4x \\
 \quad - 6x^2 - 6x + 3 \quad \quad 2x^3 - 16x^2 + 5x + 12 \\
 \quad - 8x^2 + 8x - 3 \quad \quad \underline{-x^3 + 19x^2 - 6x} \\
 \hline
 x^3 - 19x^2 + 6x \quad \quad \quad x^3 + 3x^2 - x + 12
 \end{array}$$

$$\begin{array}{r}
 x^3 + 3x^2 - x + 12 \quad | \quad x^2 - x + 3 \\
 \underline{-x^3 + x^2 - 3x} \quad \quad \quad x+4 \\
 \quad 4x^2 - 4x + 12 \\
 \quad \underline{-4x^2 + 4x - 12}
 \end{array}$$

$$\begin{aligned}
 15. & (2+x)^2(1+x^2) - (x^2-2)(x^2+x-3) = x^2(3x+10) + 2(3x-1) \\
 & (4+4x+x^2)(1+x^2) - (x^4+x^3-5x^2-2x+6) = 3x^3+10x^2+6x-2 \\
 & 4+5x^2+4x+4x^3-x^3+5x^2+2x-6 = 3x^3+10x^2+6x-2 \\
 & 3x^3+10x^2+6x-2 = 3x^3+10x^2+6x-2
 \end{aligned}$$



$$16. x = -2 \quad y = 1$$

$$\begin{aligned} & (x+y)^2(x-y)^2 + 2(x+y)(x-y) \\ &= (-2+1)^2(-2-1)^2 + 2(-2+1)(-2-1) \\ &= (-1)^2(-3)^2 + 2(-1)(-3) \\ &= 1 \cdot 9 + 2 \cdot 3 = 9 + 6 = 15 \end{aligned}$$

$$17. x^2 + 2x + 8$$

$$\begin{array}{r} x+4 \quad x^2+4x+6 \\ x-6 \quad 4x^2-8x-3 \quad Rta \\ \hline x^2+4x+6 \quad 5x^2-4x+3 \end{array}$$

$$18. -\{3a+(-b+a)-2(a+b)\} \quad -2[(a+b)-(a-b)]$$

$$\begin{aligned} &= -\{3a-b+a-2a-2b\} &= -2[a+b-a+b] \\ &= -\{2a-3b\} &= -2[2b] \\ &= -2a+3b &= -4b \\ & \quad -4b \\ & \quad \frac{2a-3b}{2a-7b} \quad Rta \end{aligned}$$

$$19. 5x + [- (3x - (x - y))] \quad 8x + [- 2x + (-x + y)]$$

$$\begin{aligned} &= 5x + [- (3x - x + y)] &= 8x + [- 2x - x + y] \\ &= 5x + [- 2x - y] &= 8x - 3x + y \\ &= 5x - 2x - y &= 5x + y \\ &= 3x - y \end{aligned}$$

$$\begin{array}{r} 5x+y \\ 3x-y \\ \hline 15x^2+3xy \\ -5xy-y^2 \\ \hline 15x^2-2xy-y^2 \quad Rta \end{array}$$

$$20. \frac{1}{4}x^3 + \frac{1}{24}x^2y + \frac{5}{12}xy^2 + \frac{1}{3}y^3 \quad \left| \frac{1}{2}x^2 - \frac{1}{4}xy + y^2 \right.$$

$$\begin{array}{r} -\frac{1}{4}x^3 + \frac{1}{8}x^2y - \frac{1}{2}xy^2 \\ \hline \frac{1}{6}x^2y - \frac{1}{12}xy^2 + \frac{1}{3}y^3 \\ -\frac{1}{6}x^2y + \frac{1}{12}xy^2 - \frac{1}{3}y^3 \\ \hline \end{array}$$

$$\begin{aligned} & 2x + [- 5x - (x - y)] &= 4x + y \\ &= 2x + [- 5x - x + y] &= \frac{1}{2}x - \frac{1}{3}y \\ &= 2x + [- 6x + y] &= \frac{8+1}{2}x + \frac{3-1}{3}y \\ &= 2x - 6x + y &= -\frac{9}{2}x + \frac{2}{3}y \quad Rta \\ &= -4x + y \end{aligned}$$

$$21. \quad [x^2 - (3x+2)][x^2 + (-x+3)] = x^2(x^2 - 4x + 4) - (7x + 6)$$

$$\begin{aligned} & [x^2 - 3x - 2][x^2 - x + 3] = x^4 - 4x^3 + 4x^2 - 7x - 6 \\ & x^4 - x^3 + 3x^2 - 3x^3 + 3x^2 - 9x - 2x^2 + 2x - 6 = x^4 - 4x^3 + 4x^2 - 7x - 6 \\ & x^4 - 4x^3 + 4x^2 - 7x - 6 = x^4 - 4x^3 + 4x^2 - 7x - 6 \end{aligned}$$

$$22. \begin{array}{r} [x(x+y) - x(x-y)] \quad [2(x^2+y^2) - 3(x^2-y^2)] \\ = x^2 + xy - x^2 - xy \quad = 2x^2 + 2y^2 - 3x^2 + 3y^2 \\ = 2xy \quad = -x^2 + 5y^2 \end{array}$$

$$\begin{array}{r} -x^2 + 5y^2 \quad -2x^3y + 10xy^3 \\ 2xy \quad 4x^3y - 7xy^3 \quad Rta \\ \hline -2x^3y + 10xy^3 \quad 2x^3y + 3xy^3 \end{array}$$

$$23. \begin{array}{r} x^3 \quad -y^3 \quad x^2 + 3xy - y^2 \\ -x^3 + x^2y \quad x^2 + xy + y^2 \quad x^2 + xy + y^2 \\ \hline x^2y \quad x^4 + 3x^3y - x^2y^2 \\ -x^2y + xy^2 \quad + x^3y + 3x^2y^2 - xy^3 \\ \hline xy^2 \quad + x^2y^2 + 3xy^3 - y^4 \\ \hline -xy^2 + y^3 \quad x^4 + 4x^3y + 3x^2y^2 + 2xy^3 - y^4 \quad Rta \end{array}$$

$$24. \begin{aligned} & (x-y)(x^2+xy+y^2) - (x+y)(x^2-xy+y^2) \\ &= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3 - [x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3] \\ &= x^3 - y^3 - [x^3 + y^3] \\ &= x^3 - y^3 - x^3 - y^3 \\ &= -2y^3 \end{aligned}$$

$$25. a=4 \quad b=9 \quad c=25$$

$$\begin{aligned} & \sqrt{\frac{ab}{c}} + 2(b-a)\sqrt{\frac{9b}{a^2}} - 3(c-b)\sqrt{\frac{c}{b}} \\ &= \sqrt{\frac{4 \cdot 9}{25}} + 2(9-4)\sqrt{\frac{9 \cdot 9}{4^2}} - 3(25-9)\sqrt{\frac{25}{9}} \\ &= \sqrt{\frac{36}{25}} + 2 \cdot 5 \sqrt{\frac{81}{16}} - 3 \cdot 16 \cdot \frac{5}{3} = \frac{6}{5} + 10 \cdot \frac{9}{4} - 48 \cdot \frac{5}{3} \\ &= \frac{6}{5} + \frac{45}{2} - 80 = \frac{12 + 225 - 800}{10} = \frac{-563}{10} = -56\frac{3}{10} \end{aligned}$$

$$26. \quad \begin{array}{r} x^3 + 3x^2 - 4x - 12 \quad | \quad x+3 \quad x^2 - 4 \quad = x - 2 \\ -x^3 - 3x^2 \\ \hline -4x - 12 \\ +4x + 12 \\ \hline \end{array}$$

$$\begin{array}{r} x^2 - 4 \quad | \quad x-2 \\ x^2 + 2x \quad x+2 \quad Rta \\ \hline 2x - 4 \\ -2x + 4 \\ \hline \end{array}$$

$$\begin{aligned}
27. \quad & 4x^2 - \{3x - (x^2 - (4+x))\} + [x^2 - \{x + (-3)\}] \\
& = 4x^2 - \{3x - (x^2 - 4 - x)\} + [x^2 - \{x - 3\}] \quad \text{Si } x = -2 \\
& = 4x^2 - \{3x - x^2 + 4 + x\} + [x^2 - x + 3] \quad 6x^2 - 5x - 1 \\
& = 4x^2 - 4x + x^2 - 4 + x^2 - x + 3 \quad = 6(-2)^2 - 5(-2) - 1 \\
& = 6x^2 - 5x - 1 \quad = 6(4) + 10 - 1 \\
& \quad \quad \quad = 24 + 10 - 1 = 33
\end{aligned}$$

$$\begin{aligned}
29. \quad & (a^2 + b^2)(a+b)(a-b) = a^4 - [3a + 2(a+2) - 4(a+1) - a + b^4] \\
& (a^2 + b^2)(a^2 - ab + ab - b^2) = a^4 - [3a + 2a + 4 - 4a - 4 - a + b^4] \\
& (a^2 + b^2)(a^2 - b^2) = a^4 - [b^4] \\
& a^4 - a^2b^2 + a^2b^2 - b^4 = a^4 - b^4 \\
& a^4 - b^4 = a^4 - b^4
\end{aligned}$$

$$\begin{aligned}
28. \quad & \frac{x^2 + 7x - 5}{x^4 + 7x^3 - 5x^2} \quad \frac{x^4 - 11x^3 + 21x}{18x^3 - 14x^2 - 84x + 45} \quad \text{Rta} \\
& \frac{-9x^2 - 63x + 45}{x^4 + 7x^3 - 14x^2 - 63x + 45}
\end{aligned}$$

$$\begin{aligned}
30. \quad & \frac{3}{x^3 + 5x^2 - 6} \quad \frac{-[x^2 + (-3x + 4) - (-x + 3)]}{x^3 + 5x^2 - 3} \\
& \quad \quad \quad = -[x^2 - 3x + 4 + x - 3] \\
& \quad \quad \quad = -[x^2 - 2x + 1] \\
& \quad \quad \quad = -x^2 + 2x - 1 \\
& \frac{x^3 + 5x^2 - 3}{x+1} \quad \frac{x^2 - x + 2}{-x^2 + 2x - 1} \\
& \frac{x^3 + 5x^2 + x - 2}{x+1} \quad \text{Rta} \quad \frac{x^2 - x + 2}{x+1}
\end{aligned}$$

## EJERCICIO 62

$$\begin{aligned}
1. \quad & (m+3)^2 = m^2 + 6m + 9 \\
2. \quad & (5+x)^2 = 25 + 10x + x^2 \\
3. \quad & (6a+b)^2 = 36a^2 + 12ab + b^2 \\
4. \quad & (9+4m)^2 = 81 + 72m + 16m^2 \\
5. \quad & (7x+11)^2 = 49x^2 + 154x + 121 \\
6. \quad & (x+y)^2 = x^2 + 2xy + y^2 \\
7. \quad & (1+3x^2)^2 = 1 + 6x^2 + 9x^4 \\
8. \quad & (2x+3y)^2 = 4x^2 + 12xy + 9y^2 \\
9. \quad & (a^2x + by^2)^2 = a^4x^2 + 2a^2xby^2 + b^2y^4
\end{aligned}$$

$$\begin{aligned}
10. \quad & (3a^3 + 8b^4)^2 = 9a^6 + 48a^3b^4 + 64b^8 \\
11. \quad & (4m^5 + 5n^6)^2 = 16m^{10} + 40m^5n^6 + 25n^{12} \\
12. \quad & (7a^2b^3 + 5x^4)^2 = 49a^4b^6 + 70a^2b^3x^4 + 25x^8 \\
13. \quad & (4ab^2 + 5xy^3)^2 = 16a^2b^4 + 40ab^2xy^3 + 25x^2y^6 \\
14. \quad & (8x^2y + 9m^3)^2 = 64x^4y^2 + 144x^2ym^3 + 81m^6 \\
15. \quad & (x^{10} + 10y^{12})^2 = x^{20} + 20x^{10}y^{12} + 100y^{24} \\
16. \quad & (a^m + a^n)^2 = a^{2m} + 2a^{m+n} + a^{2n} \\
17. \quad & (a^x + b^{x+1})^2 = a^{2x} + 2a^xb^{x+1} + b^{2x+2} \\
18. \quad & (x^{a+1} + y^{x-2})^2 = x^{2a+2} + 2x^{a+1}y^{x-2} + y^{2x-4}
\end{aligned}$$

## EJERCICIO 63

$$\begin{aligned}
1. \quad & (a-3)^2 = a^2 - 6a + 9 \\
2. \quad & (x-7)^2 = x^2 - 14x + 49 \\
3. \quad & (9-a)^2 = 81 - 18a + a^2 \\
4. \quad & (2a-3b)^2 = 4a^2 - 12ab + 9b^2 \\
5. \quad & (4ax-1)^2 = 16a^2x^2 - 8ax + 1 \\
6. \quad & (a^3 - b^3)^2 = a^6 - 2a^3b^3 + b^6 \\
7. \quad & (3a^4 - 5b^2)^2 = 9a^8 - 30a^4b^2 + 25b^4
\end{aligned}$$

$$\begin{aligned}
8. \quad & (x^2 - 1)^2 = x^4 - 2x^2 + 1 \\
9. \quad & (x^5 - 3ay^2)^2 = x^{10} - 6ax^5y^2 + 9a^2y^4 \\
10. \quad & (a^7 - b^7)^2 = a^{14} - 2a^7b^7 + b^{14} \\
11. \quad & (2m - 3n)^2 = 4m^2 - 12mn + 9n^2 \\
12. \quad & (10x^3 - 9xy^5)^2 = 100x^6 - 180x^4y^5 + 81x^2y^{10} \\
13. \quad & (x^m - y^n)^2 = x^{2m} - 2x^my^n + y^{2n} \\
14. \quad & (a^{x-2} - 5)^2 = a^{2x-4} - 10a^{x-2} + 25 \\
15. \quad & (x^{a+1} - 3x^{a-2})^2 = x^{2a+2} - 6x^{2a-1} + 9x^{2a-4}
\end{aligned}$$

## EJERCICIO 64

1.  $(x+y)(x-y) = x^2 - y^2$
2.  $(m-n)(m+n) = m^2 - n^2$
3.  $(a-x)(x+a) = a^2 - x^2$
4.  $(x^2+a^2)(x^2-a^2) = x^4 - a^4$
5.  $(2a-1)(1+2a) = 4a^2 - 1$
6.  $(n-1)(n+1) = n^2 - 1$
7.  $(1-3ax)(3ax+1) = 1 - 9a^2x^2$
8.  $(2m+9)(2m-9) = 4m^2 - 81$
9.  $(a^3-b^2)(a^3+b^2) = a^6 - b^4$
10.  $(y^2-3y)(y^2+3y) = y^4 - 9y^2$
11.  $(1-8xy)(8xy+1) = 1 - 64x^2y^2$
12.  $(6x^2-m^2x)(6x^2+m^2x) = 36x^4 - m^4x^2$
13.  $(a^m+b^n)(a^m-b^n) = a^{2m} - b^{2n}$
14.  $(3x^a-5y^m)(5y^m+3x^a) = 9x^{2a} - 25y^{2m}$
15.  $(a^{x+1}-2b^{x-1})(2b^{x-1}+a^{x+1}) = a^{2x+2} - 4b^{2x-2}$

## EJERCICIO 65

1.  $(x+y+z)(x+y-z) = x^2 + 2xy + y^2 - z^2$
2.  $(x-y+z)(x+y-z) = x^2 - y^2 + 2yz - z^2$
3.  $(x+y+z)(x-y-z) = x^2 - y^2 - 2yz - z^2$
4.  $(m+n+1)(m+n-1) = m^2 + 2mn + n^2 - 1$
5.  $(m-n-1)(m-n+1) = m^2 + n^2 - 2mn - 1$
6.  $(x+y-2)(x-y+2) = x^2 - y^2 + 4y - 4$
7.  $(n^2+2n+1)(n^2-2n-1) = n^4 - 4n^2 - 4n - 1$
8.  $(a^2-2a+3)(a^2+2a+3) = a^4 + 2a^2 + 9$
9.  $(m^2-m-1)(m^2+m-1) = m^4 - 3m^2 + 1$
10.  $(2a-b-c)(2a-b+c) = 4a^2 - 4ab + b^2 - c^2$
11.  $(2x+y-z)(2x-y+z) = 4x^2 - y^2 + 2yz - z^2$
12.  $(x^2-5x+6)(x^2+5x-6) = x^4 - 25x^2 + 60x - 36$
13.  $(a^2-ab+b^2)(a^2+b^2+ab) = a^4 + a^2b^2 + b^4$
14.  $(x^3-x^2-x)(x^3+x^2+x) = x^6 - x^4 - 2x^3 - x^2$

## EJERCICIO 66

1.  $(a+2)^3 = a^3 + 6a^2 + 12a + 8$
2.  $(x-1)^3 = x^3 - 3x^2 + 3x - 1$
3.  $(m+3)^3 = m^3 + 9m^2 + 27m + 27$
4.  $(n-4)^3 = n^3 - 12n^2 + 48n - 64$
5.  $(2x+1)^3 = 8x^3 + 12x^2 + 6x + 1$
6.  $(1-3y)^3 = 1 - 9y + 27y^2 - 27y^3$
7.  $(2+y^2)^3 = 8 + 12y^2 + 6y^4 + y^6$
8.  $(1-2n)^3 = 1 - 6n + 12n^2 - 8n^3$
9.  $(4n+3)^3 = 64n^3 + 144n^2 + 108n + 27$
10.  $(a^2-2b)^3 = a^6 - 6a^4b + 12a^2b^2 - 8b^3$
11.  $(2x+3y)^3 = 8x^3 + 36x^2y + 54xy^2 + 27y^3$
12.  $(1-a^2)^3 = 1 - 3a^2 + 3a^4 - a^6$

## EJERCICIO 67

1.  $(a+1)(a+2) = a^2 + 3a + 2$
2.  $(x+2)(x+4) = x^2 + 6x + 8$
3.  $(x+5)(x-2) = x^2 + 3x - 10$
4.  $(m-6)(m-5) = m^2 - 11m + 30$
5.  $(x+7)(x-3) = x^2 + 4x - 21$
6.  $(x+2)(x-1) = x^2 + x - 2$
7.  $(x-3)(x-1) = x^2 - 4x + 3$
8.  $(x-5)(x+4) = x^2 - x - 20$
9.  $(a-11)(a+10) = a^2 - a - 110$
10.  $(n-19)(n+10) = n^2 - 9n - 190$
11.  $(a^2+5)(a^2-9) = a^4 - 4a^2 - 45$
12.  $(x^2-1)(x^2-7) = x^4 - 8x^2 + 7$
13.  $(n^2-1)(n^2+20) = n^4 + 19n^2 - 20$
14.  $(n^3+3)(n^3-6) = n^6 - 3n^3 - 18$
15.  $(x^3+7)(x^3-6) = x^6 + x^3 - 42$
16.  $(a^4+8)(a^4-1) = a^8 + 7a^4 - 8$
17.  $(a^5-2)(a^5+7) = a^{10} + 5a^5 - 14$
18.  $(a^6+7)(a^6-9) = a^{12} - 2a^6 - 63$
19.  $(ab+5)(ab-6) = a^2b^2 - ab - 30$
20.  $(xy^2-9)(xy^2+12) = x^2y^4 + 3xy^2 - 108$
21.  $(a^2b^2-1)(a^2b^2+7) = a^4b^4 + 6a^2b^2 - 7$
22.  $(x^3y^3-6)(x^3y^3+8) = x^6y^6 + 2x^3y^3 - 48$
23.  $(a^x-3)(a^x+8) = a^{2x} + 5a^x - 24$
24.  $(a^{x+1}-6)(a^{x+1}+5) = a^{2x+2} - 11a^{x+1} + 30$

## EJERCICIO 68

- $(x+2)^2 = x^2 + 4x + 4$
- $(x+2)(x+3) = x^2 + 5x + 6$
- $(x+1)(x-1) = x^2 - 1$
- $(x-1)^2 = x^2 - 2x + 1$
- $(n+3)(n+5) = n^2 + 8n + 15$
- $(m-3)(m+3) = m^2 - 9$
- $(a+b-1)(a+b+1) = a^2 + 2ab + b^2 - 1$
- $(1+b)^3 = 1 + 3b + 3b^2 + b^3$
- $(a^2+4)(a^2-4) = a^4 - 16$
- $(3ab-5x^2)^2 = 9a^2b^2 - 30abx^2 + 25x^4$
- $(ab+3)(3-ab) = 9 - a^2b^2$
- $(1-4ax)^2 = 1 - 8ax + 16a^2x^2$
- $(a^2+8)(a^2-7) = a^4 + a^2 - 56$
- $(x+y+1)(x-y-1) = x^2 - y^2 - 2y - 1$
- $(1-a)(a+1) = 1 - a^2$
- $(m-8)(m+12) = m^2 + 4m - 96$
- $(x^2-1)(x^2+3) = x^4 + 2x^2 - 3$
- $(x^3+6)(x^3-8) = x^6 - 2x^3 - 48$
- $(5x^3+6m^4)^2 = 25x^6 + 60x^3m^4 + 36m^8$

## EJERCICIO 69

- $\frac{x^2-1}{x+1} = x-1$
- $\frac{1-x^2}{1-x} = 1+x$
- $\frac{x^2-y^2}{x+y} = x-y$
- $\frac{y^2-x^2}{y-x} = y+x$
- $\frac{x^2-4}{x+2} = x-2$
- $\frac{9-x^4}{3-x^2} = 3+x^2$
- $\frac{a^2-4b^2}{a+2b} = a-2b$
- $\frac{25-36x^4}{5-6x^2} = 5+6x^2$
- $\frac{4x^2-9m^2n^4}{2x+3mn^2} = 2x-3mn^2$
- $\frac{36m^2-49n^2x^4}{6m-7nx^2} = 6m+7nx^2$
- $\frac{81a^6-100b^8}{9a^3+10b^4} = 9a^3-10b^4$
- $\frac{a^4b^6-4x^8y^{10}}{a^2b^3+2x^4y^5} = a^2b^3+2x^4y^5$
- $\frac{x^{2n}-y^{2n}}{x^n+y^n} = x^n-y^n$

- $(x^4-2)(x^4+5) = x^8 + 3x^4 - 10$
- $(1-a+b)(b-a-1) = -1 + a^2 - 2ab + b^2$
- $(a^x+b^n)(a^x-b^n) = a^{2x} - b^{2n}$
- $(x^{a+1}-8)(x^{a+1}+9) = x^{2a+2} - x^{a+1} - 72$
- $(a^2b^2+c^2)(a^2b^2-c^2) = a^4b^4 - c^4$
- $(2a+x)^3 = 8a^3 + 12a^2x + 6ax^2 + x^3$
- $(x^2-11)(x^2-2) = x^4 - 13x^2 + 22$
- $(2a^3-5b^4)^2 = 4a^6 - 20a^3b^4 + 25b^8$
- $(a^3+12)(a^3-15) = a^6 - 3a^3 - 180$
- $(m^2-m+n)(n+m+m^2) = m^4 + 2m^2n + n^2 - m^2$
- $(x^4+7)(x^4-11) = x^8 - 4x^4 - 77$
- $(11-ab)^2 = 121 - 22ab + a^2b^2$
- $(x^2y^3-8)(x^2y^3+6) = x^4y^6 - 2x^2y^3 - 48$
- $(a+b)(a-b)(a^2-b^2) = a^4 - 2a^2b^2 + b^4$
- $(x+1)(x-1)(x^2-2) = x^4 - 3x^2 + 2$
- $(a+3)(a^2+9)(a-3) = a^4 - 81$
- $(x+5)(x-5)(x^2+1) = x^4 - 24x^2 - 25$
- $(a+1)(a-1)(a+2)(a-2) = a^4 - 5a^2 + 4$
- $(a+2)(a-3)(a-2)(a+3) = a^4 - 13a^2 + 36$

$$14. \frac{a^{2x+2}-100}{a^{x+1}-10} = a^{x+1} + 10$$

$$15. \frac{1-9x^{2m+4}}{1+3x^{m+2}} = 1-3x^{m+2}$$

$$16. \frac{(x+y)^2 - z^2}{(x+y) - z} = x + y + z$$

$$17. \frac{1-(a+b)^2}{1+(a+b)} = 1-a-b$$

$$18. \frac{4-(m+n)^2}{2+(m+n)} = 2-m-n$$

$$19. \frac{x^2-(x-y)^2}{x+(x-y)} = y$$

$$20. \frac{(a+x)^2-9}{(a+x)+3} = a+x-3$$

## EJERCICIO 70

1.  $\frac{1+a^3}{1+a} = 1-a+a^2$
2.  $\frac{1-a^3}{1-a} = 1+a+a^2$
3.  $\frac{x^3+y^3}{x+y} = x^2-xy+y^2$
4.  $\frac{8a^3-1}{2a-1} = 4a^2+2a+1$
5.  $\frac{8x^3+27y^3}{2x+3y} = 4x^2-6xy+9y^2$
6.  $\frac{27m^3-125n^3}{3m-5n} = 9m^2+15m+25n^2$
7.  $\frac{64a^3+343}{4a+7} = 16a^2-28a+49$
8.  $\frac{216-125y^3}{6-5y} = 36+30y+25y^2$
9.  $\frac{1+a^3b^3}{1+ab} = 1-ab+a^2b^2$
10.  $\frac{729-512b^3}{9-8b} = 81+72b+64b^2$
11.  $\frac{a^3x^3+b^3}{ax+b} = a^2x^2-axb+b^2$
12.  $\frac{n^3-m^3x^3}{n-mx} = n^2+nmx+m^2x^2$
13.  $\frac{x^6-27y^3}{x^2-3y} = x^4+3x^2y+9y^2$
14.  $\frac{8a^9+y^9}{2a^3+y^3} = 4a^6-2a^3y^3+y^6$
15.  $\frac{1-x^{12}}{1-x^4} = 1+x^4+x^8$
16.  $\frac{27x^6+1}{3x^2+1} = 9x^4-3x^2+1$
17.  $\frac{64a^3+b^9}{4a+b^3} = 16a^2-4ab^3+b^6$
18.  $\frac{a^6-b^6}{a^2-b^2} = a^4+a^2b^2+b^4$
19.  $\frac{125-343x^{15}}{5-7x^5} = 25+35x^5+49x^{10}$
20.  $\frac{n^6+1}{n^2+1} = n^4-n^2+1$

## EJERCICIO 71

1.  $\frac{x^4-y^4}{x-y} = x^3+x^2y+xy^2+y^3$
2.  $\frac{m^5+n^5}{m+n} = m^4-m^3n+m^2n^2-mn^3+n^4$
3.  $\frac{a^5-n^5}{a-n} = a^4+a^3n+a^2n^2+an^3+n^4$
4.  $\frac{x^6-y^6}{x+y} = x^5-x^4y+x^3y^2-x^2y^3+xy^4-y^5$
5.  $\frac{a^6-b^6}{a-b} = a^5+a^4b+a^3b^2+a^2b^3+ab^4+b^5$
6.  $\frac{x^7+y^7}{x+y} = x^6-x^5y+x^4y^2-x^3y^3+x^2y^4-xy^5+y^6$
7.  $\frac{a^7-m^7}{a-m} = a^6+a^5m+a^4m^2+a^3m^3+a^2m^4+am^5+m^6$
8.  $\frac{a^8-b^8}{a+b} = a^7-a^6b+a^5b^2-a^4b^3+a^3b^4-a^2b^5+ab^6-b^7$
9.  $\frac{x^{10}-y^{10}}{x-y} = x^9+x^8y+x^7y^2+x^6y^3+x^5y^4+x^4y^5+x^3y^6+x^2y^7+xy^8+y^9$
10.  $\frac{m^9+n^9}{m+n} = m^8-m^7n+m^6n^2-m^5n^3+m^4n^4-m^3n^5+m^2n^6+mn^7+n^8$
11.  $\frac{m^9-n^9}{m-n} = m^8+m^7n+m^6n^2+m^5n^3+m^4n^4+m^3n^5+m^2n^6+mn^7+n^8$
12.  $\frac{a^{10}-x^{10}}{a+x} = a^9-a^8x+a^7x^2-a^6x^3+a^5x^4-a^4x^5+a^3x^6-a^2x^7+ax^8-x^9$
13.  $\frac{1-n^5}{1-n} = 1+n+n^2+n^3+n^4$
14.  $\frac{1-a^6}{1-a} = 1+a+a^2+a^3+a^4+a^5$
15.  $\frac{1+a^7}{1+a} = 1-a+a^2-a^3+a^4-a^5+a^6$
16.  $\frac{1-m^8}{1+m} = 1-m+m^2-m^3+m^4-m^5+m^6-m^7$
17.  $\frac{x^4-16}{x-2} = x^3+2x^2+4x+8$
18.  $\frac{x^6-64}{x+2} = x^5-2x^4+4x^3-8x^2+16x-32$
19.  $\frac{x^7-128}{x-2} = x^6+2x^5+4x^4+8x^3+16x^2+32x+64$
20.  $\frac{a^5+243}{a+3} = a^4-3a^3+9a^2-27a+81$
21.  $\frac{x^6-729}{x-3} = x^5+3x^4+9x^3+27x^2+81x+243$

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22.  $\frac{625-x^4}{x+5} = 125 - 25x + 5x^2 - x^3$
23.  $\frac{m^8-256}{m-2} = m^7 + 2m^6 + 4m^5 + 8m^4 + 16m^3 + 32m^2 + 64m + 128$
24.  $\frac{x^{10}-1}{x-1} = x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$
25.  $\frac{x^5+243y^5}{x+3y} = x^4 - 3x^3y + 9x^2y^2 - 27xy^3 + 81y^4$
26.  $\frac{16a^4-81b^4}{2a-3b} = 8a^3 + 12a^2b + 18ab^2 + 27b^3$
27.  $\frac{64m^6-729n^6}{2m+3n} = 32m^5 - 48m^4n + 72m^3n^2 - 108m^2n^3 + 162mn^4 + 243n^5$
28.  $\frac{1.024x^{10}-1}{2x-1} = 512x^9 + 256x^8 + 128x^7 + 64x^6 + 32x^5 + 16x^4 + 8x^3 + 4x^2 + 2x + 1$
29.  $\frac{512a^9+b^9}{2a+b} = 256a^8 - 128a^7b + 64a^6b^2 - 32a^5b^3 + 16a^4b^4 - 8a^3b^5 + 4a^2b^6 - 2ab^7 + b^8$
30.  $\frac{a^6-729}{a-3} = a^5 + 3a^4 + 9a^3 + 27a^2 + 81a + 243$

## EJERCICIO 72

1.  $\frac{x^6+y^6}{x^2+y^2} = x^4 - x^2y^2 + y^4$
2.  $\frac{a^8-b^8}{a^2+b^2} = a^6 - a^4b^2 + a^2b^4 - b^6$
3.  $\frac{m^{10}-n^{10}}{m^2-n^2} = m^8 + m^6n^2 + m^4n^4 + m^2n^6 + n^8$
4.  $\frac{a^{12}-b^{12}}{a^3+b^3} = a^9 - a^6b^3 + a^3b^6 - b^9$
5.  $\frac{a^{12}-x^{12}}{a^3-x^3} = a^9 + a^6x^3 + a^3x^6 + x^9$
6.  $\frac{x^{15}+y^{15}}{x^3+y^3} = x^{12} - x^9y^3 + x^6y^6 - x^3y^9 + y^{12}$
7.  $\frac{m^{12}+1}{m^4+1} = m^8 - m^4 + 1$
8.  $\frac{m^{16}-n^{16}}{m^4-n^4} = m^{12} + m^8n^4 + m^4n^8 + n^{12}$
9.  $\frac{a^{18}-b^{18}}{a^3+b^3} = a^{15} - a^{12}b^3 + a^9b^6 - a^6b^9 + a^3b^{12} - b^{15}$
10.  $\frac{x^{20}-y^{20}}{x^5+y^5} = x^{15} - x^{10}y^5 + x^5y^{10} - y^{15}$
11.  $\frac{m^{21}+n^{21}}{m^3+n^3} = m^{18} - m^{15}n^3 + m^{12}n^6 - m^9n^9 + m^6n^{12} - m^3n^{15} + n^{18}$
12.  $\frac{x^{24}-1}{x^6-1} = x^{18} + x^{12} + x^6 + 1$
13.  $\frac{a^{25}+b^{25}}{a^5+b^5} = a^{20} - a^{15}b^5 + a^{10}b^{10} - a^5b^{15} + b^{20}$
14.  $\frac{a^{30}-m^{30}}{a^6-m^6} = a^{24} + a^{18}m^6 + a^{12}m^{12} + a^6m^{18} + m^{24}$

## EJERCICIO 73

1.  $\frac{x^4-1}{1+x^2} = x^2 - 1$
2.  $\frac{8m^3+n^6}{2m+n^2} = 4m^2 - 2mn^2 + n^4$
3.  $\frac{1-a^5}{1-a} = 1 + a + a^2 + a^3 + a^4$
4.  $\frac{x^6-27y^3}{x^2-3y} = x^4 + 3x^2y + 9y^2$
5.  $\frac{x^6-49y^6}{x^3+7y^3} = x^3 - 7y^3$
6.  $\frac{a^{14}-b^{14}}{a^2-b^2} = a^{12} + a^{10}b^2 + a^8b^4 + a^6b^6 + a^4b^8 + a^2b^{10} + b^{12}$

7.  $\frac{1+a^3}{1+a} = 1-a+a^2$
8.  $\frac{16x^2y^4-25m^6}{4xy^2+5m^3} = 4xy^2-5m^3$
9.  $\frac{x^{27}+y^{27}}{x^3+y^3} = x^{24}-x^{21}y^3+x^{18}y^6-x^{15}y^9+x^{12}y^{12}-x^9y^{15}+x^6y^{18}-x^3y^{21}+y^{24}$
10.  $\frac{a^{27}+y^{27}}{a^9+y^9} = a^{18}-a^9y^9+y^{18}$
11.  $\frac{a^4b^4-64x^6}{a^2b^2+8x^3} = a^2b^2-8x^3$
12.  $\frac{1-a^2b^4c^8}{1-ab^2c^4} = 1+ab^2c^4$
13.  $\frac{32x^5+243y^5}{2x+3y} = 16x^4-24x^3y+36x^2y^2-54xy^3+81y^4$
14.  $\frac{25-(a+1)^2}{5+(a+1)} = 4-a$
15.  $\frac{1-x^{12}}{1-x^4} = 1+x^4+x^8$
16.  $\frac{64x^6-343y^9}{4x^2-7y^3} = 16x^4+28x^2y^3+49y^6$
17.  $\frac{a^{18}-b^{18}}{a^3+b^3} = a^{15}-a^{12}b^3+a^9b^6-a^6b^9+a^3b^{12}-b^{15}$
18.  $\frac{(a+x)^2-y^2}{(a+x)-y} = a+x+y$
19.  $\frac{1+x^{11}}{x+1} = x^{10}-x^9+x^8-x^7+x^6-x^5+x^4-x^3+x^2-x+1$
20.  $\frac{x^{40}-y^{40}}{x^8-y^8} = x^{32}+x^{24}y^8+x^{16}y^{16}+x^8y^{24}+y^{32}$
21.  $\frac{9-36x^{10}}{3+6x^5} = 3-6x^5$
22.  $\frac{x^8-256}{x-2} = x^7+2x^6+4x^5+8x^4+16x^3+32x^2+64x+128$

## EJERCICIO 74

1.  $x^2-2x+3 \div x-1$   
 $= 1^2-2 \cdot 1+3$   
 $= 1-2+3=2$
4.  $a^4-5a^3+2a^2-6 \div a+3$   
 $= (-3)^4-5(-3)^3+2(-3)^2-6$   
 $= 81-5(-27)+2 \cdot 9-6$   
 $= 81+135+18-6 = 228$
7.  $a^5-2a^3+2a-4 \div a-5$   
 $= 5^5-2 \cdot 5^3+2 \cdot 5-4$   
 $= 3 \cdot 125-2 \cdot 125+10-4$   
 $= 3 \cdot 125-250+10-4 = 2.881$
10.  $15x^3-11x^2+10x+18 \div 3x+2$   
 $= 15\left(-\frac{2}{3}\right)^3-11\left(-\frac{2}{3}\right)^2+10\left(-\frac{2}{3}\right)+18$   
 $= 15\left(-\frac{8}{27}\right)-11 \cdot \frac{4}{9}-\frac{20}{3}+18$   
 $= -\frac{40}{9}-\frac{44}{9}-\frac{20}{3}+18$   
 $= \frac{-40-44-60+162}{9} = \frac{18}{9} = 2$
2.  $x^3-3x^2+2x-2 \div x+1$   
 $= (-1)^3-3(-1)^2+2(-1)-2$   
 $= -1-3-2-2 = -8$
5.  $m^4+m^3-m^2+5 \div m-4$   
 $= 4^4+4^3-4^2+5$   
 $= 256+64-16+5 = 309$
8.  $6x^3+x^2+3x+5 \div 2x+1$   
 $= 6\left(-\frac{1}{2}\right)^3+\left(-\frac{1}{2}\right)^2+3\left(-\frac{1}{2}\right)+5$   
 $= 6\left(-\frac{1}{8}\right)+\frac{1}{4}-\frac{3}{2}+5 = -\frac{3}{4}+\frac{1}{4}-\frac{3}{2}+5$   
 $= \frac{-3+1-6+20}{4} = \frac{12}{4} = 3$
11.  $5x^4-12x^3+9x^2-22x+21 \div 5x-2$   
 $= 5\left(\frac{2}{5}\right)^4-12\left(\frac{2}{5}\right)^3+9\left(\frac{2}{5}\right)^2-22\left(\frac{2}{5}\right)+21$   
 $= 5 \cdot \frac{16}{625}-12 \cdot \frac{8}{125}+9 \cdot \frac{4}{25}-\frac{44}{5}+21$   
 $= \frac{16}{125}-\frac{96}{125}+\frac{36}{25}-\frac{44}{5}+21$   
 $= -\frac{80}{125}+\frac{36}{25}-\frac{44}{5}+21$   
 $= \frac{-80+180-1.100+2.625}{125} = \frac{1.625}{125} = 13$
3.  $x^4-x^3+5 \div x-2$   
 $= 2^4-2^3+5$   
 $= 16-8+5=13$
6.  $x^5+3x^4-2x^3+4x^2-2x+2 \div x+3$   
 $= (-3)^5+3(-3)^4-2(-3)^3+4(-3)^2-2(-3)+2$   
 $= -243+3 \cdot 81-2(-27)+4 \cdot 9+6+2$   
 $= -243+243+54+36+8=98$
9.  $12x^3-21x+90 \div 3x-3$   
 $= 12 \cdot 1^3-21 \cdot 1+90$   
 $= 12-21+90=81$
12.  $a^6+a^4-8a^2+4a+1 \div 2a+3$   
 $= \left(-\frac{3}{2}\right)^6+\left(-\frac{3}{2}\right)^4-8\left(-\frac{3}{2}\right)^2+4\left(-\frac{3}{2}\right)+1$   
 $= \frac{729}{64}+\frac{81}{16}-8 \cdot \frac{9}{4}-\frac{12}{2}+1$   
 $= \frac{729}{64}+\frac{81}{16}-18-6+1 = \frac{729}{64}+\frac{81}{16}-23$   
 $= \frac{729+324-1.472}{64} = -\frac{419}{64}$

## EJERCICIO 75

1.  $x^2 - 7x + 5 \div x - 3$

$$\begin{array}{r|l} 1 & -7 & 5 & 3 \\ \hline & 3 & -12 & \end{array}$$

$$\begin{array}{r} 1 & -4 & -7 \\ \hline = & x-4 & \text{Res. } -7 \end{array}$$

2.  $a^2 - 5a + 1 \div a + 2$

$$\begin{array}{r|l} 1 & -5 & 1 & -2 \\ \hline & -2 & 14 & \end{array}$$

$$\begin{array}{r} 1 & -7 & 15 \\ \hline = & a-7 & \text{Res. } 15 \end{array}$$

3.  $x^3 - x^2 + 2x - 2 \div x + 1$

$$\begin{array}{r|l} 1 & -1 & 2 & -2 & -1 \\ \hline & -1 & 2 & -4 & \end{array}$$

$$\begin{array}{r} 1 & -2 & 4 & -6 \\ \hline = & x^2 - 2x + 4 & \text{Res. } -6 \end{array}$$

4.  $x^3 - 2x^2 + x - 2 \div x - 2$

$$\begin{array}{r|l} 1 & -2 & 1 & -2 & 2 \\ \hline & 2 & 0 & 2 & \end{array}$$

$$\begin{array}{r} 1 & 0 & 1 & 0 \\ \hline = & x^2 + 1 & \text{Res. } 0 \end{array}$$

5.  $a^3 - 3a^2 - 6a + 3$

$$\begin{array}{r|l} 1 & -3 & 0 & -6 & -3 \\ \hline & -3 & 18 & -54 & \end{array}$$

$$\begin{array}{r} 1 & -6 & 18 & -60 \\ \hline = & a^2 - 6a + 18 & \text{Res. } -60 \end{array}$$

6.  $n^4 - 5n^3 + 4n - 48 \div n + 2$

$$\begin{array}{r|l} 1 & -5 & 0 & 4 & -48 & -2 \\ \hline & -2 & 14 & -28 & 48 & \end{array}$$

$$\begin{array}{r} 1 & -7 & 14 & -24 & 0 \\ \hline = & n^3 - 7n^2 + 14n - 24 & \text{Res. } 0 \end{array}$$

7.  $x^4 - 3x + 5 \div x - 1$

$$\begin{array}{r|l} 1 & 0 & 0 & -3 & 5 & 1 \\ \hline & 1 & 1 & 1 & -2 & \end{array}$$

$$\begin{array}{r} 1 & 1 & 1 & -2 & 3 \\ \hline = & x^3 + x^2 + x - 2 & \text{Res. } 3 \end{array}$$

8.  $x^5 + x^4 - 12x^3 - x^2 - 4x - 2 \div x + 4$

$$\begin{array}{r|l} 1 & 1 & 1 & -12 & -1 & -4 & -2 & -4 \\ \hline & -4 & 12 & 0 & 4 & 0 & \end{array}$$

$$\begin{array}{r} 1 & -3 & 0 & -1 & 0 & -2 \\ \hline = & x^4 - 3x^3 - x & \text{Res. } -2 \end{array}$$

9.  $a^5 - 3a^3 + 4a - 6 \div a - 2$

$$\begin{array}{r|l} 1 & 0 & -3 & 0 & 4 & -6 & 2 \\ \hline & 2 & 4 & 2 & 4 & 16 & \end{array}$$

$$\begin{array}{r} 1 & 2 & 1 & 2 & 8 & 10 \\ \hline = & a^4 + 2a^3 + a^2 + 2a + 8 & \text{Res. } 10 \end{array}$$

10.  $x^5 - 208x^2 + 2076 \div x - 5$

$$\begin{array}{r|l} 1 & 0 & 0 & -208 & 0 & 2.076 & 5 \\ \hline & 5 & 25 & 125 & -415 & -2.075 & \end{array}$$

$$\begin{array}{r} 1 & 5 & 25 & -83 & -415 & 1 \\ \hline = & x^4 + 5x^3 + 25x^2 - 83x - 415 & \text{Res. } 1 \end{array}$$

11.  $x^6 - 3x^5 + 4x^4 - 3x^3 - x^2 + 2x + 3$

$$\begin{array}{r|l} 1 & -3 & 4 & -3 & -1 & 0 & 2 & -3 \\ \hline & -3 & 18 & -66 & 207 & -618 & 1.854 & \end{array}$$

$$\begin{array}{r} 1 & -6 & 22 & -69 & 206 & -618 & 1.856 \\ \hline = & x^5 - 6x^4 + 22x^3 - 69x^2 + 206x - 618 & \text{Res. } 1.856 \end{array}$$

12.  $2x^3 - 3x^2 + 7x - 5 \div 2x - 1$

$$\begin{array}{r|l} 2 & -3 & 7 & -5 & \frac{1}{2} \\ \hline & 1 & -1 & 3 & \end{array}$$

$$\begin{array}{r} 2 & -2 & 6 & -2 \\ \hline = & x^2 - x + 3 & \text{Res. } -2 \end{array}$$

13.  $3a^3 - 4a^2 + 5a + 6 \div 3a + 2$

$$\begin{array}{r|l} 3 & -4 & 5 & 6 & -\frac{2}{3} \\ \hline & -2 & 4 & -6 & \end{array}$$

$$\begin{array}{r} 3 & -6 & 9 & 0 \\ \hline = & a^2 - 2a + 3 & \text{Res. } 0 \end{array}$$

14.  $3x^4 - 4x^3 + 4x^2 - 10x + 8 \div 3x - 1$

$$\begin{array}{r|l} 3 & -4 & 4 & -10 & 8 & \frac{1}{3} \\ \hline & 1 & -1 & 1 & -3 & \end{array}$$

$$\begin{array}{r} 3 & -3 & 3 & -9 & 5 \\ \hline = & x^3 - x^2 + x - 3 & \text{Res. } 5 \end{array}$$

15.  $x^6 - x^4 + \frac{15}{8}x^3 + x^2 - 1 \div 2x + 3$

$$\begin{array}{r|l} 1 & 0 & -1 & \frac{15}{8} & 1 & 0 & -1 & -\frac{3}{2} \\ \hline & -\frac{3}{2} & \frac{9}{4} & -\frac{15}{8} & 0 & -\frac{3}{2} & \frac{9}{4} & \end{array}$$

$$\begin{array}{r} 1 & -\frac{3}{2} & \frac{5}{4} & 0 & 1 & -\frac{3}{2} & \frac{5}{4} \\ \hline = & \frac{1}{2}x^5 - \frac{3}{4}x^4 + \frac{5}{8}x^3 + \frac{1}{2}x^2 - \frac{3}{4} & \text{Res. } \frac{5}{4} \end{array}$$

## EJERCICIO 76

1.  $x^2 - x - 6 \div x - 3$

Exacta (6 múltiplo de 3)

2.  $x^3 + 4x^2 - x - 10 \div x + 2$

Exacta (10 múltiplo de 2)

3.  $2x^4 - 5x^3 + 7x^2 - 9x + 3 \div x - 1$

Inexacta (1 no anula el polinomio)

4.  $x^5 + x^4 - 5x^3 - 7x + 8 \div x + 3$

Inexacta (8 no es múltiplo de 3)

5.  $4x^3 - 8x^2 - 11x - 4 \div 2x - 1$

Exacta (4 múltiplo de 1)

6.  $6x^5 + 2x^4 - 3x^3 - x^2 + 3x + 3 \div 3x + 1$

Inexacta (-1 no anula el polinomio)

7.  $a + 1$  Es factor de  $a^3 - 2a^2 + 2a + 5$

$$(-1)^3 - 2(-1)^2 + 2(-1) + 5$$

$$= -1 - 2 - 2 + 5 = 0$$

No existe residuo, luego  $a + 1$  divide exactamente al polinomio, por lo que se deduce es un factor de este.

8.  $x - 5$  divide a  $x^5 + 6x^4 + 6x^3 - 5x^2 + 2x - 10$

$$5^5 + 6 \cdot 5^4 + 6 \cdot 5^3 - 5 \cdot 5^2 + 2 \cdot 5 - 10$$

$$= 3125 - 3750 + 750 + 125 + 10 - 10 = 0$$

Al sustituir  $x$  por 5 en el polinomio, este se anula, entonces  $x - 5$  divide con exactitud a

$$x^5 + 6x^4 + 6x^3 - 5x^2 + 2x - 10$$



9.  $4x-3$  divide a  $4x^4-7x^3+7x^2-7x+3$

$$\begin{array}{r} 4\left(\frac{3}{4}\right)^4 - 7\left(\frac{3}{4}\right)^3 + 7\left(\frac{3}{4}\right)^2 - 7\left(\frac{3}{4}\right) + 3 \\ = \frac{324}{256} - \frac{189}{64} + \frac{63}{16} - \frac{21}{4} + 3 \\ = \frac{324 - 756 + 1.008 - 1.344 + 768}{256} = \frac{0}{256} = 0 \end{array}$$

La variable  $x$  del dividendo se reemplaza por  $3/4$  ( $4x - 3 = 0$  luego  $x = 3/4$ ) que es el valor de la variable del divisor. Se observa su anulación, por consiguiente,  $4x - 3$  es un divisor exacto de tal polinomio.

10.  $3n + 2$  no es factor de

$3n^5 + 2n^4 - 3n^3 - 2n^2 + 6n + 7$  porque 7 no es múltiplo de  $-2/3$ , lo cual significa que al reemplazar tal valor en el polinomio resultará un residuo, por ende la división no será exacta y  $3n + 2$  no se puede concebir como factor de dicho polinomio.

11.  $2a^3 - 2a^2 - 4a + 16 \div a + 2$

$$\begin{array}{r} -4 \quad 12 \quad -16 \\ 2 \quad -6 \quad 8 \quad 0 \end{array}$$

Exacta, coc.  $2a^2 - 6a + 8$

12.  $a^4 - a^2 + 2a + 2 \div a + 1$

$$\begin{array}{r} -1 \quad 1 \quad 0 \quad -2 \\ 1 \quad -1 \quad 0 \quad 2 \quad 0 \end{array}$$

Exacta, coc.  $a^3 - a^2 + 2$

13.  $x^4 + 5x - 6 \div x - 1$

$$\begin{array}{r} 1 \quad 1 \quad 1 \quad 6 \\ 1 \quad 1 \quad 1 \quad 6 \quad 0 \end{array}$$

Exacta, coc.  $x^3 + x^2 + x + 6$

14.  $x^6 - 39x^4 + 26x^3 - 52x^2 + 29x - 30 \div x - 6$

$$\begin{array}{r} 6 \quad 36 \quad -18 \quad 48 \quad -24 \quad 30 \\ 1 \quad 6 \quad -3 \quad 8 \quad -4 \quad 5 \quad 0 \end{array}$$

Exacta, coc.  $x^5 + 6x^4 - 3x^3 + 8x^2 - 4x + 5$

15.  $a^6 - 4a^5 - a^4 + 4a^3 + a^2 - 8a + 25 \div a - 4$

$$\begin{array}{r} 4 \quad 0 \quad -4 \quad 0 \quad 4 \quad -16 \\ 1 \quad 0 \quad -1 \quad 0 \quad 1 \quad -4 \quad 9 \end{array}$$

Inexacta, coc.  $a^5 - a^3 + a - 4$ ; Res. 9

16.  $16x^4 - 24x^3 + 37x^2 - 24x + 4 \div 4x - 1$

$$\begin{array}{r} 4 \quad -5 \quad 8 \quad -4 \\ 16 \quad -20 \quad 32 \quad -16 \quad 0 \end{array}$$

Exacta, coc.  $4x^3 - 5x^2 + 8x - 4$

17.  $15n^5 + 25n^4 - 18n^3 - 18n^2 + 17n - 11 \div 3n + 5$

$$\begin{array}{r} -25 \quad 0 \quad 30 \quad -20 \quad 5 \\ 15 \quad 0 \quad -18 \quad 12 \quad -3 \quad -6 \end{array}$$

Inexacta, coc.  $5n^4 - 6n^2 + 4n - 1$ ; Res. -6

18.  $7x^2 - 5x + k \div x - 5$

$$\begin{array}{r} 35 \quad 150 \\ 7 \quad 30 \quad 0 \end{array}$$

$5k = -150$  se anula

19.  $x^3 - 3x^2 + 4x + k \div x - 2$

$$\begin{array}{r} 2 \quad -2 \quad 4 \\ 1 \quad -1 \quad 2 \quad 0 \end{array}$$

$k + 4 = 0$ , luego  $k = -4$

20.  $2a^4 + 25a + k \div a + 3$

$$\begin{array}{r} -6 \quad 18 \quad -54 \quad 87 \\ 2 \quad -6 \quad 18 \quad -29 \quad 0 \end{array}$$

Para  $k = -87$  se cumple que  $k + 87 = 0$

21.  $20x^3 - 7x^2 + 29x + k \div 4x + 1$

$$\begin{array}{r} -5 \quad 3 \quad -8 \\ 20 \quad -12 \quad 32 \quad 0 \end{array}$$

$k - 8 = 0$  entonces  $k = 8$

## EJERCICIO 77

1.  $\frac{x^5+1}{x-1}$  Inexacta Res. 2      9.  $\frac{a^5+32}{a-2}$  Inexacta Res. 64

2.  $\frac{a^4+b^4}{a+b}$  Inexacta Res.  $2b^4$

3.  $\frac{x^8-1}{x^2+1}$  Exacta      10.  $\frac{x^7-128}{x+2}$  Inexacta Res. -256

4.  $\frac{a^{11}+1}{a-1}$  Inexacta Res. 2

5.  $\frac{a^6+b^6}{a^2+b^2}$  Inexacta Res.  $2b^6$       11.  $\frac{16a^4-81b^4}{2a+3b}$  Exacta

6.  $\frac{x^7-1}{x-1}$  Exacta

7.  $\frac{x^3-8}{x+2}$  Inexacta Res. -16      12.  $\frac{a^3x^6+b^9}{ax^2+b^3}$  Exacta

8.  $\frac{x^4-16}{x+2}$  Exacta

## EJERCICIO 78

1.  $5x=8x-15$   
 $5x-8x=-15$   
 $-3x=-15$   
 $x=\frac{-15}{-3}$   
 $x=5$
2.  $4x+1=2$   
 $4x=2-1$   
 $4x=1$   
 $x=\frac{1}{4}$
3.  $y-5=3y-25$   
 $y-3y=-25+5$   
 $-2y=-20$   
 $y=\frac{-20}{-2}$   
 $y=10$
4.  $5x+6=10x+5$   
 $5x-10x=5-6$   
 $-5x=-1$   
 $x=\frac{-1}{-5}$   
 $x=\frac{1}{5}$
5.  $9y-11=-10+12y$   
 $9y-12y=-10+11$   
 $-3y=1$   
 $y=-\frac{1}{3}$
6.  $21-6x=27-8x$   
 $-6x+8x=27-21$   
 $2x=6$   
 $x=\frac{6}{2}$   
 $x=3$
7.  $11x+5x-1=65x-36$   
 $16x=65x-36+1$   
 $16x-65x=-35$   
 $-49x=-35$   
 $x=\frac{-35}{-49}$   
 $x=\frac{5}{7}$
8.  $8x-4+3x=7x+x+14$   
 $11x-4=8x+14$   
 $11x-8x=14+4$   
 $3x=18$   
 $x=\frac{18}{3}$   
 $x=6$
9.  $8x+9-12x=4x-13-5x$   
 $-4x+9=-x-13$   
 $-4x+x=-13-9$   
 $-3x=-22$   
 $x=\frac{-22}{-3}$   
 $x=\frac{22}{3}$
10.  $5y+6y-81=7y+102+65y$   
 $11y-81=72y+102$   
 $11y-72y=102+81$   
 $-61y=183$   
 $y=\frac{183}{-61}$   
 $y=-3$
11.  $16+7x-5+x=11x-3-x$   
 $8x+11=10x-3$   
 $8x-10x=-3-11$   
 $-2x=-14$   
 $x=\frac{-14}{-2}$   
 $x=7$
12.  $3x+101-4x-33=108-16x-100$   
 $-x+68=8-16x$   
 $-x+16x=8-68$   
 $15x=-60$   
 $x=\frac{-60}{15}$   
 $x=-4$
13.  $14-12x+39x-18x=256-60x-657x$   
 $9x+14=-717x+256$   
 $9x+717x=256-14$   
 $726x=242$   
 $x=\frac{242}{726}$   
 $x=\frac{1}{3}$
14.  $8x-15x-30x-51x=53x+31x-172$   
 $-88x=84x-172$   
 $-88x-84x=-172$   
 $-172x=-172$   
 $x=\frac{-172}{-172}$   
 $x=1$

## EJERCICIO 79

1.  $x-(2x+1)=8-(3x+3)$   
 $x-2x-1=8-3x-3$   
 $-x-1=5-3x$   
 $-x+3x=5+1$   
 $2x=6$   
 $x=\frac{6}{2}$   
 $x=3$
2.  $15x-10=6x-(x+2)+(-x+3)$   
 $15x-10=6x-x-2-x+3$   
 $15x-10=4x+1$   
 $15x-4x=10+1$   
 $x=\frac{11}{11}$   
 $x=1$
3.  $(5-3x)-(-4x+6)=(8x+11)-(3x-6)$   
 $5-3x+4x-6=8x+11-3x+6$   
 $x-1=5x+17$   
 $x-5x=17+1$   
 $-4x=18$   
 $x=\frac{18}{-4}$   
 $x=-\frac{9}{2}$

$$\begin{array}{ll}
 \text{4. } 30x - (-x+6) + (-5x+4) = -(5x+6) + (-8+3x) & \text{5. } 15x + (-6x+5) - 2 - (-x+3) = -(7x+23) - x + (3-2x) \\
 30x + x - 6 - 5x + 4 = -5x - 6 - 8 + 3x & 15x - 6x + 5 - 2 + x - 3 = -7x - 23 - x + 3 - 2x \\
 26x - 2 = -2x - 14 & 10x = -10x - 20 \\
 26x + 2x = -14 + 2 & 10x + 10x = -20 \\
 28x = -12 & 20x = -20 \\
 x = -\frac{12}{28} & x = -\frac{20}{20} \\
 x = -\frac{3}{7} & x = -1
 \end{array}$$

$$\begin{array}{ll}
 \text{6. } 3x + [-5x - (x+3)] = 8x + (-5x-9) & \text{7. } 16x - [3x - (6-9x)] = 30x + [- (3x+2) - (x+3)] \\
 3x + [-5x - x - 3] = 8x - 5x - 9 & 16x - [3x - 6 + 9x] = 30x + [-3x - 2 - x - 3] \\
 3x + [-6x - 3] = 3x - 9 & 16x - [12x - 6] = 30x + [-4x - 5] \\
 3x - 6x - 3 = 3x - 9 & 16x - 12x + 6 = 30x - 4x - 5 \\
 -3x - 3 = 3x - 9 & 4x + 6 = 26x - 5 \\
 -x - 1 = x - 3 & 4x - 26x = -5 - 6 \\
 -x - x = -3 + 1 & -22x = -11 \\
 -2x = -2 \Rightarrow x = \frac{-2}{-2} = 1 & x = \frac{-11}{-22} = \frac{1}{2}
 \end{array}$$

$$\begin{array}{ll}
 \text{9. } 9x - (5x+1) - \{2+8x - (7x-5)\} + 9x = 0 & \text{8. } x - [5+3x - \{5x - (6+x)\}] = -3 \\
 9x - 5x - 1 - \{2+8x - 7x + 5\} + 9x = 0 & x - [5+3x - \{5x - 6 - x\}] = -3 \\
 4x - 1 - \{x+7\} + 9x = 0 & x - [5+3x - \{4x - 6\}] = -3 \\
 4x - 1 - x - 7 + 9x = 0 & x - [5+3x - 4x + 6] = -3 \\
 12x - 8 = 0 & x - [11 - x] = -3 \\
 12x = 8 & x - 11 + x = -3 \\
 x = \frac{8}{12} & 2x = -3 + 11 \\
 x = \frac{2}{3} & 2x = 8 \Rightarrow x = \frac{8}{2} = 4 \\
 \text{11. } -\{3x+8 - [-15+6x - (-3x+2)] - (5x+4)\} - 29 = -5 & \text{10. } 71 + [-5x + (-2x+3)] = 25 - [- (3x+4) - (4x+3)] \\
 -\{3x+8 - [-15+6x+3x-2-5x-4] - 29\} = -5 & 71 + [-5x - 2x + 3] = 25 - [-3x - 4 - 4x - 3] \\
 -\{3x - 21 - [-21+4x]\} = -5 & 71 + [-7x+3] = 25 - [-7x-7] \\
 -\{3x - 21 + 21 - 4x\} = -5 & 71 - 7x + 3 = 25 + 7x + 7 \\
 -\{-x\} = -5 & 74 - 7x = 32 + 7x \\
 x = -5 & 74 - 32 = 7x + 7x \\
 & 42 = 14x \\
 & \frac{42}{14} = x \\
 & 3 = x
 \end{array}$$

## EJERCICIO 80

$$\begin{array}{lll}
 \text{1. } x + 3(x-1) = 6 - 4(2x+3) & \text{2. } 5(x-1) + 16(2x+3) = 3(2x-7) - x & \text{3. } 2(3x+3) - 4(5x-3) = x(x-3) - x(x+5) \\
 x + 3x - 3 = 6 - 8x - 12 & 5x - 5 + 32x + 48 = 6x - 21 - x & 6x + 6 - 20x + 12 = x^2 - 3x - x^2 - 5x \\
 4x - 3 = -8x - 6 & 37x + 43 = 5x - 21 & -14x + 18 = -8x \\
 4x + 8x = -6 + 3 & 37x - 5x = -21 - 43 & -14x + 8x = -18 \\
 12x = -3 & 32x = -64 & -6x = -18 \\
 x = \frac{-3}{12} \Rightarrow x = -\frac{1}{4} & x = \frac{-64}{32} \Rightarrow x = -2 & x = \frac{-18}{-6} \Rightarrow x = 3
 \end{array}$$

$$4. 184 - 7(2x + 5) = 301 + 6(x - 1) - 6$$

$$184 - 14x - 35 = 301 + 6x - 6 - 6$$

$$149 - 14x = 289 + 6x$$

$$149 - 289 = 6x + 14x$$

$$-140 = 20x$$

$$\frac{-140}{20} = x$$

$$-7 = x$$

$$5. 7(18 - x) - 6(3 - 5x) = -(7x + 9) - 3(2x + 5) - 12$$

$$126 - 7x - 18 + 30x = -7x - 9 - 6x - 15 - 12$$

$$108 + 23x = -13x - 36$$

$$23x + 13x = -36 - 108$$

$$36x = -144$$

$$x = \frac{-144}{36}$$

$$x = -4$$

$$6. 3x(x - 3) + 5(x + 7) - x(x + 1) - 2(x^2 + 7) + 4 = 0$$

$$3x^2 - 9x + 5x + 35 - x^2 - x - 2x^2 - 14 + 4 = 0$$

$$-5x + 25 = 0$$

$$-5x = -25$$

$$x = \frac{-25}{-5}$$

$$x = 5$$

$$7. -3(2x + 7) + (-5x + 6) - 8(1 - 2x) - (x - 3) = 0$$

$$-6x - 21 - 5x + 6 - 8 + 16x - x + 3 = 0$$

$$4x - 20 = 0$$

$$4x = 20$$

$$x = \frac{20}{4}$$

$$x = 5$$

$$8. (3x - 4)(4x - 3) = (6x - 4)(2x - 5)$$

$$12x^2 - 9x - 16x + 12 = 12x^2 - 38x + 20$$

$$-25x + 38x = 20 - 12$$

$$13x = 8$$

$$x = \frac{8}{13}$$

$$9. (4 - 5x)(4x - 5) = (10x - 3)(7 - 2x)$$

$$-20x^2 + 41x - 20 = -20x^2 + 76x - 21$$

$$41x - 76x = -21 + 20$$

$$-35x = -1$$

$$x = \frac{-1}{-35}$$

$$x = \frac{1}{35}$$

$$10. (x + 1)(2x + 5) = (2x + 3)(x - 4) + 5$$

$$2x^2 + 7x + 5 = 2x^2 - 5x - 12 + 5$$

$$7x + 5x = -7 - 5$$

$$12x = -12$$

$$x = \frac{-12}{12}$$

$$x = -1$$

$$11. (x - 2)^2 - (3 - x)^2 = 1$$

$$x^2 - 4x + 4 - 9 + 6x - x^2 = 1$$

$$2x - 5 = 1$$

$$2x = 6$$

$$x = \frac{6}{2}$$

$$x = 3$$

$$12. 14 - (5x - 1)(2x + 3) = 17 - (10x + 1)(x - 6)$$

$$14 - (10x^2 + 13x - 3) = 17 - (10x^2 - 59x - 6)$$

$$14 - 10x^2 - 13x + 3 = 17 - 10x^2 + 59x + 6$$

$$-13x - 59x = 6$$

$$-72x = 6$$

$$x = \frac{6}{-72}$$

$$x = -\frac{1}{12}$$

$$13. (x - 2)^2 + x(x - 3) = 3(x + 4)(x - 3) - (x + 2)(x - 1) + 2$$

$$x^2 - 4x + 4 + x^2 - 3x = 3(x^2 + x - 12) - (x^2 + x - 2) + 2$$

$$2x^2 - 7x + 4 = 3x^2 + 3x - 36 - x^2 - x + 2 + 2$$

$$2x^2 - 7x + 4 = 2x^2 + 2x - 36 + 4$$

$$-7x - 2x = -36$$

$$-9x = -36$$

$$x = \frac{-36}{-9}$$

$$x = 4$$

$$14. (3x - 1)^2 - 5(x - 2) - (2x + 3)^2 - (5x + 2)(x - 1) = 0$$

$$9x^2 - 6x + 1 - 5x + 10 - 4x^2 - 12x - 9 - 5x^2 + 3x + 2 = 0$$

$$-20x + 4 = 0$$

$$-20x = -4$$

$$x = \frac{-4}{-20}$$

$$x = \frac{1}{5}$$

$$\begin{aligned}
 15. \quad & 2(x-3)^2 - 3(x+1)^2 + (x-5)(x-3) + 4(x^2 - 5x + 1) = 4x^2 - 12 \\
 & 2x^2 - 12x + 18 - 3x^2 - 6x - 3 + x^2 - 8x + 15 + 4x^2 - 20x + 4 = 4x^2 - 12 \\
 & 4x^2 - 46x + 34 = 4x^2 - 12 \\
 & -46x = -12 - 34 \\
 & -46x = -46 \\
 & x = \frac{-46}{-46} \\
 & x = 1
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & 5(x-2)^2 - 5(x+3)^2 + (2x-1)(5x+2) - 10x^2 = 0 \\
 & 5x^2 - 20x + 20 - 5x^2 - 30x - 45 + 10x^2 - x - 2 - 10x^2 = 0 \\
 & -51x - 27 = 0 \\
 & -51x = 27 \\
 & x = \frac{27}{-51} \\
 & x = -\frac{9}{17}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & 7(x-4)^2 - 3(x+5)^2 = 4(x+1)(x-1) - 2 \\
 & 7x^2 - 56x + 112 - 3x^2 - 30x - 75 = 4x^2 - 4 - 2 \\
 & 4x^2 - 86x + 37 = 4x^2 - 6 \\
 & -86x = -6 - 37 \\
 & -86x = -43 \\
 & x = \frac{-43}{-86} \\
 & x = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & x^2 - 5x + 15 = x(x-3) - 14 + 5(x-2) + 3(13-2x) \\
 & x^2 - 5x + 15 = x^2 - 3x - 14 + 5x - 10 + 39 - 6x \\
 & -5x + 15 = -4x + 15 \\
 & -5x + 4x = 0 \\
 & -x = 0 \\
 & x = 0
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & 5(1-x)^2 - 6(x^2 - 3x - 7) = x(x-3) - 2x(x+5) - 2 \\
 & 5 - 10x + 5x^2 - 6x^2 + 18x + 42 = x^2 - 3x - 2x^2 - 10x - 2 \\
 & -x^2 + 8x + 47 = -x^2 - 13x - 2 \\
 & 8x + 13x = -2 - 47 \\
 & 21x = -49 \\
 & x = \frac{-49}{21} \\
 & x = -\frac{7}{3}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & 3(5x-6)(3x+2) - 6(3x+4)(x-1) - 3(9x+1)(x-2) = 0 \\
 & 3(15x^2 - 8x - 12) - 6(3x^2 + x - 4) - 3(9x^2 - 17x - 2) = 0 \\
 & 45x^2 - 24x - 36 - 18x^2 - 6x + 24 - 27x^2 + 51x + 6 = 0 \\
 & 21x - 6 = 0 \\
 & 21x = 6 \\
 & x = \frac{6}{21} \\
 & x = \frac{2}{7}
 \end{aligned}$$

## EJERCICIO 81

$$\begin{aligned}
 1. \quad & 14x - (3x-2) - [5x+2-(x-1)] = 0 \\
 & 14x - 3x + 2 - [5x+2-x+1] = 0 \\
 & 11x + 2 - [4x+3] = 0 \\
 & 11x + 2 - 4x - 3 = 0 \\
 & 7x - 1 = 0 \\
 & 7x = 1 \\
 & x = \frac{1}{7}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & (3x-7)^2 - 5(2x+1)(x-2) = -x^2 - [(3x+1)] \\
 & 9x^2 - 42x + 49 - 5(2x^2 - 3x - 2) = -x^2 - [-3x-1] \\
 & 9x^2 + x^2 - 42x + 49 - 10x^2 + 15x + 10 = 3x + 1 \\
 & -27x + 59 = 3x + 1 \\
 & -27x - 3x = 1 - 59 \\
 & -30x = -58 \\
 & x = \frac{-58}{-30} \Rightarrow x = \frac{29}{15}
 \end{aligned}$$

$$\begin{aligned}
3. \quad & 6x - (2x+1) = -\left\{-5x + \left[-(-2x-1)\right]\right\} \\
& 6x - 2x - 1 = -\left\{-5x + [2x+1]\right\} \\
& 4x - 1 = -\left\{-5x + 2x + 1\right\} \\
& 4x - 1 = -\left\{-3x + 1\right\} \\
& 4x - 1 = 3x - 1 \\
& 4x - 3x = -1 + 1 \\
& x = 0
\end{aligned}$$

$$\begin{aligned}
4. \quad & 2x + 3(-x^2 - 1) = -\left\{3x^2 + 2(x-1) - 3(x+2)\right\} \\
& 2x - 3x^2 - 3 = -\left\{3x^2 + 2x - 2 - 3x - 6\right\} \\
& 2x - 3x^2 - 3 = -\left\{3x^2 - x - 8\right\} \\
& 2x - 3x^2 - 3 = -3x^2 + x + 8 \\
& 2x - x = 8 + 3 \\
& x = 11
\end{aligned}$$

$$\begin{aligned}
5. \quad & x^2 - \left\{3x + [x(x+1) + 4(x^2-1) - 4x^2]\right\} = 0 \\
& x^2 - \left\{3x + [x^2 + x + 4x^2 - 4 - 4x^2]\right\} = 0 \\
& x^2 - \left\{3x + [x^2 + x - 4]\right\} = 0 \\
& x^2 - \{3x + x^2 + x - 4\} = 0 \\
& x^2 - 4x - x^2 + 4 = 0 \\
& -4x = -4 \\
& x = \frac{-4}{-4} \\
& x = 1
\end{aligned}$$

$$\begin{aligned}
6. \quad & 3(2x+1)(-x+3) - (2x+5)^2 = -\left[-3(x+5)\right] + 10x^2 \\
& 3(-2x^2 + 5x + 3) - 4x^2 - 20x - 25 = -\left[-3x - 15\right] + 10x^2 \\
& -6x^2 + 15x + 9 - 4x^2 - 20x - 25 = -\left[3x + 15 + 10x^2\right] \\
& -10x^2 - 5x - 16 = -3x - 15 - 10x^2 \\
& -5x + 3x = 16 - 15 \\
& -2x = 1 \\
& x = -\frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
7. \quad & (x+1)(x+2)(x-3) = (x-2)(x+1)(x+1) \\
& (x^2 + 3x + 2)(x-3) = (x^2 - x - 2)(x+1) \\
& x^3 - 3x^2 + 3x^2 - 9x + 2x - 6 = x^3 + x^2 - x^2 - x - 2x - 2 \\
& -7x - 6 = -3x - 2 \\
& -7x + 3x = -2 + 6 \\
& -4x = 4 \\
& x = \frac{4}{-4} \Rightarrow x = -1
\end{aligned}$$

$$\begin{aligned}
8. \quad & (x+2)(x+3)(x-1) = (x+4)(x+4)(x-4) + 7 \\
& (x^2 + 5x + 6)(x-1) = (x+4)(x^2 - 16) + 7 \\
& x^3 - x^2 + 5x^2 - 5x + 6x - 6 = x^3 - 16x + 4x^2 - 64 + 7 \\
& 4x^2 + x - 6 = 4x^2 - 16x - 57 \\
& x + 16x = -57 + 6 \\
& 17x = -51 \\
& x = \frac{-51}{17} \Rightarrow x = -3
\end{aligned}$$

$$\begin{aligned}
9. \quad & (x+1)^3 - (x-1)^3 = 6x(x-3) \\
& x^3 + 3x^2 + 3x + 1 - (x^3 - 3x^2 + 3x - 1) = 6x^2 - 18x \\
& x^3 + 3x^2 + 3x + 1 - x^3 + 3x^2 - 3x + 1 = 6x^2 - 18x \\
& 6x^2 + 2 = 6x^2 - 18x \\
& 2 = -18x \\
& \frac{2}{-18} = x \Rightarrow -\frac{1}{9} = x
\end{aligned}$$

$$\begin{aligned}
10. \quad & 3(x-2)^2(x+5) = 3(x+1)^2(x-1) + 3 \\
& 3(x^2 - 4x + 4)(x+5) = 3(x^2 + 2x + 1)(x-1) + 3 \\
& (3x^2 - 12x + 12)(x+5) = (3x^2 + 6x + 3)(x-1) + 3 \\
& 3x^3 + 15x^2 - 12x^2 - 60x + 12x + 60 = 3x^3 - 3x^2 + 6x^2 - 6x + 3x - 3 + 3 \\
& 3x^2 - 48x + 60 = 3x^2 - 3x \\
& -48x + 3x = -60 \\
& -45x = -60 \\
& x = \frac{-60}{-45} \Rightarrow x = \frac{4}{3}
\end{aligned}$$

## EJERCICIO 82

$$\begin{aligned}
1. \quad & x \rightarrow N^\circ \text{ mayor} \\
& x - 8 \rightarrow N^\circ \text{ menor} \\
& x + x - 8 = 106 \\
& 2x - 8 = 106 \\
& 2x = 106 + 8 \\
& x = \frac{114}{2} \\
& x = 57 \rightarrow N^\circ \text{ mayor}
\end{aligned}$$

$$\begin{aligned}
& x - 8 \Rightarrow 57 - 8 \\
& = 49 \rightarrow N^\circ \text{ menor}
\end{aligned}$$

$$\begin{aligned}
2. \quad & x \rightarrow \text{N}^\circ \text{ mayor} \\
& x - 32 \rightarrow \text{N}^\circ \text{ menor} \\
& x + x - 32 = 540 \\
& 2x = 540 + 32 \\
& 2x = 572 \\
& x = \frac{572}{2} \\
& x = 286 \rightarrow \text{N}^\circ \text{ mayor}
\end{aligned}$$

$$\begin{aligned}
x - 32 & \Rightarrow 286 - 32 \\
& = 254 \rightarrow \text{N}^\circ \text{ menor}
\end{aligned}$$

$$\begin{aligned}
3. \quad & A + B = 1.154 \text{ bs.} \\
& A - 506 = B \\
& A + A - 506 = 1.154 \\
& 2A = 1.154 + 506 \\
& 2A = 1.660 \\
& A = \frac{1.660}{2} \\
& A = 830 \text{ bs.}
\end{aligned}$$

$$\begin{aligned}
A - 506 & = B \\
\Rightarrow 830 - 506 & = B \\
324 \text{ bs.} & = B
\end{aligned}$$

$$\begin{aligned}
4. \quad & x \rightarrow \text{N}^\circ \text{ mayor} \\
& x - 24 \rightarrow \text{N}^\circ \text{ menor} \\
& x + x - 24 = 106 \\
& 2x = 106 + 24 \\
& x = \frac{130}{2} \\
& x = 65 \rightarrow \text{N}^\circ \text{ mayor}
\end{aligned}$$

$$\begin{aligned}
x - 24 & \Rightarrow 65 - 24 \\
& = 41 \rightarrow \text{N}^\circ \text{ menor}
\end{aligned}$$

$$\begin{aligned}
5. \quad & A + B = 56 \text{ años} \\
& A + 14 = B \\
& A + A + 14 = 56 \\
& 2A = 56 - 14 \\
& A = \frac{42}{2} \\
& A = 21 \text{ años}
\end{aligned}$$

$$\begin{aligned}
A + 14 & = B \\
21 + 14 & = B \\
35 \text{ años} & = B
\end{aligned}$$

$$\begin{aligned}
6. \quad & A + B = 1.080 \text{ soles} \\
& A - 1.014 = B \\
& A + A - 1.014 = 1.080 \\
& 2A = 1.080 + 1.014 \\
& A = \frac{2.094}{2} \\
& A = 1.047 \text{ soles}
\end{aligned}$$

$$\begin{aligned}
A - 1.014 & = B \\
\Rightarrow 1.047 - 1.014 & = B \\
33 \text{ soles} & = B
\end{aligned}$$

$$\begin{aligned}
7. \quad & x \rightarrow \text{N}^\circ \text{ mayor} \\
& x - 1 \rightarrow \text{N}^\circ \text{ menor} \\
& x + x - 1 = 103 \\
& 2x = 103 + 1 \\
& x = \frac{104}{2} \\
& x = 52 \rightarrow \text{N}^\circ \text{ mayor}
\end{aligned}$$

$$\begin{aligned}
x - 1 & \Rightarrow 52 - 1 \\
& = 51 \rightarrow \text{N}^\circ \text{ menor}
\end{aligned}$$

$$\begin{aligned}
8. \quad & x \rightarrow \text{N}^\circ \text{ menor} \\
& x + 1 \rightarrow \text{N}^\circ \text{ medio} \\
& x + 2 \rightarrow \text{N}^\circ \text{ mayor} \\
& x + x + 1 + x + 2 = 204 \\
& 3x + 3 = 204 \\
& 3x = 204 - 3 \\
& x = \frac{201}{3} \\
& x = 67 \rightarrow \text{N}^\circ \text{ menor}
\end{aligned}$$

$$\begin{aligned}
x + 1 & \Rightarrow 67 + 1 \\
& = 68 \rightarrow \text{N}^\circ \text{ medio} \\
x + 2 & \Rightarrow 67 + 2 \\
& = 69 \rightarrow \text{N}^\circ \text{ mayor}
\end{aligned}$$

$$\begin{aligned}
9. \quad & x \rightarrow \text{N}^\circ 1 \quad x + 1 \rightarrow \text{N}^\circ 2 \\
& x + 2 \rightarrow \text{N}^\circ 3 \quad x + 3 \rightarrow \text{N}^\circ 4 \\
& x + x + 1 + x + 2 + x + 3 = 74 \\
& 4x + 6 = 74
\end{aligned}$$

$$\begin{aligned}
4x & = 74 - 6 \\
4x & = 68 \\
x & = \frac{68}{4} \\
x & = 17 \rightarrow \text{N}^\circ 1
\end{aligned}$$

$$\begin{aligned}
x + 1 & \Rightarrow 17 + 1 \\
& = 18 \rightarrow \text{N}^\circ 2 \\
x + 2 & \Rightarrow 17 + 2 \\
& = 19 \rightarrow \text{N}^\circ 3 \\
x + 3 & \Rightarrow 17 + 3 \\
& = 20 \rightarrow \text{N}^\circ 4
\end{aligned}$$

$$\begin{aligned}
10. \quad & 2x \rightarrow \text{N}^\circ \text{ menor} \\
& 2x + 2 \rightarrow \text{N}^\circ \text{ mayor} \\
& 2x + 2x + 2 = 194 \\
& 4x = 194 - 2 \\
& x = \frac{192}{4} \\
& x = 48
\end{aligned}$$

$$\begin{aligned}
2x & \Rightarrow 2 \cdot 48 \\
& = 96 \rightarrow \text{N}^\circ \text{ menor} \\
2x + 2 & \Rightarrow 2 \cdot 48 + 2 \\
& = 96 + 2 \\
& = 98 \rightarrow \text{N}^\circ \text{ mayor}
\end{aligned}$$

$$\begin{aligned}
11. \quad & x \rightarrow \text{N}^\circ \text{ mayor} \\
& x - 1 \rightarrow \text{N}^\circ \text{ medio} \\
& x - 2 \rightarrow \text{N}^\circ \text{ menor} \\
& x + x - 1 + x - 2 = 186 \\
& 3x - 3 = 186 \\
& 3x = 189 \\
& x = \frac{189}{3} \\
& x = 63 \rightarrow \text{N}^\circ \text{ mayor}
\end{aligned}$$

$$\begin{aligned}
x - 1 & \Rightarrow 63 - 1 \\
& = 62 \rightarrow \text{N}^\circ \text{ medio} \\
x - 2 & \Rightarrow 63 - 2 \\
& = 61 \rightarrow \text{N}^\circ \text{ menor}
\end{aligned}$$

$$\begin{aligned}
12. \quad & \text{caballo} + \text{coche} + \text{arreas} = 325 \\
& \text{coche} + 80 = \text{caballo} \\
& \text{coche} - 25 = \text{arreas} \\
& \text{coche} + 80 + \text{coche} + \text{coche} - 25 = 325 \\
& 3 \text{ coches} + 55 = 325 \\
& 3 \text{ coches} = 270 \\
& \text{coche} = \frac{270}{3} \\
& \text{coche} = \$90
\end{aligned}$$

$$\begin{aligned}
& \text{coche} + 80 = \text{caballo} \\
& \Rightarrow 90 + 80 = \text{caballo} \\
& \$170 = \text{caballo} \\
& \text{coche} - 25 = \text{arreas} \\
& \Rightarrow 90 - 25 = \text{arreas} \\
& \$65 = \text{arreas}
\end{aligned}$$

13.  $x \rightarrow N^{\circ} \text{ mayor}$

$x - 32 \rightarrow N^{\circ} \text{ medio}$

$x - 65 \rightarrow N^{\circ} \text{ menor}$

$x + x - 32 + x - 65 = 200$

$3x - 97 = 200$

$3x = 297$

$x = \frac{297}{3}$

$x = 99 \rightarrow N^{\circ} \text{ mayor}$

$x - 32 \Rightarrow 99 - 32$

$= 67 \rightarrow N^{\circ} \text{ medio}$

$x - 65 \Rightarrow 99 - 65$

$= 34 \rightarrow N^{\circ} \text{ menor}$

14.  $x \rightarrow 1^{\circ} \text{ cesto}$

$x - 10 \rightarrow 2^{\circ} \text{ cesto}$

$x - 15 \rightarrow 3^{\circ} \text{ cesto}$

$x + x - 10 + x - 15 = 575$

$3x - 25 = 575$

$3x = 600$

$x = \frac{600}{3}$

$x = 200 \rightarrow 1^{\circ} \text{ cesto}$

$x - 10 \Rightarrow 200 - 10$

$= 190 \rightarrow 2^{\circ} \text{ cesto}$

$x - 15 \Rightarrow 200 - 15$

$= 185 \rightarrow 3^{\circ} \text{ cesto}$

15.  $x \rightarrow \text{mayor}$

$x - 55 \rightarrow \text{medio}$

$x - 70 \rightarrow \text{menor}$

$x + x - 55 + x - 70 = 454$

$3x - 125 = 454$

$3x = 454 + 125$

$3x = 579$

$x = \frac{579}{3}$

$x = 193 \rightarrow \text{mayor}$

$x - 55 \Rightarrow 193 - 55$

$= 138 \rightarrow \text{medio}$

$x - 70 \Rightarrow 193 - 70$

$= 123 \rightarrow \text{menor}$

16.  $x \rightarrow 1^{\circ}$

$x - 20 \rightarrow 2^{\circ}$

$x - 20 - 40 \rightarrow 3^{\circ}$

$\Rightarrow x - 60 \rightarrow 3^{\circ}$

$x + x - 20 + x - 60 = 310$

$3x = 310 + 80$

$3x = 390$

$x = \frac{390}{3}$

$x = 130 \rightarrow 1^{\circ}$

$x - 20 \Rightarrow 130 - 20$

$= 110 \rightarrow 2^{\circ}$

$x - 60 \Rightarrow 130 - 60$

$= 70 \rightarrow 3^{\circ}$

17.  $x \rightarrow \text{mayor}$

$x - 20 \rightarrow \text{menor}$

$x - 18 \rightarrow \text{medio}$

$x + x - 20 + x - 18 = 88$

$3x - 38 = 88$

$3x = 88 + 38$

$3x = 126$

$x = \frac{126}{3}$

$x = 42 \rightarrow \text{mayor}$

$x - 20 \Rightarrow 42 - 20$

$= 22 \rightarrow \text{menor}$

$x - 18 \Rightarrow 42 - 18$

$= 24 \rightarrow \text{medio}$

18.  $x \rightarrow \text{mayor}$

$x - 36 \rightarrow \text{menor}$

$x + x - 36 = 642$

$2x - 36 = 642$

$2x = 642 + 36$

$2x = 678$

$x = \frac{678}{2}$

$x = 339 \rightarrow \text{mayor}$

$x - 36 \Rightarrow 339 - 36$

$= 303 \rightarrow \text{menor}$

## EJERCICIO 83

1.  $x \rightarrow \text{edad Juan}$

$3x \rightarrow \text{edad Pedro}$

$x + 3x = 40$

$4x = 40$

$x = \frac{40}{4}$

$x = 10 \rightarrow \text{edad Juan}$

$3x \Rightarrow 3 \cdot 10$

$= 30 \rightarrow \text{edad Pedro}$

2.  $x \rightarrow \text{Arreos}$

$4x \rightarrow \text{Caballo}$

$x + 4x = 600$

$5x = 600$

$x = \frac{600}{5}$

$x = \$120 \rightarrow \text{Arreos}$

$4x \Rightarrow 4 \cdot 120$

$= \$480 \rightarrow \text{Caballo}$

3.  $x \rightarrow 1^{\circ} \text{ piso}$

$\frac{x}{2} \rightarrow 2^{\circ} \text{ piso}$

$x + \frac{x}{2} = 48$

$\frac{2x + x}{2} = 48$

$3x = 48 \cdot 2$

$3x = 96$

$x = \frac{96}{3}$

$x = 32 \text{ Habt.} \rightarrow 1^{\circ} \text{ piso}$

$\frac{x}{2} \Rightarrow \frac{32}{2}$

$= 16 \text{ Habt.} \rightarrow 2^{\circ} \text{ piso}$

4.  $A + B + C = 300$

$B = 2A$

$C = 3A$

$\Rightarrow A + 2A + 3A = 300$

$6A = 300$

$A = \frac{300}{6}$

$A = 50 \text{ colones}$

$B = 2A \Rightarrow 2 \cdot 50$

$= 100 \text{ colones}$

$C = 3A \Rightarrow 3 \cdot 50$

$= 150 \text{ colones}$



5.  $A+B+C=133$

$$A = \frac{B}{2}$$

$$C=2B$$

$$\Rightarrow \frac{B}{2} + B + 2B = 133$$

$$\frac{B}{2} + 3B = 133$$

$$\frac{B+6B}{2} = 133$$

$$7B = 133 \cdot 2$$

$$B = \frac{266}{7}$$

$$B = 38 \text{ Sucres}$$

$$A = \frac{B}{2} \Rightarrow \frac{38}{2}$$

$$= 19 \text{ Sucres}$$

$$C = 2B \Rightarrow 2 \cdot 38$$

$$= 76 \text{ Sucres}$$

6.  $x \rightarrow N^{\circ} \text{ mayor}$

$$\frac{x}{6} \rightarrow N^{\circ} \text{ menor}$$

$$x + \frac{x}{6} = 147$$

$$\frac{6x+x}{6} = 147$$

$$7x = 147 \cdot 6$$

$$7x = 882$$

$$x = \frac{882}{7}$$

$$x = 126 \rightarrow N^{\circ} \text{ mayor}$$

$$\frac{x}{6} \Rightarrow \frac{126}{6}$$

$$= 21 \rightarrow N^{\circ} \text{ menor}$$

7.  $A+B+C=140$

$$B = \frac{A}{2} \Rightarrow 2B = A$$

$$B = \frac{C}{4} \Rightarrow 4B = C$$

$$2B+B+4B=140$$

$$7B=140$$

$$B = \frac{140}{7}$$

$$B = 20 \text{ Quetz}$$

$$A = 2B \Rightarrow 2 \cdot 20$$

$$= 40 \text{ Quetz}$$

$$C = 4B \Rightarrow 4 \cdot 20$$

$$= 80 \text{ Quetz}$$

8.  $x \rightarrow 1^{\circ} \text{ parte}$

$$4x \rightarrow 2^{\circ} \text{ parte}$$

$$5x \rightarrow 3^{\circ} \text{ parte}$$

$$x+4x+5x=850$$

$$10x=850$$

$$x = \frac{850}{10}$$

$$x = 85 \rightarrow 1^{\circ} \text{ parte}$$

$$4x \Rightarrow 4 \cdot 85$$

$$= 340 \rightarrow 2^{\circ} \text{ parte}$$

$$5x \Rightarrow 5 \cdot 85$$

$$= 425 \rightarrow 3^{\circ} \text{ parte}$$

9.  $x \rightarrow N^{\circ} \text{ buscado}$

$$2x = x + 111$$

$$2x - x = 111$$

$$x = 111 \rightarrow N^{\circ} \text{ buscado}$$

10.  $x \rightarrow \text{edad Rosa}$

$$3x + 15 \rightarrow \text{edad María}$$

$$x + 3x + 15 = 59$$

$$4x = 59 - 15$$

$$4x = 44$$

$$x = \frac{44}{4}$$

$$x = 11 \rightarrow \text{edad Rosa}$$

$$3x + 15 \Rightarrow 3 \cdot 11 + 15$$

$$= 33 + 15$$

$$= 48 \rightarrow \text{edad María}$$

11.  $x \rightarrow N^{\circ} \text{ buscado}$

$$8x = x + 21$$

$$8x - x = 21$$

$$7x = 21$$

$$x = \frac{21}{7}$$

$$x = 3 \rightarrow N^{\circ} \text{ buscado}$$

12.  $x \rightarrow \text{Mi edad}$

$$3x + 7 = 100$$

$$3x = 100 - 7$$

$$3x = 93$$

$$x = \frac{93}{3}$$

$$x = 31 \rightarrow \text{Mi edad}$$

13.  $x \rightarrow 1^{\circ} \text{ parte}$

$$\frac{x}{3} \rightarrow 2^{\circ} \text{ parte}$$

$$\frac{4x}{3} \rightarrow 3^{\circ} \text{ parte}$$

$$x + \frac{x}{3} + \frac{4x}{3} = 96$$

$$\frac{3x + x + 4x}{3} = 96$$

$$8x = 96 \cdot 3$$

$$8x = 288$$

$$x = \frac{288}{8}$$

$$x = 36 \rightarrow 1^{\circ} \text{ parte}$$

$$\frac{x}{3} \Rightarrow \frac{36}{3} = 12 \rightarrow 2^{\circ} \text{ parte}$$

$$\frac{4x}{3} \Rightarrow \frac{4 \cdot 36}{3} = 4 \cdot 12 = 48 \rightarrow 3^{\circ} \text{ parte}$$

14.  $x \rightarrow \text{Edad Enrique}$

$$2x \rightarrow \text{Edad Pedro}$$

$$3x \rightarrow \text{Edad Juan}$$

$$6x \rightarrow \text{Edad Eugenio}$$

$$x + 2x + 3x + 6x = 132$$

$$12x = 132$$

$$x = \frac{132}{12}$$

$$x = 11 \rightarrow \text{Edad Enrique}$$

$$2x \Rightarrow 2 \cdot 11 = 22 \rightarrow \text{Edad Pedro}$$

$$3x \Rightarrow 3 \cdot 11 = 33 \rightarrow \text{Edad Juan}$$

$$6x \Rightarrow 6 \cdot 11 = 66 \rightarrow \text{Edad Eugenio}$$

## EJERCICIO 84

1.  $x \rightarrow 1^{\circ} \text{ parte}$

$$3x \rightarrow 2^{\circ} \text{ parte}$$

$$3x - 40 \rightarrow 3^{\circ} \text{ parte}$$

$$x + 3x + 3x - 40 = 254$$

$$7x = 254 + 40$$

$$7x = 294$$

$$x = \frac{294}{7}$$

$$x = 42 \rightarrow 1^{\circ} \text{ parte}$$

$$3x \Rightarrow 3 \cdot 42 = 126 \rightarrow 2^{\circ} \text{ parte}$$

$$3x - 40 \Rightarrow 3 \cdot 42 - 40$$

$$= 126 - 40 = 86 \rightarrow 3^{\circ} \text{ parte}$$

2.  $A+B+C=130$

$$C=2A$$

$$B-15=C \Rightarrow B-15=2A$$

$$B=2A+15$$

$$A+2A+15+2A=130$$

$$5A=130-15$$

$$5A=115$$

$$A=\frac{115}{5}$$

$$A=23 \text{ Balb.}$$

$$B=2A+15 \Rightarrow 2 \cdot 23+15=46+15=61 \text{ Balb.}$$

$$C=2A \Rightarrow 2 \cdot 23=46 \text{ Balb.}$$

3.  $x \rightarrow 1^\circ \text{ Número}$

$$\frac{x-8}{2} \rightarrow 2^\circ \text{ Número}$$

$$x-18 \rightarrow 3^\circ \text{ Número}$$

$$x+\frac{x-8}{2}+x-18=238$$

$$2x+\frac{x-8}{2}=256$$

$$\frac{4x+x-8}{2}=256+4$$

$$5x=260-2$$

$$x=\frac{520}{5}$$

$$x=104 \rightarrow 1^\circ \text{ Número}$$

$$\frac{x-8}{2} \Rightarrow \frac{104-8}{2}=\frac{96}{2}=48 \rightarrow 2^\circ \text{ Número}$$

$$x-18 \Rightarrow 104-18=86 \rightarrow 3^\circ \text{ Número}$$

4.  $x \rightarrow \text{Costo traje}$

$$\frac{x}{8} \rightarrow \text{Costo sombrero}$$

$$x-30 \rightarrow \text{Costo bastón}$$

$$x+\frac{x}{8}+x-30=259$$

$$2x+\frac{x}{8}=259+30$$

$$\frac{16x+x}{8}=289$$

$$17x=289 \cdot 8$$

$$x=\frac{2.312}{17}$$

$$x=\$136 \rightarrow \text{Costo traje}$$

$$\frac{x}{8} \Rightarrow \frac{136}{8}=\$17 \rightarrow \text{Costo sombrero}$$

$$x-30 \Rightarrow 136-30=\$106 \rightarrow \text{Costo bastón}$$

5.  $x \rightarrow 1^\circ \text{ Número}$

$$\frac{x-6}{5} \rightarrow 2^\circ \text{ Número}$$

$$x-6 \rightarrow 3^\circ \text{ Número}$$

$$x+\frac{x-6}{5}+x-6=72$$

$$2x+\frac{x-6}{5}=78$$

$$\frac{10x+x-6}{5}=78+\frac{6}{5}$$

$$\frac{11x-6}{5}=\frac{390+6}{5}$$

$$11x=396$$

$$x=\frac{396}{11}$$

$$x=36 \rightarrow 1^\circ \text{ Número}$$

$$\frac{x-6}{5} \Rightarrow \frac{36-6}{5}$$

$$=\frac{30}{5}=6 \rightarrow 2^\circ \text{ Número}$$

$$x-6 \Rightarrow 36-6=30 \rightarrow 3^\circ \text{ Número}$$

6.  $A+B=99$

$$B=3A+19$$

$$A+3A+19=99$$

$$4A=99-19$$

$$4A=80$$

$$A=\frac{80}{4}$$

$$A=20 \text{ bs.}$$

$$B=3A+19 \Rightarrow 3 \cdot 20+19$$

$$=60+19=79 \text{ bs.}$$

7.  $x \rightarrow \text{cm de azul}$

$$\frac{x-14}{2} \rightarrow \text{cm de blanco}$$

$$x+\frac{x-14}{2}=74$$

$$x+\frac{x-14}{2}=74$$

$$\frac{2x+x-14}{2}=74+7$$

$$3x=81-2$$

$$x=\frac{162}{3}$$

$$x=54 \rightarrow \text{cm de azul}$$

$$\frac{x-14}{2} \Rightarrow \frac{54-14}{2}$$

$$=\frac{40}{2}=20 \rightarrow \text{cm de blanco}$$

8.  $A+B+C=152$

$$B=2A-8 \Rightarrow \frac{B+8}{2}=A$$

$$B-32=C$$

$$\frac{B+8}{2}+B+B-32=152$$

$$\frac{B}{2}+\frac{8}{2}+2B=152+32$$

$$\frac{B+4B}{2}=184-4$$

$$5B=180-2$$

$$B=\frac{360}{5}$$

$$B=\$72$$

$$A=\frac{B+8}{2} \Rightarrow \frac{72+8}{2}=\frac{80}{2}=\$40$$

$$C=B-32 \Rightarrow 72-32=\$40$$

9.  $x \rightarrow N^\circ \text{ buscado}$

$$x-80=220-2x$$

$$x+2x=220+80$$

$$3x=300$$

$$x=\frac{300}{3}$$

$$x=100 \rightarrow N^\circ \text{ buscado}$$

10.  $x \rightarrow \text{Tengo ahora}$

$$2x+10=x+60$$

$$2x-x=60-10$$

$$x=50 \text{ S/.} \rightarrow \text{Tengo ahora}$$

11.  $x \rightarrow \text{Parte separada}$

$$x+80 \rightarrow \text{La otra parte}$$

$$x+x+80=910$$

$$2x=910-80$$

$$x=\frac{830}{2}$$

$$x=415 \text{ cm}$$

$$\Rightarrow 4,15 \text{ m} \rightarrow \text{Parte separada}$$

$$x+80 \Rightarrow 415+80=495 \text{ cm}$$

$$\Rightarrow 4,95 \text{ m} \rightarrow \text{La otra parte}$$

12.  $x \rightarrow \text{Edad padre}$

$$\frac{x-3}{3} \rightarrow \text{Edad hijo}$$

$$x + \frac{x-3}{3} = 83$$

$$x + \frac{x-3}{3} = 83$$

$$\frac{3x+x}{3} = 83+1$$

$$4x = 84 \cdot 3$$

$$x = \frac{252}{4}$$

$$x = 63 \rightarrow \text{Edad padre}$$

$$\frac{x-3}{3} \rightarrow \frac{63-3}{3} = \frac{60}{3} = 20 \rightarrow \text{Edad hijo}$$

13.  $A+B+C=9.000$

$$B+500=A$$

$$B-800=C$$

$$B+500+B+B-800=9.000$$

$$3B=9.000+300$$

$$B = \frac{9.300}{3}$$

$$B = 3.100 \text{ Votos}$$

$$A = B + 500 \Rightarrow 3.100 + 500 = 3.600 \text{ Votos}$$

$$C = B - 800 \Rightarrow 3.100 - 800 = 2.300 \text{ Votos}$$

14.  $x \rightarrow \text{N}^\circ \text{ buscado}$

$$8x-60=60-7x$$

$$8x+7x=60+60$$

$$15x=120$$

$$x = \frac{120}{15}$$

$$x = 8 \rightarrow \text{N}^\circ \text{ buscado}$$

15.  $x \rightarrow \text{Edad hom bre}$

$$2x-17=100-x$$

$$2x+x=100+17$$

$$3x=117$$

$$x = \frac{117}{3}$$

$$x = 39 \rightarrow \text{Edad hom bre}$$

## EJERCICIO 85

1.  $x \rightarrow \text{N}^\circ \text{ mayor}$

$$\frac{2x}{3} \rightarrow \text{N}^\circ \text{ menor}$$

$$x + \frac{2x}{3} = 100$$

$$\frac{3x+2x}{3} = 100$$

$$5x = 100 \cdot 3$$

$$x = \frac{300}{5}$$

$$x = 60 \rightarrow \text{N}^\circ \text{ mayor}$$

$$\frac{2x}{3} \Rightarrow \frac{2 \cdot 60}{3} = 40 \rightarrow \text{N}^\circ \text{ menor}$$

2.  $x \rightarrow \text{Edad padre}$

$$\frac{x-15}{2} \rightarrow \text{Edad hijo}$$

$$x + \frac{x-15}{2} = 60$$

$$x + \frac{x-15}{2} = 60$$

$$\frac{2x+x}{2} = 60 + \frac{15}{2}$$

$$\frac{3x}{2} = \frac{120+15}{2}$$

$$3x = 135$$

$$x = \frac{135}{3}$$

$$x = 45 \rightarrow \text{Edad padre}$$

$$\frac{x-15}{2} \Rightarrow \frac{45-15}{2} = 15 \rightarrow \text{Edad hijo}$$

3.  $x \rightarrow \text{Partemayor}$

$$x-232 \rightarrow \text{Partemenor}$$

$$x+x-232=1.080$$

$$2x=1.080+232$$

$$x = \frac{1.312}{2}$$

$$x = 656 \rightarrow \text{Partemayor}$$

$$x-232 \Rightarrow 656-232$$

$$= 424 \rightarrow \text{Partemenor}$$

4.  $A+B=150$

$$A-46=B$$

$$A+A-46=150$$

$$2A=150+46$$

$$A = \frac{196}{2}$$

$$A = 98 \text{ Soles}$$

$$B = A - 46 \Rightarrow 98 - 46$$

$$= 52 \text{ Soles}$$

5.  $x \rightarrow \text{Ang. mayor}$

$$\frac{x+45}{2} \rightarrow \text{Ang. menor}$$

$$x + \frac{x+45}{2} = 180$$

$$x + \frac{x+45}{2} = 180$$

$$\frac{2x+x}{2} = 180 - \frac{45}{2}$$

$$\frac{3x}{2} = \frac{360-45}{2}$$

$$3x = 315$$

$$x = \frac{315}{3}$$

$$x = 105 \rightarrow \text{Ang. mayor}$$

$$\frac{x+45}{2} \Rightarrow \frac{105+45}{2} = 75 \rightarrow \text{Ang. menor}$$

6.  $x \rightarrow \text{N}^\circ \text{ mayor}$

$$\frac{x-88}{3} \rightarrow \text{N}^\circ \text{ menor}$$

$$x + \frac{x-88}{3} = 540$$

$$x + \frac{x-88}{3} = 540$$

$$\frac{3x+x}{3} = 540 + \frac{88}{3}$$

$$\frac{4x}{3} = \frac{1.620+88}{3}$$

$$4x = 1.708$$

$$x = \frac{1.708}{4}$$

$$x = 427 \rightarrow \text{N}^\circ \text{ mayor}$$

$$\frac{x-88}{3} \Rightarrow \frac{427-88}{3}$$

$$= \frac{339}{3} = 113 \rightarrow \text{N}^\circ \text{ menor}$$

7.  $x \rightarrow \text{N}^\circ \text{ mayor}$

$$\frac{x-12}{4} \rightarrow \text{N}^\circ \text{ menor}$$

$$x - \left( \frac{x-12}{4} \right) = 36$$

$$x - \frac{x}{4} + \frac{12}{4} = 36$$

$$\frac{4x-x}{4} = 36 - 3$$

$$3x = 33 \cdot 4$$

$$3x = 132$$

$$x = \frac{132}{3}$$

$$x = 44 \rightarrow \text{N}^\circ \text{ mayor}$$

$$\frac{x-12}{4} \Rightarrow \frac{44-12}{4} = 8 \rightarrow \text{N}^\circ \text{ menor}$$

8  $x \rightarrow \text{Costo perro}$

$$\frac{x}{8} \rightarrow \text{Costo collar}$$

$$x + \frac{x}{8} = 54$$

$$\frac{8x+x}{8} = 54$$

$$9x = 54 \cdot 8$$

$$9x = 432$$

$$x = \frac{432}{9}$$

$$x = \$48 \rightarrow \text{Costo perro}$$

$$\frac{x}{8} \Rightarrow \frac{48}{8} = \$6 \rightarrow \text{Costo collar}$$

9.  $A + B = 84$

$$A - 16 = B + 20$$

$$\Rightarrow A - 16 - 20 = B$$

$$A - 36 = B$$

$$A + A - 36 = 84$$

$$2A = 84 + 36$$

$$A = \frac{120}{2}$$

$$A = \$60$$

$$B = A - 36 \Rightarrow 60 - 36 = \$24$$

12.  $x \rightarrow \text{N}^\circ \text{ mayor}$

$$\frac{x+150}{3} \rightarrow \text{N}^\circ \text{ menor}$$

$$x + \frac{x+150}{3} = 506$$

$$x + \frac{x}{3} = 506 - \frac{150}{3}$$

$$\frac{3x+x}{3} = \frac{1518-150}{3}$$

$$4x = 1368$$

$$x = \frac{1368}{4}$$

$$x = 342 \rightarrow \text{N}^\circ \text{ mayor}$$

$$\frac{x+150}{3} \Rightarrow \frac{342+150}{3}$$

$$= \frac{492}{3} = 164 \rightarrow \text{N}^\circ \text{ menor}$$

10.  $x \rightarrow \text{N}^\circ \text{ Señoritas}$

$$\frac{x-15}{2} \rightarrow \text{N}^\circ \text{ Jovenes}$$

$$x + \frac{x-15}{2} = 60$$

$$x + \frac{x}{2} = 60 + \frac{15}{2}$$

$$\frac{2x+x}{2} = \frac{120+15}{2}$$

$$3x = 135$$

$$x = \frac{135}{3}$$

$$x = 45 \rightarrow \text{N}^\circ \text{ Señoritas}$$

$$\frac{x-15}{2} \Rightarrow \frac{45-15}{2} = 15 \rightarrow \text{N}^\circ \text{ jovenes}$$

11.  $x \rightarrow \text{Parte mayor}$

$$\frac{x+16}{3} \rightarrow \text{Parte menor}$$

$$x + \frac{x+16}{3} = 160$$

$$x + \frac{x}{3} = 160 - \frac{16}{3}$$

$$\frac{3x+x}{3} = \frac{480-16}{3}$$

$$4x = 464$$

$$x = \frac{464}{4}$$

$$x = 116 \rightarrow \text{Parte mayor}$$

$$\frac{x+16}{3} \Rightarrow \frac{116+16}{3}$$

$$= \frac{132}{3} = 44 \rightarrow \text{Parte menor}$$

13.  $x \rightarrow \text{Estilográfica}$

$$x - 10 \rightarrow \text{Lapicero}$$

$$x + x - 10 = 18$$

$$2x = 28$$

$$x = \frac{28}{2}$$

$$x = 14 \text{ bs.} \rightarrow \text{Estilográfica}$$

$$x - 10 \Rightarrow 14 - 10 = 4 \text{ bs.} \rightarrow \text{Lapicero}$$

14.  $x \rightarrow \text{Parte roja}$

$$x + 4 \rightarrow \text{Parte negra}$$

$$x + x + 4 = 84$$

$$2x = 80$$

$$x = 40 \text{ cm} \rightarrow \text{Parte roja}$$

$$x + 4 \Rightarrow 40 + 4 = 44 \text{ cm} \rightarrow \text{Parte negra}$$

## EJERCICIO 86

1.  $2x \rightarrow \text{Edad actual A}$

$$x \rightarrow \text{Edad actual B}$$

$$2x - 10 = 3(x - 10)$$

$$2x - 10 = 3x - 30$$

$$2x - 3x = -30 + 10$$

$$-x = -20$$

$$x = 20 \rightarrow \text{Edad actual B}$$

$$2x \Rightarrow 2 \cdot 20 = 40 \rightarrow \text{Edad actual A}$$

2.

$$3x \rightarrow \text{Edad A}$$

$$x \rightarrow \text{Edad B}$$

$$3x + 5 = 2(x + 5)$$

$$3x + 5 = 2x + 10$$

$$3x - 2x = 10 - 5$$

$$x = 5 \rightarrow \text{Edad B}$$

$$3x \Rightarrow 3 \cdot 5 = 15 \rightarrow \text{Edad A}$$

3.  $2x \rightarrow \text{Tiene A}$

$$x \rightarrow \text{Tiene B}$$

$$2x - 30 = x - 5$$

$$2x - x = 30 - 5$$

$$x = \$25 \rightarrow \text{Tiene B}$$

$$2x \Rightarrow 2 \cdot 25 = \$50 \rightarrow \text{Tiene A}$$

4.  $\frac{x}{2} \rightarrow \text{Tiene A}$

$$x \rightarrow \text{Tiene B}$$

$$\frac{x}{2} + 66 = 2(x - 90)$$

$$\frac{x+132}{2} = 2(x-90)$$

$$x + 132 = 4(x - 90)$$

$$x + 132 = 4x - 360$$

$$-3x = -492$$

$$x = \frac{-492}{-3}$$

$$x = 164 \text{ colones} \rightarrow \text{Tiene B}$$

$$\frac{x}{2} \Rightarrow \frac{164}{2} = 82 \rightarrow \text{colones Tiene A}$$

5.  $x \rightarrow \text{N}^\circ \text{ var ones}$

$$\frac{x}{3} \rightarrow \text{N}^\circ \text{ Srtas}$$

$$\frac{x}{3} + 14 = x - 10$$

$$\frac{x}{3} - x = -10 - 14$$

$$\frac{x-3x}{3} = -24$$

$$-2x = -72$$

$$x = 36 \rightarrow \text{N}^\circ \text{ var ones}$$

$$\frac{x}{3} \Rightarrow \frac{36}{3} = 12 \rightarrow \text{N}^\circ \text{ Srtas}$$

6.  $3x \rightarrow \text{Edad padre}$

$$x \rightarrow \text{Edad hijo}$$

$$3x - 5 = 2(x + 10)$$

$$3x - 5 = 2x + 20$$

$$x = 25 \rightarrow \text{Edad hijo}$$

$$3x \Rightarrow 3 \cdot 25 = 75 \rightarrow \text{Edad padre}$$

7.  $x \rightarrow \text{N}^\circ \text{ mayor}$   
 $2x - 56 \rightarrow \text{N}^\circ \text{ menor}$   
 $x + 2x - 56 = 85$   
 $3x = 141$   
 $x = \frac{141}{3}$   
 $x = 47 \rightarrow \text{N}^\circ \text{ mayor}$   
 $2x - 56 \Rightarrow 2 \cdot 47 - 56$   
 $= 94 - 56 = 38 \rightarrow \text{N}^\circ \text{ menor}$
8.  $5x \rightarrow \text{Tiene Enrique}$   
 $x \rightarrow \text{Tiene suhno.}$   
 $5x - 0,5 = x + 0,5$   
 $5x - x = 0,5 + 0,5$   
 $4x = 1$   
 $x = \frac{1}{4}$   
 $x = \$0,25 \rightarrow \text{Tiene suhno}$
9.  $x \rightarrow \text{Bolsa 1}$   
 $x - 400 \rightarrow \text{Bolsa 2}$   
 $x + x - 400 = 1.400$   
 $2x = 1.800$   
 $x = 900 \text{ S/.} \rightarrow \text{Bolsa 1}$   
 $x - 400 \Rightarrow 900 - 400 = 500 \text{ S/.} \rightarrow \text{Bolsa 2}$
10.  $4x \rightarrow \text{Días Trab. Pedro}$   
 $x \rightarrow \text{Días Trab. Enrique}$   
 $4x - 15 = x + 21$   
 $4x - x = 21 + 15$   
 $3x = 36$   
 $x = \frac{36}{3}$   
 $x = 12 \rightarrow \text{Días Trab. Enrique}$   
 $4x \Rightarrow 4 \cdot 12$   
 $= 48 \rightarrow \text{Días Trab. Pedro}$
11.  $2x \rightarrow \text{Edad act. padre}$   
 $x \rightarrow \text{Edad act. hijo}$   
 $2x - 14 = 3(x - 14)$   
 $2x - 14 = 3x - 42$   
 $2x - 3x = -42 + 14$   
 $-x = -28$   
 $x = 28 \rightarrow \text{Edad act. hijo}$   
 $\text{Edades hace 14 años:}$   
 $2x - 14 \Rightarrow 2 \cdot 28 - 14$   
 $= 42 \text{ años} \rightarrow \text{padre}$   
 $x - 14 \Rightarrow 28 - 14$   
 $= 14 \text{ años} \rightarrow \text{hijo}$
12.  $3x \rightarrow \text{Edad actual Juan}$   
 $x \rightarrow \text{Edad actual hijo}$   
 $3x + 22 = 2(x + 22)$   
 $3x = 2x + 44 - 22$   
 $x = 22 \rightarrow \text{Edad actual hijo}$   
 $3x \Rightarrow 3 \cdot 22 = 66 \rightarrow \text{Edad actual Juan}$
13.  $A + B = 84 \Rightarrow A = 84 - B$   
 $A + 80 = 3(B + 4)$   
 $\Rightarrow 84 - B + 80 = 3B + 12$   
 $-B - 3B = 12 - 164$   
 $-4B = -152$   
 $B = \frac{-152}{-4}$   
 $B = \$38$   
 $A = 84 - B \Rightarrow 84 - 38 = \$46$

## EJERCICIO 87

1.  $2x \rightarrow \text{N}^\circ \text{ Sombreros}$   
 $x \rightarrow \text{N}^\circ \text{ trajes}$   
 $4x + 50x = 702$   
 $54x = 702$   
 $x = \frac{702}{54}$   
 $x = 13 \rightarrow \text{N}^\circ \text{ trajes}$   
 $2x \Rightarrow 2 \cdot 13$   
 $= 26 \rightarrow \text{N}^\circ \text{ Sombreros}$
2.  $x \rightarrow \text{Cab.}$   
 $x - 6 \rightarrow \text{Vacas}$   
 $600x + 800(x - 6) = 40.000$   
 $600x + 800x - 4.800 = 40.000$   
 $1.400x = 44.800$   
 $x = \frac{44.800}{1.400}$   
 $x = 32 \rightarrow \text{Cab.}$   
 $x - 6 \Rightarrow 32 - 6 = 26 \rightarrow \text{Vacas}$
3.  $16 - x \rightarrow \text{Pr ob. reslt.}$   
 $x \rightarrow \text{Pr ob. no reslt.}$   
 $12(16 - x) - 5x = 73$   
 $192 - 12x - 5x = 73$   
 $-17x = 73 - 192$   
 $x = \frac{-119}{-17}$   
 $x = 7 \rightarrow \text{Pr ob. no reslt.}$   
 $16 - x \Rightarrow 16 - 7 = 9 \rightarrow \text{Pr ob. reslt.}$
4.  $50 - x \rightarrow \text{Días trab.}$   
 $x \rightarrow \text{Días no trab.}$   
 $3(50 - x) - 2x = 90$   
 $150 - 3x - 2x = 90$   
 $-5x = -60$   
 $x = \frac{-60}{-5}$   
 $x = 12 \rightarrow \text{Días no trab.}$   
 $50 - x \Rightarrow 50 - 12 = 38 \rightarrow \text{Días trab.}$
5.  $35 - x \rightarrow \text{Trajes de 30 Q}$   
 $x \rightarrow \text{Trajes de 25 Q}$   
 $30(35 - x) + 25x = 1.015$   
 $1.050 - 30x + 25x = 1.015$   
 $-5x = 1.015 - 1.050$   
 $x = \frac{-35}{-5}$   
 $x = 7 \rightarrow \text{Trajes de 25 Q}$   
 $35 - x \Rightarrow 35 - 7 = 28 \rightarrow \text{Trajes de 30 Q}$
6.  $x \rightarrow \text{Traje Cal.}$   
 $x - 7 \rightarrow \text{Traje inf.}$   
 $32x + 18(x - 7) = 1.624$   
 $32x + 18x - 126 = 1.624$   
 $50x = 1.750$   
 $x = \frac{1.750}{50}$   
 $x = 35 \text{ balb.} \rightarrow \text{Traje Cal.}$   
 $x - 7 \Rightarrow 35 - 7 = 28 \text{ balb.} \rightarrow \text{Traje inf.}$

7.  $3x \rightarrow N^{\circ} \text{ Lápices}$   
 $x \rightarrow N^{\circ} \text{ cuad.}$   
 $0,05 \cdot 3x + 0,06x = 1,47$   
 $0,15x + 0,06x = 1,47$   
 $0,21x = 1,47$   
 $x = \frac{1,47}{0,21}$   
 $x = 7$   
 $\Rightarrow 7 \rightarrow N^{\circ} \text{ cuad.}$   
 $3x \rightarrow 3 \cdot 7 = 21 \rightarrow N^{\circ} \text{ Lápices}$
8.  $3x + 5 \rightarrow \text{Sac. Frij.}$   
 $x \rightarrow \text{Sac. azúcar.}$   
 $6(3x + 5) + 5x = 582$   
 $18x + 30 + 5x = 582$   
 $23x = 552$   
 $x = \frac{552}{23}$   
 $x = 24$   
 $\Rightarrow 24 \rightarrow \text{Sac. azúcar.}$   
 $3x + 5 \rightarrow 3 \cdot 24 + 5$   
 $= 77 \rightarrow \text{Sac. Frij.}$
9.  $80 - x \rightarrow \text{Cedro}$   
 $x \rightarrow \text{Caoba}$   
 $0,75(80 - x) + 0,90x = 68,40$   
 $60 - 0,75x + 0,90x = 68,40$   
 $0,15x = 8,40$   
 $x = \frac{8,40}{0,15}$   
 $x = 56$   
 $\Rightarrow 56 \text{ pies}^3 \rightarrow \text{Caoba}$   
 $80 - x \rightarrow 80 - 56$   
 $= 24 \text{ pies}^3 \rightarrow \text{Cedro}$
10.  $1.050 - x \rightarrow P. mayor$   
 $x \rightarrow P. menor$   
 $(1.050 - x) - 2x = 1.825$   
 $1.050 - 3x = 1.825$   
 $-5x = 1.825 - 3.150$   
 $-5x = -1.325$   
 $x = \frac{-1.325}{-5}$   
 $x = 265$   
 $\Rightarrow 265 \rightarrow P. menor$   
 $1.050 - x \rightarrow 1.050 - 265$   
 $= 785 \rightarrow P. mayor$

## EJERCICIO 88

1.  $x \rightarrow 1^{\circ}$   
 $2x \rightarrow 2^{\circ}$   
 $x + 2x - 20 \rightarrow 3^{\circ}$   
 $\Rightarrow 3x - 20 \rightarrow 3^{\circ}$   
 $x + 2x + 3x - 20 = 196$   
 $6x = 196 + 20$   
 $x = \frac{216}{6}$   
 $x = 36 \rightarrow 1^{\circ}$   
 $2x \Rightarrow 2 \cdot 36 = 72 \rightarrow 2^{\circ}$   
 $3x - 20 \Rightarrow 3 \cdot 36 - 20$   
 $= 108 - 20 = 88 \rightarrow 3^{\circ}$
2.  $3x \rightarrow \text{Edad A}$   
 $x \rightarrow \text{Edad B}$   
 $3x - 5 = 4(x - 5)$   
 $3x - 5 = 4x - 20$   
 $3x - 4x = -20 + 5$   
 $-x = -15$   
 $x = 15 \rightarrow \text{Edad B}$   
 $3x \Rightarrow 3 \cdot 15 = 45 \rightarrow \text{Edad A}$
3.  $x \rightarrow \text{Par zap.}$   
 $2x + 50 \rightarrow \text{Traje}$   
 $50(2x + 50) + 35x = 16.000$   
 $100x + 2.500 + 35x = 16.000$   
 $135x = 16.000 - 2.500$   
 $x = \frac{13.500}{135}$   
 $x = 100 \text{ soles} \rightarrow \text{Par zap.}$   
 $2x + 50 \Rightarrow 2 \cdot 100 + 50 = 250 \text{ soles} \rightarrow \text{Traje}$
4.  $x \rightarrow \text{Costo casa}$   
 $\frac{x}{4} = \frac{x}{6} + 2.000$   
 $\frac{x}{4} = \frac{x + 12.000}{6}$   
 $6x = 4x + 48.000$   
 $2x = 48.000$   
 $x = \frac{48.000}{2}$   
 $x = 24.000 \text{ bs.} \rightarrow \text{Costo casa}$
5.  $x \rightarrow N^{\circ} \text{ mayor}$   
 $\frac{2x - 156}{3} \rightarrow N^{\circ} \text{ menor}$   
 $x + \frac{2x - 156}{3} = 108$   
 $x + \frac{2x}{3} = 108 + \frac{156}{3}$   
 $\frac{3x + 2x}{3} = \frac{324 + 156}{3}$   
 $5x = 480$   
 $x = \frac{480}{5}$   
 $x = 96 \rightarrow N^{\circ} \text{ mayor}$   
 $\frac{2x - 156}{3} \Rightarrow \frac{2 \cdot 96 - 156}{3}$   
 $= \frac{36}{3} = 12 \rightarrow N^{\circ} \text{ menor}$
6.  $x \rightarrow \text{Ancho}$   
 $461 - 11 = 9x$   
 $450 = 9x$   
 $\frac{450}{9} = x$   
 $50 = x$   
 $\Rightarrow 50 \text{ pies} \rightarrow \text{Ancho}$
7.  $x \rightarrow \text{Gasté}$   
 $85 - x = 4x$   
 $85 = 5x$   
 $\frac{85}{5} = x$   
 $17 = x$   
 $\Rightarrow \$17 \rightarrow \text{Gasté}$
8.  $x \rightarrow \text{Edad act. B}$   
 $2(x - 12) \rightarrow \text{Edad A hace 12 años}$   
 $2(x - 12) + 24 + 68 = 3(x + 12)$   
 $2x - 24 + 92 = 3x + 36$   
 $2x - 3x = 36 - 68$   
 $-x = -32$   
 $x = 32 \rightarrow \text{Edad act. B}$   
 $\text{Edad actual de A:}$   
 $2(x - 12) + 12 \Rightarrow 2(32 - 12) + 12$   
 $= 2 \cdot 20 + 12 = 52 \text{ años}$
9.  $x \rightarrow \text{mon. 10 cts.}$   
 $22 - x \rightarrow \text{mon. 5 cts.}$   
 $0,10x + 0,05(22 - x) = 1,85$   
 $0,10x + 1,10 - 0,05x = 1,85$   
 $0,05x = 1,85 - 1,10$   
 $0,05x = 0,75$   
 $x = \frac{0,75}{0,05}$   
 $x = 15 \rightarrow \text{mon. 10 cts.}$   
 $22 - x \Rightarrow 22 - 15 = 7 \rightarrow \text{mon. 5 cts.}$

10.  $x \rightarrow N^{\circ} \text{ buscado}$

$$12(x-24)=24(x-27)$$

$$12x-288=24x-648$$

$$12x-24x=-648+288$$

$$-12x=-360$$

$$x=\frac{-360}{-12}$$

$$x=30 \rightarrow N^{\circ} \text{ buscado}$$

11.  $x \rightarrow c / \text{cab.}$

$$35x=40(x-10)$$

$$35x=40x-400$$

$$-5x=-400$$

$$x=\frac{-400}{-5}$$

$$x=\$80 \rightarrow c / \text{cab.}$$

12.  $x \rightarrow N^{\circ} \text{ buscado}$

$$3x-55=233-x$$

$$3x+x=233+55$$

$$4x=288$$

$$x=\frac{288}{4}$$

$$x=72 \rightarrow N^{\circ} \text{ buscado}$$

13.  $x \rightarrow N^{\circ} \text{ menor}$

$$x+1 \rightarrow N^{\circ} \text{ medio}$$

$$x+2 \rightarrow N^{\circ} \text{ mayor}$$

$$2x+3(x+1)+4(x+2)=740$$

$$2x+3x+3+4x+8=740$$

$$9x=740-11$$

$$x=\frac{729}{9}$$

$$x=81$$

$$\Rightarrow 81 \rightarrow N^{\circ} \text{ menor}$$

$$x+1 \Rightarrow 81+1=82 \rightarrow N^{\circ} \text{ medio}$$

$$x+2 \Rightarrow 81+2=83 \rightarrow N^{\circ} \text{ mayor}$$

14.  $x \rightarrow A \text{ caballo}$

$$3x \rightarrow E \text{ nauto}$$

$$x-20 \rightarrow A \text{ pie}$$

$$x+3x+x-20=150$$

$$5x=150+20$$

$$x=\frac{170}{5}$$

$$x=34 \text{ Km} \rightarrow A \text{ caballo}$$

$$3x \Rightarrow 3 \cdot 34=102 \text{ Km} \rightarrow E \text{ nauto}$$

$$x-20 \Rightarrow 34-20=14 \text{ Km} \rightarrow A \text{ pie}$$

15.  $x \rightarrow \text{Parte hijo}$

$$x+2.000 \rightarrow \text{Partehijas}$$

$$3x+2(x+2.000)=16.500$$

$$3x+2x+4.000=16.500$$

$$5x=12.500$$

$$x=\frac{12.500}{5}$$

$$x=2.500$$

$$\Rightarrow 2.500 \text{ colones} \rightarrow \text{Parte hijo}$$

$$x+2.000 \Rightarrow 2.500+2.000=4.500$$

$$\Rightarrow 4.500 \text{ colones} \rightarrow \text{Partehijas}$$

16.  $x \rightarrow N^{\circ} \text{ mayor}$

$$x-1 \rightarrow N^{\circ} \text{ menor}$$

$$x^2-(x-1)^2=31$$

$$x^2-x^2+2x-1=31$$

$$2x=32$$

$$x=16 \rightarrow N^{\circ} \text{ mayor}$$

$$x-1 \Rightarrow 16-1=15 \rightarrow N^{\circ} \text{ menor}$$

17.  $3x \rightarrow \text{Edad A}$

$$x \rightarrow \text{Edad B}$$

$$\frac{x}{5} \rightarrow \text{Edad C}$$

$$x-12 \rightarrow \text{Edad C}$$

$$\text{Luego:}$$

$$\frac{x}{5}=x-12$$

$$x=5(x-12)$$

$$x=5x-60$$

$$-4x=-60$$

$$x=15 \rightarrow \text{Edad B}$$

$$3x \Rightarrow 3 \cdot 15=45 \rightarrow \text{Edad A}$$

$$x-12 \Rightarrow 15-12=3 \rightarrow \text{Edad C}$$

18.  $x \rightarrow \text{Edad act. A}$

$$\frac{x+5}{3} \rightarrow \text{Edad B en 5 años}$$

$$\frac{x+20}{2} \rightarrow \text{Edad B en 20 años}$$

$$\text{Luego:}$$

$$\frac{x+5}{3}+15=\frac{x+20}{2}$$

$$\frac{x+5+45}{3}=\frac{x+20}{2}$$

$$2(x+50)=3(x+20)$$

$$2x+100=3x+60$$

$$-x=-40$$

$$x=40 \rightarrow \text{Edad act. A}$$

$$\text{Edad actual de B:}$$

$$\frac{x+20}{2}-20 \Rightarrow \frac{40+20}{2}-20$$

$$=30-20=10 \text{ años}$$

19.  $x \rightarrow \text{Lunes}$

$$2x \rightarrow \text{Martes}$$

$$4x \rightarrow \text{Miercoles}$$

$$8x \rightarrow \text{Jueves}$$

$$8x-30 \rightarrow \text{Viernes}$$

$$8x-20 \rightarrow \text{Sábado}$$

$$x+2x+4x+8x-30+8x-20=911$$

$$31x-50=911$$

$$31x=911+50$$

$$x=\frac{961}{31}$$

$$x=\$31 \rightarrow \text{Lunes}$$

$$2x \Rightarrow 2 \cdot 31=\$62 \rightarrow \text{Martes}$$

$$4x \Rightarrow 4 \cdot 31=\$124 \rightarrow \text{Miercoles}$$

$$8x \Rightarrow 8 \cdot 31=\$248 \rightarrow \text{Jueves}$$

$$8x-30 \Rightarrow 8 \cdot 31-30=\$218 \rightarrow \text{Viernes}$$

$$8x-20 \Rightarrow 8 \cdot 31-20=\$228 \rightarrow \text{Sábado}$$

20.  $x \rightarrow N^{\circ} 1$

$$x-18 \rightarrow N^{\circ} 2$$

$$x+x-18=3 \cdot 18$$

$$2x=54+18$$

$$x=\frac{72}{2}$$

$$x=36 \rightarrow N^{\circ} 1$$

$$x-18 \Rightarrow 36-18=18 \rightarrow N^{\circ} 2$$

21.  $x \rightarrow \text{Tiene A}$

$$3(x-16) \rightarrow \text{Tiene B}$$

$$x+3(x-16)=36$$

$$x+3x-48=36$$

$$4x=84$$

$$x=\frac{84}{4}$$

$$x=\$21 \rightarrow \text{Tiene A}$$

$$3(x-16) \Rightarrow 3(21-16)$$

$$=3 \cdot 5=\$15 \rightarrow \text{Tiene B}$$

22.  $3x \rightarrow \text{Tiene A}$

$$x \rightarrow \text{Tiene B}$$

$$\frac{x}{2} \rightarrow \text{Tiene C}$$

$$3x-1-(x-3)=2\left(\frac{x}{2}+20\right)$$

$$3x-1-x+3=x+40$$

$$2x-x=40-2$$

$$x=38 \rightarrow \text{Tiene B}$$

$$3x \Rightarrow 3 \cdot 38=\$114 \rightarrow \text{Tiene A}$$

$$\frac{x}{2} \Rightarrow \frac{38}{2}=\$19 \rightarrow \text{Tiene C}$$

23.  $x \rightarrow \text{Costo tienda}$

$$\frac{x}{5} - 800 = \frac{x}{7}$$

$$\frac{x - 4.000}{5} = \frac{x}{7}$$

$$7x - 28.000 = 5x$$

$$2x = 28.000$$

$$x = 14.000$$

$\Rightarrow 14.000 \text{ bs.} \rightarrow \text{Costo tienda}$

24.  $x \rightarrow \text{Cab. peor}$

$$2(x + 15) \rightarrow \text{Cab. mejor}$$

$$x + 2(x + 15) = 120$$

$$x + 2x + 30 = 120$$

$$3x = 90$$

$$x = \$30 \rightarrow \text{Cab. peor}$$

$$2(x + 15) = 2(30 + 15)$$

$$= 2 \cdot 45 = \$90 \rightarrow \text{Cab. mejor}$$

25.  $x \rightarrow \text{queda A}$

$$3x \rightarrow \text{queda B}$$

$$x + 3x = 160$$

$$4x = 160$$

$$x = 40$$

Como A tenía 80 Q.

$\Rightarrow \text{lo que perdió A}$

$= 80 - x \Rightarrow 80 - 40 = 40 \text{ Q.}$

26.  $2x \rightarrow \text{Emp. A}$

$x \rightarrow \text{Emp. B}$

$$2(2x - 400) = x + 400$$

$$4x - 800 = x + 400$$

$$3x = 1.200$$

$$x = \$400 \rightarrow \text{Emp. B}$$

$$2x \Rightarrow 2 \cdot 400 = \$800 \rightarrow \text{Emp. A}$$

27.  $4x \rightarrow \text{Cab.}$

$x \rightarrow \text{Vacas}$

$$4x + 5 = 3(x + 5)$$

$$4x + 5 = 3x + 15$$

$$x = 10 \rightarrow \text{Vacas}$$

$$4x \Rightarrow 4 \cdot 10 = 40 \rightarrow \text{Cab.}$$

28.  $x \rightarrow \text{Lunes}$

$x + 6 \rightarrow \text{Martes}$

$x + 12 \rightarrow \text{Miercoles}$

$x + 18 \rightarrow \text{Jueves}$

$4x \rightarrow \text{Jueves}$

$x + 18 = 4x$

$-3x = -18$

$x = \$6 \rightarrow \text{Lunes}$

$x + 6 \Rightarrow 6 + 6 = \$12 \rightarrow \text{Martes}$

$x + 12 \Rightarrow 6 + 12 = 18 \rightarrow \text{Miercoles}$

$4x \Rightarrow 4 \cdot 6 = \$24 \rightarrow \text{Jueves}$

29.  $x \rightarrow \text{Tenía ppio.}$

$$2x - 50 + 2(2x - 50) - 390 = 0$$

$$2x - 440 + 4x - 100 = 0$$

$$6x = 540$$

$$x = \frac{540}{6}$$

$$x = 90$$

$\Rightarrow 90 \text{ soles} \rightarrow \text{Tenía ppio.}$

30.  $2x \rightarrow \text{Largo}$

$x \rightarrow \text{Ancho}$

$$(2x - 6)(x + 4) = 2x^2$$

$$2x^2 + 2x - 24 = 2x^2$$

$$2x = 24$$

$x = 12 \text{ m} \rightarrow \text{Ancho}$

$2x \Rightarrow 2 \cdot 12 = 24 \text{ m} \rightarrow \text{Largo}$

31.  $3x \rightarrow \text{Padre hace 5 años}$

$x \rightarrow \text{Hijo hace 5 años}$

$$3x + 10 = 2(x + 10)$$

$$3x + 10 = 2x + 20$$

$x = 10 \text{ años}$

*Edad actual Padre:*

$3x + 5 \Rightarrow 3 \cdot 10 + 5 = 35 \text{ años}$

*Edad actual Hijo:*

$x + 5 \Rightarrow 10 + 5 = 15 \text{ años}$

32.  $3x \rightarrow \text{Edad A en 4 años}$

$x \rightarrow \text{Edad B en 4 años}$

$$3x - 6 = 5(x - 6)$$

$$3x - 6 = 5x - 30$$

$$-2x = -24$$

$x = 12 \text{ años}$

*Edad Actual A:*

$3x - 4 \Rightarrow 3 \cdot 12 - 4 = 32 \text{ años}$

*Edad Actual B:*

$x - 4 \Rightarrow 12 - 4 = 8 \text{ años}$

## EJERCICIO 89

1.  $a^2 + ab = a(a + b)$

2.  $b + b^2 = b(1 + b)$

3.  $x^2 + x = x(x + 1)$

4.  $3a^3 - a^2 = a^2(3a - 1)$

5.  $x^3 - 4x^4 = x^3(1 - 4x)$

6.  $5m^2 + 15m^3 = 5m^2(1 + 3m)$

7.  $ab - bc = b(a - c)$

8.  $x^2y + x^2z = x^2(y + z)$

9.  $2a^2x + 6ax^2 = 2ax(a + 3x)$

10.  $8m^2 - 12mn = 4m(2m - 3n)$

11.  $9a^3x^2 - 18ax^3 = 9ax^2(a^2 - 2x)$

12.  $15c^3d^2 + 60c^2d^3 = 15c^2d^2(c + 4d)$

13.  $35m^2n^3 - 70m^3 = 35m^2(n^3 - 2m)$

14.  $abc + abc^2 = abc(1 + c)$

15.  $24x^2y^2 - 36x^2y^4 = 12xy^2(2a^2 - 3xy^2)$

16.  $a^3 + a^2 + a = a(a^2 + a + 1)$

17.  $4x^2 - 8x + 2 = 2(2x^2 - 4x + 1)$

18.  $15y^3 + 20y^2 - 5y = 5y(3y^2 + 4y - 1)$

19.  $a^3 - a^2x + ax^2 = a(a^2 - ax + x^2)$

20.  $2a^2x + 2ax^2 - 3ax$

$= ax(2a + 2x - 3)$

21.  $x^3 + x^5 - x^7 = x^3(1 + x^2 - x^4)$

22.  $14x^2y^2 - 28x^3 + 56x^4$

$= 14x^2(y^2 - 2x + 4x^2)$

23.  $34ax^2 + 51a^2y - 68ay^2$

$= 17a(2x^2 + 3ay - 4y^2)$

24.  $96 - 48mn^2 + 144n^3$

$= 48(2 - mn^2 + 3n^3)$

25.  $a^2b^2c^2 - a^2c^2x^2 + a^2c^2y^2$

$= a^2c^2(b^2 - x^2 + y^2)$



$$26. 55m^2n^3x + 110m^2n^3x^2 - 220m^2y^3 \\ = 55m^2(n^3x + 2n^3x^2 - 4y^3)$$

$$27. 93a^3x^2y - 62a^2x^3y^2 - 124a^2x \\ = 31a^2x(3axy - 2x^2y^2 - 4)$$

$$28. x - x^2 + x^3 - x^4 = x(1 - x + x^2 - x^3)$$

$$29. a^6 - 3a^4 + 8a^3 - 4a^2 = a^2(a^4 - 3a^2 + 8a - 4)$$

$$30. 25x^7 - 10x^5 + 15x^3 - 5x^2 \\ = 5x^2(5x^5 - 2x^3 + 3x - 1)$$

$$31. x^{15} - x^{12} + 2x^9 - 3x^6 = x^6(x^9 - x^6 + 2x^3 - 3)$$

$$32. 9a^2 - 12ab + 15a^3b^2 - 24ab^3 \\ = 3a(3a - 4b + 5a^2b^2 - 8b^3)$$

$$33. 16x^3y^2 - 8x^2y - 24x^4y^2 - 40x^2y^3 \\ = 8x^2y(2xy - 1 - 3x^2y - 5y^2)$$

$$34. 12m^2n + 24m^3n^2 - 36m^4n^3 + 48m^5n^4 \\ = 12m^2n(1 + 2mn - 3m^2n^2 + 4m^3n^3)$$

$$35. 100a^2b^3c - 150ab^2c^2 + 50ab^3c^3 - 200abc^2 \\ = 50abc(2ab^2 - 3bc + b^2c^2 - 4c)$$

$$36. x^5 - x^4 + x^3 - x^2 + x = x(x^4 - x^3 + x^2 - x + 1)$$

$$37. a^2 - 2a^3 + 3a^4 - 4a^5 + 6a^6 \\ = a^2(1 - 2a + 3a^2 - 4a^3 + 6a^4)$$

$$38. 3a^2b + 6ab - 5a^3b^2 + 8a^2bx + 4ab^2m \\ = ab(3a + 6 - 5a^2b + 8ax + 4bm)$$

$$39. a^{20} - a^{16} + a^{12} - a^8 + a^4 - a^2 \\ = a^2(a^{18} - a^{14} + a^{10} - a^6 + a^2 - 1)$$

## EJERCICIO 90

$$1. a(x+1) + b(x+1) = (x+1)(a+b)$$

$$2. x(a+1) - 3(a+1) = (a+1)(x-3)$$

$$3. 2(x-1) + y(x-1) = (x-1)(2+y)$$

$$4. m(a-b) + (a-b)n = (a-b)(m+n)$$

$$5. 2x(n-1) - 3y(n-1) = (n-1)(2x-3y)$$

$$6. a(n+2) + n + 2 = (n+2)(a+1)$$

$$7. x(a+1) - a - 1 = (a+1)(x-1)$$

$$8. a^2 + 1 - b(a^2 + 1) = (a^2 + 1)(1-b)$$

$$9. 3x(x-2) - 2y(x-2) = (x-2)(3x-2y)$$

$$10. 1 - x + 2a(1-x) = (1-x)(1+2a)$$

$$11. 4x(m-n) + n - m = (m-n)(4x-1)$$

$$12. -m - n + x(m+n) = (m+n)(x-1)$$

$$13. a^3(a-b+1) - b^2(a-b+1) = (a-b+1)(a^3-b^2)$$

$$14. 4m(a^2+x-1) + 3n(x-1+a^2) = (a^2+x-1)(4m+3n)$$

$$15. x(2a+b+c) - 2a - b - c \\ = x(2a+b+c) - (2a+b+c) = (2a+b+c)(x-1)$$

$$16. (x+y)(n+1) - 3(n+1) = (n+1)(x+y-3)$$

$$17. (x+1)(x-2) + 3y(x-2) = (x-2)(x+1+3y)$$

$$18. (a+3)(a+1) - 4(a+1) = (a+1)(a+3-4) = (a+1)(a-1)$$

$$19. (x^2+2)(m-n) + 2(m-n) = (m-n)(x^2+4)$$

$$20. a(x-1) - (a+2)(x-1) = (x-1)(a-a-2) = -2(x-1)$$

$$21. 5x(a^2+1) + (x+1)(a^2+1) = (a^2+1)(6x+1)$$

$$22. (a+b)(a-b) - (a-b)(a-b) \\ = (a-b)(a+b-a+b) = 2b(a-b)$$

$$23. (m+n)(a-2) + (m-n)(a-2) = 2m(a-2)$$

$$24. (x+m)(x+1) - (x+1)(x-n) \\ = (x+1)(x+m-x+n) = (x+1)(m+n)$$

$$25. (x-3)(x-4) + (x-3)(x+4) = (x-3)2x$$

$$26. (a+b-1)(a^2+1) - a^2 - 1 = (a^2+1)(a+b-2)$$

$$27. (a+b-c)(x-3) - (b-c-a)(x-3) \\ = (x-3)(a+b-c+a-b+c) = (x-3)2a$$

$$28. 3x(x-1) - 2y(x-1) + z(x-1) = (x-1)(3x-2y+z)$$

$$29. a(n+1) - b(n+1) - n - 1 = (n+1)(a-b-1)$$

$$30. x(a+2) - a - 2 + 3(a+2) = (a+2)(x+2)$$

$$31. (1+3a)(x+1) - 2a(x+1) + 3(x+1) = (x+1)(a+4)$$

$$32. (3x+2)(x+y-z) - (3x+2) - (x+y-1)(3x+2) \\ = (3x+2)(x+y-z-1-x-y+1) = -z(3x+2)$$

## EJERCICIO 91

1.  $a^2 + ab + ax + bx$   
 $= (a^2 + ab) + (ax + bx)$   
 $= a(a + b) + x(a + b)$   
 $= (a + b)(a + x)$
2.  $am - bm + an - bn$   
 $= (am - bm) + (an - bn)$   
 $= m(a - b) + n(a - b)$   
 $= (a - b)(m + n)$
3.  $ax - 2bx - 2ay + 4by$   
 $= (ax - 2bx) - (2ay - 4by)$   
 $= x(a - 2b) - 2y(a - 2b)$   
 $= (a - 2b)(x - 2y)$
4.  $a^2x^2 - 3bx^2 + a^2y^2 - 3by^2$   
 $= (a^2x^2 - 3bx^2) + (a^2y^2 - 3by^2)$   
 $= x^2(a^2 - 3b) + y^2(a^2 - 3b)$   
 $= (a^2 - 3b)(x^2 + y^2)$
5.  $3m - 2n - 2nx^4 + 3mx^4$   
 $= (3m + 3mx^4) - (2n + 2nx^4)$   
 $= 3m(1 + x^4) - 2n(1 + x^4)$   
 $= (1 + x^4)(3m - 2n)$
6.  $x^2 - a^2 + x - a^2x$   
 $= -(a^2 + a^2x) + (x^2 + x)$   
 $= -a^2(1 + x) + x(x + 1)$   
 $= (x + 1)(x - a^2)$
7.  $4a^3 - 1 - a^2 + 4a$   
 $= (4a^3 + 4a) - (1 + a^2)$   
 $= 4a(a^2 + 1) - (1 + a^2)$   
 $= (a^2 + 1)(4a - 1)$
8.  $x + x^2 - xy^2 - y^2$   
 $= (x + x^2) - (xy^2 + y^2)$   
 $= x(1 + x) - y^2(x + 1)$   
 $= (x + 1)(x - y^2)$
9.  $3abx^2 - 2y^2 - 2x^2 + 3aby^2$   
 $= (3abx^2 + 3aby^2) - (2y^2 + 2x^2)$   
 $= 3ab(x^2 + y^2) - 2(x^2 + y^2)$   
 $= (x^2 + y^2)(3ab - 2)$
10.  $3a - b^2 + 2b^2x - 6ax$   
 $= (3a - 6ax) - (b^2 - 2b^2x)$   
 $= 3a(1 - 2x) - b^2(1 - 2x)$   
 $= (1 - 2x)(3a - b^2)$
11.  $4a^3x - 4a^2b + 3bm - 3amx$   
 $= (4a^3x - 4a^2b) + (3bm - 3amx)$   
 $= 4a^2(ax - b) + 3m(b - ax)$   
 $= 4a^2(ax - b) - 3m(ax - b)$   
 $= (ax - b)(4a^2 - 3m)$
12.  $6ax + 3a + 1 + 2x$   
 $= (6ax + 3a) + (1 + 2x)$   
 $= 3a(2x + 1) + (2x + 1)$   
 $= (2x + 1)(3a + 1)$
13.  $3x^3 - 9ax^2 - x + 3a$   
 $= (3x^3 - x) - (9ax^2 - 3a)$   
 $= x(3x^2 - 1) - 3a(3x^2 - 1)$   
 $= (3x^2 - 1)(x - 3a)$
14.  $2a^2x - 5a^2y + 15by - 6bx$   
 $= (2a^2x - 5a^2y) + (15by - 6bx)$   
 $= a^2(2x - 5y) + 3b(5y - 2x)$   
 $= a^2(2x - 5y) - 3b(2x - 5y)$   
 $= (2x - 5y)(a^2 - 3b)$
15.  $2x^2y + 2xz^2 + y^2z^2 + xy^3$   
 $= (2x^2y + xy^3) + (2xz^2 + y^2z^2)$   
 $= xy(2x + y^2) + z^2(2x + y^2)$   
 $= (2x + y^2)(xy + z^2)$
16.  $6m - 9n + 21nx - 14mx$   
 $= (6m - 14mx) - (9n - 21nx)$   
 $= 2m(3 - 7x) - 3n(3 - 7x)$   
 $= (3 - 7x)(2m - 3n)$
17.  $n^2x - 5a^2y^2 - n^2y^2 + 5a^2x$   
 $= (n^2x - n^2y^2) - (5a^2y^2 - 5a^2x)$   
 $= n^2(x - y^2) - 5a^2(y^2 - x)$   
 $= n^2(x - y^2) + 5a^2(x - y^2)$   
 $= (x - y^2)(n^2 + 5a^2)$
18.  $1 + a + 3ab + 3b$   
 $= (1 + a) + (3ab + 3b)$   
 $= (1 + a) + 3b(a + 1)$   
 $= (1 + a)(3b + 1)$
19.  $4am^3 - 12amn - m^2 + 3n$   
 $= (4am^3 - 12amn) - (m^2 - 3n)$   
 $= 4am(m^2 - 3n) - (m^2 - 3n)$   
 $= (m^2 - 3n)(4am - 1)$
20.  $20ax - 5bx - 2by + 8ay$   
 $= (20ax + 8ay) - (5bx + 2by)$   
 $= 4a(5x + 2y) - b(5x + 2y)$   
 $= (5x + 2y)(4a - b)$
21.  $3 - x^2 + 2abx^2 - 6ab$   
 $= (3 - x^2) + (2abx^2 - 6ab)$   
 $= (3 - x^2) + 2ab(x^2 - 3)$   
 $= (3 - x^2) - 2ab(3 - x^2)$   
 $= (3 - x^2)(1 - 2ab)$
22.  $a^3 + a^2 + a + 1$   
 $= (a^3 + a^2) + (a + 1)$   
 $= a^2(a + 1) + (a + 1)$   
 $= (a^2 + 1)(a + 1)$
23.  $3a^2 - 7b^2x + 3ax - 7ab^2$   
 $= (3a^2 + 3ax) - (7b^2x + 7ab^2)$   
 $= 3a(a + x) - 7b^2(x + a)$   
 $= (a + x)(3a - 7b^2)$
24.  $2am - 2an + 2a - m + n - 1$   
 $= (2am - 2an + 2a) - (m - n + 1)$   
 $= 2a(m - n + 1) - (m - n + 1)$   
 $= (2a - 1)(m - n + 1)$

$$25. 3ax - 2by - 2bx - 6a + 3ay + 4b$$

$$\begin{aligned} &= (3ax - 6a + 3ay) - (2by + 2bx - 4b) \\ &= 3a(x - 2 + y) - 2b(y + x - 2) \\ &= (x + y - 2)(3a - 2b) \end{aligned}$$

$$26. a^3 + a + a^2 + 1 + x^2 + a^2 x^2$$

$$\begin{aligned} &= (a^3 + a^2 + a^2 x^2) + (a + 1 + x^2) \\ &= a^2(a + 1 + x^2) + (a + 1 + x^2) \\ &= (a^2 + 1)(a + 1 + x^2) \end{aligned}$$

$$27. 3a^3 - 3a^2b + 9ab^2 - a^2 + ab - 3b^2$$

$$\begin{aligned} &= (3a^3 - 3a^2b + 9ab^2) - (a^2 - ab + 3b^2) \\ &= 3a(a^2 - ab + 3b^2) - (a^2 - ab + 3b^2) \\ &= (3a - 1)(a^2 - ab + 3b^2) \end{aligned}$$

$$28. 2x^3 - nx^2 + 2xz^2 - nz^2 - 3ny^2 + 6xy^2$$

$$\begin{aligned} &= -(nx^2 + nz^2 + 3ny^2) + (2x^3 + 2xz^2 + 6xy^2) \\ &= -n(x^2 + z^2 + 3y^2) + 2x(x^2 + z^2 + 3y^2) \\ &= (2x - n)(x^2 + 3y^2 + z^2) \end{aligned}$$

$$29. 3x^3 + 2axy + 2ay^2 - 3xy^2 - 2ax^2 - 3x^2y$$

$$\begin{aligned} &= (3x^3 - 3xy^2 - 3x^2y) + (2axy + 2ay^2 - 2ax^2) \\ &= 3x(x^2 - y^2 - xy) + 2a(xy + y^2 - x^2) \\ &= 3x(x^2 - y^2 - xy) - 2a(-xy - y^2 + x^2) \\ &= (3x - 2a)(x^2 - xy - y^2) \end{aligned}$$

$$30. a^2b^3 - n^4 + a^2b^3x^2 - n^4x^2 - 3a^2b^3x + 3n^4x$$

$$\begin{aligned} &= (a^2b^3 + a^2b^3x^2 - 3a^2b^3x) - (n^4 + n^4x^2 - 3n^4x) \\ &= a^2b^3(1 + x^2 - 3x) - n^4(1 + x^2 - 3x) \\ &= (a^2b^3 - n^4)(1 + x^2 - 3x) \end{aligned}$$

## EJERCICIO 92

$$1. a^2 - 2ab + b^2 = (a - b)^2$$

$$2. a^2 + 2ab + b^2 = (a + b)^2$$

$$3. x^2 - 2x + 1 = (x - 1)^2$$

$$4. y^4 + 2y^2 + 1 = (y^2 + 1)^2$$

$$5. a^2 - 10a + 25 = (a - 5)^2$$

$$6. 9 - 6x + x^2 = (3 - x)^2$$

$$7. 16 + 40x^2 + 25x^4 = (4 + 5x^2)^2$$

$$8. 1 - 14a + 49a^2 = (1 - 7a)^2$$

$$9. 36 + 12m^2 + m^4 = (6 + m^2)^2$$

$$10. 1 - 2a^3 + a^6 = (1 - a^3)^2$$

$$11. a^8 + 18a^4 + 81 = (a^4 + 9)^2$$

$$12. a^6 - 2a^3b^3 + b^6 = (a^3 - b^3)^2$$

$$13. 4x^2 - 12xy + 9y^2 = (2x - 3y)^2$$

$$14. 9b^2 - 30a^2b + 25a^4 = (3b - 5a^2)^2$$

$$15. 1 + 14x^2y + 49x^4y^2 = (1 + 7x^2y)^2$$

$$16. 1 - 2a^5 + a^{10} = (1 - a^5)^2$$

$$17. 49m^6 - 70am^3n^2 + 25a^2n^4$$

$$= (7m^3 - 5an^2)^2$$

$$18. 100x^{10} - 60a^4x^5y^6 + 9a^8y^{12}$$

$$= (10x^5 - 3a^4y^6)^2$$

$$19. 121 + 198x^6 + 81x^{12} = (11 + 9x^6)^2$$

$$20. a^2 - 24am^2x^2 + 144m^4x^4$$

$$= (a - 12m^2x^2)^2$$

$$21. 16 - 104x^2 + 169x^4 = (4 - 13x^2)^2$$

$$22. 400x^{10} + 40x^5 + 1 = (20x^5 + 1)^2$$

$$23. \frac{a^2}{4} - ab + b^2 = \left(\frac{a}{2} - b\right)^2$$

$$24. 1 + \frac{2b}{3} + \frac{b^2}{9} = \left(1 + \frac{b}{3}\right)^2$$

$$25. a^4 - a^2b^2 + \frac{b^4}{4} = \left(a^2 - \frac{b^2}{2}\right)^2$$

$$26. \frac{1}{25} - \frac{x^2}{3} + \frac{25x^4}{36} = \left(\frac{1}{5} - \frac{5x^2}{6}\right)^2$$

$$27. 16x^6 - 2x^3y^2 + \frac{y^4}{16} = \left(4x^3 - \frac{y^2}{4}\right)^2$$

$$28. \frac{n^2}{9} + 2mn + 9m^2 = \left(\frac{n}{3} + 3m\right)^2$$

$$29. a^2 + 2a(a + b) + (a + b)^2$$

$$= (a + a + b)^2 = (2a + b)^2$$

$$30. 4 - 4(1 - a) + (1 - a)^2$$

$$= (2 - 1 + a)^2 = (1 + a)^2$$

$$31. 4m^2 - 4m(n - m) + (n - m)^2$$

$$= (2m - n + m)^2 = (3m - n)^2$$

$$32. (m - n)^2 + 6(m - n) + 9$$

$$= (m - n + 3)^2$$

$$33. (a + x)^2 - 2(a + x)(x + y) + (x + y)^2$$

$$= (a + x - x - y)^2 = (a - y)^2$$

$$34. (m + n)^2 - 2(a - m)(m + n) + (a - m)^2$$

$$= (m + n - a + m)^2 = (2m + n - a)^2$$

$$35. 4(1 + a)^2 - 4(1 + a)(b - 1) + (b - 1)^2$$

$$= (2 + 2a - b + 1)^2 = (2a - b + 3)^2$$

$$36. 9(x - y)^2 + 12(x - y)(x + y) + 4(x + y)^2$$

$$= (3x - 3y + 2x + 2y)^2 = (5x - y)^2$$

### EJERCICIO 93

1.  $x^2 - y^2 = (x+y)(x-y)$
2.  $a^2 - 1 = (a+1)(a-1)$
3.  $a^2 - 4 = (a+2)(a-2)$
4.  $9 - b^2 = (3+b)(3-b)$
5.  $1 - 4m^2 = (1+2m)(1-2m)$
6.  $16 - n^2 = (4+n)(4-n)$
7.  $a^2 - 25 = (a+5)(a-5)$
8.  $1 - y^2 = (1+y)(1-y)$
9.  $4a^2 - 9 = (2a+3)(2a-3)$
10.  $25 - 36x^4 = (5+6x^2)(5-6x^2)$
11.  $1 - 49a^2b^2 = (1+7ab)(1-7ab)$
12.  $4x^2 - 81y^4 = (2x+9y^2)(2x-9y^2)$
13.  $a^2b^8 - c^2 = (ab^4+c)(ab^4-c)$
14.  $100 - x^2y^6 = (10+xy^3)(10-xy^3)$
15.  $a^{10} - 49b^{12} = (a^5+7b^6)(a^5-7b^6)$
16.  $25x^2y^4 - 121 = (5xy^2+11)(5xy^2-11)$
17.  $100m^2n^4 - 169y^6$   
 $= (10mn^2+13y^3)(10mn^2-13y^3)$
18.  $a^2m^4n^6 - 144 = (am^2n^3+12)(am^2n^3-12)$
19.  $196x^2y^4 - 225z^{12}$   
 $= (14xy^2+15z^6)(14xy^2-15z^6)$
20.  $256a^{12} - 289b^4m^{10}$   
 $= (16a^6+17b^2m^5)(16a^6-17b^2m^5)$
21.  $1 - 9a^2b^4c^6d^8 = (1+3ab^2c^3d^4)(1-3ab^2c^3d^4)$
22.  $361x^{14} - 1 = (19x^7+1)(19x^7-1)$
23.  $\frac{1}{4} - 9a^2 = \left(\frac{1}{2} + 3a\right)\left(\frac{1}{2} - 3a\right)$
24.  $1 - \frac{a^2}{25} = \left(1 + \frac{a}{5}\right)\left(1 - \frac{a}{5}\right)$
25.  $\frac{1}{16} - \frac{4x^2}{49} = \left(\frac{1}{4} + \frac{2x}{7}\right)\left(\frac{1}{4} - \frac{2x}{7}\right)$
26.  $\frac{a^2}{36} - \frac{x^6}{25} = \left(\frac{a}{6} + \frac{x^3}{5}\right)\left(\frac{a}{6} - \frac{x^3}{5}\right)$

27.  $\frac{x^2}{100} - \frac{y^2z^4}{81} = \left(\frac{x}{10} + \frac{yz^2}{9}\right)\left(\frac{x}{10} - \frac{yz^2}{9}\right)$
28.  $\frac{x^6}{49} - \frac{4a^{10}}{121} = \left(\frac{x^3}{7} + \frac{2a^5}{11}\right)\left(\frac{x^3}{7} - \frac{2a^5}{11}\right)$
29.  $100m^2n^4 - \frac{1}{16}x^8 = \left(10mn^2 + \frac{1}{4}x^4\right)\left(10mn^2 - \frac{1}{4}x^4\right)$
30.  $a^{2n} - b^{2n} = (a^n + b^n)(a^n - b^n)$
31.  $4x^{2n} - \frac{1}{9} = \left(2x^n + \frac{1}{3}\right)\left(2x^n - \frac{1}{3}\right)$
32.  $a^{4n} - 225b^4 = (a^{2n} + 15b^2)(a^{2n} - 15b^2)$
33.  $16x^{6m} - \frac{y^{2n}}{49} = \left(4x^{3m} + \frac{y^n}{7}\right)\left(4x^{3m} - \frac{y^n}{7}\right)$
34.  $49a^{10n} - \frac{b^{12x}}{81} = \left(7a^{5n} + \frac{b^{6x}}{9}\right)\left(7a^{5n} - \frac{b^{6x}}{9}\right)$
35.  $a^{2n}b^{4n} - \frac{1}{25} = \left(a^n b^{2n} + \frac{1}{5}\right)\left(a^n b^{2n} - \frac{1}{5}\right)$
36.  $\frac{1}{100} - x^{2n} = \left(\frac{1}{10} + x^n\right)\left(\frac{1}{10} - x^n\right)$

### EJERCICIO 94

1.  $(x+y)^2 - a^2 = (x+y+a)(x+y-a)$
2.  $4 - (a+1)^2 = (2+a+1)(2-a-1) = (3+a)(1-a)$
3.  $9 - (m+n)^2 = (3+m+n)(3-m-n)$
4.  $(m-n)^2 - 16 = (m-n+4)(m-n-4)$
5.  $(x-y)^2 - 4z^2 = (x-y+2z)(x-y-2z)$
6.  $(a+2b)^2 - 1 = (a+2b+1)(a+2b-1)$
7.  $1 - (x-2y)^2 = (1+x-2y)(1-x+2y)$
8.  $(x+2a)^2 - 4x^2$   
 $= (x+2a+2x)(x+2a-2x) = (3x+2a)(2a-x)$
9.  $(a+b)^2 - (c+d)^2 = (a+b+c+d)(a+b-c-d)$
10.  $(a-b)^2 - (c-d)^2 = (a-b+c-d)(a-b-c+d)$

11.  $(x+1)^2 - 16x^2$   
 $= (x+1+4x)(x+1-4x) = (5x+1)(1-3x)$
12.  $64m^2 - (m-2n)^2$   
 $= (8m+m-2n)(8m-m+2n) = (9m-2n)(7m+2n)$
13.  $(a-2b)^2 - (x+y)^2 = (a-2b+x+y)(a-2b-x-y)$
14.  $(2a-c)^2 - (a+c)^2$   
 $= (2a-c+a+c)(2a-c-a-c) = (3a)(a-2c)$
15.  $(x+1)^2 - 4x^2$   
 $= (x+1+2x)(x+1-2x) = (3x+1)(1-x)$
16.  $36x^2 - (a+3x)^2$   
 $= (6x+a+3x)(6x-a-3x) = (9x+a)(3x-a)$
17.  $a^6 - (a-1)^2 = (a^3+a-1)(a^3-a+1)$
18.  $(a-1)^2 - (m-2)^2$   
 $= (a-1+m-2)(a-1-m+2) = (a+m-3)(a-m+1)$
19.  $(2x-3)^2 - (x-5)^2$   
 $= (2x-3+x-5)(2x-3-x+5) = (3x-8)(x+2)$
20.  $1 - (5a+2x)^2 = (1+5a+2x)(1-5a-2x)$
21.  $(7x+y)^2 - 81 = (7x+y+9)(7x+y-9)$
22.  $m^6 - (m^2-1)^2 = (m^3+m^2-1)(m^3-m^2+1)$
23.  $16a^{10} - (2a^2+3)^2 = (4a^5+2a^2+3)(4a^5-2a^2-3)$
24.  $(x-y)^2 - (c+d)^2 = (x-y+c+d)(x-y-c-d)$
25.  $(2a+b-c)^2 - (a+b)^2$   
 $= (2a+b-c+a+b)(2a+b-c-a-b)$   
 $= (3a+2b-c)(a-c)$
26.  $100 - (x-y+z)^2 = (10+x-y+z)(10-x+y-z)$
27.  $x^2 - (y-x)^2 = (x+y-x)(x-y+x) = (y)(2x-y)$
28.  $(2x+3)^2 - (5x-1)^2$   
 $= (2x+3+5x-1)(2x+3-5x+1) = (7x+2)(4-3x)$
29.  $(x-y+z)^2 - (y-z+2x)^2$   
 $= (x-y+z+y-z+2x)(x-y+z-y+z-2x)$   
 $= (3x)(2z-2y-x)$
30.  $(2x+1)^2 - (x+4)^2$   
 $= (2x+1+x+4)(2x+1-x-4) = (3x+5)(x-3)$
31.  $(a+2x+1)^2 - (x+a-1)^2$   
 $= (a+2x+1+x+a-1)(a+2x+1-x-a+1)$   
 $= (2a+3x)(x+2)$
32.  $4(x+a)^2 - 49y^2 = (2x+2a+7y)(2x+2a-7y)$
33.  $25(x-y)^2 - 4(x+y)^2$   
 $= (5x-5y+2x+2y)(5x-5y-2x-2y)$   
 $= (7x-3y)(3x-7y)$
34.  $36(m+n)^2 - 121(m-n)^2$   
 $= (6m+6n+11m-11n)(6m+6n-11m+11n)$   
 $= (17m-5n)(17n-5m)$

## EJERCICIO 95

1.  $a^2 + 2ab + b^2 - x^2$   
 $= (a+b)^2 - x^2$   
 $= (a+b+x)(a+b-x)$
2.  $x^2 - 2xy + y^2 - m^2$   
 $= (x-y)^2 - m^2$   
 $= (x-y+m)(x-y-m)$
3.  $m^2 + 2mn + n^2 - 1$   
 $= (m+n)^2 - 1$   
 $= (m+n+1)(m+n-1)$
4.  $a^2 - 2a + 1 - b^2$   
 $= (a-1)^2 - b^2$   
 $= (a-1+b)(a-1-b)$
5.  $n^2 + 6n + 9 - c^2$   
 $= (n+3)^2 - c^2$   
 $= (n+3+c)(n+3-c)$
6.  $a^2 + x^2 + 2ax - 4$   
 $= (a+x)^2 - 4$   
 $= (a+x+2)(a+x-2)$
7.  $a^2 + 4 - 4a - 9b^2$   
 $= (a-2)^2 - 9b^2$   
 $= (a+3b-2)(a-3b-2)$
8.  $x^2 + 4y^2 - 4xy - 1$   
 $= (x-2y)^2 - 1$   
 $= (x-2y+1)(x-2y-1)$
9.  $a^2 - 6ay + 9y^2 - 4x^2$   
 $= (a-3y)^2 - 4x^2$   
 $= (a-3y+2x)(a-3y-2x)$
10.  $4x^2 + 25y^2 - 36 + 20xy$   
 $= (2x+5y)^2 - 36$   
 $= (2x+5y+6)(2x+5y-6)$
11.  $9x^2 - 1 + 16a^2 - 24ax$   
 $= (3x-4a)^2 - 1$   
 $= (3x-4a+1)(3x-4a-1)$
12.  $1 + 64a^2b^2 - x^4 - 16ab$   
 $= (8ab-1)^2 - x^4$   
 $= (8ab-1+x^2)(8ab-1-x^2)$

13.  $a^2 - b^2 - 2bc - c^2$   
 $= a^2 - (b^2 + 2bc + c^2)$   
 $= a^2 - (b + c)^2$   
 $= (a + b + c)(a - b - c)$
14.  $1 - a^2 + 2ax - x^2$   
 $= 1 - (a^2 - 2ax + x^2)$   
 $= 1 - (a - x)^2$   
 $= (1 + a - x)(1 - a + x)$
15.  $m^2 - x^2 - 2xy - y^2$   
 $= m^2 - (x^2 + 2xy + y^2)$   
 $= m^2 - (x + y)^2$   
 $= (m + x + y)(m - x - y)$
16.  $c^2 - a^2 + 2a - 1$   
 $= c^2 - (a^2 - 2a + 1)$   
 $= c^2 - (a - 1)^2$   
 $= (c + a - 1)(c - a + 1)$
17.  $9 - n^2 - 25 - 10n$   
 $= 9 - (n^2 + 10n + 25)$   
 $= 9 - (n + 5)^2$   
 $= (3 + n + 5)(3 - n - 5)$   
 $= -(n + 8)(n + 2)$
18.  $4a^2 - x^2 + 4x - 4$   
 $= 4a^2 - (x^2 - 4x + 4)$   
 $= 4a^2 - (x - 2)^2$   
 $= (2a + x - 2)(2a - x + 2)$
19.  $1 - a^2 - 9n^2 - 6an$   
 $= 1 - (a^2 + 6an + 9n^2)$   
 $= 1 - (a + 3n)^2$   
 $= (a + 3n + 1)(1 - a - 3n)$
20.  $25 - x^2 - 16y^2 + 8xy$   
 $= 25 - (x^2 - 8xy + 16y^2)$   
 $= 25 - (x - 4y)^2$   
 $= (5 + x - 4y)(5 - x + 4y)$
21.  $9x^2 - a^2 - 4m^2 + 4am$   
 $= 9x^2 - (a^2 - 4am + 4m^2)$   
 $= 9x^2 - (a - 2m)^2$   
 $= (3x + a - 2m)(3x - a + 2m)$
22.  $16x^2y^2 + 12ab - 4a^2 - 9b^2$   
 $= 16x^2y^2 - (4a^2 - 12ab + 9b^2)$   
 $= 16x^2y^2 - (2a - 3b)^2$   
 $= (4xy + 2a - 3b)(4xy - 2a + 3b)$
23.  $-a^2 + 25m^2 - 1 - 2a$   
 $= 25m^2 - (a^2 + 2a + 1)$   
 $= 25m^2 - (a + 1)^2$   
 $= (5m + a + 1)(5m - a - 1)$
24.  $49x^4 - 25x^2 - 9y^2 + 30xy$   
 $= 49x^4 - (25x^2 - 30xy + 9y^2)$   
 $= 49x^4 - (5x - 3y)^2$   
 $= (7x^2 + 5x - 3y)(7x^2 - 5x + 3y)$
25.  $a^2 - 2ab + b^2 - c^2 - 2cd - d^2$   
 $= (a - b)^2 - (c + d)^2$   
 $= (a - b + c + d)(a - b - c - d)$
26.  $x^2 + 2xy + y^2 - m^2 + 2mn - n^2$   
 $= (x + y)^2 - (m - n)^2$   
 $= (x + y + m - n)(x + y - m + n)$
27.  $a^2 + 4b^2 + 4ab - x^2 - 2ax - a^2$   
 $= (a^2 + 4ab + 4b^2) - (x^2 + 2ax + a^2)$   
 $= (a + 2b)^2 - (x + a)^2$   
 $= (a + 2b + x + a)(a + 2b - x - a)$   
 $= (2a + 2b + x)(2b - x)$
28.  $x^2 + 4a^2 - 4ax - y^2 - 9b^2 + 6by$   
 $= (x^2 - 4ax + 4a^2) - (y^2 - 6by + 9b^2)$   
 $= (x - 2a)^2 - (y - 3b)^2$   
 $= (x - 2a + y - 3b)(x - 2a - y + 3b)$
29.  $m^2 - x^2 + 9n^2 + 6mn - 4ax - 4a^2$   
 $= (m^2 + 6mn + 9n^2) - (x^2 + 4ax + 4a^2)$   
 $= (m + 3n)^2 - (x + 2a)^2$   
 $= (m + 3n + x + 2a)(m + 3n - x - 2a)$
30.  $9x^2 + 4y^2 - a^2 - 12xy - 25b^2 - 10ab$   
 $= (9x^2 - 12xy + 4y^2) - (a^2 + 10ab + 25b^2)$   
 $= (3x - 2y)^2 - (a + 5b)^2$   
 $= (3x - 2y + a + 5b)(3x - 2y - a - 5b)$
31.  $2am - x^2 - 9 + a^2 + m^2 - 6x$   
 $= (a^2 + 2am + m^2) - (x^2 + 6x + 9)$   
 $= (a + m)^2 - (x + 3)^2$   
 $= (a + m + x + 3)(a + m - x - 3)$
32.  $x^2 - 9a^4 + 6a^2b + 1 + 2x - b^2$   
 $= (x^2 + 2x + 1) - (b^2 - 6a^2b + 9a^4)$   
 $= (x + 1)^2 - (b - 3a^2)^2$   
 $= (x + 1 + b - 3a^2)(x + 1 - b + 3a^2)$
33.  $16a^2 - 1 - 10m + 9x^2 - 24ax - 25m^2$   
 $= (16a^2 - 24ax + 9x^2) - (25m^2 + 10m + 1)$   
 $= (4a - 3x)^2 - (5m + 1)^2$   
 $= (4a - 3x + 5m + 1)(4a - 3x - 5m - 1)$
34.  $9m^2 - a^2 + 2acd - c^2d^2 + 100 - 60m$   
 $= -(a^2 - 2acd + c^2d^2) + (9m^2 - 60m + 100)$   
 $= (3m - 10)^2 - (a - cd)^2$   
 $= (3m - 10 + a - cd)(3m - 10 - a + cd)$
35.  $4a^2 - 9x^2 + 49b^2 - 30xy - 25y^2 - 28ab$   
 $= (4a^2 - 28ab + 49b^2) - (9x^2 + 30xy + 25y^2)$   
 $= (2a - 7b)^2 - (3x + 5y)^2$   
 $= (2a - 7b + 3x + 5y)(2a - 7b - 3x - 5y)$
36.  $225a^2 - 169b^2 + 1 + 30a + 26bc - c^2$   
 $= (225a^2 + 30a + 1) - (169b^2 - 26bc + c^2)$   
 $= (15a + 1)^2 - (13b - c)^2$   
 $= (15a + 1 + 13b - c)(15a + 1 - 13b + c)$
37.  $x^2 - y^2 + 4 + 4x - 1 - 2y$   
 $= (x^2 + 4x + 4) - (y^2 + 2y + 1)$   
 $= (x + 2)^2 - (y + 1)^2$   
 $= (x + 2 + y + 1)(x + 2 - y - 1)$   
 $= (x + y + 3)(x - y + 1)$
38.  $a^2 - 16 - x^2 + 36 + 12a - 8x$   
 $= (a^2 + 12a + 36) - (x^2 + 8x + 16)$   
 $= (a + 6)^2 - (x + 4)^2$   
 $= (a + 6 + x + 4)(a + 6 - x - 4)$   
 $= (a + x + 10)(a - x + 2)$

## EJERCICIO 96

1.  $a^4 + a^2 + 1$

$$\frac{+a^2 \quad -a^2}{(a^4 + 2a^2 + 1) - a^2}$$
$$= (a^2 + 1)^2 - a^2$$
$$= (a^2 + a + 1)(a^2 - a + 1)$$

2.  $m^4 + m^2n^2 + n^4$

$$\frac{+m^2n^2 \quad -m^2n^2}{m^4 + 2m^2n^2 + n^4 - m^2n^2}$$
$$= (m^2 + n^2)^2 - m^2n^2$$
$$= (m^2 + n^2 + mn)(m^2 + n^2 - mn)$$

3.  $x^8 + 3x^4 + 4$

$$\frac{+x^4 \quad -x^4}{x^8 + 4x^4 + 4 - x^4}$$
$$= (x^4 + 2)^2 - x^4$$
$$= (x^4 + x^2 + 2)(x^4 - x^2 + 2)$$

4.  $a^4 + 2a^2 + 9$

$$\frac{+4a^2 \quad -4a^2}{a^4 + 6a^2 + 9 - 4a^2}$$
$$= (a^2 + 3)^2 - 4a^2$$
$$= (a^2 + 2a + 3)(a^2 - 2a + 3)$$

5.  $a^4 - 3a^2b^2 + b^4$

$$\frac{+a^2b^2 \quad -a^2b^2}{a^4 - 2a^2b^2 + b^4 - a^2b^2}$$
$$= (a^2 - b^2)^2 - a^2b^2$$
$$= (a^2 + ab - b^2)(a^2 - ab - b^2)$$

6.  $x^4 - 6x^2 + 1$

$$\frac{+4x^2 \quad -4x^2}{x^4 - 2x^2 + 1 - 4x^2}$$
$$= (x^2 - 1)^2 - 4x^2$$
$$= (x^2 + 2x - 1)(x^2 - 2x - 1)$$

7.  $4a^4 + 3a^2b^2 + 9b^4$

$$\frac{+9a^2b^2 \quad -9a^2b^2}{4a^4 + 12a^2b^2 + 9b^4 - 9a^2b^2}$$
$$= (2a^2 + 3b^2)^2 - 9a^2b^2$$
$$= (2a^2 + 3ab + 3b^2)(2a^2 - 3ab + 3b^2)$$

8.  $4x^4 - 29x^2 + 25$

$$\frac{+9x^2 \quad -9x^2}{4x^4 - 20x^2 + 25 - 9x^2}$$
$$= (2x^2 - 5)^2 - 9x^2$$
$$= (2x^2 + 3x - 5)(2x^2 - 3x - 5)$$

9.  $x^8 + 4x^4y^4 + 16y^8$

$$\frac{+4x^4y^4 \quad -4x^4y^4}{x^8 + 8x^4y^4 + 16y^8 - 4x^4y^4}$$
$$= (x^4 + 4y^4)^2 - 4x^4y^4$$
$$= (x^4 + 2x^2y^2 + 4y^4)(x^4 - 2x^2y^2 + 4y^4)$$

10.  $16m^4 - 25m^2n^2 + 9n^4$

$$\frac{+m^2n^2 \quad -m^2n^2}{16m^4 - 24m^2n^2 + 9n^4 - m^2n^2}$$
$$= (4m^2 - 3n^2)^2 - m^2n^2$$
$$= (4m^2 + mn - 3n^2)(4m^2 - mn - 3n^2)$$

11.  $25a^4 + 54a^2b^2 + 49b^4$

$$\frac{+16a^2b^2 \quad -16a^2b^2}{25a^4 + 70a^2b^2 + 49b^4 - 16a^2b^2}$$
$$= (5a^2 + 7b^2)^2 - 16a^2b^2$$
$$= (5a^2 + 4ab + 7b^2)(5a^2 - 4ab + 7b^2)$$

12.  $36x^4 - 109x^2y^2 + 49y^4$

$$\frac{+25x^2y^2 \quad -25x^2y^2}{36x^4 - 84x^2y^2 + 49y^4 - 25x^2y^2}$$
$$= (6x^2 - 7y^2)^2 - 25x^2y^2$$
$$= (6x^2 + 5xy - 7y^2)(6x^2 - 5xy - 7y^2)$$

13.  $81m^8 + 2m^4 + 1$

$$\frac{+16m^4 \quad -16m^4}{81m^8 + 18m^4 + 1 - 16m^4}$$
$$= (9m^4 + 1)^2 - 16m^4$$
$$= (9m^4 + 4m^2 + 1)(9m^4 - 4m^2 + 1)$$

14.  $c^4 - 45c^2 + 100$

$$\frac{+25c^2 \quad -25c^2}{c^4 - 20c^2 + 100 - 25c^2}$$
$$= (c^2 - 10)^2 - 25c^2$$
$$= (c^2 + 5c - 10)(c^2 - 5c - 10)$$

15.  $4a^8 - 53a^4b^4 + 49b^8$

$$\frac{+25a^4b^4 \quad -25a^4b^4}{4a^8 - 28a^4b^4 + 49b^8 - 25a^4b^4}$$
$$= (2a^4 - 7b^4)^2 - 25a^4b^4$$
$$= (2a^4 + 5a^2b^2 - 7b^4)(2a^4 - 5a^2b^2 - 7b^4)$$

16.  $49 + 76n^2 + 64n^4$

$$\frac{+36n^2 \quad -36n^2}{49 + 112n^2 + 64n^4 - 36n^2}$$
$$= (7 + 8n^2)^2 - 36n^2$$
$$= (8n^2 + 6n + 7)(8n^2 - 6n + 7)$$

17.  $25x^4 - 139x^2y^2 + 81y^4$

$$\frac{+49x^2y^2 \quad -49x^2y^2}{25x^4 - 90x^2y^2 + 81y^4 - 49x^2y^2}$$
$$= (5x^2 - 9y^2)^2 - 49x^2y^2$$
$$= (5x^2 + 7xy - 9y^2)(5x^2 - 7xy - 9y^2)$$

18.  $49x^8 + 76x^4y^4 + 100y^8$

$$\frac{+64x^4y^4 \quad -64x^4y^4}{49x^8 + 140x^4y^4 + 100y^8 - 64x^4y^4}$$
$$= (7x^4 + 10y^4)^2 - 64x^4y^4$$
$$= (7x^4 + 8x^2y^2 + 10y^4)(7x^4 - 8x^2y^2 + 10y^4)$$

19.  $4 - 108x^2 + 121x^4$

$$\frac{+64x^2 \quad -64x^2}{4 - 44x^2 + 121x^4 - 64x^2}$$
$$= (2 - 11x^2)^2 - 64x^2$$
$$= (2 + 8x - 11x^2)(2 - 8x - 11x^2)$$

20.  $121x^4 - 133x^2y^4 + 36y^8$

$$\frac{+x^2y^4 \quad -x^2y^4}{121x^4 - 132x^2y^4 + 36y^8 - x^2y^4}$$
$$= (11x^2 - 6y^4)^2 - x^2y^4$$
$$= (11x^2 + xy^2 - 6y^4)(11x^2 - xy^2 - 6y^4)$$

$$21. 144 + 23n^6 + 9n^{12}$$

$$\frac{+ 49n^6}{144 + 72n^6 + 9n^{12} - 49n^6} \quad - 49n^6$$

$$144 + 72n^6 + 9n^{12} - 49n^6$$

$$= (12 + 3n^6)^2 - 49n^6$$

$$= (12 + 7n^3 + 3n^6)(12 - 7n^3 + 3n^6)$$

$$22. 16 - 9c^4 + c^8$$

$$\frac{+ c^4}{16 - 8c^4 + c^8 - c^4} \quad - c^4$$

$$16 - 8c^4 + c^8 - c^4$$

$$= (4 - c^4)^2 - c^4$$

$$= (4 + c^2 - c^4)(4 - c^2 - c^4)$$

$$23. 64a^4 - 169a^2b^4 + 81b^8$$

$$\frac{+ 25a^2b^4}{64a^4 - 144a^2b^4 + 81b^8 - 25a^2b^4} \quad - 25a^2b^4$$

$$64a^4 - 144a^2b^4 + 81b^8 - 25a^2b^4$$

$$= (8a^2 - 9b^4)^2 - 25a^2b^4$$

$$= (8a^2 + 5ab - 9b^4)(8a^2 - 5ab - 9b^4)$$

$$24. 225 + 5m^2 + m^4$$

$$\frac{+ 25m^2}{225 + 30m^2 + m^4 - 25m^2} \quad - 25m^2$$

$$225 + 30m^2 + m^4 - 25m^2$$

$$= (15 + m^2)^2 - 25m^2$$

$$= (m^2 + 5m + 15)(m^2 - 5m + 15)$$

$$25. 1 - 126a^2b^4 + 169a^4b^8$$

$$\frac{+ 100a^2b^4}{1 - 26a^2b^4 + 169a^4b^8 - 100a^2b^4} \quad - 100a^2b^4$$

$$1 - 26a^2b^4 + 169a^4b^8 - 100a^2b^4$$

$$= (1 - 13a^2b^4)^2 - 100a^2b^4$$

$$= (1 + 10ab^2 - 13a^2b^4)(1 - 10ab^2 - 13a^2b^4)$$

$$26. x^4y^4 + 21x^2y^2 + 121$$

$$\frac{+ x^2y^2}{x^4y^4 + 22x^2y^2 + 121 - x^2y^2} \quad - x^2y^2$$

$$x^4y^4 + 22x^2y^2 + 121 - x^2y^2$$

$$= (x^2y^2 + 11)^2 - x^2y^2$$

$$= (x^2y^2 + xy + 11)(x^2y^2 - xy + 11)$$

$$27. 49c^8 + 75c^4m^2n^2 + 196m^4n^4$$

$$\frac{+ 121c^4m^2n^2}{49c^8 + 196c^4m^2n^2 + 196m^4n^4 - 121c^4m^2n^2} \quad - 121c^4m^2n^2$$

$$49c^8 + 196c^4m^2n^2 + 196m^4n^4 - 121c^4m^2n^2$$

$$= (7c^4 + 14m^2n^2)^2 - 121c^4m^2n^2$$

$$= (7c^4 + 11c^2mn + 14m^2n^2)(7c^4 - 11c^2mn + 14m^2n^2)$$

$$28. 81a^4b^8 - 292a^2b^4x^8 + 256x^{16}$$

$$\frac{+ 4a^2b^4x^8}{81a^4b^8 - 288a^2b^4x^8 + 256x^{16} - 4a^2b^4x^8} \quad - 4a^2b^4x^8$$

$$81a^4b^8 - 288a^2b^4x^8 + 256x^{16} - 4a^2b^4x^8$$

$$= (9a^2b^4 - 16x^8)^2 - 4a^2b^4x^8$$

$$= (9a^2b^4 + 2ab^2x^4 - 16x^8)(9a^2b^4 - 2ab^2x^4 - 16x^8)$$

## EJERCICIO 97

$$1. x^4 + 64y^4$$

$$\frac{+ 16x^2y^2}{x^4 + 16x^2y^2 + 64y^4 - 16x^2y^2} \quad - 16x^2y^2$$

$$x^4 + 16x^2y^2 + 64y^4 - 16x^2y^2$$

$$= (x^2 + 8y^2)^2 - 16x^2y^2$$

$$= (x^2 + 4xy + 8y^2)(x^2 - 4xy + 8y^2)$$

$$3. a^4 + 324b^4$$

$$\frac{+ 36a^2b^2}{a^4 + 36a^2b^2 + 324b^4 - 36a^2b^2} \quad - 36a^2b^2$$

$$a^4 + 36a^2b^2 + 324b^4 - 36a^2b^2$$

$$= (a^2 + 18b^2)^2 - 36a^2b^2$$

$$= (a^2 + 6ab + 18b^2)(a^2 - 6ab + 18b^2)$$

$$5. 4 + 625x^8$$

$$\frac{+ 100x^4}{4 + 100x^4 + 625x^8 - 100x^4} \quad - 100x^4$$

$$4 + 100x^4 + 625x^8 - 100x^4$$

$$= (2 + 25x^4)^2 - 100x^4$$

$$= (25x^4 + 10x^2 + 2)(25x^4 - 10x^2 + 2)$$

$$2. 4x^8 + y^8$$

$$\frac{+ 4x^4y^4}{4x^8 + 4x^4y^4 + y^8 - 4x^4y^4} \quad - 4x^4y^4$$

$$4x^8 + 4x^4y^4 + y^8 - 4x^4y^4$$

$$= (2x^4 + y^4)^2 - 4x^4y^4$$

$$= (2x^4 + 2x^2y^2 + y^4)(2x^4 - 2x^2y^2 + y^4)$$

$$4. 4m^4 + 81n^4$$

$$\frac{+ 36m^2n^2}{4m^4 + 36m^2n^2 + 81n^4 - 36m^2n^2} \quad - 36m^2n^2$$

$$4m^4 + 36m^2n^2 + 81n^4 - 36m^2n^2$$

$$= (2m^2 + 9n^2)^2 - 36m^2n^2$$

$$= (2m^2 + 6mn + 9n^2)(2m^2 - 6mn + 9n^2)$$

$$6. 64 + a^{12}$$

$$\frac{+ 16a^6}{64 + 16a^6 + a^{12} - 16a^6} \quad - 16a^6$$

$$64 + 16a^6 + a^{12} - 16a^6$$

$$= (8 + a^6)^2 - 16a^6$$

$$= (a^6 + 4a^3 + 8)(a^6 - 4a^3 + 8)$$



$$\begin{array}{r}
 7. \quad 1 \quad + 4n^4 \\
 \hline
 + 4n^2 \quad - 4n^2 \\
 1 + 4n^2 + 4n^4 - 4n^2
 \end{array}$$

$$= (1 + 2n^2)^2 - 4n^2$$

$$= (2n^2 + 2n + 1)(2n^2 - 2n + 1)$$

$$\begin{array}{r}
 8. \quad 64x^8 \quad + y^8 \\
 \hline
 + 16x^4y^4 \quad - 16x^4y^4 \\
 64x^8 + 16x^4y^4 + y^8 - 16x^4y^4
 \end{array}$$

$$= (8x^4 + y^4)^2 - 16x^4y^4$$

$$= (8x^4 + 4x^2y^2 + y^4)(8x^4 - 4x^2y^2 + y^4)$$

$$\begin{array}{r}
 9. \quad 81a^4 \quad + 64b^4 \\
 \hline
 + 144a^2b^2 \quad - 144a^2b^2 \\
 81a^4 + 144a^2b^2 + 64b^4 - 144a^2b^2
 \end{array}$$

$$= (9a^2 + 8b^2)^2 - 144a^2b^2$$

$$= (9a^2 + 12ab + 8b^2)(9a^2 - 12ab + 8b^2)$$

## EJERCICIO 98

$$1. x^2 + 7x + 10 = (x + 5)(x + 2)$$

$$2. x^2 - 5x + 6 = (x - 3)(x - 2)$$

$$3. x^2 + 3x - 10 = (x + 5)(x - 2)$$

$$4. x^2 + x - 2 = (x + 2)(x - 1)$$

$$5. a^2 + 4a + 3 = (a + 3)(a + 1)$$

$$6. m^2 + 5m - 14 = (m + 7)(m - 2)$$

$$7. y^2 - 9y + 20 = (y - 5)(y - 4)$$

$$8. x^2 - x - 6 = (x - 3)(x + 2)$$

$$9. x^2 - 9x + 8 = (x - 8)(x - 1)$$

$$10. c^2 + 5c - 24 = (c + 8)(c - 3)$$

$$11. x^2 - 3x + 2 = (x - 2)(x - 1)$$

$$12. a^2 + 7a + 6 = (a + 6)(a + 1)$$

$$13. y^2 - 4y + 3 = (y - 3)(y - 1)$$

$$14. n^2 - 8n + 12 = (n - 6)(n - 2)$$

$$15. x^2 + 10x + 21 = (x + 7)(x + 3)$$

$$16. a^2 + 7a - 18 = (a + 9)(a - 2)$$

$$17. m^2 - 12m + 11 = (m - 11)(m - 1)$$

$$18. x^2 - 7x - 30 = (x - 10)(x + 3)$$

$$19. n^2 + 6n - 16 = (n + 8)(n - 2)$$

$$20. a^2 - 21a + 20 = (a - 20)(a - 1)$$

$$21. y^2 + y - 30 = (y + 6)(y - 5)$$

$$22. a^2 - 11a + 28 = (a - 7)(a - 4)$$

$$23. n^2 - 6n - 40 = (n - 10)(n + 4)$$

$$24. x^2 - 5x - 36 = (x - 9)(x + 4)$$

$$25. a^2 - 2a - 35 = (a - 7)(a + 5)$$

$$26. x^2 + 14x + 13 = (x + 13)(x + 1)$$

$$27. a^2 - 14a + 33 = (a - 11)(a - 3)$$

$$28. m^2 + 13m - 30 = (m + 15)(m - 2)$$

$$29. c^2 - 13c - 14 = (c - 14)(c + 1)$$

$$30. x^2 + 15x + 56 = (x + 8)(x + 7)$$

$$31. x^2 - 15x + 54 = (x - 9)(x - 6)$$

$$32. a^2 + 7a - 60 = (a + 12)(a - 5)$$

$$33. x^2 - 17x - 60$$

$$\begin{array}{r|l}
 60 & 2 \\
 30 & 2 \quad 2 \cdot 2 \cdot 5 = 20 \\
 15 & 3 \quad 3 \cdot 1 = 3 \\
 5 & 5 \Rightarrow 20 - 3 = 17 \\
 1 & = (x - 20)(x + 3)
 \end{array}$$

$$34. x^2 + 8x - 180$$

$$\begin{array}{r|l}
 180 & 2 \\
 90 & 2 \quad 2 \cdot 3 \cdot 3 = 18 \\
 45 & 3 \quad 2 \cdot 5 = 10 \\
 15 & 3 \Rightarrow 18 - 10 = 8 \\
 5 & 5 = (x + 18)(x - 10) \\
 1 &
 \end{array}$$

$$35. m^2 - 20m - 300$$

$$\begin{array}{r|l}
 300 & 2 \\
 150 & 2 \quad 2 \cdot 3 \cdot 5 = 30 \\
 75 & 3 \quad 2 \cdot 5 = 10 \\
 25 & 5 \Rightarrow 30 - 10 = 20 \\
 5 & 5 = (m - 30)(m + 10) \\
 1 &
 \end{array}$$

$$36. x^2 + x - 132$$

$$\begin{array}{r|l}
 132 & 2 \\
 66 & 2 \quad 2 \cdot 2 \cdot 3 = 12 \\
 33 & 3 \quad 11 \cdot 1 = 11 \\
 11 & 11 \Rightarrow 12 - 11 = 1 \\
 1 & = (x + 12)(x - 11)
 \end{array}$$

$$37. m^2 - 2m - 168$$

$$\begin{array}{r|l}
 168 & 2 \\
 84 & 2 \quad 2 \cdot 2 \cdot 3 = 12 \\
 42 & 2 \quad 2 \cdot 7 = 14 \\
 21 & 3 \Rightarrow 14 - 12 = 2 \\
 7 & 7 = (m - 14)(m + 12) \\
 1 &
 \end{array}$$

$$38. c^2 + 24c + 135$$

$$\begin{array}{r|l}
 135 & 3 \\
 45 & 3 \quad 3 \cdot 3 = 9 \\
 15 & 3 \quad 3 \cdot 5 = 15 \\
 5 & 5 \Rightarrow 15 + 9 = 24 \\
 1 & = (c + 15)(c + 9)
 \end{array}$$

$$39. m^2 - 41m + 400$$

$$\begin{array}{r|l}
 400 & 2 \\
 200 & 2 \quad 2^4 = 16 \\
 100 & 2 \quad 5^2 = 25 \\
 50 & 2 \Rightarrow 25 + 16 = 41 \\
 25 & 5 = (m - 25)(m - 16) \\
 5 & 5 \\
 1 &
 \end{array}$$

$$40. a^2 + a - 380$$

$$\begin{array}{r|l}
 380 & 2 \\
 190 & 2 \quad 2^2 \cdot 5 = 20 \\
 95 & 5 \quad 19 \cdot 1 = 19 \\
 19 & 19 \Rightarrow 20 - 19 = 1 \\
 1 & = (a + 20)(a - 19)
 \end{array}$$

$$41. x^2 + 12x - 364$$

$$\begin{array}{r|l}
 364 & 2 \\
 182 & 2 \quad 2 \cdot 13 = 26 \\
 91 & 7 \quad 2 \cdot 7 = 14 \\
 13 & 13 \Rightarrow 26 - 14 = 12 \\
 1 & = (x + 26)(x - 14)
 \end{array}$$

42.  $a^2 + 42a + 432$

$$\begin{array}{r|l} 432 & 2 \\ 216 & 2 \quad 2^3 \cdot 3 = 24 \\ 108 & 2 \quad 2 \cdot 3^2 = 18 \\ 54 & 2 \Rightarrow 24 + 18 = 42 \\ 27 & 3 = (a+24)(a+18) \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$$

43.  $m^2 - 30m - 675$

$$\begin{array}{r|l} 675 & 3 \\ 225 & 5 \quad 3 \cdot 5 \cdot 3 = 45 \\ 45 & 3 \quad 3 \cdot 5 = 15 \\ 15 & 3 \Rightarrow 45 - 15 = 30 \\ 5 & 5 = (m-45)(m+15) \\ 1 & \end{array}$$

44.  $y^2 + 50y + 336$

$$\begin{array}{r|l} 336 & 3 \\ 112 & 2 \quad 3 \cdot 7 \cdot 2 = 42 \\ 56 & 2 \quad 2^3 = 8 \\ 28 & 2 \Rightarrow 42 + 8 = 50 \\ 14 & 2 = (y+42)(y+8) \\ 7 & 7 \\ 1 & \end{array}$$

45.  $x^2 - 2x - 528$

$$\begin{array}{r|l} 528 & 2 \\ 264 & 2 \quad 2^3 \cdot 3 = 24 \\ 132 & 2 \quad 2 \cdot 11 = 22 \\ 66 & 2 \Rightarrow 24 - 22 = 2 \\ 33 & 3 = (x-24)(x+22) \\ 11 & 11 \\ 1 & \end{array}$$

46.  $n^2 + 43n + 432$

$$\begin{array}{r|l} 432 & 2 \\ 216 & 2 \quad 2^4 = 16 \\ 108 & 2 \quad 3^3 = 27 \\ 54 & 2 \Rightarrow 16 + 27 = 43 \\ 27 & 3 = (n+16)(n+27) \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$$

47.  $c^2 - 4c - 320$

$$\begin{array}{r|l} 320 & 2 \\ 160 & 2 \quad 2^4 = 16 \\ 80 & 2 \quad 2^2 \cdot 5 = 20 \\ 40 & 2 \Rightarrow 20 - 16 = 4 \\ 20 & 2 = (c-20)(c+16) \\ 10 & 2 \\ 5 & 5 \\ 1 & \end{array}$$

48.  $m^2 - 8m - 1.008$

$$\begin{array}{r|l} 1.008 & 2 \\ 504 & 2 \quad 2^2 \cdot 7 = 28 \\ 252 & 2 \quad 2^2 \cdot 3^2 = 36 \\ 126 & 2 \Rightarrow 36 - 28 = 8 \\ 63 & 3 = (m-36)(m+28) \\ 21 & 3 \\ 7 & 7 \\ 1 & \end{array}$$

## EJERCICIO 99

1.  $x^4 + 5x^2 + 4 = (x^2 + 4)(x^2 + 1)$

2.  $x^6 - 6x^3 - 7 = (x^3 - 7)(x^3 + 1)$

3.  $x^8 - 2x^4 - 80 = (x^4 - 10)(x^4 + 8)$

4.  $x^2y^2 + xy - 12 = (xy + 4)(xy - 3)$

5.  $(4x)^2 - 2(4x) - 15 = (4x - 5)(4x + 3)$

6.  $(5x)^2 + 13(5x) + 42 = (5x + 7)(5x + 6)$

7.  $x^2 + 2ax - 15a^2 = (x + 5a)(x - 3a)$

8.  $a^2 - 4ab - 21b^2 = (a - 7b)(a + 3b)$

9.  $(x - y)^2 + 2(x - y) - 24 = (x - y + 6)(x - y - 4)$

10.  $-x^2 + 4x + 5$

$$= -(x^2 - 4x - 5) = -(x - 5)(x + 1) = (5 - x)(x + 1)$$

11.  $x^{10} + x^5 - 20 = (x^5 + 5)(x^5 - 4)$

12.  $m^2 + mn - 56n^2 = (m + 8n)(m - 7n)$

13.  $x^4 + 7ax^2 - 60a^2$

$$= (x^2 + 12a)(x^2 - 5a)$$

14.  $(2x)^2 - 4(2x) + 3$

$$= (2x - 3)(2x - 1)$$

15.  $(m - n)^2 + 5(m - n) - 24$

$$= (m - n + 8)(m - n - 3)$$

16.  $x^8 + x^4 - 240$

$$\begin{array}{r|l} 240 & 2 \\ 120 & 2 \quad 2^4 = 16 \\ 60 & 2 \quad 3 \cdot 5 = 15 \\ 30 & 2 \Rightarrow 16 - 15 = 1 \\ 15 & 3 = (x^4 + 16)(x^4 - 15) \\ 5 & 5 \\ 1 & \end{array}$$

17.  $-y^2 + 2y + 15$

$$= -(y^2 - 2y - 15)$$

$$= -(y - 5)(y + 3) = (5 - y)(y + 3)$$

18.  $a^4b^4 - 2a^2b^2 - 99$

$$\begin{array}{r|l} 99 & 3 \quad 3 \cdot 3 = 9 \\ 33 & 3 \quad 11 \cdot 1 = 11 \\ 11 & 11 \Rightarrow 11 - 9 = 2 \\ 1 & = (a^2b^2 - 11)(a^2b^2 + 9) \end{array}$$

19.  $c^2 + 11cd + 28d^2$

$$= (c + 7d)(c + 4d)$$

20.  $25x^2 - 5(5x) - 84$

$$\begin{array}{r|l} 84 & 2 \\ 42 & 2 \quad 2^2 \cdot 3 = 12 \\ 21 & 3 \quad 7 \cdot 1 = 7 \\ 7 & 7 \Rightarrow 12 - 7 = 5 \\ 1 & = (5x - 12)(5x + 7) \end{array}$$

21.  $a^2 - 21ab + 98b^2$

$$\begin{array}{r|l} 98 & 2 \quad 2 \cdot 7 = 14 \\ 49 & 7 \quad 7 \cdot 1 = 7 \\ 7 & 7 \Rightarrow 14 + 7 = 21 \\ 1 & = (a - 14b)(a - 7b) \end{array}$$

$$22. x^4 y^4 + x^2 y^2 - 132$$

$$\begin{array}{r|l} 132 & 2 \\ 66 & 2 \quad 2 \cdot 2 \cdot 3 = 12 \\ 33 & 3 \quad 11 \cdot 1 = 11 \\ 11 & 11 \Rightarrow 12 - 11 = 1 \\ 1 & = (x^2 y^2 + 12)(x^2 y^2 - 11) \end{array}$$

$$23. -x^4 + 2x^2 + 48$$

$$\begin{aligned} &= -(x^4 - 2x^2 - 48) \\ &= -(x^2 - 8)(x^2 + 6) = (8 - x^2)(x^2 + 6) \end{aligned}$$

$$24. (c+d)^2 - 18(c+d) + 65$$

$$= (c+d-13)(c+d-5)$$

$$25. a^2 + 2axy - 440x^2 y^2$$

$$\begin{array}{r|l} 440 & 2 \\ 220 & 2 \quad 2^2 \cdot 5 = 20 \\ 110 & 2 \quad 11 \cdot 2 = 22 \\ 55 & 5 \Rightarrow 22 - 20 = 2 \\ 11 & 11 = (a + 22xy)(a - 20xy) \\ 1 & \end{array}$$

$$26. m^6 n^6 - 21m^3 n^3 + 104$$

$$\begin{array}{r|l} 104 & 2 \\ 52 & 2 \quad 2^3 = 8 \\ 26 & 2 \quad 13 \cdot 1 = 13 \\ 13 & 13 \Rightarrow 13 + 8 = 21 \\ 1 & = (m^3 n^3 - 13)(m^3 n^3 - 8) \end{array}$$

$$27. -n^2 + 5n + 14$$

$$\begin{aligned} &= -(n^2 - 5n - 14) \\ &= -(n-7)(n+2) = (7-n)(n+2) \end{aligned}$$

$$28. x^6 + x^3 - 930$$

$$\begin{array}{r|l} 930 & 2 \\ 465 & 5 \quad 2 \cdot 5 \cdot 3 = 30 \\ 93 & 3 \quad 31 \cdot 1 = 31 \\ 31 & 31 \Rightarrow 31 - 30 = 1 \\ 1 & = (x^3 + 31)(x^3 - 30) \end{array}$$

$$29. (4x^2)^2 - 8(4x^2) - 105$$

$$\begin{array}{r|l} 105 & 5 \\ 21 & 3 \quad 5 \cdot 3 = 15 \\ 7 & 7 \quad 7 \cdot 1 = 7 \\ 1 & \Rightarrow 15 - 7 = 8 \\ & = (4x^2 - 15)(4x^2 + 7) \end{array}$$

$$30. x^4 + 5abx^2 - 36a^2 b^2$$

$$= (x^2 + 9ab)(x^2 - 4ab)$$

$$31. a^4 - a^2 b^2 - 156b^4$$

$$\begin{array}{r|l} 156 & 2 \\ 78 & 2 \quad 2^2 \cdot 3 = 12 \\ 39 & 3 \quad 13 \cdot 1 = 13 \\ 13 & 13 \Rightarrow 13 - 12 = 1 \\ 1 & = (a^2 - 13b^2)(a^2 + 12b^2) \end{array}$$

$$32. -x^2 + 4ax + 21a^2$$

$$\begin{aligned} &= -(x^2 - 4ax - 21a^2) \\ &= -(x-7a)(x+3a) \\ &= (7a-x)(x+3a) \end{aligned}$$

$$33. x^8 y^8 - 15x^4 y^4 - 100a^2$$

$$= (x^4 y^4 - 20a)(x^4 y^4 + 5a)$$

$$34. (a-1)^2 + 3(a-1) - 108$$

$$\begin{array}{r|l} 108 & 2 \\ 54 & 2 \quad 2^2 \cdot 3 = 12 \\ 27 & 3 \quad 3^2 = 9 \\ 9 & 3 \Rightarrow 12 - 9 = 3 \\ 3 & 3 = (a-1+12)(a-1-9) \\ 1 & = (a+11)(a-10) \end{array}$$

$$35. m^2 + abcm - 56a^2 b^2 c^2$$

$$= (m+8abc)(m-7abc)$$

$$36. (7x^2)^2 + 24(7x^2) + 128$$

$$\begin{array}{r|l} 128 & 4 \\ 32 & 4 \quad 4^2 = 16 \\ 8 & 4 \quad 4 \cdot 2 = 8 \\ 2 & 2 \Rightarrow 16 + 8 = 24 \\ 1 & = (7x^2 + 16)(7x^2 + 8) \end{array}$$

## EJERCICIO 100

$$1. 2x^2 + 3x - 2$$

$$\begin{aligned} &= 4x^2 + 3(2x) - 4 \\ &= \frac{(2x+4)(2x-1)}{2} \\ &= (x+2)(2x-1) \end{aligned}$$

$$2. 3x^2 - 5x - 2$$

$$\begin{aligned} &= 9x^2 - 5(3x) - 6 \\ &= \frac{(3x-6)(3x+1)}{3} \\ &= (x-2)(3x+1) \end{aligned}$$

$$3. 6x^2 + 7x + 2$$

$$\begin{aligned} &= 36x^2 + 7(6x) + 12 \\ &= \frac{(6x+4)(6x+3)}{2 \cdot 3} \\ &= (3x+2)(2x+1) \end{aligned}$$

$$4. 5x^2 + 13x - 6$$

$$\begin{aligned} &= 25x^2 + 13(5x) - 30 \\ &= \frac{(5x+15)(5x-2)}{5} \\ &= (x+3)(5x-2) \end{aligned}$$

$$5. 6x^2 - 5x - 6$$

$$\begin{aligned} &= 36x^2 - 5(6x) - 36 \\ &= \frac{(6x-9)(6x+4)}{3 \cdot 2} \\ &= (2x-3)(3x+2) \end{aligned}$$

$$6. 12x^2 - x - 6$$

$$\begin{aligned} &= 144x^2 - 1(12x) - 72 \\ &= \frac{(12x-9)(12x+8)}{3 \cdot 4} \\ &= (4x-3)(3x+2) \end{aligned}$$

$$7. 4a^2 + 15a + 9$$

$$\begin{aligned} &= 16a^2 + 15(4a) + 36 \\ &= \frac{(4a+12)(4a+3)}{4} \\ &= (a+3)(4a+3) \end{aligned}$$

$$8. 10a^2 + 11a + 3$$

$$\begin{aligned} &= 100a^2 + 11(10a) + 30 \\ &= \frac{(10a+6)(10a+5)}{2 \cdot 5} \\ &= (5a+3)(2a+1) \end{aligned}$$

$$9. 12m^2 - 13m - 35$$

$$\begin{aligned} &= 144m^2 - 13(12m) - 420 \\ &\begin{array}{r|l} 420 & 4 \quad 4 \cdot 7 = 28 \\ 105 & 5 \quad 5 \cdot 3 = 15 \\ 21 & 3 \Rightarrow 28 - 15 = 13 \\ 7 & 7 = \frac{(12m-28)(12m+15)}{4 \cdot 3} \\ 1 & = (3m-7)(4m+5) \end{array} \end{aligned}$$

$$10. 20y^2 + y - 1$$

$$\begin{aligned} &= 400y^2 + 1(20y) - 20 \\ &= \frac{(20y+5)(20y-4)}{5 \cdot 4} \\ &= (4y+1)(5y-1) \end{aligned}$$

$$11. 8a^2 - 14a - 15$$

$$\begin{aligned} &= 64a^2 - 14(8a) - 120 \\ &\begin{array}{r|l} 120 & 2 \\ 60 & 2 \quad 2 \cdot 2 \cdot 5 = 20 \\ 30 & 2 \quad 2 \cdot 3 = 6 \\ 15 & 3 \Rightarrow 20 - 6 = 14 \\ 5 & 5 = \frac{(8a-20)(8a+6)}{4 \cdot 2} \\ 1 & = (2a-5)(4a+3) \end{array} \end{aligned}$$

$$12. 7x^2 - 44x - 35$$

$$= 49x^2 - 44(7x) - 245$$

245	5	
49	7	$7 \cdot 7 = 49$
7	7	$5 \cdot 1 = 5$
1		$\Rightarrow 49 - 5 = 44$

$$= \frac{(7x-49)(7x+5)}{7}$$

$$= (x-7)(7x+5)$$

$$13. 15m^2 + 16m - 15$$

$$= 225m^2 + 16(15m) - 225$$

225	5	
45	5	$5 \cdot 5 = 25$
9	3	$3 \cdot 3 = 9$
3	3	$\Rightarrow 25 - 9 = 16$
1		$= \frac{(15m+25)(15m-9)}{5 \cdot 3}$

$$= (3m+5)(5m-3)$$

$$14. 2a^2 + 5a + 2$$

$$= 4a^2 + 5(2a) + 4$$

$$= \frac{(2a+4)(2a+1)}{2}$$

$$= (a+2)(2a+1)$$

$$15. 12x^2 - 7x - 12$$

$$= 144x^2 - 7(12x) - 144$$

144	2	
72	2	$2^2 = 16$
36	2	$3^2 = 9$
18	2	$\Rightarrow 16 - 9 = 7$
9	3	$= \frac{(12x-16)(12x+9)}{4 \cdot 3}$
3	3	$= (3x-4)(4x+3)$
1		

$$16. 9a^2 + 10a + 1$$

$$= 81a^2 + 10(9a) + 9$$

$$= \frac{(9a+9)(9a+1)}{9}$$

$$= (a+1)(9a+1)$$

$$17. 20n^2 - 9n - 20$$

$$= 400n^2 - 9(20n) - 400$$

400	2	
200	2	$2^2 = 16$
100	2	$5^2 = 25$
50	2	$\Rightarrow 25 - 16 = 9$
25	5	$= \frac{(20n-25)(20n+16)}{5 \cdot 4}$
5	5	$= (4n-5)(5n+4)$
1		

$$18. 21x^2 + 11x - 2$$

$$= 441x^2 + 11(21x) - 42$$

$$= \frac{(21x+14)(21x-3)}{7 \cdot 3}$$

$$= (3x+2)(7x-1)$$

$$19. 15m^2 + m - 6$$

$$= 225m^2 + 15m - 90$$

$$= \frac{(15m+10)(15m-9)}{5 \cdot 3}$$

$$= (3m+2)(5m-3)$$

$$20. 15a^2 - 8a - 12$$

$$= 225a^2 - 8(15a) - 180$$

180	2	
90	2	$2 \cdot 3^2 = 18$
45	5	$5 \cdot 2 = 10$
9	3	$\Rightarrow 18 - 10 = 8$
3	3	$= \frac{(15a-18)(15a+10)}{3 \cdot 5}$
1		$= (5a-6)(3a+2)$

$$21. 9x^2 + 37x + 4$$

$$= 81x^2 + 37(9x) + 36$$

$$= \frac{(9x+36)(9x+1)}{9}$$

$$= (x+4)(9x+1)$$

$$22. 20n^2 + 44n - 15$$

$$= 400n^2 + 44(20n) - 300$$

300	2	
150	2	$2 \cdot 5^2 = 50$
75	5	$2 \cdot 3 = 6$
15	5	$\Rightarrow 50 - 6 = 44$
3	3	$= \frac{(20n+50)(20n-6)}{10 \cdot 2}$
1		$= (2n+5)(10n-3)$

$$23. 14m^2 - 31m - 10$$

$$= 196m^2 - 31(14m) - 140$$

140	2	
70	2	$5 \cdot 7 = 35$
35	5	$2 \cdot 2 = 4$
7	7	$\Rightarrow 35 - 4 = 31$
1		$= \frac{(14m-35)(14m+4)}{7 \cdot 2}$
		$= (2m-5)(7m+2)$

$$24. 2x^2 + 29x + 90$$

$$= 4x^2 + 29(2x) + 180$$

180	2	
90	2	$2^2 \cdot 5 = 20$
45	3	$3^2 = 9$
15	3	$\Rightarrow 20 + 9 = 29$
5	5	$= \frac{(2x+20)(2x+9)}{2}$
1		$= (2x+10)(2x+9)$

$$25. 20a^2 - 7a - 40$$

$$= 400a^2 - 7(20a) - 800$$

800	4	
200	4	$4^2 \cdot 2 = 32$
50	5	$5^2 = 25$
10	5	$\Rightarrow 32 - 25 = 7$
2	2	$= \frac{(20a-32)(20a+25)}{4 \cdot 5}$
1		$= (5a-8)(4a+5)$

$$26. 4n^2 + n - 33$$

$$= 16n^2 + 4n - 132$$

132	2	
66	2	$2^2 \cdot 3 = 12$
33	3	$11 \cdot 1 = 11$
11	11	$\Rightarrow 12 - 11 = 1$
1		$= \frac{(4n+12)(4n-11)}{4}$
		$= (n+3)(4n-11)$

$$27. 30x^2 + 13x - 10$$

$$= 900x^2 + 13(30x) - 300$$

300	2	
150	2	$5^2 = 25$
75	5	$2^2 \cdot 3 = 12$
15	5	$\Rightarrow 25 - 12 = 13$
3	3	$= \frac{(30x+25)(30x-12)}{5 \cdot 6}$
1		$= (6x+5)(5x-2)$

## EJERCICIO 101

1.  $6x^4 + 5x^2 - 6$

$$= (6x^2)^2 + 5(6x^2) - 36$$

$$= \frac{(6x^2 + 9)(6x^2 - 4)}{3 \cdot 2}$$

$$= (2x^2 + 3)(3x^2 - 2)$$

2.  $5x^6 + 4x^3 - 12$

$$= (5x^3)^2 + 4(5x^3) - 60$$

$$= \frac{(5x^3 + 10)(5x^3 - 6)}{5}$$

$$= (x^3 + 2)(5x^3 - 6)$$

3.  $10x^8 + 29x^4 + 10$

$$= (10x^4)^2 + 29(10x^4) + 100$$

$$= \frac{(10x^4 + 25)(10x^4 + 4)}{5 \cdot 2}$$

$$= (2x^4 + 5)(5x^4 + 2)$$

4.  $6a^2x^2 + 5ax - 21$

$$(6ax)^2 + 5(6ax) - 126$$

126	2	
63	3	$2 \cdot 7 = 14$
21	7	$3 \cdot 3 = 9$
3	3	$\Rightarrow 14 - 9 = 5$
1		$= \frac{(6ax + 14)(6ax - 9)}{2 \cdot 3}$

$$= (3ax + 7)(2ax - 3)$$

5.  $20x^2y^2 + 9xy - 20$

$$= (20xy)^2 + 9(20xy) - 400$$

400	2	
200	2	$5^2 = 25$
100	2	$2^4 = 16$
50	2	$\Rightarrow 25 - 16 = 9$
25	5	$= \frac{(20xy + 25)(20xy - 16)}{5 \cdot 4}$
5	5	$= (4xy + 5)(5xy - 4)$
1		

6.  $15x^2 - ax - 2a^2$

$$= (15x)^2 - a(15x) - 30a^2$$

$$= \frac{(15x - 6a)(15x + 5a)}{3 \cdot 5}$$

$$= (5x - 2a)(3x + a)$$

7.  $-10x^2 - 7x + 12$

$$= -(10x^2 + 7x - 12)$$

$$= -[(10x)^2 + 7(10x) - 120]$$

120	2	
60	2	$5 \cdot 3 = 15$
30	2	$2^3 = 8$
15	5	$\Rightarrow 15 - 8 = 7$
3	3	$= \frac{-[(10x + 15)(10x - 8)]}{5 \cdot 2}$
1		$= -(2x + 3)(5x - 4)$

$$= (2x + 3)(4 - 5x)$$

8.  $21x^2 - 29xy - 72y^2$

$$= 441x^2 - 29(21xy) - 1.512y^2$$

1.512	2	
756	2	$2^3 \cdot 7 = 56$
378	2	$3^3 = 27$
189	3	$\Rightarrow 56 - 27 = 29$
63	3	$= \frac{(21x - 56y)(21x + 27y)}{7 \cdot 3}$
21	3	$= (3x - 8y)(7x + 9y)$
7	7	
1		

9.  $6m^2 - 13am - 15a^2$

$$= 36m^2 - 13(6am) - 90a^2$$

$$= \frac{(6m - 18a)(6m + 5a)}{6}$$

$$= (m - 3a)(6m + 5a)$$

10.  $14x^4 - 45x^2 - 14$

$$= (14x^2)^2 - 45(14x^2) - 196$$

196	2	
98	2	$7 \cdot 7 = 49$
49	7	$2 \cdot 2 = 4$
7	7	$\Rightarrow 49 - 4 = 45$
1		$= \frac{(14x^2 - 49)(14x^2 + 4)}{7 \cdot 2}$

$$= (2x^2 - 7)(7x^2 + 2)$$

11.  $30a^2 - 13ab - 3b^2$

$$= 900a^2 - 13(30ab) - 90b^2$$

$$= \frac{(30a - 18b)(30a + 5b)}{6 \cdot 5}$$

$$= (5a - 3b)(6a + b)$$

12.  $7x^6 - 33x^3 - 10$

$$= (7x^3)^2 - 33(7x^3) - 70$$

$$= \frac{(7x^3 - 35)(7x^3 + 2)}{7}$$

$$= (x^3 - 5)(7x^3 + 2)$$

13.  $-3a^2 + 13a + 30$

$$= -(3a^2 - 13a - 30)$$

$$= -(9a^2 - 13(3a) - 90)$$

$$= -\frac{(3a - 18)(3a + 5)}{3}$$

$$= -(a - 6)(3a + 5)$$

$$= (6 - a)(3a + 5)$$

14.  $-6x^8 + 7x^4 + 5$

$$= -(6x^8 - 7x^4 - 5)$$

$$= -((6x^4)^2 - 7(6x^4) - 30)$$

$$= -\frac{(6x^4 - 10)(6x^4 + 3)}{2 \cdot 3}$$

$$= -(3x^4 - 5)(2x^4 + 1)$$

$$= (5 - 3x^4)(2x^4 + 1)$$

15.  $6a^2 - ax - 15x^2$

$$= 36a^2 - 6ax - 90x^2$$

$$= \frac{(6a - 10x)(6a + 9x)}{2 \cdot 3}$$

$$= (3a - 5x)(2a + 3x)$$

16.  $4x^2 + 7mnx - 15m^2n^2$

$$= (4x)^2 + 7mn(4x) - 60m^2n^2$$

$$= \frac{(4x + 12mn)(4x - 5mn)}{4}$$

$$= (x + 3mn)(4x - 5mn)$$

17.  $18a^2 + 17ay - 15y^2$

$$= (18a)^2 + 17y(18a) - 270y^2$$

270	2	
135	5	$3^3 = 27$
27	3	$5 \cdot 2 = 10$
9	3	$\Rightarrow 27 - 10 = 17$
3	3	$= \frac{(18a + 27y)(18a - 10y)}{9 \cdot 2}$
1		$= (2a + 3y)(9a - 5y)$

$$18. -8x^4 + 2x^2 + 15$$

$$= -(8x^4 - 2x^2 - 15)$$

$$= -((8x^2)^2 - 2(8x^2) - 120)$$

$$= -\frac{(8x^2 - 12)(8x^2 + 10)}{4 \cdot 2}$$

$$= -(2x^2 - 3)(4x^2 + 5)$$

$$= (3 - 2x^2)(4x^2 + 5)$$

$$19. -25x^8 + 5x^4 + 6$$

$$= -(25x^8 - 5x^4 - 6)$$

$$= -((25x^4)^2 - 5(25x^4) - 150)$$

$$= -\frac{(25x^4 - 15)(25x^4 + 10)}{5 \cdot 5}$$

$$= -(5x^4 - 3)(5x^4 + 2)$$

$$= (3 - 5x^4)(5x^4 + 2)$$

$$20. 30x^{10} - 91x^5 - 30$$

$$= (30x^5)^2 - 91(30x^5) - 900$$

$$= \frac{(30x^5 - 100)(30x^5 + 9)}{10 \cdot 3}$$

$$= (3x^5 - 10)(10x^5 + 3)$$

$$21. 30m^2 + 17am - 21a^2$$

$$= (30m)^2 + 17a(30m) - 630a^2$$

$$630 \quad 2$$

$$315 \quad 5 \quad 5 \cdot 7 = 35$$

$$63 \quad 3 \quad 2 \cdot 3^2 = 18$$

$$21 \quad 3 \Rightarrow 35 - 18 = 17$$

$$7 \quad 7 \quad = \frac{(30m + 35a)(30m - 18a)}{5 \cdot 6}$$

$$1 \quad = (6m + 7a)(5m - 3a)$$

$$23. -6y^2 + 11xy - 4x^2$$

$$= -(6y^2 - 11xy + 4x^2)$$

$$= -((6y)^2 - 11x(6y) + 24x^2)$$

$$= -\frac{(6y - 8x)(6y - 3x)}{2 \cdot 3}$$

$$= -(3y - 4x)(2y - x)$$

$$= (4x - 3y)(2y - x)$$

$$24. -20a^2 + 27ab - 9b^2$$

$$= -(20a^2 - 27ab + 9b^2)$$

$$= -((20a)^2 - 27b(20a) + 180b^2)$$

$$180 \quad 2$$

$$90 \quad 2 \quad 2^2 \cdot 3 = 12$$

$$45 \quad 5 \quad 5 \cdot 3 = 15$$

$$9 \quad 3 \Rightarrow 15 + 12 = 27$$

$$3 \quad 3 \quad = -\frac{(20a - 15b)(20a - 12b)}{5 \cdot 4}$$

$$1 \quad = -(4a - 3b)(5a - 3b)$$

$$= (4a - 3b)(3b - 5a)$$

## EJERCICIO 102

$$1. a^3 + 3a^2 + 3a + 1 = (a + 1)^3$$

$$2. 27 - 27x + 9x^2 - x^3 = (3 - x)^3$$

$$3. m^3 + 3m^2n + 3mn^2 + n^3 = (m + n)^3$$

$$4. 1 - 3a + 3a^2 - a^3 = (1 - a)^3$$

$$5. 8 + 12a^2 + 6a^4 + a^6 = (2 + a^2)^3$$

$$6. 125x^3 + 75x^2 + 15x + 1 = (5x + 1)^3$$

$$7. 8a^3 - 36a^2b + 54ab^2 - 27b^3 = (2a - 3b)^3$$

$$8. 27m^3 + 108m^2n + 144mn^2 + 64n^3 = (3m + 4n)^3$$

$$9. x^3 - 3x^2 + 3x + 1 = \text{No es cubo perfecto}$$

$$10. 1 + 12a^2b - 6ab - 8a^3b^3 = \text{No es cubo perfecto}$$

$$11. 125a^3 + 150a^2b + 60ab^2 + 8b^3 = (5a + 2b)^3$$

$$12. 8 + 36x + 54x^2 + 27x^3 = (2 + 3x)^3$$

$$13. 8 - 12a^2 - 64a^4 - a^6 = \text{No es cubo perfecto}$$

$$14. a^6 + 3a^4b^3 + 3a^2b^6 + b^9 = (a^2 + b^3)^3$$

$$15. x^9 - 9x^6y^4 + 27x^3y^8 - 27y^{12} = (x^3 - 3y^4)^3$$

$$16. 64x^3 + 240x^2y + 300xy^2 + 125y^3 = (4x + 5y)^3$$

$$17. 216 - 756a^2 + 882a^4 - 343a^6 = (6 - 7a^2)^3$$

$$18. 125x^{12} + 600x^8y^5 + 960x^4y^{10} + 512y^{15} = (5x^4 + 8y^5)^3$$

$$19. a^{18} + 3a^{12} + 3a^6 + 1 = (a^6 + 1)^3$$

$$20. m^3 - 3am^2n + 3a^2mn^2 - a^3n^3 = (m - an)^3$$

$$21. 1 + 18a^2b^3 + 108a^4b^6 + 216a^6b^9 = (1 + 6a^2b^3)^3$$

$$22. 64x^9 - 240x^6y^4 + 300x^3y^8 - 125y^{12} = (4x^3 - 5y^4)^3$$

### EJERCICIO 103

1.  $1+a^3=(1+a)(1-a+a^2)$

2.  $1-a^3=(1-a)(1+a+a^2)$

3.  $x^3+y^3=(x+y)(x^2-xy+y^2)$

4.  $m^3-n^3=(m-n)(m^2+mn+n^2)$

5.  $a^3-1=(a-1)(a^2+a+1)$

6.  $y^3+1=(y+1)(y^2-y+1)$

7.  $y^3-1=(y-1)(y^2+y+1)$

8.  $8x^3-1=(2x-1)(4x^2+2x+1)$

9.  $1-8x^3=(1-2x)(1+2x+4x^2)$

10.  $x^3-27=(x-3)(x^2+3x+9)$

11.  $a^3+27=(a+3)(a^2-3a+9)$

12.  $8x^3+y^3=(2x+y)(4x^2-2xy+y^2)$

13.  $27a^3-b^3=(3a-b)(9a^2+3ab+b^2)$

14.  $64+a^6=(4+a^2)(16-4a^2+a^4)$

15.  $a^3-125=(a-5)(a^2+5a+25)$

16.  $1-216m^3=(1-6m)(1+6m+36m^2)$

17.  $8a^3+27b^6=(2a+3b^2)(4a^2-6ab^2+9b^4)$

18.  $x^6-b^9=(x^2-b^3)(x^4+x^2b^3+b^6)$

19.  $8x^3-27y^3=(2x-3y)(4x^2+6xy+9y^2)$

20.  $1+343n^3=(1+7n)(1-7n+49n^2)$

21.  $64a^3-729=(4a-9)(16a^2+36a+81)$

22.  $a^3b^3-x^6=(ab-x^2)(a^2b^2+abx^2+x^4)$

23.  $512+27a^9=(8+3a^3)(64-24a^3+9a^6)$

24.  $x^6-8y^{12}=(x^2-2y^4)(x^4+2x^2y^4+4y^8)$

25.  $1+729x^6=(1+9x^2)(1-9x^2+81x^4)$

26.  $27m^3+64n^9=(3m+4n^3)(9m^2-12mn^3+16n^6)$

27.  $343x^3+512y^6=(7x+8y^2)(49x^2-56xy^2+64y^4)$

28.  $x^3y^6-216y^9=(xy^2-6y^3)(x^2y^4+6xy^5+36y^6)$

29.  $a^3b^3x^3+1=(abx+1)(a^2b^2x^2-abx+1)$

30.  $x^9+y^9=(x^3+y^3)(x^6-x^3y^3+y^6)$

31.  $1.000x^3-1=(10x-1)(100x^2+10x+1)$

32.  $a^6+125b^{12}=(a^2+5b^4)(a^4-5a^2b^4+25b^8)$

33.  $x^{12}+y^{12}=(x^4+y^4)(x^8-x^4y^4+y^8)$

34.  $1-27a^3b^3=(1-3ab)(1+3ab+9a^2b^2)$

35.  $8x^6+729=(2x^2+9)(4x^4-18x^2+81)$

36.  $a^3+8b^{12}=(a+2b^4)(a^2-2ab^4+4b^8)$

37.  $8x^9-125y^3z^6=(2x^3-5yz^2)(4x^6+10x^3yz^2+25y^2z^4)$

38.  $27m^6+343n^9=(3m^2+7n^3)(9m^4-21m^2n^3+49n^6)$

39.  $216-x^{12}=(6-x^4)(36+6x^4+x^8)$

### EJERCICIO 104

1.  $1+(x+y)^3=(1+x+y)[1-(x+y)+(x+y)^2]$   
 $= (1+x+y)(1-x-y+x^2+2xy+y^2)$

2.  $1-(a+b)^3=[1-(a+b)][1+(a+b)+(a+b)^2]$   
 $= (1-a-b)(1+a+b+a^2+2ab+b^2)$

3.  $27+(m-n)^3=(3+m-n)[9-3(m-n)+(m-n)^2]$   
 $= (3+m-n)(9-3m+3n+m^2-2mn+n^2)$

4.  $(x-y)^3-8=(x-y-2)[(x-y)^2+2(x-y)+4]$   
 $= (x-y-2)(x^2-2xy+y^2+2x-2y+4)$

5.  $(x+2y)^3+1=(x+2y+1)[(x+2y)^2-(x+2y)+1]$   
 $= (x+2y+1)(x^2+4xy+4y^2-x-2y+1)$

6.  $1-(2a-b)^3=[1-(2a-b)][1+(2a-b)+(2a-b)^2]$   
 $= (1-2a+b)(1+2a-b+4a^2-4ab+b^2)$

7.  $a^3+(a+1)^3=(a+a+1)[a^2-a(a+1)+(a+1)^2]$   
 $= (2a+1)(a^2-a^2-a+a^2+2a+1)$   
 $= (2a+1)(a^2+a+1)$

8.  $8a^3-(a-1)^3=[2a-(a-1)][4a^2+2a(a-1)+(a-1)^2]$   
 $= (2a-a+1)(4a^2+2a^2-2a+a^2-2a+1)$   
 $= (a+1)(7a^2-4a+1)$

$$\begin{aligned}
 9. \quad & 27x^3 - (x-y)^3 \\
 &= [3x - (x-y)][9x^2 + 3x(x-y) + (x-y)^2] \\
 &= (3x - x + y)(9x^2 + 3x^2 - 3xy + x^2 - 2xy + y^2) \\
 &= (2x + y)(13x^2 - 5xy + y^2)
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & (2a-b)^3 - 27 \\
 &= (2a-b-3)[(2a-b)^2 + 3(2a-b)+9] \\
 &= (2a-b-3)(4a^2 - 4ab + b^2 + 6a - 3b + 9)
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & x^6 - (x+2)^3 \\
 &= [x^2 - (x+2)][x^4 + x^2(x+2) + (x+2)^2] \\
 &= (x^2 - x - 2)(x^4 + x^3 + 2x^2 + x^2 + 4x + 4) \\
 &= (x^2 - x - 2)(x^4 + x^3 + 3x^2 + 4x + 4)
 \end{aligned}$$

$$\begin{aligned}
 12. \quad & (a+1)^3 + (a-3)^3 \\
 &= (a+1+a-3)[(a+1)^2 - (a-3)(a+1) + (a-3)^2] \\
 &= (2a-2)(a^2 + 2a + 1 - a^2 + 2a + 3 + a^2 - 6a + 9) \\
 &= (2a-2)(a^2 - 2a + 13)
 \end{aligned}$$

$$\begin{aligned}
 13. \quad & (x-1)^3 - (x+2)^3 \\
 &= [x-1-(x+2)][(x-1)^2 + (x-1)(x+2) + (x+2)^2] \\
 &= (x-1-x-2)(x^2 - 2x + 1 + x^2 + x - 2 + x^2 + 4x + 4) \\
 &= -3(3x^2 + 3x + 3) = -9(x^2 + x + 1)
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & (x-y)^3 - (x+y)^3 \\
 &= [(x-y) - (x+y)][(x-y)^2 + (x-y)(x+y) + (x+y)^2] \\
 &= (x-y-x-y)(x^2 - 2xy + y^2 + x^2 - y^2 + x^2 + 2xy + y^2) \\
 &= -2y(3x^2 + y^2)
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & (m-2)^3 + (m-3)^3 \\
 &= [(m-2) + (m-3)][(m-2)^2 - (m-2)(m-3) + (m-3)^2] \\
 &= (m-2+m-3)(m^2 - 4m + 4 - m^2 + 5m - 6 + m^2 - 6m + 9) \\
 &= (2m-5)(m^2 - 5m + 7)
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & (2x-y)^3 + (3x+y)^3 \\
 &= [(2x-y) + (3x+y)][(2x-y)^2 - (2x-y)(3x+y) + (3x+y)^2] \\
 &= (2x-y+3x+y)(4x^2 - 4xy + y^2 - 6x^2 + xy + y^2 + 9x^2 + 6xy + y^2) \\
 &= (5x)(7x^2 + 3xy + 3y^2)
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & 8(a+b)^3 + (a-b)^3 \\
 &= [2(a+b) + (a-b)][4(a+b)^2 - 2(a+b)(a-b) + (a-b)^2] \\
 &= (2a+2b+a-b)(4a^2 + 8ab + 4b^2 - 2a^2 + 2b^2 + a^2 - 2ab + b^2) \\
 &= (3a+b)(3a^2 + 6ab + 7b^2)
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & 64(m+n)^3 - 125 \\
 &= [4(m+n) - 5][16(m+n)^2 + 4(m+n)(5) + 25] \\
 &= (4m+4n-5)(16m^2 + 32mn + 16n^2 + 20m + 20n + 25)
 \end{aligned}$$

## EJERCICIO 105

$$1. \quad a^5 + 1 = (a+1)(a^4 - a^3 + a^2 - a + 1)$$

$$2. \quad a^5 - 1 = (a-1)(a^4 + a^3 + a^2 + a + 1)$$

$$3. \quad 1 - x^5 = (1-x)(1+x+x^2+x^3+x^4)$$

$$\begin{aligned}
 4. \quad & a^7 + b^7 \\
 &= (a+b)(a^6 - a^5b + a^4b^2 - a^3b^3 + a^2b^4 - ab^5 + b^6)
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & m^7 - n^7 \\
 &= (m-n)(m^6 + m^5n + m^4n^2 + m^3n^3 + m^2n^4 + mn^5 + n^6)
 \end{aligned}$$

$$6. \quad a^5 + 243 = (a+3)(a^4 - 3a^3 + 9a^2 - 27a + 81)$$

$$7. \quad 32 - m^5 = (2-m)(16 + 8m + 4m^2 + 2m^3 + m^4)$$

$$8. \quad 1 + 243x^5 = (1+3x)(1-3x+9x^2-27x^3+81x^4)$$

$$\begin{aligned}
 9. \quad & x^7 + 128 \\
 &= (x+2)(x^6 - 2x^5 + 4x^4 - 8x^3 + 16x^2 - 32x + 64)
 \end{aligned}$$

$$10. \quad 243 - 32b^5 = (3-2b)(81 + 54b + 36b^2 + 24b^3 + 16b^4)$$

$$11. \quad a^5 + b^5c^5 = (a+bc)(a^4 - a^3bc + a^2b^2c^2 - ab^3c^3 + b^4c^4)$$

$$\begin{aligned}
 12. \quad & m^7 - a^7x^7 \\
 &= (m-ax)(m^6 + m^5ax + m^4a^2x^2 + m^3a^3x^3 + m^2a^4x^4 + ma^5x^5 + a^6x^6)
 \end{aligned}$$

$$13. \quad 1 + x^7 = (1+x)(1-x+x^2-x^3+x^4-x^5+x^6)$$

$$\begin{aligned}
 14. \quad & x^7 - y^7 \\
 &= (x-y)(x^6 + x^5y + x^4y^2 + x^3y^3 + x^2y^4 + xy^5 + y^6)
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & a^7 + 2.187 \\
 &= (a+3)(a^6 - 3a^5 + 9a^4 - 27a^3 + 81a^2 - 243a + 729)
 \end{aligned}$$

$$16. \quad 1 - 128a^7 = (1-2a)(1+2a+4a^2+8a^3+16a^4+32a^5+64a^6)$$

$$17. \quad x^{10} + 32y^5 = (x^2 + 2y)(x^8 - 2x^6y + 4x^4y^2 - 8x^2y^3 + 16y^4)$$

$$\begin{aligned}
 18. \quad & 1 + 128x^4 \\
 &= (1+2x^2)(1-2x^2+4x^4-8x^6+16x^8-32x^{10}+64x^{12})
 \end{aligned}$$



## EJERCICIO 106

$$1. 5a^2 + a = a(5a + 1)$$

$$2. m^2 + 2mx + x^2 = (m + x)^2$$

$$3. a^2 + a - ab - b = (a^2 + a) - (ab - b) \\ = a(a + 1) - b(a + 1) \\ = (a + 1)(a - b)$$

$$4. x^2 - 36 = x^2 - 6^2 = (x + 6)(x - 6)$$

$$5. 9x^2 - 6xy + y^2 = (3x - y)^2$$

$$6. x^2 - 3x - 4 = (x - 1)(x + 4)$$

$$7. 6x^2 - x - 2 \\ = 36x^2 - 6x - 12 \\ = \frac{(6x - 4)(6x + 3)}{3 \cdot 2} \\ = (3x - 2)(2x + 1)$$

$$8. 1 + x^3 = (1 + x)(1 - x + x^2)$$

$$9. 27a^3 - 1 = (3a - 1)(9a^2 + 3a + 1)$$

$$10. x^5 + m^5 = (x + m)(x^4 - x^3m + x^2m^2 - xm^3 + m^4)$$

$$11. a^3 - 3a^2b + 5ab^2 = a(a^2 - 3ab + 5b^2)$$

$$12. 2xy - 6y + xz - 3z \\ = 2y(x - 3) + z(x - 3) = (x - 3)(2y + z)$$

$$13. 1 - 4b + 4b^2 = (1 - 2b)^2$$

$$14. 4x^4 + 3x^2y^2 + y^4 \\ + x^2y^2 - x^2y^2 \\ 4x^4 + 4x^2y^2 + y^4 - x^2y^2 \\ = (2x^2 + y^2)^2 - x^2y^2 \\ = (2x^2 + xy + y^2)(2x^2 - xy + y^2)$$

$$15. x^8 - 6x^4y^4 + y^8 \\ + 2x^4y^4 - 4x^4y^4 \\ x^8 - 2x^4y^4 + y^8 - 4x^4y^4 \\ = (x^4 - y^4)^2 - 4x^4y^4 \\ = (x^4 + 2x^2y^2 - y^4)(x^4 - 2x^2y^2 - y^4)$$

$$16. a^2 - a - 30 = (a - 6)(a + 5)$$

$$17. 15m^2 + 11m - 14 \\ = 225m^2 + 11(15m) - 210 \\ = \frac{(15m + 21)(15m - 10)}{3 \cdot 5} \\ = (5m + 7)(3m - 2)$$

$$18. a^6 + 1 = (a^2 + 1)(a^4 - a^2 + 1)$$

$$19. 8m^3 - 27y^6 = (2m - 3y^2)(4m^2 + 6my^2 + 9y^4)$$

$$20. 16a^2 - 24ab + 9b^2 = (4a - 3b)^2$$

$$21. 1 + a^7 = (1 + a)(1 - a + a^2 - a^3 + a^4 - a^5 + a^6)$$

$$22. 8a^3 - 12a^2 + 6a - 1 = (2a - 1)^3$$

$$23. 1 - m^2 = (1 + m)(1 - m)$$

$$24. x^4 + 4x^2 - 21 = (x^2 + 7)(x^2 - 3)$$

$$25. 125a^6 + 1 = (5a^2 + 1)(25a^4 - 5a^2 + 1)$$

$$26. a^2 + 2ab + b^2 - m^2 \\ = (a + b)^2 - m^2 = (a + b + m)(a + b - m)$$

$$27. 8a^2b + 16a^3b - 24a^2b^2 = 8a^2b(1 + 2a - 3b)$$

$$28. x^5 - x^4 + x - 1 \\ = (x^5 - x^4) + (x - 1) = x^4(x - 1) + (x - 1) = (x^4 + 1)(x - 1)$$

$$29. 6x^2 + 19x - 20 \\ = 36x^2 + 19(6x) - 120 \\ \begin{array}{r|l} 120 & 2 \\ 60 & 2 \quad 2^3 \cdot 3 = 24 \\ 30 & 2 \quad 5 \cdot 1 = 5 \\ 15 & 3 \Rightarrow 24 - 5 = 19 \\ 5 & 5 \Rightarrow \frac{(6x + 24)(6x - 5)}{6} \\ 1 & = (x + 4)(6x - 5) \end{array}$$

$$30. 25x^4 - 81y^2 = (5x^2 + 9y)(5x^2 - 9y)$$

$$31. 1 - m^3 = (1 - m)(1 + m + m^2)$$

$$32. x^2 - a^2 + 2xy + y^2 + 2ab - b^2 \\ = (x^2 + 2xy + y^2) - (a^2 - 2ab + b^2) \\ = (x + y)^2 - (a - b)^2 \\ = (x + y + a - b)(x + y - a + b)$$

$$33. 21m^5n - 7m^4n^2 + 7m^3n^3 - 7m^2n = 7m^2n(3m^3 - m^2n + mn^2 - 1)$$

$$34. a(x + 1) - b(x + 1) + c(x + 1) = (a - b + c)(x + 1)$$

$$35. 4 + 4(x - y) + (x - y)^2 = (2 + x - y)^2$$

$$36. 1 - a^2b^4 = (1 + ab^2)(1 - ab^2)$$

$$37. b^2 + 12ab + 36a^2 = (b + 6a)^2$$

$$38. x^6 + 4x^3 - 77 = (x^3 + 11)(x^3 - 7)$$

$$\begin{aligned}
 39. \quad & 15x^4 - 17x^2 - 4 \\
 & = (15x^2)^2 - 17(15x^2) - 60 \\
 & = \frac{(15x^2 - 20)(15x^2 + 3)}{5 \cdot 3} \\
 & = (3x^2 - 4)(5x^2 + 1)
 \end{aligned}$$

$$\begin{aligned}
 40. \quad & 1 + (a - 3b)^3 \\
 & = (1 + a - 3b)[1 - (a - 3b) + (a - 3b)^2] \\
 & = (1 + a - 3b)(1 - a + 3b + a^2 - 6ab + 9b^2)
 \end{aligned}$$

$$\begin{aligned}
 41. \quad & x^4 + x^2 + 25 \\
 & \quad + 9x^2 \quad - 9x^2 \\
 & \frac{x^4 + 10x^2 + 25 - 9x^2}{x^4 + 10x^2 + 25 - 9x^2} \\
 & = (x^2 + 5)^2 - 9x^2 \\
 & = (x^2 + 3x + 5)(x^2 - 3x + 5)
 \end{aligned}$$

$$\begin{aligned}
 42. \quad & a^8 - 28a^4 + 36 \\
 & \quad + 16a^4 \quad - 16a^4 \\
 & \frac{a^8 - 12a^4 + 36 - 16a^4}{a^8 - 12a^4 + 36 - 16a^4} \\
 & = (a^4 - 6)^2 - 16a^4 \\
 & = (a^4 + 4a^2 - 6)(a^4 - 4a^2 - 6)
 \end{aligned}$$

$$43. \quad 343 + 8a^3 = (7 + 2a)(49 - 14a + 4a^2)$$

$$44. \quad 12a^2bx - 15a^2by = 3a^2b(4x - 5y)$$

$$45. \quad x^2 + 2xy - 15y^2 = (x + 5y)(x - 3y)$$

$$\begin{aligned}
 46. \quad & 6am - 4an - 2n + 3m \\
 & = (6am + 3m) - (4an + 2n) \\
 & = 3m(2a + 1) - 2n(2a + 1) \\
 & = (3m - 2n)(2a + 1)
 \end{aligned}$$

$$47. \quad 81a^6 - 4b^2c^8 = (9a^3 + 2bc^4)(9a^3 - 2bc^4)$$

$$48. \quad 16 - (2a + b)^2 = (4 + 2a + b)(4 - 2a - b) \quad 68. \quad a^2 - x^2 - a - x = (a^2 - x^2) - (a + x) = (a + x)(a - x) - (a + x) = (a + x)(a - x - 1)$$

$$\begin{aligned}
 49. \quad & 20 - x - x^2 \\
 & = -(x^2 + x - 20)
 \end{aligned}$$

$$= -(x + 5)(x - 4)$$

$$= (x + 5)(4 - x)$$

$$50. \quad n^2 + n - 42 = (n + 7)(n - 6)$$

$$\begin{aligned}
 51. \quad & a^2 - d^2 + n^2 - c^2 - 2an - 2cd \\
 & = (a^2 - 2an + n^2) - (c^2 + 2cd + d^2)
 \end{aligned}$$

$$= (a - n)^2 - (c + d)^2$$

$$= (a - n + c + d)(a - n - c - d)$$

$$52. \quad 1 + 216x^9 = (1 + 6x^3)(1 - 6x^3 + 36x^6)$$

$$53. \quad x^3 - 64 = (x - 4)(x^2 + 4x + 16)$$

$$54. \quad x^3 - 64x^4 = x^3(1 - 64x)$$

$$55. \quad 18ax^5y^3 - 36x^4y^3 - 54x^2y^8 = 18x^2y^3(ax^3 - 2x^2 - 3y^5)$$

$$56. \quad 49a^2b^2 - 14ab + 1 = (7ab - 1)$$

$$57. \quad (x + 1)^2 - 81 = (x + 1 + 9)(x + 1 - 9) = (x + 10)(x - 8)$$

$$58. \quad a^2 - (b + c)^2 = (a + b + c)(a - b - c)$$

$$59. \quad (m + n)^2 - 6(m + n) + 9 = (m + n - 3)^2$$

$$\begin{aligned}
 60. \quad & 7x^2 + 31x - 20 \\
 & = (7x)^2 + 31(7x) - 140
 \end{aligned}$$

140	2	
70	2	7 \cdot 5 = 35
35	5	2^2 = 4
7	7	\Rightarrow 35 - 4 = 31
1		= \frac{(7x + 35)(7x - 4)}{7}
		= (x + 5)(7x - 4)

$$64. \quad 1 - 27b^2 + b^4$$

$$\begin{aligned}
 & \quad + 25b^2 \quad - 25b^2 \\
 & \frac{1 - 2b^2 + b^4 - 25b^2}{1 - 2b^2 + b^4 - 25b^2} \\
 & = (1 - b^2)^2 - 25b^2 \\
 & = (1 + 5b - b^2)(1 - 5b - b^2)
 \end{aligned}$$

$$66. \quad c^4 - 4d^4 = (c^2 + 2d^2)(c^2 - 2d^2)$$

$$67. \quad 15x^4 - 15x^3 + 20x^2 = 5x^2(3x^2 - 3x + 4)$$

$$\begin{aligned}
 61. \quad & 9a^3 + 63a - 45a^2 \\
 & = 9a(a^2 + 7 - 5a)
 \end{aligned}$$

$$\begin{aligned}
 62. \quad & ax + a - x - 1 \\
 & = (ax + a) - (x + 1) \\
 & = a(x + 1) - (x + 1) \\
 & = (a - 1)(x + 1)
 \end{aligned}$$

$$\begin{aligned}
 63. \quad & 81x^4 + 25y^2 - 90x^2y \\
 & \Rightarrow 81x^4 - 90x^2y + 25y^2 \\
 & = (9x^2 - 5y)^2
 \end{aligned}$$

$$\begin{aligned}
 65. \quad & m^4 + m^2n^2 + n^4 \\
 & \quad + m^2n^2 \quad - m^2n^2 \\
 & \frac{m^4 + 2m^2n^2 + n^4 - m^2n^2}{m^4 + 2m^2n^2 + n^4 - m^2n^2} \\
 & = (m^2 + n^2)^2 - m^2n^2 \\
 & = (m^2 + mn + n^2)(m^2 - mn + n^2)
 \end{aligned}$$

$$69. \quad x^4 - 8x^2 - 240$$

240	4	
60	4	4 \cdot 5 = 20
15	3	4 \cdot 3 = 12
5	5	\Rightarrow 20 - 12 = 8
1		= (x^2 - 20)(x^2 + 12)

$$70. \quad 6m^4 + 7m^2 - 20$$

$$\begin{aligned}
 & = (6m^2)^2 + 7(6m^2) - 120 \\
 & \quad 120 \quad 4 \\
 & \quad 30 \quad 3 \quad 4 \cdot 2 = 8 \\
 & \quad 10 \quad 5 \quad 5 \cdot 3 = 15 \\
 & \quad 2 \quad 2 \quad \Rightarrow 15 - 8 = 7 \\
 & \quad 1 \quad = \frac{(6m^2 + 15)(6m^2 - 8)}{3 \cdot 2} \\
 & = (2m^2 + 5)(3m^2 - 4)
 \end{aligned}$$

$$71. 9n^2 + 4a^2 - 12an \Rightarrow 9n^2 - 12an + 4a^2 \\ = (3n - 2a)^2$$

$$72. 2x^2 + 2 = 2(x^2 + 1)$$

$$73. 7a(x + y - 1) - 3b(x + y - 1) = (7a - 3b)(x + y - 1)$$

$$74. x^2 + 3x - 18 = (x + 6)(x - 3)$$

$$75. (a + m)^2 - (b + n)^2 = (a + m + b + n)(a + m - b - n)$$

$$76. x^3 + 6x^2y + 12xy^2 + 8y^3 = (x + 2y)^3$$

$$77. 8a^2 - 22a - 21$$

$$= (8a)^2 - 22(8a) - 168$$

$$168 \quad 4$$

$$42 \quad 2 \quad 3 \cdot 2 = 6$$

$$21 \quad 3 \quad 4 \cdot 7 = 28$$

$$7 \quad 7 \Rightarrow 28 - 6 = 22$$

$$1 \quad = \frac{(8a - 28)(8a + 6)}{4 \cdot 2}$$

$$= (2a - 7)(4a + 3)$$

$$78. 1 + 18ab + 81a^2b^2 = (1 + 9ab)^2$$

$$79. 4a^6 - 1 = (2a^3 + 1)(2a^3 - 1)$$

$$80. x^6 - 4x^3 - 480$$

$$480 \quad 4$$

$$120 \quad 3 \quad 5 \cdot 4 = 20$$

$$40 \quad 4 \quad 4 \cdot 3 \cdot 2 = 24$$

$$10 \quad 5 \Rightarrow 24 - 20 = 4$$

$$2 \quad 2 = (x^3 - 24)(x^3 + 20)$$

$$1$$

$$81. ax - bx + b - a - by + ay$$

$$= (ax + ay - a) - (bx + by - b)$$

$$= a(x + y - 1) - b(x + y - 1)$$

$$= (a - b)(x + y - 1)$$

$$82. 6am - 3m - 2a + 1$$

$$= (6am - 3m) - (2a - 1)$$

$$= 3m(2a - 1) - (2a - 1) = (3m - 1)(2a - 1)$$

$$83. 15 + 14x - 8x^2$$

$$= -(8x^2 - 14x - 15)$$

$$= -[(8x)^2 - 14(8x) - 120]$$

$$120 \quad 4$$

$$30 \quad 3 \quad 4 \cdot 5 = 20$$

$$10 \quad 5 \quad 3 \cdot 2 = 6$$

$$2 \quad 2 \Rightarrow 20 - 6 = 14$$

$$1 \quad = -\frac{(8x - 20)(8x + 6)}{4 \cdot 2}$$

$$= -(2x - 5)(4x + 3)$$

$$= (5 - 2x)(4x + 3)$$

$$84. a^{10} - a^8 + a^6 + a^4 = a^4(a^6 - a^4 + a^2 + 1)$$

$$85. 2x(a - 1) - a + 1 = 2x(a - 1) - (a - 1) = (a - 1)(2x - 1)$$

$$86. (m + n)(m - n) + 3n(m - n) = (m - n)(m + n + 3n) = (m - n)(m + 4n)$$

$$87. a^2 - b^3 + 2b^3x^2 - 2a^2x^2$$

$$= -(b^3 - 2b^3x^2) + (a^2 - 2a^2x^2)$$

$$= -b^3(1 - 2x^2) + a^2(1 - 2x^2) = (1 - 2x^2)(a^2 - b^3)$$

$$88. 2am - 3b - c - cm - 3bm + 2a$$

$$= (2am - cm - 3bm) - (3b + c - 2a)$$

$$= m(2a - c - 3b) - (3b + c - 2a)$$

$$= m(2a - 3b - c) + (2a - 3b - c)$$

$$= (2a - 3b - c)(m + 1)$$

$$89. x^2 - \frac{2}{3}x + \frac{1}{9} = \left(x - \frac{1}{3}\right)^2$$

$$90. 4a^{2n} - b^{4n} \\ = (2a^n + b^{2n})(2a^n - b^{2n})$$

$$91. 81x^2 - (a + x)^2 = (9x + a + x)(9x - a - x) = (10x + a)(8x - a)$$

$$92. a^2 + 9 - 6a - 16x^2$$

$$= (a^2 - 6a + 9) - 16x^2 = (a - 3)^2 - 16x^2 = (a - 3 + 4x)(a - 3 - 4x)$$

$$93. 9a^2 - x^2 - 4 + 4x$$

$$= 9a^2 - (x^2 - 4x + 4) = 9a^2 - (x - 2)^2 = (3a + x - 2)(3a - x + 2)$$

$$94. 9x^2 - y^2 + 3x - y$$

$$= (9x^2 - y^2) + (3x - y) = (3x + y)(3x - y) + (3x - y) = (3x - y)(3x + y + 1)$$

$$95. x^2 - x - 72 = (x - 9)(x + 8)$$

$$96. 36a^4 - 120a^2b^2 + 49b^4$$

$$+ 36a^2b^2 \quad - 36a^2b^2$$

$$36a^4 - 84a^2b^2 + 49b^4 - 36a^2b^2$$

$$= (6a^2 - 7b^2)^2 - 36a^2b^2$$

$$= (6a^2 + 6ab - 7b^2)(6a^2 - 6ab - 7b^2)$$

$$100. 49x^2 - 77x + 30$$

$$= (49x)^2 - 77(49x) + 1.470$$

$$1.470 \quad 7$$

$$210 \quad 7$$

$$30 \quad 5$$

$$6 \quad 6 \quad 7 \cdot 5 = 35$$

$$1 \quad 7 \cdot 6 = 42$$

$$\Rightarrow 42 + 35 = 77$$

$$97. a^2 - m^2 - 9n^2 - 6mn + 4ab + 4b^2$$

$$= (a^2 + 4ab + 4b^2) - (m^2 + 6mn + 9n^2) = (a + 2b)^2 - (m + 3n)^2$$

$$= (a + 2b + m + 3n)(a + 2b - m - 3n)$$

$$= \frac{(49x - 35)(49x - 42)}{7 \cdot 7}$$

$$= (7x - 5)(7x - 6)$$

$$98. 1 - \frac{4}{9}a^8 = \left(1 + \frac{2}{3}a^4\right)\left(1 - \frac{2}{3}a^4\right)$$

$$99. 81a^8 + 64b^{12}$$

$$+ 144a^4b^6 \quad - 144a^4b^6$$

$$81a^8 + 144a^4b^6 + 64b^{12} - 144a^4b^6$$

$$= (9a^4 + 8b^6)^2 - 144a^4b^6$$

$$= (9a^4 - 12a^2b^3 + 8b^6)(9a^4 + 12a^2b^3 + 8b^6)$$

$$101. x^2 - 2abx - 35a^2b^2 \\ + 36a^2b^2 - 36a^2b^2$$

$$x^2 - 2abx + a^2b^2 - 36a^2b^2$$

$$= (x - ab)^2 - 36a^2b^2$$

$$= (x - ab + 6ab)(x - ab - 6ab)$$

$$= (x + 5ab)(x - 7ab)$$

$$102. 125x^3 - 225x^2 + 135x - 27 = (5x - 3)^3$$

$$103. (a-2)^2 - (a+3)^2 \\ = (a-2+a+3)(a-2-a-3) = (2a+1)(-5)$$

$$104. 4a^2m + 12a^2n - 5bm - 15bn \\ = (4a^2m + 12a^2n) - (5bm + 15bn) \\ = 4a^2(m+3n) - 5b(m+3n) = (m+3n)(4a^2 - 5b)$$

$$105. 1 + 6x^3 + 9x^6 = (1 + 3x^3)^2$$

$$106. a^4 + 3a^2b - 40b^2 = (a^2 + 8b)(a^2 - 5b)$$

$$107. m^3 + 8a^3x^3 = (m + 2ax)(m^2 - 2axm + 4a^2x^2)$$

$$108. 1 - 9x^2 + 24xy - 16y^2 \\ = -[(9x^2 - 24xy + 16y^2) - 1] \\ = -[(3x - 4y)^2 - 1] \\ = -[(3x - 4y + 1)(3x - 4y - 1)] \\ = (3x - 4y + 1)(1 + 4y - 3x)$$

$$109. 1 + 11x + 24x^2 \\ = (24x)^2 + 11(24x) + 24 \\ = \frac{(24x+8)(24x+3)}{8 \cdot 3} \\ = (3x+1)(8x+1)$$

$$110. 9x^2y^3 - 27x^3y^3 - 9x^5y^3 = 9x^2y^3(1 - 3x - x^3)$$

$$111. (a^2 + b^2 - c^2)^2 - 9x^2y^2 \\ = (a^2 + b^2 - c^2 + 3xy)(a^2 + b^2 - c^2 - 3xy)$$

$$112. 8(a+1)^3 - 1 \\ = [2(a+1) - 1][4(a+1)^2 + 2(a+1) + 1] \\ = (2a+2-1)(4a^2+8a+4+2a+2+1) \\ = (2a+1)(4a^2+10a+7)$$

$$113. 100x^4y^6 - 121m^4 \\ = (10x^2y^3 + 11m^2)(10x^2y^3 - 11m^2)$$

$$114. (a^2 + 1)^2 + 5(a^2 + 1) - 24 \\ = (a^2 + 1 + 8)(a^2 + 1 - 3) = (a^2 + 9)(a^2 - 2)$$

$$115. 1 + 1.000x^6 = (1 + 10x^2)(1 - 10x^2 + 100x^4)$$

$$116. 49a^2 - x^2 - 9y^2 + 6xy \\ = -[(x^2 - 6xy + 9y^2) - 49a^2] \\ = -[(x - 3y)^2 - 49a^2] \\ = -(7a + x - 3y)(-7a + x - 3y) \\ = (7a + x - 3y)(7a - x + 3y)$$

$$117. x^4 - y^2 + 4x^2 + 4 - 4yz - 4z^2 \\ = (x^4 + 4x^2 + 4) - (y^2 + 4yz + 4z^2) \\ = (x^2 + 2)^2 - (y + 2z)^2 \\ = (x^2 + 2 + y + 2z)(x^2 + 2 - y - 2z)$$

$$118. a^3 - 64 = (a - 4)(a^2 + 4a + 16)$$

$$119. a^5 + x^5 = (a + x)(a^4 - a^3x + a^2x^2 - ax^3 + x^4)$$

$$120. a^6 - 3a^3b - 54b^2 = (a^3 - 9b)(a^3 + 6b)$$

$$121. 165 + 4x - x^2 \\ = -(x^2 - 4x - 165) \\ \begin{array}{r|l} 165 & 5 \\ 33 & 3 \quad 5 \cdot 3 = 15 \\ 11 & 11 \quad 11 \cdot 1 = 11 \\ 1 & \Rightarrow 15 - 11 = 4 \\ & = -(x - 15)(x + 11) \\ & = (15 - x)(x + 11) \end{array}$$

$$122. a^4 + a^2 + 1 \\ \frac{+a^2 \quad -a^2}{a^4 + 2a^2 + 1 - a^2} \\ = (a^2 + 1)^2 - a^2 \\ = (a^2 + a + 1)(a^2 - a + 1)$$

$$123. \frac{x^2}{4} - \frac{y^6}{81} = \left(\frac{x}{2} + \frac{y^3}{9}\right)\left(\frac{x}{2} - \frac{y^3}{9}\right)$$

$$124. 16x^2 + \frac{8xy}{5} + \frac{y^2}{25} = \left(4x + \frac{y}{5}\right)^2$$

$$125. a^4b^4 + 4a^2b^2 - 96 \\ \frac{+100-100}{a^4b^4 + 4a^2b^2 + 4 - 100} \\ = (a^2b^2 + 2)^2 - 100 \\ = (a^2b^2 + 2 + 10)(a^2b^2 + 2 - 10) \\ = (a^2b^2 + 12)(a^2b^2 - 8)$$

$$126. 8a^2x + 7y + 21by - 7ay - 8a^3x + 24a^2bx$$

$$= (7y + 21by - 7ay) + (8a^2x - 8a^3x + 24a^2bx)$$

$$= 7y(1 + 3b - a) + 8a^2x(1 - a + 3b)$$

$$= (1 + 3b - a)(7y + 8a^2x)$$

$$127. x^4 + 11x^2 - 390$$

$$\begin{array}{r|l} 390 & 3 \\ 130 & 2 \quad 13 \cdot 2 = 26 \\ 65 & 5 \quad 5 \cdot 3 = 15 \\ 13 & 13 \Rightarrow 26 - 15 = 11 \\ 1 & = (x^2 + 26)(x^2 - 15) \end{array}$$

$$128. 7 + 33m - 10m^2$$

$$= -(10m^2 - 33m - 7)$$

$$= -[(10m)^2 - 33(10m) - 70]$$

$$= -\frac{(10m - 35)(10m + 2)}{5 \cdot 2}$$

$$= -(2m - 7)(5m + 1)$$

$$= (7 - 2m)(5m + 1)$$

$$129. 4(a+b)^2 - 9(c+d)^2$$

$$= [2(a+b) + 3(c+d)][2(a+b) - 3(c+d)]$$

$$= (2a + 2b + 3c + 3d)(2a + 2b - 3c - 3d)$$

$$130. 729 - 125x^3y^{12} = (9 - 5xy^4)(81 + 45xy^4 + 25x^2y^8)$$

$$131. (x+y)^2 + x+y = (x+y)^2 + (x+y) = (x+y)(x+y+1)$$

$$132. 4 - (a^2 + b^2) + 2ab$$

$$\begin{array}{r} 4 - a^2 - b^2 + 2ab \\ + b^2 \quad - b^2 \\ \hline 4 - a^2 \quad + 2ab - b^2 \end{array}$$

$$= 4 - (a^2 - 2ab + b^2)$$

$$= 4 - (a - b)^2$$

$$= (2 + a - b)(2 - a + b)$$

$$133. x^3 - y^3 + x - y$$

$$= (x^3 - y^3) + (x - y)$$

$$= (x - y)(x^2 + xy + y^2) + (x - y) = (x - y)(x^2 + xy + y^2 + 1)$$

$$134. a^2 - b^2 + a^3 - b^3$$

$$= (a^2 - b^2) + (a^3 - b^3)$$

$$= (a + b)(a - b) + (a - b)(a^2 + ab + b^2)$$

$$= (a - b)(a + b + a^2 + ab + b^2)$$

## EJERCICIO 107

$$1. 3ax^2 - 3a$$

$$= 3a(x^2 - 1) = 3a(x + 1)(x - 1)$$

$$2. 3x^2 - 3x - 6$$

$$= 3(x^2 - x - 2) = 3(x - 2)(x + 1)$$

$$3. 2a^2x - 4abx + 2b^2x$$

$$= 2x(a^2 - 2ab + b^2)$$

$$= 2x(a - b)^2 = 2x(a - b)(a - b)$$

$$4. 2a^3 - 2$$

$$= 2(a^3 - 1) = 2(a - 1)(a^2 + a + 1)$$

$$5. a^3 - 3a^2 - 28a$$

$$= a(a^2 - 3a - 28) = a(a - 7)(a + 4)$$

$$6. x^3 - 4x + x^2 - 4$$

$$= (x^3 + x^2) - (4x + 4)$$

$$= x^2(x + 1) - 4(x + 1)$$

$$= (x^2 - 4)(x + 1) = (x + 2)(x - 2)(x + 1)$$

$$7. 3ax^3 + 3ay^3$$

$$= 3a(x^3 + y^3)$$

$$= 3a(x + y)(x^2 - xy + y^2)$$

$$8. 4ab^2 - 4abn + an^2$$

$$= a(4b^2 - 4bn + n^2) = a(2b - n)^2$$

$$9. x^4 - 3x^2 - 4$$

$$= (x^2 - 4)(x^2 + 1)$$

$$= (x + 2)(x - 2)(x^2 + 1)$$

$$10. a^3 - a^2 - a + 1$$

$$= (a^3 - a^2) - (a - 1)$$

$$= a^2(a - 1) - (a - 1)$$

$$= (a - 1)(a^2 - 1)$$

$$= (a - 1)(a + 1)(a - 1)$$

$$11. 2ax^2 - 4ax + 2a$$

$$= 2a(x^2 - 2x + 1) = 2a(x - 1)^2$$

$$12. x^3 - x + x^2y - y$$

$$= (x^3 - x) + (x^2y - y)$$

$$= x(x^2 - 1) + y(x^2 - 1)$$

$$= (x^2 - 1)(x + y) = (x + 1)(x - 1)(x + y)$$

$$13. 2a^3 + 6a^2 - 8a$$

$$= 2a(a^2 + 3a - 4) = 2a(a + 4)(a - 1)$$

$$14. 16x^3 - 48x^2y + 36xy^2$$

$$= 4x(4x^2 - 12xy + 9y^2) = 4x(2x - 3y)^2$$

$$15. 3x^3 - x^2y - 3xy^2 + y^3$$

$$= (3x^3 - x^2y) - (3xy^2 - y^3)$$

$$= x^2(3x - y) - y^2(3x - y)$$

$$= (3x - y)(x^2 - y^2) = (3x - y)(x + y)(x - y)$$

$$16. 5a^4 + 5a$$

$$= 5a(a^3 + 1) = 5a(a + 1)(a^2 - a + 1)$$

$$17. 6ax^2 - ax - 2a$$

$$= a(6x^2 - x - 2)$$

$$= a(36x^2 - 6x - 12)$$

$$= \frac{a(6x-4)(6x+3)}{2 \cdot 3}$$

$$= a(3x-2)(2x+1)$$

$$18. n^4 - 81$$

$$= (n^2 - 9)(n^2 + 9) = (n+3)(n-3)(n^2 + 9)$$

$$19. 8ax^2 - 2a$$

$$= 2a(4x^2 - 1) = 2a(2x-1)(2x+1)$$

$$20. ax^3 + 10ax^2 + 25ax$$

$$= ax(x^2 + 10x + 25) = ax(x+5)(x+5)$$

$$21. x^3 - 6x^2 - 7x$$

$$= x(x^2 - 6x - 7)$$

$$= x(x^2 - 6x - 7 + 16 - 16)$$

$$= x(x^2 - 6x + 9 - 16)$$

$$= x[(x-3)^2 - 4^2]$$

$$= x(x-3+4)(x-3-4)$$

$$= x(x+1)(x-7)$$

$$22. m^3 + 3m^2 - 16m - 48$$

$$= (m^3 + 3m^2) - (16m + 48)$$

$$= m^2(m+3) - 16(m+3)$$

$$= (m+3)(m^2 - 16)$$

$$= (m+3)(m-4)(m+4)$$

$$23. x^3 - 6x^2y + 12xy^2 - 8y^3$$

$$= (x-2y)^3 = (x-2y)(x-2y)(x-2y)$$

$$24. (a+b)(a^2 - b^2) - (a^2 - b^2)$$

$$= (a^2 - b^2)(a+b-1)$$

$$= (a+b)(a-b)(a+b-1)$$

$$25. 32a^5x - 48a^3bx + 18ab^2x$$

$$= 2ax(16a^4 - 24a^2b + 9b^2)$$

$$= 2ax(4a^2 - 3b)^2$$

$$= 2ax(4a^2 - 3b)(4a^2 - 3b)$$

$$26. x^4 - x^3 + x^2 - x$$

$$= x(x^3 - x^2 + x - 1)$$

$$= x[x^2(x-1) + (x-1)] = x[(x-1)(x^2+1)]$$

$$27. 4x^2 + 32x - 36$$

$$= 16x^2 + 32(4x) - 144$$

$$144 \quad \left| \begin{array}{l} 2 \\ 2 \\ 4 \\ 3 \\ 3 \\ 3 \end{array} \right. \quad \begin{array}{l} 2 \\ 2 \\ 4 \cdot 3 \cdot 3 = 36 \\ 3 \Rightarrow 36 - 4 = 32 \\ 3 = \frac{(4x+36)(4x-4)}{4} \\ 3 = (x+9)(4x-4) \\ 1 = 4(x+9)(x-1) \end{array}$$

$$72 \quad \left| \begin{array}{l} 2 \\ 2 \\ 4 \end{array} \right. \quad \begin{array}{l} 2 \\ 2 \\ 4 \end{array}$$

$$36 \quad \left| \begin{array}{l} 4 \\ 4 \end{array} \right. \quad \begin{array}{l} 4 \cdot 3 \cdot 3 = 36 \\ 3 \Rightarrow 36 - 4 = 32 \end{array}$$

$$9 \quad \left| \begin{array}{l} 3 \\ 3 \end{array} \right. \quad \begin{array}{l} 3 \\ 3 \end{array}$$

$$3 \quad \left| \begin{array}{l} 3 \\ 3 \end{array} \right. \quad \begin{array}{l} 3 \\ 3 \end{array}$$

$$1 \quad \left| \begin{array}{l} 3 \\ 3 \end{array} \right. \quad \begin{array}{l} 3 \\ 3 \end{array}$$

$$= (x+9)(4x-4)$$

$$= 4(x+9)(x-1)$$

$$28. a^4 - (a+2)^2$$

$$= (a^2 + a + 2)(a^2 - a - 2)$$

$$= (a^2 + a + 2)(a-2)(a+1)$$

$$29. x^6 - 25x^3 - 54$$

$$= (x^3 - 27)(x^3 + 2)$$

$$= (x-3)(x^2 + 3x + 9)(x^3 + 2)$$

$$30. a^6 + a$$

$$= a(a^5 + 1)$$

$$= a(a+1)(a^4 - a^3 + a^2 - a + 1)$$

$$31. a^3b + 2a^2bx + abx^2 - aby^2$$

$$= ab(a^2 + 2ax + x^2 - y^2)$$

$$= ab[(a+x)^2 - y^2]$$

$$= ab(a+x+y)(a+x-y)$$

$$32. 3abm^2 - 3ab$$

$$= 3ab(m^2 - 1)$$

$$= 3ab(m+1)(m-1)$$

$$33. 81x^4y + 3xy^4$$

$$= 3xy(27x^3 + y^3)$$

$$= 3xy(3x+y)(9x^2 - 3xy + y^2)$$

$$34. a^4 - a^3 + a - 1$$

$$= (a^4 - a^3) + (a - 1)$$

$$= a^3(a-1) + (a-1)$$

$$= (a-1)(a^3 + 1)$$

$$= (a-1)(a+1)(a^2 - a + 1)$$

$$35. x - 3x^2 - 18x^3$$

$$= x(1 - 3x - 18x^2)$$

$$= -x(18x^2 + 3x - 1)$$

$$= -x[(18x)^2 + 3(18x) - 18]$$

$$= \frac{-x(18x+6)(18x-3)}{6 \cdot 3}$$

$$= -x(3x+1)(6x-1)$$

$$= x(3x+1)(1-6x)$$

$$36. 6ax - 2bx + 6ab - 2b^2$$

$$= (6ax - 2bx) + (6ab - 2b^2)$$

$$= 2x(3a - b) + 2b(3a - b)$$

$$= (3a - b)(2x + 2b)$$

$$= 2(x+b)(3a-b)$$

$$37. am^3 - 7am^2 + 12am$$

$$= am(m^2 - 7m + 12)$$

$$= am(m-4)(m-3)$$

$$38. 4a^2x^3 - 4a^2$$

$$= 4a^2(x^3 - 1)$$

$$= 4a^2(x-1)(x^2 + x + 1)$$

$$39. 28x^3y - 7xy^3$$

$$= 7xy(4x^2 - y^2)$$

$$= 7xy(2x-y)(2x+y)$$

$$40. 3abx^2 - 3abx - 18ab$$

$$= 3ab(x^2 - x - 6)$$

$$= 3ab(x-3)(x+2)$$

$$41. x^4 - 8x^2 - 128$$

$$128 \quad \left| \begin{array}{l} 4 \\ 4 \\ 4 \\ 2 \\ 2 \end{array} \right. \quad \begin{array}{l} 4 \\ 4 \\ 4 \\ 2 \\ 2 \end{array}$$

$$32 \quad \left| \begin{array}{l} 4 \\ 4 \end{array} \right. \quad \begin{array}{l} 4 \cdot 4 = 16 \\ 4 \cdot 2 = 8 \end{array}$$

$$8 \quad \left| \begin{array}{l} 4 \\ 2 \end{array} \right. \quad \begin{array}{l} 4 \\ 2 \end{array}$$

$$2 \quad \left| \begin{array}{l} 2 \\ 2 \end{array} \right. \quad \begin{array}{l} 2 \\ 2 \end{array}$$

$$1 \quad \left| \begin{array}{l} 2 \\ 2 \end{array} \right. \quad \begin{array}{l} 2 \\ 2 \end{array}$$

$$= (x^2 - 16)(x^2 + 8)$$

$$= (x+4)(x-4)(x^2 + 8)$$

$$42. 18x^2y + 60xy^2 + 50y^3$$

$$= 2y(9x^2 + 30xy + 25y^2)$$

$$= 2y(3x+5y)(3x+5y)$$

$$\begin{aligned}
 43. & (x^2 - 2xy)(a+1) + y^2(a+1) \\
 &= (a+1)(x^2 - 2xy + y^2) \\
 &= (a+1)(x-y)(x-y)
 \end{aligned}$$

$$\begin{aligned}
 44. & x^3 + 2x^2y - 3xy^2 \\
 &= x(x^2 + 2xy - 3y^2) \\
 &= x(x+3y)(x-y)
 \end{aligned}$$

$$\begin{aligned}
 45. & a^2x - 4b^2x + 2a^2y - 8b^2y \\
 &= (a^2x - 4b^2x) + (2a^2y - 8b^2y) \\
 &= x(a^2 - 4b^2) + 2y(a^2 - 4b^2) \\
 &= (a^2 - 4b^2)(x+2y) \\
 &= (a-2b)(a+2b)(x+2y)
 \end{aligned}$$

$$\begin{aligned}
 46. & 45a^2x^4 - 20a^2 \\
 &= 5a^2(9x^4 - 4) \\
 &= 5a^2(3x^2 + 2)(3x^2 - 2)
 \end{aligned}$$

$$\begin{aligned}
 47. & a^4 - (a-12)^2 \\
 &= (a^2 + a - 12)(a^2 - a + 12) \\
 &= (a+4)(a-3)(a^2 - a + 12)
 \end{aligned}$$

$$\begin{aligned}
 48. & bx^2 - b - x^2 + 1 \\
 &= (bx^2 - x^2) - (b-1) \\
 &= x^2(b-1) - (b-1) \\
 &= (b-1)(x^2 - 1) \\
 &= (b-1)(x+1)(x-1)
 \end{aligned}$$

$$\begin{aligned}
 49. & 2x^4 + 6x^3 - 56x^2 \\
 &= 2x^2(x^2 + 3x - 28) \\
 &= 2x^2(x+7)(x-4)
 \end{aligned}$$

$$\begin{aligned}
 50. & 30a^2 - 55a - 50 \\
 &= (30a)^2 - 55(30a) - 1.500 \\
 &\begin{array}{l|l} 1.500 & 2 \\ 750 & 2 \quad 5 \cdot 2^2 = 20 \\ 375 & 5 \quad 5^2 \cdot 3 = 75 \\ 75 & 5 \Rightarrow 75 - 20 = 55 \\ 15 & 5 = \frac{(30a-75)(30a+20)}{15 \cdot 2} \\ 3 & 3 = (2a-5)(15a+10) \\ 1 & = 5(2a-5)(3a+2) \end{array}
 \end{aligned}$$

$$\begin{aligned}
 51. & 9(x-y)^3 - (x-y) \\
 &= (x-y)[9(x-y)^2 - 1] \\
 &= (x-y)[3(x-y)+1][3(x-y)-1] \\
 &= (x-y)(3x-3y+1)(3x-3y-1)
 \end{aligned}$$

$$\begin{aligned}
 52. & 6a^2x - 9a^3 - ax^2 \\
 &= a(6ax - 9a^2 - x^2) \\
 &= -a(x^2 - 6ax + 9a^2) \\
 &= -a(x-3a)(x-3a) \\
 &= a(3a-x)(x-3a)
 \end{aligned}$$

$$\begin{aligned}
 53. & 64a - 125a^4 \\
 &= a(64 - 125a^3) \\
 &= a(4-5a)(16+20a+25a^2)
 \end{aligned}$$

$$\begin{aligned}
 54. & 70x^4 + 26x^3 - 24x^2 \\
 &= 2x^2(35x^2 + 13x - 12) \\
 &= 2x^2[(35x)^2 + 13(35x) - 420] \\
 &\begin{array}{l|l} 420 & 2 \\ 210 & 2 \\ 105 & 5 \\ 21 & 3 \\ 7 & 7 \\ 1 & \\ 2^2 \cdot 7 = 28 & \\ 3 \cdot 5 = 15 & \\ \Rightarrow 28 - 15 = 13 & \\ = \frac{(35x+28)(35x-15)}{7 \cdot 5} & \\ = 2x^2(5x+4)(7x-3) & \end{array}
 \end{aligned}$$

$$\begin{aligned}
 55. & a^7 + 6a^6 - 55a^3 \\
 &= a^3(a^4 + 6a^3 - 55) \\
 &= a^3(a^2 + 11)(a^2 - 5)
 \end{aligned}$$

$$\begin{aligned}
 56. & 16a^5b - 56a^3b^3 + 49ab^5 \\
 &= ab(16a^4 - 56a^2b^2 + 49b^4) \\
 &= ab(4a^2 - 7b^2)^2
 \end{aligned}$$

$$\begin{aligned}
 57. & 7x^6 + 32a^2x^4 - 15a^4x^2 \\
 &= x^2(7x^4 + 32a^2x^2 - 15a^4) \\
 &= x^2[(7x^2)^2 + 32a^2(7x^2) - 105a^4]
 \end{aligned}$$

$$\begin{array}{l|l} 105 & 5 \\ 21 & 3 \quad 5 \cdot 7 = 35 \\ 7 & 7 \quad 3 \cdot 1 = 3 \\ 1 & \Rightarrow 35 - 3 = 32 \end{array}$$

$$\begin{aligned}
 &= \frac{x^2(7x^2 + 35a^2)(7x^2 - 3a^2)}{7} \\
 &= x^2(x^2 + 5a^2)(7x^2 - 3a^2)
 \end{aligned}$$

$$\begin{aligned}
 58. & x^{2m+2} - x^2y^{2n} \\
 &= (x^{m+1} + xy^n)(x^{m+1} - xy^n) \\
 &= x(x^m + y^n)x(x^m - y^n) \\
 &= x^2(x^m + y^n)(x^m - y^n)
 \end{aligned}$$

$$\begin{aligned}
 59. & 2x^4 + 5x^3 - 54x - 135 \\
 &= (2x^4 - 54x) + (5x^3 - 135) \\
 &= 2x(x^3 - 27) + 5(x^3 - 27) \\
 &= (x^3 - 27)(2x+5) \\
 &= (x-3)(x^2 + 3x+9)(2x+5)
 \end{aligned}$$

$$\begin{aligned}
 60. & ax^3 + ax^2y + axy^2 - 2ax^2 - 2axy - 2ay^2 \\
 &= a(x^3 + x^2y + xy^2 - 2x^2 - 2xy - 2y^2) \\
 &= a[(x^3 + x^2y + xy^2) - (2x^2 + 2xy + 2y^2)] \\
 &= a[x(x^2 + xy + y^2) - 2(x^2 + xy + y^2)] \\
 &= a(x-2)(x^2 + xy + y^2)
 \end{aligned}$$

$$\begin{aligned}
 61. & (x+y)^4 - 1 \\
 &= [(x+y)^2 + 1][(x+y)^2 - 1] \\
 &= (x^2 + 2xy + y^2 + 1)(x+y+1)(x+y-1)
 \end{aligned}$$

$$\begin{aligned}
 62. & 3a^5 + 3a^3 + 3a \\
 &= 3a(a^4 + a^2 + 1) \\
 &= 3a(a^4 + a^2 + 1 + a^2 - a^2) \\
 &= 3a[(a^4 + 2a^2 + 1) - a^2] \\
 &= 3a[(a^2 + 1)^2 - a^2] \\
 &= 3a(a^2 + a + 1)(a^2 - a + 1)
 \end{aligned}$$

## EJERCICIO 108

1.  $1 - a^8$

$$= (1 + a^4)(1 - a^4)$$

$$= (1 + a^4)(1 + a^2)(1 - a^2)$$

$$= (1 + a^4)(1 + a^2)(1 + a)(1 - a)$$

2.  $a^6 - 1$

$$= (a^3 + 1)(a^3 - 1)$$

$$= (a + 1)(a^2 - a + 1)(a - 1)(a^2 + a + 1)$$

3.  $x^4 - 41x^2 + 400$

400	4	
100	5	$4^2 = 16$
20	5	$5^2 = 25$
4	4	$\Rightarrow 25 + 16 = 41$
1		$= (x^2 - 25)(x^2 - 16)$
		$= (x + 5)(x - 5)(x + 4)(x - 4)$

4.  $a^4 - 2a^2b^2 + b^4$

$$= (a^2 - b^2)^2$$

$$= [(a + b)(a - b)]^2 = (a + b)^2(a - b)^2$$

5.  $x^5 + x^3 - 2x$

$$= x(x^4 + x^2 - 2)$$

$$= x(x^2 + 2)(x^2 - 1)$$

$$= x(x^2 + 2)(x + 1)(x - 1)$$

6.  $2x^4 + 6x^3 - 2x - 6$

$$= (2x^4 + 6x^3) - (2x + 6)$$

$$= 2x^3(x + 3) - 2(x + 3)$$

$$= (2x^3 - 2)(x + 3)$$

$$= 2(x^3 - 1)(x + 3)$$

$$= 2(x - 1)(x^2 + x + 1)(x + 3)$$

7.  $3x^4 - 243$

$$= 3(x^4 - 81)$$

$$= 3(x^2 + 9)(x^2 - 9)$$

$$= 3(x^2 + 9)(x + 3)(x - 3)$$

8.  $16x^4 - 8x^2y^2 + y^4$

$$= (4x^2 - y^2)^2$$

$$= [(2x + y)(2x - y)]^2$$

$$= (2x + y)(2x - y)(2x + y)(2x - y) = 2(x^2 + 1)(4x^2 - 1) = 2(x^2 + 1)(2x + 1)(2x - 1)$$

9.  $9x^4 + 9x^3y - x^2 - xy$

$$= (9x^4 + 9x^3y) - (x^2 + xy)$$

$$= 9x^3(x + y) - x(x + y)$$

$$= (9x^3 - x)(x + y)$$

$$= x(9x^2 - 1)(x + y)$$

$$= x(3x + 1)(3x - 1)(x + y)$$

10.  $12ax^4 + 33ax^2 - 9a$

$$= 3a(4x^4 + 11x^2 - 3)$$

$$= 3a[(4x^2)^2 + 11(4x^2) - 12]$$

$$= \frac{3a(4x^2 + 12)(4x^2 - 1)}{4}$$

$$= 3a(x^2 + 3)(4x^2 - 1)$$

$$= 3a(x^2 + 3)(2x + 1)(2x - 1)$$

11.  $x^8 - y^8$

$$= (x^4 + y^4)(x^4 - y^4)$$

$$= (x^4 + y^4)(x^2 + y^2)(x^2 - y^2)$$

$$= (x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$$

12.  $x^6 - 7x^3 - 8$

$$= (x^3 - 8)(x^3 + 1)$$

$$= (x - 2)(x^2 + 2x + 4)(x^3 + 1)$$

$$= (x - 2)(x^2 + 2x + 4)(x + 1)(x^2 - x + 1)$$

13.  $64 - x^6$

$$= (8 + x^3)(8 - x^3)$$

$$= (2 + x)(4 - 2x + x^2)(2 - x)(4 + 2x + x^2)$$

14.  $a^5 - a^3b^2 - a^2b^3 + b^5$

$$= (a^5 - a^3b^2) - (a^2b^3 - b^5)$$

$$= a^3(a^2 - b^2) - b^3(a^2 - b^2)$$

$$= (a^2 - b^2)(a^3 - b^3)$$

$$= (a + b)(a - b)(a - b)(a^2 + ab + b^2)$$

15.  $8x^4 + 6x^2 - 2$

$$= 2(4x^4 + 3x^2 - 1)$$

$$= 2[(4x^2)^2 + 3(4x^2) - 4]$$

$$= \frac{2(4x^2 + 4)(4x^2 - 1)}{4}$$

16.  $a^4 - 25a^2 + 144$

144	4	
36	4	$4^2 = 16$
9	3	$3^2 = 9$
3	3	$\Rightarrow 16 + 9 = 25$
1		

$$= (a^2 - 16)(a^2 - 9)$$

$$= (a + 4)(a - 4)(a + 3)(a - 3)$$

17.  $a^2x^3 - a^2y^3 + 2ax^3 - 2ay^3$

$$= (a^2x^3 - a^2y^3) + (2ax^3 - 2ay^3)$$

$$= a^2(x^3 - y^3) + 2a(x^3 - y^3)$$

$$= (a^2 + 2a)(x^3 - y^3)$$

$$= a(a + 2)(x - y)(x^2 + xy + y^2)$$

18.  $a^4 + 2a^3 - a^2 - 2a$

$$= (a^4 + 2a^3) - (a^2 + 2a)$$

$$= a^3(a + 2) - a(a + 2)$$

$$= (a^3 - a)(a + 2)$$

$$= a(a^2 - 1)(a + 2)$$

$$= a(a + 1)(a - 1)(a + 2)$$

19.  $1 - 2a^3 + a^6$

$$= (a^3 - 1)^2$$

$$= [(a - 1)(a^2 + a + 1)]^2$$

$$= (a - 1)^2(a^2 + a + 1)^2$$

20.  $m^6 - 729$

$$= (m^2 - 9)(m^4 + 9m^2 + 81)$$

$$= (m + 3)(m - 3)(m^4 + 9m^2 + 81 + 9m^2 - 9m^2)$$

$$= (m + 3)(m - 3)[(m^4 + 18m^2 + 81) - 9m^2]$$

$$= (m + 3)(m - 3)[(m^2 + 9)^2 - 9m^2]$$

$$= (m + 3)(m - 3)(m^2 + 3m + 9)(m^2 - 3m + 9)$$

21.  $x^5 - x$

$$= x(x^4 - 1)$$

$$= x(x^2 + 1)(x^2 - 1)$$

$$= x(x^2 + 1)(x + 1)(x - 1)$$



$$22. x^5 - x^3y^2 + x^2y^3 - y^5$$

$$\begin{aligned} &= (x^5 - x^3y^2) + (x^2y^3 - y^5) \\ &= x^3(x^2 - y^2) + y^3(x^2 - y^2) \\ &= (x^3 + y^3)(x^2 - y^2) \\ &= (x + y)(x^2 - xy + y^2)(x + y)(x - y) \end{aligned}$$

$$\begin{aligned} 23. a^4b - a^3b^2 - a^2b^3 + ab^4 \\ &= (a^4b - a^3b^2) - (a^2b^3 - ab^4) \\ &= a^3b(a - b) - ab^3(a - b) \\ &= (a^3b - ab^3)(a - b) \\ &= ab(a^2 - b^2)(a - b) \\ &= ab(a + b)(a - b)(a - b) \end{aligned}$$

$$24. 5a^4 - 3.125$$

$$\begin{aligned} &= 5(a^4 - 625) \\ &= 5(a^2 + 25)(a^2 - 25) \\ &= 5(a^2 + 25)(a + 5)(a - 5) \end{aligned}$$

$$\begin{aligned} 25. (a^2 + 2a)^2 - 2(a^2 + 2a) - 3 \\ &= (a^2 + 2a)^2 - 2(a^2 + 2a) - 3 + 4 - 4 \\ &= \left\{ (a^2 + 2a)^2 - 2(a^2 + 2a) + 1 \right\} - 4 \\ &= (a^2 + 2a - 1)^2 - 4 \\ &= (a^2 + 2a - 1 + 2)(a^2 + 2a - 1 - 2) \\ &= (a^2 + 2a + 1)(a^2 + 2a - 3) \\ &= (a + 1)^2(a^2 + 2a - 3 + 4 - 4) \\ &= (a + 1)^2[(a^2 + 2a + 1) - 4] \\ &= (a + 1)^2[(a + 1)^2 - 4] \\ &= (a + 1)^2(a + 1 + 2)(a + 1 - 2) \\ &= (a + 1)^2(a + 3)(a - 1) \end{aligned}$$

$$\begin{aligned} 26. a^2x^3 + 2ax^3 - 8a^2 - 16a \\ &= (a^2x^3 + 2ax^3) - (8a^2 + 16a) \\ &= ax^3(a + 2) - 8a(a + 2) \\ &= (ax^3 - 8a)(a + 2) \\ &= a(x^3 - 8)(a + 2) \\ &= a(x - 2)(x^2 + 2x + 4)(a + 2) \end{aligned}$$

$$27. 1 - a^6b^6$$

$$\begin{aligned} &= (1 - a^3b^3)(1 + a^3b^3) \\ &= (1 - ab)(1 + ab + a^2b^2)(1 + a^3b^3) \\ &= (1 - ab)(1 + ab + a^2b^2)(1 + ab)(1 - ab + a^2b^2) \end{aligned}$$

$$28. 5ax^3 + 10ax^2 - 5ax - 10a$$

$$\begin{aligned} &= (5ax^3 + 10ax^2) - (5ax + 10a) \\ &= 5ax^2(x + 2) - 5a(x + 2) \\ &= (5ax^2 - 5a)(x + 2) \\ &= 5a(x^2 - 1)(x + 2) \\ &= 5a(x + 1)(x - 1)(x + 2) \end{aligned}$$

$$29. a^2x^2 + b^2y^2 - b^2x^2 - a^2y^2$$

$$\begin{aligned} &= (a^2x^2 - b^2x^2) + (b^2y^2 - a^2y^2) \\ &= x^2(a^2 - b^2) + y^2(b^2 - a^2) \\ &= x^2(a^2 - b^2) - y^2(a^2 - b^2) \\ &= (a^2 - b^2)(x^2 - y^2) \\ &= (a + b)(a - b)(x + y)(x - y) \end{aligned}$$

$$30. x^8 + x^4 - 2$$

$$\begin{aligned} &= (x^4 + 2)(x^4 - 1) \\ &= (x^4 + 2)(x^2 + 1)(x^2 - 1) \\ &= (x^4 + 2)(x^2 + 1)(x + 1)(x - 1) \end{aligned}$$

$$31. a^4 + a^3 - 9a^2 - 9a$$

$$\begin{aligned} &= (a^4 + a^3) - (9a^2 + 9a) \\ &= a^3(a + 1) - 9a(a + 1) \\ &= (a^3 - 9a)(a + 1) \\ &= a(a^2 - 9)(a + 1) \\ &= a(a + 3)(a - 3)(a + 1) \end{aligned}$$

$$32. a^2x^2 + a^2x - 6a^2 - x^2 - x + 6$$

$$\begin{aligned} &= (a^2x^2 + a^2x - 6a^2) - (x^2 + x - 6) \\ &= a^2(x^2 + x - 6) - (x^2 + x - 6) \\ &= (a^2 - 1)(x^2 + x - 6) \\ &= (a + 1)(a - 1)(x + 3)(x - 2) \end{aligned}$$

$$33. 16m^4 - 25m^2 + 9$$

$$\begin{aligned} &= (16m^2)^2 - 25(16m^2) + 144 \\ &\quad \begin{array}{c|c} 144 & 4 \\ 36 & 4 \quad 4^2 = 16 \\ 9 & 3 \quad 3^2 = 9 \\ 3 & 3 \Rightarrow 16 + 9 = 25 \\ 1 & \end{array} \end{aligned}$$

$$\begin{aligned} &= \frac{(16m^2 - 16)(16m^2 - 9)}{16} \\ &= (m^2 - 1)(16m^2 - 9) \\ &= (m + 1)(m - 1)(4m + 3)(4m - 3) \end{aligned}$$

$$34. 3abx^2 - 12ab + 3bx^2 - 12b$$

$$\begin{aligned} &= (3abx^2 + 3bx^2) - (12ab + 12b) \\ &= 3bx^2(a + 1) - 12b(a + 1) \\ &= (3bx^2 - 12b)(a + 1) \\ &= 3b(x^2 - 4)(a + 1) \\ &= 3b(x + 2)(x - 2)(a + 1) \end{aligned}$$

$$35. 3a^2m + 9am - 30m + 3a^2 + 9a - 30$$

$$\begin{aligned} &= (3a^2m + 9am - 30m) + (3a^2 + 9a - 30) \\ &= 3m(a^2 + 3a - 10) + 3(a^2 + 3a - 10) \\ &= (3m + 3)(a^2 + 3a - 10) \\ &= 3(m + 1)(a + 5)(a - 2) \end{aligned}$$

$$36. a^3x^2 - 5a^3x + 6a^3 + x^2 - 5x + 6$$

$$\begin{aligned} &= (a^3x^2 - 5a^3x + 6a^3) + (x^2 - 5x + 6) \\ &= a^3(x^2 - 5x + 6) + (x^2 - 5x + 6) \\ &= (a^3 + 1)(x^2 - 5x + 6) \\ &= (a + 1)(a^2 - a + 1)(x - 3)(x - 2) \end{aligned}$$

$$37. x^2(x^2 - y^2) - (2x - 1)(x^2 - y^2)$$

$$\begin{aligned} &= (x^2 - y^2)[x^2 - (2x - 1)] \\ &= (x + y)(x - y)(x^2 - 2x + 1) \\ &= (x + y)(x - y)(x - 1)^2 \end{aligned}$$

$$38. a(x^3 + 1) + 3ax(x + 1)$$

$$\begin{aligned} &= a(x + 1)(x^2 - x + 1) + 3ax(x + 1) \\ &= (x + 1)[a(x^2 - x + 1) + 3ax] \\ &= (x + 1)(ax^2 - ax + a + 3ax) \\ &= (x + 1)(ax^2 + 2ax + a) \\ &= a(x + 1)(x^2 + 2x + 1) \\ &= a(x + 1)(x + 1)^2 = a(x + 1)^3 \end{aligned}$$

## EJERCICIO 109

1.  $x^9 - xy^8$   
 $= x(x^8 - y^8)$   
 $= x(x^4 - y^4)(x^4 + y^4)$   
 $= x(x^2 + y^2)(x^2 - y^2)(x^4 + y^4)$   
 $= x(x^2 + y^2)(x + y)(x - y)(x^4 + y^4)$
2.  $x^5 - 40x^3 + 144x$   
 $= x(x^4 - 40x^2 + 144)$   
 $= x(x^2 - 36)(x^2 - 4)$   
 $= x(x - 6)(x + 6)(x + 2)(x - 2)$
3.  $a^6 + a^3b^3 - a^4 - ab^3$   
 $= (a^6 + a^3b^3) - (a^4 + ab^3)$   
 $= a^3(a^3 + b^3) - a(a^3 + b^3)$   
 $= (a^3 - a)(a^3 + b^3)$   
 $= a(a^2 - 1)(a + b)(a^2 - ab + b^2)$   
 $= a(a + 1)(a - 1)(a + b)(a^2 - ab + b^2)$
4.  $4x^4 - 8x^2 + 4$   
 $= 4(x^4 - 2x^2 + 1)$   
 $= 4(x^2 - 1)(x^2 - 1)$   
 $= 4(x + 1)^2(x - 1)^2$
5.  $a^7 - ab^6$   
 $= a(a^6 - b^6)$   
 $= a(a^3 + b^3)(a^3 - b^3)$   
 $= a(a + b)(a^2 - ab + b^2)(a - b)(a^2 + ab + b^2)$
6.  $2a^4 - 2a^3 - 4a^2 - 2a^2b^2 + 2ab^2 + 4b^2$   
 $= (2a^4 - 2a^3 - 4a^2) - (2a^2b^2 - 2ab^2 - 4b^2)$   
 $= 2a^2(a^2 - a - 2) - 2b^2(a^2 - a - 2)$   
 $= 2(a^2 - a - 2)(a^2 - b^2)$   
 $= 2(a - 2)(a + 1)(a + b)(a - b)$
7.  $x^6 + 5x^5 - 81x^2 - 405x$   
 $= (x^6 + 5x^5) - (81x^2 + 405x)$   
 $= x^5(x + 5) - 81x(x + 5)$   
 $= (x^5 - 81x)(x + 5)$   
 $= x(x^4 - 81)(x + 5)$   
 $= x(x^2 - 9)(x^2 + 9)(x + 5)$   
 $= x(x + 3)(x - 3)(x^2 + 9)(x + 5)$
8.  $3 - 3a^6$   
 $= 3(1 - a^6)$   
 $= 3(1 - a^3)(1 + a^3)$   
 $= 3(1 - a)(1 + a + a^2)(1 + a)(1 - a + a^2)$
9.  $4ax^2(a^2 - 2ax + x^2) - a^3 + 2a^2x - ax^2$   
 $= 4ax^2(a - x)^2 - a(a^2 - 2ax + x^2)$   
 $= 4ax^2(a - x)^2 - a(a - x)^2$   
 $= (4ax^2 - a)(a - x)^2$   
 $= a(4x^2 - 1)(a - x)^2$   
 $= a(2x + 1)(2x - 1)(a - x)^2$
10.  $x^7 + x^4 - 81x^3 - 81$   
 $= (x^7 + x^4) - (81x^3 + 81)$   
 $= x^4(x^3 + 1) - 81(x^3 + 1)$   
 $= (x^4 - 81)(x^3 + 1)$   
 $= (x^2 + 9)(x^2 - 9)(x + 1)(x^2 - x + 1)$   
 $= (x^2 + 9)(x + 3)(x - 3)(x + 1)(x^2 - x + 1)$
11.  $x^{17} - x$   
 $= x(x^{16} - 1)$   
 $= x(x^8 + 1)(x^8 - 1)$   
 $= x(x^8 + 1)(x^4 + 1)(x^4 - 1)$   
 $= x(x^8 + 1)(x^4 + 1)(x^2 + 1)(x^2 - 1)$   
 $= x(x^8 + 1)(x^4 + 1)(x^2 + 1)(x + 1)(x - 1)$
12.  $3x^6 - 75x^4 - 48x^2 + 1.200$   
 $= (3x^6 - 75x^4) - (48x^2 - 1.200)$   
 $= 3x^4(x^2 - 25) - 48(x^2 - 25)$   
 $= (3x^4 - 48)(x^2 - 25)$   
 $= 3(x^4 - 16)(x + 5)(x - 5)$   
 $= 3(x^2 + 4)(x^2 - 4)(x + 5)(x - 5)$   
 $= 3(x^2 + 4)(x + 2)(x - 2)(x + 5)(x - 5)$
13.  $a^6x^2 - x^2 + a^6x - x$   
 $= (a^6x^2 - x^2) + (a^6x - x)$   
 $= x^2(a^6 - 1) + x(a^6 - 1)$   
 $= (x^2 + x)(a^6 - 1)$   
 $= x(x + 1)(a^3 - 1)(a^3 + 1)$   
 $= x(x + 1)(a - 1)(a^2 + a + 1)(a + 1)(a^2 - a + 1)$
14.  $(a^2 - ax)(x^4 - 82x^2 + 81)$   
 $= a(a - x)(x^2 - 81)(x^2 - 1)$   
 $= a(a - x)(x + 9)(x - 9)(x + 1)(x - 1)$

## EJERCICIO 110

1.  $x^3 + x^2 - x - 1$   

$$\begin{array}{cccc|c} 1 & 1 & -1 & -1 & +1 \\ & 1 & 2 & 1 & \\ \hline 1 & 2 & 1 & 0 & \end{array}$$
  
 $= (x^2 + 2x + 1)(x - 1)$   
 $= (x + 1)^2(x - 1)$
2.  $x^3 - 4x^2 + x + 6$   

$$\begin{array}{cccc|c} 1 & -4 & 1 & 6 & -1 \\ & -1 & 5 & -6 & \\ \hline 1 & -5 & 6 & 0 & \end{array}$$
  
 $= (x^2 - 5x + 6)(x + 1)$   
 $= (x - 3)(x - 2)(x + 1)$
3.  $a^3 - 3a^2 - 4a + 12$   

$$\begin{array}{cccc|c} 1 & -3 & -4 & 12 & +2 \\ & 2 & -2 & -12 & \\ \hline 1 & -1 & -6 & 0 & \end{array}$$
  
 $= (a^2 - a - 6)(a - 2)$   
 $= (a - 3)(a + 2)(a - 2)$
4.  $m^3 - 12m + 16$   

$$\begin{array}{cccc|c} 1 & 0 & -12 & 16 & +2 \\ & 2 & 4 & -16 & \\ \hline 1 & 2 & -8 & 0 & \end{array}$$
  
 $= (m^2 + 2m - 8)(m - 2)$   
 $= (m + 4)(m - 2)(m - 2)$   
 $= (m + 4)(m - 2)^2$

$$\begin{array}{r|l}
 2x^3 - x^2 - 18x + 9 & \\
 2 & -1 \quad -18 \quad 9 \\
 \hline
 & 6 \quad 15 \quad 9 \\
 2 & 5 \quad -3 \quad 0 \\
 \hline
 \end{array}$$

$$= (2x^2 + 5x - 3)(x - 3)$$

$$\begin{array}{r|l}
 2 & 5 \quad -3 \quad -3 \\
 \hline
 & -6 \quad 3 \\
 2 & -1 \quad 0 \\
 \hline
 \end{array}$$

$$= (2x - 1)(x - 3)(x + 3)$$

$$\begin{array}{r|l}
 a^3 + a^2 - 13a - 28 & \\
 1 & 1 \quad -13 \quad -28 \\
 \hline
 & 4 \quad 20 \quad 28 \\
 1 & 5 \quad 7 \quad 0 \\
 \hline
 \end{array}$$

$$= (a^2 + 5a + 7)(a - 4)$$

$$\begin{array}{r|l}
 x^3 + 2x^2 + x + 2 & \\
 1 & 2 \quad 1 \quad 2 \\
 \hline
 & -2 \quad 0 \quad 2 \\
 1 & 0 \quad 1 \quad 0 \\
 \hline
 \end{array}$$

$$= (x^2 + 1)(x + 2)$$

$$\begin{array}{r|l}
 n^3 - 7n + 6 & \\
 1 & 0 \quad -7 \quad 6 \\
 \hline
 & 1 \quad 1 \quad -6 \\
 1 & 1 \quad -6 \quad 0 \\
 \hline
 \end{array}$$

$$= (n^2 + n - 6)(n - 1)$$

$$= (n + 3)(n - 2)(n - 1)$$

$$\begin{array}{r|l}
 x^3 - 6x^2 + 32 & \\
 1 & -6 \quad 0 \quad 32 \\
 \hline
 & -2 \quad 16 \quad -32 \\
 1 & -8 \quad 16 \quad 0 \\
 \hline
 \end{array}$$

$$= (x^2 - 8x + 16)(x + 2)$$

$$= (x - 4)^2(x + 2)$$

$$\begin{array}{r|l}
 6x^3 + 23x^2 + 9x - 18 & \\
 6 & 23 \quad 9 \quad -18 \\
 \hline
 & -18 \quad -15 \quad 18 \\
 6 & 5 \quad -6 \quad 0 \\
 \hline
 \end{array}$$

$$= (6x^2 + 5x - 6)(x + 3)$$

$$= [(6x)^2 + 5(6x) - 36](x + 3)$$

$$= \frac{(6x + 9)(6x - 4)(x + 3)}{3 \cdot 2}$$

$$= (2x + 3)(3x - 2)(x + 3)$$

$$\begin{array}{r|l}
 x^4 - 4x^3 + 3x^2 + 4x - 4 & \\
 1 & -4 \quad 3 \quad 4 \quad -4 \\
 \hline
 & 1 \quad -3 \quad 0 \quad 4 \\
 1 & -3 \quad 0 \quad 4 \quad 0 \\
 \hline
 \end{array}$$

$$= (x^3 - 3x^2 + 4)(x - 1)$$

$$\begin{array}{r|l}
 1 & -3 \quad 0 \quad 4 \\
 \hline
 & -1 \quad 4 \quad -4 \\
 1 & -4 \quad 4 \quad 0 \\
 \hline
 \end{array}$$

$$= (x^2 - 4x + 4)(x - 1)(x + 1)$$

$$= (x - 2)^2(x - 1)(x + 1)$$

$$\begin{array}{r|l}
 x^4 - 2x^3 - 13x^2 + 14x + 24 & \\
 1 & -2 \quad -13 \quad 14 \quad 24 \\
 \hline
 & 2 \quad 0 \quad -26 \quad -24 \\
 1 & 0 \quad -13 \quad -12 \quad 0 \\
 \hline
 \end{array}$$

$$= (x^3 - 13x - 12)(x - 2)$$

$$\begin{array}{r|l}
 1 & 0 \quad -13 \quad -12 \\
 \hline
 & -1 \quad 1 \quad 12 \\
 1 & -1 \quad -12 \quad 0 \\
 \hline
 \end{array}$$

$$= (x^2 - x - 12)(x - 2)(x + 1)$$

$$= (x - 4)(x + 3)(x - 2)(x + 1)$$

$$\begin{array}{r|l}
 a^4 - 15a^2 - 10a + 24 & \\
 1 & 0 \quad -15 \quad -10 \quad 24 \\
 \hline
 & 1 \quad 1 \quad -14 \quad -24 \\
 1 & 1 \quad -14 \quad -24 \quad 0 \\
 \hline
 \end{array}$$

$$= (a^3 + a^2 - 14a - 24)(a - 1)$$

$$\begin{array}{r|l}
 1 & 1 \quad -14 \quad -24 \\
 \hline
 & -2 \quad 2 \quad 24 \\
 1 & -1 \quad -12 \quad 0 \\
 \hline
 \end{array}$$

$$= (a^2 - a - 12)(a - 1)(a + 2)$$

$$= (a - 4)(a + 3)(a - 1)(a + 2)$$

$$\begin{array}{r|l}
 n^4 - 27n^2 - 14n + 120 & \\
 1 & 0 \quad -27 \quad -14 \quad 120 \\
 \hline
 & 2 \quad 4 \quad -46 \quad -120 \\
 1 & 2 \quad -23 \quad -60 \quad 0 \\
 \hline
 \end{array}$$

$$= (n^3 + 2n^2 - 23n - 60)(n - 2)$$

$$\begin{array}{r|l}
 1 & 2 \quad -23 \quad -60 \\
 \hline
 & -3 \quad 3 \quad 60 \\
 1 & -1 \quad -20 \quad 0 \\
 \hline
 \end{array}$$

$$= (n^2 - n - 20)(n - 2)(n + 3)$$

$$= (n - 5)(n + 4)(n - 2)(n + 3)$$

$$\begin{array}{r|l}
 x^4 + 6x^3 + 3x + 140 & \\
 1 & 6 \quad 0 \quad 3 \quad 140 \\
 \hline
 & -4 \quad -8 \quad 32 \quad -140 \\
 1 & 2 \quad -8 \quad 35 \quad 0 \\
 \hline
 \end{array}$$

$$= (x^3 + 2x^2 - 8x + 35)(x + 4)$$

$$\begin{array}{r|l}
 1 & 2 \quad -8 \quad 35 \\
 \hline
 & -5 \quad 15 \quad -35 \\
 1 & -3 \quad 7 \quad 0 \\
 \hline
 \end{array}$$

$$= (x^2 - 3x + 7)(x + 4)(x + 5)$$

$$\begin{array}{r|l}
 8a^4 - 18a^3 - 75a^2 + 46a + 120 & \\
 8 & -18 \quad -75 \quad 46 \quad 120 \\
 \hline
 & -16 \quad 68 \quad 14 \quad -120 \\
 8 & -34 \quad -7 \quad 60 \quad 0 \\
 \hline
 \end{array}$$

$$= (8a^3 - 34a^2 - 7a + 60)(a + 2)$$

$$\begin{array}{r|l}
 8 & -34 \quad -7 \quad 60 \\
 \hline
 & 32 \quad -8 \quad -60 \\
 8 & -2 \quad -15 \quad 0 \\
 \hline
 \end{array}$$

$$= (8a^2 - 2a - 15)(a + 2)(a - 4)$$

$$= [(8a)^2 - 2(8a) - 120](a + 2)(a - 4)$$

$$\begin{array}{l|l}
 120 & 4 \\
 30 & 3 \quad 4 \cdot 3 = 12 \\
 10 & 2 \quad 5 \cdot 2 = 10 \\
 5 & 5 \Rightarrow 12 - 10 = 2 \\
 1 & \\
 \hline
 \end{array}$$

$$= \frac{(8a - 12)(8a + 10)(a + 2)(a - 4)}{4 \cdot 2}$$

$$= (2a - 3)(4a + 5)(a + 2)(a - 4)$$

$$\begin{array}{r|l}
 x^4 - 22x^2 - 75 & \\
 1 & 0 \quad -22 \quad 0 \quad -75 \\
 \hline
 & 5 \quad 25 \quad 15 \quad 75 \\
 1 & 5 \quad 3 \quad 15 \quad 0 \\
 \hline
 \end{array}$$

$$= (x^3 + 5x^2 + 3x + 15)(x - 5)$$

$$\begin{array}{r|l}
 1 & 5 \quad 3 \quad 15 \\
 \hline
 & -5 \quad 0 \quad -15 \\
 1 & 0 \quad 3 \quad 0 \\
 \hline
 \end{array}$$

$$= (x^2 + 3)(x - 5)(x + 5)$$

$$18. 15x^4 + 94x^3 - 5x^2 - 164x + 60$$

$$\begin{array}{r|l} 15 & 94 & -5 & -164 & 60 & +1 \\ \hline & 15 & 109 & 104 & -60 & \\ \hline 15 & 109 & 104 & -60 & 0 & \\ \hline = & (15x^3 + 109x^2 + 104x - 60)(x-1) & \\ \hline 15 & 109 & 104 & -60 & +6 & \\ \hline & -90 & -114 & 60 & & \\ \hline 15 & 19 & -10 & 0 & & \\ \hline = & (15x^2 + 19x - 10)(x-1)(x+6) & \\ \hline = & [(15x)^2 + 19(15x) - 150](x-1)(x+6) & \\ \hline = & (15x+25)(15x-6)(x-1)(x+6) & \\ \hline & 5 \cdot 3 & \\ \hline = & (3x+5)(5x-2)(x-1)(x+6) & \end{array}$$

$$19. x^5 - 21x^3 + 16x^2 + 108x - 144$$

$$\begin{array}{r|l} 1 & 0 & -21 & 16 & 108 & -144 & +2 \\ \hline & 2 & 4 & -34 & -36 & 144 & \\ \hline 1 & 2 & -17 & -18 & 72 & 0 & \\ \hline = & (x^4 + 2x^3 - 17x^2 - 18x + 72)(x-2) & \\ \hline 1 & 2 & -17 & -18 & 72 & -3 & \\ \hline & -3 & 3 & 42 & -72 & & \\ \hline 1 & -1 & -14 & 24 & 0 & & \\ \hline = & (x^3 - x^2 - 14x + 24)(x-2)(x+3) & \\ \hline 1 & -1 & -14 & 24 & +2 & & \\ \hline & 2 & 2 & -24 & & & \\ \hline 1 & 1 & -12 & 0 & & & \\ \hline = & (x^2 + x - 12)(x-2)(x+3)(x-2) & \\ \hline = & (x+4)(x-3)(x-2)(x+3)(x-2) & \\ \hline = & (x+4)(x-3)(x+3)(x-2)^2 & \end{array}$$

$$20. a^5 - 23a^3 - 6a^2 + 112a + 96$$

$$\begin{array}{r|l} 1 & 0 & -23 & -6 & 112 & 96 & -1 \\ \hline & -1 & 1 & 22 & -16 & -96 & \\ \hline 1 & -1 & -22 & 16 & 96 & 0 & \\ \hline = & (a^4 - a^3 - 22a^2 + 16a + 96)(a+1) & \\ \hline 1 & -1 & -22 & 16 & 96 & -2 & \\ \hline & -2 & 6 & 32 & -96 & & \\ \hline 1 & -3 & -16 & 48 & 0 & & \\ \hline = & (a^3 - 3a^2 - 16a + 48)(a+1)(a+2) & \\ \hline 1 & -3 & -16 & 48 & +3 & & \\ \hline & 3 & 0 & -48 & & & \\ \hline 1 & 0 & -16 & 0 & & & \\ \hline = & (a^2 - 16)(a+1)(a+2)(a-3) & \\ \hline = & (a+4)(a-4)(a+1)(a+2)(a-3) & \end{array}$$

$$21. 4x^5 + 3x^4 - 108x^3 - 25x^2 + 522x + 360$$

$$\begin{array}{r|l} 4 & 3 & -108 & -25 & 522 & 360 & +2 \\ \hline & -8 & 10 & 196 & -342 & -360 & \\ \hline 4 & -5 & -98 & 171 & 180 & 0 & \\ \hline = & (4x^4 - 5x^3 - 98x^2 + 171x + 180)(x+2) & \\ \hline 4 & -5 & -98 & 171 & 180 & +3 & \\ \hline & 12 & 21 & -231 & -180 & & \\ \hline 4 & 7 & -77 & -60 & 0 & & \\ \hline = & (4x^3 + 7x^2 - 77x - 60)(x+2)(x-3) & \\ \hline 4 & 7 & -77 & -60 & +4 & & \\ \hline & 16 & 92 & 60 & & & \\ \hline 4 & 23 & 15 & 0 & & & \\ \hline = & (4x^2 + 23x + 15)(x+2)(x-3)(x-4) & \\ \hline = & [(4x)^2 + 23(4x) + 60](x+2)(x-3)(x-4) & \\ \hline = & [(4x+20)(4x+3)(x+2)(x-3)(x-4)] \div 4 & \\ \hline = & (x+5)(4x+3)(x+2)(x-3)(x-4) & \end{array}$$

$$22. n^5 - 30n^3 - 25n^2 - 36n - 180$$

$$\begin{array}{r|l} 1 & 0 & -30 & -25 & -36 & -180 & -2 \\ \hline & -2 & 4 & 52 & -54 & 180 & \\ \hline 1 & -2 & -26 & 27 & -90 & 0 & \\ \hline = & (n^4 - 2n^3 - 26n^2 + 27n - 90)(n+2) & \\ \hline 1 & -2 & -26 & 27 & -90 & -5 & \\ \hline & -5 & 35 & -45 & 90 & & \\ \hline 1 & -7 & 9 & -18 & 0 & & \\ \hline = & (n^3 - 7n^2 + 9n - 18)(n+2)(n+5) & \\ \hline 1 & -7 & 9 & -18 & +6 & & \\ \hline & 6 & -6 & 18 & & & \\ \hline 1 & -1 & 3 & 0 & & & \\ \hline = & (n^2 - n + 3)(n+2)(n+5)(n-6) & \end{array}$$

$$23. 6x^5 - 13x^4 - 81x^3 + 112x^2 + 180x - 144$$

$$\begin{array}{r|l} 6 & -13 & -81 & 112 & 180 & -144 & +2 \\ \hline & 12 & -2 & -166 & -108 & 144 & \\ \hline 6 & -1 & -83 & -54 & 72 & 0 & \\ \hline = & (6x^4 - x^3 - 83x^2 - 54x + 72)(x-2) & \\ \hline 6 & -1 & -83 & -54 & 72 & -3 & \\ \hline & -18 & 57 & 78 & -72 & & \\ \hline 6 & -19 & -26 & 24 & 0 & & \\ \hline = & (6x^3 - 19x^2 - 26x + 24)(x-2)(x+3) & \\ \hline 6 & -19 & -26 & 24 & +4 & & \\ \hline & 24 & 20 & -24 & & & \\ \hline 6 & 5 & -6 & 0 & & & \\ \hline = & (6x^2 + 5x - 6)(x-2)(x+3)(x-4) & \\ \hline = & [(6x)^2 + 5(6x) - 36](x-2)(x+3)(x-4) & \\ \hline = & \frac{(6x+9)(6x-4)(x-2)(x+3)(x-4)}{3 \cdot 2} & \\ \hline = & (2x+3)(3x-2)(x-2)(x+3)(x-4) & \end{array}$$

$$24. x^5 - 25x^3 + x^2 - 25$$

$$\begin{array}{r|l} 1 & 0 & -25 & 1 & 0 & -25 & +1 \\ \hline & -1 & 1 & 24 & -25 & 25 & \\ \hline 1 & -1 & -24 & 25 & -25 & 0 & \\ \hline = & (x^4 - x^3 - 24x^2 + 25x - 25)(x+1) & \\ \hline 1 & -1 & -24 & 25 & -25 & +5 & \\ \hline & 5 & 20 & -20 & 25 & & \\ \hline 1 & 4 & -4 & 5 & 0 & & \\ \hline = & (x^3 + 4x^2 - 4x + 5)(x+1)(x-5) & \\ \hline 1 & 4 & -4 & 5 & -5 & & \\ \hline & -5 & 5 & -5 & & & \\ \hline 1 & -1 & 1 & 0 & & & \\ \hline = & (x^2 - x + 1)(x+1)(x-5)(x+5) & \end{array}$$

$$25. 2a^5 - 8a^4 + 3a - 12$$

$$\begin{array}{r|l} 2 & -8 & 0 & 0 & 3 & -12 & +4 \\ \hline & 8 & 0 & 0 & 0 & 12 & \\ \hline 2 & 0 & 0 & 0 & 3 & 0 & \\ \hline = & (2a^4 + 3)(a-4) & \end{array}$$

$$26. x^5 + 2x^4 - 15x^3 - 3x^2 - 6x + 45$$

$$\begin{array}{r|l} 1 & 2 & -15 & -3 & -6 & 45 & +3 \\ \hline & 3 & 15 & 0 & -9 & -45 & \\ \hline 1 & 5 & 0 & -3 & -15 & 0 & \\ \hline = & (x^4 + 5x^3 - 3x - 15)(x-3) & \\ \hline 1 & 5 & 0 & -3 & -15 & -5 & \\ \hline & -5 & 0 & 0 & 15 & & \\ \hline 1 & 0 & 0 & -3 & 0 & & \\ \hline = & (x^3 - 3)(x-3)(x+5) & \end{array}$$

$$27. x^6 + 6x^5 + 4x^4 - 42x^3 - 113x^2 - 108x - 36$$

$$\begin{array}{r|l} 1 & 6 & 4 & -42 & -113 & -108 & -36 & -1 \\ \hline & -1 & -5 & 1 & 41 & 72 & 36 & \\ \hline 1 & 5 & -1 & -41 & -72 & -36 & 0 & \\ \hline = & (x^5 + 5x^4 - x^3 - 41x^2 - 72x - 36)(x+1) & \\ \hline 1 & 5 & -1 & -41 & -72 & -36 & -1 & \\ \hline & -1 & -4 & 5 & 36 & 36 & & \\ \hline 1 & 4 & -5 & -36 & -36 & 0 & & \\ \hline = & (x^4 + 4x^3 - 5x^2 - 36x - 36)(x+1)(x+1) & \\ \hline 1 & 4 & -5 & -36 & -36 & -2 & & \\ \hline & -2 & -4 & 18 & 36 & & & \\ \hline 1 & 2 & -9 & -18 & 0 & & & \\ \hline = & (x^3 + 2x^2 - 9x - 18)(x+1)^2(x+2) & \\ \hline 1 & 2 & -9 & -18 & -2 & & & \\ \hline & -2 & 0 & 18 & & & & \\ \hline 1 & 0 & -9 & 0 & & & & \\ \hline = & (x^2 - 9)(x+1)^2(x+2)^2 & \\ \hline = & (x+3)(x-3)(x+1)^2(x+2)^2 & \end{array}$$

$$28. a^6 - 32a^4 + 18a^3 + 247a^2 - 162a - 360$$

$$\begin{array}{r|l} 1 & 0 & -32 & 18 & 247 & -162 & -360 & -1 \\ & -1 & 1 & 31 & -49 & -198 & 360 & \end{array}$$

$$\begin{array}{r|l} 1 & -1 & -31 & 49 & 198 & -360 & 0 & \end{array}$$

$$= (a^5 - a^4 - 31a^3 + 49a^2 + 198a - 360)(a+1)$$

$$\begin{array}{r|l} 1 & -1 & -31 & 49 & 198 & -360 & +2 \\ & 2 & 2 & -58 & -18 & 360 & \end{array}$$

$$\begin{array}{r|l} 1 & 1 & -29 & -9 & 180 & 0 & \end{array}$$

$$= (a^4 + a^3 - 29a^2 - 9a + 180)(a+1)(a-2)$$

$$\begin{array}{r|l} 1 & 1 & -29 & -9 & 180 & +3 \\ & 3 & 12 & -51 & -180 & \end{array}$$

$$\begin{array}{r|l} 1 & 4 & -17 & -60 & 0 & \end{array}$$

$$= (a^3 + 4a^2 - 17a - 60)(a+1)(a-2)(a-3)$$

$$\begin{array}{r|l} 1 & 4 & -17 & -60 & -3 \\ & -3 & -3 & 60 & \end{array}$$

$$\begin{array}{r|l} 1 & 1 & -20 & 0 & \end{array}$$

$$= (a^2 + a - 20)(a+1)(a-2)(a-3)(a+3)$$

$$= (a+5)(a-4)(a+1)(a-2)(a-3)(a+3)$$

$$30. 2x^6 - 10x^5 - 34x^4 + 146x^3 + 224x^2 - 424x - 480$$

$$\begin{array}{r|l} 2 & -10 & -34 & 146 & 224 & -424 & -480 & -1 \\ & -2 & 12 & 22 & -168 & -56 & 480 & \end{array}$$

$$\begin{array}{r|l} 2 & -12 & -22 & 168 & 56 & -480 & 0 & \end{array}$$

$$= (2x^5 - 12x^4 - 22x^3 + 168x^2 + 56x - 480)(x+1)$$

$$\begin{array}{r|l} 2 & -12 & -22 & 168 & 56 & -480 & -2 \\ & -4 & 32 & -20 & -296 & 480 & \end{array}$$

$$\begin{array}{r|l} 2 & -16 & 10 & 148 & -240 & 0 & \end{array}$$

$$= (2x^4 - 16x^3 + 10x^2 + 148x - 240)(x+1)(x+2)$$

$$\begin{array}{r|l} 2 & -16 & 10 & 148 & -240 & +2 \\ & 4 & -24 & -28 & 240 & \end{array}$$

$$\begin{array}{r|l} 2 & -12 & -14 & 120 & 0 & \end{array}$$

$$= (2x^3 - 12x^2 - 14x + 120)(x+1)(x+2)(x-2)$$

$$\begin{array}{r|l} 2 & -12 & -14 & 120 & +4 \\ & 8 & -16 & -120 & \end{array}$$

$$\begin{array}{r|l} 2 & -4 & -30 & 0 & \end{array}$$

$$= (2x^2 - 4x - 30)(x+1)(x+2)(x-2)(x-4)$$

$$= [(2x)^2 - 4(2x) - 60](x+1)(x+2)(x-2)(x-4)$$

$$= \frac{(2x-10)(2x+6)(x+1)(x+2)(x-2)(x-4)}{2}$$

$$= (x-5)(2x+6)(x+1)(x+2)(x-2)(x-4)$$

$$= 2(x-5)(x+3)(x+1)(x+2)(x-2)(x-4)$$

$$29. x^6 - 41x^4 + 184x^2 - 144$$

$$\begin{array}{r|l} 1 & 0 & -41 & 0 & 184 & 0 & -144 & +1 \\ & 1 & 1 & -40 & -40 & 144 & 144 & \end{array}$$

$$\begin{array}{r|l} 1 & 1 & -40 & -40 & 144 & 144 & 0 & \end{array}$$

$$= (x^5 + x^4 - 40x^3 - 40x^2 + 144x + 144)(x-1)$$

$$\begin{array}{r|l} 1 & 1 & -40 & -40 & 144 & 144 & -1 \\ & -1 & 0 & 40 & 0 & -144 & \end{array}$$

$$\begin{array}{r|l} 1 & 0 & -40 & 0 & 144 & 0 & \end{array}$$

$$= (x^4 - 40x^2 + 144)(x-1)(x+1)$$

$$= (x^2 - 36)(x^2 - 4)(x-1)(x+1)$$

$$= (x+6)(x-6)(x+2)(x-2)(x-1)(x+1)$$

$$31. a^6 - 8a^5 + 6a^4 + 103a^3 - 344a^2 + 396a - 144$$

$$\begin{array}{r|l} 1 & -8 & 6 & 103 & -344 & 396 & -144 & +2 \\ & 2 & -12 & -12 & 182 & -324 & 144 & \end{array}$$

$$\begin{array}{r|l} 1 & -6 & -6 & 91 & -162 & 72 & 0 & \end{array}$$

$$= (a^5 - 6a^4 - 6a^3 + 91a^2 - 162a + 72)(a-2)$$

$$\begin{array}{r|l} 1 & -6 & -6 & 91 & -162 & 72 & +2 \\ & 2 & -8 & -28 & 126 & -72 & \end{array}$$

$$\begin{array}{r|l} 1 & -4 & -14 & 63 & -36 & 0 & \end{array}$$

$$= (a^4 - 4a^3 - 14a^2 + 63a - 36)(a-2)(a-2)$$

$$\begin{array}{r|l} 1 & -4 & -14 & 63 & -36 & +3 \\ & 3 & -3 & -51 & 36 & \end{array}$$

$$\begin{array}{r|l} 1 & -1 & -17 & 12 & 0 & \end{array}$$

$$= (a^3 - a^2 - 17a + 12)(a-2)^2(a-3)$$

$$\begin{array}{r|l} 1 & -1 & -17 & 12 & -4 \\ & -4 & 20 & -12 & \end{array}$$

$$\begin{array}{r|l} 1 & -5 & 3 & 0 & \end{array}$$

$$= (a^2 - 5a + 3)(a-2)^2(a-3)(a+4)$$

$$32. x^7 - 20x^5 - 2x^4 + 64x^3 + 40x^2 - 128$$

$$\begin{array}{r|l} 1 & 0 & -20 & -2 & 64 & 40 & 0 & -128 & +2 \\ & 2 & 4 & -32 & -68 & -8 & 64 & 128 & \end{array}$$

$$\begin{array}{r|l} 1 & 2 & -16 & -34 & -4 & 32 & 64 & 0 & \end{array}$$

$$= (x^6 + 2x^5 - 16x^4 - 34x^3 - 4x^2 + 32x + 64)(x-2)$$

$$\begin{array}{r|l} 1 & 2 & -16 & -34 & -4 & 32 & 64 & -2 \\ & -2 & 0 & 32 & 4 & 0 & -64 & \end{array}$$

$$\begin{array}{r|l} 1 & 0 & -16 & -2 & 0 & 32 & 0 & \end{array}$$

$$= (x^5 - 16x^3 - 2x^2 + 32)(x-2)(x+2)$$

$$\begin{array}{r|l} 1 & 0 & -16 & -2 & 0 & 32 & +4 \\ & 4 & 16 & 0 & -8 & -32 & \end{array}$$

$$\begin{array}{r|l} 1 & 4 & 0 & -2 & -8 & 0 & \end{array}$$

$$= (x^4 + 4x^3 - 2x - 8)(x-2)(x+2)(x-4)$$

$$\begin{array}{r|l} 1 & 4 & 0 & -2 & -8 & -4 \\ & -4 & 0 & 0 & 8 & \end{array}$$

$$\begin{array}{r|l} 1 & 0 & 0 & -2 & 0 & \end{array}$$

$$= (x^3 - 2)(x-2)(x+2)(x-4)(x+4)$$

## EJERCICIO 111

1.  $a^2x, ax^2$  mcd  $ax$

2.  $ab^2c, a^2bc$  mcd  $abc$

3.  $2x^2y, x^2y^3$  mcd  $x^2y$

4.  $6a^2b^3, 15a^3b^4$

$$\begin{array}{c|c|c} 6 & 3 & 15 \\ 2 & 2 & 5 \\ 1 & 1 & 1 \end{array}$$

$$3 \cdot 2 = 6 \quad 3 \cdot 5 = 15$$

$$\text{mcd } 3 \Rightarrow 3a^2b^3$$

5.  $8am^3n, 20x^2m^2$

$$\begin{array}{c|c|c} 8 & 2 & 20 \\ 4 & 2 & 10 \\ 2 & 2 & 5 \\ 1 & 1 & 1 \end{array}$$

$$2^3 = 8 \quad 2^2 \cdot 5 = 20$$

$$\text{mcd } 2^2 = 4$$

$$\Rightarrow 4m^2$$

6.  $18mn^2, 27a^2m^3n^4$

$$\begin{array}{c|c|c} 18 & 3 & 27 \\ 6 & 3 & 9 \\ 2 & 2 & 3 \\ 1 & 1 & 1 \end{array}$$

$$3^2 \cdot 2 = 18 \quad 3^3 = 27$$

$$\text{mcd } 3^2 = 9$$

$$\Rightarrow 9mn^2$$

7.  $15a^2b^3c, 24ab^2x, 36b^4x^2$

$$\begin{array}{c|c|c} 15 & 5 & 24 \\ 3 & 3 & 8 \\ 1 & 2 & 2 \\ & 1 & 1 \end{array}$$

$$5 \cdot 3 = 15 \quad 3 \cdot 4 \cdot 2 = 24 \quad 3^2 \cdot 4 = 36$$

$$\text{mcd } 3 \Rightarrow 3b^2$$

8.  $12x^2yz^3, 18xy^2z, 24x^3yz^2$

$$\begin{array}{c|c|c} 12 & 3 & 18 \\ 4 & 2 & 6 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{array}$$

$$3 \cdot 2^2 = 12 \quad 3^2 \cdot 2 = 18 \quad 3 \cdot 4 \cdot 2 = 24$$

$$\text{mcd } 3 \cdot 2 \Rightarrow 6xyz$$

9.  $28a^2b^3c^4, 35a^3b^4c^5, 42a^4b^5c^6$

$$\begin{array}{c|c|c} 28 & 7 & 35 \\ 4 & 4 & 5 \\ 1 & 1 & 1 \end{array}$$

$$7 \cdot 4 = 28 \quad 7 \cdot 5 = 35 \quad 7 \cdot 6 = 42$$

$$\text{mcd } 7 \Rightarrow 7a^2b^3c^4$$

10.  $72x^3y^4z^4, 96x^2y^2z^3, 120x^4y^5z^7$

$$\begin{array}{c|c|c} 72 & 8 & 96 \\ 9 & 3 & 12 \\ 3 & 3 & 4 \\ 1 & 1 & 1 \end{array}$$

$$8 \cdot 3^2 = 72 \quad 8 \cdot 3 \cdot 4 = 96 \quad 8 \cdot 3 \cdot 5 = 120$$

$$\text{mcd } 8 \cdot 3 \Rightarrow 24x^2y^2z^3$$

11.  $42am^2n, 56m^3n^2x, 70m^4n^2y$

$$\begin{array}{c|c|c} 42 & 14 & 56 \\ 3 & 3 & 4 \\ 1 & 1 & 1 \end{array}$$

$$14 \cdot 3 = 42 \quad 14 \cdot 4 = 56 \quad 14 \cdot 5 = 70$$

$$\text{mcd } 14 \Rightarrow 14m^2n$$

12.  $75a^4b^3c^2, 150a^5b^7x^2, 225a^3b^6y^2$

$$\begin{array}{c|c|c} 75 & 15 & 150 \\ 5 & 5 & 10 \\ 1 & 2 & 2 \\ & 1 & 1 \end{array}$$

$$15 \cdot 5 = 75 \quad 15 \cdot 5 \cdot 2 = 150 \quad 15 \cdot 5 \cdot 3 = 225$$

$$\text{mcd } 15 \cdot 5 \Rightarrow 75a^3b^3$$

13.  $4a^2b, 8a^3b^2, 2a^2bc, 10ab^3c^2$

$$\begin{array}{c|c|c} 4 & 2 & 8 \\ 2 & 2 & 4 \\ 1 & 1 & 1 \end{array}$$

$$2^2 = 4 \quad 2 \cdot 4 = 8 \quad 2 = 2 \quad 2 \cdot 5 = 10$$

$$\text{mcd } 2 \Rightarrow 2ab$$

14.  $38a^2x^6y^4, 76mx^4y^7, 95x^5y^6$

$$\begin{array}{c|c|c} 38 & 19 & 76 \\ 2 & 2 & 4 \\ 1 & 1 & 1 \end{array}$$

$$19 \cdot 2 = 38 \quad 19 \cdot 4 = 76 \quad 19 \cdot 5 = 95$$

$$\text{mcd } 19 \Rightarrow 19x^4y^4$$

## EJERCICIO 112

1.  $2a^2 + 2ab = 2a(a+b)$

$$4a^2 - 4ab = 2^2a(a-b)$$

$$\text{mcd } 2a$$

2.  $6x^3y - 6x^2y = 2 \cdot 3x^2y(x-1)$

$$9x^3y^2 + 18x^2y^2 = 3^2x^2y^2(x+2)$$

$$\text{mcd } 3x^2y$$

3.  $12a^2b^3 = 4 \cdot 3a^2b^3$

$$4a^3b^2 - 8a^2b^3 = 4a^2b^2(a-2b)$$

$$\text{mcd } 4a^2b^2$$

4.  $ab+b = b(a+1)$

$$a^2+a = a(a+1)$$

$$\text{mcd } a+1$$

5.  $x^2 - x = x(x-1)$

$$x^3 - x^2 = x^2(x-1)$$

$$\text{mcd } x(x-1)$$

6.  $30ax^2 - 15x^3 = 5 \cdot 3x^2(2a-x)$

$$10axy^2 - 20x^2y^2 = 5 \cdot 2xy^2(a-2x)$$

$$\text{mcd } 5x$$

7.  $18a^2x^3y^4 = 6 \cdot 3a^2x^3y^4$

$$6a^2x^2y^4 - 18a^2xy^4 = 6a^2xy^4(x-3)$$

$$\text{mcd } 6a^2xy^4$$

8.  $5a^2 - 15a = 5a(a-3)$

$$a^3 - 3a^2 = a^2(a-3)$$

$$\text{mcd } a(a-3)$$

9.  $3x^3 + 15x^2 = 3x^2(x+5)$

$$ax^2 + 5ax = ax(x+5)$$

$$\text{mcd } x(x+5)$$

10.  $a^2 - b^2 = (a+b)(a-b)$

$$a^2 - 2ab + b^2 = (a-b)^2$$

$$\text{mcd } (a-b)$$

11.  $m^3 + n^3 = (m+n)(m^2 - mn + n^2)$

$$3am + 3an = 3a(m+n)$$

$$\text{mcd } m+n$$

12.  $x^2 - 4 = (x+2)(x-2)$

$$x^3 - 8 = (x-2)(x^2 + 2x + 4)$$

$$\text{mcd } x-2$$

13.  $2ax^2 + 4ax = 2ax(x+2)$   
 $x^3 - x^2 - 6x = x(x^2 - x - 6) = x(x-3)(x+2)$   
 $mcd \ x(x+2)$
14.  $9x^2 - 1 = (3x+1)(3x-1)$   
 $9x^2 - 6x + 1 = (3x-1)^2$   
 $mcd \ 3x-1$
15.  $4a^2 + 4ab + b^2 = (2a+b)^2$   
 $2a^2 - 2ab + ab - b^2 = (2a^2 - 2ab) + (ab - b^2)$   
 $= 2a(a-b) + b(a-b)$   
 $= (2a+b)(a-b)$   
 $mcd \ 2a+b$
16.  $3x^2 + 3x - 60 = 3(x^2 + x - 20)$   
 $= 3(x+5)(x-4)$   
 $6x^2 - 18x - 24 = 6(x^2 - 3x - 4)$   
 $= 3 \cdot 2(x-4)(x+1)$   
 $mcd \ 3(x-4)$
17.  $8x^3 + y^3 = (2x+y)(4x^2 - 2xy + y^2)$   
 $4ax^2 - ay^2 = a(4x^2 - y^2)$   
 $= a(2x+y)(2x-y)$   
 $mcd \ 2x+y$
18.  $2a^3 - 12a^2b + 18ab^2 = 2a(a^2 - 6ab + 9b^2)$   
 $= 2a(a-3b)^2$   
 $a^3x - 9ab^2x = ax(a^2 - 9b^2)$   
 $= ax(a+3b)(a-3b)$   
 $mcd \ a(a-3b)$
19.  $ac + ad - 2bc - 2bd = (ac + ad) - (2bc + 2bd)$   
 $= a(c+d) - 2b(c+d)$   
 $= (a-2b)(c+d)$   
 $2c^2 + 4cd + 2d^2 = 2(c^2 + 2cd + d^2)$   
 $= 2(c+d)^2$   
 $mcd \ c+d$
20.  $3a^2m^2 + 6a^2m - 45a^2 = 3a^2(m^2 + 2m - 15)$   
 $= 3a^2(m+5)(m-3)$   
 $6am^2x + 24amx - 30ax = 6ax(m^2 - 4m - 5)$   
 $= 3 \cdot 2ax(m+5)(m-1)$   
 $mcd \ 3a(m+5)$
21.  $4x^4 - y^2 = (2x^2 + y)(2x^2 - y)$   
 $(2x^2 + y)^2 = (2x^2 + y)(2x^2 + y)$   
 $mcd \ 2x^2 + y$
22.  $3x^5 - 3x = 3x(x^4 - 1) = 3x(x^2 + 1)(x^2 - 1)$   
 $= 3x(x^2 + 1)(x+1)(x-1)$   
 $9x^3 - 9x = 9x(x^2 - 1) = 3^2x(x+1)(x-1)$   
 $mcd \ 3x(x+1)(x-1)$
23.  $a^2 + ab = a(a+b)$   
 $ab + b^2 = b(a+b)$   
 $a^3 + a^2b = a^2(a+b)$   
 $mcd \ a+b$
24.  $2x^3 - 2x^2 = 2x^2(x-1)$   
 $3x^2 - 3x = 3x(x-1)$   
 $4x^3 - 4x^2 = 4x^2(x-1)$   
 $mcd \ x(x-1)$
25.  $x^4 - 9x^2 = x^2(x^2 - 9) = x^2(x+3)(x-3)$   
 $x^4 - 5x^3 + 6x^2 = x^2(x^2 - 5x + 6) = x^2(x-3)(x-2)$   
 $x^4 - 6x^3 + 9x^2 = x^2(x^2 - 6x + 9) = x^2(x-3)^2$   
 $mcd \ x^2(x-3)$
26.  $a^3b + 2a^2b^2 + ab^3 = ab(a^2 + 2ab + b^2) = ab(a+b)^2$   
 $a^4b - a^2b^3 = a^2b(a^2 - b^2) = a^2b(a+b)(a-b)$   
 $mcd \ ab(a+b)$
27.  $2x^2 + 2x - 4 = 2(x^2 + x - 2) = 2(x+2)(x-1)$   
 $2x^2 - 8x + 6 = 2(x^2 - 4x + 3) = 2(x-3)(x-1)$   
 $2x^3 - 2 = 2(x^2 - 1) = 2(x+1)(x-1)$   
 $mcd \ 2(x-1)$
28.  $ax^3 - 2ax^2 - 8ax = ax(x^2 - 2x - 8) = ax(x-4)(x+2)$   
 $ax^2 - ax - 6a = a(x^2 - x - 6) = a(x-3)(x+2)$   
 $a^2x^3 - 3a^2x^2 - 10a^2x = a^2x(x^2 - 3x - 10) = a^2x(x-5)(x+2)$   
 $mcd \ a(x+2)$
29.  $2an^4 - 16an^2 + 32a = 2a(n^4 - 8n^2 + 16)$   
 $= 2a(n^2 - 4)(n^2 - 4) = 2a(n+2)^2(n-2)^2$   
 $2an^3 - 8an = 2an(n^2 - 4) = 2an(n+2)(n-2)$   
 $2a^2n^3 + 16a^2 = 2a^2(n^3 + 8) = 2a^2(n+2)(n^2 - 2n + 4)$   
 $mcd \ 2a(n+2)$

$$\begin{aligned}
30. \quad & 4a^2 + 8a - 12 = 4(a^2 + 2a - 3) = 2^2(a+3)(a-1) \\
& 2a^2 - 6a + 4 = 2(a^2 - 3a + 2) = 2(a-2)(a-1) \\
& 6a^2 + 18a - 24 = 6(a^2 + 3a - 4) = 3 \cdot 2(a+4)(a-1) \\
& \text{mcd} \quad 2(a-1)
\end{aligned}$$

$$\begin{aligned}
31. \quad & 4a^2 - b^2 = (2a+b)(2a-b) \\
& 8a^3 + b^3 = (2a+b)(4a^2 - 2ab + b^2) \\
& 4a^2 + 4ab + b^2 = (2a+b)^2 \\
& \text{mcd} \quad 2a+b
\end{aligned}$$

$$\begin{aligned}
32. \quad & x^2 - 2x - 8 = (x-4)(x+2) \\
& x^2 - x - 12 = (x-4)(x+3) \\
& x^3 - 9x^2 + 20x = x(x^2 - 9x + 20) = x(x-5)(x-4) \\
& \text{mcd} \quad x-4
\end{aligned}$$

$$\begin{aligned}
33. \quad & a^2 + a = a(a+1) \\
& a^3 - 6a^2 - 7a = a(a^2 - 6a - 7) = a(a-7)(a+1) \\
& a^6 + a = a(a^5 + 1) = a(a+1)(a^4 - a^3 + a^2 - a + 1) \\
& \text{mcd} \quad a(a+1)
\end{aligned}$$

$$\begin{aligned}
34. \quad & x^3 + 27 = (x+3)(x^2 - 3x + 9) \\
& 2x^2 - 6x + 18 = 2(x^2 - 3x + 9) \\
& x^4 - 3x^3 + 9x^2 = x^2(x^2 - 3x + 9) \\
& \text{mcd} \quad x^2 - 3x + 9
\end{aligned}$$

$$\begin{aligned}
35. \quad & x^2 + ax - 6a^2 = (x+3a)(x-2a) \\
& x^2 + 2ax - 3a^2 = (x+3a)(x-a) \\
& x^2 + 6ax + 9a^2 = (x+3a)^2 \\
& \text{mcd} \quad x+3a
\end{aligned}$$

$$\begin{aligned}
36. \quad & 54x^3 + 250 = 2(27x^3 + 125) = 2(3x+5)(9x^2 - 15x + 25) \\
& 18ax^2 - 50a = 2a(9x^2 - 25) = 2a(3x+5)(3x-5) \\
& 18x^2 + 60x + 50 = 2(9x^2 + 30x + 25) = 2(3x+5)^2 \\
& \text{mcd} \quad 2(3x+5)
\end{aligned}$$

$$\begin{aligned}
37. \quad & (x^2 - 1)^2 = [(x+1)(x-1)]^2 \\
& x^2 - 4x - 5 = (x-5)(x+1) \\
& x^4 - 1 = (x^2 + 1)(x^2 - 1) = (x^2 + 1)(x+1)(x-1) \\
& \text{mcd} \quad x+1
\end{aligned}$$

$$\begin{aligned}
38. \quad & 4ax^2 - 28ax = 4ax(x-7) \\
& a^2x^3 - 8a^2x^2 + 7a^2x = a^2x(x^2 - 8x + 7) = a^2x(x-7)(x-1) \\
& ax^4 - 15ax^3 + 56ax^2 = ax^2(x^2 - 15x + 56) = ax^2(x-8)(x-7) \\
& \text{mcd} \quad ax(x-7)
\end{aligned}$$

$$\begin{aligned}
39. \quad & 3a^2 - 6a = 3a(a-2) \\
& a^3 - 4a = a(a^2 - 4) = a(a+2)(a-2) \\
& a^2b - 2ab = ab(a-2) \\
& a^2 - a - 2 = (a-2)(a+1) \\
& \text{mcd} \quad a-2
\end{aligned}$$

$$\begin{aligned}
40. \quad & 3x^2 - x = x(3x-1) \\
& 27x^3 - 1 = (3x-1)(9x^2 + 3x + 1) \\
& 9x^2 - 6x + 1 = (3x-1)^2 \\
& 3ax - a + 6x - 2 = (3ax - a) + (6x - 2) \\
& = a(3x-1) + 2(3x-1) \\
& = (a+2)(3x-1) \\
& \text{mcd} \quad 3x-1
\end{aligned}$$

$$\begin{aligned}
41. \quad & a^4 - 1 = (a^2 + 1)(a^2 - 1) = (a^2 + 1)(a+1)(a-1) \\
& a^3 + a^2 + a + 1 = (a^3 + a^2) + (a+1) \\
& = a^2(a+1) + (a+1) = (a^2 + 1)(a+1) \\
& a^3x + a^2x + ax + x = (a^3x + a^2x) + (ax + x) \\
& = a^2x(a+1) + x(a+1) \\
& = (a^2x + x)(a+1) = x(a^2 + 1)(a+1) \\
& a^5 + a^3 + a^2 + 1 = (a^5 + a^3) + (a^2 + 1) \\
& = a^3(a^2 + 1) + (a^2 + 1) \\
& = (a^3 + 1)(a^2 + 1) = (a+1)(a^2 - a + 1)(a^2 + 1) \\
& \text{mcd} \quad (a^2 + 1)(a+1)
\end{aligned}$$

$$\begin{aligned}
42. \quad & 2m^2 + 4mn + 2n^2 = 2(m^2 + 2mn + n^2) \\
& = 2(m+n)^2 \\
& m^3 + m^2n + mn^2 + n^3 = m^2(m+n) + n^2(m+n) \\
& = (m^2 + n^2)(m+n) \\
& m^3 + n^3 = (m+n)(m^2 - mn + n^2) \\
& m^3 - mn^2 = m(m^2 - n^2) \\
& = m(m+n)(m-n) \\
& \text{mcd} \quad m+n
\end{aligned}$$



$$\begin{aligned}
 43. \quad a^3 - 3a^2 + 3a - 1 &= (a^3 - 1) - 3a(a - 1) \\
 &= (a - 1)(a^2 + a + 1) - 3a(a - 1) \\
 &= (a - 1)(a^2 + a + 1 - 3a) \\
 &= (a - 1)(a^2 - 2a + 1) = (a - 1)(a - 1)^2 \\
 a^2 - 2a + 1 &= (a - 1)^2 \\
 a^3 - a &= a(a^2 - 1) = a(a + 1)(a - 1) \\
 a^2 - 4a + 3 &= (a - 3)(a - 1) \\
 mcd \quad a - 1
 \end{aligned}$$

$$\begin{aligned}
 44. \quad 16a^3x + 54x &= 2x(8a^3 + 27) \\
 &= 2x(2a + 3)(4a^2 - 6a + 9) \\
 12a^2x^2 - 42ax^2 - 90x^2 &= 6x^2(2a^2 - 7a - 15) \\
 &= 6x^2[(2a)^2 - 7(2a) - 30] \\
 &= [6x^2(2a - 10)(2a + 3)] \div 2 \\
 &= 3 \cdot 2x^2(a - 5)(2a + 3) \\
 32a^3x + 24a^2x - 36ax &= 4ax(8a^2 + 6a - 9) \\
 &= 4ax[(8a)^2 + 6(8a) - 72] \\
 &= [4ax(8a + 12)(8a - 6)] \div 4 \\
 &= 2^2ax(2a + 3)(8a - 6) \\
 &= 2^3ax(2a + 3)(4a - 3) \\
 32a^4x - 144a^2x + 162x &= 2x(16a^4 - 72a^2 + 81) \\
 &= 2x(4a^2 - 9)^2 \\
 &= 2x[(2a + 3)(2a - 3)]^2 \\
 mcd \quad 2x(2a + 3)
 \end{aligned}$$

$$\begin{aligned}
 45. \quad (xy + y^2)^2 &= x^2y^2 + 2xy^3 + y^4 \\
 &= y^2(x^2 + 2xy + y^2) = y^2(x + y)^2 \\
 x^2y - 2xy^2 - 3y^3 &= y(x^2 - 2xy - 3y^2) = y(x - 3y)(x + y) \\
 ax^3y + ay^4 &= ay(x^3 + y^3) = ay(x + y)(x^2 - xy + y^2) \\
 x^2y - y^3 &= y(x^2 - y^2) = y(x + y)(x - y) \\
 mcd \quad y(x + y)
 \end{aligned}$$

$$\begin{aligned}
 46. \quad 2a^2 - am + 4a - 2m &= (2a^2 - am) + (4a - 2m) \\
 &= a(2a - m) + 2(2a - m) \\
 &= (a + 2)(2a - m) \\
 2am^2 - m^3 &= m^2(2a - m) \\
 6a^2 + 5am - 4m^2 &= [(6a)^2 + 5m(6a) - 24m^2] \\
 &= [(6a + 8m)(6a - 3m)] \div 2 \cdot 3 \\
 &= (3a + 4m)(2a - m) \\
 16a^2 + 72am - 40m^2 &= 8(2a^2 + 9am - 5m^2) \\
 &= 8[(2a)^2 + 9m(2a) - 10m^2] \\
 &= 8[(2a + 10m)(2a - m)] \div 2 \\
 &= 8(a + 5m)(2a - m) \\
 mcd \quad 2a - m
 \end{aligned}$$

$$\begin{aligned}
 47. \quad 12ax - 6ay + 24bx - 12by &= (12ax - 6ay) + (24bx - 12by) \\
 &= 6a(2x - y) + 12b(2x - y) \\
 &= (6a + 12b)(2x - y) \\
 &= 3 \cdot 2(a + 2b)(2x - y) \\
 3a^3 + 24b^3 &= 3(a^3 + 8b^3) \\
 &= 3(a + 2b)(a^2 - 2ab + 4b^2) \\
 9a^2 + 9ab - 18b^2 &= 9(a^2 + ab - 2b^2) \\
 &= 3^2(a + 2b)(a - b) \\
 12a^2 + 24ab &= 12a(a + 2b) = 4 \cdot 3a(a + 2b) \\
 mcd \quad 3(a + 2b)
 \end{aligned}$$

$$\begin{aligned}
 48. \quad 5a^2 + 5ax + 5ay + 5xy &= 5a(a + x) + 5y(a + x) \\
 &= (5a + 5y)(a + x) = 5(a + y)(a + x) \\
 15a^3 - 15ax^2 + 15a^2y - 15x^2y &= 15a^2(a + y) - 15x^2(a + y) \\
 &= (15a^2 - 15x^2)(a + y) \\
 &= 15(a^2 - x^2)(a + y) \\
 &= 5 \cdot 3(a + x)(a - x)(a + y) \\
 20a^3 - 20ay^2 + 20a^2x - 20xy^2 &= 20a^2(a + x) - 20y^2(a + x) \\
 &= (20a^2 - 20y^2)(a + x) \\
 &= 20(a^2 - y^2)(a + x) \\
 &= 5 \cdot 4(a + y)(a - y)(a + x) \\
 mcd \quad 5(a + y)(a + x)
 \end{aligned}$$

## EJERCICIO 113

$$\begin{array}{r}
 1. \quad \begin{array}{r} 12x^2 + 8x + 1 \quad \overline{) 2x^2 - 5x - 3} \\ -12x^2 + 30x + 18 \quad \quad 6 \\ \hline 38x + 19 \quad (+2) \\ 2x^2 - 5x - 3 \quad \overline{) 2x + 1} \\ -2x^2 - x \quad \quad x \\ \hline -6x - 3 \quad (+-3) \\ 2x + 1 \quad \overline{) 2x + 1} \\ -2x - 1 \quad \quad 1 \quad \quad \text{mcd } 2x + 1 \end{array}
 \end{array}$$

$$\begin{array}{r}
 2. \quad \begin{array}{r} 3(2a^3 - a^2 - 6a) ; (6a^2 - 2a - 20) \div 2 \\ 6a^3 - 3a^2 - 18a \quad \overline{) 3a^2 - a - 10} \\ -6a^3 + 2a^2 + 20a \quad \quad 2a \\ \hline -a^2 + 2a \quad (+-a) \\ 3a^2 - a - 10 \quad \overline{) a - 2} \\ -3a^2 + 6a \quad \quad 3a \\ \hline 5a - 10 \quad (+2) \\ a - 2 \quad \overline{) a - 2} \\ -a + 2 \quad \quad 1 \quad \quad \text{mcd } a - 2 \end{array}
 \end{array}$$

$$\begin{array}{r}
 3. \quad \begin{array}{r} 3(5a^3 - 6a^2x + ax^2) ; 3a^3 - 4a^2x + ax^2 \\ = 15a^3 - 18a^2x + 3ax^2 = a(3a^2 - 4ax + x^2) \\ = a(15a^2 - 18ax + 3x^2) \\ 15a^2 - 18ax + 3x^2 \quad \overline{) 3a^2 - 4ax + x^2} \\ -15a^2 + 20ax - 5x^2 \quad \quad 5 \\ \hline 2ax - 2x^2 \quad (+2x) \\ 3a^2 - 4ax + x^2 \quad \overline{) a - x} \\ -3a^2 + 3ax \quad \quad 3a \\ \hline -ax + x^2 \quad (+-x) \\ a - x \quad \overline{) a - x} \\ -a + x \quad \quad 1 \quad \quad \text{mcd } a(a - x) \end{array}
 \end{array}$$

$$\begin{array}{r}
 4. \quad \begin{array}{r} (2x^3 + 4x^2 - 4x + 6) \div 2 \\ x^3 + 2x^2 - 2x + 3 \quad \overline{) x^3 + x^2 - x + 2} \\ -x^3 - x^2 + x - 2 \quad \quad 1 \\ \hline x^2 - x + 1 \\ x^3 + x^2 - x + 2 \quad \overline{) x^2 - x + 1} \\ -x^3 + x^2 - x \quad \quad x \\ \hline 2x^2 - 2x + 2 \quad (+2) \\ x^2 - x + 1 \quad \overline{) x^2 - x + 1} \\ -x^2 + x - 1 \quad \quad 1 \quad \quad \text{mcd } x^2 - x + 1 \end{array}
 \end{array}$$

$$\begin{array}{r}
 5. \quad \begin{array}{r} 8a^4 - 6a^3x + 7a^2x^2 - 3ax^3 ; 2a^3 + 3a^2x - 2ax^2 \\ = a(8a^3 - 6a^2x + 7ax^2 - 3x^3) = a(2a^2 + 3ax - 2x^2) \\ 8a^3 - 6a^2x + 7ax^2 - 3x^3 \quad \overline{) 2a^2 + 3ax - 2x^2 (3x)} \\ -8a^3 + 12a^2x + 8ax^2 \quad \quad 4a \\ \hline -18a^2x + 15ax^2 - 3x^3 \quad + -3 \\ 6a^2x + 9ax^2 - 6x^3 \quad \overline{) 6a^2x - 5ax^2 + x^3} \\ -6a^2x + 5ax^2 - x^3 \quad \quad 1 \\ \hline 14ax^2 - 7x^3 \quad + 7x^2 \\ 6a^2x - 5ax^2 + x^3 \quad \overline{) 2a - x} \\ -6a^2x + 3ax^2 \quad \quad 3ax \\ \hline -2ax^2 + x^3 \quad + -x^2 \\ 2a - x \quad \overline{) 2a - x} \\ -2a + x \quad \quad 1 \quad \quad \text{mcd } a(2a - x) \end{array}
 \end{array}$$

$$\begin{array}{r}
 6. \quad \begin{array}{r} (12ax^4 - 3ax^3 + 26ax^2 - 5ax + 10a) \div a \\ 12x^4 - 3x^3 + 26x^2 - 5x + 10 \quad \overline{) 3x^4 + 3x^3 - 4x^2 + 5x - 15 (5)} \\ -12x^4 + 12x^3 + 16x^2 - 20x + 60 \quad \quad 4 \\ \hline -15x^3 + 42x^2 - 25x + 70 \quad (-1) \\ 15x^4 + 15x^3 - 20x^2 + 25x - 75 \quad \overline{) 15x^3 - 42x^2 + 25x - 70 (19)} \\ -15x^4 + 42x^3 - 25x^2 + 70x \quad \quad x \\ \hline 57x^3 - 45x^2 + 95x - 75 \quad (5) \\ 285x^3 - 798x^2 + 475x - 1.330 \quad \overline{) 285x^3 - 225x^2 + 475x - 375} \\ -285x^3 + 225x^2 - 475x + 375 \quad \quad 1 \\ \hline -573x^2 \quad - 955 \quad (+-191) \\ 285x^3 - 225x^2 + 475x - 375 \quad \overline{) 3x^2 + 5} \\ -285x^3 \quad - 475x \quad \quad 95x \\ \hline -225x^2 \quad - 375 \quad (+-75) \\ 3x^2 + 5 \quad \overline{) 3x^2 + 5} \\ -3x^2 - 5 \quad \quad 1 \quad \quad \text{mcd } 3x^2 + 5 \end{array}
 \end{array}$$

$$\begin{array}{r}
 7. \quad \begin{array}{r} 6x^4 - 4x^3y - 3x^2y^2 + 5xy^3 - 2y^4 \quad \overline{) 3x^3 - 2x^2y + 9xy^2 - 6y^3 (7)} \\ -6x^4 + 4x^3y - 18x^2y^2 + 12xy^3 \quad \quad 2x \\ \hline -21x^2y^2 + 17xy^3 - 2y^4 \quad (+-y^2) \\ 21x^3 - 14x^2y + 63xy^2 - 42y^3 \quad \overline{) 21x^2 - 17xy + 2y^2} \\ -21x^3 + 17x^2y - 2xy^2 \quad \quad x \\ \hline 3x^2y + 61xy^2 - 42y^3 \quad (+y) \\ 21x^2 - 17xy + 2y^2 \quad \overline{) 3x^2 + 61xy - 42y^2} \\ -21x^2 + 427xy + 294y^2 \quad \quad 7 \\ \hline -444xy + 296y^2 \quad (+-148y) \\ 3x^2 + 61xy - 42y^2 \quad \overline{) 3x - 2y} \\ -3x^2 + 2xy \quad \quad x \\ \hline 63xy - 42y^2 \quad (+21y) \\ 3x - 2y \quad \overline{) 3x - 2y} \\ -3x + 2y \quad \quad 1 \quad \quad \text{mcd } 3x - 2y \end{array}
 \end{array}$$

$$8. (ax^4 + 3ax^3 - 2ax^2 + 6ax - 8a) \div a ;$$

$$\begin{array}{r} (x^4 + 4x^3 - x^2 - 4x) \div x \\ x^4 + 4x^3 - 2x^2 + 6x - 8 \quad | \quad x^3 + 4x^2 - x - 4 \\ -x^4 - 4x^3 + x^2 + 4x \\ \hline -x^3 - x^2 + 10x - 8 \quad (-1) \\ x^3 + 4x^2 - x - 4 \quad | \quad x^3 + x^2 - 10x + 8 \\ -x^3 - x^2 + 10x - 8 \quad 1 \\ \hline 3x^2 + 9x - 12 \quad (\div 3) \\ x^3 + x^2 - 10x + 8 \quad | \quad x^2 + 3x - 4 \\ -x^3 - 3x^2 + 4x \quad x \\ \hline -2x^2 - 6x + 8 \quad (\div -2) \\ x^2 + 3x - 4 \quad | \quad x^2 + 3x - 4 \\ -x^2 - 3x + 4 \quad 1 \quad mcd \quad x^2 + 3x - 4 \end{array}$$

$$9. 3(2m^4 - 4m^3 - m^2 + 6m - 3) ;$$

$$\begin{array}{r} (3m^5 - 6m^4 + 8m^3 - 10m^2 + 5m) \div m \\ 6m^4 - 12m^3 - 3m^2 + 18m - 9 \quad | \quad 3m^4 - 6m^3 + 8m^2 - 10m + 5 \\ -6m^4 + 12m^3 - 16m^2 + 20m - 10 \quad 2 \\ \hline -19m^2 + 38m - 19 \quad (\div -19) \\ 3m^4 - 6m^3 + 8m^2 - 10m + 5 \quad | \quad m^2 - 2m + 1 \\ -3m^4 + 6m^3 - 3m^2 \quad 3m^2 \\ \hline 5m^2 - 10m + 5 \quad (\div 5) \\ m^2 - 2m + 1 \quad | \quad m^2 - 2m + 1 \\ -m^2 + 2m - 1 \quad 1 \quad mcd \quad m^2 - 2m + 1 \end{array}$$

10. Factor común a

$$\begin{array}{r} 7(3a^5 - 6a^4 + 16a^3 - 2a^2 + 5a) \quad ; \quad 3(7a^5 - 14a^4 + 33a^3 + 4a^2 - 10a) \\ = 21a^5 - 42a^4 + 112a^3 - 14a^2 + 35a \quad ; \quad = 21a^5 - 42a^4 + 99a^3 + 12a^2 - 30a \\ 21a^4 - 42a^3 + 112a^2 - 14a + 35 \quad | \quad 21a^4 - 42a^3 + 99a^2 + 12a - 30 \\ -21a^4 + 42a^3 - 99a^2 - 12a + 30 \quad 1 \\ \hline 13a^2 - 26a + 65 \quad (\div 13) \\ 21a^4 - 42a^3 + 99a^2 + 12a - 30 \quad | \quad a^2 - 2a + 5 \\ -21a^4 + 42a^3 - 105a^2 \quad 21a^2 \\ \hline -6a^2 + 12a - 30 \quad (\div -6) \\ a^2 - 2a + 5 \quad | \quad a^2 - 2a + 5 \\ -a^2 + 2a - 5 \quad 1 \quad mcd \quad a(a^2 - 2a + 5) \end{array}$$

$$11. (45ax^3 + 75ax^2 - 18ax - 30a) \div 3a ;$$

$$\begin{array}{r} (24ax^3 + 40ax^2 - 30ax - 50a) \div 2a \\ 4(15x^3 + 25x^2 - 6x - 10) \quad ; \quad 5(12x^3 + 20x^2 - 15x - 25) \\ 60x^3 + 100x^2 - 24x - 40 \quad | \quad 60x^3 + 100x^2 - 75x - 125 \quad (\div 5) \\ -60x^3 - 100x^2 + 75x + 125 \quad 1 \\ \hline 51x + 85 \quad (\div 17) \\ 12x^3 + 20x^2 - 15x - 25 \quad | \quad 3x + 5 \\ -12x^3 - 20x^2 \quad 4x^2 \\ \hline -15x - 25 \quad (\div -5) \\ 3x + 5 \quad | \quad 3x + 5 \\ -3x - 5 \quad 1 \quad mcd \quad a(3x + 5) \end{array}$$

12. Factor común 2:

$$\begin{array}{r} 2x^3 + 2ax^2 + 2a^2x + 2a^3 \quad ; \quad 10x^3 + 4ax^2 + 10a^2x + 4a^3 \\ 5x^3 + 2ax^2 + 5a^2x + 2a^3 \quad | \quad x^3 + ax^2 + a^2x + a^3 \\ -5x^3 - 5ax^2 - 5a^2x - 5a^3 \quad 5 \\ \hline -3ax^2 - 3a^3 \quad (\div -3a) \\ x^3 + ax^2 + a^2x + a^3 \quad | \quad x^2 + a^2 \\ -x^3 - a^2x \quad x \\ \hline ax^2 + a^3 \quad (\div a) \\ x^2 + a^2 \quad | \quad x^2 + a^2 \\ -x^2 - a^2 \quad 1 \quad mcd \quad 2(x^2 + a^2) \end{array}$$

**13. Factor común 3 :**

$$\begin{array}{r}
 9x^3 + 15ax^2 + 3a^2x - 3a^3 ; \\
 12x^3 + 21ax^2 + 6a^2x - 3a^3 \quad (3) \\
 \underline{12x^3 + 21ax^2 + 6a^2x - 3a^3} \quad \left| \begin{array}{l} 3x^3 + 5ax^2 + a^2x - a^3 \\ -12x^3 - 20ax^2 - 4a^2x + 4a^3 \end{array} \right| \quad 4 \\
 \hline
 ax^2 + 2a^2x + a^3 \quad (\div a) \\
 \underline{3x^3 + 5ax^2 + a^2x - a^3} \quad \left| \begin{array}{l} x^2 + 2ax + a^2 \\ -3x^3 - 6ax^2 - 3a^2x \end{array} \right| \quad 3x \\
 \hline
 -ax^2 - 2a^2x - a^3 \quad (\div -a) \\
 \underline{x^2 + 2ax + a^2} \quad \left| \begin{array}{l} x^2 + 2ax + a^2 \\ -x^2 - 2ax - a^2 \end{array} \right| \quad 1 \quad mcd \ 3(x^2 + 2ax + a^2)
 \end{array}$$

**14. Factor común 2ab:**

$$\begin{array}{r}
 8a^4b + 4a^3b^2 + 4ab^4 ; \quad 12a^4b - 18a^3b^2 + 12a^2b^3 - 6ab^4 \\
 3(4a^3 + 2a^2b + 2b^3) \quad 6a^3 - 9a^2b + 6ab^2 - 3b^3 \\
 \underline{12a^3 + 6a^2b + 6b^3} \quad + 6b^3 \quad \left| \begin{array}{l} 6a^3 - 9a^2b + 6ab^2 - 3b^3 \\ -12a^3 + 18a^2b - 12ab^2 + 6b^3 \end{array} \right| \quad 2 \\
 \hline
 24a^2b - 12ab^2 + 12b^3 \quad (\div 12b) \\
 \underline{6a^3 - 9a^2b + 6ab^2 - 3b^3} \quad \left| \begin{array}{l} 2a^2 - ab + b^2 \\ -6a^3 + 3a^2b - 3ab^2 \end{array} \right| \quad 3a \\
 \hline
 -6a^2b + 3ab^2 - 3b^3 \quad (\div -3b) \\
 \underline{2a^2 - ab + b^2} \quad \left| \begin{array}{l} 2a^2 - ab + b^2 \\ -2a^2 + ab - b^2 \end{array} \right| \quad 1 \quad mcd \ 2ab(2a^2 - ab + b^2)
 \end{array}$$

**15. Factor común  $3a^2n^2$ :**

$$\begin{array}{r}
 9a^5n^2 - 33a^4n^3 + 27a^3n^4 - 6a^2n^5 ; \\
 9a^5n^2 + 12a^4n^3 - 21a^3n^4 + 6a^2n^5 \\
 \underline{3a^3 - 11a^2n + 9an^2 - 2n^3} \quad \left| \begin{array}{l} 3a^3 + 4a^2n - 7an^2 + 2n^3 \\ -3a^3 - 4a^2n + 7an^2 - 2n^3 \end{array} \right| \quad 1 \quad (5) \\
 \hline
 -15a^2n + 16an^2 - 4n^3 \quad (\div -n) \\
 \underline{15a^3 + 20a^2n - 35an^2 + 10n^3} \quad \left| \begin{array}{l} 15a^2 - 16an + 4n^2 \\ -15a^3 + 16a^2n - 4an^2 \end{array} \right| \quad a \quad (12) \\
 \hline
 36a^2n - 39an^2 + 10n^3 \quad (\div n) \\
 \underline{180a^2 - 192an + 48n^2} \quad \left| \begin{array}{l} 36a^2 - 39an + 10n^2 \\ -180a^2 + 195an - 50n^2 \end{array} \right| \quad 5 \\
 \hline
 3an - 2n^2 \quad (\div n) \\
 \underline{36a^2 - 39an + 10n^2} \quad \left| \begin{array}{l} 3a - 2n \\ -36a^2 + 24an \end{array} \right| \quad 12a \\
 \hline
 -15an + 10n^2 \quad (\div -5n) \\
 \underline{3a - 2n} \quad \left| \begin{array}{l} 3a - 2n \\ -3a + 2n \end{array} \right| \quad 1 \quad mcd \ 3a^2n^2(3a - 2n)
 \end{array}$$

$$\begin{array}{r}
 16. \quad \frac{a^7 - a^6 + a^4 + 1}{-a^7 + 2a^6 - a^5 - a^3 + a^2} + 1 \quad \left| \frac{a^5 - 2a^4 + a^3 + a - 1}{a^2} \right| (a) \\
 \hline
 \frac{a^6 - a^5 + a^4 - a^3 + a^2 + 1}{a^6 - 2a^5 + a^4 + a^2 - a} + 1 \quad \left| \frac{a^6 - a^5 + a^4 - a^3 + a^2 + 1}{-a^6 + a^5 - a^4 + a^3 - a^2 - 1} \right| \quad 1 \\
 \hline
 \frac{-a^5 + a^3 - a - 1}{a^6 - a^5 + a^4 - a^3 + a^2} + 1 \quad \left| \frac{a^5 - a^3 + a + 1}{-a^6 + a^4 - a^2 - a} \right| \quad a \\
 \hline
 \frac{-a^5 + 2a^4 - a^3 - a + 1}{a^5 - a^3 + a + 1} + 1 \quad \left| \frac{a^5 - 2a^4 + a^3 + a - 1}{-a^5 + 2a^4 - a^3 - a + 1} \right| \quad 1 \\
 \hline
 \frac{2a^4 - 2a^3 + 2}{a^5 - 2a^4 + a^3 + a - 1} + 1 \quad \left| \frac{a^4 - a^3 + 1}{-a^5 + a^4 - a} \right| \quad a \\
 \hline
 \frac{-a^4 + a^3 - 1}{a^4 - a^3 + 1} + 1 \quad \left| \frac{a^4 - a^3 + 1}{-a^4 + a^3 - 1} \right| \quad (-1) \\
 \hline
 \frac{a^4 - a^3 + 1}{-a^4 + a^3 - 1} + 1 \quad \left| \frac{a^4 - a^3 + 1}{-a^4 + a^3 - 1} \right| \quad 1 \quad mcd \ a^4 - a^3 + 1
 \end{array}$$

**17. Factor común 2a :**

$$\begin{array}{r}
 6ax^4 - 4ax^3 + 6ax^2 - 10ax + 4a ; \\
 36ax^4 - 24ax^3 - 18ax^2 + 48ax - 24a \\
 \underline{18x^4 - 12x^3 - 9x^2 + 24x - 12} \quad \left| \begin{array}{l} 3x^4 - 2x^3 + 3x^2 - 5x + 2 \\ -18x^4 + 12x^3 - 18x^2 + 30x - 12 \end{array} \right| \quad 6 \quad (9) \\
 \hline
 -27x^2 + 54x - 24 \quad (-1) \\
 \underline{27x^4 - 18x^3 + 27x^2 - 45x + 18} \quad \left| \begin{array}{l} 27x^2 - 54x + 24 \\ -27x^4 + 54x^3 - 24x^2 \end{array} \right| \quad x^2 \quad (4x) \\
 \hline
 36x^3 + 3x^2 - 45x + 18 \quad (\div 3) \\
 \underline{108x^3 - 216x^2 + 96x} \quad \left| \begin{array}{l} 12x^3 + x^2 - 15x + 6 \\ -108x^3 - 9x^2 + 135x - 54 \end{array} \right| \quad 9 \quad (25) \\
 \hline
 -225x^2 + 231x - 54 \quad (\div -3) \\
 \underline{300x^3 + 25x^2 - 375x + 150} \quad \left| \begin{array}{l} 75x^2 - 77x + 18 \\ -300x^3 + 308x^2 - 72x \end{array} \right| \quad 4x \quad (111) \\
 \hline
 333x^2 - 447x + 150 \quad (\div 3) \\
 \underline{8.325x^2 - 8.547x + 1.998} \quad \left| \begin{array}{l} 111x^2 - 149x + 50 \\ -8.325x^2 + 11.175x - 3.750 \end{array} \right| \quad 75 \quad (2.628) \\
 \hline
 2.628x - 1.752 \\
 \underline{291.708x^2 - 391.572x + 131.400} \quad \left| \begin{array}{l} 2.628x - 1.752 \\ -291.708x^2 + 194.472x \end{array} \right| \quad 111x \quad (\div 876) \\
 \hline
 -197.100x + 131.400 \quad (\div -65.700) \\
 \underline{3x - 2} \quad \left| \begin{array}{l} 3x - 2 \\ -3x + 2 \end{array} \right| \quad 1 \quad mcd \ 2a(3x - 2)
 \end{array}$$

**EJERCICIO 114**     2.  $\frac{8x^3 + 6x^2y - 3xy^2 - y^3}{-8x^3 + 4x^2y + 8xy^2 - 4y^3} \mid \frac{2x^3 - x^2y - 2xy^2 + y^3}{4}$

$$\begin{array}{r}
 1. \quad 2x^3 - 5x^2 - 6x + 9 \quad \left| \begin{array}{l} 2x^2 - 5x - 3 \\ -2x^3 + 5x^2 + 3x \\ \hline -3x + 9 \end{array} \right. \quad \begin{array}{l} \phantom{0} \\ \phantom{0} \\ \div (-3) \end{array} \\
 \hline
 2x^2 - 5x - 3 \quad \left| \begin{array}{l} x - 3 \\ -2x^2 + 6x \\ \hline x - 3 \end{array} \right. \\
 \hline
 x^3 - 2x^2 - 5x + 6 \quad \left| \begin{array}{l} x - 3 \\ -x^3 + 3x^2 \\ \hline x^2 - 5x + 6 \end{array} \right. \quad (x) \\
 \hline
 x^2 - 3x \quad \left| \begin{array}{l} x^2 - 5x + 6 \\ +5x - 6 \\ \hline 2x - 6 \end{array} \right. \quad \begin{array}{l} \phantom{0} \\ 1 \\ \div (2) \end{array} \\
 \hline
 x^2 - 5x + 6 \quad \left| \begin{array}{l} x - 3 \\ -x^2 + 2x \\ \hline -3x + 6 \end{array} \right. \quad \begin{array}{l} \phantom{0} \\ x \\ \div (-3) \end{array} \\
 \hline
 x - 3 \quad \left| \begin{array}{l} x - 3 \\ -x + 3 \\ \hline 0 \end{array} \right. \quad \begin{array}{l} \phantom{0} \\ 1 \\ mcd \end{array} \quad x - 3
 \end{array}$$

$$\begin{array}{r}
4. \quad 3a^4 + 9a^3x + 9a^2x^2 - 3ax^3 - 2x^4 \quad \left| \begin{array}{l} a^4 + 3a^3x + a^2x^2 - 3ax^3 - 2x^4 \\ -3a^4 - 9a^3x - 3a^2x^2 + 9ax^3 + 6x^4 \end{array} \right. \quad \frac{a^2x^2 + 6ax^3 + 8x^4}{3} \quad \div x^2 \\
\hline
a^4 + 3a^3x + a^2x^2 - 3ax^3 - 2x^4 \quad \left| \begin{array}{l} a^2 + 6ax + 8x^2 \quad (3a) \\ -a^4 - 6a^3x - 8a^2x^2 \end{array} \right. \quad \frac{a^2}{a^2} \\
\hline
-3a^3x - 7a^2x^2 - 3ax^3 - 2x^4 \quad \div x \\
\hline
3a^3 + 18a^2x + 24ax^2 \quad \left| \begin{array}{l} 3a^3 + 7a^2x + 3ax^2 + 2x^3 \quad (11) \\ -3a^3 - 7a^2x - 3ax^2 - 2x^3 \end{array} \right. \quad 1 \\
\hline
11a^2x + 21ax^2 - 2x^3 \quad \div x \\
\hline
33a^3 + 77a^2x + 33ax^2 + 22x^3 \quad \left| \begin{array}{l} 11a^2 + 21ax - 2x^2 \quad (14) \\ -33a^3 - 63a^2x + 6ax^2 \end{array} \right. \quad 3a \\
\hline
14a^2x + 39ax^2 + 22x^3 \quad \div x \\
\hline
154a^2 + 294ax - 28x^2 \quad \left| \begin{array}{l} 14a^2 + 39ax + 22x^2 \\ -154a^2 - 429ax - 242x^2 \end{array} \right. \quad 11 \\
\hline
-135ax - 270x^2 \quad \div (-135x) \\
\hline
14a^2 + 39ax + 22x^2 \quad \left| \begin{array}{l} a + 2x \\ -14a^2 - 28ax \end{array} \right. \quad \frac{a + 2x}{14a} \quad \left| \begin{array}{l} a + 2x \\ -a - 2x \end{array} \right. \quad 1 \\
\hline
11ax + 22x^2 \quad \div (11x) \quad 0 \\
\hline
4a^3 + 8a^2x - ax^2 - 2x^3 \quad \left| \begin{array}{l} a + 2x \\ -4a^3 - 8a^2x \end{array} \right. \quad \frac{a + 2x}{4a^2} \\
\hline
-ax^2 - 2x^3 \quad \div (-x^2) \\
\hline
a + 2x \quad \left| \begin{array}{l} a + 2x \\ -a - 2x \end{array} \right. \quad 1 \quad mcd \quad a + 2x
\end{array}$$

$$\begin{array}{r} 8x^3 + 6x^2y - 3xy^2 - y^3 \\ -8x^3 + 4x^2y + 8xy^2 - 4y^3 \\ \hline 2x^3 - x^2y - 2xy^2 + y^3 \end{array}$$

$$\begin{array}{r}
10x^2y + 5xy^2 - 5y^3 \quad (\div 5y) \\
\hline
2x^3 - x^2y - 2xy^2 + y^3 \quad | \quad 2x^2 + xy - y^2 \\
-2x^3 - x^2y + xy^2 \\
\hline
-2x^2y - xy^2 + y^3 \quad (\div -y) \\
\hline
2x^2 + xy - y^2 \quad | \quad 2x^2 + xy - y^2 \\
-2x^2 - xy + y^2 \\
\hline
0 \\
\hline
6x^2 - xy - y^2 \quad | \quad 2x^2 + xy - y^2 \\
-6x^2 - 3xy + 3y^2 \quad 3 \\
\hline
-4xy + 2y^2 \quad (\div -2y) \\
\hline
2x^2 + xy - y^2 \quad | \quad 2x - y \\
-2x^2 + xy \\
\hline
2xy - y^2 \quad (\div +y) \\
\hline
2x - y \quad | \quad 2x - y \\
-2x + y \quad 1 \qquad mcd \quad 2x - y
\end{array}$$

$$\begin{array}{cc} -2x+y & 1 \\ mcd & 2x-y \end{array}$$

$$\begin{array}{r}
 3. \quad (x^4 + x^3 - x^2 - x) \div x ; \\
 (2x^3 + 2x^2 - 2x - 2) \div 2 \\
 \begin{array}{r}
 x^3 + x^2 - x - 1 \quad | \quad x^3 + x^2 - x - 1 \\
 -x^3 - x^2 + x + 1 \quad | \quad 1 \\
 \hline
 0
 \end{array} \\
 \begin{array}{r}
 5x^3 - 5x^2 + 2x - 2 \quad | \quad x^3 + x^2 - x - 1 \quad (10) \\
 -5x^3 - 5x^2 + 5x + 5 \quad | \quad 5 \\
 \hline
 -10x^2 + 7x + 3 \quad (-1) \\
 10x^3 + 10x^2 - 10x - 10 \quad | \quad 10x^2 - 7x - 3 \quad (17) \\
 -10x^3 + 7x^2 + 3x \quad | \quad x \\
 \hline
 17x^2 - 7x - 10 \\
 170x^2 - 119x - 51 \quad | \quad 17x^2 - 7x - 10 \\
 -170x^2 + 70x + 100 \quad | \quad 10 \\
 \hline
 -49x + 49 \quad (\div -49) \\
 17x^2 - 7x - 10 \quad | \quad x - 1 \\
 -17x^2 + 17x \quad | \quad 17x \\
 \hline
 10x - 10 \quad (\div 10) \\
 x - 1 \quad | \quad x - 1 \\
 -x + 1 \quad | \quad 1 \quad mcd \quad x - 1
 \end{array}
 \end{array}$$

### 5. Factor común $x$

$$\begin{array}{r}
2(2x^5 + 2x^4 - 2x^2 - 2x) \\
4x^4 + 4x^3 \quad -4x - 4 \quad \left| \begin{array}{l} 4x^3 - 4x^2 + 3x - 3 \\ x \end{array} \right. \quad (2) \\
-4x^4 + 4x^3 - 3x^2 + 3x \\
\hline
8x^3 - 3x^2 - x - 4 \\
8x^3 - 8x^2 + 6x - 6 \quad \left| \begin{array}{l} 8x^3 - 3x^2 - x - 4 \\ 8x^3 - 3x^2 - x - 4 \end{array} \right. \quad (5) \\
-8x^3 + 3x^2 + x + 4 \quad 1 \\
\hline
-5x^2 + 7x - 2 \quad (-1) \\
40x^3 - 15x^2 - 5x - 20 \quad \left| \begin{array}{l} 5x^2 - 7x + 2 \\ 5x^2 - 7x + 2 \end{array} \right. \quad (41) \\
-40x^3 + 56x^2 - 16x \quad 8x \\
\hline
41x^2 - 21x - 20 \\
205x^2 - 287x + 82 \quad \left| \begin{array}{l} 41x^2 - 21x - 20 \\ 41x^2 - 21x - 20 \end{array} \right. \\
-205x^2 + 105x + 100 \quad 5 \\
\hline
-182x + 182 \quad (\div -182) \\
41x^2 - 21x - 20 \quad \left| \begin{array}{l} x-1 \\ x-1 \end{array} \right. \quad x-1 \left| \begin{array}{l} x-1 \\ x-1 \end{array} \right. \\
-41x^2 + 41x \quad 41x \quad \frac{-x+1}{1} \quad 1 \\
\hline
20x - 20 \quad (\div 20) \quad 0 \\
3x^6 \quad -4x^4 - 3x^3 + 4x \quad \left| \begin{array}{l} x-1 \\ x-1 \end{array} \right. \quad (3x^4) \\
-3x^6 + 3x^5 \quad 3x^5 \\
\hline
3x^5 - 4x^4 - 3x^3 + 4x
\end{array}$$

**Continua.**

### Continuación.

$$\begin{array}{r}
 5. \quad 3x^5 - 3x^4 \quad \left| \begin{array}{r} 3x^5 - 4x^4 - 3x^3 + 4x^2 + x \\ -3x^5 + 4x^4 + 3x^3 - 4x^2 \\ \hline x^4 + 3x^3 - 4x^2 + x \\ -x^4 + 3x^3 - 4x^2 + x \\ \hline 3x^4 - 4x^3 - 3x^2 + 4x \\ -3x^4 + 9x^3 + 12x^2 - 4x \\ \hline -13x^3 - 3x^2 + 12x + 4 \\ -13x^3 + 39x^2 - 52x + 4 \\ \hline -13x^3 - 3x^2 + 12x + 4 \\ \hline 36x^2 + 12x - 48 \\ \hline 39x^3 + 9x^2 - 36x - 12 \\ -39x^3 - 13x^2 + 52x \\ \hline -4x^2 + 16x - 12 \\ \hline 3x^2 + x - 4 \\ -3x^2 + 12x - 9 \\ \hline 13x - 13 \\ \hline x^2 - 4x + 3 \\ -x^2 + x \\ \hline -3x + 3 \\ \hline x - 1 \\ -x + 1 \\ \hline 1 \end{array} \right. \quad \begin{array}{l} (\div x) \\ (13) \\ (\div -1) \\ (3) \\ (\div 12) \\ (3x^2 + x - 4) \\ (x^2 - 4x + 3) \\ (\div -3) \\ (x - 1) \\ mcd \quad x(x-1) \end{array}
 \end{array}$$

$$16. 3x^3, 6x^2, 9x^4y^2 \text{ mcm } 18x^4y^2$$

$$17. 9a^2bx, 12ab^2x^2, 18a^3b^3x \text{ mcm } 36a^3b^3x^2$$

$$18. 10m^2, 15mn^2, 20n^3 \text{ mcm } 60m^2n^3$$

$$19. 18a^3, 24b^2, 36ab^3 \text{ mcm } 72a^3b^3$$

$$20. 20m^2n^3, 24m^3n, 30mn^2$$

$$\begin{array}{ccc|c}
 20 & 24 & 30 & 6 \\
 20 & 4 & 5 & 5 \cdot 6 \cdot 4 = 120 \\
 4 & 4 & 1 & 4 \Rightarrow \text{mcm } 120m^3n^3 \\
 1 & 1 & & 
 \end{array}$$

$$21. ab^2, bc^2, a^2c^3, b^3c^3 \text{ mcm } a^2b^3c^3$$

$$22. 2x^2y, 8xy^3, 4a^2x^3, 12a^3 \text{ mcm } 24a^3x^3y^3$$

$$23. 6a^2, 9x, 12ay^2, 18x^3y \text{ mcm } 36a^2x^3y^2$$

$$24. 15mn^2, 10m^2, 20n^3, 25mn^4$$

$$\begin{array}{cccc|c}
 15 & 10 & 20 & 25 & 5 \\
 3 & 2 & 4 & 5 & 5 \cdot 5^2 \cdot 3 \cdot 2 = 300 \\
 3 & 2 & 4 & 1 & 3 \Rightarrow \text{mcm } 300m^2n^4 \\
 1 & 2 & 4 & & 2 \\
 & 1 & 2 & & 2 \\
 & & 1 & & 1
 \end{array}$$

## EJERCICIO 115

$$1. a^2, ab^2 \text{ mcm } a^2b^2$$

$$2. x^2y, xy^2 \text{ mcm } x^2y^2$$

$$3. ab^2c, a^2bc \text{ mcm } a^2b^2c$$

$$4. a^2x^3, a^3bx^2 \text{ mcm } a^3bx^3$$

$$5. 6m^2n, 4m^3 \text{ mcm } 12m^3n$$

$$6. 9ax^3y, 15x^2y^5 \text{ mcm } 45ax^3y^5$$

$$7. a^3, ab^2, a^2b \text{ mcm } a^3b^2$$

$$8. x^2y, xy^2, xy^3z \text{ mcm } x^2y^3z$$

$$9. 2ab^2, 4a^2b, 8a^3 \text{ mcm } 8a^3b^2$$

$$10. 3x^2y^3z, 4x^3y^3z^2, 6x^4 \text{ mcm } 12x^4y^3z^2$$

$$11. 6m^2n^2, 9m^2n^3, 12m^3n \text{ mcm } 36m^3n^3$$

$$12. 3a^2, 4b^2, 8x^2 \text{ mcm } 24a^2b^2x^2$$

$$13. 5x^2, 10xy, 15xy^2 \text{ mcm } 30x^2y^2$$

$$14. ax^3y^2, a^3xy, a^2x^2y^3 \text{ mcm } a^3x^3y^3$$

$$15. 4ab, 6a^2, 3b^2 \text{ mcm } 12a^2b^2$$

$$25. 24a^2x^3, 36a^2y^4, 40x^2y^5, 60a^3y^6$$

$$\begin{array}{cccc|c}
 24 & 36 & 40 & 60 & 2 \\
 12 & 18 & 20 & 30 & 2 \\
 6 & 9 & 10 & 15 & 3 \cdot 2^3 \cdot 3^2 \cdot 5 = 360 \\
 2 & 3 & 10 & 5 & 5 \Rightarrow \text{mcm } 360a^3x^3y^6 \\
 2 & 3 & 2 & 1 & 3 \\
 2 & 1 & 2 & & 2 \\
 1 & & 1 & & 1
 \end{array}$$

$$26. 3a^3, 8ab, 10b^2, 12a^2b^3, 16a^2b^2$$

$$\begin{array}{cccc|c}
 3 & 8 & 10 & 12 & 16 & 2 \\
 3 & 4 & 5 & 6 & 8 & 2 \\
 3 & 2 & 5 & 3 & 4 & 2 \cdot 2^4 \cdot 3 \cdot 5 = 240 \\
 3 & 1 & 5 & 3 & 2 & 2 \Rightarrow \text{mcm } 240a^3b^3 \\
 3 & & 5 & 3 & 1 & 3 \\
 1 & & 5 & 1 & & 5 \\
 & & 1 & & & 1
 \end{array}$$

## EJERCICIO 116

1.  $2a = 2a$   
 $4x - 8 = 2^2(x - 2)$   
*mcm*  $4a(x - 2)$
2.  $ab - b^2 = b(a - b)$   
 $3b^2 = 3b^2$   
*mcm*  $3b^2(a - b)$
3.  $x^2y + xy^2 = xy(x + y)$   
 $x^2y = x^2y$   
*mcm*  $x^2y(x + y)$
4.  $4 + 8a = 2^2(1 + 2a)$   
 $8 = 2^3$   
*mcm*  $2^3(1 + 2a) = 8(1 + 2a)$   
 $6a^2b = 2 \cdot 3a^2b$
5.  $3a^2b^2 + 6ab^3 = 3ab^2(a + 2b)$   
*mcm*  $6a^2b^2(a + 2b)$
6.  $14x^2 = 7 \cdot 2x^2$   
 $6x^2 + 4xy = 2x(3x + 2y)$   
*mcm*  $14x^2(3x + 2y)$
7.  $9m = 3^2m$   
 $6mn^2 - 12mn = 2 \cdot 3mn(n - 2)$   
*mcm*  $2 \cdot 3^2mn(n - 2)$   
 $\Rightarrow 18mn(n - 2)$
8.  $15 = 5 \cdot 3$   
 $3x + 6 = 3(x + 2)$   
*mcm*  $15(x + 2)$
9.  $10 = 5 \cdot 2$   
 $5 - 15b = 5(1 - 3b)$   
*mcm*  $10(1 - 3b)$
10.  $4ax - 12ay = 2^2a(x - 3y)$   
 $36a^2 = 2^2 \cdot 3^2a^2$   
*mcm*  $2^2 \cdot 3^2a^2(x - 3y)$   
 $\Rightarrow 36a^2(x - 3y)$
11.  $12xy^2 = 2^2 \cdot 3xy^2$   
 $2ax^2y^3 + 5x^2y^3 = x^2y^3(2a + 5)$   
*mcm*  $12x^2y^3(2a + 5)$
12.  $mn^3 - mn^2 = mn^2(n - 1)$   
 $mn = mn$   
 $m^2 = m^2$   
*mcm*  $m^2n^2(n - 1)$
13.  $3a^2 - 6ab = 3a(a - 2b)$   
 $2a^2 = 2a^2$   
 $6ab = 2 \cdot 3ab$   
*mcm*  $6a^2b(a - 2b)$
14.  $5x^5 - 5x^4 = 5x^4(x - 1)$   
 $xy^2 = xy^2$   
 $x^2y^3 = x^2y^3$   
*mcm*  $5x^4y^3(x - 1)$
15.  $27a^4b + 81a^3b^2 = 3^3a^3b(a + 3b)$   
 $9a^2 = 3^2a^2$   
 $18b^3 = 2 \cdot 3^2b^3$   
*mcm*  $2 \cdot 3^3a^3b^3(a + 3b)$   
 $\Rightarrow 54a^3b^3(a + 3b)$
16.  $9x^3y + 9xy^3 = 3^2xy(x^2 + y^2)$   
 $10 = 5 \cdot 2$   
 $6x^2 = 2 \cdot 3x^2$   
*mcm*  $5 \cdot 2 \cdot 3^2x^2y(x^2 + y^2)$   
 $\Rightarrow 90x^2y(x^2 + y^2)$
17.  $x^2y - xy = xy(x - 1)$   
 $x^3 + x^2 = x^2(x + 1)$   
 $4x = 2^2x$   
*mcm*  $2^2x^2y(x - 1)(x + 1)$   
 $\Rightarrow 4x^2y(x^2 - 1)$
18.  $6m^2 + 18m = 2 \cdot 3m(m + 3)$   
 $8m - 24 = 2^3(m - 3)$   
 $24 = 2^3 \cdot 3$   
*mcm*  $2^3 \cdot 3m(m + 3)(m - 3)$   
 $\Rightarrow 24m(m^2 - 9)$
19.  $3ax + 3a = 3a(x + 1)$   
 $2a^2b^2 = 2a^2b^2$   
 $6x - 18 = 2 \cdot 3(x - 3)$   
*mcm*  $6a^2b^2(x + 1)(x - 3)$
20.  $x^2 + 4x + 4 = (x + 2)^2$   
 $x^3 + x^2 - 2x = x(x^2 + x - 2)$   
 $= x(x + 2)(x - 1)$   
 $x^2 = x^2$   
*mcm*  $x^2(x + 2)^2(x - 1)$
21.  $9a^2x - 18a^2y = 3^2a^2(x - 2y)$   
 $x^2 - 4xy + 4y^2 = (x - 2y)^2$   
 $6ab = 2 \cdot 3ab$   
*mcm*  $3^2 \cdot 2a^2b(x - 2y)^2$   
 $\Rightarrow 18a^2b(x - 2y)^2$
22.  $9x^4 - 36x^2 = 3^2x^2(x^2 - 4)$   
 $= 3^2x^2(x + 2)(x - 2)$   
 $3x^3 - 3x^2 - 18x = 3x(x^2 - x - 6)$   
 $= 3x(x - 3)(x + 2)$   
 $6x^3 = 2 \cdot 3x^3$   
*mcm*  $2 \cdot 3^2x^3(x + 2)(x - 2)(x - 3)$   
 $\Rightarrow 18x^3(x^2 - 4)(x - 3)$
23.  $4x^3 - 12x^2y + 9xy^2 = x(4x^2 - 12xy + 9y^2)$   
 $= x(2x - 3y)^2$   
 $2x^4 - 3x^3y = x^3(2x - 3y)$   
 $a^2x^2 = a^2x^2$   
*mcm*  $a^2x^3(2x - 3y)^2$
24.  $9x^2 - 45x = 3^2x(x - 5)$   
 $12x^2y^2 = 3 \cdot 2^2x^2y^2$   
 $8x^3 = 2^3x^3$   
*mcm*  $3^2 \cdot 2^3x^3y^2(x - 5)$   
 $\Rightarrow 72x^3y^2(x - 5)$
25.  $n^2x^2 + n^2y^2 = n^2(x^2 + y^2)$   
 $nx^2 + 2nxy + ny^2 = n(x^2 + 2xy + y^2)$   
 $= n(x + y)^2$   
 $2n = 2n$   
 $an^3 = an^3$   
*mcm*  $2an^3(x + y)^2(x^2 + y^2)$
26.  $4x^3 + 24x^2 + 36x = 2^2x(x^2 + 6x + 9)$   
 $= 2^2x(x + 3)^2$   
 $2x^3 - 8x^2 + 8x = 2x(x^2 - 4x + 4)$   
 $= 2x(x - 2)^2$   
 $x^3 + x^2 - 6x = x(x^2 + x - 6)$   
 $= x(x + 3)(x - 2)$   
 $8x^2 = 2^3x^2$   
*mcm*  $8x^2(x + 3)^2(x - 2)^2$

$$\begin{aligned}
 27. \quad & 6x^3 + 6x^2 = 2 \cdot 3x^2(x+1) \\
 & 2x^2 - 2x + 2 = 2(x^2 - x + 1) \\
 & x^3 + 1 = (x+1)(x^2 - x + 1) \\
 & 3x^3 = 3x^3
 \end{aligned}$$

$$\begin{aligned}
 mcm \quad & 2 \cdot 3x^3(x+1)(x^2 - x + 1) \\
 & \Rightarrow 6x^3(x+1)(x^2 - x + 1)
 \end{aligned}$$

$$\begin{aligned}
 29. \quad & 5ab^3 - 5b^4 = 5b^3(a-b) \\
 & 12a^2 - 24ab + 12b^2 = 2^2 \cdot 3(a^2 - 2ab + b^2) \\
 & = 2^2 \cdot 3(a-b)^2 \\
 & 6a^2b = 2 \cdot 3a^2b \\
 & 4b = 2^2b \\
 & 2a = 2a
 \end{aligned}$$

$$mcm = 5 \cdot 2^2 \cdot 3a^2b^3(a-b)^2 \Rightarrow 60a^2b^3(a-b)^2$$

$$\begin{aligned}
 28. \quad & ax - a + bx - b = a(x-1) + b(x-1) = (a+b)(x-1) \\
 & a^2 + 2ab + b^2 = (a+b)^2
 \end{aligned}$$

$$4xy^2 = 2^2xy^2$$

$$3x^3 - 3x^2 = 3x^2(x-1)$$

$$mcm \quad 2^2 \cdot 3x^2y^2(a+b)^2(x-1) \Rightarrow 12x^2y^2(a+b)^2(x-1)$$

$$30. \quad 14x + 14 = 7 \cdot 2(x+1)$$

$$x^2 + 2x + 1 = (x+1)^2$$

$$7x^2 + 7 = 7(x^2 + 1)$$

$$28x = 7 \cdot 2^2x$$

$$x^2 + 1 = (x^2 + 1)$$

$$mcm \quad 28x(x+1)^2(x^2 + 1)$$

## EJERCICIO 117

$$1. \quad 6x - 6 = 2 \cdot 3(x-1)$$

$$3x + 3 = 3(x+1)$$

$$mcm \quad 2 \cdot 3(x-1)(x+1)$$

$$\Rightarrow 6(x^2 - 1)$$

$$2. \quad 10x^2 - 40 = 5 \cdot 2(x^2 - 4)$$

$$= 5 \cdot 2(x+2)(x-2)$$

$$5x + 10 = 5(x+2)$$

$$mcm \quad 5 \cdot 2(x+2)(x-2)$$

$$\Rightarrow 10(x^2 - 4)$$

$$3. \quad x^2 - 4y^2 = (x+2y)(x-2y)$$

$$x^3 + 2x^2y = x^2(x+2y)$$

$$mcm \quad x^2(x+2y)(x-2y)$$

$$\Rightarrow x^2(x^2 - 4y^2)$$

$$4. \quad x^2 - 6x + 9 = (x-3)^2$$

$$3a^2x - 9a^2 = 3a^2(x-3)$$

$$mcm \quad 3a^2(x-3)^2$$

$$6. \quad a^3 + 2a^2b + ab^2 = a(a^2 + 2ab + b^2)$$

$$= a(a+b)^2$$

$$a^3 + a^2b = a^2(a+b)$$

$$mcm \quad a^2(a+b)^2$$

$$7. \quad 2bx^2 + 6bx - 8b = 2b(x^2 + 3x - 4)$$

$$= 2b(x+4)(x-1)$$

$$3ax + 12a = 3a(x+4)$$

$$mcm \quad 2b \cdot 3a(x+4)(x-1)$$

$$\Rightarrow 6ab(x+4)(x-1)$$

$$8. \quad x^2 + 2x - 15 = (x+5)(x-3)$$

$$x^3 - 25x = x(x^2 - 25)$$

$$= x(x+5)(x-5)$$

$$mcm \quad x(x+5)(x-5)(x-3)$$

$$\Rightarrow x(x^2 - 25)(x-3)$$

$$9. \quad (x-1)^2 = (x-1)^2$$

$$x^2 - 1 = (x+1)(x-1)$$

$$mcm \quad (x+1)(x-1)^2$$

$$5. \quad 4a^2 - 12ab + 9b^2 = (2a - 3b)^2$$

$$4a^2 - 9b^2 = (2a + 3b)(2a - 3b)$$

$$mcm \quad (2a + 3b)(2a - 3b)^2$$

$$10. \quad (x+1)^2 = (x+1)^2$$

$$x^2 + 1 = x^2 + 1$$

$$mcm \quad (x^2 + 1)(x+1)^2$$

$$11. \quad x^3 + y^3 = (x+y)(x^2 - xy + y^2)$$

$$(x+y)^3 = (x+y)^3$$

$$mcm \quad (x+y)^3(x^2 - xy + y^2)$$

$$12. \quad x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

$$(x-y)^3 = (x-y)^3$$

$$mcm \quad (x-y)^3(x^2 + xy + y^2)$$

$$13. \quad 4x^2 - 7x - 2 = (4x)^2 - 7(4x) - 8$$

$$= [(4x-8)(4x+1)] + 4$$

$$= (x-2)(4x+1)$$

$$x^2 + 3x - 10 = (x+5)(x-2)$$

$$mcm \quad (x-2)(4x+1)(x+5)$$

$$14. \quad a^2 + a - 30 = (a+6)(a-5)$$

$$a^2 + 3a - 18 = (a+6)(a-3)$$

$$mcm \quad (a+6)(a-3)(a-5)$$

$$15. \quad x^4 + 2x^3 - 15x^2 = x^2(x^2 + 2x - 15)$$

$$= x^2(x+5)(x-3)$$

$$x^3 - 9x + 5x^2 - 45 = x(x^2 - 9) + 5(x^2 - 9)$$

$$= (x+5)(x^2 - 9)$$

$$= (x+5)(x+3)(x-3)$$

$$mcm \quad x^2(x+5)(x+3)(x-3)$$

$$\Rightarrow x^2(x+5)(x^2 - 9)$$



$$\begin{aligned}
 16. \quad x^6 - 4x^3 - 32 &= (x^3 - 8)(x^3 + 4) \\
 &= (x - 2)(x^2 + 2x + 4)(x^3 + 4) \\
 4x^4 + 2ax^3 + 4ax^2 &= ax^2(x^2 + 2x + 4) \\
 mcm \quad ax^2(x - 2)(x^2 + 2x + 4)(x^3 + 4)
 \end{aligned}$$

$$\begin{aligned}
 17. \quad 12(x^2 - y^2) &= 2^2 \cdot 3(x + y)(x - y) \\
 8(x - y)^2 &= 2^3(x - y)^2 \\
 mcm \quad 2^3 \cdot 3(x + y)(x - y)^2 &\Rightarrow 24(x + y)(x - y)^2
 \end{aligned}$$

$$\begin{aligned}
 18. \quad 5(x + y)^2 &= 5(x + y)^2 \\
 10(x^2 + y^2) &= 5 \cdot 2(x^2 + y^2) \\
 mcm \quad 10(x^2 + y^2)(x + y)^2
 \end{aligned}$$

$$\begin{aligned}
 19. \quad 6a(m + n)^3 &= 2 \cdot 3a(m + n)^3 \\
 4a^2b(m^3 + n^3) &= 2^2 a^2 b(m + n)(m^2 - mn + n^2) \\
 mcm \quad 2^2 \cdot 3a^2 b(m + n)^3(m^2 - mn + n^2) \\
 &\Rightarrow 12a^2 b(m + n)^3(m^2 - mn + n^2)
 \end{aligned}$$

$$\begin{aligned}
 20. \quad ax(m - n)^3 &= ax(m - n)^3 \\
 x^3(m^3 - n^3) &= x^3(m - n)(m^2 + mn + n^2) \\
 mcm \quad ax^3(m - n)^3(m^2 + mn + n^2)
 \end{aligned}$$

$$\begin{aligned}
 21. \quad 2a^2 + 2a &= 2a(a + 1) \\
 3a^2 - 3a &= 3a(a - 1) \\
 a^4 - a^2 &= a^2(a^2 - 1) = a^2(a + 1)(a - 1) \\
 mcm \quad 2 \cdot 3a^2(a + 1)(a - 1) &\Rightarrow 6a^2(a^2 - 1)
 \end{aligned}$$

$$\begin{aligned}
 22. \quad x^2 + 2x &= x(x + 2) \\
 x^3 - 2x^2 &= x^2(x - 2) \\
 x^2 - 4 &= (x + 2)(x - 2) \\
 mcm \quad x^2(x - 2)(x + 2) &\Rightarrow x^2(x^2 - 4)
 \end{aligned}$$

$$\begin{aligned}
 23. \quad x^2 + x - 2 &= (x + 2)(x - 1) \\
 x^2 - 4x + 3 &= (x - 3)(x - 1) \\
 x^2 - x - 6 &= (x - 3)(x + 2) \\
 mcm(x + 2)(x - 1)(x - 3)
 \end{aligned}$$

$$\begin{aligned}
 24. \quad 4 + 12a + 9a^2 &= (2 + 3a)^2 \\
 3a^2 + 14a + 8 &= (3a)^2 + 14(3a) + 24 \\
 &= [(3a + 12)(3a + 2)] \div 3 = (a + 4)(3a + 2) \\
 6a^2 + 13a + 6 &= (6a)^2 + 13(6a) + 36 \\
 &= [(6a + 9)(6a + 4)] \div 3 \cdot 2 = (2a + 3)(3a + 2) \\
 mcm \quad (a + 4)(2a + 3)(3a + 2)^2
 \end{aligned}$$

$$\begin{aligned}
 25. \quad 10x^2 + 10 &= 5 \cdot 2(x^2 + 1) \\
 15x + 15 &= 5 \cdot 3(x + 1) \\
 5x^2 - 5 &= 5(x^2 - 1) = 5(x + 1)(x - 1) \\
 mcm \quad 5 \cdot 2 \cdot 3(x^2 + 1)(x + 1)(x - 1) &\Rightarrow 30(x^2 + 1)(x^2 - 1)
 \end{aligned}$$

$$\begin{aligned}
 26. \quad ax - 2bx + ay - 2by &= x(a - 2b) + y(a - 2b) \\
 &= (x + y)(a - 2b) \\
 x^2 + xy &= x(x + y) \\
 x^2 - xy &= x(x - y) \\
 mcm \quad x(x + y)(x - y)(a - 2b) &\Rightarrow x(x^2 - y^2)(a - 2b)
 \end{aligned}$$

$$\begin{aligned}
 27. \quad 4a^2b + 4ab^2 &= 2^2 ab(a + b) \\
 6a - 6b &= 2 \cdot 3(a - b) \\
 15a^2 - 15b^2 &= 5 \cdot 3(a^2 - b^2) = 5 \cdot 3(a + b)(a - b) \\
 mcm \quad 2^2 \cdot 3 \cdot 5ab(a + b)(a - b) &\Rightarrow 60ab(a^2 - b^2)
 \end{aligned}$$

$$\begin{aligned}
 28. \quad x^2 - 25 &= (x + 5)(x - 5) \\
 x^3 - 125 &= (x - 5)(x^2 + 5x + 25) \\
 2x + 10 &= 2(x + 5) \\
 mcm \quad 2(x + 5)(x - 5)(x^2 + 5x + 25)
 \end{aligned}$$

$$\begin{aligned}
 29. \quad a^3b - 6a^2b^2 + 9ab^3 &= ab(a^2 - 6ab + 9b^2) \\
 &= ab(a - 3b)^2 \\
 a^2 - 2ab - 3b^2 &= (a - 3b)(a + b) \\
 ab^2 + b^3 &= b^2(a + b) \\
 mcm \quad ab^2(a - 3b)^2(a + b)
 \end{aligned}$$

$$\begin{aligned}
 30. \quad 2m^2 + 2mn &= 2m(m + n) \\
 4mn - 4n^2 &= 2^2 n(m - n) \\
 6m^3n - 6mn^3 &= 2 \cdot 3mn(m^2 - n^2) = 2 \cdot 3mn(m + n)(m - n) \\
 mcm \quad 2^2 \cdot 3mn(m + n)(m - n) &\Rightarrow 12mn(m^2 - n^2)
 \end{aligned}$$

$$\begin{aligned}
 31. \quad 20(x^2 - y^2) &= 2^2 \cdot 5(x + y)(x - y) \\
 15(x - y)^2 &= 5 \cdot 3(x - y)^2 \\
 12(x + y)^2 &= 2^2 \cdot 3(x + y)^2 \\
 mcm \quad 2^2 \cdot 5 \cdot 3(x + y)^2(x - y)^2 &\Rightarrow 60(x + y)^2(x - y)^2 \\
 32. \quad ax^2 + 5ax - 14a &= a(x^2 + 5x - 14) = a(x + 7)(x - 2) \\
 x^3 + 14x^2 + 49x &= x(x^2 + 14x + 49) = x(x + 7)^2 \\
 x^4 + 7x^3 - 18x^2 &= x^2(x^2 + 7x - 18) = x^2(x + 9)(x - 2) \\
 mcm \quad ax^2(x + 7)^2(x - 2)(x + 9)
 \end{aligned}$$

33.  $2x^3 - 12x^2 + 18x = 2x(x^2 - 6x + 9) = 2x(x-3)^2$   
 $3x^4 - 27x^2 = 3x^2(x^2 - 9) = 3x^2(x+3)(x-3)$   
 $5x^3 + 30x^2 + 45x = 5x(x^2 + 6x + 9) = 5x(x+3)^2$   
 $mcm \ 2 \cdot 3 \cdot 5x^2(x+3)^2(x-3)^2 \Rightarrow 30x^2(x+3)^2(x-3)^2$
34.  $3 - 3a^2 = 3(1 - a^2) = 3(1+a)(1-a)$   
 $6 + 6a = 2 \cdot 3(1+a)$   
 $12 + 12a^2 = 2^2 \cdot 3(1+a^2)$   
 $9 - 9a = 3^2(1-a)$   
 $mcm \ 2^2 \cdot 3^2(1+a^2)(1+a)(1-a)$   
 $\Rightarrow 36(1+a^2)(1-a^2) = 36(1-a^4)$
35.  $2(3n-2)^2 = 2(3n-2)^2$   
 $135n^3 - 40 = 5(27n^3 - 8) = 5(3n-2)(9n^2 + 6n + 4)$   
 $12n - 8 = 2^2(3n-2)$   
 $mcm \ 5 \cdot 2^2(3n-2)^2(9n^2 + 6n + 4)$   
 $\Rightarrow 20(3n-2)^2(9n^2 + 6n + 4)$
36.  $12mn + 8m - 3n - 2 = 4m(3n+2) - (3n+2)$   
 $= (4m-1)(3n+2)$   
 $48m^2n - 3n + 32m^2 - 2 = 16m^2(3n+2) - (3n+2)$   
 $= (16m^2-1)(3n+2)$   
 $= (4m+1)(4m-1)(3n+2)$   
 $6n^2 - 5n - 6 = (6n)^2 - 5(6n) - 36$   
 $= [(6n-9)(6n+4)] \div 3 \cdot 2$   
 $= (2n-3)(3n+2)$   
 $mcm \ (4m+1)(4m-1)(3n+2)(2n-3)$   
 $\Rightarrow (16m^2-1)(3n+2)(2n-3)$
37.  $15a^2x^5 + 16a^2x^4 - 15a^2x^3 = a^2x^3(15x^2 + 16x - 15)$   
 $= a^2x^3[(15x)^2 + 16(15x) - 225]$   
 $= a^2x^3[(15x+25)(15x-9) \div 5 \cdot 3]$   
 $= a^2x^3(3x+5)(5x-3)$   
 $18x^3 + 60x^2 + 50x = 2x(9x^2 + 30x + 25)$   
 $= 2x[(9x)^2 + 30(9x) + 225]$   
 $= 2x[(9x+15)(9x+15) \div 3 \cdot 3]$   
 $= 2x(3x+5)^2$   
 $12ax^3 + 20ax^2 = 2^2ax^2(3x+5)$   
 $mcm = 2^2a^2x^3(3x+5)^2(5x-3)$   
 $= 4a^2x^3(3x+5)^2(5x-3)$
38.  $16 + 8x^2 + x^4 = (4 + x^2)^2$   
 $16 - 8x^2 + x^4 = (4 - x^2)^2 = (2-x)^2(2+x)^2$   
 $16 - x^4 = (4 + x^2)(4 - x^2) = (4 + x^2)(2+x)(2-x)$   
 $mcm \ (4 + x^2)^2(2+x)^2(2-x)^2$
39.  $1 + a^2 = 1 + a^2$   
 $(1 + a)^2 = (1 + a)^2$   
 $1 + a^3 = (1+a)(1-a+a^2)$   
 $mcm \ (1+a)^2(1-a+a^2)(1+a^2)$
40.  $8n^2 - 10n - 3 = (8n)^2 - 10(8n) - 24$   
 $= [(8n-12)(8n+2) \div 4 \cdot 2 = (2n-3)(4n+1)]$   
 $20n^2 + 13n + 2 = (20n)^2 + 13(20n) + 40$   
 $= [(20n+8)(20n+5) \div 4 \cdot 5 = (5n+2)(4n+1)]$   
 $10n^2 - 11n - 6 = (10n)^2 - 11(10n) - 60$   
 $= [(10n-15)(10n+4) \div 5 \cdot 2 = (2n-3)(5n+2)]$   
 $mcm \ (2n-3)(5n+2)(4n+1)$
41.  $6a^2 + ab - 2b^2 = 36a^2 + 6ab - 12b^2$   
 $= [(6a+4b)(6a-3b) \div 2 \cdot 3]$   
 $= (3a+2b)(2a-b)$   
 $15a^2 + 22ab + 8b^2 = (15a)^2 + 22b(15a) + 120b^2$   
 $= [(15a+12b)(15a+10b) \div 3 \cdot 5]$   
 $= (5a+4b)(3a+2b)$   
 $10a^2 + 3ab - 4b^2 = (10a)^2 + 3b(10a) - 40b^2$   
 $= [(10a+8b)(10a-5b) \div 2 \cdot 5]$   
 $= (5a+4b)(2a-b)$   
 $mcm \ (3a+2b)(2a-b)(5a+4b)$
42.  $12x^2 + 5xy - 2y^2 = (12x)^2 + 5y(12x) - 24y^2$   
 $= [(12x+8y)(12x-3y) \div 4 \cdot 3]$   
 $= (3x+2y)(4x-y)$   
 $15x^2 + 13xy + 2y^2 = (15x)^2 + 13y(15x) + 30y^2$   
 $= [(15x+10y)(15x+3y) \div 5 \cdot 3]$   
 $= (3x+2y)(5x+y)$   
 $20x^2 - xy - y^2 = (20x)^2 - y(20x) - 20y^2$   
 $= [(20x-5y)(20x+4y) \div 5 \cdot 4]$   
 $= (4x-y)(5x+y)$   
 $mcm \ (4x-y)(5x+y)(3x+2y)$

$$\begin{aligned}
 43. \quad & 6b^2x^2 + 6b^2x^3 = 2 \cdot 3b^2x^2(1+x) \\
 & 3a^2x - 3a^2x^2 = 3a^2x(1-x) \\
 & 1-x^4 = (1+x^2)(1-x^2) = (1+x^2)(1+x)(1-x) \\
 & mcm \ 2 \cdot 3a^2b^2x^2(1+x^2)(1+x)(1-x) \\
 & \Rightarrow 6a^2b^2x^2(1+x^2)(1-x^2)
 \end{aligned}$$

$$\begin{aligned}
 44. \quad & x^4 + 8x - 4x^3 - 32 = x^3(x-4) + 8(x-4) \\
 & = (x^3+8)(x-4) \\
 & = (x+2)(x^2-2x+4)(x-4) \\
 & a^2x^4 - 2a^2x^3 - 8a^2x^2 = a^2x^2(x^2-2x-8) \\
 & = a^2x^2(x-4)(x+2) \\
 & 2x^4 - 4x^3 + 8x^2 = 2x^2(x^2-2x+4) \\
 & mcm \ 2a^2x^2(x+2)(x^2-2x+4)(x-4)
 \end{aligned}$$

$$\begin{aligned}
 45. \quad & x^4 - 10x^2 + 9 = (x^4 - 10x^2 + 9 + 4x^2 - 4x^2) \\
 & = (x^4 - 6x^2 + 9) - 4x^2 \\
 & = (x^2 - 3)^2 - 4x^2 \\
 & = (x^2 - 2x - 3)(x^2 + 2x - 3) \\
 & = (x-3)(x+1)(x+3)(x-1) \\
 & x^2 + 4x + 3 = (x^2 + 4x + 3 + 1 - 1) \\
 & = (x^2 + 4x + 4) - 1 \\
 & = (x+2)^2 - 1 \\
 & = (x+2+1)(x+2-1) = (x+3)(x+1) \\
 & x^2 - 4x + 3 = (x^2 - 4x + 3 + 1 - 1) \\
 & = (x^2 - 4x + 4) - 1 \\
 & = (x-2)^2 - 1 \\
 & = (x-2+1)(x-2-1) = (x-1)(x-3) \\
 & x^3 - 9x + x^2 - 9 = x^2(x+1) - 9(x+1) \\
 & = (x^2-9)(x+1) \\
 & = (x+3)(x-3)(x+1) \\
 & mcm \ (x+3)(x-3)(x+1)(x-1) \Rightarrow (x^2-9)(x^2-1)
 \end{aligned}$$

$$\begin{aligned}
 46. \quad & 1-a^3 = (1-a)(1+a+a^2) \\
 & 1-a = (1-a) \\
 & 1-a^2 = (1+a)(1-a) \\
 & 1-2a+a^2 = (1-a)^2 \\
 & mcm \ (1-a)^2(1+a+a^2)(1+a)
 \end{aligned}$$

$$\begin{aligned}
 47. \quad & a(ab-b^2)^2 = a[b(a-b)]^2 = ab^2(a-b)^2 \\
 & b(a^2+ab)^2 = b[a(a+b)]^2 = a^2b(a+b)^2 \\
 & a^4b^2 - a^2b^4 = a^2b^2(a^2-b^2) = a^2b^2(a+b)(a-b) \\
 & a^2b - ab^2 = ab(a-b) \\
 & mcm \ a^2b^2(a+b)^2(a-b)^2
 \end{aligned}$$

$$\begin{aligned}
 48. \quad & m^3 - 27n^3 = (m-3n)(m^2+3mn+9n^2) \\
 & m^2 - 9n^2 = (m-3n)(m+3n) \\
 & m^2 - 6mn + 9n^2 = (m-3n)^2 \\
 & m^2 + 3mn + 9n^2 = (m^2+3mn+9n^2) \\
 & mcm \ (m-3n)^2(m^2+3mn+9n^2)(m+3n)
 \end{aligned}$$

## EJERCICIO 118

$$\begin{aligned}
 1. \quad & \frac{a^2}{ab} = \frac{a}{b} & 10. \quad \frac{21mn^3x^6}{28m^4n^2x^2} = \frac{3nx^4}{4m^3} \\
 2. \quad & \frac{2a}{8a^2b} = \frac{1}{4ab} & 11. \quad \frac{42a^2c^3n}{26a^4c^5m} = \frac{21n}{13a^2c^2m} \\
 3. \quad & \frac{x^2y^2}{x^3y^3} = \frac{1}{xy} & 12. \quad \frac{7x^3y^4z^6}{34x^7y^8z^{10}} = \frac{1}{2x^4y^4z^4} \\
 4. \quad & \frac{ax^3}{4x^5y} = \frac{a}{4x^2y} & 13. \quad \frac{30x^6y^2}{45a^3x^4z^3} = \frac{2x^2y^2}{3a^3z^3} \\
 5. \quad & \frac{6m^2n^3}{3m} = 2mn^3 & 14. \quad \frac{a^5b^7}{3a^8b^9c} = \frac{1}{3a^3b^2c} \\
 6. \quad & \frac{9x^2y^3}{24a^2x^3y^4} = \frac{3}{8a^2xy} & 15. \quad \frac{21a^8b^{10}c^{12}}{63a^4bc^2} = \frac{a^4b^3c^{10}}{3} \\
 7. \quad & \frac{8m^4n^3x^2}{24mn^2x^2} = \frac{m^3n}{3} & 16. \quad \frac{54x^9y^{11}z^{13}}{63x^{10}y^{12}z^{15}} = \frac{6}{7xyz^2} \\
 8. \quad & \frac{12x^3y^4z^5}{32xy^2z} = \frac{3x^2y^2z^4}{8} & 17. \quad \frac{15a^{12}b^{15}c^{20}}{75a^{11}b^{16}c^{22}} = \frac{a}{5bc^2} \\
 9. \quad & \frac{12a^2b^3}{60a^3b^5x^6} = \frac{1}{5ab^2x^6} & 18. \quad \frac{75a^7m^5}{100a^3m^{12}n^3} = \frac{3a^4}{4m^7n^3}
 \end{aligned}$$

## EJERCICIO 119

1.  $\frac{3ab}{2a^2x+2a^3} = \frac{3ab}{2a^2(x+a)} = \frac{3b}{2a(x+a)}$
2.  $\frac{xy}{3x^2y-3xy^2} = \frac{xy}{3xy(x-y)} = \frac{1}{3(x-y)}$
3.  $\frac{2ax+4bx}{3ay+6by} = \frac{2x(a+2b)}{3y(a+2b)} = \frac{2x}{3y}$
4.  $\frac{x^2-2x-3}{x-3} = \frac{(x-3)(x+1)}{x-3} = x+1$
5.  $\frac{10a^2b^3c}{80(a^3-a^2b)} = \frac{a^2b^3c}{8a^2(a-b)} = \frac{b^3c}{8(a-b)}$
6.  $\frac{x^2-4}{5ax+10a} = \frac{(x+2)(x-2)}{5a(x+2)} = \frac{x-2}{5a}$
7.  $\frac{3x^2-4x-15}{x^2-5x+6} = \frac{(3x)^2-4(3x)-45}{(x-3)(x-2)} = \frac{[(3x-9)(3x+5)] \div 3}{(x-3)(x-2)} = \frac{(x-3)(3x+5)}{(x-3)(x-2)} = \frac{3x+5}{x-2}$
8.  $\frac{15a^2bn-45a^2bm}{10a^2b^2n-30a^2b^2m} = \frac{15a^2b(n-3m)}{10a^2b^2(n-3m)} = \frac{3}{2b}$
9.  $\frac{x^2-y^2}{x^2+2xy+y^2} = \frac{(x+y)(x-y)}{(x+y)^2} = \frac{x-y}{x+y}$
10.  $\frac{3x^2y+15xy}{x^2-25} = \frac{3xy(x+5)}{(x+5)(x-5)} = \frac{3xy}{x-5}$
11.  $\frac{a^2-4ab+4b^2}{a^3-8b^3} = \frac{(a-2b)^2}{(a-2b)(a^2+2ab+4b^2)} = \frac{a-2b}{a^2+2ab+4b^2}$
12.  $\frac{x^3+4x^2-21x}{x^3-9x} = \frac{x(x^2+4x-21)}{x(x^2-9)} = \frac{(x+7)(x-3)}{(x+3)(x-3)} = \frac{x+7}{x+3}$
13.  $\frac{6x^2+5x-6}{15x^2-7x-2} = \frac{(6x)^2+5(6x)-36}{(15x)^2-7(15x)-30} = \frac{[(6x+9)(6x-4)] \div 3 \cdot 2}{[(15x-10)(15x+3)] \div 5 \cdot 3} = \frac{(2x+3)(3x-2)}{(3x-2)(5x+1)} = \frac{2x+3}{5x+1}$
14.  $\frac{a^3+1}{a^4-a^3+a-1} = \frac{a^3+1}{a^3(a-1)+(a-1)} = \frac{a^3+1}{(a^3+1)(a-1)} = \frac{1}{a-1}$
15.  $\frac{2ax+ay-4bx-2by}{ax-4a-2bx+8b} = \frac{2x(a-2b)+y(a-2b)}{x(a-2b)-4(a-2b)} = \frac{(2x+y)(a-2b)}{(x-4)(a-2b)} = \frac{2x+y}{x-4}$
16.  $\frac{a^2-ab-6b^2}{a^3x-6a^2bx+9ab^2x} = \frac{(a-3b)(a+2b)}{ax(a^2-6b+9b^2)} = \frac{(a-3b)(a+2b)}{ax(a-3b)^2} = \frac{a+2b}{ax(a-3b)}$
17.  $\frac{m^2+n^2}{m^4-n^4} = \frac{m^2+n^2}{(m^2+n^2)(m^2-n^2)} = \frac{1}{m^2-n^2}$
18.  $\frac{x^3+y^3}{(x+y)^3} = \frac{(x+y)(x^2-xy+y^2)}{(x+y)^3} = \frac{x^2-xy+y^2}{(x+y)^2}$
19.  $\frac{(m-n)^2}{m^2-n^2} = \frac{(m-n)^2}{(m+n)(m-n)} = \frac{m-n}{m+n}$
20.  $\frac{(a-x)^3}{a^3-x^3} = \frac{(a-x)^3}{(a-x)(a^2+ax+x^2)} = \frac{(a-x)^2}{a^2+ax+x^2}$
21.  $\frac{a^2-a-20}{a^2-7a+10} = \frac{(a-5)(a+4)}{(a-5)(a-2)} = \frac{a+4}{a-2}$
22.  $\frac{(1-a^2)^2}{a^2+2a+1} = \frac{[(a+1)(1-a)]^2}{(a+1)^2} = (1-a)^2$
23.  $\frac{a^4b^2-a^2b^4}{a^4-b^4} = \frac{a^2b^2(a^2-b^2)}{(a^2+b^2)(a^2-b^2)} = \frac{a^2b^2}{a^2+b^2}$
24.  $\frac{x^2-y^2}{x^3-y^3} = \frac{(x+y)(x-y)}{(x-y)(x^2+xy+y^2)} = \frac{x+y}{x^2+xy+y^2}$
25.  $\frac{24a^3b+8a^2b^2}{36a^4+24a^3b+4a^2b^2} = \frac{8a^2b(3a+b)}{4a^2(9a^2+6ab+b^2)} = \frac{2b(3a+b)}{(3a+b)^2} = \frac{2b}{3a+b}$
26.  $\frac{n^3-n}{n^2-5n-6} = \frac{n(n^2-1)}{(n-6)(n+1)} = \frac{n(n+1)(n-1)}{(n-6)(n+1)} = \frac{n(n-1)}{n-6}$
27.  $\frac{8n^3+1}{8n^3-4n^2+2n} = \frac{(2n+1)(4n^2-2n+1)}{2n(4n^2-2n+1)} = \frac{2n+1}{2n}$

$$28. \frac{a^2 - (b-c)^2}{(a+b)^2 - c^2}$$

$$= \frac{(a+b-c)(a-b+c)}{(a+b-c)(a+b+c)} = \frac{a-b+c}{a+b+c}$$

$$29. \frac{(a+b)^2 - (c-d)^2}{(a+c)^2 - (b-d)^2}$$

$$= \frac{(a+b+c-d)(a+b-c+d)}{(a+b+c-d)(a-b+c+d)}$$

$$= \frac{a+b-c+d}{a-b+c+d}$$

$$30. \frac{3x^3 + 9x^2}{x^2 + 6x + 9} = \frac{3x^2(x+3)}{(x+3)^2} = \frac{3x^2}{x+3}$$

$$31. \frac{10a^2(a^3 + b^3)}{6a^4 - 6a^3b + 6a^2b^2}$$

$$= \frac{10a^2(a+b)(a^2 - ab + b^2)}{6a^2(a^2 - ab + b^2)} = \frac{5(a+b)}{3}$$

$$32. \frac{a(4a^2 - 8ab)}{x(3a^2 - 6ab)} = \frac{4a^2(a-2b)}{3ax(a-2b)} = \frac{4a}{3x}$$

$$33. \frac{x^3 - 6x^2}{x^2 - 12x + 36} = \frac{x^2(x-6)}{(x-6)^2} = \frac{x}{x-6}$$

$$34. \frac{(x-4y)^2}{x^5 - 64x^2y^3}$$

$$= \frac{(x-4y)^2}{x^2(x^3 - 64y^3)}$$

$$= \frac{(x-4y)^2}{x^2(x-4y)(x^2 + 4xy + 16y^2)}$$

$$= \frac{x-4y}{x^2(x^2 + 4xy + 16y^2)}$$

$$35. \frac{x^3 - 3xy^2}{x^4 - 6x^2y^2 + 9y^4} = \frac{x(x^2 - 3y^2)}{(x^2 - 3y^2)^2} = \frac{x}{x^2 - 3y^2}$$

$$36. \frac{m^3n + 3m^2n + 9mn}{m^3 - 27}$$

$$= \frac{mn(m^2 + 3m + 9)}{(m-3)(m^2 + 3m + 9)} = \frac{mn}{m-3}$$

$$37. \frac{x^4 - 8x^2 + 15}{x^4 - 9} = \frac{(x^2 - 5)(x^2 - 3)}{(x^2 + 3)(x^2 - 3)} = \frac{x^2 - 5}{x^2 + 3}$$

$$38. \frac{a^4 + 6a^2 - 7}{a^4 + 8a^2 - 9} = \frac{(a^2 + 7)(a^2 - 1)}{(a^2 + 9)(a^2 - 1)} = \frac{a^2 + 7}{a^2 + 9}$$

$$39. \frac{3x^2 + 19x + 20}{6x^2 + 17x + 12}$$

$$= \frac{(3x)^2 + 19(3x) + 60}{(6x)^2 + 17(6x) + 72}$$

$$= \frac{[(3x+15)(3x+4)] \div 3}{[(6x+8)(6x+9)] \div 2 \cdot 3}$$

$$= \frac{(x+5)(3x+4)}{(3x+4)(2x+3)} = \frac{x+5}{2x+3}$$

$$40. \frac{4a^4 - 15a^2 - 4}{a^2 - 8a - 20}$$

$$= \frac{(4a^2)^2 - 15(4a^2) - 16}{(a-10)(a+2)}$$

$$= \frac{[(4a^2 - 16)(4a^2 + 1)] \div 4}{(a-10)(a+2)}$$

$$= \frac{(a^2 - 4)(4a^2 + 1)}{(a-10)(a+2)}$$

$$= \frac{(a+2)(a-2)(4a^2 + 1)}{(a-10)(a+2)}$$

$$= \frac{(a-2)(4a^2 + 1)}{a-10}$$

$$41. \frac{125a + a^4}{2a^3 + 20a^2 + 50a}$$

$$= \frac{a(125 + a^3)}{2a(a^2 + 10a + 25)}$$

$$= \frac{(5+a)(25 - 5a + a^2)}{2(a+5)^2}$$

$$= \frac{a^2 - 5a + 25}{2(a+5)}$$

$$42. \frac{a^2n^2 - 36a^2}{an^2 + an - 30a}$$

$$= \frac{a^2(n^2 - 36)}{a(n^2 + n - 30)}$$

$$= \frac{a(n+6)(n-6)}{(n+6)(n-5)}$$

$$= \frac{a(n-6)}{n-5}$$

$$43. \frac{3m^2 + 5mn - 8n^2}{m^3 - n^3}$$

$$= \frac{(3m)^2 + 5n(3m) - 24n^2}{(m-n)(m^2 + mn + n^2)}$$

$$= \frac{[(3m+8n)(3m-3n)] \div 3}{(m-n)(m^2 + mn + n^2)}$$

$$= \frac{(3m+8n)(m-n)}{(m-n)(m^2 + mn + n^2)}$$

$$= \frac{3m+8n}{m^2 + mn + n^2}$$

$$44. \frac{15a^3b - 18a^2b}{20a^2b^2 - 24ab^2}$$

$$= \frac{3a^2b(5a-6)}{4ab^2(5a-6)} = \frac{3a}{4b}$$

$$45. \frac{9x^2 - 24x + 16}{9x^4 - 16x^2}$$

$$= \frac{(3x-4)^2}{x^2(9x^2 - 16)}$$

$$= \frac{(3x-4)^2}{x^2(3x+4)(3x-4)} = \frac{3x-4}{x^2(3x+4)}$$

$$46. \frac{16a^2x - 25x}{12a^3 - 7a^2 - 10a}$$

$$= \frac{x(16a^2 - 25)}{a(12a^2 - 7a - 10)}$$

$$= \frac{x(4a+5)(4a-5)}{a[(12a)^2 - 7(12a) - 120]}$$

$$= \frac{x(4a+5)(4a-5)}{a[(12a-15)(12a+8)] \div 3 \cdot 4}$$

$$= \frac{x(4a+5)(4a-5)}{a(4a-5)(3a+2)} = \frac{x(4a+5)}{a(3a+2)}$$

$$47. \frac{8x^4 - xy^3}{4x^4 - 4x^3y + x^2y^2}$$

$$= \frac{x(8x^3 - y^3)}{x^2(4x^2 - 4xy + y^2)}$$

$$= \frac{(2x-y)(4x^2 + 2xy + y^2)}{x(2x-y)^2}$$

$$= \frac{4x^2 + 2xy + y^2}{x(2x-y)}$$

$$\begin{aligned}
 48. \quad & \frac{3an - 4a - 6bn + 8b}{6n^2 - 5n - 4} \\
 &= \frac{3n(a - 2b) - 4(a - 2b)}{(6n)^2 - 5(6n) - 24} \\
 &= \frac{(3n - 4)(a - 2b)}{[(6n - 8)(6n + 3)] \div 2 \cdot 3} \\
 &= \frac{(3n - 4)(a - 2b)}{(3n - 4)(2n + 1)} = \frac{a - 2b}{2n + 1}
 \end{aligned}$$

$$\begin{aligned}
 49. \quad & \frac{x^4 - 49x^2}{x^3 + 2x^2 - 63x} \\
 &= \frac{(x^2 + 7x)(x^2 - 7x)}{x(x^2 + 2x - 63)} \\
 &= \frac{x^2(x + 7)(x - 7)}{x(x + 9)(x - 7)} = \frac{x(x + 7)}{x + 9}
 \end{aligned}$$

$$\begin{aligned}
 50. \quad & \frac{x^4 + x - x^3y - y}{x^3 - x - x^2y + y} \\
 &= \frac{x^3(x - y) + (x - y)}{x^2(x - y) - (x - y)} \\
 &= \frac{(x^3 + 1)(x - y)}{(x^2 - 1)(x - y)} \\
 &= \frac{(x + 1)(x^2 - x + 1)}{(x + 1)(x - 1)} \\
 &= \frac{x^2 - x + 1}{x - 1}
 \end{aligned}$$

$$\begin{aligned}
 51. \quad & \frac{2x^3 + 6x^2 - x - 3}{x^3 + 3x^2 + x + 3} \\
 &= \frac{2x^2(x + 3) - (x + 3)}{x^2(x + 3) + (x + 3)} \\
 &= \frac{(2x^2 - 1)(x + 3)}{(x^2 + 1)(x + 3)} = \frac{2x^2 - 1}{x^2 + 1}
 \end{aligned}$$

$$\begin{aligned}
 52. \quad & \frac{a^3m - 4am + a^3n - 4an}{a^4 - 4a^3 - 12a^2} \\
 &= \frac{am(a^2 - 4) + an(a^2 - 4)}{a^2(a^2 - 4a - 12)} \\
 &= \frac{a(m + n)(a^2 - 4)}{a^2(a - 6)(a + 2)} \\
 &= \frac{(m + n)(a + 2)(a - 2)}{a(a - 6)(a + 2)} \\
 &= \frac{(m + n)(a - 2)}{a(a - 6)}
 \end{aligned}$$

$$\begin{aligned}
 53. \quad & \frac{4a^2 - (x - 3)^2}{(2a + x)^2 - 9} \\
 &= \frac{(2a + x - 3)(2a - x + 3)}{(2a + x + 3)(2a + x - 3)} \\
 &= \frac{2a - x + 3}{2a + x + 3}
 \end{aligned}$$

$$\begin{aligned}
 54. \quad & \frac{m - am + n - an}{1 - 3a + 3a^2 - a^3} \\
 &= \frac{m(1 - a) + n(1 - a)}{(1 - a^3) - 3a(1 - a)} \\
 &= \frac{(m + n)(1 - a)}{(1 - a)(1 + a + a^2) - 3a(1 - a)} \\
 &= \frac{(m + n)(1 - a)}{(1 - a)(1 - 2a + a^2)} \\
 &= \frac{m + n}{a^2 - 2a + 1} = \frac{m + n}{(a - 1)^2}
 \end{aligned}$$

$$\begin{aligned}
 55. \quad & \frac{6x^2 + 3}{42x^5 - 9x^3 - 15x} \\
 &= \frac{3(2x^2 + 1)}{3x(14x^4 - 3x^2 - 5)} \\
 &= \frac{2x^2 + 1}{x[(14x^2)^2 - 3(14x^2) - 70]} \\
 &= \frac{2x^2 + 1}{x[(14x^2 - 10)(14x^2 + 7)] \div 2 \cdot 7} \\
 &= \frac{2x^2 + 1}{x(7x^2 - 5)(2x^2 + 1)} = \frac{1}{x(7x^2 - 5)}
 \end{aligned}$$

$$\begin{aligned}
 56. \quad & \frac{a^2 - a^3 - 1 + a}{a^2 + 1 - a^3 - a} \\
 &= \frac{a^2(1 - a) - (1 - a)}{a^2(1 - a) + (1 - a)} \\
 &= \frac{(a^2 - 1)(1 - a)}{(a^2 + 1)(1 - a)} = \frac{a^2 - 1}{a^2 + 1}
 \end{aligned}$$

$$\begin{aligned}
 57. \quad & \frac{8x^3 + 12x^2y + 6xy^2 + y^3}{6x^2 + xy - y^2} \\
 &= \frac{6xy(2x + y) + (8x^3 + y^3)}{(6x)^2 + y(6x) - 6y^2} \\
 &= \frac{6xy(2x + y) + (2x + y)(4x^2 - 2xy + y^2)}{[(6x + 3y)(6x - 2y)] \div 3 \cdot 2} \\
 &= \frac{(2x + y)(4x^2 + 4xy + y^2)}{(2x + y)(3x - y)} = \frac{(2x + y)^2}{3x - y}
 \end{aligned}$$

$$\begin{aligned}
 58. \quad & \frac{8n^3 - 125}{25 - 20n + 4n^2} \\
 &= \frac{(2n - 5)(4n^2 + 10n + 25)}{(2n - 5)^2} \\
 &= \frac{4n^2 + 10n + 25}{2n - 5}
 \end{aligned}$$

$$\begin{aligned}
 59. \quad & \frac{6 - x - x^2}{15 + 2x - x^2} \\
 &= \frac{-(x^2 + x - 6)}{-(x^2 - 2x - 15)} \\
 &= \frac{(x + 3)(x - 2)}{(x - 5)(x + 3)} = \frac{x - 2}{x - 5}
 \end{aligned}$$

$$\begin{aligned}
 60. \quad & \frac{3 + 2x - 8x^2}{4 + 5x - 6x^2} \\
 &= \frac{-(8x^2 - 2x - 3)}{-(6x^2 - 5x - 4)} \\
 &= \frac{(8x)^2 - 2(8x) - 24}{(6x)^2 - 5(6x) - 24} \\
 &= \frac{[(8x - 6)(8x + 4)] \div 2 \cdot 4}{[(6x - 8)(6x + 3)] \div 2 \cdot 3} \\
 &= \frac{(4x - 3)(2x + 1)}{(3x - 4)(2x + 1)} = \frac{4x - 3}{3x - 4}
 \end{aligned}$$

$$\begin{aligned}
 61. \quad & \frac{m^2n^2 + 3mn - 10}{4 - 4mn + m^2n^2} \\
 &= \frac{(mn + 5)(mn - 2)}{(mn - 2)^2} = \frac{mn + 5}{mn - 2}
 \end{aligned}$$

$$\begin{aligned}
 62. \quad & \frac{x^3 + x^2y - 4b^2x - 4b^2y}{4b^2 - 4bx + x^2} \\
 &= \frac{x^2(x + y) - 4b^2(x + y)}{(2b - x)^2} \\
 &= \frac{(x^2 - 4b^2)(x + y)}{(x - 2b)^2} \\
 &= \frac{(x + 2b)(x - 2b)(x + y)}{(x - 2b)^2} \\
 &= \frac{(x + 2b)(x + y)}{x - 2b}
 \end{aligned}$$

$$63. \frac{x^6 + x^3 - 2}{x^4 - x^3y - x + y}$$

$$= \frac{(x^3 + 2)(x^3 - 1)}{x^3(x - y) - (x - y)}$$

$$= \frac{(x^3 + 2)(x^3 - 1)}{(x^3 - 1)(x - y)} = \frac{x^3 + 2}{x - y}$$

$$64. \frac{(x^2 - x - 2)(x^2 - 9)}{(x^2 - 2x - 3)(x^2 + x - 6)}$$

$$= \frac{(x - 2)(x + 1)(x + 3)(x - 3)}{(x - 3)(x + 1)(x + 3)(x - 2)} = 1$$

$$65. \frac{(a^2 - 4a + 4)(4a^2 - 4a + 1)}{(a^2 + a - 6)(2a^2 - 5a + 2)}$$

$$= \frac{(a - 2)^2(2a - 1)^2}{(a + 3)(a - 2)[(2a^2 - 5(2a) + 4)]}$$

$$= \frac{(a - 2)(2a - 1)^2}{(a + 3)[(2a - 4)(2a - 1)] \div 2}$$

$$= \frac{(a - 2)(2a - 1)^2}{(a + 3)(a - 2)(2a - 1)} = \frac{2a - 1}{a + 3}$$

$$66. \frac{(x^3 - 3x)(x^3 - 1)}{(x^4 + x^3 + x^2)(x^2 - 1)}$$

$$= \frac{x(x^2 - 3)(x - 1)(x^2 + x + 1)}{x^2(x^2 + x + 1)(x + 1)(x - 1)}$$

$$= \frac{x^2 - 3}{x(x + 1)}$$

$$67. \frac{(4n^2 + 4n - 3)(n^2 + 7n - 30)}{(2n^2 - 7n + 3)(4n^2 + 12n + 9)}$$

$$= \frac{[(4n)^2 + 4(4n) - 12][n + 10](n - 3)}{[(2n)^2 - 7(2n) + 6](2n + 3)^2}$$

$$= \frac{(n + 10)(n - 3)[(4n + 6)(4n - 2)] \div 2 \cdot 2}{(2n + 3)^2 [(2n - 6)(2n - 1)] \div 2 \cdot 2}$$

$$= \frac{(n + 10)(n - 3)(2n + 3)(2n - 1)}{(2n + 3)^2(n - 3)(2n - 1)} = \frac{n + 10}{2n + 3}$$

$$68. \frac{(x^6 - y^6)(x + y)}{(x^3 - y^3)(x^3 + x^2y + xy^2 + y^3)}$$

$$= \frac{(x^3 + y^3)(x^3 - y^3)(x + y)}{(x^3 - y^3)[x^2(x + y) + y^2(x + y)]}$$

$$= \frac{(x^3 + y^3)(x + y)}{(x^2 + y^2)(x + y)} = \frac{x^3 + y^3}{x^2 + y^2}$$

$$69. \frac{x^3 + 3x^2 - 4}{1 \quad 3 \quad 0 \quad -4 \quad | \quad -2}$$

$$\begin{array}{r} -2 \quad -2 \quad 4 \\ \hline 1 \quad 1 \quad -2 \quad 0 \end{array}$$

$$= (x + 2)(x^2 + x - 2)$$

$$= (x + 2)(x + 2)(x - 1)$$

$$= (x + 2)^2(x - 1)$$

$$x^3 + x^2 - 8x - 12$$

$$1 \quad 1 \quad -8 \quad -12 \quad | \quad -2$$

$$\begin{array}{r} -2 \quad 2 \quad 12 \\ \hline 1 \quad -1 \quad -6 \quad 0 \end{array}$$

$$= (x + 2)(x^2 - x - 6)$$

$$= (x + 2)(x - 3)(x + 2)$$

$$= (x + 2)^2(x - 3)$$

$$\Rightarrow \frac{(x + 2)^2(x - 1)}{(x + 2)^2(x - 3)} = \frac{x - 1}{x - 3}$$

$$70. \frac{x^3 - x^2 - 8x + 12}{1 \quad -1 \quad -8 \quad 12 \quad | \quad 2}$$

$$\begin{array}{r} 2 \quad 2 \quad -12 \\ \hline 1 \quad 1 \quad -6 \quad 0 \end{array}$$

$$= (x - 2)(x^2 + x - 6)$$

$$= (x - 2)(x + 3)(x - 2)$$

$$= (x - 2)^2(x + 3)$$

$$x^4 - 2x^3 - 7x^2 + 20x - 12$$

$$1 \quad -2 \quad -7 \quad 20 \quad -12 \quad | \quad 2$$

$$\begin{array}{r} 2 \quad 0 \quad -14 \quad 12 \\ \hline 1 \quad 0 \quad -7 \quad 6 \quad 0 \end{array}$$

$$(x - 2)(x^3 - 7x + 6)$$

$$1 \quad 0 \quad -7 \quad 6 \quad | \quad 2$$

$$\begin{array}{r} 2 \quad 4 \quad -6 \\ \hline 1 \quad 2 \quad -3 \quad 0 \end{array}$$

$$= (x - 2)(x - 2)(x^2 + 2x - 3)$$

$$= (x - 2)^2(x + 3)(x - 1)$$

$$\Rightarrow \frac{(x - 2)^2(x + 3)}{(x - 2)^2(x + 3)(x - 1)}$$

$$= \frac{1}{x - 1}$$

$$71. \frac{x^4 - 7x^2 - 2x + 8}{1 \quad 0 \quad -7 \quad -2 \quad 8 \quad | \quad -2}$$

$$\begin{array}{r} -2 \quad 4 \quad 6 \quad -8 \\ \hline 1 \quad -2 \quad -3 \quad 4 \quad 0 \end{array}$$

$$(x + 2)(x^3 - 2x^2 - 3x + 4)$$

$$1 \quad -2 \quad -3 \quad 4 \quad | \quad 1$$

$$\begin{array}{r} 1 \quad -1 \quad -4 \\ \hline 1 \quad -1 \quad -4 \quad 0 \end{array}$$

$$(x + 2)(x - 1)(x^2 - x - 4)$$

$$x^4 - 2x^3 - 9x^2 + 10x + 24$$

$$1 \quad -2 \quad -9 \quad 10 \quad 24 \quad | \quad -2$$

$$\begin{array}{r} -2 \quad 8 \quad 2 \quad -24 \\ \hline 1 \quad -4 \quad -1 \quad 12 \quad 0 \end{array}$$

$$(x + 2)(x^3 - 4x^2 - x + 12)$$

$$1 \quad -4 \quad -1 \quad 12 \quad | \quad 3$$

$$\begin{array}{r} 3 \quad 3 \quad 12 \\ \hline 1 \quad -1 \quad -4 \quad 0 \end{array}$$

$$(x + 2)(x - 3)(x^2 - x - 4)$$

$$\Rightarrow \frac{(x + 2)(x - 1)(x^2 - x - 4)}{(x + 2)(x - 3)(x^2 - x - 4)} = \frac{x - 1}{x - 3}$$

$$72. \frac{a^5 - a^3 - a^2 + 1}{a^5 - a^3 - a^2 + 1}$$

$$= a^2(a^3 - 1) - (a^3 - 1)$$

$$= (a^2 - 1)(a^3 - 1)$$

$$= (a - 1)^2(a + 1)(a^2 + a + 1)$$

$$a^5 - 2a^4 - 6a^3 + 8a^2 + 5a - 6$$

$$1 \quad -2 \quad -6 \quad 8 \quad 5 \quad -6 \quad | \quad 1$$

$$\begin{array}{r} 1 \quad -1 \quad -7 \quad 1 \quad 6 \\ \hline 1 \quad -1 \quad -7 \quad 1 \quad 6 \quad 0 \end{array}$$

$$(a - 1)(a^4 - a^3 - 7a^2 + a + 6)$$

$$1 \quad -1 \quad -7 \quad 1 \quad 6 \quad | \quad 1$$

$$\begin{array}{r} 1 \quad 0 \quad -7 \quad -6 \\ \hline 1 \quad 0 \quad -7 \quad -6 \quad 0 \end{array}$$

$$(a - 1)^2(a^3 - 7a - 6)$$

$$1 \quad 0 \quad -7 \quad -6 \quad | \quad -1$$

$$\begin{array}{r} -1 \quad 1 \quad 6 \\ \hline 1 \quad -1 \quad -6 \quad 0 \end{array}$$

$$(a - 1)^2(a + 1)(a^2 - a - 6)$$

$$= (a - 1)^2(a + 1)(a - 3)(a + 2)$$

$$\Rightarrow \frac{(a - 1)^2(a + 1)(a^2 + a + 1)}{(a - 1)^2(a + 1)(a - 3)(a + 2)}$$

$$= \frac{a^2 + a + 1}{(a - 3)(a + 2)}$$

## EJERCICIO 120

$$1. \frac{4-4x}{6x-6} = \frac{4(1-x)}{6(x-1)} = -\frac{4(x-1)}{6(x-1)} = -\frac{2}{3}$$

$$2. \frac{a^2-b^2}{b^2-a^2} = -\frac{(a^2-b^2)}{(a^2-b^2)} = -1$$

$$3. \frac{m^2-n^2}{(n-m)^2} = \frac{(m+n)(m-n)}{(n-m)(n-m)} = \frac{(m+n)(m-n)}{(m-n)(m-n)} = \frac{m+n}{m-n}$$

$$4. \frac{x^2-x-12}{16-x^2} = \frac{(x-4)(x+3)}{(4-x)(4+x)} = -\frac{(x-4)(x+3)}{(x-4)(4+x)} = -\frac{x+3}{4+x}$$

$$\begin{aligned} 5. & \frac{3y-6x}{2mx-my-2nx+ny} \\ &= \frac{3(y-2x)}{2x(m-n)-y(m-n)} \\ &= \frac{3(y-2x)}{(2x-y)(m-n)} = -\frac{3(2x-y)}{(2x-y)(m-n)} = -\frac{3}{m-n} \end{aligned}$$

$$\begin{aligned} 6. & \frac{2x^2-9x-5}{10+3x-x^2} \\ &= -\frac{[(2x)^2-9(2x)-10]}{x^2-3x-10} \\ &= -\frac{(2x-10)(2x+1)}{(x-5)(x+2)} = -\frac{(x-5)(2x+1)}{(x-5)(x+2)} = -\frac{2x+1}{x+2} \end{aligned}$$

$$\begin{aligned} 7. & \frac{8-a^3}{a^2+2a-8} \\ &= \frac{(2-a)(4+2a+a^2)}{(a+4)(a-2)} = -\frac{(a-2)(4+2a+a^2)}{(a+4)(a-2)} = -\frac{a^2+2a+4}{a+4} \end{aligned}$$

$$\begin{aligned} 8. & \frac{a^2+a-2}{n-an-m+am} \\ &= \frac{(a+2)(a-1)}{n(1-a)-m(1-a)} = \frac{(a+2)(a-1)}{(n-m)(1-a)} = \frac{(a+2)(a-1)}{(m-n)(a-1)} = \frac{a+2}{m-n} \end{aligned}$$

$$9. \frac{4x^2-4xy+y^2}{5y-10x} = \frac{(2x-y)^2}{5(y-2x)} = -\frac{(2x-y)^2}{5(2x-y)} = -\frac{2x-y}{5}$$

$$\begin{aligned} 10. & \frac{3mx-nx-3my+ny}{ny^2-nx^2-3my^2+3mx^2} \\ &= \frac{3m(x-y)-n(x-y)}{-3m(y^2-x^2)+n(y^2-x^2)} \\ &= \frac{(3m-n)(x-y)}{(n-3m)(y^2-x^2)} = \frac{(3m-n)(x-y)}{(3m-n)(x^2-y^2)} = \frac{x-y}{(x+y)(x-y)} = \frac{1}{x+y} \end{aligned}$$

$$11. \frac{9-6x+x^2}{x^2-7x+12} = \frac{(x-3)^2}{(x-4)(x-3)} = \frac{x-3}{x-4}$$

$$\begin{aligned} 12. & \frac{a^2-b^2}{b^3-a^3} \\ &= \frac{(a+b)(a-b)}{(b-a)(b^2+ab+a^2)} \\ &= -\frac{(a+b)(a-b)}{(a-b)(b^2+ab+a^2)} = -\frac{a+b}{b^2+ab+a^2} \end{aligned}$$

$$\begin{aligned} 13. & \frac{3ax-3bx-6a+6b}{2b-2a-bx+ax} \\ &= \frac{3x(a-b)-6(a-b)}{2(b-a)-x(b-a)} \\ &= \frac{3(x-2)(a-b)}{(2-x)(b-a)} = \frac{3(x-2)(a-b)}{(x-2)(a-b)} = 3 \end{aligned}$$

$$\begin{aligned} 14. & \frac{a^2-x^2}{x^2-ax-3x+3a} \\ &= \frac{(a+x)(a-x)}{x(x-a)-3(x-a)} \\ &= \frac{(a+x)(a-x)}{(x-3)(x-a)} = \frac{(a+x)(a-x)}{(3-x)(a-x)} = \frac{a+x}{3-x} \end{aligned}$$

$$\begin{aligned} 15. & \frac{3bx-6x}{8-b^3} \\ &= \frac{3x(b-2)}{(2-b)(4+2b+b^2)} \\ &= -\frac{3x(b-2)}{(b-2)(4+2b+b^2)} = -\frac{3x}{4+2b+b^2} \end{aligned}$$

$$16. \frac{(1-a)^3}{a-1} = -\frac{(1-a)^3}{(1-a)} = -(1-a)^2$$

$$\begin{aligned} 17. & \frac{2x^3-2x^2y-2xy^2}{3y^3+3xy^2-3x^2y} \\ &= \frac{2x(x^2-xy-y^2)}{3y(y^2+xy-x^2)} = -\frac{2x(x^2-xy-y^2)}{3y(x^2-xy-y^2)} = -\frac{2x}{3y} \end{aligned}$$

$$18. \frac{(a-b)^3}{(b-a)^2} = \frac{(a-b)^3}{(a-b)^2} = a-b$$



$$\begin{aligned}
 19. \quad & \frac{2x^2 - 22x + 60}{75 - 3x^2} \\
 &= \frac{[(2x)^2 - 22(2x) + 120]}{3(25 - x^2)} \\
 &= \frac{(2x - 12)(2x - 10)}{3(5 - x)(5 + x)} \\
 &= \frac{(2x - 12)(x - 5)}{3(5 - x)(5 + x)} = \frac{2(6 - x)(5 - x)}{3(5 - x)(5 + x)} = \frac{2(6 - x)}{3(5 + x)}
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & \frac{6an^2 - 3b^2n^2}{b^4 - 4ab^2 + 4a^2} \\
 &= \frac{3n^2(2a - b^2)}{(b^2 - 2a)^2} = \frac{3n^2(2a - b^2)}{(2a - b^2)(2a - b^2)} = \frac{3n^2}{2a - b^2}
 \end{aligned}$$

$$\begin{aligned}
 21. \quad & \frac{(x - y)^2 - z^2}{(y + z) - x^2} \\
 &= \frac{(x - y - z)(x - y + z)}{(y + z - x)(y + z + x)} \\
 &= \frac{(z + y - x)(y - x - z)}{(z + y - x)(y + z + x)} = \frac{y - x - z}{x + y + z}
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & \frac{3a^2 - 3ab}{bd - ad - bc + ac} \\
 &= \frac{3a(a - b)}{d(b - a) - c(b - a)} \\
 &= \frac{3a(a - b)}{(d - c)(b - a)} = \frac{3a(a - b)}{(c - d)(a - b)} = \frac{3a}{c - d}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & \frac{(x - 5)^3}{125 - x^3} \\
 &= \frac{(x - 5)^3}{(5 - x)(25 + 5x + x^2)} \\
 &= -\frac{(x - 5)^3}{(x - 5)(x^2 + 5x + 25)} = -\frac{(x - 5)^2}{x^2 + 5x + 25}
 \end{aligned}$$

$$24. \quad \frac{13x - 6 - 6x^2}{6x^2 - 13x + 6} = -\frac{(6x^2 - 13x + 6)}{6x^2 - 13x + 6} = -1$$

$$\begin{aligned}
 25. \quad & \frac{2x^3 - 2xy^2 + x^2 - y^2}{2xy^2 + y^2 - 2x^3 - x^2} \\
 &= \frac{2x(x^2 - y^2) + (x^2 - y^2)}{y^2(2x + 1) - x^2(2x + 1)} \\
 &= \frac{(2x + 1)(x^2 - y^2)}{(2x + 1)(y^2 - x^2)} = -\frac{(x^2 - y^2)}{(x^2 - y^2)} = -1
 \end{aligned}$$

$$\begin{aligned}
 26. \quad & \frac{30x^2y - 45xy^2 - 20x^3}{8x^3 + 27y^3} \\
 &= \frac{5x(6xy - 9y^2 - 4x^2)}{(2x + 3y)(4x^2 - 6xy + 9y^2)} \\
 &= -\frac{5x(4x^2 - 6xy + 9y^2)}{(2x + 3y)(4x^2 - 6xy + 9y^2)} = -\frac{5x}{2x + 3y}
 \end{aligned}$$

$$\begin{aligned}
 27. \quad & \frac{n + 1 - n^3 - n^2}{n^3 - n - 2n^2 + 2} \\
 &= \frac{(n + 1) - n^2(n + 1)}{n(n^2 - 1) - 2(n^2 - 1)} \\
 &= \frac{(1 - n^2)(n + 1)}{(n - 2)(n^2 - 1)} = \frac{(1 - n^2)(n + 1)}{(2 - n)(1 - n^2)} = \frac{n + 1}{2 - n}
 \end{aligned}$$

$$\begin{aligned}
 28. \quad & \frac{(x - 2)^2(x^2 + x - 12)}{(2 - x)(3 - x)^2} \\
 &= \frac{(2 - x)^2(x + 4)(x - 3)}{(2 - x)(3 - x)^2} \\
 &= \frac{(x - 2)(x + 4)(3 - x)}{(3 - x)^2} = \frac{(x - 2)(x + 4)}{3 - x}
 \end{aligned}$$

$$\begin{aligned}
 29. \quad & \frac{5x^3 - 15x^2y}{90x^3y^2 - 10x^5} \\
 &= \frac{5x^2(x - 3y)}{10x^3(9y^2 - x^2)} \\
 &= \frac{(x - 3y)}{2x(3y - x)(3y + x)} \\
 &= -\frac{x - 3y}{2x(x - 3y)(3y + x)} = \frac{-1}{2x(3y + x)}
 \end{aligned}$$

$$\begin{aligned}
 30. \quad & \frac{(x^2 - 1)(x^2 - 8x + 16)}{(x^2 - 4x)(1 - x^2)} \\
 &= \frac{(x + 1)(x - 1)(x - 4)(x - 4)}{x(x - 4)(1 + x)(1 - x)} = \frac{(1 - x)(4 - x)}{x(1 - x)} = \frac{4 - x}{x}
 \end{aligned}$$

## EJERCICIO 121

1.  $\frac{a^4 - a^3x + a^2x^2 - ax^3}{a^4 - a^3x - 2a^2x^2 + 2ax^3}$  Factor común  $a$

$$\begin{array}{r} a^3 - a^2x + ax^2 - x^3 \\ -a^3 + a^2x + 2a^2x^2 - 2x^3 \\ \hline \end{array} \quad \begin{array}{r} a^3 - a^2x - 2ax^2 + 2x^3 \\ 1 \\ \hline \end{array}$$

$$3ax^2 - 3x^3 \div 3x^2$$

$$\begin{array}{r} a^3 - a^2x - 2ax^2 + 2x^3 \\ -a^3 + a^2x \\ \hline \end{array} \quad \begin{array}{r} a - x \\ a^2 - 2x^2 \\ \hline \end{array}$$

$$-2ax^2 + 2x^3 \quad mcd = a(a - x)$$

$$+ 2ax^2 - 2x^3 = a^2 - ax$$

$$\begin{array}{r} a^4 - a^3x + a^2x^2 - ax^3 \\ -a^4 + a^3x \\ \hline \end{array} \quad \begin{array}{r} a^2 - ax \\ a^2 + x^2 \\ \hline \end{array}$$

$$a^2x^2 - ax^3$$

$$-a^2x^2 + ax^3$$

$$\begin{array}{r} a^4 - a^3x - 2a^2x^2 + 2ax^3 \\ -a^4 + a^3x \\ \hline \end{array} \quad \begin{array}{r} a^2 - ax \\ a^2 - 2x^2 \\ \hline \end{array}$$

$$-2a^2x^2 + 2ax^3$$

$$+ 2a^2x^2 - 2ax^3$$

$$= \frac{a^2 + x^2}{a^2 - 2x^2} \text{ Rta.}$$

2.  $\frac{x^4 + 3x^3 + 4x^2 - 3x - 5}{x^4 + 3x^3 + 6x^2 + 3x + 5}$

$$\begin{array}{r} x^4 + 3x^3 + 4x^2 - 3x - 5 \\ -x^4 - 3x^3 - 6x^2 - 3x - 5 \\ \hline \end{array} \quad \begin{array}{r} x^4 + 3x^3 + 6x^2 + 3x + 5 \\ 1 \\ \hline \end{array}$$

$$-2x^2 - 6x - 10 \quad \div -2$$

$$\begin{array}{r} x^4 + 3x^3 + 6x^2 + 3x + 5 \\ -x^4 - 3x^3 - 5x^2 \\ \hline \end{array} \quad \begin{array}{r} x^2 + 3x + 5 \\ x^2 \\ \hline \end{array}$$

$$x^2 + 3x + 5$$

$$\begin{array}{r} x^2 + 3x + 5 \\ -x^2 - 3x - 5 \\ \hline \end{array} \quad \begin{array}{r} x^2 + 3x + 5 \\ 1 \\ \hline \end{array} \quad mcd \quad x^2 + 3x + 5$$

$$\Rightarrow \begin{array}{r} x^4 + 3x^3 + 4x^2 - 3x - 5 \\ -x^4 - 3x^3 - 5x^2 \\ \hline \end{array} \quad \begin{array}{r} x^2 + 3x + 5 \\ x^2 - 1 \\ \hline \end{array}$$

$$-x^2 - 3x - 5$$

$$x^2 + 3x + 5$$

$$\begin{array}{r} x^4 + 3x^3 + 6x^2 + 3x + 5 \\ -x^4 - 3x^3 - 5x^2 \\ \hline \end{array} \quad \begin{array}{r} x^2 + 3x + 5 \\ x^2 + 1 \\ \hline \end{array}$$

$$x^2 + 3x + 5$$

$$-x^2 - 3x - 5$$

$$= \frac{x^2 - 1}{x^2 + 1} \text{ Rta.}$$

3.  $\frac{2ax^4 - ax^3 - ax^2 - 2ax + 2a(3)}{3ax^4 - 4ax^3 + ax^2 + 3ax - 3a(2)}$  Factor común  $a$

$$\begin{array}{r} 6x^4 - 3x^3 - 3x^2 - 6x + 6 \\ -6x^4 + 8x^3 - 2x^2 - 6x + 6 \\ \hline \end{array} \quad \begin{array}{r} 6x^4 - 8x^3 + 2x^2 + 6x - 6 \\ 1 \\ \hline \end{array} \quad (5)$$

$$5x^3 - 5x^2 - 12x + 12 \quad (6)$$

$$\begin{array}{r} 30x^4 - 40x^3 + 10x^2 + 30x - 30 \\ -30x^4 + 30x^3 + 72x^2 - 72x \\ \hline \end{array} \quad \begin{array}{r} 30x^3 - 30x^2 - 72x + 72 \\ x \\ \hline \end{array} \quad (\div 6)$$

$$-10x^3 + 82x^2 - 42x - 30 \quad (\div -2)$$

$$\begin{array}{r} 5x^3 - 5x^2 - 12x + 12 \\ -5x^3 + 41x^2 - 21x - 15 \\ \hline \end{array} \quad \begin{array}{r} 5x^3 - 41x^2 + 21x + 15 \\ 1 \\ \hline \end{array} \quad (12)$$

$$36x^2 - 33x - 3 \quad (\div 3)$$

$$\begin{array}{r} 60x^3 - 492x^2 + 252x + 180 \\ -60x^3 + 55x^2 + 5x \\ \hline \end{array} \quad \begin{array}{r} 12x^2 - 11x - 1 \\ 5x \\ \hline \end{array} \quad (437)$$

$$-437x^2 + 257x + 180 \quad (-12)$$

$$\begin{array}{r} 5.244x^2 - 4.807x - 437 \\ -5.244x^2 + 3.084x + 2.160 \\ \hline \end{array} \quad \begin{array}{r} 5.244x^2 - 3.084x - 2.160 \\ 1 \\ \hline \end{array} \quad (\div 12)$$

$$-1723x + 1723 \quad (\div -1.723)$$

$$\begin{array}{r} 437x^2 - 257x - 180 \\ -437x^2 + 437x \\ \hline \end{array} \quad \begin{array}{r} x - 1 \\ 437x \\ \hline \end{array}$$

$$180x - 180 \quad (\div 180)$$

$$\begin{array}{r} x - 1 \\ -x + 1 \\ \hline \end{array} \quad \begin{array}{r} x - 1 \\ 1 \\ \hline \end{array} \quad mcd = a(x - 1) = ax - a$$

$$\Rightarrow \begin{array}{r} 2ax^4 - ax^3 - ax^2 - 2ax + 2a \\ -2ax^4 + 2ax^3 \\ \hline \end{array} \quad \begin{array}{r} ax - a \\ 2x^3 + x^2 - 2 \\ \hline \end{array}$$

$$ax^3 - ax^2$$

$$-ax^3 + ax^2$$

$$-2ax + 2a$$

$$+ 2ax - 2a$$

$$\begin{array}{r} 3ax^4 - 4ax^3 + ax^2 + 3ax - 3a \\ -3ax^4 + 3ax^3 \\ \hline \end{array} \quad \begin{array}{r} ax - a \\ 3x^3 - x^2 + 3 \\ \hline \end{array}$$

$$-ax^3 + ax^2$$

$$+ ax^3 - ax^2$$

$$3ax - 3a$$

$$-3ax + 3a$$

$$= \frac{2x^3 + x^2 - 2}{3x^3 - x^2 + 3} \text{ Rta.}$$

$$\begin{array}{r}
 4. \quad \frac{6x^3 - 13x^2 + 18x - 8}{10x^3 - 9x^2 + 11x + 12} \quad (10) \\
 \begin{array}{r}
 60x^3 - 130x^2 + 180x - 80 \\
 - 60x^3 + 54x^2 - 66x - 72 \\
 \hline
 - 76x^2 + 114x - 152 \quad (\div -38) \\
 10x^3 - 9x^2 + 11x + 12 \\
 - 10x^3 + 15x^2 - 20x \\
 \hline
 6x^2 - 9x + 12 \quad (\div 3) \\
 2x^2 - 3x + 4 \\
 - 2x^2 + 3x - 4 \\
 \hline
 1
 \end{array} \\
 \begin{array}{r}
 6x^3 - 13x^2 + 18x - 8 \\
 - 6x^3 + 9x^2 - 12x \\
 \hline
 - 4x^2 + 6x - 8 \\
 + 4x^2 - 6x + 8 \\
 \hline
 0
 \end{array} \\
 \text{mcd} = 2x^2 - 3x + 4
 \end{array}$$

$$\begin{array}{r}
 10x^3 - 9x^2 + 11x + 12 \\
 - 10x^3 + 15x^2 - 20x \\
 \hline
 6x^2 - 9x + 12 \\
 - 6x^2 + 9x - 12 \\
 \hline
 0
 \end{array}$$

$$\begin{array}{r}
 5. \quad \frac{x^4 - 2x^3y + 2x^2y^2 - xy^3}{2x^4 - 5x^3y + 4x^2y^2 - xy^3} \quad (2) \quad \text{Factor común } x \\
 \begin{array}{r}
 2x^3 - 4x^2y + 4xy^2 - 2y^3 \\
 - 2x^3 + 5x^2y - 4xy^2 + y^3 \\
 \hline
 x^2y - y^3 \quad (\div y) \\
 2x^3 - 5x^2y + 4xy^2 - y^3 \\
 - 2x^3 + 2xy^2 \\
 \hline
 - 5x^2y + 6xy^2 - y^3 \quad (\div -y) \\
 5x^2 - 5y^2 \\
 - 5x^2 + 6xy - y^2 \\
 \hline
 6xy - 6y^2 \quad (\div 6y) \\
 5x^2 - 6xy + y^2 \\
 - 5x^2 + 5xy \\
 \hline
 - xy + y^2 \quad (\div -y) \\
 x - y \\
 - x + y \\
 \hline
 1
 \end{array} \\
 \text{mcd } x(x-y) = x^2 - xy
 \end{array}$$

Continúa

## Continuación

$$\begin{array}{r}
 5. \quad \frac{x^4 - 2x^3y + 2x^2y^2 - xy^3}{-x^4 + x^3y} \quad \frac{x^2 - xy}{x^2 - xy + y^2} \\
 \begin{array}{r}
 -x^3y + 2x^2y^2 - xy^3 \\
 + x^3y - x^2y^2 \\
 \hline
 x^2y^2 - xy^3 \\
 -x^2y^2 + xy^3 \\
 \hline
 0
 \end{array} \\
 \begin{array}{r}
 2x^4 - 5x^3y + 4x^2y^2 - xy^3 \\
 - 2x^4 + 2x^3y \\
 \hline
 -3x^3y + 4x^2y^2 \\
 + 3x^3y - 3x^2y^2 \\
 \hline
 x^2y^2 - xy^3 \\
 -x^2y^2 + xy^3 \\
 \hline
 0
 \end{array} \\
 = \frac{x^2 - xy + y^2}{2x^2 - 3xy + y^2} \quad \text{Rta.}
 \end{array}$$

$$\begin{array}{r}
 6. \quad \frac{2a^5 - a^4 + 2a^3 + 2a^2 + 3}{3a^5 - a^4 + 3a^3 + 4a^2 + 5} \quad (2) \\
 \begin{array}{r}
 6a^5 - 3a^4 + 6a^3 + 6a^2 + 9 \\
 - 6a^5 + 2a^4 - 6a^3 - 8a^2 - 10 \\
 \hline
 -a^4 - 2a^2 - 1 \quad (\div -1) \\
 3a^5 - a^4 + 3a^3 + 4a^2 + 5 \\
 - 3a^5 + 6a^3 - 3a \\
 \hline
 -a^4 - 3a^3 + 4a^2 - 3a + 5 \quad (-1) \\
 a^4 + 2a^2 + 1 \\
 - a^4 + 3a^3 - 4a^2 + 3a - 5 \\
 \hline
 -3a^3 + 6a^2 - 3a + 6 \quad (\div -3) \\
 a^4 + 3a^3 - 4a^2 + 3a - 5 \\
 - a^4 + 2a^3 - a^2 + 2a \\
 \hline
 5a^3 - 5a^2 + 5a - 5 \quad (\div 5) \\
 a^3 - 2a^2 + a - 2 \\
 - a^3 + a^2 - a + 1 \\
 \hline
 -a^2 - 1 \quad (\div -1) \\
 a^3 - a^2 + a - 1 \\
 - a^3 + a \\
 \hline
 -a^2 - 1 \quad (\div -1) \\
 a^2 + 1 \\
 - a^2 - 1 \\
 \hline
 0
 \end{array} \\
 \text{mcd} = a^2 + 1
 \end{array}$$

Continúa

### Continuación

$$\begin{array}{r|l}
 6. \Rightarrow 2a^5 - a^4 + 2a^3 + 2a^2 + 3 & a^2 + 1 \\
 -2a^5 & -2a^3 \\
 \hline
 -a^4 & +2a^2 + 3 \\
 +a^4 & +a^2 \\
 \hline
 & 3a^2 + 3 \\
 & -3a^2 - 3 \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 3a^5 - a^4 + 3a^3 + 4a^2 + 5 & a^2 + 1 \\
 -3a^5 & -3a^3 \\
 \hline
 -a^4 & +4a^2 + 5 \\
 +a^4 & +a^2 \\
 \hline
 & 5a^2 + 5 \\
 & -5a^2 - 5 \\
 \hline
 \end{array}$$

$$= \frac{2a^3 - a^2 + 3}{3a^3 - a^2 + 5} \quad \text{Rta.}$$

$$\begin{array}{r|l}
 7. \quad 1 - x & -x^3 + x^4 \\
 -1 + 2x + x^2 + 2x^3 - x^4 & 1 \\
 \hline
 x + x^2 + x^3 & (\div x) \\
 1 - 2x - x^2 - 2x^3 + x^4 & 1 + x + x^2 \quad (3) \\
 -1 - x - x^2 & 1 \\
 \hline
 -3x - 2x^2 - 2x^3 + x^4 & (\div -x) \\
 3 + 3x + 3x^2 & 3 + 2x + 2x^2 - x^3 \\
 -3 - 2x - 2x^2 + x^3 & 1 \\
 \hline
 x + x^2 + x^3 & (\div x) \\
 3 + 2x + 2x^2 - x^3 & 1 + x + x^2 \\
 -3 - 3x - 3x^2 & 3 \\
 \hline
 -x - x^2 - x^3 & (\div -x) \\
 1 + x + x^2 & 1 + x + x^2 \\
 -1 - x - x^2 & 1 \quad \text{mcd} = 1 + x + x^2 \\
 \hline
 \Rightarrow 1 - x & -x^3 + x^4 \\
 -1 - x - x^2 & 1 - 2x + x^2 \\
 \hline
 -2x - x^2 - x^3 + x^4 & \\
 +2x + 2x^2 + 2x^3 & \\
 \hline
 x^2 + x^3 + x^4 & \\
 -x^2 - x^3 - x^4 & \\
 \hline
 \end{array}$$

Continúa

### Continuación

$$\begin{array}{r|l}
 7. \quad 1 - 2x - x^2 - 2x^3 + x^4 & 1 + x + x^2 \\
 -1 - x - x^2 & 1 - 3x + x^2 \\
 \hline
 -3x - 2x^2 - 2x^3 + x^4 & \\
 +3x + 3x^2 + 3x^3 & \\
 \hline
 x^2 + x^3 + x^4 & \\
 -x^2 - x^3 - x^4 & \\
 \hline
 \end{array}$$

$$= \frac{1 - 2x + x^2}{1 - 3x + x^2} \quad \text{Rta.}$$

$$\begin{array}{r|l}
 8. \quad 2m^3 + 2m^2n - mn^2 - n^3 & \\
 = m(2m^2 - n^2) + n(2m^2 - n^2) = (m+n)(2m^2 - n^2) & \\
 3m^3 + 3m^2n + mn + n^2 & \\
 = m(3m^2 + n) + n(3m^2 + n) = (m+n)(3m^2 + n) & \\
 \text{mcd} = m + n & \\
 2m^3 + 2m^2n - mn^2 - n^3 & m + n \\
 -2m^3 - 2m^2n & 2m^2 - n^2 \\
 \hline
 -mn^2 - n^3 & \\
 +mn^2 + n^3 & \\
 \hline
 3m^3 + 3m^2n + mn + n^2 & m + n \\
 -3m^3 - 3m^2n & 3m^2 + n \\
 \hline
 mn + n^2 & \\
 -mn - n^2 & \\
 \hline
 \end{array}$$

$$= \frac{2m^2 - n^2}{3m^2 + n} \quad \text{Rta.}$$

$$\begin{array}{r|l}
 9. \quad \frac{6a^5 + 3a^4 - 4a^3 - 2a^2 + 10a + 5}{3a^6 + 7a^4 - a^2 + 15} \quad (2) & \\
 6a^6 & +14a^4 & -2a^2 & +30 & 6a^5 + 3a^4 - 4a^3 - 2a^2 + 10a + 5 \\
 -6a^6 - 3a^5 + 4a^4 + 2a^3 - 10a^2 - 5a & & & & a \\
 \hline
 -3a^5 + 18a^4 + 2a^3 - 12a^2 - 5a + 30 & (\div -1) & \\
 6a^5 + 3a^4 - 4a^3 - 2a^2 + 10a + 5 & 3a^5 - 18a^4 - 2a^3 + 12a^2 + 5a - 30 \\
 -6a^5 + 36a^4 + 4a^3 - 24a^2 - 10a + 60 & 2 \\
 \hline
 39a^4 & -26a^2 & +65 & (\div 13) & \\
 3a^5 - 18a^4 - 2a^3 + 12a^2 + 5a - 30 & 3a^4 - 2a^2 + 5 \\
 -3a^5 & +2a^3 & -5a & & a \\
 \hline
 -18a^4 & +12a^2 & -30 & (\div -6) & \\
 3a^4 - 2a^2 + 5 & 3a^4 - 2a^2 + 5 \\
 -3a^4 + 2a^2 - 5 & 1 & \text{mcd} = 3a^4 - 2a^2 + 5 \\
 \hline
 \end{array}$$

Continúa

$$9. \quad \frac{6a^5+3a^4-4a^3-2a^2+10a+5}{-6a^5+4a^3-10a} \div \frac{3a^4-2a^2+5}{2a+1} = \frac{3a^6+7a^4-a^2+15}{-3a^6+2a^4-5a^2} \div \frac{3a^4-2a^2+5}{a^2+3}$$

$$\frac{3a^4-2a^2+5}{-3a^4+2a^2-5} = \frac{9a^4-6a^2+15}{-9a^4+6a^2-15} = \frac{2a+1}{a^2+3} \quad \text{Rta.}$$

$$\begin{array}{l}
 10. \quad \frac{5x^6 - 10x^4 + 21x^3 - 2x + 4}{3x^6 - 6x^4 + 11x^3 + 2x - 4} \quad (3) \\
 \begin{array}{r}
 15x^6 - 30x^4 + 63x^3 - 6x + 12 \\
 -15x^6 + 30x^4 - 55x^3 - 10x + 20 \\
 \hline
 8x^3 - 16x + 32
 \end{array} \quad \begin{array}{l}
 5x^6 - 30x^4 + 55x^3 + 10x - 20 \quad (\div 5) \\
 \hline
 1
 \end{array} \\
 \begin{array}{r}
 8x^3 - 16x + 32 \quad (\div 8) \\
 3x^6 - 6x^4 + 11x^3 + 2x - 4 \\
 -3x^6 + 6x^4 - 12x^3 \\
 \hline
 -x^3 + 2x - 4 \quad (\div -1) \\
 x^3 - 2x + 4 \\
 -x^3 + 2x - 4 \\
 \hline
 1
 \end{array} \quad \begin{array}{l}
 x^3 - 2x + 4 \\
 3x^3 \\
 \hline
 mcd = x^3 - 2x + 4
 \end{array} \\
 \begin{array}{r}
 5x^6 - 10x^4 + 21x^3 - 2x + 4 \\
 -5x^6 + 10x^4 - 20x^3 \\
 \hline
 x^3 - 2x + 4
 \end{array} \quad \begin{array}{r}
 x^3 - 2x + 4 \\
 5x^3 + 1 \\
 \hline
 3x^3 - 1
 \end{array} \quad \begin{array}{r}
 3x^6 - 6x^4 + 11x^3 + 2x - 4 \\
 -3x^6 + 6x^4 - 12x^3 \\
 \hline
 -x^3 + 2x - 4 \\
 + x^3 - 2x + 4 \\
 \hline
 1
 \end{array} \\
 \end{array}$$

$$\begin{aligned}
 11. \quad & n^6 - 3n^5 - n^4 + 3n^3 + 7n^2 - 21n \\
 &= n^5(n-3) - n^3(n-3) + 7n(n-3) = (n-3)(n^5 - n^3 + 7n) \\
 & n^6 + 2n^5 - n^4 - 2n^3 + 7n^2 + 14n \\
 &= n^5(n+2) - n^3(n+2) + 7n(n+2) = (n+2)(n^5 - n^3 + 7n) \\
 & \frac{n^6 - 3n^5 - n^4 + 3n^3 + 7n^2 - 21n}{-n^6 + n^4 - 7n^2} \bigg| \frac{n^5 - n^3 + 7n}{n-3} \quad \frac{n^6 + 2n^5 - n^4 - 2n^3 + 7n^2 + 14n}{-n^6 + n^4 - 7n^2} \bigg| \frac{n^5 - n^3 + 7n}{n+2} \\
 & \quad \frac{-3n^5 + 3n^3 - 21n}{+3n^5 - 3n^3 + 21n} \quad \frac{2n^5 - 2n^3 + 14n}{-2n^5 + 2n^3 - 14n} = \frac{n-3}{n+2} \quad Rta.
 \end{aligned}$$

$$\begin{array}{r|l}
 12. & \begin{array}{l} a^7 + 2a^6 - 5a^5 + 8a^4 + a^3 + 2a^2 - 5a + 8 \\ -a^7 - 2a^6 + 5a^5 - 10a^4 - 4a^3 + 10a^2 - 16a \\ \hline -2a^4 - 3a^3 + 12a^2 - 21a + 8 \end{array} & \begin{array}{l} a^6 + 2a^5 - 5a^4 + 10a^3 + 4a^2 - 10a + 16 \\ \hline a \\ \hline \end{array} & (2) \\
 & \begin{array}{l} \phantom{12.} \\ \phantom{12.} \\ \phantom{12.} \\ 2a^6 + 4a^5 - 10a^4 + 20a^3 + 8a^2 - 20a + 32 \\ -2a^6 - 3a^5 + 12a^4 - 21a^3 + 8a^2 \\ \hline a^5 + 2a^4 - a^3 + 16a^2 - 20a + 32 \end{array} & \begin{array}{l} 2a^4 + 3a^3 - 12a^2 + 21a - 8 \\ \hline a^2 \end{array} & (a) \\
 & \begin{array}{l} \phantom{12.} \\ \phantom{12.} \\ \phantom{12.} \\ 2a^5 + 3a^4 - 12a^3 + 21a^2 - 8a \\ -2a^5 - 4a^4 + 2a^3 - 32a^2 + 40a - 64 \\ \hline -a^4 - 10a^3 - 11a^2 + 32a - 64 \end{array} & \begin{array}{l} a^5 + 2a^4 - a^3 + 16a^2 - 20a + 32 \\ \hline a^5 + 2a^4 - a^3 + 16a^2 - 20a + 32 \\ \hline 2 \\ \hline \end{array} & \\
 & \phantom{12.} & \begin{array}{l} -a^4 - 10a^3 - 11a^2 + 32a - 64 \\ \phantom{12.} \end{array} & (-1)
 \end{array}$$

**Continúa**

### Continuación

$$12. \begin{array}{r|l} a^5 + 2a^4 - a^3 + 16a^2 - 20a + 32 & a^4 + 10a^3 + 11a^2 - 32a + 64 \quad (2) \\ -a^5 - 10a^4 - 11a^3 + 32a^2 - 64a & a \end{array}$$

$$-8a^4 - 12a^3 + 48a^2 - 84a + 32 \quad (\div -4)$$

$$\begin{array}{r|l} 2a^4 + 20a^3 + 22a^2 - 64a + 128 & 2a^4 + 3a^3 - 12a^2 + 21a - 8 \\ -2a^4 - 3a^3 + 12a^2 - 21a + 8 & 1 \end{array}$$

$$17a^3 + 34a^2 - 85a + 136 \quad (\div 17)$$

$$\begin{array}{r|l} 2a^4 + 3a^3 - 12a^2 + 21a - 8 & a^3 + 2a^2 - 5a + 8 \\ -2a^4 - 4a^3 + 10a^2 - 16a & 2a \end{array}$$

$$-a^3 - 2a^2 + 5a - 8 \quad (-1)$$

$$\begin{array}{r|l} a^3 + 2a^2 - 5a + 8 & a^3 + 2a^2 - 5a + 8 \\ -a^3 - 2a^2 + 5a - 8 & 1 \end{array}$$

$$mcd = a^3 + 2a^2 - 5a + 8$$

$$\begin{array}{r|l} a^7 + 2a^6 - 5a^5 + 8a^4 + a^3 + 2a^2 - 5a + 8 & a^3 + 2a^2 - 5a + 8 \\ -a^7 - 2a^6 + 5a^5 - 8a^4 & a^4 + 1 \end{array} \quad \begin{array}{r|l} a^6 + 2a^5 - 5a^4 + 10a^3 + 4a^2 - 10a + 16 & a^3 + 2a^2 - 5a + 8 \\ -a^6 - 2a^5 + 5a^4 - 8a^3 & a^3 + 2 \end{array}$$

$$\begin{array}{r} a^3 + 2a^2 - 5a + 8 \\ -a^3 - 2a^2 + 5a - 8 \\ \hline \end{array} = \frac{a^4 + 1}{a^3 + 2} \quad Rta$$

$$\begin{array}{r} 2a^3 + 4a^2 - 10a + 16 \\ -2a^3 - 4a^2 + 10a - 16 \\ \hline \end{array}$$

### EJERCICIO 122

$$1. \frac{3}{2a} \cdot \frac{2a}{2a} = \frac{6a}{4a^2}$$

$$2. \frac{5}{9x^2} \cdot \frac{4a}{4a} = \frac{20a}{36ax^2}$$

$$3. \frac{m}{ab^2} \cdot \frac{2a}{2a} = \frac{2am}{2a^2b^2}$$

$$4. \frac{3x}{8y} \cdot \frac{3xy^2}{3xy^2} = \frac{9x^2y^2}{24xy^3}$$

$$5. \frac{4m}{5n^2} \cdot \frac{n}{n} = \frac{4mn}{5n^3}$$

$$6. \frac{2x+7}{5} \cdot \frac{3}{3} = \frac{6x+21}{15}$$

$$7. \frac{2x}{x-1} \cdot \frac{x}{x} = \frac{2x^2}{x^2-x}$$

$$8. \frac{a^2}{a+2} \cdot \frac{2a}{2a} = \frac{2a^3}{2a^2+4a}$$

$$9. \frac{3a}{a+b} \cdot \frac{a+b}{a+b} = \frac{3a^2+3ab}{a^2+2ab+b^2}$$

$$10. \frac{x-4}{x+3} \cdot \frac{x+2}{x+2} = \frac{x^2-2x-8}{x^2+5x+6}$$

$$11. \frac{2a}{x+a} \cdot \frac{a^2}{a^2} = \frac{2a^3}{a^2x+a^3}$$

$$12. \frac{x-y}{6} \cdot \frac{2}{2} = \frac{2x-2y}{12}$$

$$13. \frac{5x}{a-b} \cdot \frac{a+b}{a+b} = \frac{5ax+5bx}{a^2-b^2}$$

$$14. \frac{x-5}{a} \cdot \frac{3x}{3x} = \frac{3x^2-15x}{3ax}$$

$$15. \frac{5x}{2x+y} \cdot \frac{2x+y}{2x+y} = \frac{10x^2+5xy}{4x^2+4xy+y^2}$$

$$16. \frac{x+3}{x+1} \cdot \frac{x-3}{x-3} = \frac{x^2-9}{x^2-2x-3}$$

$$17. \frac{2}{a+1} \cdot \frac{a^2-a+1}{a^2-a+1} = \frac{2a^2-2a+2}{a^3+1}$$

$$18. \frac{x-2y}{3x} \cdot \frac{3xy}{3xy} = \frac{3x^2y-6xy^2}{9x^2y}$$

$$19. \frac{x-1}{x+1} \cdot \frac{x+1}{x+1} = \frac{x^2-1}{x^2+2x+1}$$

$$20. \frac{a+b}{7a^2} \cdot \frac{9ab}{9ab} = \frac{9a^2b+9ab^2}{63a^3b}$$

$$21. \frac{x+1}{x+5} \cdot \frac{x-2}{x-2} = \frac{x^2-x-2}{x^2+3x-10}$$

### EJERCICIO 123

$$1. \frac{6a^3-10a^2}{2a} = \frac{6a^3}{2a} - \frac{10a^2}{2a} = 3a^2 - 5a$$

$$2. \frac{9x^3y-6x^2y^2+3xy^3}{3xy} = \frac{9x^3y}{3xy} - \frac{6x^2y^2}{3xy} + \frac{3xy^3}{3xy} = 3x^2 - 2xy + y^2$$

$$3. \begin{array}{r|l} x^2+3 & x \\ -x^2 & x \\ \hline 3 & \\ =x+\frac{3}{x} & \end{array}$$

$$4. \begin{array}{r|l} 10a^2+15a-2 & 5a \\ -10a^2 & 2a+3 \\ \hline 15a-2 & \\ -15a & \\ \hline -2 & \\ =2a+3-\frac{2}{5a} & \end{array}$$

$$\begin{array}{r}
 5. \quad \begin{array}{r} 9x^3 - 6x^2 + 3x - 5 \\ -9x^3 \end{array} \quad \begin{array}{r} 3x \\ 3x^2 - 2x + 1 \end{array} \\
 \hline
 \begin{array}{r} -6x^2 \\ +6x^2 \end{array} \\
 \hline
 \begin{array}{r} 3x \\ -3x \end{array} \\
 \hline
 -5 \\
 = 3x^2 - 2x + 1 - \frac{5}{3x}
 \end{array}$$

$$\begin{array}{r}
 6. \quad \begin{array}{r} x^2 - 5x - 16 \\ -x^2 - 2x \end{array} \quad \begin{array}{r} x + 2 \\ x - 7 \end{array} \\
 \hline
 \begin{array}{r} -7x - 16 \\ +7x + 14 \end{array} \\
 \hline
 -2 \\
 = x - 7 - \frac{2}{x+2}
 \end{array}$$

$$\begin{array}{r}
 7. \quad \begin{array}{r} 12x^2 - 6x - 2 \\ -12x^2 + 3x \end{array} \quad \begin{array}{r} 4x - 1 \\ 3x \end{array} \\
 \hline
 -3x - 2 \\
 = 3x + \frac{-3x - 2}{4x - 1} = 3x - \frac{3x + 2}{4x - 1}
 \end{array}$$

$$\begin{array}{r}
 8. \quad \begin{array}{r} a^3 \\ -a^3 - 2a^2b \end{array} \quad \begin{array}{r} +3b^3 \\ a^2 - 2ab + 4b^2 \end{array} \\
 \hline
 -2a^2b \\
 \begin{array}{r} 2a^2b + 4ab^2 \\ 4ab^2 + 3b^3 \\ -4ab^2 - 8b^3 \\ -5b^3 \end{array} \\
 = a^2 - 2ab + 4b^2 - \frac{5b^3}{a+2b}
 \end{array}$$

$$\begin{array}{r}
 9. \quad \begin{array}{r} x^3 - x^2 - 6x + 1 \\ -x^3 + 3x \end{array} \quad \begin{array}{r} x^2 - 3 \\ x - 1 \end{array} \\
 \hline
 \begin{array}{r} -x^2 - 3x + 1 \\ x^2 - 3 \end{array} \\
 \hline
 -3x - 2 \\
 = x - 1 + \frac{-3x - 2}{x^2 - 3} = x - 1 - \frac{3x + 2}{x^2 - 3}
 \end{array}$$

$$\begin{array}{r}
 10. \quad \begin{array}{r} 3x^3 + 4x^2y + 2xy^2 - 6y^3 \\ -3x^3 + 2x^2y \end{array} \quad \begin{array}{r} 3x - 2y \\ x^2 + 2xy + 2y^2 \end{array} \\
 \hline
 \begin{array}{r} 6x^2y + 2xy^2 \\ -6x^2y + 4xy^2 \end{array} \\
 \hline
 \begin{array}{r} 6xy^2 - 6y^3 \\ -6xy^2 + 4y^3 \\ -2y^3 \end{array} \\
 = x^2 + 2xy + 2y^2 - \frac{2y^3}{3x - 2y}
 \end{array}$$

$$\begin{array}{r}
 11. \quad \begin{array}{r} 2x^3 - 7x^2 + 6x - 8 \\ -2x^3 + x^2 - x \end{array} \quad \begin{array}{r} 2x^2 - x + 1 \\ x - 3 \end{array} \\
 \hline
 \begin{array}{r} -6x^2 + 5x - 8 \\ 6x^2 - 3x + 3 \end{array} \\
 \hline
 2x - 5 \\
 = x - 3 + \frac{2x - 5}{2x^2 - x + 1}
 \end{array}$$

$$\begin{array}{r}
 12. \quad \begin{array}{r} 2a^4 - 3a^3 + a^2 \\ -2a^4 + 2a^3 - 2a^2 \end{array} \quad \begin{array}{r} a^2 - a + 1 \\ 2a^2 - a - 2 \end{array} \\
 \hline
 \begin{array}{r} -a^3 - a^2 \\ a^3 - a^2 + a \end{array} \\
 \hline
 \begin{array}{r} -2a^2 + a \\ 2a^2 - 2a + 2 \end{array} \\
 \hline
 -a + 2 \\
 = 2a^2 - a - 2 - \frac{a - 2}{a^2 - a + 1}
 \end{array}$$

$$\begin{array}{r}
 13. \quad \begin{array}{r} x^4 - 4x^2 - 3x \\ -x^4 + 2x^2 \end{array} \quad \begin{array}{r} x^2 - 2 \\ x^2 - 2 \end{array} \\
 \hline
 \begin{array}{r} -2x^2 - 3x \\ 2x^2 - 4 \end{array} \\
 \hline
 -3x - 4 \\
 = x^2 - 2 - \frac{3x + 4}{x^2 - 2}
 \end{array}$$

$$\begin{array}{r}
 14. \quad \begin{array}{r} 10n^3 - 18n^2 - 5n + 3 \\ -10n^3 + 15n^2 - 5n \end{array} \quad \begin{array}{r} 2n^2 - 3n + 1 \\ 5n \end{array} \\
 \hline
 -3n^2 - 10n + 3 \\
 = 5n - \frac{3n^2 + 10n - 3}{2n^2 - 3n + 1}
 \end{array}$$

$$\begin{array}{r}
 15. \quad \begin{array}{r} 8x^4 \\ -8x^4 - 10x^3 - 12x^2 \end{array} \quad \begin{array}{r} 4x^2 + 5x + 6 \\ 2x^2 \end{array} \\
 \hline
 -10x^3 - 12x^2 \\
 = 2x^2 - \frac{10x^3 + 12x^2}{4x^2 + 5x + 6}
 \end{array}$$

$$\begin{array}{r}
 16. \quad \begin{array}{r} 6m^5 + 3m^4n \\ -6m^5 \end{array} \quad \begin{array}{r} 2m^3n^2 - 2m^2n^3 \\ 3m^3 - mn^2 + n^3 \end{array} \\
 \hline
 \begin{array}{r} 3m^4n + 2m^3n^2 - 2m^2n^3 \\ -3m^4n \end{array} \quad \begin{array}{r} m^2n^3 - mn^4 \\ 2m^3n^2 - m^2n^3 - mn^4 \end{array} \\
 \hline
 2m^3n^2 - m^2n^3 - mn^4 \\
 = 2m^2 + mn + \frac{2m^3n^2 - m^2n^3 - mn^4}{3m^3 - mn^2 + n^3}
 \end{array}$$

## EJERCICIO 124

$$1. a + \frac{4a}{a+2} = \frac{a(a+2)+4a}{a+2} = \frac{a^2+2a+4a}{a+2} = \frac{a^2+6a}{a+2} = \frac{a(a+6)}{a+2}$$

$$2. m - n - \frac{n^2}{m} = \frac{(m-n)m - n^2}{m} = \frac{m^2 - mn - n^2}{m}$$

$$3. x + 5 - \frac{3}{x-2} = \frac{(x+5)(x-2) - 3}{x-2} = \frac{x^2 + 3x - 10 - 3}{x-2} = \frac{x^2 + 3x - 13}{x-2}$$

$$4. a + \frac{ab}{a+b} = \frac{a(a+b) + ab}{a+b} = \frac{a^2 + ab + ab}{a+b} = \frac{a^2 + 2ab}{a+b}$$

$$5. \frac{1-a^2}{a} + a - 3 = \frac{a(a-3) + 1 - a^2}{a} = \frac{a^2 - 3a + 1 - a^2}{a} = -\frac{3a-1}{a}$$

$$6. 1 - \frac{a+x}{a-x} = \frac{(a-x) - (a+x)}{a-x} = \frac{a-x-a-x}{a-x} = -\frac{2x}{a-x}$$

$$7. \frac{2a+x}{a+x} - 1 = \frac{2a+x - (a+x)}{a+x} = \frac{2a+x-a-x}{a+x} = \frac{a}{a+x}$$

$$8. x + 2 - \frac{3}{x-1} = \frac{(x+2)(x-1) - 3}{x-1} = \frac{x^2 + x - 2 - 3}{x-1} = \frac{x^2 + x - 5}{x-1}$$

$$9. x^2 - 3x - \frac{x^2 - 6x}{x+2} = \frac{(x^2 - 3x)(x+2) - (x^2 - 6x)}{x+2} = \frac{x^3 - x^2 - 6x - x^2 + 6x}{x+2} = \frac{x^3 - 2x^2}{x+2} = \frac{x^2(x-2)}{x+2}$$

$$10. x + y + \frac{x^2 - y^2}{x-y} = \frac{(x+y)(x-y) + x^2 - y^2}{x-y} = \frac{x^2 - y^2 + x^2 - y^2}{x-y} = \frac{2x^2 - 2y^2}{x-y} = \frac{2(x^2 - y^2)}{x-y} = \frac{2(x+y)(x-y)}{x-y} = 2x + 2y = 2(x+y)$$

$$11. \frac{3mn}{m-n} + m - 2n = \frac{(m-n)(m-2n) + 3mn}{m-n} = \frac{m^2 - 3mn + 2n^2 + 3mn}{m-n} = \frac{m^2 + 2n^2}{m-n}$$

$$12. 2a - 3x - \frac{5ax - 6x^2}{a+2x} = \frac{(2a-3x)(a+2x) - (5ax - 6x^2)}{a+2x} = \frac{2a^2 + ax - 6x^2 - 5ax + 6x^2}{a+2x} = \frac{2a^2 - 4ax}{a+2x}$$

$$13. m^2 - 2m + 4 - \frac{m^3}{m+2} = \frac{(m^2 - 2m + 4)(m+2) - m^3}{m+2} = \frac{m^3 + 8 - m^3}{m+2} = \frac{8}{m+2}$$

$$14. x^2 - 5x - \frac{3x(x+2)}{x-2} = \frac{(x^2 - 5x)(x-2) - 3x(x+2)}{x-2} = \frac{x^3 - 7x^2 + 10x - 3x^2 - 6x}{x-2} = \frac{x^3 - 10x^2 + 4x}{x-2}$$

$$15. a^2 + 3ab - b^2 + \frac{7ab^2 - b^3}{2a-b} = \frac{(a^2 + 3ab - b^2)(2a-b) + 7ab^2 - b^3}{2a-b} = \frac{2a^3 + 5a^2b - 5ab^2 + b^3 + 7ab^2 - b^3}{2a-b} = \frac{2a^3 + 5a^2b + 2ab^2}{2a-b}$$

$$16. \frac{x^3 + 2}{x^2 - x + 1} - (x+1) = \frac{(x^3 + 2) - (x^2 - x + 1)(x+1)}{x^2 - x + 1} = \frac{x^3 + 2 - x^3 - 1}{x^2 - x + 1} = \frac{1}{x^2 - x + 1}$$

$$17. x + 3 - \frac{x^3 - 2x^2 + 1}{x^2 - 4x + 3} = \frac{(x+3)(x^2 - 4x + 3) - (x^3 - 2x^2 + 1)}{x^2 - 4x + 3} = \frac{x^3 - x^2 - 9x + 9 - x^3 + 2x^2 - 1}{x^2 - 4x + 3} = \frac{x^2 - 9x + 8}{x^2 - 4x + 3} = \frac{(x-8)(x-1)}{(x-3)(x-1)} = \frac{x-8}{x-3}$$



$$\begin{aligned}
 18. \quad & 3a + \frac{3a^2b + 3ab^2}{a^2 - b^2} \\
 &= \frac{3a(a^2 - b^2) + 3a^2b + 3ab^2}{a^2 - b^2} \\
 &= \frac{3a^3 - 3ab^2 + 3a^2b + 3ab^2}{a^2 - b^2} \\
 &= \frac{3a^3 + 3a^2b}{a^2 - b^2} = \frac{3a^2(a+b)}{(a+b)(a-b)} \\
 &= \frac{3a^2}{a-b}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & x - 3 - \frac{x^3 - 27}{x^2 - 6x + 9} \\
 &= \frac{(x-3)(x^2 - 6x + 9) - (x^3 - 27)}{x^2 - 6x + 9} \\
 &= \frac{x^3 - 9x^2 + 27x - 27 - x^3 + 27}{x^2 - 6x + 9} \\
 &= \frac{-9x^2 + 27x}{(x-3)^2} = \frac{-9x(x-3)}{(x-3)^2} \\
 &= \frac{-9x}{x-3} = \frac{9x}{3-x}
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & a^2 - 3a + 5 + \frac{2a^3 - 11a + 9}{a^2 + a - 2} \\
 &= \frac{(a^2 - 3a + 5)(a^2 + a - 2) + 2a^3 - 11a + 9}{a^2 + a - 2} \\
 &= \frac{a^4 - 2a^3 + 11a - 10 + 2a^3 - 11a + 9}{a^2 + a - 2} \\
 &= \frac{a^4 - 1}{(a+2)(a-1)} = \frac{(a^2-1)(a^2+1)}{(a+2)(a-1)} \\
 &= \frac{(a^2+1)(a+1)(a-1)}{(a+2)(a-1)} \\
 &= \frac{a^3 + a^2 + a + 1}{a+2}
 \end{aligned}$$

## EJERCICIO 125

$$\begin{aligned}
 1. \quad & \frac{a}{b} ; \frac{1}{ab} \quad mcm = ab \quad ab \div b = a \\
 & \quad \quad \quad ab \div ab = 1 \\
 & \frac{a}{b} = \frac{a \cdot a}{ab} = \frac{a^2}{ab} \\
 & \frac{1}{ab} = \frac{1 \cdot 1}{ab} = \frac{1}{ab} \Rightarrow \frac{a^2}{ab}, \frac{1}{ab} \quad Rta.
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \frac{x}{2a}, \frac{4}{3a^2x} \quad mcm = 6a^2x \\
 & 6a^2x \div 2a = 3ax \\
 & 6a^2x \div 3a^2x = 2 \\
 & \frac{x}{2a} = \frac{x(3ax)}{6a^2x} = \frac{3ax^2}{6a^2x} \\
 & \frac{4}{3a^2x} = \frac{4 \cdot 2}{6a^2x} = \frac{8}{6a^2x} \\
 & \Rightarrow \frac{3ax^2}{6a^2x}, \frac{8}{6a^2x} \quad Rta.
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \frac{1}{2x^2}, \frac{3}{4x}, \frac{5}{8x^3} \quad mcm = 8x^3 \\
 & 8x^3 \div 2x^2 = 4x \\
 & 8x^3 \div 4x = 2x^2 \\
 & 8x^3 \div 8x^3 = 1 \\
 & \frac{1}{2x^2} = \frac{1 \cdot 4x}{8x^3} = \frac{4x}{8x^3} \\
 & \frac{3}{4x} = \frac{3x \cdot 2x^2}{8x^3} = \frac{6x^2}{8x^3} \\
 & \frac{5}{8x^3} = \frac{5 \cdot 1}{8x^3} = \frac{5}{8x^3} \\
 & \Rightarrow \frac{4x}{8x^3}, \frac{6x^2}{8x^3}, \frac{5}{8x^3} \quad Rta.
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \frac{3x}{ab^2}, \frac{x}{a^2b}, \frac{3}{a^3} \quad mcm = a^3b^2 \\
 & a^3b^2 \div ab^2 = a^2 \quad a^3b^2 \div a^2b = ab \quad a^3b^2 \div a^3 = b^2 \\
 & \frac{3x}{ab^2} = \frac{3x(a^2)}{a^3b^2} = \frac{3a^2x}{a^3b^2}; \quad \frac{x}{a^2b} = \frac{x(ab)}{a^3b^2} = \frac{abx}{a^3b^2}; \quad \frac{3}{a^3} = \frac{3(b^2)}{a^3b^2} = \frac{3b^2}{a^3b^2} \\
 & \Rightarrow \frac{3a^2x}{a^3b^2}, \frac{abx}{a^3b^2}, \frac{3b^2}{a^3b^2} \quad Rta.
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \frac{7y}{6x^2}, \frac{1}{9xy}, \frac{5x}{12y^3} \quad mcm = 36x^2y^3 \\
 & 36x^2y^3 \div 6x^2 = 6y^3 \quad 36x^2y^3 \div 9xy = 4xy^2 \quad 36x^2y^3 \div 12y^3 = 3x^2 \\
 & \frac{7y}{6x^2} = \frac{7y(6y^3)}{36x^2y^3} = \frac{42y^4}{36x^2y^3}; \quad \frac{1}{9xy} = \frac{1(4xy^2)}{36x^2y^3} = \frac{4xy^2}{36x^2y^3}; \\
 & \frac{5x}{12y^2} = \frac{5x(3x^2)}{36x^2y^3} = \frac{15x^3}{36x^2y^3} \Rightarrow \frac{42y^4}{36x^2y^3}, \frac{4xy^2}{36x^2y^3}, \frac{15x^3}{36x^2y^3} \quad Rta.
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \frac{a-1}{3a}, \frac{5}{6a}, \frac{a+2}{a^2} \quad mcm = 6a^2 \\
 & 6a^2 \div 3a = 2a \quad 6a^2 \div 6a = a \quad 6a^2 \div a^2 = 6 \\
 & \frac{a-1}{3a} = \frac{(a-1)(2a)}{6a^2} = \frac{2a^2 - 2a}{6a^2}; \quad \frac{5}{6a} = \frac{5 \cdot 1}{6a^2} = \frac{5}{6a^2}; \\
 & \frac{a+2}{a^2} = \frac{(a+2)6}{6a^2} = \frac{6a + 12}{6a^2} \Rightarrow \frac{2a^2 - 2a}{6a^2}, \frac{5}{6a^2}, \frac{6a + 12}{6a^2} \quad Rta.
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & \frac{x-y}{x^2y}, \frac{x+y}{3xy^2}, 5 \quad mcm = 3x^2y^2 \\
 & 3x^2y^2 \div x^2y = 3y \\
 & 3x^2y^2 \div 3xy^2 = x \\
 & \frac{x-y}{x^2y} = \frac{(x-y)(3y)}{3x^2y^2} = \frac{3xy - 3y^2}{3x^2y^2}; \quad \frac{x+y}{3xy^2} = \frac{(x+y)x}{3x^2y^2} = \frac{x^2 + xy}{3x^2y^2}; \\
 & 5 = \frac{5(3x^2y^2)}{3x^2y^2} = \frac{15x^2y^2}{3x^2y^2} \Rightarrow \frac{3xy - 3y^2}{3x^2y^2}, \frac{x^2 + xy}{3x^2y^2}, \frac{15x^2y^2}{3x^2y^2} \quad Rta.
 \end{aligned}$$

$$\begin{aligned}
8. \quad & \frac{m+n}{2m}, \frac{m-n}{5m^3n}, \frac{1}{10n^2} \quad mcm = 10m^3n^2 \\
& 10m^3n^2 \div 2m = 5m^2n^2 \\
& 10m^3n^2 \div 5m^3n = 2n \\
& 10m^3n^2 \div 10n^2 = m^3 \\
& \frac{m+n}{2m} = \frac{(m+n)(5m^2n^2)}{10m^3n^2} = \frac{5m^3n^2 + 5m^2n^3}{10m^3n^2} \\
& \frac{m-n}{5m^3n} = \frac{(m-n)(2n)}{10m^3n^2} = \frac{2mn - 2n^2}{10m^3n^2} \\
& \frac{1}{10n^2} = \frac{1(m^3)}{10m^3n^2} = \frac{m^3}{10m^3n^2} \\
& \Rightarrow \frac{5m^3n^2 + 5m^2n^3}{10m^3n^2}, \frac{2mn - 2n^2}{10m^3n^2}, \frac{m^3}{10m^3n^2} \quad Rta.
\end{aligned}$$

$$\begin{aligned}
9. \quad & \frac{a+b}{6}, \frac{a-b}{2a}, \frac{a^2+b^2}{3b^2} \quad mcm = 6ab^2 \\
& 6ab^2 \div 6 = ab^2 \\
& 6ab^2 \div 2a = 3b^2 \\
& 6ab^2 \div 3b^2 = 2a \\
& \frac{a+b}{6} = \frac{(a+b)(ab^2)}{6ab^2} = \frac{a^2b^2 + ab^3}{6ab^2}; \\
& \frac{a-b}{2a} = \frac{(a-b)(3b^2)}{6ab^2} = \frac{3ab^2 - 3b^3}{6ab^2}; \\
& \frac{a^2+b^2}{3b^2} = \frac{(a^2+b^2)(2a)}{6ab^2} = \frac{2a^3 + 2ab^2}{6ab^2} \\
& \Rightarrow \frac{a^2b^2 + ab^3}{6ab^2}, \frac{3ab^2 - 3b^3}{6ab^2}, \frac{2a^3 + 2ab^2}{6ab^2} \quad Rta.
\end{aligned}$$

$$\begin{aligned}
10. \quad & \frac{2a-b}{3a^2}, \frac{3b-a}{4b^2}, \frac{a-3b}{2} \quad mcm = 12a^2b^2 \\
& 12a^2b^2 \div 3a^2 = 4b^2 \\
& 12a^2b^2 \div 4b^2 = 3a^2 \\
& 12a^2b^2 \div 2 = 6a^2b^2 \\
& \frac{2a-b}{3a^2} = \frac{(2a-b)4b^2}{12a^2b^2} = \frac{8ab^2 - 4b^3}{12a^2b^2}; \\
& \frac{3b-a}{4b^2} = \frac{(3b-a)3a^2}{12a^2b^2} = \frac{9a^2b - 3a^3}{12a^2b^2}; \\
& \frac{a-3b}{2} = \frac{(a-3b)6a^2b^2}{12a^2b^2} = \frac{6a^3b^2 - 18a^2b^3}{12a^2b^2} \\
& \Rightarrow \frac{8ab^2 - 4b^3}{12a^2b^2}, \frac{9a^2b - 3a^3}{12a^2b^2}, \frac{6a^3b^2 - 18a^2b^3}{12a^2b^2} \quad Rta.
\end{aligned}$$

$$\begin{aligned}
11. \quad & \frac{2}{5}, \frac{3}{x+1} \quad mcm = 5x+5 \\
& (5x+5) \div 5 = x+1 \quad (5x+5) \div (x+1) = 5 \\
& \frac{2}{5} = \frac{2(x+1)}{5x+5} = \frac{2x+2}{5x+5}; \quad \frac{3}{x+1} = \frac{3 \cdot 5}{5x+5} = \frac{15}{5x+5} \\
& \Rightarrow \frac{2x+2}{5x+5}, \frac{15}{5x+5} \quad Rta.
\end{aligned}$$

$$\begin{aligned}
12. \quad & \frac{a}{a+b}, \frac{b}{a^2-b^2} \quad mcm = a^2-b^2 \\
& (a^2-b^2) \div (a+b) = a-b \quad (a^2-b^2) \div (a^2-b^2) = 1 \\
& \frac{a}{a+b} = \frac{a(a-b)}{a^2-b^2} = \frac{a^2-ab}{a^2-b^2}; \\
& \frac{b}{a^2-b^2} = \frac{1b}{a^2-b^2} = \frac{b}{a^2-b^2} \\
& \Rightarrow \frac{a^2-ab}{a^2-b^2}, \frac{b}{a^2-b^2} \quad Rta.
\end{aligned}$$

$$\begin{aligned}
13. \quad & \frac{x}{x^2-1}, \frac{1}{x^2-x-2} \quad x^2-1 = (x+1)(x-1) \\
& x^2-x-2 = (x-2)(x+1) \\
& mcm = (x-1)(x-2)(x+1) \\
& = (x^2-1)(x-2) \\
& = x^3-2x^2-x+2 \\
& \begin{array}{r|l} x^3-2x^2-x+2 & x^2-1 \\ -x^3 & x-2 \\ \hline -2x^2 & +2 \\ -2x^2 & -2 \end{array} \quad \begin{array}{r|l} x^3-2x^2-x+2 & x^2-x-2 \\ -x^3+x^2+2x & x-1 \\ \hline -x^2+x+2 & \\ -x^2 & x^2-x-2 \end{array} \\
& \frac{x}{x^2-1} = \frac{x(x-2)}{x^3-2x^2-x+2} = \frac{x^2-2x}{x^3-2x^2-x+2}; \\
& \frac{1}{x^2-x-2} = \frac{1(x-1)}{x^3-2x^2-x+2} = \frac{x-1}{x^3-2x^2-x+2} \\
& \Rightarrow \frac{x^2-2x}{x^3-2x^2-x+2}; \frac{x-1}{x^3-2x^2-x+2} \quad Rta.
\end{aligned}$$

$$\begin{aligned}
14. \quad & \frac{a-3}{4(a+5)}, \frac{3a}{8} \quad mcm = 8(a+5) \\
& [8(a+5)] \div [4(a+5)] = 2 \quad 8(a+5) \div 8 = a+5 \\
& \frac{a-3}{4(a+5)} = \frac{2(a-3)}{8(a+5)} = \frac{2a-6}{8(a+5)}; \\
& \frac{3a}{8} = \frac{3a(a+5)}{8(a+5)} = \frac{3a^2+15a}{8(a+5)} \\
& \Rightarrow \frac{2a-6}{8(a+5)}, \frac{3a^2+15a}{8(a+5)} \quad Rta.
\end{aligned}$$

$$\begin{aligned}
 15. \quad & \frac{x^2}{3(a-x)}, \frac{x}{6} \quad mcm = 6(a-x) \\
 & [6(a-x)] \div [3(a-x)] = 2 \quad [6(a-x)] \div 6 = a-x \\
 & \frac{x^2}{3(a-x)} = \frac{2x^2}{6(a-x)}; \quad \frac{x}{6} = \frac{x(a-x)}{6(a-x)} = \frac{ax-x^2}{6(a-x)} \\
 & \Rightarrow \frac{2x^2}{6(a-x)}, \frac{ax-x^2}{6(a-x)} \quad Rta.
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \frac{3}{x^2}, \frac{2}{x}, \frac{x+3}{x^2-x} \quad mcm = x^2(x-1) \\
 & [x^2(x-1)] \div x^2 = x-1 \quad [x^2(x-1)] \div x = x(x-1) \\
 & [x^2(x-1)] \div (x^2-x) = x
 \end{aligned}$$

$$\frac{3}{x^2} = \frac{3(x-1)}{x^2(x-1)} = \frac{3x-3}{x^2(x-1)};$$

$$\frac{2}{x} = \frac{2(x^2-x)}{x^2(x-1)} = \frac{2x^2-2x}{x^2(x-1)};$$

$$\frac{x+3}{x^2-x} = \frac{x(x+3)}{x^2(x-1)} = \frac{x^2+3x}{x^2(x-1)}$$

$$\Rightarrow \frac{3x-3}{x^2(x-1)}, \frac{2x^2-2x}{x^2(x-1)}, \frac{x^2+3x}{x^2(x-1)} \quad Rta.$$

$$17. \quad \frac{1}{2a+2b}, \frac{a}{4a-4b}, \frac{b}{8} \quad mcm = 8(a^2-b^2)$$

$$[8(a^2-b^2)] \div (2a+2b) = 4(a-b) \quad [8(a^2-b^2)] \div 8 = a^2-b^2$$

$$[8(a^2-b^2)] \div (4a-4b) = 2(a+b)$$

$$\frac{1}{2a+2b} = \frac{4(a-b)}{8(a^2-b^2)};$$

$$\frac{a}{4a-4b} = \frac{2a(a+b)}{8(a^2-b^2)} = \frac{2a^2+2ab}{8(a^2-b^2)};$$

$$\frac{b}{8} = \frac{b(a^2-b^2)}{8(a^2-b^2)} = \frac{a^2b-b^3}{8(a^2-b^2)}$$

$$\Rightarrow \frac{4(a-b)}{8(a^2-b^2)}, \frac{2a^2+2ab}{8(a^2-b^2)}, \frac{a^2b-b^3}{8(a^2-b^2)} \quad Rta.$$

$$18. \quad \frac{x}{xy}, \frac{y}{x^2+xy}, \frac{3}{xy+y^2} \quad mcm = xy(x+y)$$

$$[xy(x+y)] \div xy = x+y \quad [xy(x+y)] \div (x^2+xy) = y$$

$$[xy(x+y)] \div (xy+y^2) = x$$

$$\frac{x}{xy} = \frac{x(x+y)}{xy(x+y)} = \frac{x^2+xy}{xy(x+y)}$$

$$\frac{y}{x^2+xy} = \frac{y^2}{xy(x+y)}; \quad \frac{3}{xy+y^2} = \frac{3x}{xy(x+y)}$$

$$\Rightarrow \frac{x^2+xy}{xy(x+y)}, \frac{y^2}{xy(x+y)}, \frac{3x}{xy(x+y)} \quad Rta.$$

$$19. \quad \frac{2}{a^2-b^2}, \frac{1}{a^2+ab}, \frac{a}{a^2-ab} \quad mcm = a(a^2-b^2)$$

$$[a(a^2-b^2)] \div (a^2-b^2) = a$$

$$[a(a^2-b^2)] \div (a^2+ab) = a-b$$

$$[a(a^2-b^2)] \div (a^2-ab) = a+b$$

$$\frac{2}{a^2-b^2} = \frac{2a}{a(a^2-b^2)}; \quad \frac{1}{a^2+ab} = \frac{a-b}{a(a^2-b^2)}$$

$$\frac{a}{a^2-ab} = \frac{a(a+b)}{a(a^2-b^2)} = \frac{a^2+ab}{a(a^2-b^2)}$$

$$\Rightarrow \frac{2a}{a(a^2-b^2)}, \frac{a-b}{a(a^2-b^2)}, \frac{a^2+ab}{a(a^2-b^2)} \quad Rta.$$

$$20. \quad \frac{3x}{x+1}, \frac{x^2}{x-1}, \frac{x^3}{x^2-1} \quad mcm = x^2-1$$

$$(x^2-1) \div (x+1) = x-1$$

$$(x^2-1) \div (x-1) = x+1 \quad (x^2-1) \div (x^2-1) = 1$$

$$\frac{3x}{x+1} = \frac{3x(x-1)}{x^2-1} = \frac{3x^2-3x}{x^2-1}$$

$$\frac{x^2}{x-1} = \frac{x^2(x+1)}{x^2-1} = \frac{x^3+x^2}{x^2-1}; \quad \frac{x^3}{x^2-1} = \frac{x^3}{x^2-1}$$

$$\Rightarrow \frac{3x^2-3x}{x^2-1}, \frac{x^3+x^2}{x^2-1}, \frac{x^3}{x^2-1} \quad Rta.$$

$$21. \quad \frac{1}{m^2-n^2}, \frac{m}{m^2+mn}, \frac{n}{m^2-mn} \quad mcm = m(m^2-n^2)$$

$$[m(m^2-n^2)] \div (m^2-n^2) = m \quad [m(m^2-n^2)] \div (m^2+mn) = m-n$$

$$[m(m^2-n^2)] \div (m^2-mn) = m+n$$

$$\frac{1}{m^2-n^2} = \frac{m}{m(m^2-n^2)};$$

$$\frac{m}{m^2+mn} = \frac{m(m-n)}{m(m^2-n^2)} = \frac{m^2-mn}{m(m^2-n^2)};$$

$$\frac{n}{m^2-mn} = \frac{n(m+n)}{m(m^2-n^2)} = \frac{mn+n^2}{m(m^2-n^2)}$$

$$\Rightarrow \frac{m}{m(m^2-n^2)}, \frac{m^2-mn}{m(m^2-n^2)}, \frac{n(m+n)}{m(m^2-n^2)} \quad Rta.$$

$$22. \quad \frac{n+1}{n-1}, \frac{n-1}{n+1}, \frac{n^2+1}{n^2-1} \quad mcm = n^2-1$$

$$(n^2-1) \div (n-1) = n+1$$

$$(n^2-1) \div (n+1) = n-1 \quad (n^2-1) \div (n^2-1) = 1$$

$$\frac{n+1}{n-1} = \frac{(n+1)(n+1)}{n^2-1} = \frac{(n+1)^2}{n^2-1}; \quad \frac{n-1}{n+1} = \frac{(n-1)^2}{n^2-1};$$

$$\frac{n^2+1}{n^2-1} = \frac{n^2+1}{n^2-1} \Rightarrow \frac{(n+1)^2}{n^2-1}, \frac{(n-1)^2}{n^2-1}, \frac{n^2+1}{n^2-1} \quad Rta.$$

$$\begin{aligned}
23. \quad & \frac{a^2-b^2}{a^2+b^2}, \frac{a^2+b^2}{a^2-b^2}, \frac{a^4+b^4}{a^4-b^4} \quad mcm = a^4 - b^4 \\
& (a^4 - b^4) \div (a^2 + b^2) = a^2 - b^2 \\
& (a^4 - b^4) \div (a^2 - b^2) = a^2 + b^2 \quad (a^4 - b^4) \div (a^4 - b^4) = 1 \\
& \frac{a^2-b^2}{a^2+b^2} = \frac{(a^2-b^2)^2}{a^4-b^4}; \quad \frac{a^2+b^2}{a^2-b^2} = \frac{(a^2+b^2)^2}{a^4-b^4}; \\
& \frac{a^4+b^4}{a^4-b^4} = \frac{a^4+b^4}{a^4-b^4} \\
& \Rightarrow \frac{(a^2-b^2)^2}{a^4-b^4}, \frac{(a^2+b^2)^2}{a^4-b^4}, \frac{a^4+b^4}{a^4-b^4} \quad Rta.
\end{aligned}$$

$$\begin{aligned}
24. \quad & \frac{3x}{x-1}, \frac{x-1}{x+2}, \frac{1}{x^2+x-2} \quad mcm = x^2 + x - 2 \\
& (x^2 + x - 2) \div (x - 1) = x + 2 \\
& (x^2 + x - 2) \div (x + 2) = x - 1 \\
& (x^2 + x - 2) \div (x^2 + x - 2) = 1 \\
& \frac{3x}{x-1} = \frac{3x(x+2)}{x^2+x-2} = \frac{3x^2+6x}{x^2+x-2}; \\
& \frac{x-1}{x+2} = \frac{(x-1)^2}{x^2+x-2}; \quad \frac{1}{x^2+x-2} = \frac{1}{x^2+x-2} \\
& \Rightarrow \frac{3x^2+6x}{x^2+x-2}, \frac{(x-1)^2}{x^2+x-2}, \frac{1}{x^2+x-2} \quad Rta.
\end{aligned}$$

$$\begin{aligned}
25. \quad & \frac{x}{2}, \frac{x}{5x+15}, \frac{x-1}{10x+30} \quad mcm = 10(x+3) \\
& [10(x+3)] \div 2 = 5(x+3) \\
& [10(x+3)] \div [5(x+3)] = 2 \\
& [10(x+3)] \div [10(x+3)] = 1 \\
& \frac{x}{2} = \frac{5x(x+3)}{10(x+3)} = \frac{5x^2+15}{10(x+3)} \\
& \frac{x}{5x+15} = \frac{2x}{10(x+3)}; \quad \frac{x-1}{10x+30} = \frac{x-1}{10(x+3)} \\
& \Rightarrow \frac{5x^2+15}{10(x+3)}, \frac{2x}{10(x+3)}, \frac{x-1}{10(x+3)} \quad Rta.
\end{aligned}$$

$$\begin{aligned}
26. \quad & \frac{2x-1}{x+4}, \frac{3x+1}{3x+12}, \frac{4x+3}{6x+24} \quad mcm = 6(x+4) \\
& [6(x+4)] \div (x+4) = 6 \quad [6(x+4)] \div [3(x+4)] = 2 \\
& [6(x+4)] \div [6(x+4)] = 1 \\
& \frac{2x-1}{x+4} = \frac{6(2x-1)}{6(x+4)} = \frac{12x-6}{6(x+4)}; \\
& \frac{3x+1}{3x+12} = \frac{2(3x+1)}{6(x+4)} = \frac{6x+2}{6(x+4)}; \\
& \frac{4x+3}{6x+24} = \frac{4x+3}{6(x+4)} \\
& \Rightarrow \frac{12x-6}{6(x+4)}, \frac{6x+2}{6(x+4)}, \frac{4x+3}{6(x+4)} \quad Rta.
\end{aligned}$$

$$\begin{aligned}
27. \quad & \frac{a}{a+4}, \frac{2}{9a^2-25}, \frac{5}{3a-5} \quad mcm = (a+4)(3a+5)(3a-5) \\
& [(a+4)(3a+5)(3a-5)] \div (a+4) = (3a+5)(3a-5) \\
& [(a+4)(3a+5)(3a-5)] \div (9a^2-25) = a+4 \\
& [(a+4)(3a+5)(3a-5)] \div (3a-5) = (a+4)(3a+5) \\
& \frac{3}{a+4} = \frac{3(3a+5)(3a-5)}{(a+4)(3a+5)(3a-5)} = \frac{27a^2-75}{(a+4)(9a^2-25)}; \\
& \frac{2}{9a^2-25} = \frac{2(a+4)}{(a+4)(3a+5)(3a-5)} = \frac{2a+8}{(a+4)(9a^2-25)}; \\
& \frac{5}{3a-5} = \frac{5(a+4)(3a+5)}{(a+4)(3a+5)(3a-5)} = \frac{15a^2+85a+100}{(a+4)(9a^2-25)} \\
& \Rightarrow \frac{27a^2-75}{(a+4)(9a^2-25)}, \frac{2a+8}{(a+4)(9a^2-25)}, \frac{15a^2+85a+100}{(a+4)(9a^2-25)} \quad Rta.
\end{aligned}$$

$$\begin{aligned}
28. \quad & \frac{x+1}{x^2-4}, \frac{x+2}{x^2+x-6}, \frac{3x}{x^2+5x+6} \quad mcm = (x+2)(x^2+x-6) \\
& [(x+2)(x^2+x-6)] \div (x^2-4) = x+3 \\
& [(x+2)(x^2+x-6)] \div (x^2+x-6) = x+2 \\
& [(x+2)(x^2+x-6)] \div (x^2+5x+6) = x-2 \\
& \frac{x+1}{x^2-4} = \frac{(x+1)(x+3)}{(x+2)(x^2+x-6)} = \frac{x^2+4x+3}{(x+2)(x^2+x-6)}; \\
& \frac{x+2}{x^2+x-6} = \frac{(x+2)(x+2)}{(x+2)(x^2+x-6)} = \frac{(x+2)^2}{(x+2)(x^2+x-6)}; \\
& \frac{3x}{x^2+5x+6} = \frac{3x(x-2)}{(x+2)(x^2+x-6)} = \frac{3x^2-6x}{(x+2)(x^2+x-6)} \\
& \Rightarrow \frac{x^2+4x+3}{(x+2)(x^2+x-6)}, \frac{(x+2)^2}{(x+2)(x^2+x-6)}, \frac{3x^2-6x}{(x+2)(x^2+x-6)} \quad Rta.
\end{aligned}$$

$$\begin{aligned}
29. \quad & \frac{a+3}{a^2+a-20}, \frac{5a}{a^2-7a+12}, \frac{a+1}{a^2+2a-15} \\
& mcm = (a+5)(a^2-7a+12) \\
& [(a+5)(a^2-7a+12)] \div (a^2+a-20) = a-3 \\
& [(a+5)(a^2-7a+12)] \div (a^2-7a+12) = a+5 \\
& [(a+5)(a^2-7a+12)] \div (a^2+2a-15) = a-4 \\
& \frac{a+3}{a^2+a-20} = \frac{(a+3)(a-3)}{(a+5)(a^2-7a+12)} = \frac{a^2-9}{(a+5)(a-4)(a-3)}; \\
& \frac{5a}{a^2-7a+12} = \frac{5a(a+5)}{(a+5)(a^2-7a+12)} = \frac{5a^2+25}{(a+5)(a-4)(a-3)}; \\
& \frac{a+1}{a^2+2a-15} = \frac{(a+1)(a-4)}{(a+5)(a^2-7a+12)} = \frac{a^2-3a-4}{(a+5)(a-4)(a-3)} \\
& \Rightarrow \frac{a^2-9}{(a+5)(a-4)(a-3)}, \frac{5a^2+25}{(a+5)(a-4)(a-3)}, \frac{a^2-3a-4}{(a+5)(a-4)(a-3)} \quad Rta.
\end{aligned}$$

$$30. \frac{a+1}{a^3-1}, \frac{2a}{a^2+a+1}, \frac{1}{x-1} \quad mcm = a^3-1$$

$$(a^3-1) \div (a^3-1) = 1 \quad (a^3-1) \div (a^2+a+1) = a-1$$

$$(a^3-1) \div (a-1) = a^2+a+1$$

$$\frac{a+1}{a^3-1} = \frac{a+1}{a^3-1}; \frac{2a}{a^2+a+1} = \frac{2a(a-1)}{a^3-1} = \frac{2a^2-2}{a^3-1};$$

$$\frac{1}{a-1} = \frac{a^2+a+1}{a^3-1} \Rightarrow \frac{a+1}{a^3-1}, \frac{2a^2-2}{a^3-1}, \frac{a^2+a+1}{a^3-1} \text{ Rta.}$$

$$31. \frac{1}{x-1}, \frac{1}{x^3-1}, \frac{2}{3} \quad mcm = 3(x^3-1)$$

$$[3(x^3-1)] \div (x-1) = 3(x^2+x+1)$$

$$[3(x^3-1)] \div (x^3-1) = 3 \quad [3(x^3-1)] \div 3 = x^3-1$$

$$\frac{1}{x-1} = \frac{3(x^2+x+1)}{3(x^3-1)} = \frac{3x^2+3x+3}{3(x^3-1)};$$

$$\frac{1}{x^3-1} = \frac{3}{3(x^3-1)}; \frac{2}{3} = \frac{2(x^3-1)}{3(x^3-1)} = \frac{2x^3-2}{3(x^3-1)}$$

$$\Rightarrow \frac{3x^2+3x+3}{3(x^3-1)}, \frac{3}{3(x^3-1)}, \frac{2x^3-2}{3(x^3-1)} \text{ Rta.}$$

$$32. \frac{3}{2a^2+2ab}, \frac{b}{a^2x+abx}, \frac{1}{4ax^2-4bx^2}$$

$$mcm = 4ax^2(a^2-b^2)$$

$$[4ax^2(a^2-b^2)] \div (2a^2+2ab) = 2x^2(a-b)$$

$$[4ax^2(a^2-b^2)] \div (a^2x+abx) = 4x(a-b)$$

$$[4ax^2(a^2-b^2)] \div (4ax^2-4bx^2) = a(a+b)$$

$$\frac{3}{2a^2+2ab} = \frac{3[2x^2(a-b)]}{4ax^2(a^2-b^2)} = \frac{6ax^2-6bx^2}{4ax^2(a^2-b^2)};$$

$$\frac{b}{a^2x+abx} = \frac{b[4x(a-b)]}{4ax^2(a^2-b^2)} = \frac{4abx-4b^2x}{4ax^2(a^2-b^2)};$$

$$\frac{1}{4ax^2-4bx^2} = \frac{a(a+b)}{4ax^2(a^2-b^2)} = \frac{a^2+ab}{4ax^2(a^2-b^2)}$$

$$\Rightarrow \frac{6ax^2-6bx^2}{4ax^2(a^2-b^2)}, \frac{4abx-4b^2x}{4ax^2(a^2-b^2)}, \frac{a^2+ab}{4ax^2(a^2-b^2)} \text{ Rta.}$$

$$33. \frac{1}{a-1}, \frac{a+1}{(a-1)^2}, \frac{3(a+1)}{(a-1)^3} \quad mcm = (a-1)^3$$

$$(a-1)^3 \div (a-1) = (a-1)^2$$

$$(a-1)^3 \div (a-1)^2 = a-1 \quad (a-1)^3 \div (a-1)^3 = 1$$

$$\frac{1}{a-1} = \frac{(a-1)^2}{(a-1)^3}; \frac{a+1}{(a-1)^2} = \frac{(a+1)(a-1)}{(a-1)^3} = \frac{a^2-1}{(a-1)^3};$$

$$\frac{3(a+1)}{(a-1)^3} = \frac{3a+3}{(a-1)^3} \Rightarrow \frac{(a-1)^2}{(a-1)^3}, \frac{a^2-1}{(a-1)^3}, \frac{3a+3}{(a-1)^3} \text{ Rta.}$$

$$34. \frac{2x-3}{6x^2+7x+2}, \frac{3}{2x+1}, \frac{2x-1}{6x+4} \quad mcm = 2(3x+2)(2x+1)$$

$$[2(3x+2)(2x+1)] \div (6x^2+7x+2) = 2$$

$$[2(3x+2)(2x+1)] \div (2x+1) = 2(3x+2)$$

$$[2(3x+2)(2x+1)] \div (6x+4) = 2x+1$$

$$\frac{2x-3}{6x^2+7x+2} = \frac{2(2x-3)}{2(3x+2)(2x+1)} = \frac{4x-6}{2(3x+2)(2x+1)};$$

$$\frac{3}{2x+1} = \frac{6(3x+2)}{2(3x+2)(2x+1)} = \frac{18x+12}{2(3x+2)(2x+1)};$$

$$\frac{2x-1}{6x+4} = \frac{(2x-1)(2x+1)}{2(3x+2)(2x+1)} = \frac{4x^2-1}{2(3x+2)(2x+1)}$$

$$\Rightarrow \frac{4x-6}{2(3x+2)(2x+1)}, \frac{18x+12}{2(3x+2)(2x+1)}, \frac{4x^2-1}{2(3x+2)(2x+1)} \text{ Rta.}$$

## EJERCICIO 126

$$1. \frac{x-2}{4} + \frac{3x+2}{6} = \frac{3x-2}{12} + \frac{2x+3}{6} = \frac{3x-2+4x+6}{12} = \frac{7x+4}{12}$$

$$2. \frac{2}{5a^2} + \frac{1}{3ab} = \frac{3b \cdot 2 + 5a}{15a^2b} = \frac{5a+6b}{15a^2b}$$

$$3. \frac{a-2b}{15a} + \frac{b-a}{20b} = \frac{4b(a-2b) + 3a(b-a)}{60ab}$$

$$= \frac{4ab-8b^2+3ab-3a^2}{60ab} = \frac{-3a^2+7ab-8b^2}{60ab}$$

$$4. \frac{a+3b}{3ab} + \frac{a^2b-4ab^2}{5a^2b^2} = \frac{5ab(a+3b) + 3(a^2b-4ab^2)}{15a^2b^2}$$

$$= \frac{5a^2b+15ab^2+3a^2b-12ab^2}{15a^2b^2}$$

$$= \frac{8a^2b+3ab^2}{15a^2b^2} = \frac{ab(8a+3b)}{15a^2b^2} = \frac{8a+3b}{15ab}$$

$$5. \frac{a-1}{3} + \frac{2a}{6} + \frac{3a+4}{12}$$

$$= \frac{4(a-1) + 2 \cdot 2a + 3a+4}{12} = \frac{4a-4+4a+3a+4}{12} = \frac{11a}{12}$$

$$6. \frac{n}{m^2} + \frac{3}{mn} + \frac{2}{m} = \frac{n \cdot n + 3m + 2mn}{m^2n} = \frac{3m+2mn+n^2}{m^2n}$$

$$7. \frac{1-x}{2x} + \frac{x+2}{x^2} + \frac{1}{3ax^2}$$

$$= \frac{3ax(1-x) + 6a(x+2) + 2}{6ax^2}$$

$$= \frac{3ax-3ax^2+6ax-12a+2}{6ax^2} = \frac{9ax+12a-3ax^2+2}{6ax^2}$$

$$8. \frac{2a-3}{3a} + \frac{3x+2}{10x} + \frac{x-a}{5ax} = \frac{10x(2a-3) + 3a(3x+2) + 6(x-a)}{30ax}$$

$$= \frac{20ax-30x+9ax+6a+6x-6a}{30ax}$$

$$= \frac{29ax-24x}{30ax} = \frac{x(29a-24)}{30ax} = \frac{29a-24}{30a}$$

$$\begin{aligned}
 9. \quad & \frac{3}{5} + \frac{x+2}{2x} + \frac{x^2+2}{6x^2} \\
 &= \frac{6x^2 \cdot 3 + 15x(x+2) + 5(x^2+2)}{30x^2} \\
 &= \frac{18x^2 + 15x^2 + 30x + 5x^2 + 10}{30x^2} \\
 &= \frac{38x^2 + 30x + 10}{30x^2} = \frac{2(19x^2 + 15x + 5)}{30x^2} = \frac{19x^2 + 15x + 5}{15x^2}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & \frac{x-y}{12} + \frac{2x+y}{15} + \frac{y-4x}{30} \\
 &= \frac{5(x-y) + 4(2x+y) + 2(y-4x)}{60} \\
 &= \frac{5x - 5y + 8x + 4y + 2y - 8x}{60} = \frac{5x + y}{60}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & \frac{m-n}{mn} + \frac{n-a}{na} + \frac{2a-m}{am} \\
 &= \frac{a(m-n) + m(n-a) + n(2a-m)}{amn} \\
 &= \frac{am - an + mn - ma + 2an - mn}{amn} = \frac{an}{amn} = \frac{1}{m}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad & \frac{x+2}{3x} + \frac{x^2-2}{5x^2} + \frac{2-x^3}{9x^3} \\
 &= \frac{15x^2(x+2) + 9x(x^2-2) + 5(2-x^3)}{45x^3} \\
 &= \frac{15x^3 + 30x^2 + 9x^3 - 18x + 10 - 5x^3}{45x^3} \\
 &= \frac{19x^3 + 30x^2 - 18x + 10}{45x^3}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad & \frac{1}{ab} + \frac{b^2-a^2}{ab^3} + \frac{ab+b^2}{a^2b^2} \\
 &= \frac{ab^2 + a(b^2-a^2) + b(ab+b^2)}{a^2b^3} \\
 &= \frac{ab^2 + ab^2 - a^3 + ab^2 + b^3}{a^2b^3} = \frac{-a^3 + 3ab^2 + b^3}{a^2b^3}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & \frac{a+3b}{ab} + \frac{2a-3m}{am} + \frac{3}{a} \\
 &= \frac{m(a+3b) + b(2a-3m) + 3bm}{abm} \\
 &= \frac{am + 3bm + 2ab - 3bm + 3bm}{abm} = \frac{am + 2ab + 3bm}{abm}
 \end{aligned}$$

## EJERCICIO 127

$$1. \quad \frac{1}{a+1} + \frac{1}{a-1} = \frac{a-1+a+1}{a^2-1} = \frac{2a}{a^2-1}$$

$$\begin{aligned}
 2. \quad & \frac{2}{x+4} + \frac{1}{x-3} \\
 &= \frac{2(x-3) + x+4}{x^2+x-12} = \frac{2x-6+x+4}{x^2+x-12} = \frac{3x-2}{x^2+x-12}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \frac{3}{1-x} + \frac{6}{2x+5} \\
 &= \frac{3(2x+5) + 6(1-x)}{(1-x)(2x+5)} = \frac{6x+15+6-6x}{(1-x)(2x+5)} = \frac{21}{(1-x)(2x+5)}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \frac{x}{x-y} + \frac{x}{x+y} \\
 &= \frac{x(x+y) + x(x-y)}{x^2-y^2} = \frac{x^2+xy+x^2-xy}{x^2-y^2} = \frac{2x^2}{x^2-y^2}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \frac{m+3}{m-3} + \frac{m+2}{m-2} \\
 &= \frac{(m-2)(m+3) + (m-3)(m+2)}{(m-3)(m-2)} \\
 &= \frac{m^2+m-6+m^2-m-6}{(m-3)(m-2)} = \frac{2m^2-12}{(m-3)(m-2)} = \frac{2(m^2-6)}{(m-3)(m-2)}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \frac{x+y}{x-y} + \frac{x-y}{x+y} = \frac{(x+y)(x+y) + (x-y)(x-y)}{x^2-y^2} \\
 &= \frac{(x+y)^2 + (x-y)^2}{x^2-y^2} \\
 &= \frac{x^2+2xy+y^2+x^2-2xy+y^2}{x^2-y^2} = \frac{2x^2+2y^2}{x^2-y^2}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & \frac{x}{x^2-1} + \frac{x+1}{(x-1)^2} = \frac{x(x-1) + (x+1)^2}{(x-1)^2(x+1)} \\
 &= \frac{x^2-x+x^2+2x+1}{(x-1)^2(x+1)} = \frac{2x^2+x+1}{(x-1)^2(x+1)}
 \end{aligned}$$

$$8. \quad \frac{2}{x-5} + \frac{3x}{x^2-25} = \frac{2(x+5) + 3x}{x^2-25} = \frac{2x+10+3x}{x^2-25} = \frac{5x+10}{x^2-25}$$

$$9. \quad \frac{1}{3x-2y} + \frac{x-y}{9x^2-4y^2} = \frac{3x+2y+x-y}{9x^2-4y^2} = \frac{4x+y}{9x^2-4y^2}$$

$$\begin{aligned}
 10. \quad & \frac{x+a}{x+3a} + \frac{3a^2-x^2}{x^2-9a^2} = \frac{(x-3a)(x+a) + 3a^2-x^2}{x^2-9a^2} \\
 &= \frac{x^2+ax-3ax-3a^2+3a^2-x^2}{x^2-9a^2} \\
 &= \frac{-2ax}{x^2-9a^2} = \frac{2ax}{9a^2-x^2}
 \end{aligned}$$

$$11. \frac{a}{1-a^2} + \frac{a}{1+a^2} = \frac{a(1+a^2) + a(1-a^2)}{(1-a^2)(1+a^2)} = \frac{a+a^3+a-a^3}{(1-a^2)(1+a^2)} = \frac{2a}{1-a^4}$$

$$12. \frac{2}{a^2-ab} + \frac{2}{ab+b^2} = \frac{2b(a+b) + 2a(a-b)}{ab(a^2-b^2)} = \frac{2ab+2b^2+2a^2-2ab}{ab(a^2-b^2)} = \frac{2a^2+2b^2}{ab(a^2-b^2)}$$

$$13. \frac{ab}{9a^2-b^2} + \frac{a}{3a+b} = \frac{ab+a(3a-b)}{9a^2-b^2} = \frac{ab+3a^2-ab}{9a^2-b^2} = \frac{3a^2}{9a^2-b^2}$$

$$14. \frac{1}{a^2-b^2} + \frac{1}{(a-b)^2} = \frac{a-b+a+b}{(a+b)(a-b)^2} = \frac{2a}{(a+b)(a-b)^2}$$

$$15. \frac{3}{x^2+y^2} + \frac{2}{(x+y)^2} = \frac{3(x+y)^2 + 2(x^2+y^2)}{(x^2+y^2)(x+y)^2} = \frac{3x^2+6xy+3y^2+2x^2+2y^2}{(x^2+y^2)(x+y)^2} = \frac{5x^2+6xy+5y^2}{(x^2+y^2)(x+y)^2}$$

$$16. \frac{x}{a^2-ax} + \frac{a+x}{ax} + \frac{a}{ax-x^2} = \frac{a^2-ax}{a^2-ax} = a(a-x) \\ ax = ax \quad mcm = ax(a-x) \\ ax-x^2 = x(a-x) \\ \frac{x^2+(a+x)(a-x)+a^2}{ax(a-x)} = \frac{x^2+a^2-x^2+a^2}{ax(a-x)} = \frac{2a^2}{ax(a-x)} = \frac{2a}{x(a-x)}$$

$$17. \frac{3}{2x+4} + \frac{x-1}{2x-4} + \frac{x+8}{x^2-4} = \frac{3(x-2) + (x-1)(x+2) + 2(x+8)}{2(x^2-4)} = \frac{3x-6+x^2+x-2+2x+16}{2(x^2-4)} = \frac{x^2+6x+8}{2(x^2-4)} = \frac{(x+2)(x+4)}{2(x+2)(x-2)} = \frac{x+4}{2(x-2)}$$

$$18. \frac{1}{x+x^2} + \frac{1}{x-x^2} + \frac{x+3}{1-x^2} = \frac{x+x^2}{x+x^2} = x(1+x) \\ x-x^2 = x(1-x) \quad mcm = x(1-x^2) \\ 1-x^2 = (1+x)(1-x) \\ = \frac{1-x+1+x+x(x+3)}{x(1-x^2)} = \frac{2+x^2+3x}{x(1-x^2)} = \frac{(x+1)(x+2)}{x(1-x)(1+x)} = \frac{x+2}{x(1-x)}$$

$$19. \frac{x-y}{x+y} + \frac{x+y}{x-y} + \frac{4xy}{x^2-y^2} \quad mcm = x^2-y^2 \\ = \frac{(x-y)(x-y) + (x+y)(x+y) + 4xy}{x^2-y^2} = \frac{x^2-2xy+y^2+x^2+2xy+y^2+4xy}{x^2-y^2} = \frac{2x^2+2y^2+4xy}{x^2-y^2} = \frac{2(x^2+2xy+y^2)}{x^2-y^2} = \frac{2(x+y)^2}{(x+y)(x-y)} = \frac{2(x+y)}{x-y}$$

$$20. \frac{1}{a-5} + \frac{a}{a^2-4a-5} + \frac{a+5}{a^2+2a+1} = \frac{a-5}{a-5} = a-5 \\ a^2-4a-5 = (a-5)(a+1) \\ a^2+2a+1 = (a+1)^2 \quad mcm = (a-5)(a+1)^2 \\ = \frac{(a+1)^2 + a(a+1) + (a+5)(a-5)}{(a-5)(a+1)^2} = \frac{a^2+2a+1+a^2+a+a^2-25}{(a-5)(a+1)^2} = \frac{3a^2+3a-24}{(a-5)(a+1)^2} = \frac{3(a^2+a-8)}{(a-5)(a+1)^2}$$

$$21. \frac{3}{a} + \frac{2}{5a-3} + \frac{1-85a}{25a^2-9} \quad mcm = a(25a^2-9) \\ = \frac{3(25a^2-9) + 2a(5a+3) + a(1-85a)}{a(25a^2-9)} = \frac{75a^2-27+10a^2+6a+a-85a^2}{a(25a^2-9)} = \frac{7a-27}{a(25a^2-9)}$$

$$22. \frac{x+1}{10} + \frac{x-3}{5x-10} + \frac{x-2}{2} \quad mcm = 10(x-2) \\ = \frac{(x+1)(x-2) + 2(x-3) + 5(x-2)(x-2)}{10(x-2)} = \frac{x^2-x-2+2x-6+5x^2-20x+20}{10(x-2)} = \frac{6x^2-19x+12}{10(x-2)}$$

$$\begin{aligned}
 23. \quad & \frac{x+5}{x^2+x-12} + \frac{x+4}{x^2+2x-15} + \frac{x-3}{x^2+9x+20} \\
 & x^2+x-12 = (x+4)(x-3) \\
 & x^2+2x-15 = (x+5)(x-3) \\
 & x^2+9x+20 = (x+5)(x+4) \\
 & mcm = (x+5)(x+4)(x-3) \\
 & = \frac{(x+5)(x+5)(x+4)(x+4)(x-3)(x-3)}{(x+5)(x+4)(x-3)} \\
 & = \frac{x^2+10x+25+x^2+8x+16+x^2-6x+9}{(x+5)(x+4)(x-3)} \\
 & = \frac{3x^2+12x+50}{(x+5)(x+4)(x-3)}
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & \frac{1}{x-2} + \frac{1-2x^2}{x^3-8} + \frac{x}{x^2+2x+4} \\
 & x^3-8 = (x-2)(x^2+2x+4) \quad mcm = x^3-8 \\
 & = \frac{x^2+2x+4+1-2x^2+x(x-2)}{x^3-8} \\
 & = \frac{-x^2+2x+5+x^2-2x}{x^3-8} = \frac{5}{x^3-8}
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & \frac{2}{a+1} + \frac{a}{(a+1)^2} + \frac{a+1}{(a+1)^3} \quad mcm = (a+1)^3 \\
 & = \frac{2(a+1)^2 + a(a+1) + a+1}{(a+1)^3} \\
 & = \frac{2a^2+4a+2+a^2+a+a+1}{(a+1)^3} \\
 & = \frac{3a^2+6a+3}{(a+1)^3} = \frac{3(a^2+2a+1)}{(a+1)^3} = \frac{3(a+1)^2}{(a+1)^3} = \frac{3}{a+1}
 \end{aligned}$$

$$\begin{aligned}
 26. \quad & \frac{2x}{3x^2+11x+6} + \frac{x+1}{x^2-9} + \frac{1}{3x+2} \\
 & 3x^2+11x+6 = (x+3)(3x+2) \\
 & x^2-9 = (x+3)(x-3) \\
 & 3x+2 = 3x+2 \quad mcm = (x^2-9)(3x+2) \\
 & = \frac{2x(x-3) + (x+1)(3x+2) + x^2-9}{(x^2-9)(3x+2)} \\
 & = \frac{2x^2-6x+3x^2+5x+2+x^2-9}{(x^2-9)(3x+2)} \\
 & = \frac{6x^2-x-7}{(x^2-9)(3x+2)}
 \end{aligned}$$

$$\begin{aligned}
 27. \quad & \frac{x^2-4}{x^3+1} + \frac{1}{x+1} + \frac{3}{x^2-x+1} \\
 & x^3+1 = (x+1)(x^2-x+1) \quad mcm = x^3+1 \\
 & = \frac{x^2-4+x^2-x+1+3x+3}{x^3+1}
 \end{aligned}$$

$$= \frac{2x^2+2x}{x^3+1} = \frac{2x(x+1)}{(x+1)(x^2-x+1)} = \frac{2x}{x^2-x+1}$$

$$\begin{aligned}
 28. \quad & \frac{1}{x-1} + \frac{1}{(x-1)(x+2)} + \frac{x+1}{(x-1)(x+2)(x+3)} \\
 & mcm = (x-1)(x+2)(x+3) \\
 & = \frac{(x+2)(x+3) + x+3 + x+1}{(x-1)(x+2)(x+3)} \\
 & = \frac{x^2+5x+6+2x+4}{(x-1)(x+2)(x+3)} = \frac{x^2+7x+10}{(x-1)(x+2)(x+3)} \\
 & = \frac{(x+5)(x+2)}{(x-1)(x+2)(x+3)} = \frac{x+5}{(x-1)(x+3)}
 \end{aligned}$$

$$\begin{aligned}
 29. \quad & \frac{x-2}{2x^2-5x-3} + \frac{x-3}{2x^2-3x-2} + \frac{2x-1}{x^2-5x+6} \\
 & 2x^2-5x-3 = (x-3)(2x+1) \\
 & 2x^2-3x-2 = (x-2)(2x+1) \\
 & x^2-5x+6 = (x-3)(x-2) \\
 & mcm = (x-3)(x-2)(2x+1) \\
 & = \frac{(x-2)(x-2) + (x-3)(x-3) + (2x-1)(2x+1)}{(x-3)(x-2)(2x+1)} \\
 & = \frac{x^2-4x+4+x^2-6x+9+4x^2-1}{(x-3)(x-2)(2x+1)} \\
 & = \frac{6x^2-10x+12}{(x-3)(x-2)(2x+1)}
 \end{aligned}$$

$$\begin{aligned}
 30. \quad & \frac{a-2}{a-1} + \frac{a+3}{a+2} + \frac{a+1}{a-3} \quad mcm = (a-1)(a+2)(a-3) \\
 & = \frac{(a+2)(a-2)(a-3) + (a-1)(a+3)(a-3) + (a-1)(a+1)(a+2)}{(a-1)(a+2)(a-3)} \\
 & = \frac{a^3-3a^2-4a+12+a^3-a^2-9a+9+a^3+2a^2-a-2}{(a-1)(a+2)(a-3)} \\
 & = \frac{3a^3-2a^2-14a+19}{(a-1)(a+2)(a-3)}
 \end{aligned}$$



## EJERCICIO 128

- $$\frac{x-3}{4} - \frac{x+2}{8} = \frac{2(x-3)-(x+2)}{8} = \frac{2x-6-x-2}{8} = \frac{x-8}{8}$$
- $$\frac{a+5b}{a^2} - \frac{b-3}{ab} = \frac{b(a+5b)-a(b-3)}{a^2b} = \frac{ab+5b^2-ab+3a}{a^2b} = \frac{3a+5b^2}{a^2b}$$
- $$\frac{2}{3mn^2} - \frac{1}{2m^2n} = \frac{2(2m)-(3n)}{6m^2n^2} = \frac{4m-3n}{6m^2n^2}$$
- $$\frac{a-3}{5ab} - \frac{4-3ab^2}{3a^2b^3} = \frac{3ab^2(a-3)-5(4-3ab^2)}{15a^2b^3} = \frac{3a^2b^2-9ab^2-20+15ab^2}{15a^2b^3} = \frac{3a^2b^2+6ab^2-20}{15a^2b^3}$$
- $$\frac{2a+3}{4a} - \frac{a-2}{8a} = \frac{2(2a+3)-(a-2)}{8a} = \frac{4a+6-a+2}{8a} = \frac{3a+8}{8a}$$
- $$\frac{y-2x}{20x} - \frac{x-3y}{24y} = \frac{6y(y-2x)-5x(x-3y)}{120xy} = \frac{6y^2-12xy-5x^2+15xy}{120xy} = \frac{6y^2+3xy-5x^2}{120xy}$$
- $$\frac{x-1}{3} - \frac{x-2}{4} - \frac{x+3}{6} = \frac{4(x-1)-3(x-2)-2(x+3)}{12} = \frac{4x-4-3x+6-2x-6}{12} = \frac{-x-4}{12} = -\frac{x+4}{12}$$
- $$\frac{3}{5} - \frac{2a+1}{10a} - \frac{4a^2+1}{20a^2} = \frac{4a^2(3)-2a(2a+1)-(4a^2+1)}{20a^2} = \frac{12a^2-4a^2-2a-4a^2-1}{20a^2} = \frac{4a^2-2a-1}{20a^2}$$
- $$\frac{3}{5x} - \frac{x-1}{3x^2} - \frac{x^2+2x+3}{15x^3} = \frac{3x^2(3)-5x(x-1)-(x^2+2x+3)}{15x^3} = \frac{9x^2-5x^2+5x-x^2-2x-3}{15x^3} = \frac{3x^2+3x-3}{15x^3} = \frac{3(x^2+x-1)}{15x^3} = \frac{x^2+x-1}{5x^3}$$
- $$\frac{1}{2a} - \frac{2+b}{3ab} - \frac{5}{6a^2b^3} = \frac{3ab^3-2ab^2(2+b)-5}{6a^2b^3} = \frac{3ab^3-4ab^2-2ab^3-5}{6a^2b^3} = \frac{ab^3-4ab^2-5}{6a^2b^3}$$

## EJERCICIO 129

- $$\frac{1}{x-4} - \frac{1}{x-3} \quad mcm=(x-4)(x-3)$$

$$= \frac{x-3-(x-4)}{(x-4)(x-3)} = \frac{x-3-x+4}{(x-4)(x-3)} = \frac{1}{(x-4)(x-3)}$$
- $$\frac{m-n}{m+n} - \frac{m+n}{m-n} \quad mcm=m^2-n^2$$

$$= \frac{(m-n)(m-n)-(m+n)(m+n)}{m^2-n^2} = \frac{m^2-2mn+n^2-m^2-2mn-n^2}{m^2-n^2} = \frac{-4mn}{m^2-n^2} = \frac{4mn}{n^2-m^2}$$
- $$\frac{1-x}{1+x} - \frac{1+x}{1-x} = \frac{(1-x)(1-x)-(1+x)(1+x)}{1-x^2} = \frac{1-2x+x^2-1-2x-x^2}{1-x^2} = \frac{-4x}{1-x^2} = \frac{4x}{x^2-1}$$
- $$\frac{a+b}{a^2+ab} - \frac{b-a}{ab+b^2}$$

$$a^2+ab=a(a+b) \quad ; \quad ab+b^2=b(a+b)$$

$$mcm=ab(a+b)$$

$$= \frac{b(a+b)-a(b-a)}{ab(a+b)} = \frac{ab+b^2-ab+a^2}{ab(a+b)} = \frac{a^2+b^2}{ab(a+b)}$$
- $$\frac{m+n}{m-n} - \frac{m^2+n^2}{m^2-n^2} = \frac{(m+n)(m+n)-(m^2+n^2)}{m^2-n^2} = \frac{m^2+2mn+n^2-m^2-n^2}{m^2-n^2} = \frac{2mn}{m^2-n^2}$$
- $$\frac{1}{x+x^2} - \frac{1}{x-x^2} \quad x+x^2=x(x+1) \quad ; \quad x-x^2=x(1-x)$$

$$mcm=x(1-x^2)$$

$$= \frac{1-x-(1+x)}{x(1-x^2)} = \frac{1-x-1-x}{x(1-x^2)} = \frac{-2x}{x(1-x^2)} = \frac{-2}{1-x^2} = \frac{2}{x^2-1}$$

$$\begin{aligned}
 7. \quad & \frac{a+x}{(a-x)^2} - \frac{x}{a^2-x^2} \\
 &= \frac{(a+x)(a+x) - x(a-x)}{(a-x)^2(a+x)} \\
 &= \frac{a^2+2ax+x^2-ax+x^2}{(a-x)^2(a+x)} = \frac{a^2+ax+2x^2}{(a-x)^2(a+x)}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & \frac{a+1}{6a+3} - \frac{1}{12a+6} \\
 & 6a+3=3(2a+1) \quad ; \quad 12a+6=6(2a+1) \\
 & \quad \quad \quad mcm=6(2a+1) \\
 &= \frac{2(a+1)-1}{6(2a+1)} = \frac{2a+2-1}{6(2a+1)} = \frac{2a+1}{6(2a+1)} = \frac{1}{6}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & \frac{a-4}{a^2-6a+9} - \frac{a+3}{a^2+a-12} \\
 & a^2-6a+9=(a-3)^2 \quad ; \quad a^2+a-12=(a+4)(a-3) \\
 & \quad \quad \quad mcm=(a-3)^2(a+4) \\
 &= \frac{(a+4)(a-4)-(a-3)(a+3)}{(a+4)(a-3)^2} \\
 &= \frac{a^2-16-a^2+9}{(a+4)(a-3)^2} = \frac{-7}{(a+4)(a-3)^2}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & \frac{a^2+4ab-3b^2}{a^2-9b^2} - \frac{b}{a+3b} \quad mcm=a^2-9b^2 \\
 &= \frac{a^2+4ab-3b^2-b(a-3b)}{a^2-9b^2} \\
 &= \frac{a^2+4ab-3b^2-ab+3b^2}{a^2-9b^2} \\
 &= \frac{a^2+3ab}{a^2-9b^2} = \frac{a(a+3b)}{(a+3b)(a-3b)} = \frac{a}{a-3b}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & \frac{x}{x^2-1} - \frac{x+1}{(x-1)^2} \quad mcm=(x-1)^2(x+1) \\
 &= \frac{x(x-1)-(x+1)(x+1)}{(x+1)(x-1)^2} \\
 &= \frac{x^2-x-x^2-2x-1}{(x+1)(x-1)^2} = \frac{-3x-1}{(x+1)(x-1)^2} = -\frac{3x+1}{(x+1)(x-1)^2}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad & \frac{1}{a^3-b^3} - \frac{1}{(a-b)^3} \quad mcm=(a-b)^3(a^2+ab+b^2) \\
 &= \frac{(a-b)^2-(a^2+ab+b^2)}{(a-b)^3(a^2+ab+b^2)} \\
 &= \frac{a^2-2ab+b^2-a^2-ab-b^2}{(a-b)^3(a^2+ab+b^2)} = \frac{-3ab}{(a-b)^3(a^2+ab+b^2)}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad & \frac{x+3}{6x^2+x-2} - \frac{1}{4x^2-4x+1} \\
 & 6x^2+x-2=(3x+2)(2x-1) \\
 & 4x^2-4x+1=(2x-1)^2 \quad mcm=(2x-1)^2(3x+2) \\
 &= \frac{(2x-1)(x+3)-(3x+2)}{(3x+2)(2x-1)^2} \\
 &= \frac{2x^2+5x-3-3x-2}{(3x+2)(2x-1)^2} = \frac{2x^2+2x-5}{(3x+2)(2x-1)^2}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & \frac{x-1}{4x+4} - \frac{x+2}{8x-8} \quad mcm=8(x^2-1) \\
 &= \frac{2(x-1)(x-1)-(x+2)(x+1)}{8(x^2-1)} \\
 &= \frac{2x^2-4x+2-x^2-3x-2}{8(x^2-1)} = \frac{x^2-7x}{8(x^2-1)}
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & \frac{x}{xy-y^2} - \frac{1}{y} \quad mcm=y(x-y) \\
 &= \frac{x-(x-y)}{y(x-y)} = \frac{x-x+y}{y(x-y)} = \frac{y}{y(x-y)} = \frac{1}{x-y}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \frac{b}{a^2-b^2} - \frac{b}{a^2+ab} \quad mcm=a(a^2-b^2) \\
 &= \frac{ab-b(a-b)}{a(a^2-b^2)} = \frac{ab-ab+b^2}{a(a^2-b^2)} = \frac{b^2}{a(a^2-b^2)}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & \frac{2a-3}{6a+9} - \frac{a-1}{4a^2+12a+9} \\
 & 6a+9=3(2a+3) \quad ; \quad 4a^2+12a+9=(2a+3)^2 \\
 & \quad \quad \quad mcm=3(2a+3)^2 \\
 &= \frac{(2a+3)(2a-3)-3(a-1)}{3(2a+3)^2} \\
 &= \frac{4a^2-9-3a+3}{3(2a+3)^2} = \frac{4a^2-3a-6}{3(2a+3)^2}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & \frac{x+1}{x^2+x+1} - \frac{x-1}{x^2-x+1} \quad mcm=(x^2+x+1)(x^2-x+1) \\
 &= \frac{x+1-(x-1)}{(x^2+x+1)(x^2-x+1)} \\
 &= \frac{x+1-x+1}{(x^2+x+1)(x^2-x+1)} = \frac{2}{(x^2+x+1)(x^2-x+1)}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & \frac{a-1}{a^2+a} - \frac{1}{2a-2} - \frac{1}{2a+2} \quad mcm = 2a(a^2-1) \\
 & = \frac{2(a-1)(a-1) - a(a+1) - a(a-1)}{2a(a^2-1)} \\
 & = \frac{2(a^2-2a+1) - a^2 - a - a^2 + a}{2a(a^2-1)} \\
 & = \frac{2a^2 - 4a + 2 - 2a^2}{2a(a^2-1)} \\
 & = -\frac{4a-2}{2a(a^2-1)} \\
 & = -\frac{2(2a-1)}{2a(a^2-1)} = -\frac{2a-1}{a(a^2-1)} = \frac{1-2a}{a(a^2-1)}
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & \frac{1}{4a+4} - \frac{1}{8a-8} - \frac{1}{12a^2+12} \\
 & \quad mcm = 24(a^2+1)(a^2-1) \\
 & = \frac{6(a^2+1)(a-1) - 3(a^2+1)(a+1) - 2(a^2-1)}{24(a^2+1)(a^2-1)} \\
 & = \frac{6(a^3 - a^2 + a - 1) - 3(a^3 + a^2 + a + 1) - 2a^2 + 2}{24(a^2+1)(a^2-1)} \\
 & = \frac{6a^3 - 6a^2 + 6a - 6 - 3a^3 - 3a^2 - 3a - 3 - 2a^2 + 2}{24(a^2+1)(a^2-1)} \\
 & = \frac{3a^3 - 11a^2 + 3a - 7}{24(a^4-1)}
 \end{aligned}$$

$$\begin{aligned}
 21. \quad & \frac{y}{x^2-xy} - \frac{1}{x} - \frac{1}{x-y} \quad mcm = x(x-y) \\
 & = \frac{y - (x-y) - x}{x(x-y)} \\
 & = \frac{y-x+y-x}{x(x-y)} \\
 & = \frac{2y-2x}{x(x-y)} = \frac{2(y-x)}{x(x-y)} = -\frac{2(x-y)}{x(x-y)} = -\frac{2}{x}
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & \frac{a}{a^2+ab} - \frac{1}{a} - \frac{1}{a+b} \quad mcm = a(a+b) \\
 & = \frac{a - (a+b) - a}{a(a+b)} = \frac{-a-b}{a(a+b)} = -\frac{a+b}{a(a+b)} = -\frac{1}{a}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & \frac{1}{x^2-xy} - \frac{1}{x^2+xy} - \frac{2y}{x^3-xy^2} \quad mcm = x(x^2-y^2) \\
 & = \frac{x+y-(x-y)-2y}{x(x^2-y^2)} = \frac{x-y-x+y}{x(x^2-y^2)} = \frac{0}{x(x^2-y^2)} = 0
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & \frac{x}{x^2+x-2} - \frac{3}{x^2+2x-3} - \frac{x}{x^2+5x+6} \\
 & \quad x^2+x-2 = (x+2)(x-1) \\
 & \quad x^2+2x-3 = (x+3)(x-1) \\
 & \quad x^2+5x+6 = (x+3)(x+2) \\
 & \quad mcm = (x+2)(x-1)(x+3) \\
 & = \frac{x(x+3) - 3(x+2) - x(x-1)}{(x+2)(x-1)(x+3)} \\
 & = \frac{x^2+3x-3x-6-x^2+x}{(x+2)(x-1)(x+3)} = \frac{x-6}{(x+2)(x-1)(x+3)}
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & \frac{3}{x^2+x+1} - \frac{x+2}{(x-1)^2} - \frac{1-9x}{(x^3-1)(x-1)} \quad mcm = (x^3-1)(x-1) \\
 & = \frac{3(x-1)^2 - (x+2)(x^2+x+1) - (1-9x)}{(x^3-1)(x-1)} \\
 & = \frac{3x^2-6x+3-x^3-x^2-x-2x^2-2x-2-1+9x}{(x^3-1)(x-1)} \\
 & = -\frac{x^3}{(x^3-1)(x-1)}
 \end{aligned}$$

$$\begin{aligned}
 26. \quad & \frac{a^2+b^2}{a^3-b^3} - \frac{a+b}{2a^2+2ab+2b^2} - \frac{1}{2a-2b} \\
 & \quad a^3-b^3 = (a-b)(a^2+ab+b^2) \\
 & \quad 2a^2+2ab+2b^2 = 2(a^2+ab+b^2) \\
 & \quad 2a-2b = 2(a-b) \\
 & \quad mcm = 2(a-b)(a^2+ab+b^2) \\
 & = \frac{2(a^2+b^2) - (a+b)(a-b) - (a^2+ab+b^2)}{2(a-b)(a^2+ab+b^2)} \\
 & = \frac{2a^2+2b^2-a^2+b^2-a^2-ab-b^2}{2(a-b)(a^2+ab+b^2)} \\
 & = \frac{2b^2-ab}{2(a-b)(a^2+ab+b^2)} = \frac{2b^2-ab}{2(a^3-b^3)}
 \end{aligned}$$

$$\begin{aligned}
 27. \quad & \frac{3a}{2a^2-2a-4} - \frac{a-1}{4a^2+8a-32} - \frac{10a-1}{8a^2+40a+32} \\
 & \quad 2a^2-2a-4 = 2(a^2-a-2) = 2(a-2)(a+1) \\
 & \quad 4a^2+8a-32 = 4(a^2+2a-8) = 4(a+4)(a-2) \\
 & \quad 8a^2+40a+32 = 8(a^2+5a+4) = 8(a+4)(a+1) \\
 & \quad mcm = 8(a+4)(a-2)(a+1) \\
 & = \frac{12a(a+4) - 2(a-1)(a+1) - (10a-1)(a-2)}{8(a+4)(a-2)(a+1)} \\
 & = \frac{12a^2+48a-2a^2+2-10a^2+20a+a-2}{8(a+4)(a-2)(a+1)} \\
 & = \frac{69a}{8(a+4)(a-2)(a+1)}
 \end{aligned}$$

$$\begin{aligned}
 28. \quad & \frac{1}{4a-12x} - \frac{a^2+9x^2}{a^3-27x^3} - \frac{a}{2(a^2+3ax+9a^2)} \\
 & 4a-12x=4(a-3x) \\
 & a^3-27x^3=(a-3x)(a^2+3ax+9a^2) \\
 & 2(a^2+3ax+9a^2)=2(a^2+3ax+9a^2) \\
 & mcm=4(a^3-27x^3) \\
 & = \frac{a^2+3ax+9x^2-4(a^2+9x^2)-2a(a-3x)}{4(a^3-27x^3)} \\
 & = \frac{a^2+3ax+9x^2-4a^2-36x^2-2a^2+6ax}{4(a^3-27x^3)} \\
 & = \frac{-5a^2+9ax-27x^2}{4(a^3-27x^3)} = -\frac{5a^2-9ax+27x^2}{4(a^3-27x^3)}
 \end{aligned}$$

$$\begin{aligned}
 29. \quad & \frac{2a^2-3}{10a+10} - \frac{a+1}{50} - \frac{9a^2-14}{50a+50} \\
 & 10a+10=10(a+1) \\
 & 50=50 \\
 & 50a+50=50(a+1) \\
 & mcm=50(a+1) \\
 & = \frac{5(2a^2-3)-(a+1)(a+1)-(9a^2-14)}{50(a+1)} \\
 & = \frac{10a^2-15-a^2-2a-1-9a^2+14}{50(a+1)} \\
 & = \frac{-2a-2}{50(a+1)} = -\frac{2(a+1)}{50(a+1)} = -\frac{1}{25}
 \end{aligned}$$

### **EJERCICIO 130**

$$\begin{aligned}
 1. \quad & \frac{2}{x-3} + \frac{3}{x+2} - \frac{4x-7}{x^2-x-6} \quad mcm=(x-3)(x+2) \\
 & = \frac{2(x+2)+3(x-3)-(4x-7)}{(x-3)(x+2)} \\
 & = \frac{2x+4+3x-9-4x+7}{(x-3)(x+2)} = \frac{x+2}{(x-3)(x+2)} = \frac{1}{x-3}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \frac{a}{3a+6} - \frac{1}{6a+12} + \frac{a+12}{12a+24} \quad mcm=12(a+2) \\
 & = \frac{4a-2+a+12}{12(a+2)} = \frac{5a+10}{12(a+2)} = \frac{5(a+2)}{12(a+2)} = \frac{5}{12}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \frac{x}{x^2+1} + \frac{1}{3x} - \frac{1}{x^2} \quad mcm=3x^2(x^2+1) \\
 & = \frac{3x^3+x(x^2+1)-3(x^2+1)}{3x^2(x^2+1)} \\
 & = \frac{3x^3+x^3+x-3x^2-3}{3x^2(x^2+1)} = \frac{4x^3+x-3x^2-3}{3x^2(x^2+1)}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \frac{a+3}{a^2-1} + \frac{a-1}{2a+2} + \frac{a-4}{4a-4} \quad mcm=4(a^2-1) \\
 & = \frac{4(a+3)+2(a-1)(a-1)+(a-4)(a+1)}{4(a^2-1)} \\
 & = \frac{4a+12+2a^2-4a+2+a^2-3a-4}{4(a^2-1)} = \frac{3a^2-3a+10}{4(a^2-1)}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \frac{a-b}{a^2+ab} + \frac{a+b}{ab} - \frac{a}{ab+b^2} \quad mcm=ab(a+b) \\
 & = \frac{b(a-b)+(a+b)(a+b)-a^2}{ab(a+b)} \\
 & = \frac{ab-b^2+a^2+2ab+b^2-a^2}{ab(a+b)} = \frac{3ab}{ab(a+b)} = \frac{3}{a+b}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \frac{x-y}{x+y} - \frac{x+y}{x-y} + \frac{4x^2}{x^2-y^2} \quad mcm=x^2-y^2 \\
 & = \frac{(x-y)(x-y)-(x+y)(x+y)+4x^2}{x^2-y^2} \\
 & = \frac{x^2-2xy+y^2-x^2-2xy-y^2+4x^2}{x^2-y^2} \\
 & = \frac{4x^2-4xy}{x^2-y^2} = \frac{4x(x-y)}{(x+y)(x-y)} = \frac{4x}{x+y}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & \frac{x}{a^2-ax} + \frac{1}{a} + \frac{1}{x} \quad mcm=ax(a-x) \\
 & = \frac{x^2+x(a-x)+a(a-x)}{ax(a-x)} \\
 & = \frac{x^2+ax-x^2+a^2-ax}{ax(a-x)} = \frac{a^2}{ax(a-x)} = \frac{a}{x(a-x)}
 \end{aligned}$$

$$\begin{aligned}
8. \quad & \frac{x+1}{x^2-x-20} - \frac{x+4}{x^2-4x-5} + \frac{x+5}{x^2+5x+4} \\
& x^2-x-20=(x-5)(x+4) \\
& x^2-4x-5=(x-5)(x+1) \\
& x^2+5x+4=(x+4)(x+1) \\
& mcm=(x-5)(x+4)(x+1) \\
& = \frac{(x+1)(x+1)-(x+4)(x+4)+(x-5)(x+5)}{(x-5)(x+4)(x+1)} \\
& = \frac{x^2+2x+1-x^2-8x-16+x^2-25}{(x-5)(x+4)(x+1)} \\
& = \frac{x^2-6x-40}{(x-5)(x+4)(x+1)} \\
& = \frac{(x-10)(x+4)}{(x-5)(x+4)(x+1)} = \frac{x-10}{(x-5)(x+1)}
\end{aligned}$$

$$\begin{aligned}
9. \quad & \frac{2x+1}{12x+8} - \frac{x^2}{6x^2+x-2} + \frac{2x}{16x-8} \\
& 12x+8=4(3x+2) \\
& 6x^2+x-2=(3x+2)(2x-1) \\
& 16x-8=8(2x-1) \\
& mcm=8(3x+2)(2x-1) \\
& = \frac{2(2x+1)(2x-1)-8x^2+2x(3x+2)}{8(3x+2)(2x-1)} \\
& = \frac{8x^2-2-8x^2+6x^2+4x}{8(3x+2)(2x-1)} \\
& = \frac{6x^2+4x-2}{8(3x+2)(2x-1)} \\
& = \frac{2(3x^2+2x-1)}{8(3x+2)(2x-1)} = \frac{3x^2+2x-1}{4(3x+2)(2x-1)}
\end{aligned}$$

$$\begin{aligned}
10. \quad & \frac{1}{ax} - \frac{1}{a^2+ax} + \frac{1}{a+x} \quad mcm=ax(a+x) \\
& = \frac{a+x-x+ax}{ax(a+x)} = \frac{ax+a}{ax(a+x)} = \frac{a(x+1)}{ax(a+x)} = \frac{x+1}{x(a+x)}
\end{aligned}$$

$$\begin{aligned}
11. \quad & \frac{1}{x+y} - \frac{1}{x-y} + \frac{2y}{x^2+y^2} \quad mcm=(x^2-y^2)(x^2+y^2) \\
& = \frac{(x-y)(x^2+y^2)-(x+y)(x^2+y^2)+2y(x^2-y^2)}{(x^2-y^2)(x^2+y^2)} \\
& = \frac{x^3+xy^2-x^2y-y^3-x^3-xy^2-x^2y-y^3+2x^2y-2y^3}{(x^2-y^2)(x^2+y^2)} \\
& = \frac{-4y^3}{x^4-y^4} = \frac{4y^3}{y^4-x^4}
\end{aligned}$$

$$\begin{aligned}
12. \quad & \frac{a-1}{3a+3} - \frac{a-2}{6a-6} + \frac{a^2+2a-6}{9a^2-9} \quad mcm=18(a^2-1) \\
& = \frac{6(a-1)(a-1)-3(a-2)(a+1)+2(a^2+2a-6)}{18(a^2-1)} \\
& = \frac{6a^2-12a+6-3a^2+3a+6+2a^2+4a-12}{18(a^2-1)} \\
& = \frac{5a^2-5a}{18(a^2-1)} = \frac{5a(a-1)}{18(a+1)(a-1)} = \frac{5a}{18(a+1)}
\end{aligned}$$

$$\begin{aligned}
13. \quad & \frac{1}{a^2+2a-24} + \frac{2}{a^2-2a-8} - \frac{3}{a^2+8a+12} \\
& a^2+2a-24=(a+6)(a-4) \\
& a^2-2a-8=(a-4)(a+2) \\
& a^2+8a+12=(a+6)(a+2) \\
& mcm=(a+6)(a+2)(a-4) \\
& = \frac{a+2+2(a+6)-3(a-4)}{(a+6)(a+2)(a-4)} \\
& = \frac{a+2+2a+12-3a+12}{(a+6)(a+2)(a-4)} = \frac{26}{(a+6)(a+2)(a-4)}
\end{aligned}$$

$$\begin{aligned}
14. \quad & \frac{x+y}{xy} - \frac{x+2y}{xy+y^2} - \frac{y}{x^2+xy} \quad mcm=xy(x+y) \\
& = \frac{(x+y)(x+y)-x(x+2y)-y^2}{xy(x+y)} \\
& = \frac{x^2+2xy+y^2-x^2-2xy-y^2}{xy(x+y)} = \frac{0}{xy(x+y)} = 0
\end{aligned}$$

$$\begin{aligned}
15. \quad & \frac{a^3}{a^3+1} + \frac{a+3}{a^2-a+1} - \frac{a-1}{a+1} \quad mcm=a^3+1 \\
& = \frac{a^3+(a+1)(a+3)-(a-1)(a^2-a+1)}{a^3+1} \\
& = \frac{a^3+a^2+4a+3-a^3+a^2-a+a^2-a+1}{a^3+1} = \frac{3a^2+2a+4}{a^3+1}
\end{aligned}$$

$$\begin{aligned}
16. \quad & \frac{1}{x-1} + \frac{2x}{x^2-1} - \frac{3x^2}{x^3-1} \quad mcm=(x^2-1)(x^2+x+1) \\
& = \frac{(x+1)(x^2+x+1)+2x(x^2+x+1)-3x^2(x+1)}{(x^2-1)(x^2+x+1)} \\
& = \frac{x^3+x^2+x+x^2+x+1+2x^3+2x^2+2x-3x^3-3x^2}{(x^2-1)(x^2+x+1)} \\
& = \frac{x^2+4x+1}{(x^2-1)(x^2+x+1)}
\end{aligned}$$

$$\begin{aligned}
 17. \quad & \frac{a+b}{a^2-ab+b^2} - \frac{1}{a+b} + \frac{3a^2}{a^3+b^3} \quad mcm=a^3+b^3 \\
 &= \frac{(a+b)(a+b) - (a^2-ab+b^2) + 3a^2}{a^3+b^3} \\
 &= \frac{a^2+2ab+b^2 - a^2+ab-b^2 + 3a^2}{a^3+b^3} \\
 &= \frac{3a^2+3ab}{a^3+b^3} = \frac{3a(a+b)}{(a+b)(a^2-ab+b^2)} = \frac{3a}{a^2-ab+b^2}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & \frac{2}{x-2} + \frac{2x+3}{x^2+2x+4} - \frac{6x+12}{x^3-8} \quad mcm=x^3-8 \\
 &= \frac{2(x^2+2x+4) + (2x+3)(x-2) - (6x+12)}{x^3-8} \\
 &= \frac{2x^2+4x+8+2x^2-4x+3x-6-6x-12}{x^3-8} \\
 &= \frac{4x^2-3x-10}{x^3-8} = \frac{(x-2)(4x+5)}{x^3-8} = \frac{4x+5}{x^2+2x+4}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & \frac{3x+2}{x^2+3x-10} - \frac{5x+1}{x^2+4x-5} + \frac{4x-1}{x^2-3x+2} \\
 & x^2+3x-10=(x+5)(x-2); \quad x^2+4x-5=(x+5)(x-1) \\
 & x^2-3x+2=(x-2)(x-1); \quad mcm=(x+5)(x-2)(x-1) \\
 &= \frac{(3x+2)(x-1) - (5x+1)(x-2) + (4x-1)(x+5)}{(x+5)(x-2)(x-1)} \\
 &= \frac{3x^2-3x+2x-2-5x^2+10x-x+2+4x^2+20x-x-5}{(x+5)(x-2)(x-1)} \\
 &= \frac{2x^2+27x-5}{(x+5)(x-2)(x-1)}
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & \frac{1}{(n+1)^2} + \frac{1}{n-1} - \frac{1}{(n-1)^3} - \frac{1}{n} \quad mcm=n(n-1)^3 \\
 &= \frac{n(n-1)+n(n-1)^2-n-(n-1)^3}{n(n-1)^3} \\
 &= \frac{n^2-n+n^3-2n^2+n-n-n^3+3n^2-3n+1}{n(n-1)^3} = \frac{2n^2-4n+1}{n(n-1)^3}
 \end{aligned}$$

$$\begin{aligned}
 21. \quad & \frac{1}{a^2+5} - \frac{a^2-5}{(a^2+5)^2} + \frac{a^2+5}{a^4-25} \quad mcm=(a^4-25)(a^2+5) \\
 &= \frac{a^4-25-(a^2-5)(a^2-5)+(a^2+5)(a^2+5)}{(a^4-25)(a^2+5)} \\
 &= \frac{a^4-25-a^4+10a^2-25+a^4+10a^2+25}{(a^4-25)(a^2+5)} = \frac{a^4+20a^2-25}{(a^4-25)(a^2+5)}
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & \frac{1-x^2}{9-x^2} - \frac{x^2}{9+6x+x^2} - \frac{6x}{9-6x+x^2} \\
 & 9-x^2=(3+x)(3-x); \quad 9+6x+x^2=(3+x)^2 \\
 & 9-6x+x^2=(3-x)^2; \quad mcm=(3+x)^2(3-x)^2 \\
 &= \frac{(1-x^2)(9-x^2) - x^2(3-x)^2 - 6x(3+x)^2}{(3+x)^2(3-x)^2} \\
 &= \frac{9-x^2-9x^2+x^4-9x^2+6x^3-x^4-54x-36x^2-6x^3}{(3+x)^2(3-x)^2} \\
 &= \frac{9-54x-55x^2}{(3+x)^2(3-x)^2}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & \frac{x}{2x+2} - \frac{x+1}{3x-3} + \frac{x-1}{6x+6} - \frac{5}{18x-18} \quad mcm=18(x^2-1) \\
 &= \frac{9x(x-1) - 6(x+1)(x+1) + 3(x-1)(x-1) - 5(x+1)}{18(x^2-1)} \\
 &= \frac{9x^2-9x-6x^2-12x-6+3x^2-6x+3-5x-5}{18(x^2-1)} \\
 &= \frac{6x^2-32x-8}{18(x^2-1)} = \frac{2(3x^2-16x-4)}{18(x^2-1)} = \frac{3x^2-16x-4}{9(x^2-1)}
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & \frac{a+2}{2a+2} - \frac{7a}{8a^2-8} - \frac{a-3}{4a-4} \quad mcm=8(a^2-1) \\
 &= \frac{4(a+2)(a-1) - 7a-2(a-3)(a+1)}{8(a^2-1)} \\
 &= \frac{4a^2+4a-8-7a-2a^2+4a+6}{8(a^2-1)} = \frac{2a^2+a-2}{8(a^2-1)}
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & \frac{a-3}{20a+10} + \frac{2a+5}{40a+20} - \frac{4a-1}{60a+30} \quad mcm=120(2a+1) \\
 &= \frac{12(a-3)+6(2a+5)-4(4a-1)}{120(2a+1)} \\
 &= \frac{12a-36+12a+30-16a+4}{120(2a+1)} \\
 &= \frac{8a-2}{120(2a+1)} = \frac{2(4a-1)}{120(2a+1)} = \frac{4a-1}{60(2a+1)}
 \end{aligned}$$

$$\begin{aligned}
 26. \quad & \frac{2}{2x^2+5x+3} - \frac{1}{2x^2-x-6} + \frac{3}{x^2-x-2} \\
 & 2x^2+5x+3=(2x+3)(x+1) \\
 & 2x^2-x-6=(2x+3)(x-2) \\
 & x^2-x-2=(x-2)(x+1) \\
 & mcm=(2x+3)(x+1)(x-2) \\
 &= \frac{2(x-2)-(x+1)+3(2x+3)}{(2x+3)(x+1)(x-2)} \\
 &= \frac{2x-4-x-1+6x+9}{(2x+3)(x+1)(x-2)} = \frac{7x+4}{(2x+3)(x+1)(x-2)}
 \end{aligned}$$

$$\begin{aligned}
 27. \frac{a-1}{a-2} - \frac{a-2}{a+3} + \frac{1}{a-1} \quad mcm &= (a-2)(a+3)(a-1) \\
 &= \frac{(a-1)^2(a+3) - (a-2)^2(a-1) + (a-2)(a+3)}{(a-2)(a+3)(a-1)} \\
 &= \frac{(a^2-2a+1)(a+3) - (a^2-4a+4)(a-1) + a^2+a-6}{(a-2)(a+3)(a-1)} \\
 &= \frac{a^3+3a^2-2a^2-6a+a+3-a^3+a^2+4a^2-4a-4a+4+a^2+a-6}{(a-2)(a+3)(a-1)} \\
 &= \frac{7a^2-12a+1}{(a-2)(a+3)(a-1)}
 \end{aligned}$$

$$\begin{aligned}
 28. \frac{2+3a}{2-3a} - \frac{2-3a}{2+3a} - \frac{a}{(2-3a)^2} \quad mcm &= (4-9a^2)(2-3a) \\
 &= \frac{(2+3a)(4-9a^2) - (2-3a)^3 - a(2+3a)}{(4-9a^2)(2-3a)} \\
 &= \frac{8-18a^2+12a-27a^3-8+36a-54a^2+27a^3-2a-3a^2}{(4-9a^2)(2-3a)} \\
 &= \frac{46a-75a^2}{(4-9a^2)(2-3a)}
 \end{aligned}$$

$$\begin{aligned}
 29. \frac{1}{5+5a} + \frac{1}{5-5a} - \frac{1}{10+10a^2} \quad mcm &= 10(1+a^2)(1-a^2) \\
 &= \frac{2(1+a^2)(1-a) + 2(1+a^2)(1+a) - (1-a^2)}{10(1+a^2)(1-a^2)} \\
 &= \frac{2-2a+2a^2-2a^3+2+2a+2a^2+2a^3-1+a^2}{10(1+a^2)(1-a^2)} \\
 &= \frac{5a^2+3}{10(1-a^4)}
 \end{aligned}$$

$$\begin{aligned}
 30. \frac{1}{3-3x} - \frac{1}{3+3x} + \frac{x}{6+6x^2} - \frac{x}{2-2x^2} \\
 3-3x = 3(1-x) \quad ; \quad 3+3x = 3(1+x) \\
 6+6x^2 = 6(1+x^2) \quad ; \quad 2-2x^2 = 2(1-x^2) \\
 mcm = 6(1+x^2)(1-x^2) = 6(1-x^4) \\
 &= \frac{2(1+x^2)(1+x) - 2(1+x^2)(1-x) + x(1-x^2) - 3x(1+x^2)}{6(1-x^4)} \\
 &= \frac{2+2x+2x^2+2x^3-2+2x-2x^2+2x^3+x-x^3-3x-3x^3}{6(1-x^4)} \\
 &= \frac{2x}{6(1-x^4)} = \frac{x}{3(1-x^4)}
 \end{aligned}$$

## EJERCICIO 131

$$\begin{aligned}
 1. \frac{1}{m-n} + \frac{m}{n^2-m^2} &= \frac{1}{m-n} - \frac{m}{m^2-n^2} \quad mcm = m^2-n^2 \\
 &= \frac{m+n-m}{m^2-n^2} = \frac{n}{m^2-n^2}
 \end{aligned}$$

$$\begin{aligned}
 2. \frac{x^2}{x^2-xy} - \frac{2x}{y-x} &= \frac{x^2}{x(x-y)} + \frac{2x}{x-y} = \frac{x}{x-y} + \frac{2x}{x-y} \\
 mcm &= x-y \\
 &= \frac{x+2x}{x-y} = \frac{3x}{x-y}
 \end{aligned}$$

$$\begin{aligned}
 3. \frac{1}{2x-x^2} + \frac{x}{x^2-4} &= \frac{1}{x(2-x)} - \frac{x}{4-x^2} \\
 mcm &= x(4-x^2) \\
 &= \frac{2+x-x^2}{x(4-x^2)} = \frac{(2-x)(1+x)}{x(2-x)(x+2)} = \frac{1+x}{x(2+x)}
 \end{aligned}$$

$$\begin{aligned}
 4. \frac{a+b}{a^2-ab} + \frac{a}{b^2-a^2} &= \frac{a+b}{a(a-b)} - \frac{a}{a^2-b^2} \\
 mcm &= a(a^2-b^2)
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{(a+b)(a+b) - a^2}{a(a^2-b^2)} = \frac{a^2+2ab+b^2-a^2}{a(a^2-b^2)} = \frac{2ab+b^2}{a(a^2-b^2)}
 \end{aligned}$$

$$\begin{aligned}
 5. \frac{x-4}{x^2-2x-3} - \frac{x}{6-2x} &= \frac{x-4}{(x-3)(x+1)} + \frac{x}{2(x-3)} \\
 mcm &= 2(x-3)(x+1) \\
 &= \frac{2(x-4) + x(x+1)}{2(x-3)(x+1)} = \frac{2x-8+x^2+x}{2(x-3)(x+1)} = \frac{x^2+3x-8}{2(x-3)(x+1)}
 \end{aligned}$$

$$\begin{aligned}
 6. \frac{1}{x^2+2x-8} + \frac{1}{(2-x)(x+3)} &= \frac{1}{(x+4)(x-2)} - \frac{1}{(x-2)(x+3)} \\
 mcm &= (x-2)(x+3)(x+4) \\
 &= \frac{x+3-(x+4)}{(x-2)(x+3)(x+4)} \\
 &= \frac{x+3-x-4}{(x-2)(x+3)(x+4)} \\
 &= \frac{-1}{(x-2)(x+3)(x+4)} = \frac{1}{(2-x)(x+3)(x+4)}
 \end{aligned}$$

$$\begin{aligned}
 7. \frac{1}{2x+2} + \frac{2}{1-x} + \frac{7}{4x-4} &= \frac{1}{2(x+1)} - \frac{2}{x-1} + \frac{7}{4(x-1)} \\
 mcm &= 4(x^2-1) \\
 &= \frac{2(x-1) - 8(x+1) + 7(x+1)}{4(x^2-1)} \\
 &= \frac{2x-2-8x-8+7x+7}{4(x^2-1)} = \frac{x-3}{4(x^2-1)}
 \end{aligned}$$

$$8. \frac{2a}{a+3} + \frac{3a}{a-3} + \frac{2a}{9-a^2} = \frac{2a}{a+3} + \frac{3a}{a-3} - \frac{2a}{a^2-9}$$

$$mcm = a^2 - 9$$

$$= \frac{2a(a-3) + 3a(a+3) - 2a}{a^2 - 9}$$

$$= \frac{2a^2 - 6a + 3a^2 + 9a - 2a}{a^2 - 9} = \frac{5a^2 + a}{a^2 - 9}$$

$$9. \frac{x+3y}{y+x} + \frac{3y^2}{x^2-y^2} - \frac{x}{y-x} = \frac{x+3y}{y+x} + \frac{3y^2}{x^2-y^2} - \frac{x}{x-y}$$

$$mcm = x^2 - y^2$$

$$= \frac{(x+3y)(x-y) + 3y^2 + x(x+y)}{x^2 - y^2}$$

$$= \frac{x^2 - xy + 3xy - 3y^2 + 3y^2 + x^2 + xy}{x^2 - y^2} = \frac{2x^2 + 3xy}{x^2 - y^2}$$

$$10. \frac{x}{x^2+2x-3} + \frac{x-3}{(1-x)(x+2)} + \frac{1}{x+2}$$

$$= \frac{x}{(x+3)(x-1)} - \frac{x-3}{(x-1)(x+2)} + \frac{1}{x+2}$$

$$mcm = (x+3)(x+2)(x-1)$$

$$= \frac{x(x+2) - (x-3)(x+3) + (x+3)(x-1)}{(x+3)(x+2)(x-1)}$$

$$= \frac{x^2 + 2x - x^2 + 9 + x^2 + 2x - 3}{(x+3)(x+2)(x-1)} = \frac{x^2 + 4x + 6}{(x+3)(x+2)(x-1)}$$

$$11. \frac{3}{2a+2} - \frac{1}{4a-4} - \frac{4}{8-8a^2} = \frac{3}{2(a+1)} - \frac{1}{4(a-1)} + \frac{4}{8(a^2-1)}$$

$$mcm = 8(a^2-1)$$

$$= \frac{12(a-1) - 2(a+1) + 4}{8(a^2-1)}$$

$$= \frac{12a - 12 - 2a - 2 + 4}{8(a^2-1)} = \frac{10a - 10}{8(a^2-1)} = \frac{10(a-1)}{8(a-1)(a+1)} = \frac{5}{4(a+1)}$$

$$12. \frac{1}{a-3} + \frac{a+1}{(3-a)(a-2)} + \frac{2}{(2-a)(1-a)}$$

$$= \frac{1}{a-3} - \frac{a+1}{(a-3)(a-2)} + \frac{2}{(a-2)(a-1)}$$

$$mcm = (a-3)(a-2)(a-1)$$

$$= \frac{(a-2)(a-1) - (a+1)(a-1) + 2(a-3)}{(a-3)(a-2)(a-1)}$$

$$= \frac{a^2 - 3a + 2 - a^2 + 1 + 2a - 6}{(a-3)(a-2)(a-1)}$$

$$= \frac{-a-3}{(a-3)(a-2)(a-1)} = \frac{a+3}{(3-a)(a-2)(a-1)}$$

$$13. \frac{2x}{x-1} + \frac{2x^3+2x^2}{1-x^3} + \frac{1}{x^2+x+1}$$

$$= \frac{2x}{x-1} - \frac{2x^2(x+1)}{x^3-1} + \frac{1}{x^2+x+1}$$

$$mcm = x^3 - 1$$

$$= \frac{2x(x^2+x+1) - 2x^2(x+1) + (x-1)}{x^3-1}$$

$$= \frac{2x^3 + 2x^2 + 2x - 2x^3 - 2x^2 + x - 1}{x^3-1} = \frac{3x-1}{x^3-1}$$

$$14. \frac{x+2}{3x-1} + \frac{x+1}{3-2x} + \frac{4x^2+6x+3}{6x^2-11x+3}$$

$$= \frac{x+2}{3x-1} - \frac{x+1}{2x-3} + \frac{4x^2+6x+3}{(2x-3)(3x-1)}$$

$$mcm = (2x-3)(3x-1)$$

$$= \frac{(x+2)(2x-3) - (x+1)(3x-1) + 4x^2+6x+3}{(2x-3)(3x-1)}$$

$$= \frac{2x^2 - 3x + 4x - 6 - 3x^2 + x - 3x + 1 + 4x^2 + 6x + 3}{(2x-3)(3x-1)}$$

$$= \frac{3x^2 + 5x - 2}{(2x-3)(3x-1)} = \frac{(x+2)(3x-1)}{(2x-3)(3x-1)} = \frac{x+2}{2x-3}$$

## EJERCICIO 132

$$1. \frac{2a^2}{3b} \cdot \frac{6b^2}{4a} = \frac{12a^2b^2}{12ab} = ab$$

$$2. \frac{x^2y}{5} \cdot \frac{10a^3}{3m^2} \cdot \frac{9m}{x^3} = \frac{90x^2ya^3m}{15x^3m^2} = \frac{6a^3y}{xm}$$

$$3. \frac{5x^2}{7y^3} \cdot \frac{4y^2}{7m^3} \cdot \frac{14m}{5x^4} = \frac{14 \cdot 5 \cdot 4x^2my^2}{49 \cdot 5x^4m^3y^3} = \frac{8}{7x^2m^2y}$$

$$4. \frac{5}{a} \cdot \frac{2a}{b^2} \cdot \frac{3b}{10} = \frac{10 \cdot 3ab}{10ab^2} = \frac{3}{b}$$

$$5. \frac{2x^3}{15a^3} \cdot \frac{3a^2}{y} \cdot \frac{5x^2}{7xy^2} = \frac{15 \cdot 2a^2x^5}{15 \cdot 7a^3xy^3} = \frac{2x^4}{7ay^3}$$

$$6. \frac{7a}{6m^2} \cdot \frac{3m}{10n^2} \cdot \frac{5n^4}{14ax} = \frac{7 \cdot 3 \cdot 5amn^4}{14 \cdot 6 \cdot 10am^2n^2x} = \frac{n^2}{8mx}$$

$$7. \frac{2x^2+x}{6} + \frac{8}{4x+2} = \frac{x(2x+1)}{3} \cdot \frac{4}{2(2x+1)} = \frac{2x}{3}$$

$$8. \frac{5x+25}{14} \cdot \frac{7x+7}{10x+50} = \frac{5(x+5)}{14} \cdot \frac{7(x+1)}{10(x+5)} = \frac{x+1}{4}$$



$$9. \frac{m+n}{mn-n^2} \cdot \frac{n^2}{m^2-n^2} \\ = \frac{m+n}{n(m-n)} \cdot \frac{n^2}{(m+n)(m-n)} = \frac{n}{(m-n)^2} = \frac{n}{m^2-2mn+n^2}$$

$$10. \frac{xy-2y^2}{x^2+xy} \cdot \frac{x^2+2xy+y^2}{x^2-2xy} \\ = \frac{y(x-2y)}{x(x+y)} \cdot \frac{(x+y)^2}{x(x-2y)} = \frac{y(x+y)}{x^2} = \frac{xy+y^2}{x^2}$$

$$11. \frac{x^2-4xy+4y^2}{x^2+2xy} \cdot \frac{x^2}{x^2-4y^2} \\ = \frac{(x-2y)^2}{x(x+2y)} \cdot \frac{x^2}{(x+2y)(x-2y)} = \frac{x(x-2y)}{(x+2y)^2} = \frac{x^2-2xy}{x^2+4xy+y^2}$$

$$12. \frac{2x^2+2x}{2x^2} \cdot \frac{x^2-3x}{x^2-2x-3} = \frac{2x(x+1)}{2x^2} \cdot \frac{x(x-3)}{(x-3)(x+1)} = 1$$

$$13. \frac{a^2-ab+a-b}{a^2+2a+1} \cdot \frac{3}{6a^2-6ab} \\ = \frac{(a+1)(a-b)}{(a+1)(a+1)} \cdot \frac{3}{6a(a-b)} = \frac{1}{2a(a+1)} = \frac{1}{2a^2+2a}$$

$$14. \frac{(x-y)^3}{x^3-1} \cdot \frac{x^2+x+1}{(x-y)^2} = \frac{(x-y)}{(x-1)(x^2+x+1)} \cdot x^2+x+1 = \frac{x-y}{x-1}$$

$$15. \frac{2a-2}{2a^2-50} \cdot \frac{a^2-4a-5}{3a+3} = \frac{2(a-1)}{2(a^2-25)} \cdot \frac{(a-5)(a+1)}{3(a+1)} \\ = \frac{(a-1)(a-5)}{3(a+5)(a-5)} = \frac{a-1}{3(a+5)} = \frac{a-1}{3a+15}$$

$$16. \frac{2x^2-3x-2}{6x+3} \cdot \frac{3x+6}{x^2-4} = \frac{(x-2)(2x+1)}{3(2x+1)} \cdot \frac{3(x+2)}{(x-2)(x+2)} = 1$$

$$17. \frac{y^2+9y+18}{y-5} \cdot \frac{5y-25}{5y+15} = \frac{(y+6)(y+3)}{y-5} \cdot \frac{5(y-5)}{5(y+3)} = y+6$$

$$18. \frac{x^3+2x^2-3x}{4x^2+8x+3} \cdot \frac{2x^2+3x}{x^2-x} \\ = \frac{x(x^2+2x-3)}{(2x+3)(2x+1)} \cdot \frac{x(2x+3)}{x(x-1)} \\ = \frac{x(x+3)(x-1)}{(2x+1)(x-1)} = \frac{x(x+3)}{2x+1} = \frac{x^2+3x}{2x+1}$$

$$19. \frac{x^3-27}{a^3-1} \cdot \frac{a^2+a+1}{x^2+3x+9} \\ = \frac{(x-3)(x^2+3x+9)}{(a-1)(a^2+a+1)} \cdot \frac{a^2+a+1}{x^2+3x+9} = \frac{x-3}{a-1}$$

$$20. \frac{a^2+4ab+4b^2}{3} \cdot \frac{2a+4b}{(a+2b)^3} = \frac{(a+2b)^2}{3} \cdot \frac{2(a+2b)}{(a+2b)^3} = \frac{2}{3}$$

$$21. \frac{1-x}{a+1} \cdot \frac{a^2+a}{x-x^2} \cdot \frac{x^2}{a} = \frac{1-x}{a+1} \cdot \frac{a(a+1)}{x(1-x)} \cdot \frac{x^2}{a} = x$$

$$22. \frac{x^2+2x}{x^2-16} \cdot \frac{x^2-2x-8}{x^3+x^2} \cdot \frac{x^2+4x}{x^2+4x+4} \\ = \frac{x(x+2)}{(x-4)(x+4)} \cdot \frac{(x-4)(x+2)}{x^2(x+1)} \cdot \frac{x(x+4)}{(x+2)(x+2)} \\ = \frac{x^2(x+2)^2(x-4)(x+4)}{x^2(x+2)^2(x-4)(x+4)(x+1)} = \frac{1}{x+1}$$

$$23. \frac{(m+n)^2-x^2}{(m+x)^2-n^2} \cdot \frac{(m-n)^2-x^2}{m^2+mn-mx} \\ = \frac{(m+n+x)(m+n-x)}{(m+n+x)(m-n+x)} \cdot \frac{(m-n+x)(m-n-x)}{m(m+n-x)} \\ = \frac{m-n-x}{m}$$

$$24. \frac{2a^3+2ab^2}{2ax^2-2ax} \cdot \frac{x^3-x}{a^2x+b^2x} \cdot \frac{x}{x+1} \\ = \frac{2a(a^2+b^2)}{2ax(x-1)} \cdot \frac{x(x^2-1)}{x(a^2+b^2)} \cdot \frac{x}{x+1} = 1$$

$$25. \frac{a^2-5a+6}{3a-15} \cdot \frac{6a}{a^2-a-30} \cdot \frac{a^2-25}{2a-4} \\ = \frac{(a-3)(a-2)}{3(a-5)} \cdot \frac{6a}{(a-6)(a+5)} \cdot \frac{(a+5)(a-5)}{2(a-2)} \\ = \frac{a(a-3)}{a-6} = \frac{a^2-3a}{a-6}$$

$$26. \frac{x^2-3xy-10y^2}{x^2-2xy-8y^2} \cdot \frac{x^2-16y^2}{x^2+4xy} \cdot \frac{x^2-6xy}{x+2y} \\ = \frac{(x-5y)(x+2y)}{(x-4y)(x+2y)} \cdot \frac{(x+4y)(x-4y)}{x(x+4y)} \cdot \frac{x(x-6y)}{x+2y} \\ = \frac{(x-5y)(x-6y)}{x+2y} = \frac{x^2-11xy+30y^2}{x+2y}$$

$$27. \frac{x^2+4ax+4a^2}{3ax-6a^2} \cdot \frac{2ax-4a^2}{ax+a} \cdot \frac{6a+6x}{x^2+3ax+2a^2} \\ = \frac{(x+2a)^2}{3a(x-2a)} \cdot \frac{2a(x-2a)}{a(x+1)} \cdot \frac{6(a+x)}{(x+2a)(x+a)} \\ = \frac{4(x+2a)}{a(x+1)} = \frac{4x+8a}{ax+a}$$

$$28. \frac{a^2-81}{2a^2+10a} \cdot \frac{a+11}{a^2-36} \cdot \frac{2a-12}{2a+18} \cdot \frac{a^3+5a^2}{2a+22} \\ = \frac{(a+9)(a-9)}{2a(a+5)} \cdot \frac{a+11}{(a+6)(a-6)} \cdot \frac{2(a-6)}{2(a+9)} \cdot \frac{a^2(a+5)}{2(a+11)} \\ = \frac{a(a-9)}{4(a+6)} = \frac{a^2-9a}{4a+24}$$

$$\begin{aligned}
 29. \quad & \frac{a^2+7a+10}{a^2-6a-7} \cdot \frac{a^2-3a-4}{a^2+2a-15} \cdot \frac{a^3-2a^2-3a}{a^2-2a-8} \\
 &= \frac{(a+5)(a+2)}{(a-7)(a+1)} \cdot \frac{(a-4)(a+1)}{(a+5)(a-3)} \cdot \frac{a(a^2-2a-3)}{(a-4)(a+2)} \\
 &= \frac{a(a-3)(a+1)}{(a-3)(a-7)} = \frac{a(a+1)}{a-7} = \frac{a^2+a}{a-7}
 \end{aligned}$$

### EJERCICIO 133

$$\begin{aligned}
 1. \quad & \left(a + \frac{a}{b}\right) \left(a - \frac{a}{b+1}\right) \\
 &\Rightarrow a + \frac{a}{b} = \frac{ab+a}{b} = \frac{a(b+1)}{b} \\
 &\Rightarrow a - \frac{a}{b+1} = \frac{a(b+1)-a}{b+1} = \frac{ab+a-a}{b+1} = \frac{ab}{b+1} \\
 &\Rightarrow \frac{a(b+1)}{b} \cdot \frac{ab}{b+1} = a^2 \\
 2. \quad & \left(x - \frac{2}{x+1}\right) \left(x + \frac{1}{x+2}\right) \\
 &\Rightarrow x - \frac{2}{x+1} = \frac{x(x+1)-2}{x+1} = \frac{x^2+x-2}{x+1} = \frac{(x+2)(x-1)}{x+1} \\
 &\Rightarrow x + \frac{1}{x+2} = \frac{x(x+2)+1}{x+2} = \frac{x^2+2x+1}{x+2} = \frac{(x+1)(x+1)}{x+2} \\
 &\Rightarrow \frac{(x+2)(x-1)}{x+1} \cdot \frac{(x+1)^2}{x+2} = (x-1)(x+1) = x^2-1 \\
 3. \quad & \left(1 - \frac{x}{a+x}\right) \left(1 + \frac{x}{a}\right) \\
 &\Rightarrow 1 - \frac{x}{a+x} = \frac{a+x-x}{a+x} = \frac{a}{a+x} \\
 &\Rightarrow 1 + \frac{x}{a} = \frac{a+x}{a} \\
 &\Rightarrow \frac{a}{a+x} \cdot \frac{a+x}{a} = 1 \\
 4. \quad & \left(a + \frac{ab}{a-b}\right) \left(1 - \frac{b^2}{a^2}\right) \\
 &\Rightarrow a + \frac{ab}{a-b} = \frac{a(a-b)+ab}{a-b} = \frac{a^2-ab+ab}{a-b} = \frac{a^2}{a-b} \\
 &\Rightarrow 1 - \frac{b^2}{a^2} = \frac{a^2-b^2}{a^2} \\
 &\Rightarrow \frac{a^2}{a-b} \cdot \frac{a^2-b^2}{a^2} = \frac{(a+b)(a-b)}{(a-b)} = a+b
 \end{aligned}$$

$$\begin{aligned}
 30. \quad & \frac{x^4+27x}{x^3-x^2+x} \cdot \frac{x^4+x}{x^4-3x^3+9x^2} \cdot \frac{1}{x(x+3)^2} \cdot \frac{x^2}{x-3} \\
 &= \frac{x(x^3+27)}{x(x^2-x+1)} \cdot \frac{x(x^3+1)}{x^2(x^2-3x+9)} \cdot \frac{x}{(x+3)^2(x-3)} \\
 &= \frac{(x+3)(x^2-3x+9)}{x^2-x+1} \cdot \frac{(x+1)(x^2-x+1)}{x^2-3x+9} \cdot \frac{1}{(x^2-9)(x+3)} \\
 &= \frac{x+1}{x^2-9}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \left(x+2 - \frac{12}{x+1}\right) \left(x-2 + \frac{10-3x}{x+5}\right) \\
 &\Rightarrow x+2 - \frac{12}{x+1} = \frac{(x+2)(x+1)-12}{x+1} \\
 &= \frac{x^2+3x+2-12}{x+1} = \frac{x^2+3x-10}{x+1} = \frac{(x+5)(x-2)}{x+1} \\
 &\Rightarrow x-2 + \frac{10-3x}{x+5} = \frac{(x-2)(x+5)+10-3x}{x+5} = \frac{x^2+3x-10+10-3x}{x+5} = \frac{x^2}{x+5} \\
 &\Rightarrow \frac{(x+5)(x-2)}{x+1} \cdot \frac{x^2}{x+5} = \frac{x^2(x-2)}{x+1} = \frac{x^3-2x^2}{x+1} \\
 6. \quad & \left(1 + \frac{x}{y}\right) \left(x - \frac{x^2}{x+y}\right) \\
 &\Rightarrow 1 + \frac{x}{y} = \frac{y+x}{y} \\
 &\Rightarrow x - \frac{x^2}{x+y} = \frac{x(x+y)-x^2}{x+y} = \frac{x^2+xy-x^2}{x+y} = \frac{xy}{x+y} \\
 &\Rightarrow \frac{y+x}{y} \cdot \frac{xy}{x+y} = x \\
 7. \quad & \left(a+x - \frac{ax+x^2}{a+2x}\right) \left(1 + \frac{x}{a+x}\right) \\
 &\Rightarrow a+x - \frac{ax+x^2}{a+2x} = \frac{(a+x)(a+2x)-(ax+x^2)}{a+2x} \\
 &= \frac{a^2+3ax+2x^2-ax-x^2}{a+2x} = \frac{a^2+2ax+x^2}{a+2x} = \frac{(a+x)^2}{a+2x} \\
 &\Rightarrow 1 + \frac{x}{a+x} = \frac{a+x+x}{a+x} = \frac{a+2x}{a+x} \\
 &\Rightarrow \frac{(a+x)^2}{a+2x} \cdot \frac{a+2x}{a+x} = a+x
 \end{aligned}$$

$$\begin{aligned}
8. & \left(x - \frac{x^3 - 6x}{x^2 - 25}\right) \left(x + 1 - \frac{8}{x+3}\right) \\
& \Rightarrow x - \frac{x^3 - 6x}{x^2 - 25} = \frac{x(x^2 - 25) - (x^3 - 6x)}{x^2 - 25} \\
& = \frac{x^3 - 25x - x^3 + 6x}{x^2 - 25} = \frac{-19x}{x^2 - 25} = \frac{19x}{25 - x^2} \\
& \Rightarrow x + 1 - \frac{8}{x+3} = \frac{(x+1)(x+3) - 8}{x+3} \\
& = \frac{x^2 + 4x + 3 - 8}{x+3} = \frac{x^2 + 4x - 5}{x+3} = \frac{(x+5)(x-1)}{x+3} \\
& \Rightarrow \frac{19x}{25 - x^2} \cdot \frac{(x+5)(x-1)}{x+3} \\
& = \frac{19x(x-1)}{(5-x)(x+3)} = \frac{19x^2 - 19x}{5x + 15 - x^2 - 3x} = \frac{19x^2 - 19x}{15 + 2x - x^2} \\
10. & \left(a + 2x - \frac{14x^2}{2a+x}\right) \left(a - x + \frac{a^2 + 5x^2}{a+4x}\right) \\
& \Rightarrow a + 2x - \frac{14x^2}{2a+x} \\
& = \frac{(a+2x)(2a+x) - 14x^2}{2a+x} \\
& = \frac{2a^2 + 5ax + 2x^2 - 14x^2}{2a+x} = \frac{2a^2 + 5ax - 12x^2}{2a+x} = \frac{(a+4x)(2a-3x)}{2a+x} \\
& \Rightarrow a - x + \frac{a^2 + 5x^2}{a+4x} \\
& = \frac{(a-x)(a+4x) + a^2 + 5x^2}{a+4x} \\
& = \frac{a^2 + 3ax - 4x^2 + a^2 + 5x^2}{a+4x} = \frac{2a^2 + 3ax + x^2}{a+4x} = \frac{(a+x)(2a+x)}{a+4x} \\
& \Rightarrow \frac{(a+4x)(2a-3x)}{2a+x} \cdot \frac{(a+x)(2a+x)}{a+4x} \\
& = (2a-3x)(a+x) = 2a^2 - ax - 3x^2
\end{aligned}$$

$$\begin{aligned}
9. & \left(m - \frac{mn}{m+n}\right) \left(1 + \frac{n^3}{m^3}\right) \\
& \Rightarrow m - \frac{mn}{m+n} = \frac{m(m+n) - mn}{m+n} = \frac{m^2 + mn - mn}{m+n} = \frac{m^2}{m+n} \\
& \Rightarrow 1 + \frac{n^3}{m^3} = \frac{m^3 + n^3}{m^3} = \frac{(m+n)(m^2 - mn + n^2)}{m^3} \\
& \Rightarrow \frac{m^2}{m+n} \cdot \frac{(m+n)(m^2 - mn + n^2)}{m^3} = \frac{m^2 - mn + n^2}{m} \\
11. & \left(1 + \frac{a}{b}\right) \left(1 - \frac{b}{a}\right) \left(1 + \frac{b^2}{a^2 - b^2}\right) \\
& \Rightarrow 1 + \frac{a}{b} = \frac{b+a}{b} \\
& \Rightarrow 1 - \frac{b}{a} = \frac{a-b}{a} \\
& \Rightarrow 1 + \frac{b^2}{a^2 - b^2} = \frac{a^2}{a^2 - b^2} \\
& \Rightarrow \frac{b+a}{b} \cdot \frac{a-b}{a} \cdot \frac{a^2}{(a+b)(a-b)} = \frac{a^2}{ab} = \frac{a}{b} \\
12. & \left(2 + \frac{2}{x+1}\right) \left(3 - \frac{6}{x+2}\right) \left(1 + \frac{1}{x}\right) \\
& \Rightarrow 2 + \frac{2}{x+1} = \frac{2(x+1) + 2}{x+1} = \frac{2x+4}{x+1} = \frac{2(x+2)}{x+1} \\
& \Rightarrow 3 - \frac{6}{x+2} = \frac{3(x+2) - 6}{x+2} = \frac{3x+6-6}{x+2} = \frac{3x}{x+2} \\
& \Rightarrow 1 + \frac{1}{x} = \frac{x+1}{x} \\
& \Rightarrow \frac{2(x+2)}{x+1} \cdot \frac{3x}{x+2} \cdot \frac{x+1}{x} = 6
\end{aligned}$$

### EJERCICIO 134

$$\begin{aligned}
1. & \frac{x^2}{3y^2} \div \frac{2x}{y^3} = \frac{x^2}{3y^2} \cdot \frac{y^3}{2x} = \frac{xy}{6} \\
2. & \frac{3a^2b}{5x^2} \div a^2b^3 = \frac{3a^2b}{5x^2} \cdot \frac{1}{a^2b^3} = \frac{3}{5b^2x^2} \\
3. & \frac{5m^2}{7n^3} \div \frac{10m^4}{14an^4} = \frac{5m^2}{7n^3} \cdot \frac{14an^4}{10m^4} = \frac{an}{m^2} \\
4. & 6a^2x^3 \div \frac{a^2x}{5} = 6a^2x^3 \cdot \frac{5}{a^2x} = 30x^2 \\
5. & \frac{15m^2}{19ax^3} \div \frac{20y^2}{38a^3x^4} = \frac{15m^2}{19ax^3} \cdot \frac{38a^3x^4}{20y^2} = \frac{3m^2a^2x}{2y^2} \\
6. & \frac{11x^2y^3}{7m^2} \div 22y^4 = \frac{11x^2y^3}{7m^2} \cdot \frac{1}{22y^4} = \frac{x^2}{14m^2y} \\
7. & \frac{x-1}{3} \div \frac{2x-2}{6} = \frac{x-1}{3} \cdot \frac{6}{2(x-1)} = 1 \\
8. & \frac{3a^2}{a^2 + 6ab + 9b^2} \div \frac{5a^3}{a^2b + 3ab^2} = \frac{3a^2}{(a+3b)^2} \cdot \frac{ab(a+3b)}{5a^3} = \frac{3b}{5(a+3b)} \\
9. & \frac{x^3 - x}{2x^2 + 6x} \div \frac{5x^2 - 5x}{2x + 6} = \frac{x(x^2 - 1)}{2x(x+3)} \cdot \frac{2(x+3)}{5x(x-1)} = \frac{x+1}{5x} \\
10. & \frac{1}{a^2 - a - 30} \div \frac{2}{a^2 + a - 42} \\
& = \frac{1}{(a-6)(a+5)} \cdot \frac{(a+7)(a-6)}{2} = \frac{a+7}{2(a+5)} \\
11. & \frac{20x^2 - 30x}{15x^3 + 15x^2} \div \frac{4x-6}{x+1} = \frac{5x(4x-6)}{15x^2(x+1)} \cdot \frac{x+1}{(4x-6)} = \frac{1}{3x}
\end{aligned}$$

$$\begin{aligned}
 12. \quad & \frac{a^2 - 6a + 5}{a^2 - 15a + 56} \div \frac{a^2 + 2a - 35}{a^2 - 5a - 24} \\
 &= \frac{(a-5)(a-1)}{(a-8)(a-7)} \cdot \frac{(a-8)(a+3)}{(a+7)(a-5)} \\
 &= \frac{(a-1)(a+3)}{(a-7)(a+7)} = \frac{a^2 + 2a - 3}{a^2 - 49}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad & \frac{8x^2 + 26x + 15}{16x^2 - 9} \div \frac{6x^2 + 13x - 5}{9x^2 - 1} \\
 &= \frac{(2x+5)(4x+3)}{(4x+3)(4x-3)} \cdot \frac{(3x+1)(3x-1)}{(2x+5)(3x-1)} = \frac{3x+1}{4x-3}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & \frac{x^3 - 121x}{x^2 - 49} \div \frac{x^2 - 11x}{x + 7} \\
 &= \frac{x(x^2 - 121)}{(x+7)(x-7)} \cdot \frac{(x+7)}{x(x-11)} = \frac{x+11}{x-7}
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & \frac{ax^2 + 5}{4a^2 - 1} \div \frac{a^3x^2 + 5a^2}{2a - 1} \\
 &= \frac{ax^2 + 5}{(2a+1)(2a-1)} \cdot \frac{2a-1}{a^2(ax^2 + 5)} = \frac{1}{a^2(2a+1)}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \frac{a^4 - 1}{a^3 + a^2} \div \frac{a^4 + 4a^2 + 3}{3a^3 + 9a} \\
 &= \frac{(a^2+1)(a^2-1)}{a^2(a+1)} \cdot \frac{3a(a^2+3)}{(a^2+3)(a^2+1)} = \frac{3(a-1)}{a}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & \frac{x^3 + 125}{x^2 - 64} \div \frac{x^3 - 5x^2 + 25x}{x^2 + x - 56} \\
 &= \frac{(x+5)(x^2 - 5x + 25)}{(x+8)(x-8)} \cdot \frac{(x+8)(x-7)}{x(x^2 - 5x + 25)} \\
 &= \frac{(x+5)(x-7)}{x(x-8)} = \frac{x^2 - 2x - 35}{x^2 - 8x}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & \frac{16x^2 - 24xy + 9y^2}{16x - 12y} \div \frac{64x^3 - 27y^3}{32x^2 + 24xy + 18y^2} \\
 &= \frac{(4x-3y)^2}{4(4x-3y)} \cdot \frac{2(16x^2 + 12xy + 9y^2)}{(4x-3y)(16x^2 + 12xy + 9y^2)} = \frac{1}{2}
 \end{aligned}$$

$$19. \quad \frac{a^2 - 6a}{a^3 + 3a^2} \div \frac{a^2 + 3a - 54}{a^2 + 9a} = \frac{a(a-6)}{a^2(a+3)} \cdot \frac{a(a+9)}{(a+9)(a-6)} = \frac{1}{a+3}$$

$$\begin{aligned}
 20. \quad & \frac{15x^2 + 7x - 2}{25x^3 - x} \div \frac{6x^2 + 13x + 6}{25x^2 + 10x + 1} \\
 &= \frac{(3x+2)(5x-1)}{x(5x-1)(5x+1)} \cdot \frac{(5x+1)^2}{(2x+3)(3x+2)} = \frac{5x+1}{x(2x+3)} = \frac{5x+1}{2x^2 + 3x}
 \end{aligned}$$

$$\begin{aligned}
 21. \quad & \frac{x^3 - 1}{2x^2 - 2x + 2} \div \frac{7x^2 + 7x + 7}{7x^3 + 7} \\
 &= \frac{(x-1)(x^2 + x + 1)}{2(x^2 - x + 1)} \cdot \frac{7(x^3 + 1)}{7(x^2 + x + 1)} \\
 &= \frac{(x-1)}{2(x^2 - x + 1)} \cdot (x+1)(x^2 - x + 1) = \frac{(x-1)(x+1)}{2} = \frac{x^2 - 1}{2}
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & \frac{2mx - 2my + nx - ny}{3x - 3y} \div 8m + 4n \\
 &= \frac{(2m+n)(x-y)}{3(x-y)} \cdot \frac{1}{4(2m+n)} = \frac{1}{12}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & \frac{x^2 - 6x + 9}{4x^2 - 1} \div \frac{x^2 + 5x - 24}{2x^2 + 17x + 8} \\
 &= \frac{(x-3)^2}{(2x+1)(2x-1)} \cdot \frac{(2x+1)(x+8)}{(x+8)(x-3)} = \frac{x-3}{2x-1}
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & \frac{2a^2 + 7ab - 15b^2}{a^3 + 4a^2b} \div \frac{a^2 - 3ab - 40b^2}{a^2 - 4ab - 32b^2} \\
 &= \frac{(a+5b)(2a-3b)}{a^2(a+4b)} \cdot \frac{(a-8b)(a+4b)}{(a-8b)(a+5b)} = \frac{2a-3b}{a^2}
 \end{aligned}$$

### EJERCICIO 135

$$\begin{aligned}
 1. \quad & \left(1 + \frac{a}{a+b}\right) \div \left(1 + \frac{2a}{b}\right) \\
 &\Rightarrow 1 + \frac{a}{a+b} = \frac{a+b+a}{a+b} = \frac{2a+b}{a+b} \\
 &\Rightarrow 1 + \frac{2a}{b} = \frac{b+2a}{b} \\
 &\Rightarrow \frac{2a+b}{a+b} \div \frac{b+2a}{b} = \frac{2a+b}{a+b} \cdot \frac{b}{b+2a} = \frac{b}{a+b}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \left(x - \frac{2}{x+1}\right) \div \left(x - \frac{x}{x+1}\right) \\
 &\Rightarrow x - \frac{2}{x+1} = \frac{x(x+1)-2}{x+1} = \frac{x^2+x-2}{x+1} \\
 &\Rightarrow x - \frac{x}{x+1} = \frac{x(x+1)-x}{x+1} = \frac{x^2}{x+1} \\
 &\Rightarrow \frac{x^2+x-2}{x+1} \div \frac{x^2}{x+1} = \frac{x^2+x-2}{x+1} \cdot \frac{x+1}{x^2} = \frac{x^2+x-2}{x^2}
 \end{aligned}$$

$$\begin{aligned}
 3. \left(1-a+\frac{a^2}{1+a}\right) \div \left(1+\frac{2}{a^2-1}\right) \\
 \Rightarrow 1-a+\frac{a^2}{1+a} = \frac{(1-a)(1+a)+a^2}{1+a} = \frac{1-a^2+a^2}{1+a} = \frac{1}{1+a} \\
 \Rightarrow 1+\frac{2}{a^2-1} = \frac{a^2-1+2}{a^2-1} = \frac{a^2+1}{a^2-1} \\
 \Rightarrow \frac{1}{1+a} \div \frac{a^2+1}{a^2-1} = \frac{1}{1+a} \cdot \frac{a^2-1}{a^2+1} = \frac{a-1}{a^2+1}
 \end{aligned}$$

$$\begin{aligned}
 4. \left(x+\frac{2}{x+3}\right) \div \left(x+\frac{3}{x+4}\right) \\
 \Rightarrow x+\frac{2}{x+3} = \frac{x(x+3)+2}{x+3} = \frac{x^2+3x+2}{x+3} = \frac{(x+1)(x+2)}{x+3} \\
 \Rightarrow x+\frac{3}{x+4} = \frac{x(x+4)+3}{x+4} = \frac{x^2+4x+3}{x+4} = \frac{(x+3)(x+1)}{x+4} \\
 \Rightarrow \frac{(x+1)(x+2)}{x+3} \div \frac{(x+3)(x+1)}{x+4} \\
 = \frac{(x+1)(x+2)}{x+3} \cdot \frac{x+4}{(x+3)(x+1)} = \frac{(x+2)(x+4)}{(x+3)^2} = \frac{x^2+6x+8}{x^2+6x+9}
 \end{aligned}$$

$$\begin{aligned}
 5. \left(a+b+\frac{b^2}{a-b}\right) \div \left(1-\frac{b}{a+b}\right) \\
 \Rightarrow a+b+\frac{b^2}{a-b} = \frac{(a+b)(a-b)+b^2}{a-b} = \frac{a^2-b^2+b^2}{a-b} = \frac{a^2}{a-b} \\
 \Rightarrow 1-\frac{b}{a+b} = \frac{a+b-b}{a+b} = \frac{a}{a+b} \\
 \Rightarrow \frac{a^2}{a-b} \div \frac{a}{a+b} = \frac{a^2}{a-b} \cdot \frac{a+b}{a} = \frac{a(a+b)}{a-b} = \frac{a^2+ab}{a-b}
 \end{aligned}$$

$$\begin{aligned}
 6. \left(1-\frac{1}{x^3+2}\right) \div \left(x+\frac{1}{x-1}\right) \\
 \Rightarrow 1-\frac{1}{x^3+2} = \frac{x^3+2-1}{x^3+2} = \frac{x^3+1}{x^3+2} = \frac{(x+1)(x^2-x+1)}{x^3+2} \\
 \Rightarrow x+\frac{1}{x-1} = \frac{x(x-1)+1}{x-1} = \frac{x^2-x+1}{x-1} \\
 \Rightarrow \frac{(x+1)(x^2-x+1)}{x^3+2} \div \frac{x^2-x+1}{x-1} \\
 = \frac{(x+1)(x^2-x+1)}{x^3+2} \cdot \frac{x-1}{x^2-x+1} = \frac{(x+1)(x-1)}{x^3-2} = \frac{x^2-1}{x^3+2}
 \end{aligned}$$

$$\begin{aligned}
 7. \left(x+\frac{1}{x+2}\right) \div \left(1+\frac{3}{x^2-4}\right) \\
 \Rightarrow x+\frac{1}{x+2} = \frac{x(x+2)+1}{x+2} = \frac{x^2+2x+1}{x+2} = \frac{(x+1)^2}{x+2} \\
 \Rightarrow 1+\frac{3}{x^2-4} = \frac{x^2-4+3}{x^2-4} = \frac{x^2-1}{x^2-4} = \frac{(x+1)(x-1)}{(x+2)(x-2)} \\
 \Rightarrow \frac{(x+1)^2}{x+2} \div \frac{(x+1)(x-1)}{(x+2)(x-2)} \\
 = \frac{(x+1)^2}{x+2} \cdot \frac{(x+2)(x-2)}{(x+1)(x-1)} = \frac{(x+1)(x-2)}{x-1} = \frac{x^2-x-2}{x-1}
 \end{aligned}$$

$$\begin{aligned}
 8. \left(n-\frac{2n-1}{n^2+2}\right) \div \left(n^2+1-\frac{n-1}{n}\right) \\
 \Rightarrow n-\frac{2n-1}{n^2+2} = \frac{n(n^2+2)-(2n-1)}{n^2+2} = \frac{n^3+2n-2n+1}{n^2+2} = \frac{n^3+1}{n^2+2} \\
 \Rightarrow n^2+1-\frac{n-1}{n} = \frac{n(n^2+1)-(n-1)}{n} = \frac{n^3+n-n+1}{n} = \frac{n^3+1}{n} \\
 \Rightarrow \frac{n^3+1}{n^2+2} \div \frac{n^3+1}{n} = \frac{n^3+1}{n^2+2} \cdot \frac{n}{n^3+1} = \frac{n}{n^2+2}
 \end{aligned}$$

## EJERCICIO 136

$$1. \frac{3x}{4y} \cdot \frac{8y}{9x} \div \frac{z^2}{3x^2} = \frac{2}{3} \cdot \frac{3x^2}{z^2} = \frac{2x^2}{z^2}$$

$$2. \frac{5a}{b} \div \left(\frac{2a}{b^2} \cdot \frac{5x}{4a^2}\right) = \frac{5a}{b} \cdot \frac{2ab^2}{5x} = \frac{2a^2b}{x}$$

$$\begin{aligned}
 3. \frac{a+1}{a-1} \cdot \frac{3a-3}{2a+2} \div \frac{a^2+a}{a^2+a-2} \\
 = \frac{a+1}{a-1} \cdot \frac{3(a-1)}{2(a+1)} \cdot \frac{(a+2)(a-1)}{a(a+1)} \\
 = \frac{3(a+2)(a-1)}{2a(a+1)} = \frac{3a^2+3a-6}{2a^2+2a}
 \end{aligned}$$

$$\begin{aligned}
 4. \frac{64a^2-81b^2}{x^2-81} \cdot \frac{(x-9)^2}{8a-9b} \div \frac{8a^2+9ab}{(x+9)^2} \\
 = \frac{(8a+9b)(8a-9b)}{(x+9)(x-9)} \cdot \frac{(x-9)^2}{8a-9b} \cdot \frac{(x+9)^2}{a(8a+9b)} \\
 = \frac{(x-9)(x+9)}{a} = \frac{x^2-81}{a}
 \end{aligned}$$

$$\begin{aligned}
 5. \frac{x^2-x-12}{x^2-49} \cdot \frac{x^2-x-56}{x^2+x-20} \div \frac{x^2-5x-24}{x+5} \\
 = \frac{(x-4)(x+3)}{(x+7)(x-7)} \cdot \frac{(x-8)(x+7)}{(x+5)(x-4)} \cdot \frac{x+5}{(x-8)(x+3)} = \frac{1}{x-7}
 \end{aligned}$$

$$\begin{aligned}
 6. \frac{a^2-8a+7}{a^2-11a+30} \cdot \frac{a^2-36}{a^2-1} \div \frac{a^2-a-42}{a^2-4a-5} \\
 = \frac{(a-7)(a-1)}{(a-6)(a-5)} \cdot \frac{(a+6)(a-6)}{(a+1)(a-1)} \cdot \frac{(a-5)(a+1)}{(a-7)(a+6)} = 1
 \end{aligned}$$

$$\begin{aligned}
7. & \frac{x^4 - 27x}{x^2 + 7x - 30} \cdot \frac{x^2 + 20x + 100}{x^3 + 3x^2 + 9x} \div \frac{x^2 - 100}{x - 3} = \frac{x(x-3)(x^2 + 3x + 9)}{(x+10)(x-3)} \cdot \frac{(x+10)^2}{x(x^2 + 3x + 9)} \cdot \frac{x-3}{(x+10)(x-10)} = \frac{x-3}{x-10} \\
8. & \frac{(a^2 + 1)}{3a - 6} \div \left( \frac{a^3 + a}{6a - 12} \cdot \frac{4x + 8}{x - 3} \right) = \frac{a^2 + 1}{3(a-2)} \div \frac{a(a^2 + 1)}{6(a-2)} \cdot \frac{4(x+2)}{x-3} = \frac{a^2 + 1}{3(a-2)} \cdot \frac{3(a-2)(x-3)}{2a(a^2 + 1)(x+2)} = \frac{x-3}{2a(x+2)} = \frac{x-3}{2ax+4a} \\
9. & \frac{8x^2 - 10x - 3}{6x^2 + 13x + 6} \cdot \frac{4x^2 - 9}{3x^2 + 2x} + \frac{8x^2 + 14x + 3}{9x^2 + 12x + 4} = \frac{(2x-3)(4x+1)}{(2x+3)(3x+2)} \cdot \frac{(2x+3)(2x-3)}{x(3x+2)} \cdot \frac{(3x+2)^2}{(2x+3)(4x+1)} = \frac{(2x-3)^2}{x(2x+3)} = \frac{4x^2 - 12x + 9}{2x^2 + 3x} \\
10. & \frac{(a+b)^2 - c^2}{(a-b)^2 - c^2} \cdot \frac{(a+c)^2 - b^2}{a^2 + ab - ac} + \frac{a+b+c}{a^2} = \frac{(a+b+c)(a+b-c)}{(a-b+c)(a-b-c)} \cdot \frac{(a+b+c)(a-b+c)}{a(a+b-c)} \cdot \frac{a^2}{a+b+c} = \frac{a(a+b+c)}{a-b-c} = \frac{a^2 + ab + ac}{a-b-c} \\
11. & \frac{a^2 - 5a}{b + b^2} \div \left( \frac{a^2 + 6a - 55}{b^2 - 1} \cdot \frac{ax + 3a}{ab^2 + 11b^2} \right) = \frac{a(a-5)}{b(1+b)} \cdot \frac{b^2(b+1)(b-1)(a+11)}{a(a+11)(a-5)(x+3)} = \frac{b(b-1)}{x+3} = \frac{b^2 - b}{x+3} \\
12. & \frac{m^3 + 6m^2n + 9mn^2}{2m^2n + 7mn^2 + 3n^3} \cdot \frac{4m^2 - n^2}{8m^2 - 2mn - n^2} \div \frac{m^3 + 27n^3}{16m^2 + 8mn + n^2} \\
& = \frac{m(m+3n)^2}{n(m+3n)(2m+n)} \cdot \frac{(2m+n)(2m-n)}{(2m-n)(4m+n)} \cdot \frac{(4m+n)^2}{(m+3n)(m^2 - 3mn - 9n^2)} = \frac{m(4m+n)}{n(m^2 - 3mn + 9n^2)} = \frac{4m^2 + mn}{m^2n - 3mn^2 + 9n^3} \\
13. & \frac{(a^2 - ax)^2}{a^2 + x^2} \cdot \frac{1}{a^3 + a^2x} \div \left( \frac{a^3 - a^2x}{a^2 + 2ax + x^2} \cdot \frac{a^2 - x^2}{a^3 + ax^2} \right) \\
& = \frac{(a^2 - ax)^2}{a^2 + x^2} \cdot \frac{1}{a^2(a+x)} \cdot \frac{a(a+x)^2(a^2 + x^2)}{a(a^2 - ax)(a+x)(a-x)} = \frac{a^2 - ax}{a^2(a-x)} = \frac{a(a-x)}{a^2(a-x)} = \frac{1}{a} \\
14. & \frac{(a^2 - 3a)^2}{9 - a^2} \cdot \frac{27 - a^3}{(a+3)^2 - 3a} \div \frac{a^4 - 9a^2}{(a^2 + 3a)^2} \\
& = \frac{(a^2 - 3a)^2}{(3+a)(3-a)} \cdot \frac{(3-a)(9+3a+a^2)}{a^2 + 3a + 9} \cdot \frac{(a^2 + 3a)^2}{(a^2 - 3a)(a^2 + 3a)} = \frac{(a^2 - 3a)(a^2 + 3a)}{3+a} = \frac{a(a^2 - 3a)(a+3)}{a+3} = a^3 - 3a^2
\end{aligned}$$

### EJERCICIO 137

$$\begin{aligned}
1. & \frac{a - \frac{a}{b}}{b - \frac{1}{b}} = \frac{\frac{ab - a}{b}}{\frac{b^2 - 1}{b}} = \frac{a(b-1)}{(b+1)(b-1)} = \frac{a}{b+1} \\
2. & \frac{x^2 - \frac{1}{x}}{1 - \frac{1}{x}} = \frac{\frac{x^3 - 1}{x}}{\frac{x-1}{x}} = \frac{(x-1)(x^2 + x + 1)}{x-1} = x^2 + x + 1 \\
3. & \frac{\frac{a}{b} - \frac{b}{a}}{1 + \frac{b}{a}} = \frac{\frac{a^2 - b^2}{ab}}{\frac{a+b}{a}} = \frac{(a+b)(a-b)}{b(a+b)} = \frac{a-b}{b} \\
4. & \frac{\frac{1}{m} + \frac{1}{n}}{\frac{1}{m} - \frac{1}{n}} = \frac{\frac{m+n}{mn}}{\frac{n-m}{mn}} = \frac{m+n}{n-m} \\
5. & \frac{x + \frac{x}{2}}{x - \frac{x}{4}} = \frac{\frac{2x+x}{2}}{\frac{4x-x}{4}} = \frac{2(3x)}{3x} = 2 \\
6. & \frac{\frac{x}{y} - \frac{y}{x}}{1 + \frac{y}{x}} = \frac{\frac{x^2 - y^2}{xy}}{\frac{x+y}{x}} = \frac{(x+y)(x-y)}{y(x+y)} = \frac{x-y}{y} \\
7. & \frac{x+4 + \frac{3}{x}}{x-4 - \frac{5}{x}} = \frac{\frac{x^2 + 4x + 3}{x}}{\frac{x^2 - 4x - 5}{x}} = \frac{(x+1)(x+3)}{(x-5)(x+1)} = \frac{x+3}{x-5} \\
8. & \frac{a-4 + \frac{4}{a}}{1 - \frac{2}{a}} = \frac{\frac{a^2 - 4a + 4}{a}}{\frac{a-2}{a}} = \frac{(a-2)(a-2)}{(a-2)} = a-2
\end{aligned}$$

$$\begin{aligned}
& \frac{2a^2 - b^2}{a} - b \\
9. & \frac{4a^2 + b^2}{4ab} + 1 \\
& \frac{2a^2 - b^2 - ab}{4ab} \\
& = \frac{a}{4a^2 + b^2 + 4ab} \\
& = \frac{4b(2a^2 - b^2 - ab)}{4a^2 + b^2 + 4ab} \\
& = \frac{4b(a-b)(2a+b)}{(2a+b)^2} \\
& = \frac{4b(a-b)}{2a+b} = \frac{4ab-4b^2}{2a+b} \\
10. & \frac{2 + \frac{3a}{5b}}{a + \frac{10b}{3}} = \frac{\frac{10b+3a}{3}}{\frac{3a+10b}{3}} = \frac{3}{5b} \\
& \frac{a-x + \frac{x^2}{a+x}}{a^2 - \frac{a^2}{a+x}} \\
11. & \frac{a^2 - x^2 + x^2}{a^2 - \frac{a^2}{a+x}} \\
& = \frac{a^2 - x^2 + x^2}{a^2 - \frac{a^2}{a+x}} \\
& = \frac{a^2 - x^2 + x^2}{a^2 - \frac{a^2}{a+x}} \\
& = \frac{a^2}{a^2(a+x-1)} = \frac{1}{a+x-1} \\
12. & \frac{a+5 - \frac{a}{8}}{1 + \frac{a}{a^2}} \\
& \frac{a^2 + 5a - 14}{a^2 + 8a + 7} \\
& = \frac{a}{a^2 + 8a + 7} \\
& = \frac{a(a+7)(a-2)}{(a+7)(a+1)} \\
& = \frac{a(a-2)}{a+1} = \frac{a^2 - 2a}{a+1} \\
13. & \frac{1}{a} - \frac{9}{a^2} + \frac{20}{a^3} \\
& \frac{16}{a} - a \\
& \frac{a^2 - 9a + 20}{16 - a^2} \\
& = \frac{a}{16 - a^2} \\
& = \frac{a(a-5)(a-4)}{a^2(4+a)(4-a)} \\
& = -\frac{a-5}{a^2(a+4)} = -\frac{a-5}{a^3+4a^2} = \frac{5-a}{a^3+4a^2}
\end{aligned}$$

$$\begin{aligned}
14. & \frac{20x^2 + 7x - 6}{\frac{4}{x^2} - 25} \\
& \frac{(4x+3)(5x-2)}{\frac{4-25x^2}{x^2}} \\
& = \frac{x(4x+3)(5x-2)}{(2+5x)(2-5x)} \\
& = -\frac{4x^2+3x}{5x+2} \\
15. & \frac{1 + \frac{1}{x-1}}{1 + \frac{1}{x^2-1}} \\
& \frac{x-1+1}{x-1} = \frac{x(x+1)}{x^2-1+1} = \frac{x+1}{x} \\
16. & \frac{a - \frac{ab}{a+b}}{a + \frac{ab}{a-b}} = \frac{a^2 + ab - ab}{a^2 - ab + ab} \\
& = \frac{a^2(a-b)}{a^2(a+b)} = \frac{a-b}{a+b} \\
17. & \frac{x-1 - \frac{5}{x+3}}{x+5 - \frac{35}{x+3}} \\
& \frac{x^2 + 2x - 8}{x^2 + 8x - 20} \\
& = \frac{(x+4)(x-2)}{(x+10)(x-2)} = \frac{x+4}{x+10} \\
18. & \frac{a+2 - \frac{7a+9}{a+3}}{a-4 + \frac{5a-11}{a+1}} \\
& \frac{a^2 + 5a + 6 - 7a - 9}{a^2 - 3a - 4 + 5a - 11} \\
& = \frac{(a+1)(a^2 - 2a - 3)}{(a+3)(a^2 + 2a - 15)} \\
& = \frac{(a+1)(a-3)(a+1)}{(a+3)(a+5)(a-3)} \\
& = \frac{(a+1)^2}{(a+3)(a+5)} = \frac{a^2 + 2a + 1}{a^2 + 8a + 15}
\end{aligned}$$

## EJERCICIO 138

$$\begin{aligned}
1. & \frac{1 + \frac{x+1}{x-1}}{\frac{1}{x-1} - \frac{1}{x+1}} \\
& = \frac{x-1+x+1}{x+1-(x-1)} = \frac{2x(x+1)}{2} = x^2 + x \\
2. & \frac{\frac{1}{x-1} + \frac{2}{x+1}}{\frac{x-2}{x} + \frac{2x+6}{x+1}} \\
& \frac{x+1+2(x-1)}{(x^2-x-2)+x(2x+6)} \\
& = \frac{x(3x-1)}{(x-1)(3x^2+5x-2)} \\
& = \frac{x(3x-1)}{(x-1)(3x-1)(x+2)} = \frac{x}{x^2+x-2} \\
3. & \frac{\frac{a}{a-b} - \frac{b}{a+b}}{\frac{a+b}{a-b} + \frac{a}{b}} \\
& \frac{a(a+b)-b(a-b)}{(a-b)(a+b)} \\
& = \frac{b(a+b)+a(a-b)}{b(a-b)} \\
& = \frac{a^2+ab-ab+b^2}{ab+b^2+a^2-ab} = \frac{a^2+b^2}{a^2+b^2} = \frac{b}{a+b} \\
4. & \frac{\frac{x+3}{x+4} - \frac{x+1}{x-3}}{\frac{x-1}{x+2} - \frac{x-3}{x+4}} \\
& \frac{(x+3)(x+2)-(x+1)(x+4)}{(x+4)(x+2)} \\
& = \frac{(x-1)(x+4)-(x-3)(x+2)}{(x+2)(x+4)} \\
& = \frac{x^2+5x+6-x^2-5x-4}{x^2+3x-4-x^2+x+6} \\
& = \frac{2}{4x+2} = \frac{2}{2(2x+1)} = \frac{1}{2x+1}
\end{aligned}$$

$$\begin{aligned}
5. \quad & \frac{\frac{m^2}{n} + \frac{m^2 - n^2}{m+n}}{\frac{m-n}{n} + \frac{n}{m}} \\
&= \frac{\frac{m^2(m+n) - n(m^2 - n^2)}{n(m+n)}}{\frac{m(m-n) + n^2}{nm}} \\
&= \frac{\frac{m^3 + m^2n - m^2n + n^3}{m+n}}{\frac{m^2 - mn + n^2}{m}} \\
&= \frac{\frac{m^3 + n^3}{m+n}}{\frac{m^2 - mn + n^2}{m}} \\
&= \frac{m}{m+n} \cdot \frac{(m+n)(m^2 - mn + n^2)}{m^2 - mn + n^2} \\
&= m^2 - mn + n^2 \cdot \frac{m}{m^2 - mn + n^2} = m
\end{aligned}$$

$$\begin{aligned}
6. \quad & \frac{\frac{a^2}{b^3} + \frac{1}{a}}{\frac{a}{b} - \frac{b-a}{a-b}} \\
&= \frac{\frac{a^3 + b^3}{ab^3}}{\frac{a(a-b) - b(b-a)}{b(a-b)}} \\
&= \frac{\frac{ab^3}{a^2 - ab - b^2 + ab}}{\frac{(a+b)(a^2 - ab + b^2)}{ab^2}} \\
&= \frac{a^2 - ab + b^2}{ab^2} \cdot \frac{a-b}{(a+b)(a-b)} \\
&= \frac{a^2 - ab + b^2}{ab^2}
\end{aligned}$$

$$\begin{aligned}
7. \quad & \frac{1 + \frac{2x}{1+x^2}}{2x + \frac{2x^5+2}{1-x^4}} \\
&= \frac{\frac{1+x^2+2x}{1+x^2}}{\frac{2x(1-x^4) + 2x^5+2}{(1+x^2)(1-x^2)}} \\
&= \frac{x^2+2x+1}{\frac{2x-2x^5+2x^5+2}{1-x^2}} \\
&= \frac{(x+1)^2}{\frac{2(x+1)}{(x+1)(1-x)}} \\
&= \frac{(x+1)^2 \cdot (x+1)(1-x)}{2(x+1)} \\
&= \frac{(x+1)^2(1-x)}{2} \\
&= \frac{(x^2+2x+1)(1-x)}{2} \\
&= \frac{x^2 - x^3 + 2x - 2x^2 + 1 - x}{2} \\
&= \frac{1+x-x^2-x^3}{2}
\end{aligned}$$

$$\begin{aligned}
8. \quad & \frac{\frac{x+y}{x} - \frac{x-y}{x+2y}}{\frac{x-y}{x} - \frac{x+2y}{x+y}} \\
&= \frac{\frac{(x+y)^2 - (x-y)^2}{(x-y)(x+y)}}{\frac{(x+y)^2 - x(x+2y)}{x(x+y)}} \\
&= \frac{x^2 + 2xy + y^2 - x^2 - 2xy - y^2}{\frac{x^2 + 2xy + y^2 - x^2 - 2xy}{x}} \\
&= \frac{4xy}{\frac{x-y}{y^2} \cdot \frac{x}{y^2}} = \frac{4x^2}{y(x-y)} = \frac{4x^2}{xy-y^2}
\end{aligned}$$

$$\begin{aligned}
9. \quad & \frac{\frac{a+x}{a-x} - \frac{b+x}{b-x}}{\frac{a-x}{a-x} - \frac{b-x}{b-x}} \\
&= \frac{\frac{(a+x)(b-x) - (b+x)(a-x)}{(a-x)(b-x)}}{\frac{2(b-x) - 2(a-x)}{(a-x)(b-x)}} \\
&= \frac{ab - ax + bx - x^2 - ab + bx - ax + x^2}{2b - 2x - 2a + 2x} \\
&= \frac{2bx - 2ax}{2(b-a)} = \frac{2x(b-a)}{2(b-a)} = x \\
10. \quad & \frac{\frac{a}{a+x} - \frac{a}{2a+2x}}{\frac{a}{a-x} + \frac{a}{a+x}} = \frac{\frac{2a-a}{2(a+x)}}{\frac{a(a+x) + a(a-x)}{(a-x)(a+x)}} \\
&= \frac{\frac{a}{2}}{\frac{a^2 + ax + a^2 - ax}{a-x}} \\
&= \frac{\frac{a}{2}}{\frac{a}{2a^2}} = \frac{a}{2} \cdot \frac{a-x}{2a^2} = \frac{a-x}{4a}
\end{aligned}$$

$$\begin{aligned}
11. \quad & \frac{\frac{a+2b}{a-b} + \frac{b}{a}}{\frac{a+b}{a} + \frac{3b}{a-b}} = \frac{\frac{a(a+2b) + b(a-b)}{a(a-b)}}{\frac{(a+b)(a-b) + 3ab}{a(a-b)}} \\
&= \frac{a^2 + 2ab + ab - b^2}{a^2 - b^2 + 3ab} \\
&= \frac{a^2 + 3ab - b^2}{a^2 + 3ab - b^2} = 1 \\
12. \quad & \frac{1 - \frac{7}{x} + \frac{12}{x^2}}{x - \frac{16}{x}} \\
&= \frac{\frac{x^2 - 7x + 12}{x^2}}{\frac{x^2 - 16}{x}} \\
&= \frac{x}{(x-4)(x-3)} \\
&= \frac{x}{(x-4)(x+4)} \cdot \frac{1}{(x-4)(x+4)} \\
&= \frac{x-3}{x(x+4)} = \frac{x-3}{x^2+4x}
\end{aligned}$$



$$\begin{aligned}
 13. \quad & \frac{\frac{a^2}{b} - \frac{b^2}{a}}{\frac{1}{b} + \frac{1}{a} + \frac{b}{a^2}} \\
 &= \frac{\frac{a^3 - b^3}{ab}}{\frac{a^3 - b^3}{a^2b}} \\
 &= \frac{a^3 - b^3}{a^2 + ab + b^2} \cdot \frac{a^2b}{a^2b} \\
 &= \frac{a^3 - b^3}{a^2 + ab + b^2} \cdot \frac{a}{a} \\
 &= (a-b)(a^2 + ab + b^2) \cdot \frac{a}{a^2 + ab + b^2} \\
 &= a(a-b) = a^2 - ab
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & \frac{x-2y - \frac{4y^2}{x+y}}{x-3y - \frac{5y^2}{x+y}} \\
 &= \frac{(x-2y)(x+y) - 4y^2}{(x-3y)(x+y) - 5y^2} \\
 &= \frac{x^2 - xy - 6y^2}{x^2 - 2xy - 8y^2} \\
 &= \frac{(x-3y)(x+2y)}{(x-4y)(x+2y)} = \frac{x-3y}{x-4y}
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & \frac{\frac{2}{1-a} + \frac{2}{1+a}}{\frac{2}{1+a} - \frac{2}{1-a}} \\
 &= \frac{\frac{2(1+a) + 2(1-a)}{(1-a)(1+a)}}{\frac{2(1+a) - 2(1-a)}{(1+a)(1-a)}} \\
 &= \frac{2+2a+2-2a}{2-2a-2-2a} = -\frac{4}{4a} = -\frac{1}{a}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \frac{\frac{1}{x+y+z} - \frac{1}{x-y+z}}{\frac{1}{x-y+z} - \frac{1}{x+y+z}} \\
 &= \frac{\frac{x-y+z-x-y-z}{(x+y+z)(x-y+z)}}{\frac{x-y+z-x+y-z}{(x-y+z)(x+y+z)}} = \frac{-2y}{2y} = -1
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & \frac{1 + \frac{2b+c}{a-b-c}}{1 - \frac{c-2b}{a-b+c}} \\
 &= \frac{\frac{a-b-c+2b+c}{a-b-c}}{\frac{a-b+c-c+2b}{a-b+c}} \\
 &= \frac{a+b}{a-b-c} \cdot \frac{a-b+c}{a+b} = \frac{a-b+c}{a-b-c}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & \frac{\frac{a}{1-a} + \frac{1-a}{a}}{\frac{1-a}{a} - \frac{a}{1-a}} \\
 &= \frac{\frac{a^2 + (1-a)^2}{a(1-a)}}{\frac{(1-a)^2 - a^2}{a(1-a)}} \\
 &= \frac{a^2 + 1 - 2a + a^2}{1 - 2a + a^2 - a^2} = \frac{2a^2 - 2a + 1}{1 - 2a}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & \frac{x+1 - \frac{6x+12}{x+2}}{x-4 + \frac{11x-22}{x-2}} \\
 &= \frac{x+1 - \frac{6(x+2)}{x+2}}{x-4 + \frac{11(x-2)}{x-2}} = \frac{x-5}{x-4+11} = \frac{x-5}{x+7}
 \end{aligned}$$

$$20. \quad \frac{1}{1+\frac{1}{x}} = \frac{1}{\frac{x+1}{x}} = 1 \cdot \frac{x}{x+1} = \frac{x}{x+1}$$

$$\begin{aligned}
 21. \quad & \frac{1}{1+\frac{1}{1-\frac{1}{x}}} = \frac{1}{1+\frac{1}{\frac{x-1}{x}}} \\
 &= \frac{1}{1+\frac{x}{x-1}} = \frac{1}{\frac{x-1+x}{x-1}} \\
 &= \frac{1}{\frac{2x-1}{x-1}} = 1 \cdot \frac{x-1}{2x-1} = \frac{x-1}{2x-1}
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & 1 - \frac{1}{2 + \frac{1}{\frac{x}{3} - 1}} \\
 &= 1 - \frac{1}{2 + \frac{1}{\frac{x-3}{3}}} \\
 &= 1 - \frac{1}{2 + \frac{3}{x-3}} \\
 &= 1 - \frac{1}{\frac{2x-3-x+3}{x-3}} \\
 &= 1 - \frac{x-3}{2x-3} = \frac{2x-3-x+3}{2x-3} = \frac{x}{2x-3}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & \frac{2}{1 + \frac{2}{1 + \frac{2}{x}}} \\
 &= \frac{2}{1 + \frac{2}{\frac{x+2}{x}}} \\
 &= \frac{2}{1 + \frac{2x}{x+2}} \\
 &= \frac{2}{\frac{x+2+2x}{x+2}} \\
 &= \frac{2}{3x+2} = 2 \cdot \frac{x+2}{3x+2} = \frac{2x+4}{3x+2}
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & \frac{1}{x - \frac{x}{x - \frac{x^2}{x+1}}} \\
 &= \frac{1}{x - \frac{x}{\frac{x^2+x-x^2}{x+1}}} \\
 &= \frac{1}{x - \frac{x}{\frac{x}{x+1}}} \\
 &= \frac{1}{x - \frac{x^2+x}{x}} \\
 &= \frac{1}{\frac{x^2-x^2-x}{x}} = \frac{1}{-\frac{x}{x}} = -1
 \end{aligned}$$

$$\begin{aligned}
25. \quad & \frac{1}{a+2-\frac{a+1}{a-\frac{1}{a}}} \\
&= \frac{1}{a+2-\frac{a+1}{\frac{a^2-1}{a}}} = \frac{1}{a+2-\frac{a(a+1)}{(a+1)(a-1)}} = \frac{1}{a+2-\frac{a}{a-1}} = \frac{1}{\frac{(a+2)(a-1)-a}{a-1}} = \frac{1}{\frac{a^2+a-2-a}{a-1}} = 1 \cdot \frac{a-1}{a^2-2} = \frac{a-1}{a^2-2} \\
26. \quad & \frac{x-1}{x+2-\frac{x^2+2}{x-\frac{x-2}{x+1}}} = \frac{x-1}{x+2-\frac{x^2+2}{\frac{x^2+x-x+2}{x+1}}} = \frac{x-1}{x+2-\frac{x^2+2}{\frac{x^2+2}{x+1}}} = \frac{x-1}{x+2-\frac{(x^2+2)(x+1)}{x^2+2}} = \frac{x-1}{x+2-x-1} = x-1
\end{aligned}$$

### EJERCICIO 139

$$\begin{aligned}
1. \quad & \frac{x-2}{x+3} \text{ Para } x \rightarrow 2 \quad \frac{2-2}{2+3} = \frac{0}{5} = 0 \\
2. \quad & \frac{x-2}{x-3} \text{ Para } x \rightarrow 3 \quad \frac{3-2}{3-3} = \frac{1}{0} = \infty \\
3. \quad & \frac{x^2-a^2}{x^2+a^2} \text{ Para } x \rightarrow a \quad \frac{a^2-a^2}{a^2+a^2} = \frac{0}{2a^2} = 0 \\
4. \quad & \frac{x^2+y^2}{x^2-y^2} \text{ Para } x \rightarrow y \quad \frac{y^2+y^2}{y^2-y^2} = \frac{2y^2}{0} = \infty \\
5. \quad & \frac{x-1}{x-2} \text{ Para } x \rightarrow 2 \quad \frac{2-1}{2-2} = \frac{1}{0} = \infty = 0 \\
6. \quad & \frac{x^2-9}{x^2+x-12} \text{ Para } x \rightarrow 3 \\
& \frac{(x+3)(x-3)}{(x+4)(x-3)} = \frac{x+3}{x+4} = \frac{3+3}{3+4} = \frac{6}{7} \\
7. \quad & \frac{a^2-a-6}{a^2+2a-15} \text{ Para } a \rightarrow 3 \\
& \frac{(a-3)(a+2)}{(a+5)(a-3)} = \frac{a+2}{a+5} = \frac{3+2}{3+5} = \frac{5}{8} \\
8. \quad & \frac{x^2-7x+10}{x^3-2x^2-x+2} \text{ Para } x \rightarrow 2 \\
& \frac{(x-5)(x-2)}{x^2(x-2)-(x-2)} \\
&= \frac{(x-5)(x-2)}{(x^2-1)(x-2)} = \frac{x-5}{x^2-1} = \frac{2-5}{4-1} = \frac{-3}{3} = -1 \\
9. \quad & \frac{x^2-2x+1}{x^3-2x^2-x+2} \text{ Para } x \rightarrow 1 \\
& \frac{(x-1)^2}{x^2(x-2)-(x-2)} \\
&= \frac{(x-1)^2}{(x^2-1)(x-2)} = \frac{(x-1)^2}{(x-1)(x+1)(x-2)} = \frac{x-1}{(x+1)(x-2)} = \frac{1-1}{1^2-1-2} = \frac{0}{-2} = 0 \\
10. \quad & \frac{a^3-8}{a^2+11a-26} \text{ Para } a \rightarrow 2 \\
& \frac{(a-2)(a^2+2a+4)}{(a+13)(a-2)} = \frac{a^2+2a+4}{a+13} = \frac{2^2+2 \cdot 2+4}{2+13} = \frac{12}{15} = \frac{4}{5} \\
11. \quad & \frac{x^2-7x+6}{x^2-2x+1} \text{ Para } x \rightarrow 1 \quad \frac{(x-6)(x-1)}{(x-1)^2} = \frac{x-6}{x-1} = \frac{1-6}{1-1} = \frac{-5}{0} = \infty \\
12. \quad & \frac{x^3-3x-2}{x^3-7x+6} \text{ Para } x \rightarrow 2 \\
& x^3-3x-2 = \begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ & 2 & 4 & 2 \\ \hline 1 & 2 & 1 & 0 \end{array} \\
& \Rightarrow (x-2)(x^2+2x+1) \\
& x^3-7x+6 = \begin{array}{ccc|c} 1 & 0 & -7 & 6 \\ & 2 & 4 & -6 \\ \hline 1 & 2 & -3 & 0 \end{array} \\
& \Rightarrow (x-2)(x^2+2x-3) \\
& \frac{(x-2)(x+1)^2}{(x-2)(x^2+2x-3)} = \frac{(x+1)^2}{x^2+2x-3} = \frac{(2+1)^2}{2^2+2 \cdot 2-3} = \frac{9}{5}
\end{aligned}$$

$$13. \frac{x^2-16}{x^3-4x^2-x+4} \quad \text{Para } x \rightarrow 4$$

$$\frac{(x-4)(x+4)}{x^2(x-4)-(x-4)} = \frac{(x-4)(x+4)}{(x^2-1)(x-4)} = \frac{x+4}{x^2-1} = \frac{4+4}{4^2-1} = \frac{8}{15}$$

$$14. \frac{4x^2-4x+1}{4x^2+8x-5} \quad \text{Para } x \rightarrow \frac{1}{2}$$

$$\frac{(2x-1)^2}{(2x+5)(2x-1)} = \frac{2x-1}{2x+5} = \frac{2 \cdot \frac{1}{2} - 1}{2 \cdot \frac{1}{2} + 5} = \frac{1-1}{1+5} = \frac{0}{6} = 0$$

$$15. \frac{8x^2-6x+1}{4x^3+12x^2-15x+4} \quad \text{Para } x \rightarrow \frac{1}{2}$$

$$\Rightarrow 8x^2-6x+1 = (8x)^2 - 6(8x) + 8$$

$$= (8x-4)(8x-2) = (2x-1)(4x-1)$$

$$\Rightarrow 4x^3+12x^2-15x+4 = \begin{array}{rrrr|r} 4 & 12 & -15 & 4 & -4 \\ & -16 & 16 & -4 & \\ \hline & 4 & -4 & 1 & 0 \end{array}$$

$$= (x+4)(4x^2-4x+1) = (x+4)(2x-1)^2$$

$$\Rightarrow \frac{(2x-1)(4x-1)}{(2x-1)^2(x+4)} = \frac{4x-1}{(2x-1)(x+4)} = \frac{4 \cdot \frac{1}{2} - 1}{\left(2 \cdot \frac{1}{2} - 1\right)\left(\frac{1}{2} + 4\right)} = \frac{1}{0} = \infty$$

$$16. \frac{x^3-9x+10}{x^4-x^3-11x^2+9x+18} \quad \text{Para } x \rightarrow 2$$

$$\Rightarrow x^3-9x+10$$

$$\begin{array}{rrrr|r} 1 & 0 & -9 & 10 & 2 \\ & 2 & 4 & -10 & \\ \hline 1 & 2 & -5 & 0 & \end{array} = (x-2)(x^2+2x-5)$$

$$\Rightarrow x^4-x^3-11x^2+9x+18$$

$$\begin{array}{rrrrr|r} 1 & -1 & -11 & 9 & 18 & 2 \\ & 2 & 2 & -18 & -18 & \\ \hline 1 & 1 & -9 & -9 & 0 & \end{array} = (x-2)(x^3+x^2-9x-9)$$

$$\Rightarrow \frac{(x-2)(x^2+2x-5)}{(x-2)(x^3+x^2-9x-9)}$$

$$= \frac{x^2+2x-5}{x^3+x^2-9x-9} = \frac{2^2+2 \cdot 2-5}{2^3+2^2-9 \cdot 2-9} = \frac{3}{-15} = -\frac{1}{5}$$

$$17. \frac{x^3-a^3}{x-a} \quad \text{Para } x \rightarrow a$$

$$\frac{(x-a)(x^2+ax+a^2)}{x-a} = x^2+ax+a^2 = a^2+a^2+a^2 = 3a^2$$

$$18. \frac{a^2-2ab+b^2}{a^2-ab} \quad \text{Para } b \rightarrow a \quad \frac{(a-b)^2}{a(a-b)} = \frac{a-b}{a} = \frac{a-a}{a} = 0$$

$$19. \frac{x^2-y^2}{xy-y^2} \quad \text{Para } y \rightarrow x$$

$$\frac{(x+y)(x-y)}{y(x-y)} = \frac{x+y}{y} = \frac{x+x}{x} = \frac{2x}{x} = 2$$

$$20. \frac{x^3-a^3}{a^2x-a^3} \quad \text{Para } x \rightarrow a$$

$$\frac{(x-a)(x^2+ax+a^2)}{a^2(x-a)} = \frac{x^2+ax+a^2}{a^2} = \frac{a^2+a^2+a^2}{a^2} = \frac{3a^2}{a^2} = 3$$

$$21. \frac{x^3-3x+2}{2x^3-6x^2+6x-2} \quad \text{Para } x \rightarrow 1$$

$$\Rightarrow x^3-3x+2$$

$$\begin{array}{rrrr|r} 1 & 0 & -3 & 2 & 1 \\ & 1 & 1 & -2 & \\ \hline 1 & 1 & -2 & 0 & \end{array} = (x-1)(x^2+x-2)$$

$$= (x-1)(x+2)(x-1)$$

$$= (x-1)^2(x+2)$$

$$\Rightarrow \frac{x^3-3x+2}{2x^3-6x^2+6x-2}$$

$$\begin{array}{rrrr|r} 2 & -6 & 6 & -2 & 1 \\ & 2 & -4 & 2 & \\ \hline 2 & -4 & 2 & 0 & \end{array} = (x-1)(2x^2-4x+2)$$

$$= (x-1)(x-1)(2x-2)$$

$$= (x-1)^2(2x-2)$$

$$\Rightarrow \frac{(x-1)^2(x+2)}{(x-1)^2(2x-2)} = \frac{x+2}{2x-2} = \frac{1+2}{2 \cdot 1-2} = \frac{3}{0} = \infty$$

$$22. \frac{x^4-x^3-7x^2+x+6}{x^4-3x^3-3x^2+11x-6} \quad \text{Para } x \rightarrow 3$$

$$\Rightarrow x^4-x^3-7x^2+x+6$$

$$\begin{array}{rrrrr|r} 1 & -1 & -7 & 1 & 6 & -2 \\ & -2 & 6 & 2 & -6 & \\ \hline 1 & -3 & -1 & 3 & 0 & \end{array}$$

$$(x+2)(x^3-3x^2-x+3)$$

$$\begin{array}{rrrr|r} 1 & -3 & -1 & 3 & 1 \\ & 1 & -2 & -3 & \\ \hline 1 & -2 & -3 & 0 & \end{array}$$

$$= (x+2)(x-1)(x^2-2x-3)$$

$$= (x+2)(x-1)(x-3)(x+1)$$

Continúa

### Continuación

$$\begin{aligned}
 22. & \Rightarrow x^4 - 3x^3 - 3x^2 + 11x - 6 \\
 & \begin{array}{r|l}
 1 & -3 & -3 & 11 & -6 & 2 \\
 & -2 & 10 & -14 & 6 & \\
 \hline
 1 & -5 & 7 & -3 & 0 & \\
 \hline
 & (x+2)(x^3 - 5x^2 + 7x - 3) \\
 & \begin{array}{r|l}
 1 & -5 & 7 & -3 & 1 \\
 & 1 & -4 & 3 & \\
 \hline
 1 & -4 & 3 & 0 & \\
 \hline
 & (x+2)(x-1)(x^2 - 4x + 3) \\
 & = (x+2)(x-1)(x-3)(x-1) \\
 & \Rightarrow \frac{(x+2)(x-1)(x-3)(x+1)}{(x+2)(x-1)(x-3)(x-1)} \\
 & = \frac{x+1}{x-1} = \frac{3+1}{3-1} = \frac{4}{2} = 2
 \end{array}
 \end{array}
 \end{aligned}$$

$$\begin{aligned}
 23. & \frac{3x^3 - 5x^2 - 4x + 4}{x^4 + 2x^3 - 3x^2 - 8x - 4} \text{ Para } x \rightarrow 2 \\
 & \Rightarrow 3x^3 - 5x^2 - 4x + 4 \\
 & \begin{array}{r|l}
 3 & -5 & -4 & 4 & 2 \\
 & 6 & 2 & -4 & \\
 \hline
 3 & 1 & -2 & 0 & \\
 \hline
 & (x-2)(3x^2 + x - 2) \\
 & \Rightarrow x^4 + 2x^3 - 3x^2 - 8x - 4 \\
 & \begin{array}{r|l}
 1 & 2 & -3 & -8 & -4 & 2 \\
 & 2 & 8 & 10 & 4 & \\
 \hline
 1 & 4 & 5 & 2 & 0 & \\
 \hline
 & (x-2)(x^3 + 4x^2 + 5x + 2) \\
 & \begin{array}{r|l}
 1 & 4 & 5 & 2 & -2 \\
 & -2 & -4 & -2 & \\
 \hline
 1 & 2 & 1 & 0 & \\
 \hline
 & (x-2)(x+2)(x^2 + 2x + 1) \\
 & \Rightarrow \frac{(x-2)(3x^2 + x - 2)}{(x-2)(x+2)(x+1)^2} \\
 & = \frac{3x^2 + x - 2}{(x+2)(x+1)^2} \\
 & = \frac{3 \cdot 2^2 + 2 - 2}{(2+2)(2+1)^2} = \frac{12}{36} = \frac{1}{3}
 \end{array}
 \end{array}
 \end{aligned}$$

$$\begin{aligned}
 24. & \frac{x^2 - 5x + 4}{x^4 - 2x^3 - 9x^2 + 2x + 8} \text{ Para } x \rightarrow 1 \\
 & \Rightarrow x^2 - 5x + 4 = (x-1)(x-4) \\
 & \Rightarrow x^4 - 2x^3 - 9x^2 + 2x + 8 \\
 & \begin{array}{r|l}
 1 & -2 & -9 & 2 & 8 & 1 \\
 & 1 & -1 & -10 & -8 & \\
 \hline
 1 & -1 & -10 & -8 & 0 & \\
 \hline
 \end{array}
 \end{aligned}$$

Continúa

### Continuación

$$\begin{aligned}
 24. & \frac{(x-1)(x^3 - x^2 - 10x - 8)}{1 - 1 - 10 - 8} \begin{array}{r|l} & 4 \\ & 4 & 12 & 8 \\ \hline & 1 & 3 & 2 & 0 \end{array} \\
 & = (x-1)(x-4)(x^2 + 3x + 2) \\
 & = \frac{(x-1)(x-4)}{(x-1)(x-4)(x^2 + 3x + 2)} \\
 & = \frac{1}{x^2 + 3x + 2} = \frac{1}{1^2 + 3 \cdot 1 + 2} = \frac{1}{6}
 \end{aligned}$$

$$\begin{aligned}
 25. & \frac{x^5 - 4x^3 + 8x^2 - 32}{x^5 - 3x^3 + 10x^2 - 4x - 40} \text{ Para } x \rightarrow 2 \\
 & \Rightarrow x^5 - 4x^3 + 8x^2 - 32 \\
 & \begin{array}{r|l}
 1 & 0 & -4 & 8 & 0 & -32 & 2 \\
 & 2 & 4 & 0 & 16 & 32 & \\
 \hline
 1 & 2 & 0 & 8 & 16 & 0 & \\
 \hline
 & (x-2)(x^4 + 2x^3 + 8x + 16) \\
 & \begin{array}{r|l}
 1 & 2 & 0 & 8 & 16 & -2 \\
 & -2 & 0 & 0 & -16 & \\
 \hline
 1 & 0 & 0 & 8 & 0 & \\
 \hline
 & = (x-2)(x+2)(x^3 + 8) \\
 & \Rightarrow x^5 - 3x^3 + 10x^2 - 4x - 40 \\
 & \begin{array}{r|l}
 1 & 0 & -3 & 10 & -4 & -40 & 2 \\
 & 2 & 4 & 2 & 24 & 40 & \\
 \hline
 1 & 2 & 1 & 12 & 20 & 0 & \\
 \hline
 & (x-2)(x^4 + 2x^3 + x^2 + 12x + 20) \\
 & \begin{array}{r|l}
 1 & 2 & 1 & 12 & 20 & -2 \\
 & -2 & 0 & -2 & -20 & \\
 \hline
 1 & 0 & 1 & 10 & 0 & \\
 \hline
 & (x-2)(x+2)(x^3 + x + 10) \\
 & \begin{array}{r|l}
 1 & 0 & 1 & 10 & -2 \\
 & -2 & 4 & -10 & \\
 \hline
 1 & -2 & 5 & 0 & \\
 \hline
 & = (x-2)(x+2)^2(x^2 - 2x + 5) \\
 & \Rightarrow \frac{(x-2)(x+2)(x^3 + 8)}{(x-2)(x+2)^2(x^2 - 2x + 5)} \\
 & = \frac{x^3 + 8}{(x+2)(x^2 - 2x + 5)} \\
 & = \frac{2^3 + 8}{(2+2)(2^2 - 2 \cdot 2 + 5)} = \frac{16}{20} = \frac{4}{5}
 \end{array}
 \end{array}
 \end{array}
 \end{aligned}$$

$$\begin{aligned}
 26. & \frac{8x^2 + 6x - 9}{12x^2 - 13x + 3} \text{ Para } x \rightarrow \frac{3}{4} \\
 & \Rightarrow \frac{(8x)^2 + 6(8x) - 72}{(12x)^2 - 13(12x) + 36} \\
 & = \frac{(8x+12)(8x-6)}{(12x-9)(12x-4)} \\
 & = \frac{(2x+3)(4x-3)}{(4x-3)(3x-1)} \\
 & = \frac{2x+3}{3x-1} = \frac{2 \cdot \frac{3}{4} + 3}{3 \cdot \frac{3}{4} - 1} = \frac{\frac{9}{2} + 3}{\frac{9}{4} - 1} = \frac{\frac{15}{2}}{\frac{5}{4}} = \frac{36}{5} = \frac{18}{5}
 \end{aligned}$$

$$\begin{aligned}
 27. & \frac{x^3 + 6x^2 + 12x + 8}{x^4 - 8x^2 + 16} \text{ Para } x \rightarrow -2 \\
 & \Rightarrow x^3 + 6x^2 + 12x + 8 \\
 & \begin{array}{r|l}
 1 & 6 & 12 & 8 & 2 \\
 & -2 & -8 & -8 & \\
 \hline
 1 & 4 & 4 & 0 & \\
 \hline
 & = (x+2)(x^2 + 4x + 4) = (x+2)^3 \\
 & \Rightarrow x^4 - 8x^2 + 16 \\
 & \begin{array}{r|l}
 1 & 0 & -8 & 0 & 16 & -2 \\
 & -2 & 4 & 8 & -16 & \\
 \hline
 1 & -2 & -4 & 8 & 0 & \\
 \hline
 & (x+2)(x^3 - 2x^2 - 4x + 8) \\
 & \begin{array}{r|l}
 1 & -2 & -4 & 8 & -2 \\
 & -2 & 8 & -8 & \\
 \hline
 1 & -4 & 4 & 0 & \\
 \hline
 & = (x+2)^2(x-4x+4) \\
 & = (x+2)^2(x-2)^2 \\
 & \Rightarrow \frac{(x+2)^3}{(x+2)^2(x-2)^2} \\
 & = \frac{x+2}{(x-2)^2} = \frac{-2+2}{(-2-2)^2} = \frac{0}{16} = 0
 \end{array}
 \end{array}
 \end{aligned}$$

$$\begin{aligned}
 28. & \frac{9x^3 + 3x^2 + 3x + 1}{27x^3 + 1} \text{ Para } x \rightarrow -\frac{1}{3} \\
 & = \frac{3x^2(3x+1) + (3x+1)}{27x^3 + 1} \\
 & = \frac{(3x^2+1)(3x+1)}{(3x+1)(9x^2 - 3x + 1)} \\
 & = \frac{3x^2 + 1}{9x^2 - 3x + 1} \\
 & = \frac{3\left(-\frac{1}{3}\right)^2 + 1}{9\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right) + 1} = \frac{\frac{4}{3} + 1}{\frac{4}{3} + 1} = \frac{\frac{7}{3}}{\frac{7}{3}} = 1
 \end{aligned}$$

$$29. \frac{1}{x-1} - \frac{3}{x^3-1} \quad \text{Para } x \rightarrow 1 \quad \frac{x^2+x+1-3}{x^3-1} = \frac{x^2+x-2}{x^3-1} = \frac{(x-1)(x+2)}{(x-1)(x^2+x+1)} = \frac{x+2}{x^2+x+1} = \frac{1+2}{1^2+1+1} = \frac{3}{3} = 1$$

$$30. (x^2+3x-10) \left( 1 + \frac{1}{x-2} \right) \quad \text{Para } x \rightarrow 2 \quad (x+5)(x-2) \left( \frac{x-1}{x-2} \right) = (x+5)(x-1) \Rightarrow (2+5)(2-1) = 7$$

## EJERCICIO 140

$$1. \frac{12x^2+31x+20}{18x^2+21x-4} = \frac{(12x)^2+31(12x)+240}{(18x)^2+21(18x)-72} = \frac{(12x+16)(12x+15)}{(18x+24)(18x-3)} = \frac{(3x+4)(4x+5)}{(3x+4)(6x-1)} = \frac{4x+5}{6x-1}$$

$$2. \left( \frac{1}{a} + \frac{2}{a^2} + \frac{1}{a^3} \right) \div \left( a + 2 - \frac{2a+1}{a} \right) = \left( \frac{a^2+2a+1}{a^3} \right) \div \left( \frac{a^2+2a-2a-1}{a} \right) = \frac{(a+1)^2}{a^3} \cdot \frac{a}{(a+1)(a-1)} = \frac{a+1}{a^2(a-1)} = \frac{a+1}{a^3-a^2}$$

$$3. \frac{x^3+3x^2+9x}{x^5-27x^2} = \frac{x(x^2+3x+9)}{x^2(x^3-27)} = \frac{x^2+3x+9}{x(x-3)(x^2+3x+9)} = \frac{1}{x(x-3)}$$

$$4. \frac{(x+y)^2}{y} - \frac{x(x-y)^2}{xy} = \frac{x(x+y)^2 - x(x-y)^2}{xy} = \frac{x^3+2x^2y+xy^2 - x^3+2x^2y-xy^2}{xy} = \frac{4x^2y}{xy} = 4x$$

$$5. \frac{a^4-2b^3+a^2b(b-2)}{a^4-a^2b-2b^2} = \frac{a^4-2b^3+a^2b^2-2a^2b}{(a^2-2b)(a^2+b)} = \frac{a^2(a^2+b^2)-2b(b^2+a^2)}{(a^2-2b)(a^2+b)} = \frac{(a^2-2b)(a^2+b^2)}{(a^2-2b)(a^2+b)} = \frac{a^2+b^2}{a^2+b}$$

$$6. a + \frac{1+5a}{a^2-5} = \frac{a^3-5a+1+5a}{a^2-5} = \frac{a^3+1}{a^2-5} = \frac{(a+1)(a^2-a+1)}{a^2-5} \\ a - \frac{a+5}{a+1} = \frac{a^2+a-a-5}{a+1} = \frac{a^2-5}{a+1} \\ \Rightarrow \frac{(a+1)(a^2-a+1)}{a^2-5} \cdot \frac{a^2-5}{a+1} = a^2-a+1$$

$$7. x^2+5x-4 - \frac{x^3-29}{x-5} = \frac{(x-5)(x^2+5x-4) - x^3+29}{x-5} = \frac{x^3-29x+20-x^3+29}{x-5} = \frac{49-29x}{x-5}$$

$$x+34 + \frac{170-x^2}{x-5} = \frac{x^2+29x-170+170-x^2}{x-5} = \frac{29x}{x-5}$$

$$\Rightarrow \frac{49-29x}{x-5} \div \frac{29x}{x-5} = \frac{49-29x}{x-5} \cdot \frac{x-5}{29x} = \frac{49-29x}{29x}$$

$$8. \frac{4x^2-5xy+y^2}{3x} = \frac{4x^2}{3x} - \frac{5xy}{3x} + \frac{y^2}{3x} = \frac{4x}{3} - \frac{5y}{3} + \frac{y^2}{3x}$$

$$9. \frac{m-n-x}{mnx} = \frac{m}{mnx} - \frac{n}{mnx} - \frac{x}{mnx} = \frac{1}{nx} - \frac{1}{mx} - \frac{1}{mn}$$

$$10. \frac{x^3-xy^2}{x-y} = x^2+xy$$

$$\Rightarrow \frac{x(x^2-y^2)}{x-y} = x(x+y)$$

$$\Rightarrow \frac{x(x+y)(x-y)}{x-y} = x(x+y) \Rightarrow x(x+y) = x(x+y)$$

$$11. x^2-2x+1 - \frac{9x-3x^2}{x-3} = \frac{x^3-1}{x-1}$$

$$\Rightarrow \frac{(x-1)^2(x-3)-3x(3-x)}{x-3} = \frac{(x-1)(x^2+x+1)}{x-1}$$

$$\Rightarrow \frac{x^3-5x^2+7x-3-9x+3x^2}{x-3} = x^2+x+1$$

$$\Rightarrow \frac{x^3-2x^2-2x-3}{x-3} = x^2+x+1$$

$$\Rightarrow \frac{(x-3)(x^2+x+1)}{x-3} = x^2+x+1 \Rightarrow x^2+x+1 = x^2+x+1$$

$$12. \frac{a^4-5a^2+4}{a^3+a^2-4a-4} = a-3 + \frac{2+4a}{2a+1}$$

$$\Rightarrow \frac{(a-2)(a-1)(a+2)(a+1)}{(a-2)(a+1)(a+2)} = \frac{2a^2-5a-3+2+4a}{2a+1}$$

$$\Rightarrow a-1 = \frac{2a^2-a-1}{2a+1} \Rightarrow a-1 = \frac{(2a+1)(a-1)}{2a+1} \Rightarrow a-1 = a-1$$

$$\begin{aligned}
 13. \quad & \frac{1}{a-b} + \frac{1}{a+b} + \frac{2a}{a^2-ab+b^2} \\
 &= \frac{a^3+b^3+(a-b)(a^2-ab+b^2)+2a(a^2-b^2)}{(a^3+b^3)(a-b)} \\
 &= \frac{a^3+b^3+a^3-2a^2b+2ab^2-b^3+2a^3-2ab^2}{(a^3+b^3)(a-b)} \\
 &= \frac{4a^3-2a^2b}{(a^3+b^3)(a-b)}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & \left( \frac{a^2}{1-a^2} - \frac{a^4}{1-a^4} \right) \cdot \left( 1-a + \frac{1+a^3}{a^2} \right) \\
 &= \left( \frac{a^2(1+a^2)-a^4}{1-a^4} \right) \cdot \left( \frac{a^2(1-a)+1+a^3}{a^2} \right) \\
 &= \left( \frac{a^2+a^4-a^4}{1-a^4} \right) \cdot \left( \frac{a^2-a^3+1+a^3}{a^2} \right) \\
 &= \frac{a^2}{1-a^4} \cdot \frac{a^2+1}{a^2} = \frac{1}{1-a^2}
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & \left( \frac{x^2-9}{x^2-x-12} \div \frac{x-3}{x^2+3x} \right) \cdot \frac{a^2x^2-16a^2}{2x^2+7x+3} \cdot \left( \frac{2}{a^2x} + \frac{1}{a^2x^2} \right) \\
 &= \left( \frac{(x+3)(x-3)}{(x-4)(x+3)} \cdot \frac{x(x+3)}{x-3} \right) \cdot \frac{a^2(x-4)(x+4)}{(x+3)(2x+1)} \cdot \left( \frac{2x+1}{a^2x^2} \right) = \frac{x+4}{x}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \frac{3x^3-x^2-12x+4}{6x^4+x^3-25x^2-4x+4} \\
 & \Rightarrow 3x^3-x^2-12x+4 \quad \left| \begin{array}{rrrr} 3 & -1 & -12 & 4 \\ -6 & 14 & -4 & \end{array} \right| \\
 & \quad \quad \quad \left| \begin{array}{rrrr} 3 & -7 & 2 & 0 \end{array} \right| \\
 &= (x+2)(3x^2-7x+2) = (x+2)(x-2)(3x-1) \\
 & 6x^4+x^3-25x^2-4x+4 \quad \left| \begin{array}{rrrr} 6 & 1 & -25 & -4 \\ -12 & 22 & 6 & -4 \end{array} \right| \\
 & \quad \quad \quad \left| \begin{array}{rrrr} 6 & -11 & -3 & 2 \\ 0 & 2 & -2 & \end{array} \right| \\
 &= (x-2)(x+2)(6x^2+x-1) \\
 &= (x-2)(x+2)(2x+1)(3x-1) \\
 & \Rightarrow \frac{(x+2)(x-2)(3x-1)}{(x-2)(x+2)(2x+1)(3x-1)} = \frac{1}{2x+1}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & \frac{16-81x^2}{72x^2-5x-12} \\
 &= \frac{(4-9x)(4+9x)}{(72x^2-5(72x)-864)} \\
 &= \frac{(4-9x)(4+9x)}{(72x-32)(72x+27)} \\
 &= \frac{(4-9x)(4+9x)}{(9x-4)(8x+3)} = -\frac{(4-9x)(4+9x)}{(4-9x)(8x+3)} = -\frac{9x+4}{8x+3} \\
 18. \quad & \left( \frac{1}{x} - \frac{2}{x+2} + \frac{3}{x+3} \right) \div \left( \frac{x}{x+2} + \frac{x}{x+3} + \frac{6}{x^2+5x+6} \right) \\
 &= \frac{x^2+5x+6-2x(x+3)+3x(x+2)}{x(x^2+5x+6)} \div \frac{x(x+3)+x(x+2)+6}{x^2+5x+6} \\
 &= \frac{x^2+5x+6-2x^2-6x+3x^2+6x}{x(x^2+5x+6)} \cdot \frac{x^2+5x+6}{x^2+3x+x^2+2x+6} \\
 &= \frac{2x^2+5x+6}{x} \cdot \frac{1}{2x^2+5x+6} = \frac{1}{x}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & \frac{\frac{b}{a}}{1-\frac{b^2}{a^2}} + \frac{1+\frac{b}{a-b}}{2-\frac{a-3b}{a-b}} \\
 &= \frac{\frac{b}{a}}{\frac{a^2-b^2}{a^2}} + \frac{\frac{a-b+b}{a-b}}{\frac{2a-2b-a+3b}{a-b}} \\
 &= \frac{\frac{b}{a}}{\frac{a^2-b^2}{a^2}} + \frac{\frac{a-b}{a-b}}{\frac{a-b}{a-b}} \\
 &= \frac{a^2b}{a(a^2-b^2)} + \frac{a-b}{a-b} \\
 &= \frac{a^2b}{a(a^2-b^2)} + \frac{a}{a+b} \\
 &= \frac{a^2b+a^2(a-b)}{a(a^2-b^2)} = \frac{a^2b+a^3-a^2b}{a(a^2-b^2)} \\
 &= \frac{a^3}{a(a^2-b^2)} = \frac{a^2}{a^2-b^2}
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & \frac{1}{3} \left( \frac{x^2-36}{x} + \frac{x}{x^2-4} \right) \cdot \frac{1}{x-\frac{36}{x}} \cdot \frac{1}{x-\frac{4}{x}} \\
 &= \frac{1}{3} \left( \frac{(x+6)(x-6)}{x} + \frac{(x+2)(x-2)}{x} \right) \cdot \frac{1}{\frac{x^2-36}{x}} \cdot \frac{1}{\frac{x^2-4}{x}} \\
 &= \frac{(x+6)(x-6)}{3x} \cdot \frac{(x+2)(x-2)}{x} \cdot \frac{x}{x^2-36} \cdot \frac{x}{x^2-4} \\
 &= \frac{x^2(x+6)(x-6)(x+2)(x-2)}{3x^2(x+6)(x-6)(x+2)(x-2)} = \frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 21. \quad & \frac{\frac{3a}{(a-2b)^2} + \frac{5}{a-5b} + \frac{1}{a-2b}}{\frac{3a^2-14ab+10b^2}{a^2-4ab+4b^2}} \\
 &= \frac{\frac{3a(a-5b)+5(a-2b)^2+(a-2b)(a-5b)}{(a-2b)^2(a-5b)}}{\frac{3a^2-14ab+10b^2}{(a-2b)^2}} \\
 &= \frac{3a^2-15ab+5a^2-20ab+20b^2+a^2-7ab+10b^2}{(a-5b)(3a^2-14ab+10b^2)} \\
 &= \frac{9a^2-42ab+30b^2}{(a-5b)(3a^2-14ab+10b^2)} = \frac{3(3a^2-14ab+10b^2)}{(a-5b)(3a^2-14ab+10b^2)} = \frac{3}{a-5b}
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & \frac{\frac{x+1}{x-1} - \frac{x-1}{x+1}}{\frac{x-1}{x+1} + \frac{x+1}{x-1}} \cdot \frac{x^2+1}{2a^2-2b} \div \frac{2x}{a^2-b} \\
 &= \frac{\frac{(x+1)^2 - (x-1)^2}{x^2-1}}{\frac{(x-1)^2 + (x+1)^2}{x^2-1}} \cdot \frac{x^2+1}{2(a^2-b)} \cdot \frac{a^2-b}{2x} \\
 &= \frac{x^2+2x+1-x^2+2x-1}{x^2-2x+1+x^2+2x+1} \cdot \frac{x^2+1}{4x} = \frac{4x}{2(x^2+1)} \cdot \frac{x^2+1}{4x} = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & \frac{1}{3x-9} - \frac{1}{6x+12} - \frac{1}{2(x-3)^2} + \frac{1}{x-6} + \frac{9}{x} \\
 &= \frac{1}{3(x-3)} - \frac{1}{6(x+2)} - \frac{1}{2(x-3)^2} + \frac{x}{x^2-6x+9} \\
 &= \frac{2(x-3)(x+2) - (x-3)^2 - 3(x+2) + 6(x+2)}{6(x-3)^2(x+2)} \\
 &= \frac{2x^2-2x-12-x^2+6x-9-3x-6+6x^2+12x}{6(x-3)^2(x+2)} \\
 &= \frac{7x^2+13x-27}{6(x-3)^2(x+2)}
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & \frac{a-b+\frac{a^2+b^2}{a+b}}{a+b-\frac{a^2-2b^2}{a-b}} \cdot \frac{b+\frac{b^2}{a}}{a-b} \cdot \frac{1}{1+\frac{2a-b}{b}} \\
 &= \frac{\frac{a^2-b^2+a^2+b^2}{a+b}}{\frac{a^2-b^2-a^2+2b^2}{a-b}} \cdot \frac{\frac{ab+b^2}{a-b}}{\frac{b+2a-b}{b}} \cdot \frac{1}{b} \\
 &= \frac{2a^2}{a+b} \cdot \frac{b(a+b)}{a-b} \cdot \frac{1}{\frac{2a}{b}} = \frac{2a^2(a-b)}{b^2(a+b)} \cdot \frac{b(a+b)}{a(a-b)} \cdot \frac{b}{2a} = 1
 \end{aligned}$$

## EJERCICIO 141

$$\begin{aligned}
 1. \quad & \frac{x}{6} + 5 = \frac{1}{3} - x \\
 \Rightarrow & \frac{x+30}{6} = \frac{1-3x}{3} \\
 & x+30 = 2(1-3x) \\
 & x+30 = 2-6x \\
 & x+6x = 2-30 \\
 & 7x = -28 \\
 & x = \frac{-28}{7} \\
 & x = -4
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \frac{3x}{5} - \frac{2x}{3} + \frac{1}{5} = 0 \quad mcm=15 \\
 & 9x-10x+3=0 \\
 & -x=-3 \\
 & x=3
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \frac{1}{2x} + \frac{1}{4} - \frac{1}{10x} = \frac{1}{5} \quad mcm=20x \\
 & 10+5x-2=4x \\
 & 5x-4x=-10+2 \\
 & x=-8
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \frac{x}{2} + 2 - \frac{x}{12} = \frac{x}{6} - \frac{5}{4} \quad mcm=12 \\
 & 6x+24-x=2x-15 \\
 & 6x-x-2x=-15-24 \\
 & 3x=-39 \\
 & x=\frac{-39}{3} \\
 & x=-13
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \frac{3x}{4} - \frac{1}{5} + 2x = \frac{5}{4} - \frac{3x}{20} \quad mcm=20 \\
 & 15x-4+40x=25-3x \\
 & 55x+3x=25+4 \\
 & 58x=29 \\
 & x=\frac{29}{58} \\
 & x=\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \frac{2}{3x} - \frac{5}{x} = \frac{7}{10} - \frac{3}{2x} + 1 \quad mcm=30x \\
 & 20-150=21x-45+30x \\
 & -130+45=51x \\
 & \frac{-85}{51} = x \\
 & \frac{-5}{3} = x
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & \frac{x-4}{3} - 5 = 0 \quad mcm=3 \\
 & x-4-15=0 \\
 & x-19=0 \\
 & x=19
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & x - \frac{x+2}{12} = \frac{5x}{2} \quad mcm=12 \\
 & 12x-x-2=30x \\
 & 11x-30x=2 \\
 & -19x=2 \\
 & x=-\frac{2}{19}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & x - \frac{5x-1}{3} = 4x - \frac{3}{5} \quad mcm=15 \\
 & 15x-25x+5=60x-9 \\
 & -10x-60x=-9-5 \\
 & -70x=-14 \\
 & x=\frac{-14}{-70} \\
 & x=\frac{1}{5}
 \end{aligned}$$

$$10. 10x - \frac{8x-3}{4} = 2(x-3) \quad mcm=4$$

$$40x - 8x + 3 = 8(x-3)$$

$$40x - 8x = 8x - 24 - 3$$

$$40x - 16x = -27$$

$$x = \frac{-27}{24}$$

$$x = -\frac{9}{8}$$

$$11. \frac{x-2}{3} - \frac{x-3}{4} = \frac{x-4}{5} \quad mcm=60$$

$$20(x-2) - 15(x-3) = 12(x-4)$$

$$20x - 40 - 15x + 45 = 12x - 48$$

$$5x - 12x = -48 - 5$$

$$-7x = -53$$

$$x = \frac{53}{7}$$

$$12. \frac{x-1}{2} - \frac{x-2}{3} - \frac{x-3}{4} = -\frac{x-5}{5}$$

$$mcm=60$$

$$30(x-1) - 20(x-2) - 15(x-3) = -12(x-5)$$

$$30x - 30 - 20x + 40 - 15x + 45 = -12x + 60$$

$$-5x + 55 = -12x + 60$$

$$-5x + 12x = 60 - 55$$

$$7x = 5$$

$$x = \frac{5}{7}$$

$$13. x - (5x-1) - \frac{7-5x}{10} = 1 \quad mcm=10$$

$$10x - 10(5x-1) - (7-5x) = 10$$

$$10x - 50x + 10 - 7 + 5x = 10$$

$$-35x + 3 = 10$$

$$-35x = 7$$

$$x = \frac{-7}{35}$$

$$x = -\frac{1}{5}$$

$$14. 2x - \frac{5x-6}{4} + \frac{1}{3}(x-5) = -5x$$

$$mcm=12$$

$$24x - 3(5x-6) + 4(x-5) = -60x$$

$$24x - 15x + 18 + 4x - 20 = -60x$$

$$13x - 2 = -60x$$

$$13x + 60x = 2$$

$$x = \frac{2}{73}$$

$$15. 4 - \frac{10x+1}{6} = 4x - \frac{16x+3}{4} \quad mcm=12$$

$$48 - 2(10x+1) = 48x - 3(16x+3)$$

$$48 - 20x - 2 = 48x - 48x - 9$$

$$-20x + 46 = -9$$

$$-20x = -55$$

$$x = \frac{-55}{-20}$$

$$x = \frac{11}{4}$$

$$16. \frac{1}{2}(x-1) - (x-3) = \frac{1}{3}(x+3) + \frac{1}{6} \quad mcm=6$$

$$3(x-1) - 6(x-3) = 2(x+3) + 1$$

$$3x - 3 - 6x + 18 = 2x + 6 + 1$$

$$-3x + 15 = 2x + 7$$

$$-5x = -8$$

$$x = \frac{8}{5}$$

$$17. \frac{6x+1}{3} - \frac{11x-2}{9} - \frac{1}{4}(5x-2) = \frac{5}{6}(6x+1) \quad mcm=36$$

$$12(6x+1) - 4(11x-2) - 9(5x-2) = 30(6x+1)$$

$$72x + 12 - 44x + 8 - 45x + 18 = 180x + 30$$

$$-17x + 38 = 180x + 30$$

$$-197x = -8$$

$$x = \frac{8}{197}$$

$$18. \frac{4x+1}{3} = \frac{1}{3}(4x-1) - \frac{13+2x}{6} - \frac{1}{2}(x-3) \quad mcm=6$$

$$2(4x+1) = 2(4x-1) - (13+2x) - 3(x-3)$$

$$8x + 2 = 8x - 2 - 13 - 2x - 3x + 9$$

$$2 = -6 - 5x$$

$$8 = -5x$$

$$-\frac{8}{5} = x$$

$$19. \frac{2}{5}(5x-1) + \frac{3}{10}(10x-3) = -\frac{1}{2}(x-2) - \frac{6}{5} \quad mcm=10$$

$$4(5x-1) + 3(10x-3) = -5(x-2) - 12$$

$$20x - 4 + 30x - 9 = -5x + 10 - 12$$

$$50x - 13 = -5x + 10 - 12$$

$$50x - 13 = -5x - 2$$

$$55x = 11$$

$$x = \frac{1}{5}$$



$$20. \frac{3x-1}{2} - \frac{5x+4}{3} - \frac{x+2}{8} = \frac{2x-3}{5} - \frac{1}{10}$$

$$mcm=120$$

$$60(3x-1) - 40(5x+4) - 15(x+2) = 24(2x-3) - 12$$

$$180x - 60 - 200x - 160 - 15x - 30 = 48x - 72 - 12$$

$$-35x - 250 = 48x - 84$$

$$-83x = 166$$

$$x = -2$$

$$21. \frac{7x-1}{3} - \frac{5-2x}{2x} = \frac{4x-3}{4} + \frac{1+4x^2}{3x}$$

$$mcm=12x$$

$$4x(7x-1) - 6(5-2x) = 3x(4x-3) + 4(1+4x^2)$$

$$28x^2 - 4x - 30 + 12x = 12x^2 - 9x + 4 + 16x^2$$

$$8x - 30 = -9x + 4$$

$$17x = 34$$

$$x = 2$$

$$22. \frac{2x+7}{3} - \frac{2(x^2-4)}{5x} - \frac{4x^2-6}{15x} = \frac{7x^2+6}{3x^2}$$

$$mcm=15x^2$$

$$5x^2(2x+7) - 6x(x^2-4) - x(4x^2-6) = 5(7x^2+6)$$

$$10x^3 + 35x^2 - 6x^3 + 24x - 4x^3 + 6x = 35x^2 + 30$$

$$30x = 30$$

$$x = \frac{30}{30}$$

$$x = 1$$

$$23. \frac{2}{3} \left( \frac{x+1}{5} \right) = \frac{3}{4} \left( \frac{x-6}{3} \right) \quad mcm=60$$

$$8(x+1) = 15(x-6)$$

$$8x + 8 = 15x - 90$$

$$8x - 15x = -90 - 8$$

$$-7x = -98$$

$$x = \frac{-98}{-7}$$

$$x = 14$$

$$24. \frac{3}{5} \left( \frac{2x-1}{6} \right) - \frac{4}{3} \left( \frac{3x+2}{4} \right) - \frac{1}{5} \left( \frac{x-2}{3} \right) + \frac{1}{5} = 0$$

$$mcm=60$$

$$6(2x-1) - 20(3x+2) - 4(x-2) + 12 = 0$$

$$12x - 6 - 60x - 40 - 4x + 8 + 12 = 0$$

$$-52x - 26 = 0$$

$$x = -\frac{26}{52} = -\frac{1}{2}$$

$$25. 10 - \frac{3x+5}{6} = 3\frac{11}{12} - \frac{x}{4}$$

$$\Rightarrow \frac{60-3x-5}{6} = \frac{47}{12} - \frac{x}{8} \quad mcm=24$$

$$4(55-3x) = 94 - 3x$$

$$220 - 12x = 94 - 3x$$

$$-12x + 3x = 94 - 220$$

$$-9x = -126$$

$$x = \frac{-126}{-9}$$

$$x = 14$$

$$26. 9x - 2 - 7x \left( \frac{1}{x} - \frac{1}{2} \right) = \frac{1 + \frac{x}{2}}{2} + 2\frac{3}{4}$$

$$\Rightarrow 9x - 2 - 7x \left( \frac{2-x}{2} \right) = \frac{x+2}{4} + \frac{11}{4}$$

$$\Rightarrow 9x - 2 - 7 \left( \frac{2-x}{2} \right) = \frac{x+13}{4} \quad mcm=4$$

$$36x - 8 - 14(2-x) = x + 13$$

$$36x - 8 - 28 + 14x = x + 13$$

$$50x - 36 = x + 13$$

$$49x = 49$$

$$x = 1$$

$$27. \frac{3x}{8} - \frac{7}{10} - \frac{12x-5}{16} - \frac{2x-3}{20} + \frac{4x+9}{4} + \frac{7}{80} = 0 \quad mcm=160$$

$$60x - 112 - 10(12x-5) - 8(2x-3) + 40(4x+9) + 14 = 0$$

$$60x - 112 - 120x + 50 - 16x + 24 + 160x + 360 + 14 = 0$$

$$84x + 336 = 0$$

$$84x = -336$$

$$x = \frac{-336}{84}$$

$$x = -4$$

$$28. \frac{5x}{4} - \frac{3}{17}(x-20) - (2x-1) = \frac{x+24}{34} \quad mcm=68$$

$$85x - 12(x-20) - 68(2x-1) = 2(x+24)$$

$$85x - 12x + 240 - 136x + 68 = 2x + 48$$

$$-63x + 308 = 2x + 48$$

$$-65x = -260$$

$$x = \frac{-260}{-65}$$

$$x = 4$$

$$\begin{aligned}
 29. \quad 5 + \frac{x}{4} &= \frac{1}{3} \left( 2 - \frac{x}{2} \right) - \frac{2}{3} + \frac{1}{4} \left( 10 - \frac{5x}{3} \right) \\
 \Rightarrow \frac{20+x}{4} &= \frac{1}{3} \left( \frac{4-x}{2} \right) - \frac{2}{3} + \frac{1}{4} \left( \frac{30-5x}{3} \right) \\
 \Rightarrow \frac{20+x}{4} &= \frac{4-x}{6} - \frac{2}{3} + \frac{30-5x}{12}
 \end{aligned}$$

$$mcm = 12$$

$$\begin{aligned}
 3(20+x) &= 2(4-x) - 8 + 30 - 5x \\
 60 + 3x &= 8 - 2x - 8 + 30 - 5x \\
 60 + 3x &= -7x + 30 \\
 10x &= -30 \\
 x &= -3
 \end{aligned}$$

$$\begin{aligned}
 30. \quad \frac{5(x+2)}{12} + \frac{4}{9} - \frac{22-x}{36} &= 3x - 20 - \frac{8-x}{12} - \frac{20-3x}{18} \quad mcm = 36 \\
 15(x+2) + 16 - 22 + x &= 108x - 720 - 3(8-x) - 2(20-3x) \\
 15x + 30 - 6 + x &= 108x - 720 - 24 + 3x - 40 + 6x \\
 16x + 24 &= 117x - 784 \\
 -101x &= -808 \\
 x &= 8
 \end{aligned}$$

$$\begin{aligned}
 31. \quad \left( 3 - \frac{x}{2} \right) - \left( 1 - \frac{x}{3} \right) &= 7 - \left( x - \frac{x}{2} \right) \\
 \Rightarrow \frac{6-x}{2} - \frac{3-x}{3} &= 7 - \frac{x}{2} \quad mcm = 6 \\
 3(6-x) - 2(3-x) &= 42 - 3x \\
 18 - 3x - 6 + 2x &= 42 - 3x \\
 12 - x &= 42 - 3x \\
 2x &= 30 \\
 x &= 15
 \end{aligned}$$

$$\begin{aligned}
 32. \quad (x+3)(x-3) - x^2 - \frac{5}{4} &= \left( x - \frac{x}{5} \right) - \left( 3x - \frac{3}{4} \right) \\
 \Rightarrow x^2 - 9 - x^2 - \frac{5}{4} &= \frac{4x}{5} - \frac{12x-3}{4} \\
 \Rightarrow -\frac{41}{4} &= \frac{16x-60x+15}{20} \\
 \Rightarrow -\frac{41}{4} &= \frac{-44x+15}{20} \quad mcm = 20 \\
 -205 &= -44x + 15 \\
 -220 &= -44x \\
 5 &= x
 \end{aligned}$$

$$\begin{aligned}
 33. \quad 2x - \left( 2x - \frac{3x-1}{8} \right) &= \frac{2}{3} \left( \frac{x+2}{6} \right) - \frac{1}{4} \\
 \Rightarrow 2x - \frac{16x-3x+1}{8} &= \frac{2x+4}{18} - \frac{1}{4} \quad mcm = 72 \\
 144x - 9(13x+1) &= 4(2x+4) - 18 \\
 144x - 117x - 9 &= 8x + 16 - 18 \\
 27x - 9 &= 8x - 2 \\
 19x &= 7 \\
 x &= \frac{7}{19}
 \end{aligned}$$

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$$\begin{aligned}
 1. \quad \frac{3}{5} + \frac{3}{2x-1} &= 0 \quad mcm = 5(2x-1) \\
 3(2x-1) + 15 &= 0 \\
 6x - 3 + 15 &= 0 \\
 6x &= -12 \\
 x &= -2
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \frac{2}{4x-1} &= \frac{3}{4x+1} \quad mcm = 16x^2 - 1 \\
 2(4x+1) &= 3(4x-1) \\
 8x + 2 &= 12x - 3 \\
 -4x &= -5 \\
 x &= \frac{5}{4}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \frac{5}{x^2-1} &= \frac{1}{x-1} \quad mcm = x^2 - 1 \\
 5 &= x + 1 \\
 4 &= x
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \frac{3}{x+1} - \frac{1}{x^2-1} &= 0 \quad mcm = x^2 - 1 \\
 3(x-1) - 1 &= 0 \\
 3x - 3 - 1 &= 0 \\
 3x &= 4 \\
 x &= \frac{4}{3}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \frac{5x+8}{3x+4} &= \frac{5x+2}{3x-4} \quad mcm = (3x+4)(3x-4) \\
 (3x-4)(5x+8) &= (5x+2)(3x+4) \\
 15x^2 + 4x - 32 &= 15x^2 + 26x + 8 \\
 -26x + 4x &= 8 + 32 \\
 -22x &= 40 \\
 x &= -\frac{40}{22} \\
 x &= -\frac{20}{11}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \frac{10x^2-5x+8}{5x^2+9x-19} &= 2 \quad mcm = 5x^2 + 9x - 19 \\
 10x^2 - 5x + 8 &= 2(5x^2 + 9x - 19) \\
 10x^2 - 5x + 8 &= 10x^2 + 18x - 38 \\
 -5x - 18x &= -38 - 8 \\
 -23x &= -46 \\
 x &= \frac{-46}{-23} \\
 x &= 2
 \end{aligned}$$

$$7. \frac{1}{3x-3} + \frac{1}{4x+4} = \frac{1}{12x-12} \quad mcm=12(x^2-1)$$

$$4(x+1)+3(x-1)=x+1$$

$$4x+4+3x-3=x+1$$

$$7x-x=0$$

$$6x=0$$

$$x=0$$

$$8. \frac{x}{4} - \frac{x^2-8x}{4x-5} = \frac{7}{4} \quad mcm=4(4x-5)$$

$$x(4x-5)-4(x^2-8x)=7(4x-5)$$

$$4x^2-5x-4x^2+32x=28x-35$$

$$27x=28x-35$$

$$-x=-35$$

$$x=35$$

$$9. \frac{2x-9}{10} + \frac{2x-3}{2x-1} = \frac{x}{5} \quad mcm=10(2x-1)$$

$$(2x-9)(2x-1)+10(2x-3)=2x(2x-1)$$

$$4x^2-20x+9+20x-30=4x^2-2x$$

$$-21=-2x$$

$$\frac{-21}{-2}=x$$

$$10\frac{1}{2}=x$$

$$10. \frac{(3x-1)^2}{x-1} = \frac{18x-1}{2} \quad mcm=2(x-1)$$

$$2(9x^2-6x+1)=(x-1)(18x-1)$$

$$18x^2-12x+2=18x^2-19x+1$$

$$-12x+19x=1-2$$

$$7x=-1$$

$$x=-\frac{1}{7}$$

$$11. \frac{2x+7}{5x+2} - \frac{2x-1}{5x-4} = 0 \quad mcm=(5x+2)(5x-4)$$

$$(2x+7)(5x-4)-(2x-1)(5x+2)=0$$

$$10x^2+27x-28-10x^2+x+2=0$$

$$28x=26$$

$$x=\frac{13}{14}$$

$$12. \frac{(5x-2)(7x+3)}{7x(5x-1)} - 1 = 0 \quad mcm=7x(5x-1)$$

$$(5x-2)(7x+3)-7x(5x-1)=0$$

$$35x^2+x-6-35x^2+7x=0$$

$$8x=6$$

$$x=\frac{3}{4}$$

$$13. \frac{3}{x-4} = \frac{2}{x-3} + \frac{8}{x^2-7x+12} \quad mcm=x^2-7x+12$$

$$3(x-3)=2(x-4)+8$$

$$3x-9=2x-8+8$$

$$3x-2x=9$$

$$x=9$$

$$14. \frac{6x-1}{18} - \frac{3(x+2)}{5x-6} = \frac{1+3x}{9} \quad mcm=18(5x-6)$$

$$(6x-1)(5x-6)-54(x+2)=2(1+3x)(5x-6)$$

$$30x^2-41x+6-54x-108=-26x-12+30x^2$$

$$-95x-102=-26x-12$$

$$-69x=90$$

$$x=-\frac{90}{69}$$

$$x=-1\frac{7}{23}$$

$$15. \frac{5}{1+x} - \frac{3}{1-x} - \frac{6}{1-x^2} = 0 \quad mcm=1-x^2$$

$$5(1-x)-3(1+x)-6=0$$

$$5-5x-3-3x-6=0$$

$$-8x-4=0$$

$$-8x=4$$

$$x=-\frac{1}{2}$$

$$16. \frac{1+2x}{1+3x} - \frac{1-2x}{1-3x} = -\frac{3x-14}{1-9x^2} \quad mcm=1-9x^2$$

$$(1+2x)(1-3x)-(1-2x)(1+3x)=-3x+14$$

$$1-x-6x^2-1-x+6x^2=-3x+14$$

$$-2x=-3x+14$$

$$x=14$$

$$17. \frac{3x-1}{x^2+7x+12} = \frac{1}{2x+6} + \frac{7}{6x+24}$$

$$\Rightarrow x^2+7x+12=(x+3)(x+4)$$

$$\Rightarrow 2x+6=2(x+3)$$

$$\Rightarrow 6x+24=6(x+4) \quad mcm=6(x+3)(x+4)$$

$$6(3x-1)=3(x+4)+7(x+3)$$

$$18x-6=3x+12+7x+21$$

$$18x-6=10x+33$$

$$8x=39$$

$$x=\frac{39}{8}$$

$$x=4\frac{7}{8}$$

$$18. \frac{1}{(x-1)^2} - \frac{3}{2x-2} = -\frac{3}{2x+2} \quad mcm = 2(x-1)^2(x+1)$$

$$2(x+1) - 3(x-1)(x+1) = -3(x-1)^2$$

$$2x+2-3x^2+3=-3x^2+6x-3$$

$$2x+5=6x-3$$

$$-4x=-8$$

$$x=2$$

$$19. \frac{5x+13}{15} - \frac{4x+5}{5x-15} = \frac{x}{3} \quad mcm = 15(x-3)$$

$$(5x+13)(x-3) - 3(4x+5) = 5x(x-3)$$

$$5x^2 - 2x - 39 - 12x - 15 = 5x^2 - 15x$$

$$-14x - 54 = -15x$$

$$x = 54$$

$$20. \frac{2x-1}{2x+1} - \frac{x-4}{3x-2} = \frac{2}{3} \quad mcm = 3(2x+1)(3x-2)$$

$$3(2x-1)(3x-2) - 3(x-4)(2x+1) = 2(2x+1)(3x-2)$$

$$18x^2 - 21x + 6 - 6x^2 + 21x + 12 = 12x^2 - 2x - 4$$

$$18 = -2x - 4$$

$$22 = -2x$$

$$-11 = x$$

$$21. \frac{4x+3}{2x-5} - \frac{3x+8}{3x-7} = 1 \quad mcm = (2x-5)(3x-7)$$

$$(4x+3)(3x-7) - (3x+8)(2x-5) = (2x-5)(3x-7)$$

$$12x^2 - 19x - 21 - 6x^2 - x + 40 = 6x^2 - 29x + 35$$

$$-20x + 19 = -29x + 35$$

$$9x = 16$$

$$x = \frac{16}{9}$$

$$x = 1\frac{7}{9}$$

$$22. \frac{10x-7}{15x+3} = \frac{3x+8}{12} - \frac{5x^2-4}{20x+4} \quad mcm = 12(5x+1)$$

$$4(10x-7) = (3x+8)(5x+1) - 3(5x^2-4)$$

$$40x - 28 = 15x^2 + 43x + 8 - 15x^2 + 12$$

$$40x - 28 = 43x + 20$$

$$-3x = 48$$

$$x = -16$$

$$23. \frac{4x-1}{5} + \frac{x-2}{2x-7} = \frac{8x-3}{10} - 1\frac{3}{10} \quad mcm = 10(2x-7)$$

$$2(4x-1)(2x-7) + 10(x-2) = (8x-3)(2x-7) - 13(2x-7)$$

$$16x^2 - 60x + 14 + 10x - 20 = 16x^2 - 62x + 21 - 26x + 91$$

$$-50x - 6 = -88x + 112$$

$$38x = 118$$

$$x = \frac{118}{38}$$

$$x = 3\frac{2}{19}$$

$$24. \frac{1}{x-1} - \frac{2}{x-2} = \frac{3}{2x-2} - \frac{2\frac{1}{3}}{2x-4}$$

$$\frac{1}{x-1} - \frac{2}{x-2} = \frac{3}{2x-2} - \frac{\frac{7}{3}}{2x-4}$$

$$\frac{1}{x-1} - \frac{2}{x-2} = \frac{3}{2x-2} - \frac{7}{6x-12}$$

$$mcm = 6(x-1)(x-2)$$

$$6(x-2) - 12(x-1) = 9(x-2) - 7(x-1)$$

$$6x - 12 - 12x + 12 = 9x - 18 - 7x + 7$$

$$-6x = 2x - 11$$

$$-8x = -11$$

$$x = \frac{-11}{-8} = 1\frac{3}{8}$$

$$25. \frac{1}{x+3} - \frac{2}{5x-20} = \frac{1\frac{1}{2}}{3x-12} - \frac{2}{x+3}$$

$$mcm = 15(x-4)(x+3)$$

$$15(x-4) - 6(x+3) = \frac{15(x+3)}{2} - 30(x-4)$$

$$15x - 60 - 6x - 18 = \frac{15x+45}{2} - 30x - 120$$

$$2(9x - 78) = 15x + 45 - 2(30x - 120)$$

$$18x - 156 = 15x + 45 - 60x + 240$$

$$18x - 156 = -45x + 285$$

$$63x = 441$$

$$x = 7$$

$$26. \frac{1}{6-2x} - \frac{4}{5-5x} = \frac{10}{12-4x} - \frac{3}{10-10x}$$

$$mcm = 20(1-x)(3-x)$$

$$10(1-x) - 16(3-x) = 50(1-x) - 6(3-x)$$

$$10 - 10x - 48 + 16x = 50 - 50x - 18 + 6x$$

$$6x - 38 = -44x + 32$$

$$50x = 70$$

$$x = \frac{7}{5}$$

$$x = 1\frac{2}{5}$$

$$27. \frac{2}{3} - \frac{6x^2}{9x^2-1} = \frac{2}{3x-1} \quad mcm = 3(9x^2-1)$$

$$2(9x^2-1) - 18x^2 = 6(3x-1)$$

$$18x^2 - 2 - 18x^2 = 18x + 6$$

$$-2 = 18x + 6$$

$$-8 = 18x$$

$$-\frac{4}{9} = x$$

$$28. \frac{5x^2 - 27x}{5x + 3} - \frac{1}{x} = x - 6 \quad mcm = x(5x + 3)$$

$$\begin{aligned} x(5x^2 - 27x) - (5x + 3) &= x(5x + 3)(x - 6) \\ 5x^3 - 27x^2 - 5x - 3 &= 5x^3 - 27x^2 - 18x \\ -5x - 3 &= -18x \\ 13x &= 3 \\ x &= \frac{3}{13} \end{aligned}$$

$$29. \frac{4x + 1}{4x - 1} - \frac{6}{16x^2 - 1} = \frac{4x - 1}{4x + 1} \quad mcm = 16x^2 - 1$$

$$\begin{aligned} (4x + 1)^2 - 6 &= (4x - 1)^2 \\ 16x^2 + 8x + 1 - 6 &= 16x^2 - 8x + 1 \\ 16x - 5 &= 1 \\ 16x &= 6 \\ x &= \frac{3}{8} \end{aligned}$$

$$30. 3\left(\frac{x-1}{x+1}\right) + 2\left(\frac{x+1}{x-4}\right) = \frac{5x(x-1)}{x^2 - 3x - 4}$$

$$\begin{aligned} mcm &= x^2 - 3x - 4 \\ 3(x-1)(x-4) + 2(x+1)^2 &= 5x(x-1) \\ 3x^2 - 15x + 12 + 2x^2 + 4x + 2 &= 5x^2 - 5x \\ -11x + 14 &= -5x \\ -6x &= -14 \\ x &= \frac{-7}{-3} \\ x &= 2\frac{1}{3} \end{aligned}$$

$$31. 2\left(\frac{x+2}{x-2}\right) - 3\left(\frac{x-2}{2x+3}\right) = \frac{x^2 + 78}{2x^2 - x - 6}$$

$$\begin{aligned} mcm &= (x-2)(2x+3) \\ 2(x+2)(2x+3) - 3(x-2)^2 &= x^2 + 78 \\ 4x^2 + 14x + 12 - 3x^2 + 12x - 12 &= x^2 + 78 \\ 26x &= 78 \\ x &= 3 \end{aligned}$$

$$32. \frac{1}{x^2 + 3x - 28} - \frac{1}{x^2 + 12x + 35} = \frac{3}{x^2 + x - 20}$$

$$\begin{aligned} \Rightarrow x^2 + 3x - 28 &= (x+7)(x-4) \\ \Rightarrow x^2 + 12x + 35 &= (x+7)(x+5) \\ \Rightarrow x^2 + x - 20 &= (x+5)(x-4) \\ mcm &= (x+7)(x+5)(x-4) \\ x+5 - (x-4) &= 3(x+7) \\ x+5 - x+4 &= 3x+21 \\ 9 &= 3x+21 \\ -12 &= 3x \\ -4 &= x \end{aligned}$$

$$33. \frac{x-2}{x^2 + 8x + 7} = \frac{2x-5}{x^2 - 49} - \frac{x-2}{x^2 - 6x - 7}$$

$$\begin{aligned} \Rightarrow x^2 + 8x + 7 &= (x+7)(x+1) \\ \Rightarrow x^2 - 49 &= (x+7)(x-7) \\ \Rightarrow x^2 - 6x - 7 &= (x-7)(x+1) \\ mcm &= (x+7)(x-7)(x+1) = (x^2 - 49)(x+1) \\ (x-2)(x-7) &= (2x-5)(x+1) - (x-2)(x+7) \\ x^2 - 9x + 14 &= 2x^2 - 3x - 5 - x^2 - 5x + 14 \\ -9x + 14 &= -8x + 9 \\ -x &= -5 \\ x &= 5 \end{aligned}$$

$$34. \frac{4x+5}{15x^2 + 7x - 2} - \frac{2x+3}{12x^2 - 7x - 10} - \frac{2x-5}{20x^2 - 29x + 5} = 0$$

$$\begin{aligned} \Rightarrow 15x^2 + 7x - 2 &= (3x+2)(5x-1) \\ \Rightarrow 12x^2 - 7x - 10 &= (4x-5)(3x+2) \\ \Rightarrow 20x^2 - 29x + 5 &= (4x-5)(5x-1) \\ mcm &= (3x+2)(5x-1)(4x-5) \\ (4x+5)(4x-5) - (2x+3)(5x-1) - (2x-5)(3x+2) &= 0 \\ 16x^2 - 25 - 10x^2 - 13x + 3 - 6x^2 + 11x + 10 &= 0 \\ -2x - 12 &= 0 \\ -2x &= 12 \\ x &= -6 \end{aligned}$$

$$35. \frac{7}{2x+1} - \frac{3}{x+4} = \frac{2}{x+1} - \frac{3(x+1)}{2x^2 + 9x + 4}$$

$$\begin{aligned} mcm &= (2x+1)(x+4)(x+1) \\ 7(x+4)(x+1) - 3(2x+1)(x+1) &= 2(2x+1)(x+4) - 3(x+1)^2 \\ 7x^2 + 35x + 28 - 6x^2 - 9x - 3 &= 4x^2 + 18x + 8 - 3x^2 - 6x - 3 \\ 26x + 25 &= 12x + 5 \\ 14x &= -20 \\ x &= -\frac{10}{7} \\ x &= -1\frac{3}{7} \end{aligned}$$

$$36. \frac{(x+3)^2}{(x-3)^2} = \frac{x-1}{x+1} + \frac{2(7x+1)}{x^2 - 2x - 3}$$

$$\begin{aligned} mcm &= (x-3)^2(x+1) \\ (x+3)^2(x+1) &= (x-3)^2(x-1) + 2(7x+1)(x-3) \\ x^3 + 7x^2 + 15x + 9 &= x^3 - 7x^2 + 15x - 9 + 14x^2 - 40x - 6 \\ 9 &= -15 - 40x \\ 24 &= -40x \\ -\frac{3}{5} &= x \end{aligned}$$

$$37. \frac{x-4}{x+5} - \frac{x+1}{x-2} = -\frac{12(x+3)}{(x+5)^2} \quad mcm = (x+5)^2(x-2)$$

$$(x+5)(x-2)(x-4) - (x+5)^2(x+1) = -12(x+3)(x-2)$$

$$x^3 - x^2 - 22x + 40 - x^3 - 11x^2 - 35x - 25 = -12x^2 - 12x + 72$$

$$-57x + 15 = -12x + 72$$

$$-45x = 57$$

$$x = -\frac{57}{45}$$

$$x = -1\frac{4}{15}$$

$$38. \frac{x-3}{x-4} - \frac{x-2}{x-3} = \frac{x+2}{x+1} - \frac{x+3}{x+2} \quad mcm = (x-4)(x-3)(x+1)(x+2)$$

$$(x-3)^2(x+1)(x+2) - (x^2-4)(x-4)(x+1) = (x+2)^2(x-4)(x-3) - (x^2-9)(x-4)(x+1)$$

$$x^4 - 3x^3 - 7x^2 + 15x + 18 - x^4 + 3x^3 + 8x^2 - 12x - 16 = x^4 - 3x^3 - 12x^2 + 20x + 48 - x^4 + 3x^3 + 13x^2 - 27x - 36$$

$$3x + 2 = -7x + 12$$

$$10x = 10$$

$$x = 1$$

$$39. \frac{x+6}{x+2} - \frac{x+1}{x-3} = \frac{x-5}{x-1} - \frac{x}{x+4} \quad mcm = (x+2)(x-3)(x-1)(x+4)$$

$$(x+6)(x-3)(x-1)(x+4) - (x^2-1)(x+2)(x+4) = (x-5)(x+2)(x-3)(x+4) - x(x+2)(x-3)(x-1)$$

$$x^4 + 6x^3 - 13x^2 - 66x + 72 - x^4 - 6x^3 - 7x^2 + 6x + 8 = x^4 - 2x^3 - 25x^2 + 26x + 120 - x^4 + 2x^3 + 5x^2 - 6x$$

$$-60x + 80 = 20x + 120$$

$$-80x = 40$$

$$x = -\frac{1}{2}$$

## EJERCICIO 143

$$1. a(x+1)=1$$

$$x+1=\frac{1}{a}$$

$$-1+x+1=\frac{1}{a}-1$$

$$x=\frac{1-a}{a}$$

$$4. 3(2a-x)+ax=a^2+9$$

$$6a-3x+ax=a^2+9$$

$$x(a-3)=a^2+9-6a$$

$$x(a-3)=(a-3)^2$$

$$x=a-3$$

$$5. a(x+b)+x(b-a)=2b(2a-x)$$

$$ax+ba+xb-ax=4ab-2bx$$

$$xb+2bx=4ab-ab$$

$$3bx=3ab$$

$$x=\frac{3ab}{3b}$$

$$x=a$$

$$6. (x-a)^2 - (x+a)^2 = a(a-7x)$$

$$x^2 - 2ax + a^2 - x^2 - 2ax - a^2 = a^2 - 7ax$$

$$-4ax + 7ax = a^2$$

$$3ax = a^2$$

$$x = \frac{a^2}{3a}$$

$$x = \frac{a}{3}$$

$$7. ax - a(a+b) = -x - (1+ab)$$

$$ax - a^2 - ab = -x - 1 - ab$$

$$ax + x = a^2 - 1$$

$$x(a+1) = (a+1)(a-1)$$

$$x = a - 1$$

$$8. a^2(a-x) - b^2(x-b) = b^2(x-b)$$

$$a^3 - a^2x - b^2x + b^3 = b^2x - b^3$$

$$-a^2x - 2b^2x = -a^3 - 2b^3$$

$$a^2x + 2b^2x = a^3 + 2b^3$$

$$x(a^2 + 2b^2) = a^3 + 2b^3$$

$$x = \frac{a^3 + 2b^3}{a^2 + 2b^2}$$

$$2. ax - 4 = bx - 2$$

$$ax - bx = 4 - 2$$

$$x(a-b) = 2$$

$$x = \frac{2}{a-b}$$

$$3. ax + b^2 = a^2 - bx$$

$$ax + bx = a^2 - b^2$$

$$x(a+b) = (a+b)(a-b)$$

$$x = a - b$$

$$(x+a)(x-b) - (x+b)(x-2a) = b(a-2) + 3a$$

$$x^2 - bx + ax - ab - x^2 + 2ax - bx + 2ab = ab - 2b + 3a$$

$$-2bx + 3ax + ab = ab - 2b + 3a$$

$$x(3a - 2b) = 3a - 2b$$

$$9.$$

$$x = 1$$

$$\begin{aligned}
10. \quad x^2 + a^2 &= (a+x)^2 - a(a-1) \\
x^2 + a^2 &= a^2 + 2ax + x^2 - a^2 + a \\
a^2 &= 2ax + a \\
\frac{a(a-1)}{2a} &= x \\
\frac{a-1}{2} &= x
\end{aligned}$$

$$\begin{aligned}
11. \quad m(n-x) - m(n-1) &= m(mx-a) \\
mn - mx - mn + m &= m^2x - am \\
-mx - m^2x &= -am - m \\
-xm(1+m) &= -m(a+1) \\
x &= \frac{a+1}{m+1}
\end{aligned}$$

$$\begin{aligned}
12. \quad x - a + 2 &= 2ax - 3(a+x) - 2(a-5) \\
x - a + 2 &= 2ax - 3a - 3x - 2a + 10 \\
x - a &= 2ax - 5a - 3x + 8 \\
x + 4a - 8 &= 2ax - 3x \\
4(a-2) &= 2ax - 4x \\
4(a-2) &= 2x(a-2) \\
2 &= x
\end{aligned}$$

$$\begin{aligned}
13. \quad a(x-a) - 2bx &= b(b-2a-x) \\
ax - a^2 - 2bx &= b^2 - 2ab - bx \\
ax - bx &= b^2 - 2ab + a^2 \\
x(a-b) &= (b-a)(b-a) \\
x &= -\frac{(a-b)(b-a)}{a-b} \\
x &= -(b-a) \\
x &= a-b
\end{aligned}$$

$$\begin{aligned}
14. \quad ax + bx &= (x+a-b)^2 - (x-2b)(x+2a) \\
x(a+b) &= x^2 + 2ax - 2bx + a^2 - 2ab + b^2 - x^2 - 2ax + 2bx + 4ab \\
x(a+b) &= a^2 + 2ab + b^2 \\
x(a+b) &= (a+b)^2 \\
x &= a+b
\end{aligned}$$

$$\begin{aligned}
15. \quad x(a+b) - 3 - a(a-2) &= 2(x-1) - x(a-b) \\
ax + bx - 3 - a^2 + 2a &= 2x - 2 - ax + bx \\
2ax - 2x &= a^2 - 2a + 1 \\
2x(a-1) &= (a-1)^2 \\
x &= \frac{a-1}{2}
\end{aligned}$$

$$\begin{aligned}
16. \quad (m+4x)(3m+x) &= (2x-m)^2 + m(15x-m) \\
3m^2 + 13mx + 4x^2 &= 4x^2 - 4mx + m^2 + 15mx - m^2 \\
2mx &= -3m^2 \\
x &= -\frac{3m^2}{2m} \\
x &= -\frac{3m}{2}
\end{aligned}$$

$$\begin{aligned}
17. \quad a^2(a-x) - a^2(a+1) - b^2(b-x) - b(1-b^2) + a(1+a) &= 0 \\
a^3 - a^2x - a^3 - a^2 - b^3 + b^2x - b + b^3 + a + a^2 &= 0 \\
-a^2x + b^2x - b + a^2 &= 0 \\
x(b^2 - a^2) - (b-a) &= 0 \\
x(b-a)(a+b) &= b-a \\
x(a+b) &= 1 \\
x &= \frac{1}{a+b}
\end{aligned}$$

$$\begin{aligned}
18. \quad (ax-b)^2 &= (bx-a)(a-x) - x^2(b-a^2) + a^2 + b(1-2b) \\
a^2x^2 - 2abx + b^2 &= abx + bx^2 - a^2 - ax - bx^2 + a^2x^2 + a^2 + b - 2b^2 \\
ax(1-3b) &= b(1-3b) \\
x &= \frac{b}{a}
\end{aligned}$$

$$\begin{aligned}
19. \quad (x+b)^2 - (x-a)^2 - (a+b)^2 &= 0 \\
x^2 + 2bx + b^2 - x^2 + 2ax - a^2 - a^2 - 2ab - b^2 &= 0 \\
2bx + 2ax - 2a^2 - 2ab &= 0 \\
2x(b+a) - 2a(a+b) &= 0 \\
2(x-a)(a+b) &= 0 \\
2x &= 2a \\
x &= a
\end{aligned}$$

$$\begin{aligned}
20. \quad (x+m)^3 - 12m^3 &= -(x-m)^3 + 2x^3 \\
x^3 + 3x^2m + 3xm^2 + m^3 - 12m^3 &= -x^3 + 3x^2m - 3xm^2 + m^3 + 2x^3 \\
2x^3 + 6xm^2 - 2x^3 &= 12m^3 \\
6xm^2 &= 12m^3 \\
x &= \frac{12m^3}{6m^2} \\
x &= 2m
\end{aligned}$$

## EJERCICIO 144

$$1. \frac{m}{x} - \frac{1}{m} = \frac{2}{m} \quad mcm = mx$$

$$m^2 - x = 2x$$

$$m^2 = 3x$$

$$\frac{m^2}{3} = x$$

$$2. \frac{a}{x} + \frac{b}{2} = \frac{4a}{x} \quad mcm = 2x$$

$$2a + bx = 8a$$

$$bx = 6a$$

$$x = \frac{6a}{b}$$

$$3. \frac{x}{2a} - \frac{1-x}{a^2} = \frac{1}{2a} \quad mcm = 2a^2$$

$$ax - 2(1-x) = a$$

$$ax - 2 + 2x = a$$

$$x(a+2) = a+2$$

$$x = \frac{a+2}{a+2}$$

$$x = 1$$

$$4. \frac{m}{x} + \frac{n}{m} = \frac{n}{x} + 1 \quad mcm = mx$$

$$m^2 + nx = mn + mx$$

$$m^2 - mn = mx - nx$$

$$m(m-n) = x(m-n)$$

$$m = x$$

$$5. \frac{a-1}{a} + \frac{1}{2} = \frac{3a-2}{x}$$

$$mcm = 2ax$$

$$2x(a-1) + ax = 2a(3a-2)$$

$$2ax - 2x + ax = 6a^2 - 4a$$

$$3ax - 2x = 2a(3a-2)$$

$$x(3a-2) = 2a(3a-2)$$

$$x = 2a$$

$$6. \frac{a-x}{a} - \frac{b-x}{b} = \frac{2(a-b)}{ab}$$

$$mcm = ab$$

$$b(a-x) - a(b-x) = 2(a-b)$$

$$ab - bx - ab + ax = 2(a-b)$$

$$x(a-b) = 2(a-b)$$

$$x = 2$$

$$7. \frac{x-3a}{a^2} - \frac{2a-x}{ab} = -\frac{1}{a} \quad mcm = a^2b$$

$$b(x-3a) - a(2a-x) = -ab$$

$$bx - 3ab - 2a^2 + ax = -ab$$

$$bx + ax = 2ab + 2a^2$$

$$x(a+b) = 2a(a+b)$$

$$x = 2a$$

$$8. \frac{x+m}{m} - \frac{x+n}{n} = \frac{m^2+n^2}{mn} - 2 \quad mcm = mn$$

$$n(x+m) - m(x+n) = m^2 + n^2 - 2mn$$

$$xn + mn - mx - mn = m^2 - 2mn + n^2$$

$$x(n-m) = (m-n)^2$$

$$x(n-m) = -(n-m)(m-n)$$

$$x = -(m-n)$$

$$x = n-m$$

$$9. \frac{x-b}{a} = 2 - \frac{x-a}{b} \quad mcm = ab$$

$$b(x-b) = 2ab - a(x-a)$$

$$bx - b^2 = 2ab - ax + a^2$$

$$bx + ax = a^2 + 2ab + b^2$$

$$x(a+b) = (a+b)^2$$

$$x = a+b$$

$$10. \frac{4x}{2a+b} - 3 = -\frac{3}{2} \quad mcm = 2(2a+b)$$

$$8x - 6(2a+b) = -3(2a+b)$$

$$8x - 12a - 6b = -6a - 3b$$

$$8x = 6a + 3b$$

$$x = \frac{3(2a+b)}{8}$$

$$11. \frac{2a+3x}{x+a} = \frac{2(6x-a)}{4x+a} \quad mcm = (x+a)(4x+a)$$

$$(2a+3x)(a+4x) = (2x+2a)(6x-a)$$

$$2a^2 + 11ax + 12x^2 = 12x^2 + 10ax - 2a^2$$

$$ax = -4a^2$$

$$x = -4a$$

$$12. \frac{2(x-c)}{4x-b} = \frac{2x+c}{4(x-b)} \quad mcm = 4(4x-b)(x-b)$$

$$8(x-c)(x-b) = (2x+c)(4x-b)$$

$$8x^2 - 8xb - 8xc + 8bc = 8x^2 - 2xb + 4xc - bc$$

$$-6xb - 12xc = -9bc$$

$$-6x(b+2c) = -9bc$$

$$x = \frac{3bc}{2(b+2c)}$$



$$13. \frac{1}{n} - \frac{m}{x} = \frac{1}{mn} - \frac{1}{x} \quad mcm = xmn$$

$$xm - m^2n = x - mn$$

$$xm - x = m^2n - mn$$

$$x(m-1) = mn(m-1)$$

$$x = mn$$

$$14. \frac{(x-2b)(2x+a)}{(x-a)(a-2b+x)} = 2 \quad mcm = (x-a)(a-2b+x)$$

$$(x-2b)(2x+a) = 2(x-a)(a-2b+x)$$

$$2x^2 + ax - 4bx - 2ab = -4bx + 2x^2 - 2a^2 + 4ab$$

$$ax = 6ab - 2a^2$$

$$ax = 2a(3b-a)$$

$$x = 2(3b-a)$$

$$15. \frac{x+m}{x-n} = \frac{n+x}{m+x} \quad mcm = (m+x)(x-n)$$

$$(x+m)^2 = (x+n)(x-n)$$

$$x^2 + 2mx + m^2 = x^2 - n^2$$

$$2mx = -m^2 - n^2$$

$$x = -\frac{m^2 + n^2}{2m}$$

$$16. \frac{x(2x+3b)(x+b)}{x+3b} = 2x^2 - bx + b^2 \quad mcm = x+3b$$

$$x(2x+3b)(x+b) = (x+3b)(2x^2 - bx + b^2)$$

$$2x^3 + 5x^2b + 3b^2x = 2x^3 + 5bx^2 - 2b^2x + 3b^3$$

$$3b^2x + 2b^2x = 3b^3$$

$$x = \frac{3b}{5}$$

$$17. \frac{3}{4} \left( \frac{x}{b} + \frac{x}{a} \right) = \frac{1}{3} \left( \frac{x}{b} - \frac{x}{a} \right) + \frac{5a+13b}{12a}$$

$$\Rightarrow \frac{3}{4} \left( \frac{ax+bx}{ab} \right) = \frac{1}{3} \left( \frac{ax-bx}{ab} \right) + \frac{5a+13b}{12a}$$

$$\Rightarrow \frac{3ax+3bx}{4ab} = \frac{ax-bx}{3ab} + \frac{5a+13b}{12a} \quad mcm = 12ab$$

$$3(3ax+3bx) = 4(ax-bx) + b(5a+13b)$$

$$9ax+9bx = 4ax-4bx+5ab+13b^2$$

$$5ax+13bx = 5ab+13b^2$$

$$x(5a+13b) = b(5a+13b)$$

$$x = b$$

$$18. \frac{x+a}{3} = \frac{(x-b)^2}{3x-a} + \frac{3ab-3b^2}{9x-3a} \quad mcm = 3(3x-a)$$

$$(x+a)(3x-a) = 3(x-b)^2 + 3ab - 3b^2$$

$$3x^2 + 2ax - a^2 = 3x^2 - 6bx + 3b^2 + 3ab - 3b^2$$

$$2ax + 6bx = a^2 + 3ab$$

$$2x(a+3b) = a(a+3b)$$

$$x = \frac{a}{2}$$

$$19. \frac{5x+a}{3x+b} = \frac{5x-b}{3x-a} \quad mcm = (3x+b)(3x-a)$$

$$(3x-a)(5x+a) = (5x-b)(3x+b)$$

$$15x^2 - 2ax - a^2 = 15x^2 + 2bx - b^2$$

$$-2ax - 2bx = a^2 - b^2$$

$$-2x(a+b) = (a+b)(a-b)$$

$$x = \frac{b-a}{2}$$

$$20. \frac{x+a}{x-a} - \frac{x-a}{x+a} = \frac{a(2x+ab)}{x^2-a^2} \quad mcm = (x-a)(x+a)$$

$$(x+a)^2 - (x-a)^2 = a(2x+ab)$$

$$x^2 + 2ax + a^2 - x^2 + 2ax - a^2 = 2ax + a^2b$$

$$4ax = 2ax + a^2b$$

$$2ax = a^2b$$

$$x = \frac{ab}{2}$$

$$21. \frac{2x-3a}{x+4a} - 2 = \frac{11a}{x^2-16a^2} \quad mcm = x^2-16a^2$$

$$(2x-3a)(x-4a) - 2(x^2-16a^2) = 11a$$

$$2x^2 - 11ax + 12a^2 - 2x^2 + 32a^2 = 11a$$

$$-11ax + 44a^2 = 11a$$

$$-11ax = 11a - 44a^2$$

$$-11ax = 11a(1-4a)$$

$$x = -(1-4a)$$

$$x = 4a-1$$

$$22. \frac{1}{x+a} + \frac{x^2}{a^2+ax} = \frac{x+a}{a} \quad mcm = a(x+a)$$

$$a+x^2 = (x+a)^2$$

$$a+x^2 = x^2 + 2ax + a^2$$

$$a-a^2 = 2ax$$

$$\frac{a(1-a)}{2a} = x$$

$$\frac{1-a}{2} = x$$

$$23. \frac{2(a+x)}{b} - \frac{3(b+x)}{a} = \frac{6(a^2-2b^2)}{ab} \quad mcm = ab$$

$$2a(a+x) - 3b(b+x) = 6(a^2-2b^2)$$

$$2a^2 + 2ax - 3b^2 - 3bx = 6a^2 - 12b^2$$

$$2ax - 3bx = 4a^2 - 9b^2$$

$$x(2a-3b) = (2a-3b)(2a+3b)$$

$$x = 2a+3b$$

$$24. m(n-x) - (m-n)(m+x) = n^2 - \frac{1}{n}(2mn^2 - 3m^2n) \quad mcm = n$$

$$\begin{aligned} nm(n-x) - n(m-n)(m+x) &= n^3 - (2mn^2 - 3m^2n) \\ mn^2 - mnx - m^2n + mn^2 - mnx + n^2x &= n^3 - 2mn^2 + 3m^2n \\ -2mnx + n^2x &= -4mn^2 + 4m^2n + n^3 \\ nx(n-2m) &= n(n^2 - 4mn + 4m^2) \\ x(n-2m) &= (n-2m)^2 \\ x &= n-2m \end{aligned}$$

## EJERCICIO 145

1.  $x \rightarrow N^\circ \text{ buscado}$

$$\begin{aligned} x - \frac{3x}{8} &= 2x - 11 \\ 8x - 3x &= 8(2x - 11) \\ 5x &= 16x - 88 \\ -11x &= -88 \\ x &= \frac{-88}{-11} \\ x &= 8 \end{aligned}$$

2.  $x \rightarrow N^\circ \text{ buscado}$

$$\begin{aligned} x + \frac{5x}{6} &= 3x - 14 \\ 6x + 5x &= 6(3x - 14) \\ 11x &= 18x - 84 \\ -7x &= -84 \\ x &= \frac{-84}{-7} \\ x &= 12 \end{aligned}$$

3.  $x \rightarrow N^\circ \text{ que se resta}$

$$\begin{aligned} 22 - x &= 11 + \frac{6x}{5} \\ 5(22 - x) &= 55 + 6x \\ 110 - 5x &= 55 + 6x \\ -11x &= -55 \\ x &= 5 \end{aligned}$$

4.  $x \rightarrow N^\circ \text{ que tiene diferencia}$

$$\begin{aligned} \frac{5}{4}x - \frac{7}{8}x &= 30 \quad mcm = 8 \\ 10x - 7x &= 240 \\ 3x &= 240 \\ x &= 80 \end{aligned}$$

5.  $x \rightarrow N^\circ \text{ buscado}$

$$\begin{aligned} x - 17 &= \frac{3}{5}x - \frac{1}{6}x \quad mcm = 30 \\ 30x - 510 &= 18x - 5x \\ -510 &= -17x \\ 30 &= x \end{aligned}$$

6.  $x \rightarrow N^\circ \text{ buscado}$

$$\begin{aligned} \frac{x}{5} + \frac{3}{8}x - 49 &= 2\left(\frac{1}{6}x - \frac{1}{12}x\right) \\ \Rightarrow \frac{x}{5} + \frac{3}{8}x - 49 &= \frac{x}{6} \quad mcm = 120 \\ 24x + 45x - 5.880 &= 20x \\ 49x &= 5.880 \\ x &= 120 \end{aligned}$$

7.  $x \rightarrow \text{Edad de A}$

$$\begin{aligned} \frac{3x}{5} &\rightarrow \text{Edad de B} \\ x + \frac{3x}{5} - 4 &= 2\left(\frac{3x}{5}\right) \quad mcm = 5 \\ 5x + 3x - 20 &= 6x \\ 2x &= 20 \\ x &= 10 \text{ años} \rightarrow \text{Edad de A} \end{aligned}$$

$$\Rightarrow \frac{3 \cdot 10}{5} = \frac{30}{5} = 6 \text{ años} \rightarrow \text{Edad de B}$$

8.  $x \rightarrow \text{Lo que tiene A}$

$$\begin{aligned} \frac{7x}{8} &\rightarrow \text{Lo que tiene B} \\ x + 90 &= 2\left(\frac{7x}{8}\right) \quad mcm = 8 \end{aligned}$$

$$\begin{aligned} 8x + 720 &= 14x \\ -6x &= -720 \\ x &= \$120 \rightarrow \text{Tiene A} \\ \Rightarrow \frac{7 \cdot 120}{8} &= \$105 \rightarrow \text{Tiene B} \end{aligned}$$

9.  $x \rightarrow \text{Long. pieza}$

$$\begin{aligned} x - \frac{3x}{5} &= 40 \quad mcm = 5 \\ 5x - 3x &= 200 \\ 2x &= 200 \\ x &= 100m \end{aligned}$$

10.  $x \rightarrow \text{Lo que tenía}$

$$\begin{aligned} x - \left(\frac{1}{3}x + \frac{1}{8}x\right) &= 39 \quad mcm = 24 \\ 24x - 8x - 3x &= 936 \\ 13x &= 936 \\ x &= 72 \text{ bs.} \end{aligned}$$

11.  $x \rightarrow N^\circ \text{ buscado}$

$$\begin{aligned} 3x - 48 &= \frac{x}{3} \quad mcm = 3 \\ 9x - 144 &= x \end{aligned}$$

$$\begin{aligned} 8x &= 144 \\ x &= 18 \end{aligned}$$

12.  $x \rightarrow N^\circ \text{ buscado}$

$$\begin{aligned} 4x - 19 &= \frac{x}{2} + 30 \quad mcm = 2 \\ 8x - 38 &= x + 60 \\ 7x &= 98 \\ x &= 14 \end{aligned}$$

13.  $x \rightarrow N^\circ \text{ buscado}$

$$\begin{aligned} 80 - \frac{x}{2} &= x - 10 \quad mcm = 2 \\ 160 - x &= 2x - 20 \\ -3x &= -180 \\ x &= 60 \end{aligned}$$

14.  $x \rightarrow N^\circ \text{ buscado}$

$$\begin{aligned} \frac{7x}{8} - 2 &= \frac{4x}{5} \quad mcm = 40 \\ 35x - 80 &= 32x \\ 3x &= 80 \\ x &= \frac{80}{3} \\ x &= 26\frac{2}{3} \end{aligned}$$

15.  $x \rightarrow \text{Ancho del buque}$

$$\begin{aligned} 800 - 744 &= \frac{8x}{9} \\ 56 &= \frac{8x}{9} \\ 504 &= 8x \\ \frac{504}{8} &= x \\ 63 \text{ Pies} &= x \end{aligned}$$

## EJERCICIO 146

1.  $x \rightarrow N^{\circ} \text{ menor}$   
 $x+1 \rightarrow N^{\circ} \text{ mayor}$   
 $\frac{4(x+1)}{5} = x-4$   
 $\frac{4x+4}{5} = x-4$   
 $4x+4 = 5(x-4)$   
 $4x+4 = 5x-20$   
 $-x = -24$   
 $x = 24 \rightarrow N^{\circ} \text{ menor}$   
 $\Rightarrow x+1$   
 $= 24+1 = 25 \rightarrow N^{\circ} \text{ mayor}$
2.  $x \rightarrow N^{\circ} \text{ menor}$   
 $x+1 \rightarrow N^{\circ} \text{ mayor}$   
 $\frac{7x}{8} - 17 = \frac{3(x+1)}{5} \quad mcm = 40$   
 $35x - 680 = 24(x+1)$   
 $35x - 680 = 24x + 24$   
 $11x = 704$   
 $x = 64 \rightarrow N^{\circ} \text{ menor}$   
 $\Rightarrow x+1$   
 $= 64+1 = 65 \rightarrow N^{\circ} \text{ mayor}$
3.  $x \rightarrow N^{\circ} \text{ menor}$   
 $x+1 \rightarrow N^{\circ} \text{ mayor}$   
 $x-81 = \frac{3x}{4} - \frac{2(x+1)}{5} \quad mcm = 20$   
 $20x - 1.620 = 15x - 8(x+1)$   
 $20x - 1.620 = 15x - 8x - 8$   
 $13x = 1.612$   
 $x = 124 \rightarrow N^{\circ} \text{ menor}$   
 $\Rightarrow x+1$   
 $= 124+1 = 125 \quad x \rightarrow N^{\circ} \text{ mayor}$
4.  $x \rightarrow N^{\circ} \text{ menor}$   
 $x+1 \rightarrow N^{\circ} \text{ mayor}$   
 $\frac{1}{5}(x+1) + \frac{1}{33}x - 8 = \frac{3(x+1)}{20} \quad mcm = 660$   
 $132(x+1) + 20x - 5.280 = 99(x+1)$   
 $132x + 132 + 20x - 5.280 = 99x + 99$   
 $152x - 99x = 5.148 + 99$   
 $53x = 5.247$   
 $x = 99 \rightarrow N^{\circ} \text{ menor}$   
 $\Rightarrow x+1 = 99+1 = 100 \rightarrow N^{\circ} \text{ mayor}$
5.  $2x \rightarrow N^{\circ} \text{ menor}$   
 $2x+2 \rightarrow N^{\circ} \text{ mayor}$   
 $(2x+2)^2 - (2x)^2 = 324$   
 $4x^2 + 8x + 4 - 4x^2 = 324$   
 $8x = 320$   
 $x = 40$   
 $\Rightarrow 2x = 2 \cdot 40 = 80 \rightarrow N^{\circ} \text{ menor}$   
 $\Rightarrow 2x+2 = 80+2 = 82 \rightarrow N^{\circ} \text{ mayor}$
6.  $x \rightarrow \text{Tiene B}$   
 $x+1 \rightarrow \text{Tiene A}$   
 $x-8 = \frac{4(x+1)}{5} - 4 \quad mcm = 5$   
 $5x - 40 = 4x + 4 - 20$   
 $5x - 4x = 40 - 16$   
 $x = \$24 \rightarrow \text{Tiene B}$   
 $\Rightarrow x+1 = 24+1 = \$25 \rightarrow \text{Tiene A}$
7.  $x \rightarrow \text{Gane ayer}$   
 $x+1 \rightarrow \text{Gane hoy}$   
 $x + x+1 - 25 = \frac{2x}{5}$   
 $5(2x-24) = 2x$   
 $10x - 120 = 2x$   
 $8x = 120$   
 $x = \$15 \rightarrow \text{Gane ayer}$   
 $\Rightarrow x+1 = 15+1 = \$16 \rightarrow \text{Gane hoy}$
8.  $x \rightarrow N^{\circ} \text{ menor}$   
 $x+1 \rightarrow N^{\circ} \text{ medio}$   
 $x+2 \rightarrow N^{\circ} \text{ mayor}$   
 $\frac{x}{20} + \frac{x+1}{27} + \frac{x+2}{41} = 9 \quad mcm = 22.140$   
 $1.107x + 820(x+1) + 540(x+2) = 199.260$   
 $1.107x + 820x + 820 + 540x + 1.080 = 199.260$   
 $2.467x = 197.360$   
 $x = 80 \rightarrow N^{\circ} \text{ menor}$   
 $\Rightarrow x+1 = 80+1 = 81 \rightarrow N^{\circ} \text{ medio}$   
 $\Rightarrow x+2 = 80+2 = 82 \rightarrow N^{\circ} \text{ mayor}$
9.  $x \rightarrow N^{\circ} \text{ menor}$   
 $x+1 \rightarrow N^{\circ} \text{ medio}$   
 $x+2 \rightarrow N^{\circ} \text{ mayor}$   
 $\frac{3x}{5} + \frac{5(x+2)}{6} - 31 = x+1 \quad mcm = 30$   
 $18x + 25(x+2) - 930 = 30x + 30$   
 $18x + 25x + 50 - 930 = 30x + 30$   
 $43x - 30x = 880 + 30$   
 $13x = 910$   
 $x = 70 \quad x \rightarrow N^{\circ} \text{ menor}$   
 $\Rightarrow x+1 = 70+1 = 71 \rightarrow N^{\circ} \text{ medio}$   
 $\Rightarrow x+2 = 70+2 = 72 \rightarrow N^{\circ} \text{ mayor}$
10.  $x \rightarrow N^{\circ} \text{ menor}$   
 $x+1 \rightarrow N^{\circ} \text{ medio}$   
 $x+2 \rightarrow N^{\circ} \text{ mayor}$   
 $\frac{3(x+1)}{7} - \frac{3x}{10} - 1 = \frac{1}{11}(x+2) \quad mcm = 770$   
 $330(x+1) - 231x - 770 = 70(x+2)$   
 $330x + 330 - 231x - 770 = 70x + 140$   
 $29x = 580$   
 $x = 20 \rightarrow N^{\circ} \text{ menor}$   
 $\Rightarrow x+1 = 20+1 = 21 \rightarrow N^{\circ} \text{ medio}$   
 $\Rightarrow x+2 = 20+2 = 22 \rightarrow N^{\circ} \text{ mayor}$

11.  $x \rightarrow \text{Edad } B$

$x+2 \rightarrow \text{Edad } A$

$x-2 \rightarrow \text{Edad } C$

$$x+x-2-12=\frac{7(x+2)}{8}$$

$$8(2x-14)=7(x+2)$$

$$16x-112=7x+14$$

$$9x=126$$

$$x=14 \rightarrow \text{Edad } B$$

$$\Rightarrow x+2=14+2=16 \rightarrow \text{Edad } A$$

$$\Rightarrow x-2=14-2=12 \rightarrow \text{Edad } C$$

## EJERCICIO 147

1.  $x \rightarrow N^\circ \text{ mayor}$

$59-x \rightarrow N^\circ \text{ menor}$

$$\frac{x-5}{59-x}=2$$

$$x-5=2(59-x)$$

$$x-5=118-2x$$

$$3x=123$$

$$x=41 \rightarrow N^\circ \text{ mayor}$$

$$\Rightarrow 59-x=59-41=18 \rightarrow N^\circ \text{ menor}$$

2.  $x \rightarrow N^\circ \text{ mayor}$

$436-x \rightarrow N^\circ \text{ menor}$

$$\frac{x-73}{436-x}=2$$

$$x-73=2(436-x)$$

$$x-73=872-2x$$

$$3x=945$$

$$x=315 \rightarrow N^\circ \text{ mayor}$$

$$\Rightarrow 436-x=436-315=121 \rightarrow N^\circ \text{ menor}$$

3.  $x \rightarrow N^\circ \text{ mayor}$

$x-44 \rightarrow N^\circ \text{ menor}$

$$\frac{x-2}{x-44}=3$$

$$x-2=3(x-44)$$

$$x-2=3x-132$$

$$-2x=-130$$

$$x=65 \rightarrow N^\circ \text{ mayor}$$

$$\Rightarrow x-44=65-44=21 \rightarrow N^\circ \text{ menor}$$

4.  $x \rightarrow N^\circ \text{ mayor}$

$x-56 \rightarrow N^\circ \text{ menor}$

$$\frac{x-8}{x-56}=3$$

$$x-8=3(x-56)$$

$$x-8=3x-168$$

$$-2x=-160$$

$$x=80 \rightarrow N^\circ \text{ mayor}$$

$$\Rightarrow x-56=80-56=24 \rightarrow N^\circ \text{ menor}$$

12.  $x \rightarrow \text{Edad } B$

$x-1 \rightarrow \text{Edad } A$

$x+1 \rightarrow \text{Edad } C$

$$(x+1)^2 - x^2 = \frac{17(x-1)}{5} - 4 \quad mcm=5$$

$$5(x^2+2x+1)-5x^2=17(x-1)-20$$

$$5x^2+10x+5-5x^2=17x-17-20$$

$$-7x=-42$$

$$x=6 \rightarrow \text{Edad } B$$

$$\Rightarrow x-1=6-1=5 \rightarrow \text{Edad } A$$

$$\Rightarrow x+1=6+1=7 \rightarrow \text{Edad } C$$

5.  $x \rightarrow \text{Parte mayor}$

$260-x \rightarrow \text{Parte menor}$

$$\frac{2x-40}{3(260-x)}=2$$

$$2x-40=6(260-x)$$

$$2x-40=1.560-6x$$

$$8x=1.600$$

$$x=200 \rightarrow \text{Parte mayor}$$

$$\Rightarrow 260-x$$

$$=260-200=60 \rightarrow \text{Parte menor}$$

6.  $x \rightarrow \text{Parte de } A$

$196-x \rightarrow \text{Parte de } B$

$$\frac{\frac{3x}{8}-16}{\frac{196-x}{5}}=1$$

$$\frac{3x-128}{8} \cdot \frac{5}{196-x}=1$$

$$5(3x-128)=8(196-x)$$

$$15x-640=1.568-8x$$

$$23x=2.208$$

$$x=96 \text{ soles} \rightarrow \text{Parte de } A$$

$$\Rightarrow 196-x$$

$$=196-96=100 \text{ soles} \rightarrow \text{Parte de } B$$

## EJERCICIO 148

1.  $x \rightarrow \text{Ganó el } 1^{\text{er}} \text{ día}$

$\frac{x}{2} \rightarrow \text{Ganó el } 2^{\text{o}} \text{ día}$

$\frac{x}{4} \rightarrow \text{Ganó el } 3^{\text{er}} \text{ día}$

$$x+\frac{x}{2}+\frac{x}{4}=175 \quad mcm=4$$

$$4x+2x+x=700$$

$$7x=700$$

$$x=\$100 \rightarrow \text{Ganó el } 1^{\text{er}} \text{ día}$$

$$\Rightarrow \frac{x}{2}=\frac{100}{2}=\$50 \rightarrow \text{Ganó el } 2^{\text{o}} \text{ día}$$

$$\Rightarrow \frac{x}{4}=\frac{100}{4}=\$25 \rightarrow \text{Ganó el } 3^{\text{er}} \text{ día}$$

2.  $x \rightarrow \text{Perdió } \text{miercoles}$

$\frac{3}{5}x \rightarrow \text{Perdió } \text{jueves}$

$\frac{1}{2}x \rightarrow \text{Perdió } \text{viernes}$

$$x+\frac{3}{5}x+\frac{1}{2}x=252 \quad mcm=10$$

$$10x+6x+5x=2.520$$

$$21x=2.520$$

$$x=\$120 \rightarrow \text{Perdió } \text{miercoles}$$

$$\Rightarrow \frac{3}{5} \cdot 120=\$72 \rightarrow \text{Perdió } \text{jueves}$$

$$\Rightarrow \frac{1}{2} \cdot 120=\$60 \rightarrow \text{Perdió } \text{viernes}$$

3.  $x \rightarrow \text{Tiene } A$

$\frac{2}{3}x \rightarrow \text{Tiene } B$

$\frac{2}{5}x \rightarrow \text{Tiene } C$

$$x+\frac{2x}{3}+\frac{2x}{5}=248 \quad mcm=15$$

$$15x+10x+6x=3.720$$

$$31x=3.720$$

$$x=120 \text{ suc.} \rightarrow \text{Tiene } A$$

$$\Rightarrow \frac{2}{3} \cdot 120=80 \text{ sucres} \rightarrow \text{Tiene } B$$

$$\Rightarrow \frac{2}{5} \cdot 120=48 \text{ sucres} \rightarrow \text{Tiene } C$$

4.  $x \rightarrow \text{Edad A}$

$$\frac{3}{5}x \rightarrow \text{Edad B}$$

$$\frac{9}{40}x \rightarrow \text{Edad C}$$

$$x + \frac{3x}{5} + \frac{9x}{40} = 73 \quad \text{mcm}=40$$

$$40x + 24x + 9x = 2.920$$

$$73x = 2.920$$

$$x = 40$$

$$40 \text{ años} \rightarrow \text{edad de A}$$

$$\Rightarrow \frac{3}{5} \cdot 40 = 24 \text{ años} \rightarrow \text{edad de B}$$

$$\Rightarrow \frac{9}{40} \cdot 40 = 9 \text{ años} \rightarrow \text{edad de C}$$

5.  $x \rightarrow 1^{\text{er}} \text{ día}$

$$\frac{1}{3}x \rightarrow 2^{\text{o}} \text{ día}$$

$$\frac{1}{9}x \rightarrow 3^{\text{o}} \text{ día}$$

$$\frac{1}{27}x \rightarrow 4^{\text{o}} \text{ día}$$

$$x + \frac{1}{3}x + \frac{1}{9}x + \frac{1}{27}x = 120 \quad \text{mcm}=27$$

$$27x + 9x + 3x + x = 3.240$$

$$40x = 3.240$$

$$x = 81 \text{ Km.}$$

$$81 \text{ Km.} \rightarrow 1^{\text{er}} \text{ día}$$

$$\Rightarrow \frac{1}{3} \cdot 81 = 27 \text{ Km.} \rightarrow 2^{\text{o}} \text{ día}$$

$$\Rightarrow \frac{1}{9} \cdot 81 = 9 \text{ Km.} \rightarrow 3^{\text{o}} \text{ día}$$

$$\Rightarrow \frac{1}{27} \cdot 81 = 3 \text{ Km.} \rightarrow 4^{\text{o}} \text{ día}$$

6.  $x \rightarrow 1^{\text{a}} \text{ semana}$

$$\frac{11}{10}x \rightarrow 2^{\text{a}} \text{ semana}$$

$$\frac{121}{100}x \rightarrow 3^{\text{a}} \text{ semana}$$

$$\frac{1.331}{1.000}x \rightarrow 4^{\text{a}} \text{ semana}$$

$$x + \frac{11}{10}x + \frac{121}{100}x + \frac{1.331}{1.000}x = 4.641 \quad \text{mcm}=1.000$$

$$1.000x + 1.100x + 1.210x + 1.331x = 4'641.000$$

$$4.641x = 4'641.000$$

$$x = 1.000 \text{ Km.}$$

## Continuación

6.  $1.000 \text{ Km.} \rightarrow 1^{\text{a}} \text{ semana}$

$$\Rightarrow \frac{11}{10} \cdot 1.000 = 1.100 \text{ Km.} \rightarrow 2^{\text{a}} \text{ semana}$$

$$\Rightarrow \frac{121}{100} \cdot 1.000 = 1.210 \text{ Km.} \rightarrow 3^{\text{a}} \text{ semana}$$

$$\Rightarrow \frac{1.331}{1.000} \cdot 1.000 = 1.331 \text{ Km.} \rightarrow 4^{\text{a}} \text{ semana}$$

7.  $x \rightarrow 1^{\text{a}} \text{ Persona}$  ;  $\frac{x}{2} \rightarrow 2^{\text{a}} \text{ Persona}$

$$\frac{x}{8} \rightarrow 3^{\text{a}} \text{ Persona} ; \frac{x}{40} \rightarrow 4^{\text{a}} \text{ Persona}$$

$$\frac{x}{400} \rightarrow 5^{\text{a}} \text{ Persona}$$

$$x + \frac{x}{2} + \frac{x}{8} + \frac{x}{40} + \frac{x}{400} = 330.500 \quad \text{mcm}=400$$

$$400x + 200x + 50x + 10x + x = 132'200.000$$

$$661x = 132'200.000$$

$$x = 200.000 \text{ colones}$$

$$200.000 \text{ colones} \rightarrow 1^{\text{a}} \text{ Persona}$$

$$\Rightarrow \frac{x}{2} = \frac{200.000}{2} = 100.000 \text{ colones} \rightarrow 2^{\text{a}} \text{ Persona}$$

$$\Rightarrow \frac{x}{8} = \frac{200.000}{8} = 25.000 \text{ colones} \rightarrow 3^{\text{a}} \text{ Persona}$$

$$\Rightarrow \frac{x}{40} = \frac{200.000}{40} = 5.000 \text{ colones} \rightarrow 4^{\text{a}} \text{ Persona}$$

$$\Rightarrow \frac{x}{400} = \frac{200.000}{400} = 500 \text{ colones} \rightarrow 5^{\text{a}} \text{ Persona}$$

8.  $x \rightarrow \text{Rec. en barco}$

$$\frac{4}{9}x \rightarrow \text{Rec. en tren}$$

$$\frac{5}{18}x \rightarrow \text{Rec. en avión}$$

$$x + \frac{4x}{9} + \frac{5x}{18} = 9.362 \quad \text{mcm}=18$$

$$18x + 8x + 5x = 168.516$$

$$31x = 168.516$$

$$x = 5.436 \text{ Km.}$$

$$5.436 \text{ Km.} \rightarrow \text{Rec. en barco}$$

$$\Rightarrow \frac{4}{9}x = \frac{4}{9} \cdot 5.436 = 2.416 \text{ Km.} \rightarrow \text{Rec. en tren}$$

$$\Rightarrow \frac{5}{18}x = \frac{5}{18} \cdot 5.436 = 1.510 \text{ Km.} \rightarrow \text{Rec. en avión}$$

Continúa

## EJERCICIO 149

1.  $x \rightarrow$  Tenía al principio

$$x - 20 - \frac{2(x-20)}{3} = 10 \quad mcm = 3$$

$$3x - 60 - 2(x - 20) = 30$$

$$3x - 60 - 2x + 40 = 30$$

$$x - 20 = 30$$

$$x = 50$$

\$50  $\rightarrow$  Tenía al principio

2.  $x \rightarrow$  Tenía al principio

$$\frac{x}{2} - \frac{x}{4} = 21 \quad mcm = 4$$

$$2x - x = 84$$

$$x = 84$$

84 Q.  $\rightarrow$  Tenía al principio

3.  $x \rightarrow$  Tengo ahora

$$x + 7 - \frac{4(x+7)}{5} = 20 \quad mcm = 5$$

$$5x + 35 - 4(x + 7) = 100$$

$$5x + 35 - 4x - 28 = 100$$

$$x + 7 = 100$$

$$x = 93$$

\$93  $\rightarrow$  Tengo ahora

4.  $x \rightarrow$  Tenía al principio

$$x - \frac{2x}{5} = \frac{3}{5}x \rightarrow \text{quedó}$$

$$\frac{5}{6} \cdot \frac{3}{5}x = \frac{1}{2}x \rightarrow \text{prestó}$$

Luego

$$x - \frac{2}{5}x - \frac{1}{2}x = 500 \quad mcm = 10$$

$$10x - 4x - 5x = 5.000$$

$$x = 5.000$$

5.000 bs.  $\rightarrow$  Tenía al principio

5.  $x \rightarrow$  Aves en la granja

$$\frac{4}{5}x \rightarrow \text{Palomas}$$

$$x - \frac{4}{5}x = \frac{x}{5} \quad \text{Re sto}$$

$$\frac{x}{5} \cdot \frac{3}{4} = \frac{3}{20}x \rightarrow \text{Gallinas}$$

$$x - \frac{4x}{5} - \frac{3x}{20} = 4 \quad mcm = 20$$

$$20x - 16x - 3x = 80$$

$$x = 80$$

80  $\rightarrow$  Aves en la granja

$$\Rightarrow \frac{4}{5}x = \frac{4}{5} \cdot 80 = 64 \rightarrow \text{Palomas}$$

$$\Rightarrow \frac{3}{20}x = \frac{3}{20} \cdot 80 = 12 \rightarrow \text{Gallinas}$$

4  $\rightarrow$  Gallos

6.  $x \rightarrow$  Tenía al principio

$$x - \frac{4}{5}x = \frac{x}{5} \rightarrow \text{quedó}$$

$$\frac{2}{3} \cdot \frac{x}{5} = \frac{2}{15}x \rightarrow \text{Perdí}$$

$$x - \frac{4}{5}x - \frac{2}{15}x - 8 = 0 \quad mcm = 15$$

$$15x - 12x - 2x - 120 = 0$$

$$x = 120$$

120 soles  $\rightarrow$  Tenía al principio

7.  $x \rightarrow$  Tenía al principio

$$x - \frac{5}{12}x + 42 = x + 2 \quad mcm = 12$$

$$12x - 5x + 504 = 12x + 24$$

$$-5x = -480$$

$$x = 96$$

\$96  $\rightarrow$  Tenía al principio

8.  $x \rightarrow$  Tenía al principio

$$x - \frac{x}{2} - 15 = 30 \quad mcm = 2$$

$$2x - x - 30 = 60$$

$$x = 90$$

\$90  $\rightarrow$  Tenía al principio

9.  $x \rightarrow$  Tenía al principio

$$x - \frac{3}{4}x + 1.300 = x + 100$$

$$mcm = 4$$

$$4x - 3x + 5.200 = 4x + 400$$

$$-3x = -4.800$$

$$x = 1.600$$

1.600 suc.  $\rightarrow$  Tenía al principio

10.  $x \rightarrow$  Tenía al principio

$$x - \frac{3}{4}x = \frac{x}{4} \rightarrow \text{quedó}$$

$$\frac{2}{3} \cdot \frac{x}{4} = \frac{x}{6} \rightarrow \text{Libros}$$

$$x - \frac{3}{4}x - \frac{x}{6} = \frac{2}{5}x - 38$$

$$mcm = 60$$

$$60x - 45x - 10x = 24x - 2.280$$

$$-19x = -2.280$$

$$x = 120$$

\$120  $\rightarrow$  Tenía al principio

## EJERCICIO 150

1.  $x \rightarrow$  Edad actual A

$3x \rightarrow$  Edad actual B

$$x - 15 = \frac{1}{6}(3x - 15) \quad mcm = 6$$

$$6x - 90 = 3x - 15$$

$$3x = 75$$

$x = 25$  años  $\rightarrow$  Edad actual A

$\Rightarrow 3x = 25 \cdot 3 = 75$  años  $\rightarrow$  Edad actual B

2.  $x \rightarrow$  Edad actual B

$3x \rightarrow$  Edad actual A

$$3x + 20 = 2(x + 20)$$

$$3x + 20 = 2x + 40$$

$x = 20$  años  $\rightarrow$  Edad actual B

$\Rightarrow 3x = 3 \cdot 20 = 60$  años  $\rightarrow$  Edad actual A

3.  $x \rightarrow \text{Edad actual A}$

$$x-5 = \frac{9}{11}(x+5) \quad mcm=11$$

$$11x-55=9x+45$$

$$2x=100$$

$$x=50$$

$$50 \text{ años} \rightarrow \text{Edad actual A}$$

4.  $x \rightarrow \text{Edad actual A}$

$$2(x-6)=x+24$$

$$2x-12=x+24$$

$$x=36$$

$$36 \text{ años} \rightarrow \text{Edad actual A}$$

5.  $x \rightarrow \text{Edad hijo}$

$$3x \rightarrow \text{Edad padre}$$

$$2(x+16)=3x+16$$

$$2x+32=3x+16$$

$$16=x$$

$$16 \text{ años} \rightarrow \text{Edad hijo}$$

$$\Rightarrow 3x=3 \cdot 16=48 \text{ años} \rightarrow \text{Edad padre}$$

6.  $x \rightarrow \text{Edad padre}$

$$\frac{2}{5}x \rightarrow \text{Edad hijo}$$

$$\frac{2(x-8)}{7} = \frac{2x}{5} - 8 \quad mcm=35$$

$$10x-80=14x-280$$

$$200=4x$$

$$50=x$$

$$50 \text{ años} \rightarrow \text{Edad padre}$$

$$\Rightarrow \frac{2}{5}x = \frac{2}{5} \cdot 50 = 20 \text{ años} \rightarrow \text{Edad hijo}$$

7.  $x \rightarrow \text{Edad actual de A}$

$$65-x \rightarrow \text{Edad actual de B}$$

$$65-x+10 = \frac{5(x+10)}{12} \quad mcm=12$$

$$900-12x=5x+50$$

$$850=17x$$

$$50=x$$

$$50 \text{ años} \rightarrow \text{Edad actual de A}$$

$$\Rightarrow 65-x=65-50=15 \text{ años} \rightarrow \text{Edad actual de B}$$

8.  $x \rightarrow \text{Edad padre}$

$$x-25 \rightarrow \text{Edad hijo}$$

$$x-25-15 = \frac{3(x-15)}{8} \quad mcm=8$$

$$8x-320=3x-45$$

$$5x=275$$

$$x=55$$

$$55 \text{ años} \rightarrow \text{Edad padre}$$

$$\Rightarrow x-25=55-25=30 \text{ años} \rightarrow \text{Edad hijo}$$

9.  $2x \rightarrow \text{Edad padre hace 10 años}$

$$x \rightarrow \text{Edad hijo hace 10 años}$$

$$2x+10 \rightarrow \text{Edad actual padre}$$

$$x+10 \rightarrow \text{Edad actual hijo}$$

$$2x+10+10 = \frac{3(x+10+10)}{2} \quad mcm=2$$

$$4x+40=3x+60$$

$$x=20$$

$$\Rightarrow 2x+10=2 \cdot 20+10=50 \text{ años} \rightarrow \text{Ed. act. padre}$$

$$\Rightarrow x+10=20+10=30 \text{ años} \rightarrow \text{Ed. act. hijo}$$

10.  $x+18 \rightarrow \text{Edad de A}$

$$x \rightarrow \text{Edad de B}$$

$$x+18-18 = \frac{5(x-18)}{2} \quad mcm=2$$

$$2x=5x-90$$

$$90=3x$$

$$30=x$$

$$30 \text{ años} \rightarrow \text{Edad de B}$$

$$\Rightarrow x+18=30+18=48 \text{ años} \rightarrow \text{Edad A}$$

11.  $3x \rightarrow \text{Edad A}$

$$x \rightarrow \text{Edad B}$$

$$3x-4+x-4=x+16$$

$$4x-8=x+16$$

$$3x=24$$

$$x=8$$

$$8 \text{ años} \rightarrow \text{Edad B}$$

$$\Rightarrow 3x=3 \cdot 8=24 \text{ años} \rightarrow \text{Edad A}$$

## EJERCICIO 151

- $x \rightarrow \text{Tiene B}$   
 $2x \rightarrow \text{Tiene A}$   
 $2x - 20 = \frac{4(x+20)}{5} \quad mcm=5$   
 $10x - 100 = 4x + 80$   
 $6x = 180$   
 $x = 30$   
 $30 \text{ soles} \rightarrow \text{Tiene B}$   
 $\Rightarrow 2x = 2 \cdot 30 = 60 \text{ soles} \rightarrow \text{Tiene A}$
- $x \rightarrow \text{Tiene B}$   
 $\frac{x}{2} \rightarrow \text{Tiene A}$   
 $x - 24 = \frac{x}{2} + 24 \quad mcm=2$   
 $2x - 48 = x + 48$   
 $x = 96$   
 $96 \text{ colones} \rightarrow \text{Tiene B}$   
 $\Rightarrow \frac{x}{2} = \frac{96}{2} = 48 \text{ colones} \rightarrow \text{Tiene A}$
- $x \rightarrow \text{Tiene A}$   
 $2x \rightarrow \text{Tiene B}$   
 $\frac{3}{5}(2x - 6) = x + 6 \quad mcm=5$   
 $6x - 18 = 5x + 30$   
 $x = 48$   
 $\$48 \rightarrow \text{Tiene A}$   
 $\Rightarrow 2x = 2 \cdot 48 = \$96 \rightarrow \text{Tiene B}$
- $x \rightarrow \text{Tiene A}$   
 $\frac{3}{5}x \rightarrow \text{Tiene B}$   
 $\frac{3}{5}x + 30 = \frac{9}{5}(x - 30) \quad mcm=5$   
 $3x + 150 = 9x - 270$   
 $420 = 6x$   
 $70 = x$   
 $\$70 \rightarrow \text{Tiene A}$   
 $\Rightarrow \frac{3}{5}x = \frac{3}{5} \cdot 70 = \$42 \rightarrow \text{Tiene B}$
- $x \rightarrow \text{Tiene A y B}$   
 $x - 30 = \frac{x+30}{2} \quad mcm=2$   
 $2x - 60 = x + 30$   
 $x = 90$   
 $90 \text{ suc.} \rightarrow \text{Tiene A y B}$

- $x \rightarrow \text{Tiene A}$   
 $\frac{2}{3}x \rightarrow \text{Tiene B}$   
 $\frac{2}{3}x + 22 = \frac{7}{5}(x - 22) \quad mcm=15$   
 $10x + 330 = 21x - 462$   
 $792 = 11x$   
 $72 = x \quad \$72 \rightarrow \text{Tiene A}$   
 $\Rightarrow \frac{2}{3}x = \frac{2}{3} \cdot 72 = \$48 \rightarrow \text{Tiene B}$
- $x \rightarrow \text{Tiene B}$   
 $\frac{4}{5}x \rightarrow \text{Tiene A}$   
 $\frac{4}{5}x + 13 = x - 5 \quad mcm=5$   
 $4x + 65 = 5x - 25$   
 $90 = x \quad \$90 \rightarrow \text{Tiene B}$   
 $\Rightarrow \frac{4}{5}x = \frac{4}{5} \cdot 90 = \$72 \rightarrow \text{Tiene A}$
- $x \rightarrow \text{Tiene A}$   
 $\frac{x}{2} \rightarrow \text{Tiene B}$   
 $\frac{x}{2} + \frac{1}{3}x = x + 5 \quad mcm=6$   
 $3x + 2x = 6x + 30$   
 $5x = 6x + 30$   
 $30 = x \quad \$30 \rightarrow \text{Tiene A}$   
 $\Rightarrow \frac{x}{2} = \frac{30}{2} = \$15 \rightarrow \text{Tiene B}$
- $x \rightarrow \text{Empezaron A y B}$   
 $x - \frac{3}{5}x = x - 24 \quad mcm=5$   
 $5x - 3x = 5x - 120$   
 $-3x = -120$   
 $x = 40$   
 $40 \text{ balboas} \rightarrow \text{Empezaron A y B}$
- $x \rightarrow \text{Empezaron A y B}$   
 $x - \frac{3}{4}x = \frac{x}{4} \rightarrow \text{Le queda B}$   
 $\frac{x}{12} \rightarrow 3^{\text{a}} \text{ parte de lo queda B}$   
 $\frac{x}{12} + 24 = \frac{x+288}{12} \rightarrow \text{ha ganado A}$   
 $\frac{x}{4} = x - \left(\frac{x+288}{12}\right) \quad mcm=12$   
 $3x = 12x - x - 288$   
 $3x = 11x - 288$   
 $-8x = -288$   
 $x = 36$   
 $36 \text{ soles} \rightarrow \text{Empezaron A y B}$

## EJERCICIO 152

- $x \rightarrow N^{\circ} \text{ años que pasan}$   
 $28 + x = \frac{3}{4}(x + 38) \quad mcm=4$   
 $112 + 4x = 3x + 114$   
 $x = 2$   
 $2 \rightarrow \text{años pasan}$
- $x \rightarrow N^{\circ} \text{ años que pasan}$   
 $30 + x = \frac{7}{6}(x + 25) \quad mcm=6$   
 $180 + 6x = 7x + 175$   
 $5 = x$   
 $5 \rightarrow \text{años pasan}$
- $x \rightarrow N^{\circ} \text{ años hace}$   
 $48 - x = \frac{9}{10}(52 - x)$   
 $mcm=10$   
 $480 - 10x = 468 - 9x$   
 $12 = x$   
 $12 \rightarrow \text{años hace}$
- $x \rightarrow N^{\circ} \text{ años hace}$   
 $18 - x = \frac{1}{4}(27 - x)$   
 $mcm=4$   
 $72 - 4x = 27 - x$   
 $45 = 3x$   
 $15 = x$   
 $15 \rightarrow \text{años hace}$
- $x \rightarrow \text{La suma de dinero}$   
 $\frac{3}{5}(50 + x) = x + 22 \quad mcm=5$   
 $150 + 3x = 5x + 110$   
 $40 = 2x$   
 $20 = x$   
 $\$20 \rightarrow \text{La suma de dinero}$
- $x \rightarrow \text{Gastó cada uno}$   
 $\frac{3}{11}(90 - x) = 50 - x \quad mcm=11$   
 $270 - 3x = 550 - 11x$   
 $8x = 280$   
 $x = 35$   
 $35Q. \rightarrow \text{Gastó cada uno}$



7.  $x \rightarrow$  Edad actual hermano

$$\begin{aligned}\frac{3}{4}x &\rightarrow \text{Edad actual persona} \\ (x+x) + \frac{3}{4}x + x &= 75 \\ 3x + \frac{3}{4}x &= 75 \quad mcm=4 \\ 12x + 3x &= 300 \\ x &= 20\end{aligned}$$

20 años  $\rightarrow$  Edad actual hermano

$$\Rightarrow \frac{3}{4}x = \frac{3}{4} \cdot 20 = 15 \text{ años} \rightarrow \text{Edad actual persona}$$

## EJERCICIO 153

1.  $x+3 \rightarrow$  Long. rect.

$x \rightarrow$  Ancho rect.

$$\begin{aligned}(x+4)(x+1) - 22 &= x(x+3) \\ x^2 + 5x + 4 - 22 &= x^2 + 3x \\ 2x &= 18 \\ x &= 9\end{aligned}$$

9m  $\rightarrow$  Ancho rect.

$\Rightarrow x+3 = 9+3 = 12m \rightarrow$  Long. rect.

2.  $x \rightarrow$  Una de las dimensiones

2x  $\rightarrow$  La otra dimensión

$$\begin{aligned}(x+5)(2x+5) - 160 &= 2x^2 \\ 2x^2 + 15x + 25 - 160 &= 2x^2 \\ 15x &= 135 \\ x &= 9\end{aligned}$$

9m  $\cdot$  18m  $\rightarrow$  Dimensiones

3.  $x+2 \rightarrow$  Una dimensión

$x \rightarrow$  Otra dimensión

$$\begin{aligned}(x-5)(x-3) + 115 &= x(x+2) \\ x^2 - 8x + 15 + 115 &= x^2 + 2x \\ -10x &= -130 \\ x &= 13\end{aligned}$$

15m  $\cdot$  13m  $\rightarrow$  Dimensiones

4.  $\frac{x}{2} + 24 \rightarrow$  Long. del rect.

$\frac{x}{2} - 12 \rightarrow$  Ancho del rect.

$$\left(\frac{x}{2} + 24\right)\left(\frac{x}{2} - 12\right) = x\left(\frac{x}{2} - 12\right)$$

**Continúa**

8.  $x \rightarrow$  ganó cada uno

$$\begin{aligned}54 + x + 32 + x &= 4x + 66 \\ 2x + 86 &= 4x + 66 \\ 20 &= 2x \\ 10 &= x \\ \$10 &\rightarrow \text{ganó cada uno}\end{aligned}$$

9.  $x \rightarrow$  Le dió A a B

$$\begin{aligned}153 - x &= \frac{1}{4}(12 + x) \quad mcm=4 \\ 612 - 4x &= 12 + x \\ 600 &= 5x \\ 120 &= x \\ 120bs. &\rightarrow \text{Le dió A a B}\end{aligned}$$

## EJERCICIO 154

1.  $x \rightarrow$  Numerador

$x-2 \rightarrow$  Denominador

$$\begin{aligned}\frac{x}{x-2+7} &= \frac{1}{2} \\ \frac{x}{x+5} &= \frac{1}{2} \\ 2x &= x+5 \\ x &= 5\end{aligned}$$

5  $\rightarrow$  Numerador

5-2=3  $\rightarrow$  Denominador

$\frac{5}{3} \rightarrow$  fracción

2.  $x \rightarrow$  Numerador

$x+1 \rightarrow$  Denominador

$$\begin{aligned}\frac{x}{x+1+15} &= \frac{1}{3} \\ \frac{x}{x+16} &= \frac{1}{3} \\ 3x &= x+16 \\ 2x &= 16 \\ x &= 8\end{aligned}$$

8  $\rightarrow$  Numerador

8+1=9  $\rightarrow$  Denominador

$\frac{8}{9} \rightarrow$  Fracción

3.  $x-8 \rightarrow$  Numerador

$x \rightarrow$  Denominador

$$\begin{aligned}\frac{x-8+1}{x+1} &= \frac{3}{4} \\ \frac{x-7}{x+1} &= \frac{3}{4}\end{aligned}$$

$$4(x-7) = 3(x+1)$$

$$4x - 28 = 3x + 3$$

$$x = 31$$

31-8=23  $\rightarrow$  Numerador

31  $\rightarrow$  Denominador

$\frac{23}{31} \rightarrow$  Fracción

### Continuación

4.  $\frac{x}{2} + 24 = x \quad mcm=2$

$$\begin{aligned}x + 48 &= 2x \\ 48 &= x\end{aligned}$$

48m  $\cdot$  12m  $\rightarrow$  Dimensiones

5.  $x \rightarrow$  Long. del cuadrado

Equiv. al rect.

$x+7 \rightarrow$  Long. del rect.

$x-6 \rightarrow$  Ancho del rect.

$$(x+7)(x-6) = x^2$$

$$x^2 + x - 42 = x^2$$

$$x = 42$$

$\Rightarrow x+7 = 42+7 = 49 \rightarrow$  Long. del rect.

$\Rightarrow x-6 = 42-6 = 36 \rightarrow$  Ancho del rect.

49m  $\cdot$  36m  $\rightarrow$  Dimensiones

6.  $x \rightarrow$  Ancho

$x+30 \rightarrow$  Longitud

$$(x+30-20)(x+15) + 150 = x(x+30)$$

$$(x+10)(x+15) + 150 = x^2 + 30x$$

$$x^2 + 25x + 150 + 150 = x^2 + 30x$$

$$300 = 5x$$

$$60 = x$$

60m  $\rightarrow$  Ancho

$\Rightarrow x+30 = 60+30 = 90m \rightarrow$  Long.

90m  $\cdot$  60m  $\rightarrow$  Dimensiones

7.  $x \rightarrow$  Ancho

$x+10 \rightarrow$  Longitud

$$(x+8)(x+1) = x(x+10)$$

$$x^2 + 9x + 8 = x^2 + 10x$$

$$8 = x \rightarrow \text{Ancho}$$

$\Rightarrow x+10 = 8+10 = 18 \rightarrow$  Long.

18m  $\cdot$  8m  $\rightarrow$  Dimensiones

4.  $x \rightarrow$  Numerador

$2x+1 \rightarrow$  Denominador

$$\frac{x-4}{2x+1} = \frac{1}{3}$$

$$3(x-4) = 2x+1$$

$$3x-12=2x+1$$

$$x=13$$

13  $\rightarrow$  Numerador

$2 \cdot 13+1=27 \rightarrow$  Denominador

$$\frac{13}{27} \rightarrow \text{Fracción}$$

7.  $x \rightarrow$  Numerador

$3x-1 \rightarrow$  Denominador

$$\frac{x+8}{3x-1+4} = \frac{11}{12}$$

$$\frac{x+8}{3x+3} = \frac{11}{12}$$

$$12(x+8) = 33(x+1)$$

$$12x+96 = 33x+33$$

$$63 = 21x$$

$$3 = x$$

3  $\rightarrow$  Numerador

$3 \cdot 3-1=8 \rightarrow$  Denominador

$$\frac{3}{8} \rightarrow \text{Fracción}$$

5.  $x \rightarrow$  Numerador

$2x+6 \rightarrow$  Denominador

$$\frac{x+15}{2x+6-1} = \frac{4}{3}$$

$$\frac{x+15}{2x+5} = \frac{4}{3}$$

$$3(x+15) = 4(2x+5)$$

$$3x+45=8x+20$$

$$25=5x$$

$$5=x$$

5  $\rightarrow$  Numerador

$2 \cdot 5+6=16 \rightarrow$  Denominador

$$\frac{5}{16} \rightarrow \text{Fracción}$$

8.  $x \rightarrow$  Numerador

$x-22 \rightarrow$  Denominador

$$\frac{x}{x-22} - \frac{x-15}{x-22} = 3 \quad \text{mcm} = x-22$$

$$x - (x-15) = 3(x-22)$$

$$15 = 3x - 66$$

$$81 = 3x$$

$$27 = x$$

27  $\rightarrow$  Numerador

$27-22=5 \rightarrow$  Denominador

$$\frac{27}{5} \rightarrow \text{Fracción}$$

6.  $x \rightarrow$  Numerador

$x+1 \rightarrow$  Denominador

$$\frac{x}{x+1+4} = 3 \left( \frac{x}{x+1} \right) - 2$$

$$\frac{x}{x+5} = \frac{3x}{x+1} - 2 \quad \text{mcm} = (x+5)(x+1)$$

$$x(x+1) = 3x(x+5) - 2(x+5)(x+1)$$

$$x^2+x = 3x^2+15x-2x^2-12x-10$$

$$x = 3x-10$$

$$10 = 2x$$

$$5 = x$$

5  $\rightarrow$  Numerador

$5+1=6 \rightarrow$  Denominador

$$\frac{5}{6} \rightarrow \text{Fracción}$$

## EJERCICIO 155

1.  $x \rightarrow$  cifra de unidades

$x+2 \rightarrow$  cifra de decenas

$$10(x+2)+x$$

$$10x+20+x$$

$11x+20 \rightarrow$  El Número

$$\frac{11x+20}{2(x+1)} = 7$$

$$11x+20 = 14(x+1)$$

$$11x+20 = 14x+14$$

$$6 = 3x$$

$$2 = x$$

2  $\rightarrow$  cifra de unidades

$2+2=4 \rightarrow$  cifra de decenas

42  $\rightarrow$  N° buscado

2.  $x \rightarrow$  cifra de unidades

$x-4 \rightarrow$  cifra de decenas

$$10(x-4)+x$$

$$10x-40+x$$

$11x-40 \rightarrow$  El Número

$$\frac{11x-40}{2(x-2)} = 4$$

$$11x-40 = 8x-16$$

$$3x = 24$$

$$x = 8$$

8  $\rightarrow$  cifra de unidades

$8-4=4 \rightarrow$  cifra de decenas

48  $\rightarrow$  N° buscado

3.  $x \rightarrow$  cifra de unidades

$2x \rightarrow$  cifra de decenas

$$10(2x)+x-9$$

$21x-9 \rightarrow$  N° disminuido en 9

$$\frac{21x-9}{3x} = 6$$

$$21x-9 = 18x$$

$$3x = 9$$

$$x = 3$$

63  $\rightarrow$  N° buscado

4.  $x \rightarrow$  cifra de unidades

$x+1 \rightarrow$  cifra de decenas

$$10(x+1)+x$$

$$3(11x+10)$$

$$33x+30 \rightarrow N^{\circ} \text{ multip. } *3$$

$$33x+30=21(2x+1)$$

$$33x+30=42x+21$$

$$9=9x$$

$$1=x$$

1  $\rightarrow$  Cifra de unidades

1+1=2  $\rightarrow$  Cifra de decenas

21  $\rightarrow$  N<sup>o</sup> buscado

5.  $x \rightarrow$  cifra de unidades

$7-x \rightarrow$  cifra de decenas

$$10(7-x)+x+8$$

$$78-9x \rightarrow N^{\circ} \text{ aumentado en } 8$$

$$\frac{78-9x}{2(7-x)}=6$$

$$78-9x=12(7-x)$$

$$78-9x=84-12x$$

$$3x=6$$

$$x=2$$

52  $\rightarrow$  N<sup>o</sup> buscado

6.  $x \rightarrow$  cifra de unidades

$x+2 \rightarrow$  cifra de decenas

$$10(x+2)+x$$

$$11x+20 \rightarrow \text{El Número}$$

$$11x+20-27=10x$$

$$11x-7=10x$$

$$x=7$$

7  $\rightarrow$  cifra de unidades

7+2=9  $\rightarrow$  cifra de decenas

97  $\rightarrow$  N<sup>o</sup> buscado

7.  $x \rightarrow$  cifra de unidades

$2x \rightarrow$  cifra de decenas

$$10(2x)+x-4$$

$$21x-4 \rightarrow N^{\circ} \text{ dis min en } 4$$

$$\frac{21x-4}{2x-x}=20$$

$$21x-4=20x$$

$$x=4$$

84  $\rightarrow$  N<sup>o</sup> buscado

## EJERCICIO 156

1.  $x \rightarrow$  Días trabaj. juntos

$$\frac{1}{3} + \frac{1}{6} = \frac{1}{x} \quad mcm=6x$$

$$2x+x=6$$

$$3x=6$$

$$x=2$$

2 días  $\rightarrow$  Hacen la obra

2.  $x \rightarrow$  Tiempo en llenar depósito

$$\frac{1}{10} + \frac{1}{20} = \frac{1}{x} \quad mcm=20x$$

$$2x+x=20$$

$$3x=20$$

$$x=\frac{20}{3}$$

$$x=6\frac{2}{3}$$

$6\frac{2}{3}$  min.  $\rightarrow$  En llenar depósito

3.  $x \rightarrow$  Hacen obra juntos

$$\frac{1}{4} + \frac{1}{6} + \frac{1}{12} = \frac{1}{x} \quad mcm=12x$$

$$3x+2x+x=12$$

$$6x=12$$

$$x=2$$

2 días  $\rightarrow$  Hacen obra juntos

4.  $x \rightarrow$  Hacen obra juntos

$$\frac{1}{1\frac{1}{2}} + \frac{1}{6} + \frac{1}{2\frac{2}{5}} = \frac{1}{x}$$

$$\frac{2}{3} + \frac{1}{6} + \frac{5}{12} = \frac{1}{x} \quad mcm=12x$$

### Continuación

$$4. 8x+2x+5x=12$$

$$15x=12$$

$$x=\frac{12}{15}$$

$$x=\frac{4}{5}$$

$\frac{4}{5}$  de día hacen la obra

5.  $x \rightarrow$  Tiempo en llenar depósito

$$\frac{1}{5} + \frac{1}{6} + \frac{1}{12} = \frac{1}{x} \quad mcm=60x$$

$$12x+10x+5x=60$$

$$27x=60$$

$$x=\frac{60}{27}$$

$$x=2\frac{2}{9}$$

$2\frac{2}{9}$  min  $\rightarrow$  Llenar el depósito

6.  $x \rightarrow$  Tiempo en llenar depósito

$$\frac{1}{4} + \frac{1}{8} - \frac{1}{20} = \frac{1}{x} \quad mcm=40x$$

$$10x+5x-2x=40$$

$$13x=40$$

$$x=\frac{40}{13}$$

$$x=3\frac{1}{13}$$

$3\frac{1}{13}$  min.  $\rightarrow$  Llenar el depósito

### Continúa

## EJERCICIO 157

1.



$$ABCD=x$$

$$AB=5$$

$$BC=\frac{x}{12}$$

$$CD=30$$

$$x=5+30+\frac{x}{12}$$

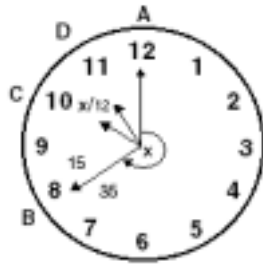
$$x=35+\frac{x}{12}$$

$$12x=420+x$$

$$11x=420$$

$$x=\frac{420}{11}=38\frac{2}{11}$$

2.



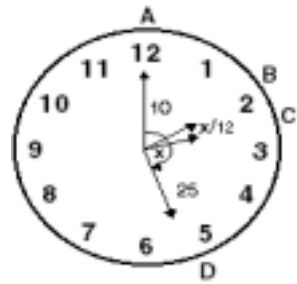
$$\begin{aligned}
 AB &= x \\
 BD &= 15 \\
 AC &= 50 \\
 CD &= \frac{x}{12} \\
 AB + BD &= AC + CD \\
 x + 15 &= 50 + \frac{x}{12} \quad mcm=12 \\
 12x + 180 &= 600 + x \\
 11x &= 420 \\
 x &= \frac{420}{11} \\
 x &= 38\frac{2}{11}
 \end{aligned}$$

3.

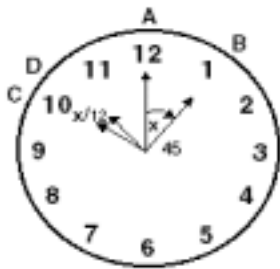


$$\begin{aligned}
 AB &= x \\
 BD &= 30 \\
 AC &= 40 \\
 CD &= \frac{x}{12} \\
 AB + BD &= AC + CD \\
 x + 30 &= 40 + \frac{x}{12} \quad mcm=12 \\
 12x + 360 &= 480 + x \\
 11x &= 120 \\
 x &= \frac{120}{11} \\
 x &= 10\frac{10}{11}
 \end{aligned}$$

5.

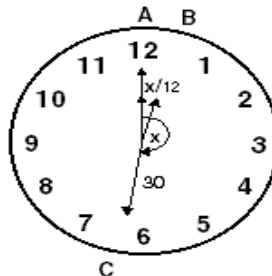


$$\begin{aligned}
 ABCD &= x \\
 AB &= 10 \\
 BC &= \frac{x}{12} \\
 CD &= 15 \\
 ABCD &= AB + BC + CD \\
 x &= 10 + \frac{x}{12} + 15 \\
 x &= 25 + \frac{x}{12} \\
 mcm &= 12 \\
 12x &= 300 + x \\
 11x &= 300 \\
 x &= \frac{300}{11} \\
 x &= 27\frac{3}{11}
 \end{aligned}$$



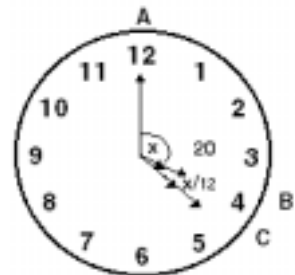
$$\begin{aligned}
 AB &= x \\
 BD &= 45 \\
 AC &= 50 \\
 CD &= \frac{x}{12} \\
 AC + CD &= AB + BD \\
 50 + \frac{x}{12} &= x + 45 \quad mcm=12 \\
 600 + x &= 12x + 540 \\
 60 &= 11x \\
 \frac{60}{11} &= x \\
 5\frac{5}{11} &= x
 \end{aligned}$$

4.



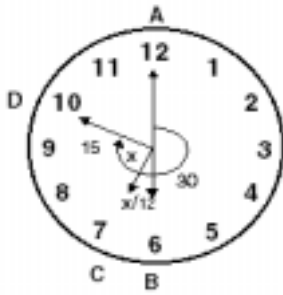
$$\begin{aligned}
 AB &= \frac{x}{12} \\
 BC &= 30 \\
 ABC &= x \\
 ABC &= AB + BC \\
 x &= \frac{x}{12} + 30 \quad mcm=12 \\
 12x &= x + 360 \\
 11x &= 360 \\
 x &= \frac{360}{11} \\
 x &= 32\frac{8}{11}
 \end{aligned}$$

6.



$$\begin{aligned}
 ABC &= x \\
 AB &= 20 \\
 BC &= \frac{x}{12} \\
 ABC &= AB + BC \\
 x &= 20 + \frac{x}{12} \quad mcm=12 \\
 12x &= 240 + x \\
 11x &= 240 \\
 x &= \frac{240}{11} \\
 x &= 21\frac{9}{11}
 \end{aligned}$$

7.



$$ABCD = x$$

$$AB = 30$$

$$BC = \frac{x}{12}$$

$$CD = 15$$

$$ABCD = AB + BC + CD$$

$$x = 30 + \frac{x}{12} + 15 \quad mcm = 12$$

$$12x = 360 + x + 180$$

$$11x = 540$$

$$x = \frac{540}{11}$$

$$x = 49\frac{1}{11}$$

8.



$$ABC = x$$

$$AB = 50$$

$$BC = \frac{x}{12}$$

$$ABC = AB + BC$$

$$x = 50 + \frac{x}{12} \quad mcm = 12$$

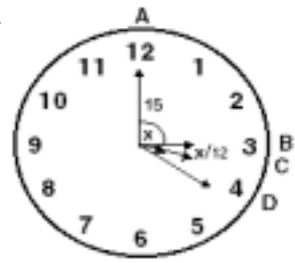
$$12x = 600 + x$$

$$11x = 600$$

$$x = \frac{600}{11}$$

$$x = 54\frac{6}{11}$$

10.



$$ABCD = x$$

$$AB = 15$$

$$BC = \frac{x}{12}$$

$$CD = 5$$

$$ABCD = AB + BC + CD$$

$$x = 15 + \frac{x}{12} + 5$$

$$x = 20 + \frac{x}{12} \quad mcm = 12$$

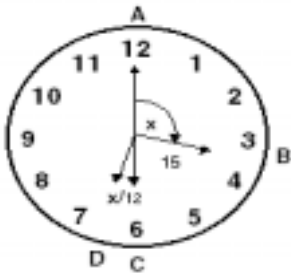
$$12x = 240 + x$$

$$11x = 240$$

$$x = \frac{240}{11}$$

$$x = 21\frac{9}{11}$$

9.



$$AB = x$$

$$BD = 15$$

$$AC = 30$$

$$CD = \frac{x}{12}$$

$$AB + BD = AC + CD$$

$$x + 15 = 30 + \frac{x}{12}$$

$$x - \frac{x}{12} = 15 \quad mcm = 12$$

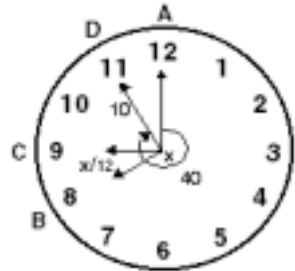
$$12x - x = 180$$

$$11x = 180$$

$$x = \frac{180}{11}$$

$$x = 16\frac{4}{11}$$

11.



$$ABCD = x$$

$$AB = 40$$

$$BC = \frac{x}{12}$$

$$CD = 10$$

$$ABCD = AB + BC + CD$$

$$x = 40 + \frac{x}{12} + 10$$

$$x = 50 + \frac{x}{12} \quad mcm = 12$$

$$12x = 600 + x$$

$$11x = 600$$

$$x = \frac{600}{11}$$

$$x = 54\frac{6}{11}$$

Continúa



5.  $x \rightarrow$  Parte mayor  
 $84 - x \rightarrow$  Parte menor  
 $\frac{1}{10}x = \frac{1}{4}(84 - x) \quad mcm = 20$   
 $2x = 5(84 - x)$   
 $2x = 420 - 5x$   
 $7x = 420$   
 $x = 60$   
 $60 \rightarrow$  Parte mayor  
 $84 - 60 = 24 \rightarrow$  Parte menor

6.  $x \rightarrow$  Parte mayor  
 $120 - x \rightarrow$  Parte menor  
 $120 - x = \frac{3}{5}x \quad mcm = 5$   
 $600 - 5x = 3x$   
 $600 = 8x$   
 $75 = x \rightarrow$  Parte mayor  
 $120 - 75 = 45 \rightarrow$  Parte menor

7.  $x \rightarrow$  Sueldo mensual  
 $\frac{x}{2} + \frac{3}{8}x = \frac{7}{8}x \rightarrow$  mensual  
 $15x - 15\left(\frac{7}{8}x\right) = 300 \quad mcm = 8$   
 $120x - 105x = 2.400$   
 $15x = 2.400$   
 $x = 160$   
 $\$160 \rightarrow$  Sueldo mensual

8.  $x \rightarrow$  Lo que tenía  
 $\frac{1}{5}x \rightarrow$  Gastó en ropa  
 $\frac{3}{8}x \rightarrow$  Gastó en libros  
 $x - \frac{1}{5}x - \frac{3}{8}x - 102 = 0 \quad mcm = 40$   
 $40x - 8x - 15x - 4.080 = 0$   
 $17x = 4.080$   
 $x = 240$   
 $\frac{1}{5} \cdot 240 = \$48 \rightarrow$  Gastó en ropa  
 $\frac{3}{8} \cdot 240 = \$90 \rightarrow$  Gastó en libros

9.  $x \rightarrow$  Edad de A  
 $\frac{2}{5}x \rightarrow$  Edad de B  
 $\frac{2}{3} \cdot \frac{2}{5}x = \frac{4}{15}x \rightarrow$  Edad de C  
 $x + \frac{2}{5}x + \frac{4}{15}x = 25 \quad mcm = 15$   
 $15x + 6x + 4x = 375$   
 $25x = 375$   
 $x = 15$   
 $15 \text{ años} \rightarrow$  Edad de A  
 $\frac{2}{5} \cdot 15 = 6 \text{ años} \rightarrow$  Edad de B  
 $\frac{4}{15} \cdot 15 = 4 \text{ años} \rightarrow$  Edad de C

10.  $x \rightarrow$  Costo auto  
 $8.000 + \frac{x}{3} = x + 2.000 \quad mcm = 3$   
 $24.000 + x = 3x + 6.000$   
 $18.000 = 2x$   
 $9.000 = x$   
 $9.000 \text{ bs} \rightarrow$  Costo auto

11.  $x \rightarrow$  Libros compre  
 $\frac{7x}{2} - \frac{5x}{2} = 8 \quad mcm = 2$   
 $7x - 5x = 16$   
 $2x = 16$   
 $x = 8 \rightarrow$  Lib. compre

12.  $x \rightarrow$  Cierta N° de libros  
 $\frac{3x}{4} \rightarrow$  Vr. cierta N° de libros  
 $\frac{3x}{4} \left(\frac{7}{10}\right) = \frac{21x}{40} \rightarrow$  Vr.  $\frac{3}{4}$  del N°  
de lib. ant.  
 $\frac{3x}{4} + \frac{21x}{40} = \frac{51x}{40} \rightarrow$  Vr. total de  
lib. compré

$x + \frac{3x}{4} = \frac{7x}{4} \rightarrow$  total lib. comp.

## 12. Continuación

$\frac{7x}{4} \cdot \frac{3}{2} = \frac{21x}{8} \rightarrow$  Vr. total vta.  
de todos los lib.

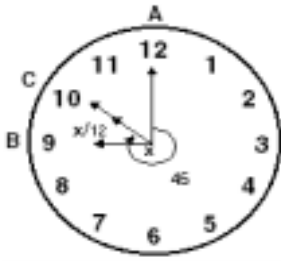
Luego  
 $\frac{21x}{8} - \frac{51x}{40} = 54 \quad mcm = 40$   
 $105x - 51x = 2.160$   
 $54x = 2.160$   
 $x = 40$   
 $\frac{7x}{4} = \frac{7 \cdot 40}{4} = 70$   
 $70 \rightarrow$  Total libros comprados

13.  $x \rightarrow$  1ª parte  
 $\frac{5x}{6} \rightarrow$  2ª parte  
 $\frac{3}{5} \cdot \frac{5x}{6} = \frac{1}{2}x \rightarrow$  3ª parte  
 $\frac{1}{3} \cdot \frac{1}{2}x = \frac{1}{6}x \rightarrow$  4ª parte  
 $x + \frac{5x}{6} + \frac{1}{2}x + \frac{1}{6}x = 150$   
 $mcm = 6$   
 $6x + 5x + 3x + x = 900$   
 $15x = 900$   
 $x = 60$   
 $60 \rightarrow$  1ª parte

$\frac{5}{6} \cdot 60 = 50 \rightarrow$  2ª parte  
 $\frac{1}{2} \cdot 60 = 30 \rightarrow$  3ª parte  
 $\frac{1}{6} \cdot 60 = 10 \rightarrow$  4ª parte

Continúa

14.



$$ABC = x$$

$$AB = 45$$

$$BC = \frac{x}{12}$$

$$ABC = AB + BC$$

$$x = 45 + \frac{x}{12} \quad mcm = 12$$

$$12x = 540 + x$$

$$11x = 540$$

$$x = \frac{540}{11}$$

$$x = 49\frac{1}{11}$$

15.  $x \rightarrow$  Edad A $x - 10 \rightarrow$  Edad B

$$x - 10 - 15 = \frac{3}{4}(x - 15) \quad mcm = 4$$

$$4x - 40 - 60 = 3x - 45$$

$$4x - 100 = 3x - 45$$

$$x = 55$$

55 años  $\rightarrow$  Edad A55 - 10 = 45 años  $\rightarrow$  Edad B16.  $x \rightarrow$  Puede hacerla A

$$\frac{1}{10} + \frac{1}{x} = \frac{1}{6} \quad mcm = 60x$$

$$6x + 60 = 10x$$

$$60 = 4x$$

$$15 = x$$

15 días  $\rightarrow$  Puede hacerla A17.  $x \rightarrow$  Parte mayor $650 - x \rightarrow$  Parte menor

$$\frac{x}{5} = 650 - x - 50$$

$$\frac{x}{5} = 600 - x \quad mcm = 5$$

$$x = 3.000 - 5x$$

$$6x = 3.000$$

$$x = 500$$

500  $\rightarrow$  Parte mayor $\Rightarrow 650 - 500 = 150 \rightarrow$  Parte menor18.  $x \rightarrow$  Edad actual B

$$\frac{x}{4} \rightarrow$$
 Edad actual A

$$\frac{x}{4} - 10 = \frac{1}{10}(x - 10) \quad mcm = 40$$

$$10x - 400 = 4x - 40$$

$$6x = 360$$

$$x = 60$$

60 años  $\rightarrow$  Edad actual B

$$\frac{60}{4} = 15 \text{ años} \rightarrow$$
 Edad actual A

19.  $x \rightarrow$  N° mayor $x - 1 \rightarrow$  N° menor

$$x^2 - (x - 1)^2 - 43 = \frac{1}{11}(x - 1)$$

$$x^2 - x^2 + 2x - 1 - 43 = \frac{1}{11}(x - 1)$$

$$2x - 44 = \frac{1}{11}(x - 1)$$

$$11(2x - 44) = x - 1$$

$$22x - 484 = x - 1$$

$$21x = 483$$

$$x = 23$$

23  $\rightarrow$  N° mayor23 - 1 = 22  $\rightarrow$  N° menor20.  $x \rightarrow$  Vr. sortija

$$7\left(\frac{3.000 + x}{12}\right) = 1.500 + x \quad mcm = 12$$

$$21.000 + 7x = 18.000 + 12x$$

$$3.000 = 5x$$

$$600 = x$$

600 sures  $\rightarrow$  Vr. sortija21.  $x \rightarrow$  N° personas

$$x + \frac{1}{5}x = \frac{6x}{5} \rightarrow$$
 N° más de pers.

$$\frac{120}{6x} = \frac{100}{x}$$

$$\frac{100}{x} = \frac{120}{x} - 2 \quad mcm = x$$

$$100 = 120 - 2x$$

$$2x = 20$$

$$x = 10$$

10 personas  $\rightarrow$  Se repartió \$22.  $x \rightarrow$  libros que compró

$$x + \frac{x}{4} = \frac{5x}{4} \rightarrow$$
 Libros más

$$\frac{400}{5x} = \frac{320}{x}$$

$$\frac{4}{4}$$

$$\frac{320}{x} = \frac{400}{x} - 2 \quad mcm = x$$

$$320 = 400 - 2x$$

$$2x = 80$$

$$x = 40$$

40 libros  $\rightarrow$  compró

$$\frac{400}{40} = \$10 \rightarrow$$
 Pagó por c/u

23.  $x \rightarrow$  Suma repartida

$$\frac{x}{2} - 30 \rightarrow$$
 Recibió A

$$\frac{3x}{7} + 20 \rightarrow$$
 Recibió B

$$\frac{x}{2} - 30 + \frac{3x}{7} + 20 + 30 = x$$

$$mcm = 14$$

$$7x + 6x + 280 = 14x$$

$$280 = x$$

$$\frac{280}{2} - 30 = \$110 \rightarrow$$
 Rec. A

$$\frac{3(280)}{7} + 20 = \$140 \rightarrow$$
 Rec. B

24.  $x \rightarrow$  Lib. comprad.

$$\frac{6x}{5} \rightarrow$$
 Vr. de x lib.

$$x - \frac{x}{3} = \frac{2x}{3} \rightarrow$$
 Resto

$$\frac{2x}{3} \cdot \frac{9}{4} = \frac{3x}{2} \rightarrow$$
 Vr. vta resto

$$\frac{3x}{2} \cdot \frac{6x}{5} = 9 \quad mcm = 10$$

$$15x - 12x = 90$$

$$3x = 90$$

$$x = 30$$

30  $\rightarrow$  Libros comprados25.  $x \rightarrow$  Fortuna

$$x - \frac{x}{2} - \frac{x}{4} - \frac{x}{6} = 2.500$$

$$mcm = 12$$

$$12x - 6x - 3x - 2x = 30.000$$

$$x = 30.000$$

30.000 colones  $\rightarrow$  Fortuna



26.  $x \rightarrow$  sueldo anual

$$x - \frac{3x}{5} - \frac{x}{8} - \frac{x}{20} = 810$$

$$mcm = 40$$

$$40x - 24x - 5x - 2x = 32.400$$

$$9x = 32.400$$

$$x = 3.600$$

$$3.600 \text{ balboas} \rightarrow \text{sueldo anual}$$

27.  $x \rightarrow$  Ahorros

$$x - \frac{3x}{8} - \frac{5x}{12} = \frac{5x}{24} \rightarrow \text{Le quedaba}$$

$$\frac{3}{5} \cdot \frac{5x}{24} = \frac{x}{8}$$

$$\frac{5x}{24} - \frac{x}{8} = 400 \quad mcm = 24$$

$$5x - 3x = 9.600$$

$$2x = 9.600$$

$$x = 4.800$$

$$\$4.800 \rightarrow \text{ahorros}$$

28.  $x \rightarrow$  Parte mayor

$350 - x \rightarrow$  Parte menor

$$350 - x - \frac{3x}{5} = x - \frac{17}{15}(350 - x)$$

$$mcm = 15$$

$$5.250 - 15x - 9x = 15x - 5.950 + 17x$$

$$-24x - 32x = -11.200$$

$$-56x = -11.200$$

$$x = 200$$

$$200 \rightarrow \text{Parte mayor}$$

$$350 - 200 = 150 \rightarrow \text{Parte menor}$$

29.  $x \rightarrow$  Suma repartida

$$\$15 \rightarrow \text{Recibió A}$$

$$x - 15 = B + C$$

$$x - 15 - C = B$$

$$C = A + B$$

$$C = 15 + (x - 15 - C)$$

$$C = x - C$$

$$2C = x$$

$$C = \frac{x}{2}$$

$$15 + 15 + \frac{2}{3} \cdot \frac{x}{2} + \frac{x}{2} = x$$

$$30 + \frac{x}{3} + \frac{x}{2} = x \quad mcm = 6$$

$$180 + 2x + 3x = 6x$$

$$180 = x$$

$$\$180 \rightarrow \text{Suma recibida}$$

30.  $x \rightarrow$  Pesos

$$\frac{3}{4}x \rightarrow \text{N}^\circ \text{ piezas de } 20 \text{ ctvs}$$

$$\frac{2}{3} \cdot \frac{3x}{4} = \frac{x}{2} \rightarrow \text{N}^\circ \text{ piezas de } 10 \text{ ctvs}$$

$$x + \frac{3}{4}(0,20x) + \frac{0,10x}{2} = 9,60$$

$$mcm = 4$$

$$4x + 0,6x + 0,2x = 38,4$$

$$4,8x = 38,4$$

$$x = 8$$

$$8 \rightarrow \text{Pesos}$$

$$\frac{3}{4} \cdot 8 = 6 \rightarrow \text{piezas de } 20 \text{ ctvs}$$

$$\frac{8}{2} = 4 \rightarrow \text{piezas de } 10 \text{ ctvs}$$

31.  $x \rightarrow$  Capital primitivo

$$\frac{1}{5}x \rightarrow 1^{\text{er}} \text{ año}$$

$$x - \frac{x}{5} = \frac{4x}{5} \rightarrow \text{Le quedaba}$$

$$\frac{3}{10} \cdot \frac{4x}{5} = \frac{6x}{25} \rightarrow 2^{\text{o}} \text{ año}$$

$$\frac{4x}{5} + \frac{6x}{25} = \frac{26x}{25} \rightarrow \text{Tenía al term.}$$

$$\text{el } 2^{\text{o}} \text{ año}$$

$$\frac{3}{5} \cdot \frac{26x}{25} = \frac{78x}{125} \rightarrow 3^{\text{er}} \text{ año}$$

$$x - \frac{x}{5} + \frac{6x}{25} + \frac{78x}{125} = 13.312$$

$$125x - 25x + 30x + 78x = 1'664.000$$

$$208x = 1'664.000$$

$$x = 8.000$$

$$8.000 \text{ Q.} \rightarrow \text{Capital primitivo}$$

32.  $x \rightarrow$  Edad de A

$$x - 10 = \frac{2}{3}(x + 5) \quad mcm = 3$$

$$3x - 30 = 2x + 10$$

$$x = 40$$

$$40 \text{ años} \rightarrow \text{Edad de A}$$

33.  $x \rightarrow$  Cant. hombres

$$\text{en el lado del } 1^{\text{er}} \text{ cuadrado}$$

$$x^2 + 36 \rightarrow \text{Hombres en la tropa}$$

$$(x + 1)^2 = x^2 + 36 + 75$$

Continúa

Continuación

$$33. x^2 + 2x + 1 = x^2 + 111$$

$$2x = 110$$

$$x = 55$$

$$55 \rightarrow \text{hombres en el lado}$$

$$\text{del } 1^{\text{er}} \text{ cuadrado}$$

$$(55)^2 + 36$$

$$= 3.061 \rightarrow \text{Homb. en la tropa}$$

34.  $x \rightarrow$  Lo que tenía

$$x - \frac{5x}{8} - 20 = \frac{x}{4} + 16$$

$$mcm = 8$$

$$8x - 5x - 160 = 2x + 128$$

$$x = 288$$

$$\$288 \rightarrow \text{tenía}$$

35.  $x \rightarrow$  Empezó a jugar

$$2x - 60 \rightarrow \text{Le quedaba}$$

$$\frac{3}{10}(2x - 60) = \frac{6x - 180}{10}$$

$$x + x - 60 - \left( \frac{6x - 180}{10} \right) - \frac{7x}{8} = 0$$

$$mcm = 40$$

$$80x - 2.400 - 24x + 720 - 35x = 0$$

$$21x = 1.680$$

$$x = 80$$

$$80 \text{ Lempiras} \rightarrow \text{Empezó a jugar}$$

36.  $x \rightarrow$  Cifra unidades

$$x + 5 \rightarrow \text{Cifra decenas}$$

$$10(x + 5) + x - 18 = 6(x + 5 + x)$$

$$10x + 50 + x - 18 = 12x + 30$$

$$2 = x$$

$$2 \rightarrow \text{Cifra unidades}$$

$$2 + 5 = 7 \rightarrow \text{Cifra decenas}$$

$$72 \rightarrow \text{N}^\circ \text{ buscado}$$

37.  $x \rightarrow$  Unidades

$$9 - x \rightarrow \text{Decenas}$$

$$10(9 - x) + x - 27 = 10x + 9 - x$$

$$90 - 10x + x - 27 = 10x + 9 - x$$

$$54 = 18x$$

$$3 = x$$

$$3 \rightarrow \text{Unidades}$$

$$9 - 3 = 6 \rightarrow \text{Decenas}$$

$$63 \rightarrow \text{N}^\circ \text{ buscado}$$

38.  $x \rightarrow$  Mangos que había

$$x - \frac{x}{3} - 4 = \frac{2x-12}{3} \rightarrow \text{Quedaban}$$

$$\frac{1}{3} \left( \frac{2x-12}{3} \right) + 6 = \frac{2x+42}{9}$$

$$\frac{2x+42}{9} + 9 = \frac{x+102}{9}$$

$$x - \frac{x}{3} - 4 - \left( \frac{2x+42}{9} \right) - \left( \frac{x+102}{9} \right) = 0$$

$$mcm = 9$$

$$9x - 3x - 36 - 2x - 42 - x - 102 = 0$$

$$3x = 180$$

$$x = 60$$

60  $\rightarrow$  Mangos había

39.  $x \rightarrow$  Ganó c / u

$$\frac{7}{10}(80+x) = 50+x \quad mcm = 10$$

$$560+7x = 500+10x$$

$$60 = 3x$$

$$20 = x$$

\$20  $\rightarrow$  Ganó c / u

40.  $x \rightarrow$  Pluma fuente

$$\frac{3}{5}x \rightarrow \text{Lapicero}$$

$$\frac{5}{6}(x-0,20) = \frac{3}{5}x + 0,30$$

$$mcm = 30$$

$$25x-5 = 18x+9$$

$$7x = 14$$

$$x = 2$$

\$2  $\rightarrow$  Costo la pluma

$$\frac{3 \cdot 2}{5} = \$1,20 \rightarrow \text{Costo lapicero}$$

41.  $x \rightarrow$  Tenía el lunes antes de

$$\frac{x}{2} + 2 = \frac{x+4}{2} \rightarrow \text{Lunes}$$

$$x - \left( \frac{x+4}{2} \right) = \frac{x-4}{2} \rightarrow \text{Quedaba}$$

$$\frac{x-4}{2} + 2 = \frac{x+4}{4} \rightarrow \text{Martes}$$

$$\frac{x-4}{2} - \left( \frac{x+4}{4} \right) = \frac{x-12}{4} \rightarrow \text{Quedaba}$$

$$\frac{x-12}{4} + 2 = \frac{x+4}{8} \rightarrow \text{Miercoles}$$

Continúa

## Continuación

$$41. \frac{x-12}{4} - \left( \frac{x+4}{8} \right) = 0 \quad mcm = 8$$

$$2x-24-x-4=0$$

$$x-28=0$$

$$x=28$$

$\Rightarrow$  \$28  $\rightarrow$  Tenía el lunes antes de

42.  $x \rightarrow$  Capital primitivo

$$x + \frac{x}{2} - 6.000 = \frac{3x-12.000}{2} \rightarrow \text{Tenía}$$

$$\frac{3x-12.000}{2}$$

$$\frac{2}{2} - 6.000 = \frac{3x-36.000}{4} \rightarrow 2^\circ \text{ año}$$

$$\frac{3x-12.000}{2} + \frac{3x-36.000}{4} = \frac{9x-60.000}{4} \rightarrow \text{Tenía}$$

$$\frac{9x-60.000}{4}$$

$$\frac{4}{2} - 6.000 = \frac{9x-108.000}{8} \rightarrow 3^\circ \text{ año}$$

$$\frac{9x-60.000}{4} + \frac{9x-108.000}{8} = 32.250 \quad mcm = 8$$

$$18x-120.000+9x-108.000=258.000$$

$$27x=486.000$$

$$x=18.000$$

\$18.000  $\rightarrow$  Capital primitivo

43.  $x \rightarrow$  Precio traje

\$15  $\rightarrow$  Precio bastón

$$s+15 = \frac{3x}{4}$$

$$x+15-5=2s$$

$$\frac{x+10}{2} \rightarrow \text{Precio sombrero}$$

$$\frac{x+10}{2} + 15 = \frac{3x}{4} \quad mcm = 4$$

$$2x+20+60=3x$$

$$80=x$$

\$80  $\rightarrow$  Precio traje

$$\frac{80+10}{2} = \$45 \rightarrow \text{Precio sombrero}$$

44.  $x \rightarrow$  Espacio recorrido por perro y conejo

Perro Conejo

sup uesto 2 5

Pr egunta ? 50

Luego:  $\frac{2:50}{5} \rightarrow$  Saltos del perro con respecto a la ventaja

Perro Conejo

sup uesto 3 8

Pr egunta x ?

Luego:  $\frac{8x}{3} \rightarrow$  Saltos que avanza el conejo

al ser alcanzado

Continúa

### Continuación

44. 

	Perro	Conejo
sup uesto	2	5
Pr egunta	x	?

Luego:  $\frac{5x}{2} \rightarrow$  Saltos que da el perro  
para alcanzar al conejo

Entonces:

$$\frac{8x}{3} - \frac{2 \cdot 50}{5} = \frac{5x}{2} \quad mcm=6$$

$$16x - 120 = 15x$$

$$x = 120$$

$$\frac{5 \cdot 120}{2} = 300 \rightarrow \text{Saltos del perro}$$

para alcanzar al conejo

45.  $x \rightarrow$  Espacio recorrido  
por perro y liebre

	Perro	Liebre
sup uesto	3	4
Pr egunta	?	60

Luego:  $\frac{3 \cdot 60}{4} \rightarrow$  Saltos del perro con  
respecto a la ventaja

	Perro	Liebre
sup uesto	5	8
Pr egunta	x	?

Luego:  $\frac{8x}{5} \rightarrow$  Saltos que avanza la  
liebre al ser alcanzada

	Perro	Liebre
sup uesto	3	4
Pr egunta	x	?

Luego:  $\frac{4x}{3} \rightarrow$  Saltos que da el perro  
para alcanzar a la liebre

Entonces:

$$\frac{8x}{5} - \frac{3 \cdot 60}{4} = \frac{4x}{3}$$

$$\frac{8x}{5} - 45 = \frac{4x}{3} \quad mcm=15$$

$$24x - 675 = 20x$$

$$4x = 675$$

$$x = 168,75$$

$$\frac{4}{3} \cdot 168,75 = 225 \rightarrow \text{Saltos del perro}$$

para alcanzar a la liebre

46.



$$AB = x$$

$$AC = 50$$

$$CD = \frac{x}{12}$$

$$BD = 6$$

$$AB + BD = AC + CD$$

$$x + 6 = 50 + \frac{x}{12}$$

$$x - \frac{x}{12} = 44 \quad mcm=12$$

$$12x - x = 528$$

$$11x = 528$$

$$x = \frac{528}{11}$$

$$x = 48$$

47.  $x \rightarrow$  Aporta A

$$\frac{3}{4}x \rightarrow \text{Aporta B}$$

$$x - \frac{x}{5} = \frac{4x}{5} \rightarrow 1^{\text{er}} \text{ año A}$$

$$\frac{3x}{4} + 3.000 = \frac{3x + 12.000}{4} \rightarrow 1^{\text{er}} \text{ año B}$$

$$\frac{4x}{5} + 1.600 = \frac{4x + 8.000}{5} \rightarrow 2^{\text{o}} \text{ año A}$$

$$\frac{3x + 12.000}{4} - \frac{1}{9} \left( \frac{3x + 12.000}{4} \right) \quad mcm=36$$

$$\frac{27x + 108.000 - 3x - 12.000}{36}$$

$$\frac{24x + 96.000}{36} = \frac{3(8x + 32.000)}{36}$$

$$\frac{8x + 32.000}{12} \rightarrow 2^{\text{o}} \text{ año B}$$

Entonces:

$$\frac{4x + 8.000}{5} = \frac{8x + 32.000}{12} \quad mcm=60$$

$$48x + 96.000 = 40x + 160.000$$

$$8x = 64.000$$

$$x = 8.000$$

$$8.000bs \rightarrow \text{Aportó A}$$

$$\frac{3 \cdot 8.000}{4} = 6.000bs \rightarrow \text{Aportó B}$$

48.  $x \rightarrow$  Años para que la edad del padre sea = a la suma de sus hijos

$$60 + x = 16 + x + 14 + x$$

$$60 + x = 30 + 2x$$

$$30 = x$$

30 años  $\rightarrow$  Edad del padre sea igual a la suma de sus hijos

49.  $x \rightarrow$  Km. recorridos

$$\frac{x}{50} + \frac{x}{10} = 12 \quad mcm = 50$$

$$x + 5x = 600$$

$$6x = 600$$

$$x = 100$$

100  $\rightarrow$  Km. recorridos

50. Buey  $\rightarrow$  \$80

$$P + 80 = 2C$$

$$C + 80 = \frac{13P}{2}$$

Luego:

$$\frac{P + 80}{2} = C$$

$$\frac{P + 80}{2} + 80 = \frac{13P}{2} \quad mcm = 2$$

$$P + 80 + 160 = 13P$$

$$240 = 12P$$

$$20 = P$$

\$20  $\rightarrow$  Costo el perro

$$\frac{20 + 80}{2} = C \quad mcm = 2$$

$$20 + 80 = 2C$$

$$100 = 2C$$

$$50 = C$$

\$50  $\rightarrow$  Costo el caballo

## EJERCICIO 159

1.  $a = 30m$

$$v = 8 \frac{m}{s}$$

$$v' = 5 \frac{m}{s}$$

$$x = \frac{30 \cdot 8}{8 - 5}$$

$$x = \frac{240}{3}$$

$$x = 80m$$

2.  $a = 160 Km$

$$v = 50 \frac{Km}{h}$$

$$v' = -30 \frac{Km}{h}$$

$$x = \frac{160 \cdot 50}{50 - (-30)}$$

$$x = \frac{8.000}{80}$$

$$x = 100 Km.$$

3.  $a = 200 Km$

$$v = 90 \frac{Km}{h}$$

$$v' = 40 \frac{Km}{h}$$

$$x = \frac{200 \cdot 90}{90 - 40}$$

Continúa

### Continuación

3.  $x = \frac{18.000}{50}$

$$x = 360 Km \text{ de A}$$

De B.

Como la distancia entre A y B

es 200 Km

Entonces B recorrió

$$360 Km - 200 Km = 160 Km$$

4.  $a = 80 Km$

$$v = 90 \frac{Km}{h}$$

$$v' = 70 \frac{Km}{h}$$

$$x = \frac{80 \cdot 90}{90 - 70}$$

$$x = \frac{7.200}{20}$$

$$x = 360 Km$$

El tiempo empleado en encontrarse será.

$$\frac{360 Km}{90 \frac{Km}{h}} = 4 \text{ horas}$$

5.  $a = 550 Km$

$$v = 100 \frac{Km}{h}$$

$$v' = -120 \frac{Km}{h}$$

De A.

$$x = \frac{550 \cdot 100}{100 - (-120)}$$

$$x = \frac{55.000}{220}$$

$$x = 250 Km$$

Como los trenes pasan

por A y B a las 8am

Entonces De A.

$$x = 250 Km$$

$$v = 100 \frac{Km}{h}$$

Tiempo empleado para los 250 Km. es

$$\frac{250 Km}{100 \frac{Km}{h}} = 2,5 \text{ horas}$$

$$2,5 \text{ horas} \rightarrow 2 h 30 \text{ min.}$$

Luego:

$$8am + 2 h 30 \text{ min} = 10:30$$

10:30am  $\rightarrow$  Se encuentran

6.  $a = 70 \text{ Km}$

$$v = 9 \text{ Km/h}$$

$$v' = -5 \text{ Km/h}$$

$x \rightarrow$  Distancia recorrida por  
A al encontrarse

$$x = \frac{70 \cdot 9}{9 - (-5)}$$

$$x = \frac{630}{14}$$

$$x = 45 \text{ Km} \rightarrow A$$

Como A recorrió 45 Km

Entonces B

$$70 \text{ Km} - 45 \text{ Km} = 25 \text{ Km} \rightarrow B$$

9.  $a = 186 \text{ Km}$

$$v = v$$

$$v' = -v$$

$$x = \frac{186v}{v - (-v)}$$

$$x = \frac{186v}{2v}$$

$$x = 93 \text{ Km.}$$

7. A y B distantes  $29\frac{1}{2} \text{ Km}$

$$\Rightarrow 29\frac{1}{2} \text{ Km} - 2\frac{1}{2} \text{ Km} = 27 \text{ Km}$$

$$a = 27 \text{ Km}$$

$$v = 5 \text{ Km/h}$$

$$v' = -4 \text{ Km/h}$$

$$x = \frac{27 \cdot 5}{5 - (-4)}$$

$$x = \frac{135}{9}$$

$$x = 15 \text{ Km}$$

Luego A recorre

$$15 \text{ Km} + 2\frac{1}{2} \text{ Km} = 17\frac{1}{2} \text{ Km}$$

B recorre

$$29\frac{1}{2} \text{ Km} - 17\frac{1}{2} \text{ Km} = 12 \text{ Km}$$

8. Recorrido tren de carga  
en 3 horas es

$$42 \frac{\text{Km}}{\text{h}} (3\text{h}) = 126 \text{ Km}$$

$$a = 126 \text{ Km}$$

$$v = 60 \text{ Km/h}$$

$$v' = 42 \text{ Km/h}$$

$$x = \frac{126 \cdot 60}{60 - 42}$$

$$x = \frac{7.560}{18}$$

$$x = 420 \text{ Km}$$

420 Km  $\rightarrow$  Alcanza el tren de  
pasajeros al de carga

El tiempo empleado es:

$$\frac{420 \text{ Km}}{60 \text{ Km/h}} = 7 \text{ horas}$$

## EJERCICIO 162

1.  $b = 10 \text{ cm}$

$$h = 8 \text{ cm}$$

$$A = \frac{b \cdot h}{2}$$

$$A = \frac{10 \text{ cm} \cdot 8 \text{ cm}}{2}$$

$$A = 40 \text{ cm}^2$$

2.  $d = 8 \text{ m}$

$$A = \frac{d^2}{2}$$

$$A = \frac{(8 \text{ m})^2}{2}$$

$$A = 32 \text{ m}^2$$

3.  $t = 15 \text{ s}$

$$v = 9 \text{ m/s}$$

$$e = v \cdot t$$

$$e = (15 \text{ s}) (9 \text{ m/s})$$

$$e = 135 \text{ m}$$

4.  $v = 9 \text{ m/s}$

$$e = 108 \text{ m}$$

$$t = \frac{e}{v} \Rightarrow t = \frac{108 \text{ m}}{9 \text{ m/s}}$$

$$t = 12 \text{ s}$$

5.  $b = 4 \text{ m}$

$$c = 3 \text{ m}$$

$$a^2 = b^2 + c^2$$

$$a^2 = (4 \text{ m})^2 + (3 \text{ m})^2$$

$$a^2 = 16 \text{ m}^2 + 9 \text{ m}^2$$

$$a^2 = 25 \text{ m}^2$$

$$a = \sqrt{25 \text{ m}^2}$$

$$a = 5 \text{ m}$$

6.  $a = 13 \text{ m}$

$$c = 5 \text{ m}$$

$$b^2 = a^2 - c^2$$

$$b^2 = (13 \text{ m})^2 - (5 \text{ m})^2$$

$$b^2 = 169 \text{ m}^2 - 25 \text{ m}^2$$

$$b^2 = 144 \text{ m}^2$$

$$b = \sqrt{144 \text{ m}^2}$$

$$b = 12 \text{ m}$$

7.  $r = 5 \text{ m}$

$$\pi = 3,14$$

$$A = \pi \cdot r^2$$

$$A = 3,14 (5 \text{ m})^2$$

$$A = 3,14 \cdot 25 \text{ m}^2$$

$$A = 78,5 \text{ m}^2$$

8.  $r = 5 \text{ m}$

$$\pi = 3,14$$

$$C = 2\pi \cdot r$$

$$C = 2 \cdot 3,14 \cdot 5 \text{ m}$$

$$C = 10 \text{ m} \cdot 3,14$$

$$C = 31,4 \text{ m}$$

9.  $h = 9 \text{ m}$

$$r = 2 \text{ m}$$

$$\pi = 3,14$$

$$v = \frac{h \cdot \pi \cdot r^2}{3}$$

$$v = \frac{9 \text{ m} \cdot 3,14 (2 \text{ m})^2}{3}$$

$$v = 12 \text{ m}^2 \cdot 3,14$$

$$v = 37,68 \text{ m}^2$$

10.  $P = 8,24 \text{ g}$

$$V = 8 \text{ cm}^3$$

$$D = \frac{P}{V}$$

$$D = \frac{8,24 \text{ g}}{8 \text{ cm}^3}$$

$$D = 1,03 \text{ g/cm}^3$$

11.  $L = 4 \text{ m}$

$$A = \frac{L^2 \sqrt{3}}{4}$$

$$A = \frac{(4 \text{ m})^2 \sqrt{3}}{4}$$

$$A = 4 \text{ m}^2 \sqrt{3}$$

$$A = 6,92 \text{ m}^2$$

12.  $N = 6$

$$S = 180^\circ (6 - 2)$$

$$S = 180^\circ (4)$$

$$S = 720^\circ$$

## EJERCICIO 163

1.  $e = v \cdot t$

$$v = \frac{e}{t} \quad Rta$$

$$t = \frac{e}{v} \quad Rta$$

2.  $A = h \left( \frac{b+b'}{2} \right)$

$$2A = h(b+b')$$

$$\frac{2A}{b+b'} = h \quad Rta$$

3.  $e = \frac{1}{2} a \cdot t^2$

$$2e = a \cdot t^2$$

$$\frac{2e}{t^2} = a \quad Rta$$

4.  $A = \frac{1}{2} a \cdot l \cdot n$

$$2A = a \cdot l \cdot n$$

$$\frac{2A}{l \cdot n} = a \quad Rta$$

$$\frac{2A}{n \cdot a} = l \quad Rta$$

$$\frac{2A}{l \cdot a} = n \quad Rta$$

5.  $A = \pi \cdot r^2$

$$\frac{A}{\pi} = r^2$$

$$\sqrt{\frac{A}{\pi}} = r \quad Rta$$

6.  $a^2 = b^2 + c^2 - 2bx$

$$a^2 - b^2 - c^2 = -2bx$$

$$-\frac{a^2 - b^2 - c^2}{2b} = x$$

$$\frac{b^2 + c^2 - a^2}{2b} = x \quad Rta$$

7.  $\bar{V} = V_0 + a \cdot t$

$$\bar{V} - a \cdot t = V_0 \quad Rta$$

$$\bar{V} - V_0 = a \cdot t$$

$$\frac{\bar{V} - V_0}{t} = a \quad Rta$$

$$\frac{\bar{V} - V_0}{a} = t \quad Rta$$

8.  $\bar{V} = V_0 - a \cdot t$

$$\bar{V} + a \cdot t = V_0 \quad Rta$$

$$a \cdot t = V_0 - \bar{V}$$

$$a = \frac{V_0 - \bar{V}}{t} \quad Rta$$

$$t = \frac{V_0 - \bar{V}}{a} \quad Rta$$

9.  $D = \frac{P}{V}$

$$DV = P$$

$$V = \frac{P}{D} \quad Rta$$

$$P = VD \quad Rta$$

10.  $a^2 = b^2 + c^2$

$$a^2 - c^2 = b^2$$

$$\sqrt{a^2 - c^2} = b \quad Rta$$

$$a^2 - b^2 = c^2$$

$$\sqrt{a^2 - b^2} = c \quad Rta$$

11.  $V = a \cdot t$

$$\frac{V}{a} = t \quad Rta$$

$$a = \frac{V}{t} \quad Rta$$

12.  $\frac{1}{f} = \frac{1}{p'} - \frac{1}{p}$

$$\frac{p' \cdot p}{f \cdot p' \cdot p} = \frac{f \cdot p - f \cdot p'}{f \cdot p' \cdot p}$$

$$p' \cdot p = f(p - p')$$

$$p' = \frac{f(p - p')}{p}$$

$$p' = \frac{f \cdot p}{p} - \frac{f \cdot p'}{p}$$

$$p' = f - \frac{f \cdot p'}{p}$$

$$p' + \frac{f \cdot p'}{p} = f$$

$$\frac{p' \cdot p + f \cdot p'}{p} = f$$

$$p'(p + f) = p \cdot f$$

$$p' = \frac{p \cdot f}{p + f} \quad Rta$$

Continúa

## Continuación

12.  $\frac{p' \cdot p + f \cdot p'}{p} = f$

$$\frac{p' \cdot p}{p} + \frac{f \cdot p'}{p} = f$$

$$p' + \frac{f \cdot p'}{p} = f$$

$$\frac{f \cdot p'}{p} = f - p'$$

$$f \cdot p' = p(f - p')$$

$$\frac{f \cdot p'}{f - p'} = p \quad Rta$$

13.  $v = \sqrt{\frac{e}{d}}$

$$v^2 = \frac{e}{d}$$

$$v^2 \cdot d = e \quad Rta$$

$$d = \frac{e}{v^2} \quad Rta$$

14.  $e = V_0 \cdot t + \frac{1}{2} a \cdot t^2$

$$2e = 2V_0 \cdot t + a \cdot t^2$$

$$2e - a \cdot t^2 = 2V_0 \cdot t$$

$$\frac{2e - a \cdot t^2}{2 \cdot t} = V_0 \quad Rta$$

15.  $e = V_0 \cdot t - \frac{1}{2} a \cdot t^2$

$$2e = 2V_0 \cdot t - a \cdot t^2$$

$$2e + a \cdot t^2 = 2V_0 \cdot t$$

$$\frac{2e + a \cdot t^2}{2 \cdot t} = V_0 \quad Rta$$

$$a \cdot t^2 = 2V_0 \cdot t - 2e$$

$$a = \frac{2V_0 \cdot t - 2e}{t^2}$$

$$a = \frac{2(V_0 \cdot t - e)}{t^2} \quad Rta$$

16.  $V = \frac{h \cdot \pi \cdot r^2}{3}$

$$\frac{3V}{\pi \cdot r^2} = h \quad Rta$$

$$\frac{3V}{\pi \cdot h} = r^2$$

$$\sqrt{\frac{3V}{\pi \cdot h}} = r \quad Rta$$

17.  $l = \frac{c \cdot t \cdot r}{100}$

$$100l = c \cdot t \cdot r$$

$$\frac{100l}{t \cdot r} = c \quad Rta$$

$$\frac{100l}{r \cdot c} = t \quad Rta$$

$$\frac{100l}{c \cdot t} = r \quad Rta$$

18.  $E = I \cdot R$

$$\frac{E}{I} = R \quad Rta$$

$$\frac{E}{R} = I \quad Rta$$

19.  $e = \frac{V^2}{2a}$

$$2ae = V^2$$

$$\sqrt{2ae} = V \quad Rta$$

20.  $u = a + (n-1)r$

$$u - (n-1)r = a \quad Rta$$

$$u - a = (n-1)r$$

$$\frac{u - a}{r} = n - 1$$

$$\frac{u - a}{r} + 1 = n$$

$$\frac{u - a + r}{r} = n \quad Rta$$

$$u - a + r = nr$$

$$u - a = nr - r$$

$$u - a = r(n-1)$$

$$\frac{u - a}{n - 1} = r \quad Rta$$

21.  $u = a \cdot r^{n-1}$

$$\frac{u}{r^{n-1}} = a \quad Rta$$

$$\frac{u}{a} = r^{n-1}$$

$$\sqrt[n-1]{\frac{u}{a}} = r \quad Rta$$

22.  $l = \frac{Q}{t}$

$$Q = l \cdot t \quad Rta$$

$$\frac{Q}{l} = t \quad Rta$$

## EJERCICIO 164

1.  $x-5 < 2x-6$   
 $-5+6 < 2x-x$   
 $1 < x$   
 $1 \rightarrow \text{Lim. inferior de } x$
2.  $5x-12 > 3x-4$   
 $5x-3x > 12-4$   
 $2x > 8$   
 $x > 4$   
 $4 \rightarrow \text{Lim. inferior de } x$
3.  $x-6 > 21-8x$   
 $x+8x > 21+6$   
 $9x > 27$   
 $x > 3$   
 $3 \rightarrow \text{Lim. inferior de } x$
4.  $3x-14 < 7x-2$   
 $-14+2 < 7x-3x$   
 $-12 < 4x$   
 $-3 < x$
5.  $2x - \frac{5}{3} > \frac{x}{3} + 10$   
 $6x-5 > x+30$   
 $6x-x > 30+5$   
 $5x > 35$   
 $x > 7$
6.  $3x-4 + \frac{x}{4} < \frac{5x}{2} + 2$   
 $12x-16+x < 10x+8$   
 $12x+x-10x < 16+8$   
 $3x < 24$   
 $x < 8$
7.  $(x-1)^2 - 7 > (x-2)^2$   
 $x^2-2x+1-7 > x^2-4x+4$   
 $-6-4 > -4x+4$   
 $-10 > -2x$   
 $10 < 2x$   
 $5 < x$
8.  $(x+2)(x-1) + 26 < (x+4)(x+5)$   
 $x^2+x-2+26 < x^2+9x+20$   
 $24-20 < 9x-x$   
 $4 < 8x$   
 $\frac{1}{2} < x$
9.  $3(x-2) + 2x(x+3) > (2x-1)(x+4)$   
 $3x-6+2x^2+6x > 2x^2+7x-4$   
 $9x-6 > 7x-4$   
 $9x-7x > -4+6$   
 $2x > 2$   
 $x > 1$
10.  $6(x^2+1) - (2x-4)(3x+2) < 3(5x+21)$   
 $6x^2+6-6x^2+8x+8 < 15x+63$   
 $8x+14 < 15x+63$   
 $8x-15x < 63-14$   
 $-7x < 49$   
 $7x > -49$   
 $x > -7$
11.  $(x-4)(x+5) < (x-3)(x-2)$   
 $x^2+x-20 < x^2-5x+6$   
 $5x+x < 20+6$   
 $6x < 26$   
 $x < \frac{26}{6}$   
 $x < \frac{13}{3}$
12.  $(2x-3)^2 + 4x^2(x-7) < 4(x-2)^3$   
 $4x^2-12x+9+4x^3-28x^2 < 4x^3-24x^2+48x-32$   
 $-12x-48x < -9-32$   
 $-60x < -41$   
 $60x > 41$   
 $x > \frac{41}{60}$
13.  $\frac{2x+1}{3x-1} > \frac{2x+5}{3x+2}$   
 $(3x+2)(2x+1) > (2x+5)(3x-1)$   
 $6x^2+7x+2 > 6x^2+13x-5$   
 $2+5 > 13x-7x$   
 $7 > 6x$   
 $\frac{7}{6} > x$
14.  $\frac{x+3}{3} - \frac{4}{x+2} > \frac{x}{3}$   
 $(x+2)(x+3) - 12 > x(x+2)$   
 $x^2+5x+6-12 > x^2+2x$   
 $5x-2x > 6$   
 $3x > 6$   
 $x > 2$
15.  $\frac{5}{3x+1} - \frac{20}{9x^2-1} < \frac{2}{3x-1}$   
 $5(3x-1) - 20 < 2(3x+1)$   
 $15x-5-20 < 6x+2$   
 $15x-6x < 25+2$   
 $9x < 27$   
 $x < 3$
16.  $\frac{1}{x^2+x} > \frac{1}{x^2-x} - \frac{1}{x^2-1}$   
 $x-1 > x+1-x$   
 $x > 2$
17.  $x \rightarrow \mathbb{N}^0 \text{ s. enteros}$   
 $\frac{x}{3} + 15 > \frac{x}{2} + 1$   
 $2x+90 > 3x+6$   
 $90-6 > 3x-2x$   
 $84 > x$   
 $\mathbb{N}^0 \text{ s. enteros menores que } 84 \text{ Rta}$

## EJERCICIO 165

1.  $x-3 > 5$  y  $2x+5 > 17$   
 $x-3 > 5$   $2x+5 > 17$   
 $x > 5+3$   $2x > 12$   
 $x > 8$   $x > 6$   
 $x > 8 \text{ Rta}$
2.  $5-x > -6$  y  $2x+9 > 3x$   
 $-x > -6-5$   $-x > -9$   
 $x < 11$   $x < 9$   
 $x < 9 \text{ Rta}$
3.  $6x+5 > 4x+11$   $4-2x > 10-5x$   
 $2x > 6$   $3x > 6$   
 $x > 3$   $x > 2$   
 $x > 3 \text{ Rta}$

$$4. 5x-4 > 7x-16 \text{ y } 8-7x < 16-15x$$

$$12 > 2x$$

$$8x < 8$$

$$6 > x$$

$$x < 1$$

$$x < 1 \text{ Rta}$$

$$5. \frac{x}{2} - 3 > \frac{x}{4} + 2 \text{ y } 2x + \frac{3}{5} < 6x - 23\frac{2}{5}$$

$$2x - 12 > x + 8$$

$$10x + 3 < 30x - 117$$

$$x > 20$$

$$120 < 20x$$

$$6 < x$$

$$x > 20 \text{ Rta}$$

$$6. 2x - 3 < x + 10 \text{ y } 6x - 4 > 5x + 6$$

$$2x - x < 10 + 3$$

$$6x - 5x > 6 + 4$$

$$x < 13$$

$$x > 10$$

$$10 < x < 13 \text{ Rta}$$

$$7. \frac{x}{4} - 1 > \frac{x}{3} - 1\frac{1}{2} \text{ y } 2x - 3\frac{3}{5} > x + \frac{2}{5}$$

$$3x - 12 > 4x - 18$$

$$10x - 18 > 5x + 2$$

$$18 - 12 > 4x - 3x$$

$$5x > 20$$

$$6 > x$$

$$x > 4$$

$$4 < x < 6 \text{ Rta}$$

$$8. (x-1)(x+2) < (x+2)(x-3) \text{ y } (x+3)(x+5) > (x+4)(x+3)$$

$$x^2 + x - 2 < x^2 - x - 6$$

$$x^2 + 8x + 15 > x^2 + 7x + 12$$

$$2x < -4$$

$$x > -3$$

$$x < -2$$

$$-3 < x < -2 \text{ Rta}$$

$$9. \frac{x+2}{x+8} > \frac{x-2}{x+3} \text{ y } \frac{x-1}{x+4} < \frac{x-5}{x-1}$$

$$(x+2)(x+3) > (x-2)(x+8)$$

$$(x-1)^2 < (x-5)(x+4)$$

$$x^2 + 5x + 6 > x^2 + 6x - 16$$

$$x^2 - 2x + 1 < x^2 - x - 20$$

$$22 > x$$

$$21 < x$$

$$21 < x < 22 \text{ Rta}$$

$$10. x \rightarrow \mathbb{N}^0 \text{ s. enteros}$$

$$3x - 6 > \frac{x}{2} + 4 \text{ y } 4x + 8 < 3x + 15$$

$$6x - 12 > x + 8$$

$$x < 7$$

$$5x > 20$$

$$x > 4$$

$$4 < x < 7 \text{ Luego son 5 y 6 Rta}$$

## EJERCICIO 166

$$1. x = Ky$$

$$9 = K6$$

$$\frac{9}{6} = K$$

$$x = \frac{3}{2} \cdot 8$$

$$x = 12$$

$$2. x = Ky$$

$$2 = K3$$

$$\frac{2}{3} = K$$

$$24 = \frac{2}{3}y$$

$$\frac{3 \cdot 24}{2} = y$$

$$36 = y$$

$$3. A = KBC$$

$$30 = K(2)(5)$$

$$\frac{30}{10} = K$$

$$3 = K$$

$$A = 3(7)(4)$$

$$A = 84$$

$$4. x = KYZ$$

$$4 = K(3)(6)$$

$$\frac{4}{18} = K$$

$$\frac{2}{9} = K$$

$$10 = Ky9$$

$$10 = \frac{2}{9}y9$$

$$10 = 2y$$

$$5 = y$$

$$5. A = \frac{K}{B}$$

$$3 = \frac{K}{5}$$

$$15 = K$$

$$A = \frac{K}{7}$$

$$A = \frac{15}{7} = 2\frac{1}{7}$$

$$6. B = \frac{K}{A}$$

$$\frac{1}{3} = \frac{K}{\frac{1}{2}}$$

$$\frac{1}{3} = 2K$$

$$\frac{1}{6} = K$$

$$\frac{1}{12} = \frac{6}{A} \Rightarrow \frac{1}{12} = \frac{1}{6A}$$

$$\Rightarrow 6A = 12 \Rightarrow A = 2$$

$$7. A = \frac{KB}{C}$$

$$8 = \frac{K12}{3}$$

$$8 = 4K$$

$$2 = K$$

$$A = \frac{2 \cdot 7}{14}$$

$$A = 1$$

$$8. x = \frac{Ky}{Z}$$

$$3 = \frac{4K}{8}$$

$$24 = 4K$$

$$6 = K$$

$$10 = \frac{7 \cdot 6}{Z}$$

$$Z = \frac{42}{10}$$

$$Z = 4\frac{1}{5}$$

$$9. x = K(y^2 - 1)$$

$$48 = K(5^2 - 1)$$

$$48 = K(25 - 1)$$

$$\frac{48}{24} = K$$

$$2 = K$$

$$x = 2(7^2 - 1)$$

$$x = 2 \cdot 48$$

$$x = 96$$

$$10. x = \frac{K}{y^2 - 1}$$

$$9 = \frac{K}{(3^2 - 1)}$$

$$9 = \frac{K}{8}$$

$$72 = K$$

$$x = \frac{72}{(5^2 - 1)}$$

$$x = \frac{72}{24}$$

$$x = 3$$

$$11. A \rightarrow \text{Area del cuadrado}$$

$$d \rightarrow \text{Diagonal}$$

$$A = Kd^2$$

$$18m^2 = K(6m)^2$$

$$\frac{18m^2}{36m^2} = K$$

$$\frac{1}{2} = K$$

$$A = \frac{1}{2}(10m)^2$$

$$A = \frac{100m^2}{2}$$

$$A = 50m^2$$

$$12. Ap \rightarrow \text{Area piramide}$$

$$a \rightarrow \text{apotema}$$

$$pb \rightarrow \text{perim. de la b.}$$

$$Ap = Kapb$$

$$480m^2 = K(12m)(80m)$$

$$480m^2 = K(960m^2)$$

$$\frac{480m^2}{960m^2} = K$$

$$\frac{1}{2} = K$$

$$Ap = \frac{1}{2}(6m)(40m)$$

$$Ap = \frac{240m^2}{2}$$

$$Ap = 120m^2$$



13.  $Vp \rightarrow \text{Vol. piramide}$

$h \rightarrow \text{Altura}$

$Ab \rightarrow \text{Area base}$

$Vp = KhAb$

$96m^3 = K \cdot 8m(36m^2)$

$96m^3 = K(288m^3)$

$\frac{1}{3} = K$

$Vp = \frac{1}{3} \cdot 12m(64m^2)$

$Vp = \frac{768m^3}{3}$

$Vp = 256m^3$

14.  $Ao \rightarrow \text{Area circulo}$

$r \rightarrow \text{Radio}$

$Ao = Kr^2$

$616cm^2 = K(14cm)^2$

$\frac{616cm^2}{196cm^2} = K$

$\frac{22}{7} = K$

$Ao = \frac{22}{7}(7cm)^2$

$Ao = 22 \cdot 7cm^2$

$Ao = 154cm^2$

15.  $Lo \rightarrow \text{Long. circunf.}$

$r \rightarrow \text{Radio}$

$Lo = Kr$

$44cm = K \cdot 7cm$

$\frac{44}{7} = K$

$66cm = \frac{44r}{7}$

$462cm = 44r$

$\frac{462}{44}cm = r$

$10\frac{1}{2}cm = r$

16.  $x = \frac{K}{y^2}$

$4 = \frac{K}{6^2}$

$36 \cdot 4 = K$

$144 = K$

$9 = \frac{144}{y^2}$

$y^2 = \frac{144}{9}$

$y = \pm \sqrt{\frac{144}{9}}$

$y = \pm \frac{12}{3}$

$y = \pm 4$

## EJERCICIO 167

1.  $A = KB$

$10 = K5$

$2 = K \Rightarrow A = 2B$

2.  $e \rightarrow \text{Espacio}$

$V \rightarrow \text{Velocidad}$

$t \rightarrow \text{Tiempo}$

$e = Kvt$

Si  $K=1 \Rightarrow e = Vt$

3.  $Ar \rightarrow \text{Area rombo}$

$D \rightarrow \text{Diag mayor}$

$D' \rightarrow \text{Diag menor}$

$A = KDD'$

$24cm^2 = K \cdot 8cm \cdot 6cm$

$24cm^2 = K \cdot 48cm^2$

$\frac{1}{2} = K$

$A = \frac{1}{2}DD'$

4.  $A = \frac{KB}{C}$

$K=3 \Rightarrow A = \frac{3B}{C}$

5.  $C = Kr$

$132cm = K \cdot 21cm$

$\frac{132cm}{21cm} = K$

$\frac{44}{7} = K$

$C = \frac{44}{7}r$

6.  $C \rightarrow \text{Espacio (h)}$

$t \rightarrow \text{Tiempo}$

$h \rightarrow \text{Altura (e)}$

$e = K \cdot t^2$

$19,6m = K(2s)^2$

$\frac{19,6m}{4s^2} = K$

$4,9\frac{m}{s^2} = K$

$e = 4,9t^2$

7.  $F = \frac{KmV^2}{r}$

8.  $y \rightarrow \text{Función}$

$x \rightarrow \text{Vble. indep.}$

$y = 2x + 3$

9.  $L \rightarrow \text{Lado de un cuad.}$

$\text{inscrito en un circulo}$

$r \rightarrow \text{Radio}$

$L = K \cdot r$

$K = \sqrt{2}$

$L = \sqrt{2} \cdot r$

10.  $y \rightarrow \text{Función}$

$x \rightarrow \text{Vble. indep.}$

$y = \frac{x^2}{2} + 2$

11.  $y = \frac{5-2x}{3}$

12.  $F \rightarrow \text{Fuerza de atracc.}$

$m \rightarrow \text{Masa cuerpo1}$

$m' \rightarrow \text{Masa cuerpo2}$

$d \rightarrow \text{Distancia}$

$F = \frac{K \cdot m \cdot m'}{d^2}$

13.  $h \rightarrow \text{Altura } \Delta$

$A \rightarrow \text{Area del } \Delta$

$B \rightarrow \text{Base}$

$h = \frac{K \cdot A}{B}$

$10cm = \frac{K \cdot 20cm^2}{4cm}$

$10cm = K \cdot 5cm$

$2 = K$

$h = \frac{2A}{B}$

14.  $W \rightarrow \text{Energ. cinetica}$

$m \rightarrow \text{Masa}$

$V \rightarrow \text{Velocidad}$

$W = KmV^2$

$K = \frac{1}{2}$

$W = \frac{1}{2}mV^2$

15.  $B \rightarrow$  Area base piramide

$V \rightarrow$  Volumen

$h \rightarrow$  Altura

$$B = \frac{KV}{h}$$

$$100 = \frac{K \cdot 400}{12}$$

$$1.200 = K \cdot 400$$

$$3 = K$$

$$B = \frac{3V}{h}$$

16.  $x = \frac{K}{y}$

$$2 = \frac{K}{5}$$

$$10 = K$$

$$x = \frac{10}{y}$$

17.  $x = \frac{K}{y^2}$

$$3 = \frac{K}{2^2}$$

$$12 = K$$

$$x = \frac{12}{y^2}$$

18.  $A = \frac{KB}{C}$

$$3 = \frac{K \cdot 24}{4}$$

$$12 = K \cdot 24$$

$$\frac{1}{2} = K$$

$$A = \frac{\frac{1}{2}B}{C}$$

$$A = \frac{B}{2C}$$

## EJERCICIO 173

1.  $x + y = 5$

$$y = 5 - x$$

$5 - x \rightarrow$  Entero  $y +$

Para que  $y$  sea entero

$y$  + el mayor valor que

podemos dar a  $x$  es 4.

Por tanto las soluciones

enteras  $y +$  son:

$$x=1 \quad y=4$$

$$x=2 \quad y=3$$

$$x=3 \quad y=2$$

$$x=4 \quad y=1$$

2.  $2x + 3y = 37$

$$2x = 37 - 3y$$

$$x = \frac{37 - 3y}{2}$$

$$x = \frac{36}{2} + \frac{1}{2} - \frac{2y}{2} - \frac{y}{2}$$

$$x = 18 - y + \frac{1 - y}{2}$$

$$x - 18 + y = \frac{1 - y}{2}$$

$$\frac{1 - y}{2} \rightarrow \text{Entero}$$

$$\frac{3 - 3y}{2} \rightarrow \text{Entero}$$

$$\frac{2}{2} + \frac{1}{2} - \frac{2y}{2} - \frac{y}{2}$$

$$1 - y + \frac{1 - y}{2} \rightarrow \text{Entero}$$

Continúa

2. Continuación

$$\frac{1 - y}{2} = m$$

$$1 - y = 2m$$

$$-y = 2m - 1$$

$$y = 1 - 2m$$

$$\Rightarrow 2x + 3(1 - 2m) = 37$$

$$2x + 3 - 6m = 37$$

$$2x = 34 + 6m$$

$$x = \frac{2(17 + 3m)}{2}$$

$$x = 17 + 3m$$

$$y = 1 - 2m \rightarrow m \text{ entero}$$

$$x = 17 + 3m$$

$$m=0 \quad y=1 \quad x=17 \quad \text{sol}$$

$$m=1 \quad y=-1 \rightarrow \text{no}$$

$$m=-1 \quad y=3 \quad x=14 \quad \text{sol}$$

$$m=-2 \quad y=5 \quad x=11 \quad \text{sol}$$

$$m=-3 \quad y=7 \quad x=8 \quad \text{sol}$$

$$m=-4 \quad y=9 \quad x=5 \quad \text{sol}$$

$$m=-5 \quad y=11 \quad x=2 \quad \text{sol}$$

$$m=-6 \quad y=13 \quad x=-1 \rightarrow \text{no}$$

3.  $3x + 5y = 43$

$$3x = 43 - 5y$$

$$x = \frac{43 - 5y}{3}$$

$$x = \frac{42}{3} + \frac{1}{3} - \frac{3y}{3} - \frac{2y}{3}$$

$$x = 14 - y + \frac{1 - 2y}{3}$$

Continúa

3. Continuación

$$x - 14 + y = \frac{1 - 2y}{3}$$

$$\frac{1 - 2y}{3} \rightarrow \text{Entero}$$

$$\frac{2 - 4y}{3} \rightarrow \text{Entero}$$

$$\frac{2}{3} - \frac{3y}{3} - \frac{1y}{3}$$

$$-y + \frac{2 - y}{3}$$

$$\frac{2 - y}{3} = m$$

$$2 - y = 3m$$

$$-y = 3m - 2$$

$$y = 2 - 3m$$

$$3x + 5(2 - 3m) = 43$$

$$3x + 10 - 15m = 43$$

$$3x = 33 + 15m$$

$$x = \frac{3(11 + 5m)}{3}$$

$$x = 11 + 5m$$

$$y = 2 - 3m$$

$$x = 11 + 5m$$

$$m=0 \quad x=11 \quad y=2 \quad \text{sol}$$

$$m=1 \quad x=16 \quad y=-1 \rightarrow \text{No}$$

$$m=-1 \quad x=6 \quad y=5 \quad \text{sol}$$

$$m=-2 \quad x=1 \quad y=8 \quad \text{sol}$$

$$m=-3 \quad x=-4 \rightarrow \text{No}$$

4.  $x+3y=9$

$$x=9-3y$$

$$9-3y \rightarrow \text{Entero}$$

$$3-y \rightarrow \text{Entero}$$

$$3-y=m$$

$$-y=m-3$$

$$y=3-m$$

$$x+3(3-m)=9$$

$$x+9-3m=9$$

$$x=3m$$

$$y=3-m$$

$$x=3m$$

$$m=0 \quad x=0 \quad y=3 \rightarrow \text{No}$$

$$m=1 \quad x=3 \quad y=2 \quad \text{sol}$$

$$m=2 \quad x=6 \quad y=1 \quad \text{sol}$$

$$m=3 \quad x=9 \quad y=0 \rightarrow \text{No}$$

$$m=4 \quad x=12 \quad y=-1 \rightarrow \text{No}$$

5.  $7x+8y=115$

$$7x=115-8y$$

$$x=\frac{115-8y}{7}$$

$$x=\frac{112}{7}+\frac{3}{7}-\frac{7y}{7}-\frac{y}{7}$$

$$x=16-y+\frac{3-y}{7}$$

$$\frac{3-y}{7} \rightarrow \text{Entero}$$

$$\frac{24-8y}{7} \rightarrow \text{Entero}$$

$$\frac{24}{7}-\frac{7y}{7}-\frac{y}{7}$$

$$-y+\frac{24-y}{7}$$

$$\frac{24-y}{7}=m$$

$$24-y=7m$$

$$-y=7m-24$$

$$y=24-7m$$

$$7x+8(24-7m)=115$$

$$7x+192-56m=115$$

$$7x-56m=-77$$

$$x=\frac{7(8m-11)}{7}$$

$$x=8m-11$$

Continúa

## 5. Continuación

$$y=24-7m$$

$$x=8m-11$$

$$m=0 \quad x=-11 \rightarrow \text{No}$$

$$m=1 \quad x=-3 \rightarrow \text{No}$$

$$m=2 \quad x=5 \quad y=10 \quad \text{sol}$$

$$m=3 \quad x=13 \quad y=3 \quad \text{sol}$$

$$m=4 \quad x=21 \quad y=-4 \rightarrow \text{No}$$

$$m=-1 \quad x=-19 \rightarrow \text{No}$$

6.  $15x+7y=136$

$$7y=136-15x$$

$$y=\frac{136-15x}{7}$$

$$y=\frac{133}{7}+\frac{3}{7}-\frac{14x}{7}-\frac{x}{7}$$

$$y=19-2x+\frac{3-x}{7}$$

$$y-19+2x=\frac{3-x}{7}$$

$$\frac{3-x}{7} \rightarrow \text{Entero}$$

$$\frac{24-8x}{7} \rightarrow \text{Entero}$$

$$\frac{24}{7}-\frac{7x}{7}-\frac{x}{7}$$

$$-x+\frac{24-x}{7}$$

$$\frac{24-x}{7}=m$$

$$24-x=7m$$

$$-x=7m-24$$

$$x=24-7m$$

$$15(24-7m)+7y=136$$

$$360-105m+7y=136$$

$$-105m+7y=-224$$

$$y=\frac{7(15m-32)}{7}$$

$$y=15m-32$$

$$x=24-7m$$

$$y=15m-32$$

$$m=0 \quad x=24 \quad y=-32 \rightarrow \text{No}$$

$$m=1 \quad x=17 \quad y=-17 \rightarrow \text{No}$$

$$m=2 \quad x=10 \quad y=-2 \rightarrow \text{No}$$

$$m=3 \quad x=3 \quad y=13 \quad \text{sol}$$

$$m=4 \quad x=-4 \rightarrow \text{No}$$

$$m=-1 \quad x=-31 \rightarrow \text{No}$$

$$x=31 \quad y=-47 \rightarrow \text{No}$$

7.  $x+5y=24$

$$x=24-5y$$

$$x=24+1-1-5y$$

$$x=\frac{25}{5}-\frac{1}{5}-\frac{5y}{5}$$

$$x+\frac{1}{5}=5-y$$

$$5-y=m$$

$$-y=m-5$$

$$y=5-m$$

$$x+5(5-m)=24$$

$$x+25-5m=24$$

$$x=5m-1$$

$$y=5-m$$

$$x=5m-1$$

$$m=0 \quad x=-1 \rightarrow \text{No}$$

$$m=1 \quad x=4 \quad y=4 \quad \text{sol}$$

$$m=2 \quad x=9 \quad y=3 \quad \text{sol}$$

$$m=3 \quad x=14 \quad y=2 \quad \text{sol}$$

$$m=4 \quad x=19 \quad y=1 \quad \text{sol}$$

$$m=5 \quad x=24 \quad y=0 \rightarrow \text{No}$$

$$m=6 \quad x=29 \quad y=-1 \rightarrow \text{No}$$

$$m=-1 \quad x=-6 \rightarrow \text{No}$$

8.  $9x+11y=203$

$$9x=203-11y$$

$$x=\frac{203-11y}{9}$$

$$x=\frac{198}{9}+\frac{5}{9}-\frac{9y}{9}-\frac{2y}{9}$$

$$x=22-y+\frac{5-2y}{9}$$

$$x-22+y=\frac{5-2y}{9}$$

$$\frac{5-2y}{9} \rightarrow \text{Entero}$$

$$\frac{25-10y}{9} \rightarrow \text{Entero}$$

$$\frac{18}{9}+\frac{7}{9}-\frac{9y}{9}-\frac{y}{9}$$

$$2-y+\frac{7-y}{9}$$

$$\frac{7-y}{9}=m$$

$$7-y=9m$$

$$-y=9m-7$$

$$y=7-9m$$

Continúa

## 8. Continuación

$$9x + 11(7 - 9m) = 203$$

$$9x + 77 - 99m = 203$$

$$x = \frac{9(14 + 11m)}{9}$$

$$x = 14 + 11m$$

$$y = 7 - 9m$$

$$x = 14 + 11m$$

$$m = 0 \quad x = 14 \quad y = 7 \quad \text{sol}$$

$$m = 1 \quad x = 25 \quad y = -2 \rightarrow \text{No}$$

$$m = -1 \quad x = 3 \quad y = 16 \quad \text{sol}$$

$$m = -2 \quad x = -8 \rightarrow \text{No}$$

## 9. $5x + 2y = 73$

$$2y = 73 - 5x$$

$$y = \frac{73 - 5x}{2}$$

$$y = \frac{72}{2} + \frac{1}{2} - \frac{4x}{2} - \frac{x}{2}$$

$$y = 36 - 2x + \frac{1 - x}{2}$$

$$y - 36 + 2x = \frac{1 - x}{2}$$

$$\frac{1 - x}{2} = m$$

$$1 - x = 2m$$

$$-x = 2m - 1$$

$$x = 1 - 2m$$

$$5(1 - 2m) + 2y = 73$$

$$5 - 10m + 2y = 73$$

$$y = \frac{2(34 + 5m)}{2}$$

$$y = 34 + 5m$$

$$x = 1 - 2m$$

$$y = 34 + 5m$$

$$m = 0 \quad x = 1 \quad y = 34 \quad \text{sol}$$

$$m = 1 \quad x = -1 \rightarrow \text{No}$$

$$m = -1 \quad x = 3 \quad y = 29 \quad \text{sol}$$

$$m = -2 \quad x = 5 \quad y = 24 \quad \text{sol}$$

$$m = -3 \quad x = 7 \quad y = 19 \quad \text{sol}$$

$$m = -4 \quad x = 9 \quad y = 14 \quad \text{sol}$$

$$m = -5 \quad x = 11 \quad y = 9 \quad \text{sol}$$

$$m = -6 \quad x = 13 \quad y = 4 \quad \text{sol}$$

$$m = -7 \quad x = 15 \quad y = -1 \rightarrow \text{No}$$

## 10. $8x + 13y = 162$

$$8x = 162 - 13y$$

$$x = \frac{162 - 13y}{8}$$

$$x = \frac{160}{8} + \frac{2}{8} - \frac{8y}{8} - \frac{5y}{8}$$

$$x = 20 - y + \frac{2 - 5y}{8}$$

$$x - 20 + y = \frac{2 - 5y}{8}$$

$$\frac{2 - 5y}{8} \rightarrow \text{Entero}$$

$$\frac{10 - 25y}{8} \rightarrow \text{Entero}$$

$$\frac{10}{8} - \frac{24y}{8} - \frac{y}{8}$$

$$-3y + \frac{10 - y}{8}$$

$$\frac{10 - y}{8} = m$$

$$10 - y = 8m$$

$$-y = 8m - 10$$

$$y = 10 - 8m$$

$$8x + 13(10 - 8m) = 162$$

$$8x + 130 - 104m = 162$$

$$x = \frac{8(4 + 13m)}{8}$$

$$x = 4 + 13m$$

$$y = 10 - 8m$$

$$x = 4 + 13m$$

$$m = 0 \quad x = 4 \quad y = 10 \quad \text{sol}$$

$$m = 1 \quad x = 17 \quad y = 2 \quad \text{sol}$$

$$m = 2 \quad x = 30 \quad y = -6 \rightarrow \text{No}$$

$$m = -1 \quad x = -9 \rightarrow \text{No}$$

## 11. $7x + 5y = 104$

$$5y = 104 - 7x$$

$$y = \frac{104 - 7x}{5}$$

$$y = \frac{100}{5} + \frac{4}{5} - \frac{5x}{5} - \frac{2x}{5}$$

$$y = 20 - x + \frac{4 - 2x}{5}$$

$$y - 20 + x = \frac{4 - 2x}{5}$$

$$\frac{4 - 2x}{5} \rightarrow \text{Entero}$$

$$\frac{12 - 6x}{5} \rightarrow \text{Entero}$$

Continúa

## 11. Continuación

$$\frac{10}{5} + \frac{2}{5} - \frac{5x}{5} - \frac{x}{5}$$

$$2 - x + \frac{2 - x}{5}$$

$$2 - x + \frac{2 - x}{5}$$

$$\frac{2 - x}{5} = m$$

$$2 - x = 5m$$

$$-x = 5m - 2$$

$$x = 2 - 5m$$

$$7(2 - 5m) + 5y = 104$$

$$14 - 35m + 5y = 104$$

$$y = \frac{5(18 + 7m)}{5}$$

$$y = 18 + 7m$$

$$x = 12 - 5m$$

$$y = 18 + 7m$$

$$m = 0 \quad x = 12 \quad y = 18 \quad \text{sol}$$

$$m = 1 \quad x = 7 \rightarrow \text{No}$$

$$m = -1 \quad x = 17 \quad y = 11 \quad \text{sol}$$

$$m = -2 \quad x = 12 \quad y = 4 \quad \text{sol}$$

$$m = -3 \quad x = 17 \quad y = -3 \rightarrow \text{No}$$

## 12. $10x + y = 32$

$$y = 32 - 10x$$

$$y = 32 + 2 - 2 - 10x$$

$$y = \frac{30}{10} + \frac{2}{10} - \frac{10x}{10}$$

$$y = 3 + \frac{2}{10} - x$$

$$y - \frac{2}{10} = 3 - x$$

$$3 - x = m$$

$$-x = m - 3$$

$$x = 3 - m$$

$$10(3 - m) + y = 32$$

$$30 - 10m + y = 32$$

$$y = 2 + 10m$$

$$m = 0 \quad x = 3 \quad y = 2 \quad \text{sol}$$

$$m = 1 \quad x = 2 \quad y = 12 \quad \text{sol}$$

$$m = 2 \quad x = 1 \quad y = 22 \quad \text{sol}$$

$$m = 3 \quad x = 0 \quad y = 32 \rightarrow \text{No}$$

$$m = 4 \quad x = -1 \rightarrow \text{No}$$

$$m = -1 \quad x = 4 \quad y = -8 \rightarrow \text{No}$$

13.  $9x + 4y = 86$

$$4y = 86 - 9x$$

$$y = \frac{86 - 9x}{4}$$

$$y = \frac{84}{4} + \frac{2}{4} - \frac{8x}{4} - \frac{x}{4}$$

$$y = 21 - 2x + \frac{2 - x}{4}$$

$$y - 21 + 2x = \frac{2 - x}{4}$$

$$\frac{2 - x}{4} = m$$

$$2 - x = 4m$$

$$-x = 4m - 2$$

$$x = 2 - 4m$$

$$9(2 - 4m) + 4y = 86$$

$$18 - 36m + 4y = 86$$

$$y = \frac{4(17 + 9m)}{4}$$

$$y = 17 + 9m$$

$$x = 2 - 4m$$

$$m = 0 \quad x = 2 \quad y = 17 \quad \text{sol}$$

$$m = 1 \quad x = -2 \rightarrow \text{No}$$

$$m = -1 \quad x = 6 \quad y = 8 \quad \text{sol}$$

$$m = -2 \quad x = 10 \quad y = -1 \rightarrow \text{No}$$

14.  $9x + 11y = 207$

$$9x = 207 - 11y$$

$$x = \frac{207 - 11y}{9}$$

$$x = \frac{198}{9} + \frac{9}{9} - \frac{9y}{9} - \frac{2y}{9}$$

$$x = 22 - y + \frac{9 - 2y}{9}$$

$$x - 22 + y = \frac{9 - 2y}{9}$$

$$\frac{9 - 2y}{9} \rightarrow \text{Entero}$$

$$\frac{45 - 10y}{9} \rightarrow \text{Entero}$$

$$\frac{45}{9} - \frac{9y}{9} - \frac{y}{9}$$

$$\frac{45 - y}{9} = m$$

$$45 - y = 9m$$

$$-y = 9m - 45$$

$$y = 45 - 9m$$

Continúa

#### 14. Continuación

$$9x + 11(45 - 9m) = 207$$

$$9x + 495 - 99m = 207$$

$$x = \frac{9(11m - 32)}{9}$$

$$x = 11m - 32$$

$$y = 45 - 9m$$

$$x = 11m - 32$$

$$m = 0 \quad x = -32 \rightarrow \text{No}$$

$$m = 1 \quad x = -21 \rightarrow \text{No}$$

$$m = 2 \quad x = -10 \rightarrow \text{No}$$

$$m = 3 \quad x = 1 \quad y = 18 \quad \text{sol}$$

$$m = 4 \quad x = 12 \quad y = 9 \quad \text{sol}$$

$$m = 5 \quad x = 23 \quad y = 0 \rightarrow \text{No}$$

$$m = 6 \quad x = 34 \quad y = -9 \rightarrow \text{No}$$

$$m = -1 \quad x = -43 \rightarrow \text{No}$$

15.  $11x + 12y = 354$

$$11x = 354 - 12y$$

$$x = \frac{354 - 12y}{11}$$

$$x = \frac{352}{11} + \frac{2}{11} - \frac{11y}{11} - \frac{y}{11}$$

$$x = 32 - y + \frac{2 - y}{11}$$

$$x - 32 + y = \frac{2 - y}{11}$$

$$\frac{2 - y}{11} = m$$

$$2 - y = 11m$$

$$-y = 11m - 2$$

$$y = 2 - 11m$$

$$11x + 12(2 - 11m) = 354$$

$$11x + 24 - 132m = 354$$

$$x = \frac{11(30 + 12m)}{11}$$

$$x = 30 + 12m$$

$$y = 2 - 11m$$

$$m = 0 \quad x = 30 \quad y = 2 \quad \text{sol}$$

$$m = 1 \quad x = 42 \quad y = -9 \rightarrow \text{No}$$

$$m = -1 \quad x = 18 \quad y = 13 \quad \text{sol}$$

$$m = -2 \quad x = 6 \quad y = 24 \quad \text{sol}$$

$$m = -3 \quad x = -6 \rightarrow \text{No}$$

16.  $10x + 13y = 294$

$$10x = 294 - 13y$$

$$x = \frac{294 - 13y}{10}$$

$$x = \frac{290}{10} + \frac{4}{10} - \frac{10y}{10} - \frac{3y}{10}$$

$$x - 29 + y = \frac{4 - 3y}{10}$$

$$\frac{4 - 3y}{10} \rightarrow \text{Entero}$$

$$\frac{28 - 21y}{10} \rightarrow \text{Entero}$$

$$\frac{28}{10} - \frac{20y}{10} - \frac{y}{10}$$

$$-2y + \frac{28 - y}{10}$$

$$\frac{28 - y}{10} = m$$

$$28 - y = 10m$$

$$-y = 10m - 28$$

$$y = 28 - 10m$$

$$10x + 13(28 - 10m) = 294$$

$$10x + 364 - 130m = 294$$

$$x = \frac{10(13m - 7)}{10}$$

$$x = 13m - 7$$

$$y = 28 - 10m$$

$$m = 0 \quad x = -7 \rightarrow \text{No}$$

$$m = 1 \quad x = 6 \quad y = 18 \quad \text{sol}$$

$$m = 2 \quad x = 19 \quad y = 8 \quad \text{sol}$$

$$m = 3 \quad x = 32 \quad y = -2 \rightarrow \text{No}$$

$$m = -1 \quad x = -20 \rightarrow \text{No}$$

17.  $11x + 8y = 300$

$$8y = 300 - 11x$$

$$y = \frac{300 - 11x}{8}$$

$$y = \frac{296}{8} + \frac{4}{8} - \frac{3x}{8} - \frac{8x}{8}$$

$$y = 37 - x + \frac{4 - 3x}{8}$$

$$y - 37 + x = \frac{4 - 3x}{8}$$

$$\frac{4 - 3x}{8} \rightarrow \text{Entero}$$

$$\frac{12 - 9x}{8} \rightarrow \text{Entero}$$

Continúa

## Continuación

$$\begin{aligned}
 17. \quad & \frac{12}{8} - \frac{8x}{8} - \frac{x}{8} \\
 & \frac{12-x}{8} = m \\
 & 12-x = 8m \\
 & -x = 8m-12 \\
 & x = 12-8m \\
 & 11(12-8m) + 8y = 300 \\
 & 132-88m+8y = 300 \\
 & y = \frac{8(21+11m)}{8} \\
 & y = 21+11m \\
 & x = 12-8m \\
 & m=0 \quad x=12 \quad y=21 \quad \text{sol} \\
 & m=1 \quad x=4 \quad y=32 \quad \text{sol} \\
 & m=2 \quad x=-4 \quad \rightarrow \text{No} \\
 & m=-1 \quad x=20 \quad y=10 \quad \text{sol} \\
 & m=-2 \quad x=28 \quad y=-1 \quad \rightarrow \text{No}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & 21x+25y=705 \\
 & 21x=705-25y \\
 & x = \frac{705-25y}{21} \\
 & x = \frac{693}{21} + \frac{12}{21} - \frac{21y}{21} - \frac{4y}{21} \\
 & x = 33 - y + \frac{12-4y}{21} \\
 & \frac{12-4y}{21} \rightarrow \text{Entero} \\
 & \frac{192-64y}{21} \rightarrow \text{Entero} \\
 & \frac{192}{21} - \frac{63y}{21} - \frac{y}{21} \\
 & -3y + \frac{192-y}{21} \\
 & \frac{192-y}{21} = m \\
 & 192-y = 21m \\
 & -y = 21m-192 \\
 & y = 192-21m \\
 & 21x+25(192-21m)=705 \\
 & 21x+4.800-525m=705 \\
 & x = \frac{21(25m-195)}{21} \\
 & x = 25m-195
 \end{aligned}$$

Continúa

## 18. Continuación

$$\begin{aligned}
 & y=192-21m \\
 & m=0 \quad x=-195 \rightarrow \text{No} \\
 & m=7 \quad x=-20 \rightarrow \text{No} \\
 & m=8 \quad x=5 \quad y=24 \quad \text{sol} \\
 & m=9 \quad x=30 \quad y=3 \quad \text{sol} \\
 & m=10 \quad x=55 \quad y=-18 \rightarrow \text{No} \\
 & m=-1 \quad x=-220 \rightarrow \text{No}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & 3x-4y=5 \\
 & 3x=5+4y \\
 & x = \frac{5+4y}{3} \\
 & x = \frac{2}{3} + \frac{3}{3} + \frac{3y}{3} + \frac{y}{3} \\
 & x = 1 + y + \frac{2+y}{3}
 \end{aligned}$$

$$\begin{aligned}
 & x-1 = \frac{2+y}{3} \\
 & \frac{2+y}{3} = m \\
 & 2+y = 3m \\
 & y = 3m-2 \\
 & 3x-4(3m-2)=5 \\
 & 3x-12m+8=5 \\
 & x = \frac{3(4m-1)}{3} \\
 & x = 4m-1
 \end{aligned}$$

$$\begin{aligned}
 & y = 3m-2 \\
 & m=1 \quad x=3 \quad y=1 \quad \text{sol} \\
 & m=2 \quad x=7 \quad y=4 \quad \text{sol} \\
 & m=3 \quad x=11 \quad y=7 \quad \text{sol}
 \end{aligned}$$

## 20. 5x-8y=1

$$\begin{aligned}
 & 5x=1+8y \\
 & x = \frac{1+8y}{5} \\
 & \frac{1+8y}{5} \rightarrow \text{Entero} \\
 & \frac{2+16y}{5} \rightarrow \text{Entero} \\
 & \frac{2}{5} + \frac{15y}{5} + \frac{y}{5} \\
 & 3y + \frac{2+y}{5} \\
 & \frac{2+y}{5} = m
 \end{aligned}$$

Continúa

## 20. Continuación

$$\begin{aligned}
 & 2+y=5m \\
 & y=5m-2 \\
 & 5x-8(5m-2)=1 \\
 & 5x-40m+16=1 \\
 & x = \frac{5(8m-3)}{5} \\
 & x = 8m-3
 \end{aligned}$$

$$\begin{aligned}
 & y=5m-2 \\
 & m=1 \quad x=5 \quad y=3 \quad \text{sol} \\
 & m=2 \quad x=13 \quad y=8 \quad \text{sol} \\
 & m=3 \quad x=21 \quad y=13 \quad \text{sol}
 \end{aligned}$$

## 21. 7x-13y=43

$$\begin{aligned}
 & 7x=43+13y \\
 & x = \frac{43+13y}{7} \\
 & x = \frac{42}{7} + \frac{1}{7} + \frac{7y}{7} + \frac{6y}{7} \\
 & x = 6 + y + \frac{1+6y}{7}
 \end{aligned}$$

$$\begin{aligned}
 & x-6-y = \frac{1+6y}{7} \\
 & \frac{1+6y}{7} \rightarrow \text{Entero} \\
 & \frac{6+36y}{7} \rightarrow \text{Entero} \\
 & \frac{6}{7} + \frac{35y}{7} + \frac{y}{7} \\
 & 5y + \frac{6+y}{7}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{6+y}{7} = m \\
 & 6+y=7m \\
 & y=7m-6 \\
 & 7x-13(7m-6)=43 \\
 & 7x-91m+78=43
 \end{aligned}$$

$$\begin{aligned}
 & x = \frac{7(13m-5)}{7} \\
 & x = 13m-5
 \end{aligned}$$

$$\begin{aligned}
 & y=7m-6 \\
 & m=1 \quad x=8 \quad y=1 \quad \text{sol} \\
 & m=2 \quad x=21 \quad y=8 \quad \text{sol} \\
 & m=3 \quad x=34 \quad y=15 \quad \text{sol}
 \end{aligned}$$

22.  $11x - 12y = 0$

$$11x = 12y$$

$$x = \frac{12y}{11}$$

$$x = \frac{11y}{11} + \frac{y}{11}$$

$$x - y = \frac{y}{11}$$

$$\frac{y}{11} = m$$

$$y = 11m$$

$$11x - 12(11m) = 0$$

$$11x - 132m = 0$$

$$x = 12m$$

$$y = 11m$$

$$m=1 \quad x=12 \quad y=11 \quad \text{sol}$$

$$m=2 \quad x=24 \quad y=22 \quad \text{sol}$$

$$m=3 \quad x=36 \quad y=33 \quad \text{sol}$$

23.  $14x - 17y = 32$

$$14x = 32 + 17y$$

$$x = \frac{32 + 17y}{14}$$

$$x = \frac{28}{14} + \frac{4}{14} + \frac{14y}{14} + \frac{3y}{14}$$

$$x = 2 + y + \frac{4 + 3y}{14}$$

$$\frac{4 + 3y}{14} \rightarrow \text{Entero}$$

$$\frac{20 + 15y}{14} \rightarrow \text{Entero}$$

$$\frac{14}{14} + \frac{6}{14} + \frac{14y}{14} + \frac{y}{14}$$

$$1 + y + \frac{6 + y}{14}$$

$$\frac{6 + y}{14} = m$$

$$6 + y = 14m$$

$$y = 14m - 6$$

$$14x - 17(14m - 6) = 32$$

$$14x - 238m + 102 = 32$$

$$x = \frac{14(17m - 5)}{14}$$

$$x = 17m - 5$$

$$y = 14m - 6$$

$$m=1 \quad x=12 \quad y=8 \quad \text{sol}$$

$$m=2 \quad x=29 \quad y=22 \quad \text{sol}$$

$$m=3 \quad x=46 \quad y=36 \quad \text{sol}$$

24.  $7x - 11y = 83$

$$7x = 83 + 11y$$

$$x = \frac{83 + 11y}{7}$$

$$x = \frac{77}{7} + \frac{6}{7} + \frac{7y}{7} + \frac{4y}{7}$$

$$x = 11 + y + \frac{6 + 4y}{7}$$

$$x - 11 - y = \frac{6 + 4y}{7}$$

$$\frac{6 + 4y}{7} \rightarrow \text{Entero}$$

$$\frac{12 + 8y}{7} \rightarrow \text{Entero}$$

$$\frac{7}{7} + \frac{5}{7} + \frac{7y}{7} + \frac{y}{7}$$

$$1 + y + \frac{5 + y}{7}$$

$$\frac{5 + y}{7} = m$$

$$5 + y = 7m$$

$$y = 7m - 5$$

$$7x - 11(7m - 5) = 83$$

$$7x - 77m + 55 = 83$$

$$x = \frac{7(11m + 4)}{7}$$

$$x = 11m + 4$$

$$y = 7m - 5$$

$$m=1 \quad x=15 \quad y=2 \quad \text{sol}$$

$$m=2 \quad x=26 \quad y=9 \quad \text{sol}$$

$$m=3 \quad x=37 \quad y=16 \quad \text{sol}$$

25.  $8x - 13y = 407$

$$8x = 407 + 13y$$

$$x = \frac{407 + 13y}{8}$$

$$x = \frac{400}{8} + \frac{7}{8} + \frac{8y}{8} + \frac{5y}{8}$$

$$x = 50 + y + \frac{7 + 5y}{8}$$

$$x - 50 - y = \frac{7 + 5y}{8}$$

$$\frac{7 + 5y}{8} \rightarrow \text{Entero}$$

$$\frac{35 + 25y}{8} \rightarrow \text{Entero}$$

$$\frac{32}{8} + \frac{3}{8} + \frac{24y}{8} + \frac{y}{8}$$

Continúa

## 25. Continuación

$$4 + 3y + \frac{3 + y}{8}$$

$$\frac{3 + y}{8} = m$$

$$3 + y = 8m$$

$$y = 8m - 3$$

$$8x - 13(8m - 3) = 407$$

$$8x - 104m + 39 = 407$$

$$x = \frac{8(13m + 46)}{8}$$

$$x = 13m + 46$$

$$y = 8m - 3$$

$$m=1 \quad x=59 \quad y=5 \quad \text{sol}$$

$$m=2 \quad x=72 \quad y=13 \quad \text{sol}$$

$$m=3 \quad x=85 \quad y=21 \quad \text{sol}$$

26.  $20y - 23x = 411$

$$20y = 411 + 23x$$

$$y = \frac{411 + 23x}{20}$$

$$y = \frac{400}{20} + \frac{11}{20} + \frac{20x}{20} + \frac{3x}{20}$$

$$y = 20 + x + \frac{11 + 3x}{20}$$

$$y - 20 - x = \frac{11 + 3x}{20}$$

$$\frac{11 + 3x}{20} \rightarrow \text{Entero}$$

$$\frac{77 + 21x}{20} \rightarrow \text{Entero}$$

$$\frac{60}{20} + \frac{17}{20} + \frac{20x}{20} + \frac{x}{20}$$

$$3 + x + \frac{17 + x}{20}$$

$$\frac{17 + x}{20} = m$$

$$17 + x = 20m$$

$$x = 20m - 17$$

$$20y - 23(20m - 17) = 411$$

$$20y - 460m + 391 = 411$$

$$y = \frac{20(23m + 1)}{20}$$

$$x = 20m - 17$$

$$y = 23m + 1$$

$$m=1 \quad x=3 \quad y=24 \quad \text{sol}$$

$$m=2 \quad x=23 \quad y=47 \quad \text{sol}$$

$$m=3 \quad x=43 \quad y=70 \quad \text{sol}$$

27.  $5y - 7x = 312$

$$5y = 312 + 7x$$

$$y = \frac{312 + 7x}{5}$$

$$y = \frac{310}{5} + \frac{2}{5} + \frac{5x}{5} + \frac{2x}{5}$$

$$y = 62 + x + \frac{2+2x}{5}$$

$$y - 62 - x = \frac{2+2x}{5}$$

$$\frac{2+2x}{5} \rightarrow \text{Entero}$$

$$\frac{6+6x}{5} \rightarrow \text{Entero}$$

$$\frac{5}{5} + \frac{1}{5} + \frac{5x}{5} + \frac{x}{5}$$

$$1 + x + \frac{1+x}{5}$$

$$\frac{1+x}{5} = m$$

$$1 + x = 5m$$

$$x = 5m - 1$$

$$5y - 7(5m - 1) = 312$$

$$5y - 35m + 7 = 312$$

$$y = \frac{5(7m + 61)}{5}$$

$$y = 7m + 61$$

$$x = 5m - 1$$

$$m = 1 \quad x = 4 \quad y = 68$$

$$m = 2 \quad x = 9 \quad y = 75$$

$$m = 3 \quad x = 14 \quad y = 82$$

## EJERCICIO 174

1.  $x \rightarrow \text{Billetes } \$2$

$y \rightarrow \text{Billetes } \$5$

$$2x + 5y = 42$$

$$2x = 42 - 5y$$

$$x = \frac{42 - 5y}{2}$$

$$x = \frac{40}{2} + \frac{2}{2} - \frac{3y}{2} - \frac{2y}{2}$$

$$x = 20 - y + \frac{2-3y}{2}$$

$$x - 20 + y = \frac{2-3y}{2}$$

$$\frac{2-3y}{2} \rightarrow \text{Entero}$$

Continúa

## 1. Continuación

$$\frac{2}{2} - \frac{2y}{2} - \frac{y}{2}$$

$$-y + \frac{2-y}{2}$$

$$\frac{2-y}{2} = m$$

$$2 - y = 2m$$

$$-y = 2m - 2$$

$$y = 2 - 2m$$

$$2x + 5(2 - 2m) = 42$$

$$2x + 10 - 10m = 42$$

$$x = \frac{2(5m + 16)}{2}$$

$$x = 5m + 16$$

$$y = 2 - 2m$$

$$m = 0 \quad x = 16 \quad y = 2 \quad \text{sol}$$

$$m = 1 \quad x = 21 \quad y = 0 \rightarrow \text{No}$$

$$m = 2 \quad x = 26 \quad y = -2 \rightarrow \text{No}$$

$$m = -1 \quad x = 11 \quad y = 4 \quad \text{sol}$$

$$m = -2 \quad x = 6 \quad y = 6 \quad \text{sol}$$

$$m = -3 \quad x = 1 \quad y = 8 \quad \text{sol}$$

$$m = -4 \quad x = -4 \rightarrow \text{No}$$

$$1 \text{ de } \$2 \quad y \quad 8 \text{ de } \$5$$

$$6 \text{ de } \$2 \quad y \quad 6 \text{ de } \$5$$

$$11 \text{ de } \$2 \quad y \quad 4 \text{ de } \$5 \quad \text{ó}$$

$$16 \text{ de } \$2 \quad y \quad 2 \text{ de } \$5$$

2.  $x \rightarrow \text{Monedas de } \$5$

$y \rightarrow \text{Monedas de } \$10$

$$5x + 10y = 45$$

$$5x = 45 - 10y$$

$$x = \frac{45 - 10y}{5}$$

$$x = \frac{40}{5} + \frac{5}{5} - \frac{5y}{5} - \frac{5y}{5}$$

$$x = 8 - y + \frac{5-5y}{5}$$

$$x - 8 + y = 1 - y$$

$$1 - y = m$$

$$-y = m - 1$$

$$y = 1 - m$$

$$5x + 10(1 - m) = 45$$

$$5x + 10 - 10m = 45$$

$$x = \frac{5(2m + 7)}{5}$$

$$x = 2m + 7$$

Continúa

## 2. Continuación

$$y = 1 - m$$

$$m = 0 \quad x = 7 \quad y = 1 \quad \text{sol}$$

$$m = 1 \quad x = 9 \quad y = 0 \rightarrow \text{No}$$

$$m = -1 \quad x = 5 \quad y = 2 \quad \text{sol}$$

$$m = -2 \quad x = 3 \quad y = 3 \quad \text{sol}$$

$$m = -3 \quad x = 1 \quad y = 4 \quad \text{sol}$$

$$m = -4 \quad x = -1 \rightarrow \text{No}$$

$$1 \text{ de } \$5 \quad y \quad 4 \text{ de } \$10$$

$$3 \text{ de } \$5 \quad y \quad 3 \text{ de } \$10$$

$$5 \text{ de } \$5 \quad y \quad 2 \text{ de } \$10$$

$$7 \text{ de } \$5 \quad y \quad 1 \text{ de } \$10$$

3.  $5x + 3y = 62$

$$3y = 62 - 5x$$

$$y = \frac{62 - 5x}{3}$$

$$y = \frac{60}{3} + \frac{2}{3} - \frac{3x}{3} - \frac{2x}{3}$$

$$y = 20 - x + \frac{2-2x}{3}$$

$$y - 20 + x = \frac{2-2x}{3}$$

$$\frac{2-2x}{3} \rightarrow \text{Entero}$$

$$\frac{10-10x}{3} \rightarrow \text{Entero}$$

$$\frac{9}{3} + \frac{1}{3} - \frac{9x}{3} - \frac{x}{3}$$

$$3 - 3x + \frac{1-x}{3}$$

$$\frac{1-x}{3} = m$$

$$1 - x = 3m$$

$$-x = 3m - 1$$

$$x = 1 - 3m$$

$$5(1 - 3m) + 3y = 62$$

$$5 - 15m + 3y = 62$$

$$y = \frac{3(5m + 19)}{3}$$

$$y = 5m + 19$$

$$x = 1 - 3m$$

$$m = 0 \quad x = 1 \quad y = 19 \quad \text{sol}$$

$$m = 1 \quad x = -2 \rightarrow \text{No}$$

$$m = -1 \quad x = 4 \quad y = 14 \quad \text{sol}$$

$$m = -2 \quad x = 7 \quad y = 9 \quad \text{sol}$$

$$m = -3 \quad x = 10 \quad y = 4 \quad \text{sol}$$

$$1 \text{ y } 19 ; 4 \text{ y } 14 ; 7 \text{ y } 9 ;$$

$$10 \text{ y } 4$$



4.  $x \rightarrow \text{Sombreros}$

$y \rightarrow \text{Pares de zapatos}$

$$8x + 15y = 340$$

$$8x = 340 - 15y$$

$$x = \frac{340 - 15y}{8}$$

$$x = \frac{336}{8} + \frac{4}{8} - \frac{8y}{8} - \frac{7y}{8}$$

$$x = 42 - y + \frac{4 - 7y}{8}$$

$$x - 42 + y = \frac{4 - 7y}{8}$$

$$\frac{4 - 7y}{8} \rightarrow \text{Entero}$$

$$\frac{28 - 49y}{8} \rightarrow \text{Entero}$$

$$\frac{24}{8} + \frac{4}{8} - \frac{48y}{8} - \frac{y}{8}$$

$$3 - 6y + \frac{4 - y}{8}$$

$$\frac{4 - y}{8} = m$$

$$4 - y = 8m$$

$$-y = 8m - 4$$

$$y = 4 - 8m$$

$$8x + 15(4 - 8m) = 340$$

$$8x + 60 - 120m = 340$$

$$x = \frac{8(15m + 35)}{8}$$

$$x = 15m + 35$$

$$y = 4 - 8m$$

$$m = 0 \quad x = 35 \quad y = 4 \quad \text{sol}$$

$$m = 1 \quad x = 50 \quad y = -4 \rightarrow \text{No}$$

$$m = -1 \quad x = 20 \quad y = 12 \quad \text{sol}$$

$$m = -2 \quad x = 5 \quad y = 20 \quad \text{sol}$$

$$m = -3 \quad x = -10 \rightarrow \text{No}$$

$$5 \text{ sombreros y } 20 \text{ pares de zap.}$$

$$20 \text{ sombreros y } 12 \text{ pares de zap.}$$

$$35 \text{ sombreros y } 4 \text{ pares de zap.}$$

5.  $x \rightarrow \text{Metros de lana}$

$y \rightarrow \text{Metros de seda}$

$$1,50x + 2,50y = 42$$

$\rightarrow \text{Si multiplico por 2 la}$

*ecuación inicial no se*

*altera. Luego*

$$3x + 5y = 84$$

$$3x = 84 - 5y$$

$$x = \frac{84 - 5y}{3}$$

$$x = \frac{81}{3} + \frac{3}{3} - \frac{3y}{3} - \frac{2y}{3}$$

$$x = 27 - y + \frac{3 - 2y}{3}$$

$$x - 27 + y = \frac{3 - 2y}{3}$$

$$\frac{3 - 2y}{3} \rightarrow \text{Entero}$$

$$\frac{15 - 10y}{3} \rightarrow \text{Entero}$$

$$\frac{12}{3} + \frac{3}{3} - \frac{9y}{3} - \frac{y}{3}$$

$$4 - 3y + \frac{3 - y}{3}$$

$$\frac{3 - y}{3} = m$$

$$3 - y = 3m$$

$$-y = 3m - 3$$

$$y = 3 - 3m$$

$$3x + 5(3 - 3m) = 84$$

$$3x + 15 - 15m = 84$$

$$x = \frac{3(5m + 23)}{3}$$

$$x = 5m + 23$$

$$y = 3 - 3m$$

$$m = 0 \quad x = 23 \quad y = 3 \quad \text{sol}$$

$$m = 1 \quad x = 28 \quad y = 0 \rightarrow \text{No}$$

$$m = -1 \quad x = 18 \quad y = 6 \quad \text{sol}$$

$$m = -2 \quad x = 13 \quad y = 9 \quad \text{sol}$$

$$m = -3 \quad x = 8 \quad y = 12 \quad \text{sol}$$

$$m = -4 \quad x = 3 \quad y = 15 \quad \text{sol}$$

$$m = -5 \quad x = -2 \rightarrow \text{No}$$

$$3m \text{ de lana y } 15m \text{ de seda}$$

$$8m \text{ de lana y } 12m \text{ de seda}$$

$$13m \text{ de lana y } 9m \text{ de seda}$$

$$18m \text{ de lana y } 6m \text{ de seda}$$

$$23m \text{ de lana y } 3m \text{ de seda}$$

6.  $x \rightarrow \text{niños}$

$y \rightarrow \text{adultos}$

$$0,45x + y = 17 \quad (20)$$

$$9x + 20y = 340$$

$$9x = 340 - 20y$$

$$x = \frac{340 - 20y}{9}$$

$$x = \frac{333}{9} + \frac{7}{9} - \frac{18y}{9} - \frac{2y}{9}$$

$$x = 37 - 2y + \frac{7 - 2y}{9}$$

$$x - 37 + 2y = \frac{7 - 2y}{9}$$

$$\frac{7 - 2y}{9} \rightarrow \text{Entero}$$

$$\frac{98 - 28y}{9} \rightarrow \text{Entero}$$

$$\frac{90}{9} + \frac{8}{9} - \frac{27y}{9} - \frac{y}{9}$$

$$10 - 3y + \frac{8 - y}{9}$$

$$\frac{8 - y}{9} = m$$

$$8 - y = 9m$$

$$-y = 9m - 8$$

$$y = 8 - 9m$$

$$9x + 20(8 - 9m) = 340$$

$$9x + 160 - 180m = 340$$

$$x = \frac{9(20m + 20)}{9}$$

$$x = 20m + 20$$

$$y = 8 - 9m$$

$$m = 0 \quad x = 20 \quad y = 8 \quad \text{sol}$$

$$m = 1 \quad x = 0 \quad y = -1 \rightarrow \text{No}$$

$$m = -1 \quad x = 0 \quad y = 17 \rightarrow \text{No}$$

$$m = -2 \quad x = -20 \rightarrow \text{No}$$

$$20 \text{ niños y } 8 \text{ adultos}$$

7.  $x \rightarrow \text{Caballos}$

$y \rightarrow \text{Vacas}$

$$460x + 440y = 41.000 \quad (+20)$$

$$23x + 22y = 2.050$$

$$22y = 2.050 - 23x$$

$$y = \frac{2.050 - 23x}{22}$$

$$y = \frac{2.046}{22} + \frac{4}{22} - \frac{22x}{22} - \frac{x}{22}$$

**Continúa**

## 7. Continuación

$$y = 93 - x + \frac{4-x}{22}$$

$$\frac{4-x}{22} = m$$

$$4-x = 22m$$

$$-x = 22m - 4$$

$$x = 4 - 22m$$

$$23(4 - 22m) + 22y = 2.050$$

$$92 - 506m + 22y = 2.050$$

$$y = \frac{22(23m + 89)}{22}$$

$$y = 23m + 89$$

$$x = 4 - 22m$$

$$m=0 \quad x=4 \quad y=89 \quad \text{sol}$$

$$m=1 \quad x=-18 \rightarrow \text{No}$$

$$m=-1 \quad x=26 \quad y=66 \quad \text{sol}$$

$$m=-2 \quad x=48 \quad y=43 \quad \text{sol}$$

$$m=-3 \quad x=70 \quad y=20 \quad \text{sol}$$

$$m=-4 \quad x=92 \quad y=-3 \rightarrow \text{No}$$

$$4 \text{ caballos} \quad y \quad 89 \text{ vacas}$$

$$26 \text{ caballos} \quad y \quad 66 \text{ vacas}$$

$$48 \text{ caballos} \quad y \quad 43 \text{ vacas}$$

$$70 \text{ caballos} \quad y \quad 20 \text{ vacas}$$

## 8. $3x+3=5y+5$

$$3x-5y=2$$

$$3x=2+5y$$

$$x = \frac{2+5y}{3}$$

$$\frac{2+5y}{3} \rightarrow \text{Entero}$$

$$\frac{4+10y}{3} \rightarrow \text{Entero}$$

$$\frac{3}{3} + \frac{1}{3} + \frac{9y}{3} + \frac{y}{3}$$

$$1+3y+\frac{1+y}{3}$$

$$\frac{1+y}{3} = m$$

$$1+y=3m$$

$$y=3m-1$$

$$3x-5(3m-1)=2$$

$$3x-15m+5=2$$

$$x = \frac{3(5m-1)}{3}$$

$$x=5m-1$$

Continúa

## 8. Continuación

$$y=3m-1$$

$$m=0 \quad x=-1 \rightarrow \text{No}$$

$$m=1 \quad x=4 \quad y=2 \quad \text{sol}$$

$$\text{Los } N^{\circ}\text{s son } 4 \text{ y } 2$$

## 9. $x \rightarrow \text{Monedas de } 0,25$

$$y \rightarrow \text{Monedas de } 0,10$$

$$0,25x+0,10y=2,10 \quad (100)$$

$$25x+10y=210$$

$$10y=210-25x$$

$$y = \frac{210-25x}{10}$$

$$y = \frac{200}{10} + \frac{10}{10} - \frac{20x}{10} - \frac{5x}{10}$$

$$y = 20 - 2x + \frac{10-5x}{10}$$

$$y - 20 + 2x = \frac{2-x}{2}$$

$$\frac{2-x}{2} = m$$

$$2-x=2m$$

$$-x=2m-2$$

$$x=2-2m$$

$$25(2-2m)+10y=210$$

$$50-50m+10y=210$$

$$y = \frac{10(5m+16)}{10}$$

$$y=5m+16$$

$$x=2-2m$$

$$m=0 \quad x=2 \quad y=16 \quad \text{sol}$$

$$m=1 \quad x=0 \rightarrow \text{No}$$

$$m=-1 \quad x=4 \quad y=11 \quad \text{sol}$$

$$m=-2 \quad x=6 \quad y=6 \quad \text{sol}$$

$$m=-3 \quad x=8 \quad y=1 \quad \text{sol}$$

$$m=-4 \quad x=10 \quad y=-4 \rightarrow \text{No}$$

$$2 \text{ mon. de } 0,25 \quad y \quad 16 \text{ mon. de } 0,10$$

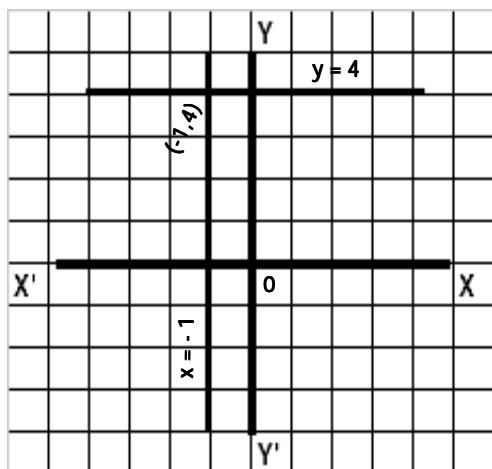
$$4 \text{ mon. de } 0,25 \quad y \quad 11 \text{ mon. de } 0,10$$

$$6 \text{ mon. de } 0,25 \quad y \quad 6 \text{ mon. de } 0,10$$

$$8 \text{ mon. de } 0,25 \quad y \quad 1 \text{ mon. de } 0,10$$

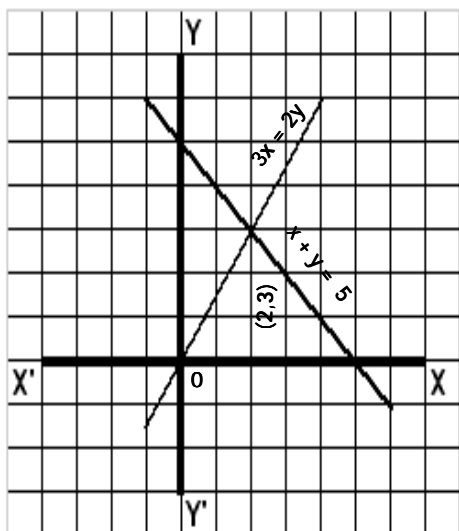
## EJERCICIO 175

21.  $x+1=0$        $y-4=0$   
 $x=-1$        $y=4$



Intersección  $\rightarrow (-1, 4)$

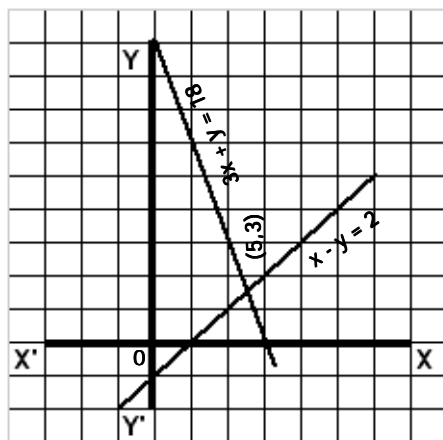
22.  $3x=2y$       con       $x+y=5$   
 $3x-2y=0$        $x=0$        $y=5$   
 $x=1$        $y=1\frac{1}{2}$        $x=5$        $y=0$



Intersección  $\rightarrow (2, 3)$

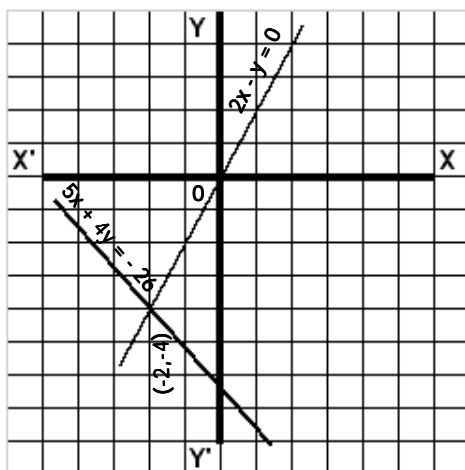
23.  $x-y=2$       con       $3x+y=18$   
 $x=0$        $y=-2$        $x=0$        $y=18$   
 $x=2$        $y=0$        $x=6$        $y=0$

Escala 1:2



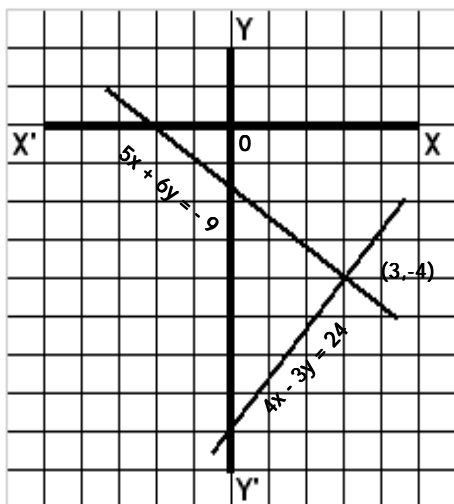
Intersección  $\rightarrow (5, 3)$

24.  $2x-y=0$       con       $5x+4y=-26$   
 $x=1$        $y=2$        $x=0$        $y=-6\frac{1}{2}$   
 $x=-2$        $y=-4$



Intersección  $\rightarrow (-2, -4)$

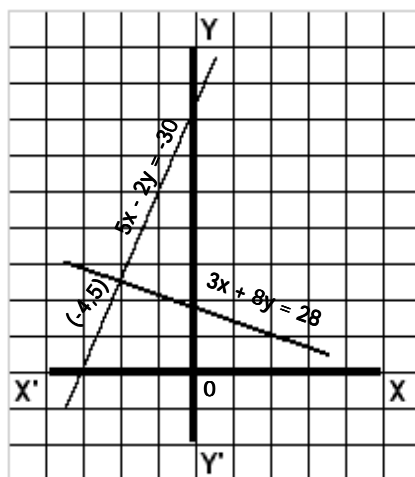
25.  $5x+6y=-9$  con  $4x-3y=24$   
 $x=0$   $y=-1\frac{1}{2}$   $x=0$   $y=-8$   
 $x=3$   $y=-4$   $x=3$   $y=-4$



Intersección  $\rightarrow (3, -4)$

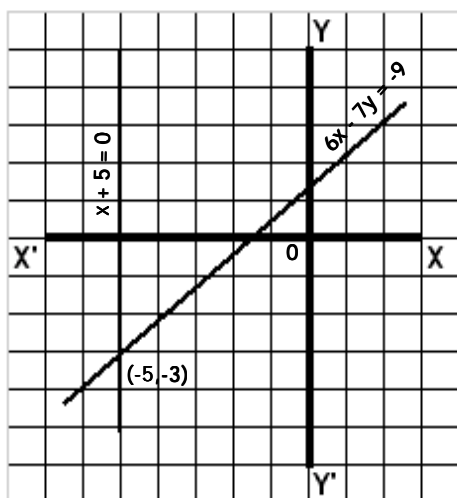
27.  $3x+8y=28$  con  $5x-2y=-30$   
 $x=4$   $y=2$   $x=0$   $y=15$   
 $x=0$   $y=3\frac{1}{2}$   $x=-2$   $y=10$

Escala 1:2



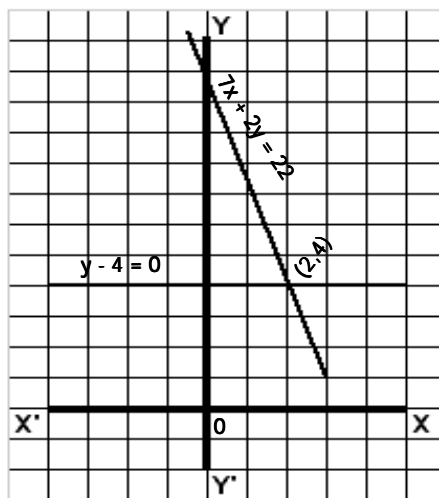
Intersección  $\rightarrow (-4, 5)$

26.  $x+5=0$  con  $6x-7y=-9$   
 $x=-5$   $x=2$   $y=3$   
 $x=-5$   $y=-3$



Intersección  $\rightarrow (-5, -3)$

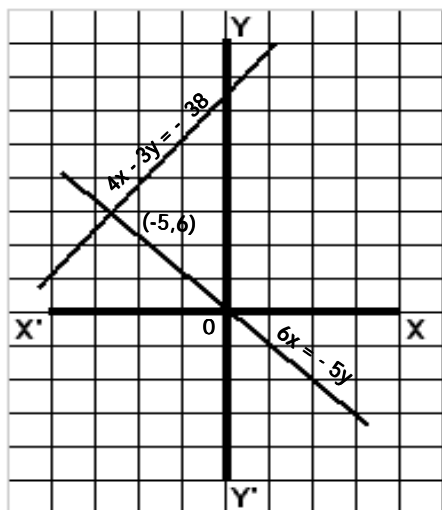
28.  $y-4=0$  con  $7x+2y=22$   
 $y=4$   $x=0$   $y=11$   
 $x=2$   $y=4$



Intersección  $\rightarrow (2, 4)$

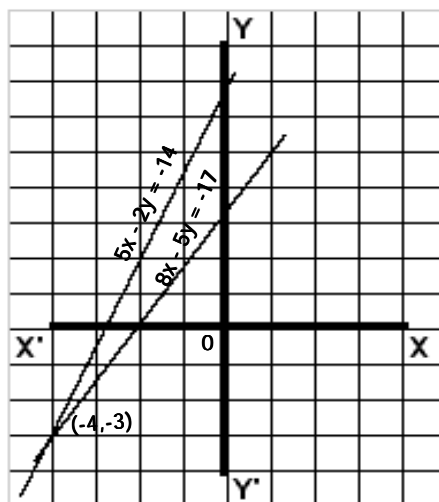
29.  $6x = -5y$       *con*       $4x - 3y = -38$   
 $6x + 5y = 0$        $x = -2$      $y = 10$   
 $x = 5$      $y = -6$        $x = 1$      $y = 14$

Escala 1:2



Intersección  $\rightarrow (-5, 6)$

30.  $5x - 2y = -14$       *con*       $8x - 5y = -17$   
 $x = 0$      $y = 7$        $x = 1$      $y = 5$   
 $x = -2$      $y = 2$        $x = -4$      $y = -3$



Intersección  $\rightarrow (-4, -3)$

## EJERCICIO 176

1.  $x + 6y = 27$  ;  $7x - 3y = 9$   
 $x = 27 - 6y$      $7x = 9 + 3y$   
 $x = \frac{9 + 3y}{7}$

$$27 - 6y = \frac{9 + 3y}{7}$$

$$7(27 - 6y) = 9 + 3y$$

$$189 - 42y = 9 + 3y$$

$$180 = 45y$$

$$4 = y$$

$$x + 6(4) = 27$$

$$x + 24 = 27$$

$$x = 3$$

$$\text{sol: } x = 3 \quad y = 4$$

2.  $3x - 2y = -2$  ;  $5x + 8y = -60$   
 $3x = -2 + 2y$      $5x = -60 - 8y$   
 $x = -\frac{2 - 2y}{3}$      $x = -\frac{60 + 8y}{5}$

$$-\frac{2 - 2y}{3} = -\frac{60 + 8y}{5}$$

$$5(2 - 2y) = 3(60 + 8y)$$

$$10 - 10y = 180 + 24y$$

$$-170 = 34y$$

$$-5 = y$$

$$3x - 2(-5) = -2$$

$$3x + 10 = -2$$

$$3x = -12$$

$$x = -4$$

$$\text{sol: } x = -4 \quad y = -5$$

3.  $3x + 5y = 7$  ;     $2x - y = -4$   
 $5y = 7 - 3x$      $-y = -4 - 2x$   
 $y = \frac{7 - 3x}{5}$      $y = 4 + 2x$   
 $\frac{7 - 3x}{5} = 4 + 2x$

$$7 - 3x = 5(4 + 2x)$$

$$7 - 3x = 20 + 10x$$

$$-13 = 13x$$

$$-1 = x$$

$$2(-1) - y = -4$$

$$-2 - y = -4$$

$$-y = -2$$

$$y = 2$$

$$\text{sol: } x = -1 \quad y = 2$$

$$4. 7x - 4y = 5 \quad ; \quad 9x + 8y = 13$$

$$7x = 5 + 4y \quad 9x = 13 - 8y$$

$$x = \frac{5 + 4y}{7} \quad x = \frac{13 - 8y}{9}$$

$$9(5 + 4y) = 7(13 - 8y)$$

$$45 + 36y = 91 - 56y$$

$$92y = 46$$

$$y = \frac{46}{92} = \frac{1}{2}$$

$$9x + 8\left(\frac{1}{2}\right) = 13$$

$$9x + 4 = 13$$

$$9x = 9$$

$$x = 1$$

$$\text{sol: } x = 1 \quad y = \frac{1}{2}$$

$$5. 9x + 16y = 7 \quad ; \quad 4y - 3x = 0$$

$$16y = 7 - 9x \quad 4y = 3x$$

$$y = \frac{7 - 9x}{16} \quad y = \frac{3x}{4}$$

$$4(7 - 9x) = 16(3x)$$

$$28 - 36x = 48x$$

$$28 = 84x$$

$$x = \frac{28}{84} = \frac{1}{3}$$

$$4y - 1 = 0$$

$$4y = 1$$

$$y = \frac{1}{4}$$

$$\text{sol: } x = \frac{1}{3} \quad y = \frac{1}{4}$$

$$6. 14x - 11y = -29 \quad ; \quad 13y - 8x = 30$$

$$14x = 11y - 29 \quad -8x = 30 - 13y$$

$$x = \frac{11y - 29}{14} \quad x = \frac{13y - 30}{8}$$

$$8(11y - 29) = 14(13y - 30)$$

$$88y - 232 = 182y - 420$$

$$188 = 94y$$

$$2 = y$$

$$13(2) - 8x = 30$$

$$26 - 8x = 30$$

$$-8x = 4$$

$$x = -\frac{1}{2}$$

$$\text{sol: } x = -\frac{1}{2} \quad y = 2$$

$$7. 15x - 11y = -87$$

$$15x = 11y - 87$$

$$x = \frac{11y - 87}{15}$$

$$-12x - 5y = -27$$

$$12x = 27 - 5y$$

$$x = \frac{27 - 5y}{12}$$

$$12(11y - 87) = 15(27 - 5y)$$

$$132y - 1.044 = 405 - 75y$$

$$207y = 1.449$$

$$y = \frac{1.449}{207} = 7$$

$$12x + 5(7) = 27$$

$$12x + 35 = 27$$

$$12x = -8$$

$$x = -\frac{8}{12} = -\frac{2}{3}$$

$$\text{sol: } x = -\frac{2}{3} \quad y = 7$$

$$8. 7x + 9y = 42$$

$$9y = 42 - 7x$$

$$y = \frac{42 - 7x}{9}$$

$$12x + 10y = -4$$

$$10y = -4 - 12x$$

$$y = -\frac{4 + 12x}{10}$$

$$10(42 - 7x) = -9(4 + 12x)$$

$$420 - 70x = -36 - 108x$$

$$38x = -456$$

$$x = -12$$

$$12(-12) + 10y = -4$$

$$-144 + 10y = -4$$

$$10y = 140$$

$$y = 14$$

$$\text{sol: } x = -12 \quad y = 14$$

$$\begin{aligned}
9. \quad & 6x - 18y = -85 ; \quad 24x - 5y = -5 \\
& -18y = -6x - 85 \quad -5y = -5 - 24x \\
& y = \frac{6x + 85}{18} \quad y = \frac{5 + 24x}{5} \\
& 5(6x + 85) = 18(5 + 24x) \\
& 30x + 425 = 90 + 432x \\
& 335 = 402x \\
& x = \frac{335}{402} = \frac{5}{6} \\
& 6\left(\frac{5}{6}\right) - 18y = -85 \\
& 5 - 18y = -85 \\
& -18y = -90 \\
& y = 5 \\
& \text{sol: } x = \frac{5}{6} \quad y = 5
\end{aligned}$$

## EJERCICIO 177

$$\begin{aligned}
1. \quad & x + 3y = 6 ; \quad 5x - 2y = 13 \\
& x = 6 - 3y \\
& 5(6 - 3y) - 2y = 13 \\
& 30 - 15y - 2y = 13 \\
& -17y = -17 \\
& y = 1 \\
& x + 3(1) = 6 \\
& x + 3 = 6 \\
& x = 3 \\
& \text{sol: } x = 3 \quad y = 1
\end{aligned}$$

$$\begin{aligned}
2. \quad & 5x + 7y = -1 ; \quad -3x + 4y = -24 \\
& 5x = -1 - 7y \\
& x = -\frac{1 + 7y}{5} \\
& 3\left(\frac{1 + 7y}{5}\right) + 4y = -24 \\
& \frac{3 + 21y}{5} + 4y = -24 \\
& 3 + 21y + 20y = -120 \\
& 41y = -123 \\
& y = -3 \\
& -3x + 4(-3) = -24 \\
& -3x - 12 = -24 \\
& -3x = -12 \\
& x = 4 \\
& \text{sol: } x = 4 \quad y = -3
\end{aligned}$$

$$\begin{aligned}
3. \quad & 3x + 4y = 8 ; \quad 8x - 9y = -77 \\
& 4y = 8 - 3x \\
& y = \frac{8 - 3x}{4} \\
& 8x - 9\left(\frac{8 - 3x}{4}\right) = -77 \\
& 32x - 9(8 - 3x) = -308 \\
& 32x - 72 + 27x = -308 \\
& 59x = -236 \\
& x = -4 \\
& 3(-4) + 4y = 8 \\
& -12 + 4y = 8 \\
& 4y = 20 \\
& y = 5 \\
& \text{sol: } x = -4 \quad y = 5
\end{aligned}$$

$$4. \quad x - 5y = 8 ; \quad -7x + 8y = 25$$

$$\begin{aligned}
& 8y = 25 + 7x \\
& y = \frac{25 + 7x}{8}
\end{aligned}$$

$$x - 5\left(\frac{25 + 7x}{8}\right) = 8$$

$$8x - 5(25 + 7x) = 64$$

$$8x - 125 - 35x = 64$$

$$-27x = 189$$

$$x = -7$$

$$-7 - 5y = 8$$

$$-5y = 15$$

$$y = -3$$

$$\text{sol: } x = -7 \quad y = -3$$

$$5. \quad 15x + 11y = 32 ; \quad 7y - 9x = 8$$

$$7y = 8 + 9x$$

$$y = \frac{8 + 9x}{7}$$

$$15x + 11\left(\frac{8 + 9x}{7}\right) = 32$$

$$105x + 11(8 + 9x) = 224$$

$$105x + 88 + 99x = 224$$

$$204x = 136$$

$$x = \frac{136}{204} = \frac{2}{3}$$

$$15\left(\frac{2}{3}\right) + 11y = 32$$

$$10 + 11y = 32$$

$$11y = 22$$

$$y = 2$$

$$\text{sol: } x = \frac{2}{3} \quad y = 2$$

$$6. \quad 10x + 18y = -11 ; \quad 16x - 9y = -5$$

$$10x = -11 - 18y$$

$$x = -\frac{11 + 18y}{10}$$

$$-16\left(\frac{11 + 18y}{10}\right) - 9y = -5$$

$$-16(11 + 18y) - 90y = -50$$

$$-176 - 288y - 90y = -50$$

$$-378y = 126$$

$$y = -\frac{126}{378} = -\frac{1}{3}$$

$$10x + 18\left(-\frac{1}{3}\right) = -11$$

$$10x - 6 = -11$$

$$10x = -5$$

$$x = -\frac{1}{2}$$

$$\text{sol: } x = -\frac{1}{2} \quad y = -\frac{1}{3}$$

---

7.  $4x + 5y = 5$  ;  $-4x - 10y = -7$

$$10y = 7 - 4x$$

$$y = \frac{7-4x}{10}$$

$$4x + 5\left(\frac{7-4x}{10}\right) = 5$$

$$8x + 7 - 4x = 10$$

$$4x = 3$$

$$x = \frac{3}{4}$$

$$4\left(\frac{3}{4}\right) + 5y = 5$$

$$3 + 5y = 5$$

$$5y = 2$$

$$y = \frac{2}{5}$$

sol:  $x = \frac{3}{4}$   $y = \frac{2}{5}$

8.  $32x - 25y = 13$  ;  $16x + 15y = 1$

$$16x = 1 - 15y$$

$$x = \frac{1-15y}{16}$$

$$32\left(\frac{1-15y}{16}\right) - 25y = 13$$

$$2(1-15y) - 25y = 13$$

$$2 - 30y - 25y = 13$$

$$2 - 55y = 13$$

$$-55y = 11$$

$$y = -\frac{1}{5}$$

$$16x + 15\left(-\frac{1}{5}\right) = 1$$

$$16x - 3 = 1$$

$$16x = 4$$

$$x = \frac{1}{4}$$

sol:  $x = \frac{1}{4}$   $y = -\frac{1}{5}$

9.  $11x - 13y = -163$  ;  $-8x + 7y = 94$

$$11x = -163 + 13y$$

$$x = -\frac{163-13y}{11}$$

$$8\left(\frac{163-13y}{11}\right) + 7y = 94$$

$$8(163-13y) + 77y = 1.034$$

$$1.304 - 104y + 77y = 1.034$$

$$-27y = -270$$

$$y = 10$$

$$11x - 13(10) = -163$$

$$11x - 130 = -163$$

$$11x = -33$$

$$x = -3$$

sol:  $x = -3$   $y = 10$

## EJERCICIO 178

1.  $6x - 5y = -9$  (2)

$$4x + 3y = 13$$

$$(-3)$$

$$\frac{12x - 10y = -18}{-12x - 9y = -39}$$

$$-19y = -57$$

$$y = 3$$

$$4x + 3(3) = 13$$

$$4x + 9 = 13$$

$$4x = 4$$

$$x = 1$$

sol:  $x = 1$   $y = 3$

2.  $7x - 15y = 1$

$$-x - 6y = 8$$

$$(7)$$

$$\frac{7x - 15y = 1}{-7x - 42y = 56}$$

$$-57y = 57$$

$$y = -1$$

$$7x - 15(-1) = 1$$

$$7x + 15 = 1$$

$$7x = -14$$

$$x = -2$$

sol:  $x = -2$   $y = -1$

3.  $3x - 4y = 41$  (3)

$$11x + 6y = 47$$

$$(2)$$

$$\frac{9x - 12y = 123}{22x + 12y = 94}$$

$$31x = 217$$

$$x = 7$$

$$3(7) - 4y = 41$$

$$21 - 4y = 41$$

$$-4y = 20$$

$$y = -5$$

sol:  $x = 7$   $y = -5$

4.  $9x + 11y = -14$  (4)

$$6x - 5y = -34$$

$$(-6)$$

$$\frac{36x + 44y = -56}{-36x + 30y = 204}$$

$$74y = 148$$

$$y = 2$$

$$6x - 5(2) = -34$$

$$6x - 10 = -34$$

$$6x = -24$$

$$x = -4$$

sol:  $x = -4$   $y = 2$

5.  $10x - 3y = 36$

$$2x + 5y = -4$$

$$(-5)$$

$$\frac{10x - 3y = 36}{-10x - 25y = 20}$$

$$-28y = 56$$

$$y = -2$$

$$10x - 3(-2) = 36$$

$$10x = 30$$

$$x = 3$$

sol:  $x = 3$   $y = -2$

6.  $11x - 9y = 2$  (5)

$$13x - 15y = -2$$

$$(-3)$$

$$\frac{55x - 45y = 10}{-39x + 45y = 6}$$

$$16x = 16$$

$$x = 1$$

$$11(1) - 9y = 2$$

$$11 - 9y = 2$$

$$-9y = -9$$

$$y = 1$$

sol:  $x = 1$   $y = 1$



$$7. \quad \begin{array}{r} 18x + 5y = -11 \quad (2) \\ 12x + 11y = 31 \quad (-3) \\ \hline 36x + 10y = -22 \\ -36x - 33y = -93 \\ \hline -23y = -115 \\ y = 5 \end{array}$$

$$\begin{array}{r} 18x + 5(5) = -11 \\ 18x + 25 = -11 \\ 18x = -36 \\ x = -2 \end{array}$$

$$\text{sol: } x = -2 \quad y = 5$$

$$8. \quad \begin{array}{r} 9x + 7y = -4 \quad (-11) \\ 11x - 13y = -48 \quad (9) \\ \hline -99x - 77y = 44 \\ 99x - 117y = -432 \\ \hline -194y = -388 \\ y = 2 \end{array}$$

$$\begin{array}{r} 9x + 7(2) = -4 \\ 9x + 14 = -4 \\ 9x = -18 \\ x = -2 \end{array}$$

$$\text{sol: } x = -2 \quad y = 2$$

$$9. \quad \begin{array}{r} 12x - 14y = 20 \quad (7) \\ -14x + 12y = -19 \quad (6) \\ \hline 84x - 98y = 140 \\ -84x + 72y = -114 \\ \hline -26y = 26 \\ y = -1 \end{array}$$

$$\begin{array}{r} 12x - 14(-1) = 20 \\ 12x + 14 = 20 \\ 12x = 6 \\ x = \frac{1}{2} \end{array}$$

$$\text{sol: } x = \frac{1}{2} \quad y = -1$$

$$10. \quad \begin{array}{r} 15x - y = 40 \quad (8) \\ 19x + 8y = 236 \\ \hline 120x - 8y = 320 \\ 19x + 8y = 236 \\ \hline 139x = 556 \\ x = 4 \end{array}$$

$$\begin{array}{r} 15(4) - y = 40 \\ 60 - y = 40 \\ -y = -20 \\ y = 20 \end{array}$$

$$\text{sol: } x = 4 \quad y = 20$$

$$11. \quad \begin{array}{r} 36x - 11y = -14 \quad (2) \\ 24x - 17y = 10 \quad (-3) \\ \hline 72x - 22y = -28 \\ -72x + 51y = -30 \\ \hline 29y = -58 \\ y = -2 \end{array}$$

$$\begin{array}{r} 36x - 11(-2) = -14 \\ 36x + 22 = -14 \\ 36x = -36 \\ x = -1 \end{array}$$

$$\text{sol: } x = -1 \quad y = -2$$

$$12. \quad \begin{array}{r} 12x - 17y = 104 \quad (5) \\ 15x + 19y = -31 \quad (-4) \\ \hline 60x - 85y = 520 \\ -60x - 76y = 124 \\ \hline -161y = 644 \\ y = -4 \end{array}$$

$$\begin{array}{r} 12x - 17(-4) = 104 \\ 12x + 68 = 104 \\ 12x = 36 \\ x = 3 \end{array}$$

$$\text{sol: } x = 3 \quad y = -4$$

## EJERCICIO 179

$$1. \quad \begin{array}{r} 8x - 5 = 7y - 9 \\ 8x - 7y = -9 + 5 \\ 8x - 7y = -4 \\ 6x = 3y + 6 \end{array}$$

$$\begin{array}{r} 6x - 3y = 6 \\ 8x - 7y = -4 \quad (3) \\ 6x - 3y = 6 \quad (-4) \\ \hline 24x - 21y = -12 \\ -24x + 12y = -24 \\ \hline -9y = -36 \\ y = 4 \end{array}$$

$$\begin{array}{r} 6x - 3(4) = 6 \\ 6x - 12 = 6 \\ 6x = 18 \\ x = 3 \end{array}$$

$$\text{sol: } x = 3 \quad y = 4$$

$$2. \quad \begin{array}{r} x - 1 = y + 1 \\ x - y = 2 \\ x - 3 = 3y - 7 \\ x - 3y = -4 \\ x - y = 2 \\ x - 3y = -4 \quad (-1) \\ \hline x - y = 2 \\ -x + 3y = 4 \\ \hline 2y = 6 \\ y = 3 \\ x - 3 = 2 \\ x = 5 \end{array}$$

$$\text{sol: } x = 5 \quad y = 3$$

$$3. \quad \begin{array}{r} 3(x+2) = 2y \quad 2(y+5) = 7x \\ 3x + 6 = 2y \quad 2y + 10 = 7x \\ 3x - 2y = -6 \quad 7x - 2y = 10 \\ 3x - 2y = -6 \quad ; \quad 7x - 2y = 10 \\ 3x = -6 + 2y \quad 7x = 10 + 2y \\ x = -\frac{6-2y}{3} \quad x = \frac{10+2y}{7} \end{array}$$

$$\begin{array}{r} -7(6-2y) = 3(10+2y) \\ -42 + 14y = 30 + 6y \\ 8y = 72 \\ y = 9 \end{array}$$

$$\begin{array}{r} 7x - 2(9) = 10 \\ 7x - 18 = 10 \\ 7x = 28 \\ x = 4 \end{array}$$

$$\text{sol: } x = 4 \quad y = 9$$

$$4. \quad x-1=2(y+6)$$

$$x-1=2y+12$$

$$x-2y=13$$

$$x=13+2y$$

$$x+6=3(1-2y)$$

$$x+6=3-6y$$

$$x+6y=-3$$

$$x=-3-6y$$

$$13+2y=-3-6y$$

$$-3-6y-2y=13$$

$$-8y=16$$

$$y=-2$$

$$x-2(-2)=13$$

$$x+4=13$$

$$x=9$$

$$\text{sol: } x=9 \quad y=-2$$

$$5. \quad 30-(8-x)=2y+30$$

$$30-8+x=2y+30$$

$$x-2y=8$$

$$5x-29=x-(5-4y)$$

$$5x-29=x-5+4y$$

$$4x-4y=24$$

$$x-y=6$$

$$x-2y=8$$

$$\frac{x-y=6}{x-2y=8} \quad (-1)$$

$$x-2y=8$$

$$-x+y=-6$$

$$-y=2$$

$$y=-2$$

$$x-2(-2)=8$$

$$x+4=8$$

$$x=4$$

$$\text{sol: } x=4 \quad y=-2$$

$$6. \quad 3x-(9x+y)=5y-(2x+9y)$$

$$3x-9x-y=5y-2x-9y$$

$$-4x+3y=0$$

$$4x-(3y+7)=5y-47$$

$$4x-3y-7=5y-47$$

$$4x-8y=-40$$

$$x-2y=-10$$

$$-4x+3y=0$$

$$\frac{x-2y=-10}{-4x+3y=0} \quad (4)$$

$$-4x+3y=0$$

$$4x-8y=-40$$

Continúa

## 6. Continuación

$$-5y=-40$$

$$y=8$$

$$x-2(8)=-10$$

$$x-16=-10$$

$$x=6$$

$$\text{sol: } x=6 \quad y=8$$

$$7. \quad (x-y)-(6x+8y)=- (10x+5y+3)$$

$$x-y-6x-8y=-10x-5y-3$$

$$-5x-9y=-10x-5y-3$$

$$5x-4y=-3$$

$$(x+y)-(9y-11x)=2y-2x$$

$$x+y-9y+11x=2y-2x$$

$$12x-8y=2y-2x$$

$$14x-10y=0$$

$$5x-4y=-3 \quad (5)$$

$$\frac{14x-10y=0}{25x-20y=-15} \quad (-2)$$

$$25x-20y=-15$$

$$-28x+20y=0$$

$$-3x=-15$$

$$x=5$$

$$5(5)-4y=-3$$

$$25-4y=-3$$

$$-4y=-28$$

$$y=7$$

$$\text{sol: } x=5 \quad y=7$$

$$8. \quad 5(x+3y)-(7x+8y)=-6$$

$$5x+15y-7x-8y=-6$$

$$-2x+7y=-6$$

$$7x-9y-2(x-18y)=0$$

$$7x-9y-2x+36y=0$$

$$5x+27y=0$$

$$-2x+7y=-6 \quad (5)$$

$$\frac{5x+27y=0}{-10x+35y=-30} \quad (2)$$

$$-10x+35y=-30$$

$$10x+54y=0$$

$$89y=-30$$

$$y=-\frac{30}{89}$$

Continúa

## 8. Continuación

$$-2x+7\left(-\frac{30}{89}\right)=-6$$

$$-178x-210=-534$$

$$-178x=-324$$

$$x=\frac{324}{178}=1\frac{73}{89}$$

$$\text{sol: } x=1\frac{73}{89} \quad y=-\frac{30}{89}$$

$$9. \quad 2(x+5)=4(y-4x)$$

$$2x+10=4y-16x$$

$$18x-4y=-10$$

$$10(y-x)=11y-12x$$

$$10y-10x=11y-12x$$

$$2x-y=0$$

$$18x-4y=-10$$

$$\frac{2x-y=0}{18x-4y=-10} \quad (-9)$$

$$18x-4y=-10$$

$$-18x+9y=0$$

$$5y=-10$$

$$y=-2$$

$$2x-(-2)=0$$

$$2x+2=0$$

$$2x=-2$$

$$x=-1$$

$$\text{sol: } x=-1 \quad y=-2$$

$$10. \quad 3x-4y-2(2x-7)=0$$

$$3x-4y-4x+14=0$$

$$-x-4y=-14$$

$$x+4y=14$$

$$5(x-1)-(2y-1)=0$$

$$5x-5-2y+1=0$$

$$5x-2y=4$$

$$x+4y=14$$

$$\frac{5x-2y=4}{x+4y=14} \quad (2)$$

$$x+4y=14$$

$$10x-4y=8$$

$$11x=22$$

$$x=2$$

$$2+4y=14$$

$$4y=12$$

$$y=3$$

$$\text{sol: } x=2 \quad y=3$$

$$11. 12(x+2y)-8(2x+y)=2(5x-6y)$$

$$12x+24y-16x-8y=10x-12y$$

$$-4x+16y=10x-12y$$

$$-14x+28y=0$$

$$-x+2y=0$$

$$20(x-4y)=-10$$

$$20x-80y=-10$$

$$2x-8y=-1$$

$$-x+2y=0 \quad (2)$$

$$\frac{2x-8y=-1}{-2x+4y=0}$$

$$-4y=-1$$

$$y=\frac{1}{4}$$

$$-x+2\left(\frac{1}{4}\right)=0$$

$$-4x+2=0$$

$$-4x=-2$$

$$x=\frac{1}{2}$$

$$sol: x=\frac{1}{2} \quad y=\frac{1}{4}$$

$$12. x(y-2)-y(x-3)=-14$$

$$xy-2x-xy+3y=-14$$

$$-2x+3y=-14$$

$$y(x-6)-x(y+9)=54$$

$$xy-6y-xy-9x=54$$

$$-9x-6y=54$$

$$3x+2y=-18$$

$$-2x+3y=-14 \quad (3)$$

$$\frac{3x+2y=-18 \quad (2)}{-6x+9y=-42}$$

$$6x+4y=-36$$

$$13y=-78$$

$$y=-6$$

$$-2x+3(-6)=-14$$

$$-2x-18=-14$$

$$-2x=4$$

$$x=-2$$

$$sol: x=-2 \quad y=-6$$

## EJERCICIO 180

$$1. \frac{3x}{2}+y=11$$

$$3x+2y=22$$

$$x+\frac{y}{2}=7$$

$$2x+y=14$$

$$3x+2y=22 \quad ; \quad 2x+y=14$$

$$3x=22-2y \quad 2x=14-y$$

$$x=\frac{22-2y}{3} \quad x=\frac{14-y}{2}$$

$$2(22-2y)=3(14-y)$$

$$44-4y=42-3y$$

$$-y=-2$$

$$y=2$$

$$2x+2=14$$

$$2x=12$$

$$x=6$$

$$sol: x=6 \quad y=2$$

$$2. \frac{5x}{12}-y=9 \quad ; \quad x-\frac{3y}{4}=15$$

$$5x-12y=108 \quad ; \quad 4x-3y=60$$

$$5x=108+12y$$

$$x=\frac{108+12y}{5}$$

$$4\left(\frac{108+12y}{5}\right)-3y=60$$

$$4(108+12y)-15y=300$$

$$432+48y-15y=300$$

$$33y=-132$$

$$y=-4$$

$$4x-3(-4)=60$$

$$4x+12=60$$

$$4x=48$$

$$x=12$$

$$sol: x=12 \quad y=-4$$

$$3. \frac{x}{7}+\frac{y}{3}=5 \quad ; \quad 3y-\frac{x}{14}=26$$

$$3x+7y=105 \quad ; \quad 42y-x=364$$

$$3x+7y=105$$

$$\frac{-x+42y=364 \quad (3)}{3x+7y=105}$$

$$-3x+126y=1.092$$

$$133y=1.197$$

$$y=9$$

$$3x+7(9)=105$$

$$3x+63=105$$

$$3x=42$$

$$x=14$$

$$sol: x=14 \quad y=9$$

$$4. \quad \frac{x}{5} = \frac{y}{4} ; \quad \frac{y}{3} = \frac{x}{3} - 1$$

$$4x = 5y \quad y = x - 3$$

$$4x - 5y = 0 \quad x - y = 3$$

$$4x - 5y = 0$$

$$\frac{x - y = 3}{4x - 5y = 0} \quad (-4)$$

$$-4x + 4y = -12$$

$$-y = -12$$

$$y = 12$$

$$x - 12 = 3$$

$$x = 15$$

$$\text{sol: } x = 15 \quad y = 12$$

$$5. \quad \frac{3x}{5} - \frac{1}{4}y = 2 ; \quad 2x = \frac{5y}{2}$$

$$12x - 5y = 40 \quad 4x = 5y$$

$$4x - 5y = 0$$

$$4x - 5y = 0$$

$$\frac{12x - 5y = 40}{4x - 5y = 0} \quad (-1)$$

$$-8x = -40$$

$$-12x + 5y = -40$$

$$-8x = -40$$

$$x = 5$$

$$4(5) - 5y = 0$$

$$20 - 5y = 0$$

$$-5y = -20$$

$$y = 4$$

$$\text{sol: } x = 5 \quad y = 4$$

$$6. \quad \frac{2x}{3} - \frac{3y}{4} = 1 ; \quad \frac{1}{8}y - \frac{5x}{6} = 2$$

$$8x - 9y = 12 ; \quad 3y - 20x = 48$$

$$8x - 9y = 12$$

$$-20x + 3y = 48 \quad (3)$$

$$8x - 9y = 12$$

$$-60x + 9y = 144$$

$$-52x = 156$$

$$x = -3$$

$$8(-3) - 9y = 12$$

$$-24 - 9y = 12$$

$$-9y = 36$$

$$y = -4$$

$$\text{sol: } x = -3 \quad y = -4$$

$$7. \quad \frac{x}{8} - \frac{y}{5} = -\frac{11}{10} ; \quad \frac{x}{5} + \frac{y}{4} = -\frac{59}{40}$$

$$5x - 8y = -44 ; \quad 8x + 10y = -59$$

$$5x - 8y = -44 \quad (5)$$

$$8x + 10y = -59 \quad (4)$$

$$25x - 40y = -220$$

$$32x + 40y = -236$$

$$57x = -456$$

$$x = -8$$

$$5(-8) - 8y = -44$$

$$-40 - 8y = -44$$

$$-8y = -4$$

$$y = \frac{1}{2}$$

$$\text{sol: } x = -8 \quad y = \frac{1}{2}$$

$$8. \quad \frac{x}{7} + \frac{y}{8} = 0 ; \quad \frac{1}{7}x - \frac{3y}{4} = 7$$

$$8x + 7y = 0 ; \quad 4x - 21y = 196$$

$$8x = -7y$$

$$4x = 196 + 21y$$

$$x = \frac{-7y}{8}$$

$$x = \frac{196 + 21y}{4}$$

$$-28y = 8(196 + 21y)$$

$$-28y = 1.568 + 168y$$

$$-196y = 1.568$$

$$y = -8$$

$$x = \frac{-7(-8)}{8}$$

$$x = 7$$

$$\text{sol: } x = 7 \quad y = -8$$

$$9. \quad \frac{2x+1}{5} = \frac{y}{4} ; \quad 2x - 3y = -8$$

$$4(2x+1) = 5y$$

$$8x + 4 = 5y$$

$$8x - 5y = -4$$

$$2x - 3y = -8 \quad (-4)$$

$$8x - 5y = -4$$

$$-8x + 12y = 32$$

$$8x - 5y = -4$$

$$7y = 28$$

$$y = 4$$

Continúa

## 9. Continuación

$$8x - 5(4) = -4$$

$$8x - 20 = -4$$

$$8x = 16$$

$$x = 2$$

$$\text{sol: } x = 2 \quad y = 4$$

$$10. \quad 12x + 5y + 6 = 0$$

$$12x + 5y = -6$$

$$\frac{5x}{3} - \frac{7y}{6} = -12$$

$$10x - 7y = -72$$

$$12x + 5y = -6 \quad (5)$$

$$\frac{10x - 7y = -72}{60x + 25y = -30} \quad (-6)$$

$$-60x + 42y = 432$$

$$67y = 402$$

$$y = 6$$

$$10x - 7(6) = -72$$

$$10x - 42 = -72$$

$$10x = -30$$

$$x = -3$$

$$\text{sol: } x = -3 \quad y = 6$$

$$11. \quad \frac{x}{5} = 3(y+2)$$

$$x = 15(y+2)$$

$$x = 15y + 30$$

$$\frac{y}{5} + 3x = \frac{224}{5}$$

$$y + 15x = 224$$

$$15x = 224 - y$$

$$x = \frac{224 - y}{15}$$

$$15(15y + 30) = 224 - y$$

$$225y + 450 = 224 - y$$

$$226y = -226$$

$$y = -1$$

$$x = 15(-1) + 30$$

$$x = -15 + 30$$

$$x = 15$$

$$\text{sol: } x = 15 \quad y = -1$$

$$12. \quad \frac{x}{5} - \frac{y}{6} = -\frac{1}{30} ; \quad \frac{x}{3} - \frac{y}{20} = \frac{13}{12}$$

$$6x - 5y = -1 ; \quad 20x - 3y = 65$$

$$6x - 5y = -1 \quad (3)$$

$$20x - 3y = 65 \quad (-5)$$

$$\hline 18x - 15y = -3$$

$$-100x + 15y = -325$$

$$\hline -82x = -328$$

$$x = 4$$

$$6(4) - 5y = -1$$

$$-5y = -25$$

$$y = 5$$

$$\text{sol: } x = 4 \quad y = 5$$

$$13. \quad \frac{x-3}{3} - \frac{y-4}{4} = 0 ; \quad \frac{x-4}{2} + \frac{y+2}{5} = 3$$

$$4(x-3) - 3(y-4) = 0 ; \quad 5(x-4) + 2(y+2) = 30$$

$$4x - 12 - 3y + 12 = 0 ; \quad 5x - 20 + 2y + 4 = 30$$

$$4x - 3y = 0 \quad 5x + 2y = 46$$

$$x = \frac{3y}{4}$$

$$5\left(\frac{3y}{4}\right) + 2y = 46$$

$$15y + 8y = 184$$

$$23y = 184$$

$$y = 8$$

$$x = \frac{3(8)}{4}$$

$$x = 6$$

$$\text{sol: } x = 6 \quad y = 8$$

$$14. \quad \frac{x-1}{2} - \frac{y-1}{3} = -\frac{13}{36} ; \quad \frac{x+1}{3} - \frac{y+1}{2} = -\frac{2}{3}$$

$$18(x-1) - 12(y-1) = -13 ; \quad 2x + 2 - 3y - 3 = -4$$

$$18x - 12y = -7 \quad 2x - 3y = -3$$

$$18x - 12y = -7$$

$$2x - 3y = -3 \quad (-4)$$

$$\hline 18x - 12y = -7$$

$$-8x + 12y = 12$$

$$\hline 10x = 5$$

$$x = \frac{1}{2}$$

$$2\left(\frac{1}{2}\right) - 3y = -3$$

$$-3y = -4$$

$$y = \frac{4}{3}$$

$$\text{sol: } x = \frac{1}{2} \quad y = \frac{4}{3}$$

$$15. \quad \frac{x+1}{10} = \frac{y-4}{5} ; \quad \frac{x-4}{5} = \frac{y-2}{10}$$

$$x+1 = 2(y-4) ; \quad 2(x-4) = y-2$$

$$x+1 = 2y-8 ; \quad 2x-8 = y-2$$

$$x-2y = -9 \quad 2x-y = 6$$

$$x = 2y-9$$

$$2(2y-9) - y = 6$$

$$4y - 18 - y = 6$$

$$3y = 24$$

$$y = 8$$

$$x = 2(8) - 9$$

$$x = 7$$

$$\text{sol: } x = 7 \quad y = 8$$

$$16. \quad x = -\frac{3y+3}{4} ; \quad y = -\frac{1+5x}{4}$$

$$4x = -3y-3 ; \quad 4y = -1-5x$$

$$4x + 3y = -3 \quad 4y + 5x = -1$$

$$4x + 3y = -3 \quad (4)$$

$$5x + 4y = -1 \quad (-3)$$

$$\hline 16x + 12y = -12$$

$$-15x - 12y = 3$$

$$\hline x = -9$$

$$4(-9) + 3y = -3$$

$$-36 + 3y = -3$$

$$3y = 33$$

$$y = 11$$

$$\text{sol: } x = -9 \quad y = 11$$

$$17. \quad \frac{x+y}{6} = \frac{x-y}{12} ; \quad \frac{2x}{3} = y+3$$

$$2(x+y) = x-y ; \quad 2x = 3(y+3)$$

$$2x + 2y = x - y ; \quad 2x = 3y + 9$$

$$x + 3y = 0 ; \quad 2x - 3y = 9$$

$$x = -3y$$

$$2(-3y) - 3y = 9$$

$$-6y - 3y = 9$$

$$-9y = 9$$

$$y = -1$$

$$x = -3(-1)$$

$$x = 3$$

$$\text{sol: } x = 3 \quad y = -1$$

$$18. 3x - \frac{y-3}{5} = 6$$

$$15x - y + 3 = 30$$

$$15x - y = 27$$

$$15x = 27 + y$$

$$x = \frac{27+y}{15}$$

$$3y - \frac{x-2}{7} = 9$$

$$21y - x + 2 = 63$$

$$-x + 21y = 61$$

$$-x = 61 - 21y$$

$$x = 21y - 61$$

$$27 + y = 15(21y - 61)$$

$$27 + y = 315y - 915$$

$$-314y = -942$$

$$y = 3$$

$$x = \frac{27+3}{15}$$

$$x = 2$$

$$\text{sol: } x = 2 \quad y = 3$$

$$19. \frac{x+y}{6} - \frac{y-x}{3} = \frac{7}{24}$$

$$4(x+y) - 8(y-x) = 7$$

$$4x + 4y - 8y + 8x = 7$$

$$12x - 4y = 7$$

$$\frac{x}{2} + \frac{x-y}{6} = \frac{5}{12}$$

$$6x + 2(x-y) = 5$$

$$6x + 2x - 2y = 5$$

$$8x - 2y = 5$$

$$12x - 4y = 7$$

$$\frac{8x - 2y = 5}{12x - 4y = 7} \quad (-2)$$

$$12x - 4y = 7$$

$$-16x + 4y = -10$$

$$-4x = -3$$

$$x = \frac{3}{4}$$

$$12\left(\frac{3}{4}\right) - 4y = 7$$

$$9 - 4y = 7$$

$$-4y = -2$$

$$y = \frac{1}{2}$$

$$\text{sol: } x = \frac{3}{4} \quad y = \frac{1}{2}$$

$$20. \frac{x-2}{4} - \frac{y-x}{2} = x-7$$

$$x - 2 - 2(y-x) = 4(x-7)$$

$$3x - 4x - 2y = -28 + 2$$

$$x + 2y = 26$$

$$\frac{3x-y}{8} - \frac{3y-x}{6} = y-13$$

$$3(3x-y) - 4(3y-x) = 24(y-13)$$

$$13x - 15y - 24y = -312$$

$$13x - 39y = -312$$

$$x + 2y = 26 \quad (-13)$$

$$\frac{13x - 39y = -312}{-13x - 26y = -338}$$

$$-13x - 26y = -338$$

$$\frac{13x - 39y = -312}{-65y = -650}$$

$$-65y = -650$$

$$y = 10$$

$$x + 2(10) = 26$$

$$x + 20 = 26$$

$$x = 6$$

$$\text{sol: } x = 6 \quad y = 10$$

$$21. 12 - \frac{3x-2y}{6} = 3y+2$$

$$72 - 3x + 2y = 6(3y+2)$$

$$-3x + 2y - 18y = 12 - 72$$

$$3x + 16y = 60$$

$$\frac{5y-3x}{3} = x-y$$

$$5y - 3x = 3(x-y)$$

$$-3x - 3x + 5y + 3y = 0$$

$$6x - 8y = 0$$

$$3x + 16y = 60$$

$$\frac{6x - 8y = 0}{3x + 16y = 60} \quad (2)$$

$$3x + 16y = 60$$

$$\frac{12x - 16y = 0}{15x = 60}$$

$$15x = 60$$

$$x = 4$$

$$6(4) - 8y = 0$$

$$-8y = -24$$

$$y = 3$$

$$\text{sol: } x = 4 \quad y = 3$$

$$22. y(x-4) = x(y-6)$$

$$xy - 4y = xy - 6x$$

$$6x - 4y = 0$$

$$\frac{5}{x-3} - \frac{11}{y-1} = 0$$

$$5(y-1) - 11(x-3) = 0$$

$$5y - 5 - 11x + 33 = 0$$

$$11x - 5y = 28$$

$$6x - 4y = 0 \quad (5)$$

$$\frac{11x - 5y = 28}{30x - 20y = 0} \quad (-4)$$

$$30x - 20y = 0$$

$$-44x + 20y = -112$$

$$\frac{-44x + 20y = -112}{-14x = -112}$$

$$x = 8$$

$$11(8) - 5y = 28$$

$$-5y = 28 - 88$$

$$-5y = -60$$

$$y = 12$$

$$\text{sol: } x = 8 \quad y = 12$$

$$23. \frac{3(x+3y)}{5x+6y} = \frac{21}{17}$$

$$51(x+3y) = 21(5x+6y)$$

$$51x + 153y = 105x + 126y$$

$$-54x + 27y = 0$$

$$2x - y = 0$$

$$\frac{4x-7y}{2y+1} = -2$$

$$4x - 7y = -2(2y+1)$$

$$4x - 7y = -4y - 2$$

$$4x - 3y = -2$$

$$2x - y = 0 \quad (-2)$$

$$\frac{4x - 3y = -2}{-4x + 2y = 0}$$

$$-4x + 2y = 0$$

$$\frac{4x - 3y = -2}{y = 2}$$

$$y = 2$$

$$2x - 2 = 0$$

$$2x = 2$$

$$x = 1$$

$$\text{sol: } x = 1 \quad y = 2$$

$$\begin{aligned}
24. \quad & \frac{7}{2x-3y+6} = -\frac{7}{3x-2y-1} \\
& 7(3x-2y-1) = -7(2x-3y+6) \\
& 21x-14y-7 = -14x+21y-42 \\
& 35x-35y = -35 \\
& x-y = -1 \\
& \frac{6}{x-y+4} = \frac{10}{y+2} \\
& 6(y+2) = 10(x-y+4) \\
& 6y+12 = 10x-10y+40 \\
& -28 = 10x-16y \\
& -14 = 5x-8y \\
& x-y = -1 \quad (-8) \\
& \frac{5x-8y = -14}{-8x+8y = 8} \\
& \frac{5x-8y = -14}{-3x} = -6 \\
& x = 2 \\
& 2-y = -1 \\
& -y = -3 \\
& y = 3 \\
& \text{sol: } x=2 \quad y=3
\end{aligned}$$

$$\begin{aligned}
25. \quad & \frac{x+y}{x-y} = -7 \\
& x+y = -7(x-y) \\
& x+y = -7x+7y \\
& 8x-6y = 0 \\
& \frac{x+y+1}{x+y-1} = \frac{3}{4} \\
& 4(x+y+1) = 3(x+y-1) \\
& 4x+4y+4 = 3x+3y-3 \\
& x+y = -7 \\
& 8x-6y = 0 \\
& \frac{x+y}{x-y} = -7 \quad (6) \\
& 8x-6y = 0 \\
& \frac{6x+6y = -42}{14x} = -42 \\
& x = -3 \\
& -3+y = -7 \\
& y = -4 \\
& \text{sol: } x=-3 \quad y=-4
\end{aligned}$$

$$\begin{aligned}
26. \quad & \frac{x}{4} - 8 = \frac{3y}{2} - \frac{33}{4} \\
& x-32 = 6y-33 \\
& x-6y = -1 \\
& \frac{y-x}{3} - \frac{2x+y}{2} = -\frac{17}{24} \\
& 8(y-x) - 12(2x+y) = -17 \\
& 8y-8x-24x-12y = -17 \\
& 32x+4y = 17 \\
& x-6y = -1 \quad (2) \\
& 32x+4y = 17 \quad (3) \\
& 2x-12y = -2 \\
& \frac{96x+12y = 51}{98x} = 49 \\
& x = \frac{1}{2} \\
& \frac{1}{2} - 6y = -1 \\
& 1-12y = -2 \\
& -12y = -3 \\
& y = \frac{1}{4} \\
& \text{sol: } x = \frac{1}{2} \quad y = \frac{1}{4}
\end{aligned}$$

$$\begin{aligned}
27. \quad & \frac{x-2}{x+2} = \frac{y-7}{y-5} \\
& (y-5)(x-2) = (y-7)(x+2) \\
& xy-2y-5x+10 = xy+2y-7x-14 \\
& 2x-4y = -24 \\
& x-2y = -12 \\
& x = -12+2y \\
& \frac{x+1}{x-1} = \frac{y-3}{y-5} \\
& (y-5)(x+1) = (x-1)(y-3) \\
& xy+y-5x-5 = xy-3x-y+3 \\
& -2x+2y = 8 \\
& -x+y = 4 \\
& x = y-4 \\
& -12+2y = y-4 \\
& y = 8 \\
& x = 8-4 \\
& x = 4 \\
& \text{sol: } x=4 \quad y=8
\end{aligned}$$

$$\begin{aligned}
28. \quad & \frac{x-y-1}{x+y+1} = -\frac{3}{17} \\
& 17(x-y-1) = -3(x+y+1) \\
& 17x-17y-17 = -3x-3y-3 \\
& 20x-14y = 14 \\
& 10x-7y = 7 \\
& \frac{x+y-1}{x-y+1} = -15 \\
& x+y-1 = -15(x-y+1) \\
& x+y-1 = -15x+15y-15 \\
& 16x-14y = -14 \\
& 8x-7y = -7 \\
& 10x-7y = 7 \\
& \frac{8x-7y = -7}{10x-7y = 7} \quad (-1) \\
& \frac{-8x+7y = 7}{2x} = 14 \\
& x = 7 \\
& 10(7)-7y = 7 \\
& -7y = -63 \\
& y = 9 \\
& \text{sol: } x=7 \quad y=9
\end{aligned}$$

$$\begin{aligned}
29. \quad & \frac{6x+9y-4}{4x-6y+5} = \frac{2}{5} \\
& 5(6x+9y-4) = 2(4x-6y+5) \\
& 30x+45y-20 = 8x-12y+10 \\
& 22x+57y = 30 \\
& \frac{2x+3y-3}{3x+2y-4} = \frac{6}{11} \\
& 11(2x+3y-3) = 6(3x+2y-4) \\
& 22x+33y-33 = 18x+12y-24 \\
& 4x+21y = 9 \\
& 22x+57y = 30 \quad (2) \\
& \frac{4x+21y = 9}{44x+114y = 60} \quad (-11) \\
& \frac{-44x-231y = -99}{-117y = -39} \\
& y = \frac{1}{3} \\
& 4x+21\left(\frac{1}{3}\right) = 9 \\
& 4x+7 = 9 \\
& 4x = 2 \\
& x = \frac{1}{2} \\
& \text{sol: } x = \frac{1}{2} \quad y = \frac{1}{3}
\end{aligned}$$

$$\begin{aligned}
 30. \quad & \frac{3x+2y}{x+y-15} = -9 \\
 & 3x+2y = -9(x+y-15) \\
 & 3x+2y = -9x-9y+135 \\
 & 12x+11y = 135 \\
 & \frac{4x}{3} - \frac{5(y-1)}{8} = -1 \\
 & 32x-15(y-1) = -24 \\
 & 32x-15y+15 = -24 \\
 & 32x-15y = -39 \\
 & 12x+11y = 135 \quad (8) \\
 & 32x-15y = -39 \quad (-3) \\
 & \hline
 & 96x+88y = 1.080 \\
 & -96x+45y = 117 \\
 & \hline
 & 133y = 1.197 \\
 & y = 9 \\
 & 12x+11(9) = 135 \\
 & 12x = 135-99 \\
 & 12x = 36 \\
 & x = 3 \\
 & \text{sol: } x=3 \quad y=9
 \end{aligned}$$

$$\begin{aligned}
 31. \quad & \frac{2x+5}{17} - (5-y) = -60 \\
 & 2x+5-17(5-y) = -1.020 \\
 & 2x-85+17y = -1.025 \\
 & 2x+17y = -940 \\
 & \frac{y+62}{2} - (1-x) = 40 \\
 & y+62-2(1-x) = 80 \\
 & y-2+2x = 18 \\
 & 2x+y = 20 \\
 & 2x+17y = -940 \\
 & 2x+y = 20 \quad (-1) \\
 & \hline
 & 2x+17y = -940 \\
 & -2x-y = -20 \\
 & \hline
 & 16y = -960 \\
 & y = -60 \\
 & 2x-60 = 20 \\
 & 2x = 80 \\
 & x = 40 \\
 & \text{sol: } x=40 \quad y=-60
 \end{aligned}$$

$$\begin{aligned}
 32. \quad & \frac{3x+4y}{x-6y} = -\frac{30}{23} \\
 & 23(3x+4y) = -30(x-6y) \\
 & 69x+92y = -30x+180y \\
 & 99x-88y = 0 \\
 & 9x-8y = 0 \\
 & \frac{9x-y}{3+x-y} = -\frac{63}{37} \\
 & 37(9x-y) = -63(3+x-y) \\
 & 333x-37y = -189-63x+63y \\
 & 396x-100y = -189 \\
 & 396x-100y = -189 \\
 & \hline
 & 9x-8y = 0 \quad (-44) \\
 & 396x-100y = -189 \\
 & -396x+352y = 0 \\
 & \hline
 & 252y = -189 \\
 & y = -\frac{3}{4} \\
 & 9x-8\left(-\frac{3}{4}\right) = 0 \\
 & 9x+6 = 0 \\
 & 9x = -6 \\
 & x = -\frac{2}{3} \\
 & \text{sol: } x = -\frac{2}{3} \quad y = -\frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 33. \quad & x - \frac{4x+1}{9} = \frac{2y-5}{3} \\
 & 9x - (4x+1) = 3(2y-5) \\
 & 9x-4x-1 = 6y-15 \\
 & 5x-6y = -14 \\
 & y - \frac{3y+2}{7} = \frac{x+18}{10} \\
 & 70y-10(3y+2) = 7(x+18) \\
 & 70y-30y-20 = 7x+126 \\
 & -146 = 7x-40y \\
 & 7x-40y = -146 \quad (-5) \\
 & \hline
 & 5x-6y = -14 \quad (7) \\
 & -35x+200y = 730 \\
 & 35x-42y = -98 \\
 & \hline
 & 158y = 632 \\
 & y = 4 \\
 & 5x-6(4) = -14 \\
 & 5x = -14+24 \\
 & x = 2 \\
 & \text{sol: } x=2 \quad y=4
 \end{aligned}$$

## EJERCICIO 181

$$\begin{aligned}
 1. \quad & x+y=a+b; \quad x-y=a-b \\
 & x+y=a+b \\
 & \underline{x-y=a-b} \\
 & x = a \\
 & a+y=a+b \\
 & y=b \\
 & \text{sol: } x=a \quad y=b
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & 2x+y=b+2 \\
 & 2bx+by=b^2+2b \\
 & \underline{bx-y=0} \\
 & -2bx+2y=0 \\
 & -2bx+2y=0 \\
 & \hline
 & 2bx+by=b^2+2b \\
 & 2y+by=b^2+2b \\
 & y(2+b)=b(b+2) \\
 & y=b \\
 & 2x+b=b+2 \\
 & 2x=2 \\
 & x=1 \\
 & \text{sol: } x=1 \quad y=b
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & 2x-y=3a; \\
 & x-2y=0 \\
 & -2x+4y=0 \\
 & \hline
 & 2x-y=3a \\
 & -2x+4y=0 \\
 & \hline
 & 3y=3a \\
 & y=a \\
 & 2x-a=3a \\
 & 2x=4a \\
 & x=2a \\
 & \text{sol: } x=2a \quad y=a
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & x-y=1-a; \quad x+y=1+a \\
 & x-y=1-a \\
 & \underline{x+y=1+a} \\
 & 2x = 2 \\
 & x=1 \\
 & 1+y=1+a \\
 & y=a \\
 & \text{sol: } x=1 \quad y=a
 \end{aligned}$$



$$\begin{aligned}
5. \quad \frac{x}{a} + y &= 2b \quad ; \quad \frac{x}{b} - y = a - b \\
x + ay &= 2ab \quad ; \quad x - by = ab - b^2 \\
x + ay &= 2ab \\
\frac{-x}{-x} + \frac{by}{by} &= \frac{b^2 - ab}{b^2 - ab} \\
ay + by &= b^2 + ab \\
y(a + b) &= b(b + a) \\
y &= b \\
x + ab &= 2ab \\
x &= ab \\
\text{sol: } x &= ab \quad y = b
\end{aligned}$$

$$\begin{aligned}
6. \quad \frac{x}{b} + \frac{y}{a} &= 2 \\
ax + by &= 2ab \\
abx + b^2y &= 2ab^2 \\
\frac{x}{a} + \frac{y}{b} &= \frac{a^2 + b^2}{ab} \\
bx + ay &= a^2 + b^2 \\
-abx - a^2y &= -a^3 - ab^2 \\
abx + b^2y &= 2ab^2 \\
\frac{-abx}{-abx} - \frac{a^2y}{-a^2y} &= \frac{-a^3 - ab^2}{-a^3 - ab^2} \\
b^2y - a^2y &= -a^3 + ab^2 \\
y(b^2 - a^2) &= a(b^2 - a^2) \\
y &= a \\
ax + ba &= 2ab \\
a(x + b) &= 2ab \\
x + b &= 2b \\
x &= b \\
\text{sol: } x &= b \quad y = a
\end{aligned}$$

$$\begin{aligned}
7. \quad x + y &= a + b \\
-ax - ay &= -a^2 - ab \\
ax + by &= a^2 + b^2 \\
-ax - ay &= -a^2 - ab \\
\frac{ax + by}{ax + by} &= \frac{a^2 + b^2}{a^2 + b^2} \\
by - ay &= b^2 - ab \\
y(b - a) &= b(b - a) \\
y &= b \\
x + b &= a + b \\
x &= a \\
\text{sol: } x &= a \quad y = b
\end{aligned}$$

$$\begin{aligned}
8. \quad ax - by &= 0 \quad ; \quad x + y = \frac{a+b}{ab} \\
abx - b^2y &= 0 \quad abx + aby = a + b \\
abx - b^2y &= 0 \\
\frac{-abx}{-abx} - \frac{aby}{-aby} &= \frac{-a - b}{-a - b} \\
-b^2y - aby &= -(a + b) \\
-yb(b + a) &= -(a + b) \\
-yb &= -1 \Rightarrow y = \frac{1}{b} \\
ax - b\left(\frac{1}{b}\right) &= 0 \\
ax = 1 &\Rightarrow x = \frac{1}{a} \\
\text{sol: } x &= \frac{1}{a} \quad y = \frac{1}{b}
\end{aligned}$$

$$\begin{aligned}
9. \quad mx - ny &= m^2 + n^2 \quad ; \quad nx + my = m^2 + n^2 \\
mnx - n^2y &= m^2n + n^3 \quad mnx + m^2y = m^3 + mn^2 \\
-mnx + n^2y &= -m^2n - n^3 \\
\frac{mnx}{mnx} + \frac{m^2y}{m^2y} &= \frac{m^3}{m^3} + \frac{mn^2}{mn^2} \\
n^2y + m^2y &= m^3 - m^2n + mn^2 - n^3 \\
y(m^2 + n^2) &= m^2(m - n) + n^2(m - n) \\
y(m^2 + n^2) &= (m^2 + n^2)(m - n) \\
y &= m - n \\
m^2x - mny &= m^3 + mn^2 \\
\frac{n^2x + mny}{n^2x + mny} &= \frac{m^2n + n^3}{m^2n + n^3} \\
m^2x + n^2x &= m^3 + mn^2 + m^2n + n^3 \\
x(m^2 + n^2) &= m(m^2 + n^2) + n(m^2 + n^2) \\
x(m^2 + n^2) &= (m + n)(m^2 + n^2) \\
x &= m + n \\
\text{sol: } x &= m + n \quad y = m - n
\end{aligned}$$

$$\begin{aligned}
10. \quad \frac{x}{m} + \frac{y}{n} &= 2m \quad ; \quad mx - ny = m^3 - mn^2 \\
xn + ym &= 2m^2n \quad xmn - n^2y = m^3n - mn^3 \\
xmn + ym^2 &= 2m^3n \\
\frac{-xmn}{-xmn} + \frac{n^2y}{n^2y} &= \frac{mn^3 - m^3n}{mn^3 - m^3n} \\
ym^2 + yn^2 &= mn^3 + m^3n \\
y(m^2 + n^2) &= mn(n^2 + m^2) \\
y &= mn \\
xn + m^2n &= 2m^2n \\
xn = m^2n &\Rightarrow x = m^2 \\
\text{sol: } x &= m^2 \quad y = mn
\end{aligned}$$

11.  $x + y = a$

$$ax + ay = a^2$$

$$ax - by = a(a + b) + b^2$$

$$ax - by = a^2 + ab + b^2$$

$$-ax - ay = -a^2$$

$$\frac{ax \quad -by = a^2 + ab + b^2}{-ay - by = ab + b^2}$$

$$-y(a + b) = b(a + b)$$

$$-y = b$$

$$y = -b$$

$$x - b = a$$

$$x = a + b$$

$$\text{sol: } x = a + b \quad y = -b$$

12.  $x - y = m - n$

$$mx - my = m^2 - mn$$

$$\frac{mx \quad -ny = m^2 - mn}{-mx + my = -m^2 + mn}$$

$$my - ny = mn - n^2$$

$$y(m - n) = n(m - n)$$

$$y = n$$

$$x - n = m - n$$

$$x = m$$

$$\text{sol: } x = m \quad y = n$$

13.  $\frac{x}{a} + \frac{y}{b} = 0$  ;  $\frac{x}{b} + \frac{2y}{a} = \frac{2b^2 - a^2}{ab}$

$$bx + ay = 0 \quad ax + 2by = 2b^2 - a^2$$

$$-abx \quad -a^2y = 0$$

$$\frac{abx + 2b^2y = 2b^3 - a^2b}{2b^2y - a^2y = 2b^3 - a^2b}$$

$$y(2b^2 - a^2) = b(2b^2 - a^2)$$

$$y = b$$

$$\frac{x}{a} + \frac{b}{b} = 0$$

$$bx + ab = 0$$

$$bx = -ab$$

$$x = -a$$

$$\text{sol: } x = -a \quad y = b$$

14.  $x + y = 2c$  ;  $a^2(x - y) = 2a^3$

$$-a^2x - a^2y = -2a^2c \quad ; \quad a^2x - a^2y = 2a^3$$

$$-a^2x - a^2y = -2a^2c$$

$$\frac{a^2x - a^2y = 2a^3}{-2a^2y = 2a^3 - 2a^2c}$$

$$-2a^2y = 2a^2(a - c)$$

$$-y = a - c$$

$$y = c - a$$

$$x + c - a = 2c$$

$$x - a = c \Rightarrow x = c + a$$

$$\text{sol: } x = a + c \quad y = c - a$$

15.  $ax - by = 0$  ;  $ay - bx = \frac{a^2 - b^2}{ab}$

$$b^2ax - b^3y = 0 \quad a^2by - b^2ax = a^2 - b^2$$

$$b^2ax \quad -b^3y = 0$$

$$\frac{-b^2ax + a^2by = a^2 - b^2}{a^2by - b^3y = a^2 - b^2}$$

$$by(a^2 - b^2) = a^2 - b^2$$

$$by = 1$$

$$y = \frac{1}{b}$$

$$ax - b\left(\frac{1}{b}\right) = 0$$

$$ax = 1$$

$$x = \frac{1}{a}$$

$$\text{sol: } x = \frac{1}{a} \quad y = \frac{1}{b}$$

16.  $\frac{x}{b^2} + \frac{y}{a^2} = a + b$  ;  $x - y = ab(b - a)$

$$a^2x + b^2y = a^3b^2 + a^2b^3 \quad ; \quad x - y = ab^2 - a^2b$$

$$a^2x - a^2y = a^3b^2 - a^4b$$

$$-a^2x - b^2y = -a^3b^2 - a^2b^3$$

$$\frac{a^2x \quad -a^2y = a^3b^2 - a^4b}{-b^2y - a^2y = -a^2b^3 - a^4b}$$

$$-y(a^2 + b^2) = -a^2b(b^2 + a^2)$$

$$-y = -a^2b$$

$$y = a^2b$$

$$\frac{x}{b^2} + \frac{a^2b}{a^2} = a + b$$

$$x + b^3 = ab^2 + b^3$$

$$x = ab^2$$

$$\text{sol: } x = ab^2 \quad y = a^2b$$

$$\begin{aligned}
 17. \quad & nx + my = m + n \quad ; \quad mx - ny = \frac{m^3 - n^3}{mn} \\
 & m^2 nx + m^3 y = m^3 + m^2 n \quad m^2 nx - mn^2 y = m^3 - n^3 \\
 & -m^2 nx - m^3 y = -m^3 - m^2 n \\
 & \frac{m^2 nx}{-m^3 y - mn^2 y} = \frac{m^3}{-m^2 n - n^3} \\
 & -m^3 y - mn^2 y = -m^2 n - n^3 \\
 & -ym(m^2 + n^2) = -n(m^2 + n^2) \\
 & \quad \quad \quad ym = n \\
 & \quad \quad \quad y = \frac{n}{m} \\
 & nx + m\left(\frac{n}{m}\right) = m + n \\
 & \quad \quad \quad nx + n = m + n \\
 & \quad \quad \quad nx = m \\
 & \quad \quad \quad x = \frac{m}{n} \\
 \text{sol: } & x = \frac{m}{n} \quad y = \frac{n}{m}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & (a-b)x - (a+b)y = b^2 - 3ab \\
 & \quad \quad \quad ax - bx - ay - by = b^2 - 3ab \\
 & (a+b)x - (a-b)y = ab - b^2 \\
 & \quad \quad \quad ax + bx - ay + by = ab - b^2 \\
 & -ax + bx + ay + by = -b^2 + 3ab \\
 & \frac{ax + bx - ay + by = -b^2 + ab}{2bx + 2by = -2b^2 + 4ab} \\
 & \quad \quad \quad 2b(x+y) = 2b(2a-b) \\
 & \quad \quad \quad x+y = 2a-b \\
 & \quad \quad \quad ax - bx - ay - by = b^2 - 3ab \\
 & \frac{ax + bx - ay + by = -b^2 + ab}{2ax - 2ay = -2ab} \\
 & \quad \quad \quad 2a(x-y) = -2ab \\
 & \quad \quad \quad x-y = -b \\
 & \quad \quad \quad x+y = 2a-b \\
 & \quad \quad \quad x-y = -b \\
 & \frac{2x}{2x} = \frac{2(a-b)}{2(a-b)} \\
 & \quad \quad \quad x = a-b \\
 & a-b-y = -b \\
 & \quad \quad \quad -y = -a \\
 & \quad \quad \quad y = a \\
 \text{sol: } & x = a-b \quad y = a
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & \frac{x+b}{a} + \frac{y-b}{b} = \frac{a+b}{b} \\
 & b(x+b) + a(y-b) = a(a+b) \\
 & \quad \quad \quad bx + b^2 + ay - ab = a^2 + ab \\
 & abx + ab^2 + a^2 y - a^2 b = a^3 + a^2 b \\
 & \quad \quad \quad -abx - ab^2 - a^2 y = -a^3 - 2a^2 b \\
 & \frac{x-a}{b} - \frac{y-a}{a} = -\frac{a+b}{a} \\
 & a(x-a) - b(y-a) = -b(a+b) \\
 & \quad \quad \quad ax - a^2 - by + ba = -ba - b^2 \\
 & abx - a^2 b - b^2 y + b^2 a = -b^2 a - b^3 \\
 & \quad \quad \quad abx - a^2 b - b^2 y = -2b^2 a - b^3 \\
 & -abx - ab^2 - a^2 y = -a^3 - 2a^2 b \\
 & \frac{abx}{-ab^2 - a^2 b - a^2 y - b^2 y} = \frac{-2b^2 a - b^3}{-a^3 - 2a^2 b - 2b^2 a - b^3} \\
 & \quad \quad \quad ab^2 + a^2 b + a^3 + b^3 = a^2 y + b^2 y \\
 & \quad \quad \quad b^2(a+b) + a^2(b+a) = y(a^2 + b^2) \\
 & \quad \quad \quad (b^2 + a^2)(a+b) = y(a^2 + b^2) \\
 & \quad \quad \quad \quad \quad \quad \quad a+b = y
 \end{aligned}$$

$$\begin{aligned}
 & \frac{x+b}{a} + \frac{a+b-b}{b} = \frac{a+b}{b} \\
 & \quad \quad \quad b(x+b) + a^2 = a(a+b) \\
 & \quad \quad \quad bx + b^2 + a^2 = a^2 + ab \\
 & \quad \quad \quad \quad \quad \quad \quad bx = ab - b^2 \Rightarrow bx = b(a-b) \Rightarrow x = a-b \\
 \text{sol: } & x = a-b \quad y = a+b
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & \frac{x}{a+b} + \frac{y}{a+b} = \frac{1}{ab} \quad ; \quad \frac{x}{b} + \frac{y}{a} = \frac{a^2 + b^2}{a^2 b^2} \\
 & \quad \quad \quad abx + aby = a + b \quad \quad \quad a^2 bx + ab^2 y = a^2 + b^2 \\
 & -a^2 bx - a^2 by = -a^2 - ab \\
 & \frac{-a^2 bx}{a^2 bx + ab^2 y} = \frac{-a^2 - ab}{a^2 + b^2} \\
 & \quad \quad \quad ab^2 y - a^2 by = b^2 - ab \\
 & \quad \quad \quad ayb(b-a) = b(b-a) \\
 & \quad \quad \quad \quad \quad \quad \quad ayb = b \\
 & \quad \quad \quad \quad \quad \quad \quad ay = 1 \\
 & \quad \quad \quad \quad \quad \quad \quad y = \frac{1}{a} \\
 & abx + ab\left(\frac{1}{a}\right) = a + b \\
 & \quad \quad \quad abx + b = a + b \\
 & \quad \quad \quad abx = a \Rightarrow bx = 1 \Rightarrow x = \frac{1}{b}
 \end{aligned}$$

$$\text{sol: } x = \frac{1}{b} \quad y = \frac{1}{a}$$

## EJERCICIO 182

$$1. \frac{1}{x} + \frac{2}{y} = \frac{7}{6}; \quad \frac{2}{x} + \frac{1}{y} = \frac{4}{3}$$

$$\frac{2}{x} - \frac{4}{y} = -\frac{7}{3}$$

$$\frac{2}{x} + \frac{1}{y} = \frac{4}{3}$$

$$\frac{-3}{y} = -1$$

$$3=y$$

$$\frac{2}{x} + \frac{1}{3} = \frac{4}{3}$$

$$6+x=4x$$

$$6=3x \Rightarrow 2=x$$

$$\text{sol: } x=2 \quad y=3$$

$$2. \frac{3}{x} - \frac{2}{y} = \frac{1}{2}; \quad \frac{2}{x} + \frac{5}{y} = \frac{23}{12}$$

$$-\frac{6}{x} + \frac{4}{y} = -1$$

$$\frac{6}{x} + \frac{15}{y} = \frac{23}{4}$$

$$\frac{19}{y} = \frac{19}{4}$$

$$4=y$$

$$\frac{3}{x} - \frac{2}{4} = \frac{1}{2}$$

$$6-x=x$$

$$6=2x$$

$$3=x$$

$$\text{sol: } x=3 \quad y=4$$

$$3. \frac{5}{x} + \frac{4}{y} = 7; \quad \frac{7}{x} - \frac{6}{y} = 4$$

$$\frac{15}{x} + \frac{12}{y} = 21$$

$$\frac{14}{x} - \frac{12}{y} = 8$$

$$\frac{29}{x} = 29$$

$$1=x$$

$$\frac{5}{1} + \frac{4}{y} = 7$$

$$5y+4=7y$$

$$4=2y$$

$$2=y$$

$$\text{sol: } x=1 \quad y=2$$

$$4. \frac{12}{x} + \frac{5}{y} = -\frac{13}{2}; \quad \frac{18}{x} + \frac{7}{y} = -\frac{19}{2}$$

$$\frac{36}{x} - \frac{15}{y} = \frac{39}{2}$$

$$\frac{36}{x} + \frac{14}{y} = -\frac{38}{2}$$

$$-\frac{1}{y} = \frac{1}{2}$$

$$-2=y$$

$$36-7x=-19x$$

$$12x=-36$$

$$x=-3$$

$$\text{sol: } x=-3 \quad y=-2$$

$$5. \frac{9}{x} + \frac{3}{y} = 27; \quad \frac{5}{x} + \frac{4}{y} = 22$$

$$\frac{3}{x} + \frac{1}{y} = 9$$

$$-\frac{12}{x} - \frac{4}{y} = -36$$

$$\frac{5}{x} + \frac{4}{y} = 22$$

$$-\frac{7}{x} = -14$$

$$\frac{1}{2}=x$$

$$\frac{5}{\frac{1}{2}} + \frac{4}{y} = 22 \Rightarrow 10 + \frac{4}{y} = 22$$

$$10y+4=22y$$

$$4=12y$$

$$\frac{1}{3}=y$$

$$\text{sol: } x=\frac{1}{2} \quad y=\frac{1}{3}$$

$$6. \frac{6}{x} - \frac{8}{y} = -23; \quad \frac{4}{x} + \frac{11}{y} = 50$$

$$-\frac{12}{x} + \frac{16}{y} = 46$$

$$\frac{12}{x} + \frac{33}{y} = 150$$

$$\frac{49}{y} = 196$$

$$\frac{1}{4}=y$$

Continúa

## 6. Continuación

$$\frac{4}{x} + \frac{11}{y} = 50$$

$$\frac{4}{x} + 44 = 50$$

$$4+44x=50x$$

$$4=6x$$

$$\frac{2}{3}=x$$

$$\text{sol: } x=\frac{2}{3} \quad y=\frac{1}{4}$$

$$7. \frac{9}{x} + \frac{10}{y} = -11; \quad \frac{7}{x} - \frac{15}{y} = -4$$

$$\frac{27}{x} + \frac{30}{y} = -33$$

$$\frac{14}{x} - \frac{30}{y} = -8$$

$$\frac{41}{x} = -41$$

$$-1=x$$

$$-\frac{7}{1} - \frac{15}{y} = -4$$

$$-7y-15=-4y$$

$$-15=3y$$

$$-5=y$$

$$\text{sol: } x=-1 \quad y=-5$$

$$8. \frac{1}{2x} - \frac{3}{y} = \frac{3}{4}; \quad \frac{1}{x} + \frac{5}{2y} = -\frac{4}{3}$$

$$\frac{1}{2x} - \frac{3}{y} = \frac{3}{4}$$

$$-\frac{1}{2x} - \frac{5}{4y} = \frac{2}{3}$$

$$-\frac{17}{4y} = \frac{17}{12}$$

$$-12=4y$$

$$-3=y$$

$$\frac{1}{x} + \frac{5}{2(-3)} = -\frac{4}{3}$$

$$6-5x=-8x$$

$$3x=-6$$

$$x=-2$$

$$\text{sol: } x=-2 \quad y=-3$$

$$\begin{array}{l}
9. \quad \frac{2}{5x} - \frac{1}{3y} = -\frac{11}{45}; \quad \frac{1}{10x} - \frac{3}{5y} = \frac{4}{5} \\
\quad -\frac{2}{10x} + \frac{1}{6y} = \frac{11}{90} \\
\quad \frac{2}{10x} - \frac{6}{5y} = \frac{8}{5} \\
\quad -\frac{31}{30y} = \frac{31}{18} \\
\quad -18 = 30y \\
\quad -\frac{3}{5} = y \\
\quad \frac{1}{10x} + \frac{3}{5\left(\frac{3}{5}\right)} = \frac{4}{5} \\
\quad 3 + 30x = 24x \\
\quad 6x = -3 \Rightarrow x = -\frac{1}{2} \\
\text{sol: } x = -\frac{1}{2} \quad y = -\frac{3}{5}
\end{array}$$

$$\begin{array}{l}
10. \quad \frac{3}{x} - \frac{7}{3y} = \frac{2}{3}; \quad \frac{1}{4x} + \frac{8}{y} = \frac{103}{84} \\
\quad \frac{3}{4x} - \frac{7}{12y} = \frac{1}{6} \\
\quad -\frac{3}{4x} - \frac{24}{y} = -\frac{309}{84} \\
\quad -\frac{295}{12y} = -\frac{295}{84} \\
\quad y = 7 \\
\quad \frac{3}{x} - \frac{7}{3(7)} = \frac{2}{3} \\
\quad 9 - x = 2x \\
\quad 9 = 3x \\
\quad 3 = x \\
\text{sol: } x = 3 \quad y = 7
\end{array}$$

$$\begin{array}{l}
11. \quad \frac{3}{10x} + \frac{1}{3y} = \frac{107}{60}; \quad \frac{6}{5x} + \frac{1}{4y} = \frac{14}{5} \\
\quad \frac{3}{5x} + \frac{2}{3y} = \frac{107}{30} \\
\quad -\frac{3}{5x} - \frac{1}{8y} = -\frac{7}{5} \\
\quad \frac{13}{24y} = \frac{13}{6} \\
\quad \frac{1}{4} = y \\
\quad \frac{6}{5x} + \frac{1}{4\left(\frac{1}{4}\right)} = \frac{14}{5} \\
\quad 6 + 5x = 14x \\
\quad 6 = 9x \\
\quad \frac{2}{3} = x \\
\text{sol: } x = \frac{2}{3} \quad y = \frac{1}{4}
\end{array}$$

$$\begin{array}{l}
12. \quad \frac{1}{x} + \frac{1}{y} = a; \quad \frac{1}{x} - \frac{1}{y} = b \\
\quad \frac{1}{x} + \frac{1}{y} = a \\
\quad -\frac{1}{x} + \frac{1}{y} = -b \\
\quad \frac{2}{y} = a - b \Rightarrow \frac{2}{a - b} = y \\
\quad \frac{1}{x} - \frac{1}{\frac{2}{a - b}} = b \\
\quad 2 - x(a - b) = 2bx \\
\quad 2 - ax + xb = 2bx \\
\quad -ax = bx - 2 \\
\quad -ax - bx = -2 \\
\quad x(a + b) = 2 \Rightarrow x = \frac{2}{a + b} \\
\text{sol: } x = \frac{2}{a + b} \quad y = \frac{2}{a - b}
\end{array}$$

$$\begin{array}{l}
13. \quad \frac{2}{x} - \frac{3b}{y} = \frac{2 - 3a}{a}; \quad \frac{a}{x} + \frac{b}{y} = 2 \\
\quad \frac{2a}{x} - \frac{3ab}{y} = 2 - 3a \\
\quad -\frac{2a}{x} - \frac{2b}{y} = -4 \\
\quad \frac{b(-3a - 2)}{y} = -3a - 2 \\
\quad b = y \\
\quad \frac{a}{x} + \frac{b}{b} = 2 \\
\quad a + x = 2x \\
\quad a = x \\
\text{sol: } x = a \quad y = b
\end{array}$$

$$\begin{array}{l}
14. \quad \frac{2}{x} + \frac{2}{y} = \frac{m + n}{mn}; \quad \frac{m}{x} - \frac{n}{y} = 0 \\
\quad \frac{2n}{x} + \frac{2n}{y} = \frac{m + n}{m} \\
\quad \frac{2m}{x} - \frac{2n}{y} = 0 \\
\quad \frac{2(m + n)}{x} = \frac{m + n}{m} \\
\quad 2m = x \\
\quad \frac{m}{2m} - \frac{n}{y} = 0 \\
\quad y - 2n = 0 \\
\quad y = 2n \\
\text{sol: } x = 2m \quad y = 2n
\end{array}$$

### EJERCICIO 183

$$\begin{array}{l}
1. \quad \left| \begin{array}{cc} 4 & 5 \\ 2 & 3 \end{array} \right| 4 \cdot 3 - 2 \cdot 5 = 12 - 10 = 2 \\
2. \quad \left| \begin{array}{cc} 2 & 7 \\ 3 & 5 \end{array} \right| 2 \cdot 5 - 3 \cdot 7 = 10 - 21 = -11 \\
3. \quad \left| \begin{array}{cc} -2 & 5 \\ 4 & 3 \end{array} \right| -2 \cdot 3 - 4 \cdot 5 = -6 - 20 = -26 \\
4. \quad \left| \begin{array}{cc} 7 & 9 \\ 5 & -2 \end{array} \right| 7(-2) - 9 \cdot 5 = -14 - 45 = -59 \\
5. \quad \left| \begin{array}{cc} 5 & -3 \\ -2 & -8 \end{array} \right| 5(-8) - (-3)(-2) = -40 - 6 = -46 \\
6. \quad \left| \begin{array}{cc} 9 & -11 \\ -3 & 7 \end{array} \right| 9 \cdot 7 - (-11)(-3) = 63 - 33 = 30 \\
7. \quad \left| \begin{array}{cc} -15 & -1 \\ 13 & 2 \end{array} \right| -15 \cdot 2 - 13(-1) = -30 + 13 = -17 \\
8. \quad \left| \begin{array}{cc} 12 & -1 \\ 13 & -9 \end{array} \right| 12(-9) - (13)(-1) = -108 + 13 = -95
\end{array}$$

$$9. \begin{vmatrix} 10 & 3 \\ 17 & 13 \end{vmatrix} 10 \cdot 13 - 3 \cdot 17 = 130 - 51 = 79$$

$$11. \begin{vmatrix} 8 & 2 \\ -3 & 0 \end{vmatrix} 8(0) - 2(-3) = 0 + 6 = 6$$

$$10. \begin{vmatrix} -5 & -8 \\ -19 & -21 \end{vmatrix} (-5)(-21) - (-8)(-19) = 105 - 152 = -47$$

$$12. \begin{vmatrix} 31 & -85 \\ -20 & 43 \end{vmatrix} 31 \cdot 43 - (-85)(-20) = 1.333 - 1.700 = -367$$

## EJERCICIO 184

1.  $7x + 8y = 29$

$5x + 11y = 26$

$$x = \frac{\begin{vmatrix} 29 & 8 \\ 26 & 11 \end{vmatrix}}{\begin{vmatrix} 7 & 8 \\ 5 & 11 \end{vmatrix}} = \frac{319 - 208}{77 - 40} = \frac{111}{37} = 3$$

$$y = \frac{\begin{vmatrix} 7 & 29 \\ 5 & 26 \end{vmatrix}}{\begin{vmatrix} 7 & 8 \\ 5 & 11 \end{vmatrix}} = \frac{182 - 145}{37} = \frac{37}{37} = 1$$

sol:  $x=3$   $y=1$

2.  $3x - 4y = 13$

$8x - 5y = -5$

$$x = \frac{\begin{vmatrix} 13 & -4 \\ -5 & -5 \end{vmatrix}}{\begin{vmatrix} 3 & -4 \\ 8 & -5 \end{vmatrix}} = \frac{-65 - 20}{-15 + 32} = \frac{-85}{17} = -5$$

$$y = \frac{\begin{vmatrix} 3 & 13 \\ 8 & -5 \end{vmatrix}}{\begin{vmatrix} 3 & -4 \\ 8 & -5 \end{vmatrix}} = \frac{-15 - 104}{17} = \frac{-119}{17} = -7$$

sol:  $x=-5$   $y=-7$

3.  $13x - 31y = -326$

$25x + 37y = 146$

$$x = \frac{\begin{vmatrix} -326 & -31 \\ 146 & 37 \end{vmatrix}}{\begin{vmatrix} 13 & -31 \\ 25 & 37 \end{vmatrix}} = \frac{-12.062 + 4.526}{481 + 775} = \frac{-7.536}{1.256} = -6$$

$$y = \frac{\begin{vmatrix} 13 & -326 \\ 25 & 146 \end{vmatrix}}{\begin{vmatrix} 13 & -31 \\ 25 & 37 \end{vmatrix}} = \frac{1.898 + 8.150}{1.256} = \frac{10.048}{1.256} = 8$$

sol:  $x=-6$   $y=8$

4.  $15x - 44y = -6$

$-27x + 32y = -1$

$$x = \frac{\begin{vmatrix} -6 & -44 \\ -1 & 32 \end{vmatrix}}{\begin{vmatrix} 15 & -44 \\ -27 & 32 \end{vmatrix}} = \frac{-192 - 44}{480 - 1.188} = \frac{-236}{-708} = \frac{1}{3}$$

$$y = \frac{\begin{vmatrix} 15 & -6 \\ -27 & -1 \end{vmatrix}}{\begin{vmatrix} 15 & -44 \\ -27 & 32 \end{vmatrix}} = \frac{-15 - 162}{-708} = \frac{-177}{-708} = \frac{1}{4}$$

sol:  $x=\frac{1}{3}$   $y=\frac{1}{4}$

5.  $8x = -9y$

$8x + 9y = 0$

$2x + 5 + 3y = \frac{7}{2}$

$4x + 10 + 6y = 7$

$4x + 6y = -3$

$$x = \frac{\begin{vmatrix} 0 & 9 \\ -3 & 6 \end{vmatrix}}{\begin{vmatrix} 8 & 9 \\ 4 & 6 \end{vmatrix}} = \frac{0 + 27}{48 - 36} = \frac{27}{12} = \frac{9}{4} = 2\frac{1}{4}$$

$$y = \frac{\begin{vmatrix} 8 & 0 \\ 4 & -3 \end{vmatrix}}{\begin{vmatrix} 8 & 9 \\ 4 & 6 \end{vmatrix}} = \frac{-24 - 0}{12} = \frac{-24}{12} = -2$$

sol:  $x=2\frac{1}{4}$   $y=-2$

6.  $ax - by = -1$

$ax + by = 7$

$$x = \frac{\begin{vmatrix} -1 & -b \\ 7 & b \end{vmatrix}}{\begin{vmatrix} a & -b \\ a & b \end{vmatrix}}$$

$$x = \frac{-b + 7b}{ab + ab} = \frac{6b}{2ab} = \frac{3}{a}$$

$$y = \frac{\begin{vmatrix} a & -1 \\ a & 7 \end{vmatrix}}{\begin{vmatrix} a & -b \\ a & b \end{vmatrix}}$$

$$y = \frac{7a + a}{2ab} = \frac{8a}{2ab} = \frac{4}{b}$$

sol:  $x=\frac{3}{a}$   $y=\frac{4}{b}$

7.  $3x - (y + 2) = 2y + 1$

$3x - y - 2 = 2y + 1$

$3x - 3y = 3$

$x - y = 1$

$5y - (x + 3) = 3x + 1$

$5y - x - 3 = 3x + 1$

$-4x + 5y = 4$

$$x = \frac{\begin{vmatrix} 1 & -1 \\ 4 & 5 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ -4 & 5 \end{vmatrix}} = \frac{5 + 4}{5 - 4} = 9$$

$$y = \frac{\begin{vmatrix} 1 & 1 \\ -4 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ -4 & 5 \end{vmatrix}} = \frac{4 + 4}{1} = 8$$

sol:  $x=9$   $y=8$

8.  $ax+2y=2$

$$\frac{ax}{2} - 3y = -1$$

$$ax - 6y = -2$$

$$x = \frac{\begin{vmatrix} 2 & 2 \\ -2 & -6 \end{vmatrix}}{\begin{vmatrix} a & 2 \\ a & -6 \end{vmatrix}} = \frac{-12+4}{-6a-2a} = \frac{-8}{-8a} = \frac{1}{a}$$

$$y = \frac{\begin{vmatrix} a & 2 \\ a & -2 \end{vmatrix}}{\begin{vmatrix} a & 2 \\ a & -6 \end{vmatrix}} = \frac{-2a-2a}{-8a} = \frac{-4a}{-8a} = \frac{1}{2}$$

$$\text{sol: } x = \frac{1}{a} \quad y = \frac{1}{2}$$

9.  $\frac{x}{4} + \frac{y}{6} = -4$

$$3x + 2y = -48$$

$$\frac{x}{8} - \frac{y}{12} = 0$$

$$3x - 2y = 0$$

$$x = \frac{\begin{vmatrix} -48 & 2 \\ 0 & -2 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 3 & -2 \end{vmatrix}} = \frac{96-0}{-6-6} = \frac{96}{-12} = -8$$

$$y = \frac{\begin{vmatrix} 3 & -48 \\ 3 & 0 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 3 & -2 \end{vmatrix}} = \frac{0+144}{-12} = -12$$

$$\text{sol: } x = -8 \quad y = -12$$

10.  $3x+ay=3a+1$

$$\frac{x}{a} + ay = 2$$

$$x + a^2y = 2a$$

$$x = \frac{\begin{vmatrix} 3a+1 & a \\ 2a & a^2 \end{vmatrix}}{\begin{vmatrix} 3 & a \\ 1 & a^2 \end{vmatrix}}$$

$$x = \frac{3a^3 + a^2 - 2a^2}{3a^2 - a} = \frac{a^2(3a-1)}{a(3a-1)} = a$$

$$y = \frac{\begin{vmatrix} 3 & 3a+1 \\ 1 & 2a \end{vmatrix}}{\begin{vmatrix} 3 & a \\ 1 & a^2 \end{vmatrix}}$$

$$y = \frac{6a-3a-1}{a(3a-1)} = \frac{3a-1}{a(3a-1)} = \frac{1}{a}$$

$$\text{sol: } x = a \quad y = \frac{1}{a}$$

11.  $\frac{x+2}{3} - \frac{y-3}{8} = \frac{5}{6}$

$$8(x+2) - 3(y-3) = 20$$

$$8x + 16 - 3y + 9 = 20$$

$$8x - 3y + 25 = 20$$

$$8x - 3y = -5$$

$$\frac{y-5}{6} - \frac{2x-3}{5} = 0$$

$$5(y-5) - 6(2x-3) = 0$$

$$5y - 25 - 12x + 18 = 0$$

$$-12x + 5y - 7 = 0$$

$$-12x + 5y = 7$$

$$x = \frac{\begin{vmatrix} -5 & -3 \\ 7 & 5 \end{vmatrix}}{\begin{vmatrix} 8 & -3 \\ -12 & 5 \end{vmatrix}}$$

$$x = \frac{-25+21}{40-36} = \frac{-4}{4} = -1$$

$$y = \frac{\begin{vmatrix} 8 & -5 \\ -12 & 7 \end{vmatrix}}{\begin{vmatrix} 8 & -3 \\ -12 & 5 \end{vmatrix}}$$

$$y = \frac{56-60}{40-36} = \frac{-4}{4} = -1$$

$$\text{sol: } x = -1 \quad y = -1$$

12.  $3x-2y=5$

$$mx+4y=2(m+1)$$

$$x = \frac{\begin{vmatrix} 5 & -2 \\ 2(m+1) & 4 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ m & 4 \end{vmatrix}}$$

$$x = \frac{20+4(m+1)}{12+2m} = \frac{4(6+m)}{2(6+m)} = 2$$

$$y = \frac{\begin{vmatrix} 3 & 5 \\ m & 2(m+1) \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ m & 4 \end{vmatrix}}$$

$$y = \frac{6(m+1)-5m}{2(6+m)} = \frac{m+6}{2(6+m)} = \frac{1}{2}$$

$$\text{sol: } x = 2 \quad y = \frac{1}{2}$$

13.  $2x - \frac{2y+3}{17} = y+2$

$$34x - (2y+3) = 17(y+2)$$

$$34x - 2y - 3 = 17y + 34$$

$$34x - 19y = 37$$

$$3y - \frac{4x+1}{21} = 3x+5$$

$$63y - (4x+1) = 21(3x+5)$$

$$63y - 4x - 1 = 63x + 105$$

$$-67x + 63y = 106$$

$$x = \frac{\begin{vmatrix} 37 & -19 \\ 106 & 63 \end{vmatrix}}{\begin{vmatrix} 34 & -19 \\ -67 & 63 \end{vmatrix}}$$

$$x = \frac{2.331+2.014}{2.142-1.273} = \frac{4.345}{869} = 5$$

$$y = \frac{\begin{vmatrix} 34 & 37 \\ -67 & 106 \end{vmatrix}}{\begin{vmatrix} 34 & -19 \\ -67 & 63 \end{vmatrix}}$$

$$y = \frac{3.604+2.479}{869} = 7$$

$$\text{sol: } x = 5 \quad y = 7$$

14.  $\frac{x+y}{x-y} = 4$

$$x+y = 4(x-y)$$

$$x+y = 4x-4y$$

$$-3x+5y=0$$

$$\frac{x-y-1}{x+y+1} = \frac{1}{9}$$

$$9(x-y-1) = x+y+1$$

$$9x-9y-9 = x+y+1$$

$$8x-10y=10$$

$$4x-5y=5$$

$$x = \frac{\begin{vmatrix} 0 & 5 \\ 5 & -5 \end{vmatrix}}{\begin{vmatrix} -3 & 5 \\ 4 & -5 \end{vmatrix}} = \frac{0-25}{-15-20} = \frac{-25}{-5} = 5$$

$$y = \frac{\begin{vmatrix} -3 & 0 \\ 4 & 5 \end{vmatrix}}{\begin{vmatrix} -3 & 5 \\ 4 & -5 \end{vmatrix}} = \frac{-15-0}{-5} = 3$$

$$\text{sol: } x = 5 \quad y = 3$$

15.  $x - y = 2b$

$$\frac{x}{a+b} + \frac{y}{a-b} = 2$$

$$x(a-b) + y(a+b) = 2(a^2 - b^2)$$

$$x = \begin{vmatrix} 2b & -1 \\ 2(a^2 - b^2) & a+b \\ 1 & -1 \\ a-b & a+b \end{vmatrix}$$

$$x = \frac{2b(a+b) + 2(a^2 - b^2)}{a+b+a-b} = \frac{2a(b+a)}{2a} = b+a$$

$$y = \begin{vmatrix} 1 & 2b \\ a-b & 2(a^2 - b^2) \\ 1 & -1 \\ a-b & a+b \end{vmatrix}$$

$$y = \frac{2(a^2 - b^2) - 2b(a-b)}{2a} = \frac{2a(a-b)}{2a} = a-b$$

sol:  $x = a+b$   $y = a-b$

16.  $\frac{x+9}{x-9} = \frac{y+21}{y+39}$

$$(x+9)(y+39) = (x-9)(y+21)$$

$$xy + 39x + 9y + 351 = xy + 21x - 9y - 189$$

$$18x + 18y = -540$$

$$x + y = -30$$

$$\frac{x+8}{x-8} = \frac{y+19}{y+11}$$

$$(y+11)(x+8) = (y+19)(x-8)$$

$$xy + 8y + 11x + 88 = xy - 8y + 19x - 152$$

$$-8x + 16y = -240$$

$$-x + 2y = -30$$

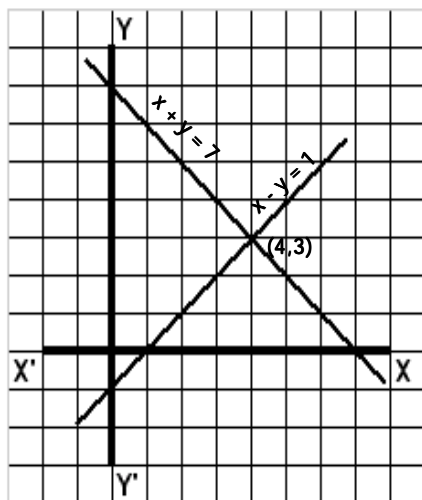
$$x = \begin{vmatrix} -30 & 1 \\ -30 & 2 \\ 1 & 1 \\ -1 & 2 \end{vmatrix} = \frac{-60 + 30}{2+1} = \frac{-30}{3} = -10$$

$$y = \begin{vmatrix} 1 & -30 \\ -1 & -30 \\ 1 & 1 \\ -1 & 2 \end{vmatrix} = \frac{-30 - 30}{3} = \frac{-60}{3} = -20$$

sol:  $x = -10$   $y = -20$

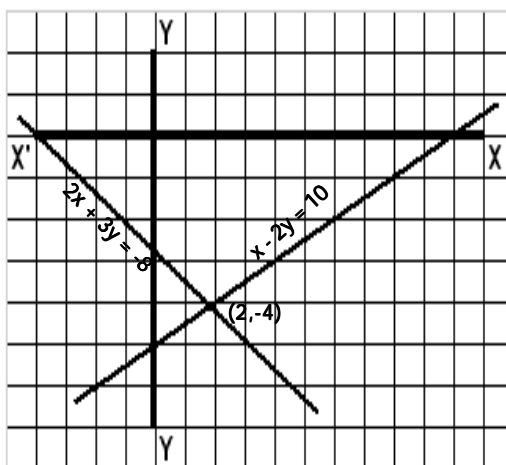
## EJERCICIO 185

1.  $x - y = 1$   $x + y = 7$   
 $x=0$   $y=-1$   $x=0$   $y=7$   
 $y=0$   $x=1$   $y=0$   $x=7$



sol:  $x = 4$   $y = 3$

2.  $x - 2y = 10$   $2x + 3y = -8$   
 $x=0$   $y=-5$   $x=0$   $y=-2\frac{2}{3}$   
 $y=0$   $x=10$   $y=0$   $x=-4$

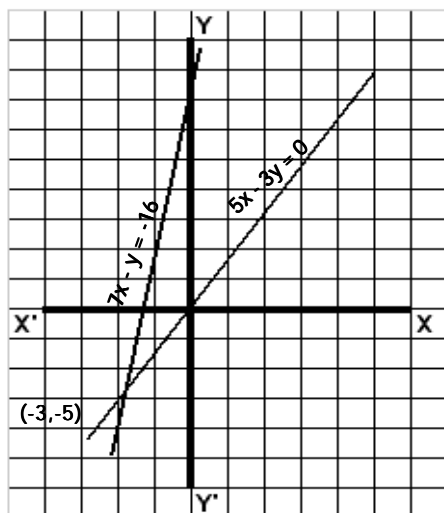


sol:  $x = 2$   $Y = -4$



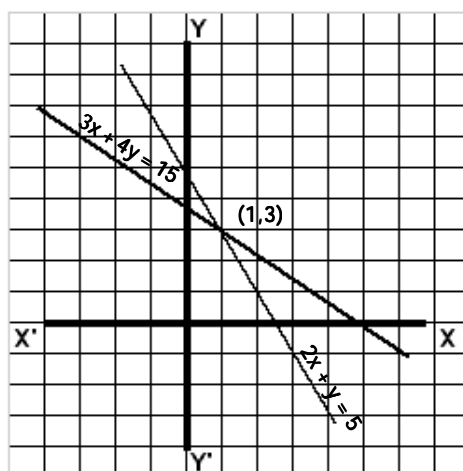
3.  $5x - 3y = 0$        $7x - y = -16$   
 $x = 3$     $y = 5$        $x = 0$     $y = 16$   
 $y = 0$     $x = -2\frac{2}{7}$

Escala 1:2



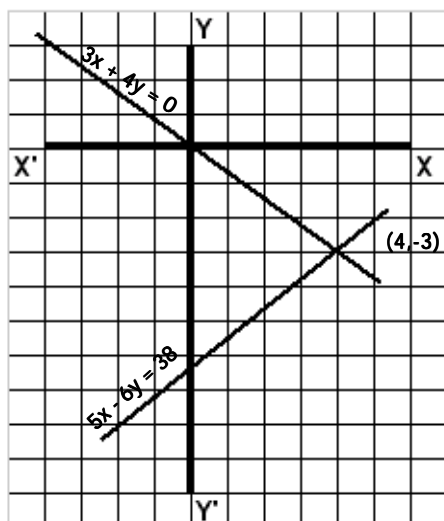
sol :  $x = -3$     $y = -5$

5.  $3x + 4y = 15$        $2x + y = 5$   
 $x = 5$     $y = 0$        $x = 0$     $y = 5$   
 $x = 1$     $y = 3$        $x = 2$     $y = 1$



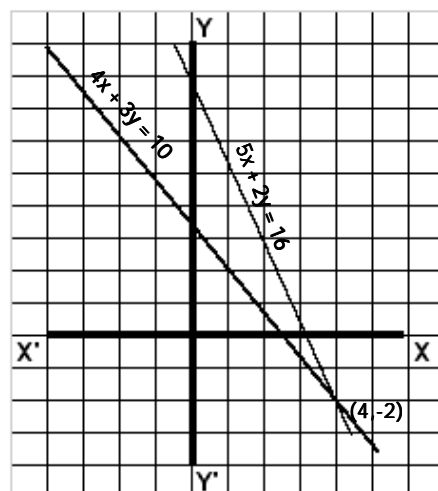
sol :  $x = 1$     $y = 3$

4.  $3x = -4y$        $5x - 6y = 38$   
 $3x + 4y = 0$        $x = -2$     $y = -8$   
 $x = 4$     $y = -3$        $y = -3$     $x = 4$



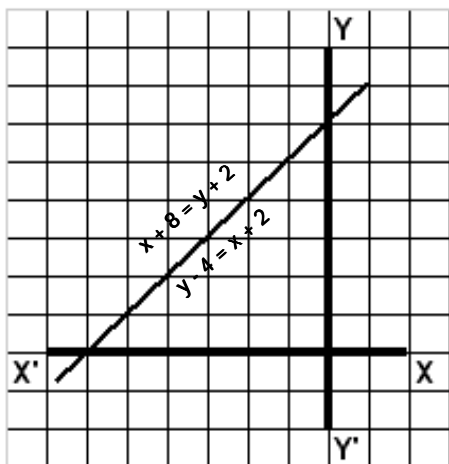
sol :  $x = 4$     $y = -3$

6.  $5x + 2y = 16$        $4x + 3y = 10$   
 $x = 2$     $y = 3$        $x = -2$     $y = 6$   
 $x = 4$     $y = -2$        $x = 1$     $y = 2$



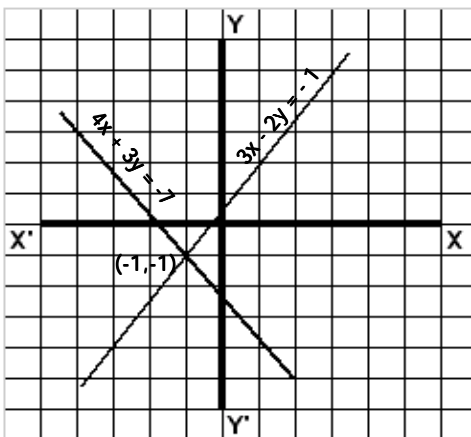
sol :  $x = 4$     $y = -2$

7.  $x+8=y+2$        $y-4=x+2$   
 $x-y=-6$        $-x+y=6$   
 $x=0$   $y=6$        $x=0$   $y=6$   
 $x=-6$   $y=0$        $x=-6$   $y=0$



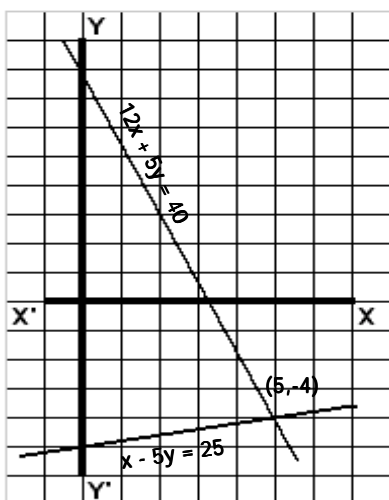
sol : Equivalentes

9.  $\frac{x}{2} - \frac{y}{3} = -\frac{1}{6}$        $3x-2y=-1$   
 $x=3$   $y=5$  ;  $x=-3$   $y=-4$   
 $\frac{x}{3} + \frac{y}{4} = -\frac{7}{12}$        $4x+3y=-7$   
 $x=-1$   $y=-1$  ;  $x=-4$   $y=3$



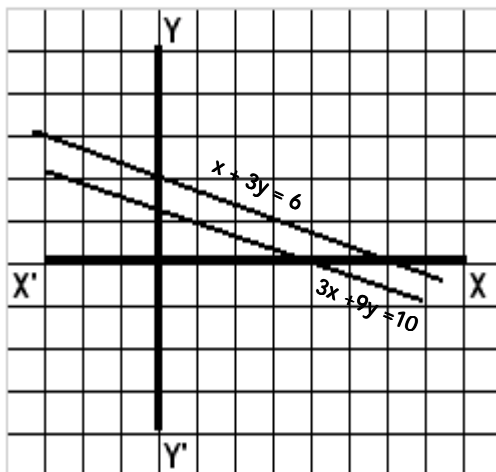
sol :  $x = -1$      $y = -1$

8.  $\frac{3x}{5} + \frac{y}{4} = 2$        $x-5y=25$   
 $12x+5y=40$        $x=0$   $y=-5$   
 $x=0$   $y=8$        $x=5$   $y=-4$   
 $x=5$   $y=-4$



sol :  $x = 5$      $y = -4$

10.  $x+3y=6$        $3x+9y=10$   
 $x=0$   $y=2$        $x=0$   $y=1\frac{1}{9}$   
 $x=6$   $y=0$        $x=3\frac{1}{3}$   $y=0$



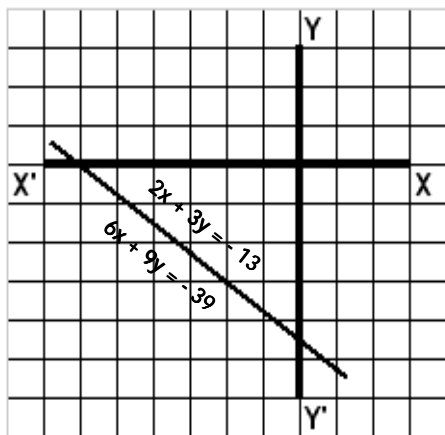
sol : Incompatibles

11.  $2x+3y=-13$

$x=-2 \quad y=-3; x=-5 \quad y=-1$

$6x+9y=-39$

$x=-2 \quad y=-3, \quad x=-5 \quad y=-1$



sol : Equivalentes

12.  $\frac{x-2}{2} - \frac{y-3}{3} = 4$        $\frac{y-2}{2} + \frac{x-3}{3} = -\frac{11}{3}$

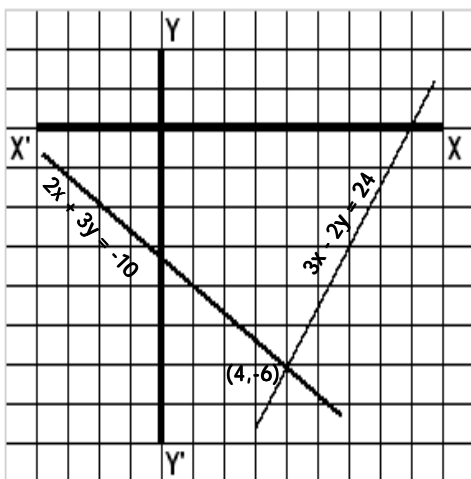
$3(x-2) - 2(y-3) = 24$        $3(y-2) + 2(x-3) = -22$

$3x - 6 - 2y + 6 = 24$        $3y - 6 + 2x - 6 = -22$

$3x - 2y = 24$        $2x + 3y = -10$

$x = 6 \quad y = -3$        $x = 4 \quad y = -6$

$x = 4 \quad y = -6$        $x = -2 \quad y = -2$



sol :  $x = 4 \quad y = -6$

13.  $x+y=9$

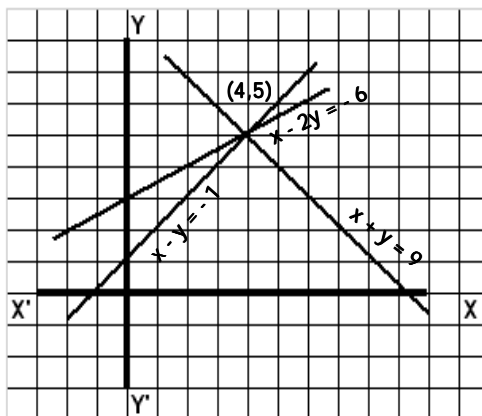
$x=6 \quad y=3; x=4 \quad y=5$

$x-y=-1$

$x=0 \quad y=1; x=2 \quad y=3$

$x-2y=-6$

$x=0 \quad y=3; x=2 \quad y=4$



sol :  $x = 4 \quad y = 5$

14.  $x+y=5$

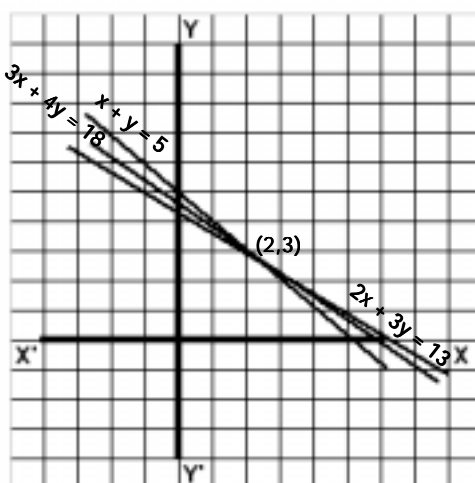
$x=0 \quad y=5; x=2 \quad y=3$

$3x+4y=18$

$x=2 \quad y=3; x=-2 \quad y=6$

$2x+3y=13$

$x=2 \quad y=3; x=5 \quad y=1$



sol :  $x = 2 \quad y = 3$

15.  $2x+y=-1$

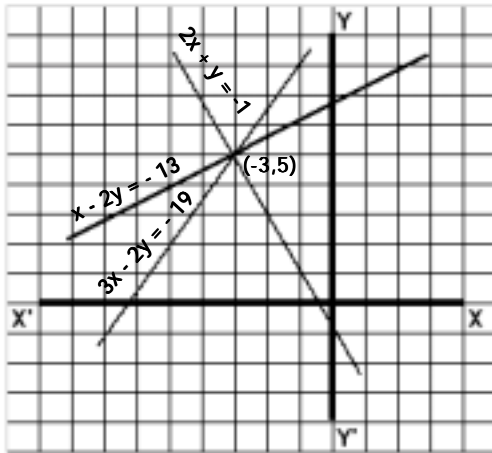
$x=0 \quad y=-1$  ;  $x=-1 \quad y=1$

$x-2y=-13$

$x=-5 \quad y=4$  ;  $x=-3 \quad y=5$

$3x-2y=-19$

$x=-5 \quad y=2$  ;  $x=-3 \quad y=5$

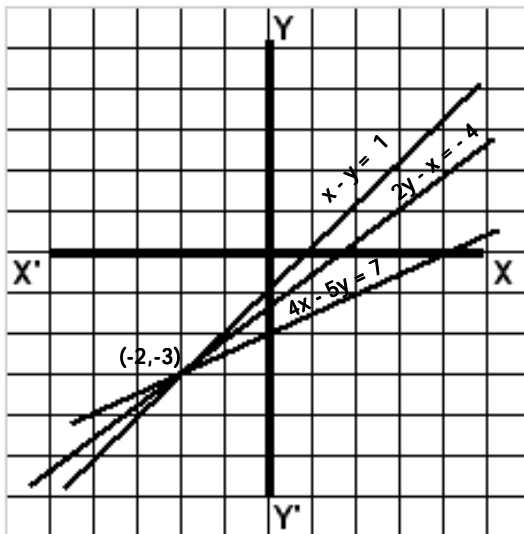


sol :  $x = -3 \quad y = 5$

16.  $x-y=1 \quad 2y-x=-4 \quad 4x-5y=7$

$x=0 \quad y=-1 \quad x=0 \quad y=-2 \quad x=-2 \quad y=-3$

$x=1 \quad y=0 \quad x=2 \quad y=-1 \quad x=3 \quad y=1$



sol :  $x = -2 \quad y = -3$

## EJERCICIO 186

1.  $x+y+z=6$

$x-y+2z=5$

$x-y-3z=-10$

$x+y+z=6$

$x-y+2z=5$

$2x+3z=11$

$x+y+z=6$

$x-y-3z=-10$

$2x-2z=-4$

$x-z=-2$

$2x+3z=11$

$x-z=-2 \quad (3)$

$2x+3z=11$

$3x-3z=-6$

$5x=5$

$x=1$

$2(1)+3z=11$

$2+3z=11$

$3z=9$

$z=3$

$1+y+3=6$

$y+4=6$

$y=2$

sol:  $x=1 \quad y=2 \quad z=3$

2.  $x+y+z=12$

$2x-y+z=7$

$x+2y-z=6$

$x+y+z=12$

$2x-y+z=7$

$3x+2z=19$

$2x-y+z=7 \quad (2)$

$x+2y-z=6$

$4x-2y+2z=14$

$x+2y-z=6$

$5x+z=20$

$3x+2z=19$

$5x+z=20 \quad (-2)$

$3x+2z=19$

$-10x-2z=-40$

$-7x=-21$

$x=3$

Continúa

## 2. Continuación

$$5(3) + z = 20$$

$$15 + z = 20$$

$$z = 5$$

$$3 + y + 5 = 12$$

$$y + 8 = 12$$

$$y = 4$$

$$\text{sol: } x=3 \ y=4 \ z=5$$

$$3. \ x - y + z = 2$$

$$x + y + z = 4$$

$$2x + 2y - z = -4$$

$$x - y + z = 2$$

$$x + y + z = 4$$

$$2x + 2z = 6$$

$$x + z = 3$$

$$x - y + z = 2 \quad (2)$$

$$2x + 2y - z = -4$$

$$2x - 2y + 2z = 4$$

$$2x + 2y - z = -4$$

$$4x + z = 0$$

$$x + z = 3 \quad (-1)$$

$$4x + z = 0$$

$$-x - z = -3$$

$$4x + z = 0$$

$$3x = -3$$

$$x = -1$$

$$-1 + z = 3$$

$$z = 4$$

$$-1 - y + 4 = 2$$

$$-y + 3 = 2$$

$$-y = -1$$

$$y = 1$$

$$\text{sol: } x=-1 \ y=1 \ z=4$$

$$4. \ 2x + y - 3z = -1$$

$$x - 3y - 2z = -12$$

$$3x - 2y - z = -5$$

$$2x + y - 3z = -1 \quad (2)$$

$$3x - 2y - z = -5$$

$$4x + 2y - 6z = -2$$

$$3x - 2y - z = -5$$

$$7x - 7z = -7$$

$$x - z = -1$$

Continúa

## 4. Continuación

$$2x + y - 3z = -1 \quad (3)$$

$$x - 3y - 2z = -12$$

$$6x + 3y - 9z = -3$$

$$x - 3y - 2z = -12$$

$$7x - 11z = -15$$

$$x - z = -1 \quad (-7)$$

$$7x - 11z = -15$$

$$-7x + 7z = 7$$

$$7x - 11z = -15$$

$$-4z = -8$$

$$z = 2$$

$$x - 2 = -1$$

$$x = 1$$

$$2(1) + y - 3(2) = -1$$

$$2 + y - 6 = -1$$

$$y - 4 = -1$$

$$y = 3$$

$$\text{sol: } x=1 \ y=3 \ z=2$$

$$5. \ 2x + 3y + z = 1$$

$$6x - 2y - z = -14$$

$$3x + y - z = 1$$

$$2x + 3y + z = 1$$

$$3x + y - z = 1$$

$$5x + 4y = 2$$

$$2x + 3y + z = 1$$

$$6x - 2y - z = -14$$

$$8x + y = -13$$

$$5x + 4y = 2$$

$$8x + y = -13 \quad (-4)$$

$$5x + 4y = 2$$

$$-32x - 4y = 52$$

$$-27x = 54$$

$$x = -2$$

$$8(-2) + y = -13$$

$$-16 + y = -13$$

$$y = 3$$

$$2(-2) + 3(3) + z = 1$$

$$-4 + 9 + z = 1$$

$$5 + z = 1$$

$$z = -4$$

$$\text{sol: } x=-2 \ y=3 \ z=-4$$

$$6. \ 5x - 2y + z = 24$$

$$2x + 5y - 2z = -14$$

$$x - 4y + 3z = 26$$

$$5x - 2y + z = 24 \quad (2)$$

$$2x + 5y - 2z = -14$$

$$10x - 4y + 2z = 48$$

$$2x + 5y - 2z = -14$$

$$12x + y = 34$$

$$5x - 2y + z = 24 \quad (-3)$$

$$x - 4y + 3z = 26$$

$$-15x + 6y - 3z = -72$$

$$x - 4y + 3z = 26$$

$$-14x + 2y = -46$$

$$12x + y = 34 \quad (-2)$$

$$-14x + 2y = -46$$

$$-24x - 2y = -68$$

$$-14x + 2y = -46$$

$$-38x = -114$$

$$x = 3$$

$$12(3) + y = 34$$

$$36 + y = 34$$

$$y = -2$$

$$3 - 4(-2) + 3z = 26$$

$$3 + 8 + 3z = 26$$

$$3z = 15$$

$$z = 5$$

$$\text{sol: } x=3 \ y=-2 \ z=5$$

$$7. \ 4x + 2y + 3z = 8$$

$$3x + 4y + 2z = -1$$

$$2x - y + 5z = 3$$

$$4x + 2y + 3z = 8 \quad (-2)$$

$$3x + 4y + 2z = -1$$

$$-8x - 4y - 6z = -16$$

$$3x + 4y + 2z = -1$$

$$-5x - 4z = -17$$

$$5x + 4z = 17$$

$$4x + 2y + 3z = 8$$

$$2x - y + 5z = 3 \quad (2)$$

$$4x + 2y + 3z = 8$$

$$4x - 2y + 10z = 6$$

$$8x + 13z = 14$$

Continúa

**7. Continuación**

$$8x + 13z = 14 \quad (-5)$$

$$\underline{5x + 4z = 17 \quad (8)}$$

$$-40x - 65z = -70$$

$$\underline{40x + 32z = 136}$$

$$-33z = 66$$

$$z = -2$$

$$5x + 4(-2) = 17$$

$$5x - 8 = 17$$

$$5x = 25$$

$$x = 5$$

$$2(5) - y + 5(-2) = 3$$

$$10 - y - 10 = 3$$

$$-y = 3$$

$$y = -3$$

$$\text{sol: } x = 5 \quad y = -3 \quad z = -2$$

$$\mathbf{8.} \quad 6x + 3y + 2z = 12$$

$$9x - y + 4z = 37$$

$$10x + 5y + 3z = 21$$

$$6x + 3y + 2z = 12$$

$$\underline{9x - y + 4z = 37 \quad (3)}$$

$$6x + 3y + 2z = 12$$

$$\underline{27x - 3y + 12z = 111}$$

$$33x + 14z = 123$$

$$9x - y + 4z = 37 \quad (5)$$

$$\underline{10x + 5y + 3z = 21}$$

$$45x - 5y + 20z = 185$$

$$\underline{10x + 5y + 3z = 21}$$

$$55x + 23z = 206$$

$$33x + 14z = 123 \quad (5)$$

$$\underline{55x + 23z = 206 \quad (-3)}$$

$$165x + 70z = 615$$

$$\underline{-165x - 69z = -618}$$

$$z = -3$$

$$33x + 14(-3) = 123$$

$$33x - 42 = 123$$

$$33x = 165$$

$$x = 5$$

$$6(5) + 3y + 2(-3) = 12$$

$$30 + 3y - 6 = 12$$

$$3y + 24 = 12$$

$$3y = -12$$

$$y = -4$$

$$\text{sol: } x = 5 \quad y = -4 \quad z = -3$$

$$\mathbf{9.} \quad 2x + 4y + 3z = 3$$

$$10x - 8y - 9z = 0$$

$$4x + 4y - 3z = 2$$

$$2x + 4y + 3z = 3 \quad (3)$$

$$\underline{10x - 8y - 9z = 0}$$

$$6x + 12y + 9z = 9$$

$$\underline{10x - 8y - 9z = 0}$$

$$16x + 4y = 9$$

$$10x - 8y - 9z = 0$$

$$4x + 4y - 3z = 2 \quad (-3)$$

$$10x - 8y - 9z = 0$$

$$\underline{-12x - 12y + 9z = -6}$$

$$-2x - 20y = -6$$

$$x + 10y = 3$$

$$16x + 4y = 9 \quad (5)$$

$$\underline{x + 10y = 3 \quad (-2)}$$

$$80x + 20y = 45$$

$$\underline{-2x - 20y = -6}$$

$$78x = 39$$

$$x = \frac{1}{2}$$

$$16\left(\frac{1}{2}\right) + 4y = 9$$

$$8 + 4y = 9$$

$$4y = 1$$

$$y = \frac{1}{4}$$

$$2\left(\frac{1}{2}\right) + 4\left(\frac{1}{4}\right) + 3z = 3$$

$$1 + 1 + 3z = 3$$

$$3z = 1$$

$$z = \frac{1}{3}$$

$$\text{sol: } x = \frac{1}{2} \quad y = \frac{1}{4} \quad z = \frac{1}{3}$$

$$\mathbf{10.} \quad 3x + y + z = 1$$

$$x + 2y - z = 1$$

$$x + y + 2z = -17$$

$$3x + y + z = 1$$

$$\underline{x + 2y - z = 1}$$

$$4x + 3y = 2$$

$$x + 2y - z = 1 \quad (2)$$

$$\underline{x + y + 2z = -17}$$

**Continúa**

**10. Continuación**

$$2x + 4y - 2z = 2$$

$$\underline{x + y + 2z = -17}$$

$$3x + 5y = -15$$

$$4x + 3y = 2 \quad (3)$$

$$\underline{3x + 5y = -15 \quad (-4)}$$

$$12x + 9y = 6$$

$$\underline{-12x - 20y = 60}$$

$$-11y = 66$$

$$y = -6$$

$$4x + 3(-6) = 2$$

$$4x - 18 = 2$$

$$4x = 20$$

$$x = 5$$

$$5 - 6 + 2z = -17$$

$$2z = -16$$

$$z = -8$$

$$\text{sol: } x = 5 \quad y = -6 \quad z = -8$$

$$\mathbf{11.} \quad 7x + 3y - 4z = -35$$

$$3x - 2y + 5z = 38$$

$$x + y - 6z = -27$$

$$7x + 3y - 4z = -35$$

$$\underline{x + y - 6z = -27 \quad (-3)}$$

$$7x + 3y - 4z = -35$$

$$\underline{-3x - 3y + 18z = 81}$$

$$4x + 14z = 46$$

$$2x + 7z = 23$$

$$3x - 2y + 5z = 38$$

$$\underline{x + y - 6z = -27 \quad (2)}$$

$$3x - 2y + 5z = 38$$

$$\underline{2x + 2y - 12z = -54}$$

$$5x - 7z = -16$$

$$2x + 7z = 23$$

$$\underline{5x - 7z = -16}$$

$$7x = 7$$

$$x = 1$$

$$2(1) + 7z = 23$$

$$7z = 21$$

$$z = 3$$

$$1 + y - 6(3) = -27$$

$$y - 17 = -27$$

$$y = -10$$

$$\text{sol: } x = 1 \quad y = -10 \quad z = 3$$

$$\mathbf{12.} \quad 4x - y + 5z = -6$$

$$3x + 3y - 4z = 30$$

$$6x + 2y - 3z = 33$$

$$4x - y + 5z = -6 \quad (3)$$

$$\underline{3x + 3y - 4z = 30}$$

$$12x - 3y + 15z = -18$$

$$\underline{3x + 3y - 4z = 30}$$

$$15x + 11z = 12$$

$$4x - y + 5z = -6 \quad (2)$$

$$\underline{6x + 2y - 3z = 33}$$

$$8x - 2y + 10z = -12$$

$$\underline{6x + 2y - 3z = 33}$$

$$14x + 7z = 21$$

$$2x + z = 3$$

$$15x + 11z = 12$$

$$\underline{2x + z = 3 \quad (-11)}$$

$$15x + 11z = 12$$

$$\underline{-22x - 11z = -33}$$

$$-7x = -21$$

$$x = 3$$

$$2(3) + z = 3$$

$$z = -3$$

$$4(3) - y + 5(-3) = -6$$

$$12 - y - 15 = -6$$

$$-y - 3 = -6$$

$$-y = -3$$

$$y = 3$$

$$\text{sol: } x = 3 \quad y = 3 \quad z = -3$$

$$\mathbf{13.} \quad 9x + 4y - 10z = 6$$

$$6x - 8y + 5z = -1$$

$$12x + 12y - 15z = 10$$

$$6x - 8y + 5z = -1 \quad (-2)$$

$$\underline{12x + 12y - 15z = 10}$$

$$-12x + 16y - 10z = 2$$

$$\underline{12x + 12y - 15z = 10}$$

$$28y - 25z = 12$$

$$9x + 4y - 10z = 6 \quad (2)$$

$$\underline{6x - 8y + 5z = -1 \quad (-3)}$$

$$18x + 8y - 20z = 12$$

$$\underline{-18x + 24y - 15z = 3}$$

$$32y - 35z = 15$$

**Continúa**

**13. Continuación**

$$28y - 25z = 12 \quad (-8)$$

$$32y - 35z = 15 \quad (7)$$

$$-224y + 200z = -96$$

$$224y - 245z = 105$$

$$-45z = 9$$

$$z = -\frac{1}{5}$$

$$28y - 25\left(-\frac{1}{5}\right) = 12$$

$$28y + 5 = 12$$

$$28y = 7$$

$$y = \frac{1}{4}$$

$$6x - 8\left(\frac{1}{4}\right) + 5\left(-\frac{1}{5}\right) = -1$$

$$6x - 2 - 1 = -1$$

$$6x = 2$$

$$x = \frac{1}{3}$$

$$\text{sol: } x = \frac{1}{3} \quad y = \frac{1}{4} \quad z = -\frac{1}{5}$$

**14.  $5x + 3y - z = -11$** 

$$10x - y + z = 10$$

$$15x + 2y - z = -7$$

$$5x + 3y - z = -11$$

$$10x - y + z = 10$$

$$15x + 2y - z = -11$$

$$10x - y + z = 10$$

$$15x + 2y - z = -7$$

$$25x + y = 3$$

$$15x + 2y = -1$$

$$25x + y = 3 \quad (-2)$$

$$15x + 2y = -1$$

$$-50x - 2y = -6$$

$$-35x = -7$$

$$x = \frac{1}{5}$$

$$15\left(\frac{1}{5}\right) + 2y = -1$$

$$3 + 2y = -1$$

$$2y = -4$$

$$y = -2$$

**Continúa****14. Continuación**

$$10x - (-2) + z = 10$$

$$10\left(\frac{1}{5}\right) - (-2) + z = 10$$

$$2 + 2 + z = 10$$

$$z = 6$$

$$\text{sol: } x = \frac{1}{5} \quad y = -2 \quad z = 6$$

**15.  $x + y = 1$** 

$$y + z = -1$$

$$z + x = -6$$

$$x + y = 1$$

$$y + z = -1 \quad (-1)$$

$$x + y = 1$$

$$-y - z = 1$$

$$x - z = 2$$

$$x - z = 2$$

$$x + z = -6$$

$$2x = -4$$

$$x = -2$$

$$-2 + y = 1$$

$$y = 3$$

$$3 + z = -1$$

$$z = -4$$

$$\text{sol: } x = -2 \quad y = 3 \quad z = -4$$

**16.  $x + 2y = -1$** 

$$2y + z = 0$$

$$x + 2z = 11$$

$$x + 2y = -1$$

$$2y + z = 0 \quad (-1)$$

$$x + 2y = -1$$

$$-2y - z = 0$$

$$x - z = -1$$

$$x - z = -1 \quad (-1)$$

$$x + 2z = 11$$

$$-x + z = 1$$

$$x + 2z = 11$$

$$3z = 12$$

$$z = 4$$

$$x - 4 = -1 \Rightarrow x = 3$$

$$3 + 2y = -1$$

$$2y = -4 \Rightarrow y = -2$$

$$\text{sol: } x = 3 \quad y = -2 \quad z = 4$$

**17.  $y + z = -8$** 

$$2x + z = 9$$

$$3y + 2x = -3$$

$$y + z = -8$$

$$2x + z = 9 \quad (-1)$$

$$y + z = -8$$

$$-2x - z = -9$$

$$-2x + y = -17$$

$$-2x + y = -17$$

$$2x + 3y = -3$$

$$4y = -20$$

$$y = -5$$

$$-5 + z = -8$$

$$z = -3$$

$$2x - 3 = 9$$

$$2x = 12$$

$$x = 6$$

$$\text{sol: } x = 6 \quad y = -5 \quad z = -3$$

**18.  $3x - 2y = 0$** 

$$3y - 4z = 25$$

$$z - 5x = -14$$

$$3y - 4z = 25$$

$$-5x + z = -14 \quad (4)$$

$$3y - 4z = 25$$

$$-20x + 4z = -56$$

$$-20x + 3y = -31$$

$$-20x + 3y = -31 \quad (2)$$

$$3x - 2y = 0 \quad (3)$$

$$-40x + 6y = -62$$

$$9x - 6y = 0$$

$$-31x = -62$$

$$x = 2$$

$$3(2) - 2y = 0$$

$$6 - 2y = 0$$

$$-2y = -6$$

$$y = 3$$

$$3(3) - 4z = 25$$

$$-4z = 16$$

$$z = -4$$

$$\text{sol: } x = 2 \quad y = 3 \quad z = -4$$

**19.  $3z - 5x = 10$** 

$$5x - 3y = -7$$

$$3y - 5z = -13$$

$$-5x + 3z = 10$$

$$5x - 3y = -7$$

$$-3y + 3z = 3$$

$$-y + z = 1$$

$$3y - 5z = -13$$

$$-y + z = 1 \quad (3)$$

$$3y - 5z = -13$$

$$-3y + 3z = 3$$

$$-2z = -10$$

$$z = 5$$

$$-y + 5 = 1$$

$$-y = -4$$

$$y = 4$$

$$5x - 3(4) = -7$$

$$5x - 12 = -7$$

$$5x = 5$$

$$x = 1$$

$$\text{sol: } x = 1 \quad y = 4 \quad z = 5$$

**20.  $x - 2y = 0$** 

$$y - 2z = 5$$

$$x + y + z = 8$$

$$x + y + z = 8$$

$$x - 2y = 0 \quad (-1)$$

$$x + y + z = 8$$

$$-x + 2y = 0$$

$$3y + z = 8$$

$$3y + z = 8 \quad (2)$$

$$y - 2z = 5$$

$$6y + 2z = 16$$

$$y - 2z = 5$$

$$7y = 21$$

$$y = 3$$

$$x - 2(3) = 0$$

$$x - 6 = 0$$

$$x = 6$$

$$6 + 3 + z = 8$$

$$z = -1$$

$$\text{sol: } x = 6 \quad y = 3 \quad z = -1$$

21.  $5x - 3z = 2$

$$2z - y = -5$$

$$x + 2y - 4z = 8$$

$$x + 2y - 4z = 8$$

$$\underline{-y + 2z = -5 \quad (2)}$$

$$x + 2y - 4z = 8$$

$$\underline{-2y + 4z = -10}$$

$$x = -2$$

$$5(-2) - 3z = 2$$

$$-10 - 3z = 2$$

$$-3z = 12$$

$$z = -4$$

$$2(-4) - y = -5$$

$$-8 - y = -5$$

$$-y = 3$$

$$y = -3$$

$$\text{sol: } x = -2 \quad y = -3 \quad z = -4$$

22.  $2x - z = 14$

$$4x + y - z = 41$$

$$3x - y + 5z = 53$$

$$4x + y - z = 41$$

$$\underline{3x - y + 5z = 53}$$

$$7x + 4z = 94$$

$$7x + 4z = 94$$

$$\underline{2x - z = 14 \quad (4)}$$

$$7x + 4z = 94$$

$$8x - 4z = 56$$

$$15x = 150$$

$$x = 10$$

$$2(10) - z = 14$$

$$20 - z = 14$$

$$-z = -6$$

$$z = 6$$

$$4(10) + y - 6 = 41$$

$$y + 34 = 41$$

$$y = 7$$

$$\text{sol: } x = 10 \quad y = 7 \quad z = 6$$

23.  $x + y - z = 1$

$$z + x - y = 3$$

$$z - x + y = 7$$

$$z + x - y = 3$$

$$\underline{z - x + y = 7}$$

$$2z = 10 \Rightarrow z = 5$$

Continúa

23. Continuación

$$x - y + z = 3$$

$$\underline{x + y - z = 1}$$

$$2x = 4$$

$$x = 2$$

$$2 - y + 5 = 3$$

$$-y + 7 = 3$$

$$-y = -4$$

$$y = 4$$

$$\text{sol: } x = 2 \quad y = 4 \quad z = 5$$

24.  $\frac{x}{2} + \frac{y}{2} - \frac{z}{3} = 3$

$$3x + 3y - 2z = 18$$

$$\frac{x}{3} + \frac{y}{6} - \frac{z}{2} = -5$$

$$2x + y - 3z = -30$$

$$\frac{x}{6} - \frac{y}{3} + \frac{z}{6} = 0$$

$$x - 2y + z = 0$$

$$3x + 3y - 2z = 18 \quad (2)$$

$$\underline{x - 2y + z = 0 \quad (3)}$$

$$6x + 6y - 4z = 36$$

$$\underline{3x - 6y + 3z = 0}$$

$$9x - z = 36$$

$$2x + y - 3z = -30 \quad (2)$$

$$\underline{x - 2y + z = 0}$$

$$4x + 2y - 6z = -60$$

$$\underline{x - 2y + z = 0}$$

$$5x - 5z = -60$$

$$x - z = -12$$

$$9x - z = 36$$

$$\underline{x - z = -12 \quad (-1)}$$

$$9x - z = 36$$

$$\underline{-x + z = 12}$$

$$8x = 48$$

$$x = 6$$

$$6 - z = -12$$

$$-z = -18$$

$$z = 18$$

Continúa

24. Continuación

$$6 - 2y + 18 = 0$$

$$-2y + 24 = 0$$

$$-2y = -24$$

$$y = 12$$

$$\text{sol: } x = 6 \quad y = 12 \quad z = 18$$

25.  $\frac{x}{3} + \frac{y}{4} + \frac{z}{3} = 21$

$$4x + 3y + 4z = 252$$

$$\frac{x}{5} + \frac{y}{6} - \frac{z}{3} = 0$$

$$6x + 5y - 10z = 0$$

$$\frac{x}{10} + \frac{y}{3} - \frac{z}{6} = 3$$

$$3x + 10y - 5z = 90$$

$$6x + 5y - 10z = 0 \quad (-2)$$

$$\underline{3x + 10y - 5z = 90}$$

$$-12x - 10y + 20z = 0$$

$$\underline{3x + 10y - 5z = 90}$$

$$-9x + 15z = 90$$

$$-3x + 5z = 30$$

$$4x + 3y + 4z = 252 \quad (10)$$

$$\underline{3x + 10y - 5z = 90 \quad (-3)}$$

$$40x + 30y + 40z = 2.520$$

$$\underline{-9x - 30y + 15z = -270}$$

$$31x + 55z = 2.250$$

$$-3x + 5z = 30 \quad (-11)$$

$$\underline{31x + 55z = 2.250}$$

$$33x - 55z = -330$$

$$\underline{31x + 55z = 2.250}$$

$$64x = 1.920$$

$$x = 30$$

$$-3(30) + 5z = 30$$

$$-90 + 5z = 30$$

$$5z = 120$$

$$z = 24$$

$$6(30) + 5y - 10(24) = 0$$

$$180 - 240 + 5y = 0$$

$$5y = 60$$

$$y = 12$$

$$\text{sol: } x = 30 \quad y = 12 \quad z = 24$$



$$26. \quad x - \frac{y+z}{3} = 4$$

$$3x - y - z = 12$$

$$y - \frac{x+z}{8} = 10$$

$$8y - x - z = 80$$

$$z - \frac{y-x}{2} = 5$$

$$2z - y + x = 10$$

$$3x - y - z = 12 \quad (-1)$$

$$-x + 8y - z = 80$$

$$-3x + y + z = -12$$

$$-x + 8y - z = 80$$

$$-4x + 9y = 68$$

$$-x + 8y - z = 80 \quad (2)$$

$$x - y + 2z = 10$$

$$-2x + 16y - 2z = 160$$

$$x - y + 2z = 10$$

$$-x + 15y = 170$$

$$-4x + 9y = 68$$

$$-x + 15y = 170 \quad (-4)$$

$$-4x + 9y = 68$$

$$4x - 60y = -680$$

$$-51y = -612$$

$$y = 12$$

$$-4x + 9(12) = 68$$

$$-4x + 108 = 68$$

$$-4x = -40$$

$$x = 10$$

$$3(10) - 12 - z = 12$$

$$30 - 12 - z = 12$$

$$18 - z = 12$$

$$z = 6$$

$$\text{sol: } x=10 \quad y=12 \quad z=6$$

$$27. \quad \frac{x+y}{7} = \frac{y+4}{5}$$

$$5(x+y) = 7(y+4)$$

$$5x + 5y = 7y + 28$$

$$5x - 2y = 28$$

$$\frac{x-z}{5} = \frac{y-4}{2}$$

$$2(x-z) = 5(y-4)$$

$$2x - 2z = 5y - 20$$

$$2x - 5y - 2z = -20$$

**Continúa**

## 27. Continuación

$$\frac{y-z}{3} = \frac{x+2}{10}$$

$$10(y-z) = 3(x+2)$$

$$10y - 10z = 3x + 6$$

$$3x - 10y + 10z = -6$$

$$2x - 5y - 2z = -20 \quad (5)$$

$$3x - 10y + 10z = -6$$

$$10x - 25y - 10z = -100$$

$$3x - 10y + 10z = -6$$

$$13x - 35y = -106$$

$$13x - 35y = -106 \quad (2)$$

$$5x - 2y = 28 \quad (-35)$$

$$26x - 70y = -212$$

$$-175x + 70y = -980$$

$$-149x = -1.192$$

$$x = 8$$

$$5(8) - 2y = 28$$

$$40 - 2y = 28$$

$$-2y = -12$$

$$y = 6$$

$$2(8) - 5(6) - 2z = -20$$

$$16 - 30 - 2z = -20$$

$$-2z = -6$$

$$z = 3$$

$$\text{sol: } x=8 \quad y=6 \quad z=3$$

$$28. \quad y - \frac{z+4}{2} = x - 6$$

$$2y - z - 4 = 2x - 12$$

$$-2x + 2y - z = -8$$

$$z - \frac{x-7}{3} = y - 5$$

$$3z - x + 7 = 3y - 15$$

$$-x - 3y + 3z = -22$$

$$x - \frac{y+2}{5} = z + 4$$

$$5x - y - 2 = 5z + 20$$

$$5x - y - 5z = 22$$

$$-2x + 2y - z = -8$$

$$-x - 3y + 3z = -22 \quad (-2)$$

$$-2x + 2y - z = -8$$

$$2x + 6y - 6z = 44$$

$$8y - 7z = 36$$

**Continúa**

## 28. Continuación

$$5x - y - 5z = 22$$

$$-x - 3y + 3z = -22 \quad (5)$$

$$5x - y - 5z = 22$$

$$-5x - 15y + 15z = -110$$

$$-16y + 10z = -88$$

$$-8y + 5z = -44$$

$$-8y + 5z = -44$$

$$8y - 7z = 36$$

$$-2z = -8$$

$$z = 4$$

$$8y - 7(4) = 36$$

$$8y - 28 = 36$$

$$8y = 64$$

$$y = 8$$

$$-x - 3(8) + 3(4) = -22$$

$$-x - 12 = -22$$

$$x = 10$$

$$\text{sol: } x=10 \quad y=8 \quad z=4$$

$$29. \quad x - y + \frac{y-z}{2} = 3$$

$$2x - 2y + y - z = 6$$

$$2x - y - z = 6$$

$$\frac{x-y}{2} - \frac{x-z}{4} = 0$$

$$2x - 2y - x + z = 0$$

$$x - 2y + z = 0$$

$$\frac{y-z}{2} - x = -5$$

$$y - z - 2x = -10$$

$$2x - y - z = 6$$

$$-2x + y - z = -10$$

$$-2z = -4$$

$$z = 2$$

$$x - 2y + z = 0$$

$$2x - y - z = 6 \quad (-2)$$

$$x - 2y + z = 0$$

$$-4x + 2y + 2z = -12$$

$$-3x + 3z = -12$$

$$-3x + 3(2) = -12$$

$$-3x + 6 = -12$$

$$-3x = -18$$

$$x = 6$$

**Continúa**

### 29. Continuación

$$6 - 2y + 2 = 0$$

$$-2y + 8 = 0$$

$$-2y = -8$$

$$y = 4$$

$$\text{sol: } x=6 \quad y=4 \quad z=2$$

$$30. \quad \frac{1}{x} + \frac{1}{y} = 5$$

$$\frac{1}{x} + \frac{1}{z} = 6 \quad (-1)$$

$$\frac{1}{x} + \frac{1}{y} = 5$$

$$-\frac{1}{x} - \frac{1}{z} = -6$$

$$\frac{1}{y} - \frac{1}{z} = -1$$

$$\frac{1}{y} - \frac{1}{z} = -1$$

$$\frac{1}{y} + \frac{1}{z} = 7$$

$$\frac{2}{y} = 6$$

$$\frac{1}{3} = y$$

$$\frac{1}{x} + \frac{1}{\frac{1}{3}} = 5$$

$$\frac{1}{x} + 3 = 5$$

$$\frac{1}{x} = 2$$

$$\frac{1}{2} = x$$

$$\frac{1}{\frac{1}{2}} + \frac{1}{z} = 6$$

$$2 + \frac{1}{z} = 6$$

$$\frac{1}{z} = 4$$

$$\frac{1}{4} = z$$

$$\text{sol: } x = \frac{1}{2} \quad y = \frac{1}{3} \quad z = \frac{1}{4}$$

$$31. \quad \frac{3}{x} + \frac{2}{y} = 2$$

$$\frac{2}{y} + \frac{2}{z} = \frac{3}{2} \quad (-1)$$

$$\frac{3}{x} + \frac{2}{y} = 2$$

$$-\frac{2}{y} - \frac{2}{z} = -\frac{3}{2}$$

$$\frac{3}{x} - \frac{2}{z} = \frac{1}{2}$$

$$\frac{3}{x} - \frac{2}{z} = \frac{1}{2} \quad (2)$$

$$\frac{1}{x} + \frac{4}{z} = \frac{4}{3}$$

$$\frac{6}{x} - \frac{4}{z} = 1$$

$$\frac{1}{x} + \frac{4}{z} = \frac{4}{3}$$

$$\frac{7}{x} = \frac{7}{3}$$

$$x = 3$$

$$\frac{3}{3} + \frac{2}{y} = 2$$

$$1 + \frac{2}{y} = 2$$

$$\frac{2}{y} = 1$$

$$2 = y$$

$$\frac{3}{3} - \frac{2}{z} = \frac{1}{2}$$

$$1 - \frac{2}{z} = \frac{1}{2}$$

$$-\frac{2}{z} = -\frac{1}{2}$$

$$4 = z$$

$$\text{sol: } x=3 \quad y=2 \quad z=4$$

$$32. \quad \frac{1}{x} + \frac{4}{y} + \frac{2}{z} = -6$$

$$\frac{3}{x} + \frac{2}{y} + \frac{4}{z} = 3 \quad (-2)$$

$$\frac{1}{x} + \frac{4}{y} + \frac{2}{z} = -6$$

$$-\frac{6}{x} - \frac{4}{y} - \frac{8}{z} = -6$$

$$-\frac{5}{x} - \frac{6}{z} = -12$$

### 32. Continuación

$$\frac{1}{x} + \frac{4}{y} + \frac{2}{z} = -6 \quad (5)$$

$$\frac{6}{x} - \frac{5}{y} - \frac{6}{z} = 31 \quad (4)$$

$$\frac{5}{x} + \frac{20}{y} + \frac{10}{z} = -30$$

$$\frac{24}{x} - \frac{20}{y} - \frac{24}{z} = 124$$

$$\frac{29}{x} - \frac{14}{z} = 94$$

$$-\frac{5}{x} - \frac{6}{z} = 12 \quad (-7)$$

$$\frac{29}{x} - \frac{14}{z} = 94 \quad (3)$$

$$\frac{35}{x} + \frac{42}{z} = 84$$

$$\frac{87}{x} - \frac{42}{z} = 282$$

$$\frac{122}{x} = 366$$

$$122 = 366x$$

$$\frac{1}{3} = x$$

$$-\frac{5}{\frac{1}{3}} - \frac{6}{z} = -12$$

$$-15 - \frac{6}{z} = -12$$

$$-\frac{6}{z} = 3$$

$$-2 = z$$

$$\frac{1}{\frac{1}{3}} + \frac{4}{y} + \frac{2}{-2} = -6$$

$$3 + \frac{4}{y} - 1 = -6$$

$$\frac{4}{y} = -8$$

$$-\frac{1}{2} = y$$

$$\text{sol: } x = \frac{1}{3} \quad y = -\frac{1}{2} \quad z = -2$$

Continúa

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## ***EJERCICIO 187***

$$1. \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 4 \\ 1 & 0 & 2 \\ 1 & 2 & 1 \\ 1 & 3 & 4 \end{vmatrix}$$

$$\Rightarrow 6+0+8-3-0-4=7$$

$$2. \begin{vmatrix} 1 & 2 & -2 \\ 1 & -3 & 3 \\ -1 & 4 & 5 \\ 1 & 2 & -2 \\ 1 & -3 & 3 \end{vmatrix}$$

$$\Rightarrow -15-8-6+6-12-10=-45$$

$$3. \begin{vmatrix} -3 & 4 & 1 \\ 2 & -3 & 0 \\ 1 & 2 & 7 \\ -3 & 4 & 1 \\ 2 & -3 & 0 \end{vmatrix}$$

$$\Rightarrow 63+4+0+3+0-56=14$$

$$4. \begin{vmatrix} 2 & 5 & -1 \\ 3 & -4 & 3 \\ 6 & 2 & 4 \\ 2 & 5 & -1 \\ 3 & -4 & 3 \end{vmatrix}$$

$$\Rightarrow -32-6+90-24-12-60=-44$$

$$5. \begin{vmatrix} 5 & -1 & -6 \\ -2 & 5 & 3 \\ 3 & 4 & 2 \\ 5 & -1 & -6 \\ -2 & 5 & 3 \end{vmatrix}$$

$$\Rightarrow 50+48-9+90-60-4=115$$

$$6. \begin{vmatrix} 4 & 1 & 5 \\ 3 & 2 & -6 \\ 12 & 3 & 2 \\ 4 & 1 & 5 \\ 3 & 2 & -6 \end{vmatrix}$$

$$\Rightarrow 16+45-72-120+72-6=-65$$

$$7. \begin{vmatrix} 5 & 2 & -8 \\ -3 & -7 & 3 \\ 4 & 0 & -1 \\ 5 & 2 & -8 \\ -3 & -7 & 3 \end{vmatrix}$$

$$\Rightarrow 35+0+24-224+0-6=-171$$

$$8. \begin{vmatrix} 3 & 2 & 5 \\ -1 & -3 & 4 \\ 3 & 2 & 5 \\ 3 & 2 & 5 \\ -1 & -3 & 4 \end{vmatrix}$$

$$\Rightarrow -45-10+24+45-24+10=0$$

$$9. \begin{vmatrix} 5 & 2 & 3 \\ 6 & 1 & 2 \\ 3 & 4 & 5 \\ 5 & 2 & 3 \\ 6 & 1 & 2 \end{vmatrix}$$

$$\Rightarrow 25+72+12-9-40-60=0$$

$$10. \begin{vmatrix} 12 & 5 & 10 \\ 8 & -6 & 9 \\ 7 & 4 & -2 \\ 12 & 5 & 10 \\ 8 & -6 & 9 \end{vmatrix}$$

$$\Rightarrow 144+320+315+420-432+80=847$$

$$11. \begin{vmatrix} -9 & 3 & -4 \\ 7 & -5 & -3 \\ 4 & 6 & 1 \\ -9 & 3 & -4 \\ 7 & -5 & -3 \end{vmatrix}$$

$$\Rightarrow 45-168-36-80-162-21=-422$$

$$12. \begin{vmatrix} 11 & -5 & 7 \\ -12 & 3 & 8 \\ -13 & 1 & 9 \\ 11 & -5 & 7 \\ -12 & 3 & 8 \end{vmatrix}$$

$$\Rightarrow 297-84+520+273-88-540=378$$

## EJERCICIO 188

1.  $x+y+z=11$

$x-y+3z=13$

$2x+2y-z=7$

$$x = \frac{\begin{vmatrix} 11 & 1 & 1 \\ 13 & -1 & 3 \\ 7 & 2 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 3 \\ 2 & 2 & -1 \end{vmatrix}}$$

$$x = \frac{11+26+21+7-66+13}{1+2+6+2-6+1} = \frac{12}{6} = 2$$

$$y = \frac{\begin{vmatrix} 1 & 11 & 1 \\ 1 & 13 & 3 \\ 2 & 7 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 3 \\ 2 & 2 & -1 \end{vmatrix}}$$

$$y = \frac{-13+7+66-26-21+11}{6} = \frac{24}{6} = 4$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 11 \\ 1 & -1 & 13 \\ 2 & 2 & 7 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 3 \\ 2 & 2 & -1 \end{vmatrix}}$$

$$z = \frac{-7+22+26+22-26-7}{6} = \frac{30}{6} = 5$$

sol:  $x=2$   $y=4$   $z=5$

2.  $x+y+z=-6$

$2x+y-z=-1$

$x-2y+3z=-6$

$$x = \frac{\begin{vmatrix} -6 & 1 & 1 \\ -1 & 1 & -1 \\ -6 & -2 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 1 & -2 & 3 \end{vmatrix}}$$

$$x = \frac{\begin{vmatrix} -6 & 1 & 1 \\ -1 & 1 & -1 \\ -6 & -2 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 1 & -2 & 3 \end{vmatrix}}$$

## 2. Continuación

$$x = \frac{-18+2+6+6+12+3}{3-4-1-1-2-6} = \frac{11}{-11} = -1$$

$$y = \frac{\begin{vmatrix} 1 & -6 & 1 \\ 2 & -1 & -1 \\ 1 & -6 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & -6 \\ 2 & 1 & -1 \\ 1 & -2 & -6 \end{vmatrix}}$$

$$y = \frac{-3-12+6+1-6+36}{-11} = \frac{22}{-11} = -2$$

$$z = \frac{\begin{vmatrix} 1 & 1 & -6 \\ 2 & 1 & -1 \\ 1 & -2 & -6 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & -6 \\ 2 & 1 & -1 \\ 1 & -2 & -6 \end{vmatrix}}$$

$$z = \frac{-6+24-1+6-2+12}{-11} = \frac{33}{-11} = -3$$

sol:  $x=-1$   $y=-2$   $z=-3$

3.  $2x+3y+4z=3$

$2x+6y+8z=5$

$4x+9y-4z=4$

$$x = \frac{\begin{vmatrix} 3 & 3 & 4 \\ 5 & 6 & 8 \\ 4 & 9 & -4 \end{vmatrix}}{\begin{vmatrix} 2 & 3 & 4 \\ 2 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix}}$$

$$x = \frac{\begin{vmatrix} 3 & 3 & 4 \\ 5 & 6 & 8 \\ 4 & 9 & -4 \end{vmatrix}}{\begin{vmatrix} 2 & 3 & 4 \\ 2 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix}}$$

$$x = \frac{-72+180+96-96-216+60}{-48+72+96-96-144+24} = \frac{-48}{-96} = \frac{1}{2}$$

$$y = \frac{\begin{vmatrix} 2 & 3 & 4 \\ 2 & 5 & 8 \\ 2 & 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 3 & 4 \\ 2 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix}}$$

$$y = \frac{-40+32+96-80-64+24}{-96} = \frac{-32}{-96} = \frac{1}{3}$$

$$z = \frac{\begin{vmatrix} 2 & 3 & 3 \\ 2 & 6 & 5 \\ 2 & 3 & 3 \end{vmatrix}}{\begin{vmatrix} 2 & 3 & 4 \\ 2 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix}}$$

$$z = \frac{48+54+60-72-90-24}{-96} = \frac{-24}{-96} = \frac{1}{4}$$

sol:  $x=\frac{1}{2}$   $y=\frac{1}{3}$   $z=\frac{1}{4}$

4.  $4x - y + z = 4$

$2x + 2y - z = 2$

$6x - 2y + 3z = 12$

$$x = \begin{vmatrix} 4 & -1 & 1 \\ 2 & 2 & -1 \\ 12 & -2 & 3 \\ 4 & -1 & 1 \\ 2 & 2 & -1 \end{vmatrix}$$

$$x = \begin{vmatrix} 4 & -1 & 1 \\ 2 & 2 & -1 \\ 6 & -2 & 3 \\ 4 & -1 & 1 \\ 2 & 2 & -1 \end{vmatrix}$$

$$x = \frac{24 - 4 + 12 - 24 - 8 + 6}{24 - 4 + 6 - 12 - 8 + 6} = \frac{6}{12} = \frac{1}{2}$$

$$y = \begin{vmatrix} 4 & 4 & 1 \\ 2 & 2 & -1 \\ 6 & 12 & 3 \\ 4 & 4 & 1 \\ 2 & 2 & -1 \end{vmatrix}$$

$$y = \frac{24 + 24 - 24 - 12 + 48 - 24}{12} = \frac{36}{12} = 3$$

$$z = \begin{vmatrix} 4 & -1 & 4 \\ 2 & 2 & 2 \\ 6 & -2 & 12 \\ 4 & -1 & 4 \\ 2 & 2 & 2 \end{vmatrix}$$

$$z = \frac{96 - 16 - 12 - 48 + 16 + 24}{12} = \frac{60}{12} = 5$$

sol:  $x = \frac{1}{2}$   $y = 3$   $z = 5$

5.  $x + 4y + 5z = 11$

$3x - 2y + z = 5$

$4x + y - 3z = -26$

$$x = \begin{vmatrix} 11 & 4 & 5 \\ 5 & -2 & 1 \\ -26 & 1 & -3 \\ 11 & 4 & 5 \\ 5 & -2 & 1 \end{vmatrix}$$

$$x = \begin{vmatrix} 1 & 4 & 5 \\ 3 & -2 & 1 \\ 4 & 1 & -3 \\ 1 & 4 & 5 \\ 3 & -2 & 1 \end{vmatrix}$$

Continúa

## 5. Continuación

$$x = \frac{66 + 25 - 104 - 260 - 11 + 60}{6 + 15 + 16 + 40 - 1 + 36} = \frac{-224}{112} = -2$$

$$y = \begin{vmatrix} 1 & 11 & 5 \\ 3 & 5 & 1 \\ 4 & -26 & -3 \\ 1 & 11 & 5 \\ 3 & 5 & 1 \end{vmatrix}$$

$$y = \frac{-15 - 390 + 44 - 100 + 26 + 99}{112} = \frac{-336}{112} = -3$$

$$z = \begin{vmatrix} 1 & 4 & 11 \\ 3 & -2 & 5 \\ 4 & 1 & -26 \\ 1 & 4 & 11 \\ 3 & -2 & 5 \end{vmatrix}$$

$$z = \frac{52 + 33 + 80 + 88 - 5 + 312}{112} = \frac{560}{112} = 5$$

sol:  $x = -2$   $y = -3$   $z = 5$

6.  $7x + 10y + 4z = -2$

$5x - 2y + 6z = 38$

$3x + y - z = 21$

$$x = \begin{vmatrix} -2 & 10 & 4 \\ 38 & -2 & 6 \\ 21 & 1 & -1 \\ -2 & 10 & 4 \\ 38 & -2 & 6 \end{vmatrix}$$

$$x = \begin{vmatrix} 7 & 10 & 4 \\ 5 & -2 & 6 \\ 3 & 1 & -1 \\ 7 & 10 & 4 \\ 5 & -2 & 6 \end{vmatrix}$$

$$x = \frac{-4 + 152 + 1.260 + 168 + 12 + 380}{14 + 20 + 180 + 24 - 42 + 50} = \frac{1.968}{246} = 8$$

$$y = \begin{vmatrix} 7 & -2 & 4 \\ 5 & 38 & 6 \\ 3 & 21 & -1 \\ 7 & -2 & 4 \\ 5 & 38 & 6 \end{vmatrix}$$

$$y = \frac{-266 + 420 - 36 - 456 - 882 - 10}{246} = \frac{-1.230}{246} = -5$$

$$z = \begin{vmatrix} 7 & 10 & -2 \\ 5 & -2 & 38 \\ 3 & 1 & 21 \\ 7 & 10 & -2 \\ 5 & -2 & 38 \end{vmatrix}$$

$$z = \frac{-294 - 10 + 1.140 - 12 - 266 - 1.050}{246} = \frac{-492}{246} = -2$$

sol:  $x = 8$   $y = -5$   $z = -2$

7.  $4x+7y+5z=-2$

$6x+3y+7z=6$

$x-y+9z=-21$

$$x = \frac{\begin{vmatrix} -2 & 7 & 5 \\ 6 & 3 & 7 \\ -21 & -1 & 9 \end{vmatrix}}{\begin{vmatrix} 4 & 7 & 5 \\ 6 & 3 & 7 \\ 1 & -1 & 9 \end{vmatrix}} = \frac{\begin{vmatrix} -2 & 7 & 5 \\ 6 & 3 & 7 \\ 4 & 7 & 5 \end{vmatrix}}{\begin{vmatrix} 4 & 7 & 5 \\ 6 & 3 & 7 \\ 6 & 3 & 7 \end{vmatrix}}$$

$$x = \frac{-54 - 30 - 1.029 + 315 - 14 - 378}{108 - 30 + 49 - 15 + 28 - 378} = \frac{-1.190}{-238} = 5$$

$$y = \frac{\begin{vmatrix} 4 & -2 & 5 \\ 6 & 6 & 7 \\ 1 & -21 & 9 \end{vmatrix}}{\begin{vmatrix} 4 & 7 & 5 \\ 6 & 3 & 7 \\ 1 & -1 & 9 \end{vmatrix}} = \frac{\begin{vmatrix} 4 & -2 & 5 \\ 6 & 6 & 7 \\ 4 & -2 & 5 \end{vmatrix}}{\begin{vmatrix} 4 & 7 & 5 \\ 6 & 3 & 7 \\ 6 & 6 & 7 \end{vmatrix}}$$

$$y = \frac{216 - 630 - 14 - 30 + 588 + 108}{-238} = \frac{238}{-238} = -1$$

$$z = \frac{\begin{vmatrix} 4 & 7 & -2 \\ 6 & 3 & 6 \\ 1 & -1 & -21 \end{vmatrix}}{\begin{vmatrix} 4 & 7 & -2 \\ 6 & 3 & 6 \\ 4 & 7 & -2 \end{vmatrix}} = \frac{\begin{vmatrix} 4 & 7 & -2 \\ 6 & 3 & 6 \\ 6 & 3 & 6 \end{vmatrix}}{\begin{vmatrix} 4 & 7 & -2 \\ 6 & 3 & 6 \\ 6 & 3 & 6 \end{vmatrix}}$$

$$z = \frac{-252 + 12 + 42 + 6 + 24 + 882}{-238} = \frac{714}{-238} = -3$$

sol:  $x=5$   $y=-1$   $z=-3$

8.  $3x-5y+2z=-22$

$2x-y+6z=32$

$8x+3y-5z=-33$

$$x = \frac{\begin{vmatrix} -22 & -5 & 2 \\ 32 & -1 & 6 \\ -33 & 3 & -5 \end{vmatrix}}{\begin{vmatrix} -22 & -5 & 2 \\ 32 & -1 & 6 \\ 2 & -1 & 6 \end{vmatrix}} = \frac{\begin{vmatrix} -22 & -5 & 2 \\ 32 & -1 & 6 \\ -33 & 3 & -5 \end{vmatrix}}{\begin{vmatrix} -22 & -5 & 2 \\ 32 & -1 & 6 \\ 8 & 3 & -5 \end{vmatrix}}$$

Continúa

## 8. Continuación

$$x = \frac{-110 + 192 + 990 - 66 + 396 - 800}{15 + 12 - 240 + 16 - 54 - 50} = \frac{602}{-301} = -2$$

$$y = \frac{\begin{vmatrix} 3 & -22 & 2 \\ 2 & 32 & 6 \\ 8 & -33 & -5 \end{vmatrix}}{\begin{vmatrix} 3 & -22 & 2 \\ 2 & 32 & 6 \\ 3 & -22 & 2 \end{vmatrix}}$$

$$y = \frac{-480 - 132 - 1.056 - 512 + 594 - 220}{-301} = \frac{-1.806}{-301} = 6$$

$$z = \frac{\begin{vmatrix} 3 & -5 & -22 \\ 2 & -1 & 32 \\ 8 & 3 & -33 \end{vmatrix}}{\begin{vmatrix} 3 & -5 & -22 \\ 2 & -1 & 32 \\ 3 & -5 & -22 \end{vmatrix}}$$

$$z = \frac{99 - 132 - 1.280 - 176 - 288 - 330}{-301} = \frac{-2.107}{-301} = 7$$

sol:  $x=-2$   $y=6$   $z=7$

9.  $x+y+z=3$

$x+2y=6$

$2x+3y=6$

$$x = \frac{\begin{vmatrix} 3 & 1 & 1 \\ 6 & 2 & 0 \\ 6 & 3 & 0 \end{vmatrix}}{\begin{vmatrix} 3 & 1 & 1 \\ 6 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix}}$$

$$x = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 2 & 3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 2 & 0 \end{vmatrix}}$$

$$x = \frac{0 + 18 + 0 - 12 + 0 + 0}{0 + 3 + 0 - 4 + 0 + 0} = \frac{6}{-1} = -6$$

$$y = \frac{\begin{vmatrix} 1 & 3 & 1 \\ 1 & 6 & 0 \\ 2 & 6 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & 1 \\ 1 & 6 & 0 \\ 1 & 6 & 0 \end{vmatrix}}$$

$$y = \frac{0 + 6 + 0 - 12 + 0 + 0}{-1} = \frac{-6}{-1} = 6$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 3 \\ 1 & 2 & 6 \\ 2 & 3 & 6 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 3 \\ 1 & 2 & 6 \\ 1 & 2 & 6 \end{vmatrix}}$$

$$z = \frac{12 + 9 + 12 - 12 - 18 - 6}{-1} = \frac{-3}{-1} = 3$$

sol:  $x=-6$   $y=6$   $z=3$

10.  $3x - 2y = -1$

$4x + z = -28$

$x + 2y + 3z = -43$

$$x = \frac{\begin{vmatrix} -1 & -2 & 0 \\ -28 & 0 & 1 \\ -43 & 2 & 3 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & 0 \\ 4 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}}$$

$$x = \frac{0 + 0 + 86 + 0 + 2 - 168}{0 + 0 - 2 + 0 - 6 + 24} = \frac{-80}{16} = -5$$

$$y = \frac{\begin{vmatrix} 3 & -1 & 0 \\ 4 & -28 & 1 \\ 1 & -43 & 3 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & 0 \\ 4 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}}$$

$$y = \frac{-252 + 0 - 1 + 0 + 129 + 12}{16} = \frac{-112}{16} = -7$$

$$z = \frac{\begin{vmatrix} 3 & -2 & -1 \\ 4 & 0 & -28 \\ 1 & 2 & -43 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & 0 \\ 4 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}}$$

$$z = \frac{0 - 8 + 56 + 0 + 168 - 344}{16} = \frac{-128}{16} = -8$$

$$\text{sol: } x = -5 \quad y = -7 \quad z = -8$$

11.  $\frac{x}{3} - \frac{y}{4} + \frac{z}{4} = 1$

$4x - 3y + 3z = 12$

$\frac{x}{6} + \frac{y}{2} - z = 1$

$2x + 6y - 12z = 12$

$x + 3y - 6z = 6$

$\frac{x}{2} - \frac{y}{8} - \frac{z}{2} = 0$

$4x - y - 4z = 0$

$$x = \frac{\begin{vmatrix} 12 & -3 & 3 \\ 6 & 3 & -6 \\ 0 & -1 & -4 \end{vmatrix}}{\begin{vmatrix} 4 & -3 & 3 \\ 1 & 3 & -6 \\ 4 & -1 & -4 \end{vmatrix}}$$

$$x = \frac{12(-3)(-4) + 3(12) + 0}{4(-12) - 3(-16) + 3(4)} = \frac{48 + 36}{-48 + 48 + 12} = \frac{84}{12} = 7$$

Continúa

## 11. Continuación

$$x = \frac{-144 - 18 + 0 + 0 - 72 - 72}{-48 - 3 + 72 - 36 - 24 - 12} = \frac{-306}{-51} = 6$$

$$y = \frac{\begin{vmatrix} 4 & 12 & 3 \\ 1 & 6 & -6 \\ 4 & 0 & -4 \end{vmatrix}}{\begin{vmatrix} 4 & -3 & 12 \\ 1 & 3 & 6 \\ 4 & -1 & 0 \end{vmatrix}}$$

$$y = \frac{-96 + 0 - 288 - 72 + 0 + 48}{-51} = \frac{-408}{-51} = 8$$

$$z = \frac{\begin{vmatrix} 4 & -3 & 12 \\ 1 & 3 & 6 \\ 4 & -1 & 0 \end{vmatrix}}{\begin{vmatrix} 4 & -3 & 12 \\ 1 & 3 & 6 \\ 4 & -1 & 0 \end{vmatrix}}$$

$$z = \frac{0 - 12 - 72 - 144 + 24 + 0}{-51} = \frac{-204}{-51} = 4$$

sol:  $x = 6 \quad y = 8 \quad z = 4$

12.  $\frac{x}{3} + y = 2z + 3$  ;  $x - y = 1$  ;  $x + z = \frac{y}{4} + 11$

$x + 3y = 6z + y$  ;  $4x + 4z = y + 44$

$x + 3y - 6z = 9$

$4x - y + 4z = 44$

$$x = \frac{\begin{vmatrix} 9 & 3 & -6 \\ 1 & -1 & 0 \\ 44 & -1 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & -6 \\ 1 & -1 & 0 \\ 4 & -1 & 4 \end{vmatrix}}$$

$$x = \frac{-36 + 6 + 0 - 264 + 0 - 12}{-4 + 6 + 0 - 24 + 0 - 12} = \frac{-306}{-34} = 9$$

$$y = \frac{\begin{vmatrix} 1 & 9 & -6 \\ 1 & 1 & 0 \\ 4 & 44 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & -6 \\ 1 & -1 & 0 \\ 4 & -1 & 4 \end{vmatrix}}$$

$$y = \frac{4(-264 + 0 + 24 + 0 - 36)}{-34} = \frac{-272}{-34} = 8$$

$$z = \frac{\begin{vmatrix} 1 & 3 & 9 \\ 1 & -1 & 1 \\ 4 & -1 & 44 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & 9 \\ 1 & -1 & 1 \\ 4 & -1 & 4 \end{vmatrix}}$$

$$z = \frac{-44 - 9 + 12 + 36 + 1 - 132}{-34} = \frac{-136}{-34} = 4$$

sol:  $x = 9 \quad y = 8 \quad z = 4$

## EJERCICIO 191

1.  $x+2y+z=8$

Plano ABC  $x+2y+z=8$

Para  $y=0$   $z=0$   $x=8$

Para  $x=0$   $z=0$   $y=4$

Para  $x=0$   $y=0$   $z=8$

$2x+2y+z=9$

Plano DEF  $2x+2y+z=9$

Para  $y=0$   $z=0$   $x=4\frac{1}{2}$

Para  $x=0$   $z=0$   $y=4\frac{1}{2}$

Para  $x=0$   $y=0$   $z=9$

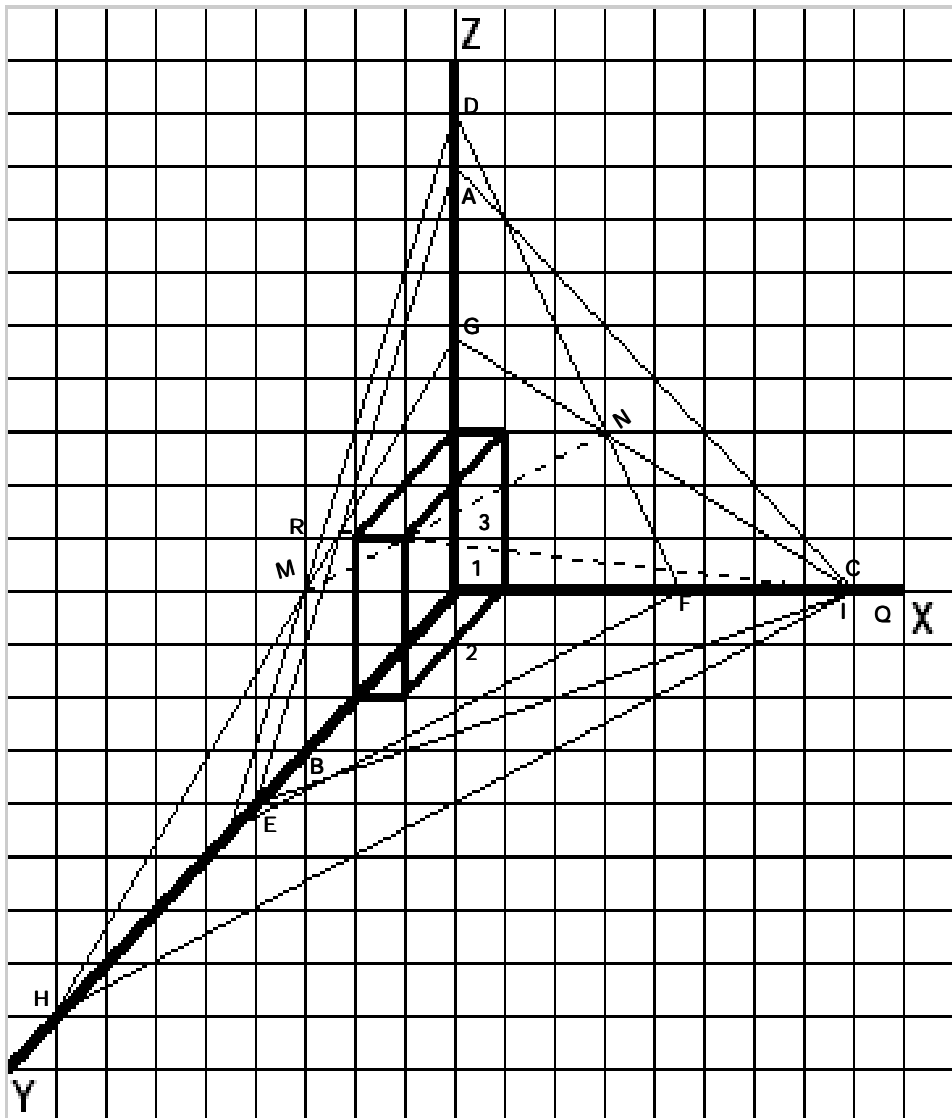
$3x+3y+5z=24$

Plano GHI  $3x+3y+5z=24$

Para  $y=0$   $z=0$   $x=8$

Para  $x=0$   $z=0$   $y=8$

Para  $x=0$   $y=0$   $z=4\frac{4}{5}$



Sol :  $x = 1$   $y = 2$   $z = 3$



2.  $x+y+z=5$

Plano ABC  $x+y+z=5$

Para  $y=0$   $z=0$   $x=5$

Para  $x=0$   $z=0$   $y=5$

Para  $x=0$   $y=0$   $z=5$

$3x+2y+z=8$

Plano DEF  $3x+2y+z=8$

Para  $y=0$   $z=0$   $x=2\frac{2}{3}$

Para  $x=0$   $z=0$   $y=4$

Para  $x=0$   $y=0$   $z=8$

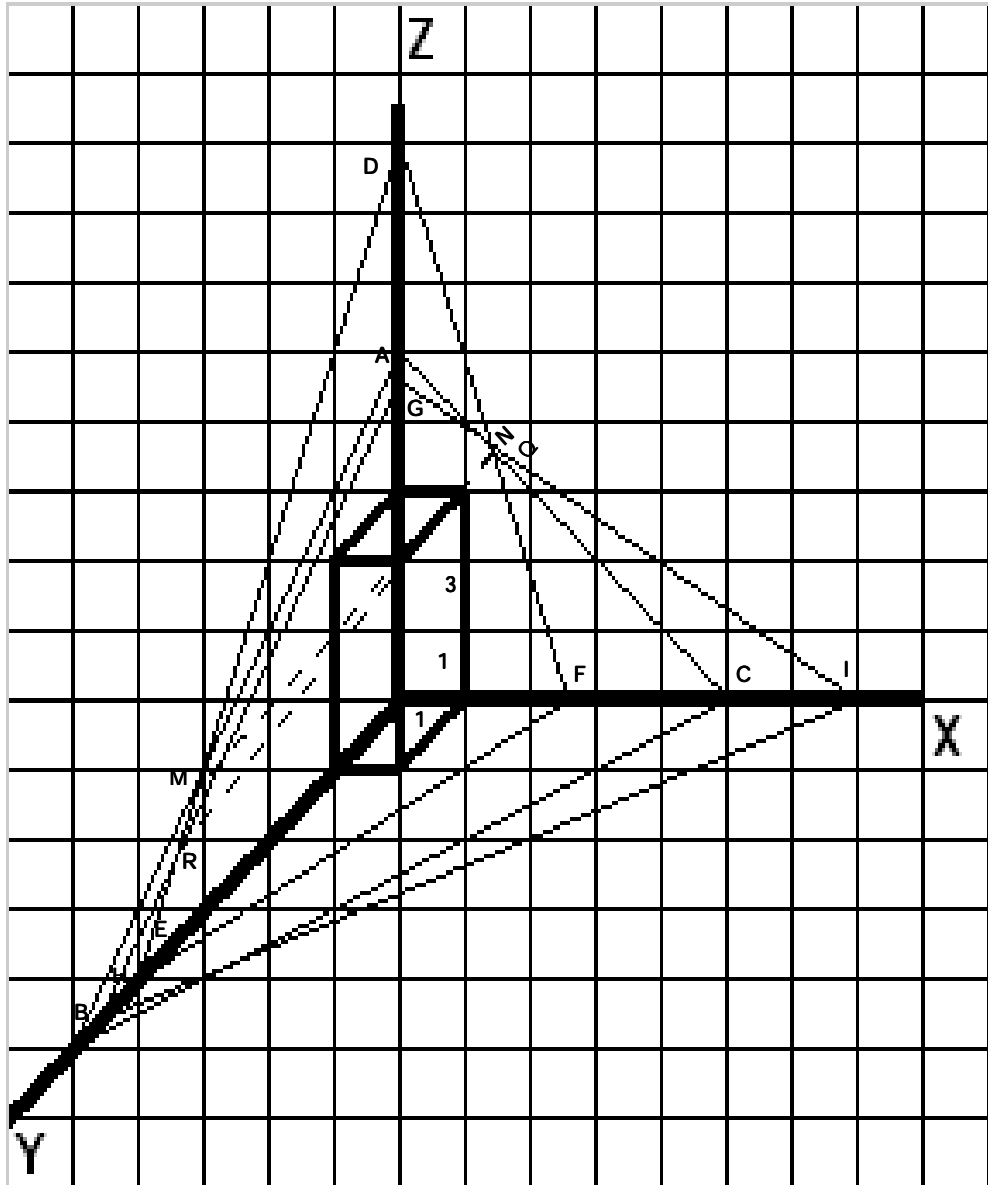
$2x+3y+3z=14$

Plano GHI  $2x+3y+3z=14$

Para  $y=0$   $z=0$   $x=7$

Para  $x=0$   $z=0$   $y=4\frac{2}{3}$

Para  $x=0$   $y=0$   $z=4\frac{2}{3}$



sol :  $x = 1$   $y = 1$   $z = 3$

3.  $2x + 2y + 3z = 23$

Plano ABC  $2x + 2y + 3z = 23$

Para  $y=0$   $z=0$   $x=11\frac{1}{2}$

Para  $x=0$   $z=0$   $y=11\frac{1}{2}$

Para  $x=0$   $y=0$   $z=7\frac{2}{3}$

$2x + 3y + 2z = 20$

Plano DEF  $2x + 3y + 2z = 20$

Para  $y=0$   $z=0$   $x=10$

Para  $x=0$   $z=0$   $y=6\frac{2}{3}$

Para  $x=0$   $y=0$   $z=10$

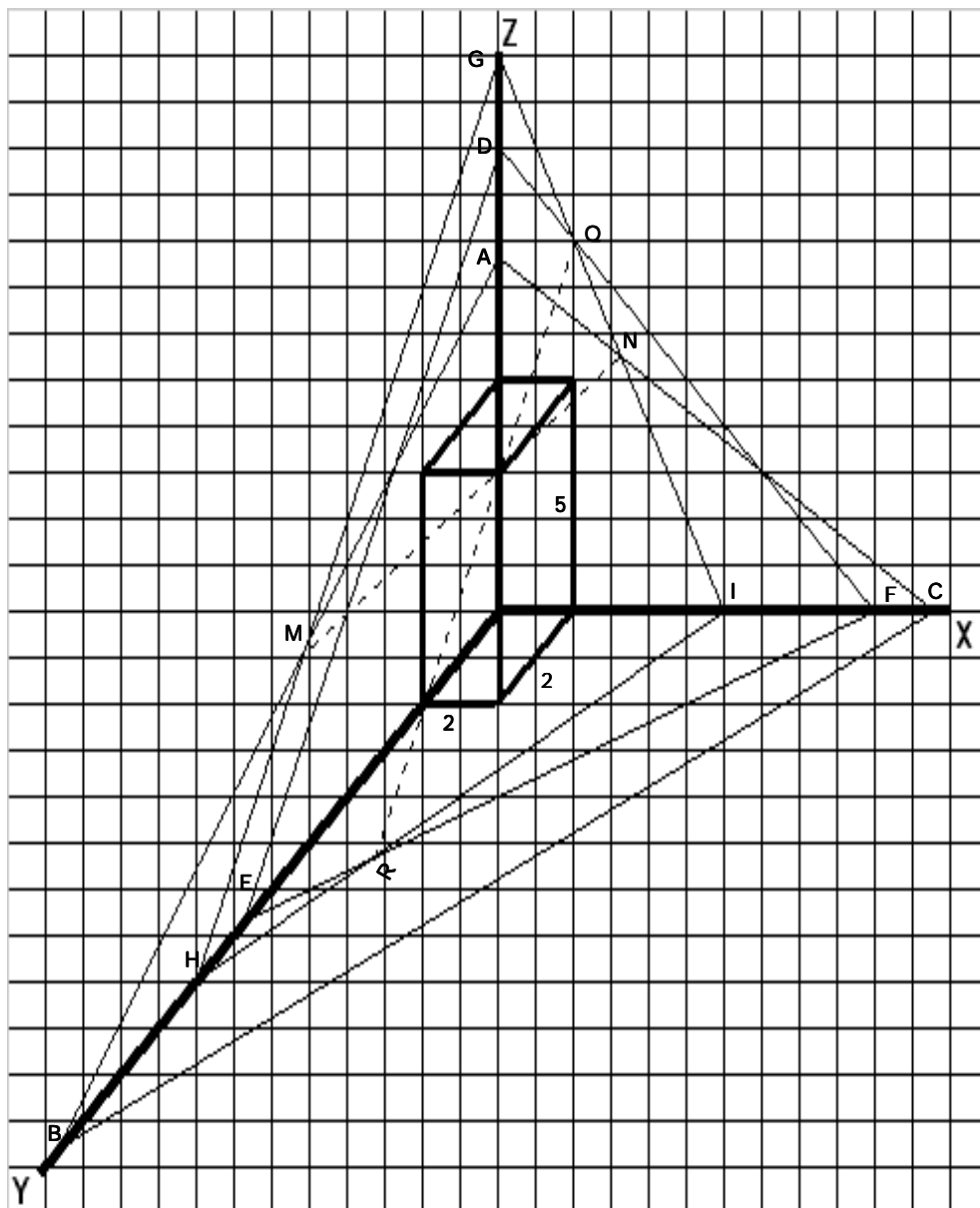
$4x + 3y + 2z = 24$

Plano GHI  $4x + 3y + 2z = 24$

Para  $y=0$   $z=0$   $x=6$

Para  $x=0$   $z=0$   $y=8$

Para  $x=0$   $y=0$   $z=12$



sol :  $x = 2$   $y = 2$   $z = 5$

4.  $2x + 2y + 3z = 24$

Plano ABC  $2x + 2y + 3z = 24$

Para  $y=0$   $z=0$   $x=12$

Para  $x=0$   $z=0$   $y=12$

Para  $x=0$   $y=0$   $z=8$

$4x + 5y + 2z = 35$

Plano DEF  $4x + 5y + 2z = 35$

Para  $y=0$   $z=0$   $x=8\frac{3}{4}$

Para  $x=0$   $z=0$   $y=7$

Para  $x=0$   $y=0$   $z=17\frac{1}{2}$

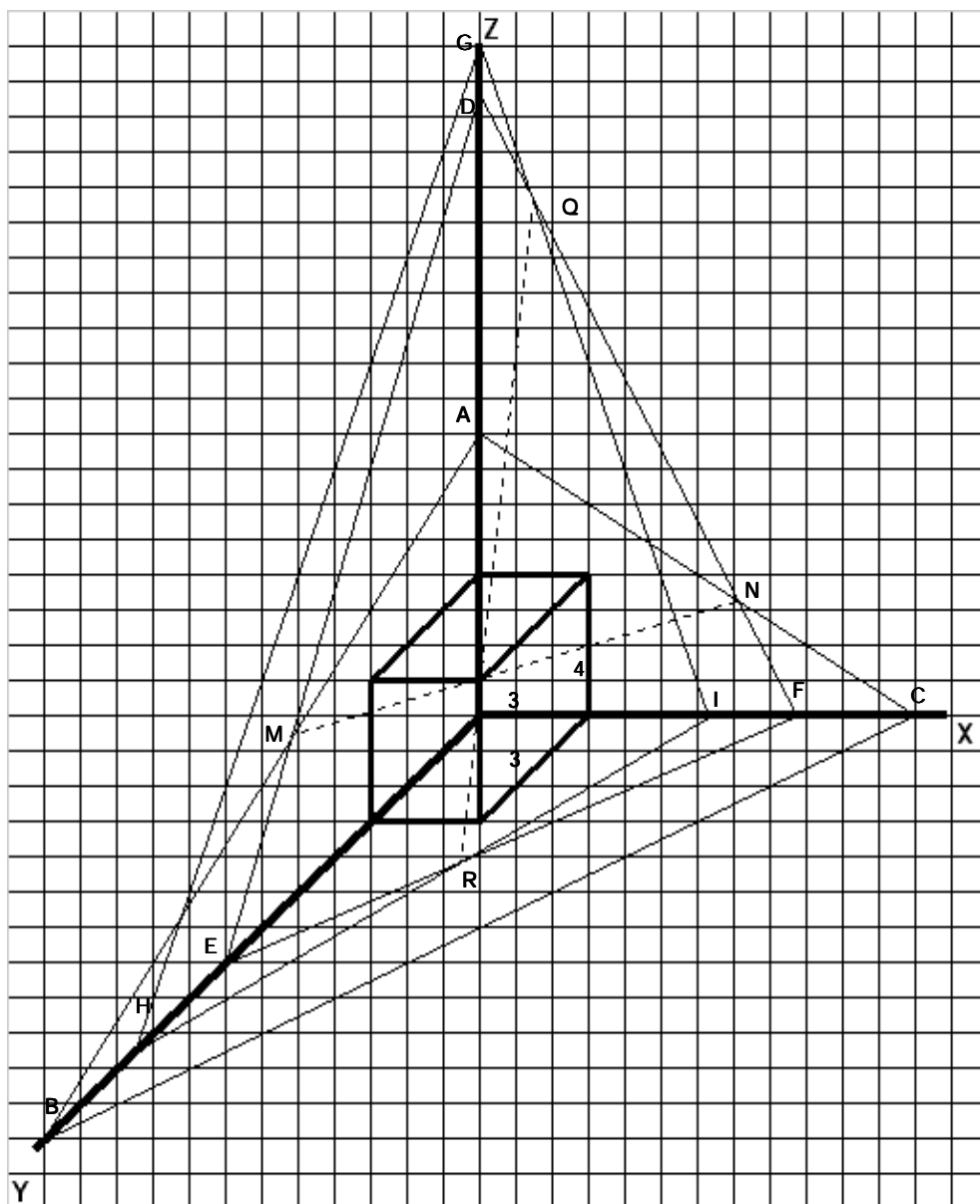
$3x + 2y + z = 19$

Plano GHI  $3x + 2y + z = 19$

Para  $y=0$   $z=0$   $x=6\frac{1}{3}$

Para  $x=0$   $z=0$   $y=9\frac{1}{2}$

Para  $x=0$   $y=0$   $z=19$



sol :  $x = 3$   $y = 3$   $z = 4$

5.  $3x+4y+5z=35$

Plano ABC  $3x+4y+5z=35$

Para  $y=0$   $z=0$   $x=11\frac{2}{3}$

Para  $x=0$   $z=0$   $y=8\frac{3}{4}$

Para  $x=0$   $y=0$   $z=7$

$2x+5y+3z=27$

Plano DEF  $2x+5y+3z=27$

Para  $y=0$   $z=0$   $x=13\frac{1}{2}$

Para  $x=0$   $z=0$   $y=5\frac{2}{5}$

Para  $x=0$   $y=0$   $z=9$

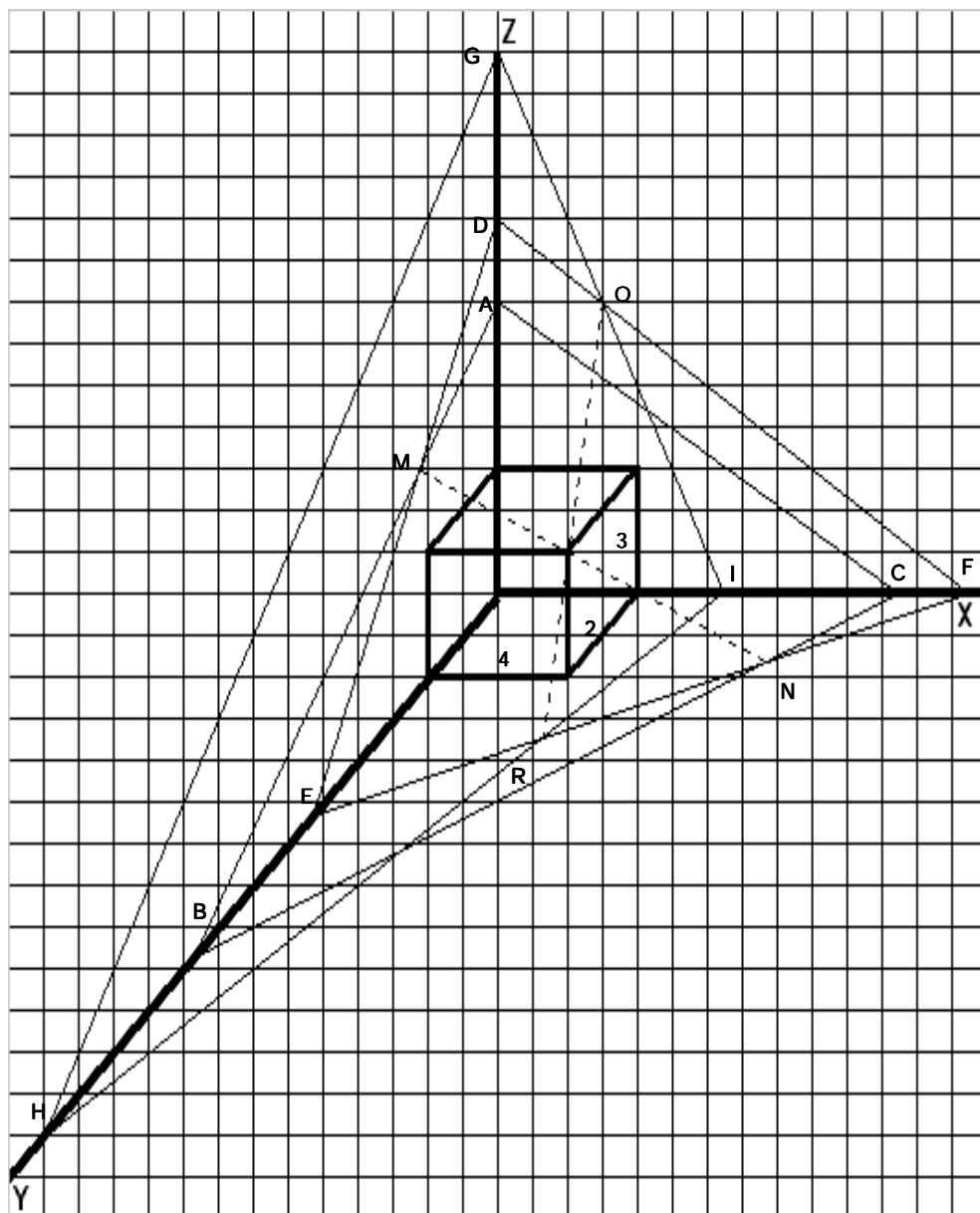
$2x+y+z=13$

Plano GHI  $2x+y+z=13$

Para  $y=0$   $z=0$   $x=6\frac{1}{2}$

Para  $x=0$   $z=0$   $y=13$

Para  $x=0$   $y=0$   $z=13$



sol :  $x = 4$   $y = 2$   $z = 3$

6.  $4x+3y+5z=42$

Plano ABC  $4x+3y+5z=42$

Para  $y=0$   $z=0$   $x=10\frac{1}{2}$

Para  $x=0$   $z=0$   $y=14$

Para  $x=0$   $y=0$   $z=8\frac{2}{5}$

$3x+4y+3z=33$

Plano DEF  $3x+4y+3z=33$

Para  $y=0$   $z=0$   $x=11$

Para  $x=0$   $z=0$   $y=8\frac{1}{4}$

Para  $x=0$   $y=0$   $z=11$

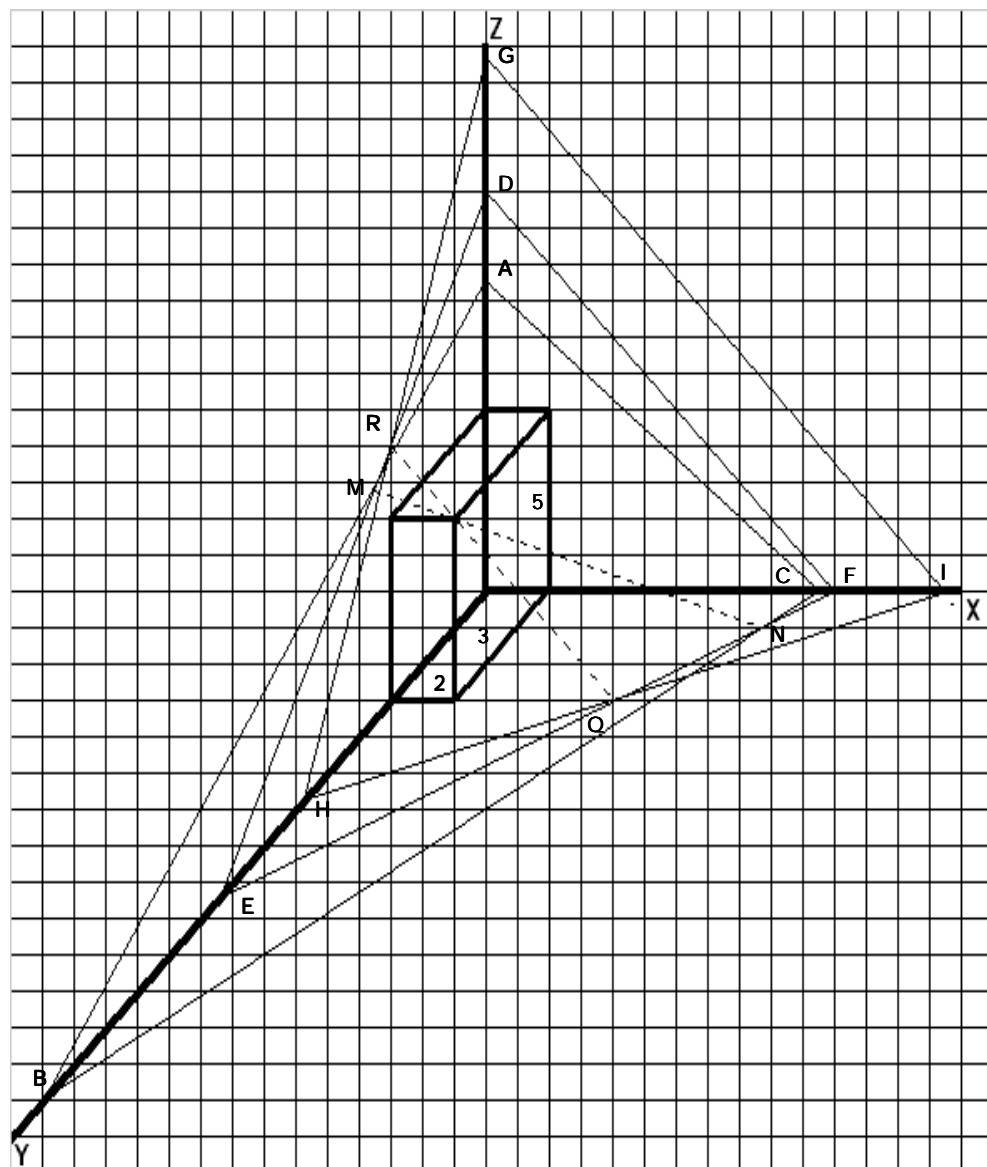
$2x+5y+2z=29$

Plano GHI  $2x+5y+2z=29$

Para  $y=0$   $z=0$   $x=14\frac{1}{2}$

Para  $x=0$   $z=0$   $y=5\frac{4}{5}$

Para  $x=0$   $y=0$   $z=14\frac{1}{2}$



sol :  $x = 2$   $y = 3$   $z = 5$

5.  $x \rightarrow$  Cifra decenas

$y \rightarrow$  Cifra unidades

$$\frac{10x+y-17}{x+y}=5$$

$$10x+y-17=5(x+y)$$

$$5x-4y=17$$

$$\frac{10x+y-2}{y-2}=19$$

$$10x+y-2=19(y-2)$$

$$10x-18y=-36$$

$$5x-9y=-18$$

$$5x-4y=17$$

$$5x-9y=-18 \quad (-1)$$

$$5x-4y=17$$

$$-5x+9y=18$$

$$5y=35 \Rightarrow y=7$$

$$5x-4(7)=17$$

$$5x-28=17$$

$$5x=45 \Rightarrow x=9 \quad \text{sol: } 97$$

6.  $x \rightarrow$  Cifra decenas

$y \rightarrow$  Cifra unidades

$$10x+y+9=10y+x$$

$$9x-9y=-9$$

$$x-y=-1$$

$$\frac{10y+x-1}{7}=6$$

$$x+10y=43$$

$$x-y=-1 \quad (-1)$$

$$x+10y=43$$

$$-x+y=1$$

$$x+10y=43$$

$$11y=44 \Rightarrow y=4$$

$$x-4=-1 \Rightarrow x=3 \quad \text{sol: } 34$$

7.  $x \rightarrow$  Cifra decenas

$y \rightarrow$  Cifra unidades

$$x+y=9$$

$$10(x+1)+y-1=10y+x$$

$$10x+10+y-1=10y+x$$

$$9x-9y=-9$$

$$x-y=-1$$

$$x+y=9$$

$$x-y=-1$$

$$2x=8 \Rightarrow x=4$$

$$4+y=9 \Rightarrow y=5 \quad \text{sol: } 45$$

## EJERCICIO 199

1.  $x \rightarrow$  Monedas de 20 cts.

$y \rightarrow$  Monedas de 10 cts.

$$x+y=78 \quad (-10)$$

$$20x+10y=1.130$$

$$-10x-10y=-780$$

$$20x+10y=1.130$$

$$10x=350$$

$$x=35$$

$$35+y=78$$

$$y=43$$

$$\text{sol: } 35 \text{ Mon. de 20 cts.}$$

$$43 \text{ Mon. de 10 cts.}$$

2.  $x \rightarrow$  Monedas de \$5

$y \rightarrow$  Monedas de \$4

$$x+y=91 \quad (-4)$$

$$5x+4y=404$$

$$-4x-4y=-364$$

$$5x+4y=404$$

$$x=40$$

$$40+y=91$$

$$y=51$$

$$\text{sol: } 40 \text{ Mon. de \$5}$$

$$51 \text{ Mon. de \$4}$$

3.  $x \rightarrow$  N° de adultos

$y \rightarrow$  N° de niños

$$x+y=700 \quad (-15)$$

$$40x+15y=18.000$$

$$-15x-15y=-10.500$$

$$40x+15y=18.000$$

$$25x=7.500$$

$$x=300$$

$$300+y=700$$

$$y=400$$

$$\text{sol: } 300 \text{ adultos y } 400 \text{ niños}$$

4.  $x \rightarrow$  Monedas de 20 cts.

$y \rightarrow$  Monedas de 25 cts.

$$x+y=44 \quad (-20)$$

$$20x+25y=995$$

$$-20x-20y=-880$$

$$20x+25y=995$$

Continúa

4. Continuación

$$5y=115$$

$$y=23$$

$$x+23=44$$

$$x=21$$

$$\text{sol: } 21 \text{ Monedas de 20 cts.}$$

$$23 \text{ Monedas de 25 cts.}$$

5.  $x \rightarrow$  Billetes de \$1

$y \rightarrow$  Billetes de \$2

$$x+y=287 \quad (-1)$$

$$x+2y=419$$

$$-x-y=-287$$

$$x+2y=419$$

$$y=132$$

$$x+132=287$$

$$x=155$$

$$\text{sol: } 155 \text{ Billetes de \$1}$$

$$132 \text{ Billetes de \$2}$$

6.  $x \rightarrow$  Libros de 3 col.

$y \rightarrow$  Libros de 7 col.

$$x+y=34 \quad (-3)$$

$$3x+7y=174$$

$$-3x-3y=-102$$

$$3x+7y=174$$

$$4y=72$$

$$y=18$$

$$x+18=34$$

$$x=16$$

$$\text{sol: } 16 \text{ libros de 3 col.}$$

$$18 \text{ libros de 7 col.}$$

7.  $x \rightarrow$  N° de trajes

$y \rightarrow$  N° de sombreros

$$x+y=54 \quad (-45)$$

$$375x+45y=6.720$$

$$-45x-45y=-2.430$$

$$375x+45y=6.720$$

$$330x=4.290$$

$$x=13$$

$$13+y=54 \Rightarrow y=41$$

$$\text{sol: } 13 \text{ trajes}$$

$$41 \text{ sombreros}$$

## EJERCICIO 200

1.  $A - 1 = B + 1$

$$A - B = 2$$

$$A + 1 = 3(B - 1)$$

$$A - 3B = -4$$

$$A - B = 2 \quad (-1)$$

$$A - 3B = -4$$

$$-A + B = -2$$

$$A - 3B = -4$$

$$-2B = -6$$

$$B = 3$$

$$A - 3 = 2$$

$$A = 5$$

$$\text{sol: } A \rightarrow \$5 \quad B \rightarrow \$3$$

2.  $B - 2 = A + 2$

$$A - B = -4$$

$$2(A - 2) = B + 2$$

$$2A - B = 6$$

$$A - B = -4 \quad (-1)$$

$$2A - B = 6$$

$$-A + B = 4$$

$$2A - B = 6$$

$$A = 10$$

$$10 - B = -4$$

$$-B = -14$$

$$B = 14$$

$$\text{sol: } A \rightarrow 10 \text{ soles}$$

$$B \rightarrow 14 \text{ soles}$$

3.  $P \rightarrow \text{Pedro}$

$$J \rightarrow \text{Juan}$$

$$P - 3 = J + 3$$

$$P - J = 6$$

$$4(J - 3) = P + 3$$

$$P - 4J = -15$$

$$P - J = 6 \quad (-1)$$

$$P - 4J = -15$$

$$-P + J = -6$$

$$P - 4J = -15$$

$$-3J = -21$$

$$J = 7$$

$$P - 7 = 6 \Rightarrow P = 13$$

$$\text{sol: } \text{Pedro} \rightarrow \$13$$

$$\text{Juan} \rightarrow \$7$$

4.  $A - 10 = 2(B - 10)$

$$A - 2B = -10$$

$$\frac{3}{4}(A + 10) = B + 10$$

$$3(A + 10) = 4(B + 10)$$

$$3A - 4B = 10$$

$$A - 2B = -10 \quad (-2)$$

$$3A - 4B = 10$$

$$-2A + 4B = 20$$

$$3A - 4B = 10$$

$$A = 30$$

$$30 - 2B = -10$$

$$-2B = -40$$

$$B = 20$$

$$\text{sol: } A \rightarrow 30 \text{ Años}$$

$$B \rightarrow 20 \text{ Años}$$

5.  $A - 6 = 2(B - 6)$

$$A - 2B = -6$$

$$A + 6 = \frac{8}{5}(B + 6)$$

$$5(A + 6) = 8(B + 6)$$

$$5A - 8B = 18$$

$$A - 2B = -6 \quad (-4)$$

$$5A - 8B = 18$$

$$-4A + 8B = 24$$

$$5A - 8B = 18$$

$$A = 42$$

$$42 - 2B = -6$$

$$-2B = -48$$

$$B = 24$$

$$\text{sol: } A \rightarrow 42 \text{ Años}$$

$$B \rightarrow 24 \text{ Años}$$

6.  $A - 5 = \frac{3}{2}(B - 5)$

$$2(A - 5) = 3(B - 5)$$

$$2A - 3B = -5$$

$$\frac{7}{9}(A + 10) = B + 10$$

$$7(A + 10) = 9(B + 10)$$

$$7A - 9B = 20$$

$$2A - 3B = -5 \quad (-3)$$

$$7A - 9B = 20$$

Continúa

## 6. Continuación

$$-6A + 9B = 15$$

$$7A - 9B = 20$$

$$A = 35$$

$$2(35) - 3B = -5$$

$$70 - 3B = -5$$

$$B = 25$$

$$\text{sol: } A \rightarrow 35 \text{ Años}$$

$$B \rightarrow 25 \text{ Años}$$

7.  $x \rightarrow \text{Edad Hombre}$

$$y \rightarrow \text{Edad Esposa}$$

$$x = \frac{9y}{5}$$

$$\frac{3}{5}(x + 4) = y + 4$$

$$3x + 12 = 5y + 20$$

$$3x - 5y = 8$$

$$5x - 9y = 0 \quad (3)$$

$$3x - 5y = 8 \quad (-5)$$

$$15x - 27y = 0$$

$$-15x + 25y = -40$$

$$y = 20$$

$$x = \frac{9}{5}(20)$$

$$x = 9(4)$$

$$x = 36$$

$$\text{sol: } 36 \text{ Años} \rightarrow \text{tiene el hombre}$$

$$20 \text{ Años} \rightarrow \text{tiene la esposa}$$

8.  $A - 25 = B + 25$

$$A - B = 50$$

$$\frac{5}{17}(A + 35) = B - 35$$

$$5A + 175 = 17B - 595$$

$$5A - 17B = -770$$

$$A - B = 50 \quad (-5)$$

$$5A - 17B = -770$$

$$-5A + 5B = -250$$

$$5A - 17B = -770$$

$$-12B = -1.020$$

$$B = 85$$

$$A - 85 = 50$$

$$A = 135$$

$$\text{sol: } A \rightarrow 135 \text{ Lempiras}$$

$$B \rightarrow 85 \text{ Lempiras}$$

9.  $x \rightarrow$  Edad padre

$y \rightarrow$  Edad hijo

$$\frac{1}{5}(x-6) = y-6$$

$$x-6 = 5y-30$$

$$x-5y = -24$$

$$\frac{2}{5}(x+9) = y+9$$

$$2x+18 = 5y+45$$

$$2x-5y = 27$$

$$x-5y = -24 \quad (-1)$$

$$2x-5y = 27$$

$$-x+5y = 24$$

$$2x-5y = 27$$

$$x = 51$$

$$51-5y = -24$$

$$-5y = -75$$

$$y = 15$$

sol: 51 Años  $\rightarrow$  Edad padre

15 Años  $\rightarrow$  Edad hijo

10.  $P+15=5(J-15)$

$$P-5J = -90$$

$$3(P-20) = J+20$$

$$3P-J = 80$$

$$P-5J = -90 \quad (-3)$$

$$3P-J = 80$$

$$-3P+15J = 270$$

$$3P-J = 80$$

$$14J = 350$$

$$J = 25$$

$$3P-25 = 80$$

$$3P = 105$$

$$P = 35$$

sol: 35 cts.  $\rightarrow$  tenia Pedro

25 cts.  $\rightarrow$  tenia Juan

11.  $A + \frac{B}{2} + 60 = 4\left(B - \frac{B}{2} - 60\right)$

$$2A + B + 120 = 8B - 4B - 480$$

$$2A + B + 120 = 4B - 480$$

$$2A - 3B = -600$$

$$B + 80 - 310 = A - 80$$

$$B - 230 = A - 80$$

$$A - B = -150$$

$$2A - 3B = -600$$

$$A - B = -150 \quad (-2)$$

$$2A - 3B = -600$$

$$-2A + 2B = 300$$

$$B = 300$$

$$A - 300 = -150$$

$$A = 150 \text{ cts.}$$

sol: \$1,50  $\rightarrow$  tiene A

\$3  $\rightarrow$  tiene B

12.  $x \rightarrow$  Edad de Enrique

$y \rightarrow$  Edad hermana

$$x-6 = \frac{3}{2}(y-6)$$

$$2x-12 = 3y-18$$

$$2x-3y = -6$$

$$4(x+6) = 5(y+6)$$

$$4x+24 = 5y+30$$

$$4x-5y = 6$$

$$2x-3y = -6 \quad (-2)$$

$$4x-5y = 6$$

$$-4x+6y = 12$$

$$4x-5y = 6$$

$$y = 18$$

$$2x-3(18) = -6$$

$$2x-54 = -6$$

$$2x = 48$$

$$x = 24$$

sol: 24 Añ.  $\rightarrow$  Ed. Enrique

18 Añ.  $\rightarrow$  Ed. herm.

## EJERCICIO 201

1.  $x \rightarrow$  Veloc. bote agua tranq.

$y \rightarrow$  Veloc. del rio

$x+y \rightarrow$  Veloc. bote a favor  
del agua

$x-y \rightarrow$  Veloc. bote contra  
la corriente

$$\frac{10}{x+y} = 1$$

$$x+y = 10$$

$$\frac{4}{x-y} = 1$$

$$x-y = 4$$

$$x+y = 10$$

$$x-y = 4$$

$$2x = 14$$

$$x = 7$$

$$7-y = 4$$

$$y = 3$$

sol:  $7 \text{ Km/h} \rightarrow$  Veloc.

bote agua tranq.

$3 \text{ Km/h} \rightarrow$  Veloc. del rio

2.  $x \rightarrow$  Veloc. bote agua tranq.

$y \rightarrow$  Veloc. del rio

$x+y \rightarrow$  Veloc. bote a favor  
del agua

$x-y \rightarrow$  Veloc. bote contra  
agua

$$\frac{28}{x+y} = \frac{7}{4}$$

$$7x+7y = 112$$

$$x+y = 16$$

$$\frac{24}{x-y} = 3$$

$$3x-3y = 24$$

$$x-y = 8$$

$$x+y = 16$$

$$x-y = 8$$

$$2x = 24$$

$$x = 12$$

$$12-y = 8$$

$$y = 4$$

sol:  $12 \text{ Km/h} \rightarrow$  Bote

$4 \text{ Km/h} \rightarrow$  Rio

3.  $x \rightarrow$  Tiempo ida

$y \rightarrow$  Tiempo vuelta

$$x+y = 5$$

$$\frac{8}{x} = \frac{12}{y}$$

$$12x-8y = 0$$

$$x+y = 5 \quad (8)$$

$$12x-8y = 0$$

$$8x+8y = 40$$

$$20x = 40$$

$$x = 2$$

$$12(2)-8y = 0$$

$$-8y = -24$$

$$y = 3$$

sol:  $2h \rightarrow$  Tiempo ida

$3h \rightarrow$  Tiempo vuelta



4.  $x \rightarrow$  Veloc. bote agua tranq.

$y \rightarrow$  Veloc. río

$x+y \rightarrow$  Veloc. bote a favor

de la cte

$x-y \rightarrow$  Veloc. bote en contra

de la cte

$$\frac{40}{x+y} = \frac{5}{2}$$

$$5x+5y=80$$

$$x+y=16$$

$$\frac{40}{x-y} = 5$$

$$5x-5y=40$$

$$x-y=8$$

$$x+y=16$$

$$x-y=8$$

$$2x = 24 \Rightarrow x=12$$

$$12-y=8 \Rightarrow y=4$$

sol:  $12h \rightarrow$  Veloc. bote

$4h \rightarrow$  Veloc. río

5.  $x \rightarrow$  tiempo ida

$y \rightarrow$  tiempo vuelta

$$x+y=6$$

$$\frac{20}{x} = \frac{40}{y}$$

$$40x-20y=0$$

$$2x-y=0$$

$$x+y=6$$

$$2x-y=0$$

$$3x = 6 \Rightarrow x=2$$

$$2(2)-y=0$$

$$-y=-4 \Rightarrow y=4$$

sol:  $2h \rightarrow$  tiempo ida

$4h \rightarrow$  tiempo vuelta

6.  $x \rightarrow$  tiempo ida

$y \rightarrow$  tiempo vuelta

$$x+y=5$$

$$\frac{8}{x} = \frac{12}{y}$$

$$12x-8y=0$$

$$3x-2y=0$$

$$x+y=5 \quad (2)$$

$$3x-2y=0$$

$$2x+2y=10$$

$$3x-2y=0$$

$$5x = 10$$

$$x=2$$

$$2+y=5$$

$$y=3$$

sol:  $2h \rightarrow$  tiempo ida

$3h \rightarrow$  tiempo vuelta

Entonces

$$\frac{32Km}{2h} = 16 Km/h \rightarrow \text{Veloc. río abajo}$$

$$\frac{12Km}{3h} = 4 Km/h \rightarrow \text{Veloc. río arriba}$$

Luego

$x \rightarrow$  Veloc. bote

$y \rightarrow$  Veloc. río

$$x+y=16$$

$$x-y=4$$

$$2x = 20$$

$$x=10$$

$$10+y=16$$

$$y=6$$

sol:  $10 Km/h \rightarrow$  Veloc. bote

$6 Km/h \rightarrow$  Veloc. río

## EJERCICIO 202

1.  $x \rightarrow$  N° mayor

$y \rightarrow$  N° medio

$z \rightarrow$  N° menor

$$z-1 = \frac{x+y}{3}$$

$$3z-3 = x+y$$

$$x+y-3z = -3$$

$$x+y+z = 37$$

$$y-z = x-13$$

$$x-y+z = 13$$

$$x+y+z = 37 \quad (-1)$$

$$x+y-3z = -3$$

$$-x-y-z = -37$$

$$x+y-3z = -3$$

$$-4z = -40$$

$$z=10$$

$$x+y+z = 37$$

$$x-y+z = 13$$

$$2x + 2z = 50$$

$$2x+2(10)=50$$

$$2x=30$$

$$x=15$$

$$15+y+10=37$$

$$y+25=37$$

$$y=12$$

sol: 15, 12, 10

### 2. Continuación

$$2x+7(0,2)=1,52$$

$$2x+1,4=1,52$$

$$2x=0,12$$

$$x=0,06$$

$$2(0,06)+0,2+2z=0,46$$

$$0,12+0,2+2z=0,46$$

$$2z=0,14$$

$$z=0,07$$

sol: 6 cts.  $\rightarrow$  Kilo Az.

20 cts.  $\rightarrow$  Kilo café

7 cts.  $\rightarrow$  Kilo frijoles

2.  $x \rightarrow$  Precio kilo azúcar

$y \rightarrow$  Precio kilo café

$z \rightarrow$  Precio kilo frijoles

$$5x+3y+4z=1,18$$

$$4x+5y+3z=1,45$$

$$2x+y+2z=0,46$$

$$5x+3y+4z=1,18 \quad (-3)$$

$$4x+5y+3z=1,45 \quad (4)$$

Continúa

### 2. Continuación

$$-15x-9y-12z=-3,54$$

$$16x+20y+12z=5,80$$

$$x+11y = 2,26$$

$$4x+5y+3z=1,45 \quad (2)$$

$$2x+y+2z=0,46 \quad (-3)$$

$$8x+10y+6z=2,90$$

$$-6x-3y-6z=-1,38$$

Continúa

### 2. Continuación

$$2x+7y = 1,52$$

$$x+11y = 2,26 \quad (-2)$$

$$2x+7y = 1,52$$

$$-2x-22y = -4,52$$

$$2x+7y = 1,52$$

$$-15y = -3$$

$$y = \frac{1}{5}$$

$$y = 0,2$$

Continúa

3.  $x \rightarrow$  Cifra centenas

$y \rightarrow$  Cifra decenas

$z \rightarrow$  Cifra unidades

$$x + y + z = 15$$

$$x + y = \frac{3z}{2}$$

$$2x + 2y = 3z$$

$$2x + 2y - 3z = 0$$

$$100x + 10y + z - 99 = 100z + 10y + x$$

$$99x - 99z = 99$$

$$x - z = 1$$

$$x + y + z = 15 \quad (-2)$$

$$2x + 2y - 3z = 0$$

$$-2x - 2y - 2z = -30$$

$$2x + 2y - 3z = 0$$

$$-5z = -30$$

$$z = 6$$

$$x - 6 = 1$$

$$x = 7$$

$$7 + y + 6 = 15$$

$$y + 13 = 15$$

$$y = 2 \quad \text{sol: } 726$$

4.  $x \rightarrow$  N° mayor

$y \rightarrow$  N° medio

$z \rightarrow$  N° menor

$$x + y + z = 127$$

$$\frac{z}{2} + \frac{y}{3} + \frac{x}{9} = 39$$

$$2x + 6y + 9z = 702$$

$$x - 4 = \frac{y + z}{2}$$

$$2x - 8 = y + z$$

$$2x - y - z = 8$$

$$x + y + z = 127$$

$$2x - y - z = 8$$

$$3x = 135$$

$$x = 45$$

$$2x + 6y + 9z = 702$$

$$2x - y - z = 8 \quad (9)$$

$$2x + 6y + 9z = 702$$

$$18x - 9y - 9z = 72$$

$$20x - 3y = 774$$

Continúa

4. Continuación

$$20(45) - 3y = 774$$

$$900 - 3y = 774$$

$$-3y = -126$$

$$y = 42$$

$$45 + 42 + z = 127$$

$$z = 40$$

$$\text{sol: } 45, 42, 40$$

5.  $x \rightarrow$  Cifra centenas

$y \rightarrow$  Cifra decenas

$z \rightarrow$  Cifra unidades

$$x + y + z = 6$$

$$\frac{100x + 10y + z}{x + y} = 41$$

$$100x + 10y + z = 41x + 41y$$

$$59x - 31y + z = 0$$

$$100x + 10y + z + 198 = 100z + 10y + x$$

$$99x - 99z = -198$$

$$x - z = -2$$

$$x + y + z = 6 \quad (31)$$

$$59x - 31y + z = 0$$

$$31x + 31y + 31z = 186$$

$$59x - 31y + z = 0$$

$$90x + 32z = 186$$

$$45x + 16z = 93$$

$$x - z = -2 \quad (-45)$$

$$45x + 16z = 93$$

$$-45x + 45z = 90$$

$$45x + 16z = 93$$

$$61z = 183$$

$$z = 3$$

$$x - 3 = -2$$

$$x = 1$$

$$1 + y + 3 = 6$$

$$y + 4 = 6$$

$$y = 2 \quad \text{sol: } 123$$

6.  $x \rightarrow$  Angulo mayor

$y \rightarrow$  Angulo medio

$z \rightarrow$  Angulo menor

$$x + y + z = 180$$

$$x - 35 = z$$

$$x - z = 35$$

Continúa

6. Continuación

$$z - 20 = x - y$$

$$x - y - z = -20$$

$$x + y + z = 180$$

$$x - y - z = -20$$

$$2x = 160$$

$$x = 80$$

$$80 - z = 35$$

$$z = 45$$

$$80 + y + 45 = 180$$

$$y + 125 = 180$$

$$y = 55$$

$$\text{sol: } 80^\circ, 55^\circ, 45^\circ$$

7.  $x \rightarrow$  N° de vacas

$y \rightarrow$  N° de caballos

$z \rightarrow$  N° de terneros

$$x + y + z = 110$$

$$\frac{x}{8} + \frac{y}{9} + \frac{z}{5} = 15$$

$$45x + 40y + 72z = 5.400$$

$$z + x = 65$$

$$x + y + z = 110$$

$$x + z = 65 \quad (-1)$$

$$x + y + z = 110$$

$$-x - z = -65$$

$$y = 45$$

$$45x + 40y + 72z = 5.400$$

$$x + z = 65 \quad (-45)$$

$$45x + 40y + 72z = 5.400$$

$$-45x - 45z = -2.925$$

$$40y + 27z = 2.475$$

$$40(45) + 27z = 2.475$$

$$1.800 + 27z = 2.475$$

$$27z = 675$$

$$z = 25$$

$$x + 45 + 25 = 110$$

$$x + 70 = 110$$

$$x = 40$$

$$\text{sol: } 40 \text{ vacas;}$$

$$45 \text{ caballos;}$$

$$25 \text{ terneros}$$

8.  $x \rightarrow$  Cifra centenas

$y \rightarrow$  Cifra decenas

$z \rightarrow$  Cifra unidades

$$x + y + z = 10$$

$$x + y - 4 = z$$

$$x + y - z = 4$$

$$x + z - 6 = y$$

$$x - y + z = 6$$

$$x + y + z = 10$$

$$x + y - z = 4 \quad (-1)$$

$$x + y + z = 10$$

$$-x - y + z = -4$$

$$2z = 6$$

$$z = 3$$

$$x + y + z = 10$$

$$x - y + z = 6$$

$$2x + 2z = 16$$

$$x + z = 8$$

$$x + 3 = 8$$

$$x = 5$$

$$5 + y + 3 = 10$$

$$y = 2 \quad \text{sol: } 523$$

9.  $x \rightarrow$  Angulo mayor

$y \rightarrow$  Angulo medio

$z \rightarrow$  Angulo menor

$$x + y + z = 180$$

$$x + y = 135$$

$$y + z = 110$$

$$x + y + z = 180$$

$$y + z = 110 \quad (-1)$$

$$x + y + z = 180$$

$$-y - z = -110$$

$$x = 70$$

$$70 + y = 135$$

$$y = 65$$

$$70 + 65 + z = 180$$

$$135 + z = 180$$

$$z = 45$$

$$\text{sol: } 70^\circ, 65^\circ, 45^\circ$$

10.  $A + B + C = 140$

$$C = \frac{A}{2}$$

$$A - 2C = 0$$

$$A - 10 = B$$

$$A - B = 10$$

$$A + B + C = 140$$

$$A - B = 10$$

$$2A + C = 150$$

$$2A + C = 150 \quad (2)$$

$$A - 2C = 0$$

$$4A + 2C = 300$$

$$A - 2C = 0$$

$$5A = 300$$

$$A = 60$$

$$2(60) + C = 150$$

$$C = 30$$

$$60 + B + 30 = 140$$

$$B = 50$$

$$\text{sol: } A \rightarrow 60\text{bs. } B \rightarrow 50\text{bs.}$$

$$C \rightarrow 30\text{bs.}$$

11.  $A - 1 = C + 1$

$$A - C = 2$$

$$B - 1 = C$$

$$B - C = 1$$

$$A + 5 = 2C$$

$$A - 2C = -5$$

$$A - C = 2$$

$$B - C = 1$$

$$A + B - 2C = 3$$

$$A + B - 2C = 3$$

$$A - 2C = -5 \quad (-1)$$

$$A + B - 2C = 3$$

$$-A + 2C = 5$$

$$B = 8$$

$$8 - C = 1$$

$$C = 7$$

$$A - 7 = 2$$

$$A = 9$$

$$\text{sol: } A \rightarrow \$9$$

$$B \rightarrow \$8 \quad C \rightarrow \$7$$

12.  $x + y + z = 6$

$$100z + 10y + x = \frac{41(100x + 10y + z)}{107}$$

$$10.700z + 1.070y + 107x = 4.100x + 410y + 41z$$

$$3.993x - 660y - 10.659z = 0$$

$$1.331x - 220y - 3.553z = 0$$

$$100x = 300$$

$$x = 3$$

$$1.331x - 220y - 3.553z = 0$$

$$x + y + z = 6 \quad (220)$$

$$1.331x - 220y - 3.553z = 0$$

$$220x + 220y + 220z = 1.320$$

$$1.551x - 3.333z = 1.320$$

$$1.551(3) - 3.333z = 1.320$$

$$4.653 - 3.333z = 1.320$$

$$z = 1$$

$$3 + y + 1 = 6$$

$$y + 4 = 6$$

$$y = 2 \quad \text{sol: } 321$$

13.  $A - 2 = B + 2$

$$A - B = 4$$

$$B - 1 = C + 1$$

$$B - C = 2$$

$$A = \frac{8}{5}C$$

$$5A - 8C = 0$$

$$A - B = 4$$

$$B - C = 2$$

$$A - C = 6$$

$$A - C = 6 \quad (-8)$$

$$5A - 8C = 0$$

$$-8A + 8C = -48$$

$$5A - 8C = 0$$

$$-3A = -48$$

$$A = 16$$

$$16 - B = 4$$

$$B = 12$$

$$12 - C = 2$$

$$C = 10$$

$$\text{sol: } A \rightarrow 16Q$$

$$B \rightarrow 12Q \quad C \rightarrow 10Q$$

14.  $x \rightarrow$  Cifra centena

$y \rightarrow$  Cifra decena

$z \rightarrow$  Cifra unidades

$$100x = 400$$

$$x = 4$$

$$x + y + z = 9$$

$$100z + 10y + x = \frac{16(100x + 10y + z)}{49}$$

$$4.900z + 490y + 49x = 1.600x + 160y + 16z$$

$$1.551x - 330y - 4.884z = 0$$

$$517x - 110y - 1.628z = 0$$

$$517x - 110y - 1.628z = 0$$

$$\underline{x + y + z = 9 \quad (110)}$$

$$517x - 110y - 1.628z = 0$$

$$\underline{110x + 110y + 110z = 990}$$

$$627x - 1.518z = 990$$

$$627(4) - 1.518z = 990$$

$$2.508 - 1.518z = 990$$

$$-1.518z = -1.518$$

$$z = 1$$

$$4 + y + 1 = 9$$

$$y + 5 = 9$$

$$y = 4 \quad \text{sol: } 441$$

15.  $2A + B = C + 32$

$$2A + B - C = 32$$

$$\frac{B}{3} + 2C = A + 9$$

$$B + 6C = 3A + 27$$

$$3A - B - 6C = -27$$

$$\frac{A+B}{3} = C - 1$$

$$A + B = 3C - 3$$

$$A + B - 3C = -3$$

$$2A + B - C = 32$$

$$3A - B - 6C = -27$$

$$5A - 7C = 5$$

$$3A - B - 6C = -27$$

$$\underline{A + B - 3C = -3}$$

$$4A - 9C = -30$$

$$5A - 7C = 5 \quad (-4)$$

$$4A - 9C = -30 \quad (5)$$

$$-20A + 28C = -20$$

$$\underline{20A - 45C = -150}$$

$$-17C = -170$$

15. Continuación

$$C = 10$$

$$5A - 7(10) = 5$$

$$5A = 75$$

$$A = 15$$

$$2(15) + B - 10 = 32$$

$$B + 20 = 32$$

$$B = 12$$

$$\text{sol: } 15, 12, 10$$

## EJERCICIO 203

1.  $x \rightarrow$  Ancho

$y \rightarrow$  Largo

$$2(x + y) = 18$$

$$4y = 5x$$

$$5x - 4y = 0$$

$$x + y = 9 \quad (4)$$

$$\underline{5x - 4y = 0}$$

$$4x + 4y = 36$$

$$\underline{5x - 4y = 0}$$

$$9x = 36$$

$$x = 4$$

$$4 + y = 9$$

$$y = 5$$

$$\text{sol: } 5m \cdot 4m$$

2.  $A = 2B$

$$A - 2B = 0$$

$$A - 12 = B + 12$$

$$A - B = 24$$

Continúa

2. Continuación

$$A - 2B = 0$$

$$\underline{A - B = 24 \quad (-1)}$$

$$A - 2B = 0$$

$$\underline{-A + B = -24}$$

$$B = 24$$

$$A - 24 = 24$$

$$A = 48$$

$$\text{sol: } A \rightarrow 48 \text{ balboas}$$

$$B \rightarrow 24 \text{ balboas}$$

3.  $x \rightarrow$  Ancho

$y \rightarrow$  Largo

$$(x+1)(y+1) = 26 + xy$$

$$xy + x + y + 1 = 26 + xy$$

$$x + y = 25$$

$$(x+2)(y-3) = 19 + xy$$

$$xy - 3x + 2y - 6 = 19 + xy$$

$$-3x + 2y = 25$$

Continúa

3. Continuación

$$x + y = 25 \quad (3)$$

$$\underline{-3x + 2y = 25}$$

$$3x + 3y = 75$$

$$\underline{-3x + 2y = 25}$$

$$5y = 100$$

$$y = 20$$

$$x + 20 = 25$$

$$x = 5$$

$$\text{sol: } 20m \cdot 5m$$

4.  $x \rightarrow$  Precio carro

$y \rightarrow$  Precio caballo

$z \rightarrow$  Precio arcos

$$x + y + z = 200$$

$$x + z = y + 20$$

$$x - y + z = 20$$

$$y + z = x + 40$$

$$x - y - z = -40$$

Continúa

4. Continuación

$$x + y + z = 200$$

$$\underline{x - y - z = -40}$$

$$2x = 160$$

$$x = 80$$

$$x + y + z = 200$$

$$\underline{x - y + z = 20}$$

$$2x + 2z = 220$$

$$x + z = 110$$

$$80 + z = 110$$

$$z = 30$$

$$80 + y + 30 = 200$$

$$y + 110 = 200$$

$$y = 90$$

$$\text{sol: } \$80 \rightarrow \text{Costo carro}$$

$$\$90 \rightarrow \text{Costo caballo}$$

$$\$30 \rightarrow \text{Costo arcos}$$

5.  $x \rightarrow 1^{\text{er}} \text{ Número}$

$y \rightarrow 2^{\text{o}} \text{ Número}$

$z \rightarrow 3^{\text{er}} \text{ Número}$

$$x+y=z+18$$

$$x+y-z=18$$

$$x+z-78=y$$

$$x-y+z=78$$

$$y+z=x+102$$

$$x-y-z=-102$$

$$x+y-z=18$$

$$x-y+z=78$$

$$2x = 96$$

$$x=48$$

$$x-y+z=78$$

$$x-y-z=-102$$

$$2x-2y=-24$$

$$x-y=-12$$

$$48-y=-12$$

$$y=60$$

$$48+60-z=18$$

$$108-18=z$$

$$90=z$$

sol: 48, 60, 90

6.  $x \rightarrow \text{Cifra decenas}$

$y \rightarrow \text{Cifra unidades}$

$$x+y=6$$

$$10x+y-36=10y+x$$

$$9x-9y=36$$

$$x-y=4$$

$$x+y=6$$

$$x-y=4$$

$$2x=10$$

$$x=5$$

$$5+y=6$$

$$y=1 \quad \text{sol: } 51$$

7.  $x \rightarrow \text{Veloc. pájaro}$

$y \rightarrow \text{Veloc. viento}$

$x+y \rightarrow \text{Veloc. pájaro a favor}$

$x-y \rightarrow \text{Veloc. pájaro contra}$

$$\frac{55}{x+y}=1$$

$$x+y=55$$

$$\frac{25}{x-y}=1$$

$$x-y=25$$

Continúa

7. Continuación

$$x-y=25$$

$$x+y=55$$

$$2x=80$$

$$x=40$$

$$40-y=25$$

$$y=15$$

sol:  $40 \text{ Km/h} \rightarrow \text{Veloc. del}$

pájaro tranq.

$15 \text{ Km/h} \rightarrow \text{Veloc. del}$

viento

8.  $x \rightarrow N^{\circ} \text{ de libros}$

$y \rightarrow \text{Precio de } c/u$

$$xy=(x+5)(y-2)$$

$$xy=xy-2x+5y-10$$

$$2x-5y=-10$$

$$xy=(x-5)(y+4)$$

$$xy=xy+4x-5y-20$$

$$4x-5y=20$$

$$2x-5y=-10$$

$$\frac{4x-5y=20}{2x-5y=-10} \quad (-1)$$

$$2x-5y=-10$$

$$-4x+5y=-20$$

$$-2x=-30$$

$$x=15$$

$$2(15)-5y=-10$$

$$30-5y=-10$$

$$-5y=-40$$

$$y=8$$

sol: 15  $\rightarrow$  Libros compró

\$ 8  $\rightarrow$  costó  $c/u$

9.  $x \rightarrow \text{Precio kilo café}$

$y \rightarrow \text{Precio kilo té}$

$$7x+6y=4,8 \quad (-9)$$

$$8x+9y=6,45 \quad (6)$$

$$-63x-54y=-43,2$$

$$48x+54y=38,7$$

$$-15x=-4,5$$

$$x=0,3$$

Continúa

9. Continuación

$$7(0,3)+6y=4,8$$

$$2,1+6y=4,8$$

$$6y=2,7$$

$$y=0,45$$

sol: 30 cts.  $\rightarrow$  Costo kilo café

45 cts.  $\rightarrow$  Costo kilo té

10.  $x \rightarrow \text{trajes de } \$40$

$y \rightarrow \text{trajes de } \$35$

$$x+y=50 \quad (-40)$$

$$40x+35y=1.910$$

$$-40x-40y=-2.000$$

$$40x+35y=1.910$$

$$-5y=-90$$

$$y=18$$

$$x+18=50$$

$$x=32$$

sol: 32  $\rightarrow$  trajes de \$40

18  $\rightarrow$  trajes de \$35

11.  $x \rightarrow \text{Numerador}$

$y \rightarrow \text{Denominador}$

$$\frac{x-1}{y}=\frac{1}{3}$$

$$3x-3=y$$

$$3x-y=3$$

$$\frac{x}{y-2}=\frac{1}{2}$$

$$2x=y-2$$

$$-2x+y=2$$

$$3x-y=3$$

$$-2x+y=2$$

$$x=5$$

$$3(5)-y=3$$

$$-y=-12$$

$$y=12 \quad \text{sol: } \frac{5}{12}$$

12.  $x \rightarrow 1^{\text{a}} \text{ bolsa}$

$y \rightarrow 2^{\text{a}} \text{ bolsa}$

$$x+y=200$$

$$x-15=y+15$$

$$x-y=30$$

Continúa

**12. Continuación**

$$x + y = 200$$

$$x - y = 30$$

$$2x = 230$$

$$x = 115$$

$$115 + y = 200$$

$$y = 85$$

sol: 115 soles  $\rightarrow$  1ª bolsa

85 soles  $\rightarrow$  2ª bolsa

**13.  $x \rightarrow$  Precio caballo**

$y \rightarrow$  Precio coche

$$x + 20 = 3y$$

$$x - 3y = -20$$

$$20 + y = \frac{3x}{5}$$

$$3x = 100 + 5y$$

$$3x - 5y = 100$$

$$x - 3y = -20 \quad (-3)$$

$$3x - 5y = 100$$

$$-3x + 9y = 60$$

$$3x - 5y = 100$$

$$4y = 160$$

$$y = 40$$

$$x - 3(40) = -20$$

$$x = 100$$

sol: 100 soles  $\rightarrow$  Costó el caballo

40 soles  $\rightarrow$  Costó el coche

**14.  $x \rightarrow$  Cifra decenas**

$y \rightarrow$  Cifra unidades

$$10x + y = 6(x + y)$$

$$10x + y = 6x + 6y$$

$$4x - 5y = 0$$

$$10x + y - 9 = 10y + x$$

$$9x - 9y = 9$$

$$x - y = 1$$

$$4x - 5y = 0$$

$$x - y = 1 \quad (-5)$$

$$4x - 5y = 0$$

$$-5x + 5y = -5$$

$$-x = -5$$

$$x = 5$$

$$5 - y = 1$$

$$y = 4 \quad \text{sol: } 54$$

**15.  $x \rightarrow$  N° de personas**

$y \rightarrow$  Precio  $c/u$

$$xy = (x + 10)(y - 5)$$

$$xy = xy - 5x + 10y - 50$$

$$5x - 10y = -50$$

$$x - 2y = -10$$

$$xy = (x - 6)(y + 5)$$

$$xy = xy + 5x - 6y - 30$$

$$5x - 6y = 30$$

$$5x - 6y = 30$$

$$x - 2y = -10 \quad (-3)$$

$$5x - 6y = 30$$

$$-3x + 6y = 30$$

$$2x = 60$$

$$x = 30$$

$$30 - 2y = -10$$

$$-2y = -40$$

$$y = 20$$

sol: 30  $\rightarrow$  Personas

20bs.  $\rightarrow$   $c/u$

**16.  $A + B = 1.080$** 

$$A - \frac{2A}{5} = B - \frac{B}{4}$$

$$20A - 8A = 20B - 5B$$

$$12A - 15B = 0$$

$$4A - 5B = 0$$

$$A + B = 1.080 \quad (5)$$

$$4A - 5B = 0$$

$$5A + 5B = 5.400$$

$$4A - 5B = 0$$

$$9A = 5.400$$

$$A = 600$$

$$600 + B = 1.080$$

$$B = 480$$

sol:  $A \rightarrow$  600 sucres

$B \rightarrow$  480 sucres

**17.  $x \rightarrow$  gané ayer**

$y \rightarrow$  gané hoy

$$y + 10 = x$$

$$y = \frac{5x}{6}$$

$$5x - 6y = 0$$

$$5x - 6y = 0$$

$$x - y = 10 \quad (-6)$$

Continúa

**17. Continuación**

$$5x - 6y = 0$$

$$-6x + 6y = -60$$

$$-x = -60$$

$$x = 60$$

$$60 - y = 10$$

$$y = 50$$

sol: \$ 60  $\rightarrow$  gané ayer

\$ 50  $\rightarrow$  gané hoy

**18.  $x \rightarrow$  1ª número**

$y \rightarrow$  2ª número

$$\frac{x}{y} = \frac{3}{5}$$

$$\frac{x - 10}{y - 10} = \frac{1}{2}$$

$$5x - 3y = 0$$

$$2x - y = 10 \quad (-3)$$

$$5x - 3y = 0$$

$$-6x + 3y = -30$$

$$-x = -30$$

$$x = 30$$

$$2(30) - y = 10$$

$$60 - y = 10$$

$$y = 50$$

sol: 30 , 50

**19.  $A + 4 = B - 4$** 

$$A - B = -8$$

$$B + 4 = \frac{9}{5}(A - 4)$$

$$5B + 20 = 9A - 36$$

$$9A - 5B = 56$$

$$A - B = -8 \quad (-5)$$

$$9A - 5B = 56$$

$$-5A + 5B = 40$$

$$9A - 5B = 56$$

$$4A = 96$$

$$A = 24$$

$$24 - B = -8$$

$$B = 32$$

sol:  $A \rightarrow$  24 lempiras

$B \rightarrow$  32 lempiras

$$20. A - 20 = 2(B - 20)$$

$$A - 2B = -20$$

$$A + 30 = \frac{9}{7}(B + 30)$$

$$7A + 210 = 9B + 270$$

$$7A - 9B = 60$$

$$A - 2B = -20 \quad (-7)$$

$$7A - 9B = 60$$

$$-7A + 14B = 140$$

$$7A - 9B = 60$$

$$5B = 200$$

$$B = 40$$

$$A - 2(40) = -20$$

$$A - 80 = -20$$

$$A = 60$$

$$\text{sol: } A \rightarrow 60 \text{ Años}$$

$$B \rightarrow 40 \text{ Años}$$

$$21. x \rightarrow \text{tiempo ida}$$

$$y \rightarrow \text{tiempo vuelta}$$

$$x + y = 3$$

$$\frac{4}{x} = \frac{8}{y}$$

$$8x - 4y = 0$$

$$2x - y = 0$$

$$x + y = 3$$

$$2x - y = 0$$

$$3x = 3$$

$$x = 1$$

$$1 + y = 3$$

$$y = 2$$

$$\text{Si } 1h \rightarrow \text{tiempo de ida}$$

$$2h \rightarrow \text{tiempo de vuelta}$$

$$\text{Entonces}$$

$$\frac{16 \text{ Km}}{1h} = 16 \text{ Km/h} \rightarrow \text{Veloc. a favor}$$

$$\frac{16 \text{ Km}}{2h} = 8 \text{ Km/h} \rightarrow \text{Veloc. en contra}$$

$$\text{Luego}$$

$$x \rightarrow \text{Veloc. del bote}$$

$$y \rightarrow \text{Veloc. del rio}$$

$$x + y = 16$$

$$x - y = 8$$

$$2x = 24$$

$$x = 12$$

$$\text{Continúa}$$

$$21. \text{ Continuación}$$

$$12 + y = 16$$

$$y = 4$$

$$\text{sol: } 12 \text{ Km/h} \rightarrow \text{Veloc. bote}$$

$$4 \text{ Km/h} \rightarrow \text{Veloc. rio}$$

$$22. \frac{A}{9} - 2 = \frac{B}{5}$$

$$5A - 90 = 9B$$

$$5A - 9B = 90$$

$$2B = A - 15$$

$$A - 2B = 15$$

$$5A - 9B = 90$$

$$\frac{A - 2B = 15 \quad (-5)}{5A - 9B = 90}$$

$$-5A + 10B = -75$$

$$B = 15$$

$$A - 2(15) = 15$$

$$A - 30 = 15$$

$$A = 45$$

$$\text{sol: } A \rightarrow 45 \text{ Años}$$

$$B \rightarrow 15 \text{ Años}$$

$$23. 5A - 4 = 4B$$

$$5A - 4B = 4$$

$$7A - 2 = 6B$$

$$7A - 6B = 2$$

$$5A - 4B = 4 \quad (6)$$

$$\frac{7A - 6B = 2 \quad (-4)}{30A - 24B = 24}$$

$$-28A + 24B = -8$$

$$2A = 16$$

$$A = 8$$

$$5(8) - 4B = 4$$

$$40 - 4B = 4$$

$$-4B = -36$$

$$B = 9$$

$$\text{sol: } A \rightarrow 8 \text{ Km } B \rightarrow 9 \text{ Km}$$

$$24. x \rightarrow \text{Cifra decenas}$$

$$y \rightarrow \text{Cifra unidades}$$

$$y - x = 4$$

$$10x + y + 10y + x = 66$$

$$11x + 11y = 66$$

$$x + y = 6$$

$$\text{Continúa}$$

$$24. \text{ Continuación}$$

$$-x + y = 4$$

$$\frac{x + y = 6}{2y = 10}$$

$$2y = 10$$

$$y = 5$$

$$x + 5 = 6$$

$$x = 1$$

$$\text{sol: } 15$$

$$25. x \rightarrow \text{Largo}$$

$$y \rightarrow \text{Ancho}$$

$$2(x + y) = 58$$

$$2x + 2y = 58$$

$$x + y = 29$$

$$(x + 2)(y - 2) = xy - 46$$

$$xy - 2x + 2y - 4 = xy - 46$$

$$-2x + 2y = -42$$

$$x - y = 21$$

$$x + y = 29$$

$$\frac{x - y = 21}{2x = 50}$$

$$2x = 50$$

$$x = 25$$

$$25 + y = 29$$

$$y = 4$$

$$\text{sol: } 25m \cdot 4m$$

$$26. x \rightarrow \text{Largo}$$

$$y \rightarrow \text{Ancho}$$

$$2x + 2y = 56$$

$$x + y = 28$$

$$(y - 2)(x + 2) = xy$$

$$xy + 2y - 2x - 4 = xy$$

$$2x - 2y = -4$$

$$x - y = -4$$

$$x + y = 28$$

$$\frac{x - y = -4}{2x = 24}$$

$$2x = 24$$

$$x = 12$$

$$12 + y = 28$$

$$y = 16$$

$$\text{sol: } 16m \cdot 12m$$

## EJERCICIO 204

1.  $m=6 \quad n=3$

$${}^3A_6 = 6 \cdot 5 \cdot \dots (6-3+1)$$

$${}^3A_6 = 6 \cdot 5 \cdot 4$$

$${}^3A_6 = 120$$

2.  $P_5 = 5!$

$$P_5 = 5 \cdot 4 \cdot 3 \cdot 2$$

$$P_5 = 120$$

3.  $m=7 \quad n=5$

$${}^5C_7 = \frac{{}^5A_7}{P_5} = \frac{7 \cdot 6 \cdot \dots (7-5+1)}{5!}$$

$${}^5C_7 = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$${}^5C_7 = \frac{2 \cdot 520}{120}$$

$${}^5C_7 = 21$$

4.  $m=6 \quad n=2$

$${}^2A_6 = 6 \cdot 5 \cdot \dots (6-2+1)$$

$${}^2A_6 = 6 \cdot 5$$

$${}^2A_6 = 30$$

5.  $m=5 \quad n=3$

$${}^3A_5 = 5 \cdot 4 \cdot \dots (5-3+1)$$

$${}^3A_5 = 5 \cdot 4 \cdot 3$$

$${}^3A_5 = 60$$

6.  $m=12 \quad n=5$

$${}^5C_{12} = \frac{{}^5A_{12}}{P_5} = \frac{12 \cdot 11 \cdot \dots (12-5+1)}{5!}$$

$${}^5C_{12} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$${}^5C_{12} = 3 \cdot 11 \cdot 2 \cdot 3 \cdot 4$$

$${}^5C_{12} = 792$$

7.  $P_7 = 7!$

$$P_7 = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$$

$$P_7 = 5.040$$

8.  $m=7 \quad n=4$

$${}^4C_7 = \frac{{}^4A_7}{P_4} = \frac{7 \cdot 6 \cdot \dots (7-4+1)}{4!}$$

$${}^4C_7 = \frac{7 \cdot 6 \cdot 5 \cdot 4}{4 \cdot 3 \cdot 2 \cdot 1}$$

$${}^4C_7 = 7 \cdot 5$$

$${}^4C_7 = 35$$

9.  $P_{5-1} = P_4$

$$P_4 = 4!$$

$$P_4 = 4 \cdot 3 \cdot 2$$

$$P_4 = 24$$

10.  $P_6 = 6!$

$$P_6 = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$$

$$P_6 = 720$$

11. Siendo el 1º

$$P_6 = 6!$$

$$P_6 = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$$

$$P_6 = 720$$

sin lugar fijo

$$P_7 = 7!$$

$$P_7 = 7 P_6$$

$$P_7 = 7 \cdot 720$$

$$P_7 = 5.040$$

12. En un banco

$$P_6 = 720$$

En una mesa

redonda

$$P_{6-1} = P_5$$

$$P_5 = 5!$$

$$P_5 = 120$$

13.  $m=9 \quad n=3$

$${}^3A_9 = 9 \cdot 8 \cdot \dots (9-3+1)$$

$${}^3A_9 = 9 \cdot 8 \cdot 7$$

$${}^3A_9 = 504$$

14.  $P_{4-1} = P_3$

$$P_3 = 3!$$

$$P_3 = 6$$

15.  $m=5 \quad n=3$

$${}^3C_5 = \frac{{}^3A_5}{P_3} = \frac{5 \cdot 4 \cdot \dots (5-3+1)}{3!}$$

$${}^3C_5 = \frac{5 \cdot 4 \cdot 3}{3 \cdot 2}$$

$${}^3C_5 = 5 \cdot 2$$

$${}^3C_5 = 10$$

16.  $P_{5-2} = P_3$

$$P_3 = 3!$$

$$P_3 = 3 \cdot 2$$

$$P_3 = 6$$

17.  $m=5 \quad n=3$

$${}^3A_5 = 5 \cdot 4 \cdot \dots (5-3+1)$$

$${}^3A_5 = 5 \cdot 4 \cdot 3$$

$${}^3A_5 = 60$$

18.  $P_{11-1} = P_{10}$

$$P_{10} = 10!$$

$$P_{10} = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$$

$$P_{10} = 3.628.800$$

19.  $m=8 \quad n=3$

$${}^3C_8 = \frac{{}^3A_8}{P_3} = \frac{8 \cdot 7 \cdot \dots (8-3+1)}{3!}$$

$${}^3C_8 = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2}$$

$${}^3C_8 = 4 \cdot 7 \cdot 2$$

$${}^3C_8 = 56$$

20.  $m=6 \quad n=3$

$${}^5A_6 = 6 \cdot 5 \cdot \dots (6-3+1)$$

$${}^5A_6 = 6 \cdot 5 \cdot 4$$

$${}^5A_6 = 120$$

21. Palabras distintas

$$P_8 = 8!$$

$$P_8 = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$$

$$P_8 = 40.320$$

Vocales fijas

$$P_5 = 5!$$

$$P_5 = 120$$

22.  $P_{5-1} = P_4$

$$P_4 = 4!$$

$$P_4 = 4 \cdot 3 \cdot 2$$

$$P_4 = 24$$



## EJERCICIO 205

1.  $(4a^2)^2 = 16a^4$
2.  $(-5a)^3 = -125a^3$
3.  $(3xy)^3 = 27x^3y^3$
4.  $(-6a^2b)^2 = 36a^4b^2$
5.  $(-2x^2y^3)^3 = -8x^6y^9$
6.  $(4a^2b^3c^4)^3 = 64a^6b^9c^{12}$
7.  $(-6x^4y^5)^2 = 36x^8y^{10}$
8.  $(-7ab^3c^4)^3 = -343a^3b^9c^{12}$
9.  $(a^mb^n)^x = a^{mx}b^{nx}$
10.  $(-2x^3y^5z^6)^4 = 16x^{12}y^{20}z^{24}$
11.  $(-3m^3n)^3 = -27m^9n^3$
12.  $(a^2b^3c)^m = a^{2m}b^{3m}c^m$
13.  $(-m^2nx^3)^4 = m^8n^4x^{12}$
14.  $(-3a^2b)^5 = -243a^{10}b^5$
15.  $(7x^5y^6z^8)^2 = 49x^{10}y^{12}z^{16}$
16.  $\left(-\frac{x}{2y}\right)^2 = \frac{x^2}{4y^2}$
17.  $\left(-\frac{2m}{n^2}\right)^3 = -\frac{8m^3}{n^6}$
18.  $\left(\frac{ab^2}{5}\right)^3 = \frac{a^3b^6}{125}$
19.  $\left(-\frac{3x^2}{4y}\right)^2 = \frac{9x^4}{16y^2}$
20.  $\left(-\frac{2ab^2}{3m^3}\right)^4 = \frac{16a^4b^8}{81m^{12}}$
21.  $\left(\frac{2m^3n}{3x^4}\right)^5 = \frac{32m^{15}n^5}{243x^{20}}$
22.  $\left(-\frac{3a^3b^2}{4}\right)^2 = \frac{9a^6b^4}{16}$
23.  $\left(-\frac{mn^2}{3}\right)^4 = \frac{m^4n^8}{81}$
24.  $\left(-\frac{a^2b^4}{2}\right)^5 = -\frac{a^{10}b^{20}}{32}$

## EJERCICIO 206

1.  $(a^5 + 7b^4)^2$   
 $= a^{10} + 2a^5(7b^4) + 49b^8$   
 $= a^{10} + 14a^5b^4 + 49b^8$
2.  $(3x^4 - 5xy^3)^2$   
 $= 9x^8 - 2(3x^4)(5xy^3) + 25x^2y^6$   
 $= 9x^8 - 30x^5y^3 + 25x^2y^6$
3.  $(a^2b^3 - a^5)^2$   
 $= a^4b^6 - 2a^2b^3(a^5) + a^{10}$   
 $= a^4b^6 - 2a^7b^3 + a^{10}$
4.  $(7x^5 - 8x^3y^4)^2$   
 $= 49x^{10} - 2(7x^5)(8x^3y^4) + 64x^6y^8$   
 $= 49x^{10} - 112x^8y^4 + 64x^6y^8$
5.  $(9ab^2 + 5a^2b^3)^2$   
 $= 81a^2b^4 + 2(9ab^2)(5a^2b^3) + 25a^4b^6$   
 $= 81a^2b^4 + 90a^3b^5 + 25a^4b^6$
6.  $(3x^2y^3 - 7x^3y^2)^2$   
 $= 9x^4y^6 - 2(3x^2y^3)(7x^3y^2) + 49x^6y^4$   
 $= 9x^4y^6 - 42x^5y^5 + 49x^6y^4$
7.  $(xy - a^2b^2)^2$   
 $= x^2y^2 - 2(xy)(a^2b^2) + a^4b^4$   
 $= x^2y^2 - 2a^2b^2xy + a^4b^4$
8.  $\left(\frac{x}{2} + \frac{2y}{3}\right)^2$   
 $= \frac{x^2}{4} + 2\left(\frac{x}{2}\right)\left(\frac{2y}{3}\right) + \frac{4y^2}{9}$   
 $= \frac{x^2}{4} + \frac{2xy}{3} + \frac{4y^2}{9}$
9.  $\left(\frac{3a^2}{4} - \frac{2b^2}{5}\right)^2$   
 $= \frac{9a^4}{16} - 2\left(\frac{3a^2}{4}\right)\left(\frac{2b^2}{5}\right) + \frac{4b^4}{25}$   
 $= \frac{9a^4}{16} - \frac{3a^2b^2}{5} + \frac{4b^4}{25}$
10.  $\left(\frac{5x^3}{6} + \frac{3xy^2}{5}\right)^2$   
 $= \frac{25x^6}{36} + 2\left(\frac{5x^3}{6}\right)\left(\frac{3xy^2}{5}\right) + \frac{9x^2y^4}{25}$   
 $= \frac{25x^6}{36} + x^4y^2 + \frac{9x^2y^4}{25}$
11.  $\left(\frac{a^5}{9} - \frac{3a^3b^7}{7}\right)^2$   
 $= \frac{a^{10}}{81} - 2\left(\frac{a^5}{9}\right)\left(\frac{3a^3b^7}{7}\right) + \frac{9a^6b^{14}}{49}$   
 $= \frac{a^{10}}{81} - \frac{2a^8b^7}{21} + \frac{9a^6b^{14}}{49}$
12.  $\left(\frac{2m^4}{5} - \frac{5n^3}{4}\right)^2$   
 $= \frac{4m^8}{25} - 2\left(\frac{2m^4}{5}\right)\left(\frac{5n^3}{4}\right) + \frac{25n^6}{16}$   
 $= \frac{4m^8}{25} - m^4n^3 + \frac{25n^6}{16}$
13.  $\left(\frac{x}{3} + \frac{y}{4}\right)^2$   
 $= \frac{x^2}{9} + 2\left(\frac{x}{3}\right)\left(\frac{y}{4}\right) + \frac{y^2}{16}$   
 $= \frac{x^2}{9} + \frac{xy}{6} + \frac{y^2}{16}$
14.  $\left(\frac{2x}{3} - \frac{3y}{5}\right)^2$   
 $= \frac{4x^2}{9} - 2\left(\frac{2x}{3}\right)\left(\frac{3y}{5}\right) + \frac{9y^2}{25}$   
 $= \frac{4x^2}{9} - \frac{4xy}{5} + \frac{9y^2}{25}$

$$15. \left( \frac{a^3}{8} + \frac{4a^2}{7b} \right)^2 = \frac{a^6}{64} + 2 \left( \frac{a^3}{8} \right) \left( \frac{4a^2}{7b} \right) + \frac{16a^4}{49b^2} = \frac{a^6}{64} + \frac{a^5}{7b} + \frac{16a^4}{49b^2}$$

$$16. \left( \frac{3}{2x} - \frac{2x^4}{3} \right)^2 = \frac{9}{4x^2} - 2 \left( \frac{3}{2x} \right) \left( \frac{2x^4}{3} \right) + \frac{4x^8}{9} = \frac{9}{4x^2} - 2x^3 + \frac{4x^8}{9}$$

$$17. \left( \frac{5x^7}{6y^4} - \frac{3y^6}{10x^2} \right)^2 = \frac{25x^{14}}{36y^8} - 2 \left( \frac{5x^7}{6y^4} \right) \left( \frac{3y^6}{10x^2} \right) + \frac{9y^{12}}{100x^4} = \frac{25x^{14}}{36y^8} - \frac{x^5y^2}{2} + \frac{9y^{12}}{100x^4}$$

$$18. \left( \frac{3a^6}{8} - \frac{4a^2}{9b^5} \right)^2 = \frac{9a^{12}}{64} - 2 \left( \frac{3a^6}{8} \right) \left( \frac{4a^2}{9b^5} \right) + \frac{16a^4}{81b^{10}} = \frac{9a^{12}}{64} - \frac{a^8}{3b^5} + \frac{16a^4}{81b^{10}}$$

## EJERCICIO 207

$$1. (2a + 3b)^3 = 8a^3 + 3(4a^2)(3b) + 3(2a)(9b^2) + 27b^3 = 8a^3 + 36a^2b + 54ab^2 + 27b^3$$

$$2. (4a - 3b^2)^3 = 64a^3 - 3(16a^2)(3b^2) + 3(4a)(9b^4) - 27b^6 = 64a^3 - 144a^2b^2 + 108ab^4 - 27b^6$$

$$3. (5x^2 + 6y^3)^3 = 125x^6 + 3(25x^4)(6y^3) + 3(5x^2)(36y^6) + 216y^9 = 125x^6 + 450x^4y^3 + 540x^2y^6 + 216y^9$$

$$4. (4x^3 - 3xy^2)^3 = 64x^9 - 3(16x^6)(3xy^2) + 3(4x^3)(9x^2y^4) - 27x^3y^6 = 64x^9 - 144x^7y^2 + 108x^5y^4 - 27x^3y^6$$

$$5. (7a^4 - 5a^2b^3)^3 = 343a^{12} - 3(49a^8)(5a^2b^3) + 3(7a^4)(25a^4b^6) - 125a^6b^9 = 343a^{12} - 735a^{10}b^3 + 525a^8b^6 - 125a^6b^9$$

$$6. (a^8 + 9a^5x^4)^3 = a^{24} + 3(a^{16})(9a^5x^4) + 3(a^8)(81a^{10}x^8) + 729a^{15}x^{12} = a^{24} + 27a^{21}x^4 + 243a^{18}x^8 + 729a^{15}x^{12}$$

$$7. (8x^4 - 7x^2y^4)^3 = 512x^{12} - 3(64x^8)(7x^2y^4) + 3(8x^4)(49x^4y^8) - 343x^6y^{12} = 512x^{12} - 1.344x^{10}y^4 + 1.176x^8y^8 - 343x^6y^{12}$$

$$8. (3a^2b - 5a^3b^2)^3 = 27a^6b^3 - 3(9a^4b^2)(5a^3b^2) + 3(3a^2b)(25a^6b^4) - 125a^9b^6 = 27a^6b^3 - 135a^7b^4 + 225a^8b^5 - 125a^9b^6$$

$$9. \left( \frac{1}{2}a + \frac{2b^2}{3} \right)^3 = \frac{a^3}{8} + 3 \left( \frac{a^2}{4} \right) \left( \frac{2b^2}{3} \right) + 3 \left( \frac{a}{2} \right) \left( \frac{4b^4}{9} \right) + \frac{8b^6}{27} = \frac{a^3}{8} + \frac{a^2b^2}{2} + \frac{2ab^4}{3} + \frac{8b^6}{27}$$

$$10. \left( \frac{3a^2}{4} - \frac{4b^2}{5} \right)^3 = \frac{27a^6}{64} - 3 \left( \frac{9a^4}{16} \right) \left( \frac{4b^2}{5} \right) + 3 \left( \frac{3a^2}{4} \right) \left( \frac{16b^4}{25} \right) - \frac{64b^6}{125} = \frac{27a^6}{64} - \frac{27a^4b^2}{20} + \frac{36a^2b^4}{25} - \frac{64b^6}{125}$$

$$11. \left( \frac{5a^2b}{6} - \frac{3b^4}{10} \right)^3 = \frac{125a^6b^3}{216} - 3 \left( \frac{25a^4b^2}{36} \right) \left( \frac{3b^4}{10} \right) + 3 \left( \frac{5a^2b}{6} \right) \left( \frac{9b^8}{100} \right) - \frac{27b^{12}}{1.000} = \frac{125a^6b^3}{216} - \frac{5a^4b^6}{8} + \frac{9a^2b^9}{40} - \frac{27b^{12}}{1.000}$$

$$12. \left( \frac{7x^5}{8} - \frac{4y^6}{7} \right)^3 = \frac{343x^{15}}{512} - 3 \left( \frac{49x^{10}}{64} \right) \left( \frac{4y^6}{7} \right) + 3 \left( \frac{7x^5}{8} \right) \left( \frac{16y^{12}}{49} \right) - \frac{64y^{18}}{343} = \frac{343x^{15}}{512} - \frac{21x^{10}y^6}{16} + \frac{6x^5y^{12}}{7} - \frac{64y^{18}}{343}$$

---


$$\begin{aligned}
13. \left(\frac{x}{2y} + \frac{3y}{x^2}\right)^3 &= \frac{x^3}{8y^3} + 3\left(\frac{x^2}{4y^2}\right)\left(\frac{3y}{x^2}\right) + 3\left(\frac{x}{2y}\right)\left(\frac{9y^2}{x^4}\right) + \frac{27y^3}{x^6} = \frac{x^3}{8y^3} + \frac{9}{4y} + \frac{27y}{2x^3} + \frac{27y^3}{x^6} \\
14. \left(\frac{2a^2}{5} - \frac{5}{2b^3}\right)^3 &= \frac{8a^6}{125} - 3\left(\frac{4a^4}{25}\right)\left(\frac{5}{2b^3}\right) + 3\left(\frac{2a^2}{5}\right)\left(\frac{25}{4b^6}\right) - \frac{125}{8b^9} = \frac{8a^6}{125} - \frac{6a^4}{5b^3} + \frac{15a^2}{2b^6} - \frac{125}{8b^9} \\
15. \left(4x^4 - \frac{3x}{y^3}\right)^3 &= 64x^{12} - 3(16x^8)\left(\frac{3x}{y^3}\right) + 3(4x^4)\left(\frac{9x^2}{y^6}\right) - \frac{27x^3}{y^9} = 64x^{12} - \frac{144x^9}{y^3} + \frac{108x^6}{y^6} - \frac{27x^3}{y^9} \\
16. \left(\frac{3a}{2b} + \frac{4b^2}{5}\right)^3 &= \frac{27a^3}{8b^3} + 3\left(\frac{9a^2}{4b^2}\right)\left(\frac{4b^2}{5}\right) + 3\left(\frac{3a}{2b}\right)\left(\frac{16b^4}{25}\right) + \frac{64b^6}{125} = \frac{27a^3}{8b^3} + \frac{27a^2}{5} + \frac{72ab^3}{25} + \frac{64b^6}{125} \\
17. \left(\frac{7}{8} - x^4y^5\right)^3 &= \frac{343}{512} - 3\left(\frac{49}{64}\right)x^4y^5 + 3\left(\frac{7}{8}\right)x^8y^{10} - x^{12}y^{15} = \frac{343}{512} - \frac{147x^4y^5}{64} + \frac{21x^8y^{10}}{8} - x^{12}y^{15} \\
18. \left(\frac{m^3}{6} - \frac{6n^2}{m^2}\right)^3 &= \frac{m^9}{216} - 3\left(\frac{m^6}{36}\right)\left(\frac{6n^2}{m^2}\right) + 3\left(\frac{m^3}{6}\right)\left(\frac{36n^4}{m^4}\right) - \frac{216n^6}{m^6} = \frac{m^9}{216} - \frac{m^4n^2}{2} + \frac{18n^4}{m} - \frac{216n^6}{m^6}
\end{aligned}$$

## EJERCICIO 208

$$\begin{aligned}
1. x^2 - 2x + 1 &= x^4 + 4x^2 + 1 + 2x^2(-2x) + 2x^2(1) + 2(-2x)(1) = x^4 + 4x^2 + 1 - 4x^3 + 2x^2 - 4x = x^4 - 4x^3 + 6x^2 - 4x + 1 \\
2. 2x^2 + x + 1 &= 4x^4 + x^2 + 1 + 2(2x^2)(x) + 2(2x^2)(1) + 2(x)(1) = 4x^4 + x^2 + 1 + 4x^3 + 4x^2 + 2x = 4x^4 + 4x^3 + 5x^2 + 2x + 1 \\
3. x^2 - 5x + 2 &= x^4 + 25x^2 + 4 + 2(x^2)(-5x) + 2(x^2)(2) + 2(-5x)(2) = x^4 + 25x^2 + 4 - 10x^3 + 4x^2 - 20x \\
&= x^4 - 10x^3 + 29x^2 - 20x + 4 \\
4. x^3 - 5x^2 + 6 &= x^6 + 25x^4 + 36 + 2(x^3)(-5x^2) + 2(x^3)(6) + 2(-5x^2)(6) = x^6 + 25x^4 + 36 - 10x^5 + 12x^3 - 60x^2 \\
&= x^6 - 10x^5 + 25x^4 + 12x^3 - 60x^2 + 36 \\
5. 4a^4 - 3a^2 + 5 &= 16a^8 + 9a^4 + 25 + 2(4a^4)(-3a^2) + 2(4a^4)(5) + 2(-3a^2)(5) \\
&= 16a^8 + 9a^4 + 25 - 24a^6 + 40a^4 - 30a^2 = 16a^8 - 24a^6 + 49a^4 - 30a^2 + 25 \\
6. x + 2y - z &= x^2 + 4y^2 + z^2 + 2(x)(2y) + 2(x)(-z) + 2(2y)(-z) = x^2 + 4y^2 + z^2 + 4xy - 2xz - 4yz \\
7. 3 - x^3 - x^6 &= 9 + x^6 + x^{12} + 2(3)(-x^3) + 2(3)(-x^6) + 2(-x^3)(-x^6) = 9 + x^6 + x^{12} - 6x^3 - 6x^6 + 2x^9 = x^{12} + 2x^9 - 5x^6 - 6x^3 + 9 \\
8. 5x^4 - 7x^2 + 3x &= 25x^8 + 49x^4 + 9x^2 + 2(5x^4)(-7x^2) + 2(5x^4)(3x) + 2(-7x^2)(3x) \\
&= 25x^8 + 49x^4 + 9x^2 - 70x^6 + 30x^5 - 42x^3 = 25x^8 - 70x^6 + 30x^5 + 49x^4 - 42x^3 + 9x^2 \\
9. 2a^2 + 2ab - 3b^2 &= 4a^4 + 4a^2b^2 + 9b^4 + 2(2a^2)(2ab) + 2(2a^2)(-3b^2) + 2(2ab)(-3b^2) \\
&= 4a^4 + 4a^2b^2 + 9b^4 + 8a^3b - 12a^2b^2 - 12ab^3 = 4a^4 + 8a^3b - 8a^2b^2 - 12ab^3 + 9b^4 \\
10. m^3 - 2m^2n + 2n^4 &= m^6 + 4m^4n^2 + 4n^8 + 2(m^3)(-2m^2n) + 2(m^3)(2n^4) + 2(-2m^2n)(2n^4) \\
&= m^6 + 4m^4n^2 + 4n^8 - 4m^5n + 4m^3n^4 - 8m^2n^5 = m^6 - 4m^5n + 4m^4n^2 + 4m^3n^4 - 8m^2n^5 + 4n^8 \\
11. \frac{a}{2} - b + \frac{c}{4} &= \frac{a^2}{4} + b^2 + \frac{c^2}{16} + 2\left(\frac{a}{2}\right)(-b) + 2\left(\frac{a}{2}\right)\left(\frac{c}{4}\right) + 2(-b)\left(\frac{c}{4}\right) = \frac{a^2}{4} + b^2 + \frac{c^2}{16} - ab + \frac{ac}{4} - \frac{bc}{2}
\end{aligned}$$

$$12. \frac{x}{5} - 5y + \frac{5}{3} = \frac{x^2}{25} + 25y^2 + \frac{25}{9} + 2\left(\frac{x}{5}\right)(-5y) + 2\left(\frac{x}{5}\right)\left(\frac{5}{3}\right) + 2(-5y)\left(\frac{5}{3}\right) = \frac{x^2}{25} + 25y^2 + \frac{25}{9} - 2xy + \frac{2x}{3} - \frac{50y}{3}$$

$$= \frac{x^2}{25} - 2xy + \frac{2x}{3} + 25y^2 - \frac{50y}{3} + \frac{25}{9}$$

$$13. \frac{x^2}{2} - x + \frac{2}{3} = \frac{x^4}{4} + x^2 + \frac{4}{9} + 2\left(\frac{x^2}{2}\right)(-x) + 2\left(\frac{x^2}{2}\right)\left(\frac{2}{3}\right) + 2\left(\frac{2}{3}\right)(-x) = \frac{x^4}{4} + x^2 + \frac{4}{9} - x^3 + \frac{2x^2}{3} - \frac{4x}{3}$$

$$= \frac{x^4}{4} - x^3 + \frac{5x^2}{3} - \frac{4x}{3} + \frac{4}{9}$$

$$14. \frac{a}{x} - \frac{1}{3} + \frac{x}{a} = \frac{a^2}{x^2} + \frac{1}{9} + \frac{x^2}{a^2} + 2\left(\frac{a}{x}\right)\left(-\frac{1}{3}\right) + 2\left(\frac{a}{x}\right)\left(\frac{x}{a}\right) + 2\left(-\frac{1}{3}\right)\left(\frac{x}{a}\right) = \frac{a^2}{x^2} + \frac{1}{9} + \frac{x^2}{a^2} - \frac{2a}{3x} + 2 - \frac{2x}{3a}$$

$$= \frac{a^2}{x^2} - \frac{2a}{3x} - \frac{2x}{3a} + \frac{x^2}{a^2} + \frac{19}{9}$$

$$15. \frac{3a^2}{4} - \frac{a}{2} + \frac{4}{5} = \frac{9a^4}{16} + \frac{a^2}{4} + \frac{16}{25} + 2\left(\frac{3a^2}{4}\right)\left(-\frac{a}{2}\right) + 2\left(\frac{3a^2}{4}\right)\left(\frac{4}{5}\right) + 2\left(-\frac{a}{2}\right)\left(\frac{4}{5}\right)$$

$$= \frac{9a^4}{16} + \frac{a^2}{4} + \frac{16}{25} - \frac{3a^3}{4} + \frac{6a^2}{5} - \frac{4a}{5} = \frac{9a^4}{16} - \frac{3a^3}{4} + \frac{29a^2}{20} - \frac{4a}{5} + \frac{16}{25}$$

$$16. \frac{a^2}{4} - \frac{3}{5} + \frac{b^2}{9} = \frac{a^4}{16} + \frac{9}{25} + \frac{b^4}{81} + 2\left(\frac{a^2}{4}\right)\left(-\frac{3}{5}\right) + 2\left(\frac{a^2}{4}\right)\left(\frac{b^2}{9}\right) + 2\left(-\frac{3}{5}\right)\left(\frac{b^2}{9}\right)$$

$$= \frac{a^4}{16} + \frac{9}{25} + \frac{b^4}{81} - \frac{3a^2}{10} + \frac{a^2b^2}{18} - \frac{2b^2}{15} = \frac{a^4}{16} - \frac{3a^2}{10} + \frac{a^2b^2}{18} + \frac{9}{25} - \frac{2b^2}{15} + \frac{b^4}{81}$$

$$17. x^3 - x^2 + x + 1 = x^6 + x^4 + x^2 + 1 + 2(x^3)(-x^2) + 2(x^3)(x) + 2x^3 + 2(-x^2)(x) + 2(-x^2) + 2x$$

$$= x^6 + x^4 + x^2 + 1 - 2x^5 + 2x^4 + 2x^3 - 2x^3 - 2x^2 + 2x = x^6 - 2x^5 + 3x^4 - x^2 + 2x + 1$$

$$18. x^3 - 3x^2 - 2x + 2$$

$$= x^6 + 9x^4 + 4x^2 + 4 + 2(x^3)(-3x^2) + 2(x^3)(-2x) + 2(x^3)(2) + 2(-3x^2)(-2x) + 2(-3x^2)(2) + 2(-2x)(2)$$

$$= x^6 + 9x^4 + 4x^2 + 4 - 6x^5 - 4x^4 + 4x^3 + 12x^3 - 12x^2 - 8x = x^6 - 6x^5 + 5x^4 + 16x^3 - 8x^2 - 8x + 4$$

$$19. x^4 + 3x^2 - 4x + 5$$

$$= x^8 + 9x^4 + 16x^2 + 25 + 2(x^4)(3x^2) + 2(x^4)(-4x) + 2(x^4)(5) + 2(3x^2)(-4x) + 2(3x^2)(5) + 2(-4x)(5)$$

$$= x^8 + 9x^4 + 16x^2 + 25 + 6x^6 - 8x^5 + 10x^4 - 24x^3 + 30x^2 - 40x = x^8 + 6x^6 - 8x^5 + 19x^4 - 24x^3 + 46x^2 - 40x + 25$$

$$20. x^4 - 4x^3 + 2x - 3$$

$$= x^8 + 16x^6 + 4x^2 + 9 + 2(x^4)(-4x^3) + 2(x^4)(2x) + 2(x^4)(-3) + 2(-4x^3)(2x) + 2(-4x^3)(-3) + 2(2x)(-3)$$

$$= x^8 + 16x^6 + 4x^2 + 9 - 8x^7 + 4x^5 - 6x^4 - 16x^4 + 24x^3 - 12x = x^8 - 8x^7 + 16x^6 + 4x^5 - 22x^4 + 24x^3 + 4x^2 - 12x + 9$$

$$21. 3 - 6a + a^2 - a^3$$

$$= 9 + 36a^2 + a^4 + a^6 + 2(3)(-6a) + 2(3)(a^2) + 2(3)(-a^3) + 2(-6a)(a^2) + 2(-6a)(-a^3) + 2(a^2)(-a^3)$$

$$= 9 + 36a^2 + a^4 + a^6 - 36a + 6a^2 - 6a^3 - 12a^3 + 12a^4 - 2a^5 = a^6 - 2a^5 + 13a^4 - 18a^3 + 42a^2 - 36a + 9$$

$$22. \frac{x^3}{2} - x^2 + \frac{2x}{3} + 2$$

$$= \frac{x^6}{4} + x^4 + \frac{4x^2}{9} + 4 + 2\left(\frac{x^3}{2}\right)(-x^2) + 2\left(\frac{x^3}{2}\right)\left(\frac{2x}{3}\right) + 2\left(\frac{x^3}{2}\right)(2) + 2(-x^2)\left(\frac{2x}{3}\right) + 2(-x^2)(2) + 2\left(\frac{2x}{3}\right)(2)$$

$$= \frac{x^6}{4} + x^4 + \frac{4x^2}{9} + 4 - x^5 + \frac{2x^4}{3} + 2x^3 - \frac{4x^3}{3} - 4x^2 + \frac{8x}{3} = \frac{x^6}{4} - x^5 + \frac{5x^4}{3} + \frac{2x^3}{3} - \frac{32x^2}{9} + \frac{8x}{3} + 4$$

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$$\begin{aligned}
 23. & \frac{a^3}{2} - \frac{2a^2}{3} + \frac{3a}{4} - \frac{1}{2} \\
 &= \frac{a^6}{4} + \frac{4a^4}{9} + \frac{9a^2}{16} + \frac{1}{4} + 2\left(\frac{a^3}{2}\right)\left(-\frac{2a^2}{3}\right) + 2\left(\frac{a^3}{2}\right)\left(\frac{3a}{4}\right) + 2\left(\frac{a^3}{2}\right)\left(-\frac{1}{2}\right) + 2\left(-\frac{2a^2}{3}\right)\left(\frac{3a}{4}\right) + 2\left(-\frac{2a^2}{3}\right)\left(-\frac{1}{2}\right) + 2\left(\frac{3a}{4}\right)\left(-\frac{1}{2}\right) \\
 &= \frac{a^6}{4} + \frac{4a^4}{9} + \frac{9a^2}{16} + \frac{1}{4} - \frac{2a^5}{3} + \frac{3a^4}{4} - \frac{a^3}{2} - a^3 + \frac{2a^2}{3} - \frac{3a}{4} = \frac{a^6}{4} - \frac{2a^5}{3} + \frac{43a^4}{36} - \frac{3a^3}{2} + \frac{59a^2}{48} - \frac{3a}{4} + \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 24. & x^5 - x^4 + x^3 - x^2 + x - 2 \\
 &= x^{10} + x^8 + x^6 + x^4 + x^2 + 4 + 2(x^5)(-x^4) + 2(x^5)(x^3) + 2(x^5)(-x^2) + 2(x^5)(x) + 2(x^5)(-2) + 2(-x^4)(x^3) + 2(-x^4)(-x^2) \\
 &+ 2(-x^4)(x) + 2(-x^4)(-2) + 2(x^3)(-x^2) + 2(x^3)(x) + 2(x^3)(-2) + 2(-x^2)(x) + 2(-x^2)(-2) + 2(x)(-2) \\
 &= x^{10} + x^8 + x^6 + x^4 + x^2 + 4 - 2x^9 + 2x^8 - 2x^7 + 2x^6 - 4x^5 - 2x^7 + 2x^6 - 2x^5 + 4x^4 - 2x^5 + 2x^4 - 4x^3 - 2x^3 + 4x^2 - 4x \\
 &= x^{10} - 2x^9 + 3x^8 - 4x^7 + 5x^6 - 8x^5 + 7x^4 - 6x^3 + 5x^2 - 4x + 4
 \end{aligned}$$

## EJERCICIO 209

1.  $x^2 + x + 1 = x^6 + x^3 + 1 + 3(x^2)^2(x) + 3(x^2)^2(1) + 3(x)^2(x^2) + 3(x)^2(1) + 3(x^2) + 3(x) + 6(x^2)(x)(1)$   
 $= x^6 + x^3 + 1 + 3x^5 + 3x^4 + 3x^4 + 3x^2 + 3x^2 + 3x + 6x^3 = x^6 + 3x^5 + 6x^4 + 7x^3 + 6x^2 + 3x + 1$
2.  $2x^2 - x - 1 = 8x^6 - x^3 - 1 + 3(2x^2)^2(-x) + 3(2x^2)^2(-1) + 3(-x)^2(-1) + 3(-x)^2(2x^2) + 3(-1)^2(2x^2) + 3(-1)^2(-x)$   
 $+ 6(2x^2)(-x)(-1)$   
 $= 8x^6 - x^3 - 1 - 12x^5 - 12x^4 - 3x^2 + 6x^4 + 6x^2 - 3x + 12x^3 = 8x^6 - 12x^5 - 6x^4 + 11x^3 + 3x^2 - 3x - 1$
3.  $1 - 3x + 2x^2$   
 $= 1 - 27x^3 + 8x^6 + 3(1)^2(-3x) + 3(1^2)(2x^2) + 3(-3x)^2(1) + 3(-3x)^2(2x^2) + 3(2x^2)^2(1) + 3(2x^2)^2(-3x)$   
 $+ 6(1)(-3x)(2x^2)$   
 $= 1 - 27x^3 + 8x^6 - 9x + 6x^2 + 27x^2 + 54x^4 + 12x^4 - 36x^5 - 36x^3 = 1 - 9x + 33x^2 - 63x^3 + 66x^4 - 36x^5 + 8x^6$
4.  $2 - 3x + x^2$   
 $= 8 - 27x^3 + x^6 + 3(2)^2(-3x) + 3(2)^2(x^2) + 3(-3x)^2(2) + 3(-3x)^2(x^2) + 3(x^2)^2(2) + 3(x^2)^2(-3x) + 6(2)(-3x)(x^2)$   
 $= 8 - 27x^3 + x^6 - 36x + 12x^2 + 54x^2 + 27x^4 + 6x^4 - 9x^5 - 36x^3 = x^6 - 9x^5 + 33x^4 - 63x^3 + 66x^2 - 36x + 8$
5.  $x^3 - 2x^2 - 4$   
 $= x^9 - 8x^6 - 64 + 3(x^3)^2(-2x^2) + 3(x^3)^2(-4) + 3(-2x^2)^2(x^3) + 3(-2x^2)^2(-4) + 3(-4)^2(x^3) + 3(-4)^2(-2x^2)$   
 $+ 6(x^3)(-2x^2)(-4)$   
 $= x^9 - 8x^6 - 64 - 6x^8 - 12x^6 + 12x^7 - 48x^4 + 48x^3 - 96x^2 + 48x^5$   
 $= x^9 - 6x^8 + 12x^7 - 20x^6 + 48x^5 - 48x^4 + 48x^3 - 96x^2 - 64$
6.  $x^4 - x^2 - 2$   
 $= x^{12} - x^6 - 8 + 3(x^4)^2(-x^2) + 3(x^4)^2(-2) + 3(-x^2)^2(x^4) + 3(-x^2)^2(-2) + 3(-2)^2(x^4) + 3(-2)^2(-x^2)$   
 $+ 6(x^4)(-x^2)(-2)$   
 $= x^{12} - x^6 - 8 - 3x^{10} - 6x^8 + 3x^8 - 6x^4 + 12x^4 - 12x^2 + 12x^6 = x^{12} - 3x^{10} - 3x^8 + 11x^6 + 6x^4 - 12x^2 - 8$

$$\begin{aligned}
7. \quad a^3 + \frac{a^2}{2} - \frac{a}{3} &= a^9 + \frac{a^6}{8} - \frac{a^3}{27} + 3(a^3)^2 \left( \frac{a^2}{2} \right) + 3(a^3)^2 \left( -\frac{a}{3} \right) + 3 \left( \frac{a^2}{2} \right)^2 (a^3) + 3 \left( \frac{a^2}{2} \right)^2 \left( -\frac{a}{3} \right) + 3 \left( -\frac{a}{3} \right)^2 (a^3) \\
&\quad + 3 \left( -\frac{a}{3} \right)^2 \left( \frac{a^2}{2} \right) + 6a^3 \left( \frac{a^2}{2} \right) \left( -\frac{a}{3} \right) \\
&= a^9 + \frac{a^6}{8} - \frac{a^3}{27} + \frac{3a^8}{2} - a^7 + \frac{3a^7}{4} - \frac{a^5}{4} + \frac{a^5}{3} + \frac{a^4}{6} - a^6 = a^9 + \frac{3a^8}{2} - \frac{a^7}{4} - \frac{7a^6}{8} + \frac{a^5}{12} + \frac{a^4}{6} - \frac{a^3}{27}
\end{aligned}$$

$$\begin{aligned}
8. \quad \frac{x^2}{2} - \frac{x}{3} + 2 &= \frac{x^6}{8} - \frac{x^3}{27} + 8 + 3 \left( \frac{x^2}{2} \right)^2 \left( -\frac{x}{3} \right) + 3(x^2)^2 (2) + 3 \left( -\frac{x}{3} \right)^2 \left( \frac{x^2}{2} \right) + 3 \left( -\frac{x}{3} \right)^2 (2) + 3(2)^2 \left( \frac{x^2}{2} \right) \\
&\quad + 3(2)^2 \left( -\frac{x}{3} \right) + 6 \left( \frac{x^2}{2} \right) \left( -\frac{x}{3} \right) (2) \\
&= \frac{x^6}{8} - \frac{x^3}{27} + 8 - \frac{x^5}{4} + \frac{3x^4}{2} + \frac{x^4}{6} + \frac{2x^2}{3} + 6x^2 - 4x - 2x^3 = \frac{x^6}{8} - \frac{x^5}{4} + \frac{5x^4}{3} - \frac{55x^3}{27} + \frac{20x^2}{3} - 4x + 8
\end{aligned}$$

$$\begin{aligned}
9. \quad a^3 - a^2 + a - 1 &= a^9 - a^6 + a^3 - 1 + 3(a^3)^2 (-a^2) + 3(a^3)^2 (a) + 3(a^3)^2 (-1) + 3(-a^2)^2 (a^3) + 3(-a^2)^2 (a) \\
&\quad + 3(-a^2)^2 (-1) + 3a^2 (a^3) + 3a^2 (-a^2) + 3a^2 (-1) + 3(-1)^2 (a^3) + 3(-1)^2 (-a^2) + 3(-1)^2 (a) \\
&\quad + 6(a^3)(-a^2)(a) + 6(a^3)(-a^2)(-1) + 6(a^3)(a)(-1) + 6(-a^2)(a)(-1) \\
&= a^9 - a^6 + a^3 - 1 - 3a^8 + 3a^7 - 3a^6 + 3a^7 + 3a^5 - 3a^4 + 3a^5 - 3a^4 - 3a^2 + 3a^3 - 3a^2 + 3a - 6a^6 + 6a^5 - 6a^4 + 6a^3 \\
&= a^9 - 3a^8 + 6a^7 - 10a^6 + 12a^5 - 12a^4 + 10a^3 - 6a^2 + 3a - 1
\end{aligned}$$

$$\begin{aligned}
10. \quad x^3 - 2x^2 + x - 3 &= x^9 - 8x^6 + x^3 - 27 + 3(x^3)^2 (-2x^2) + 3(x^3)^2 (x) + 3(x^3)^2 (-3) + 3(-2x^2)^2 (x^3) + 3(-2x^2)^2 (x) \\
&\quad + 3(-2x^2)^2 (-3) + 3x^2 (x^3) + 3x^2 (-2x^2) + 3x^2 (-3) + 6(x^3)(-2x^2)(x) + 6(x^3)(-2x^2)(-3) \\
&\quad + 6(x^3)(x)(-3) + 6(-2x^2)(x)(-3) + 3(-3)^2 (x^3) + 3(-3)^2 (-2x^2) + 3(-3)^2 x \\
&= x^9 - 8x^6 + x^3 - 27 - 6x^8 + 3x^7 - 9x^6 + 12x^7 + 12x^5 - 36x^4 + 3x^5 - 6x^4 - 9x^2 - 12x^6 + 36x^5 \\
&\quad - 18x^4 + 36x^3 + 27x^3 - 54x^2 + 27x \\
&= x^9 - 6x^8 + 15x^7 - 29x^6 + 51x^5 - 60x^4 + 64x^3 - 63x^2 + 27x - 27
\end{aligned}$$

$$\begin{aligned}
11. \quad x^3 - 4x^2 + 2x - 3 &= x^9 - 64x^6 + 8x^3 - 27 + 3(x^3)^2 (-4x^2) + 3(x^3)^2 (2x) + 3(x^3)^2 (-3) + 3(-4x^2)^2 (x^3) + 3(-4x^2)^2 (2x) \\
&\quad + 3(-4x^2)^2 (-3) + 3(2x)^2 (x^3) + 3(2x)^2 (-4x^2) + 3(2x)^2 (-3) + 3(-3)^2 (x^3) + 3(-3)^2 (-4x^2) \\
&\quad + 3(-3)^2 (2x) + 6(x^3)(-4x^2)(2x) + 6(x^3)(-4x^2)(-3) + 6(x^3)(2x)(-3) + 6(-4x^2)(2x)(-3) \\
&= x^9 - 64x^6 + 8x^3 - 27 - 12x^8 + 6x^7 - 9x^6 + 48x^7 + 96x^5 - 144x^4 + 12x^5 - 48x^4 - 36x^2 + 27x^3 - 108x^2 \\
&\quad + 54x - 48x^6 + 72x^5 - 36x^4 + 144x^3 \\
&= x^9 - 12x^8 + 54x^7 - 121x^6 + 180x^5 - 228x^4 + 179x^3 - 144x^2 + 54x - 27
\end{aligned}$$

$$\begin{aligned}
12. \quad 1 - x^2 + 2x^4 - x^6 &= 1 - x^6 + 8x^{12} - x^{18} + 3(-x^2) + 3(2x^4) + 3(-x^6) + 3(-x^2)^2 + 3(-x^2)^2 (2x^4) + 3(-x^2)^2 (-x^6) \\
&\quad + 3(2x^4)^2 + 3(2x^4)^2 (-x^2) + 3(2x^4)^2 (-x^6) + 3(-x^6)^2 + 3(-x^6)^2 (-x^2) + 3(-x^6)^2 (2x^4) \\
&\quad + 6(-x^2)(2x^4) + 6(2x^4)(-x^6) + 6(-x^2)(-x^6) + 6(-x^2)(2x^4)(-x^6) \\
&= 1 - x^6 + 8x^{12} - x^{18} - 3x^2 + 6x^4 - 3x^6 + 3x^4 + 6x^8 - 3x^{10} + 12x^8 - 12x^{10} - 12x^{14} + 3x^{12} \\
&\quad - 3x^{14} + 6x^{16} - 12x^6 - 12x^{10} + 6x^8 + 12x^{12} \\
&= 1 - 3x^2 + 9x^4 - 16x^6 + 24x^8 - 27x^{10} + 23x^{12} - 15x^{14} + 6x^{16} - x^{18}
\end{aligned}$$

## EJERCICIO 210

1.  $(x-2)^4 = x^4 - 4x^3(2) + \frac{4 \cdot 3}{2}x^2(2)^2 - \frac{6 \cdot 2}{3}x(2)^3 + (2)^4 = x^4 - 8x^3 + 24x^2 - 32x + 16$
2.  $(a+3)^4 = a^4 + 4a^3(3) + \frac{4 \cdot 3}{2}a^2(3)^2 + \frac{6 \cdot 2}{3}a(3)^3 + (3)^4 = a^4 + 12a^3 + 54a^2 + 108a + 81$
3.  $(2-x)^5 = (2)^5 - 5(2)^4x + \frac{5 \cdot 4}{2}(2)^3x^2 - \frac{10 \cdot 3}{3}(2)^2x^3 + \frac{10 \cdot 2}{4}(2)x^4 - x^5 = 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$
4.  $(2x+5y)^4$   
 $= (2x)^4 + 4(2x)^3(5y) + \frac{4 \cdot 3}{2}(2x)^2(5y)^2 + \frac{6 \cdot 2}{3}(2x)(5y)^3 + (5y)^4 = 16x^4 + 160x^3y + 600x^2y^2 + 1.000xy^3 + 625y^4$
5.  $(a-3)^6 = a^6 - 6a^5(3) + \frac{6 \cdot 5}{2}a^4(3)^2 - \frac{15 \cdot 4}{3}a^3(3)^3 + \frac{20 \cdot 3}{4}a^2(3)^4 - \frac{15 \cdot 2}{5}a(3)^5 + (3)^6$   
 $= a^6 - 18a^5 + 135a^4 - 540a^3 + 1.215a^2 - 1.458a + 729$
6.  $(2a-b)^6 = (2a)^6 - 6(2a)^5b + \frac{6 \cdot 5}{2}(2a)^4b^2 - \frac{15 \cdot 4}{3}(2a)^3b^3 + \frac{20 \cdot 3}{4}(2a)^2b^4 - \frac{15 \cdot 2}{5}(2a)b^5 + b^6$   
 $= 64a^6 - 192a^5b + 240a^4b^2 - 160a^3b^3 + 60a^2b^4 - 12ab^5 + b^6$
7.  $(x^2+2y^3)^5 = (x^2)^5 + 5(x^2)^4(2y^3) + \frac{5 \cdot 4}{2}(x^2)^3(2y^3)^2 + \frac{10 \cdot 3}{3}(x^2)^2(2y^3)^3 + \frac{10 \cdot 2}{4}(x^2)(2y^3)^4 + (2y^3)^5$   
 $= x^{10} + 10x^8y^3 + 40x^6y^6 + 80x^4y^9 + 80x^2y^{12} + 32y^{15}$
8.  $(x^3+1)^6 = (x^3)^6 + 6(x^3)^5(1) + \frac{6 \cdot 5}{2}(x^3)^4(1)^2 + \frac{15 \cdot 4}{3}(x^3)^3(1)^3 + \frac{20 \cdot 3}{4}(x^3)^2(1)^4 + \frac{15 \cdot 2}{5}(x^3)(1)^5 + (1)^6$   
 $= x^{18} + 6x^{15} + 15x^{12} + 20x^9 + 15x^6 + 6x^3 + 1$
9.  $(2a-3b)^5 = (2a)^5 - 5(2a)^4(3b) + \frac{5 \cdot 4}{2}(2a)^3(3b)^2 - \frac{10 \cdot 3}{3}(2a)^2(3b)^3 + \frac{10 \cdot 2}{4}(2a)(3b)^4 - (3b)^5$   
 $= 32a^5 - 240a^4b + 720a^3b^2 - 1.080a^2b^3 + 810ab^4 - 243b^5$
10.  $(x^4-5y^3)^6$   
 $= (x^4)^6 - 6(x^4)^5(5y^3) + \frac{6 \cdot 5}{2}(x^4)^4(5y^3)^2 - \frac{15 \cdot 4}{3}(x^4)^3(5y^3)^3 + \frac{20 \cdot 3}{4}(x^4)^2(5y^3)^4 - \frac{15 \cdot 2}{5}(x^4)(5y^3)^5 + (5y^3)^6$   
 $= x^{24} - 30x^{20}y^3 + 375x^{16}y^6 - 2.500x^{12}y^9 + 9.375x^8y^{12} - 18.750x^4y^{15} + 15.625y^{18}$
11.  $\left(2x - \frac{y}{2}\right)^6$   
 $= (2x)^6 - 6(2x)^5\left(\frac{y}{2}\right) + \frac{6 \cdot 5}{2}(2x)^4\left(\frac{y}{2}\right)^2 - \frac{15 \cdot 4}{3}(2x)^3\left(\frac{y}{2}\right)^3 + \frac{20 \cdot 3}{4}(2x)^2\left(\frac{y}{2}\right)^4 - \frac{15 \cdot 2}{5}(2x)\left(\frac{y}{2}\right)^5 + \left(\frac{y}{2}\right)^6$   
 $= 64x^6 - 96x^5y + 60x^4y^2 - 20x^3y^3 + \frac{15}{4}x^2y^4 - \frac{3}{8}xy^5 + \frac{y^6}{64}$
12.  $\left(3 - \frac{x^2}{3}\right)^5 = (3)^5 - 5(3)^4\left(\frac{x^2}{3}\right) + \frac{5 \cdot 4}{2}(3)^3\left(\frac{x^2}{3}\right)^2 - \frac{10 \cdot 3}{3}(3)^2\left(\frac{x^2}{3}\right)^3 + \frac{10 \cdot 2}{4}(3)\left(\frac{x^2}{3}\right)^4 - \left(\frac{x^2}{3}\right)^5$   
 $= 243 - 135x^2 + 30x^4 - \frac{10}{3}x^6 + \frac{5}{27}x^8 - \frac{x^{10}}{243}$
13.  $(2m^3-3n^4)^6 = (2m^3)^6 - 6(2m^3)^5(3n^4) + \frac{6 \cdot 5}{2}(2m^3)^4(3n^4)^2 - \frac{15 \cdot 4}{3}(2m^3)^3(3n^4)^3 + \frac{20 \cdot 3}{4}(2m^3)^2(3n^4)^4$   
 $- \frac{15 \cdot 2}{5}(2m^3)(3n^4)^5 + (3n^4)^6$   
 $= 64m^{18} - 576m^{15}n^4 + 2.160m^{12}n^8 - 4.320m^9n^{12} + 4.860m^6n^{16} - 2.916m^3n^{20} + 729n^{24}$

- 
14.  $(x^2 - 3)^7$   

$$= (x^2)^7 - 7(x^2)^6(3) + \frac{7 \cdot 6}{2}(x^2)^5(3)^2 - \frac{21 \cdot 5}{3}(x^2)^4(3)^3 + \frac{35 \cdot 4}{4}(x^2)^3(3)^4 - \frac{35 \cdot 3}{5}(x^2)^2(3)^5 + \frac{21 \cdot 2}{6}(x^2)(3)^6 - (3)^7$$

$$= x^{14} - 21x^{12} + 189x^{10} - 945x^8 + 2.835x^6 - 5.103x^4 + 5.103x^2 - 2.187$$
15.  $\left(3a - \frac{b^2}{3}\right)^5 = (3a)^5 - 5(3a)^4\left(\frac{b^2}{3}\right) + \frac{5 \cdot 4}{2}(3a)^3\left(\frac{b^2}{3}\right)^2 - \frac{10 \cdot 3}{3}(3a)^2\left(\frac{b^2}{3}\right)^3 + \frac{10 \cdot 2}{4}(3a)\left(\frac{b^2}{3}\right)^4 - \left(\frac{b^2}{3}\right)^5$   

$$= 243a^5 - 135a^4b^2 + 30a^3b^4 - \frac{10}{3}a^2b^6 + \frac{5}{27}ab^8 - \frac{b^{10}}{243}$$
16.  $(x^2 + 2y^2)^7 = (x^2)^7 + 7(x^2)^6(2y^2) + \frac{7 \cdot 6}{2}(x^2)^5(2y^2)^2 + \frac{21 \cdot 5}{3}(x^2)^4(2y^2)^3 + \frac{35 \cdot 4}{4}(x^2)^3(2y^2)^4$   

$$+ \frac{35 \cdot 3}{5}(x^2)^2(2y^2)^5 + \frac{21 \cdot 2}{6}(x^2)(2y^2)^6 + (2y^2)^7$$

$$= x^{14} + 14x^{12}y^2 + 84x^{10}y^4 + 280x^8y^6 + 560x^6y^8 + 672x^4y^{10} + 448x^2y^{12} + 128y^{14}$$
17.  $(x^3 - 1)^8 = (x^3)^8 - 8(x^3)^7(1) + \frac{8 \cdot 7}{2}(x^3)^6(1)^2 - \frac{28 \cdot 6}{3}(x^3)^5(1)^3 + \frac{56 \cdot 5}{4}(x^3)^4(1)^4 - \frac{70 \cdot 4}{5}(x^3)^3(1)^5$   

$$+ \frac{56 \cdot 3}{6}(x^3)^2(1)^6 - \frac{28 \cdot 2}{7}(x^3)(1)^7 + (1)^8$$

$$= x^{24} - 8x^{21} + 28x^{18} - 56x^{15} + 70x^{12} - 56x^9 + 28x^6 - 8x^3 + 1$$
18.  $\left(x^2 - \frac{y}{2}\right)^9 = (x^2)^9 - 9(x^2)^8\left(\frac{y}{2}\right) + \frac{9 \cdot 8}{2}(x^2)^7\left(\frac{y}{2}\right)^2 - \frac{36 \cdot 7}{3}(x^2)^6\left(\frac{y}{2}\right)^3 + \frac{84 \cdot 6}{4}(x^2)^5\left(\frac{y}{2}\right)^4 - \frac{126 \cdot 5}{5}(x^2)^4\left(\frac{y}{2}\right)^5$   

$$+ \frac{126 \cdot 4}{6}(x^2)^3\left(\frac{y}{2}\right)^6 - \frac{84 \cdot 3}{7}(x^2)^2\left(\frac{y}{2}\right)^7 + \frac{36 \cdot 2}{8}(x^2)\left(\frac{y}{2}\right)^8 - \left(\frac{y}{2}\right)^9$$

$$= x^{18} - \frac{9}{2}x^{16}y + 9x^{14}y^2 - \frac{21}{2}x^{12}y^3 + \frac{63}{8}x^{10}y^4 - \frac{63}{16}x^8y^5 + \frac{21}{16}x^6y^6 - \frac{9}{32}x^4y^7 + \frac{9}{256}x^2y^8 - \frac{y^9}{512}$$
19.  $(2m^3 - n^4)^7 = (2m^3)^7 - 7(2m^3)^6(n^4) + \frac{7 \cdot 6}{2}(2m^3)^5(n^4)^2 - \frac{21 \cdot 5}{3}(2m^3)^4(n^4)^3 + \frac{35 \cdot 4}{4}(2m^3)^3(n^4)^4$   

$$- \frac{35 \cdot 3}{5}(2m^3)^2(n^4)^5 + \frac{21 \cdot 2}{6}(2m^3)(n^4)^6 - (n^4)^7$$

$$= 128m^{21} - 448m^{18}n^4 + 672m^{15}n^8 - 560m^{12}n^{12} + 280m^9n^{16} - 84m^6n^{20} + 14m^3n^{24} - n^{28}$$
20.  $\left(\frac{x^2}{2} + \frac{2y^2}{3}\right)^5$   

$$= \left(\frac{x^2}{2}\right)^5 + 5\left(\frac{x^2}{2}\right)^4\left(\frac{2y^2}{3}\right) + \frac{5 \cdot 4}{2}\left(\frac{x^2}{2}\right)^3\left(\frac{2y^2}{3}\right)^2 + \frac{10 \cdot 3}{3}\left(\frac{x^2}{2}\right)^2\left(\frac{2y^2}{3}\right)^3 + \frac{10 \cdot 2}{4}\left(\frac{x^2}{2}\right)\left(\frac{2y^2}{3}\right)^4 + \left(\frac{2y^2}{3}\right)^5$$

$$= \frac{x^{10}}{32} + \frac{5x^8y^2}{24} + \frac{5x^6y^4}{9} + \frac{20x^4y^6}{27} + \frac{40x^2y^8}{81} + \frac{32y^{10}}{243}$$
21.  $\left(\frac{1}{5} - \frac{5a}{2}\right)^6 = \left(\frac{1}{5}\right)^6 - 6\left(\frac{1}{5}\right)^5\left(\frac{5a}{2}\right) + \frac{6 \cdot 5}{2}\left(\frac{1}{5}\right)^4\left(\frac{5a}{2}\right)^2 - \frac{15 \cdot 4}{3}\left(\frac{1}{5}\right)^3\left(\frac{5a}{2}\right)^3 + \frac{20 \cdot 3}{4}\left(\frac{1}{5}\right)^2\left(\frac{5a}{2}\right)^4$   

$$- \frac{15 \cdot 2}{5}\left(\frac{1}{5}\right)\left(\frac{5a}{2}\right)^5 + \left(\frac{5a}{2}\right)^6$$

$$= \frac{1}{15.625} - \frac{3a}{625} + \frac{3a^2}{20} - \frac{5a^3}{2} + \frac{375a^4}{16} - \frac{1.875a^5}{16} + \frac{15.625a^6}{64}$$



**NOTA:** Triángulo necesario para el desarrollo de los problemas del presente ejercicio

$$\begin{aligned}
9. \left(\frac{2}{3x} - \frac{3}{2y}\right)^7 &= \left(\frac{2}{3x}\right)^7 - 7\left(\frac{2}{3x}\right)^6\left(\frac{3}{2y}\right) + 21\left(\frac{2}{3x}\right)^5\left(\frac{3}{2y}\right)^2 - 35\left(\frac{2}{3x}\right)^4\left(\frac{3}{2y}\right)^3 + 35\left(\frac{2}{3x}\right)^3\left(\frac{3}{2y}\right)^4 \\
&\quad - 21\left(\frac{2}{3x}\right)^2\left(\frac{3}{2y}\right)^5 + 7\left(\frac{2}{3x}\right)\left(\frac{3}{2y}\right)^6 - \left(\frac{3}{2y}\right)^7 \\
&= \frac{128}{2.187x^7} - \frac{224}{243x^6y} + \frac{56}{9x^5y^2} - \frac{70}{3x^4y^3} + \frac{105}{2x^3y^4} - \frac{567}{8x^2y^5} + \frac{1.701}{32xy^6} - \frac{2.187}{128y^7}
\end{aligned}$$

$$\begin{aligned}
10. \left(\frac{2}{m} - \frac{m^2}{2}\right)^7 &= \left(\frac{2}{m}\right)^7 - 7\left(\frac{2}{m}\right)^6\left(\frac{m^2}{2}\right) + 21\left(\frac{2}{m}\right)^5\left(\frac{m^2}{2}\right)^2 - 35\left(\frac{2}{m}\right)^4\left(\frac{m^2}{2}\right)^3 + 35\left(\frac{2}{m}\right)^3\left(\frac{m^2}{2}\right)^4 \\
&\quad - 21\left(\frac{2}{m}\right)^2\left(\frac{m^2}{2}\right)^5 + 7\left(\frac{2}{m}\right)\left(\frac{m^2}{2}\right)^6 - \left(\frac{m^2}{2}\right)^7 \\
&= \frac{128}{m^7} - \frac{224}{m^4} + \frac{168}{m} - 70m^2 + \frac{35m^5}{2} - \frac{21m^8}{8} + \frac{7m^{11}}{32} - \frac{m^{14}}{128}
\end{aligned}$$

$$\begin{aligned}
11. (x^3 + mn)^8 &= (x^3)^8 + 8(x^3)^7 mn + 28(x^3)^6 (mn)^2 + 56(x^3)^5 (mn)^3 + 70(x^3)^4 (mn)^4 + 56(x^3)^3 (mn)^5 \\
&\quad + 28(x^3)^2 (mn)^6 + 8(x^3)(mn)^7 + (mn)^8 \\
&= x^{24} + 8x^{21}mn + 28x^{18}m^2n^2 + 56x^{15}m^3n^3 + 70x^{12}m^4n^4 + 56x^9m^5n^5 + 28x^6m^6n^6 + 8x^3m^7n^7 + m^8n^8
\end{aligned}$$

$$\begin{aligned}
12. \left(3 - \frac{b^2}{3}\right)^9 &= (3)^9 - 9(3)^8\left(\frac{b^2}{3}\right) + 36(3)^7\left(\frac{b^2}{3}\right)^2 - 84(3)^6\left(\frac{b^2}{3}\right)^3 + 126(3)^5\left(\frac{b^2}{3}\right)^4 - 126(3)^4\left(\frac{b^2}{3}\right)^5 \\
&\quad + 84(3)^3\left(\frac{b^2}{3}\right)^6 - 36(3)^2\left(\frac{b^2}{3}\right)^7 + 9(3)\left(\frac{b^2}{3}\right)^8 - \left(\frac{b^2}{3}\right)^9 \\
&= 19.683 - 19.683b^2 + 8.748b^4 - 2.268b^6 + 378b^8 - 42b^{10} + \frac{28b^{12}}{9} - \frac{4b^{14}}{27} + \frac{b^{16}}{243} - \frac{b^{18}}{19.683}
\end{aligned}$$

$$\begin{aligned}
13. \left(1 - \frac{1}{x}\right)^{10} &= (1)^{10} - 10(1)^9\left(\frac{1}{x}\right) + 45(1)^8\left(\frac{1}{x}\right)^2 - 120(1)^7\left(\frac{1}{x}\right)^3 + 210(1)^6\left(\frac{1}{x}\right)^4 - 252(1)^5\left(\frac{1}{x}\right)^5 + 210(1)^4\left(\frac{1}{x}\right)^6 \\
&\quad - 120(1)^3\left(\frac{1}{x}\right)^7 + 45(1)^2\left(\frac{1}{x}\right)^8 - 10(1)\left(\frac{1}{x}\right)^9 + \left(\frac{1}{x}\right)^{10} \\
&= 1 - \frac{10}{x} + \frac{45}{x^2} - \frac{120}{x^3} + \frac{210}{x^4} - \frac{252}{x^5} + \frac{210}{x^6} - \frac{120}{x^7} + \frac{45}{x^8} - \frac{10}{x^9} + \frac{1}{x^{10}}
\end{aligned}$$

$$\begin{aligned}
14. (2m^2 - 5n^5)^6 &= (2m^2)^6 - 6(2m^2)^5(5n^5) + 15(2m^2)^4(5n^5)^2 - 20(2m^2)^3(5n^5)^3 + 15(2m^2)^2(5n^5)^4 \\
&\quad - 6(2m^2)(5n^5)^5 + (5n^5)^6 \\
&= 64m^{12} - 960m^{10}n^5 + 6.000m^8n^{10} - 20.000m^6n^{15} + 37.500m^4n^{20} - 37.500m^2n^{25} + 15.625n^{30}
\end{aligned}$$

$$\begin{aligned}
15. \left(4 - \frac{x^5}{4}\right)^7 &= (4)^7 - 7(4)^6\left(\frac{x^5}{4}\right) + 21(4)^5\left(\frac{x^5}{4}\right)^2 - 35(4)^4\left(\frac{x^5}{4}\right)^3 + 35(4)^3\left(\frac{x^5}{4}\right)^4 - 21(4)^2\left(\frac{x^5}{4}\right)^5 + 7(4)\left(\frac{x^5}{4}\right)^6 - \left(\frac{x^5}{4}\right)^7 \\
&= 16.384 - 7.168x^5 + 1.344x^{10} - 140x^{15} + \frac{35x^{20}}{4} - \frac{21x^{25}}{64} + \frac{7x^{30}}{1.024} - \frac{x^{35}}{16.384}
\end{aligned}$$

## EJERCICIO 212

1.  $(x-y)^5$  donde  $r=3$   

$$= \frac{5 \cdot 4}{1 \cdot 2} (x)^{5-2} (-y)^2 = 10x^3y^2$$
2.  $(a-4b)^7$  donde  $r=4$   

$$= \frac{7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3} (a)^{7-3} (-4b)^{4-1}$$

$$= 35a^4(-64b^3)$$

$$= -2.240a^4b^3$$
3.  $(1+x)^{11}$  donde  $r=5$   

$$= \frac{11 \cdot 10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3 \cdot 4} (1)^{11-4} (x)^{5-1}$$

$$= 330x^4$$
4.  $(3x-2y)^6$  donde  $r=4$   

$$= \frac{6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3} (3x)^{6-3} (-2y)^{4-1}$$

$$= 20(27x^3)(-8y^3)$$

$$= -4.320x^3y^3$$
5.  $(a^2-2b)^9$  donde  $r=5$   

$$= \frac{9 \cdot 8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3 \cdot 4} (a^2)^{9-4} (-2b)^{5-1}$$

$$= 126(a^{10})(16b^4)$$

$$= 2.016a^{10}b^4$$
6.  $\left(2a - \frac{b}{2}\right)^8$  donde  $r=6$   

$$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} (2a)^{8-5} \left(-\frac{b}{2}\right)^{6-1}$$

$$= 56(8a^3) \left(-\frac{b^5}{32}\right)$$

$$= -14a^3b^5$$
7.  $(x^2-2y)^{10}$  donde  $r=7$   

$$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} (x^2)^{10-6} (-2y)^{7-1}$$

$$= 210(x^8)(64y^6)$$

$$= 13.440x^8y^6$$
8.  $(x-y^2)^{11}$  donde  $r=8$   

$$= \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7} (x)^{11-7} (-y^2)^{8-1}$$

$$= 330x^4(-y^{14})$$

$$= -330x^4y^{14}$$
9.  $(a^2+b)^{15}$  donde  $r=10$   

$$= \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9} (a^2)^{15-9} (b)^{10-1}$$

$$= 5.005a^{12}b^9$$
10.  $(1-x^2)^{12}$  donde  $r=9$   

$$= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8} (1)^{12-8} (-x^2)^{9-1}$$

$$= 495x^{16}$$
11.  $(2a-b^2)^6$  donde  $r=6$   

$$= \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} (2a)^{6-5} (-b^2)^{6-1}$$

$$= 6(2a)(-b^{10})$$

$$= -12ab^{10}$$
12.  $(3x^2-y^2)^8$  donde  $r=5$   

$$= \frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4} (3x^2)^{8-4} (-y^2)^{5-1}$$

$$= 70(81x^8)(y^8)$$

$$= 5.670x^8y^8$$

## EJERCICIO 213

1.  $\sqrt{4a^2b^4} = \pm 2ab^2$
2.  $\sqrt{25x^6y^8} = \pm 5x^3y^4$
3.  $\sqrt[3]{27a^3b^9} = 3ab^3$
4.  $\sqrt[3]{-8a^3b^6x^{12}} = -2ab^2x^4$
5.  $\sqrt{64x^8y^{10}} = \pm 8x^4y^5$
6.  $\sqrt[4]{16a^8b^{16}} = \pm 2a^2b^4$
7.  $\sqrt[5]{x^{15}y^{20}z^{25}} = x^3y^4z^5$
8.  $\sqrt[3]{-64a^3x^6y^{18}} = -4ax^2y^6$
9.  $\sqrt{-243m^5n^{15}} = -3mn^3$
10.  $\sqrt{81x^6y^8z^{20}} = \pm 9x^3y^4z^{10}$
11.  $\sqrt[3]{1.000x^9y^{18}} = 10x^3y^6$
12.  $\sqrt[4]{81a^{12}b^{24}} = \pm 3a^3b^6$
13.  $\sqrt[6]{64a^{12}b^{18}c^{30}} = \pm 2a^2b^3c^5$
14.  $\sqrt{49a^{2n}b^{4n}} = \pm 7a^nb^{2n}$
15.  $\sqrt[5]{-x^{5n}y^{10n}} = -x^ny^{2n}$
16.  $\sqrt{\frac{9a^2}{25x^4}} = \pm \frac{3a}{5x^2}$
17.  $\sqrt[3]{\frac{27a^3}{64x^9}} = \pm \frac{3a}{4x^3}$
18.  $\sqrt[5]{\frac{a^5b^{10}}{32x^{15}}} = \pm \frac{ab^2}{2x^3}$
19.  $\sqrt[4]{\frac{a^8}{81b^4c^{12}}} = \pm \frac{a^2}{3bc^3}$
20.  $\sqrt[7]{\frac{128}{x^{14}}} = \frac{2}{x^2}$
21.  $\sqrt{\frac{x^{2m}}{121y^{4n}}} = \pm \frac{x^m}{11y^{2n}}$
22.  $\sqrt[3]{\frac{125x^9}{216m^{12}}} = \pm \frac{5x^3}{6m^4}$
23.  $\sqrt[9]{\frac{a^{18}}{b^9c^{27}}} = \frac{a^2}{bc^3}$
24.  $\sqrt[10]{\frac{x^{20}}{1.024y^{30}}} = \pm \frac{x^2}{2y^3}$

## EJERCICIO 214

$$\begin{array}{r|l}
 \sqrt{16x^2 - 24xy^2 + 9y^4} & 4x - 3y^2 \\
 \hline
 -16x^2 & (8x - 3y^2)(-3y^2) \\
 \hline
 -24xy^2 + 9y^4 & = -24xy^2 + 9y^4 \\
 \hline
 24xy^2 - 9y^4 & \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{r|l}
 \sqrt{25a^4 - 70a^3x + 49a^2x^2} & 5a^2 - 7ax \\
 \hline
 -25a^4 & (10a^2 - 7ax)(-7ax) \\
 \hline
 -70a^3x + 49a^2x^2 & = -70a^3x + 49a^2x^2 \\
 \hline
 70a^3x - 49a^2x^2 & \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{r|l}
 \sqrt{x^4 - 4x^3 + 6x^2 - 4x + 1} & x^2 - 2x + 1 \\
 \hline
 -x^4 & (2x^2 - 2x)(-2x) \\
 \hline
 -4x^3 + 6x^2 & = -4x^3 + 4x^2 \\
 \hline
 4x^3 - 4x^2 & \\
 \hline
 2x^2 - 4x + 1 & (2x^2 - 4x + 1)(1) \\
 \hline
 -2x^2 + 4x - 1 & = 2x^2 - 4x + 1 \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{r|l}
 \sqrt{4a^4 + 4a^3 + 5a^2 + 2a + 1} & 2a^2 + a + 1 \\
 \hline
 -4a^4 & (4a^2 + a)(a) \\
 \hline
 4a^3 + 5a^2 & = 4a^3 + a^2 \\
 \hline
 -4a^3 - a^2 & \\
 \hline
 4a^2 + 2a + 1 & (4a^2 + 2a + 1)(1) \\
 \hline
 -4a^2 - 2a - 1 & = 4a^2 + 2a + 1 \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{r|l}
 \sqrt{n^4 - 10n^3 + 29n^2 - 20n + 4} & n^2 - 5n + 2 \\
 \hline
 -n^4 & (2n^2 - 5n)(-5n) \\
 \hline
 -10n^3 + 29n^2 & = -10n^3 + 25n^2 \\
 \hline
 10n^3 - 25n^2 & \\
 \hline
 4n^2 - 20n + 4 & (2n^2 - 10n + 2)(2) \\
 \hline
 -4n^2 + 20n - 4 & = 4n^2 - 20n + 4 \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{l|l}
6. \sqrt{x^6 - 10x^5 + 25x^4 + 12x^3 - 60x^2 + 36} & x^3 - 5x^2 + 6 \\
\hline
- x^6 & (2x^3 - 5x^2)(-5x^2) \\
\hline
- 10x^5 + 25x^4 & = -10x^5 + 25x^4 \\
10x^5 - 25x^4 & \\
\hline
12x^3 - 60x^2 + 36 & (2x^3 - 10x^2 + 6)(6) \\
- 12x^3 + 60x^2 - 36 & = 12x^3 - 60x^2 + 36 \\
\hline
0 &
\end{array}$$

$$\begin{array}{l|l}
7. \sqrt{16a^8 - 24a^6 + 49a^4 - 30a^2 + 25} & 4a^4 - 3a^2 + 5 \\
\hline
- 16a^8 & (8a^4 - 3a^2)(-3a^2) \\
\hline
- 24a^6 + 49a^4 & = -24a^6 + 9a^4 \\
24a^6 - 9a^4 & \\
\hline
40a^4 - 30a^2 + 25 & (8a^4 - 6a^2 + 5)(5) \\
- 40a^4 + 30a^2 - 25 & = 40a^4 - 30a^2 + 25 \\
\hline
0 &
\end{array}$$

$$\begin{array}{l|l}
8. \sqrt{x^2 + 4xy - 2xz - 4yz + 4y^2 + z^2} & x + 2y - z \\
\hline
- x^2 & (2x + 2y)(2y) \\
\hline
4xy - 2xz & + 4y^2 \\
& = 4xy + 4y^2 \\
- 4xy & - 4y^2 \\
\hline
- 2xz - 4yz & + z^2 \\
2xz + 4yz & - z^2 \\
\hline
0 &
\end{array}$$

$$\begin{array}{l|l}
9. \sqrt{x^{12} + 2x^9 - 5x^6 - 6x^3 + 9} & x^6 + x^3 - 3 \\
\hline
- x^{12} & (2x^6 + x^3)(x^3) \\
\hline
2x^9 - 5x^6 & = 2x^9 + x^6 \\
- 2x^9 - x^6 & \\
\hline
- 6x^6 - 6x^3 + 9 & (2x^6 + 2x^3 - 3)(-3) \\
6x^6 + 6x^3 - 9 & = -6x^6 - 6x^3 + 9 \\
\hline
0 &
\end{array}$$

$$\begin{array}{l|l}
10. \sqrt{25x^8 - 70x^6 + 30x^5 + 49x^4 - 42x^3 + 9x^2} & 5x^4 - 7x^2 + 3x \\
\hline
- 25x^8 & (10x^4 - 7x^2)(-7x^2) \\
\hline
- 70x^6 + 30x^5 + 49x^4 & = -70x^6 + 49x^4 \\
70x^6 & - 49x^4 \\
\hline
30x^5 & - 42x^3 + 9x^2 \\
- 30x^5 & + 42x^3 - 9x^2 \\
\hline
0 &
\end{array}$$

$$\begin{array}{l|l}
 11. \sqrt{4a^4 + 8a^3b - 8a^2b^2 - 12ab^3 + 9b^4} & 2a^2 + 2ab - 3b^2 \\
 \hline
 -4a^4 & (4a^2 + 2ab)(2ab) \\
 \hline
 8a^3b - 8a^2b^2 & = 8a^3b + 4a^2b^2 \\
 -8a^3b - 4a^2b^2 & \\
 \hline
 -12a^2b^2 - 12ab^3 + 9b^4 & (4a^2 + 4ab - 3b^2)(-3b^2) \\
 12a^2b^2 + 12ab^3 - 9b^4 & = -12a^2b^2 - 12ab^3 + 9b^4 \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{l|l}
 12. \sqrt{x^6 - 2x^5 + 3x^4 - x^2 + 2x + 1} & x^3 - x^2 + x + 1 \\
 \hline
 -x^6 & (2x^3 - x^2)(-x^2) \\
 \hline
 -2x^5 + 3x^4 & = -2x^5 + x^4 \\
 2x^5 - x^4 & \\
 \hline
 2x^4 - x^2 + 2x & (2x^3 - 2x^2 + x)(x) \\
 2x^3 - 2x^4 - x^2 & = 2x^4 - 2x^3 + x^2 \\
 \hline
 2x^3 - 2x^2 + 2x + 1 & \\
 -2x^3 + 2x^2 - 2x - 1 & (2x^3 - 2x^2 + 2x + 1)(1) \\
 \hline
 0 & = 2x^3 - 2x^2 + 2x + 1
 \end{array}$$

$$\begin{array}{l|l}
 13. \sqrt{x^6 - 6x^5 + 5x^4 + 16x^3 - 8x^2 - 8x + 4} & x^3 - 3x^2 - 2x + 2 \\
 \hline
 -x^6 & (2x^3 - 3x^2)(-3x^2) \\
 \hline
 -6x^5 + 5x^4 & = -6x^5 + 9x^4 \\
 6x^5 - 9x^4 & \\
 \hline
 -4x^4 + 16x^3 - 8x^2 & (2x^3 - 6x^2 - 2x)(-2x) \\
 4x^4 - 12x^3 - 4x^2 & = -4x^4 + 12x^3 + 4x^2 \\
 \hline
 4x^3 - 12x^2 - 8x + 4 & \\
 -4x^3 + 12x^2 + 8x - 4 & (2x^3 - 6x^2 - 4x + 2)(2) \\
 \hline
 0 & = 4x^3 - 12x^2 - 8x + 4
 \end{array}$$

$$\begin{array}{l|l}
 14. \sqrt{x^8 + 6x^6 - 8x^5 + 19x^4 - 24x^3 + 46x^2 - 40x + 25} & x^4 + 3x^2 - 4x + 5 \\
 \hline
 -x^8 & (2x^4 + 3x^2)(3x^2) = 6x^6 + 9x^4 \\
 \hline
 6x^6 - 8x^5 + 19x^4 & (2x^4 + 6x^2 - 4x)(-4x) \\
 -6x^6 - 9x^4 & = -8x^5 - 24x^3 + 16x^2 \\
 \hline
 -8x^5 + 10x^4 - 24x^3 + 46x^2 & \\
 8x^5 + 24x^3 - 16x^2 & \\
 \hline
 10x^4 + 30x^2 - 40x + 25 & (2x^4 + 6x^2 - 8x + 5)(5) \\
 -10x^4 - 30x^2 + 40x - 25 & = 10x^4 + 30x^2 - 40x + 25 \\
 \hline
 0 & 
 \end{array}$$

<p>15. <math>\sqrt{x^8 - 8x^7 + 16x^6 + 4x^5 - 22x^4 + 24x^3 + 4x^2 - 12x + 9}</math></p> $  \begin{array}{r}  -x^8 \\  \hline  -8x^7 + 16x^6 \\  8x^7 - 16x^6 \\  \hline  4x^5 - 22x^4 + 24x^3 + 4x^2 \\  -4x^5 + 16x^4 \qquad -4x^2 \\  \hline  -6x^4 + 24x^3 \qquad -12x + 9 \\  6x^4 - 24x^3 \qquad +12x - 9 \\  \hline  0  \end{array}  $	$  \begin{array}{l}  x^4 - 4x^3 + 2x - 3 \\  \hline  (2x^4 - 4x^3)(-4x^3) = -8x^7 + 16x^6 \\  \\  (2x^4 - 8x^3 + 2x)(2x) \\  \qquad = 4x^5 - 16x^4 + 4x^2 \\  \\  (2x^4 - 8x^3 + 4x - 3)(-3) \\  \qquad = -6x^4 + 24x^3 - 12x + 9  \end{array}  $
<p>16. <math>\sqrt{9 - 36a + 42a^2 - 18a^3 + 13a^4 - 2a^5 + a^6}</math></p> $  \begin{array}{r}  -9 \\  \hline  -36a + 42a^2 \\  36a - 36a^2 \\  \hline  6a^2 - 18a^3 + 13a^4 \\  -6a^2 + 12a^3 - a^4 \\  \hline  -6a^3 + 12a^4 - 2a^5 + a^6 \\  6a^3 - 12a^4 + 2a^5 - a^6 \\  \hline  0  \end{array}  $	$  \begin{array}{l}  3 - 6a + a^2 - a^3 \\  \hline  (6 - 6a)(-6a) = -36a + 36a^2 \\  \\  (6 - 12a + a^2)(a^2) = 6a^2 - 12a^3 + a^4 \\  \\  (6 - 12a + 2a^2 - a^3)(-a^3) \\  \qquad = -6a^3 + 12a^4 - 2a^5 + a^6  \end{array}  $
<p>17. <math>\sqrt{9x^6 - 24x^5 + 28x^4 - 22x^3 + 12x^2 - 4x + 1}</math></p> $  \begin{array}{r}  -9x^6 \\  \hline  -24x^5 + 28x^4 \\  24x^5 - 16x^4 \\  \hline  12x^4 - 22x^3 + 12x^2 \\  -12x^4 + 16x^3 - 4x^2 \\  \hline  -6x^3 + 8x^2 - 4x + 1 \\  6x^3 - 8x^2 + 4x - 1 \\  \hline  0  \end{array}  $	$  \begin{array}{l}  3x^3 - 4x^2 + 2x - 1 \\  \hline  (6x^3 - 4x^2)(-4x^2) = -24x^5 + 16x^4 \\  \\  (6x^3 - 8x^2 + 2x)(2x) \\  \qquad = 12x^4 - 16x^3 + 4x^2 \\  \\  (6x^3 - 8x^2 + 4x - 1)(-1) \\  \qquad = -6x^3 + 8x^2 - 4x + 1  \end{array}  $
<p>18. <math>\sqrt{16x^6 - 40x^5 + 73x^4 - 84x^3 + 66x^2 - 36x + 9}</math></p> $  \begin{array}{r}  -16x^6 \\  \hline  -40x^5 + 73x^4 \\  40x^5 - 25x^4 \\  \hline  48x^4 - 84x^3 + 66x^2 \\  -48x^4 + 60x^3 - 36x^2 \\  \hline  -24x^3 + 30x^2 - 36x + 9 \\  24x^3 - 30x^2 + 36x - 9 \\  \hline  0  \end{array}  $	$  \begin{array}{l}  4x^3 - 5x^2 + 6x - 3 \\  \hline  (8x^3 - 5x^2)(-5x^2) = -40x^5 + 25x^4 \\  \\  (8x^3 - 10x^2 + 6x)(6x) \\  \qquad = 48x^4 - 60x^3 + 36x^2 \\  \\  (8x^3 - 10x^2 + 12x - 3)(-3) \\  \qquad = -24x^3 + 30x^2 - 36x + 9  \end{array}  $

$$\begin{array}{l|l}
 19. \sqrt{m^6 - 4m^5n + 4m^4n^2 + 4m^3n^4 - 8m^2n^5 + 4n^8} & m^3 - 2m^2n + 2n^4 \\
 \hline
 -m^6 & (2m^3 - 2m^2n)(-2m^2n) \\
 \hline
 -4m^5n + 4m^4n^2 & = -4m^5n + 4m^4n^2 \\
 \hline
 4m^5n - 4m^4n^2 & \\
 \hline
 4m^3n^4 - 8m^2n^5 + 4n^8 & (2m^3 - 4m^2n + 2n^4)(2n^4) \\
 -4m^3n^4 + 8m^2n^5 - 4n^8 & = 4m^3n^4 - 8m^2n^5 + 4n^8 \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{l|l}
 20. \sqrt{9x^6 - 6x^5y + 13x^4y^2 - 16x^3y^3 + 8x^2y^4 - 8xy^5 + 4y^6} & 3x^3 - x^2y + 2xy^2 - 2y^3 \\
 \hline
 -9x^6 & (6x^3 - x^2y)(-x^2y) = -6x^5y + x^4y^2 \\
 \hline
 -6x^5y + 13x^4y^2 & (6x^3 - 2x^2y + 2xy^2)(2xy^2) \\
 \hline
 6x^5y - x^4y^2 & = 12x^4y^2 - 4x^3y^3 + 4x^2y^4 \\
 \hline
 12x^4y^2 - 16x^3y^3 + 8x^2y^4 & \\
 -12x^4y^2 + 4x^3y^3 - 4x^2y^4 & \\
 \hline
 -12x^3y^3 + 4x^2y^4 - 8xy^5 + 4y^6 & (6x^3 - 2x^2y + 4xy^2 - 2y^3)(-2y^3) \\
 12x^3y^3 - 4x^2y^4 + 8xy^5 - 4y^6 & = -12x^3y^3 + 4x^2y^4 - 8xy^5 + 4y^6 \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{l|l}
 21. \sqrt{16a^6 - 24a^5b + 25a^4b^2 - 20a^3b^3 + 10a^2b^4 - 4ab^5 + b^6} & 4a^3 - 3a^2b + 2ab^2 - b^3 \\
 \hline
 -16a^6 & (8a^3 - 3a^2b)(-3a^2b) = -24a^5b + 9a^4b^2 \\
 \hline
 -24a^5b + 25a^4b^2 & (8a^3 - 6a^2b + 2ab^2)(2ab^2) \\
 \hline
 24a^5b - 9a^4b^2 & = 16a^4b^2 - 12a^3b^3 + 4a^2b^4 \\
 \hline
 16a^4b^2 - 20a^3b^3 + 10a^2b^4 & \\
 -16a^4b^2 + 12a^3b^3 - 4a^2b^4 & \\
 \hline
 -8a^3b^3 + 6a^2b^4 - 4ab^5 + b^6 & (8a^3 - 6a^2b + 4ab^2 - b^3)(-b^3) \\
 8a^3b^3 - 6a^2b^4 + 4ab^5 - b^6 & = -8a^3b^3 + 6a^2b^4 - 4ab^5 + b^6 \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{l|l}
 22. \sqrt{36x^8 - 36x^6y^2 + 48x^5y^3 - 15x^4y^4 - 24x^3y^5 + 28x^2y^6 - 16xy^7 + 4y^8} & 6x^4 - 3x^2y^2 + 4xy^3 - 2y^4 \\
 \hline
 -36x^8 & (12x^4 - 3x^2y^2)(-3x^2y^2) = -36x^6y^2 + 9x^4y^4 \\
 \hline
 -36x^6y^2 + 48x^5y^3 - 15x^4y^4 & (12x^4 - 6x^2y^2 + 4xy^3)(4xy^3) \\
 \hline
 36x^6y^2 - 9x^4y^4 & = 48x^5y^3 - 24x^3y^5 + 16x^2y^6 \\
 \hline
 48x^5y^3 - 24x^4y^4 - 24x^3y^5 + 28x^2y^6 & (12x^4 - 6x^2y^2 + 8xy^3 - 2y^4)(-2y^4) \\
 -48x^5y^3 + 24x^3y^5 - 16x^2y^6 & = -24x^4y^4 + 12x^2y^6 - 16xy^7 + 4y^8 \\
 \hline
 -24x^4y^4 + 12x^2y^6 - 16xy^7 + 4y^8 & \\
 24x^4y^4 - 12x^2y^6 + 16xy^7 - 4y^8 & \\
 \hline
 0 & 
 \end{array}$$



$  \begin{array}{r}  23. \sqrt{25a^6 - 40a^5x + 26a^4x^2 - 28a^3x^3 + 17a^2x^4 - 4ax^5 + 4x^6} \\  \hline  -25a^6 \\  \hline  -40a^5x + 26a^4x^2 \\  \hline  40a^5x - 16a^4x^2 \\  \hline  10a^4x^2 - 28a^3x^3 + 17a^2x^4 \\  \hline  -10a^4x^2 + 8a^3x^3 - a^2x^4 \\  \hline  -20a^3x^3 + 16a^2x^4 - 4ax^5 + 4x^6 \\  \hline  20a^3x^3 - 16a^2x^4 + 4ax^5 - 4x^6 \\  \hline  0  \end{array}  $	$  \begin{array}{r}  5a^3 - 4a^2x + ax^2 - 2x^3 \\  \hline  (10a^3 - 4a^2x)(-4a^2x) = -40a^5x + 16a^4x^2 \\  \\  (10a^3 - 8a^2x + ax^2)(ax^2) = 10a^4x^2 - 8a^3x^3 + a^2x^4 \\  \\  (10a^3 - 8a^2x + 2ax^2 - 2x^3)(-2x^3) \\  = -20a^3x^3 + 16a^2x^4 - 4ax^5 + 4x^6  \end{array}  $
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$  \begin{array}{r}  24. \sqrt{4a^8 - 12a^7 + 17a^6 - 16a^5 + 14a^4 - 10a^3 + 5a^2 - 2a + 1} \\  \hline  -4a^8 \\  \hline  -12a^7 + 17a^6 \\  \hline  12a^7 - 9a^6 \\  \hline  8a^6 - 16a^5 + 14a^4 \\  \hline  -8a^6 + 12a^5 - 4a^4 \\  \hline  -4a^5 + 10a^4 - 10a^3 + 5a^2 \\  \hline  4a^5 - 6a^4 + 4a^3 - a^2 \\  \hline  4a^4 - 6a^3 + 4a^2 - 2a + 1 \\  \hline  -4a^4 + 6a^3 - 4a^2 + 2a - 1 \\  \hline  0  \end{array}  $	$  \begin{array}{r}  2a^4 - 3a^3 + 2a^2 - a + 1 \\  \hline  (4a^4 - 3a^3)(-3a^3) = -12a^7 + 9a^6 \\  \\  (4a^4 - 6a^3 + 2a^2)(2a^2) \\  = 8a^6 - 12a^5 + 4a^4 \\  \\  (4a^4 - 6a^3 + 4a^2 - a)(-a) \\  = -4a^5 + 6a^4 - 4a^3 + a^2 \\  \\  (4a^4 - 6a^3 + 4a^2 - 2a + 1)(1) \\  = 4a^4 - 6a^3 + 4a^2 - 2a + 1  \end{array}  $
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$  \begin{array}{r}  25. \sqrt{x^{10} - 2x^9 + 3x^8 - 4x^7 + 5x^6 - 8x^5 + 7x^4 - 6x^3 + 5x^2 - 4x + 4} \\  \hline  -x^{10} \\  \hline  -2x^9 + 3x^8 \\  \hline  2x^9 - x^8 \\  \hline  2x^8 - 4x^7 + 5x^6 \\  \hline  -2x^8 + 2x^7 - x^6 \\  \hline  -2x^7 + 4x^6 - 8x^5 + 7x^4 \\  \hline  2x^7 - 2x^6 + 2x^5 - x^4 \\  \hline  2x^6 - 6x^5 + 6x^4 - 6x^3 + 5x^2 \\  \hline  -2x^6 + 2x^5 - 2x^4 + 2x^3 - x^2 \\  \hline  -4x^5 + 4x^4 - 4x^3 + 4x^2 - 4x + 4 \\  \hline  4x^5 - 4x^4 + 4x^3 - 4x^2 + 4x - 4 \\  \hline  0  \end{array}  $	$  \begin{array}{r}  x^5 - x^4 + x^3 - x^2 + x - 2 \\  \hline  (2x^5 - x^4)(-x^4) = -2x^9 + x^8 \\  \\  (2x^5 - 2x^4 + x^3)(x^3) = 2x^8 - 2x^7 + x^6 \\  \\  (2x^5 - 2x^4 + 2x^3 - x^2)(-x^2) \\  = -2x^7 + 2x^6 - 2x^5 + x^4 \\  \\  (2x^5 - 2x^4 + 2x^3 - 2x^2 + x)(x) \\  = 2x^6 - 2x^5 + 2x^4 - 2x^3 + x^2 \\  \\  (2x^5 - 2x^4 + 2x^3 - 2x^2 + 2x - 2)(-2) \\  = -4x^5 + 4x^4 - 4x^3 + 4x^2 - 4x + 4  \end{array}  $
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## EJERCICIO 215

$$\begin{array}{l|l}
 1. \sqrt{\frac{x^4}{4} - x^3 + \frac{5x^2}{3} - \frac{4x}{3} + \frac{4}{9}} & \frac{x^2}{2} - x + \frac{2}{3} \\
 \hline
 -\frac{x^4}{4} & (x^2 - x)(-x) = -x^3 + x^2 \\
 \hline
 -x^3 + \frac{5x^2}{3} & \\
 \hline
 x^3 - x^2 & \left(x^2 - 2x + \frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{2x^2}{3} - \frac{4x}{3} + \frac{4}{9} \\
 \hline
 \frac{2x^2}{3} - \frac{4x}{3} + \frac{4}{9} & \\
 \hline
 -\frac{2x^2}{3} + \frac{4x}{3} - \frac{4}{9} & \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{l|l}
 2. \sqrt{\frac{a^2}{x^2} - \frac{2a}{3x} + \frac{19}{9} - \frac{2x}{3a} + \frac{x^2}{a^2}} & \frac{a}{x} - \frac{1}{3} + \frac{x}{a} \\
 \hline
 -\frac{a^2}{x^2} & \left(\frac{2a}{x} - \frac{1}{3}\right)\left(-\frac{1}{3}\right) = -\frac{2a}{3x} + \frac{1}{9} \\
 \hline
 -\frac{2a}{3x} + \frac{19}{9} & \\
 \hline
 \frac{2a}{3x} - \frac{1}{9} & \left(\frac{2a}{x} - \frac{2}{3} + \frac{x}{a}\right)\left(\frac{x}{a}\right) = 2 - \frac{2x}{3a} + \frac{x^2}{a^2} \\
 \hline
 2 - \frac{2x}{3a} + \frac{x^2}{a^2} & \\
 \hline
 -2 + \frac{2x}{3a} - \frac{x^2}{a^2} & \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{l|l}
 3. \sqrt{\frac{a^2}{4} - ab + b^2 + \frac{ac}{4} - \frac{bc}{2} + \frac{c^2}{16}} & \frac{a}{2} - b + \frac{c}{4} \\
 \hline
 -\frac{a^2}{4} & (a - b)(-b) = -ab + b^2 \\
 \hline
 -ab + b^2 & \\
 \hline
 ab - b^2 & \left(a - 2b + \frac{c}{4}\right)\left(\frac{c}{4}\right) = \frac{ac}{4} - \frac{bc}{2} + \frac{c^2}{16} \\
 \hline
 \frac{ac}{4} - \frac{bc}{2} + \frac{c^2}{16} & \\
 \hline
 -\frac{ac}{4} + \frac{bc}{2} - \frac{c^2}{16} & \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{l|l}
 4. \sqrt{\frac{9a^4}{16} - \frac{3a^3}{4} + \frac{29a^2}{20} - \frac{4a}{5} + \frac{16}{25}} & \frac{3a^2}{4} - \frac{a}{2} + \frac{4}{5} \\
 \hline
 -\frac{9a^4}{16} & \left(\frac{3a^2}{2} - \frac{a}{2}\right)\left(-\frac{a}{2}\right) = -\frac{3a^3}{4} + \frac{a^2}{4} \\
 \hline
 -\frac{3a^3}{4} + \frac{29a^2}{20} & \\
 \hline
 \frac{3a^3}{4} - \frac{a^2}{4} & \left(\frac{3a^2}{2} - a + \frac{4}{5}\right)\left(\frac{4}{5}\right) = \frac{6a^2}{5} - \frac{4a}{5} + \frac{16}{25} \\
 \hline
 \frac{6a^2}{5} - \frac{4a}{5} + \frac{16}{25} & \\
 \hline
 -\frac{6a^2}{5} + \frac{4a}{5} - \frac{16}{25} & \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{l|l}
 5. \sqrt{\frac{a^4}{16} + \frac{a^3b}{2} + \frac{3a^2b^2}{4} - ab^3 + \frac{b^4}{4}} & \frac{a^2}{4} + ab - \frac{b^2}{2} \\
 \hline
 -\frac{a^4}{16} & \left(\frac{a^2}{2} + ab\right)(ab) = \frac{a^3b}{2} + a^2b^2 \\
 \hline
 \frac{a^3b}{2} + \frac{3a^2b^2}{4} & \\
 \hline
 -\frac{a^3b}{2} - a^2b^2 & \left(\frac{a^2}{2} + 2ab - \frac{b^2}{2}\right)\left(-\frac{b^2}{2}\right) = -\frac{a^2b^2}{4} - ab^3 + \frac{b^4}{4} \\
 \hline
 -\frac{a^2b^2}{4} - ab^3 + \frac{b^4}{4} & \\
 \hline
 \frac{a^2b^2}{4} + ab^3 - \frac{b^4}{4} & \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{l|l}
 6. \sqrt{\frac{x^2}{25} + \frac{2x}{3} - 2xy + \frac{25}{9} - \frac{50y}{3} + 25y^2} & \frac{x}{5} + \frac{5}{3} - 5y \\
 \hline
 -\frac{x^2}{25} & \left(\frac{2x}{5} + \frac{5}{3}\right)\left(\frac{5}{3}\right) = \frac{2x}{3} + \frac{25}{9} \\
 \hline
 \frac{2x}{3} - 2xy + \frac{25}{9} & \\
 \hline
 -\frac{2x}{3} - \frac{25}{9} & \left(\frac{2x}{5} + \frac{10}{3} - 5y\right)(-5y) = -2xy - \frac{50y}{3} + 25y^2 \\
 \hline
 -2xy - \frac{50y}{3} + 25y^2 & \\
 \hline
 2xy + \frac{50y}{3} - 25y^2 & \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{l|l} 7. \sqrt{\frac{x^4}{9} - \frac{4x^3y}{3} + \frac{62x^2y^2}{15} - \frac{4xy^3}{5} + \frac{y^4}{25}} & \frac{x^2}{3} - 2xy + \frac{y^2}{5} \\ \hline -\frac{x^4}{9} & \left(\frac{2x^2}{3} - 2xy\right)(-2xy) = -\frac{4x^3y}{3} + 4x^2y^2 \\ \hline -\frac{4x^3y}{3} + \frac{62x^2y^2}{15} - \frac{4xy^3}{5} & \\ \hline \frac{4x^3y}{3} - 4x^2y^2 & \left(\frac{2x^2}{3} - 4xy + \frac{y^2}{5}\right)\left(\frac{y^2}{5}\right) = \frac{2x^2y^2}{15} - \frac{4xy^3}{5} + \frac{y^4}{25} \\ \hline \frac{2x^2y^2}{15} - \frac{4xy^3}{5} + \frac{y^4}{25} & \\ \hline -\frac{2x^2y^2}{15} + \frac{4xy^3}{5} - \frac{y^4}{25} & \\ \hline 0 & \end{array}$$

$$\begin{array}{l} \text{8. } \sqrt{\frac{a^4}{16} - \frac{3a^2}{10} + \frac{9}{25} + \frac{a^2b^2}{18} - \frac{2b^2}{15} + \frac{b^4}{81}} \\ \quad - \frac{a^4}{16} \\ \quad - \frac{3a^2}{10} + \frac{9}{25} \\ \quad \frac{3a^2}{10} - \frac{9}{25} \\ \quad \frac{a^2b^2}{18} - \frac{2b^2}{15} + \frac{b^4}{81} \\ \quad - \frac{a^2b^2}{18} + \frac{2b^2}{15} - \frac{b^4}{81} \\ \quad 0 \end{array} \quad \left| \begin{array}{l} \frac{a^2}{4} - \frac{3}{5} + \frac{b^2}{9} \\ \left( \frac{a^2}{2} - \frac{3}{5} \right) \left( -\frac{3}{5} \right) = -\frac{3a^2}{10} + \frac{9}{25} \\ \left( \frac{a^2}{2} - \frac{6}{5} + \frac{b^2}{9} \right) \left( \frac{b^2}{9} \right) = \frac{a^2b^2}{18} - \frac{2b^2}{15} + \frac{b^4}{81} \end{array} \right.$$

$$\begin{array}{r} 9. \sqrt{x^2 + 4x + 2 - \frac{4}{y} + \frac{1}{x^2}} \\ \underline{-x^2} \\ 4x + 2 \\ \underline{-4x - 4} \\ -2 - \frac{4}{y} + \frac{1}{x^2} \\ \underline{2 + \frac{4}{y} - \frac{1}{x^2}} \\ 0 \end{array} \quad \begin{array}{l} x + 2 - \frac{1}{x} \\ (2x + 2)(2) = 4x + 4 \\ \left(2x + 4 - \frac{1}{x}\right)\left(-\frac{1}{x}\right) = -2 - \frac{4}{x} + \frac{1}{x^2} \end{array}$$

$$\begin{array}{l|l}
 10. \sqrt{\frac{x^2}{9} - \frac{10x}{3} + \frac{79}{3} - \frac{20}{x} + \frac{4}{x^2}} & \frac{x}{3} - 5 + \frac{2}{x} \\
 \hline
 -\frac{x^2}{9} & \left(\frac{2x}{3} - 5\right)(-5) = -\frac{10x}{3} + 25 \\
 \hline
 -\frac{10x}{3} + \frac{79}{3} & \\
 \hline
 \frac{10x}{3} - 25 & \left(\frac{2x}{3} - 10 + \frac{2}{x}\right)\left(\frac{2}{x}\right) = \frac{4}{3} - \frac{20}{x} + \frac{4}{x^2} \\
 \hline
 \frac{4}{3} - \frac{20}{x} + \frac{4}{x^2} & \\
 \hline
 -\frac{4}{3} + \frac{20}{x} - \frac{4}{x^2} & \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{l|l}
 11. \sqrt{\frac{a^4}{4} - 5a^2 + 28 - \frac{30}{a^2} + \frac{9}{a^4}} & \frac{a^2}{2} - 5 + \frac{3}{a^2} \\
 \hline
 -\frac{a^4}{4} & (a^2 - 5)(-5) = -5a^2 + 25 \\
 \hline
 -5a^2 + 28 & \\
 \hline
 5a^2 - 25 & \left(a^2 - 10 + \frac{3}{a^2}\right)\left(\frac{3}{a^2}\right) = 3 - \frac{30}{a^2} + \frac{9}{a^4} \\
 \hline
 3 - \frac{30}{a^2} + \frac{9}{a^4} & \\
 \hline
 -3 + \frac{30}{a^2} - \frac{9}{a^4} & \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{l|l}
 12. \sqrt{\frac{a^4}{9} + \frac{2a^3}{3x} + \frac{a^2}{x^2} - \frac{2ax}{3} - 2 + \frac{x^2}{a^2}} & \frac{a^2}{3} + \frac{a}{x} - \frac{x}{a} \\
 \hline
 -\frac{a^4}{9} & \left(\frac{2a^2}{3} + \frac{a}{x}\right)\left(\frac{a}{x}\right) = \frac{2a^3}{3x} + \frac{a^2}{x^2} \\
 \hline
 \frac{2a^3}{3x} + \frac{a^2}{x^2} & \\
 \hline
 -\frac{2a^3}{3x} - \frac{a^2}{x^2} & \left(\frac{2a^2}{3} + \frac{2a}{x} - \frac{x}{a}\right)\left(-\frac{x}{a}\right) = -\frac{2ax}{3} - 2 + \frac{x^2}{a^2} \\
 \hline
 -\frac{2ax}{3} - 2 + \frac{x^2}{a^2} & \\
 \hline
 \frac{2ax}{3} + 2 - \frac{x^2}{a^2} & \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{l|l}
 13. \sqrt{\frac{9a^2}{x^2} - \frac{3a}{2x} + \frac{65}{16} - \frac{x}{3a} + \frac{4x^2}{9a^2}} & \frac{3a}{x} - \frac{1}{4} + \frac{2x}{3a} \\
 \hline
 -\frac{9a^2}{x^2} & \left(\frac{6a}{x} - \frac{1}{4}\right)\left(-\frac{1}{4}\right) = -\frac{3a}{2x} + \frac{1}{16} \\
 \hline
 -\frac{3a}{2x} + \frac{65}{16} & \\
 \hline
 \frac{3a}{2x} - \frac{1}{16} & \left(\frac{6a}{x} - \frac{1}{2} + \frac{2x}{3a}\right)\left(\frac{2x}{3a}\right) = 4 - \frac{x}{3a} + \frac{4x^2}{9a^2} \\
 \hline
 4 - \frac{x}{3a} + \frac{4x^2}{9a^2} & \\
 \hline
 -4 + \frac{x}{3a} - \frac{4x^2}{9a^2} & \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{l|l}
 14. \sqrt{9x^4 + 30x^2 + 55 + \frac{50}{x^2} + \frac{25}{x^4}} & 3x^2 + 5 + \frac{5}{x^2} \\
 \hline
 -9x^4 & (6x^2 + 5)(5) = 30x^2 + 25 \\
 \hline
 30x^2 + 55 & \\
 \hline
 -30x^2 - 25 & \left(6x^2 + 10 + \frac{5}{x^2}\right)\left(\frac{5}{x^2}\right) = 30 + \frac{50}{x^2} + \frac{25}{x^4} \\
 \hline
 30 + \frac{50}{x^2} + \frac{25}{x^4} & \\
 \hline
 -30 - \frac{50}{x^2} - \frac{25}{x^4} & \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{l|l}
 15. \sqrt{\frac{4a^2}{25x^2} - \frac{2a}{5x} + \frac{19}{12} - \frac{5x}{3a} + \frac{25x^2}{9a^2}} & \frac{2a}{5x} - \frac{1}{2} + \frac{5x}{3a} \\
 \hline
 -\frac{4a^2}{25x^2} & \left(\frac{4a}{5x} - \frac{1}{2}\right)\left(-\frac{1}{2}\right) = -\frac{2a}{5x} + \frac{1}{4} \\
 \hline
 -\frac{2a}{5x} + \frac{19}{12} & \\
 \hline
 \frac{2a}{5x} - \frac{1}{4} & \left(\frac{4a}{5x} - 1 + \frac{5x}{3a}\right)\left(\frac{5x}{3a}\right) = \frac{4}{3} - \frac{5x}{3a} + \frac{25x^2}{9a^2} \\
 \hline
 \frac{4}{3} - \frac{5x}{3a} + \frac{25x^2}{9a^2} & \\
 \hline
 -\frac{4}{3} + \frac{5x}{3a} - \frac{25x^2}{9a^2} & \\
 \hline
 0 & 
 \end{array}$$

$$\begin{array}{l|l}
 16. \sqrt{\frac{x^4}{16} - \frac{x^3y}{4} + \frac{3x^2y^2}{20} + \frac{xy^3}{5} + \frac{y^4}{25}} & \frac{x^2}{4} - \frac{xy}{2} - \frac{y^2}{5} \\
 -\frac{x^4}{16} & \left(\frac{x^2}{2} - \frac{xy}{2}\right)\left(-\frac{xy}{2}\right) = -\frac{x^3y}{4} + \frac{x^2y^2}{4} \\
 -\frac{x^3y}{4} + \frac{3x^2y^2}{20} & \\
 \frac{x^3y}{4} - \frac{x^2y^2}{4} & \left(\frac{x^2}{2} - xy - \frac{y^2}{5}\right)\left(-\frac{y^2}{5}\right) = -\frac{x^2y^2}{10} + \frac{xy^3}{5} + \frac{y^4}{25} \\
 -\frac{x^2y^2}{10} + \frac{xy^3}{5} + \frac{y^4}{25} & \\
 \frac{x^2y^2}{10} - \frac{xy^3}{5} - \frac{y^4}{25} & \\
 0 & 
 \end{array}$$

$$\begin{array}{l|l}
 17. \sqrt{\frac{4a^2b^2}{49x^2y^2} - \frac{2ab}{7xy} + \frac{21}{20} - \frac{7xy}{5ab} + \frac{49x^2y^2}{25a^2b^2}} & \frac{2ab}{7xy} - \frac{1}{2} + \frac{7xy}{5ab} \\
 -\frac{4a^2b^2}{49x^2y^2} & \left(\frac{4ab}{7xy} - \frac{1}{2}\right)\left(-\frac{1}{2}\right) = -\frac{2ab}{7xy} + \frac{1}{4} \\
 -\frac{2ab}{7xy} + \frac{21}{20} & \\
 \frac{2ab}{7xy} - \frac{1}{4} & \left(\frac{4ab}{7xy} - 1 + \frac{7xy}{5ab}\right)\left(\frac{7xy}{5ab}\right) \\
 \frac{4}{5} - \frac{7xy}{5ab} + \frac{49x^2y^2}{25a^2b^2} & = \frac{4}{5} - \frac{7xy}{5ab} + \frac{49x^2y^2}{25a^2b^2} \\
 -\frac{4}{5} + \frac{7xy}{5ab} - \frac{49x^2y^2}{25a^2b^2} & \\
 0 & 
 \end{array}$$

$$\begin{array}{l|l}
 18. \sqrt{\frac{9a^2x^2}{25m^2n^2} - \frac{6ax}{25mn} + \frac{23}{75} - \frac{4mn}{45ax} + \frac{4m^2n^2}{81a^2x^2}} & \frac{3ax}{5mn} - \frac{1}{5} + \frac{10mn}{45ax} \\
 -\frac{9a^2x^2}{25m^2n^2} & \left(\frac{6ax}{5mn} - \frac{1}{5}\right)\left(-\frac{1}{5}\right) = -\frac{6ax}{25mn} + \frac{1}{25} \\
 -\frac{6ax}{25mn} + \frac{23}{75} & \\
 \frac{6ax}{25mn} - \frac{1}{25} & \left(\frac{6ax}{5mn} - \frac{2}{5} + \frac{10mn}{45ax}\right)\left(\frac{10mn}{45ax}\right) \\
 \frac{20}{75} - \frac{4mn}{45ax} + \frac{4m^2n^2}{81a^2x^2} & = \frac{20}{75} - \frac{4mn}{45ax} + \frac{4m^2n^2}{81a^2x^2} \\
 -\frac{20}{75} + \frac{4mn}{45ax} - \frac{4m^2n^2}{81a^2x^2} & \\
 0 & 
 \end{array}$$

<p>19. <math display="block">\begin{array}{r} \sqrt{\frac{x^6}{4} - x^5 + \frac{5x^4}{3} + \frac{2x^3}{3} - \frac{32x^2}{9} + \frac{8x}{3} + 4} \\ - \frac{x^6}{4} \\ \hline -x^5 + \frac{5x^4}{3} \\ -x^5 - x^4 \\ \hline \frac{2x^4}{3} + \frac{2x^3}{3} - \frac{32x^2}{9} \\ - \frac{2x^4}{3} + \frac{4x^3}{3} - \frac{4x^2}{9} \\ \hline 2x^3 - 4x^2 + \frac{8x}{3} + 4 \\ - 2x^3 + 4x^2 - \frac{8x}{3} - 4 \\ \hline 0 \end{array}</math></p>	<p><math display="block">\frac{x^3}{2} - x^2 + \frac{2x}{3} + 2</math></p> <hr/> <p><math display="block">(x^3 - x^2)(-x^2) = -x^5 + x^4</math></p> <p><math display="block">\left(x^3 - 2x^2 + \frac{2x}{3}\right)\left(\frac{2x}{3}\right) = \frac{2x^4}{3} - \frac{4x^3}{3} + \frac{4x^2}{9}</math></p> <p><math display="block">\left(x^3 - 2x^2 + \frac{4x}{3} + 2\right)(2) = 2x^3 - 4x^2 + \frac{8x}{3} + 4</math></p>
<p>20. <math display="block">\begin{array}{r} \sqrt{\frac{1}{4} - \frac{3a}{4} + \frac{59a^2}{48} - \frac{3a^3}{2} + \frac{43a^4}{36} - \frac{2a^5}{3} + \frac{1a^6}{4}} \\ - \frac{1}{4} \\ \hline -\frac{3a}{4} + \frac{59a^2}{48} \\ \frac{3a}{4} - \frac{9a^2}{16} \\ \hline \frac{2a^2}{3} - \frac{3a^3}{2} + \frac{43a^4}{36} \\ - \frac{2a^2}{3} + a^3 - \frac{4a^4}{9} \\ \hline -\frac{a^3}{2} + \frac{3a^4}{4} - \frac{2a^5}{3} + \frac{a^6}{4} \\ \frac{a^3}{2} - \frac{3a^4}{4} + \frac{2a^5}{3} - \frac{a^6}{4} \\ \hline 0 \end{array}</math></p>	<p><math display="block">\frac{1}{2} - \frac{3a}{4} + \frac{2a^2}{3} - \frac{a^3}{2}</math></p> <hr/> <p><math display="block">\left(1 - \frac{3a}{4}\right)\left(-\frac{3a}{4}\right) = -\frac{3a}{4} + \frac{9a^2}{16}</math></p> <p><math display="block">\left(1 - \frac{3a}{2} + \frac{2a^2}{3}\right)\left(\frac{2a^2}{3}\right) = \frac{2a^2}{3} - a^3 + \frac{4a^4}{9}</math></p> <p><math display="block">\left(1 - \frac{3a}{2} + \frac{4a^2}{3} - \frac{a^3}{2}\right)\left(-\frac{a^3}{2}\right) = -\frac{a^3}{2} + \frac{3a^4}{4} - \frac{2a^5}{3} + \frac{a^6}{4}</math></p>

## EJERCICIO 216

<p>1. <math display="block">\begin{array}{r} \sqrt[3]{8 - 36y + 54y^2 - 27y^3} \\ - 8 \\ \hline -36y + 54y^2 - 27y^3 \\ 36y - 54y^2 + 27y^3 \\ \hline 0 \end{array}</math></p>	<p><math display="block">2 - 3y</math></p> <hr/> <p><math display="block">3(2)^2 = 12</math></p> <p><math display="block">3(2)^2(-3y) = -36y</math></p> <p><math display="block">3(2)(-3y)^2 = 54y^2</math></p> <p><math display="block">(-3y)^3 = -27y^3</math></p>
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$ \begin{array}{r} 2. \sqrt[3]{64a^6 + 240a^4b^2 + 300a^2b^4 + 125b^6} \\ \underline{-64a^6} \\ 240a^4b^2 + 300a^2b^4 + 125b^6 \\ \underline{-240a^4b^2 - 300a^2b^4 - 125b^6} \\ 0 \end{array} $	$ \begin{array}{r} 4a^2 + 5b^2 \\ \underline{3(4a^2)^2 = 48a^4} \\ 3(4a^2)^2(5b^2) = 240a^4b^2 \\ 3(4a^2)(5b^2)^2 = 300a^2b^4 \\ (5b^2)^3 = 125b^6 \end{array} $
$ \begin{array}{r} 3. \sqrt[3]{x^6 + 3x^5 + 6x^4 + 7x^3 + 6x^2 + 3x + 1} \\ \underline{-x^6} \\ 3x^5 + 6x^4 + 7x^3 \\ \underline{-3x^5 - 3x^4 - x^3} \\ 3x^4 + 6x^3 + 6x^2 + 3x + 1 \\ \underline{-3x^4 - 6x^3 - 6x^2 - 3x - 1} \\ 0 \end{array} $	$ \begin{array}{r} x^2 + x + 1 \\ \underline{3(x^2)^2 = 3x^4} \\ 3(x^2)^2(x) = 3x^5 \\ 3(x^2)(x^2) = 3x^4 \\ x^3 = x^3 \\ 3(x^2 + x)^2 = 3x^4 + 6x^3 + 3x^2 \\ 3(x^2 + x)^2(1) = 3x^4 + 6x^3 + 3x^2 \\ 3(x^2 + x)(1)^2 = 3x^2 + 3x \\ (1)^3 = 1 \end{array} $
$ \begin{array}{r} 4. \sqrt[3]{8x^6 - 12x^5 - 6x^4 + 11x^3 + 3x^2 - 3x - 1} \\ \underline{-8x^6} \\ -12x^5 - 6x^4 + 11x^3 \\ \underline{12x^5 - 6x^4 + x^3} \\ -12x^4 + 12x^3 + 3x^2 - 3x - 1 \\ \underline{12x^4 - 12x^3 - 3x^2 + 3x + 1} \\ 0 \end{array} $	$ \begin{array}{r} 2x^2 - x - 1 \\ \underline{3(2x^2)^2 = 12x^4} \\ 3(2x^2)^2(-x) = -12x^5 \\ 3(2x^2)(-x)^2 = 6x^4 \\ (-x)^3 = -x^3 \\ 3(2x^2 - x)^2 = 12x^4 - 12x^3 + 3x^2 \\ 3(2x^2 - x)^2(-1) = -12x^4 + 12x^3 - 3x^2 \\ 3(2x^2 - x)(-1)^2 = 6x^2 - 3x \\ (-1)^3 = -1 \end{array} $
$ \begin{array}{r} 5. \sqrt[3]{1 - 9x + 33x^2 - 63x^3 + 66x^4 - 36x^5 + 8x^6} \\ \underline{-1} \\ -9x + 33x^2 - 63x^3 \\ \underline{9x - 27x^2 + 27x^3} \\ 6x^2 - 36x^3 + 66x^4 - 36x^5 + 8x^6 \\ \underline{-6x^2 + 36x^3 - 66x^4 + 36x^5 - 8x^6} \\ 0 \end{array} $	$ \begin{array}{r} 1 - 3x + 2x^2 \\ \underline{3(1)^2 = 3} \\ 3(1)^2(-3x) = -9x \\ 3(1)(-3x)^2 = 27x^2 \\ (-3x)^3 = -27x^3 \\ 3(1 - 3x)^2 = 3 - 18x + 27x^2 \\ 3(1 - 3x)^2(2x^2) = 6x^2 - 36x^3 + 54x^4 \\ 3(1 - 3x)(2x^2)^2 = 12x^4 - 36x^5 \\ (2x^2)^3 = 8x^6 \end{array} $

6.	$\begin{array}{r} \sqrt[3]{x^6 - 9x^5 + 33x^4 - 63x^3 + 66x^2 - 36x + 8} \\ - x^6 \\ \hline -9x^5 + 33x^4 - 63x^3 \\ 9x^5 - 27x^4 + 27x^3 \\ \hline 6x^4 - 36x^3 + 66x^2 - 36x + 8 \\ - 6x^4 + 36x^3 - 66x^2 + 36x - 8 \\ \hline 0 \end{array}$	$\begin{array}{l} x^2 - 3x + 2 \\ \hline 3(x^2)^2 = 3x^4 \\ 3(x^2)^2(-3x) = -9x^5 \\ 3(x^2)(-3x)^2 = 27x^4 \\ (-3x)^3 = -27x^3 \\ \\ 3(x^2 - 3x)^2 = 3x^4 - 18x^3 + 27x^2 \\ 3(x^2 - 3x)^2(2) = 6x^4 - 36x^3 + 54x^2 \\ 3(x^2 - 3x)(2)^2 = 12x^2 - 36x \\ (2)^3 = 8 \end{array}$
7.	$\begin{array}{r} \sqrt[3]{x^9 - 6x^8 + 12x^7 - 20x^6 + 48x^5 - 48x^4 + 48x^3 - 96x^2 - 64} \\ - x^9 \\ \hline -6x^8 + 12x^7 - 20x^6 \\ 6x^8 - 12x^7 + 8x^6 \\ \hline -12x^6 + 48x^5 - 48x^4 + 48x^3 - 96x^2 - 64 \\ 12x^6 - 48x^5 + 48x^4 - 48x^3 + 96x^2 + 64 \\ \hline 0 \end{array}$	$\begin{array}{l} x^3 - 2x^2 - 4 \\ \hline 3(x^3)^2 = 3x^6 \\ 3(x^3)^2(-2x^2) = -6x^8 \\ 3(x^3)^3(-2x^2)^2 = 12x^7 \\ (-2x^2)^3 = -8x^6 \\ \\ 3(x^3 - 2x^2)^2 = 3x^6 - 12x^5 + 12x^4 \\ 3(x^3 - 2x^2)^2(-4) = -12x^6 + 48x^5 - 48x^4 \\ 3(x^3 - 2x^2)(-4)^2 = 48x^3 - 96x^2 \\ (-4)^3 = -64 \end{array}$
8.	$\begin{array}{r} \sqrt{x^{12} - 3x^{10} - 3x^8 + 11x^6 + 6x^4 - 12x^2 - 8} \\ - x^{12} \\ \hline -3x^{10} - 3x^8 + 11x^6 \\ 3x^{10} - 3x^8 + x^6 \\ \hline -6x^8 + 12x^6 + 6x^4 - 12x^2 - 8 \\ 6x^8 - 12x^6 - 6x^4 + 12x^2 + 8 \\ \hline 0 \end{array}$	$\begin{array}{l} x^4 - x^2 - 2 \\ \hline 3(x^4)^2 = 3x^8 \\ 3(x^4)^2(-x^2) = -3x^{10} \\ 3(x^4)(-x^2)^2 = 3x^8 \\ (-x^2)^3 = -x^6 \\ \\ 3(x^4 - x^2)^2 = 3x^8 - 6x^6 + 3x^4 \\ 3(x^4 - x^2)^2(-2) = -6x^8 + 12x^6 - 6x^4 \\ 3(x^4 - x^2)(-2)^2 = 12x^4 - 12x^2 \\ (-2)^3 = -8 \end{array}$

<p>9. <math>\sqrt[3]{8x^6 - 36x^5 + 66x^4 - 63x^3 + 33x^2 - 9x + 1}</math></p> <hr/> $-8x^6$ <hr/> $-36x^5 + 66x^4 - 63x^3$ <hr/> $36x^5 - 54x^4 + 27x^3$ <hr/> $12x^4 - 36x^3 + 33x^2 - 9x + 1$ <hr/> $-12x^4 + 36x^3 - 33x^2 + 9x - 1$ <hr/> <p>0</p>	<p><math>2x^2 - 3x + 1</math></p> <hr/> $3(2x^2)^2 = 12x^4$ $3(2x^2)^2(-3x) = -36x^5$ $3(2x^2)(-3x)^2 = 54x^4$ $(-3x)^3 = -27x^3$  $3(2x^2 - 3x)^2 = 12x^4 - 36x^3 + 27x^2$ $3(2x^2 - 3x)^2(1) = 12x^4 - 36x^3 + 27x^2$ $3(2x^2 - 3x)(1)^2 = 6x^2 - 9x$ $(1)^3 = 1$
<p>10. <math>\sqrt[3]{27a^6 - 135a^5 + 117a^4 + 235a^3 - 156a^2 - 240a - 64}</math></p> <hr/> $-27a^6$ <hr/> $-135a^5 + 117a^4 + 235a^3$ <hr/> $135a^5 - 225a^4 + 125a^3$ <hr/> $-108a^4 + 360a^3 - 156a^2 - 240a - 64$ <hr/> $108a^4 - 360a^3 + 156a^2 + 240a + 64$ <hr/> <p>0</p>	<p><math>3a^2 - 5a - 4</math></p> <hr/> $3(3a^2)^2 = 27a^4$ $3(3a^2)^2(-5a) = -135a^5$ $3(3a^2)(-5a)^2 = 225a^4$ $(-5a)^3 = -125a^3$  $3(3a^2 - 5a)^2 = 27a^4 - 90a^3 + 75a^2$ $3(3a^2 - 5a)^2(-4) = -108a^4 + 360a^3 - 300a^2$ $3(3a^2 - 5a)(-4)^2 = 144a^2 - 240a$ $(-4)^3 = -64$
<p>11. <math>\sqrt[3]{a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6}</math></p> <hr/> $-a^6$ <hr/> $-6a^5b + 15a^4b^2 - 20a^3b^3$ <hr/> $6a^5b - 12a^4b^2 + 8a^3b^3$ <hr/> $3a^4b^2 - 12a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$ <hr/> $-3a^4b^2 + 12a^3b^3 - 15a^2b^4 + 6ab^5 - b^6$ <hr/> <p>0</p>	<p><math>a^2 - 2ab + b^2</math></p> <hr/> $3(a^2)^2 = 3a^4$ $3(a^2)^2(-2ab) = -6a^5b$ $3(a^2)(-2ab)^2 = 12a^4b^2$ $(-2ab)^3 = -8a^3b^3$  $3(a^2 - 2ab)^2 = 3a^4 - 12a^3b + 12a^2b^2$ $3(a^2 - 2ab)^2(b)^2 = 3a^4b^2 - 12a^3b^3 + 12a^2b^4$ $3(a^2 - 2ab)(b^2)^2 = 3a^2b^4 - 6ab^5$ $(b^2)^3 = b^6$

<p>12. <math>\sqrt[3]{x^6 - 9x^5y + 42x^4y^2 - 117x^3y^3 + 210x^2y^4 - 225xy^5 + 125y^6}</math></p> <hr/> $-x^6$ <hr/> $-9x^5y + 42x^4y^2 - 117x^3y^3$ <hr/> $9x^5y - 27x^4y^2 + 27x^3y^3$ <hr/> $15x^4y^2 - 80x^3y^3 + 210x^2y^4 - 225xy^5 + 125y^6$ <hr/> $-15x^4y^2 + 80x^3y^3 - 210x^2y^4 + 225xy^5 - 125y^6$ <hr/> <p>0</p>	<p><math>x^2 - 3xy + 5y^2</math></p> <hr/> $3(x^2)^2 = 3x^4$ $3(x^2)^2(-3xy) = -9x^5y$ $3(x^2)(-3xy)^2 = 27x^4y^2$ $(-3xy)^3 = -27x^3y^3$ $3(x^2 - 3xy)^2 = 3x^4 - 18x^3y + 27x^2y^2$ $3(x^2 - 3xy)^2(5y^2) = 15x^4y^2 - 80x^3y^3 + 135x^2y^4$ $3(x^2 - 3xy)(5y^2)^2 = 75x^2y^4 - 225xy^5$ $(5y^2)^3 = 125y^6$
<p>13. <math>\sqrt{a^{12} - 3a^{10} + 15a^8 - 25a^6 + 60a^4 - 48a^2 + 64}</math></p> <hr/> $-a^{12}$ <hr/> $-3a^{10} + 15a^8 - 25a^6$ <hr/> $3a^{10} - 3a^8 + a^6$ <hr/> $12a^8 - 24a^6 + 60a^4 - 48a^2 + 64$ <hr/> $-12a^8 + 24a^6 - 60a^4 + 48a^2 - 64$ <hr/> <p>0</p>	<p><math>a^4 - a^2 + 4</math></p> <hr/> $3(a^4)^2 = 3a^8$ $3(a^4)^2(-a^2) = -3a^{10}$ $3(a^4)(-a^2)^2 = 3a^8$ $(-a^2)^3 = -a^6$ $3(a^4 - a^2)^2 = 3a^8 - 6a^6 + 3a^4$ $3(a^4 - a^2)^2(4) = 12a^8 - 24a^6 + 12a^4$ $3(a^4 - a^2)(4)^2 = 48a^4 - 48a^2$ $(4)^3 = 64$
<p>14. <math>\sqrt[3]{a^9 - 9a^8x + 27a^7x^2 - 21a^6x^3 - 36a^5x^4 + 54a^4x^5 + 12a^3x^6 - 36a^2x^7 + 8x^9}</math></p> <hr/> $-a^9$ <hr/> $-9a^8x + 27a^7x^2 - 21a^6x^3$ <hr/> $9a^8x - 27a^7x^2 + 27a^6x^3$ <hr/> $6a^6x^3 - 36a^5x^4 + 54a^4x^5 + 12a^3x^6 - 36a^2x^7 + 8x^9$ <hr/> $-6a^6x^3 + 36a^5x^4 - 54a^4x^5 - 12a^3x^6 + 36a^2x^7 - 8x^9$ <hr/> <p>0</p>	<p><math>a^3 - 3a^2x + 2x^3</math></p> <hr/> $3(a^3)^2 = 3a^6$ $3(a^3)^2(-3a^2x) = -9a^8x$ $3(a^3)(-3a^2x)^2 = 27a^7x^2$ $(-3a^2x)^3 = -27a^6x^3$ $3(a^3 - 3a^2x)^2 = 3a^6 - 18a^5x + 27a^4x^2$ $3(a^3 - 3a^2x)^2(2x^3) = 6a^6x^3 - 36a^5x^4 + 54a^4x^5$ $3(a^3 - 3a^2x)(2x^3)^2 = 12a^3x^6 - 36a^2x^7$ $(2x^3)^3 = 8x^9$

$$15. \sqrt[3]{a^9 - 3a^8 + 6a^7 - 10a^6 + 12a^5 - 12a^4 + 10a^3 - 6a^2 + 3a - 1}$$

$$-a^9$$

$$-3a^8 + 6a^7 - 10a^6$$

$$3a^8 - 3a^7 + a^6$$

$$3a^7 - 9a^6 + 12a^5 - 12a^4 + 10a^3$$

$$-3a^7 + 6a^6 - 6a^5 + 3a^4 - a^3$$

$$-3a^6 + 6a^5 - 9a^4 + 9a^3 - 6a^2 + 3a - 1$$

$$3a^6 - 6a^5 + 9a^4 - 9a^3 + 6a^2 - 3a + 1$$

$$0$$

$$a^3 - a^2 + a - 1$$

$$3(a^3)^2 = 3a^6$$

$$3(a^3)^2(-a^2) = -3a^8$$

$$3(a^3)(-a^2)^2 = 3a^7$$

$$(-a^2)^3 = -a^6$$

$$3(a^3 - a^2)^2 = 3a^6 - 6a^5 + 3a^4$$

$$3(a^3 - a^2)^2(a) = 3a^7 - 6a^6 + 3a^5$$

$$3(a^3 - a^2)(a^2) = 3a^5 - 3a^4$$

$$(a^3) = a^3$$

$$3(a^3 - a^2 + a)^2 = 3a^6 + 3a^4 + 3a^2 - 6a^5 + 6a^4 - 6a^3$$

$$3(a^3 - a^2 + a)^2(-1) = -3a^6 + 6a^5 - 9a^4 + 6a^3 - 3a^2$$

$$3(a^3 - a^2 + a)(-1)^2 = 3a^3 - 3a^2 + 3a$$

$$(-1)^3 = -1$$

$$16. \sqrt[3]{x^9 - 12x^8 + 54x^7 - 121x^6 + 180x^5 - 228x^4 + 179x^3 - 144x^2 + 54x - 27}$$

$$-x^9$$

$$-12x^8 + 54x^7 - 121x^6$$

$$12x^8 - 48x^7 + 64x^6$$

$$6x^7 - 57x^6 + 180x^5 - 228x^4 + 179x^3$$

$$-6x^7 + 48x^6 - 108x^5 + 48x^4 - 8x^3$$

$$-9x^6 + 72x^5 - 180x^4 + 171x^3 - 144x^2 + 54x - 27$$

$$9x^6 - 72x^5 + 180x^4 - 171x^3 + 144x^2 - 54x + 27$$

$$0$$

$$x^3 - 4x^2 + 2x - 3$$

$$3(x^3)^2 = 3x^6 \quad 3(x^3)^2(-4x^2) = -12x^8$$

$$3(x^3)(-4x^2)^2 = 48x^7 \quad (-4x^2)^3 = -64x^6$$

$$3(x^3 - 4x^2)^2 = 3x^6 - 24x^5 + 48x^4$$

$$3(x^3 - 4x^2)^2(2x) = 6x^7 - 48x^6 + 96x^5$$

$$3(x^3 - 4x^2)(2x)^2 = 12x^5 - 48x^4$$

$$(2x)^3 = 8x^3$$

$$3(x^3 - 4x^2 + 2x)^2 = 3x^6 + 48x^4 + 12x^2 - 24x^5 + 12x^4 - 48x^3$$

$$3(x^3 - 4x^2 + 2x)^2(-3) = -9x^6 - 144x^4 - 36x^2 + 72x^5 - 36x^4 + 144x^3$$

$$3(x^3 - 4x^2 + 2x)(-3)^2 = 27x^3 - 108x^2 + 54x$$

$$(-3)^3 = -27$$

## EJERCICIO 217

$1. \sqrt[3]{\frac{x^6}{8} - \frac{x^5}{4} + \frac{5x^4}{3} - \frac{55x^3}{27} + \frac{20x^2}{3} - 4x + 8}$ <hr style="border: 0.5px solid black; margin: 5px 0;"/> $-\frac{x^6}{8}$ <hr style="border: 0.5px solid black; margin: 5px 0;"/> $-\frac{x^5}{4} + \frac{5x^4}{3} - \frac{55x^3}{27}$ <hr style="border: 0.5px solid black; margin: 5px 0;"/> $\frac{x^5}{4} - \frac{x^4}{6} + \frac{x^3}{27}$ <hr style="border: 0.5px solid black; margin: 5px 0;"/> $\frac{3x^4}{2} - 2x^3 + \frac{20x^2}{3} - 4x + 8$ <hr style="border: 0.5px solid black; margin: 5px 0;"/> $-\frac{3x^4}{2} + 2x^3 - \frac{20x^2}{3} + 4x - 8$ <hr style="border: 0.5px solid black; margin: 5px 0;"/> $0$	$\frac{x^2}{2} - \frac{x}{3} + 2$ <hr style="border: 0.5px solid black; margin: 5px 0;"/> $3\left(\frac{x^2}{2}\right)^2 = \frac{3x^4}{4}$ $3\left(\frac{x^2}{2}\right)^2\left(-\frac{x}{3}\right) = -\frac{x^5}{4}$ $3\left(\frac{x^2}{2}\right)\left(-\frac{x}{3}\right)^2 = \frac{x^4}{6}$ $\left(-\frac{x}{3}\right)^3 = -\frac{x^3}{27}$ $3\left(\frac{x^2}{2} - \frac{x}{3}\right)^2 = \frac{3x^4}{4} - x^3 + \frac{x^2}{3}$ $3\left(\frac{x^2}{2} - \frac{x}{3}\right)^2(2) = \frac{3x^4}{2} - 2x^3 + \frac{2x^2}{3}$ $3\left(\frac{x^2}{2} - \frac{x}{3}\right)(2)^2 = 6x^2 - 4x$ $(2)^3 = 8$
$2. \sqrt[3]{a^9 + \frac{3a^8}{2} - \frac{a^7}{4} - \frac{7a^6}{8} + \frac{a^5}{12} + \frac{a^4}{6} - \frac{a^3}{27}}$ <hr style="border: 0.5px solid black; margin: 5px 0;"/> $-a^9$ <hr style="border: 0.5px solid black; margin: 5px 0;"/> $\frac{3a^8}{2} - \frac{a^7}{4} - \frac{7a^6}{8}$ <hr style="border: 0.5px solid black; margin: 5px 0;"/> $-\frac{3a^8}{2} + \frac{3a^7}{4} - \frac{a^6}{8}$ <hr style="border: 0.5px solid black; margin: 5px 0;"/> $-a^7 - a^6 + \frac{a^5}{12} + \frac{a^4}{6} - \frac{a^3}{27}$ <hr style="border: 0.5px solid black; margin: 5px 0;"/> $a^7 + a^6 - \frac{a^5}{12} - \frac{a^4}{6} + \frac{a^3}{27}$ <hr style="border: 0.5px solid black; margin: 5px 0;"/> $0$	$a^3 + \frac{a^2}{2} - \frac{a}{3}$ <hr style="border: 0.5px solid black; margin: 5px 0;"/> $3(a^3)^2 = 3a^6$ $3(a^3)^2\left(\frac{a^2}{2}\right) = \frac{3a^8}{2}$ $3(a^3)\left(\frac{a^2}{2}\right)^2 = \frac{3a^7}{4}$ $\left(\frac{a^2}{2}\right)^3 = \frac{a^6}{8}$ $3\left(a^3 + \frac{a^2}{2}\right)^2 = 3a^6 + 3a^5 + \frac{3a^4}{4}$ $3\left(a^3 + \frac{a^2}{2}\right)^2\left(-\frac{a}{3}\right) = -a^7 - a^6 - \frac{a^5}{4}$ $3\left(a^3 + \frac{a^2}{2}\right)\left(-\frac{a}{3}\right)^2 = \frac{a^5}{3} + \frac{a^4}{6}$ $\left(-\frac{a}{3}\right)^3 = -\frac{a^3}{27}$

$3. \sqrt[3]{\frac{x^3}{8} - \frac{9x^2}{4} + 15x - 45 + \frac{60}{x} - \frac{36}{x^2} + \frac{8}{x^3}}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $-\frac{x^3}{8}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $-\frac{9x^2}{4} + 15x - 45$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $\frac{9x^2}{4} - \frac{27x}{2} + 27$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $\frac{3x}{2} - 18 + \frac{60}{x} - \frac{36}{x^2} + \frac{8}{x^3}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $-\frac{3x}{2} + 18 - \frac{60}{x} + \frac{36}{x^2} - \frac{8}{x^3}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $0$	$\frac{x}{2} - 3 + \frac{2}{x}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $3\left(\frac{x}{2}\right)^2 = \frac{3x^2}{4}$ $3\left(\frac{x}{2}\right)^2(-3) = -\frac{9x^2}{4}$ $3\left(\frac{x}{2}\right)(-3)^2 = \frac{27x}{2}$ $(-3)^3 = -27$ $3\left(\frac{x}{2} - 3\right)^2 = \frac{3x^2}{4} - 9x + 27$ $3\left(\frac{x}{2} - 3\right)\left(\frac{2}{x}\right) = \frac{3x}{2} - 18 + \frac{54}{x}$ $3\left(\frac{x}{2} - 3\right)\left(\frac{2}{x}\right)^2 = \frac{6}{x} - \frac{36}{x^2}$ $\left(\frac{2}{x}\right)^3 = \frac{8}{x^3}$
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$4. \sqrt[3]{\frac{a^3}{8b^3} - \frac{3a^2}{4b^2} + \frac{15a}{8b} - \frac{5}{2} + \frac{15b}{8a} - \frac{3b^2}{4a^2} + \frac{b^3}{8a^3}}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $-\frac{a^3}{8b^3}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $-\frac{3a^2}{4b^2} + \frac{15a}{8b} - \frac{5}{2}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $\frac{3a^2}{4b^2} - \frac{3a}{2b} + 1$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $\frac{3a}{8b} - \frac{3}{2} + \frac{15b}{8a} - \frac{3b^2}{4a^2} + \frac{b^3}{8a^3}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $-\frac{3a}{8b} + \frac{3}{2} - \frac{15b}{8a} + \frac{3b^2}{4a^2} - \frac{b^3}{8a^3}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $0$	$\frac{a}{2b} - 1 + \frac{b}{2a}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $3\left(\frac{a}{2b}\right)^2 = \frac{3a^2}{4b^2}$ $3\left(\frac{a}{2b}\right)^2(-1) = -\frac{3a^2}{4b^2}$ $3\left(\frac{a}{2b}\right)(-1)^2 = \frac{3a}{2b}$ $(-1)^3 = -1$  $3\left(\frac{a}{2b} - 1\right)^2 = \frac{3a^2}{4b^2} - \frac{3a}{b} + 3$ $3\left(\frac{a}{2b} - 1\right)\left(\frac{b}{2a}\right) = \frac{3a}{8b} - \frac{3}{2} + \frac{3b}{2a}$ $3\left(\frac{a}{2b} - 1\right)\left(\frac{b}{2a}\right)^2 = \frac{3b}{8a} - \frac{3b^2}{4a^2}$ $\left(\frac{b}{2a}\right)^3 = \frac{b^3}{8a^3}$
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$5. \sqrt[3]{\frac{8a^3}{27x^3} - \frac{2a^2}{3x^2} + \frac{a}{18x} + \frac{13}{24} - \frac{x}{36a} - \frac{x^2}{6a^2} - \frac{x^3}{27a^3}}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $- \frac{8a^3}{27x^3}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $- \frac{2a^2}{3x^2} + \frac{a}{18x} + \frac{13}{24}$ $\frac{2a^2}{3x^2} - \frac{a}{2x} + \frac{1}{8}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $- \frac{4a}{9x} + \frac{2}{3} - \frac{x}{36a} - \frac{x^2}{6a^2} - \frac{x^3}{27a^3}$ $\frac{4a}{9x} - \frac{2}{3} + \frac{x}{36a} + \frac{x^2}{6a^2} + \frac{x^3}{27a^3}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $0$	$\frac{2a}{3x} - \frac{1}{2} - \frac{x}{3a}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $3\left(\frac{2a}{3x}\right)^2 = \frac{4a^2}{3x^2}$ $3\left(\frac{2a}{3x}\right)^2\left(-\frac{1}{2}\right) = -\frac{2a^2}{3x^2}$ $3\left(\frac{2a}{3x}\right)\left(-\frac{1}{2}\right)^2 = \frac{a}{2x}$ $\left(-\frac{1}{2}\right)^3 = -\frac{1}{8}$ $3\left(\frac{2a}{3x} - \frac{1}{2}\right)^2 = \frac{4a^2}{3x^2} - \frac{2a}{x} + \frac{3}{4}$ $3\left(\frac{2a}{3x} - \frac{1}{2}\right)^2\left(-\frac{x}{3a}\right) = -\frac{4a}{9x} + \frac{2}{3} - \frac{x}{4a}$ $3\left(\frac{2a}{3x} - \frac{1}{2}\right)\left(-\frac{x}{3a}\right)^2 = \frac{2x}{9a} - \frac{x^2}{6a^2}$ $\left(-\frac{x}{3a}\right)^3 = -\frac{x^3}{27a^3}$
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$6. \sqrt[3]{\frac{8a^3}{27b^3} + \frac{4a^2}{3b^2} + \frac{3a}{b} + 4 + \frac{27b}{8a} + \frac{27b^2}{16a^2} + \frac{27b^3}{64a^3}}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $- \frac{8a^3}{27b^3}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $\frac{4a^2}{3b^2} + \frac{3a}{b} + 4$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $- \frac{4a^2}{3b^2} - \frac{2a}{b} - 1$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $\frac{a}{b} + 3 + \frac{27b}{8a} + \frac{27b^2}{16a^2} + \frac{27b^3}{64a^3}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $- \frac{a}{b} - 3 - \frac{27b}{8a} - \frac{27b^2}{16a^2} - \frac{27b^3}{64a^3}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $0$	$\frac{2a}{3b} + 1 + \frac{3b}{4a}$ <hr style="border: 0; border-top: 1px solid black; margin: 5px 0;"/> $3\left(\frac{2a}{3b}\right)^2 = \frac{4a^2}{3b^2}$ $3\left(\frac{2a}{3b}\right)^2(1) = \frac{4a^2}{3b^2}$ $3\left(\frac{2a}{3b}\right)(1)^2 = \frac{2a}{b}$ $(1)^3 = 1$ $3\left(\frac{2a}{3b} + 1\right)^2 = \frac{4a^2}{3b^2} + \frac{4a}{b} + 3$ $3\left(\frac{2a}{3b} + 1\right)^2\left(\frac{3b}{4a}\right) = \frac{a}{b} + 3 + \frac{9b}{4a}$ $3\left(\frac{2a}{3b} + 1\right)\left(\frac{3b}{4a}\right)^2 = \frac{9b}{8a} + \frac{27b^2}{16a^2}$ $\left(\frac{3b}{4a}\right)^3 = \frac{27b^3}{64a^3}$
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## EJERCICIO 218

1.  $x^{\frac{1}{3}} = \sqrt[3]{x}$
2.  $m^{\frac{3}{5}} = \sqrt[5]{m^3}$
3.  $4a^{\frac{3}{4}} = 4\sqrt[4]{a^3}$
4.  $xy^{\frac{1}{2}} = x\sqrt{y}$
5.  $a^{\frac{4}{5}}b^{\frac{3}{2}} = \sqrt[5]{a^4} \sqrt[2]{b^3}$   
 $= \sqrt[5]{a^4} \sqrt[2]{b^2 b}$   
 $= b\sqrt[5]{a^4} \sqrt{b}$
6.  $x^{\frac{3}{2}}y^{\frac{1}{4}}z^{\frac{1}{5}} = \sqrt{x^3} \sqrt[4]{y} \sqrt[5]{z}$   
 $= \sqrt{x^2 x} \sqrt[4]{y} \sqrt[5]{z}$   
 $= x\sqrt{x} \sqrt[4]{y} \sqrt[5]{z}$
7.  $2a^{\frac{4}{5}}b^{\frac{5}{2}} = 2\sqrt[5]{a^4} \sqrt[2]{b^5}$   
 $= 2\sqrt[5]{a^4} \sqrt[2]{b^2 b^2 b}$   
 $= 2b^2 \sqrt[5]{a^4} \sqrt{b}$
8.  $3x^{\frac{2}{7}}y^{\frac{4}{5}}z^{\frac{2}{7}} = 3\sqrt[7]{x^2} \sqrt[5]{y^4} \sqrt[7]{z^2}$
9.  $a^{\frac{1}{4}}b^{\frac{5}{4}}c^{\frac{7}{4}} = \sqrt[4]{ab^5c^7}$   
 $= \sqrt[4]{ab^4bc^4c^3}$   
 $= bc\sqrt[4]{abc^3}$
10.  $8mn^{\frac{8}{3}} = 8m\sqrt[3]{n^8}$   
 $= 8m\sqrt[3]{n^3n^3n^2}$   
 $= 8mn^2\sqrt[3]{n^2}$
11.  $4a^2b^{\frac{7}{3}}c^{\frac{5}{6}} = 4a^2\sqrt[3]{b^7} \sqrt[6]{c^5}$   
 $= 4a^2\sqrt[3]{b^3b^3b} \sqrt[6]{c^5}$   
 $= 4a^2b^2\sqrt[3]{b} \sqrt[6]{c^5}$
12.  $5m^{\frac{2}{5}}n^{\frac{3}{5}}x^{\frac{4}{5}} = 5\sqrt[5]{m^2n^3x^4}$
13.  $\sqrt{a^5} = a^2\sqrt{a}$
14.  $\sqrt[3]{x^7} = x^2\sqrt[3]{x}$
15.  $\sqrt{x} = x^{\frac{1}{2}}$
16.  $\sqrt[3]{m} = m^{\frac{1}{3}}$
17.  $2\sqrt{x^5} = 2x^2\sqrt{x}$
18.  $\sqrt{a^3} \sqrt[3]{b^5} = a^{\frac{3}{2}}b^{\frac{5}{3}}$
19.  $3\sqrt{x^7} \sqrt[5]{y^6} = 3x^{\frac{7}{2}}y^{\frac{6}{5}}$
20.  $2\sqrt[4]{ab^3c^5} = 2a^{\frac{1}{4}}b^{\frac{3}{4}}c^{\frac{5}{4}}$
21.  $5a\sqrt{x^2y^3z^9} = 5ax^{\frac{2}{2}}y^{\frac{3}{2}}z^{\frac{9}{2}}$
22.  $3\sqrt[6]{m^7} \sqrt[5]{n^8} = 3m^{\frac{7}{6}}n^{\frac{8}{5}}$
23.  $3\sqrt{a^m} \sqrt[3]{b^n} = 3a^{\frac{m}{2}}b^{\frac{n}{3}}$
24.  $\sqrt[m]{a} \sqrt[n]{b^3} \sqrt[r]{c^x} = a^{\frac{1}{m}}b^{\frac{3}{n}}c^{\frac{x}{r}}$

## EJERCICIO 219

1.  $a^2b^{-3} = \frac{a^2}{b^3}$
2.  $3x^{-5} = \frac{3}{x^5}$
3.  $a^{-4}b^{-\frac{1}{2}} = \frac{1}{a^4b^{\frac{1}{2}}} = \frac{1}{a^4\sqrt{b}}$
4.  $3x^{-2}y^{-\frac{1}{3}} = \frac{3}{x^2y^{\frac{1}{3}}} = \frac{3}{x^2\sqrt[3]{y}}$
5.  $m^{-\frac{1}{2}}n^{-5} = \frac{1}{m^{\frac{1}{2}}n^5}$
6.  $a^2b^{-1}c = \frac{a^2c}{b}$
7.  $4x^2y^{\frac{3}{5}} = \frac{4x^2}{y^{\frac{2}{5}}}$
8.  $5a^{-\frac{1}{3}}b^{-\frac{3}{4}}c^{-1} = \frac{5}{a^{\frac{1}{3}}b^{\frac{3}{4}}c}$
9.  $\frac{1}{2x^{-2}} = \frac{x^2}{2}$
10.  $\frac{3}{x^{-1}y^{-5}} = 3xy^5$
11.  $\frac{2a^{-2}b^{-3}}{a^{-4}c^{-1}} = \frac{2a^4c}{a^2b^3} = \frac{2a^2c}{b^3}$
12.  $\frac{x^{-1}y^{-2}z^{-3}}{a^{-2}b^{-5}c^{-8}} = \frac{a^2b^5c^8}{xy^2z^3}$
13.  $\frac{3m^{-4}n^{-\frac{1}{2}}}{8m^{-3}n^{-4}} = \frac{3m^3n^{-\frac{1}{2}}n^4}{8m^4} = \frac{3n^{\frac{7}{2}}}{8m}$
14.  $\frac{4a^{\frac{1}{2}}}{7a^{-4}b^2c^{-\frac{2}{3}}} = \frac{4a^{\frac{1}{2}}a^4c^{\frac{2}{3}}}{7b^2} = \frac{4a^{\frac{9}{2}}c^{\frac{2}{3}}}{7b^2}$
15.  $\frac{2m^{-5}n^{-7}}{a^2m^3n^{-4}} = \frac{2}{a^2m^8n^3}$
16.  $\frac{a^{-\frac{1}{2}}x^{-2}}{3a^3x^2y^{-1}} = \frac{y}{3a^{\frac{7}{2}}x^{\frac{5}{2}}} = \frac{y}{3a^2x^2}$
17.  $\frac{c^2}{4b^{-\frac{1}{2}}x^3} = \frac{b^{\frac{1}{2}}c^2}{4x^3}$
18.  $\frac{1}{3a^{-\frac{3}{4}}b^{-\frac{2}{5}}c^4} = \frac{a^{\frac{3}{4}}b^{\frac{2}{5}}}{3c^4}$
19.  $\frac{3a^2mn}{a^{-3}m^{-\frac{1}{2}}n^{-4}} = 3a^{\frac{13}{2}}a^3mm^{\frac{1}{2}}nn^4 = 3a^{\frac{25}{2}}m^{\frac{3}{2}}n^{\frac{9}{2}}$
20.  $\frac{x^{\frac{2}{3}}y^{\frac{1}{4}}}{x^2yz^{-\frac{1}{2}}} = \frac{y^{\frac{1}{4}}}{x^{\frac{4}{3}}x^{\frac{2}{3}}yy^{\frac{1}{4}}} = \frac{y^{\frac{1}{2}}}{x^{\frac{8}{3}}y^{\frac{5}{4}}}$

## EJERCICIO 220

1.  $\frac{a^2}{b^2} = \frac{1}{a^{-2}b^2}$
2.  $\frac{3x^{-1}}{y^2} = \frac{3}{xy^2}$
3.  $\frac{4mr^2}{x^3} = \frac{4}{m^{-1}n^{-2}x^3}$
4.  $\frac{a^{-1}b^{-3}}{3} = \frac{1}{3ab^3}$
5.  $\frac{3c^{-\frac{2}{3}}}{7} = \frac{3}{7c^{\frac{2}{3}}}$
6.  $\frac{2x^{\frac{1}{4}}}{5y^2} = \frac{2}{5x^{-\frac{1}{4}}y^2}$
7.  $\frac{m^{-3}}{5} = \frac{1}{5m^3}$
8.  $\frac{3a^{-2}b^3}{c^4} = \frac{3}{a^2c^4b^{-3}}$
9.  $x^{-\frac{1}{2}}y^2 = \frac{1}{x^{\frac{1}{2}}y^{-2}}$
10.  $a^{-\frac{2}{3}}b^3c^{-2} = \frac{1}{a^{\frac{2}{3}}c^2b^{-3}}$
11.  $\frac{3x^{-1}y^{-\frac{1}{2}}}{y^3} = \frac{3}{xy^3y^{\frac{1}{2}}} = \frac{3}{xy^{\frac{7}{2}}}$
12.  $\frac{2m^{-2}n^2}{9} = \frac{2}{9m^2n^{-\frac{1}{2}}}$
13.  $\frac{2}{a} = 2a^{-1}$
14.  $\frac{3a}{b^2} = 3ab^{-2}$
15.  $\frac{x^2y}{y^{-2}} = x^2yy^2 = x^2y^3$
16.  $\frac{4}{x^{-\frac{1}{2}}y^2} = 4x^{\frac{1}{2}}y^{-2}$
17.  $\frac{3a^5}{7x^{-5}y^{-\frac{3}{4}}} = \frac{3a^5x^5y^{\frac{3}{4}}}{7}$
18.  $\frac{1}{a^{-4}b^{-\frac{1}{3}}} = a^4b^{\frac{1}{3}}$
19.  $\frac{2m^2}{3m^{-3}n^{-\frac{1}{4}}} = \frac{2m^2m^3n^{\frac{1}{4}}}{3} = \frac{2m^5n^{\frac{1}{4}}}{3}$
20.  $\frac{a^3}{x^2y^{-\frac{1}{2}}} = a^3x^{-2}y^{\frac{1}{2}}$
21.  $\frac{3a^2b^3}{a^{-1}x} = 3a^2ab^3x^{-1} = 3a^3b^3x^{-1}$
22.  $\frac{3xy^2z^3}{x^{-1}y^{-2}z^{-3}} = 3xy^2y^2z^3z^3 = 3x^2y^4z^6$
23.  $\frac{m^{-2}n^{-1}x^{-\frac{1}{2}}}{m^{-4}n^{-5}x^{-2}} = m^{-2}m^4n^{-1}n^5x^{-\frac{1}{2}}x^2 = m^2n^4x^{\frac{3}{2}}$

## EJERCICIO 221

1.  $x^{-\frac{1}{2}} = \frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}}$
2.  $\frac{1}{a^{-\frac{1}{2}}b^{\frac{2}{3}}} = \frac{a^{\frac{1}{2}}}{\sqrt[3]{b^2}} = \frac{\sqrt{a}}{\sqrt[3]{b^2}}$
3.  $5a^{\frac{5}{7}}b^{-\frac{1}{3}} = \frac{5\sqrt[7]{a^5}}{b^{\frac{1}{3}}} = \frac{5\sqrt[7]{a^5}}{\sqrt[3]{b}}$
4.  $\frac{3x^{-1}}{x^{-\frac{1}{2}}} = \frac{3}{x^{\frac{1}{2}}x} = \frac{3}{x^{\frac{3}{2}}} = \frac{3}{\sqrt{x}}$
5.  $2m^{-\frac{2}{5}}n^{\frac{3}{4}} = \frac{2\sqrt[4]{n^3}}{m^{\frac{2}{5}}} = \frac{2\sqrt[4]{n^3}}{\sqrt[5]{m^2}}$
6.  $\frac{1}{4x^{\frac{1}{3}}} = \frac{1}{4\sqrt[3]{x}}$
7.  $\frac{x^{\frac{3}{5}}}{y^{-\frac{2}{3}}} = \sqrt[5]{x^3}y^{\frac{2}{3}} = \sqrt[5]{x^3}\sqrt[3]{y^2}$
8.  $\frac{3a^{-\frac{3}{2}}}{x^{\frac{1}{4}}} = \frac{3}{a^{\frac{3}{2}}\sqrt[4]{x}} = \frac{3}{\sqrt[2]{a^2a}\sqrt[4]{x}} = \frac{3}{a\sqrt{a}\sqrt[4]{x}}$
9.  $\frac{a^{-\frac{1}{2}}}{4a^2} = \frac{1}{4a^{\frac{1}{2}}a^2} = \frac{1}{4a^{\frac{5}{2}}} = \frac{1}{4\sqrt[2]{a^5}} = \frac{1}{4a^2\sqrt{a}}$
10.  $x^{-\frac{2}{3}}y^{\frac{3}{5}}z^{-\frac{4}{7}} = \frac{\sqrt[5]{y^3}}{x^{\frac{2}{3}}z^{\frac{4}{7}}} = \frac{\sqrt[5]{y^3}}{\sqrt[3]{x^2}\sqrt[7]{z^4}}$
11.  $x^{-2}m^{-3}n^{-\frac{2}{5}} = \frac{1}{x^2m^3n^{\frac{2}{5}}} = \frac{1}{x^2m^3\sqrt[5]{n^2}}$
12.  $\left(a^{-\frac{1}{2}}\right)^3 = \frac{1}{a^{\frac{3}{2}}} = \frac{1}{\sqrt{a^3}} = \frac{1}{a\sqrt{a}}$
13.  $\left(x^{\frac{2}{3}}\right)^{-2} = x^{-\frac{4}{3}} = \frac{1}{x^{\frac{4}{3}}} = \frac{1}{\sqrt[3]{x^4x}} = \frac{1}{x\sqrt[3]{x}}$
14.  $\left(\frac{a}{b}\right)^{-\frac{3}{2}} = \frac{a^{-\frac{3}{2}}}{b^{-\frac{3}{2}}} = \frac{b^{\frac{3}{2}}}{a^{\frac{3}{2}}} = \frac{\sqrt[2]{b^3b}}{\sqrt[2]{a^3a}} = \frac{b\sqrt{b}}{a\sqrt{a}}$
15.  $\left(x^{-\frac{1}{2}}\right)^{\frac{1}{3}} = x^{-\frac{1}{6}} = \frac{1}{x^{\frac{1}{6}}} = \frac{1}{\sqrt[6]{x}}$
16.  $\sqrt{a^{-3}} = a^{-\frac{3}{2}} = \frac{1}{a^{\frac{3}{2}}}$
17.  $2\sqrt{x^{-3}y^{-4}} = 2x^{-\frac{3}{2}}y^{-\frac{4}{2}} = \frac{2}{x^{\frac{3}{2}}y^2}$

$$18. \frac{a^{\frac{2}{3}}}{\sqrt{x^{-5}}} = \frac{a^{\frac{2}{3}}}{x^{-\frac{5}{2}}} = a^{\frac{2}{3}} x^{\frac{5}{2}}$$

$$19. \frac{3\sqrt[3]{m^2}}{5\sqrt[4]{n^{-3}}} = \frac{3m^{\frac{2}{3}}}{5n^{-\frac{3}{4}}} = \frac{3m^{\frac{2}{3}}n^{\frac{3}{4}}}{5}$$

$$20. a^{-\frac{3}{5}}\sqrt[4]{b^{-3}} = \frac{b^{-\frac{3}{4}}}{a^{\frac{3}{5}}} = \frac{1}{\frac{3}{3} \frac{3}{3} a^{\frac{3}{5}} b^{\frac{3}{4}}}$$

$$21. x^2\sqrt{x^{-1}} = x^2x^{-\frac{1}{2}} = x^{\frac{3}{2}}$$

$$22. \frac{1}{\sqrt{a^{-7}b^{-6}}} = \frac{1}{a^{-\frac{7}{2}}b^{-\frac{6}{2}}} = a^{\frac{7}{2}}b^3$$

$$23. \frac{3x^{-\frac{2}{3}}}{\sqrt{y^{-4}}} = \frac{3y^{\frac{4}{2}}}{x^{\frac{2}{3}}} = \frac{3y^2}{x^{\frac{2}{3}}}$$

$$24. \sqrt{m^{-1}}\sqrt[3]{n^{-3}} = m^{-\frac{1}{2}}n^{-\frac{3}{3}} = \frac{1}{m^{\frac{1}{2}}n}$$

$$25. 16^{\frac{3}{2}} = \sqrt{(16)^3}$$

$$= \sqrt{(16)^2(16)} = 16\sqrt{16} = 64$$

$$26. 8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$$

$$27. 81^{\frac{3}{4}} = 81^{\frac{1}{2}} 81^{\frac{1}{4}}$$

$$= \sqrt{81} \sqrt[4]{81} = 9 \cdot 3 = 27$$

$$28. 9^{\frac{5}{2}} = \frac{1}{9^{\frac{5}{2}}}$$

$$= \frac{1}{\sqrt{9^5}}$$

$$= \frac{1}{\sqrt{9^2 9^2 9}} = \frac{1}{81\sqrt{9}} = \frac{1}{243}$$

$$29. (-27)^{\frac{2}{3}} = \sqrt[3]{(-27)^2}$$

$$= \sqrt[3]{-27} \sqrt[3]{-27}$$

$$= -3 \cdot -3 = 9$$

$$30. (-32)^{\frac{2}{5}} = \sqrt[5]{(-32)^2}$$

$$= \sqrt[5]{-32} \sqrt[5]{-32}$$

$$= -2 \cdot -2 = 4$$

$$31. 49^{-\frac{3}{2}} = \frac{1}{\sqrt{49^3}} = \frac{1}{\sqrt{49^2} \sqrt{49}}$$

$$= \frac{1}{49 \cdot 7} = \frac{1}{343}$$

$$32. \left(\frac{4}{9}\right)^{\frac{5}{2}} = \frac{\sqrt{4^5}}{\sqrt{9^5}}$$

$$= \frac{\sqrt{4^2 4^2 4}}{\sqrt{9^2 9^2 9}}$$

$$= \frac{16\sqrt{4}}{81\sqrt{9}}$$

$$= \frac{16 \cdot 2}{81 \cdot 3} = \frac{32}{243}$$

$$33. \left(\frac{8}{27}\right)^{-\frac{1}{3}} = \frac{27^{\frac{1}{3}}}{8^{\frac{1}{3}}}$$

$$= \frac{\sqrt[3]{27}}{\sqrt[3]{8}} = \frac{3}{2} = 1\frac{1}{2}$$

$$34. \left(\frac{25}{36}\right)^{-\frac{1}{2}} = \frac{36^{\frac{1}{2}}}{25^{\frac{1}{2}}}$$

$$= \frac{\sqrt{36}}{\sqrt{25}} = \frac{6}{5} = 1\frac{1}{5}$$

$$35. \left(\frac{32}{243}\right)^{-\frac{1}{5}} = \frac{243^{\frac{1}{5}}}{32^{\frac{1}{5}}}$$

$$= \frac{\sqrt[5]{243}}{\sqrt[5]{32}} = \frac{3}{2} = 1\frac{1}{2}$$

$$36. \left(-\frac{27}{64}\right)^{\frac{2}{3}} = \frac{64^{\frac{2}{3}}}{-27^{\frac{2}{3}}}$$

$$= \frac{\sqrt[3]{64^2}}{\sqrt[3]{(-27)^2}}$$

$$= \frac{\sqrt[3]{64} \sqrt[3]{64}}{\sqrt[3]{-27} \sqrt[3]{-27}}$$

$$= \frac{4 \cdot 4}{-3 \cdot -3} = \frac{16}{9} = 1\frac{7}{9}$$

$$37. \frac{1}{9^{-3}} = 9^3 = 729$$

$$38. \left(\frac{16}{81}\right)^{-\frac{5}{4}} = \frac{81^{\frac{5}{4}}}{16^{\frac{5}{4}}}$$

$$= \frac{\sqrt[4]{81^4} \sqrt[4]{81}}{\sqrt[4]{16^4} \sqrt[4]{16}}$$

$$= \frac{81 \cdot 3}{16 \cdot 2} = \frac{243}{32} = 7\frac{19}{32}$$

$$39. \left(-\frac{32}{243}\right)^{-\frac{2}{5}} = \frac{243^{\frac{2}{5}}}{-32^{\frac{2}{5}}}$$

$$= \frac{\sqrt[5]{243^2} \sqrt[5]{243}}{\sqrt[5]{-32} \sqrt[5]{-32}}$$

$$= \frac{3 \cdot 3}{-2 \cdot -2} = \frac{9}{4} = 2\frac{1}{4}$$

$$40. (2\frac{7}{9})^{-\frac{3}{2}} = \left(\frac{25}{9}\right)^{-\frac{3}{2}}$$

$$= \frac{9^{\frac{3}{2}}}{25^{\frac{3}{2}}}$$

$$= \frac{\sqrt{9^2} \sqrt{9}}{\sqrt{25^2} \sqrt{25}}$$

$$= \frac{9 \cdot 3}{25 \cdot 5} = \frac{27}{125}$$

$$41. (5\frac{1}{16})^{-\frac{1}{4}} = \left(\frac{81}{16}\right)^{-\frac{1}{4}}$$

$$= \frac{16^{\frac{1}{4}}}{81^{\frac{1}{4}}} = \frac{\sqrt[4]{16}}{\sqrt[4]{81}} = \frac{2}{3}$$

$$42. 8^{\frac{2}{3}} \cdot 4^{\frac{3}{2}} = \sqrt[3]{8} \sqrt[3]{8} \sqrt{4^2} \sqrt{4}$$

$$= 2 \cdot 2 \cdot 4 \cdot 2 = 32$$

$$43. 9^{\frac{5}{2}} \cdot 27^{-\frac{1}{3}} = \frac{\sqrt{9^5}}{\sqrt[3]{27}}$$

$$= \frac{\sqrt{9^2} \sqrt{9^2} \sqrt{9}}{3} = 81$$

$$44. 243^{\frac{1}{5}} \cdot 128^{\frac{3}{7}}$$

$$= \frac{\sqrt[5]{128^3}}{\sqrt[5]{243}} = \frac{\sqrt[5]{128} \sqrt[5]{128} \sqrt[5]{128}}{3}$$

$$= \frac{2 \cdot 2 \cdot 2}{3} = \frac{8}{3} = 2\frac{2}{3}$$

## EJERCICIO 222

1.  $a^{-2} + a^{-1}b^{\frac{1}{2}} + x^0$  para  $a=3$   $b=4$

$$\begin{aligned} &= 3^{-2} + 3^{-1} \cdot 4^{\frac{1}{2}} + x^0 \\ &= \frac{1}{3^2} + \frac{1\sqrt{4}}{3} + 1 \\ &= \frac{1}{9} + \frac{2}{3} + 1 = \frac{1+6+9}{9} = \frac{16}{9} = 1\frac{7}{9} \end{aligned}$$

2.  $3x^{\frac{1}{2}} + x^2y^{-3} + x^0y^{\frac{1}{3}}$  para  $x=4$   $y=1$

$$\begin{aligned} &= 3(4)^{\frac{1}{2}} + 4^2(1)^{-3} + 1(1)^{\frac{1}{3}} \\ &= \frac{3}{\sqrt{4}} + 16 + 1 = \frac{3}{2} + 17 = \frac{3+34}{2} = \frac{37}{2} = 18\frac{1}{2} \end{aligned}$$

3.  $2a^{-3}b + \frac{a^{-4}}{b^{-1}} + a^{\frac{1}{2}}b^{-\frac{3}{4}}$  para  $a=4$   $b=16$

$$\begin{aligned} &= 2(4)^{-3} \cdot 16 + \frac{4^{-4}}{16^{-1}} + 4^{\frac{1}{2}} \cdot 16^{-\frac{3}{4}} \\ &= \frac{2}{4^3} \cdot 16 + \frac{16}{4^4} + \frac{\sqrt{4}}{\sqrt[4]{(16)^2 \cdot 16}} \\ &= \frac{1}{2} + \frac{1}{16} + \frac{2}{16^2 \sqrt[4]{16}} = \frac{1}{2} + \frac{1}{16} + \frac{1}{4} = \frac{8+1+4}{16} = \frac{13}{16} \end{aligned}$$

4.  $\frac{x^{\frac{3}{4}}}{y^{-2}} + x^{-\frac{1}{2}}y^{-\frac{1}{3}} - x^0y^0 + \frac{x}{y^{\frac{4}{3}}}$  para  $x=16$   $y=8$

$$\begin{aligned} &= \frac{16^{\frac{3}{4}}}{8^{-2}} + 16^{-\frac{1}{2}} \cdot 8^{-\frac{1}{3}} - 16^0 \cdot 8^0 + \frac{16}{8^{\frac{4}{3}}} \\ &= \sqrt[4]{16^2} \cdot \sqrt[4]{16} \cdot 8^2 + \frac{1}{\sqrt{16} \sqrt[3]{8}} - 1 + \frac{16}{\sqrt[3]{(8^3)8}} \\ &= 4 \cdot 2 \cdot 64 + \frac{1}{4 \cdot 2} - 1 + \frac{16}{8 \sqrt[3]{8}} = 512 + \frac{1}{8} - 1 + 1 \\ &= 512\frac{1}{8} \end{aligned}$$

5.  $\frac{x^0}{x^{-1}} + \frac{y^{-3}}{y^0} + 2x^0 + x^{\frac{3}{4}}y^{-2}$  para  $x=81$   $y=3$

$$\begin{aligned} &= \frac{1}{81^{-1}} + \frac{3^{-3}}{1} + 2(1) + 81^{\frac{3}{4}} \cdot 3^{-2} \\ &= 81 + \frac{1}{3^3} + 2 + \frac{\sqrt[4]{81^2} \cdot \sqrt[4]{81}}{3^2} \\ &= 83 + \frac{1}{27} + \frac{9 \cdot 3}{9} = 83 + \frac{1}{27} + 3 = 86 + \frac{1}{27} = 86\frac{1}{27} \end{aligned}$$

6.  $a^{\frac{1}{2}}x^{\frac{1}{3}} + a^{-\frac{1}{2}}x^{-\frac{1}{3}} + \frac{1}{a^{-\frac{1}{4}}x^{-1}} + 3x^0$  para  $a=16$   $x=8$

$$\begin{aligned} &= 16^{\frac{1}{2}} \cdot 8^{\frac{1}{3}} + 16^{-\frac{1}{2}} \cdot 8^{-\frac{1}{3}} + \frac{1}{16^{-\frac{1}{4}} \cdot 8^{-1}} + 3(1) \\ &= \sqrt{16} \sqrt[3]{8} + \frac{1}{\sqrt{16} \sqrt[3]{8}} + \sqrt[4]{16} \cdot 8 + 3 \\ &= 8 + \frac{1}{4 \cdot 2} + 16 + 3 = 27 + \frac{1}{8} = 27\frac{1}{8} \end{aligned}$$

7.  $\frac{a^{-2}}{b^{-1}} + 3a^{-1}b^2c^{-3} - \frac{a^{-2}}{b^{\frac{1}{2}}c^{-1}} + b^4 + c^0$

para  $b=16$   $c=2$   $a=3$

$$\begin{aligned} &= \frac{3^{-2}}{16^{-1}} + 3(3)^{-1} \cdot 16^2(2)^{-3} - \frac{3^{-2}}{16^{\frac{1}{2}} \cdot 2^{-1}} + 16^4 + 1 \\ &= \frac{16}{3^2} + \frac{3}{3} \cdot \frac{256}{2^3} - \frac{2}{3^2 \sqrt{16}} + \sqrt[4]{16} + 1 \\ &= \frac{16}{9} + 32 - \frac{1}{18} + 2 + 1 \\ &= \frac{16}{9} + 35 - \frac{1}{18} = \frac{32+630-1}{18} = \frac{661}{18} = 36\frac{13}{18} \end{aligned}$$

8.  $\frac{x^0}{3y^0} + x^{\frac{2}{3}} - y^{\frac{1}{5}} + \frac{x^{-2}}{y^{-1}} + y^0$  para  $x=8$   $y=32$

$$\begin{aligned} &= \frac{1}{3(1)} + 8^{\frac{2}{3}} - 32^{\frac{1}{5}} + \frac{8^{-2}}{32^{-1}} + 1 \\ &= \frac{1}{3} + \sqrt[3]{(2^3)^2} - \sqrt[5]{32} + \frac{32}{8^2} + 1 \\ &= \frac{1}{3} + 2^2 - 2 + \frac{1}{2} + 1 = \frac{1}{3} + 3 + \frac{1}{2} = \frac{2+18+3}{6} = \frac{23}{6} = 3\frac{5}{6} \end{aligned}$$

9.  $a^{-\frac{1}{3}} - \frac{1}{b^{\frac{4}{5}}} + a^0b - \sqrt[3]{ab^{\frac{2}{5}}} - \frac{1}{a^{-\frac{2}{3}}}$  para  $a=27$   $b=243$

$$\begin{aligned} &= 27^{-\frac{1}{3}} - \frac{1}{243^{\frac{4}{5}}} + (1)243 - \sqrt[3]{27 \cdot 243^{\frac{2}{5}}} - \frac{1}{27^{-\frac{2}{3}}} \\ &= \frac{1}{\sqrt[3]{27}} - \sqrt[5]{(3^5)^4} + 243 - 3 \sqrt[3]{(3^5)^2} - \sqrt[3]{(3^3)^2} \\ &= \frac{1}{3} - 3^4 + 243 - 3(3)^2 - 3^2 \\ &= \frac{1}{3} + 162 - 27 - 9 = \frac{1}{3} + 126 = 126\frac{1}{3} \end{aligned}$$

## EJERCICIO 223

- $x^2x^{-3}=x^{2+(-3)}=x^{-1}$
- $a^{-2}a^{-3}=a^{-2-3}=a^{-5}$
- $x^3x^{-3}=x^{3-3}=x^0=1$
- $a^{\frac{1}{2}}a=a^{\frac{1}{2}+1}=a^{\frac{3}{2}}$
- $x^{\frac{1}{2}}x^{\frac{1}{4}}=x^{\frac{1}{2}+\frac{1}{4}}=x^{\frac{3}{4}}$
- $a^{\frac{3}{4}}a^{\frac{1}{4}}=a^{\frac{3}{4}+\frac{1}{4}}=a^1=a$
- $3m^{\frac{2}{5}}m^{-\frac{3}{5}}=3m^{\frac{2}{5}-\frac{3}{5}}=3m^{-\frac{1}{5}}$
- $2a^{\frac{3}{4}}a^{-\frac{1}{2}}=2a^{\frac{3}{4}-\frac{1}{2}}=2a^{\frac{1}{4}}$
- $x^{-2}x^{-\frac{1}{3}}=x^{-2-\frac{1}{3}}=x^{-\frac{7}{3}}$
- $3r^{\frac{2}{3}}n^{-\frac{2}{3}}=3n^{-\frac{2}{3}}=3n^{\frac{4}{3}}$
- $4a^{-2}a^{-\frac{1}{2}}=4a^{-2-\frac{1}{2}}=4a^{-\frac{5}{2}}$
- $a^{-1}b^{-2}ab^2=a^{-1+1}b^{-2+2}=a^0b^0=1$
- $x^{-3}y^{\frac{1}{2}}x^{-2}y^{-\frac{1}{2}}=x^{-3-2}y^{\frac{1}{2}-\frac{1}{2}}=x^{-5}y^0=x^{-5}$
- $3a^2b^{\frac{1}{2}}2a^{-2}b^{-\frac{1}{2}}=3\cdot 2a^{2-2}b^{\frac{1}{2}-\frac{1}{2}}=6a^0b^0=6$
- $a^3b^{-1}a^{-2}b^{-2}=a^{3-2}b^{-1-2}=ab^{-3}$
- $a^{-\frac{1}{2}}b^{\frac{3}{4}}a^{\frac{1}{2}}b^{\frac{1}{4}}=a^{-\frac{1}{2}+\frac{1}{2}}b^{\frac{3}{4}+\frac{1}{4}}=a^0b^1=b$
- $m^{-\frac{2}{3}}n^{\frac{1}{3}}m^{-\frac{1}{3}}n^{\frac{2}{3}}=m^{-\frac{2}{3}-\frac{1}{3}}n^{\frac{1}{3}+\frac{2}{3}}=m^{-1}n$
- $2a^{-1}b^{\frac{3}{4}}ab^{-2}=2a^{-1+1}b^{\frac{3}{4}-2}=2a^0b^{-\frac{5}{4}}=2b^{-\frac{5}{4}}$

## EJERCICIO 224

- $$\begin{array}{r} a^{-4}+3a^{-2}+2 \\ \hline a^{-4}-a^{-2}+1 \\ \hline a^{-8}+3a^{-6}+2a^{-4} \\ -a^{-6}-3a^{-4}-2a^{-2} \\ \hline a^{-4}+3a^{-2}+2 \\ \hline a^{-8}+2a^{-6} \quad +a^{-2}+2 \end{array}$$
- $$\begin{array}{r} x^2-1+x^{-2} \\ \hline x^2+2-x^{-2} \\ \hline x^4-x^2+1 \\ 2x^2-2+2x^{-2} \\ -1+x^{-2}-x^{-4} \\ \hline x^4+x^2-2+3x^{-2}-x^{-4} \end{array}$$
- $$\begin{array}{r} x+2x^{\frac{2}{3}}+x^{\frac{1}{3}} \\ \hline x^{\frac{1}{3}}-2+x^{\frac{1}{3}} \\ \hline x^{\frac{4}{3}}+2x+x^{\frac{2}{3}} \\ -2x-4x^{\frac{2}{3}}-2x^{\frac{1}{3}} \\ \hline x^{\frac{2}{3}}+2x^{\frac{1}{3}}+1 \\ \hline x^{\frac{4}{3}}-2x^{\frac{2}{3}}+1 \end{array}$$
- $$\begin{array}{r} a^{\frac{2}{3}}-2+2a^{-\frac{2}{3}} \\ \hline 3+a^{-\frac{2}{3}}-4a^{-\frac{4}{3}} \\ \hline 3a^{\frac{2}{3}}-6+6a^{-\frac{2}{3}} \\ +1-2a^{-\frac{2}{3}}+2a^{-\frac{4}{3}} \\ -4a^{-\frac{2}{3}}+8a^{-\frac{4}{3}}-8a^{-2} \\ \hline 3a^{\frac{2}{3}}-5 \quad +10a^{-\frac{4}{3}}-8a^{-2} \end{array}$$
- $$\begin{array}{r} x^{\frac{3}{4}}+2x^{\frac{1}{4}}-x^{-\frac{1}{4}} \\ \hline x^{\frac{1}{2}}-2+x^{-\frac{1}{2}} \\ \hline x^{\frac{5}{4}}+2x^{\frac{3}{4}}-x^{\frac{1}{4}} \\ -2x^{\frac{3}{4}}-4x^{\frac{1}{4}}+2x^{-\frac{1}{4}} \\ \hline x^{\frac{1}{4}}+2x^{-\frac{1}{4}}-x^{-\frac{3}{4}} \\ \hline x^{\frac{5}{4}}-4x^{\frac{1}{4}}+4x^{-\frac{1}{4}}-x^{-\frac{3}{4}} \end{array}$$
- $$\begin{array}{r} a^2b^{-1}+a+b \\ \hline a^{-2}b^{-2}-a^{-3}b^{-1}+a^{-4} \\ \hline b^{-3}+a^{-1}b^{-2}+a^{-2}b^{-1} \\ -a^{-1}b^{-2}-a^{-2}b^{-1}-a^{-3} \\ \hline a^{-2}b^{-1}+a^{-3}+a^{-4}b \\ \hline b^{-3} \quad +a^{-2}b^{-1} \quad +a^{-4}b \end{array}$$
- $$\begin{array}{r} x^{-1}y^{-1}+x^{-3}y^{-3}+x^{-5}y^{-5} \\ \hline x^{-3}y^{-2}-x^{-5}y^{-4}+x^{-7}y^{-6} \\ \hline x^{-4}y^{-3}+x^{-6}y^{-5}+x^{-8}y^{-7} \\ -x^{-6}y^{-5}-x^{-8}y^{-7}-x^{-10}y^{-9} \\ \hline x^{-8}y^{-7}+x^{-10}y^{-9}+x^{-12}y^{-11} \\ \hline x^{-4}y^{-3} \quad +x^{-8}y^{-7} \quad +x^{-12}y^{-11} \end{array}$$

$$9. \frac{a^{\frac{3}{4}}b^{-3} + a^{\frac{1}{4}}b^{-2} - a^{-\frac{1}{4}}b^{-1}}{a^{\frac{1}{2}}b^{-1} - 2 + 3a^{-\frac{1}{2}}b}$$

$$\frac{a^{\frac{5}{4}}b^{-4} + a^{\frac{3}{4}}b^{-3} - a^{\frac{1}{4}}b^{-2} - 2a^{\frac{3}{4}}b^{-3} - 2a^{\frac{1}{4}}b^{-2} + 2a^{-\frac{1}{4}}b^{-1}}{3a^{\frac{1}{4}}b^{-2} + 3a^{-\frac{1}{4}}b^{-1} - 3a^{-\frac{3}{4}}}$$

$$\frac{a^{\frac{5}{4}}b^{-4} - a^{\frac{3}{4}}b^{-3} + 5a^{-\frac{1}{4}}b^{-1} - 3a^{-\frac{3}{4}}}{}$$

$$10. \frac{a^{-1} + 2a^{-\frac{1}{2}}b^{-\frac{1}{2}} + 2b^{-1}}{a^{-1} - a^{-\frac{1}{2}}b^{-\frac{1}{2}} + b^{-1}}$$

$$\frac{a^{-2} + 2a^{-\frac{3}{2}}b^{-\frac{1}{2}} + 2a^{-1}b^{-1} - a^{-\frac{3}{2}}b^{-\frac{1}{2}} - 2a^{-1}b^{-1} - 2a^{-\frac{1}{2}}b^{-\frac{3}{2}}}{+ a^{-1}b^{-1} + 2a^{-\frac{1}{2}}b^{-\frac{3}{2}} + 2b^{-2}}$$

$$\frac{a^{-2} + a^{-\frac{3}{2}}b^{-\frac{1}{2}} + a^{-1}b^{-1} + 2b^{-2}}{}$$

$$11. \frac{4x^2 - x^{\frac{3}{2}}y^{\frac{1}{2}} + xy - x^{\frac{1}{2}}y^{\frac{3}{2}}}{x^{\frac{1}{2}} + y^{\frac{1}{2}}}$$

$$\frac{4x^2 - x^{\frac{3}{2}}y^{\frac{1}{2}} + x^{\frac{3}{2}}y - xy^{\frac{3}{2}}}{4x^2y^{\frac{1}{2}} - x^{\frac{3}{2}}y + xy^{\frac{3}{2}} - x^{\frac{1}{2}}y^2}$$

$$\frac{4x^2 + 3x^{\frac{3}{2}}y^{\frac{1}{2}} - x^{\frac{1}{2}}y^2}{}$$

$$12. \frac{x - 2a^{\frac{1}{3}}x^{\frac{2}{3}} + a^{\frac{2}{3}}x^{\frac{1}{3}} - 3a}{x^{\frac{4}{3}} + 3a^{\frac{2}{3}}x^{\frac{2}{3}} + 2a^{\frac{1}{3}}x}$$

$$\frac{x^{\frac{7}{3}} - 2a^{\frac{1}{3}}x^{\frac{4}{3}} + a^{\frac{2}{3}}x^{\frac{5}{3}} - 3ax^{\frac{4}{3}} + 3a^{\frac{2}{3}}x^{\frac{5}{3}} - 6ax^{\frac{4}{3}} + 3a^{\frac{4}{3}}x - 9a^{\frac{5}{3}}x^{\frac{2}{3}}}{+ 2a^{\frac{1}{3}}x^2 - 4a^{\frac{2}{3}}x^{\frac{5}{3}} + 2ax^{\frac{4}{3}} - 6a^{\frac{4}{3}}x}$$

$$\frac{x^{\frac{7}{3}} - 7ax^{\frac{4}{3}} - 3a^{\frac{4}{3}}x - 9a^{\frac{5}{3}}x^{\frac{2}{3}}}{}$$

$$13. \frac{5a^2 - 3a + 4 - 2a^{-1}}{3a + 2 - 5a^{-1}}$$

$$\frac{15a^3 - 9a^2 + 12a - 6}{10a^2 - 6a + 8 - 4a^{-1} - 25a + 15 - 20a^{-1} + 10a^{-2}}$$

$$\frac{15a^3 + a^2 - 19a + 17 - 24a^{-1} + 10a^{-2}}{}$$

$$14. \frac{2x - 3 + x^{-1} + 4x^{-2}}{x^{-1} - 2x^{-2} + x^{-3}}$$

$$\frac{2 - 3x^{-1} + x^{-2} + 4x^{-3} - 4x^{-1} + 6x^{-2} - 2x^{-3} - 8x^{-4} + 2x^{-2} - 3x^{-3} + x^{-4} + 4x^{-5}}{2 - 7x^{-1} + 9x^{-2} - x^{-3} - 7x^{-4} + 4x^{-5}}$$

$$15. \frac{m - m^{\frac{1}{2}}n^{\frac{1}{2}} + n - m^{-\frac{1}{2}}n^{\frac{3}{2}}}{m^{\frac{1}{2}} + n^{\frac{1}{2}} + m^{-\frac{1}{2}}n}$$

$$\frac{m^{\frac{3}{2}} - mn^{\frac{1}{2}} + m^{\frac{1}{2}}n - n^{\frac{3}{2}} + mn^{\frac{1}{2}} - m^{\frac{1}{2}}n + n^{\frac{3}{2}} - m^{-\frac{1}{2}}n^2 + m^{\frac{1}{2}}n - n^{\frac{3}{2}} + m^{-\frac{1}{2}}n^2 - m^{-1}n^{\frac{5}{2}}}{m^{\frac{3}{2}} + m^{\frac{1}{2}}n - n^{\frac{3}{2}} - m^{-1}n^{\frac{5}{2}}}$$

$$16. \frac{a^{\frac{3}{5}} + 2a^{\frac{1}{5}} - a^{-\frac{1}{5}}}{-2 + a^{\frac{2}{5}} - a^{-\frac{2}{5}}}$$

$$\frac{-2a^{\frac{3}{5}} - 4a^{\frac{1}{5}} + 2a^{-\frac{1}{5}} + 2a^{\frac{3}{5}} - a^{\frac{1}{5}} + a - a^{\frac{1}{5}} - 2a^{-\frac{1}{5}} + a^{-\frac{3}{5}}}{-6a^{\frac{1}{5}} + a + a^{-\frac{3}{5}}}$$

$$17. \frac{m + 3m^{\frac{2}{3}} + 2m^{\frac{1}{3}}}{2 - 2m^{\frac{1}{3}} + 2m^{-\frac{2}{3}}}$$

$$\frac{2m + 6m^{\frac{2}{3}} + 4m^{\frac{1}{3}} - 2m^{\frac{2}{3}} - 6m^{\frac{1}{3}} - 4 + 2m^{\frac{1}{3}} + 6 + 4m^{-\frac{1}{3}}}{2m + 4m^{\frac{2}{3}} + 2 + 4m^{-\frac{1}{3}}}$$

$$18. \frac{x^{-\frac{3}{4}}y^{\frac{3}{4}} + 3x^{-\frac{1}{4}}y - x^{\frac{1}{4}}y^{\frac{1}{2}}}{x^{-\frac{5}{4}}y^{\frac{1}{2}} - 3x^{-\frac{3}{4}} - x^{-\frac{1}{4}}y^{\frac{1}{2}}}$$

$$\frac{x^{-2}y^2 + 3x^{-\frac{3}{2}}y^{\frac{3}{2}} - x^{-1}y - 3x^{-\frac{3}{2}}y^{\frac{3}{2}} - 9x^{-1}y + 3x^{-\frac{1}{2}}y^{\frac{1}{2}} - x^{-1}y - 3x^{-\frac{1}{2}}y^{\frac{1}{2}} + 1}{x^{-2}y^2 - 11x^{-1}y + 1}$$

$$19. \frac{x^2y^{-1} + 5x^3y^{-3} + 2x^4y^{-5}}{x^{-3}y^3 - x^{-2}y + 3x^{-1}y^{-1}}$$

$$\frac{x^{-1}y^2 + 5 + 2xy^{-2} - 1 - 5xy^{-2} - 2x^2y^{-4} + 3xy^{-2} + 15x^2y^{-4} + 6x^3y^{-6}}{x^{-1}y^2 + 4 + 13x^2y^{-4} + 6x^3y^{-6}}$$

$$20. \frac{a^{-\frac{2}{3}}b^{\frac{1}{2}} + 2a^{-\frac{4}{3}}b - a^{-2}b^{\frac{3}{2}}}{3a^{\frac{2}{3}}b^{-\frac{1}{2}} + 1 + a^{-\frac{2}{3}}b^{\frac{1}{2}}}$$

$$\frac{3 + 6a^{-\frac{2}{3}}b^{\frac{1}{2}} - 3a^{-\frac{4}{3}}b + a^{-\frac{2}{3}}b^{\frac{1}{2}} + 2a^{-\frac{4}{3}}b - a^{-2}b^{\frac{3}{2}}}{a^{-\frac{4}{3}}b + 2a^{-2}b^{\frac{3}{2}} - a^{-\frac{8}{3}}b^2}$$

$$3 + 7a^{-\frac{2}{3}}b^{\frac{1}{2}} + a^{-2}b^{\frac{3}{2}} - a^{-\frac{8}{3}}b^2$$

## EJERCICIO 225

$$1. a^2 \div a^{-2} = a^{2-(-2)} = a^{2+2} = a^4$$

$$2. x^{-3} \div x^2 = x^{-3-2} = x^{-5}$$

$$3. m^{\frac{1}{2}} \div m^{-\frac{1}{4}} = m^{\frac{1}{2}-(-\frac{1}{4})} = m^{\frac{1}{2}+\frac{1}{4}} = m^{\frac{3}{4}}$$

$$4. a^2 \div a^5 = a^{2-5} = a^{-3}$$

$$5. x^{-3} \div x^{-7} = x^{-3-(-7)} = x^{-3+7} = x^4$$

$$6. a^{\frac{1}{2}} \div a = a^{\frac{1}{2}-1} = a^{-\frac{1}{2}}$$

$$7. x^{-\frac{2}{3}} \div x^{-\frac{1}{3}} = x^{-\frac{2}{3}-(-\frac{1}{3})} = x^{-\frac{2}{3}+\frac{1}{3}} = x^{-\frac{1}{3}}$$

$$8. a^{\frac{2}{5}} \div a^{-\frac{1}{5}} = a^{\frac{2}{5}-(-\frac{1}{5})} = a^{\frac{2}{5}+\frac{1}{5}} = a^{\frac{3}{5}}$$

$$9. m^{-\frac{3}{4}} \div m^2 = m^{-\frac{3}{4}-2} = m^{-\frac{5}{4}}$$

$$10. a^{\frac{1}{3}} \div a = a^{\frac{1}{3}-1} = a^{-\frac{2}{3}}$$

$$11. 4x^{\frac{2}{5}} \div 2x^{-\frac{1}{5}} = \frac{4}{2}x^{\frac{2}{5}-(-\frac{1}{5})} = 2x^{\frac{2}{5}+\frac{1}{5}} = 2x^{\frac{3}{5}}$$

$$12. a^{-3} \div a^{-\frac{7}{4}} = a^{-3-(-\frac{7}{4})} = a^{-3+\frac{7}{4}} = a^{-\frac{5}{4}}$$

$$13. x^{-2}y^{-1} \div x^{-3}y^{-2} = x^{-2+3}y^{-1+2} = xy$$

$$14. a^{\frac{1}{2}}b^{\frac{1}{3}} \div ab = a^{\frac{1}{2}-1}b^{\frac{1}{3}-1} = a^{-\frac{1}{2}}b^{-\frac{2}{3}}$$

$$15. a^2b^{-3} \div a^{-1}b = a^{2+1}b^{-3-1} = a^3b^{-4}$$

$$16. x^{-\frac{1}{2}}y^{-\frac{2}{3}} \div x^{-\frac{1}{2}}y^{-1} = x^{-\frac{1}{2}+\frac{1}{2}}y^{-\frac{2}{3}+1} = x^0y^{\frac{1}{3}} = y^{\frac{1}{3}}$$

$$17. m^{\frac{3}{4}}n^{-\frac{3}{4}} \div m^{-\frac{1}{2}}n^{\frac{3}{4}} = m^{\frac{3}{4}+\frac{1}{2}}n^{-\frac{3}{4}-\frac{3}{4}} = m^{\frac{5}{4}}n^{-\frac{3}{2}}$$

$$18. 8x^{-2}y^{\frac{2}{5}} \div 4xy^{-\frac{1}{5}} = \frac{8}{4}x^{-2-1}y^{\frac{2}{5}+\frac{1}{5}} = 2x^{-3}y^{\frac{3}{5}}$$

$$19. a^{\frac{1}{3}}b \div a^{-\frac{1}{4}}b^{-3} = a^{\frac{1}{3}+\frac{1}{4}}b^{1+3} = a^{\frac{7}{12}}b^4$$

$$20. x^{-4}y^{-5} \div x^2y^{-1} = x^{-4-2}y^{-5+1} = x^{-6}y^{-4}$$

## EJERCICIO 226

$$1. \frac{x^{-8} + 2x^{-6}}{-x^{-8} + x^{-6} - x^{-4}} + x^{-2} + 2 \frac{x^{-4} - x^{-2} + 1}{x^4 + 3x^{-2} + 2}$$

$$\frac{3x^{-6} - x^{-4} + x^{-2}}{-3x^{-6} + 3x^{-4} - 3x^{-2}} + 2$$

$$\frac{2x^{-4} - 2x^{-2} + 2}{-2x^{-4} + 2x^{-2} - 2}$$

$$2. \frac{a^{\frac{4}{3}}}{-a^{\frac{4}{3}} - 2a} - \frac{2a^{\frac{2}{3}}}{a^{\frac{2}{3}}} + 1 \frac{a + 2a^{\frac{2}{3}} + a^{\frac{1}{3}}}{a^{\frac{1}{3}} - 2 + a^{-\frac{1}{3}}}$$

$$\frac{-2a - 3a^{\frac{2}{3}}}{+2a + 4a^{\frac{2}{3}} + 2a^{\frac{1}{3}}} + 1$$

$$\frac{a^{\frac{2}{3}} + 2a^{\frac{1}{3}} + 1}{-a^{\frac{2}{3}} - 2a^{\frac{1}{3}} - 1}$$

$$\begin{array}{r}
 3. \quad m^4 + m^2 - 2 + 3m^{-2} - m^{-4} \quad \left| \begin{array}{l} m^2 - 1 + m^{-2} \\ m^2 + 2 - m^{-2} \end{array} \right. \\
 \hline
 -m^4 + m^2 - 1 \\
 2m^2 - 3 + 3m^{-2} \\
 \hline
 -2m^2 + 2 - 2m^{-2} \\
 \hline
 -1 + m^{-2} - m^{-4} \\
 \hline
 +1 - m^{-2} + m^{-4} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 4. \quad 2x + x^{\frac{3}{4}} - x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 2 \quad \left| \begin{array}{l} x^{\frac{1}{4}} + 1 - x^{-\frac{1}{4}} \\ 2x^{\frac{3}{4}} - x^{\frac{1}{2}} + 2x^{\frac{1}{4}} \end{array} \right. \\
 \hline
 -2x - 2x^{\frac{3}{4}} + 2x^{\frac{1}{2}} \\
 \hline
 -x^{\frac{3}{4}} + x^{\frac{1}{2}} + 3x^{\frac{1}{4}} \\
 \hline
 +x^{\frac{3}{4}} + x^{\frac{1}{2}} - x^{\frac{1}{4}} \\
 \hline
 2x^{\frac{1}{2}} + 2x^{\frac{1}{4}} - 2 \\
 \hline
 -2x^{\frac{1}{2}} - 2x^{\frac{1}{4}} + 2 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 5. \quad 3m^{\frac{2}{3}} - 5 + 10m^{\frac{4}{3}} - 8m^{-2} \quad \left| \begin{array}{l} 3 + m^{-\frac{2}{3}} - 4m^{-\frac{4}{3}} \\ m^{\frac{2}{3}} - 2 + 2m^{-\frac{2}{3}} \end{array} \right. \\
 \hline
 -3m^{\frac{2}{3}} - 1 + 4m^{-\frac{2}{3}} \\
 \hline
 -6 + 4m^{-\frac{2}{3}} + 10m^{-\frac{4}{3}} \\
 \hline
 +6 + 2m^{-\frac{2}{3}} - 8m^{-\frac{4}{3}} \\
 \hline
 6m^{-\frac{2}{3}} + 2m^{-\frac{4}{3}} - 8m^{-2} \\
 \hline
 -6m^{-\frac{2}{3}} - 2m^{-\frac{4}{3}} + 8m^{-2} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 6. \quad a^{\frac{5}{4}} - 4a^{\frac{1}{4}} + 4a^{-\frac{1}{4}} - a^{-\frac{3}{4}} \quad \left| \begin{array}{l} a^{\frac{1}{2}} - 2 + a^{-\frac{1}{2}} \\ a^{\frac{3}{4}} + 2a^{\frac{1}{4}} - a^{-\frac{1}{4}} \end{array} \right. \\
 \hline
 -a^{\frac{5}{4}} + 2a^{\frac{3}{4}} - a^{\frac{1}{4}} \\
 \hline
 2a^{\frac{3}{4}} - 5a^{\frac{1}{4}} + 4a^{-\frac{1}{4}} \\
 \hline
 -2a^{\frac{3}{4}} + 4a^{\frac{1}{4}} - 2a^{-\frac{1}{4}} \\
 \hline
 -a^{\frac{1}{4}} + 2a^{-\frac{1}{4}} - a^{-\frac{3}{4}} \\
 \hline
 +a^{\frac{1}{4}} - 2a^{-\frac{1}{4}} + a^{-\frac{3}{4}} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 7. \quad 4x^{-5} - 7x^{-4} - x^{-3} + 9x^{-2} - 7x^{-1} + 2 \quad \left| \begin{array}{l} 4x^{-2} + x^{-1} - 3 + 2x \\ x^{-3} - 2x^{-2} + x^{-1} \end{array} \right. \\
 \hline
 -4x^{-5} - x^{-4} + 3x^{-3} - 2x^{-2} \\
 \hline
 -8x^{-4} + 2x^{-3} + 7x^{-2} - 7x^{-1} \\
 \hline
 8x^{-4} + 2x^{-3} - 6x^{-2} + 4x^{-1} \\
 \hline
 4x^{-3} + x^{-2} - 3x^{-1} + 2 \\
 \hline
 -4x^{-3} - x^{-2} + 3x^{-1} - 2 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 8. \quad a^{-12}b^{-11} + a^{-8}b^{-7} + a^{-4}b^{-3} \quad \left| \begin{array}{l} a^{-7}b^{-6} - a^{-5}b^{-4} + a^{-3}b^{-2} \\ a^{-5}b^{-5} + a^{-3}b^{-3} + a^{-1}b^{-1} \end{array} \right. \\
 \hline
 -a^{-12}b^{-11} + a^{-10}b^{-9} - a^{-8}b^{-7} \\
 \hline
 a^{-10}b^{-9} \\
 \hline
 -a^{-10}b^{-9} + a^{-8}b^{-7} - a^{-6}b^{-5} \\
 \hline
 a^{-8}b^{-7} - a^{-6}b^{-5} + a^{-4}b^{-3} \\
 \hline
 -a^{-8}b^{-7} + a^{-6}b^{-5} - a^{-4}b^{-3} \\
 \hline
 \end{array}$$



---


$$\begin{array}{r}
 9. \quad \frac{m^{-4}n + m^{-2}n^{-1}}{-m^{-4}n + m^{-3} - m^{-2}n^{-1}} + n^{-3} \left| \frac{m^{-4} - m^{-3}n^{-1} + m^{-2}n^{-2}}{n + m + m^2n^{-1}} \right. \\
 \hline
 m^{-3} \\
 -m^{-3} + m^{-2}n^{-1} - m^{-1}n^{-2} \\
 \hline
 m^{-2}n^{-1} - m^{-1}n^{-2} + n^{-3} \\
 -m^{-2}n^{-1} + m^{-1}n^{-2} - n^{-3} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 10. \quad \frac{15a^3 + a^2 - 19a + 17 - 24a^{-1} + 10a^{-2}}{-15a^3 - 10a^2 + 25a} \left| \frac{3a + 2 - 5a^{-1}}{5a^2 - 3a + 4 - 2a^{-1}} \right. \\
 \hline
 -9a^2 + 6a + 17 \\
 + 9a^2 + 6a - 15 \\
 \hline
 12a + 2 - 24a^{-1} \\
 -12a - 8 + 20a^{-1} \\
 \hline
 -6 - 4a^{-1} + 10a^{-2} \\
 + 6 + 4a^{-1} - 10a^{-2} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 11. \quad \frac{a^{\frac{5}{4}}b^{-4} - a^{\frac{3}{4}}b^{-3}}{-a^{\frac{5}{4}}b^{-4} + 2a^{\frac{3}{4}}b^{-3} - 3a^{\frac{1}{4}}b^{-2}} + 5a^{\frac{1}{4}}b^{-1} - 3a^{\frac{3}{4}} \left| \frac{a^{\frac{1}{2}}b^{-1} - 2 + 3a^{\frac{1}{2}}b}{a^{\frac{3}{4}}b^{-3} + a^{\frac{1}{4}}b^{-2} - a^{-\frac{1}{4}}b^{-1}} \right. \\
 \hline
 a^{\frac{3}{4}}b^{-3} - 3a^{\frac{1}{4}}b^{-2} + 5a^{\frac{1}{4}}b^{-1} \\
 -a^{\frac{3}{4}}b^{-3} + 2a^{\frac{1}{4}}b^{-2} - 3a^{\frac{1}{4}}b^{-1} \\
 \hline
 -a^{\frac{1}{4}}b^{-2} + 2a^{\frac{1}{4}}b^{-1} - 3a^{\frac{3}{4}} \\
 + a^{\frac{1}{4}}b^{-2} - 2a^{\frac{1}{4}}b^{-1} + 3a^{\frac{3}{4}} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 12. \quad \frac{x^{-2} + x^{\frac{3}{2}}y^{\frac{1}{2}} + x^{-1}y^{-1}}{-x^{-2} + x^{\frac{3}{2}}y^{\frac{1}{2}} - x^{-1}y^{-1}} + 2y^{-2} \left| \frac{x^{-1} - x^{\frac{1}{2}}y^{\frac{1}{2}} + y^{-1}}{x^{-1} + 2x^{\frac{1}{2}}y^{\frac{1}{2}} + 2y^{-1}} \right. \\
 \hline
 2x^{\frac{3}{2}}y^{\frac{1}{2}} \\
 -2x^{\frac{3}{2}}y^{\frac{1}{2}} + 2x^{-1}y^{-1} - 2x^{\frac{1}{2}}y^{\frac{3}{2}} \\
 \hline
 2x^{-1}y^{-1} - 2x^{\frac{1}{2}}y^{\frac{3}{2}} + 2y^{-2} \\
 -2x^{-1}y^{-1} + 2x^{\frac{1}{2}}y^{\frac{3}{2}} - 2y^{-2} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 13. \quad m \quad -6m^{\frac{1}{5}} \quad +m^{-\frac{3}{5}} \quad \left| \begin{array}{l} m^{\frac{3}{5}}+2m^{\frac{1}{5}}-m^{-\frac{1}{5}} \\ m^{\frac{2}{5}}-2-m^{-\frac{2}{5}} \end{array} \right. \\
 \hline
 -m-2m^{\frac{3}{5}}+m^{\frac{1}{5}} \\
 -2m^{\frac{3}{5}}-5m^{\frac{1}{5}} \\
 \hline
 2m^{\frac{3}{5}}+4m^{\frac{1}{5}}-2m^{-\frac{1}{5}} \\
 -m^{\frac{1}{5}}-2m^{-\frac{1}{5}}+m^{-\frac{3}{5}} \\
 \hline
 +m^{\frac{1}{5}}+2m^{-\frac{1}{5}}-m^{-\frac{3}{5}} \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 14. \quad 2x+4x^{\frac{2}{3}} \quad +2+4x^{-\frac{1}{3}} \quad \left| \begin{array}{l} x+3x^{\frac{2}{3}}+2x^{\frac{1}{3}} \\ 2-2x^{-\frac{1}{3}}+2x^{-\frac{2}{3}} \end{array} \right. \\
 \hline
 -2x-6x^{\frac{2}{3}}-4x^{\frac{1}{3}} \\
 -2x^{\frac{2}{3}}-4x^{\frac{1}{3}}+2 \\
 \hline
 +2x^{\frac{2}{3}}+6x^{\frac{1}{3}}+4 \\
 \hline
 2x^{\frac{1}{3}}+6+4x^{-\frac{1}{3}} \\
 \hline
 -2x^{\frac{1}{3}}-6-4x^{-\frac{1}{3}} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 15. \quad 4x^{\frac{5}{2}}+3x^2y^{\frac{1}{2}} \quad -x^{\frac{1}{2}}y^2 \quad \left| \begin{array}{l} x^{\frac{1}{2}}+y^{\frac{1}{2}} \\ 4x^2-x^{\frac{3}{2}}y^{\frac{1}{2}}-xy-x^{\frac{1}{2}}y^{\frac{3}{2}} \end{array} \right. \\
 \hline
 -4x^{\frac{5}{2}}-4x^2y^{\frac{1}{2}} \\
 -x^2y^{\frac{1}{2}} \\
 \hline
 +x^2y^{\frac{1}{2}}-x^{\frac{3}{2}}y \\
 -x^{\frac{3}{2}}y \\
 \hline
 +x^{\frac{3}{2}}y-xy^{\frac{3}{2}} \\
 -xy^{\frac{3}{2}}-x^{\frac{1}{2}}y^2 \\
 \hline
 +xy^{\frac{3}{2}}+x^{\frac{1}{2}}y^2 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 16. \quad x^{\frac{7}{3}} \quad -7ax^{\frac{4}{3}}-3a^{\frac{4}{3}}x-9a^{\frac{5}{3}}x^{\frac{2}{3}} \quad \left| \begin{array}{l} x^{\frac{4}{3}}+2a^{\frac{1}{3}}x+3a^{\frac{2}{3}}x^{\frac{2}{3}} \\ x-2a^{\frac{1}{3}}x^{\frac{2}{3}}+a^{\frac{2}{3}}x^{\frac{1}{3}}-3a \end{array} \right. \\
 \hline
 -x^{\frac{7}{3}}-2a^{\frac{1}{3}}x^2-3a^{\frac{2}{3}}x^{\frac{5}{3}} \\
 -2a^{\frac{1}{3}}x^2-3a^{\frac{2}{3}}x^{\frac{5}{3}}-7ax^{\frac{4}{3}} \\
 \hline
 +2a^{\frac{1}{3}}x^2+4a^{\frac{2}{3}}x^{\frac{5}{3}}+6ax^{\frac{4}{3}} \\
 \hline
 a^{\frac{2}{3}}x^{\frac{5}{3}}-ax^{\frac{4}{3}}-3a^{\frac{4}{3}}x \\
 -a^{\frac{2}{3}}x^{\frac{5}{3}}-2ax^{\frac{4}{3}}-3a^{\frac{4}{3}}x \\
 \hline
 -3ax^{\frac{4}{3}}-6a^{\frac{4}{3}}x-9a^{\frac{5}{3}}x^{\frac{2}{3}} \\
 \hline
 +3ax^{\frac{4}{3}}+6a^{\frac{4}{3}}x+9a^{\frac{5}{3}}x^{\frac{2}{3}} \\
 \hline
 \end{array}$$

$$\begin{array}{l}
 17. \quad \begin{array}{r}
 a^{\frac{3}{2}} + a^{\frac{1}{2}}b - b^{\frac{3}{2}} \\
 -a^{\frac{3}{2}} - ab^{\frac{1}{2}} - a^{\frac{1}{2}}b \\
 \hline
 -ab^{\frac{1}{2}} - b^{\frac{3}{2}} \\
 +ab^{\frac{1}{2}} + a^{\frac{1}{2}}b + b^{\frac{3}{2}} \\
 \hline
 a^{\frac{1}{2}}b \\
 -a^{\frac{1}{2}}b - b^{\frac{3}{2}} - a^{-\frac{1}{2}}b^2 \\
 \hline
 -b^{\frac{3}{2}} - a^{-\frac{1}{2}}b^2 - a^{-1}b^{\frac{5}{2}} \\
 +b^{\frac{3}{2}} + a^{-\frac{1}{2}}b^2 + a^{-1}b^{\frac{5}{2}} \\
 \hline
 \end{array}
 \quad \left| \begin{array}{l}
 a^{\frac{1}{2}} + b^{\frac{1}{2}} + a^{-\frac{1}{2}}b \\
 a - a^{\frac{1}{2}}b^{\frac{1}{2}} + b - a^{-\frac{1}{2}}b^{\frac{3}{2}}
 \end{array} \right.
 \end{array}$$

$$\begin{array}{l}
 18. \quad \begin{array}{r}
 m^{-2}n^2 - 11m^{-1}n \\
 -m^{-2}n^2 - 3m^{-\frac{3}{2}}n^{\frac{3}{2}} + m^{-1}n \\
 \hline
 -3m^{-\frac{3}{2}}n^{\frac{3}{2}} - 10m^{-1}n \\
 +3m^{-\frac{3}{2}}n^{\frac{3}{2}} + 9m^{-1}n - 3m^{-\frac{1}{2}}n^{\frac{1}{2}} \\
 \hline
 -m^{-1}n - 3m^{-\frac{1}{2}}n^{\frac{1}{2}} + 1 \\
 +m^{-1}n + 3m^{-\frac{1}{2}}n^{\frac{1}{2}} - 1 \\
 \hline
 \end{array}
 \quad +1 \left| \begin{array}{l}
 m^{-\frac{3}{4}}n^{\frac{3}{2}} + 3m^{-\frac{1}{4}}n - m^{\frac{1}{4}}n^{\frac{1}{2}} \\
 m^{-\frac{5}{4}}n^{\frac{1}{2}} - 3m^{-\frac{3}{4}} - m^{-\frac{1}{4}}n^{-\frac{1}{2}}
 \end{array} \right.
 \end{array}$$

$$\begin{array}{l}
 19. \quad \begin{array}{r}
 x^{-1}y^2 + 4 + 13x^2y^{-4} + 6x^3y^{-6} \\
 -x^{-1}y^2 + 1 - 3xy^{-2} \\
 \hline
 5 - 3xy^{-2} + 13x^2y^{-4} \\
 -5 + 5xy^{-2} - 15x^2y^{-4} \\
 \hline
 2xy^{-2} - 2x^2y^{-4} + 6x^3y^{-6} \\
 -2xy^{-2} + 2x^2y^{-4} - 6x^3y^{-6} \\
 \hline
 \end{array}
 \quad \left| \begin{array}{l}
 x^{-3}y^3 - x^{-2}y + 3x^{-1}y^{-1} \\
 x^2y^{-1} + 5x^3y^{-3} + 2x^4y^{-5}
 \end{array} \right.
 \end{array}$$

$$\begin{array}{l}
 20. \quad \begin{array}{r}
 3 + 7a^{-\frac{2}{3}}b^{\frac{1}{2}} + a^{-2}b^{\frac{3}{2}} - a^{-\frac{8}{3}}b^2 \\
 -3 - a^{-\frac{2}{3}}b^{\frac{1}{2}} - a^{-\frac{4}{3}}b \\
 \hline
 6a^{-\frac{2}{3}}b^{\frac{1}{2}} - a^{-\frac{4}{3}}b + a^{-2}b^{\frac{3}{2}} \\
 -6a^{-\frac{2}{3}}b^{\frac{1}{2}} - 2a^{-\frac{4}{3}}b - 2a^{-2}b^{\frac{3}{2}} \\
 \hline
 -3a^{-\frac{4}{3}}b - a^{-2}b^{\frac{3}{2}} - a^{-\frac{8}{3}}b^2 \\
 +3a^{-\frac{4}{3}}b + a^{-2}b^{\frac{3}{2}} + a^{-\frac{8}{3}}b^2 \\
 \hline
 \end{array}
 \quad \left| \begin{array}{l}
 3a^{\frac{2}{3}}b^{-\frac{1}{2}} + 1 + a^{-\frac{2}{3}}b^{\frac{1}{2}} \\
 a^{-\frac{2}{3}}b^{\frac{1}{2}} + 2a^{-\frac{4}{3}}b - a^{-2}b^{\frac{3}{2}}
 \end{array} \right.
 \end{array}$$

## EJERCICIO 227

1.  $(a^{-1})^2 = a^{-1 \cdot 2} = a^{-2}$
2.  $(a^{-2}b^{-1})^3 = a^{-2 \cdot 3}b^{-1 \cdot 3} = a^{-6}b^{-3}$
3.  $\left(a^{\frac{3}{2}}\right)^2 = a^{\frac{3}{2} \cdot 2} = a^{\frac{6}{2}} = a^3$
4.  $\left(x^{\frac{3}{4}}\right)^3 = x^{\frac{3}{4} \cdot 3} = x^{\frac{9}{4}}$
5.  $\left(m^{\frac{3}{4}}\right)^2 = m^{\frac{3}{4} \cdot 2} = m^{\frac{6}{4}} = m^{\frac{3}{2}}$
6.  $\left(a^{-\frac{2}{3}}\right)^3 = a^{-\frac{2}{3} \cdot 3} = a^{-\frac{6}{3}} = a^{-2}$
7.  $\left(x^{-4}y^{\frac{1}{4}}\right)^2 = x^{-4 \cdot 2}y^{\frac{1}{4} \cdot 2} = x^{-8}y^{\frac{2}{4}} = x^{-8}y^{\frac{1}{2}}$
8.  $\left(2a^{\frac{1}{2}}b^{\frac{1}{3}}\right)^2 = 2^2a^{\frac{1}{2} \cdot 2}b^{\frac{1}{3} \cdot 2} = 4a^{\frac{2}{2}}b^{\frac{2}{3}} = 4ab^{\frac{2}{3}}$
9.  $(a^{-3}b^{-1})^4 = a^{-3 \cdot 4}b^{-1 \cdot 4} = a^{-12}b^{-4}$
10.  $\left(x^{\frac{2}{3}}y^{-\frac{1}{2}}\right)^6 = x^{\frac{2}{3} \cdot 6}y^{-\frac{1}{2} \cdot 6} = x^{\frac{12}{3}}y^{-\frac{6}{2}} = x^4y^{-3}$
11.  $\left(3a^{\frac{2}{5}}b^{-3}\right)^5 = 3^5a^{\frac{2}{5} \cdot 5}b^{-3 \cdot 5} = 243a^{\frac{10}{5}}b^{-15} = 243a^2b^{-15}$
12.  $\left(2m^{-\frac{1}{2}}n^{-\frac{1}{3}}\right)^3 = 2^3m^{-\frac{1}{2} \cdot 3}n^{-\frac{1}{3} \cdot 3} = 8m^{-\frac{3}{2}}n^{-\frac{3}{3}} = 8m^{-\frac{3}{2}}n^{-1}$

## EJERCICIO 228

1.  $\left(a^{\frac{1}{2}} + b^{\frac{1}{2}}\right)^2 = \left(a^{\frac{1}{2}}\right)^2 + 2\left(a^{\frac{1}{2}}\right)\left(b^{\frac{1}{2}}\right) + \left(b^{\frac{1}{2}}\right)^2$   
 $= a + 2a^{\frac{1}{2}}b^{\frac{1}{2}} + b$
2.  $\left(x^{\frac{3}{4}} - y^{\frac{1}{3}}\right)^2 = \left(x^{\frac{3}{4}}\right)^2 - 2\left(x^{\frac{3}{4}}\right)\left(y^{\frac{1}{3}}\right) + \left(y^{\frac{1}{3}}\right)^2$   
 $= x^{\frac{3}{2}} - 2x^{\frac{3}{4}}y^{\frac{1}{3}} + y^{\frac{2}{3}}$
3.  $\left(m^{-\frac{1}{2}} + 2m\right)^2 = \left(m^{-\frac{1}{2}}\right)^2 + 2(2m)\left(m^{-\frac{1}{2}}\right) + (2m)^2$   
 $= m^{-1} + 4m^{1-\frac{1}{2}} + 4m^2 = m^{-1} + 4m^{\frac{1}{2}} + 4m^2$
4.  $(a^{-2}b^3 - a^3b^{-2})^2$   
 $= (a^{-2}b^3)^2 - 2(a^{-2}b^3)(a^3b^{-2}) + (a^3b^{-2})^2$   
 $= a^{-4}b^6 - 2a^{-2+3}b^{3-2} + a^6b^{-4}$   
 $= a^{-4}b^6 - 2ab + a^6b^{-4}$
5.  $\left(a^{-1} - 3b^{-\frac{3}{4}}\right)^2 = (a^{-1})^2 - 2(a^{-1})\left(3b^{-\frac{3}{4}}\right) + \left(3b^{-\frac{3}{4}}\right)^2$   
 $= a^{-2} - 6a^{-1}b^{-\frac{3}{4}} + 9b^{-\frac{3}{2}}$
6.  $(a^{-2} + \sqrt{b})^2 = (a^{-2})^2 + 2(a^{-2})\left(b^{\frac{1}{2}}\right) + \left(b^{\frac{1}{2}}\right)^2$   
 $= a^{-4} + 2a^{-2}b^{\frac{1}{2}} + b$
7.  $\left(\sqrt[4]{x^3} - y^{-\frac{1}{2}}\right)^2 = \left(x^{\frac{3}{4}}\right)^2 - 2\left(x^{\frac{3}{4}}\right)\left(y^{-\frac{1}{2}}\right) + \left(y^{-\frac{1}{2}}\right)^2$   
 $= x^{\frac{3}{2}} - 2x^{\frac{3}{4}}y^{-\frac{1}{2}} + y^{-1}$
8.  $\left(m^{-2}n^{\frac{1}{4}} - m^{\frac{1}{2}}n^{-1}\right)^2$   
 $= \left(m^{-2}n^{\frac{1}{4}}\right)^2 - 2\left(m^{-2}n^{\frac{1}{4}}\right)\left(m^{\frac{1}{2}}n^{-1}\right) + \left(m^{\frac{1}{2}}n^{-1}\right)^2$   
 $= m^{-4}n^{\frac{1}{2}} - 2m^{-2+\frac{1}{2}}n^{\frac{1}{4}-1} + mn^{-2}$   
 $= m^{-4}n^{\frac{1}{2}} - 2m^{-\frac{3}{2}}n^{-\frac{3}{4}} + mn^{-2}$
9.  $\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\right)^3$   
 $= \left(a^{\frac{1}{3}}\right)^3 + 3\left(a^{\frac{1}{3}}\right)^2\left(b^{\frac{1}{3}}\right) + 3\left(a^{\frac{1}{3}}\right)\left(b^{\frac{1}{3}}\right)^2 + \left(b^{\frac{1}{3}}\right)^3$   
 $= a + 3a^{\frac{2}{3}}b^{\frac{1}{3}} + 3a^{\frac{1}{3}}b^{\frac{2}{3}} + b$
10.  $\left(\sqrt[3]{x^2} - 3y^{-1}\right)^3$   
 $= \left(x^{\frac{2}{3}}\right)^3 - 3\left(x^{\frac{2}{3}}\right)^2\left(3y^{-1}\right) + 3\left(x^{\frac{2}{3}}\right)\left(3y^{-1}\right)^2 - \left(3y^{-1}\right)^3$   
 $= x^2 - 9x^{\frac{4}{3}}y^{-1} + 27x^{\frac{2}{3}}y^{-2} - 27y^{-3}$

$$11. \left(m^{\frac{2}{3}} + 4n^{-\frac{3}{2}}\right)^3 = \left(m^{\frac{2}{3}}\right)^3 + 3\left(m^{\frac{2}{3}}\right)^2\left(4n^{-\frac{3}{2}}\right) + 3\left(m^{\frac{2}{3}}\right)\left(4n^{-\frac{3}{2}}\right)^2 + \left(4n^{-\frac{3}{2}}\right)^3 = m^2 + 12m^{\frac{4}{3}}n^{-\frac{3}{2}} + 48m^{\frac{2}{3}}n^{-3} + 64n^{-\frac{9}{2}}$$

$$12. \left(2a^{-4} - 3b^{-\frac{1}{2}}\right)^3 = (2a^{-4})^3 - 3(2a^{-4})^2\left(3b^{-\frac{1}{2}}\right) + 3(2a^{-4})\left(3b^{-\frac{1}{2}}\right)^2 - \left(3b^{-\frac{1}{2}}\right)^3 = 8a^{-12} - 36a^{-8}b^{-\frac{1}{2}} + 54a^{-4}b^{-1} - 27b^{-\frac{3}{2}}$$

$$13. \left(\sqrt{x} - \sqrt[3]{y}\right)^3 = \left(x^{\frac{1}{2}}\right)^3 - 3\left(x^{\frac{1}{2}}\right)^2\left(y^{\frac{1}{3}}\right) + 3\left(x^{\frac{1}{2}}\right)\left(y^{\frac{1}{3}}\right)^2 - \left(y^{\frac{1}{3}}\right)^3 = x^{\frac{3}{2}} - 3xy^{\frac{1}{3}} + 3x^{\frac{1}{2}}y^{\frac{2}{3}} - y$$

$$14. \left(a^{\frac{1}{2}} + b^{\frac{2}{3}}\right)^4 = \left(a^{\frac{1}{2}}\right)^4 + 4\left(a^{\frac{1}{2}}\right)^3\left(b^{\frac{2}{3}}\right) + 6\left(a^{\frac{1}{2}}\right)^2\left(b^{\frac{2}{3}}\right)^2 + 4\left(a^{\frac{1}{2}}\right)\left(b^{\frac{2}{3}}\right)^3 + \left(b^{\frac{2}{3}}\right)^4 = a^2 + 4a^{\frac{3}{2}}b^{\frac{2}{3}} + 6ab^{\frac{4}{3}} + 4a^{\frac{1}{2}}b^2 + b^{\frac{8}{3}}$$

$$15. \left(x^{-2} - y^{-\frac{1}{3}}\right)^4 = (x^{-2})^4 - 4(x^{-2})^3\left(y^{-\frac{1}{3}}\right) + 6(x^{-2})^2\left(y^{-\frac{1}{3}}\right)^2 - 4(x^{-2})\left(y^{-\frac{1}{3}}\right)^3 + \left(y^{-\frac{1}{3}}\right)^4 \\ = x^{-8} - 4x^{-6}y^{-\frac{1}{3}} + 6x^{-4}y^{-\frac{2}{3}} - 4x^{-2}y^{-1} + y^{-\frac{4}{3}}$$

$$16. \left(x^{\frac{1}{3}} + y^{-\frac{3}{4}}\right)^5 = \left(x^{\frac{1}{3}}\right)^5 + 5\left(x^{\frac{1}{3}}\right)^4\left(y^{-\frac{3}{4}}\right) + 10\left(x^{\frac{1}{3}}\right)^3\left(y^{-\frac{3}{4}}\right)^2 + 10\left(x^{\frac{1}{3}}\right)^2\left(y^{-\frac{3}{4}}\right)^3 + 5\left(x^{\frac{1}{3}}\right)\left(y^{-\frac{3}{4}}\right)^4 + \left(y^{-\frac{3}{4}}\right)^5 \\ = x^{\frac{5}{3}} + 5x^{\frac{4}{3}}y^{-\frac{3}{4}} + 10xy^{-\frac{3}{2}} + 10x^{\frac{2}{3}}y^{-\frac{9}{4}} + 5x^{\frac{1}{3}}y^{-3} + y^{-\frac{15}{4}}$$

$$17. \left(\sqrt{m} - \sqrt[3]{n}\right)^5 = \left(m^{\frac{1}{2}}\right)^5 - 5\left(m^{\frac{1}{2}}\right)^4\left(n^{\frac{1}{3}}\right) + 10\left(m^{\frac{1}{2}}\right)^3\left(n^{\frac{1}{3}}\right)^2 - 10\left(m^{\frac{1}{2}}\right)^2\left(n^{\frac{1}{3}}\right)^3 + 5\left(m^{\frac{1}{2}}\right)\left(n^{\frac{1}{3}}\right)^4 - \left(n^{\frac{1}{3}}\right)^5 \\ = m^{\frac{5}{2}} - 5m^2n^{\frac{1}{3}} + 10m^{\frac{3}{2}}n^{\frac{2}{3}} - 10mn + 5m^{\frac{1}{2}}n^{\frac{4}{3}} - n^{\frac{5}{3}}$$

$$18. \left(a^2 - 2\sqrt{m}\right)^6 \\ = (a^2)^6 - 6(a^2)^5\left(2m^{\frac{1}{2}}\right) + 15(a^2)^4\left(2m^{\frac{1}{2}}\right)^2 - 20(a^2)^3\left(2m^{\frac{1}{2}}\right)^3 + 15(a^2)^2\left(2m^{\frac{1}{2}}\right)^4 - 6(a^2)\left(2m^{\frac{1}{2}}\right)^5 + \left(2m^{\frac{1}{2}}\right)^6 \\ = a^{12} - 12a^{10}m^{\frac{1}{2}} + 60a^8m - 160a^6m^{\frac{3}{2}} + 240a^4m^2 - 192a^2m^{\frac{5}{2}} + 64m^3$$

$$19. \left(x^{-3} + \sqrt[4]{y}\right)^5 = (x^{-3})^5 + 5(x^{-3})^4\left(y^{\frac{1}{4}}\right) + 10(x^{-3})^3\left(y^{\frac{1}{4}}\right)^2 + 10(x^{-3})^2\left(y^{\frac{1}{4}}\right)^3 + 5(x^{-3})\left(y^{\frac{1}{4}}\right)^4 + \left(y^{\frac{1}{4}}\right)^5 \\ = x^{-15} + 5x^{-12}y^{\frac{1}{4}} + 10x^{-9}y^{\frac{1}{2}} + 10x^{-6}y^{\frac{3}{4}} + 5x^{-3}y + y^{\frac{5}{4}}$$

$$20. \left(a^{-2} + 3a^{-1} + 2\right)^2 = (a^{-2})^2 + (3a^{-1})^2 + (2)^2 + 2(a^{-2})(3a^{-1}) + 2(a^{-2})(2) + 2(3a^{-1})(2) \\ = a^{-4} + 9a^{-2} + 4 + 6a^{-3} + 4a^{-2} + 12a^{-1} = a^{-4} + 6a^{-3} + 13a^{-2} + 12a^{-1} + 4$$

$$21. \left(x^{\frac{1}{2}} - x^{\frac{1}{4}} + 2x^{-\frac{1}{4}}\right)^2 = \left(x^{\frac{1}{2}}\right)^2 + \left(-x^{\frac{1}{4}}\right)^2 + \left(2x^{-\frac{1}{4}}\right)^2 + 2\left(x^{\frac{1}{2}}\right)\left(-x^{\frac{1}{4}}\right) + 2\left(x^{\frac{1}{2}}\right)\left(2x^{-\frac{1}{4}}\right) + 2\left(-x^{\frac{1}{4}}\right)\left(2x^{-\frac{1}{4}}\right) \\ = x + x^{\frac{1}{2}} + 4x^{-\frac{1}{2}} - 2x^{\frac{3}{4}} + 4x^{\frac{1}{4}} - 4 = x - 2x^{\frac{3}{4}} + x^{\frac{1}{2}} + 4x^{\frac{1}{4}} - 4 + 4x^{-\frac{1}{2}}$$

$$22. \left(a^{-\frac{1}{2}} + 3 + a^{\frac{1}{2}}\right)^2 = \left(a^{-\frac{1}{2}}\right)^2 + (3)^2 + \left(a^{\frac{1}{2}}\right)^2 + 2\left(a^{-\frac{1}{2}}\right)(3) + 2\left(a^{-\frac{1}{2}}\right)\left(a^{\frac{1}{2}}\right) + 2(3)\left(a^{\frac{1}{2}}\right) \\ = a^{-1} + 9 + a + 6a^{-\frac{1}{2}} + 2 + 6a^{\frac{1}{2}} = a + 6a^{\frac{1}{2}} + 11 + 6a^{-\frac{1}{2}} + a^{-1}$$

---


$$\begin{aligned}
23. \quad & \left(m + 2m^{\frac{3}{4}} - 3m^{\frac{1}{2}}\right)^2 = (m)^2 + \left(2m^{\frac{3}{4}}\right)^2 + \left(-3m^{\frac{1}{2}}\right)^2 + 2(m)\left(2m^{\frac{3}{4}}\right) + 2(m)\left(-3m^{\frac{1}{2}}\right) + 2\left(2m^{\frac{3}{4}}\right)\left(-3m^{\frac{1}{2}}\right) \\
& = m^2 + 4m^{\frac{3}{2}} + 9m + 4m^{\frac{7}{4}} - 6m^{\frac{3}{2}} - 12m^{\frac{5}{4}} = m^2 + 4m^{\frac{7}{4}} - 2m^{\frac{3}{2}} - 12m^{\frac{5}{4}} + 9m \\
24. \quad & \left(a^{\frac{1}{2}}b^{-\frac{1}{3}} - 2 + a^{-\frac{1}{2}}b^{\frac{1}{3}}\right)^2 = \left(a^{\frac{1}{2}}b^{-\frac{1}{3}}\right)^2 + (-2)^2 + \left(a^{-\frac{1}{2}}b^{\frac{1}{3}}\right)^2 + 2\left(a^{\frac{1}{2}}b^{-\frac{1}{3}}\right)(-2) + 2\left(a^{\frac{1}{2}}b^{-\frac{1}{3}}\right)\left(a^{-\frac{1}{2}}b^{\frac{1}{3}}\right) + 2(-2)\left(a^{-\frac{1}{2}}b^{\frac{1}{3}}\right) \\
& = ab^{-\frac{2}{3}} + 4 + a^{-1}b^{\frac{2}{3}} - 4a^{\frac{1}{2}}b^{-\frac{1}{3}} + 2 - 4a^{-\frac{1}{2}}b^{\frac{1}{3}} = ab^{-\frac{2}{3}} - 4a^{\frac{1}{2}}b^{-\frac{1}{3}} + 6 - 4a^{-\frac{1}{2}}b^{\frac{1}{3}} + a^{-1}b^{\frac{2}{3}} \\
25. \quad & \left(x^{\frac{1}{2}} + x^{\frac{1}{4}} - 1\right)^3 = \left(x^{\frac{1}{2}}\right)^3 + \left(x^{\frac{1}{4}}\right)^3 + (-1)^3 + 3\left(x^{\frac{1}{2}}\right)^2\left(x^{\frac{1}{4}}\right) + 3\left(x^{\frac{1}{2}}\right)^2(-1) + 3\left(x^{\frac{1}{4}}\right)^2\left(x^{\frac{1}{2}}\right) + 3\left(x^{\frac{1}{4}}\right)^2(-1) \\
& \quad + 3(-1)^2\left(x^{\frac{1}{2}}\right) + 3(-1)^2\left(x^{\frac{1}{4}}\right) + 6\left(x^{\frac{1}{2}}\right)\left(x^{\frac{1}{4}}\right)(-1) \\
& = x^{\frac{3}{2}} + x^{\frac{3}{4}} - 1 + 3x^{\frac{5}{4}} - 3x + 3x - 3x^{\frac{1}{2}} + 3x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 6x^{\frac{3}{4}} = x^{\frac{3}{2}} + 3x^{\frac{5}{4}} - 5x^{\frac{3}{4}} + 3x^{\frac{1}{4}} - 1 \\
26. \quad & \left(a^{\frac{2}{3}} - 2 + a^{-\frac{2}{3}}\right)^3 = \left(a^{\frac{2}{3}}\right)^3 + (-2)^3 + \left(a^{-\frac{2}{3}}\right)^3 + 3\left(a^{\frac{2}{3}}\right)^2(-2) + 3\left(a^{\frac{2}{3}}\right)^2\left(a^{-\frac{2}{3}}\right) + 3(-2)^2\left(a^{\frac{2}{3}}\right) + 3(-2)^2\left(a^{-\frac{2}{3}}\right) \\
& \quad + 3\left(a^{-\frac{2}{3}}\right)^2\left(a^{\frac{2}{3}}\right) + 3\left(a^{-\frac{2}{3}}\right)^2(-2) + 6\left(a^{\frac{2}{3}}\right)(-2)\left(a^{-\frac{2}{3}}\right) \\
& = a^2 - 8 + a^{-2} - 6a^{\frac{4}{3}} + 3a^{\frac{2}{3}} + 12a^{\frac{2}{3}} + 12a^{-\frac{2}{3}} + 3a^{-\frac{2}{3}} - 6a^{-\frac{4}{3}} - 12 \\
& = a^2 - 6a^{\frac{4}{3}} + 15a^{\frac{2}{3}} - 20 + 15a^{-\frac{2}{3}} - 6a^{-\frac{4}{3}} + a^{-2} \\
27. \quad & \left(m^{\frac{1}{6}} + 2m^{\frac{1}{3}} + m^{\frac{1}{2}}\right)^3 = \left(m^{\frac{1}{6}}\right)^3 + \left(2m^{\frac{1}{3}}\right)^3 + \left(m^{\frac{1}{2}}\right)^3 + 3\left(m^{\frac{1}{6}}\right)^2\left(2m^{\frac{1}{3}}\right) + 3\left(m^{\frac{1}{6}}\right)^2\left(m^{\frac{1}{2}}\right) + 3\left(2m^{\frac{1}{3}}\right)^2\left(m^{\frac{1}{6}}\right) \\
& \quad + 3\left(2m^{\frac{1}{3}}\right)^2\left(m^{\frac{1}{2}}\right) + 3\left(m^{\frac{1}{2}}\right)^2\left(m^{\frac{1}{6}}\right) + 3\left(m^{\frac{1}{2}}\right)^2\left(2m^{\frac{1}{3}}\right) + 6\left(m^{\frac{1}{6}}\right)\left(2m^{\frac{1}{3}}\right)\left(m^{\frac{1}{2}}\right) \\
& = m^{\frac{1}{2}} + 8m + m^{\frac{3}{2}} + 6m^{\frac{2}{3}} + 3m^{\frac{5}{6}} + 12m^{\frac{5}{6}} + 12m^{\frac{7}{6}} + 3m^{\frac{7}{6}} + 6m^{\frac{4}{3}} + 12m \\
& = m^{\frac{3}{2}} + 6m^{\frac{4}{3}} + 15m^{\frac{7}{6}} + 20m + 15m^{\frac{5}{6}} + 6m^{\frac{2}{3}} + m^{\frac{1}{2}}
\end{aligned}$$

## EJERCICIO 229

$$\begin{array}{l|l}
1. \quad \sqrt{x^{-4} + 6x^{-3} + 13x^{-2} + 12x^{-1} + 4} & x^{-2} + 3x^{-1} + 2 \\
\hline
-x^{-4} & (2x^{-2} + 3x^{-1})(3x^{-1}) = 6x^{-3} + 9x^{-2} \\
\hline
+ 6x^{-3} + 13x^{-2} & \\
\hline
- 6x^{-3} - 9x^{-2} & (2x^{-2} + 6x^{-1} + 2)(2) \\
\hline
4x^{-2} + 12x^{-1} + 4 & = 4x^{-2} + 12x^{-1} + 4 \\
\hline
- 4x^{-2} - 12x^{-1} - 4 & \\
\hline
\end{array}$$

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$$\begin{array}{l|l}
 2. \sqrt{m+6m^{\frac{1}{2}}+11+6m^{-\frac{1}{2}}+m^{-1}} & m^{\frac{1}{2}}+3+m^{-\frac{1}{2}} \\
 \hline
 -m & \left(2m^{\frac{1}{2}}+3\right)(3)=6m^{\frac{1}{2}}+9 \\
 \hline
 6m^{\frac{1}{2}}+11 & \\
 -6m^{\frac{1}{2}}-9 & \left(2m^{\frac{1}{2}}+6+m^{-\frac{1}{2}}\right)\left(m^{-\frac{1}{2}}\right) \\
 \hline
 2+6m^{-\frac{1}{2}}+m^{-1} & =2+6m^{-\frac{1}{2}}+m^{-1} \\
 -2-6m^{-\frac{1}{2}}-m^{-1} & \\
 \hline
 \end{array}$$

$$\begin{array}{l|l}
 3. \sqrt{9a^{\frac{4}{3}}-6a+25a^{\frac{2}{3}}-8a^{\frac{1}{3}}+16} & 3a^{\frac{2}{3}}-a^{\frac{1}{3}}+4 \\
 \hline
 -9a^{\frac{4}{3}} & \left(6a^{\frac{2}{3}}-a^{\frac{1}{3}}\right)\left(-a^{\frac{1}{3}}\right) \\
 \hline
 -6a+25a^{\frac{2}{3}} & =-6a+a^{\frac{2}{3}} \\
 +6a-a^{\frac{2}{3}} & \\
 \hline
 24a^{\frac{2}{3}}-8a^{\frac{1}{3}}+16 & \left(6a^{\frac{2}{3}}-2a^{\frac{1}{3}}+4\right)(4) \\
 -24a^{\frac{2}{3}}+8a^{\frac{1}{3}}-16 & =24a^{\frac{2}{3}}-8a^{\frac{1}{3}}+16 \\
 \hline
 \end{array}$$

$$\begin{array}{l|l}
 4. \sqrt{a^2+4a^{\frac{7}{4}}-2a^{\frac{3}{2}}-12a^{\frac{5}{4}}+9a} & a+2a^{\frac{3}{4}}-3a^{\frac{1}{2}} \\
 \hline
 -a^2 & \left(2a+2a^{\frac{3}{4}}\right)\left(2a^{\frac{3}{4}}\right) \\
 \hline
 +4a^{\frac{7}{4}}-2a^{\frac{3}{2}} & =4a^{\frac{7}{4}}+4a^{\frac{3}{2}} \\
 -4a^{\frac{7}{4}}-4a^{\frac{3}{2}} & \\
 \hline
 -6a^{\frac{3}{2}}-12a^{\frac{5}{4}}+9a & \left(2a+4a^{\frac{3}{4}}-3a^{\frac{1}{2}}\right)\left(-3a^{\frac{1}{2}}\right) \\
 +6a^{\frac{3}{2}}+12a^{\frac{5}{4}}-9a & =-6a^{\frac{3}{2}}-12a^{\frac{5}{4}}+9a \\
 \hline
 \end{array}$$

$$\begin{array}{l|l}
 5. \sqrt{mn^{-\frac{2}{3}} - 4m^{\frac{1}{2}}n^{-\frac{1}{3}} + 6 - 4m^{-\frac{1}{2}}n^{\frac{1}{3}} + m^{-1}n^{\frac{2}{3}}} & m^{\frac{1}{2}}n^{-\frac{1}{3}} - 2 + m^{-\frac{1}{2}}n^{\frac{1}{3}} \\
 \hline
 -mn^{-\frac{2}{3}} & \left(2m^{\frac{1}{2}}n^{-\frac{1}{3}} - 2\right)(-2) \\
 & = -4m^{\frac{1}{2}}n^{-\frac{1}{3}} + 4 \\
 \\ 
 -4m^{\frac{1}{2}}n^{-\frac{1}{3}} + 6 & \\
 +4m^{\frac{1}{2}}n^{-\frac{1}{3}} - 4 & \\
 \hline
 2 - 4m^{-\frac{1}{2}}n^{\frac{1}{3}} + m^{-1}n^{\frac{2}{3}} & \left(2m^{\frac{1}{2}}n^{-\frac{1}{3}} - 4 + m^{-\frac{1}{2}}n^{\frac{1}{3}}\right)\left(m^{-\frac{1}{2}}n^{\frac{1}{3}}\right) \\
 -2 + 4m^{-\frac{1}{2}}n^{\frac{1}{3}} - m^{-1}n^{\frac{2}{3}} & = 2 - 4m^{-\frac{1}{2}}n^{\frac{1}{3}} + m^{-1}n^{\frac{2}{3}}
 \end{array}$$

$$\begin{array}{l|l}
 6. \sqrt{a^{\frac{4}{5}} - 8a^{\frac{3}{5}} + 10a^{\frac{2}{5}} + 24a^{\frac{1}{5}} + 9} & a^{\frac{2}{5}} - 4a^{\frac{1}{5}} - 3 \\
 \hline
 -a^{\frac{4}{5}} & \left(2a^{\frac{2}{5}} - 4a^{\frac{1}{5}}\right)\left(-4a^{\frac{1}{5}}\right) \\
 & = -8a^{\frac{3}{5}} + 16a^{\frac{2}{5}} \\
 \\ 
 -8a^{\frac{3}{5}} + 10a^{\frac{2}{5}} & \\
 +8a^{\frac{3}{5}} - 16a^{\frac{2}{5}} & \\
 \hline
 -6a^{\frac{2}{5}} + 24a^{\frac{1}{5}} + 9 & \left(2a^{\frac{2}{5}} - 8a^{\frac{1}{5}} - 3\right)(-3) \\
 +6a^{\frac{2}{5}} - 24a^{\frac{1}{5}} - 9 & = -6a^{\frac{2}{5}} + 24a^{\frac{1}{5}} + 9
 \end{array}$$

$$\begin{array}{l|l}
 7. \sqrt[3]{a^{-3} - 6a^{-\frac{5}{2}} + 21a^{-2} - 44a^{-\frac{3}{2}} + 63a^{-1} - 54a^{-\frac{1}{2}} + 27} & a^{-1} - 2a^{-\frac{1}{2}} + 3 \\
 \hline
 -a^{-3} & 3(a^{-1})^2 = 3a^{-2} \\
 \\ 
 -6a^{-\frac{5}{2}} + 21a^{-2} - 44a^{-\frac{3}{2}} & 3(a^{-1})^2\left(-2a^{-\frac{1}{2}}\right) = -6a^{-\frac{5}{2}} \\
 \\ 
 +6a^{-\frac{5}{2}} - 12a^{-2} + 8a^{-\frac{3}{2}} & 3(a^{-1})\left(-2a^{-\frac{1}{2}}\right)^2 = 12a^{-2} \\
 \\ 
 9a^{-2} - 36a^{-\frac{3}{2}} + 63a^{-1} - 54a^{-\frac{1}{2}} + 27 & \left(-2a^{-\frac{1}{2}}\right)^3 = -8a^{-\frac{3}{2}} \\
 \\ 
 -9a^{-2} + 36a^{-\frac{3}{2}} - 63a^{-1} + 54a^{-\frac{1}{2}} - 27 & 3\left(a^{-1} - 2a^{-\frac{1}{2}}\right)^2 = 3a^{-2} - 12a^{-\frac{3}{2}} + 12a^{-1} \\
 & 3\left(a^{-1} - 2a^{-\frac{1}{2}}\right)^2(3) = 9a^{-2} - 36a^{-\frac{3}{2}} + 36a^{-1} \\
 & 3\left(a^{-1} - 2a^{-\frac{1}{2}}\right)(3)^2 = 27a^{-1} - 54a^{-\frac{1}{2}} \\
 & (3)^3 = 27
 \end{array}$$



$8. \sqrt[3]{x^2 - 6x^{\frac{4}{3}} + 15x^{\frac{2}{3}} - 20 + 15x^{-\frac{2}{3}} - 6x^{-\frac{4}{3}} + x^{-2}}$	$x^{\frac{2}{3}} - 2 + x^{-\frac{2}{3}}$
$-x^2$	$3\left(x^{\frac{2}{3}}\right)^2 = 3x^{\frac{4}{3}}$
$-6x^{\frac{4}{3}} + 15x^{\frac{2}{3}} - 20$	$3\left(x^{\frac{2}{3}}\right)^2(-2) = -6x^{\frac{4}{3}}$
$+6x^{\frac{4}{3}} - 12x^{\frac{2}{3}} + 8$	$3\left(x^{\frac{2}{3}}\right)^2(-2)^2 = 12x^{\frac{2}{3}}$
$3x^{\frac{2}{3}} - 12 + 15x^{-\frac{2}{3}} - 6x^{-\frac{4}{3}} + x^{-2}$	$(-2)^3 = -8$
$-3x^{\frac{2}{3}} + 12 - 15x^{-\frac{2}{3}} + 6x^{-\frac{4}{3}} - x^{-2}$	$3\left(x^{\frac{2}{3}} - 2\right)^2 = 3x^{\frac{4}{3}} - 12x^{\frac{2}{3}} + 12$
	$3\left(x^{\frac{2}{3}} - 2\right)^2\left(x^{-\frac{2}{3}}\right) = 3x^{\frac{2}{3}} - 12 + 12x^{-\frac{2}{3}}$
	$3\left(x^{\frac{2}{3}} - 2\right)\left(x^{-\frac{2}{3}}\right)^2 = 3x^{-\frac{2}{3}} - 6x^{-\frac{4}{3}}$
	$\left(x^{-\frac{2}{3}}\right)^3 = x^{-2}$

$9. \sqrt[3]{a^{\frac{3}{2}} + 3a^{\frac{5}{4}} - 5a^{\frac{3}{4}} + 3a^{\frac{1}{4}} - 1}$	$a^{\frac{1}{2}} + a^{\frac{1}{4}} - 1$
$-a^{\frac{3}{2}}$	$3\left(a^{\frac{1}{2}}\right)^2 = 3a$
$+3a^{\frac{5}{4}} - 5a^{\frac{3}{4}} + 3a^{\frac{1}{4}}$	$3\left(a^{\frac{1}{2}}\right)^2\left(a^{\frac{1}{4}}\right) = 3a^{\frac{5}{4}}$
$-3a^{\frac{5}{4}} - 3a - a^{\frac{3}{4}}$	$3\left(a^{\frac{1}{2}}\right)\left(a^{\frac{1}{4}}\right)^2 = 3a$
$-3a - 6a^{\frac{3}{4}} + 3a^{\frac{1}{4}} - 1$	$\left(a^{\frac{1}{4}}\right)^3 = a^{\frac{3}{4}}$
$+3a + 6a^{\frac{3}{4}} - 3a^{\frac{1}{4}} + 1$	$3\left(a^{\frac{1}{2}} + a^{\frac{1}{4}}\right)^2 = 3a + 6a^{\frac{3}{4}} + 3a^{\frac{1}{2}}$
	$3\left(a^{\frac{1}{2}} + a^{\frac{1}{4}}\right)^2(-1) = -3a - 6a^{\frac{3}{4}} - 3a^{\frac{1}{2}}$
	$3\left(a^{\frac{1}{2}} + a^{\frac{1}{4}}\right)(-1)^2 = 3a^{\frac{1}{2}} + 3a^{\frac{1}{4}}$
	$(-1)^3 = -1$

## EJERCICIO 230

$$\begin{array}{l|l}
 1. \sqrt{a^2x^{-2} - \frac{2ax^{-1}}{3} + \frac{19}{9} - \frac{2xa^{-1}}{3} + x^2a^{-2}} & ax^{-1} - \frac{1}{3} + a^{-1}x \\
 \hline
 -a^2x^{-2} & \left(2ax^{-1} - \frac{1}{3}\right)\left(-\frac{1}{3}\right) \\
 \hline
 -\frac{2ax^{-1}}{3} + \frac{19}{9} & = -\frac{2ax^{-1}}{3} + \frac{1}{9} \\
 +\frac{2ax^{-1}}{3} - \frac{1}{9} & \\
 \hline
 2 - \frac{2a^{-1}x}{3} + x^2a^{-2} & \left(2ax^{-1} - \frac{2}{3} + a^{-1}x\right)(a^{-1}x) \\
 -2 + \frac{2a^{-1}x}{3} - a^{-2}x^2 & = 2 - \frac{2a^{-1}x}{3} + a^{-2}x^2
 \end{array}$$

$$\begin{array}{l|l}
 2. \sqrt{x^2 - 4 + 2x^{-1} + 4x^{-2} - 4x^{-3} + x^{-4}} & x - 2x^{-1} + x^{-2} \\
 \hline
 -x^2 & (2x - 2x^{-1})(-2x^{-1}) = -4 + 4x^{-2} \\
 \hline
 -4 + 2x^{-1} + 4x^{-2} & \\
 +4 & -4x^{-2} \\
 \hline
 2x^{-1} & -4x^{-3} + x^{-4} \\
 -2x^{-1} & +4x^{-3} - x^{-4}
 \end{array}$$

$$\begin{array}{l|l}
 3. \sqrt{a^4 - 10a + 4 + 25a^{-2} - 20a^{-3} + 4a^{-4}} & a^2 - 5a^{-1} + 2a^{-2} \\
 \hline
 -a^4 & (2a^2 - 5a^{-1})(-5a^{-1}) \\
 \hline
 -10a + 4 + 25a^{-2} & = -10a + 25a^{-2} \\
 +10a & -25a^{-2} \\
 \hline
 +4 & -20a^{-3} + 4a^{-4} \\
 -4 & +20a^{-3} - 4a^{-4}
 \end{array}$$

$$\begin{array}{l|l}
 4. \sqrt{\frac{m^4}{4} - 5m^2 + 28 - 30m^{-2} + 9m^{-4}} & \frac{m^2}{2} - 5 + 3m^{-2} \\
 \hline
 -\frac{m^4}{4} & (m^2 - 5)(-5) = -5m^2 + 25 \\
 \hline
 -5m^2 + 28 & (m^2 - 10 + 3m^{-2})(3m^{-2}) \\
 +5m^2 - 25 & = 3 - 30m^{-2} + 9m^{-4} \\
 \hline
 3 - 30m^{-2} + 9m^{-4} & \\
 -3 + 30m^{-2} - 9m^{-4} &
 \end{array}$$

---

5.  $\sqrt{\frac{4x^2y^{-2}}{25} - \frac{2xy^{-1}}{5} + \frac{19}{12} - \frac{5x^{-1}y}{3} + \frac{25x^{-2}y^2}{9}}$   $\left| \frac{2xy^{-1}}{5} - \frac{1}{2} + \frac{5x^{-1}y}{3} \right.$

---

$-\frac{4x^2y^{-2}}{25}$   $\left( \frac{4xy^{-1}}{5} - \frac{1}{2} \right) \left( -\frac{1}{2} \right) = -\frac{2xy^{-1}}{5} + \frac{1}{4}$

---

$-\frac{2xy^{-1}}{5} + \frac{19}{12}$   $\left( \frac{4xy^{-1}}{5} - 1 + \frac{5x^{-1}y}{3} \right) \left( \frac{5x^{-1}y}{3} \right)$

---

$+\frac{2xy^{-1}}{5} - \frac{1}{4}$   $= \frac{4}{3} - \frac{5x^{-1}y}{3} + \frac{25x^{-2}y^2}{9}$

---

$\frac{4}{3} - \frac{5x^{-1}y}{3} + \frac{25x^{-2}y^2}{9}$

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$-\frac{4}{3} + \frac{5x^{-1}y}{3} - \frac{25x^{-2}y^2}{9}$

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6.  $\sqrt{\frac{a^4}{9} + \frac{2a^3x^{-1}}{3} + a^2x^{-2} - \frac{2ax}{3} - 2 + a^{-2}x^2}$   $\left| \frac{a^2}{3} + ax^{-1} - a^{-1}x \right.$

---

$-\frac{a^4}{9}$   $\left( \frac{2a^2}{3} + ax^{-1} \right) (ax^{-1}) = \frac{2a^3x^{-1}}{3} + a^2x^{-2}$

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$\frac{2a^3x^{-1}}{3} + a^2x^{-2}$   $\left( \frac{2a^2}{3} + 2ax^{-1} - a^{-1}x \right) (-a^{-1}x)$

---

$-\frac{2a^3x^{-1}}{3} - a^2x^{-2}$   $= -\frac{2ax}{3} - 2 + a^{-2}x^2$

---

$-\frac{2ax}{3} - 2 + a^{-2}x^2$

---

$+\frac{2ax}{3} + 2 - a^{-2}x^2$

---

7.  $\sqrt{9m^4 + 30m^2 + 55 + 50m^{-2} + 25m^{-4}}$   $3m^2 + 5 + 5m^{-2}$

---

$-9m^4$   $(6m^2 + 5)(5) = 30m^2 + 25$

---

$+30m^2 + 55$   $(6m^2 + 10 + 5m^{-2})(5m^{-2})$

---

$-30m^2 - 25$   $= 30 + 50m^{-2} + 25m^{-4}$

---

$30 + 50m^{-2} + 25m^{-4}$

---

$-30 - 50m^{-2} - 25m^{-4}$

---

8.  $\sqrt{\frac{4a^2b^2x^{-2}y^{-2}}{49} - \frac{2abx^{-1}y^{-1}}{7} + \frac{21}{20} - \frac{7xya^{-1}b^{-1}}{5} + \frac{49x^2y^2a^{-2}b^{-2}}{25}}$   $\left| \frac{2abx^{-1}y^{-1}}{7} - \frac{1}{2} + \frac{7a^{-1}b^{-1}xy}{5} \right.$

---

$-\frac{4a^2b^2x^{-2}y^{-2}}{49}$   $\left( \frac{4abx^{-1}y^{-1}}{7} - \frac{1}{2} \right) \left( -\frac{1}{2} \right)$

---

$-\frac{2abx^{-1}y^{-1}}{7} + \frac{21}{20}$   $= -\frac{2abx^{-1}y^{-1}}{7} + \frac{1}{4}$

---

$\frac{2abx^{-1}y^{-1}}{7} - \frac{1}{4}$

---

$\frac{4}{5} - \frac{7xya^{-1}b^{-1}}{5} + \frac{49x^2y^2a^{-2}b^{-2}}{25}$   $\left( \frac{4abx^{-1}y^{-1}}{7} - 1 + \frac{7a^{-1}b^{-1}xy}{5} \right) \left( \frac{7a^{-1}b^{-1}xy}{5} \right)$

---

$-\frac{4}{5} + \frac{7xya^{-1}b^{-1}}{5} - \frac{49a^{-2}b^{-2}x^2y^2}{25}$   $= \frac{4}{5} - \frac{7a^{-1}b^{-1}xy}{5} + \frac{49a^{-2}b^{-2}x^2y^2}{25}$

---

$$\begin{array}{l|l}
 9. \sqrt{ab^{\frac{2}{3}} - 4a^{\frac{1}{2}}b^{\frac{1}{3}} + 6 - 4b^{\frac{1}{3}}a^{\frac{1}{2}} + a^{-1}b^{\frac{2}{3}}} & a^{\frac{1}{2}}b^{-\frac{1}{3}} - 2 + a^{-\frac{1}{2}}b^{\frac{1}{3}} \\
 \hline
 -ab^{\frac{2}{3}} & \left(2a^{\frac{1}{2}}b^{-\frac{1}{3}} - 2\right)(-2) = -4a^{\frac{1}{2}}b^{-\frac{1}{3}} + 4 \\
 \hline
 -4a^{\frac{1}{2}}b^{-\frac{1}{3}} + 6 & \\
 \hline
 +4a^{\frac{1}{2}}b^{-\frac{1}{3}} - 4 & \left(2a^{\frac{1}{2}}b^{-\frac{1}{3}} - 4 + a^{-\frac{1}{2}}b^{\frac{1}{3}}\right)\left(a^{-\frac{1}{2}}b^{\frac{1}{3}}\right) \\
 \hline
 2 - 4a^{-\frac{1}{2}}b^{\frac{1}{3}} + a^{-1}b^{\frac{2}{3}} & = 2 - 4a^{-\frac{1}{2}}b^{\frac{1}{3}} + a^{-1}b^{\frac{2}{3}} \\
 \hline
 -2 + 4a^{-\frac{1}{2}}b^{\frac{1}{3}} - a^{-1}b^{\frac{2}{3}} & 
 \end{array}$$

$$\begin{array}{l|l}
 10. \sqrt{a^4b^4 + 6a^2b^2 + 7 - 6a^{-2}b^{-2} + a^{-4}b^{-4}} & a^2b^2 + 3 - a^{-2}b^{-2} \\
 \hline
 -a^4b^4 & (2a^2b^2 + 3)(3) = 6a^2b^2 + 9 \\
 \hline
 +6a^2b^2 + 7 & \\
 \hline
 -6a^2b^2 - 9 & (2a^2b^2 + 6 - a^{-2}b^{-2})(-a^{-2}b^{-2}) \\
 \hline
 -2 - 6a^{-2}b^{-2} + a^{-4}b^{-4} & = -2 - 6a^{-2}b^{-2} + a^{-4}b^{-4} \\
 \hline
 +2 + 6a^{-2}b^{-2} - a^{-4}b^{-4} & 
 \end{array}$$

$$\begin{array}{l|l}
 11. \sqrt{xy^{\frac{2}{3}} - 8x^{\frac{1}{2}}y^{\frac{1}{3}} + 18 - 8x^{-\frac{1}{2}}y^{-\frac{1}{3}} + x^{-1}y^{-\frac{2}{3}}} & x^{\frac{1}{2}}y^{\frac{1}{3}} - 4 + x^{-\frac{1}{2}}y^{-\frac{1}{3}} \\
 \hline
 -xy^{\frac{2}{3}} & \left(2x^{\frac{1}{2}}y^{\frac{1}{3}} - 4\right)(-4) = -8x^{\frac{1}{2}}y^{\frac{1}{3}} + 16 \\
 \hline
 -8x^{\frac{1}{2}}y^{\frac{1}{3}} + 18 & \\
 \hline
 +8x^{\frac{1}{2}}y^{\frac{1}{3}} - 16 & \left(2x^{\frac{1}{2}}y^{\frac{1}{3}} - 8 + x^{-\frac{1}{2}}y^{-\frac{1}{3}}\right)\left(x^{-\frac{1}{2}}y^{-\frac{1}{3}}\right) \\
 \hline
 2 - 8x^{-\frac{1}{2}}y^{-\frac{1}{3}} + x^{-1}y^{-\frac{2}{3}} & = 2 - 8x^{-\frac{1}{2}}y^{-\frac{1}{3}} + x^{-1}y^{-\frac{2}{3}} \\
 \hline
 -2 + 8x^{-\frac{1}{2}}y^{-\frac{1}{3}} - x^{-1}y^{-\frac{2}{3}} & 
 \end{array}$$

## EJERCICIO 231

$$1. \sqrt{18} = \sqrt{2 \cdot 3^2} = 3\sqrt{2}$$

$$2. 3\sqrt{48} = 3\sqrt{3 \cdot 2^4} = 3 \cdot 2^2 \sqrt{3} = 12\sqrt{3}$$

$$3. \sqrt[3]{16} = \sqrt[3]{2 \cdot 2^3} = 2\sqrt[3]{2}$$

$$4. \frac{1}{2}\sqrt[3]{128} = \frac{1}{2}\sqrt[3]{2^6 \cdot 2} = \frac{1}{2} \cdot 2^2 \sqrt[3]{2} = 2\sqrt[3]{2}$$

$$5. 2\sqrt[4]{243} = 2\sqrt[4]{3^4 \cdot 3} = 2 \cdot 3 \sqrt[4]{3} = 6\sqrt[4]{3}$$

$$6. \sqrt{50a^2b} = \sqrt{2 \cdot 5^2 a^2 b} = 5a\sqrt{2b}$$

$$7. 3\sqrt{81x^3y^4} = 3\sqrt{9^2 \cdot x^2 xy^4} = 3 \cdot 9xy^2 \sqrt{x} = 27xy^2 \sqrt{x}$$

$$\begin{aligned}
 8. \frac{1}{2}\sqrt{108a^5b^7} &= \frac{1}{2}\sqrt{2^2 \cdot 3^2 \cdot 3 \cdot a^4 ab^6 b} \\
 &= \frac{2}{2} \cdot 3a^2 b^3 \sqrt{3ab} = 3a^2 b^3 \sqrt{3ab}
 \end{aligned}$$

$$9. \frac{3}{5}\sqrt{125mn^6} = \frac{3}{5}\sqrt{5^2 \cdot 5mn^6} = \frac{3 \cdot 5}{5} n^3 \sqrt{5m} = 3n^3 \sqrt{5m}$$

$$\begin{aligned}
 10. 2a\sqrt{44a^3b^7c^9} &= 2a\sqrt{2^2 \cdot 11a^2 ab^6 bc^8 c} \\
 &= 2 \cdot 2a \cdot ab^3 c^4 \sqrt{11abc} = 4a^2 b^3 c^4 \sqrt{11abc}
 \end{aligned}$$

$$\begin{aligned}
11. \quad & 2\sqrt[3]{16x^2y^7} \\
& = 2\sqrt[3]{2^3 \cdot 2x^2y^6}y = 2 \cdot 2y^2\sqrt[3]{2x^2y} = 4y^2\sqrt[3]{2x^2y} \\
12. \quad & \frac{2}{3}\sqrt[3]{27m^2n^8} \\
& = \frac{2}{3}\sqrt[3]{3^3m^2n^6n^2} = \frac{2 \cdot 3}{3}n^2\sqrt[3]{m^2n^2} = 2n^2\sqrt[3]{m^2n^2} \\
13. \quad & 5a\sqrt[3]{160x^7y^9z^{13}} \\
& = 5a\sqrt[3]{2^3 \cdot 20x^6xy^9z^{12}z} \\
& = 5 \cdot 2ax^2y^3z^4\sqrt[3]{20xz} = 10ax^2y^3z^4\sqrt[3]{20xz} \\
14. \quad & \sqrt[4]{80a^4b^5c^{12}} = \sqrt[4]{2^4 \cdot 5a^4b^4bc^{12}} = 2abc^3\sqrt[4]{5b} \\
15. \quad & 3\sqrt[4]{5x^8y^{14}z^{16}} = 3\sqrt[4]{5x^8y^{12}y^2z^{16}} = 3x^2y^3z^4\sqrt[4]{5y^2} \\
16. \quad & \frac{2}{5}\sqrt[5]{32x^2y^{11}} \\
& = \frac{2}{5}\sqrt[5]{2^5x^2y^{10}y} = \frac{2 \cdot 2}{5}y^2\sqrt[5]{x^2y} = \frac{4y^2}{5}\sqrt[5]{x^2y} \\
17. \quad & 2xy\sqrt[3]{128x^2y^8} \\
& = 2xy\sqrt[3]{2^6 \cdot 2x^2y^6y^2} \\
& = 2 \cdot 2^2xyy^2\sqrt[3]{2x^2y^2} = 8xy^3\sqrt[3]{2x^2y^2} \\
18. \quad & \frac{1}{3a}\sqrt{27a^3m^7} \\
& = \frac{1}{3a}\sqrt{3^2 \cdot 3a^2am^6m} = \frac{3am^3}{3a}\sqrt{3am} = m^3\sqrt{3am}
\end{aligned}$$

## EJERCICIO 232

$$\begin{aligned}
1. \quad & \sqrt{\frac{1}{5}} = \sqrt{\frac{1 \cdot 5}{5 \cdot 5}} = \sqrt{\frac{5}{5^2}} = \frac{1}{5}\sqrt{5} \\
2. \quad & \sqrt{\frac{3}{8}} = \sqrt{\frac{3 \cdot 2}{8 \cdot 2}} = \sqrt{\frac{6}{2^4}} = \frac{1}{2^2}\sqrt{6} = \frac{1}{4}\sqrt{6} \\
3. \quad & 2\sqrt{\frac{1}{2}} = 2\sqrt{\frac{2 \cdot 1}{2 \cdot 2}} = 2\sqrt{\frac{2}{2^2}} = \frac{2}{2}\sqrt{2} = \sqrt{2} \\
4. \quad & 3\sqrt{\frac{1}{6}} = 3\sqrt{\frac{1 \cdot 6}{6 \cdot 6}} = 3\sqrt{\frac{6}{6^2}} = \frac{3}{6}\sqrt{6} = \frac{1}{2}\sqrt{6} \\
5. \quad & \frac{1}{2}\sqrt{\frac{2}{3}} = \frac{1}{2}\sqrt{\frac{2 \cdot 3}{3 \cdot 3}} = \frac{1}{2}\sqrt{\frac{6}{3^2}} = \frac{1}{2 \cdot 3}\sqrt{6} = \frac{1}{6}\sqrt{6} \\
6. \quad & \sqrt{\frac{a^2}{8x}} = \sqrt{\frac{a^2x}{2^2 \cdot 2x^2}} = \frac{a}{2x}\sqrt{\frac{x}{2}} = \frac{a}{2x}\sqrt{\frac{2x}{2^2}} = \frac{a}{4x}\sqrt{2x}
\end{aligned}$$

$$\begin{aligned}
19. \quad & \frac{3}{5x}\sqrt[3]{375a^8b} \\
& = \frac{3}{5x}\sqrt[3]{5^3 \cdot 3a^6a^2b} = \frac{3 \cdot 5a^2}{5x}\sqrt[3]{3a^2b} = \frac{3a^2}{x}\sqrt[3]{3a^2b} \\
20. \quad & \frac{1}{3}\sqrt[4]{81a^4b} = \frac{1}{3}\sqrt[4]{3^4a^4b} = \frac{3}{3}a\sqrt[4]{b} = a\sqrt[4]{b} \\
21. \quad & \sqrt{9a+18b} = \sqrt{9(a+2b)} = \sqrt{3^2(a+2b)} = 3\sqrt{a+2b} \\
22. \quad & \sqrt{3a^3b^2-3a^2b^2} \\
& = \sqrt{3a^2b^2(a-1)} = ab\sqrt{3(a-1)} = ab\sqrt{3a-3} \\
23. \quad & \sqrt{8x^2y^4+16xy^4} \\
& = \sqrt{2^2 \cdot 2xy^4(x+2)} = 2y^2\sqrt{2x(x+2)} = 2y^2\sqrt{2x^2+4x} \\
24. \quad & \sqrt{2x^2-4xy+2y^2} \\
& = \sqrt{2(x^2-2xy+y^2)} = \sqrt{2(x-y)^2} = (x-y)\sqrt{2} \\
25. \quad & \sqrt{(a-b)(a^2-b^2)} \\
& = \sqrt{(a-b)(a-b)(a+b)} \\
& = \sqrt{(a-b)^2(a+b)} = (a-b)\sqrt{a+b} \\
26. \quad & \sqrt{2am^2+4amn+2an^2} \\
& = \sqrt{2a(m^2+2mn+n^2)} = \sqrt{2a(m+n)^2} = (m+n)\sqrt{2a} \\
27. \quad & \sqrt{9a^3-36a^2+36a} \\
& = \sqrt{3^2a(a^2-2a+4)} \\
& = 3\sqrt{a(a-2)^2} = 3(a-2)\sqrt{a} = (3a-6)\sqrt{a}
\end{aligned}$$

$$\begin{aligned}
7. \quad & \frac{3}{2}\sqrt{\frac{4a^2}{27y^3}} = \frac{3}{2}\sqrt{\frac{3 \cdot 2^2a^2y}{3^4y^4}} = \frac{3 \cdot 2a}{2 \cdot 3^2y^2}\sqrt{3y} = \frac{a}{3y^2}\sqrt{3y} \\
8. \quad & 5\sqrt{\frac{9n}{5m^3}} = 5\sqrt{\frac{3^2 \cdot 5mn}{5^2m^4}} = \frac{5 \cdot 3}{5m^2}\sqrt{5mn} = \frac{3}{m^2}\sqrt{5mn} \\
9. \quad & 6\sqrt{\frac{5a^3}{24x^2}} = 6\sqrt{\frac{5 \cdot 6a^3}{2^4 \cdot 3^2x^2}} = \frac{6}{2^2 \cdot 3x}\sqrt{30a^2a} = \frac{a}{2x}\sqrt{30a} \\
10. \quad & \sqrt[3]{\frac{2}{3}} = \sqrt[3]{\frac{2 \cdot 3^2}{3^3}} = \frac{1}{3}\sqrt[3]{18} \\
11. \quad & 5\sqrt[3]{\frac{1}{5}} = 5\sqrt[3]{\frac{2^3 \cdot 5^2}{5^3}} = \frac{5}{5}\sqrt[3]{5^2} = \sqrt[3]{25} \\
12. \quad & \sqrt[3]{\frac{8}{9x^2}} = \sqrt[3]{\frac{2^3 \cdot 3x}{3^2x^3}} = \frac{2}{3x}\sqrt[3]{3x}
\end{aligned}$$

$$13. 2b^2 \sqrt[3]{\frac{125}{4b^5}} = 2b^2 \sqrt[3]{\frac{5^3 \cdot 2b}{2^3 b^6}} = \frac{2 \cdot 5b^2}{2b^2} \sqrt[3]{2b} = 5 \sqrt[3]{2b}$$

$$14. \frac{2}{3} \sqrt[3]{\frac{27x^2}{16a^2b^4}} = \frac{2}{3} \sqrt[3]{\frac{3^3 \cdot 4ab^2x^2}{2^6 a^2b^6}} = \frac{2 \cdot 3}{3 \cdot 2^2 ab^2} \sqrt[3]{4ab^2x^2} = \frac{1}{2ab^2} \sqrt[3]{4ab^2x^2}$$

$$15. 2xy \sqrt[4]{\frac{81a^2}{4x^3y}} = 2xy \sqrt[4]{\frac{3^4 \cdot 2^2 a^2 xy^3}{2^4 x^4 y^4}} = \frac{2 \cdot 3xy}{2xy} \sqrt[4]{4a^2 xy^3} = 3 \sqrt[4]{4a^2 xy^3}$$

### EJERCICIO 233

$$1. \sqrt[4]{9} = \sqrt[4]{3^2} = 3^{\frac{2}{4}} = 3^{\frac{1}{2}} = \sqrt{3}$$

$$2. \sqrt[6]{4} = \sqrt[6]{2^2} = 2^{\frac{2}{6}} = 2^{\frac{1}{3}} = \sqrt[3]{2}$$

$$3. \sqrt[9]{27} = \sqrt[9]{3^3} = 3^{\frac{3}{9}} = 3^{\frac{1}{3}} = \sqrt[3]{3}$$

$$4. \sqrt[8]{16} = \sqrt[8]{2^4} = 2^{\frac{4}{8}} = 2^{\frac{1}{2}} = \sqrt{2}$$

$$5. 3 \sqrt[12]{64} = 3 \sqrt[12]{2^6} = 3 \cdot 2^{\frac{6}{12}} = 3 \cdot 2^{\frac{1}{2}} = 3\sqrt{2}$$

$$6. \sqrt[4]{25a^2b^2} = \sqrt[4]{5^2 a^2 b^2} = 5^{\frac{2}{4}} a^{\frac{2}{4}} b^{\frac{2}{4}} = 5^{\frac{1}{2}} a^{\frac{1}{2}} b^{\frac{1}{2}} = \sqrt{5ab}$$

$$7. 5 \sqrt[6]{49a^2b^4} = 5 \sqrt[6]{7^2 a^2 b^4} = 5 \cdot 7^{\frac{2}{6}} a^{\frac{2}{6}} b^{\frac{4}{6}} = 5 \sqrt[3]{7ab^2}$$

$$8. \sqrt[8]{81x^4y^8} = \sqrt[8]{3^4 x^4 y^8} = 3^{\frac{4}{8}} x^{\frac{4}{8}} y^{\frac{8}{8}} = 3^{\frac{1}{2}} x^{\frac{1}{2}} y = y\sqrt{3x}$$

$$9. \sqrt[10]{32x^{10}y^{15}} = \sqrt[10]{2^5 x^{10} y^{15}} = 2^{\frac{5}{10}} x^{\frac{10}{10}} y^{\frac{15}{10}} = 2^{\frac{1}{2}} xy^{\frac{3}{2}} = x\sqrt{2y^3} = x\sqrt{2y^2y} = xy\sqrt{2y}$$

$$10. \sqrt[12]{64m^6n^{18}} = \sqrt[12]{2^6 m^6 n^{18}} = 2^{\frac{6}{12}} m^{\frac{6}{12}} n^{\frac{18}{12}} = 2^{\frac{1}{2}} m^{\frac{1}{2}} n^{\frac{3}{2}} = \sqrt{2mn^3} = \sqrt{2mn^2n} = n\sqrt{2mn}$$

$$11. \sqrt[6]{343a^9x^{12}} = \sqrt[6]{7^3 a^9 x^{12}} = 7^{\frac{3}{6}} a^{\frac{9}{6}} x^{\frac{12}{6}} = 7^{\frac{1}{2}} a^{\frac{3}{2}} x^2 = x^2 \sqrt{7a^3} = x^2 \sqrt{7a^2a} = ax^2 \sqrt{7a}$$

$$12. \sqrt[15]{m^{10}n^{15}x^{20}} = m^{\frac{10}{15}} n^{\frac{15}{15}} x^{\frac{20}{15}} = m^{\frac{2}{3}} nx^{\frac{4}{3}} = n \sqrt[3]{m^2x^4} = n \sqrt[3]{m^2x^3x} = nx \sqrt[3]{m^2x}$$

### EJERCICIO 234

$$1. 2\sqrt{3} = \sqrt{2^2 \cdot 3} = \sqrt{12}$$

$$2. 3\sqrt{5} = \sqrt{3^2 \cdot 5} = \sqrt{45}$$

$$3. 5a\sqrt{b} = \sqrt{(5a)^2 b} = \sqrt{25a^2b}$$

$$4. \frac{1}{2}\sqrt{2} = \sqrt{\left(\frac{1}{2}\right)^2 \cdot 2} = \sqrt{\frac{2}{4}} = \sqrt{\frac{1}{2}}$$

$$5. 3a\sqrt{2a^2} = \sqrt{(3a)^2 \cdot 2a^2} = \sqrt{9a^2(2a^2)} = \sqrt{18a^4}$$

$$6. 5x^2y\sqrt{3} = \sqrt{(5x^2y)^2 \cdot 3} = \sqrt{(25x^4y^2) \cdot 3} = \sqrt{75x^4y^2}$$

$$7. ab^2 \sqrt[3]{a^2b} = \sqrt[3]{(ab^2)^3 (a^2b)} = \sqrt[3]{(a^3b^6)(a^2b)} = \sqrt[3]{a^5b^7}$$

$$8. 4m \sqrt[3]{2m^2} = \sqrt[3]{(4m)^3 (2m^2)} = \sqrt[3]{(64m^3)(2m^2)} = \sqrt[3]{128m^5}$$

$$9. 2a \sqrt[4]{8ab^3} = \sqrt[4]{(2a)^4 (8ab^3)} = \sqrt[4]{(16a^4)(8ab^3)} = \sqrt[4]{128a^5b^3}$$

$$10. (a+b) \sqrt{\frac{a}{a+b}} = \sqrt{\frac{(a+b)^2 (a)}{a+b}} = \sqrt{(a+b)(a)} = \sqrt{a^2 + ab}$$

$$11. (x+1) \sqrt{\frac{2x}{x+1}} = \sqrt{\frac{(x+1)^2 2x}{x+1}} = \sqrt{(x+1)2x} = \sqrt{2x^2 + 2x}$$

$$12. (x-1) \sqrt{\frac{x-2}{x-1}} = \sqrt{\frac{(x-1)^2 (x-2)}{x-1}} = \sqrt{(x-1)(x-2)} = \sqrt{x^2 - 3x + 2}$$

### EJERCICIO 235

$$1. \sqrt{5} = \sqrt[6]{5^3} = \sqrt[6]{125}$$

$$2. \sqrt{2} = \sqrt[4]{2^2} = \sqrt[4]{4}$$

$$3. \sqrt{3} = \sqrt[12]{3^6} = \sqrt[12]{729}$$

$$\sqrt[3]{2} = \sqrt[6]{2^2} = \sqrt[6]{4}$$

$$\sqrt[4]{3} = \sqrt[4]{3}$$

$$\sqrt[3]{4} = \sqrt[12]{4^4} = \sqrt[12]{256}$$

$$\sqrt[4]{8} = \sqrt[12]{8^3} = \sqrt[12]{512}$$

$$4. \sqrt{2} = \sqrt[12]{2^6} = \sqrt[12]{64}$$

$$\sqrt[3]{3} = \sqrt[12]{3^4} = \sqrt[12]{81}$$

$$\sqrt[4]{5} = \sqrt[12]{5^3} = \sqrt[12]{125}$$

$$\sqrt[6]{7} = \sqrt[12]{7^2} = \sqrt[12]{49}$$

$$5. \sqrt{5x} = \sqrt[6]{(5x)^3} = \sqrt[6]{125x^3}$$

$$\sqrt[3]{4x^2y} = \sqrt[6]{(4x^2y)^2} = \sqrt[6]{16x^4y^2}$$

$$\sqrt[6]{7a^3b} = \sqrt[6]{7a^3b}$$

$$6. \sqrt[3]{2ab} = \sqrt[15]{(2ab)^5} = \sqrt[15]{32a^5b^5}$$

$$\sqrt[5]{3a^2x} = \sqrt[15]{(3a^2x)^3} = \sqrt[15]{27a^6x^3}$$

$$\sqrt[15]{5a^3x^2} = \sqrt[15]{5a^3x^2}$$

$$7. \sqrt[4]{8a^2x^3} = \sqrt[12]{(8a^2x^3)^3} = \sqrt[12]{512a^6x^9}$$

$$\sqrt[6]{3a^5m^4} = \sqrt[12]{(3a^5m^4)^2} = \sqrt[12]{9a^{10}m^8}$$

$$8. \sqrt[3]{x^2} = \sqrt[18]{(x^2)^6} = \sqrt[18]{x^{12}}$$

$$\sqrt[6]{2y^3} = \sqrt[18]{(2y^3)^3} = \sqrt[18]{8y^9}$$

$$\sqrt[9]{5m^7} = \sqrt[18]{(5m^7)^2} = \sqrt[18]{25m^{14}}$$

$$9. \sqrt[4]{3a} = \sqrt[20]{(3a)^5} = \sqrt[20]{243a^5}$$

$$\sqrt[5]{2b^2} = \sqrt[20]{(2b^2)^4} = \sqrt[20]{16b^8}$$

$$\sqrt[10]{7x^3} = \sqrt[20]{(7x^3)^2} = \sqrt[20]{49x^6}$$

$$10. 2\sqrt[3]{a} = 2\sqrt[12]{a^4} = 2\sqrt[12]{a^4}$$

$$3\sqrt{2b} = 3\sqrt[12]{(2b)^6} = 3\sqrt[12]{64b^6}$$

$$4\sqrt[4]{5x^2} = 4\sqrt[12]{(5x^2)^3} = 4\sqrt[12]{125x^6}$$

$$11. 3\sqrt[3]{a^2} = 3\sqrt[18]{(a^2)^6} = 3\sqrt[18]{a^{12}}$$

$$\frac{1}{2}\sqrt[6]{b^3} = \frac{1}{2}\sqrt[18]{(b^3)^3} = \frac{1}{2}\sqrt[18]{b^9}$$

$$4\sqrt[9]{x^5} = 4\sqrt[18]{(x^5)^2} = 4\sqrt[18]{x^{10}}$$

$$12. \sqrt{2m} = \sqrt[10]{(2m)^5} = \sqrt[10]{32m^5}$$

$$3\sqrt[5]{a^3x^4} = 3\sqrt[10]{(a^3x^4)^2} = 3\sqrt[10]{a^6x^8}$$

$$2\sqrt[10]{x^7y^2} = 2\sqrt[10]{x^7y^2}$$

## EJERCICIO 236

$$1. \sqrt{5} = \sqrt[6]{5^3} = \sqrt[6]{125}$$

$$\sqrt[3]{2} = \sqrt[6]{2^2} = \sqrt[6]{4}$$

Luego el orden es:  $\sqrt{5}$ ,  $\sqrt[3]{2}$

$$2. \sqrt[6]{15} = \sqrt[12]{(15)^2} = \sqrt[12]{225}$$

$$\sqrt[4]{7} = \sqrt[12]{7^3} = \sqrt[12]{343}$$

Luego el orden es:  $\sqrt[4]{7}$ ,  $\sqrt[6]{15}$

$$3. \sqrt{11} = \sqrt[6]{(11)^3} = \sqrt[6]{1.331}$$

$$\sqrt[3]{43} = \sqrt[6]{(43)^2} = \sqrt[6]{1.849}$$

Luego el orden es:  $\sqrt[3]{43}$ ,  $\sqrt{11}$

$$4. \sqrt{3} = \sqrt[6]{3^3} = \sqrt[6]{27}$$

$$\sqrt[3]{5} = \sqrt[6]{5^2} = \sqrt[6]{25}$$

$$\sqrt[6]{32} = \sqrt[6]{32}$$

Luego el orden es:  $\sqrt[6]{32}$ ,  $\sqrt{3}$ ,  $\sqrt[3]{5}$

$$5. \sqrt[4]{3} = \sqrt[20]{3^5} = \sqrt[20]{243}$$

$$\sqrt[5]{4} = \sqrt[20]{4^4} = \sqrt[20]{256}$$

$$\sqrt[10]{15} = \sqrt[20]{15^2} = \sqrt[20]{225}$$

Luego el orden es:  $\sqrt[5]{4}$ ,  $\sqrt[4]{3}$ ,  $\sqrt[10]{15}$

$$6. \sqrt[3]{2} = \sqrt[18]{2^6} = \sqrt[18]{64}$$

$$\sqrt[6]{3} = \sqrt[18]{3^3} = \sqrt[18]{27}$$

$$\sqrt[9]{9} = \sqrt[18]{9^2} = \sqrt[18]{81}$$

Luego el orden es:  $\sqrt[9]{9}$ ,  $\sqrt[3]{2}$ ,  $\sqrt[6]{3}$

## EJERCICIO 237

$$1. 7\sqrt{2} - 15\sqrt{2} = (7-15)\sqrt{2} = -8\sqrt{2}$$

$$2. 4\sqrt{3} - 20\sqrt{3} + 19\sqrt{3} = (4-20+19)\sqrt{3} = 3\sqrt{3}$$

$$3. \sqrt{5} - 22\sqrt{5} - 8\sqrt{5} = (1-22-8)\sqrt{5} = -29\sqrt{5}$$

$$4. \sqrt{2} - 9\sqrt{2} + 30\sqrt{2} - 40\sqrt{2} = (1-9+30-40)\sqrt{2} = -18\sqrt{2}$$

$$5. \frac{3}{4}\sqrt{2} - \frac{1}{2}\sqrt{2} = \left(\frac{3}{4} - \frac{1}{2}\right)\sqrt{2} = \left(\frac{3-2}{4}\right)\sqrt{2} = \frac{1}{4}\sqrt{2}$$

$$6. \frac{3}{5}\sqrt{3} - \sqrt{3} = \left(\frac{3}{5} - 1\right)\sqrt{3} = \left(\frac{3-5}{5}\right)\sqrt{3} = -\frac{2}{5}\sqrt{3}$$

$$7. 2\sqrt{5} - \frac{1}{2}\sqrt{5} + \frac{3}{4}\sqrt{5} \\ = \left(2 - \frac{1}{2} + \frac{3}{4}\right)\sqrt{5} = \left(\frac{8-2+3}{4}\right)\sqrt{5} = \frac{9}{4}\sqrt{5}$$

$$8. \frac{1}{4}\sqrt{3} + 5\sqrt{3} - \frac{1}{8}\sqrt{3} \\ = \left(\frac{1}{4} + 5 - \frac{1}{8}\right)\sqrt{3} = \left(\frac{2+40-1}{8}\right)\sqrt{3} = \frac{41}{8}\sqrt{3}$$

$$9. a\sqrt{b} - 3a\sqrt{b} + 7a\sqrt{b} = (a - 3a + 7a)\sqrt{b} = 5a\sqrt{b}$$

$$10. 3x\sqrt{y} + (a-x)\sqrt{y} - 2x\sqrt{y} \\ = (3x + a - x - 2x)\sqrt{y} = a\sqrt{y}$$

$$11. (x-1)\sqrt{3} + (x-3)\sqrt{3} + 4\sqrt{3} \\ = (x-1+x-3+4)\sqrt{3} = 2x\sqrt{3}$$

$$12. \frac{1}{3}\sqrt[3]{2} - \frac{2}{3}\sqrt[3]{2} + 2\sqrt[3]{2} \\ = \left(\frac{1}{3} - \frac{2}{3} + 2\right)\sqrt[3]{2} = \left(\frac{1-2+6}{3}\right)\sqrt[3]{2} = \frac{5}{3}\sqrt[3]{2}$$

$$13. \frac{3}{5}\sqrt[3]{2} - \frac{1}{4}\sqrt[3]{2} + \frac{1}{6}\sqrt[3]{2} = \left(\frac{36-15+10}{60}\right)\sqrt[3]{2} = \frac{31}{60}\sqrt[3]{2}$$

$$14. x\sqrt[3]{a^2} - (a-2x)\sqrt[3]{a^2} + (2a-3x)\sqrt[3]{a^2} \\ = (x-a+2x+2a-3x)\sqrt[3]{a^2} = a\sqrt[3]{a^2}$$

### EJERCICIO 238

$$1. \sqrt{45} = \sqrt{3^2 \cdot 5} = 3\sqrt{5} \\ -\sqrt{27} = -\sqrt{3^2 \cdot 3} = -3\sqrt{3} \\ -\sqrt{20} = -\sqrt{2^2 \cdot 5} = -2\sqrt{5}$$

Entonces:

$$= 3\sqrt{5} - 3\sqrt{3} - 2\sqrt{5} \\ = (3-2)\sqrt{5} - 3\sqrt{3} = \sqrt{5} - 3\sqrt{3}$$

$$2. \sqrt{175} = \sqrt{5^2 \cdot 7} = 5\sqrt{7} \\ \sqrt{243} = \sqrt{3^4 \cdot 3} = 9\sqrt{3} \\ -\sqrt{63} = -\sqrt{3^2 \cdot 7} = -3\sqrt{7} \\ -2\sqrt{75} = -2\sqrt{5^2 \cdot 3} = -10\sqrt{3}$$

Entonces:

$$= 5\sqrt{7} + 9\sqrt{3} - 3\sqrt{7} - 10\sqrt{3} \\ = (5-3)\sqrt{7} + (9-10)\sqrt{3} = 2\sqrt{7} - \sqrt{3}$$

$$3. \sqrt{80} = \sqrt{2^4 \cdot 5} = 2^2\sqrt{5} = 4\sqrt{5} \\ -2\sqrt{252} = -2\sqrt{2^2 \cdot 3^2 \cdot 7} = -2 \cdot 2 \cdot 3\sqrt{7} = -12\sqrt{7} \\ 3\sqrt{405} = 3\sqrt{3^4 \cdot 5} = 3 \cdot 3^2\sqrt{5} = 27\sqrt{5} \\ -3\sqrt{500} = -3\sqrt{2^2 \cdot 5^2 \cdot 5} = -3 \cdot 2 \cdot 5\sqrt{5} = -30\sqrt{5}$$

Entonces:

$$= 4\sqrt{5} + 27\sqrt{5} - 30\sqrt{5} - 12\sqrt{7} \\ = (4+27-30)\sqrt{5} - 12\sqrt{7} = \sqrt{5} - 12\sqrt{7}$$

$$4. 7\sqrt{450} = 7\sqrt{3^2 \cdot 5^2 \cdot 2} = 7 \cdot 3 \cdot 5\sqrt{2} = 105\sqrt{2} \\ -4\sqrt{320} = -4\sqrt{2^6 \cdot 5} = -4 \cdot 2^3\sqrt{5} = -32\sqrt{5} \\ 3\sqrt{80} = 3\sqrt{2^4 \cdot 5} = 3 \cdot 2^2\sqrt{5} = 12\sqrt{5} \\ -5\sqrt{800} = -5\sqrt{2^4 \cdot 2 \cdot 5^2} = -5 \cdot 2^2 \cdot 5\sqrt{2} = -100\sqrt{2}$$

Entonces:

$$= 105\sqrt{2} - 100\sqrt{2} - 32\sqrt{5} + 12\sqrt{5} \\ = (105-100)\sqrt{2} + (-32+12)\sqrt{5} = 5\sqrt{2} - 20\sqrt{5}$$

$$5. \frac{1}{2}\sqrt{12} = \frac{1}{2}\sqrt{2^2 \cdot 3} = \sqrt{3} \\ -\frac{1}{3}\sqrt{18} = -\frac{1}{3}\sqrt{3^2 \cdot 2} = -\sqrt{2} \\ \frac{3}{4}\sqrt{48} = \frac{3}{4}\sqrt{2^4 \cdot 3} = 3\sqrt{3} \\ \frac{1}{6}\sqrt{72} = \frac{1}{6}\sqrt{2^3 \cdot 2 \cdot 3^2} = \sqrt{2}$$

Entonces:

$$= \sqrt{3} + 3\sqrt{3} - \sqrt{2} + \sqrt{2} = (1+3)\sqrt{3} = 4\sqrt{3}$$

$$6. \frac{3}{4}\sqrt{176} = \frac{3}{4}\sqrt{2^4 \cdot 11} = \frac{2^2 \cdot 3}{4}\sqrt{11} = 3\sqrt{11} \\ -\frac{2}{3}\sqrt{45} = -\frac{2}{3}\sqrt{3^2 \cdot 5} = -\frac{2 \cdot 3}{3}\sqrt{5} = -2\sqrt{5} \\ \frac{1}{8}\sqrt{320} = \frac{1}{8}\sqrt{2^6 \cdot 5} = \frac{2^3 \cdot 3}{8}\sqrt{5} = \sqrt{5} \\ \frac{1}{5}\sqrt{275} = \frac{1}{5}\sqrt{5^2 \cdot 11} = \frac{5}{5}\sqrt{11} = \sqrt{11}$$

Entonces:

$$= 3\sqrt{11} + \sqrt{11} - 2\sqrt{5} + \sqrt{5} \\ = (3+1)\sqrt{11} - (2+1)\sqrt{5} = 4\sqrt{11} - \sqrt{5}$$



$$\begin{aligned}
 7. \quad \frac{1}{7}\sqrt{147} &= \frac{1}{7}\sqrt{3 \cdot 7^2} = \sqrt{3} \\
 -\frac{1}{5}\sqrt{700} &= -\frac{1}{5}\sqrt{2^2 \cdot 5^2 \cdot 7} = -2\sqrt{7} \\
 \frac{1}{10}\sqrt{28} &= \frac{1}{10}\sqrt{2^2 \cdot 7} = \frac{1}{5}\sqrt{7} \\
 \frac{1}{3}\sqrt{2 \cdot 187} &= \frac{1}{3}\sqrt{3^6 \cdot 3} = 9\sqrt{3}
 \end{aligned}$$

Entonces:

$$\begin{aligned}
 &= \sqrt{3} + 9\sqrt{3} - 2\sqrt{7} + \frac{1}{5}\sqrt{7} \\
 &= (1+9)\sqrt{3} + \left(-2 + \frac{1}{5}\right)\sqrt{7} \\
 &= 10\sqrt{3} - \frac{9}{5}\sqrt{7}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad \sqrt{\frac{1}{3}} &= \sqrt{\frac{3}{3^2}} = \frac{1}{3}\sqrt{3} \\
 -\sqrt{\frac{1}{2}} &= -\sqrt{\frac{2}{2^2}} = -\frac{1}{2}\sqrt{2} \\
 \sqrt{\frac{3}{4}} &= \sqrt{\frac{3}{2^2}} = \frac{1}{2}\sqrt{3}
 \end{aligned}$$

Entonces:

$$\begin{aligned}
 &= \frac{1}{3}\sqrt{3} + \frac{1}{2}\sqrt{3} - \frac{1}{2}\sqrt{2} \\
 &= \left(\frac{1}{3} + \frac{1}{2}\right)\sqrt{3} - \frac{1}{2}\sqrt{2} \\
 &= \frac{5}{6}\sqrt{3} - \frac{1}{2}\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad \sqrt{\frac{9}{5}} &= \sqrt{\frac{3^2 \cdot 5}{5^2}} = \frac{3}{5}\sqrt{5} \\
 -\sqrt{\frac{1}{6}} &= -\sqrt{-\frac{6}{6^2}} = -\frac{1}{6}\sqrt{6} \\
 -\sqrt{\frac{1}{20}} &= -\sqrt{\frac{5}{2^2 \cdot 5^2}} = -\frac{1}{10}\sqrt{5} \\
 &\quad \sqrt{6} = \sqrt{6}
 \end{aligned}$$

Entonces:

$$\begin{aligned}
 &= \frac{3}{5}\sqrt{5} - \frac{1}{10}\sqrt{5} - \frac{1}{6}\sqrt{6} + \sqrt{6} \\
 &= \left(\frac{3}{5} - \frac{1}{10}\right)\sqrt{5} + \left(-\frac{1}{6} + 1\right)\sqrt{6} \\
 &= \frac{1}{2}\sqrt{5} + \frac{5}{6}\sqrt{6}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad \frac{5}{3}\sqrt{\frac{3}{5}} &= \frac{5}{3}\sqrt{\frac{15}{5^2}} = \frac{1}{3}\sqrt{15} \\
 -\frac{1}{2}\sqrt{\frac{3}{4}} &= -\frac{1}{2}\sqrt{\frac{3}{2^2}} = -\frac{1}{4}\sqrt{3} \\
 -5\sqrt{\frac{1}{15}} &= -5\sqrt{\frac{15}{3^2 \cdot 5^2}} = -\frac{1}{3}\sqrt{15} \\
 3\sqrt{\frac{1}{12}} &= 3\sqrt{\frac{3}{2^2 \cdot 3^2}} = \frac{1}{2}\sqrt{3}
 \end{aligned}$$

Entonces:

$$\begin{aligned}
 &= \frac{1}{3}\sqrt{15} - \frac{1}{3}\sqrt{15} - \frac{1}{4}\sqrt{3} + \frac{1}{2}\sqrt{3} \\
 &= \left(-\frac{1}{4} + \frac{1}{2}\right)\sqrt{3} = \frac{1}{4}\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad 5\sqrt{128} &= 5\sqrt{2^6 \cdot 2} = 40\sqrt{2} \\
 -\frac{1}{3}\sqrt{\frac{1}{3}} &= -\frac{1}{3}\sqrt{\frac{3}{3^2}} = -\frac{1}{9}\sqrt{3} \\
 -5\sqrt{98} &= -5\sqrt{7^2 \cdot 2} = -35\sqrt{2} \\
 \sqrt{\frac{1}{27}} &= \sqrt{\frac{3}{3^4}} = \frac{1}{9}\sqrt{3}
 \end{aligned}$$

Entonces:

$$\begin{aligned}
 &= 40\sqrt{2} - 35\sqrt{2} - \frac{1}{9}\sqrt{3} + \frac{1}{9}\sqrt{3} \\
 &= (40-35)\sqrt{2} = 5\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad 2\sqrt{700} &= 2\sqrt{2^2 \cdot 5^2 \cdot 7} = 20\sqrt{7} \\
 -15\sqrt{\frac{1}{45}} &= -15\sqrt{\frac{5}{3^2 \cdot 5^2}} = -\sqrt{5} \\
 4\sqrt{\frac{5}{16}} &= 4\sqrt{\frac{5}{2^4}} = \sqrt{5} \\
 -56\sqrt{\frac{1}{7}} &= -56\sqrt{\frac{7}{7^2}} = -8\sqrt{7}
 \end{aligned}$$

Entonces:

$$\begin{aligned}
 &= 20\sqrt{7} - 8\sqrt{7} - \sqrt{5} + \sqrt{5} \\
 &= (20-8)\sqrt{7} = 12\sqrt{7}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad \sqrt{25ax^2} &= \sqrt{5^2 ax^2} = 5x\sqrt{a} \\
 \sqrt{49b} &= \sqrt{7^2 b} = 7\sqrt{b} \\
 -\sqrt{9ax^2} &= -\sqrt{3^2 ax^2} = -3x\sqrt{a} \\
 \text{entonces:} \\
 &= 5x\sqrt{a} - 3x\sqrt{a} + 7\sqrt{b} \\
 &= (5x-3x)\sqrt{a} + 7\sqrt{b} \\
 &= 2x\sqrt{a} + 7\sqrt{b}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad 2\sqrt{m^2 n} &= 2m\sqrt{n} \\
 -\sqrt{9m^2 n} &= -\sqrt{3^2 m^2 n} = -3m\sqrt{n} \\
 \sqrt{16mn^2} &= \sqrt{2^4 mn^2} = 4n\sqrt{m} \\
 -\sqrt{4mn^2} &= -\sqrt{2^2 mn^2} = -2n\sqrt{m} \\
 \text{Entonces:} \\
 &= 2m\sqrt{n} - 3m\sqrt{n} + 4n\sqrt{m} - 2n\sqrt{m} \\
 &= (2m-3m)\sqrt{n} + (4n-2n)\sqrt{m} \\
 &= -m\sqrt{n} + 2n\sqrt{m}
 \end{aligned}$$

$$\begin{aligned}
 15. \quad a\sqrt{320x} &= a\sqrt{2^6 \cdot 5x} = 8a\sqrt{5x} \\
 -7\sqrt{5a^2 x} &= -7a\sqrt{5x} \\
 &\quad -(a-4b)\sqrt{5x}
 \end{aligned}$$

Entonces:

$$\begin{aligned}
 &= 8a\sqrt{5x} - 7a\sqrt{5x} - (a-4b)\sqrt{5x} \\
 &= (8a-7a-a+4b)\sqrt{5x} = 4b\sqrt{5x}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad \sqrt{9x-9} &= \sqrt{3^2(x-1)} = 3\sqrt{x-1} \\
 \sqrt{4x-4} &= \sqrt{2^2(x-1)} = 2\sqrt{x-1} \\
 &\quad -5\sqrt{x-1}
 \end{aligned}$$

Entonces:

$$\begin{aligned}
 &= 3\sqrt{x-1} + 2\sqrt{x-1} - 5\sqrt{x-1} \\
 &= (3+2-5)\sqrt{x-1} = 0
 \end{aligned}$$

$$\begin{aligned}
 17. \quad 2\sqrt{a^4x+3a^4y} &= 2\sqrt{a^4(x+3y)} = 2a^2\sqrt{x+3y} \\
 -a^2\sqrt{9x+27y} &= -a^2\sqrt{3^2(x+3y)} = -3a^2\sqrt{x+3y} \\
 \sqrt{25a^4x+75a^4y} &= \sqrt{5^2a^4(x+3y)} = 5a^2\sqrt{x+3y}
 \end{aligned}$$

Entonces:

$$\begin{aligned}
 &= 2a^2\sqrt{x+3y} - 3a^2\sqrt{x+3y} + 5a^2\sqrt{x+3y} \\
 &= (2a^2 - 3a^2 + 5a^2)\sqrt{x+3y} = 4a^2\sqrt{x+3y}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad 3a\sqrt{\frac{a+1}{a^2}} &= \frac{3a}{a}\sqrt{a+1} = 3\sqrt{a+1} \\
 -\sqrt{4a+4} &= -\sqrt{2^2(a+1)} = -2\sqrt{a+1} \\
 (a+1)\sqrt{\frac{1}{a+1}} &= (a+1)\sqrt{\frac{a+1}{(a+1)^2}} = \sqrt{a+1}
 \end{aligned}$$

Entonces:

$$= 3\sqrt{a+1} - 2\sqrt{a+1} + \sqrt{a+1} = (3-2+1)\sqrt{a+1} = 2\sqrt{a+1}$$

$$\begin{aligned}
 19. \quad (a-b)\sqrt{\frac{a+b}{a-b}} &= (a-b)\sqrt{\frac{a^2-b^2}{(a-b)^2}} = \sqrt{a^2-b^2} \\
 -(a+b)\sqrt{\frac{a-b}{a+b}} &= -(a+b)\sqrt{\frac{a^2-b^2}{(a+b)^2}} = -\sqrt{a^2-b^2} \\
 (2a-2b)\sqrt{\frac{1}{a-b}} &= 2(a-b)\sqrt{\frac{a-b}{(a-b)^2}} = 2\sqrt{a-b}
 \end{aligned}$$

entonces:

$$= \sqrt{a^2-b^2} - \sqrt{a^2-b^2} + 2\sqrt{a-b} = 2\sqrt{a-b}$$

$$\begin{aligned}
 4. \quad 5\sqrt[3]{48} &= 5\sqrt[3]{2^3 \cdot 6} = 10\sqrt[3]{6} \\
 -3\sqrt[3]{3 \cdot 645} &= -3\sqrt[3]{3^6 \cdot 5} = -27\sqrt[3]{5} \\
 -2\sqrt[3]{384} &= -2\sqrt[3]{2^6 \cdot 6} = -8\sqrt[3]{6} \\
 4\sqrt[3]{1.715} &= 4\sqrt[3]{7^3 \cdot 5} = 28\sqrt[3]{5}
 \end{aligned}$$

Entonces:

$$\begin{aligned}
 &= 10\sqrt[3]{6} - 8\sqrt[3]{6} - 27\sqrt[3]{5} + 28\sqrt[3]{5} \\
 &= (10-8)\sqrt[3]{6} + (-27+28)\sqrt[3]{5} = 2\sqrt[3]{6} + \sqrt[3]{5}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \frac{1}{2}\sqrt[3]{24} &= \frac{1}{2}\sqrt[3]{2^3 \cdot 3} = \sqrt[3]{3} \\
 -\frac{2}{3}\sqrt[3]{54} &= -\frac{2}{3}\sqrt[3]{3^3 \cdot 2} = -2\sqrt[3]{2} \\
 \frac{3}{5}\sqrt[3]{375} &= \frac{3}{5}\sqrt[3]{5^3 \cdot 3} = 3\sqrt[3]{3} \\
 -\frac{1}{4}\sqrt[3]{128} &= -\frac{1}{4}\sqrt[3]{2^9 \cdot 2} = -\sqrt[3]{2}
 \end{aligned}$$

Entonces:

$$\begin{aligned}
 &= \sqrt[3]{3} + 3\sqrt[3]{3} - 2\sqrt[3]{2} - \sqrt[3]{2} \\
 &= (1+3)\sqrt[3]{3} + (-2-1)\sqrt[3]{2} = 4\sqrt[3]{3} - 3\sqrt[3]{2}
 \end{aligned}$$

## EJERCICIO 239

$$\begin{aligned}
 1. \quad \sqrt[3]{54} &= \sqrt[3]{3^3 \cdot 2} = \sqrt[3]{2} \\
 -\sqrt[3]{24} &= -\sqrt[3]{2^3 \cdot 3} = -2\sqrt[3]{3} \\
 -\sqrt[3]{16} &= -\sqrt[3]{2^3 \cdot 2} = -2\sqrt[3]{2}
 \end{aligned}$$

Entonces:

$$= 3\sqrt[3]{2} - 2\sqrt[3]{2} - 2\sqrt[3]{3} = \sqrt[3]{2} - 2\sqrt[3]{3}$$

$$\begin{aligned}
 2. \quad \sqrt[3]{40} &= \sqrt[3]{2^3 \cdot 5} = 2\sqrt[3]{5} \\
 \sqrt[3]{1.029} &= \sqrt[3]{7^3 \cdot 3} = 7\sqrt[3]{3} \\
 -\sqrt[3]{625} &= -\sqrt[3]{5^3 \cdot 5} = -5\sqrt[3]{5}
 \end{aligned}$$

Entonces:

$$\begin{aligned}
 &= 2\sqrt[3]{5} - 5\sqrt[3]{5} + 7\sqrt[3]{3} \\
 &= (2-5)\sqrt[3]{5} + 7\sqrt[3]{3} = 7\sqrt[3]{3} - 3\sqrt[3]{5}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad 2\sqrt[3]{250} &= 2\sqrt[3]{5^3 \cdot 2} = 10\sqrt[3]{2} \\
 -4\sqrt[3]{24} &= -4\sqrt[3]{2^3 \cdot 3} = -8\sqrt[3]{3} \\
 -6\sqrt[3]{16} &= -6\sqrt[3]{2^3 \cdot 2} = -12\sqrt[3]{2} \\
 \sqrt[3]{2.187} &= \sqrt[3]{3^6 \cdot 3} = 9\sqrt[3]{3} \\
 \text{Entonces:} \\
 &= 10\sqrt[3]{2} - 12\sqrt[3]{2} + 9\sqrt[3]{3} - 8\sqrt[3]{3} \\
 &= (10-12)\sqrt[3]{2} + (9-8)\sqrt[3]{3} = \sqrt[3]{3} - 2\sqrt[3]{2}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \sqrt[3]{81} &= \sqrt[3]{3^3 \cdot 3} = 3\sqrt[3]{3} \\
 -3\sqrt[3]{375} &= -3\sqrt[3]{5^3 \cdot 3} = -15\sqrt[3]{3} \\
 \sqrt[3]{686} &= \sqrt[3]{7^3 \cdot 2} = 7\sqrt[3]{2} \\
 2\sqrt[3]{648} &= 2\sqrt[3]{2^3 \cdot 3^3 \cdot 3} = 12\sqrt[3]{3}
 \end{aligned}$$

Entonces:

$$= 3\sqrt[3]{3} + 12\sqrt[3]{3} - 15\sqrt[3]{3} + 7\sqrt[3]{2} = 7\sqrt[3]{2}$$

$$\begin{aligned}
 7. \quad \frac{3}{5}\sqrt[3]{625} &= \frac{3}{5}\sqrt[3]{5^3 \cdot 5} = 3\sqrt[3]{5} \\
 -\frac{3}{2}\sqrt[3]{192} &= -\frac{3}{2}\sqrt[3]{2^6 \cdot 3} = -6\sqrt[3]{3} \\
 \frac{1}{7}\sqrt[3]{1.715} &= \frac{1}{7}\sqrt[3]{7^3 \cdot 5} = \sqrt[3]{5} \\
 -\frac{3}{8}\sqrt[3]{1.536} &= -\frac{3}{8}\sqrt[3]{2^9 \cdot 3} = -3\sqrt[3]{3}
 \end{aligned}$$

Entonces:

$$\begin{aligned}
 &= 3\sqrt[3]{5} + \sqrt[3]{5} - 6\sqrt[3]{3} - 3\sqrt[3]{3} \\
 &= (3+1)\sqrt[3]{5} + (-6-3)\sqrt[3]{3} = 4\sqrt[3]{5} - 9\sqrt[3]{3}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad \sqrt[3]{\frac{1}{4}} &= \sqrt[3]{\frac{2}{2^3}} = \frac{1}{2}\sqrt[3]{2} \\
 \sqrt[3]{\frac{1}{3}} &= \sqrt[3]{\frac{9}{3^3}} = \frac{1}{3}\sqrt[3]{9} \\
 -\sqrt[3]{\frac{2}{27}} &= -\sqrt[3]{\frac{2}{3^3}} = -\frac{1}{3}\sqrt[3]{2}
 \end{aligned}$$

Entonces:

$$\begin{aligned}
 &= \frac{1}{2}\sqrt[3]{2} - \frac{1}{3}\sqrt[3]{2} + \frac{1}{3}\sqrt[3]{9} \\
 &= \left(\frac{1}{2} - \frac{1}{3}\right)\sqrt[3]{2} + \frac{1}{3}\sqrt[3]{9} = \frac{1}{6}\sqrt[3]{2} + \frac{1}{3}\sqrt[3]{9}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad 6\sqrt[3]{\frac{1}{24}} &= 6\sqrt[3]{\frac{9}{2^3 \cdot 3^3}} = \sqrt[3]{9} \\
 \sqrt[3]{\frac{1}{25}} &= \sqrt[3]{\frac{5}{5^3}} = \frac{1}{5}\sqrt[3]{5} \\
 -2\sqrt[3]{\frac{5}{64}} &= -2\sqrt[3]{\frac{5}{2^3 \cdot 2^3}} = -\frac{1}{2}\sqrt[3]{5}
 \end{aligned}$$

Entonces:

$$\begin{aligned}
 &= \frac{1}{5}\sqrt[3]{2} - \frac{1}{2}\sqrt[3]{5} + \sqrt[3]{9} \\
 &= \left(\frac{1}{5} - \frac{1}{2}\right)\sqrt[3]{5} + \sqrt[3]{9} = -\frac{3}{10}\sqrt[3]{5} + \sqrt[3]{9}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad 7\sqrt[3]{\frac{1}{49}} &= 7\sqrt[3]{\frac{7}{7^3}} = \sqrt[3]{7} \\
 \sqrt[3]{\frac{1}{16}} &= \sqrt[3]{\frac{4}{2^6}} = \frac{1}{4}\sqrt[3]{4} \\
 \sqrt[3]{\frac{1}{2}} &= \sqrt[3]{\frac{4}{2^3}} = \frac{1}{2}\sqrt[3]{4} \\
 -2\sqrt[3]{\frac{7}{8}} &= -2\sqrt[3]{\frac{7}{2^3}} = -\sqrt[3]{7}
 \end{aligned}$$

Entonces:

$$= \frac{1}{4}\sqrt[3]{4} + \frac{1}{2}\sqrt[3]{4} + \sqrt[3]{7} - \sqrt[3]{7} = \left(\frac{1}{4} + \frac{1}{2}\right)\sqrt[3]{4} = \frac{3}{4}\sqrt[3]{4}$$

$$\begin{aligned}
 11. \quad \frac{2}{3}\sqrt[3]{135} &= \frac{2}{3}\sqrt[3]{3^3 \cdot 5} = 2\sqrt[3]{5} \\
 \frac{1}{2}\sqrt[3]{\frac{1}{32}} &= \frac{1}{2}\sqrt[3]{\frac{2}{2^6}} = \frac{1}{8}\sqrt[3]{2} \\
 \frac{7}{4}\sqrt[3]{\frac{1}{4}} &= \frac{7}{4}\sqrt[3]{\frac{2}{2^3}} = \frac{7}{8}\sqrt[3]{2} \\
 -20\sqrt[3]{\frac{1}{200}} &= -20\sqrt[3]{\frac{5}{2^3 \cdot 5^3}} = -2\sqrt[3]{5}
 \end{aligned}$$

Entonces:

$$= 2\sqrt[3]{5} - 2\sqrt[3]{5} + \frac{1}{8}\sqrt[3]{2} + \frac{7}{8}\sqrt[3]{2} = \left(\frac{1}{8} + \frac{7}{8}\right)\sqrt[3]{2} = \sqrt[3]{2}$$

$$\begin{aligned}
 12. \quad 3\sqrt[3]{-24} &= 3\sqrt[3]{(-2)^3 \cdot 3} = -6\sqrt[3]{3} \\
 -4\sqrt[3]{-81} &= -4\sqrt[3]{(-3)^3 \cdot 3} = 12\sqrt[3]{3} \\
 -\sqrt[3]{-375} &= -\sqrt[3]{(-5)^3 \cdot 3} = 5\sqrt[3]{3}
 \end{aligned}$$

Entonces:

$$\begin{aligned}
 &= -6\sqrt[3]{3} + 12\sqrt[3]{3} + 5\sqrt[3]{3} \\
 &= (-6 + 12 + 5)\sqrt[3]{3} = 11\sqrt[3]{3}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad 4\sqrt[3]{-320} &= 4\sqrt[3]{(2)^6(-5)} = 16\sqrt[3]{-5} \\
 -10\sqrt[3]{-40} &= -10\sqrt[3]{(2)^3(-5)} = -20\sqrt[3]{-5} \\
 -2\sqrt[3]{-54} &= -2\sqrt[3]{(3)^3(-2)} = -6\sqrt[3]{-2} \\
 3\sqrt[3]{-1.024} &= 3\sqrt[3]{(2)^9(-2)} = 24\sqrt[3]{-2}
 \end{aligned}$$

Entonces:

$$\begin{aligned}
 &= 16\sqrt[3]{-5} - 20\sqrt[3]{-5} - 6\sqrt[3]{-2} + 24\sqrt[3]{-2} \\
 &= (16 - 20)\sqrt[3]{-5} + (-6 + 24)\sqrt[3]{-2} \\
 &= -4\sqrt[3]{-5} + 18\sqrt[3]{-2}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad 3\sqrt[3]{2a^3} &= 3a\sqrt[3]{2} \\
 -b\sqrt[3]{128} &= -b\sqrt[3]{2^6 \cdot 2} = -4b\sqrt[3]{2} \\
 &\quad (4b - 3a)\sqrt[3]{2}
 \end{aligned}$$

Entonces:

$$\begin{aligned}
 &= 3a\sqrt[3]{2} - 4b\sqrt[3]{2} + (4b - 3a)\sqrt[3]{2} \\
 &= (3a - 4b + 4b - 3a)\sqrt[3]{2} = 0
 \end{aligned}$$

$$\begin{aligned}
 15. \quad a\sqrt[3]{250b} &= a\sqrt[3]{5^3 \cdot 2b} = 5a\sqrt[3]{2b} \\
 -\sqrt[3]{3ab^3} &= -b\sqrt[3]{3a} \\
 -5\sqrt[3]{2a^3b} &= -5a\sqrt[3]{2b} \\
 3b\sqrt[3]{3a} &
 \end{aligned}$$

Entonces:

$$\begin{aligned}
 &5a\sqrt[3]{2b} - 5a\sqrt[3]{2b} - b\sqrt[3]{3a} + 3b\sqrt[3]{3a} \\
 &= (-b + 3b)\sqrt[3]{3a} = 2b\sqrt[3]{3a}
 \end{aligned}$$

## EJERCICIO 240

- $\sqrt{3} \cdot \sqrt{6} = \sqrt{18} = \sqrt{3^2 \cdot 2} = 3\sqrt{2}$
- $5\sqrt{21} \cdot 2\sqrt{3} = 10\sqrt{63} = 10\sqrt{3^2 \cdot 7} = 30\sqrt{7}$
- $\frac{1}{2}\sqrt{14} \cdot \frac{2}{7}\sqrt{21} = \frac{1}{7}\sqrt{294} = \frac{1}{7}\sqrt{7^2 \cdot 6} = \sqrt{6}$
- $\sqrt[3]{12} \cdot \sqrt[3]{9} = \sqrt[3]{108} = \sqrt[3]{3^3 \cdot 4} = 3\sqrt[3]{4}$
- $\frac{5}{6}\sqrt[3]{15} \cdot 12\sqrt[3]{50} = 10\sqrt[3]{750} = 10\sqrt[3]{5^3 \cdot 6} = 50\sqrt[3]{6}$
- $x\sqrt{2a} \cdot \frac{1}{a}\sqrt{5a} = \frac{x}{a}\sqrt{10a^2} = x\sqrt{10}$
- $5\sqrt{12} \cdot 3\sqrt{75} = 15\sqrt{900} = 15\sqrt{2^2 \cdot 3^2 \cdot 5^2} = 450$
- $\frac{3}{4}\sqrt[3]{9a^2} \cdot 8\sqrt[3]{3ab} = 6\sqrt[3]{27a^3b} = 6a\sqrt[3]{3^3b} = 18a\sqrt[3]{b}$
- $3\sqrt{6} \cdot \sqrt{14} \cdot 2\sqrt{35} = 6\sqrt{2 \cdot 940} = 6\sqrt{2^2 \cdot 7^2 \cdot 15} = 84\sqrt{15}$
- $\frac{1}{2}\sqrt{21} \cdot \frac{2}{3}\sqrt{42} \cdot \frac{3}{7}\sqrt{22} = \frac{1}{7}\sqrt{19 \cdot 404} = \frac{1}{7}\sqrt{2^2 \cdot 7^2 \cdot 3^2 \cdot 11} = 6\sqrt{11}$
- $3\sqrt[3]{45} \cdot \frac{1}{6}\sqrt[3]{15} \cdot 4\sqrt[3]{20} = 2\sqrt[3]{13 \cdot 500} = 2\sqrt[3]{5^3 \cdot 3^3 \cdot 4} = 30\sqrt[3]{4}$
- $\frac{5}{6}\sqrt{\frac{7}{8}} \cdot \frac{3}{5}\sqrt{\frac{4}{7}} = \frac{1}{2}\sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{\frac{2}{2^2}} = \frac{1}{4}\sqrt{2}$
- $\frac{2}{x}\sqrt{a^2x} \cdot \frac{3}{2}\sqrt{\frac{1}{a^3}} = \frac{3}{x}\sqrt{\frac{x}{a}} = \frac{3}{x}\sqrt{\frac{ax}{a^2}} = \frac{3}{ax}\sqrt{ax}$
- $\frac{1}{3}\sqrt{\frac{x}{y^2}} \cdot 6\sqrt{\frac{2}{y}} = 2\sqrt{\frac{2x}{y^3}} = 2\sqrt{\frac{2xy}{y^4}} = \frac{2}{y^2}\sqrt{2xy}$

## EJERCICIO 241

- $$\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}}$$

$$= \sqrt{4}-\sqrt{6}$$

$$= 2-\sqrt{6}$$
- $$\frac{7\sqrt{5}+5\sqrt{3}}{2\sqrt{3}}$$

$$= 14\sqrt{15}+10\sqrt{9}$$

$$= 14\sqrt{15}+30$$
- $$\frac{2\sqrt{3}+\sqrt{5}-5\sqrt{2}}{4\sqrt{15}}$$

$$= 8\sqrt{45}+4\sqrt{75}-20\sqrt{30}$$

$$= 4(2\sqrt{3^2 \cdot 5}+\sqrt{5^2 \cdot 3}-5\sqrt{30})$$

$$= 4(6\sqrt{5}+5\sqrt{3}-5\sqrt{30})$$
- $$\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+2\sqrt{3}}$$

$$\frac{\sqrt{4}-\sqrt{6}}{+2\sqrt{6}-2\sqrt{9}}$$

$$= \frac{\sqrt{4}+\sqrt{6}}{+2\sqrt{9}} - 2\sqrt{9}$$

$$= \frac{2+\sqrt{6}}{6} - 6$$

$$= \frac{\sqrt{6}-4}{6}$$
- $$\frac{\sqrt{5}+5\sqrt{3}}{2\sqrt{5}+3\sqrt{3}}$$

$$\frac{2\sqrt{25}+10\sqrt{15}}{+3\sqrt{15}+15\sqrt{9}}$$

$$= \frac{2\sqrt{25}+13\sqrt{15}+15\sqrt{9}}{+10+13\sqrt{15}+45}$$

$$= \frac{55+13\sqrt{15}}{55+13\sqrt{15}}$$
- $$\frac{3\sqrt{7}-2\sqrt{3}}{5\sqrt{3}+4\sqrt{7}}$$

$$\frac{15\sqrt{21}-10\sqrt{9}}{-8\sqrt{21}+12\sqrt{49}}$$

$$= \frac{7\sqrt{21}-10\sqrt{9}+12\sqrt{49}}{7\sqrt{21}-30+84}$$

$$= \frac{7\sqrt{21}+54}{7\sqrt{21}+54}$$
- $$\frac{\sqrt{a}-2\sqrt{x}}{3\sqrt{a}+\sqrt{x}}$$

$$\frac{3\sqrt{a^2}-6\sqrt{ax}}{+ \sqrt{ax}-2\sqrt{x^2}}$$

$$= \frac{3\sqrt{a^2}-5\sqrt{ax}-2\sqrt{x^2}}{3a-5\sqrt{ax}-2x}$$
- $$\frac{7\sqrt{5}-11\sqrt{7}}{5\sqrt{5}-8\sqrt{7}}$$

$$\frac{35\sqrt{25}-55\sqrt{35}}{-56\sqrt{35}+88\sqrt{49}}$$

$$= \frac{35\sqrt{25}-111\sqrt{35}+88\sqrt{49}}{175-111\sqrt{35}+616}$$

$$= \frac{791-111\sqrt{35}}{791-111\sqrt{35}}$$
- $$\frac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{\sqrt{2}-\sqrt{3}}$$

$$\frac{\sqrt{4}+\sqrt{6}+\sqrt{10}}{-\sqrt{6}-\sqrt{9}-\sqrt{15}}$$

$$= \frac{\sqrt{4}+\sqrt{10}-\sqrt{9}-\sqrt{15}}{2+\sqrt{10}-3-\sqrt{15}}$$

$$= \frac{\sqrt{10}-\sqrt{15}-1}{\sqrt{10}-\sqrt{15}-1}$$
- $$\frac{\sqrt{2}-3\sqrt{3}+\sqrt{5}}{\sqrt{2}+2\sqrt{3}-\sqrt{5}}$$

$$\frac{\sqrt{4}-3\sqrt{6}+\sqrt{10}}{+2\sqrt{6}-6\sqrt{9}+2\sqrt{15}}$$

$$= \frac{\sqrt{4}-\sqrt{6}}{-\sqrt{10}+3\sqrt{15}-\sqrt{25}}$$

$$= \frac{2-\sqrt{6}}{5\sqrt{15}-\sqrt{6}-21}$$

$$\begin{aligned}
 11. \quad & \frac{2\sqrt{3}-\sqrt{6}+\sqrt{5}}{\sqrt{3}+\sqrt{6}+3\sqrt{5}} \\
 & \frac{2\sqrt{9}-\sqrt{18}+\sqrt{15}}{+2\sqrt{18} \quad -\sqrt{36}+\sqrt{30}} \\
 & \frac{+6\sqrt{15} \quad -3\sqrt{30}+3\sqrt{25}}{=2\sqrt{9}+\sqrt{3^2 \cdot 2}+7\sqrt{15}-\sqrt{36}-2\sqrt{30}+3\sqrt{25}} \\
 & =6+3\sqrt{2}+7\sqrt{15}-6-2\sqrt{30}+15 \\
 & =7\sqrt{15}+3\sqrt{2}-2\sqrt{30}+15
 \end{aligned}$$

$$\begin{aligned}
 12. \quad & \frac{\sqrt{a}+\sqrt{a+1}}{\sqrt{a}+2\sqrt{a+1}} \\
 & \frac{\sqrt{a^2}+\sqrt{a^2+a}}{+2\sqrt{a^2+a}+2\sqrt{(a+1)^2}} \\
 & =\sqrt{a^2}+3\sqrt{a^2+a}+2\sqrt{(a+1)^2} \\
 & =a+3\sqrt{a^2+a}+2a+2 \\
 & =3a+3\sqrt{a^2+a}+2
 \end{aligned}$$

$$\begin{aligned}
 13. \quad & \frac{2\sqrt{a}-3\sqrt{a-b}}{3\sqrt{a}+\sqrt{a-b}} \\
 & \frac{6\sqrt{a^2}-9\sqrt{a^2-ab}}{2\sqrt{a^2-ab}-3\sqrt{(a-b)^2}} \\
 & =6\sqrt{a^2}-7\sqrt{a^2-ab}-3\sqrt{(a-b)^2} \\
 & =6a-7\sqrt{a^2-ab}-3a+3b \\
 & =3a+3b-7\sqrt{a^2-ab}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & \frac{\sqrt{1-x^2}+x}{2x+\sqrt{1-x^2}} \\
 & \frac{2x\sqrt{1-x^2}+2x^2}{x\sqrt{1-x^2}+\sqrt{(1-x^2)^2}} \\
 & =3x\sqrt{1-x^2}+2x^2+1-x^2 \\
 & =3x\sqrt{1-x^2}+x^2+1
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & \frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}+2\sqrt{a-1}} \\
 & \frac{\sqrt{(a+1)^2}+\sqrt{a^2-1}}{+2\sqrt{a^2-1}+2\sqrt{(a-1)^2}} \\
 & =\frac{a+1+3\sqrt{a^2-1}+2a-2}{=3a+3\sqrt{a^2-1}-1}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \frac{2\sqrt{x+2}-2}{\sqrt{x+2}-3} \\
 & \frac{2\sqrt{(x+2)^2}-2\sqrt{x+2}}{-6\sqrt{x+2}+6} \\
 & =\frac{2\sqrt{(x+2)^2}-8\sqrt{x+2}+6}{=2x+4-8\sqrt{x+2}+6} \\
 & =2x+10-8\sqrt{x+2}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & \frac{3\sqrt{a}-2\sqrt{a+x}}{2\sqrt{a}+3\sqrt{a+x}} \\
 & \frac{6\sqrt{a^2}-4\sqrt{a^2+ax}}{+9\sqrt{a^2+ax}-6\sqrt{(a+x)^2}} \\
 & =\frac{6\sqrt{a^2}+5\sqrt{a^2+ax}-6\sqrt{(a+x)^2}}{=6a+5\sqrt{a^2+ax}-6a-6x} \\
 & =5\sqrt{a^2+ax}-6x
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & \frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}-2\sqrt{a-x}} \\
 & \frac{\sqrt{(a+x)^2}-\sqrt{a^2-x^2}}{-2\sqrt{a^2-x^2}+2\sqrt{(a-x)^2}} \\
 & =\frac{\sqrt{(a+x)^2}-3\sqrt{a^2-x^2}+2\sqrt{(a-x)^2}}{=a+x-3\sqrt{a^2-x^2}+2a-2x} \\
 & =3a-x-3\sqrt{a^2-x^2}
 \end{aligned}$$

## EJERCICIO 242

$$\begin{aligned}
 1. \quad \sqrt{x} &= \sqrt[6]{x^3} \\
 \sqrt[3]{2x^2} &= \sqrt[6]{(2x^2)^2} = \sqrt[6]{4x^4} \\
 \sqrt[6]{x^3} \cdot \sqrt[6]{4x^4} &= \sqrt[6]{4x^7} \\
 &= \sqrt[6]{4x^6 x} \\
 &= x \sqrt[6]{4x}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad 3\sqrt{2ab} &= 3\sqrt[4]{(2ab)^2} = 3\sqrt[4]{4a^2b^2} \\
 3\sqrt[4]{4a^2b^2} \cdot 4\sqrt[4]{8a^3} &= 12\sqrt[4]{32a^5b^2} \\
 &= 12\sqrt[4]{2^4 \cdot 2a^4ab^2} \\
 &= 24a\sqrt[4]{2ab^2}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \sqrt[3]{9x^2y} &= \sqrt[6]{(9x^2y)^2} \\
 &= \sqrt[6]{81x^4y^2} \\
 \sqrt[6]{81x^4y^2} \cdot \sqrt[6]{81x^5} &= \sqrt[6]{6.561x^9y^2} \\
 &= \sqrt[6]{3^6 \cdot 3^2 x^6 x^3 y^2} \\
 &= 3x \sqrt[6]{9x^3y^2}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \sqrt[3]{a^2b^2} &= \sqrt[12]{(a^2b^2)^4} \\
 &= \sqrt[12]{a^8b^8} \\
 2\sqrt[4]{3a^3b} &= 2\sqrt[12]{(3a^3b)^3} \\
 &= 2\sqrt[12]{27a^9b^3} \\
 \sqrt[12]{a^8b^8} \cdot 2\sqrt[12]{27a^9b^3} &= 2\sqrt[12]{27a^{17}b^{11}} \\
 &= 2\sqrt[12]{27a^{12}a^5b^{11}} \\
 &= 2a\sqrt[12]{27a^5b^{11}}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \sqrt[4]{25x^2y^3} &= \sqrt[12]{(25x^2y^3)^3} \\
 &= \sqrt[12]{15.625x^6y^9} \\
 \sqrt[6]{125x^2} &= \sqrt[12]{(125x^2)^2} \\
 &= \sqrt[12]{15.625x^4} \\
 \sqrt[12]{15.625x^6y^9} \cdot \sqrt[12]{15.625x^4} &= \sqrt[12]{244.140.625x^{10}y^9} \\
 &= \sqrt[12]{5^{12}x^{10}y^9} = 5\sqrt[12]{x^{10}y^9}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \frac{2}{3}\sqrt[3]{4m^2} &= \frac{2}{3}\sqrt[15]{(4m^2)^5} \\
 &= \frac{2}{3}\sqrt[15]{4^5m^{10}} \\
 \frac{3}{4}\sqrt[5]{16m^4n} &= \frac{3}{4}\sqrt[15]{(4^2m^4n)^3} \\
 &= \frac{3}{4}\sqrt[15]{4^6m^{12}n^3} \\
 \frac{2}{3}\sqrt[15]{4^5m^{10}} \cdot \frac{3}{4}\sqrt[15]{4^6m^{12}n^3} &= \frac{1}{2}\sqrt[15]{4^{11}m^{22}n^3} \\
 &= \frac{1}{2}\sqrt[15]{2^{22}m^{22}n^3} \\
 &= \frac{1}{2}\sqrt[15]{2^{15} \cdot 2^7 m^{15} m^7 n^3} \\
 &= m\sqrt[15]{2^7 m^7 n^3} = m\sqrt[15]{128m^7n^3}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \sqrt{\frac{1}{2x}} &= \sqrt[6]{\left(\frac{1}{2x}\right)^3} = \sqrt[6]{\frac{1}{8x^3}} \\
 \sqrt[3]{2} &= \sqrt[6]{(x^2)^2} = \sqrt[6]{x^4} \\
 \sqrt[6]{\frac{x^{-3}}{8}} \cdot \sqrt[6]{x^4} &= \sqrt[6]{\frac{x}{8}} = \sqrt[6]{\frac{8x}{2^6}} \\
 &= \frac{1}{2}\sqrt[6]{8x}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad \sqrt{2x} &= \sqrt[10]{(2x)^5} = \sqrt[10]{2^5x^5} \\
 \sqrt[5]{4x} &= \sqrt[10]{(2^2x)^2} = \sqrt[10]{2^4x^2} \\
 \sqrt[10]{2^5x^5} \cdot \sqrt[10]{2^4x^2} &= \sqrt[10]{\frac{1}{16x^2}}
 \end{aligned}$$

$$\begin{aligned}
 &= \sqrt[10]{\frac{2^5 \cdot 2^4 x^2 x^5}{2^4 x^2}} \\
 &= \sqrt[10]{2^5 x^5} = (2x)^{\frac{5}{10}} = (2x)^{\frac{1}{2}} = \sqrt{2x}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad \frac{2}{3}\sqrt{\frac{2b}{a}} &= \frac{2}{3}\sqrt[6]{\left(\frac{2b}{a}\right)^3} = \frac{2}{3}\sqrt[6]{\frac{8b^3}{a^3}} \\
 \frac{3}{8}\sqrt[3]{\frac{a^2}{4b^2}} &= \frac{3}{8}\sqrt[6]{\left(\frac{a^2}{2^2b^2}\right)^2} = \frac{3}{8}\sqrt[6]{\frac{a^4}{2^4b^4}} \\
 \frac{2}{3}\sqrt[6]{\frac{2^3b^3}{a^3}} \cdot \frac{3}{8}\sqrt[6]{\frac{a^4}{2^4b^4}} &= \frac{1}{4}\sqrt[6]{\frac{a}{2b}} = \frac{1}{4}\sqrt[6]{\frac{2^5ab^5}{2^6b^6}} = \frac{1}{8b}\sqrt[6]{32ab^5}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad \frac{1}{2}\sqrt{\frac{1}{3}} &= \frac{1}{2}\sqrt[6]{\left(\frac{1}{3}\right)^3} = \frac{1}{2}\sqrt[6]{\frac{1}{3^3}} \\
 \frac{3}{2}\sqrt[3]{\frac{1}{9}} &= \frac{3}{2}\sqrt[6]{\left(\frac{1}{9}\right)^2} = \frac{3}{2}\sqrt[6]{\frac{1}{3^4}} \\
 \frac{1}{2}\sqrt[6]{\frac{1}{3^3}} \cdot \frac{3}{2}\sqrt[6]{\frac{1}{3^4}} &= \sqrt[6]{243} \\
 &= \frac{3}{4}\sqrt[6]{\frac{3^5}{3^7}} \\
 &= \frac{3}{4}\sqrt[6]{\frac{1}{9}} \\
 &= \frac{3}{4}\sqrt[6]{\frac{3^4}{3^6}} \\
 &= \frac{1}{4}(3)^{\frac{4}{6}} = \frac{1}{4}(3)^{\frac{2}{3}} = \frac{1}{4}\sqrt[3]{3^2} = \frac{1}{4}\sqrt[3]{9}
 \end{aligned}$$

## EJERCICIO 243

$$1. \quad \frac{4\sqrt{6}}{2\sqrt{3}} = 2\sqrt{\frac{6}{3}} = 2\sqrt{2}$$

$$2. \quad \frac{2\sqrt{3a}}{10\sqrt{a}} = \frac{1}{5}\sqrt{\frac{3a}{a}} = \frac{1}{5}\sqrt{3}$$

$$3. \quad \frac{\frac{1}{2}\sqrt{3xy}}{\frac{3}{4}\sqrt{x}} = \frac{4}{6}\sqrt{\frac{3xy}{x}} = \frac{2}{3}\sqrt{3y}$$

$$4. \quad \frac{\sqrt{75x^2y^3}}{5\sqrt{3xy}} = \frac{1}{5}\sqrt{\frac{75x^2y^3}{3xy}} = \frac{1}{5}\sqrt{25xy^2} = \frac{1}{5}\sqrt{5^2xy^2} = y\sqrt{x}$$

$$5. \quad \frac{3\sqrt[3]{16a^5}}{4\sqrt[3]{2a^2}} = \frac{3}{4}\sqrt[3]{\frac{16a^5}{2a^2}} = \frac{3}{4}\sqrt[3]{8a^3} = \frac{3}{4}\sqrt[3]{2^3a^3} = \frac{3}{2}a$$

$$6. \quad \frac{\frac{5}{6}\sqrt{\frac{1}{2}}}{\frac{10}{3}\sqrt{\frac{2}{3}}} = \frac{15}{60}\sqrt{\frac{\frac{1}{2}}{\frac{2}{3}}} = \frac{1}{4}\sqrt{\frac{3}{4}} = \frac{1}{4}\sqrt{\frac{3}{2^2}} = \frac{1}{8}\sqrt{3}$$

$$7. \quad \frac{4x\sqrt{a^3x^2}}{2\sqrt{a^2x^3}} = 2x\sqrt{\frac{a^3x^2}{a^2x^3}} = 2x\sqrt{\frac{a}{x}} = 2x\sqrt{\frac{ax}{x^2}} = 2\sqrt{ax}$$

$$8. \frac{\frac{2a}{3} \sqrt[3]{x^2}}{\frac{a}{3x^2} \sqrt[3]{x^3}} = \frac{6ax^2}{3a} \sqrt[3]{\frac{x^2}{x^3}} = 2x^2 \sqrt[3]{\frac{1}{x}} = 2x^2 \sqrt[3]{\frac{x^2}{x^3}} = 2x \sqrt[3]{x^2}$$

$$9. \frac{\frac{1}{3} \sqrt[3]{\frac{1}{2}}}{\frac{1}{6} \sqrt[3]{\frac{1}{3}}} = \frac{6}{3} \sqrt[3]{\frac{\frac{1}{2}}{\frac{1}{3}}} = 2 \sqrt[3]{\frac{3}{2}} = 2 \sqrt[3]{\frac{3 \cdot 2^2}{2^3}} = \sqrt[3]{12}$$

## EJERCICIO 244

$$1. \sqrt[3]{2} = \sqrt[6]{2^2} \\ \sqrt{2} = \sqrt[6]{2^3} \\ = \frac{\sqrt[6]{2^2}}{\sqrt[6]{2^3}} = \sqrt[6]{\frac{2^2}{2^3}} = \sqrt[6]{\frac{1}{2}} = \sqrt[6]{\frac{2^5}{2^6}} = \frac{1}{2} \sqrt[6]{32}$$

$$2. \sqrt{9x} = \sqrt{(3^2 x)^3} = \sqrt[6]{3^6 x^3} = 3 \sqrt[6]{x^3} \\ \sqrt[3]{3x^2} = \sqrt[6]{(3x^2)^2} = \sqrt[6]{9x^4} \\ = \frac{3 \sqrt[6]{x^3}}{\sqrt[6]{9x^4}} = 3 \sqrt[6]{\frac{x^3}{9x^4}} = 3 \sqrt[6]{\frac{1}{9x}} = 3 \sqrt[6]{\frac{3^4 x^5}{3^6 x^6}} = \frac{1}{x} \sqrt[6]{81x^5}$$

$$3. \sqrt[3]{8a^3 b} = \sqrt[12]{(2^3 a^3 b)^4} = \sqrt[12]{2^{12} a^{12} b^4} = 2a \sqrt[12]{b^4} \\ \sqrt[4]{4a^2} = \sqrt[12]{(4a^2)^3} = \sqrt[12]{2^6 a^6} \\ = \frac{2a \sqrt[12]{b^4}}{\sqrt[12]{2^6 a^6}} \\ = 2a \sqrt[12]{\frac{2^6 a^6 b^4}{2^{12} a^{12}}} = \sqrt[12]{2^4 \cdot 2^2 a^4 a^2 b^4} \\ = (2ab)^{\frac{4}{12}} \cdot (2a)^{\frac{2}{12}} \\ = (2ab)^{\frac{1}{3}} \cdot (2a)^{\frac{1}{6}} = \sqrt[3]{2ab} \cdot \sqrt[6]{2a} \\ \Rightarrow \sqrt[3]{2ab} = \sqrt[6]{(2ab)^2} = \sqrt[6]{4a^2 b^2} \\ = \sqrt[6]{4a^2 b^2} \cdot \sqrt[6]{2a} = \sqrt[6]{8a^3 b^2}$$

$$4. \frac{1}{2} \sqrt{2x} = \frac{1}{2} \sqrt[6]{(2x)^3} = \frac{1}{2} \sqrt[6]{2^3 x^3}$$

$$\frac{\frac{1}{2} \sqrt[6]{2^3 x^3}}{\frac{1}{4} \sqrt[6]{2^4 x^4}} = 2 \sqrt[6]{\frac{1}{2x}} = 2 \sqrt[6]{\frac{2^5 x^5}{2^6 x^6}} = \frac{1}{x} \sqrt[6]{32x^5}$$

$$5. \sqrt[3]{5m^2 n} = \sqrt[15]{(5m^2 n)^5} = \sqrt[15]{5^5 m^{10} n^5} \\ \sqrt[5]{m^3 n^2} = \sqrt[15]{(m^3 n^2)^3} = \sqrt[15]{m^9 n^6} \\ = \frac{\sqrt[15]{5^5 m^{10} n^5}}{\sqrt[15]{m^9 n^6}} = \sqrt[15]{\frac{5^5 m}{n}} = \sqrt[15]{\frac{5^5 mn^{14}}{n^{15}}} = \frac{1}{n} \sqrt[15]{3.125mn^{14}}$$

$$6. \sqrt[6]{18x^3 y^4 z^5} = \sqrt[12]{(3^2 \cdot 2x^3 y^4 z^5)^2} = \sqrt[12]{3^4 \cdot 2^2 x^6 y^8 z^{10}} \\ \sqrt[4]{3x^2 y^2 z^3} = \sqrt[12]{(3x^2 y^2 z^3)^3} = \sqrt[12]{3^3 x^6 y^6 z^9} \\ = \frac{\sqrt[12]{3^4 \cdot 2^2 x^6 y^8 z^{10}}}{\sqrt[12]{3^3 x^6 y^6 z^9}} = \sqrt[12]{12y^2 z}$$

$$7. \sqrt[3]{3m^4} = \sqrt[9]{(3m^4)^3} = \sqrt[9]{27m^{12}} = m \sqrt[9]{27m^3} \\ = \frac{m \sqrt[9]{27m^3}}{\sqrt[9]{27m^2}} = m \sqrt[9]{m}$$

$$8. \frac{4}{5} \sqrt[3]{4ab} = \frac{4}{5} \sqrt[6]{(2^2 ab)^2} = \frac{4}{5} \sqrt[6]{2^4 a^2 b^2} \\ \frac{1}{10} \sqrt{2a^2} = \frac{1}{10} \sqrt[6]{(2a^2)^3} = \frac{1}{10} \sqrt[6]{2^3 a^6} \\ = \frac{\frac{4}{5} \sqrt[6]{2^4 a^2 b^2}}{\frac{1}{10} \sqrt[6]{2^3 a^6}} = 8 \sqrt[6]{\frac{2b^2}{a^4}} = 8 \sqrt[6]{\frac{2a^2 b^2}{a^6}} = \frac{8}{a} \sqrt[6]{2a^2 b^2}$$

## EJERCICIO 245

$$1. (4\sqrt{2})^2 = 4^2 \sqrt{2}^2 = 16 \cdot 2 = 32$$

$$2. (2\sqrt{3})^2 = 2^2 \sqrt{3}^2 = 4 \cdot 3 = 12$$

$$3. (5\sqrt{7})^2 = 5^2 \sqrt{7}^2 = 25 \cdot 7 = 175$$

$$4. (2\sqrt[3]{4})^2 = 2^2 \sqrt[3]{4}^2 = 4 \sqrt[3]{2^3 \cdot 2} = 8 \sqrt[3]{2}$$

$$5. (3\sqrt[3]{2a^2 b})^4 = 3^4 \sqrt[3]{(2a^2 b)^4} = 81 \sqrt[3]{2^3 \cdot 2 \cdot a^3 a^3 a^2 b^3 b} = 162a^2 b \sqrt{2a^2 b}$$

$$6. (\sqrt[4]{8x^3})^2 = \sqrt[4]{(2^3 x^3)^2} = \sqrt[4]{2^4 \cdot 2^2 x^4 x^2} = 2x \sqrt[4]{2^2 x^2} = 2x(2x)^{\frac{2}{4}} \\ = 2x(2x)^{\frac{1}{2}} = 2x\sqrt{2x}$$

$$7. (\sqrt[5]{81ab^3})^3 = \sqrt[5]{(3^4 ab^3)^3} = \sqrt[5]{3^5 \cdot 3^5 \cdot 3^2 a^3 b^5 b^4} = 9b \sqrt[5]{9a^3 b^4}$$

8.  $(\sqrt[6]{18})^3 = \sqrt[6]{18^3} = (18)^{\frac{3}{6}} = (18)^{\frac{1}{2}} = \sqrt{18} = \sqrt{3^2 \cdot 2} = 3\sqrt{2}$
9.  $(4a\sqrt{2x})^2 = (4a)^2 \sqrt{(2x)^2} = 16a^2(2x) = 32a^2x$
10.  $(2\sqrt{x+1})^2 = 2^2 \sqrt{(x+1)^2} = 4(x+1) = 4x+4$
11.  $(3\sqrt{x-a})^2 = 3^2 \sqrt{(x-a)^2} = 9(x-a) = 9x-9a$
12.  $(4\sqrt[6]{9a^3b^4})^3$   
 $= 4^3 \sqrt[6]{(3^2a^3b^4)^3}$   
 $= 64 \sqrt[6]{3^6a^9b^{12}}$   
 $= 192ab^2 \sqrt[6]{a^3}$   
 $= 192ab^2 (a)^{\frac{3}{6}} = 192ab^2 (a)^{\frac{1}{2}} = 192ab^2 \sqrt{a}$
13.  $(\sqrt{2}-\sqrt{3})^2 = \sqrt{2}^2 - 2\sqrt{2}\sqrt{3} + \sqrt{3}^2$   
 $= 2 - 2\sqrt{6} + 3 = 5 - 2\sqrt{6}$
14.  $(4\sqrt{2}+\sqrt{3})^2 = 4^2\sqrt{2}^2 + 2 \cdot 4\sqrt{2}\sqrt{3} + \sqrt{3}^2$   
 $= 32 + 8\sqrt{6} + 3 = 35 + 8\sqrt{6}$
15.  $(\sqrt{5}-\sqrt{7})^2 = \sqrt{5}^2 - 2\sqrt{5}\sqrt{7} + \sqrt{7}^2$   
 $= 5 - 2\sqrt{35} + 7 = 12 - 2\sqrt{35}$
16.  $(5\sqrt{7}-6)^2 = 5^2\sqrt{7}^2 - 2 \cdot 5 \cdot 6\sqrt{7} + 6^2$   
 $= 175 - 60\sqrt{7} + 36 = 211 - 60\sqrt{7}$
17.  $(\sqrt{x}+\sqrt{x-1})^2 = \sqrt{x}^2 + 2\sqrt{x}\sqrt{x-1} + \sqrt{(x-1)^2}$   
 $= x + 2\sqrt{x^2-x} + x - 1$   
 $= 2x + 2\sqrt{x^2-x} - 1$
18.  $(\sqrt{x+1}-4\sqrt{x})^2$   
 $= \sqrt{(x+1)^2} - 2 \cdot 4\sqrt{x+1}\sqrt{x} + 4^2\sqrt{x}^2$   
 $= x+1 - 8\sqrt{x^2+x} + 16x = 17x+1 - 8\sqrt{x^2+x}$
19.  $(\sqrt{a+1}-\sqrt{a-1})^2$   
 $= \sqrt{(a+1)^2} - 2\sqrt{a+1}\sqrt{a-1} + \sqrt{(a-1)^2}$   
 $= a+1 - 2\sqrt{a^2-1} + a-1 = 2a - 2\sqrt{a^2-1}$
20.  $(2\sqrt{2x-1}+\sqrt{2x+1})^2$   
 $= 2^2\sqrt{(2x-1)^2} + 2 \cdot 2\sqrt{2x-1}\sqrt{2x+1} + \sqrt{(2x+1)^2}$   
 $= 4(2x-1) + 4\sqrt{4x^2-1} + 2x+1$   
 $= 8x-4 + 4\sqrt{4x^2-1} + 2x+1$   
 $= 10x-3 + 4\sqrt{4x^2-1}$

## EJERCICIO 246

1.  $\sqrt[3]{a^2} = \sqrt[6]{a^2} = (a)^{\frac{2}{6}} = a^{\frac{1}{3}} = \sqrt[3]{a}$
2.  $\sqrt[3]{\sqrt{8}} = \sqrt[6]{8} = (2)^{\frac{3}{6}} = (2)^{\frac{1}{2}} = \sqrt{2}$
3.  $\sqrt[4]{\sqrt{81}} = \sqrt[8]{3^4} = (3)^{\frac{4}{8}} = (3)^{\frac{1}{2}} = \sqrt{3}$
4.  $\sqrt{\sqrt{3a}} = \sqrt[4]{3a}$
5.  $\sqrt[3]{4a^2} = \sqrt[6]{2^2a^2} = (2a)^{\frac{2}{6}} = (2a)^{\frac{1}{3}} = \sqrt[3]{2a}$
6.  $\sqrt[3]{2\sqrt{2}} = \sqrt[6]{2^2 \cdot 2} = \sqrt[6]{2^3} = (2)^{\frac{3}{6}} = (2)^{\frac{1}{2}} = \sqrt{2}$
7.  $\sqrt[4]{25a^2} = \sqrt[8]{5^2a^2} = (5a)^{\frac{2}{8}} = (5a)^{\frac{1}{4}} = \sqrt[4]{5a}$
8.  $\sqrt[3]{\sqrt[4]{27a^3}} = \sqrt[12]{3^3a^3} = (3a)^{\frac{3}{12}} = (3a)^{\frac{1}{4}} = \sqrt[4]{3a}$
9.  $\sqrt{3\sqrt[3]{3}} = \sqrt[6]{3^2 \cdot 3} = \sqrt[6]{3^3} = (3)^{\frac{3}{6}} = (3)^{\frac{1}{2}} = \sqrt{3}$
10.  $\sqrt[4]{\sqrt{a^4b^6}} = \sqrt[8]{a^4b^4b^2} = (ab)^{\frac{4}{8}}(b)^{\frac{2}{8}} = (ab)^{\frac{1}{2}}(b)^{\frac{1}{4}}$   
 $= \sqrt{ab} \cdot \sqrt[4]{b}$   
 $\Rightarrow \sqrt{ab} = \sqrt[4]{(ab)^2} = \sqrt[4]{a^2b^2}$   
 $= \sqrt[4]{a^2b^2} \sqrt[4]{b} = \sqrt[4]{a^2b^3}$
11.  $\sqrt[5]{\sqrt[3]{x^{10}}} = \sqrt[15]{x^{10}} = x^{\frac{10}{15}} = x^{\frac{2}{3}} = \sqrt[3]{x^2}$
12.  $\sqrt[3]{\sqrt{(a+b)^2}} = \sqrt[6]{(a+b)^2} = (a+b)^{\frac{2}{6}} = (a+b)^{\frac{1}{3}} = \sqrt[3]{a+b}$

## EJERCICIO 247

1.  $\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3^2}} = \frac{\sqrt{3}}{3}$
2.  $\frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{2^2}} = \frac{5\sqrt{2}}{2}$
3.  $\frac{3}{4\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{4\sqrt{5^2}} = \frac{3\sqrt{5}}{4 \cdot 5} = \frac{3\sqrt{5}}{20}$
4.  $\frac{2a}{\sqrt{2ax}} \cdot \frac{\sqrt{2ax}}{\sqrt{2ax}} = \frac{2a\sqrt{2ax}}{\sqrt{2^2a^2x^2}} = \frac{2a\sqrt{2ax}}{2ax} = \frac{\sqrt{2ax}}{x}$
5.  $\frac{5}{\sqrt[3]{4a^2}} \cdot \frac{\sqrt[3]{2a}}{\sqrt[3]{2a}} = \frac{5\sqrt[3]{2a}}{\sqrt[3]{2^3a^3}} = \frac{5\sqrt[3]{2a}}{2a}$
6.  $\frac{1}{\sqrt[3]{9x}} \cdot \frac{\sqrt[3]{3x^2}}{\sqrt[3]{3x^2}} = \frac{\sqrt[3]{3x^2}}{\sqrt[3]{3^3x^3}} = \frac{\sqrt[3]{3x^2}}{3x}$



$$7. \frac{3}{\sqrt[4]{9a}} \cdot \frac{\sqrt[4]{3^2 a^3}}{\sqrt[4]{3^2 a^3}} = \frac{3\sqrt[4]{3^2 a^3}}{\sqrt[4]{3^4 a^4}} = \frac{3\sqrt[4]{9a^3}}{3a} = \frac{\sqrt[4]{9a^3}}{a}$$

$$8. \frac{6}{5\sqrt[3]{3x}} \cdot \frac{\sqrt[3]{3^2 x^2}}{\sqrt[3]{3^2 x^2}} = \frac{6\sqrt[3]{3^2 x^2}}{5\sqrt[3]{3^3 x^3}} = \frac{6\sqrt[3]{9x^2}}{15x} = \frac{2\sqrt[3]{9x^2}}{5x}$$

$$9. \frac{x}{\sqrt[4]{27x^2}} \cdot \frac{\sqrt[4]{3x^2}}{\sqrt[4]{3x^2}} = \frac{x\sqrt[4]{3x^2}}{\sqrt[4]{3^4 x^4}} = \frac{x\sqrt[4]{3x^2}}{3x} = \frac{\sqrt[4]{3x^2}}{3}$$

$$10. \frac{1}{\sqrt[5]{8a^4}} \cdot \frac{\sqrt[5]{2^2 a}}{\sqrt[5]{2^2 a}} = \frac{\sqrt[5]{4a}}{\sqrt[5]{2^6 a^5}} = \frac{\sqrt[5]{4a}}{2a}$$

$$11. \frac{5n^2}{3\sqrt{mn}} \cdot \frac{\sqrt{mn}}{\sqrt{mn}} = \frac{5n^2 \sqrt{mn}}{3\sqrt{m^2 n^2}} = \frac{5n^2 \sqrt{mn}}{3mn} = \frac{5n\sqrt{mn}}{3m}$$

$$12. \frac{1}{5a\sqrt[4]{25x^3}} \cdot \frac{\sqrt[4]{5^2 x}}{\sqrt[4]{5^2 x}} = \frac{\sqrt[4]{25x}}{5a\sqrt[4]{5^4 x^4}} = \frac{\sqrt[4]{25x}}{25ax}$$

## EJERCICIO 248

$$1. \frac{3-\sqrt{2}}{1+\sqrt{2}} \cdot \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{3-3\sqrt{2}-\sqrt{2}+\sqrt{2^2}}{1-\sqrt{2}^2} = \frac{3-4\sqrt{2}+2}{1-2} = \frac{5-4\sqrt{2}}{-1} = 4\sqrt{2}-5$$

$$2. \frac{5+2\sqrt{3}}{4-\sqrt{3}} \cdot \frac{4+\sqrt{3}}{4+\sqrt{3}} = \frac{20+5\sqrt{3}+8\sqrt{3}+2\sqrt{3^2}}{4^2-\sqrt{3}^2} = \frac{20+13\sqrt{3}+6}{16-3} = \frac{26+13\sqrt{3}}{13} = 2+\sqrt{3}$$

$$3. \frac{\sqrt{2}-\sqrt{5}}{\sqrt{2}+\sqrt{5}} \cdot \frac{\sqrt{2}-\sqrt{5}}{\sqrt{2}-\sqrt{5}} = \frac{\sqrt{2^2}-2\sqrt{2}\sqrt{5}+\sqrt{5^2}}{\sqrt{2^2}-\sqrt{5^2}} = \frac{2-2\sqrt{10}+5}{2-5} = \frac{7-2\sqrt{10}}{-3} = \frac{2\sqrt{10}-7}{3}$$

$$4. \frac{\sqrt{7}+2\sqrt{5}}{\sqrt{7}-\sqrt{5}} \cdot \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{\sqrt{7^2}+\sqrt{35}+2\sqrt{35}+2\sqrt{5^2}}{\sqrt{7^2}-\sqrt{5^2}} = \frac{7+3\sqrt{35}+10}{7-5} = \frac{17+3\sqrt{35}}{2}$$

$$5. \frac{\sqrt{2}-3\sqrt{5}}{2\sqrt{2}+\sqrt{5}} \cdot \frac{2\sqrt{2}-\sqrt{5}}{2\sqrt{2}-\sqrt{5}} = \frac{2\sqrt{2^2}-\sqrt{10}-6\sqrt{10}+3\sqrt{5^2}}{2^2\sqrt{2^2}-\sqrt{5^2}} = \frac{4-7\sqrt{10}+15}{8-5} = \frac{19-7\sqrt{10}}{3}$$

$$6. \frac{19}{5\sqrt{2}-4\sqrt{3}} \cdot \frac{5\sqrt{2}+4\sqrt{3}}{5\sqrt{2}+4\sqrt{3}} = \frac{95\sqrt{2}+76\sqrt{3}}{5^2\sqrt{2^2}-4^2\sqrt{3^2}} = \frac{95\sqrt{2}+76\sqrt{3}}{50-48} = \frac{95\sqrt{2}+76\sqrt{3}}{2}$$

$$7. \frac{3\sqrt{2}}{7\sqrt{2}-6\sqrt{3}} \cdot \frac{7\sqrt{2}+6\sqrt{3}}{7\sqrt{2}+6\sqrt{3}} = \frac{21\sqrt{2^2}+18\sqrt{6}}{7^2\sqrt{2^2}-6^2\sqrt{3^2}} = \frac{42+18\sqrt{6}}{98-108} = \frac{42+18\sqrt{6}}{-10} = -\frac{21+9\sqrt{6}}{5}$$

$$8. \frac{4\sqrt{3}-3\sqrt{7}}{2\sqrt{3}+3\sqrt{7}} \cdot \frac{2\sqrt{3}-3\sqrt{7}}{2\sqrt{3}-3\sqrt{7}} = \frac{8\sqrt{3^2}-12\sqrt{21}-6\sqrt{21}+9\sqrt{7^2}}{2^2\sqrt{3^2}-3^2\sqrt{7^2}} = \frac{24-18\sqrt{21}+63}{12-63} = \frac{87-18\sqrt{21}}{-51} = \frac{6\sqrt{21}-29}{17}$$

$$9. \frac{5\sqrt{2}-6\sqrt{3}}{4\sqrt{2}-3\sqrt{3}} \cdot \frac{4\sqrt{2}+3\sqrt{3}}{4\sqrt{2}+3\sqrt{3}} = \frac{20\sqrt{2^2}+15\sqrt{6}-24\sqrt{6}-18\sqrt{3^2}}{4^2\sqrt{2^2}-3^2\sqrt{3^2}} = \frac{40-9\sqrt{6}-54}{32-27} = \frac{-14-9\sqrt{6}}{5} = -\frac{14+9\sqrt{6}}{5}$$

$$10. \frac{\sqrt{7}+3\sqrt{11}}{5\sqrt{7}+4\sqrt{11}} \cdot \frac{5\sqrt{7}-4\sqrt{11}}{5\sqrt{7}-4\sqrt{11}} = \frac{5\sqrt{7^2}-4\sqrt{77}+15\sqrt{77}-12\sqrt{11^2}}{5^2\sqrt{7^2}-4^2\sqrt{11^2}} = \frac{35+11\sqrt{77}-132}{175-176} = \frac{-97+11\sqrt{77}}{-1} = 97-11\sqrt{77}$$

$$11. \frac{\sqrt{5}+\sqrt{2}}{7+2\sqrt{10}} \cdot \frac{7-2\sqrt{10}}{7-2\sqrt{10}} = \frac{7\sqrt{5}-2\sqrt{50}+7\sqrt{2}-2\sqrt{20}}{7^2-2^2\sqrt{10^2}} = \frac{7\sqrt{5}-2\sqrt{5^2 \cdot 2}+7\sqrt{2}-2\sqrt{2^2 \cdot 5}}{49-40} = \frac{7\sqrt{5}-10\sqrt{2}+7\sqrt{2}-4\sqrt{5}}{9} = \frac{3\sqrt{5}-3\sqrt{2}}{9} = \frac{\sqrt{5}-\sqrt{2}}{3}$$

$$12. \frac{9\sqrt{3}-3\sqrt{2}}{6-\sqrt{6}} \cdot \frac{6+\sqrt{6}}{6+\sqrt{6}} = \frac{54\sqrt{3}+9\sqrt{18}-18\sqrt{2}-3\sqrt{12}}{6^2-\sqrt{6}^2} = \frac{54\sqrt{3}+9\sqrt{3^2 \cdot 2}-18\sqrt{2}-3\sqrt{2^2 \cdot 3}}{36-6} = \frac{54\sqrt{3}+27\sqrt{2}-18\sqrt{2}-6\sqrt{3}}{30} = \frac{48\sqrt{3}+9\sqrt{2}}{30} = \frac{16\sqrt{3}+3\sqrt{2}}{10}$$

$$13. \frac{\sqrt{a}+\sqrt{x}}{2\sqrt{a}+\sqrt{x}} \cdot \frac{2\sqrt{a}-\sqrt{x}}{2\sqrt{a}-\sqrt{x}} \\ = \frac{2\sqrt{a^2}-\sqrt{ax}+2\sqrt{ax}-\sqrt{x^2}}{2^2\sqrt{a^2}-\sqrt{x^2}} = \frac{2a+\sqrt{ax}-x}{4a-x}$$

$$14. \frac{\sqrt{x}-\sqrt{x-1}}{\sqrt{x}+\sqrt{x-1}} \cdot \frac{\sqrt{x}-\sqrt{x-1}}{\sqrt{x}-\sqrt{x-1}} \\ = \frac{\sqrt{x^2}-2\sqrt{x}\sqrt{x-1}+\sqrt{(x-1)^2}}{\sqrt{x^2}-\sqrt{(x-1)^2}} \\ = \frac{x-2\sqrt{x^2-x}+x-1}{x-(x-1)} = 2x-1-2\sqrt{x^2-x}$$

$$15. \frac{\sqrt{a}-\sqrt{a+1}}{\sqrt{a}+\sqrt{a+1}} \cdot \frac{\sqrt{a}-\sqrt{a+1}}{\sqrt{a}-\sqrt{a+1}} \\ = \frac{\sqrt{a^2}-2\sqrt{a}\sqrt{a+1}+\sqrt{(a+1)^2}}{\sqrt{a^2}-\sqrt{(a+1)^2}} \\ = \frac{a-2\sqrt{a^2+a}+a+1}{a-(a+1)} \\ = \frac{2a+1-2\sqrt{a^2+a}}{-1} = 2\sqrt{a^2+a}-2a-1$$

$$16. \frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}-\sqrt{2}} \cdot \frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}} \\ = \frac{\sqrt{(x+2)^2}+2\sqrt{x+2}\sqrt{2}+\sqrt{2^2}}{\sqrt{(x+2)^2}-\sqrt{2^2}} \\ = \frac{x+2+2\sqrt{2x+4}+2}{x+2-2} = \frac{x+4+2\sqrt{2x+4}}{x}$$

$$17. \frac{\sqrt{a+4}-\sqrt{a}}{\sqrt{a+4}+\sqrt{a}} \cdot \frac{\sqrt{a+4}-\sqrt{a}}{\sqrt{a+4}-\sqrt{a}} \\ = \frac{\sqrt{(a+4)^2}-2\sqrt{a+4}\sqrt{a}+\sqrt{a^2}}{\sqrt{(a+4)^2}-\sqrt{a^2}} \\ = \frac{a+4-2\sqrt{a^2+4a}+a}{a+4-a} = \frac{2a+4-2\sqrt{a^2+4a}}{4} \\ = \frac{a+2-\sqrt{a^2+4a}}{2}$$

$$18. \frac{\sqrt{a+b}-\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}} \cdot \frac{\sqrt{a+b}-\sqrt{a-b}}{\sqrt{a+b}-\sqrt{a-b}} \\ = \frac{\sqrt{(a+b)^2}-2\sqrt{a+b}\sqrt{a-b}+\sqrt{(a-b)^2}}{\sqrt{(a+b)^2}-\sqrt{(a-b)^2}} \\ = \frac{a+b-2\sqrt{a^2-b^2}+a-b}{a+b-a+b} = \frac{2a-2\sqrt{a^2-b^2}}{2b} = \frac{a-\sqrt{a^2-b^2}}{b}$$

## EJERCICIO 249

$$1. \frac{\sqrt{3}}{\sqrt{2}+\sqrt{3}-\sqrt{5}} \cdot \frac{(\sqrt{2}+\sqrt{3})+\sqrt{5}}{(\sqrt{2}+\sqrt{3})+\sqrt{5}} \\ = \frac{\sqrt{6}+\sqrt{9}+\sqrt{15}}{(\sqrt{2}+\sqrt{3})^2-(\sqrt{5})^2} \\ = \frac{3+\sqrt{6}+\sqrt{15}}{2+2\sqrt{6}+3-5} = \frac{3+\sqrt{6}+\sqrt{15}}{2\sqrt{6}} \cdot \frac{2\sqrt{6}}{2\sqrt{6}} = \frac{6\sqrt{6}+2\sqrt{36}+2\sqrt{90}}{(2\sqrt{6})^2} \\ = \frac{6\sqrt{6}+12+6\sqrt{10}}{24} = \frac{6(\sqrt{6}+2+\sqrt{10})}{24} = \frac{2+\sqrt{6}+\sqrt{10}}{4}$$

$$2. \frac{\sqrt{2}}{\sqrt{2}+\sqrt{3}+\sqrt{6}} \cdot \frac{(\sqrt{2}+\sqrt{3})-\sqrt{6}}{(\sqrt{2}+\sqrt{3})-\sqrt{6}} \\ = \frac{2+\sqrt{6}-\sqrt{12}}{(\sqrt{2}+\sqrt{3})^2-(\sqrt{6})^2} \\ = \frac{2+\sqrt{6}-2\sqrt{3}}{2+2\sqrt{6}+3-6} = \frac{2+\sqrt{6}-2\sqrt{3}}{2\sqrt{6}-1} \cdot \frac{2\sqrt{6}+1}{2\sqrt{6}+1} = \frac{4\sqrt{6}+2+12+\sqrt{6}-4\sqrt{18}-2\sqrt{3}}{(2\sqrt{6})^2-1} \\ = \frac{5\sqrt{6}+14-12\sqrt{2}-2\sqrt{3}}{23}$$

$$\begin{aligned}
 3. \quad & \frac{2-\sqrt{3}}{2+\sqrt{3}+\sqrt{5}} \cdot \frac{(2+\sqrt{3})-\sqrt{5}}{(2+\sqrt{3})-\sqrt{5}} \\
 &= \frac{4-3-2\sqrt{5}+\sqrt{15}}{(2+\sqrt{3})^2-(\sqrt{5})^2} \\
 &= \frac{1-2\sqrt{5}+\sqrt{15}}{4+4\sqrt{3}+3-5} \\
 &= \frac{1-2\sqrt{5}+\sqrt{15}}{2+4\sqrt{3}} \cdot \frac{2-4\sqrt{3}}{2-4\sqrt{3}} \\
 &= \frac{2-4\sqrt{3}-4\sqrt{5}+8\sqrt{15}+2\sqrt{15}-4\sqrt{45}}{2^2-(4\sqrt{3})^2} \\
 &= \frac{2-4\sqrt{3}-4\sqrt{5}+10\sqrt{15}-12\sqrt{5}}{-44} \\
 &= \frac{2(2\sqrt{3}+8\sqrt{5}-5\sqrt{15}-1)}{44} = \frac{2\sqrt{3}+8\sqrt{5}-5\sqrt{15}-1}{22}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \frac{\sqrt{6}+\sqrt{3}+\sqrt{2}}{\sqrt{6}+\sqrt{3}-\sqrt{2}} \cdot \frac{(\sqrt{6}+\sqrt{3})+\sqrt{2}}{(\sqrt{6}+\sqrt{3})+\sqrt{2}} \\
 &= \frac{6+3+2+2\sqrt{18}+2\sqrt{12}+2\sqrt{6}}{(\sqrt{6}+\sqrt{3})^2-(\sqrt{2})^2} \\
 &= \frac{11+6\sqrt{2}+4\sqrt{3}+2\sqrt{6}}{6+2\sqrt{18}+3-2} \\
 &= \frac{11+6\sqrt{2}+4\sqrt{3}+2\sqrt{6}}{2\sqrt{18}+7} \cdot \frac{2\sqrt{18}-7}{2\sqrt{18}-7} \\
 &= \frac{22\sqrt{18}-77+12\sqrt{36}-42\sqrt{2}+8\sqrt{54}-28\sqrt{3}+4\sqrt{108}-14\sqrt{6}}{(2\sqrt{18})^2-49} \\
 &= \frac{66\sqrt{2}-77+72-42\sqrt{2}+24\sqrt{6}-28\sqrt{3}+24\sqrt{3}-14\sqrt{6}}{72-49} \\
 &= \frac{24\sqrt{2}-4\sqrt{3}+10\sqrt{6}-5}{23}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \frac{\sqrt{3}+\sqrt{5}}{\sqrt{2}+\sqrt{3}+\sqrt{5}} \cdot \frac{(\sqrt{2}+\sqrt{3})-\sqrt{5}}{(\sqrt{2}+\sqrt{3})-\sqrt{5}} \\
 &= \frac{\sqrt{6}+\sqrt{10}+3-5}{(\sqrt{2}+\sqrt{3})^2-(\sqrt{5})^2} \\
 &= \frac{\sqrt{10}+\sqrt{6}-2}{2+2\sqrt{6}+3-5} \\
 &= \frac{\sqrt{10}+\sqrt{6}-2}{2\sqrt{6}} \cdot \frac{2\sqrt{6}}{2\sqrt{6}} \\
 &= \frac{2\sqrt{60}+2\sqrt{36}-4\sqrt{6}}{(2\sqrt{6})^2} \\
 &= \frac{4\sqrt{15}+12-4\sqrt{6}}{24} \\
 &= \frac{4(\sqrt{15}+3-\sqrt{6})}{24} = \frac{\sqrt{15}+3-\sqrt{6}}{6}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \frac{\sqrt{2}-\sqrt{5}}{\sqrt{2}+\sqrt{5}-\sqrt{10}} \cdot \frac{(\sqrt{2}+\sqrt{5})+\sqrt{10}}{(\sqrt{2}+\sqrt{5})+\sqrt{10}} \\
 &= \frac{2-5+\sqrt{20}-\sqrt{50}}{2+2\sqrt{10}+5-10} \\
 &= \frac{2\sqrt{5}-5\sqrt{2}-3}{2\sqrt{10}-3} \cdot \frac{2\sqrt{10}+3}{2\sqrt{10}+3} \\
 &= \frac{4\sqrt{50}+6\sqrt{5}-10\sqrt{20}-15\sqrt{2}-6\sqrt{10}-9}{40-9} \\
 &= \frac{20\sqrt{2}+6\sqrt{5}-20\sqrt{5}-15\sqrt{2}-6\sqrt{10}-9}{31} \\
 &= \frac{5\sqrt{2}-14\sqrt{5}-6\sqrt{10}-9}{31}
 \end{aligned}$$

## EJERCICIO 250

$$1. \quad \frac{\sqrt{2}}{\sqrt{2}+\sqrt{3}} \cdot \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{2-\sqrt{6}}{2-3} = \frac{2-\sqrt{6}}{-1} = \sqrt{6}-2$$

$$\begin{aligned}
 2. \quad & \frac{\sqrt{3}}{\sqrt{3}-2\sqrt{5}} \cdot \frac{\sqrt{3}+2\sqrt{5}}{\sqrt{3}+2\sqrt{5}} \\
 &= \frac{3+2\sqrt{15}}{3-20} = \frac{3+2\sqrt{15}}{-17} = -\frac{3+2\sqrt{15}}{17}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \frac{2+\sqrt{5}}{1-\sqrt{5}} \cdot \frac{1+\sqrt{5}}{1+\sqrt{5}} \\
 &= \frac{2+2\sqrt{5}+\sqrt{5}+5}{1-5} = \frac{7+3\sqrt{5}}{-4} = -\frac{7+3\sqrt{5}}{4}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}-\sqrt{5}} \cdot \frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}+\sqrt{5}} \\
 &= \frac{2+2\sqrt{10}+5}{2-5} = \frac{7+2\sqrt{10}}{-3} = -\frac{7+2\sqrt{10}}{3}
 \end{aligned}$$

$$5. \frac{2\sqrt{3}-\sqrt{7}}{\sqrt{3}+\sqrt{7}} \cdot \frac{\sqrt{3}-\sqrt{7}}{\sqrt{3}-\sqrt{7}} = \frac{6-2\sqrt{21}-\sqrt{21}+7}{3-7} = \frac{13-3\sqrt{21}}{-4} = \frac{3\sqrt{21}-13}{4}$$

$$6. \frac{\sqrt{6}+2\sqrt{5}}{2\sqrt{6}-\sqrt{5}} \cdot \frac{2\sqrt{6}+\sqrt{5}}{2\sqrt{6}+\sqrt{5}} = \frac{12+5\sqrt{30}+10}{24-5} = \frac{22+5\sqrt{30}}{19}$$

$$7. \frac{5\sqrt{2}+3\sqrt{3}}{3\sqrt{2}-4\sqrt{3}} \cdot \frac{3\sqrt{2}+4\sqrt{3}}{3\sqrt{2}+4\sqrt{3}} = \frac{30+29\sqrt{6}+36}{18-48} = \frac{66+29\sqrt{6}}{-30} = -\frac{66+29\sqrt{6}}{30}$$

$$8. \frac{\sqrt{7}-2\sqrt{11}}{2\sqrt{7}+\sqrt{11}} \cdot \frac{2\sqrt{7}-\sqrt{11}}{2\sqrt{7}-\sqrt{11}} = \frac{14-5\sqrt{77}+22}{28-11} = \frac{36-5\sqrt{77}}{17}$$

## EJERCICIO 251

$$1. \sqrt{x-8}=2 \\ (\sqrt{x-8})^2=2^2 \\ x-8=4 \\ x=12$$

$$2. 5-\sqrt{3x+1}=0 \\ 5=\sqrt{3x+1} \\ 5^2=(\sqrt{3x+1})^2 \\ 25=3x+1 \\ 24=3x \\ 8=x$$

$$3. 7+\sqrt[3]{5x-2}=9 \\ \sqrt[3]{5x-2}=2 \\ (\sqrt[3]{5x-2})^3=2^3 \\ 5x-2=8 \\ 5x=10 \\ x=2$$

$$4. \sqrt{9x^2-5}-3x=-1 \\ \sqrt{9x^2-5}=3x-1 \\ (\sqrt{9x^2-5})^2=(3x-1)^2 \\ 9x^2-5=9x^2-6x+1 \\ -6=-6x \\ 1=x$$

$$5. \sqrt{x^2-2x+1}=9-x \\ \sqrt{(x-1)^2}=9-x \\ x-1=9-x \\ 2x=10 \\ x=5$$

$$6. 15-\sqrt[3]{7x-1}=12 \\ 3=\sqrt[3]{7x-1} \\ 3^3=(\sqrt[3]{7x-1})^3 \\ 27=7x-1 \\ 28=7x \\ 4=x$$

$$7. \sqrt{x}+\sqrt{x+7}=7 \\ (\sqrt{x+7})^2=(7-\sqrt{x})^2 \\ x+7=49-14\sqrt{x}+x \\ 7=49-14\sqrt{x} \\ 1=7-2\sqrt{x} \\ -6=-2\sqrt{x} \\ 3=\sqrt{x} \\ 3^2=(\sqrt{x})^2 \\ 9=x$$

$$8. \sqrt{3x-5}+\sqrt{3x-14}=9 \\ (\sqrt{3x-5})^2=(9-\sqrt{3x-14})^2 \\ 3x-5=81-18\sqrt{3x-14}+3x-14 \\ -72=-18\sqrt{3x-14} \\ 4^2=(\sqrt{3x-14})^2 \\ 16=3x-14 \\ 30=3x \\ 10=x$$

$$9. \sqrt{x+10}-\sqrt{x+19}=-1 \\ (\sqrt{x+10})^2=(\sqrt{x+19}-1)^2 \\ x+10=x+19-2\sqrt{x+19}+1 \\ -10=-2\sqrt{x+19} \\ 5=\sqrt{x+19} \\ 5^2=(\sqrt{x+19})^2 \\ 25=x+19 \\ 6=x$$

$$10. \sqrt{4x-11}=7\sqrt{2x-29} \\ (\sqrt{4x-11})^2=(7\sqrt{2x-29})^2 \\ 4x-11=49(2x-29) \\ 4x-11=98x-1.421 \\ 1.410=94x \\ 15=x$$

$$11. \sqrt{5x-19}-\sqrt{5x}=-1 \\ (\sqrt{5x-19})^2=(\sqrt{5x}-1)^2 \\ 5x-19=5x-2\sqrt{5x}+1 \\ -20=-2\sqrt{5x} \\ 10=\sqrt{5x} \\ 100=5x \\ 20=x$$

$$12. \sqrt{x-2}+5=\sqrt{x+53} \\ (\sqrt{x-2}+5)^2=(\sqrt{x+53})^2 \\ x-2+10\sqrt{x-2}+25=x+53 \\ 10\sqrt{x-2}=30 \\ \sqrt{x-2}=3 \\ x-2=9 \\ x=11$$

$$\begin{aligned}
13. \quad & \sqrt{9x-14} = 3\sqrt{x+10} - 4 \\
& (\sqrt{9x-14})^2 = (3\sqrt{x+10} - 4)^2 \\
& 9x-14 = 9(x+10) - 24\sqrt{x+10} + 16 \\
& -14 = 90 - 24\sqrt{x+10} + 16 \\
& -120 = -24\sqrt{x+10} \\
& 5 = \sqrt{x+10} \\
& 25 = x+10 \\
& 15 = x
\end{aligned}$$

$$\begin{aligned}
14. \quad & \sqrt{x-16} - \sqrt{x+8} = -4 \\
& (\sqrt{x-16})^2 = (\sqrt{x+8} - 4)^2 \\
& x-16 = x+8 - 8\sqrt{x+8} + 16 \\
& -40 = -8\sqrt{x+8} \\
& 5 = \sqrt{x+8} \\
& 25 = x+8 \\
& 17 = x
\end{aligned}$$

$$\begin{aligned}
15. \quad & \sqrt{5x-1} + 3 = \sqrt{5x+26} \\
& (\sqrt{5x-1} + 3)^2 = (\sqrt{5x+26})^2 \\
& 5x-1 + 6\sqrt{5x-1} + 9 = 5x+26 \\
& 6\sqrt{5x-1} = 18 \\
& \sqrt{5x-1} = 3 \\
& 5x-1 = 9 \\
& 5x = 10 \\
& x = 2
\end{aligned}$$

$$\begin{aligned}
16. \quad & 13 - \sqrt{13+4x} = 2\sqrt{x} \\
& (13 - 2\sqrt{x})^2 = (\sqrt{13+4x})^2 \\
& 169 - 52\sqrt{x} + 4x = 13 + 4x \\
& -52\sqrt{x} = -156 \\
& \sqrt{x} = 3 \\
& x = 9
\end{aligned}$$

$$\begin{aligned}
17. \quad & \sqrt{x-4} + \sqrt{x+4} = 2\sqrt{x-1} \\
& (\sqrt{x-4} + \sqrt{x+4})^2 = (2\sqrt{x-1})^2 \\
& x-4 + 2\sqrt{x^2-16} + x+4 = 4(x-1) \\
& 2x + 2\sqrt{x^2-16} = 4x-4 \\
& (2\sqrt{x^2-16})^2 = (2x-4)^2 \\
& 4x^2 - 64 = 4x^2 - 16x + 16 \\
& -80 = -16x \\
& 5 = x
\end{aligned}$$

$$\begin{aligned}
18. \quad & \sqrt{9x+7} - \sqrt{x} - \sqrt{16x-7} = 0 \\
& (\sqrt{9x+7} - \sqrt{x})^2 = (\sqrt{16x-7})^2 \\
& 9x+7 - 2\sqrt{9x^2+7x} + x = 16x-7 \\
& -2\sqrt{9x^2+7x} = 6x-14 \\
& (-2\sqrt{9x^2+7x})^2 = (6x-14)^2 \\
& 4(9x^2+7x) = 36x^2 - 168x + 196 \\
& 36x^2 + 28x = 36x^2 - 168x + 196 \\
& 196x = 196 \\
& x = 1
\end{aligned}$$

$$\begin{aligned}
19. \quad & \sqrt{9x+10} - 2\sqrt{x+3} = \sqrt{x-2} \\
& (\sqrt{9x+10} - 2\sqrt{x+3})^2 = (\sqrt{x-2})^2 \\
& 9x+10 - 4\sqrt{9x^2+37x+30} + 4(x+3) = x-2 \\
& 13x+22 - x+2 = 4\sqrt{9x^2+37x+30} \\
& 12x+24 = 4\sqrt{9x^2+37x+30} \\
& 3x+6 = \sqrt{9x^2+37x+30} \\
& (3x+6)^2 = (\sqrt{9x^2+37x+30})^2 \\
& 9x^2+36x+36 = 9x^2+37x+30 \\
& 36-30 = 37x-36x \\
& 6 = x
\end{aligned}$$

$$\begin{aligned}
20. \quad & \sqrt{18x-8} - \sqrt{2x-4} - 2\sqrt{2x+1} = 0 \\
& (\sqrt{18x-8} - \sqrt{2x-4})^2 = (2\sqrt{2x+1})^2 \\
& 18x-8 - 2x-4 + 4\sqrt{4x^2-6x-4} + 4(2x+1) \\
& 18x-8 = 10x+4 + 4\sqrt{4x^2-6x-4} \\
& 8x-8 = 4\sqrt{4x^2-6x-4} \\
& 2x-2 = \sqrt{4x^2-6x-4} \\
& (2x-2)^2 = (\sqrt{4x^2-6x-4})^2 \\
& 4x^2-8x+4 = 4x^2-6x-4 \\
& -2x = -8 \\
& x = 4
\end{aligned}$$

$$\begin{aligned}
21. \quad & \sqrt{8x+9} - \sqrt{18x+34} + \sqrt{2x+7} = 0 \\
& (\sqrt{8x+9} + \sqrt{2x+7})^2 = (\sqrt{18x+34})^2 \\
& 8x+9+2\sqrt{16x^2+74x+63}+2x+7=18x+34 \\
& 10x+16+2\sqrt{16x^2+74x+63}=18x+34 \\
& 2\sqrt{16x^2+74x+63}=8x+18 \\
& \sqrt{16x^2+74x+63}=4x+9 \\
& (\sqrt{16x^2+74x+63})^2 = (4x+9)^2 \\
& 16x^2+74x+63=16x^2+72x+81 \\
& 2x=18 \\
& x=9
\end{aligned}$$

$$\begin{aligned}
22. \quad & \sqrt{x-2} - \sqrt{x-5} = \sqrt{4x-23} \\
& (\sqrt{x-2} - \sqrt{x-5})^2 = (\sqrt{4x-23})^2 \\
& x-2-2\sqrt{x^2-7x+10}+x-5=4x-23 \\
& 2x-7-4x+23=2\sqrt{x^2-7x+10} \\
& -2x+16=2\sqrt{x^2-7x+10} \\
& -x+8=\sqrt{x^2-7x+10} \\
& (8-x)^2 = (\sqrt{x^2-7x+10})^2 \\
& 64-16x+x^2=x^2-7x+10 \\
& 54=9x \\
& 6=x
\end{aligned}$$

$$\begin{aligned}
23. \quad & \sqrt{x+6} - \sqrt{9x+70} = -2\sqrt{x+9} \\
& (\sqrt{x+6} + 2\sqrt{x+9})^2 = (\sqrt{9x+70})^2 \\
& x+6+4\sqrt{x^2+15x+54}+4(x+9)=9x+70 \\
& 5x+42+4\sqrt{x^2+15x+54}=9x+70 \\
& 4\sqrt{x^2+15x+54}=4x+28 \\
& \sqrt{x^2+15x+54}=x+7 \\
& (\sqrt{x^2+15x+54})^2 = (x+7)^2 \\
& x^2+15x+54=x^2+14x+49 \\
& x=-5
\end{aligned}$$

$$\begin{aligned}
24. \quad & \sqrt{x-a} + \sqrt{x+a} = \sqrt{4x-2a} \\
& (\sqrt{x-a} + \sqrt{x+a})^2 = (\sqrt{4x-2a})^2 \\
& x-a+2\sqrt{x^2-a^2}+x+a=4x-2a \\
& 2x+2\sqrt{x^2-a^2}=4x-2a \\
& 2\sqrt{x^2-a^2}=2x-2a \\
& \sqrt{x^2-a^2}=x-a \\
& (\sqrt{x^2-a^2})^2 = (x-a)^2 \\
& x^2-a^2=x^2-2ax+a^2 \\
& 2ax=2a^2 \\
& x=a
\end{aligned}$$

$$\begin{aligned}
25. \quad & \sqrt{x-4ab} = -2b + \sqrt{x} \\
& (\sqrt{x-4ab})^2 = (\sqrt{x} + 2b)^2 \\
& x-4ab=x-4b\sqrt{x}+4b^2 \\
& 4b\sqrt{x}=4b^2+4ab \\
& \sqrt{x}=b+a \\
& (\sqrt{x})^2 = (a+b)^2 \\
& x=(a+b)^2
\end{aligned}$$

$$\begin{aligned}
26. \quad & \sqrt{x+4a} - \sqrt{x+2a-1} = 1 \\
& (\sqrt{x+4a} - 1)^2 = (\sqrt{x+2a-1})^2 \\
& x+4a-2\sqrt{x+4a}+1=x+2a-1 \\
& 2a+2=2\sqrt{x+4a} \\
& a+1=\sqrt{x+4a} \\
& (a+1)^2 = (\sqrt{x+4a})^2 \\
& a^2+2a+1=x+4a \\
& a^2-2a+1=x \\
& (a-1)^2 = x
\end{aligned}$$

## EJERCICIO 252

$$1. \quad \sqrt{x} + \sqrt{x+5} = \frac{10}{\sqrt{x}}$$

$$\sqrt{x^2} + \sqrt{x^2 + 5x} = 10$$

$$\left(\sqrt{x^2 + 5x}\right)^2 = \left(10 - \sqrt{x^2}\right)^2$$

$$x^2 + 5x = 100 - 20x + x^2$$

$$25x = 100$$

$$x = 4$$

$$2. \quad \sqrt{4x-11} + 2\sqrt{x} = \frac{55}{\sqrt{4x-11}}$$

$$\sqrt{(4x-11)^2} + 2\sqrt{x(4x-11)} = 55$$

$$4x - 11 + 2\sqrt{4x^2 - 11x} = 55$$

$$2\sqrt{4x^2 - 11x} = 66 - 4x$$

$$\sqrt{4x^2 - 11x} = 33 - 2x$$

$$\left(\sqrt{4x^2 - 11x}\right)^2 = (33 - 2x)^2$$

$$4x^2 - 11x = 1.089 - 132x + 4x^2$$

$$121x = 1.089$$

$$x = 9$$

$$3. \quad \sqrt{x} - \sqrt{x-7} = \frac{4}{\sqrt{x}}$$

$$\sqrt{x^2} - \sqrt{x^2 - 7x} = 4$$

$$(x-4)^2 = \left(\sqrt{x^2 - 7x}\right)^2$$

$$x^2 - 8x + 16 = x^2 - 7x$$

$$16 = x$$

$$4. \quad \frac{\sqrt{x}-2}{\sqrt{x}+4} = \frac{\sqrt{x}+1}{\sqrt{x}+13}$$

$$(\sqrt{x}-2)(\sqrt{x}+13) = (\sqrt{x}+1)(\sqrt{x}+4)$$

$$\sqrt{x^2} + 11\sqrt{x} - 26 = \sqrt{x^2} + 5\sqrt{x} + 4$$

$$6\sqrt{x} = 30$$

$$\sqrt{x} = 5$$

$$(\sqrt{x})^2 = 5^2$$

$$x = 25$$

$$5. \quad \frac{6}{\sqrt{x+8}} = \sqrt{x+8} - \sqrt{x}$$

$$6 = \sqrt{(x+8)^2} - \sqrt{x(x+8)}$$

$$6 = x + 8 - \sqrt{x^2 + 8x}$$

$$\sqrt{x^2 + 8x} = x + 2$$

$$\left(\sqrt{x^2 + 8x}\right)^2 = (x+2)^2$$

$$x^2 + 8x = x^2 + 4x + 4$$

$$4x = 4$$

$$x = 1$$

$$6. \quad \sqrt{x-3} + \frac{8}{\sqrt{x+9}} = \sqrt{x+9}$$

$$\sqrt{(x-3)(x+9)} + 8 = \sqrt{(x+9)^2}$$

$$\sqrt{x^2 + 6x - 27} + 8 = x + 9$$

$$\left(\sqrt{x^2 + 6x - 27}\right)^2 = (x+1)^2$$

$$x^2 + 6x - 27 = x^2 + 2x + 1$$

$$4x = 28$$

$$x = 7$$

$$7. \quad \frac{\sqrt{x}+4}{\sqrt{x}-2} = \frac{\sqrt{x}+11}{\sqrt{x}-1}$$

$$(\sqrt{x}+4)(\sqrt{x}-1) = (\sqrt{x}+11)(\sqrt{x}-2)$$

$$\sqrt{x^2} + 3\sqrt{x} - 4 = \sqrt{x^2} + 9\sqrt{x} - 22$$

$$18 = 6\sqrt{x}$$

$$3 = \sqrt{x}$$

$$(3)^2 = (\sqrt{x})^2$$

$$9 = x$$

$$8. \quad 2\sqrt{x+6} - \sqrt{4x-3} = \frac{9}{\sqrt{4x-3}}$$

$$2\sqrt{(x+6)(4x-3)} - \sqrt{(4x-3)^2} = 9$$

$$2\sqrt{4x^2 + 21x - 18} - (4x-3) = 9$$

$$2\sqrt{4x^2 + 21x - 18} - 4x + 3 = 9$$

$$\left(2\sqrt{4x^2 + 21x - 18}\right)^2 = (4x+6)^2$$

$$4(4x^2 + 21x - 18) = 16x^2 + 48x + 36$$

$$16x^2 + 84x - 72 = 16x^2 + 48x + 36$$

$$36x = 108$$

$$x = 3$$

$$\begin{array}{ll}
 9. & \frac{\sqrt{x}-2}{\sqrt{x}+2} = \frac{2\sqrt{x}-5}{2\sqrt{x}-1} \\
 & (\sqrt{x}-2)(2\sqrt{x}-1) = (2\sqrt{x}-5)(\sqrt{x}+2) \\
 & 2\sqrt{x^2} - 5\sqrt{x} + 2 = 2\sqrt{x^2} - \sqrt{x} - 10 \\
 & 12 = 4\sqrt{x} \\
 & 3 = \sqrt{x} \\
 & (3)^2 = (\sqrt{x})^2 \\
 & 9 = x
 \end{array}
 \qquad
 \begin{array}{ll}
 10. & \sqrt{x+14} - \sqrt{x-7} = \frac{6}{\sqrt{x-7}} \\
 & \sqrt{(x+14)(x-7)} - \sqrt{(x-7)^2} = 6 \\
 & \sqrt{x^2+7x-98} - (x-7) = 6 \\
 & \sqrt{x^2+7x-98} - x + 7 = 6 \\
 & (\sqrt{x^2+7x-98})^2 = (x-1)^2 \\
 & x^2+7x-98 = x^2-2x+1 \\
 & 9x = 99 \\
 & x = 11
 \end{array}$$

### EJERCICIO 253

$$\begin{array}{ll}
 1. & \sqrt{-a^2} = \sqrt{a^2(-1)} = a\sqrt{-1} = ai \\
 2. & \sqrt{-2} = \sqrt{2(-1)} = \sqrt{2}\sqrt{-1} = \sqrt{2}i \\
 3. & 2\sqrt{-9} = 2\sqrt{3^2(-1)} = 6\sqrt{-1} = 6i \\
 4. & \sqrt{-81} = \sqrt{9^2(-1)} = 9\sqrt{-1} = 9i \\
 5. & \sqrt{-6} = \sqrt{6(-1)} = \sqrt{6}\sqrt{-1} = \sqrt{6}i \\
 6. & 3\sqrt{-b^4} = 3\sqrt{b^2b^2(-1)} = 3b^2\sqrt{-1} = 3b^2i \\
 7. & \sqrt{-12} = \sqrt{2^2 \cdot 3(-1)} = 2\sqrt{3}\sqrt{-1} = 2i\sqrt{3} \\
 8. & \sqrt{-7} = \sqrt{7(-1)} = \sqrt{7}\sqrt{-1} = \sqrt{7}i \\
 9. & \sqrt{-27} = \sqrt{3^2 \cdot 3(-1)} = 3\sqrt{3}\sqrt{-1} = 3i\sqrt{3} \\
 10. & \sqrt{-4m^4} = \sqrt{2^2 m^2 m^2 (-1)} = 2m^2\sqrt{-1} = 2im^2 \\
 11. & \sqrt{-\frac{1}{16}} = \sqrt{\frac{1}{4^2}(-1)} = \frac{1}{4}\sqrt{-1} = \frac{1}{4}i \\
 12. & \sqrt{-a^2-b^2} = \sqrt{-1(a^2+b^2)} = \sqrt{-1}\sqrt{a^2+b^2} = i\sqrt{a^2+b^2}
 \end{array}$$

### EJERCICIO 254

$$\begin{array}{ll}
 1. & \sqrt{-4} = \sqrt{2^2(-1)} = 2i \\
 & \sqrt{-16} = \sqrt{4^2(-1)} = 4i \\
 & = 2i + 4i = 6i \\
 2. & \sqrt{-25} = \sqrt{5^2(-1)} = 5i \\
 & \sqrt{-81} = \sqrt{9^2(-1)} = 9i \\
 & -\sqrt{-49} = -\sqrt{7^2(-1)} = -7i \\
 & = 5i + 9i - 7i = 7i \\
 3. & 2\sqrt{-9} = 2\sqrt{3^2(-1)} = 6i \\
 & 3\sqrt{-100} = 3\sqrt{10^2(-1)} = 30i \\
 & = 6i + 30i = 36i \\
 4. & 3\sqrt{-64} = 3\sqrt{8^2(-1)} = 24i \\
 & -5\sqrt{-49} = -5\sqrt{7^2(-1)} = -35i \\
 & 3\sqrt{-121} = 3\sqrt{11^2(-1)} = 33i \\
 & = 24i - 35i + 33i = 22i \\
 5. & 2\sqrt{-a^2} = 2\sqrt{a^2(-1)} = 2ai \\
 & \sqrt{-a^4} = \sqrt{a^2a^2(-1)} = a^2i \\
 & \sqrt{-a^6} = \sqrt{a^4a^2(-1)} = a^3i \\
 & = 2ai + a^2i + a^3i = i(2a + a^2 + a^3) \\
 6. & \sqrt{-18} = \sqrt{3^2 \cdot 2(-1)} = 3\sqrt{2}i \\
 & \sqrt{-8} = \sqrt{2^2 \cdot 2(-1)} = 2\sqrt{2}i \\
 & 2\sqrt{-50} = 2\sqrt{5^2 \cdot 2(-1)} = 10\sqrt{2}i \\
 & = 3\sqrt{2}i + 2\sqrt{2}i + 10\sqrt{2}i = 15\sqrt{2}i \\
 7. & 3\sqrt{-20} = 3\sqrt{2^2 \cdot 5(-1)} = 6\sqrt{5}i \\
 & -2\sqrt{-45} = -2\sqrt{3^2 \cdot 5(-1)} = -6\sqrt{5}i \\
 & 3\sqrt{-125} = 3\sqrt{5^2 \cdot 5(-1)} = 15\sqrt{5}i \\
 & = 6\sqrt{5}i - 6\sqrt{5}i + 15\sqrt{5}i = 15\sqrt{5}i \\
 8. & \sqrt{-a^4} = \sqrt{a^4(-1)} = a^2i \\
 & 4\sqrt{-9a^4} = 4\sqrt{3^2a^4(-1)} = 12a^2i \\
 & -3\sqrt{-4a^4} = -3\sqrt{2^2a^4(-1)} = -6a^2i \\
 & = a^2i + 12a^2i - 6a^2i = 7a^2i
 \end{array}$$



## EJERCICIO 255

$$\begin{aligned} 1. \sqrt{-16} \cdot \sqrt{-25} \\ = 4\sqrt{-1} \cdot 5\sqrt{-1} \\ = 20(\sqrt{-1})^2 \\ = 20(-1) \\ = -20 \end{aligned}$$

$$\begin{aligned} 2. \sqrt{-81} \cdot \sqrt{-49} \\ = 9\sqrt{-1} \cdot 7\sqrt{-1} \\ = 63(\sqrt{-1})^2 \\ = 63(-1) \\ = -63 \end{aligned}$$

$$\begin{aligned} 3. 5\sqrt{-36} \cdot 4\sqrt{-64} \\ = 5 \cdot 6\sqrt{-1} \cdot 4 \cdot 8\sqrt{-1} \\ = 30\sqrt{-1} \cdot 32\sqrt{-1} \\ = 960(\sqrt{-1})^2 \\ = 960(-1) \\ = -960 \end{aligned}$$

$$\begin{aligned} 4. \sqrt{-3} \cdot \sqrt{-2} \\ = \sqrt{3}\sqrt{-1} \cdot \sqrt{2}\sqrt{-1} \\ = \sqrt{6}(\sqrt{-1})^2 \\ = \sqrt{6}(-1) \\ = -\sqrt{6} \end{aligned}$$

$$\begin{aligned} 13. \frac{\sqrt{-2} + 3\sqrt{-5}}{2\sqrt{-2} - 6\sqrt{-5}} \\ \frac{2\sqrt{4}(\sqrt{-1})^2 + 6\sqrt{10}(\sqrt{-1})^2}{-6\sqrt{10}(\sqrt{-1})^2 - 18\sqrt{25}(\sqrt{-1})^2} \\ \frac{4(-1) - 90(-1)}{-4 + 90} \\ = 86 \end{aligned}$$

$$\begin{aligned} 5. 2\sqrt{-5} \cdot 3\sqrt{-7} \\ = 2\sqrt{5}\sqrt{-1} \cdot 3\sqrt{7}\sqrt{-1} \\ = 6\sqrt{35}(\sqrt{-1})^2 \\ = 6\sqrt{35}(-1) \\ = -6\sqrt{35} \end{aligned}$$

$$\begin{aligned} 6. \sqrt{-3} \cdot \sqrt{-75} \\ = \sqrt{3}\sqrt{-1} \cdot \sqrt{25}\sqrt{3}\sqrt{-1} \\ = 5\sqrt{3^2}(\sqrt{-1})^2 \\ = 15(-1) \\ = -15 \end{aligned}$$

$$\begin{aligned} 7. 2\sqrt{-7} \cdot 3\sqrt{-28} \\ = 2\sqrt{7}\sqrt{-1} \cdot 3\sqrt{4}\sqrt{7}\sqrt{-1} \\ = 6\sqrt{4}\sqrt{7^2}(\sqrt{-1})^2 \\ = 12 \cdot 7(-1) \\ = -84 \end{aligned}$$

$$\begin{aligned} 8. \sqrt{-49} \cdot \sqrt{-4} \cdot \sqrt{-9} \\ = 7\sqrt{-1} \cdot 2\sqrt{-1} \cdot 3\sqrt{-1} \\ = 42\sqrt{-1}^3 \\ = 42\sqrt{-1}j \\ = -42\sqrt{-1} \\ = -42i \end{aligned}$$

$$\begin{aligned} 9. \sqrt{-2} \cdot 3\sqrt{-5} \cdot \sqrt{-10} \\ = \sqrt{2}\sqrt{-1} \cdot 3\sqrt{5}\sqrt{-1} \cdot \sqrt{10}\sqrt{-1} \\ = 3\sqrt{2 \cdot 5 \cdot 10}(\sqrt{-1})^3 \\ = 3\sqrt{100}(-\sqrt{-1}) \\ = -30\sqrt{-1} \\ = -30i \end{aligned}$$

$$\begin{aligned} 10. \sqrt{-12} \cdot \sqrt{-27} \cdot \sqrt{-8} \cdot \sqrt{-50} \\ = 2\sqrt{3}\sqrt{-1} \cdot 3\sqrt{3}\sqrt{-1} \cdot 2\sqrt{2}\sqrt{-1} \cdot 5\sqrt{2}\sqrt{-1} \\ = 60\sqrt{9 \cdot 4}(\sqrt{-1})^4 \\ = 60 \cdot 3 \cdot 2(1) \\ = 360 \end{aligned}$$

$$\begin{aligned} 11. -5\sqrt{-x} \cdot 3\sqrt{-y} \\ = -5\sqrt{x}\sqrt{-1} \cdot 3\sqrt{y}\sqrt{-1} \\ = -15\sqrt{xy}(\sqrt{-1})^2 \\ = -15\sqrt{xy}(-1) \\ = 15\sqrt{xy} \end{aligned}$$

$$\begin{aligned} 12. \frac{\sqrt{-4} + \sqrt{-9}}{\sqrt{-25} - \sqrt{-16}} \\ \frac{10(\sqrt{-1})^2 + 15(\sqrt{-1})^2}{-8(\sqrt{-1})^2 - 12(\sqrt{-1})^2} \\ \frac{10(-1) + 15(-1)}{-8(-1) - 12(-1)} \\ = \frac{-25}{20} = -\frac{5}{4} \end{aligned}$$

$$\begin{aligned} 14. \frac{2\sqrt{-2} + 5\sqrt{-3}}{\sqrt{-2} - 4\sqrt{-3}} \\ \frac{2\sqrt{4}(\sqrt{-1})^2 + 5\sqrt{6}(\sqrt{-1})^2}{-8\sqrt{6}(\sqrt{-1})^2 - 20\sqrt{9}(\sqrt{-1})^2} \\ \frac{4(-1) - 3\sqrt{6}(-1)}{-8(-1) - 60(-1)} \\ = \frac{-4 + 3\sqrt{6}}{68} \\ = \frac{-4 + 3\sqrt{6}}{68} \end{aligned}$$

## EJERCICIO 256

$$\begin{aligned} 1. \frac{\sqrt{-16}}{\sqrt{-4}} &= \frac{4\sqrt{-1}}{2\sqrt{-1}} = 2 & 2. \frac{\sqrt{-10}}{\sqrt{-2}} &= \frac{\sqrt{10}\sqrt{-1}}{\sqrt{2}\sqrt{-1}} = \sqrt{\frac{10}{2}} = \sqrt{5} & 3. \frac{\sqrt{-81}}{\sqrt{-3}} &= \frac{9\sqrt{-1}}{\sqrt{3}\sqrt{-1}} = \frac{9}{\sqrt{3}} = \frac{9\sqrt{3}}{\sqrt{9}} = \frac{9\sqrt{3}}{3} = 3\sqrt{3} \end{aligned}$$

$$\begin{array}{ll}
 4. \frac{\sqrt{-90}}{\sqrt{-5}} = \frac{3\sqrt{10}\sqrt{-1}}{\sqrt{5}\sqrt{-1}} = 3\sqrt{\frac{10}{5}} = 3\sqrt{2} & 7. \frac{2\sqrt{-18}}{\sqrt{-6}} = \frac{6\sqrt{2}\sqrt{-1}}{\sqrt{6}\sqrt{-1}} = \frac{6\sqrt{2}}{\sqrt{6}} = \frac{6\sqrt{12}}{\sqrt{36}} = \frac{6\sqrt{2^2 \cdot 3}}{6} = 2\sqrt{3} \\
 5. \frac{\sqrt{-150}}{\sqrt{-3}} = \frac{5\sqrt{6}\sqrt{-1}}{\sqrt{3}\sqrt{-1}} = 5\sqrt{\frac{6}{3}} = 5\sqrt{2} & 8. \frac{\sqrt{-315}}{\sqrt{-7}} = \frac{3\sqrt{35}\sqrt{-1}}{\sqrt{7}\sqrt{-1}} = 3\sqrt{\frac{35}{7}} = 3\sqrt{5} \\
 6. \frac{10\sqrt{-36}}{5\sqrt{-4}} = \frac{60\sqrt{-1}}{10\sqrt{-1}} = 6 & 9. \frac{\sqrt[4]{-27}}{\sqrt[4]{-3}} = \frac{\sqrt[4]{27}\sqrt{-1}}{\sqrt[4]{3}\sqrt{-1}} = \sqrt[4]{\frac{27}{3}} = \sqrt[4]{9} = \sqrt[4]{3^2} = 3^{\frac{2}{4}} = 3^{\frac{1}{2}} = \sqrt{3} \\
 & 10. \frac{\sqrt[4]{-300}}{\sqrt[4]{-12}} = \frac{\sqrt[4]{300}\sqrt{-1}}{\sqrt[4]{12}\sqrt{-1}} = \sqrt[4]{\frac{300}{12}} = \sqrt[4]{25} = \sqrt[4]{5^2} = 5^{\frac{2}{4}} = 5^{\frac{1}{2}} = \sqrt{5}
 \end{array}$$

### ***EJERCICIO 257***

$$\begin{array}{lll}
 1. \frac{2+3\sqrt{-1}}{5-2\sqrt{-1}} \cdot \frac{7+\sqrt{-1}}{7+i} & 4. \frac{5+\sqrt{-1}}{7+2\sqrt{-1}} \cdot \frac{9+7\sqrt{-1}}{21+10\sqrt{-1}} & 7. \frac{2+\sqrt{-2}}{4-\sqrt{-3}} \cdot \frac{6+(\sqrt{2}\sqrt{-1}-\sqrt{3}\sqrt{-1})}{6+(\sqrt{2}i-\sqrt{3}i)} \\
 2. \frac{-4-5\sqrt{-1}}{-2+8\sqrt{-1}} \cdot \frac{-6+3\sqrt{-1}}{-6+3i} & 5. \frac{3-2i}{5-8i} \cdot \frac{-10+13i}{-2+3i} & 8. \frac{7+\sqrt{-5}}{\sqrt{2}-\sqrt{-9}} \cdot \frac{-4+\sqrt{-16}}{(3+\sqrt{2})+(\sqrt{5}\sqrt{-1}-\sqrt{9}\sqrt{-1}+\sqrt{16}\sqrt{-1})} \\
 3. \frac{12-11\sqrt{-1}}{8+7\sqrt{-1}} \cdot \frac{20-4\sqrt{-1}}{20-4i} & 6. \frac{1-i}{4+3i} \cdot \frac{\sqrt{2}+5i}{5+\sqrt{2}+7i} & \frac{(3+\sqrt{2})+(\sqrt{5}i-3i+4i)}{(3+\sqrt{2})+(\sqrt{5}i+i)} \\
 & & \frac{(3+\sqrt{2})+(\sqrt{5}+1)i}{(3+\sqrt{2})+(\sqrt{5}+1)i}
 \end{array}$$

### ***EJERCICIO 258***

$$\begin{array}{lll}
 1. (7-2\sqrt{-1})+(7+2\sqrt{-1}) & 3. (9+i\sqrt{3})+(9-i\sqrt{3}) & 5. (8-3\sqrt{-2})+(8+3\sqrt{-2}) \\
 = (7+7)+(-2+2)\sqrt{-1} & = (9+9)+(i-i)\sqrt{3} & = (8+8)+(-3+3)\sqrt{2}\sqrt{-1} \\
 = 7 \cdot 2 & = 9 \cdot 2 & = 8 \cdot 2 \\
 = 14 & = 18 & = 16 \\
 2. (-5-3\sqrt{-1})+(-5+3\sqrt{-1}) & 4. (-7-5\sqrt{-1})+(-7+5\sqrt{-1}) & 6. (\sqrt{2}+i\sqrt{3})+(\sqrt{2}-i\sqrt{3}) \\
 = (-5-5)+(-3+3)\sqrt{-1} & = (-7-7)+(-5+5)\sqrt{-1} & = (\sqrt{2}+\sqrt{2})+(i-i)\sqrt{3} \\
 = -5 \cdot 2 & = -7 \cdot 2 & = 2\sqrt{2} \\
 = -10 & = -14 &
 \end{array}$$

## EJERCICIO 259

$$\begin{array}{r}
 1. (3-2\sqrt{-1})-(5+3\sqrt{-1}) \\
 3-2\sqrt{-1} \\
 \underline{-5-3\sqrt{-1}} \\
 -2-5\sqrt{-1} \\
 -2-5i
 \end{array}$$

$$\begin{array}{r}
 2. (8+4\sqrt{-1})-(3-10\sqrt{-1}) \\
 8+4\sqrt{-1} \\
 \underline{-3+10\sqrt{-1}} \\
 5+14\sqrt{-1} \\
 5+14i
 \end{array}$$

$$\begin{array}{r}
 3. (-1-\sqrt{-1})-(-7-8\sqrt{-1}) \\
 -1-\sqrt{-1} \\
 \underline{+7+8\sqrt{-1}} \\
 6+7\sqrt{-1} \\
 6+7i
 \end{array}$$

$$\begin{array}{r}
 4. (4-7\sqrt{-1})-(5-3\sqrt{-1}) \\
 4-7\sqrt{-1} \\
 \underline{-5+3\sqrt{-1}} \\
 -1-4\sqrt{-1} \\
 -1-4i
 \end{array}$$

$$\begin{array}{r}
 5. (15-4\sqrt{-1})-(8-7\sqrt{-1}) \\
 15-4\sqrt{-1} \\
 \underline{-8+7\sqrt{-1}} \\
 7+3\sqrt{-1} \\
 7+3i
 \end{array}$$

$$\begin{array}{r}
 6. (11+80\sqrt{-1})-(3-50\sqrt{-1}) \\
 11+80\sqrt{-1} \\
 \underline{-3+50\sqrt{-1}} \\
 8+130\sqrt{-1} \\
 8+130i
 \end{array}$$

$$\begin{array}{r}
 7. (5-\sqrt{-25})-(3+6i) \\
 5-\sqrt{-25} \\
 \underline{-3-6i} \\
 2+(-\sqrt{25}\sqrt{-1}-6i) \\
 2+(-5i-6i) \\
 2+(-11i) \\
 2-11i
 \end{array}$$

$$\begin{array}{r}
 8. (4+\sqrt{-5})-(2+\sqrt{-3}) \\
 4+\sqrt{-5} \\
 \underline{-2-\sqrt{-3}} \\
 2+\sqrt{5}\sqrt{-1}-\sqrt{3}\sqrt{-1} \\
 2+(\sqrt{5}i-\sqrt{3}i) \\
 2+(\sqrt{5}-\sqrt{3})i
 \end{array}$$

$$\begin{array}{r}
 9. (\sqrt{2}-5\sqrt{-1})-(\sqrt{3}+6\sqrt{-1}) \\
 \sqrt{2}-5i \\
 \underline{-\sqrt{3}-6i} \\
 (\sqrt{2}-\sqrt{3})+(-5i-6i) \\
 (\sqrt{2}-\sqrt{3})+(-11i) \\
 (\sqrt{2}-\sqrt{3})-11i
 \end{array}$$

$$\begin{array}{r}
 10. (8-\sqrt{-7})-(-7+\sqrt{-3}) \\
 8-\sqrt{-7} \\
 \underline{7-\sqrt{-3}} \\
 15+(-\sqrt{7}\sqrt{-1}-\sqrt{3}\sqrt{-1}) \\
 15+(-\sqrt{7}i-\sqrt{3}i) \\
 15-(\sqrt{7}+\sqrt{3})i
 \end{array}$$

## EJERCICIO 260

$$\begin{array}{r}
 1. (2-\sqrt{-1})-(2+\sqrt{-1}) \\
 2-\sqrt{-1}-2-\sqrt{-1} \\
 =(2-2)+(-1-1)\sqrt{-1} \\
 =-2\sqrt{-1} \\
 =-2i
 \end{array}$$

$$\begin{array}{r}
 2. (7+3\sqrt{-1})-(7-3\sqrt{-1}) \\
 7+3\sqrt{-1}-7+3\sqrt{-1} \\
 =(7-7)+(3+3)\sqrt{-1} \\
 =6\sqrt{-1} \\
 =6i
 \end{array}$$

$$\begin{array}{r}
 3. (-3-7\sqrt{-1})-(-3+7\sqrt{-1}) \\
 -3-7\sqrt{-1}+3-7\sqrt{-1} \\
 =(-3+3)+(-7-7)\sqrt{-1} \\
 =-7\cdot 2\sqrt{-1} \\
 =-14i
 \end{array}$$

$$\begin{array}{r}
 4. (-5+\sqrt{-2})-(-5-\sqrt{-2}) \\
 -5+\sqrt{-2}+5+\sqrt{-2} \\
 =(-5+5)+(1+1)\sqrt{2}\sqrt{-1} \\
 =2\sqrt{2}\sqrt{-1} \\
 =2\sqrt{2}i
 \end{array}$$

$$\begin{array}{r}
 5. (\sqrt{2}+\sqrt{-3})-(\sqrt{2}-\sqrt{-3}) \\
 \sqrt{2}+\sqrt{-3}-\sqrt{2}+\sqrt{-3} \\
 =(\sqrt{2}-\sqrt{2})+(1+1)\sqrt{3}\sqrt{-1} \\
 =2\sqrt{3}\sqrt{-1} \\
 =2\sqrt{3}i
 \end{array}$$

$$\begin{array}{r}
 6. (-\sqrt{5}-4\sqrt{-2})-(-\sqrt{5}+4\sqrt{-2}) \\
 -\sqrt{5}-4\sqrt{-2}+\sqrt{5}-4\sqrt{-2} \\
 -\sqrt{5}-4\sqrt{-2} \\
 \underline{+\sqrt{5}-4\sqrt{-2}} \\
 -8\sqrt{-2}=-8\sqrt{2}\sqrt{-1} \\
 =-8\sqrt{2}i
 \end{array}$$

## EJERCICIO 261

1. 
$$\frac{3-4\sqrt{-1}}{5-3\sqrt{-1}} \cdot \frac{15-20\sqrt{-1}}{-9\sqrt{-1}+12(\sqrt{-1})^2}$$
$$\frac{15-29\sqrt{-1}+12(-1)}{15-29i-12}$$
$$\frac{3-29i}{3-29i}$$
2. 
$$\frac{4+7\sqrt{-1}}{-3-2\sqrt{-1}} \cdot \frac{-12-21\sqrt{-1}}{-8\sqrt{-1}-14(\sqrt{-1})^2}$$
$$\frac{-12-29\sqrt{-1}-14(-1)}{-12-29i+14}$$
$$\frac{2-29i}{2-29i}$$
3. 
$$\frac{7-\sqrt{-4}}{5+\sqrt{-9}} \cdot \frac{35-5\sqrt{4}\sqrt{-1}}{+7\sqrt{9}\sqrt{-1}-\sqrt{4}\sqrt{9}(\sqrt{-1})^2}$$
$$\frac{35-10\sqrt{-1}+21\sqrt{-1}-\sqrt{36}(-1)}{35-10i+21i+6}$$
$$\frac{41+11i}{41+11i}$$
4. 
$$\frac{8-\sqrt{-9}}{11+\sqrt{-25}} \cdot \frac{88-11\sqrt{9}\sqrt{-1}}{+8\sqrt{25}\sqrt{-1}-\sqrt{9}\sqrt{25}(\sqrt{-1})^2}$$
$$\frac{88-33i+40i-15(-1)}{88+7i+15}$$
$$\frac{103+7i}{103+7i}$$

5. 
$$\frac{3+\sqrt{-2}}{5-\sqrt{-2}} \cdot \frac{15+5\sqrt{2}\sqrt{-1}}{-3\sqrt{2}\sqrt{-1}-\sqrt{2}\sqrt{2}(\sqrt{-1})^2}$$
$$\frac{15+2\sqrt{2}\sqrt{-1}-\sqrt{4}(-1)}{15+2\sqrt{2}i+2}$$
$$\frac{17+2\sqrt{2}i}{17+2\sqrt{2}i}$$
6. 
$$\frac{4+\sqrt{-3}}{5-\sqrt{-2}} \cdot \frac{20+5\sqrt{3}\sqrt{-1}}{-4\sqrt{2}\sqrt{-1}-\sqrt{2}\sqrt{3}(\sqrt{-1})^2}$$
$$\frac{20+5\sqrt{3}i-4\sqrt{2}i-\sqrt{6}(-1)}{20+(5\sqrt{3}-4\sqrt{2})i+\sqrt{6}}$$
$$\frac{(20+\sqrt{6})+(5\sqrt{3}-4\sqrt{2})i}{(20+\sqrt{6})+(5\sqrt{3}-4\sqrt{2})i}$$
7. 
$$\frac{\sqrt{2}+\sqrt{-5}}{\sqrt{3}+\sqrt{-2}} \cdot \frac{\sqrt{6}+\sqrt{3}\sqrt{5}\sqrt{-1}}{+\sqrt{2}\sqrt{2}\sqrt{-1}+\sqrt{5}\sqrt{2}(\sqrt{-1})^2}$$
$$\frac{\sqrt{6}+\sqrt{15}i+\sqrt{4}i+\sqrt{10}(-1)}{\sqrt{6}+(\sqrt{15}i+2i)-\sqrt{10}}$$
$$\frac{(\sqrt{6}-\sqrt{10})+(\sqrt{15}+2)i}{(\sqrt{6}-\sqrt{10})+(\sqrt{15}+2)i}$$
8. 
$$\frac{\sqrt{5}+\sqrt{-3}}{\sqrt{5}+2\sqrt{-3}} \cdot \frac{\sqrt{25}+\sqrt{5}\sqrt{3}\sqrt{-1}}{+2\sqrt{5}\sqrt{3}\sqrt{-1}+2\sqrt{9}(\sqrt{-1})^2}$$
$$\frac{5+\sqrt{15}i+2\sqrt{15}i+6(-1)}{5+3\sqrt{15}i-6}$$
$$\frac{3\sqrt{15}i-1}{3\sqrt{15}i-1}$$

## EJERCICIO 262

1. 
$$(1-i)(1+i)$$
$$= 1-i^2$$
$$= 1-(\sqrt{-1})^2$$
$$= 1-(-1)$$
$$= 1+1=2$$
2. 
$$(3+2\sqrt{-1})(3-2\sqrt{-1})$$
$$= 3^2-(2\sqrt{-1})^2$$
$$= 9-[4(-1)]$$
$$= 9+4=13$$
3. 
$$(\sqrt{2}-5i)(\sqrt{2}+5i)$$
$$= (\sqrt{2})^2-(5i)^2$$
$$= 2-[25(\sqrt{-1})^2]$$
$$= 2-[25(-1)]$$
$$= 2+25=27$$
4. 
$$(2\sqrt{3}+4i)(2\sqrt{3}-4i)$$
$$= (2\sqrt{3})^2-(4i)^2$$
$$= 4(3)-[16(\sqrt{-1})^2]$$
$$= 12-[16(-1)]$$
$$= 12+16=28$$
5. 
$$(5-\sqrt{-2})(5+\sqrt{-2})$$
$$= 25-(\sqrt{-2})^2$$
$$= 25-(\sqrt{2}\sqrt{-1})^2$$
$$= 25-[2(\sqrt{-1})^2]$$
$$= 25-[2(-1)]$$
$$= 25+2=27$$
6. 
$$(-9-\sqrt{-5})(-9+\sqrt{-5})$$
$$= 81-(\sqrt{-5})^2$$
$$= 81-(\sqrt{5}\sqrt{-1})^2$$
$$= 81-[5(-1)]$$
$$= 81+5=86$$

## EJERCICIO 263

- $$\frac{1+\sqrt{-1}}{1-\sqrt{-1}} \cdot \frac{1+\sqrt{-1}}{1+\sqrt{-1}} = \frac{1+2\sqrt{-1}+(\sqrt{-1})^2}{1-(\sqrt{-1})^2} = \frac{1+2i-1}{1+1} = \frac{2i}{2} = i$$
- $$\frac{3+\sqrt{-1}}{3-\sqrt{-1}} \cdot \frac{3+\sqrt{-1}}{3+\sqrt{-1}} = \frac{9+6\sqrt{-1}+(-1)}{9-(\sqrt{-1})^2} = \frac{9+6i-1}{9+1} = \frac{8+6i}{10} = \frac{4+3i}{5}$$
- $$\frac{5-3\sqrt{-1}}{3+4\sqrt{-1}} \cdot \frac{3-4\sqrt{-1}}{3-4\sqrt{-1}} = \frac{15-29\sqrt{-1}+12(-1)}{9-(4\sqrt{-1})^2} = \frac{15-29i-12}{9-16(-1)} = \frac{3-29i}{25}$$
- $$\frac{8-5i}{7+6i} \cdot \frac{7-6i}{7-6i} = \frac{56-83i+30i^2}{49-36i^2} = \frac{56-83i-30}{49+36} = \frac{26-83i}{85}$$
- $$\frac{4+\sqrt{-3}}{5-4\sqrt{-3}} \cdot \frac{5+4\sqrt{-3}}{5+4\sqrt{-3}} = \frac{20+21\sqrt{3}\sqrt{-1}+4(\sqrt{3})^2(-1)}{25-(4\sqrt{3}\sqrt{-1})^2} = \frac{20+21\sqrt{3}i-12}{25-(-48)} = \frac{8+21\sqrt{3}i}{73}$$
- $$\frac{\sqrt{2}+2\sqrt{-5}}{4\sqrt{2}-\sqrt{-5}} \cdot \frac{4\sqrt{2}+\sqrt{-5}}{4\sqrt{2}+\sqrt{-5}} = \frac{4(\sqrt{2})^2+9\sqrt{2}\sqrt{5}\sqrt{-1}+2(\sqrt{5})^2(-1)}{16(\sqrt{2})^2-(\sqrt{5}\sqrt{-1})^2} = \frac{8+9\sqrt{10}i-10}{32+5} = \frac{9\sqrt{10}i-2}{37}$$

## EJERCICIO 265

- $$3x^2-5x+2=0$$

$$x^2-\frac{5x}{3}+\frac{2}{3}=0$$

$$x^2-\frac{5x}{3}=-\frac{2}{3}$$

$$x^2-\frac{5x}{3}+\frac{25}{36}=\frac{25}{36}-\frac{2}{3}$$

$$\left(x-\frac{5}{6}\right)^2=\frac{25-24}{36}$$

$$x-\frac{5}{6}=\pm\sqrt{\frac{1}{36}}$$

$$x=\frac{5}{6}\pm\frac{1}{6}$$

$$x_1=\frac{5+1}{6}=1$$

$$x_2=\frac{5-1}{6}=\frac{4}{6}=\frac{2}{3}$$
- $$4x^2+3x-22=0$$

$$x^2+\frac{3x}{4}=\frac{22}{4}$$

$$x^2+\frac{3x}{4}+\frac{9}{64}=\frac{11}{2}+\frac{9}{64}$$

$$\left(x+\frac{3}{8}\right)^2=\frac{352+9}{64}$$

**Continúa**
- 2. Continuación**

$$x+\frac{3}{8}=\pm\sqrt{\frac{361}{64}}$$

$$x=-\frac{3}{8}\pm\frac{19}{8}$$

$$x_1=\frac{-3+19}{8}=\frac{16}{8}=2$$

$$x_2=\frac{-3-19}{8}=-\frac{22}{8}=-\frac{11}{4}$$
- $$x^2+11x=-24$$

$$x^2+11x+\frac{121}{4}=\frac{121}{4}-24$$

$$\left(x+\frac{11}{2}\right)^2=\frac{121-96}{4}$$

$$x+\frac{11}{2}=\pm\sqrt{\frac{25}{4}}$$

$$x=-\frac{11}{2}\pm\frac{5}{2}$$

$$x_1=\frac{-11+5}{2}=-\frac{6}{2}=-3$$

$$x_2=\frac{-11-5}{2}=-\frac{16}{2}=-8$$
- $$x^2=16x-63$$

$$x^2-16x=-63$$

$$x^2-16x+64=64-63$$

$$(x-8)^2=1$$

$$x-8=\pm 1$$

$$x=8\pm 1$$

$$x_1=8+1=9$$

$$x_2=8-1=7$$
- $$12x-4-9x^2=0$$

$$-9x^2+12x=4$$

$$9x^2-12x=-4$$

$$x^2-\frac{12x}{9}=-\frac{4}{9}$$

$$x^2-\frac{4x}{3}+\frac{4}{9}=\frac{4}{9}-\frac{4}{9}$$

$$\left(x-\frac{2}{3}\right)^2=0$$

$$x-\frac{2}{3}=0$$

$$x_1=x_2=\frac{2}{3}$$

$$6. 5x^2 - 7x - 90 = 0$$

$$x = \frac{7 \pm \sqrt{7^2 - 4(5)(-90)}}{2(5)}$$

$$x = \frac{7 \pm \sqrt{49 + 1.800}}{10}$$

$$x = \frac{7 \pm \sqrt{1.849}}{10} = \frac{7 \pm 43}{10}$$

$$x_1 = \frac{7 + 43}{10} = \frac{50}{10} = 5$$

$$x_2 = \frac{7 - 43}{10} = -\frac{36}{10} = -3\frac{3}{5}$$

$$7. 6x^2 = x + 222$$

$$6x^2 - x - 222 = 0$$

$$x = \frac{1 \pm \sqrt{1^2 + 4(6)(222)}}{2(6)}$$

$$x = \frac{1 \pm \sqrt{5.329}}{12}$$

$$x = \frac{1 \pm 73}{12}$$

$$x_1 = \frac{1 + 73}{12} = \frac{74}{12} = 6\frac{1}{6}$$

$$x_2 = \frac{1 - 73}{12} = \frac{-72}{12} = -6$$

$$8. x + 11 = 10x^2$$

$$10x^2 - x - 11 = 0$$

$$x = \frac{1 \pm \sqrt{1^2 - 4(10)(-11)}}{2(10)}$$

$$x = \frac{1 \pm \sqrt{1 + 440}}{20}$$

$$x = \frac{1 \pm 21}{20}$$

$$x_1 = \frac{1 + 21}{20} = \frac{22}{20} = 1\frac{1}{10}$$

$$x_2 = \frac{1 - 21}{20} = \frac{-20}{20} = -1$$

$$9. 49x^2 - 70x + 25 = 0$$

$$x = \frac{70 \pm \sqrt{(70)^2 - 4(49)(25)}}{2(49)}$$

$$x = \frac{70 \pm \sqrt{4.900 - 4.900}}{98}$$

$$x_1 = x_2 = \frac{70}{98} = \frac{5}{7}$$

$$10. 12x - 7x^2 + 64 = 0$$

$$-7x^2 + 12x + 64 = 0$$

$$x = \frac{-12 \pm \sqrt{(12)^2 - 4(-7)(64)}}{2(-7)}$$

$$x = \frac{-12 \pm \sqrt{144 + 1.792}}{-14}$$

$$x = \frac{-12 \pm \sqrt{1.936}}{-14}$$

$$x = \frac{-12 \pm 44}{-14}$$

$$x_1 = \frac{-12 + 44}{-14} = -\frac{32}{14} = -2\frac{2}{7}$$

$$x_2 = \frac{-12 - 44}{-14} = \frac{-56}{-14} = 4$$

$$11. x^2 = 15x - 56$$

$$x^2 + 15x + 56 = 0$$

$$x = \frac{-15 \pm \sqrt{(15)^2 - 4(1)(56)}}{2(1)}$$

$$x = \frac{-15 \pm \sqrt{225 - 224}}{2}$$

$$x = \frac{-15 \pm 1}{2}$$

$$x_1 = \frac{-15 + 1}{2} = \frac{-14}{2} = -7$$

$$x_2 = \frac{-15 - 1}{2} = \frac{-16}{2} = -8$$

$$12. 32x^2 + 18x - 17 = 0$$

$$x = \frac{-18 \pm \sqrt{(18)^2 - 4(32)(-17)}}{2(32)}$$

$$x = \frac{-18 \pm \sqrt{324 + 2.176}}{64}$$

$$x = \frac{-18 \pm \sqrt{2.500}}{64}$$

$$x = \frac{-18 \pm 50}{64}$$

$$x_1 = \frac{-18 + 50}{64} = \frac{32}{64} = \frac{1}{2}$$

$$x_2 = \frac{-18 - 50}{64} = \frac{-68}{64} = -1\frac{1}{16}$$

$$13. 176x = 121 + 64x^2$$

$$64x^2 - 176x + 121 = 0$$

$$x = \frac{176 \pm \sqrt{(176)^2 - 4(64)(121)}}{2(64)}$$

$$= \frac{176 \pm \sqrt{30.976 - 30.976}}{128}$$

$$x_1 = x_2 = \frac{176}{128} = 1\frac{3}{8}$$

$$14. 8x + 5 = 36x^2$$

$$-36x^2 + 8x + 5 = 0$$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(-36)(5)}}{2(-36)}$$

$$x = \frac{-8 \pm \sqrt{64 + 720}}{-72}$$

$$x = \frac{-8 \pm \sqrt{784}}{-72}$$

$$x = \frac{-8 \pm 28}{-72}$$

$$x_1 = \frac{-8 + 28}{-72} = -\frac{20}{72} = -\frac{5}{18}$$

$$x_2 = \frac{-8 - 28}{-72} = \frac{-36}{-72} = \frac{1}{2}$$

$$15. 27x^2 + 12x - 7 = 0$$

$$x = \frac{-12 \pm \sqrt{(12)^2 - 4(27)(-7)}}{2(27)}$$

$$x = \frac{-12 \pm \sqrt{144 + 756}}{54}$$

$$x = \frac{-12 \pm \sqrt{900}}{54}$$

$$x_1 = \frac{-12 + 30}{54} = \frac{18}{54} = \frac{1}{3}$$

$$x_2 = \frac{-12 - 30}{54} = \frac{-42}{54} = -\frac{7}{9}$$

$$\begin{aligned}
 16. \quad & 15x = 25x^2 + 2 \\
 & 25x^2 - 15x + 2 = 0 \\
 & x = \frac{15 \pm \sqrt{(15)^2 - 4(25)(2)}}{2(25)} \\
 & x = \frac{15 \pm \sqrt{225 - 200}}{50} \\
 & x = \frac{15 \pm \sqrt{25}}{50} \\
 & x_1 = \frac{15+5}{50} = \frac{20}{50} = \frac{2}{5} \\
 & x_2 = \frac{15-5}{50} = \frac{10}{50} = \frac{1}{5}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & 8x^2 - 2x - 3 = 0 \\
 & x = \frac{2 \pm \sqrt{(2)^2 - 4(8)(-3)}}{2(8)} \\
 & x = \frac{2 \pm \sqrt{4+96}}{16} \\
 & x = \frac{2 \pm \sqrt{100}}{16} \\
 & x = \frac{2 \pm 10}{16} \\
 & x_1 = \frac{2+10}{16} = \frac{12}{16} = \frac{3}{4} \\
 & x_2 = \frac{2-10}{16} = \frac{-8}{16} = -\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & 105 = 2x^2 + x \\
 & 2x^2 + x - 105 = 0 \\
 & x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-105)}}{2(2)} \\
 & x = \frac{-1 \pm \sqrt{1+840}}{4} \\
 & x = \frac{-1 \pm \sqrt{841}}{4} \\
 & x = \frac{-1 \pm 29}{4} \\
 & x_1 = \frac{-1+29}{4} = \frac{28}{4} = 7 \\
 & x_2 = \frac{-1-29}{4} = \frac{-30}{4} = -7\frac{1}{2}
 \end{aligned}$$

## EJERCICIO 266

$$\begin{aligned}
 1. \quad & x(x+3) = 5x+3 \\
 & x^2 + 3x = 5x+3 \\
 & x^2 + 3x - 5x - 3 = 0 \\
 & x^2 - 2x - 3 = 0 \\
 & x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)} \\
 & x = \frac{2 \pm \sqrt{4+12}}{2} \\
 & x = \frac{2 \pm 4}{2} \\
 & x_1 = \frac{2+4}{2} = \frac{6}{2} = 3 \\
 & x_2 = \frac{2-4}{2} = \frac{-2}{2} = -1
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & 3(3x-2) = (x+4)(4-x) \\
 & 9x-6 = -x^2+16 \\
 & x^2+9x-22=0 \\
 & x = \frac{-9 \pm \sqrt{(9)^2 - 4(1)(-22)}}{2(1)} \\
 & x = \frac{9 \pm \sqrt{169}}{2} \\
 & x_1 = \frac{-9+13}{2} = \frac{4}{2} = 2 \\
 & x_2 = \frac{-9-13}{2} = \frac{-22}{2} = -11
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & 9x+1 = 3(x^2-5) - (x-3)(x+2) \\
 & 9x+1 = 3x^2-15-x^2+x+6 \\
 & 9x+1 = 2x^2+x-9 \\
 & 0 = 2x^2-8x-10 \\
 & x = \frac{8 \pm \sqrt{(-8)^2 - 4(2)(-10)}}{2(2)} \\
 & x = \frac{8 \pm \sqrt{64+80}}{4} \\
 & x = \frac{8 \pm 12}{4} \\
 & x_1 = \frac{8+12}{4} = \frac{20}{4} = 5 \\
 & x_2 = \frac{8-12}{4} = \frac{-4}{4} = -1
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & (2x-3)^2 - (x+5)^2 = -23 \\
 & 4x^2 - 12x + 9 - x^2 - 10x - 25 + 23 = 0 \\
 & 3x^2 - 22x + 7 = 0 \\
 & x = \frac{22 \pm \sqrt{(22)^2 - 4(3)(7)}}{2(3)} \\
 & x = \frac{22 \pm \sqrt{400}}{6} \\
 & x = \frac{22 \pm 20}{6} \\
 & x_1 = \frac{22+20}{6} = \frac{42}{6} = 7 \\
 & x_2 = \frac{22-20}{6} = \frac{2}{6} = \frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & 25(x+2)^2 = (x-7)^2 - 81 \\
 & 25x^2 + 100x + 100 = x^2 - 14x + 49 - 81 \\
 & 24x^2 + 114x + 132 = 0 \\
 & 4x^2 + 19x + 22 = 0 \\
 & x = \frac{-19 \pm \sqrt{(19)^2 - 4(4)(22)}}{2(4)} \\
 & x = \frac{-19 \pm \sqrt{361-352}}{8} \\
 & x = \frac{-19 \pm 3}{8} \\
 & x_1 = \frac{-19+3}{8} = \frac{-16}{8} = -2 \\
 & x_2 = \frac{-19-3}{8} = \frac{-22}{8} = -2\frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & 3x(x-2) - (x-6) = 23(x-3) \\
 & 3x^2 - 6x - x + 6 = 23x - 69 \\
 & 3x^2 - 30x + 75 = 0 \\
 & x^2 - 10x + 25 = 0 \\
 & x = \frac{10 \pm \sqrt{(-10)^2 - 4(1)(25)}}{2(1)} \\
 & x = \frac{10 \pm \sqrt{100-100}}{2} \\
 & x_1 = x_2 = \frac{10}{2} = 5
 \end{aligned}$$

$$7. 7(x-3) - 5(x^2-1) = x^2 - 5x(x+2)$$

$$7x - 21 - 5x^2 + 5 = x^2 - 5x^2 - 10x$$

$$7x - 16 - 5x^2 = -4x^2 - 10x$$

$$-x^2 + 17x - 16 = 0$$

$$x = \frac{-17 \pm \sqrt{(17)^2 - 4(-1)(-16)}}{2(-1)}$$

$$x = \frac{-17 \pm \sqrt{225}}{-2}$$

$$x = \frac{-17 \pm 15}{-2}$$

$$x_1 = \frac{-17 + 15}{-2} = \frac{-2}{-2} = 1$$

$$x_2 = \frac{-17 - 15}{-2} = \frac{-32}{-2} = 16$$

$$8. (x-5)^2 - (x-6)^2 = (2x-3)^2 - 118$$

$$x^2 - 10x + 25 - x^2 + 12x - 36 = 4x^2 - 12x + 9 - 118$$

$$2x - 11 = 4x^2 - 12x - 109$$

$$4x^2 - 14x - 98 = 0$$

$$2x^2 - 7x - 49 = 0$$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(2)(-49)}}{2(2)}$$

$$x = \frac{7 \pm \sqrt{441}}{4}$$

$$x_1 = \frac{7 + 21}{4} = \frac{28}{4} = 7$$

$$x_2 = \frac{7 - 21}{4} = \frac{-14}{4} = -3\frac{1}{2}$$

$$9. (5x-2)^2 - (3x+1)^2 - x^2 - 60 = 0$$

$$25x^2 - 20x + 4 - 9x^2 - 6x - 1 - x^2 - 60 = 0$$

$$15x^2 - 26x - 57 = 0$$

$$x = \frac{26 \pm \sqrt{(-26)^2 - 4(15)(-57)}}{2(15)}$$

$$x = \frac{26 \pm \sqrt{676 + 3.420}}{30}$$

$$x = \frac{26 \pm 64}{30}$$

$$x_1 = \frac{26 + 64}{30} = \frac{90}{30} = 3$$

$$x_2 = \frac{26 - 64}{30} = \frac{-38}{30} = -1\frac{4}{15}$$

$$10. (x+4)^3 - (x-3)^3 = 343$$

$$x^3 + 12x^2 + 48x + 64 - x^3 + 9x^2 - 27x + 27 - 343 = 0$$

$$21x^2 + 21x - 252 = 0$$

$$-x^2 - x + 12 = 0$$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(-1)(12)}}{2(-1)}$$

$$x = \frac{1 \pm \sqrt{49}}{-2}$$

$$x = \frac{1 \pm 7}{-2}$$

$$x_1 = \frac{1 + 7}{-2} = \frac{8}{-2} = -4$$

$$x_2 = \frac{1 - 7}{-2} = \frac{-6}{-2} = 3$$

$$11. (x+2)^3 - (x-1)^3 = x(3x+4) + 8$$

$$x^3 + 6x^2 + 12x + 8 - x^3 + 3x^2 - 3x + 1 = 3x^2 + 4x + 8$$

$$6x^2 + 5x + 1 = 0$$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(6)(1)}}{2(6)}$$

$$x = \frac{-5 \pm \sqrt{25 - 24}}{12}$$

$$x_1 = \frac{-5 + 1}{12} = \frac{-4}{12} = -\frac{1}{3}$$

$$x_2 = \frac{-5 - 1}{12} = \frac{-6}{12} = -\frac{1}{2}$$

$$12. (5x-4)^2 - (3x+5)(2x-1) = 20x(x-2) + 27$$

$$25x^2 - 40x + 16 - 6x^2 - 7x + 5 = 20x^2 - 40x + 27$$

$$19x^2 - 47x + 21 = 20x^2 - 40x + 27$$

$$-x^2 - 7x - 6 = 0$$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(-1)(-6)}}{2(-1)}$$

$$x = \frac{7 \pm \sqrt{49 - 24}}{-2}$$

$$x = \frac{7 \pm \sqrt{25}}{-2}$$

$$x_1 = \frac{7 + 5}{-2} = \frac{12}{-2} = -6$$

$$x_2 = \frac{7 - 5}{-2} = \frac{2}{-2} = -1$$



## EJERCICIO 267

1.  $x^2 - 3x + 2 = 0$

$$x = -\frac{(-3)}{2} \pm \sqrt{\frac{(-3)^2}{4} - 2}$$

$$x = \frac{3}{2} \pm \sqrt{\frac{9}{4} - 2}$$

$$x = \frac{3}{2} \pm \sqrt{\frac{1}{4}}$$

$$x_1 = \frac{3}{2} + \frac{1}{2} = 2$$

$$x_2 = \frac{3}{2} - \frac{1}{2} = 1$$

2.  $x^2 - 2x - 15 = 0$

$$x = -\frac{(-2)}{2} \pm \sqrt{\frac{(-2)^2}{4} - (-15)}$$

$$x = 1 \pm \sqrt{1 + 15}$$

$$x = 1 \pm 4$$

$$x_1 = 1 + 4 = 5$$

$$x_2 = 1 - 4 = -3$$

3.  $x^2 = 19x - 88$

$$x^2 - 19x + 88 = 0$$

$$x = -\frac{(-19)}{2} \pm \sqrt{\frac{(-19)^2}{4} - 88}$$

$$x = \frac{19}{2} \pm \sqrt{\frac{361}{4} - 88}$$

$$x = \frac{19}{2} \pm \sqrt{\frac{9}{4}}$$

$$x_1 = \frac{19}{2} + \frac{3}{2} = \frac{22}{2} = 11$$

$$x_2 = \frac{19}{2} - \frac{3}{2} = \frac{16}{2} = 8$$

4.  $x^2 + 4x = 285$

$$x^2 + 4x - 285 = 0$$

$$x = -\frac{4}{2} \pm \sqrt{\frac{(4)^2}{4} - (-285)}$$

$$x = -2 \pm \sqrt{4 + 285}$$

$$x = -2 \pm \sqrt{289}$$

$$x_1 = -2 + 17 = 15$$

$$x_2 = -2 - 17 = -19$$

5.  $5x(x-1) - 2(2x^2 - 7x) = -8$

$$5x^2 - 5x - 4x^2 + 14x + 8 = 0$$

$$x^2 + 9x + 8 = 0$$

$$x = -\frac{9}{2} \pm \sqrt{\frac{(9)^2}{4} - 8}$$

$$x = -\frac{9}{2} \pm \sqrt{\frac{81}{4} - 8}$$

$$x = -\frac{9}{2} \pm \sqrt{\frac{49}{4}}$$

$$x_1 = -\frac{9}{2} + \frac{7}{2} = \frac{-2}{2} = -1$$

$$x_2 = -\frac{9}{2} - \frac{7}{2} = \frac{-16}{2} = -8$$

6.  $x^2 - (7x + 6) = x + 59$

$$x^2 - 7x - 6 - x - 59 = 0$$

$$x^2 - 8x - 65 = 0$$

$$x = -\frac{(-8)}{2} \pm \sqrt{\frac{(-8)^2}{4} - (-65)}$$

$$x = 4 \pm \sqrt{16 + 65}$$

$$x = 4 \pm 9$$

$$x_1 = 4 + 9 = 13$$

$$x_2 = 4 - 9 = -5$$

7.  $(x-1)^2 + 11x + 199 = 3x^2 - (x-2)^2$

$$x^2 - 2x + 1 + 11x + 199 = 3x^2 - x^2 + 4x - 4$$

$$x^2 + 9x + 200 = 2x^2 + 4x - 4$$

$$x^2 - 5x - 204 = 0$$

$$x = -\frac{(-5)}{2} \pm \sqrt{\frac{(5)^2}{4} - (-204)}$$

$$x = \frac{5}{2} \pm \sqrt{\frac{25}{4} + 204}$$

$$x = \frac{5}{2} \pm \sqrt{\frac{841}{4}}$$

$$x_1 = \frac{5}{2} + \frac{29}{2} = \frac{34}{2} = 17$$

$$x_2 = \frac{5}{2} - \frac{29}{2} = \frac{-24}{2} = -12$$

8.  $(x-2)(x+2) - 7(x-1) = 21$

$$x^2 - 4 - 7x + 7 - 21 = 0$$

$$x^2 - 7x - 18 = 0$$

$$x = -\frac{(-7)}{2} \pm \sqrt{\frac{(-7)^2}{4} - (-18)}$$

$$x = \frac{7}{2} \pm \sqrt{\frac{49}{4} + 18}$$

$$x = \frac{7}{2} \pm \sqrt{\frac{121}{4}}$$

$$x_1 = \frac{7}{2} + \frac{11}{2} = \frac{18}{2} = 9$$

$$x_2 = \frac{7}{2} - \frac{11}{2} = \frac{-4}{2} = -2$$

9.  $2x^2 - (x-2)(x+5) = 7(x+3)$

$$2x^2 - x^2 - x^2 - 3x + 10 = 7x + 21$$

$$x^2 - 10x - 11 = 0$$

$$x = -\frac{(-10)}{2} \pm \sqrt{\frac{(-10)^2}{4} - (-11)}$$

$$x = 5 \pm \sqrt{25 + 11}$$

$$x = 5 \pm \sqrt{36}$$

$$x_1 = 5 + 6 = 11$$

$$x_2 = 5 - 6 = -1$$

10.  $(x-1)(x+2) - (2x-3)(x+4) - x + 14 = 0$

$$x^2 + x - 2 - 2x^2 - 5x + 12 - x + 14 = 0$$

$$-x^2 - 5x + 24 = 0$$

$$x^2 + 5x - 24 = 0$$

$$x = -\frac{5}{2} \pm \sqrt{\frac{(5)^2}{4} - (-24)}$$

$$x = -\frac{5}{2} \pm \sqrt{\frac{25}{4} + 24}$$

$$x = -\frac{5}{2} \pm \sqrt{\frac{121}{4}}$$

$$x_1 = -\frac{5}{2} + \frac{11}{2} = \frac{6}{2} = 3$$

$$x_2 = -\frac{5}{2} - \frac{11}{2} = \frac{-16}{2} = -8$$

## **EJERCICIO 268**

$$1. \frac{x^2}{5} - \frac{x}{2} = \frac{3}{10} \quad mcm=10$$

$$2x^2 - 5x = 3$$

$$2x^2 - 5x - 3 = 0$$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{5 \pm \sqrt{25+24}}{4}$$

$$x = \frac{5 \pm 7}{4}$$

$$x_1 = \frac{5+7}{4} = \frac{12}{4} = 3$$

$$x_2 = \frac{5-7}{4} = \frac{-2}{4} = -\frac{1}{2}$$

$$2. 4x - \frac{13}{x} = \frac{3}{2} \quad mcm=2x$$

$$8x^2 - 26 = 3x$$

$$8x^2 - 3x - 26 = 0$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(8)(-26)}}{2(8)}$$

$$x = \frac{3 \pm \sqrt{9+832}}{16}$$

$$x = \frac{3 \pm 29}{16}$$

$$x_1 = \frac{3+29}{16} = \frac{32}{16} = 2$$

$$x_2 = \frac{3-29}{16} = \frac{-26}{16} = -1\frac{5}{8}$$

$$3. \frac{x^2}{6} - \frac{x}{2} = 3(x-5) \quad mcm=6$$

$$x^2 - 3x = 18(x-5)$$

$$x^2 - 3x = 18x - 90$$

$$x^2 - 21x + 90 = 0$$

$$x = -\frac{(-21)}{2} \pm \sqrt{\frac{(-21)^2}{4} - 90}$$

$$x = \frac{21}{2} \pm \sqrt{\frac{81}{4}}$$

$$x_1 = \frac{21}{2} + \frac{9}{2} = \frac{30}{2} = 15$$

$$x_2 = \frac{21}{2} - \frac{9}{2} = \frac{12}{2} = 6$$

$$4. \frac{1}{4}(x-4) + \frac{2}{5}(x-5) = \frac{1}{5}(x^2 - 53)$$

$$mcm=20$$

$$5(x-4) + 8(x-5) = 4(x^2 - 53)$$

$$5x - 20 + 8x - 40 - 4x^2 + 212 = 0$$

$$-4x^2 + 13x + 152 = 0$$

$$x = \frac{-13 \pm \sqrt{(13)^2 - 4(-4)(152)}}{2(-4)}$$

$$x = \frac{-13 \pm \sqrt{169 + 2.432}}{-8}$$

$$x = \frac{-13 \pm 51}{8}$$

$$x_1 = \frac{-13+51}{8} = \frac{38}{8} = -4\frac{3}{4}$$

$$x_2 = \frac{-13-51}{-8} = \frac{-64}{-8} = 8$$

$$5. \frac{5}{x} - \frac{1}{x+2} = 1 \quad mcm=x+2$$

$$5\left(1 + \frac{2}{x}\right) - 1 = x + 2 \quad mcm=x$$

$$5x + 10 - x = x^2 + 2x$$

$$x^2 - 2x - 10 = 0$$

$$x = -\frac{(-2)}{2} \pm \sqrt{\frac{(-2)^2}{4} - (-10)}$$

$$x = 1 \pm \sqrt{1+10}$$

$$x = 1 \pm \sqrt{11}$$

$$x_1 = 1 + \sqrt{11}$$

$$x_2 = 1 - \sqrt{11}$$

$$6. \frac{15}{x} - \frac{11x+5}{x^2} = -1 \quad mcm=x^2$$

$$15x - (11x+5) = -x^2$$

$$x^2 + 4x - 5 = 0$$

$$x = -\frac{4}{2} \pm \sqrt{\frac{(4)^2}{4} - (-5)}$$

$$x = -2 \pm \sqrt{4+5}$$

$$x = -2 \pm 3$$

$$x_1 = -2 + 3 = 1$$

$$x_2 = -2 - 3 = -5$$

$$7. \frac{8x}{3x+5} + \frac{5x-1}{x+1} = 3 \quad mcm=(3x+5)(x+1)$$

$$8x(x+1) + (5x-1)(3x+5) = 3(3x+5)(x+1)$$

$$8x^2 + 8x + 15x^2 + 22x - 5 = 9x^2 + 24x + 15$$

$$23x^2 + 30x - 5 = 9x^2 + 24x + 15$$

$$14x^2 + 6x - 20 = 0$$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(14)(-20)}}{2(14)}$$

$$x = \frac{-6 \pm \sqrt{1.156}}{28} = \frac{-6 \pm 34}{28}$$

$$x_1 = \frac{-6+34}{28} = \frac{28}{28} = 1$$

$$x_2 = \frac{-6-34}{28} = \frac{-40}{28} = -1\frac{3}{7}$$

$$8. \frac{1}{x-2} - \frac{1}{x-1} = \frac{1}{6}$$

$$mcm=6(x-2)(x-1)$$

$$6(x-1) - 6(x-2) = (x-2)(x-1)$$

$$6x - 6 - 6x + 12 = x^2 - 3x + 2$$

$$6 = x^2 - 3x + 2$$

$$0 = x^2 - 3x - 4$$

$$x = -\frac{(-3)}{2} \pm \sqrt{\frac{(-3)^2}{4} - (-4)}$$

$$x = \frac{3}{2} \pm \sqrt{\frac{9}{4} + 4}$$

$$x = \frac{3}{2} \pm \sqrt{\frac{25}{4}} = \frac{3}{2} \pm \frac{5}{2}$$

$$x_1 = \frac{3}{2} + \frac{5}{2} = \frac{8}{2} = 4$$

$$x_2 = \frac{3}{2} - \frac{5}{2} = \frac{-2}{2} = -1$$

$$9. 1 - \frac{2x-3}{x+5} = \frac{x-2}{10} \quad mcm=10(x+5)$$

$$10(x+5) - 10(2x-3) = (x+5)(x-2)$$

$$10x + 50 - 20x + 30 = x^2 + 3x - 10$$

$$-10x + 80 = x^2 + 3x - 10$$

$$0 = x^2 + 13x - 90$$

$$x = -\frac{13}{2} \pm \sqrt{\frac{(13)^2}{4} - (-90)}$$

**Continúa**

## 9. Continuación

$$x = -\frac{13}{2} \pm \sqrt{\frac{169}{4} + 90}$$

$$x = -\frac{13}{2} \pm \sqrt{\frac{529}{4}}$$

$$x = \frac{-13}{2} \pm \frac{23}{2}$$

$$x_1 = \frac{-13}{2} + \frac{23}{2} = \frac{10}{2} = 5$$

$$x_2 = \frac{-13}{2} - \frac{23}{2} = \frac{-36}{2} = -18$$

$$10. \frac{x-13}{x} = 5 - \frac{10(5x+3)}{x^2} \quad mcm = x^2$$

$$x(x-13) = 5x^2 - 10(5x+3)$$

$$x^2 - 13x = 5x^2 - 50x - 30$$

$$-4x^2 + 37x + 30 = 0$$

$$x = \frac{-37 \pm \sqrt{(37)^2 - 4(-4)(30)}}{2(-4)}$$

$$x = \frac{-37 \pm \sqrt{1.369 + 480}}{-8}$$

$$x = \frac{37 \pm \sqrt{1.849}}{-8}$$

$$x = \frac{-37 \pm 43}{-8}$$

$$x_1 = \frac{-37 + 43}{-8} = \frac{6}{-8} = -\frac{3}{4}$$

$$x_2 = \frac{-37 - 43}{-8} = \frac{-80}{-8} = 10$$

$$11. \frac{x}{x-2} - \frac{x-2}{x} = \frac{5}{2} \quad mcm = 2x(x-2)$$

$$2x(x) - 2(x-2)(x-2) = 5x(x-2)$$

$$2x^2 - 2(x^2 - 4x + 4) = 5x^2 - 10x$$

$$2x^2 - 2x^2 + 8x - 8 = 5x^2 - 10x$$

$$0 = 5x^2 - 18x + 8$$

$$x = \frac{-(-18) \pm \sqrt{(-18)^2 - 4(5)(8)}}{2(5)}$$

$$x = \frac{18 \pm \sqrt{324 - 160}}{10}$$

$$x = \frac{18 \pm \sqrt{164}}{10}$$

$$x = \frac{18 \pm \sqrt{2^2 \cdot 41}}{10} = \frac{18 \pm 2\sqrt{41}}{10} = \frac{9 \pm \sqrt{41}}{5}$$

$$x_1 = \frac{9 + \sqrt{41}}{5} ; \quad x_2 = \frac{9 - \sqrt{41}}{5}$$

$$12. \frac{4x^2}{x-1} - \frac{1-3x}{4} = \frac{20x}{3} \quad mcm = 12(x-1)$$

$$12(4x^2) - 3(x-1)(1-3x) = 4(x-1)(20x)$$

$$48x^2 + 9x^2 - 12x + 3 = 80x^2 - 80x$$

$$-23x^2 + 68x + 3 = 0$$

$$x = \frac{-68 \pm \sqrt{(68)^2 - 4(-23)(3)}}{2(-23)}$$

$$x = \frac{-68 \pm \sqrt{4.624 + 276}}{-46} = \frac{-68 \pm \sqrt{4.900}}{-46} = \frac{-68 \pm 70}{-46}$$

$$x_1 = \frac{-68 + 70}{-46} = \frac{2}{-46} = -\frac{1}{23} ; \quad x_2 = \frac{-68 - 70}{-46} = \frac{-138}{-46} = 3$$

$$13. \frac{3x-1}{x} - \frac{2x}{2x-1} - \frac{7}{6} = 0 \quad mcm = 6x(2x-1)$$

$$6(2x-1)(3x-1) - 6x(2x) - 7x(2x-1) = 0$$

$$6(6x^2 - 5x + 1) - 12x^2 - 14x^2 + 7x = 0$$

$$36x^2 - 30x + 6 - 12x^2 - 14x^2 + 7x = 0$$

$$10x^2 - 23x + 6 = 0$$

$$x = \frac{23 \pm \sqrt{(23)^2 - 4(10)(6)}}{2(10)}$$

$$x = \frac{23 \pm \sqrt{529 - 240}}{20} = \frac{23 \pm \sqrt{289}}{20} = \frac{23 \pm 17}{20}$$

$$x_1 = \frac{23 + 17}{20} = \frac{40}{20} = 2 ; \quad x_2 = \frac{23 - 17}{20} = \frac{6}{20} = \frac{3}{10}$$

$$14. \frac{5x-8}{x-1} = \frac{7x-4}{x+2}$$

$$(x+2)(5x-8) = (7x-4)(x-1)$$

$$5x^2 + 2x - 16 = 7x^2 - 11x + 4$$

$$0 = 2x^2 - 13x + 20$$

$$x = \frac{13 \pm \sqrt{(13)^2 - 4(2)(20)}}{2(2)}$$

$$x = \frac{13 \pm \sqrt{169 - 160}}{4} = \frac{13 \pm \sqrt{9}}{4} = \frac{13 \pm 3}{4}$$

$$x_1 = \frac{13 + 3}{4} = \frac{16}{4} = 4 ; \quad x_2 = \frac{13 - 3}{4} = \frac{10}{4} = \frac{5}{2} = 2\frac{1}{2}$$

$$15. \frac{x+3}{2x-1} - \frac{5x-1}{4x+7} = 0$$

$$mcm = (2x-1)(4x+7)$$

$$(x+3)(4x+7) - (5x-1)(2x-1) = 0$$

$$4x^2 + 19x + 21 - 10x^2 + 7x - 1 = 0$$

$$-6x^2 + 26x + 20 = 0$$

$$3x^2 - 13x - 10 = 0$$

$$x = \frac{13 \pm \sqrt{(-13)^2 - 4(3)(-10)}}{2(3)}$$

$$x = \frac{13 \pm \sqrt{169 + 120}}{6} = \frac{13 \pm \sqrt{289}}{6} = \frac{13 \pm 17}{6}$$

$$x_1 = \frac{13+17}{6} = \frac{30}{6} = 5 ; x_2 = \frac{13-17}{6} = \frac{-4}{6} = -\frac{2}{3}$$

$$18. \frac{5}{x^2-1} - \frac{6}{x+1} = 3\frac{5}{8} \quad mcm = 8(x^2-1)$$

$$40 - 48(x-1) = 29(x^2-1)$$

$$40 - 48x + 48 = 29x^2 - 29$$

$$-29x^2 - 48x + 117 = 0$$

$$x = \frac{-(-48) \pm \sqrt{(-48)^2 - 4(-29)(117)}}{2(-29)}$$

$$x = \frac{48 \pm \sqrt{2.304 + 13.572}}{-58} = \frac{48 \pm 126}{-58}$$

$$x_1 = \frac{48+126}{-58} = \frac{174}{-58} = -3 ; x_2 = \frac{48-126}{-58} = \frac{-78}{-58} = 1\frac{10}{29}$$

$$16. \frac{1}{4-x} - \frac{1}{6} = \frac{1}{x+1}$$

$$mcm = 6(4-x)(x+1)$$

$$6(x+1) - (4-x)(x+1) = 6(4-x)$$

$$6x + 6 + x^2 - 3x - 4 = 24 - 6x$$

$$x^2 + 3x + 2 = 24 - 6x$$

$$x^2 + 9x - 22 = 0$$

$$x = -\frac{9}{2} \pm \sqrt{\frac{(9)^2}{4} - (-22)}$$

$$x = -\frac{9}{2} \pm \sqrt{\frac{81}{4} + 22} = -\frac{9}{2} \pm \sqrt{\frac{169}{4}} = -\frac{9}{2} \pm \frac{13}{2}$$

$$x_1 = \frac{-9}{2} + \frac{13}{2} = \frac{4}{2} = 2 ; x_2 = \frac{-9}{2} - \frac{13}{2} = \frac{-22}{2} = -11$$

$$19. \frac{x-1}{x+1} + \frac{x+1}{x-1} = \frac{2x+9}{x+3} \quad mcm = (x+1)(x-1)(x+3)$$

$$(x-1)(x-1)(x+3) + (x+1)(x+1)(x+3) = (2x+9)(x+1)(x-1)$$

$$(x^2 - 2x + 1)(x+3) + (x^2 + 2x + 1)(x+3) = (2x+9)(x^2 - 1)$$

$$x^3 + 3x^2 - 2x^2 - 6x + x + 3 + x^3 + 3x^2 + 2x^2 + 6x + x + 3 = 2x^3 - 2x + 9x^2 - 9$$

$$2x^3 + 6x^2 + 2x + 6 = 2x^3 + 9x^2 - 2x - 9$$

$$0 = 3x^2 - 4x - 15$$

$$x = \frac{4 \pm \sqrt{(4)^2 - 4(3)(-15)}}{2(3)}$$

$$x = \frac{4 \pm \sqrt{16 + 180}}{6} = \frac{4 \pm \sqrt{196}}{6} = \frac{4 \pm 14}{6}$$

$$x_1 = \frac{4+14}{6} = \frac{18}{6} = 3 ; x_2 = \frac{4-14}{6} = \frac{-10}{6} = -1\frac{2}{3}$$

$$17. \frac{x+4}{x+5} - \frac{x+2}{x+3} = \frac{1}{24} \quad mcm = 24(x+5)(x+3)$$

$$24(x+3)(x+4) - 24(x+2)(x+5) = (x+5)(x+3)$$

$$24x^2 + 168x + 288 - 24x^2 - 168x - 240 = x^2 + 8x + 15$$

$$48 = x^2 + 8x + 15$$

$$0 = x^2 + 8x - 33$$

$$x = -\frac{8}{2} \pm \sqrt{\frac{(8)^2}{4} - (-33)}$$

$$x = -4 \pm \sqrt{16 + 33} = -4 \pm \sqrt{49} = -4 \pm 7$$

$$x_1 = -4 + 7 = 3 ; x_2 = -4 - 7 = -11$$

$$20. \frac{3}{x+2} - \frac{1}{x-2} = \frac{1}{x+1} \quad mcm = (x+2)(x-2)(x+1)$$

$$3(x-2)(x+1) - (x+2)(x+1) = (x+2)(x-2)$$

$$3x^2 - 3x - 6 - x^2 - 3x - 2 = x^2 - 4$$

$$2x^2 - 6x - 8 = x^2 - 4$$

$$x^2 - 6x - 4 = 0$$

$$x = -\frac{(-6)}{2} \pm \sqrt{\frac{(-6)^2}{4} - (-4)} = 3 \pm \sqrt{9 + 4} = 3 \pm \sqrt{13}$$

$$x_1 = 3 + \sqrt{13} ; x_2 = 3 - \sqrt{13}$$

## EJERCICIO 269

1.  $x^2 - x - 6 = 0$

$$(x-3)(x+2) = 0$$

$$x_1 = 3 \quad x_2 = -2$$

2.  $x^2 + 7x = 18$

$$x^2 + 7x - 18 = 0$$

$$(x+9)(x-2) = 0$$

$$x_1 = -9 \quad x_2 = 2$$

3.  $8x - 65 = -x^2$

$$x^2 + 8x - 65 = 0$$

$$(x+13)(x-5) = 0$$

$$x_1 = -13 \quad x_2 = 5$$

4.  $x^2 = 108 - 3x$

$$x^2 + 3x - 108 = 0$$

$$108 \quad 2$$

$$54 \quad 2 \quad 2^2 \cdot 3 - 3^2$$

$$27 \quad 3 \quad 12 - 9 = 3$$

$$9 \quad 3$$

$$3 \quad 3$$

$$1$$

$$(x+12)(x-9) = 0$$

$$x_1 = -12 \quad x_2 = 9$$

5.  $2x^2 + 7x - 4 = 0$

$$(2x)^2 + 7(2x) - 8 = 0$$

$$\frac{(2x+8)(2x-1)}{2} = 0$$

$$(x+4)(2x-1) = 0$$

$$x+4=0 \quad 2x-1=0$$

$$x_1 = -4 \quad x_2 = \frac{1}{2}$$

6.  $6x^2 = 10 - 11x$

$$6x^2 + 11x - 10 = 0$$

$$(6x)^2 + 11(6x) - 60 = 0$$

$$\frac{(6x+15)(6x-4)}{3 \cdot 2} = 0$$

$$(2x+5)(3x-2) = 0$$

$$2x+5=0 \quad 3x-2=0$$

$$x_1 = -\frac{5}{2} = -2\frac{1}{2} \quad x_2 = \frac{2}{3}$$

7.  $20x^2 - 27x = 14$

$$20x^2 - 27x - 14 = 0$$

$$(20x)^2 - 27(20x) - 280 = 0$$

$$\frac{(20x-35)(20x+8)}{5 \cdot 4} = 0$$

$$(4x-7)(5x+2) = 0$$

$$x_1 = \frac{7}{4} = 1\frac{3}{4} \quad x_2 = -\frac{2}{5}$$

8.  $7x = 15 - 30x^2$

$$30x^2 + 7x - 15 = 0$$

$$(30x)^2 + 7(30x) - 450 = 0$$

$$\frac{(30x+25)(30x-18)}{5 \cdot 6} = 0$$

$$(6x+5)(5x-3) = 0$$

$$6x+5=0 \quad 5x-3=0$$

$$6x=-5 \quad 5x=3$$

$$x_1 = \frac{-5}{6} \quad x_2 = \frac{3}{5}$$

9.  $60 = 8x^2 + 157x$

$$8x^2 + 157x - 60 = 0$$

$$(8x)^2 + 157(8x) - 480 = 0$$

$$\frac{(8x+160)(8x-3)}{8} = 0$$

$$(x+20)(8x-3) = 0$$

$$x+20=0 \quad 8x-3=0$$

$$x_1 = -20 \quad 8x=3$$

$$x_2 = \frac{3}{8}$$

10.  $x(x-1) - 5(x-2) = 2$

$$x^2 - x - 5x + 10 - 2 = 0$$

$$x^2 - 6x + 8 = 0$$

$$(x-4)(x-2) = 0$$

$$x-4=0 \quad x-2=0$$

$$x_1 = 4 \quad x_2 = 2$$

11.  $(x-2)^2 - (2x+3)^2 = -80$

$$x^2 - 4x + 4 - 4x^2 - 12x - 9 + 80 = 0$$

$$-3x^2 - 16x + 75 = 0$$

$$3x^2 + 16x - 75 = 0$$

$$(3x)^2 + 16(3x) - 225 = 0$$

$$\frac{(3x+25)(3x-9)}{3} = 0$$

Continúa

## 11. Continuación

$$(3x+25)(x-3) = 0$$

$$3x+25=0 \quad x-3=0$$

$$3x=-25 \quad x_2=3$$

$$x_1 = -\frac{25}{3} = -8\frac{1}{3}$$

12.  $\frac{6}{x^2} - \frac{9}{x} = -\frac{4}{3} \quad mcm = 3x^2$

$$18 - 27x = -4x^2$$

$$4x^2 - 27x + 18 = 0$$

$$(4x)^2 - 27(4x) + 72 = 0$$

$$\frac{(4x-24)(4x-3)}{4} = 0$$

$$(x-6)(4x-3) = 0$$

$$x-6=0 \quad 4x-3=0$$

$$x_1 = 6 \quad 4x=3$$

$$x_2 = \frac{3}{4}$$

13.  $\frac{x+2}{x} + x = \frac{74}{x} \quad mcm = x$

$$x+2+x^2 = 74$$

$$x^2 + x - 72 = 0$$

$$(x+9)(x-8) = 0$$

$$x+9=0 \quad x-8=0$$

$$x_1 = -9 \quad x_2 = 8$$

14.  $(x+2)^2 - \frac{2x-5}{3} = 3 \quad mcm = 3$

$$3(x+2)^2 - (2x-5) = 9$$

$$3x^2 + 12x + 12 - 2x + 5 - 9 = 0$$

$$3x^2 + 10x + 8 = 0$$

$$(3x)^2 + 10(3x) + 24 = 0$$

$$\frac{(3x+6)(3x+4)}{3} = 0$$

$$(x+2)(3x+4) = 0$$

$$x+2=0 \quad 3x+4=0$$

$$x_1 = -2 \quad 3x=-4$$

$$x_2 = \frac{-4}{3} = -1\frac{1}{3}$$

15.  $\frac{x}{x-2} + x = \frac{3x+15}{4} \quad mcm = 4(x-2)$

$$4x + 4x(x-2) = (3x+15)(x-2)$$

$$4x + 4x^2 - 8x = 3x^2 + 9x - 30$$

$$x^2 - 13x + 30 = 0$$

$$(x-10)(x-3) = 0$$

$$x-10=0 \quad x-3=0$$

$$x_1 = 10 \quad x_2 = 3$$

$$16. \frac{6}{x-4} - \frac{4}{x} = \frac{5}{12} \quad mcm = 12x(x-4)$$

$$72x - 48(x-4) = 5x(x-4)$$

$$72x - 48x + 192 = 5x^2 - 20x$$

$$5x^2 - 44x - 192 = 0$$

$$(5x)^2 - 44(5x) - 960 = 0$$

$$\frac{(5x-60)(5x+16)}{5} = 0$$

$$(x-12)(5x+16) = 0$$

$$x-12=0 \quad 5x+16=0$$

$$x_1 = 12 \quad 5x = -16$$

$$x_2 = \frac{-16}{5} = -3\frac{1}{5}$$

$$17. (x-2)^3 - (x-3)^3 = 37$$

$$x^3 - 6x^2 + 12x - 8 - x^3 + 9x^2 - 27x + 27 - 37 = 0$$

$$3x^2 - 15x - 18 = 0$$

$$(3x)^2 - 15(3x) - 54 = 0$$

$$(3x-18)(3x+3) = 0$$

$$3x-18=0$$

$$3x+3=0$$

$$3x=18$$

$$3x=-3$$

$$x_1 = 6$$

$$x_2 = -1$$

$$18. \frac{x-1}{x+1} - 2 = \frac{x+3}{3} \quad mcm = 3(x+1)$$

$$3(x-1) - 6(x+1) = (x+3)(x+1)$$

$$3x - 3 - 6x - 6 = x^2 + 4x + 3$$

$$x^2 + 7x + 12 = 0$$

$$(x+4)(x+3) = 0$$

$$x+4=0 \quad x+3=0$$

$$x_1 = -4 \quad x_2 = -3$$

$$19. \frac{4x-1}{2x+3} = \frac{2x+1}{6x+5}$$

$$(6x+5)(4x-1) = (2x+1)(2x+3)$$

$$24x^2 + 14x - 5 = 4x^2 + 8x + 3$$

$$20x^2 + 6x - 8 = 0$$

$$10x^2 + 3x - 4 = 0$$

$$(10x)^2 + 3(10x) - 40 = 0$$

$$\frac{(10x+8)(10x-5)}{2 \cdot 5} = 0$$

$$(5x+4)(2x-1) = 0$$

$$5x+4=0 \quad 2x-1=0$$

$$5x=-4 \quad 2x=1$$

$$x_1 = \frac{-4}{5} \quad x_2 = \frac{1}{2}$$

$$20. \frac{3x+2}{4} = 5 - \frac{9x+14}{12x} \quad mcm = 12x$$

$$3x(3x+2) = 60x - (9x+14)$$

$$9x^2 + 6x = 60x - 9x - 14$$

$$9x^2 - 45x + 14 = 0$$

$$(9x)^2 - 45(9x) + 126 = 0$$

$$\frac{(9x-42)(9x-3)}{3 \cdot 3} = 0$$

$$(3x-14)(3x-1) = 0$$

$$3x-14=0 \quad 3x-1=0$$

$$3x=14 \quad 3x=1$$

$$x_1 = \frac{14}{3} \quad x_2 = \frac{1}{3}$$

$$x_1 = 4\frac{2}{3}$$

## EJERCICIO 270

$$1. x^2 + 2ax - 35a^2 = 0$$

$$(x+7a)(x-5a) = 0$$

$$x+7a=0 \quad x-5a=0$$

$$x_1 = -7a \quad x_2 = 5a$$

$$2. 10x^2 = 36a^2 - 37ax$$

$$10x^2 + 37ax - 36a^2 = 0$$

$$x = \frac{-37a \pm \sqrt{(37a)^2 - 4(10)(-36a^2)}}{2(10)}$$

$$x = \frac{-37a \pm \sqrt{2.809a^2}}{20} = \frac{-37a \pm 53a}{20}$$

$$x_1 = \frac{-37a + 53a}{20} = \frac{16a}{20} = \frac{4a}{5}$$

$$x_2 = \frac{-37a - 53a}{20} = \frac{-90a}{20} = \frac{-9a}{2}$$

$$3. a^2x^2 + abx - 2b^2 = 0$$

$$(a^2x)^2 + ab(a^2x) - 2a^2b^2 = 0$$

$$(a^2x + 2ab)(a^2x - ab) = 0$$

$$a^2x + 2ab = 0 \quad a^2x - ab = 0$$

$$a^2x = -2ab \quad a^2x = ab$$

$$x_1 = \frac{-2ab}{a^2} \quad x_2 = \frac{ab}{a^2}$$

$$x_1 = -\frac{2b}{a} \quad x_2 = \frac{b}{a}$$

$$4. \quad 89bx = 42x^2 + 22b^2$$

$$42x^2 - 89bx + 22b^2 = 0$$

$$x = \frac{-(-89b) \pm \sqrt{(-89b)^2 - 4(42)(22b^2)}}{2(42)}$$

$$x = \frac{89 \pm \sqrt{4 \cdot 225b^2}}{84} = \frac{89b \pm 65b}{84}$$

$$x_1 = \frac{89b - 65b}{84} = \frac{24b}{84} = \frac{2b}{7}$$

$$x_2 = \frac{89b + 65b}{84} = \frac{154b}{84} = \frac{11b}{6}$$

$$5. \quad x^2 + ax = 20a^2$$

$$x^2 + ax - 20a^2 = 0$$

$$(x + 5a)(x - 4a) = 0$$

$$x + 5a = 0 \quad x - 4a = 0$$

$$x_1 = -5a \quad x_2 = 4a$$

$$6. \quad 2x^2 = abx + 3a^2b^2$$

$$2x^2 - abx - 3a^2b^2 = 0$$

$$(2x)^2 - ab(2x) - 6a^2b^2 = 0$$

$$\frac{(2x - 3ab)(2x + 2ab)}{2} = 0$$

$$(2x - 3ab)(x + ab) = 0$$

$$2x - 3ab = 0 \quad x + ab = 0$$

$$2x = 3ab \quad x = -ab$$

$$x_1 = \frac{3ab}{2} \quad x_2 = -ab$$

$$7. \quad b^2x^2 + 2abx = 3a^2$$

$$b^2x^2 + 2abx - 3a^2 = 0$$

$$(b^2x)^2 + 2ab(b^2x) - 3a^2b^2 = 0$$

$$\frac{(b^2x + 3ab)(b^2x - ab)}{b \cdot b} = 0$$

$$(bx + 3a)(bx - a) = 0$$

$$bx + 3a = 0 \quad bx - a = 0$$

$$bx = -3a \quad bx = a$$

$$x_1 = -\frac{3a}{b} \quad x_2 = \frac{a}{b}$$

$$8. \quad x^2 + ax - bx = ab$$

$$x^2 + ax - bx - ab = 0$$

$$x(x + a) - b(x + a) = 0$$

$$(x - b)(x + a) = 0$$

$$x - b = 0 \quad x + a = 0$$

$$x_1 = b \quad x_2 = -a$$

$$9. \quad x^2 - 2ax = 6ab - 3bx$$

$$x^2 + 3bx - 2ax - 6ab = 0$$

$$x(x + 3b) - 2a(x + 3b) = 0$$

$$(x + 3b)(x - 2a) = 0$$

$$x + 3b = 0 \quad x - 2a = 0$$

$$x_1 = -3b \quad x_2 = 2a$$

$$10. \quad 3(2x^2 - mx) + 4nx - 2mn = 0$$

$$3x(2x - m) + 2n(2x - m) = 0$$

$$(2x - m)(3x + 2n) = 0$$

$$2x - m = 0 \quad 3x + 2n = 0$$

$$2x = m \quad 3x = -2n$$

$$x_1 = \frac{m}{2} \quad x_2 = -\frac{2n}{3}$$

$$11. \quad x^2 - a^2 - bx - ab = 0$$

$$(x^2 - a^2) - b(x + a) = 0$$

$$(x + a)(x - a) - b(x + a) = 0$$

$$(x - a - b)(x + a) = 0$$

$$x - a - b = 0 \quad x + a = 0$$

$$x_1 = a + b \quad x_2 = -a$$

$$12. \quad abx^2 - x(b - 2a) = 2$$

$$abx^2 - bx + 2ax - 2 = 0$$

$$abx^2 + 2ax - bx - 2 = 0$$

$$ax(bx + 2) - (bx + 2) = 0$$

$$(bx + 2)(ax - 1) = 0$$

$$bx + 2 = 0 \quad ax - 1 = 0$$

$$bx = -2 \quad ax = 1$$

$$x_1 = \frac{-2}{b} \quad x_2 = \frac{1}{a}$$

$$13. \quad x^2 - 2ax + a^2 - b^2 = 0$$

$$x = \frac{-(-2a) \pm \sqrt{(2a)^2 - 4(a^2 - b^2)}}{2}$$

$$x = \frac{2a \pm \sqrt{4a^2 - 4a^2 + 4b^2}}{2} = \frac{2a \pm 2b}{2}$$

$$x_1 = \frac{2a + 2b}{2} = \frac{2(a + b)}{2} = a + b$$

$$x_2 = \frac{2a - 2b}{2} = \frac{2(a - b)}{2} = a - b$$

$$14. 4x(x-b)+b^2=4m^2$$

$$4x^2-4bx+b^2-4m^2=0$$

$$x = \frac{-(-4b) \pm \sqrt{(-4b)^2 - 4(4)(b^2 - 4m^2)}}{2(4)}$$

$$x = \frac{4b \pm \sqrt{16b^2 - 16b^2 + 64m^2}}{8}$$

$$x = \frac{4b \pm \sqrt{64m^2}}{8} = \frac{4b \pm 8m}{8}$$

$$x_1 = \frac{4b+8m}{8} = \frac{4(b+2m)}{8} = \frac{b+2m}{2}$$

$$x_2 = \frac{4b-8m}{8} = \frac{4(b-2m)}{8} = \frac{b-2m}{2}$$

$$15. x^2 - b^2 + 4a^2 - 4ax = 0$$

$$x^2 - 4ax - b^2 + 4a^2 = 0$$

$$x = \frac{-(-4a) \pm \sqrt{(-4a)^2 - 4(-b^2 + 4a^2)}}{2}$$

$$x = \frac{4a \pm \sqrt{16a^2 + 4b^2 - 16a^2}}{2} = \frac{4a \pm 2b}{2}$$

$$x_1 = \frac{4a+2b}{2} = \frac{2(2a+b)}{2} = 2a+b$$

$$x_2 = \frac{4a-2b}{2} = \frac{2(2a-b)}{2} = 2a-b$$

$$16. x^2 - (a+2)x = -2a$$

$$x^2 - ax - 2x + 2a = 0$$

$$x(x-a) - 2(x-a) = 0$$

$$(x-a)(x-2) = 0$$

$$x-a=0 \quad x-2=0$$

$$x_1 = a \quad x_2 = 2$$

$$17. x^2 + 2x(4-3a) = 48a$$

$$x^2 + 8x - 6ax - 48a = 0$$

$$x(x+8) - 6a(x+8) = 0$$

$$(x+8)(x-6a) = 0$$

$$x+8=0 \quad x-6a=0$$

$$x_1 = -8 \quad x_2 = 6a$$

$$18. x^2 - 2x = m^2 + 2m$$

$$x^2 - m^2 - 2x - 2m = 0$$

$$(x+m)(x-m) - 2(x+m) = 0$$

$$(x-m-2)(x+m) = 0$$

$$x-m-2=0 \quad x+m=0$$

$$x_1 = m+2 \quad x_2 = -m$$

$$19. x^2 + m^2x(m-2) = 2m^5$$

$$x^2 + xm^3 - 2xm^2 - 2m^5 = 0$$

$$x(x+m^3) - 2m^2(x+m^3) = 0$$

$$(x+m^3)(x-2m^2) = 0$$

$$x+m^3=0 \quad x-2m^2=0$$

$$x_1 = -m^3 \quad x_2 = 2m^2$$

$$20. 6x^2 - 15ax = 2bx - 5ab$$

$$6x^2 - 2bx - 15ax + 5ab = 0$$

$$2x(3x-b) - 5a(3x-b) = 0$$

$$(3x-b)(2x-5a) = 0$$

$$3x-b=0 \quad 2x-5a=0$$

$$3x=b \quad 2x=5a$$

$$x_1 = \frac{b}{3} \quad x_2 = \frac{5a}{2}$$

$$21. \frac{3x}{4} + \frac{a}{2} - \frac{x^2}{2a} = 0 \quad mcm = 4a$$

$$3ax + 2a^2 - 2x^2 = 0$$

$$2x^2 - 3ax - 2a^2 = 0$$

$$(2x)^2 - 3a(2x) - 4a^2 = 0$$

$$\frac{(2x-4a)(2x+a)}{2} = 0$$

$$(x-2a)(2x+a) = 0$$

$$x-2a=0 \quad 2x+a=0$$

$$x_1 = 2a \quad 2x = -a$$

$$x_2 = -\frac{a}{2}$$

$$22. \frac{2x-b}{2} = \frac{2bx-b^2}{3x}$$

$$3x(2x-b) = 2(2bx-b^2)$$

$$6x^2 - 3bx - 4bx + 2b^2 = 0$$

$$3x(2x-b) - 2b(2x-b) = 0$$

$$(2x-b)(3x-2b) = 0$$

$$2x-b=0 \quad 3x-2b=0$$

$$2x=b \quad 3x=2b$$

$$x_1 = \frac{b}{2} \quad x_2 = \frac{2b}{3}$$



$$\begin{aligned}
 23. \quad & \frac{a+x}{a-x} + \frac{a-2x}{a+x} = -4 \quad mcm = a^2 - x^2 \\
 & (a+x)(a+x) + (a-2x)(a-x) = -4(a^2 - x^2) \\
 & a^2 + 2ax + x^2 + a^2 - 3ax + 2x^2 = -4a^2 + 4x^2 \\
 & 6a^2 - ax - x^2 = 0 \\
 & x^2 + ax - 6a^2 = 0 \\
 & (x+3a)(x-2a) = 0 \\
 & x+3a=0 \quad x-2a=0 \\
 & x_1 = -3a \quad x_2 = 2a
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & \frac{x^2}{x-1} = \frac{a^2}{2(a-2)} \\
 & 2(a-2)x^2 = a^2(x-1) \\
 & x^2(2a-4) = a^2x - a^2 \\
 & x^2(2a-4) - a^2x + a^2 = 0 \\
 & x = \frac{-(-a^2) \pm \sqrt{(-a^2)^2 - 4(2a-4)(a^2)}}{2(2a-4)} \\
 & x = \frac{a^2 \pm \sqrt{a^4 - 8a^3 + 16a^2}}{2(2a-4)} \\
 & x = \frac{a^2 \pm \sqrt{a^2(a^2 - 8a + 16)}}{4a-8} \\
 & x = \frac{a^2 \pm \sqrt{a^2(a-4)^2}}{4(a-2)} = \frac{a^2 \pm a^2 - 4a}{4(a-2)} \\
 & x_1 = \frac{a^2 + a^2 - 4a}{4(a-2)} = \frac{2a^2 - 4a}{4(a-2)} = \frac{2a(a-2)}{4(a-2)} = \frac{a}{2} \\
 & x_2 = \frac{a^2 - (a^2 - 4a)}{4(a-2)} = \frac{4a}{4(a-2)} = \frac{a}{a-2}
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & x + \frac{2}{x} = \frac{1}{a} + 2a \quad mcm = ax \\
 & ax^2 + 2a = x + 2a^2x \\
 & ax^2 - x + 2a - 2a^2x = 0 \\
 & x(ax-1) - 2a(ax-1) = 0 \\
 & (ax-1)(x-2a) = 0 \\
 & ax-1=0 \quad x-2a=0 \\
 & ax=1 \quad x=2a \\
 & x_1 = \frac{1}{a} \quad x_2 = 2a
 \end{aligned}$$

$$\begin{aligned}
 26. \quad & \frac{2x-b}{b} - \frac{x}{x+b} = \frac{2x}{4b} \quad mcm = 4b(x+b) \\
 & 4(2x-b)(x+b) - 4bx = 2x(x+b) \\
 & 8x^2 + 4bx - 4b^2 - 4bx = 2x^2 + 2bx \\
 & 6x^2 - 2bx - 4b^2 = 0 \\
 & 3x^2 - bx - 2b^2 = 0 \\
 & (3x)^2 - b(3x) - 6b^2 = 0 \\
 & \frac{(3x-3b)(3x+2b)}{3} = 0 \\
 & (x-b)(3x+2b) = 0 \\
 & x-b=0 \quad 3x+2b=0 \\
 & x_1 = b \quad 3x = -2b \\
 & x_2 = -\frac{2b}{3}
 \end{aligned}$$

## EJERCICIO 271

$$\begin{aligned}
 1. \quad & 3x^2 = 48 \\
 & x^2 = 16 \\
 & x = \sqrt{16} \\
 & x = \pm 4 \\
 2. \quad & 5x^2 - 9 = 46 \\
 & 5x^2 = 55 \\
 & x^2 = 11 \\
 & x = \sqrt{11} \\
 & x = \pm \sqrt{11} \\
 3. \quad & 7x^2 + 14 = 0 \\
 & x^2 + 2 = 0 \\
 & x^2 = -2 \\
 & x = \sqrt{-2} \\
 & x = \sqrt{2} \sqrt{-1} \\
 & x = \pm \sqrt{2} i \\
 4. \quad & 9x^2 - a^2 = 0 \\
 & 9x^2 = a^2 \\
 & x^2 = \frac{a^2}{9} \\
 & x = \sqrt{\frac{a^2}{9}} \\
 & x = \pm \frac{a}{3} \\
 5. \quad & (x+5)(x-5) = -7 \\
 & x^2 - 25 = -7 \\
 & x^2 = 18 \\
 & x = \sqrt{18} \\
 & x = \sqrt{3^2 \cdot 2} \\
 & x = \pm 3\sqrt{2}
 \end{aligned}$$

$$6. (2x-3)(2x+3)-135=0$$

$$4x^2-9-135=0$$

$$4x^2=144$$

$$x^2=36$$

$$x=\sqrt{36}$$

$$x=\pm 6$$

$$7. 3(x+2)(x-2)=(x-4)^2+8x$$

$$3x^2-12=x^2-8x+16+8x$$

$$2x^2-28=0$$

$$2x^2=28$$

$$x^2=14$$

$$x=\pm\sqrt{14}$$

$$8. \left(x+\frac{1}{3}\right)\left(x-\frac{1}{3}\right)=\frac{1}{3}$$

$$x^2-\frac{1}{9}-\frac{1}{3}=0$$

$$x^2=\frac{4}{9}$$

$$x=\pm\sqrt{\frac{4}{9}}$$

$$x=\pm\frac{2}{3}$$

$$9. (2x-1)(x+2)-(x+4)(x-1)+5=0$$

$$2x^2+3x-2-x^2-3x+4+5=0$$

$$x^2+7=0$$

$$x^2=-7$$

$$x=\sqrt{7}\sqrt{-1}$$

$$x=\pm\sqrt{7}i$$

$$10. \frac{5}{2x^2}-\frac{1}{6x^2}=\frac{7}{12} \quad mcm=12x^2$$

$$30-2=7x^2$$

$$28=7x^2$$

$$4=x^2$$

$$\sqrt{4}=x$$

$$\pm 2=x$$

$$11. \frac{2x-3}{x-3}=\frac{x-2}{x-1} \quad mcm=(x-3)(x-1)$$

$$(2x-3)(x-1)=(x-2)(x-3)$$

$$2x^2-5x+3=x^2-5x+6$$

$$x^2-3=0$$

$$x^2=3$$

$$x=\pm\sqrt{3}$$

$$12. \frac{x^2-5}{3}+\frac{4x^2-1}{5}-\frac{14x^2-1}{15}=0$$

$$5x^2-25+12x^2-3-14x^2+1=0$$

$$3x^2-27=0$$

$$3x^2=27$$

$$x^2=9$$

$$x=\sqrt{9}$$

$$x=\pm 3$$

$$13. 2x-3-\frac{x^2+1}{x-2}=-7 \quad mcm=x-2$$

$$2x(x-2)-3(x-2)-(x^2+1)=-7(x-2)$$

$$2x^2-4x-3x+6-x^2-1=-7x+14$$

$$x^2+5=14$$

$$x^2=9$$

$$x=\sqrt{9}$$

$$x=\pm 3$$

$$14. 3-\frac{3}{4x^2-1}=2 \quad mcm=4x^2-1$$

$$3(4x^2-1)-3=2(4x^2-1)$$

$$12x^2-6=8x^2-2$$

$$4x^2=4$$

$$x^2=1$$

$$x=\sqrt{1}$$

$$x=\pm 1$$

## EJERCICIO 272

$$1. x^2=5x$$

$$x^2-5x=0$$

$$x(x-5)=0$$

$$x_1=0 \quad x-5=0$$

$$x_2=5$$

$$2. 4x^2=-32x$$

$$4x^2+32x=0$$

$$4x(x+8)=0$$

$$4x=0 \quad x+8=0$$

$$x_1=0 \quad x_2=-8$$

$$3. x^2-3x=3x^2-4x$$

$$0=2x^2-x$$

$$0=x(2x-1)$$

$$0=x_1 \quad 0=2x-1$$

$$1=2x \Rightarrow \frac{1}{2}=x_2$$

4.  $5x^2 + 4 = 2(x + 2)$   
 $5x^2 + 4 = 2x + 4$   
 $5x^2 - 2x = 0$   
 $x(5x - 2) = 0$   
 $x_1 = 0 \quad 5x - 2 = 0$   
 $5x = 2$   
 $x_2 = \frac{2}{5}$
5.  $(x - 3)^2 - (2x + 5)^2 = -16$   
 $x^2 - 6x + 9 - 4x^2 - 20x - 25 + 16 = 0$   
 $-3x^2 - 26x = 0$   
 $x(3x + 26) = 0$   
 $x_1 = 0 \quad 3x + 26 = 0$   
 $3x = -26$   
 $x_2 = \frac{-26}{3} = -8\frac{2}{3}$
6.  $\frac{x^2}{3} - \frac{x - 9}{6} = \frac{3}{2} \quad mcm = 12$   
 $4x^2 - 2x + 18 = 18$   
 $2x(2x - 1) = 0$   
 $2x = 0 \quad 2x - 1 = 0$   
 $x_1 = 0 \quad 2x = 1$   
 $x_2 = \frac{1}{2}$
7.  $(4x - 1)(2x + 3) = (x + 3)(x - 1)$   
 $8x^2 + 10x - 3 = x^2 + 2x - 3$   
 $7x^2 + 8x = 0$   
 $x(7x + 8) = 0$   
 $x_1 = 0 \quad 7x + 8 = 0$   
 $7x = -8$   
 $x_2 = -\frac{8}{7}$   
 $x_2 = -1\frac{1}{7}$
8.  $\frac{x + 1}{x - 1} - \frac{x + 4}{x - 2} = 1 \quad mcm = (x - 1)(x - 2)$   
 $(x + 1)(x - 2) - (x + 4)(x - 1) = (x - 1)(x - 2)$   
 $x^2 - x - 2 - x^2 - 3x + 4 = x^2 - 3x + 2$   
 $0 = x^2 + x$   
 $0 = x(x + 1)$   
 $x_1 = 0 \quad x + 1 = 0$   
 $x_2 = -1$

## EJERCICIO 273

1.  $x + \sqrt{4x + 1} = 5$   
 $\sqrt{4x + 1} = 5 - x$   
 $(\sqrt{4x + 1})^2 = (5 - x)^2$   
 $4x + 1 = 25 - 10x + x^2$   
 $0 = x^2 - 14x + 24$   
 $0 = (x - 12)(x - 2)$   
 $x - 12 = 0$   
 $x_1 = 12 \rightarrow \text{Rechazo sol extraña}$   
 $x - 2 = 0$   
 $x_2 = 2 \rightarrow \text{sol que satisface}$
2.  $2x - \sqrt{x - 1} = 3x - 7$   
 $7 - x = \sqrt{x - 1}$   
 $(7 - x)^2 = \sqrt{(x - 1)^2}$   
 $49 - 14x + x^2 = x - 1$   
 $x^2 - 15x + 50 = 0$   
 $(x - 10)(x - 5) = 0$   
 $x - 10 = 0$   
 $x_1 = 10 \rightarrow \text{Rechazo sol extraña}$   
 $x - 5 = 0$   
 $x_2 = 5 \rightarrow \text{sol que satisface}$
3.  $\sqrt{5x - 1} + \sqrt{x + 3} = 4$   
 $(\sqrt{5x - 1} + \sqrt{x + 3})^2 = 16$   
 $5x - 1 + 2\sqrt{5x^2 + 14x - 3} + x + 3 = 16$   
 $2\sqrt{5x^2 + 14x - 3} = 14 - 6x$   
 $(\sqrt{5x^2 + 14x - 3})^2 = (7 - 3x)^2$   
 $5x^2 + 14x - 3 = 49 - 42x + 9x^2$   
 $0 = 4x^2 - 56x + 52$   
 $0 = x^2 - 14x + 13$   
 $0 = (x - 13)(x - 1)$   
 $x - 13 = 0$   
 $x_1 = 13 \rightarrow \text{Rechazo por sol extraña}$   
 $x - 1 = 0$   
 $x_2 = 1 \rightarrow \text{sol que satisface}$

4.  $2\sqrt{x} - \sqrt{x+5} = 1$   
 $(2\sqrt{x})^2 = (1 + \sqrt{x+5})^2$   
 $4x = 1 + 2\sqrt{x+5} + x + 5$   
 $3x - 6 = 2\sqrt{x+5}$   
 $(3x - 6)^2 = (2\sqrt{x+5})^2$   
 $9x^2 - 36x + 36 = 4x + 20$   
 $9x^2 - 40x + 16 = 0$   
 $(9x)^2 - 40(9x) + 144 = 0$   
 $\frac{(9x - 36)(9x - 4)}{9} = 0$   
 $(x - 4)(9x - 4) = 0$   
 $x - 4 = 0$   
 $x_1 = 4 \rightarrow \text{sol que satisface}$   
 $9x - 4 = 0$   
 $9x = 4$   
 $x_2 = \frac{4}{9} \rightarrow \text{Rechazo por sol extraña}$

5.  $\sqrt{2x-1} + \sqrt{x+3} = 3$   
 $(\sqrt{2x-1})^2 = (3 - \sqrt{x+3})^2$   
 $2x - 1 = 9 - 6\sqrt{x+3} + x + 3$   
 $x - 13 = -6\sqrt{x+3}$   
 $(6\sqrt{x+3})^2 = (13 - x)^2$   
 $36(x+3) = 169 - 26x + x^2$   
 $36x + 108 = 169 - 26x + x^2$   
 $0 = x^2 - 62x + 61$   
 $0 = (x - 61)(x - 1)$   
 $x - 61 = 0$   
 $x_1 = 61 \rightarrow \text{Rechazo por sol extraña}$   
 $x - 1 = 0$   
 $x_2 = 1 \rightarrow \text{sol que satisface}$

6.  $\sqrt{x-3} + \sqrt{2x+1} - 2\sqrt{x} = 0$   
 $(\sqrt{x-3} + \sqrt{2x+1})^2 = (2\sqrt{x})^2$   
 $x - 3 + 2\sqrt{2x^2 - 5x - 3} + 2x + 1 = 4x$   
 $(2\sqrt{2x^2 - 5x - 3})^2 = (x + 2)^2$   
 $4(2x^2 - 5x - 3) = x^2 + 4x + 4$   
 $8x^2 - 20x - 12 = x^2 + 4x + 4$

**Continúa**

## 6. Continuación

$$7x^2 - 24x - 16 = 0$$

$$(7x)^2 - 24(7x) - 112 = 0$$

$$\frac{(7x - 28)(7x + 4)}{7} = 0$$

$$(x - 4)(7x + 4) = 0$$

$$x - 4 = 0 \Rightarrow x_1 = 4 \rightarrow \text{sol factible}$$

$$7x + 4 = 0$$

$$7x = -4 \Rightarrow x_2 = -\frac{4}{7} \rightarrow \text{sol inadmisibile}$$

7.  $\sqrt{5x-1} - \sqrt{3-x} = \sqrt{2x}$   
 $(\sqrt{5x-1} - \sqrt{2x})^2 = (\sqrt{3-x})^2$   
 $5x - 1 - 2\sqrt{10x^2 - 2x} + 2x = 3 - x$   
 $-2\sqrt{10x^2 - 2x} = -8x + 4$   
 $8x - 4 = 2\sqrt{10x^2 - 2x}$   
 $4(2x - 1) = 2\sqrt{10x^2 - 2x}$   
 $(4x - 2)^2 = (\sqrt{10x^2 - 2x})^2$   
 $16x^2 - 16x + 4 = 10x^2 - 2x$   
 $6x^2 - 14x + 4 = 0$   
 $(6x)^2 - 14(6x) + 24 = 0$   
 $\frac{(6x - 12)(6x - 2)}{6} = 0$   
 $(x - 2)(6x - 2) = 0$   
 $x - 2 = 0$   
 $x_1 = 2 \rightarrow \text{sol factible}$   
 $6x - 2 = 0$   
 $6x = 2$   
 $x_2 = \frac{1}{3} \rightarrow \text{sol inadmisibile}$

8.  $\sqrt{3x+1} + \sqrt{5x} = \sqrt{16x+1}$   
 $3x + 1 + 2\sqrt{15x^2 + 5x} + 5x = 16x + 1$   
 $2\sqrt{15x^2 + 5x} = 8x$   
 $(\sqrt{15x^2 + 5x})^2 = (4x)^2$   
 $15x^2 + 5x = 16x^2$   
 $0 = x^2 - 5x$   
 $0 = x(x - 5)$   
 $x_1 = 0 \rightarrow \text{sol admisible}$   
 $x - 5 = 0$   
 $x_2 = 5 \rightarrow \text{sol admisible}$

$$\begin{aligned}
9. \quad & \sqrt{2x+\sqrt{4x-3}}=3 \\
& 2x+\sqrt{4x-3}=9 \\
& (\sqrt{4x-3})^2=(9-2x)^2 \\
& 4x-3=81-36x+4x^2 \\
& 0=4x^2-40x+84 \\
& 0=x^2-10x+21 \\
& 0=(x-7)(x-3) \\
& x-7=0 \\
& x_1=7 \rightarrow \text{Rechazo por} \\
& \quad \text{sol inadmisibile} \\
& x-3=0 \\
& x_2=3 \rightarrow \text{sol aceptada}
\end{aligned}$$

$$\begin{aligned}
10. \quad & \sqrt{x+3}+\frac{6}{\sqrt{x+3}}=5 \\
& (\sqrt{x+3})^2+6=5\sqrt{x+3} \\
& (x+9)^2=(5\sqrt{x+3})^2 \\
& x^2-7x+6=0 \\
& (x-1)(x-6)=0 \\
& x-1=0 \\
& x_1=1 \rightarrow \text{sol admisible} \\
& x-6=0 \\
& x_2=6 \rightarrow \text{sol admisible}
\end{aligned}$$

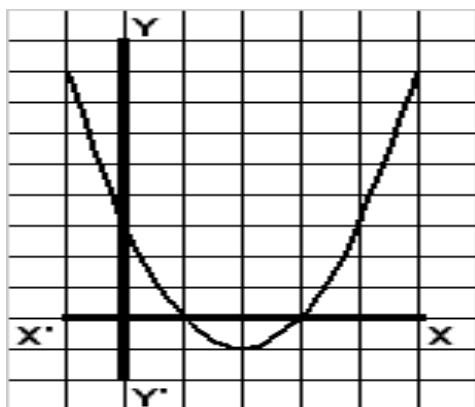
$$\begin{aligned}
11. \quad & \sqrt{x}+\frac{4}{\sqrt{x}}=5 \\
& (\sqrt{x})^2+4=5\sqrt{x} \\
& (x+4)^2=(5\sqrt{x})^2 \\
& x^2+8x+16=25x \\
& x^2-17x+16=0 \\
& (x-16)(x-1)=0 \\
& x-16=0 \\
& x_1=16 \rightarrow \text{sol admisible} \\
& x-1=0 \\
& x_2=1 \rightarrow \text{sol admisible}
\end{aligned}$$

$$\begin{aligned}
12. \quad & 2\sqrt{x}=\sqrt{x+7}+\frac{8}{\sqrt{x+7}} \\
& 2\sqrt{x^2+7x}=\sqrt{(x+7)^2}+8 \\
& (2\sqrt{x^2+7x})^2=(x+15)^2 \\
& 4x^2+28x=x^2+30x+225 \\
& 3x^2-2x-225=0 \\
& (3x)^2-2(3x)-675=0 \\
& \frac{(3x-27)(3x+25)}{3}=0 \\
& (x-9)(3x+25)=0 \\
& x-9=0 \Rightarrow x_1=9 \rightarrow \text{sol aceptada} \\
& 3x+25=0 \Rightarrow x_2=-\frac{25}{3} \rightarrow \text{sol inadmisibile} \\
13. \quad & \sqrt{x+\sqrt{x+8}}=2\sqrt{x} \\
& x+\sqrt{x+8}=4x \\
& (\sqrt{x+8})^2=(3x)^2 \\
& x+8=9x^2 \\
& 9x^2-x-8=0 \\
& (9x)^2-1(9x)-72=0 \\
& \frac{(9x-9)(9x+8)}{9}=0 \\
& x-1=0 \Rightarrow x_1=1 \rightarrow \text{sol aceptada} \\
& 9x+8=0 \Rightarrow x_2=-\frac{8}{9} \rightarrow \text{sol rechazada} \\
14. \quad & \sqrt{6-x}+\sqrt{x+7}-\sqrt{12x+1}=0 \\
& (\sqrt{6-x}+\sqrt{x+7})^2=(\sqrt{12x+1})^2 \\
& 6-x+2\sqrt{42-x-x^2}+x+7=12x+1 \\
& 2\sqrt{42-x-x^2}=12x-12 \\
& (\sqrt{42-x-x^2})^2=(6x-6)^2 \\
& 42-x-x^2=36x^2-72x+36 \\
& 0=37x^2-71x-6 \\
& (37x)^2-71(37x)-222=0 \\
& \frac{(37x-74)(37x+3)}{37}=0 \\
& (x-2)(37x+3)=0 \\
& x-2=0 \Rightarrow x_1=2 \rightarrow \text{sol aceptada} \\
& 37x+3=0 \\
& 37x=-3 \Rightarrow x_2=-\frac{3}{37} \rightarrow \text{sol inadmisibile}
\end{aligned}$$

## EJERCICIO 274

11.  $x^2 - 4x + 3 = 0$

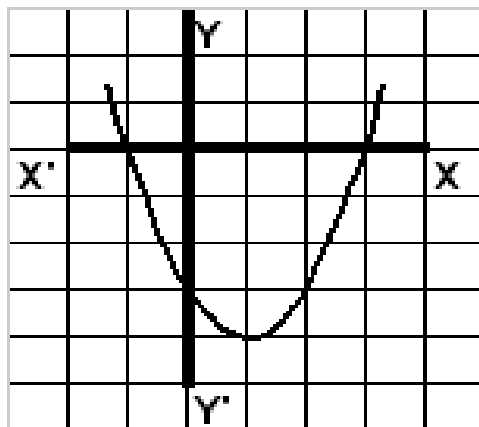
$x$	$y$
0	3
1	0
2	-1
3	0
-1	8



sol:  $x_1 = 1$   $x_2 = 3$

13.  $x^2 - 2x - 3 = 0$

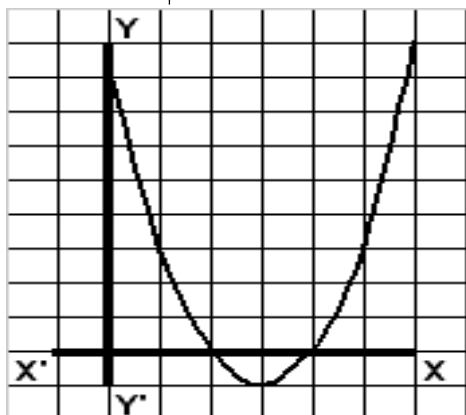
$x$	$y$
0	-3
1	-4
2	-3
3	0
-1	0



sol:  $x_1 = -1$   $x_2 = 3$

12.  $x^2 - 6x + 8 = 0$

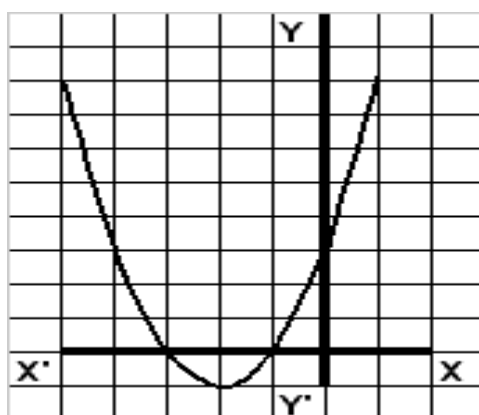
$x$	$y$
0	8
1	3
2	0
3	-1
4	0



sol:  $x_1 = 2$   $x_2 = 4$

14.  $x^2 + 4x + 3 = 0$

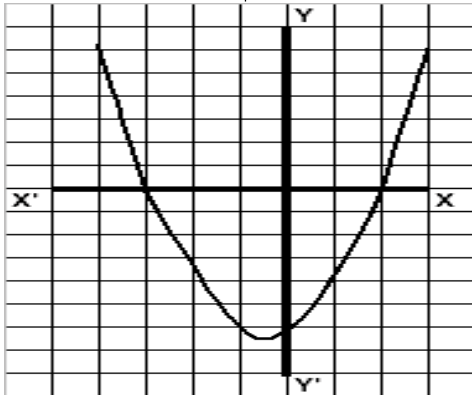
$x$	$y$
0	3
-1	0
-2	-1
-3	0
1	8



sol:  $x_1 = -3$   $x_2 = -1$

15.  $x^2 = 6 - x$   
 $x^2 + x - 6 = 0$

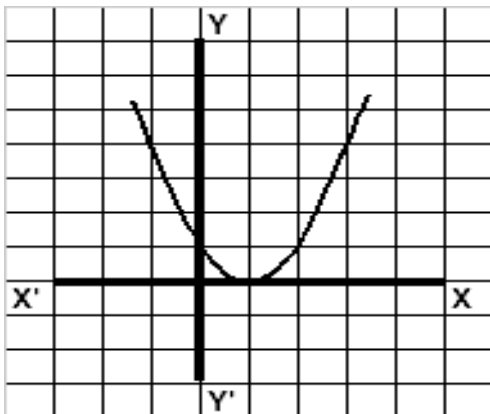
x	y
0	-6
1	-4
2	0
3	6
-1	-6
-3	0



sol:  $x_1 = -3$   $x_2 = 2$

16.  $x^2 = 2x - 1$   
 $x^2 - 2x + 1 = 0$

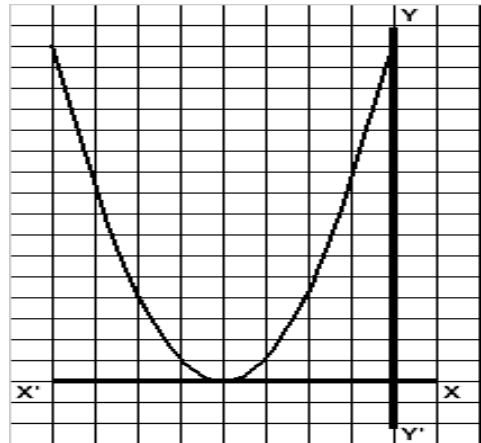
x	y
0	1
1	0
2	1
3	4
-1	4



sol:  $x_1 = x_2 = 1$

17.  $x^2 + 8x + 16 = 0$

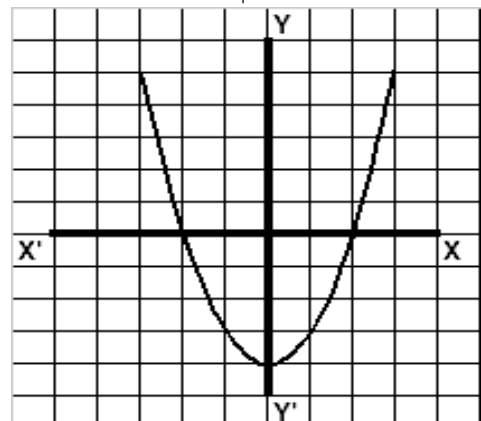
x	y
0	16
-2	4
-4	0
-6	4



sol:  $x_1 = x_2 = -4$

18.  $x^2 - 4 = 0$

x	y
1	-3
2	0
3	5
-1	-3
-2	0
-3	5
0	-4

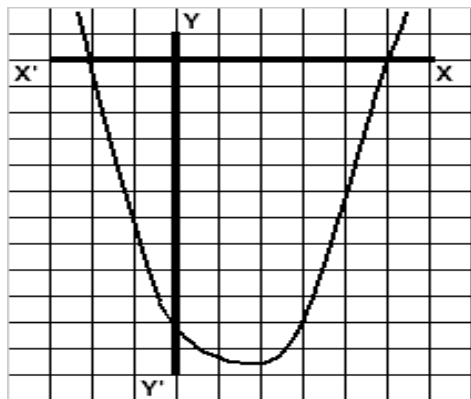


sol:  $x_1 = -2$   $x_2 = 2$

19.  $x^2 = 3x + 10$

$$x^2 - 3x - 10 = 0$$

x	y
0	-10
3	-10
5	0
-1	-6
-2	0

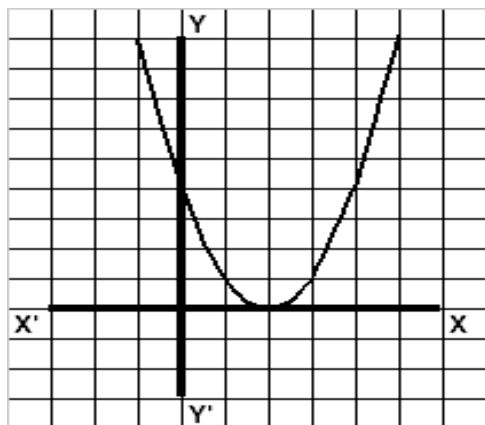


sol:  $x_1 = -2$   $x_2 = 5$

20.  $x^2 - 4x = -4$

$$x^2 - 4x + 4 = 0$$

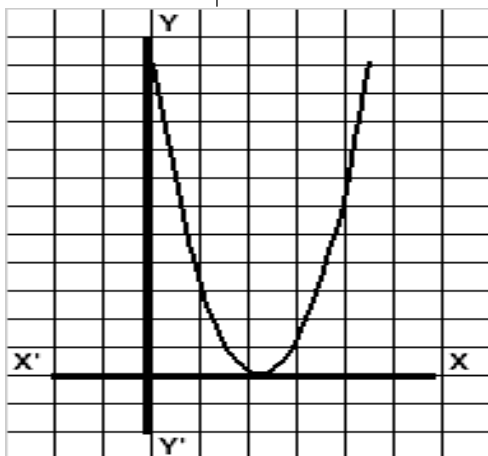
x	y
0	4
1	1
2	0
3	1
-1	9



sol:  $x_1 = x_2 = 2$

21.  $2x^2 - 9x + 10 = 0$

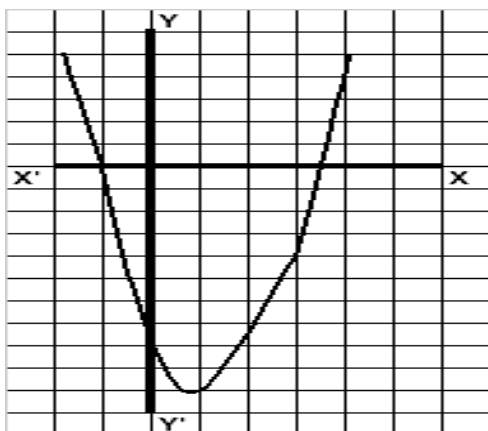
x	y
0	10
1	3
2	0
3	1
4	6



sol:  $x_1 = 2$   $x_2 = 2\frac{1}{2}$

22.  $2x^2 - 5x - 7 = 0$

x	y
0	-7
1	-10
3	-4
4	5
-1	0



sol:  $x_1 = -1$   $x_2 = 3\frac{1}{2}$



## EJERCICIO 275

1.  $x \rightarrow N^{\circ} \text{ mayor}$

$9 - x \rightarrow N^{\circ} \text{ menor}$

$$x^2 + (9 - x)^2 = 53$$

$$x^2 + 81 - 18x + x^2 = 53$$

$$2x^2 - 18x + 28 = 0$$

$$x^2 - 9x + 14 = 0$$

$$(x - 7)(x - 2) = 0$$

$$x_1 = 7 \quad x_2 = 2$$

$$7 \rightarrow N^{\circ} \text{ mayor}$$

$$9 - 7 = 2 \rightarrow N^{\circ} \text{ menor}$$

2.  $x \rightarrow N^{\circ} \text{ positivo}$

$$\frac{3x}{5} \rightarrow \text{Otro Número}$$

$$x\left(\frac{3x}{5}\right) = 2.160$$

$$3x^2 = 10.800$$

$$x^2 = 3.600$$

$$x = \pm \sqrt{3.600}$$

$$x_1 = 60 \quad x_2 = -60$$

$$60 \rightarrow N^{\circ} \text{ positivo}$$

$$\frac{3(60)}{5} = 36 \rightarrow \text{Otro Número}$$

3.  $x \rightarrow \text{Edad de A}$

$$x - 3 \rightarrow \text{Edad de B}$$

$$x^2 + (x - 3)^2 = 317$$

$$x^2 + x^2 - 6x + 9 = 317$$

$$2x^2 - 6x - 308 = 0$$

$$x^2 - 3x - 154 = 0$$

$$(x - 14)(x + 11) = 0$$

$$x_1 = 14 \quad x_2 = -11$$

$$14 \text{ años} \rightarrow \text{Edad de A}$$

$$14 - 3 = 11 \text{ años} \rightarrow \text{Edad de B}$$

4.  $x \rightarrow \text{Un Número}$

$$3x \rightarrow \text{El otro Número}$$

$$(3x)^2 - x^2 = 1.800$$

$$9x^2 - x^2 = 1.800$$

$$8x^2 = 1.800$$

$$x^2 = 225$$

$$x = \pm \sqrt{225}$$

$$x_1 = 15 \quad x_2 = -15$$

$$15 \rightarrow \text{Un Número}$$

$$3(15) = 45 \rightarrow \text{El otro Número}$$

5.  $x \rightarrow N^{\circ} \text{ buscado}$

$$x^2 - 9 = 8(x - 2)$$

$$x^2 - 9 = 8x - 16$$

$$x^2 - 8x + 7 = 0$$

$$(x - 7)(x - 1) = 0$$

$$x_1 = 7 \quad x_2 = 1$$

$$7 \rightarrow N^{\circ} \text{ buscado}$$

6.  $x + 1 \rightarrow N^{\circ} \text{ mayor}$

$$x \rightarrow N^{\circ} \text{ menor}$$

$$(x + 1)^2 - 57 = 3x$$

$$x^2 + 2x + 1 - 57 = 3x$$

$$x^2 - x - 56 = 0$$

$$(x - 8)(x + 7) = 0$$

$$x_1 = 8 \quad x_2 = -7$$

$$8 \rightarrow N^{\circ} \text{ menor}$$

$$8 + 1 = 9 \rightarrow N^{\circ} \text{ mayor}$$

7.  $x + 4 \rightarrow \text{Largo}$

$$x \rightarrow \text{Ancho}$$

$$\text{Area} = x(x + 4)$$

$$= x^2 + 4x$$

$$\text{Doble del area} = 2x^2 + 8x$$

$$(x + 8)(x + 4) = 2x^2 + 8x$$

$$x^2 + 12x + 32 = 2x^2 + 8x$$

$$x^2 - 4x - 32 = 0$$

$$(x - 8)(x + 4) = 0$$

$$x_1 = 8 \quad x_2 = -4$$

$$8m + 4m = 12m \rightarrow \text{Largo}$$

$$8m \rightarrow \text{Ancho}$$

8.  $x \rightarrow N^{\circ} \text{ sacos comp.}$

$$\frac{1.000}{x} \text{ bs.} \rightarrow \text{costo cada saco}$$

$$\frac{1.000}{x + 10} = \frac{1.000}{x} - 5$$

$$1.000x = (x + 10)1.000 - 5x(x + 10)$$

$$10^3 x = 10^3 x + 10^4 - 5x^2 - 50x$$

$$5x^2 + 50x - 10^4 = 0$$

$$x^2 + 10x - 2.000 = 0$$

$$(x + 50)(x - 40) = 0$$

$$x_1 = -50 \quad x_2 = 40$$

$$40 \rightarrow N^{\circ} \text{ sacos comp.}$$

$$\frac{1.000}{40} \text{ bs.} = 25 \text{ bs.} \rightarrow \text{costo cada saco}$$

9.  $4x \rightarrow \text{Costo caballo}$

$$x \rightarrow \text{Costo arreos}$$

$$(4x)^2 + x^2 = 860.625$$

$$16x^2 + x^2 = 860.625$$

$$17x^2 = 860.625$$

$$x^2 = 50.625$$

$$x = \pm \sqrt{50.625}$$

$$x_1 = 225 \quad x_2 = -225$$

$$4(225) = 900$$

$$900 \text{ sucres} \rightarrow \text{Costo caballo}$$

$$225 \text{ sucres} \rightarrow \text{Costo arreos}$$

10.  $x - 7 \rightarrow N^{\circ} \text{ menor}$

$$x \rightarrow N^{\circ} \text{ mayor}$$

$$(x - 7 + x)(x - 7) = 184$$

$$(2x - 7)(x - 7) = 184$$

$$2x^2 - 21x + 49 = 184$$

$$2x^2 - 21x - 135 = 0$$

$$(2x)^2 - 21(2x) - 270 = 0$$

$$(2x - 30)(2x + 9) = 0$$

$$(x - 15)(2x + 9) = 0$$

$$x_1 = 15 \quad x_2 = -\frac{9}{2}$$

$$15 \rightarrow N^{\circ} \text{ mayor}$$

$$15 - 7 = 8 \rightarrow N^{\circ} \text{ menor}$$

11.  $x \rightarrow \text{Edad de A}$

$$23 - x \rightarrow \text{Edad de B}$$

$$x(23 - x) = 102$$

$$23x - x^2 = 102$$

$$x^2 - 23x + 102 = 0$$

$$(x - 17)(x - 6) = 0$$

$$x_1 = 17 \quad x_2 = 6$$

$$17 \text{ años} \rightarrow \text{Edad de A}$$

$$6 \text{ años} \rightarrow \text{Edad de B}$$

12.  $x \rightarrow N^\circ \text{ de libros}$

$$\frac{180}{x} \rightarrow \text{Costo } c/u$$

$$\frac{180}{x-6} = \frac{180}{x} + 1$$

$$180x = 180(x-6) + x(x-6)$$

$$180x = 180x - 1.080 + x^2 - 6x$$

$$0 = x^2 - 6x - 1.080$$

$$0 = (x-36)(x+30)$$

$$x_1 = 36 \quad x_2 = -30$$

$$36 \rightarrow \text{Libros}$$

$$\frac{180}{36} = \$5 \rightarrow \text{Costo } c/u$$

13.  $x \rightarrow N^\circ \text{ de filas}$

$$\frac{180}{x} \rightarrow N^\circ \text{ soldados cada fila}$$

$$\frac{180}{x} - 8 = x$$

$$180 - 8x = x^2$$

$$0 = x^2 + 8x - 180$$

$$0 = (x+18)(x-10)$$

$$x_1 = -18 \quad x_2 = 10$$

$$10 \rightarrow N^\circ \text{ de filas}$$

$$\frac{180}{10} = 18 \rightarrow N^\circ \text{ soldados}$$

cada fila

14.  $x \rightarrow \text{Costo del reloj}$

$$x\% \text{ de } x = \frac{x^2}{100}$$

$$\frac{x^2}{100} + x = 75$$

$$x^2 + 100x = 7.500$$

$$x^2 + 100x - 7.500 = 0$$

$$(x+150)(x-50) = 0$$

$$x_1 = -150 \quad x_2 = 50$$

$$50 \text{ soles} \rightarrow \text{Costo del reloj}$$

15.  $x \rightarrow \text{Pers. comp.}$

$$\frac{1.200}{x} - 194 = x$$

$$1.200 - 194x = x^2$$

$$x^2 + 194x - 1.200 = 0$$

$$(x+200)(x-6) = 0$$

$$x_1 = -200 \quad x_2 = 6$$

$$6 \rightarrow \text{Pers. comp. el auto}$$

16.  $x \rightarrow \text{Relojes comp.}$

$$\frac{192}{x} \rightarrow \text{Costo cada reloj}$$

$$\frac{192}{x} = \frac{3x}{4}$$

$$768 = 3x^2$$

$$256 = x^2$$

$$\pm \sqrt{256} = x$$

$$x_1 = 16 \quad x_2 = -16$$

$$16 \rightarrow \text{Relojes comp.}$$

$$\frac{192}{16} = \$12 \rightarrow \text{Costo cada reloj}$$

17.  $x \rightarrow \text{Libros comp.}$

$$\frac{150}{x} \rightarrow \text{Costo cada lib.}$$

$$\frac{150}{x} + 1 = \frac{150}{x-5}$$

$$150(x-5) + x(x-5) = 150x$$

$$150x - 750 + x^2 - 5x = 150x$$

$$x^2 - 5x - 750 = 0$$

$$(x-30)(x+25) = 0$$

$$x_1 = 30 \quad x_2 = -25$$

$$30 \rightarrow \text{Libros comp.}$$

$$\frac{150}{30} = \$5 \rightarrow \text{Costo cada lib.}$$

18.  $x \rightarrow \text{Libros comp.}$

$$\frac{200}{x} - 10 = x$$

$$200 - 10x = x^2$$

$$x^2 + 10x - 200 = 0$$

$$(x+20)(x-10) = 0$$

$$x_1 = -20 \quad x_2 = 10$$

$$10 \rightarrow \text{Libros comp.}$$

19.  $x \rightarrow \text{Plumas comp.}$

$$\frac{24}{x} \rightarrow \text{Precio } c/u$$

$$\frac{24}{x} - 1 = \frac{24}{x+4}$$

$$24(x+4) - x(x+4) = 24x$$

$$24x + 96 - x^2 - 4x = 24x$$

$$x^2 + 4x - 96 = 0$$

$$(x+12)(x-8) = 0$$

$$x_1 = -12 \quad x_2 = 8$$

$$8 \rightarrow \text{Plumas comp.}$$

$$\frac{24}{8} = \$3 \rightarrow \text{Precio } c/u$$

20.  $x \rightarrow \text{Tiempo de recorrido}$

$$\frac{240}{x} + 20 = \frac{240}{x-2}$$

$$240(x-2) + 20x(x-2) = 240x$$

$$240x - 480 + 20x^2 - 40x = 240x$$

$$20x^2 - 40x - 480 = 0$$

$$x^2 - 2x - 24 = 0$$

$$(x-6)(x+4) = 0$$

$$x_1 = 6 \quad x_2 = -4$$

$$6 \text{ horas} \rightarrow \text{Tiempo de recorrido}$$

21.  $x \rightarrow \text{Caballos comp.}$

$$\frac{2.000}{x} \rightarrow \text{Precio cada caballo}$$

$$\frac{2.000}{x} + 60 = \frac{2.000 + 80}{x-2}$$

$$2.000(x-2) + 60x(x-2) = 2.080x$$

$$2.000x - 4.000 + 60x^2 - 120x = 2.080x$$

$$60x^2 - 200x - 4.000 = 0$$

$$3x^2 - 10x - 200 = 0$$

$$(3x)^2 - 10(3x) - 600 = 0$$

$$(3x-30)(3x+20) = 0$$

$$(x-10)(3x+20) = 0$$

$$x_1 = 10 \quad x_2 = -\frac{20}{3}$$

$$10 \rightarrow \text{Caballos comp.}$$

$$\frac{2.000}{10} = \$200 \rightarrow \text{Precio } c/\text{caballo}$$

22.  $x \rightarrow N^\circ \text{ menor}$

$$x+1 \rightarrow N^\circ \text{ intermedio}$$

$$x+2 \rightarrow N^\circ \text{ mayor}$$

$$\frac{x+2}{x+1} = \frac{3x}{10}$$

$$10(x+2) = 3x(x+1)$$

$$10x + 20 = 3x^2 + 3x$$

$$0 = 3x^2 - 7x - 20$$

$$0 = (3x)^2 - 7(3x) - 60$$

$$0 = (3x-12)(3x+5)$$

$$0 = (x-4)(3x+5)$$

$$x_1 = 4 \quad x_2 = -\frac{5}{3}$$

$$4 \rightarrow N^\circ \text{ menor}$$

$$4+1=5 \rightarrow N^\circ \text{ intermedio}$$

$$4+2=6 \rightarrow N^\circ \text{ mayor}$$

23.  $x \rightarrow \text{Un } N^{\circ}.$

$$\frac{180}{x} \rightarrow \text{El otro } N^{\circ}.$$

$$\frac{180}{x} = 1\frac{1}{4}$$

$$\frac{180}{x^2} = \frac{5}{4}$$

$$720 = 5x^2$$

$$144 = x^2$$

$$\pm\sqrt{144} = x$$

$$x_1 = 12 \quad x_2 = -12$$

$$12 \rightarrow \text{Un } N^{\circ}.$$

$$\frac{180}{12} = 15 \rightarrow \text{El otro } N^{\circ}.$$

24.  $x \rightarrow N^{\circ} \text{ naranjas comp.}$

$$\frac{150}{x} \rightarrow \text{Costo } c / \text{naranja}$$

$$\left(\frac{150}{x} + 1\right)(x-5) = 150$$

$$(150+x)(x-5) = 150x$$

$$150x - 750 + x^2 - 5x = 150x$$

$$x^2 - 5x - 750 = 0$$

$$(x-30)(x+25) = 0$$

$$x_1 = 30 \quad x_2 = -25$$

$$30 \rightarrow N^{\circ} \text{ naranjas comp.}$$

$$\frac{150}{30} = 5 \text{ ctvs.} \rightarrow \text{Costo } c / \text{naranja}$$

25.  $x \rightarrow \text{Costo del caballo}$

$$\frac{x^2}{100} + x = 171$$

$$x^2 + 100x = 17.100$$

$$x^2 + 100x - 17.100 = 0$$

$$(x+190)(x-90) = 0$$

$$x_1 = -190 \quad x_2 = 90$$

$$90 \text{ Q.} \rightarrow \text{Costo del caballo}$$

26.  $x \rightarrow N^{\circ} \text{ mayor}$

$$\frac{352}{x} \rightarrow N^{\circ} \text{ menor}$$

$$\frac{x}{352} = 2 + \frac{10}{352}$$

$$\frac{x^2}{352} = 2 + \frac{10x}{352}$$

$$x^2 = 704 + 10x$$

**Continúa**

26. **Continuación**

$$x^2 - 10x - 704 = 0$$

$$(x-32)(x+22) = 0$$

$$x_1 = 32 \quad x_2 = -22$$

$$32 \rightarrow N^{\circ} \text{ mayor}$$

$$\frac{352}{32} = 11 \rightarrow N^{\circ} \text{ menor}$$

27.  $x \rightarrow \text{Long. pieza mayor}$

$$20 - x \rightarrow \text{Long. pieza menor}$$

$$x \cdot x = 9(20-x)(20-x)$$

$$x^2 = 9(400 - 40x + x^2)$$

$$x^2 = 3.600 - 360x + 9x^2$$

$$0 = 8x^2 - 360x + 3.600$$

$$0 = x^2 - 45x + 450$$

$$0 = (x-30)(x-15)$$

$$x_1 = 30 \quad x_2 = 15$$

$$15m \rightarrow \text{Long. pieza mayor}$$

$$20 - 15 = 5m \rightarrow \text{Long. pieza menor}$$

28.  $x \rightarrow \text{Tiempo en horas}$

$$\frac{200}{x} \rightarrow \text{Veloc. del tren}$$

$$\frac{200}{x} = \frac{200}{x-1} + 10$$

$$200(x-1) = 200x + 10x(x-1)$$

$$200x - 200 = 200x + 10x^2 - 10x$$

$$0 = 10x^2 - 10x + 200$$

$$0 = x^2 - x + 20$$

$$0 = (x-5)(x+4)$$

$$x_1 = 5 \quad x_2 = -4$$

$$\frac{200}{5} = 40 \text{ Km/h} \rightarrow \text{Veloc. del tren}$$

29.  $x \rightarrow \text{Días trabaj.}$

$$\frac{84}{x} \rightarrow \text{Valor del jornal}$$

$$\frac{84}{x} - 1 = \frac{84}{x+2}$$

$$84(x+2) - x(x+2) = 84x$$

$$84x + 168 - x^2 - 2x = 84x$$

$$x^2 + 2x - 168 = 0$$

$$(x+14)(x-12) = 0$$

$$x_1 = -14 \quad x_2 = 12$$

$$12 \rightarrow \text{Días trabaj.}$$

$$\frac{84}{12} = 7 \text{ col} \rightarrow \text{Valor del jornal}$$

30.  $x \rightarrow \text{Personas que van}$

$$\frac{90}{x} \rightarrow \text{Valor } c / u$$

$$\frac{90}{x-3} - 1 = \frac{90}{x}$$

$$90x - x(x-3) = 90(x-3)$$

$$90x - x^2 + 3x = 90x - 270$$

$$x^2 - 3x - 270 = 0$$

$$(x-18)(x+15) = 0$$

$$x_1 = 18 \quad x_2 = -15$$

$$18 \rightarrow \text{Personas que van}$$

$$\frac{90}{18} = \$5 \rightarrow \text{Valor } c / u$$

31.  $x \rightarrow N^{\circ} \text{ buscado}$

$$\frac{84}{x} - 5 = x$$

$$84 - 5x = x^2$$

$$x^2 + 5x - 84 = 0$$

$$(x+12)(x-7) = 0$$

$$x_1 = -12 \quad x_2 = 7$$

$$7 \rightarrow N^{\circ} \text{ buscado}$$

32.  $x \rightarrow \text{Edad actual } A$

$$x-6 = \sqrt{x+6}$$

$$(x-6)^2 = (\sqrt{x+6})^2$$

$$x^2 - 12x + 36 = x + 6$$

$$x^2 - 13x + 30 = 0$$

$$(x-10)(x-3) = 0$$

$$x_1 = 10 \quad x_2 = 3$$

$$10 \text{ años} \rightarrow \text{Edad actual } A$$

33.  $x \rightarrow \text{Libros comp.}$

$$\frac{40}{x} \rightarrow \text{Precio de } c / \text{libro}$$

$$x-2 \rightarrow N^{\circ} \text{ de plumas}$$

$$\frac{40}{x} = \frac{40}{x-2} - 1$$

$$40(x-2) = 40x - x(x-2)$$

$$40x - 80 = 40x - x^2 + 2x$$

$$x^2 - 2x - 80 = 0$$

$$(x-10)(x+8) = 0$$

$$x_1 = 10 \quad x_2 = -8$$

$$10 \rightarrow \text{Libros comp.}$$

$$\frac{40}{10} = \$4 \rightarrow \text{Precio } c / \text{libro}$$

## EJERCICIO 276

1.  $3x^2 + 5x - 2 = 0$

$$b^2 - 4ac = 5^2 - 4(3)(-2) = 25 + 24 = 49$$

Reales y desiguales, racionales

2.  $2x^2 - 4x + 1 = 0$

$$b^2 - 4ac = (-4)^2 - 4(2)(1) = 16 - 8 = 8$$

Reales y desiguales, irracionales

3.  $4x^2 - 4x + 1 = 0$

$$b^2 - 4ac = (-4)^2 - 4(4)(1) = 16 - 16 = 0$$

Reales e iguales

4.  $3x^2 - 2x + 5 = 0$

$$b^2 - 4ac = (-2)^2 - 4(3)(5) = 4 - 60 = -56$$

Imaginarias

5.  $x^2 - 10x + 25 = 0$

$$b^2 - 4ac = (-10)^2 - 4(1)(25) = 100 - 100 = 0$$

Reales e iguales

6.  $x^2 - 5x - 5 = 0$

$$b^2 - 4ac = (-5)^2 - 4(1)(-5) = 25 + 20 = 45$$

Reales y desiguales, irracionales

7.  $2x^2 - 9x + 7 = 0$

$$b^2 - 4ac = (-9)^2 - 4(2)(7) = 81 - 56 = 25$$

Reales y desiguales, racionales

8.  $36x^2 + 12x + 1 = 0$

$$b^2 - 4ac = (12)^2 - 4(36)(1) = 144 - 144 = 0$$

Reales e iguales

9.  $4x^2 - 5x + 3 = 0$

$$b^2 - 4ac = (-5)^2 - 4(4)(3) = 25 - 48 = -23$$

Imaginarias

10.  $x^2 + x - 1 = 0$

$$b^2 - 4ac = (1)^2 - 4(1)(-1) = 1 + 4 = 5$$

Reales y desiguales, irracionales

11.  $5x^2 - 7x + 8 = 0$

$$b^2 - 4ac = (-7)^2 - 4(5)(8) = 49 - 160 = -111$$

Imaginarias

12.  $x^2 - 10x - 11 = 0$

$$b^2 - 4ac = (-10)^2 - 4(1)(-11) = 100 + 44 = 144$$

Reales y desiguales, racionales

## EJERCICIO 277

1.  $x^2 + x - 6 = 0$

Suma  $2 + (-3) = -1$

Producto  $2(-3) = -6$

Si son raíces

2.  $x^2 - 4x - 5 = 0$

Suma  $1 + 5 = 6$

No son raíces

3.  $2x^2 - x - 1 = 0$

$$x^2 - \frac{x}{2} - \frac{1}{2} = 0$$

Suma  $1 - \frac{1}{2} = \frac{1}{2}$

Producto  $1\left(-\frac{1}{2}\right) = -\frac{1}{2}$

Si son raíces

4.  $3x^2 + 8x - 3 = 0$

$$x^2 + \frac{8x}{3} - 1 = 0$$

Suma  $-3 + \frac{1}{3} = -\frac{8}{3}$

Continúa

### 4. Continuación

Producto  $-3\left(\frac{1}{3}\right) = -1$

Si son raíces

5.  $5x^2 - 11x + 2 = 0$

$$x^2 - \frac{11x}{5} + \frac{2}{5} = 0$$

Suma  $2 - \frac{1}{5} = \frac{9}{5}$

No son raíces

6.  $4x^2 + 17x + 4 = 0$

$$x^2 + \frac{17x}{4} + 1 = 0$$

Suma  $-4 - \frac{1}{4} = -\frac{17}{4}$

Producto  $-4\left(-\frac{1}{4}\right) = 1$

Si son raíces

7.  $5x^2 + 24x - 5 = 0$

$$x^2 + \frac{24}{5}x - 1 = 0$$

Continúa

### 7. Continuación

Suma  $-5 - \frac{1}{5} = -\frac{26}{5}$

No son raíces

8.  $x^2 + 3x - 28 = 0$

Suma  $4 - 7 = -3$

Producto  $4(-7) = -28$

Si son raíces

9.  $6x^2 + x - 2 = 0$

$$x^2 + \frac{x}{6} - \frac{1}{3} = 0$$

Suma  $\frac{1}{2} - \frac{2}{3} = -\frac{1}{6}$

Producto  $\left(\frac{1}{2}\right)\left(-\frac{2}{3}\right) = -\frac{1}{3}$

Si son raíces

10.  $8x^2 - 2x - 3 = 0$

$$x^2 - \frac{1}{4}x - \frac{3}{8} = 0$$

Suma  $\frac{1}{2} - \frac{3}{4} = -\frac{1}{4}$

No son raíces

## EJERCICIO 278

1. Suma  $3+4=7$

Producto  $3(4)=12$

Luego  $x^2-7x+12=0$

2. Suma  $-1+3=2$

Producto  $-1(3)=-3$

Luego  $x^2-2x-3=0$

3. Suma  $-5-7=-12$

Producto  $-5(-7)=35$

Luego  $x^2+12x+35=0$

4. Suma  $-10+11=1$

Producto  $-10(11)=-110$

Luego  $x^2-x-110=0$

5. Suma  $1+\frac{1}{2}=\frac{3}{2}$

Producto  $1\left(\frac{1}{2}\right)=\frac{1}{2}$

Luego  $x^2-\frac{3x}{2}+\frac{1}{2}=0$

$\Rightarrow 2x^2-3x+1=0$

6. Suma  $-2-\frac{1}{5}=-\frac{11}{5}$

Producto  $-2\left(-\frac{1}{5}\right)=\frac{2}{5}$

Luego  $x^2+\frac{11x}{5}+\frac{2}{5}=0$

$\Rightarrow 5x^2+11x+2=0$

7. Suma  $3-\frac{2}{3}=\frac{7}{3}$

Producto  $3\left(-\frac{2}{3}\right)=-2$

Luego  $x^2-\frac{7x}{3}-2=0$

$\Rightarrow 3x^2-7x-6=0$

8. Suma  $-2-\frac{3}{2}=-\frac{7}{2}$

Producto  $-2\left(-\frac{3}{2}\right)=3$

Luego  $x^2+\frac{7x}{2}+3=0$

$\Rightarrow 2x^2+7x+6=0$

9. Suma  $-\frac{1}{2}+\frac{3}{4}=\frac{1}{4}$

Producto  $\left(-\frac{1}{2}\right)\left(\frac{3}{4}\right)=-\frac{3}{8}$

Luego  $x^2-\frac{x}{4}-\frac{3}{8}=0$

$\Rightarrow 4x^2-x-\frac{3}{2}=0$

$\Rightarrow 8x^2-2x-3=0$

10. Suma  $-5+\frac{2}{7}=-\frac{33}{7}$

Producto  $-5\left(\frac{2}{7}\right)=-\frac{10}{7}$

Luego  $x^2+\frac{33x}{7}-\frac{10}{7}=0$

$\Rightarrow 7x^2+33x-10=0$

11. Suma  $6-\frac{5}{3}=\frac{13}{3}$

Producto  $6\left(-\frac{5}{3}\right)=-10$

Luego  $x^2-\frac{13}{3}x-10=0$

$\Rightarrow 3x^2-13x-30=0$

12. Suma  $-2-\frac{1}{8}=-\frac{17}{8}$

Producto  $-2\left(-\frac{1}{8}\right)=\frac{1}{4}$

Luego  $x^2+\frac{17}{8}x+\frac{1}{4}=0$

$\Rightarrow 8x^2+17x+2=0$

13. Suma  $18-52=-34$

Producto  $18(-52)=-936$

Luego  $x^2+34x-936=0$

14. Suma  $-15-11=-26$

Producto  $-15(-11)=165$

Luego  $x^2+26x+165=0$

15. Suma  $0+2=2$

Producto  $0(2)=0$

Luego  $x^2-2x=0$

16. Suma  $0-\frac{1}{3}=-\frac{1}{3}$

Producto  $0\left(-\frac{1}{3}\right)=0$

Luego  $x^2+\frac{x}{3}=0$

$\Rightarrow 3x^2+x=0$

17. Suma  $5-5=0$

Producto  $5(-5)=-25$

Luego  $x^2-25=0$

18. Suma  $\frac{1}{2}-\frac{1}{2}=0$

Producto  $\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)=-\frac{1}{4}$

Luego  $x^2-\frac{1}{4}=0$

$\Rightarrow 4x^2-1=0$

19. Suma  $7+7=14$

Producto  $7(7)=49$

Luego  $x^2-14x+49=0$

20. Suma  $8-\frac{11}{3}=\frac{13}{3}$

Producto  $8\left(-\frac{11}{3}\right)=-\frac{88}{3}$

Luego  $x^2-\frac{13}{3}x-\frac{88}{3}=0$

$\Rightarrow 3x^2-13x-88=0$

21. Suma  $-\frac{5}{6}-\frac{9}{2}=-\frac{16}{3}$

Producto  $\left(-\frac{5}{6}\right)\left(-\frac{9}{2}\right)=\frac{15}{4}$

Luego  $x^2+\frac{16}{3}x+\frac{15}{4}=0$

$\Rightarrow 12x^2+64x+45=0$

22. Suma  $-\frac{11}{2}+\frac{2}{7}=-\frac{73}{14}$

Producto  $\left(-\frac{11}{2}\right)\left(\frac{2}{7}\right)=-\frac{11}{7}$

Luego  $x^2+\frac{73}{14}x-\frac{11}{7}=0$

$\Rightarrow 14x^2+73x-22=0$

23. Suma  $2a-a=a$

Producto  $2a(-a)=-2a^2$

Luego  $x^2-ax-2a^2=0$

$$24. \text{ Suma } -\frac{2b}{3} + \frac{b}{4} = -\frac{5b}{12}$$

$$\text{Pr oducto } \left(-\frac{2b}{3}\right)\left(\frac{b}{4}\right) = -\frac{b^2}{6}$$

$$\text{Luego } x^2 + \frac{5b}{12}x - \frac{b^2}{6} = 0$$

$$\Rightarrow 12x^2 + 5bx - 2b^2 = 0$$

$$25. \text{ Suma } m - \frac{m}{2} = \frac{m}{2}$$

$$\text{Pr oducto } m\left(-\frac{m}{2}\right) = -\frac{m^2}{2}$$

$$\text{Luego } x^2 - \frac{m}{2}x - \frac{m^2}{2} = 0$$

$$\Rightarrow 2x^2 - mx - m^2 = 0$$

$$26. \text{ Suma } b + a - b = a$$

$$\text{Pr oducto } b(a - b) = ab - b^2$$

$$\text{Luego } x^2 - ax + ab - b^2 = 0$$

$$27. \text{ Suma } \frac{a}{2} - \frac{b}{3} = \frac{3a - 2b}{6}$$

$$\text{Pr oducto } \left(\frac{a}{2}\right)\left(-\frac{b}{3}\right) = -\frac{ab}{6}$$

$$\text{Luego } x^2 - \frac{3a - 2b}{6}x - \frac{ab}{6} = 0$$

$$\Rightarrow 6x^2 - (3a - 2b)x - ab = 0$$

$$28. \text{ Suma } 1 + \sqrt{2} + 1 - \sqrt{2} = 2$$

$$\text{Pr oducto } (1 + \sqrt{2})(1 - \sqrt{2}) = -1$$

$$\text{Luego } x^2 - 2x - 1 = 0$$

$$29. \text{ Suma } 2 + \sqrt{5} + 2 - \sqrt{5} = 4$$

$$\text{Pr oducto } (2 + \sqrt{5})(2 - \sqrt{5}) = -1$$

$$\text{Luego } x^2 - 4x - 1 = 0$$

$$30. \text{ Suma } 3 + \sqrt{-1} + 3 - \sqrt{-1} = 6$$

$$\text{Pr oducto } (3 + \sqrt{-1})(3 - \sqrt{-1}) = 10$$

$$\text{Luego } x^2 - 6x + 10 = 0$$

## EJERCICIO 279

$$1. x^2 - 11x + 30 = 0$$

$$(x - 6)(x - 5) = 0$$

$$x_1 = 6 \quad x_2 = 5$$

$$2. x^2 + 33x + 260 = 0$$

$$x = -\frac{33}{2} \pm \sqrt{\frac{(33)^2}{4} - 260}$$

$$x = -\frac{33}{2} \pm \sqrt{\frac{1.089 - 1.040}{4}}$$

$$x = \frac{-33}{2} \pm \sqrt{\frac{49}{4}}$$

$$x = \frac{-33}{2} \pm \frac{7}{2}$$

$$x_1 = -\frac{33}{2} + \frac{7}{2} = -\frac{26}{2} = -13$$

$$x_2 = -\frac{33}{2} - \frac{7}{2} = -\frac{40}{2} = -20$$

$$3. x^2 + x - 306 = 0$$

$$(x + 18)(x - 17) = 0$$

$$x_1 = -18 \quad x_2 = 17$$

$$4. x^2 + 49x + 294 = 0$$

$$(x + 42)(x + 7) = 0$$

$$x_1 = -42 \quad x_2 = -7$$

$$5. x^2 - 6x - 247 = 0$$

$$(x - 19)(x + 13) = 0$$

$$x_1 = 19 \quad x_2 = -13$$

$$6. x^2 - \frac{3}{2}x - 1 = 0$$

$$2x^2 - 3x - 2 = 0$$

$$(2x)^2 - 3(2x) - 4 = 0$$

$$(2x - 4)(2x + 1) = 0$$

$$(x - 2)(2x + 1) = 0$$

$$x_1 = 2 \quad x_2 = -\frac{1}{2}$$

$$7. x^2 + \frac{22}{3}x + 8 = 0$$

$$3x^2 + 22x + 24 = 0$$

$$(3x)^2 + 22(3x) + 72 = 0$$

$$(3x + 18)(3x + 4) = 0$$

$$(x + 6)(3x + 4) = 0$$

$$x_1 = -6 \quad x_2 = -\frac{4}{3}$$

$$8. x^2 - \frac{x}{4} - \frac{3}{8} = 0$$

$$8x^2 - 2x - 3 = 0$$

$$(8x)^2 - 2(8x) - 24 = 0$$

$$(8x - 6)(8x + 4) = 0$$

$$(4x - 3)(2x + 1) = 0$$

$$x_1 = \frac{3}{4} \quad x_2 = -\frac{1}{2}$$

$$9. x^2 + \frac{95}{7}x - 6 = 0$$

$$7x^2 + 95x - 42 = 0$$

$$(7x)^2 + 95(7x) - 294 = 0$$

$$(7x + 98)(7x - 3) = 0$$

$$(x + 14)(7x - 3) = 0$$

$$x_1 = -14 \quad x_2 = \frac{3}{7}$$

$$10. x^2 + \frac{10}{3}x + 1 = 0$$

$$3x^2 + 10x + 3 = 0$$

$$x = \frac{-10 \pm \sqrt{(10)^2 - 4(3)(3)}}{2(3)}$$

$$x = \frac{-10 \pm \sqrt{64}}{6} = \frac{-10 \pm 8}{6}$$

$$x_1 = \frac{-10 + 8}{6} = \frac{-2}{6} = -\frac{1}{3}$$

$$x_2 = \frac{-10 - 8}{6} = \frac{-18}{6} = -3$$

$$11. x^2 - \frac{31}{40}x + \frac{3}{20} = 0$$

$$40x^2 - 31x + 6 = 0$$

$$(40x)^2 - 31(40x) + 240 = 0$$

$$(40x - 16)(40x - 15) = 0$$

$$(5x - 2)(8x - 3) = 0$$

$$x_1 = \frac{2}{5} \quad x_2 = \frac{3}{8}$$

$$12. \quad x^2 + \frac{x}{6} - \frac{5}{9} = 0$$

$$18x^2 + 3x - 10 = 0$$

$$(18x)^2 + 3(18x) - 180 = 0$$

$$(18x+15)(18x-12) = 0$$

$$(6x+5)(3x-2) = 0$$

$$x_1 = -\frac{5}{6} \quad x_2 = \frac{2}{3}$$

$$13. \quad x^2 - \frac{7}{20}x - \frac{3}{10} = 0$$

$$20x^2 - 7x - 6 = 0$$

$$(20x)^2 - 7(20x) - 120 = 0$$

$$(20x-15)(20x+8) = 0$$

$$(4x-3)(5x+2) = 0$$

$$x_1 = \frac{3}{4} \quad x_2 = -\frac{2}{5}$$

$$14. \quad x^2 - \frac{21x}{5} - 4 = 0$$

$$5x^2 - 21x - 20 = 0$$

$$(5x)^2 - 21(5x) - 100 = 0$$

$$(5x-25)(5x+4) = 0$$

$$(x-5)(5x+4) = 0$$

$$x_1 = 5 \quad x_2 = -\frac{4}{5}$$

$$15. \quad x^2 - \frac{59}{72}x + \frac{1}{6} = 0$$

$$72x^2 - 59x + 12 = 0$$

$$(72x)^2 - 59(72x) + 864 = 0$$

$$(72x-32)(72x-27) = 0$$

$$(9x-4)(8x-3) = 0$$

$$x_1 = \frac{4}{9} \quad x_2 = \frac{3}{8}$$

$$16. \quad x^2 - 2x - 4 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{20}}{2} = \frac{2 \pm 2\sqrt{5}}{2}$$

$$x_1 = \frac{2+2\sqrt{5}}{2} = \frac{2(1+\sqrt{5})}{2} = 1+\sqrt{5}$$

$$x_2 = \frac{2-2\sqrt{5}}{2} = \frac{2(1-\sqrt{5})}{2} = 1-\sqrt{5}$$

$$17. \quad x^2 - x - \frac{11}{4} = 0$$

$$4x^2 - 4x - 11 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(-11)}}{2(4)}$$

$$x = \frac{4 \pm \sqrt{64 \cdot 3}}{8} = \frac{4 \pm 8\sqrt{3}}{8}$$

$$x_1 = \frac{4+8\sqrt{3}}{8} = \frac{4(1+2\sqrt{3})}{8} = \frac{1+2\sqrt{3}}{2} = \frac{1}{2} + \sqrt{3}$$

$$x_2 = \frac{4-8\sqrt{3}}{8} = \frac{4(1-2\sqrt{3})}{8} = \frac{1-2\sqrt{3}}{2} = \frac{1}{2} - \sqrt{3}$$

$$18. \quad x^2 + \frac{4}{3}x - \frac{59}{9} = 0$$

$$9x^2 + 12x - 59 = 0$$

$$x = \frac{-12 \pm \sqrt{(12)^2 - 4(9)(-59)}}{2(9)}$$

$$x = \frac{-12 \pm \sqrt{2 \cdot 268}}{18} = \frac{-12 \pm \sqrt{2^2 \cdot 3^4 \cdot 7}}{18} = \frac{-12 \pm 18\sqrt{7}}{18}$$

$$x_1 = \frac{-12+18\sqrt{7}}{18} = \frac{6(-2+3\sqrt{7})}{18} = \frac{-2+3\sqrt{7}}{3} = -\frac{2}{3} + \sqrt{7}$$

$$x_2 = \frac{-12-18\sqrt{7}}{18} = \frac{-6(2+3\sqrt{7})}{18} = \frac{-2-3\sqrt{7}}{3} = -\frac{2}{3} - \sqrt{7}$$

$$19. \quad x^2 - ax - 2a^2 = 0$$

$$(x-2a)(x+a) = 0$$

$$x_1 = 2a \quad x_2 = -a$$

$$20. \quad x^2 + 7bx + 10b^2 = 0$$

$$(x+5b)(x+2b) = 0$$

$$x_1 = -5b \quad x_2 = -2b$$

$$21. \quad x^2 - \frac{mx}{2} - \frac{m^2}{9} = 0$$

$$18x^2 - 9mx - 2m^2 = 0$$

$$(18x)^2 - 9m(18x) - 36m^2 = 0$$

$$(18x-12m)(18x+3m) = 0$$

$$(6x-4m)(6x+m) = 0$$

$$x_1 = \frac{4m}{6} = \frac{2m}{3} \quad x_2 = -\frac{m}{6}$$

## EJERCICIO 280

1.  $x^2 - 16x + 63 = 0$

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(63)}}{2}$$

$$x = \frac{16 \pm \sqrt{4}}{2} = \frac{16 \pm 2}{2}$$

$$x_1 = \frac{16+2}{2} = \frac{18}{2} = 9$$

$$x_2 = \frac{16-2}{2} = \frac{14}{2} = 7$$

$$x^2 - 16x + 63 = (x-9)(x-7)$$

2.  $x^2 + 24x + 143 = 0$

$$x = \frac{-24 \pm \sqrt{(24)^2 - 4(143)}}{2}$$

$$x = \frac{-24 \pm \sqrt{4}}{2} = \frac{-24 \pm 2}{2}$$

$$x_1 = \frac{-24+2}{2} = \frac{-22}{2} = -11$$

$$x_2 = \frac{-24-2}{2} = \frac{-26}{2} = -13$$

$$x^2 + 24x + 143 = [x - (-11)][x - (-13)] \\ = (x+11)(x+13)$$

3.  $x^2 - 26x - 155 = 0$

$$x = \frac{(-26) \pm \sqrt{(26)^2 - 4(-155)}}{2}$$

$$x = 13 \pm \sqrt{324} = 13 \pm 18$$

$$x_1 = 13+18 = 31$$

$$x_2 = 13-18 = -5$$

$$x^2 - 26x - 155 = (x-31)(x+5)$$

4.  $2x^2 + x - 6 = 0$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-6)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{49}}{4} = \frac{-1 \pm 7}{4}$$

$$x_1 = \frac{-1+7}{4} = \frac{6}{4} = \frac{3}{2}$$

$$x_2 = \frac{-1-7}{4} = \frac{-8}{4} = -2$$

$$2x^2 + x - 6 = 2\left(x - \frac{3}{2}\right)(x+2) \\ = 2\left(\frac{2x-3}{2}\right)(x+2) \\ = (2x-3)(x+2)$$

5.  $12x^2 + 5x - 2 = 0$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(12)(-2)}}{2(12)}$$

$$x = \frac{-5 \pm \sqrt{121}}{24} = \frac{-5 \pm 11}{24}$$

$$x_1 = \frac{-5+11}{24} = \frac{6}{24} = \frac{1}{4}$$

$$x_2 = \frac{-5-11}{24} = \frac{-16}{24} = -\frac{2}{3}$$

$$12x^2 + 5x - 2 = 12\left(x - \frac{1}{4}\right)\left(x + \frac{2}{3}\right) \\ = 12\left(\frac{4x-1}{4}\right)\left(\frac{3x+2}{3}\right) \\ = \frac{12(4x-1)(3x+2)}{12} \\ = (4x-1)(3x+2)$$

6.  $5x^2 + 41x + 8$

$$x = \frac{-41 \pm \sqrt{(41)^2 - 4(5)(8)}}{2(5)}$$

$$x = \frac{-41 \pm \sqrt{1.521}}{10} = \frac{-41 \pm 39}{10}$$

$$x_1 = \frac{-41+39}{10} = -\frac{2}{10} = -\frac{1}{5}$$

$$x_2 = \frac{-41-39}{10} = \frac{-80}{10} = -8$$

$$5x^2 + 41x + 8 = 5\left(x + \frac{1}{5}\right)(x+8) \\ = (5x+1)(x+8)$$

8.  $12x^2 - 25x + 12$

$$x = \frac{-(-25) \pm \sqrt{(-25)^2 - 4(12)(12)}}{2(12)}$$

$$x = \frac{25 \pm \sqrt{49}}{24} = \frac{25 \pm 7}{24}$$

$$x_1 = \frac{25+7}{24} = \frac{32}{24} = \frac{4}{3}$$

$$x_2 = \frac{25-7}{24} = \frac{18}{24} = \frac{3}{4}$$

$$12x^2 - 25x + 12 = 12\left(x - \frac{4}{3}\right)\left(x - \frac{3}{4}\right) \\ = \frac{12(3x-4)(4x-3)}{12} \\ = (3x-4)(4x-3)$$

9.  $8x^2 + 50x + 63$

$$x = \frac{50 \pm \sqrt{(50)^2 - 4(8)(63)}}{2(8)}$$

$$x = \frac{-50 \pm \sqrt{484}}{16} = \frac{-50 \pm 22}{16}$$

$$x_1 = \frac{-50+22}{16} = \frac{-28}{16} = -\frac{7}{4}$$

$$x_2 = \frac{-50-22}{16} = \frac{-72}{16} = -\frac{9}{2}$$

$$8x^2 + 50x + 63 = 8\left(x + \frac{7}{4}\right)\left(x + \frac{9}{2}\right) \\ = (4x+7)(2x+9)$$

7.  $6x^2 + 7x - 10$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(6)(-10)}}{2(6)}$$

$$x = \frac{-7 \pm \sqrt{289}}{12} = \frac{-7 \pm 17}{12}$$

$$x_1 = \frac{-7+17}{12} = \frac{10}{12} = \frac{5}{6}$$

$$x_2 = \frac{-7-17}{12} = \frac{-24}{12} = -2$$

$$6x^2 + 7x - 10 = 6\left(x - \frac{5}{6}\right)(x+2) \\ = (6x-5)(x+2)$$

10.  $27x^2 + 30x + 7$

$$x = \frac{-30 \pm \sqrt{(30)^2 - 4(27)(7)}}{2(27)}$$

$$x = \frac{-30 \pm \sqrt{144}}{54} = \frac{-30 \pm 12}{54}$$

$$x_1 = \frac{-30+12}{54} = -\frac{18}{54} = -\frac{1}{3}$$

$$x_2 = \frac{-30-12}{54} = -\frac{42}{54} = -\frac{7}{9}$$

$$27x^2 + 30x + 7 = 27\left(x + \frac{1}{3}\right)\left(x + \frac{7}{9}\right) \\ = (3x+1)(9x+7)$$



11.  $30x^2 - 61x + 30$

$$x = \frac{-(-61) \pm \sqrt{(-61)^2 - 4(30)(30)}}{2(30)}$$

$$x = \frac{61 \pm \sqrt{121}}{60} = \frac{61 \pm 11}{60}$$

$$x_1 = \frac{61+11}{60} = \frac{72}{60} = \frac{6}{5}$$

$$x_2 = \frac{61-11}{60} = \frac{50}{60} = \frac{5}{6}$$

$$30x^2 - 61x + 30 = 30\left(x - \frac{6}{5}\right)\left(x - \frac{5}{6}\right) \\ = (5x - 6)(6x - 5)$$

12.  $11x^2 - 153x - 180$

$$x = \frac{-(-153) \pm \sqrt{(-153)^2 - 4(11)(-180)}}{2(11)}$$

$$x = \frac{153 \pm \sqrt{31.329}}{22} = \frac{153 \pm 177}{22}$$

$$x_1 = \frac{153+177}{22} = \frac{330}{22} = 15$$

$$x_2 = \frac{153-177}{22} = -\frac{24}{22} = -\frac{12}{11}$$

$$11x^2 - 153x - 180 = 11\left(x - 15\right)\left(x + \frac{12}{11}\right) \\ = (x - 15)(11x + 12)$$

13.  $6 - x - x^2$

$$x^2 + x - 6$$

$$x = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - (-6)} = -\frac{1}{2} \pm \sqrt{\frac{25}{4}}$$

$$x_1 = -\frac{1}{2} + \frac{5}{2} = \frac{4}{2} = 2$$

$$x_2 = -\frac{1}{2} - \frac{5}{2} = -\frac{6}{2} = -3$$

$$x^2 + x - 6 = (x - 2)(x + 3)$$

$$6 - x - x^2 = (2 - x)(3 + x)$$

14.  $5 - 9x - 2x^2$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(-2)(5)}}{2(-2)}$$

$$x = \frac{9 \pm \sqrt{121}}{-4} = \frac{9 \pm 11}{-4}$$

$$x_1 = \frac{9+11}{-4} = \frac{20}{-4} = -5$$

$$x_2 = \frac{9-11}{-4} = \frac{-2}{-4} = \frac{1}{2}$$

Continúa

#### 14. Continuación

$$5 - 9x - 2x^2 = -2\left(x + 5\right)\left(x - \frac{1}{2}\right) \\ = -(x + 5)(2x - 1) \\ = (x + 5)(1 - 2x)$$

15.  $15 + 4x - 4x^2$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(15)(-4)}}{2(-4)} \quad x = \frac{-4 \pm \sqrt{256}}{-8}$$

$$x_1 = \frac{-4+16}{-8} = \frac{12}{-8} = -\frac{3}{2}$$

$$x_2 = \frac{-4-16}{-8} = \frac{-20}{-8} = \frac{5}{2}$$

$$15 + 4x - 4x^2 = -4\left(x + \frac{3}{2}\right)\left(x - \frac{5}{2}\right) \\ = -(2x + 3)(2x - 5) \\ = (3 + 2x)(5 - 2x)$$

16.  $4 + 13x - 12x^2$

$$12x^2 - 13x - 4$$

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(12)(-4)}}{2(12)} \quad x = \frac{13 \pm \sqrt{361}}{24}$$

$$x_1 = \frac{13+19}{24} = \frac{32}{24} = \frac{4}{3}$$

$$x_2 = \frac{13-19}{24} = \frac{-6}{24} = -\frac{1}{4}$$

$$12x^2 - 13x - 4 = 12\left(x - \frac{4}{3}\right)\left(x + \frac{1}{4}\right) \\ = (3x - 4)(1 + 4x)$$

$$4 + 13x - 12x^2 = (4 - 3x)(1 + 4x)$$

17.  $72x^2 - 55x - 7$

$$x = \frac{-(-55) \pm \sqrt{(-55)^2 - 4(72)(-7)}}{2(72)} \quad x = \frac{55 \pm \sqrt{5.041}}{144}$$

$$x_1 = \frac{55+71}{144} = \frac{126}{144} = \frac{7}{8}$$

$$x_2 = \frac{55-71}{144} = \frac{-16}{144} = -\frac{1}{9}$$

$$72x^2 - 55x - 7 = 72\left(x - \frac{7}{8}\right)\left(x + \frac{1}{9}\right) \\ = (8x - 7)(9x + 1)$$

18.  $6 + 31x - 30x^2$

$$x = \frac{-31 \pm \sqrt{(31)^2 - 4(-30)(6)}}{2(-30)}$$

$$x = \frac{-31 \pm \sqrt{1.681}}{-60}$$

$$x_1 = \frac{-31 + 41}{-60} = \frac{10}{-60} = -\frac{1}{6}$$

$$x_2 = \frac{-31 - 41}{-60} = \frac{-72}{-60} = \frac{6}{5}$$

$$\begin{aligned} 6 + 31x - 30x^2 &= -30 \left( x + \frac{1}{6} \right) \left( x - \frac{6}{5} \right) \\ &= -(6x + 1)(5x - 6) \\ &= (1 + 6x)(6 - 5x) \end{aligned}$$

19.  $10x^2 + 207x - 63$

$$x = \frac{-207 \pm \sqrt{(207)^2 - 4(10)(-63)}}{2(10)}$$

$$x = \frac{-207 \pm \sqrt{45.369}}{20}$$

$$x_1 = \frac{-207 + 213}{20} = \frac{6}{20} = \frac{3}{10}$$

$$x_2 = \frac{-207 - 213}{20} = \frac{-420}{20} = -21$$

$$\begin{aligned} 10x^2 + 207x - 63 &= 10 \left( x - \frac{3}{10} \right) (x + 21) \\ &= (10x - 3)(x + 21) \end{aligned}$$

20.  $100 - 15x - x^2$

$$x^2 + 15x - 100$$

$$x = -\frac{15}{2} \pm \sqrt{\left(\frac{15}{2}\right)^2 - (-100)}$$

$$x = -\frac{15}{2} \pm \sqrt{\frac{625}{4}}$$

$$x_1 = \frac{-15}{2} + \frac{25}{2} = \frac{10}{2} = 5$$

$$x_2 = \frac{-15}{2} - \frac{25}{2} = \frac{-40}{2} = -20$$

$$\begin{aligned} x^2 + 15x - 100 &= (x - 5)(x + 20) \\ -x^2 - 15x + 100 &= -(x - 5)(x + 20) \\ &= (5 - x)(20 + x) \end{aligned}$$

21.  $18x^2 + 31x - 49$

$$x = \frac{-31 \pm \sqrt{(31)^2 - 4(18)(-49)}}{2(18)}$$

$$x = \frac{-31 \pm \sqrt{4.489}}{36}$$

$$x_1 = \frac{-31 + 67}{36} = \frac{36}{36} = 1$$

$$x_2 = \frac{-31 - 67}{36} = \frac{-98}{36} = -\frac{49}{18}$$

$$\begin{aligned} 18x^2 + 31x - 49 &= 18(x - 1) \left( x + \frac{49}{18} \right) \\ &= (x - 1)(18x + 49) \end{aligned}$$

22.  $6x^2 - ax - 2a^2$

$$x = \frac{-(-a) \pm \sqrt{(-a)^2 - 4(6)(-2a^2)}}{2(6)} = \frac{a \pm \sqrt{49a^2}}{12}$$

$$x_1 = \frac{a + 7a}{12} = \frac{8a}{12} = \frac{2a}{3}$$

$$x_2 = \frac{a - 7a}{12} = \frac{-6a}{12} = -\frac{1}{2}a$$

$$\begin{aligned} 6x^2 - ax - 2a^2 &= 6 \left( x - \frac{2}{3}a \right) \left( x + \frac{a}{2} \right) \\ &= (3x - 2a)(2x + a) \end{aligned}$$

23.  $5x^2 + 22xy - 15y^2$

$$x = \frac{-22y \pm \sqrt{(22y)^2 - 4(5)(-15y^2)}}{2(5)}$$

$$x = \frac{-22y \pm \sqrt{784y^2}}{10} = \frac{-22 \pm 28y}{10}$$

$$x_1 = \frac{-22y + 28y}{10} = \frac{6y}{10} = \frac{3}{5}y$$

$$x_2 = \frac{-22y - 28y}{10} = \frac{-50y}{10} = -5y$$

$$\begin{aligned} 5x^2 + 22xy - 15y^2 &= 5 \left( x - \frac{3y}{5} \right) (x + 5y) \\ &= (5x - 3y)(x + 5y) \end{aligned}$$

24.  $15x^2 - 32mx - 7m^2$

$$x = \frac{-(-32m) \pm \sqrt{(-32m)^2 - 4(15)(-7m^2)}}{2(15)}$$

$$x = \frac{32m \pm \sqrt{1.444m^2}}{30} = \frac{32m \pm 38m}{30}$$

$$x_1 = \frac{32m + 38m}{30} = \frac{70m}{30} = \frac{7m}{3}$$

$$x_2 = \frac{32m - 38m}{30} = \frac{-6m}{30} = -\frac{m}{5}$$

$$\begin{aligned} 15x^2 - 32mx - 7m^2 &= 15 \left( x + \frac{m}{5} \right) \left( x - \frac{7m}{3} \right) \\ &= (5x + m)(3x - 7m) \end{aligned}$$

## EJERCICIO 282

1.  $x^4 - 1 = 0$

$$(x^2 + 1)(x^2 - 1) = 0$$

$$x^2 + 1 = 0 \quad x^2 - 1 = 0$$

$$x^2 = -1 \quad x^2 = 1$$

$$x = \pm \sqrt{-1} \quad x = \pm \sqrt{1}$$

$$x = \pm i \quad x = \pm 1$$

2.  $x^3 + 1 = 0$

$$(x+1)(x^2 - x + 1) = 0$$

$$x+1=0$$

$$x_1 = -1$$

$$x^2 - x + 1 = 0$$

$$x = -\frac{(-1)}{2} \pm \sqrt{\frac{(-1)^2}{4} - 1}$$

$$x = \frac{1}{2} \pm \sqrt{-\frac{3}{4}} = \frac{1}{2} \pm \frac{\sqrt{3}\sqrt{-1}}{2}$$

$$x_2 = \frac{1}{2} + \frac{i\sqrt{3}}{2} = \frac{1+i\sqrt{3}}{2}$$

$$x_3 = \frac{1}{2} - \frac{i\sqrt{3}}{2} = \frac{1-i\sqrt{3}}{2}$$

3.  $x^4 = 81$

$$x^4 - 81 = 0$$

$$(x^2 - 9)(x^2 + 9) = 0$$

$$(x-3)(x+3)(x^2 + 9) = 0$$

$$x-3=0 \quad x+3=0$$

$$x_1 = 3 \quad x_2 = -3$$

$$x^2 + 9 = 0$$

$$x^2 = -9$$

$$x = \pm \sqrt{9}\sqrt{-1} = \pm \sqrt{9}i$$

$$x_3 = 3i \quad x_4 = -3i$$

4.  $x^4 - 256 = 0$

$$(x^2 - 16)(x^2 + 16) = 0$$

$$(x+4)(x-4)(x^2 + 16) = 0$$

$$x+4=0 \quad x-4=0$$

$$x_1 = -4 \quad x_2 = 4$$

$$x^2 + 16 = 0$$

$$x^2 = -16$$

$$x = \pm \sqrt{16}\sqrt{-1} = \pm 4i$$

$$x_3 = 4i \quad x_4 = -4i$$

5.  $(x^3 + 8) = 0$

$$(x+2)(x^2 - 2x + 4) = 0$$

$$x+2=0$$

$$x_1 = -2$$

$$x^2 - 2x + 4 = 0$$

$$x = \frac{-(-2)}{2} \pm \sqrt{\frac{(-2)^2}{4} - 4}$$

$$x = 1 \pm \sqrt{-\frac{12}{4}} = 1 \pm \frac{2\sqrt{3}\sqrt{-1}}{2} = 1 \pm i\sqrt{3}$$

$$x_2 = 1 + i\sqrt{3} \quad x_3 = 1 - i\sqrt{3}$$

6.  $x^4 - 625 = 0$

$$(x^2 - 25)(x^2 + 25) = 0$$

$$(x+5)(x-5)(x^2 + 25) = 0$$

$$x+5=0 \quad x-5=0$$

$$x_1 = -5 \quad x_2 = 5$$

$$x^2 + 25 = 0$$

$$x^2 = -25$$

$$x = \pm \sqrt{25}\sqrt{-1} = \pm 5i$$

$$x_3 = 5i \quad x_4 = -5i$$

7.  $x^3 + 64 = 0$

$$(x+4)(x^2 - 4x + 16) = 0$$

$$x+4=0$$

$$x_1 = -4$$

$$x^2 - 4x + 16 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(16)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 64}}{2}$$

$$x_2 = \frac{4 + \sqrt{-48}}{2} = \frac{4 + 4\sqrt{3}\sqrt{-1}}{2} = 2 + 2\sqrt{3}i$$

$$x_3 = \frac{4 - \sqrt{-48}}{2} = \frac{4 - 4\sqrt{3}\sqrt{-1}}{2} = 2 - 2\sqrt{3}i$$

8.  $x^6 - 729 = 0$

$$(x^3 - 27)(x^3 + 27) = 0$$

$$(x-3)(x^2 + 3x + 9)(x+3)(x^2 - 3x + 9) = 0$$

$$x-3=0 \quad x+3=0$$

$$x_1 = 3 \quad x_2 = -3$$

$$x^2 + 3x + 9 = 0$$

Continúa

## 8. Continuación

$$x = \frac{-3}{2} \pm \sqrt{\frac{(3)^2}{4} - 9}$$

$$x = \frac{-3}{2} \pm \frac{\sqrt{27}\sqrt{-1}}{\sqrt{4}} = \frac{-3}{2} \pm \frac{3\sqrt{3}i}{2}$$

$$x_3 = \frac{-3 - 3\sqrt{3}i}{2} \quad x_4 = \frac{-3 + 3\sqrt{3}i}{2}$$

$$x^2 - 3x + 9 = 0$$

$$x = \frac{-(-3)}{2} \pm \sqrt{\frac{(-3)^2}{4} - 9}$$

$$x = \frac{3}{2} \pm \frac{\sqrt{27}\sqrt{-1}}{\sqrt{4}} = \frac{3}{2} \pm \frac{3\sqrt{3}i}{2}$$

$$x_5 = \frac{3 + 3\sqrt{3}i}{2} \quad x_6 = \frac{3 - 3\sqrt{3}i}{2}$$

9.  $x^3 = 8$

$$x^3 - 8 = 0$$

$$(x-2)(x^2 + 2x + 4) = 0$$

$$x-2=0$$

$$x_1 = 2$$

$$x^2 + 2x + 4 = 0$$

$$x = \frac{-2}{2} \pm \sqrt{\frac{(2)^2}{4} - 4}$$

$$x = -1 \pm \sqrt{3}\sqrt{-1}$$

$$x = -1 \pm \sqrt{3}i$$

$$x_2 = -1 + \sqrt{3}i$$

$$x_3 = -1 - \sqrt{3}i$$

10.  $x^4 = 64$

$$x^4 - 64 = 0$$

$$(x^2 - 8)(x^2 + 8) = 0$$

$$x^2 - 8 = 0 \quad x^2 + 8 = 0$$

$$x = \sqrt{2^2 \cdot 2} \quad x = \sqrt{8}\sqrt{-1}$$

$$x = \pm 2\sqrt{2} \quad x = \pm 2\sqrt{2}i$$

$$x_1 = 2\sqrt{2} \quad x_3 = 2\sqrt{2}i$$

$$x_2 = -2\sqrt{2} \quad x_4 = -2\sqrt{2}i$$

## EJERCICIO 283

1.  $x^4 - 10x^2 + 9 = 0$   
 $(x^2 - 9)(x^2 - 1) = 0$   
 $(x+3)(x-3)(x+1)(x-1) = 0$   
 $x+3=0 \quad x-3=0$   
 $x_1 = -3 \quad x_2 = 3$   
 $x+1=0 \quad x-1=0$   
 $x_3 = -1 \quad x_4 = 1$
2.  $x^4 - 13x^2 + 36 = 0$   
 $(x^2 - 4)(x^2 - 9) = 0$   
 $x^2 - 4 = 0 \quad x^2 - 9 = 0$   
 $x^2 = 4 \quad x^2 = 9$   
 $x = \pm \sqrt{4} \quad x = \pm \sqrt{9}$   
 $x_1 = 2 \quad x_3 = 3$   
 $x_2 = -2 \quad x_4 = -3$
3.  $x^4 - 29x^2 + 100 = 0$   
 $(x^2 - 25)(x^2 - 4) = 0$   
 $(x+5)(x-5)(x+2)(x-2) = 0$   
 $x+5=0 \quad x-5=0$   
 $x_1 = -5 \quad x_2 = 5$   
 $x+2=0 \quad x-2=0$   
 $x_3 = -2 \quad x_4 = 2$
4.  $x^4 - 61x^2 + 900 = 0$   
 $(x^2 - 36)(x^2 - 25) = 0$   
 $x^2 - 36 = 0 \quad x^2 - 25 = 0$   
 $x^2 = 36 \quad x^2 = 25$   
 $x = \pm \sqrt{36} \quad x = \pm \sqrt{25}$   
 $x_1 = 6 \quad x_3 = 5$   
 $x_2 = -6 \quad x_4 = -5$
5.  $x^4 + 3x^2 - 4 = 0$   
 $(x^2 + 4)(x^2 - 1) = 0$   
 $(x^2 + 4)(x+1)(x-1) = 0$   
 $x+1=0 \quad x-1=0$   
 $x_1 = -1 \quad x_2 = 1$   
 $x^2 + 4 = 0$   
 $x^2 = -4$   
 $x = \sqrt{4} \sqrt{-1} = \pm 2i$   
 $x_3 = 2i \quad x_4 = -2i$
6.  $x^4 + 16x^2 - 225 = 0$   
 $(x^2 + 25)(x^2 - 9) = 0$   
 $(x^2 + 25)(x+3)(x-3) = 0$   
 $x+3=0 \quad x-3=0$   
 $x_1 = -3 \quad x_2 = 3$   
 $x^2 + 25 = 0$   
 $x = \pm \sqrt{25} \sqrt{-1}$   
 $x_3 = 5i \quad x_4 = -5i$
7.  $x^4 - 45x^2 - 196 = 0$   
 $(x^2 - 49)(x^2 + 4) = 0$   
 $(x+7)(x-7)(x^2 + 4) = 0$   
 $x+7=0 \quad x-7=0$   
 $x_1 = -7 \quad x_2 = 7$   
 $x^2 + 4 = 0$   
 $x^2 = -4$   
 $x = \sqrt{4} \sqrt{-1} = \pm 2i$   
 $x_3 = 2i \quad x_4 = -2i$
8.  $x^4 - 6x^2 + 5 = 0$   
 $(x^2 - 5)(x^2 - 1) = 0$   
 $(x^2 - 5)(x+1)(x-1) = 0$   
 $x+1=0 \quad x-1=0$   
 $x_1 = -1 \quad x_2 = 1$   
 $x^2 - 5 = 0$   
 $x^2 = 5$   
 $x = \pm \sqrt{5}$   
 $x_3 = \sqrt{5} \quad x_4 = -\sqrt{5}$
9.  $4x^4 - 37x^2 + 9 = 0$   
 $(4x^2)^2 - 37(4x^2) + 36 = 0$   
 $(4x^2 - 36)(4x^2 - 1) = 0$   
 $(x^2 - 9)(4x^2 - 1) = 0$   
 $x^2 - 9 = 0 \quad 4x^2 - 1 = 0$   
 $x^2 = 9 \quad x = \pm \sqrt{\frac{1}{4}}$   
 $x = \pm \sqrt{9}$   
 $x_1 = 3 \quad x_3 = \frac{1}{2}$   
 $x_2 = -3 \quad x_4 = -\frac{1}{2}$
10.  $9x^4 - 40x^2 + 16 = 0$   
 $(9x^2)^2 - 40(9x^2) + 144 = 0$   
 $(9x^2 - 36)(9x^2 - 4) = 0$   
 $(x^2 - 4)(9x^2 - 4) = 0$   
 $(x+2)(x-2)(9x^2 - 4) = 0$   
 $x+2=0 \quad x-2=0$   
 $x_1 = -2 \quad x_2 = 2$   
 $9x^2 - 4 = 0$   
 $x = \pm \sqrt{\frac{4}{9}}$   
 $x_3 = \frac{2}{3} \quad x_4 = -\frac{2}{3}$
11.  $25x^4 + 9x^2 - 16 = 0$   
 $25(x^2)^2 + 9x^2 - 16 = 0$   
 $x^2 = \frac{-9 \pm \sqrt{(9)^2 - 4(25)(-16)}}{2(25)}$   
 $x^2 = \frac{-9 \pm \sqrt{1.681}}{50} = \frac{-9 \pm 41}{50}$   
 $x^2 = \frac{-9 + 41}{50} = \frac{32}{50} = \frac{16}{25}$   
 $x = \pm \sqrt{\frac{16}{25}} \quad x_1 = \frac{4}{5} \quad x_2 = -\frac{4}{5}$   
 $x^2 = \frac{-9 - 41}{50} = \frac{-50}{50} = -1$   
 $x = \pm \sqrt{-1} \quad x_3 = i \quad x_4 = -i$
12.  $4x^4 + 11x^2 - 3 = 0$   
 $(4x^2)^2 + 11(4x^2) - 12 = 0$   
 $(4x^2 + 12)(4x^2 - 1) = 0$   
 $(x^2 + 3)(4x^2 - 1) = 0$   
 $x^2 + 3 = 0$   
 $x^2 = -3$   
 $x = \pm \sqrt{-3} = \pm \sqrt{3} \sqrt{-1} = \pm \sqrt{3}i$   
 $x_1 = \sqrt{3}i \quad x_2 = -\sqrt{3}i$   
 $4x^2 - 1 = 0$   
 $(2x+1)(2x-1) = 0$   
 $2x+1=0 \quad 2x-1=0$   
 $x_3 = -\frac{1}{2} \quad x_4 = \frac{1}{2}$

$$13. (2x^2 + 1)^2 - (x^2 - 3)^2 = 80$$

$$4x^4 + 4x^2 + 1 - x^4 + 6x^2 - 9 = 80$$

$$3x^4 + 10x^2 - 88 = 0$$

$$(3x^2)^2 + 10(3x^2) - 264 = 0$$

$$(3x^2 + 22)(3x^2 - 12) = 0$$

$$(3x^2 + 22)(x^2 - 4) = 0$$

$$(3x^2 + 22)(x + 2)(x - 2) = 0$$

$$x + 2 = 0 \quad x - 2 = 0$$

$$x_1 = -2 \quad x_2 = 2$$

$$3x^2 + 22 = 0$$

$$x^2 = \frac{-22}{3}$$

$$x = \pm \frac{\sqrt{22} \sqrt{-1}}{\sqrt{3}}$$

$$x_3 = i \frac{\sqrt{22}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{i\sqrt{66}}{3}$$

$$x_4 = -i \frac{\sqrt{22}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-i\sqrt{66}}{3}$$

$$14. x^2(3x^2 + 2) = 4(x^2 - 3) + 13$$

$$3x^4 + 2x^2 = 4x^2 - 12 + 13$$

$$3x^4 - 2x^2 - 1 = 0$$

$$(3x^2)^2 - 2(3x^2) - 3 = 0$$

$$(3x^2 - 3)(3x^2 + 1) = 0$$

$$(x^2 - 1)(3x^2 + 1) = 0$$

$$(x + 1)(x - 1)(3x^2 + 1) = 0$$

$$x + 1 = 0 \quad x - 1 = 0$$

$$x_1 = -1 \quad x_2 = 1$$

$$3x^2 + 1 = 0$$

$$x = \frac{\sqrt{-1}}{\sqrt{3}} = \pm \frac{i}{\sqrt{3}}$$

$$x_3 = \frac{i}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{i\sqrt{3}}{3}$$

$$x_4 = -\frac{i}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{i\sqrt{3}}{3}$$

## EJERCICIO 284

$$1. x^6 - 7x^3 - 8 = 0$$

$$(x^3 - 8)(x^3 + 1) = 0$$

$$x^3 - 8 = 0 \quad x^3 + 1 = 0$$

$$x^3 = 8 \quad x^3 = -1$$

$$x = \sqrt[3]{8} \quad x = \sqrt[3]{-1}$$

$$x_1 = 2 \quad x_2 = -1$$

$$(x^3 - 8) = 0$$

$$(x - 2)(x^2 + 2x + 4) = 0$$

$$x^2 + 2x + 4 = 0$$

$$(\text{ver ejerc. 282 prob.9})$$

$$x_3 = -1 + \sqrt{3}i$$

$$x_4 = -1 - \sqrt{3}i$$

$$(x^3 + 1) = 0$$

$$(x + 1)(x^2 - x + 1)$$

$$(\text{ver ejerc. 282 prob.2})$$

$$x_5 = \frac{1 + \sqrt{3}i}{2} \quad x_6 = \frac{1 - \sqrt{3}i}{2}$$

$$2. x^6 + 30x^3 + 81 = 0$$

$$(x^3 + 27)(x^3 + 3) = 0$$

$$x^3 + 27 = 0 \quad x^3 + 3 = 0$$

$$x^3 = -27 \quad x^3 = -3$$

$$x = \sqrt[3]{-27} \quad x = \sqrt[3]{-3}$$

$$x = -3 \quad x = -\sqrt[3]{3}$$

$$3. 8x^6 + 15x^3 - 2 = 0$$

$$(8x^3)^2 + 15(8x^3) - 16 = 0$$

$$(8x^3 + 16)(8x^3 - 1) = 0$$

$$(x^3 + 2)(8x^3 - 1) = 0$$

$$x^3 + 2 = 0 \quad 8x^3 - 1 = 0$$

$$x^3 = -2 \quad x = \sqrt[3]{\frac{1}{8}}$$

$$x = \sqrt[3]{-2} \quad x = \frac{1}{2}$$

$$x = -\sqrt[3]{2}$$

$$4. x^8 - 41x^4 + 400 = 0$$

$$(x^4 - 25)(x^4 - 16) = 0$$

$$(x^2 + 5)(x^2 - 5)(x^2 + 4)(x^2 - 4) = 0$$

$$x^2 - 5 = 0 \quad x^2 - 4 = 0$$

$$x = \pm\sqrt{5} \quad x = \pm\sqrt{4}$$

$$x = \pm 2$$

$$5. x^{10} - 33x^5 + 32 = 0$$

$$(x^5 - 32)(x^5 - 1) = 0$$

$$x^5 - 32 = 0 \quad x^5 - 1 = 0$$

$$x = \sqrt[5]{32} \quad x = \sqrt[5]{1}$$

$$x = 2 \quad x = 1$$

$$6. x^{-4} - 13x^{-2} + 36 = 0$$

$$(x^{-2} - 9)(x^{-2} - 4) = 0$$

$$x^{-2} - 9 = 0 \quad x^{-2} - 4 = 0$$

$$x^{-2} = 9 \quad x^{-2} = 4$$

$$\frac{1}{x^2} = 9 \quad \frac{1}{x^2} = 4$$

$$\frac{1}{9} = x^2 \quad \frac{1}{4} = x^2$$

$$\pm \sqrt{\frac{1}{9}} = x \quad \pm \sqrt{\frac{1}{4}} = x$$

$$x_1 = \frac{1}{3}; \quad x_2 = -\frac{1}{3}; \quad x_3 = \frac{1}{2}; \quad x_4 = -\frac{1}{2}$$

$$\begin{aligned}
 7. \quad & x^{-6} + 35x^{-3} = -216 \\
 & x^{-6} + 35x^{-3} + 216 = 0 \\
 & (x^{-3} + 27)(x^{-3} + 8) = 0 \\
 & x^{-3} + 27 = 0 \quad x^{-3} + 8 = 0 \\
 & x^{-3} = -27 \quad x^{-3} = -8 \\
 & \frac{1}{x^3} = -27 \quad \frac{1}{x^3} = -8 \\
 & \sqrt[3]{\frac{1}{-27}} = x \quad \sqrt[3]{\frac{1}{-8}} = x \\
 & x = -\frac{1}{3} \quad x = -\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & x + x^{\frac{1}{2}} = 6 \\
 & x + x^{\frac{1}{2}} - 6 = 0 \\
 & \left(x^{\frac{1}{2}} + 3\right)\left(x^{\frac{1}{2}} - 2\right) = 0 \\
 & x^{\frac{1}{2}} + 3 = 0 \quad x^{\frac{1}{2}} - 2 = 0 \\
 & \sqrt{x} = -3 \quad \sqrt{x} = 2 \\
 & x = (-3)^2 \quad x = (2)^2 \\
 & x = 9 \quad x = 4
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & x^{-10} = 242x^{-5} + 243 \\
 & x^{-10} - 242x^{-5} - 243 = 0 \\
 & (x^{-5} - 243)(x^{-5} + 1) = 0 \\
 & x^{-5} - 243 = 0 \quad x^{-5} + 1 = 0 \\
 & x^{-5} = 243 \quad x^{-5} = -1 \\
 & \frac{1}{x^5} = 243 \quad \frac{1}{x^5} = -1 \\
 & \sqrt[5]{\frac{1}{243}} = x \quad \sqrt[5]{\frac{1}{-1}} = x \\
 & x = \frac{1}{3} \quad x = -1
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & 3x = 16\sqrt{x} - 5 \\
 & 3x - 16x^{\frac{1}{2}} + 5 = 0 \\
 & 9x - 16\left(3x^{\frac{1}{2}}\right) + 15 = 0 \\
 & \left(3x^{\frac{1}{2}} - 15\right)\left(3x^{\frac{1}{2}} - 1\right) = 0 \\
 & 3x^{\frac{1}{2}} - 15 = 0 \quad 3x^{\frac{1}{2}} - 1 = 0 \\
 & \sqrt{x} = \frac{15}{3} \quad \sqrt{x} = \frac{1}{3} \\
 & x = (5)^2 \quad x = \left(\frac{1}{3}\right)^2 \\
 & x = 25 \quad x = \frac{1}{9}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & x^3 - 9x^{\frac{3}{2}} + 8 = 0 \\
 & \left(x^{\frac{3}{2}} - 8\right)\left(x^{\frac{3}{2}} - 1\right) = 0 \\
 & x^{\frac{3}{2}} - 8 = 0 \quad x^{\frac{3}{2}} - 1 = 0 \\
 & \sqrt{x^3} = 8 \quad \sqrt{x^3} = 1 \\
 & x^3 = 64 \quad x^3 = 1 \\
 & x = \sqrt[3]{64} \quad x = 1 \\
 & x = 4
 \end{aligned}$$

$$\begin{aligned}
 12. \quad & 2x^{\frac{1}{2}} - 5x^{\frac{1}{4}} + 2 = 0 \\
 & 4x^{\frac{1}{2}} - 5\left(2x^{\frac{1}{4}}\right) + 4 = 0 \\
 & \left(2x^{\frac{1}{4}} - 4\right)\left(2x^{\frac{1}{4}} - 1\right) = 0 \\
 & \left(x^{\frac{1}{4}} - 2\right)\left(2x^{\frac{1}{4}} - 1\right) = 0 \\
 & x^{\frac{1}{4}} - 2 = 0 \quad 2x^{\frac{1}{4}} - 1 = 0 \\
 & x^{\frac{1}{4}} = 2 \quad 2\sqrt[4]{x} = 1 \\
 & \sqrt[4]{x} = 2 \quad x = \left(\frac{1}{2}\right)^4 \\
 & x = (2)^4 \quad x = \frac{1}{16} \\
 & x = 16
 \end{aligned}$$

## **EJERCICIO 285**

$$\begin{aligned}
 1. \quad & \sqrt{5 + \sqrt{24}} \\
 & m = \sqrt{(5)^2 - 24} \\
 & m = \sqrt{1} \\
 & m = 1 \\
 & = \sqrt{\frac{5+1}{2}} + \sqrt{\frac{5-1}{2}} \\
 & = \sqrt{3} + \sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \sqrt{8 - \sqrt{60}} \\
 & m = \sqrt{(8)^2 - 60} \\
 & m = \sqrt{4} \\
 & m = 2 \\
 & = \sqrt{\frac{8+2}{2}} - \sqrt{\frac{8-2}{2}} \\
 & = \sqrt{5} - \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \sqrt{8 + \sqrt{28}} \\
 & m = \sqrt{(8)^2 - 28} \\
 & m = \sqrt{36} \\
 & m = 6 \\
 & = \sqrt{\frac{8+6}{2}} + \sqrt{\frac{8-6}{2}} \\
 & = \sqrt{7} + 1
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \sqrt{32 - \sqrt{700}} \\
 & m = \sqrt{(32)^2 - 700} \\
 & m = \sqrt{324} = 18 \\
 & = \sqrt{\frac{32+18}{2}} - \sqrt{\frac{32-18}{2}} \\
 & = \sqrt{25} - \sqrt{7} \\
 & = 5 - \sqrt{7}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \sqrt{14 + \sqrt{132}} \\
 & m = \sqrt{(14)^2 - 132} \\
 & m = \sqrt{64} = 8 \\
 & = \sqrt{\frac{14+8}{2}} + \sqrt{\frac{14-8}{2}} \\
 & = \sqrt{11} + \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \sqrt{13 + \sqrt{88}} \\
 & m = \sqrt{(13)^2 - 88} \\
 & m = \sqrt{81} = 9 \\
 & = \sqrt{\frac{13+9}{2}} + \sqrt{\frac{13-9}{2}} \\
 & = \sqrt{11} + \sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & \sqrt{11 + 2\sqrt{30}} \\
 & \sqrt{11 + \sqrt{120}} \\
 & m = \sqrt{(11)^2 - 120} = 1 \\
 & = \sqrt{\frac{11+1}{2}} + \sqrt{\frac{11-1}{2}} \\
 & = \sqrt{6} + \sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & \sqrt{84 - 18\sqrt{3}} \\
 & \sqrt{84 - \sqrt{972}} \\
 & m = \sqrt{(84)^2 - 972} \\
 & m = \sqrt{6.084} = 78 \\
 & = \sqrt{\frac{84+78}{2}} - \sqrt{\frac{84-78}{2}} \\
 & = \sqrt{81} - \sqrt{3} = 9 - \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 9. & \sqrt{21+6\sqrt{10}} \\
 & \sqrt{21+\sqrt{360}} \\
 & m = \sqrt{(21)^2 - 360} \\
 & m = \sqrt{81} = 9 \\
 & = \sqrt{\frac{21+9}{2}} + \sqrt{\frac{21-9}{2}} = \sqrt{15} + \sqrt{6}
 \end{aligned}$$

$$\begin{aligned}
 10. & \sqrt{28+14\sqrt{3}} \\
 & \sqrt{28+\sqrt{588}} \\
 & m = \sqrt{(28)^2 - 588} \\
 & m = \sqrt{196} = 14 \\
 & = \sqrt{\frac{28+14}{2}} + \sqrt{\frac{28-14}{2}} \\
 & = \sqrt{21} + \sqrt{7}
 \end{aligned}$$

$$\begin{aligned}
 11. & \sqrt{14-4\sqrt{6}} \\
 & \sqrt{14-\sqrt{96}} \\
 & m = \sqrt{(14)^2 - 96} \\
 & m = \sqrt{100} = 10 \\
 & = \sqrt{\frac{14+10}{2}} - \sqrt{\frac{14-10}{2}} \\
 & = \sqrt{12} - \sqrt{2} \\
 & = \sqrt{2^2 \cdot 3} - \sqrt{2} = 2\sqrt{3} - \sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 12. & \sqrt{55+30\sqrt{2}} \\
 & \sqrt{55+\sqrt{1.800}} \\
 & m = \sqrt{(55)^2 - 1.800} \\
 & m = \sqrt{1.225} = 35 \\
 & = \sqrt{\frac{55+35}{2}} + \sqrt{\frac{55-35}{2}} \\
 & = \sqrt{45} + \sqrt{10} = 3\sqrt{5} + \sqrt{10}
 \end{aligned}$$

$$\begin{aligned}
 13. & \sqrt{73-12\sqrt{35}} \\
 & \sqrt{73-\sqrt{5.040}} \\
 & m = \sqrt{(73)^2 - 5.040} \\
 & m = \sqrt{289} = 17 \\
 & = \sqrt{\frac{73+17}{2}} - \sqrt{\frac{73-17}{2}} \\
 & = \sqrt{45} - \sqrt{28} = 3\sqrt{5} - 2\sqrt{7}
 \end{aligned}$$

$$\begin{aligned}
 14. & \sqrt{253-60\sqrt{7}} \\
 & \sqrt{253-\sqrt{25.200}} \\
 & m = \sqrt{(253)^2 - 25.200} \\
 & m = \sqrt{38.809} = 197 \\
 & = \sqrt{\frac{253+197}{2}} - \sqrt{\frac{253-197}{2}} \\
 & = \sqrt{225} - \sqrt{28} = 15 - 2\sqrt{7}
 \end{aligned}$$

$$\begin{aligned}
 15. & \sqrt{293-30\sqrt{22}} \\
 & \sqrt{293-\sqrt{19.800}} \\
 & m = \sqrt{(293)^2 - 19.800} \\
 & m = \sqrt{66.049} = 257 \\
 & = \sqrt{\frac{293+257}{2}} - \sqrt{\frac{293-257}{2}} \\
 & = \sqrt{275} - \sqrt{18} = 5\sqrt{11} - 3\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 16. & \sqrt{\frac{5}{6} + \sqrt{\frac{2}{3}}} \\
 & m = \sqrt{\left(\frac{5}{6}\right)^2 - \frac{2}{3}} \\
 & m = \sqrt{\frac{1}{36}} = \frac{1}{6} \\
 & = \sqrt{\frac{\frac{5}{6} + \frac{1}{6}}{2}} + \sqrt{\frac{\frac{5}{6} - \frac{1}{6}}{2}} \\
 & = \sqrt{\frac{1}{2}} + \sqrt{\frac{1}{3}} = \frac{\sqrt{1}\sqrt{2}}{\sqrt{2}\sqrt{2}} + \frac{\sqrt{1}\sqrt{3}}{\sqrt{3}\sqrt{3}} \\
 & = \frac{1}{2}\sqrt{2} + \frac{1}{3}\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 17. & \sqrt{\frac{3}{4} - \sqrt{\frac{1}{2}}} \\
 & m = \sqrt{\left(\frac{3}{4}\right)^2 - \frac{1}{2}} \\
 & m = \sqrt{\frac{1}{16}} = \frac{1}{4} \\
 & = \sqrt{\frac{\frac{3}{4} + \frac{1}{4}}{2}} - \sqrt{\frac{\frac{3}{4} - \frac{1}{4}}{2}} \\
 & = \sqrt{\frac{1}{2}} - \sqrt{\frac{1}{4}} = \frac{\sqrt{2}}{2} - \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 18. & \sqrt{\frac{9}{16} + \sqrt{\frac{1}{8}}} \\
 & m = \sqrt{\left(\frac{9}{16}\right)^2 - \frac{1}{8}} \\
 & m = \sqrt{\frac{49}{256}} = \frac{7}{16} \\
 & = \sqrt{\frac{\frac{9}{16} + \frac{7}{16}}{2}} + \sqrt{\frac{\frac{9}{16} - \frac{7}{16}}{2}} \\
 & = \sqrt{\frac{1}{2}} + \sqrt{\frac{1}{16}} = \frac{\sqrt{2}}{2} + \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 19. & \sqrt{6+4\sqrt{2}} \\
 & \sqrt{6+\sqrt{32}} \\
 & m = \sqrt{(6)^2 - 32} \\
 & m = \sqrt{4} = 2 \\
 & = \sqrt{\frac{6+2}{2}} + \sqrt{\frac{6-2}{2}} \\
 & = \sqrt{4} + \sqrt{2} = 2 + \sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 20. & \sqrt{7+4\sqrt{3}} \\
 & \sqrt{7+\sqrt{48}} \\
 & m = \sqrt{(7)^2 - 48} = 1 \\
 & = \sqrt{\frac{7+1}{2}} + \sqrt{\frac{7-1}{2}} \\
 & = \sqrt{4} + \sqrt{3} = 2 + \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 21. & \sqrt{8+2\sqrt{7}} \\
 & \sqrt{8+\sqrt{28}} \\
 & m = \sqrt{(8)^2 - 28} \\
 & m = \sqrt{36} = 6 \\
 & = \sqrt{\frac{8+6}{2}} + \sqrt{\frac{8-6}{2}} \\
 & = \sqrt{7} + 1
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & \sqrt{10+2\sqrt{21}} \\
 & \sqrt{10+\sqrt{84}} \\
 & m = \sqrt{(10)^2 - 84} \\
 & m = \sqrt{16} = 4 \\
 & = \sqrt{\frac{10+4}{2}} + \sqrt{\frac{10-4}{2}} \\
 & = \sqrt{7} + \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & \sqrt{18+6\sqrt{5}} \\
 & \sqrt{18+\sqrt{180}} \\
 & m = \sqrt{(18)^2 - 180} \\
 & m = \sqrt{144} = 12 \\
 & = \sqrt{\frac{18+12}{2}} + \sqrt{\frac{18-12}{2}} \\
 & = \sqrt{15} + \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & \sqrt{24-2\sqrt{143}} \\
 & \sqrt{24-\sqrt{572}} \\
 & m = \sqrt{(24)^2 - 572} \\
 & m = \sqrt{4} = 2 \\
 & = \sqrt{\frac{24+2}{2}} - \sqrt{\frac{24-2}{2}} \\
 & = \sqrt{13} - \sqrt{11}
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & \sqrt{30-20\sqrt{2}} \\
 & \sqrt{30-\sqrt{800}} \\
 & m = \sqrt{(30)^2 - 800} \\
 & m = \sqrt{100} = 10 \\
 & = \sqrt{\frac{30+10}{2}} - \sqrt{\frac{30-10}{2}} \\
 & = \sqrt{20} - \sqrt{10} = 2\sqrt{5} - \sqrt{10}
 \end{aligned}$$

$$\begin{aligned}
 26. \quad & \sqrt{9+6\sqrt{2}} \\
 & \sqrt{9+\sqrt{72}} \\
 & m = \sqrt{(9)^2 - 72} \\
 & m = \sqrt{9} = 3 \\
 & = \sqrt{\frac{9+3}{2}} + \sqrt{\frac{9-3}{2}} \\
 & = \sqrt{6} + \sqrt{3} \\
 27. \quad & \sqrt{98-24\sqrt{5}} \\
 & \sqrt{98-\sqrt{2.880}} \\
 & m = \sqrt{(98)^2 - 2.880} \\
 & m = \sqrt{6.724} = 82 \\
 & = \sqrt{\frac{98+82}{2}} - \sqrt{\frac{98-82}{2}} \\
 & = \sqrt{90} - \sqrt{8} = 3\sqrt{10} - 2\sqrt{2}
 \end{aligned}$$

## EJERCICIO 286

Los problemas de este ejercicio se resuelven aplicando la siguiente formula:

$u = a + (n - 1)r$  donde:

$u$  es el término enésimo ;  $a$  es el primer termino de la progresión ;  $n$  es la cantidad de términos ;  $r$  es la razón de cambio.

$$1. \quad a=7 \quad n=9 \quad r=10-7=3$$

$$u = 7 + 8(3) = 7 + 24 = 31$$

$$2. \quad a=5 \quad n=12 \quad r=10-5=5$$

$$u = 5 + 11(5) = 5 + 55 = 60$$

$$3. \quad a=9 \quad n=48 \quad r=15-12=3$$

$$u = 9 + 47(3) = 9 + 141 = 150$$

$$4. \quad a=3 \quad n=63 \quad r=17-10=7$$

$$u = 3 + 62(7) = 3 + 434 = 437$$

$$5. \quad a=11 \quad n=12 \quad r=6-11=-5$$

$$u = 11 + 11(-5) = 11 - 55 = -44$$

$$6. \quad a=19 \quad n=28 \quad r=5-12=-7$$

$$u = 19 + 27(-7) = 19 - 189 = -170$$

$$7. \quad a=3 \quad n=13 \quad r=-5-(-1)=-4$$

$$u = 3 + 12(-4) = 3 - 48 = -45$$

$$8. \quad a=8 \quad n=54 \quad r=0-8=-8$$

$$u = 8 + 53(-8) = 8 - 424 = -416$$

$$9. \quad a=-7 \quad n=31 \quad r=1-(-3)=4$$

$$u = -7 + 30(4) = -7 + 120 = 113$$

$$10. \quad a=-8 \quad n=17 \quad r=12-2=10$$

$$u = -8 + 16(10) = -8 + 160 = 152$$

$$11. \quad a=\frac{1}{2} \quad n=12 \quad r=\frac{3}{4}-\frac{1}{2}=\frac{1}{4}$$

$$u = \frac{1}{2} + 11\left(\frac{1}{4}\right) = \frac{1}{2} + \frac{11}{4} = \frac{13}{4} = 3\frac{1}{4}$$

$$12. \quad a=\frac{2}{3} \quad n=17 \quad r=\frac{5}{6}-\frac{2}{3}=\frac{1}{6}$$

$$u = \frac{2}{3} + 16\left(\frac{1}{6}\right)$$

$$u = \frac{2}{3} + \frac{8}{3} = \frac{10}{3} = 3\frac{1}{3}$$

$$13. \quad a=\frac{3}{8} \quad n=25 \quad r=\frac{11}{24}-\frac{3}{8}=\frac{1}{12}$$

$$u = \frac{3}{8} + 24\left(\frac{1}{12}\right)$$

$$u = \frac{3}{8} + 2 = \frac{19}{8} = 2\frac{3}{8}$$

$$14. \quad a=\frac{1}{3} \quad n=19 \quad r=\frac{7}{8}-\frac{1}{3}=\frac{13}{24}$$

$$u = \frac{1}{3} + 18\left(\frac{13}{24}\right)$$

$$u = \frac{1}{3} + \frac{39}{4} = \frac{121}{12} = 10\frac{1}{12}$$

$$15. \quad a=\frac{7}{2} \quad n=27 \quad r=\frac{21}{4}-\frac{7}{2}=\frac{7}{4}$$

$$u = \frac{7}{2} + 26\left(\frac{7}{4}\right) = \frac{7}{2} + \frac{91}{2} = \frac{98}{2} = 49$$

$$16. \quad a=\frac{7}{9} \quad n=36 \quad r=\frac{1}{3}-\frac{7}{9}=-\frac{4}{9}$$

$$u = \frac{7}{9} + 35\left(-\frac{4}{9}\right)$$

$$u = \frac{7}{9} - \frac{140}{9} = -\frac{133}{9} = -14\frac{7}{9}$$

$$17. \quad a=\frac{2}{7} \quad n=15 \quad r=\frac{1}{8}-\frac{2}{7}=-\frac{9}{56}$$

$$u = \frac{2}{7} + 14\left(-\frac{9}{56}\right)$$

$$u = \frac{2}{7} - \frac{126}{56} = -\frac{110}{56} = -1\frac{27}{28}$$

$$18. \quad a=-\frac{3}{5} \quad n=21 \quad r=-\frac{14}{15}+\frac{3}{5}=-\frac{1}{3}$$

$$u = -\frac{3}{5} + 20\left(-\frac{1}{3}\right)$$

$$u = -\frac{3}{5} - \frac{20}{3} = -\frac{109}{15} = -7\frac{4}{15}$$



$$19. a = -\frac{1}{4} \quad n = 13 \quad r = -\frac{9}{4} + \frac{1}{4} = -2$$

$$u = -\frac{1}{4} + 12(-2) = -\frac{1}{4} - 24 = -\frac{97}{4} = -24\frac{1}{4}$$

$$20. a = -\frac{5}{6} \quad n = 19 \quad r = -\frac{1}{3} + \frac{5}{6} = \frac{1}{2}$$

$$u = -\frac{5}{6} + 18\left(\frac{1}{2}\right) = -\frac{5}{6} + 9 = \frac{49}{6} = 8\frac{1}{6}$$

$$21. a = \frac{11}{3} \quad n = 33 \quad r = \frac{35}{12} - \frac{11}{3} = -\frac{3}{4}$$

$$u = \frac{11}{3} + 32\left(-\frac{3}{4}\right) = \frac{11}{3} - 24 = -\frac{61}{3} = -20\frac{1}{3}$$

$$22. a = \frac{14}{5} \quad n = 41 \quad r = \frac{27}{10} - \frac{14}{5} = -\frac{1}{10}$$

$$u = \frac{14}{5} + 40\left(-\frac{1}{10}\right)$$

$$u = \frac{14}{5} - 4 = -\frac{6}{5} = -1\frac{1}{5}$$

$$23. a = -\frac{3}{5} \quad n = 26 \quad r = \frac{3}{10} + \frac{3}{5} = \frac{9}{10}$$

$$u = -\frac{3}{5} + 25\left(\frac{9}{10}\right)$$

$$u = -\frac{3}{5} + \frac{45}{2} = \frac{219}{10} = 21\frac{9}{10}$$

$$24. a = -4 \quad n = 19$$

$$r = -\frac{2}{3} + 4 = \frac{10}{3}$$

$$u = -4 + 18\left(\frac{10}{3}\right)$$

$$u = -4 + 60 = 56$$

$$25. a = 3 \quad n = 39$$

$$r = -\frac{5}{4} - 3 = -\frac{17}{4}$$

$$u = 3 + 38\left(-\frac{17}{4}\right)$$

$$u = 3 - \frac{323}{2} = -\frac{317}{2} = -158\frac{1}{2}$$

## EJERCICIO 287

Para resolver los problemas de este ejercicio se utilizan las siguientes formulas:

$$a = u - (n-1)r \quad r = \frac{u-a}{n-1} \quad n = \frac{u-a+r}{r}$$

Donde  $u$  es el término enésimo ;  $r$  es la razón de cambio ;  $n$  es la cantidad de términos ;  $a$  es el primer término de la progresión.

$$1. u = 20 \quad r = \frac{2}{7} \quad n = 15$$

$$a = 20 - (14)\left(\frac{2}{7}\right)$$

$$a = 20 - 4 = 16$$

$$2. u = -18 \quad n = 32 \quad r = 3$$

$$a = -18 - (31)(3)$$

$$a = -18 - 93 = -111$$

$$3. u = 1 \quad n = 9 \quad r = 1 - \frac{3}{4} = \frac{1}{4}$$

$$a = 1 - (8)\left(\frac{1}{4}\right) = 1 - 2 = -1$$

$$4. u = \frac{25}{3} \quad n = 7$$

Se plantea la ecuación para encontrar la razón

$$\frac{25}{3} - 2r = 7$$

$$-2r = 7 - \frac{25}{3}$$

$$-2r = -\frac{4}{3}$$

Continúa

### 4. Continuación

$$r = \frac{4}{6} = \frac{2}{3}$$

$$a = \frac{25}{3} - (6)\left(\frac{2}{3}\right)$$

$$a = \frac{25}{3} - \frac{12}{3} = \frac{13}{3} = 4\frac{1}{3}$$

$$5. a = 3 \quad n = 6 \quad u = 8$$

$$r = \frac{8-3}{6-1} = \frac{5}{5} = 1$$

$$6. a = -1 \quad n = 10 \quad u = -4$$

$$r = \frac{-4 - (-1)}{10-1} = -\frac{3}{9} = -\frac{1}{3}$$

$$7. a = \frac{1}{2} \quad n = 17 \quad u = -\frac{3}{8}$$

$$r = \frac{\frac{3}{8} - \frac{1}{2}}{17-1} = \frac{-\frac{1}{8}}{16} = -\frac{1}{128}$$

$$8. a = 5 \quad n = 18 \quad u = -80$$

$$r = \frac{-80-5}{18-1} = \frac{-85}{17} = -5$$

$$9. a = -42 \quad n = 92 \quad u = 1.050$$

$$r = \frac{1.050 - (-42)}{92-1} = \frac{1.092}{91} = 12$$

$$10. a = 4 \quad u = 30$$

$$r = 6 - 4 = 2$$

$$n = \frac{30-4+2}{2} = \frac{28}{2} = 14$$

$$11. a = 5 \quad u = 18$$

$$r = \frac{16}{3} - 5 = \frac{1}{3}$$

$$n = \frac{18-5+\frac{1}{3}}{\frac{1}{3}}$$

$$n = \frac{13+\frac{1}{3}}{\frac{1}{3}} = \frac{40}{\frac{1}{3}} = 40$$

$$12. a = \frac{26}{5} \quad u = 18$$

$$r = 6 - \frac{26}{5} = \frac{4}{5}$$

$$n = \frac{18 - \frac{26}{5} + \frac{4}{5}}{\frac{4}{5}}$$

$$n = \frac{18 - \frac{22}{5} + \frac{68}{5}}{\frac{4}{5}} = \frac{68}{4} = 17$$

## EJERCICIO 288

1.  $r = 19 - 15 = 4$

$a = 15 \quad n = 8$

$u = 15 + (7)4$

$u = 15 + 28 = 43$

$s = \frac{(15 + 43)8}{2}$

$s = (58)(4) = 232$

2.  $a = 31 \quad n = 19$

$r = 7$

$u = 31 + (18)7$

$u = 31 + 126 = 157$

$s = \frac{(31 + 157)(19)}{2}$

$s = \frac{3.572}{2} = 1.786$

3.  $a = 42 \quad n = 24 \quad r = -10$

$u = 42 + (23)(-10)$

$u = -188$

$s = \frac{(42 - 188)(24)}{2}$

$s = (-146)(12) = -1.752$

4.  $a = -10 \quad n = 80 \quad r = 4$

$u = -10 + (79)(4)$

$u = -10 + 316 = 306$

$s = \frac{(-10 + 306)(80)}{2}$

$s = (296)(40) = 11.840$

5.  $a = 11 \quad n = 60 \quad r = -10$

$u = 11 + (59)(-10)$

$u = 11 - 590 = -579$

$s = \frac{(11 - 579)(60)}{2}$

$s = (-568)(30) = -17.040$

6.  $a = -5 \quad n = 50 \quad r = -8$

$u = -5 + (49)(-8)$

$u = -5 - 392 = -397$

$s = \frac{(-5 - 397)(50)}{2}$

$s = (-402)(25) = -10.050$

7.  $a = \frac{1}{2} \quad n = 9 \quad r = \frac{1}{2}$

$u = \frac{1}{2} + (8)\left(\frac{1}{2}\right)$

$u = \frac{1}{2} + 4 = \frac{9}{2}$

$s = \frac{\left(\frac{1}{2} + \frac{9}{2}\right)(9)}{2}$

$s = \frac{45}{2} = 22\frac{1}{2}$

8.  $a = \frac{3}{10} \quad n = 14 \quad r = \frac{1}{10}$

$u = \frac{3}{10} + (13)\left(\frac{1}{10}\right)$

$u = \frac{3}{10} + \frac{13}{10} = \frac{8}{5}$

$s = \frac{\left(\frac{3}{10} + \frac{8}{5}\right)(14)}{2}$

$s = \frac{133}{10} = 13\frac{3}{10}$

9.  $a = \frac{3}{4} \quad n = 19 \quad r = \frac{3}{4}$

$u = \frac{3}{4} + 18\left(\frac{3}{4}\right)$

$u = \frac{3}{4} + \frac{27}{2} = \frac{57}{4}$

$s = \frac{19\left(\frac{3}{4} + \frac{57}{4}\right)}{2}$

$s = \frac{19(15)}{2} = \frac{285}{2} = 142\frac{1}{2}$

10.  $a = \frac{2}{5} \quad n = 34 \quad r = -\frac{3}{11}$

$u = \frac{2}{5} + 33\left(-\frac{3}{11}\right)$

$u = \frac{2}{5} - 9 = -\frac{43}{5}$

$s = \frac{34\left(\frac{2}{5} - \frac{43}{5}\right)}{2}$

$s = -\frac{1.394}{10} = -139\frac{2}{5}$

11.  $a = \frac{7}{3} \quad n = 11 \quad r = \frac{4}{5}$

$r = \frac{47}{15} - \frac{7}{3} = \frac{47 - 35}{15} = \frac{4}{5}$

$u = \frac{7}{3} + 10\left(\frac{4}{5}\right)$

$u = \frac{7}{3} + 8 = \frac{31}{3}$

$s = \frac{11\left(\frac{7}{3} + \frac{31}{3}\right)}{2}$

$s = \frac{418}{6} = 69\frac{2}{3}$

12.  $a = \frac{13}{4} \quad n = 46$

$r = \frac{73}{20} - \frac{13}{4} = \frac{8}{20} = \frac{2}{5}$

$u = \frac{13}{4} + 45\left(\frac{2}{5}\right)$

$u = \frac{13}{4} + 18 = \frac{85}{4}$

$s = \frac{46\left(\frac{13}{4} + \frac{85}{4}\right)}{2}$

$s = \frac{4.508}{8} = 563\frac{1}{2}$

13.  $a = -2 \quad n = 17 \quad r = \frac{1}{4} + 2 = \frac{9}{4}$

$u = -2 + 16\left(\frac{9}{4}\right) = -2 + 36 = 34$

$s = \frac{17(-2 + 34)}{2} = 17(16) = 272$

14.  $a = -5 \quad n = 12 \quad r = -\frac{37}{8} + 5 = \frac{3}{8}$

$u = -5 + 11\left(\frac{3}{8}\right) = -\frac{7}{8}$

$s = \frac{12\left(-5 - \frac{7}{8}\right)}{2}$

$s = 6\left(-\frac{47}{8}\right) = -\frac{141}{4} = -35\frac{1}{4}$

## EJERCICIO 289

1.  $u=11$   $a=3$   $n=5$

$$r = \frac{11-3}{5-1} = \frac{8}{4} = 2$$

$$\Rightarrow 3+2=5$$

$$5+2=7$$

$$7+2=9$$

$$\text{Luego } \div 3. 5. 7. 9. 11.$$

2.  $u=-5$   $a=19$   $m=7$

$$r = \frac{-5-19}{7+1} = \frac{-24}{8} = -3$$

$$\Rightarrow 19-3=16$$

$$16-3=13$$

$$13-3=10$$

$$10-3=7$$

$$7-3=4$$

$$4-3=1$$

$$1-3=-2$$

$$\text{Luego } \div 19. 16. 13. 10. 7. 4. 1. -2. -5.$$

3.  $u=-73$   $a=-13$   $n=7$

$$r = \frac{-73+13}{7-1} = \frac{-60}{6} = -10$$

$$\Rightarrow -13-10=-23$$

$$-23-10=-33$$

$$-33-10=-43$$

$$-43-10=-53$$

$$-53-10=-63$$

$$\text{Luego } \div -13. -23. -33. -43. -53. -63. -73.$$

4.  $u=53$   $a=-42$   $m=4$

$$r = \frac{53+42}{4+1} = \frac{95}{5} = 19$$

$$\Rightarrow -42+19=-23$$

$$-23+19=-4$$

$$-4+19=15$$

$$15+19=34$$

$$\text{Luego } \div -42. -23. -4. 15. 34. 53.$$

5.  $u=-9$   $a=-81$   $n=7$

$$r = \frac{-9+81}{7-1} = \frac{72}{6} = 12$$

$$\Rightarrow -81+12=-69$$

$$-69+12=-57$$

$$-57+12=-45$$

$$-45+12=-33$$

$$-33+12=-21$$

$$\text{Luego } \div -81. -69. -57. -45. -33. -21. -9.$$

6.  $u=3$   $a=1$   $m=3$

$$r = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$$

$$\Rightarrow 1+\frac{1}{2}=\frac{3}{2}$$

$$\frac{3}{2}+\frac{1}{2}=2$$

$$2+\frac{1}{2}=\frac{5}{2}$$

$$\text{Luego } \div 1. \frac{3}{2}. 2. \frac{5}{2}. 3.$$

$$\div 1. 1\frac{1}{2}. 2. 2\frac{1}{2}. 3.$$

7.  $u=12$   $a=5$   $n=6$

$$r = \frac{12-5}{6-1} = \frac{7}{5}$$

$$\Rightarrow 5+\frac{7}{5}=\frac{32}{5}$$

$$\frac{32}{5}+\frac{7}{5}=\frac{39}{5}$$

$$\frac{39}{5}+\frac{7}{5}=\frac{46}{5}$$

$$\frac{46}{5}+\frac{7}{5}=\frac{53}{5}$$

$$\text{Luego } \div 5. \frac{32}{5}. \frac{39}{5}. \frac{46}{5}. \frac{53}{5}. 12.$$

$$\div 5. 6\frac{2}{5}. 7\frac{4}{5}. 9\frac{1}{5}. 10\frac{3}{5}. 12.$$

8.  $u=3$   $a=-4$   $m=5$

$$r = \frac{3+4}{5+1} = \frac{7}{6}$$

$$\Rightarrow -4+\frac{7}{6}=-\frac{17}{6}$$

$$-\frac{17}{6}+\frac{7}{6}=-\frac{5}{3}$$

$$-\frac{5}{3}+\frac{7}{6}=-\frac{1}{2}$$

$$-\frac{1}{2}+\frac{7}{6}=\frac{2}{3}$$

$$\frac{2}{3}+\frac{7}{6}=\frac{11}{6}$$

$$\text{Luego } \div -4. -\frac{17}{6}. -\frac{5}{3}. -\frac{1}{2}. \frac{2}{3}. \frac{11}{6}. 3.$$

$$\div -4. -2\frac{5}{6}. -1\frac{2}{3}. -\frac{1}{2}. \frac{2}{3}. -1\frac{5}{6}. 3.$$

$$9. u = \frac{1}{8} \quad a = \frac{3}{4} \quad n = 7$$

$$r = \frac{\frac{1}{8} - \frac{3}{4}}{7 - 1} = \frac{-\frac{5}{8}}{6} = -\frac{5}{48}$$

$$\Rightarrow \frac{3}{4} - \frac{5}{48} = \frac{31}{48}$$

$$\frac{31}{48} - \frac{5}{48} = \frac{13}{24}$$

$$\frac{13}{24} - \frac{5}{48} = \frac{7}{16}$$

$$\frac{7}{16} - \frac{5}{48} = \frac{1}{3}$$

$$\frac{1}{3} - \frac{5}{48} = \frac{11}{48}$$

$$\text{Luego } \div \frac{3}{4} \cdot \frac{31}{48} \cdot \frac{13}{24} \cdot \frac{7}{16} \cdot \frac{1}{3} \cdot \frac{11}{48} \cdot \frac{1}{8}.$$

$$10. u = 3 \quad a = -1 \quad m = 6$$

$$r = \frac{3 + 1}{6 + 1} = \frac{4}{7}$$

$$\Rightarrow -1 + \frac{4}{7} = -\frac{3}{7}$$

$$-\frac{3}{7} + \frac{4}{7} = \frac{1}{7}$$

$$\frac{1}{7} + \frac{4}{7} = \frac{5}{7}$$

$$\frac{5}{7} + \frac{4}{7} = \frac{9}{7}$$

$$\frac{9}{7} + \frac{4}{7} = \frac{13}{7}$$

$$\frac{13}{7} + \frac{4}{7} = \frac{17}{7}$$

$$\text{Luego } \div -1 \cdot \frac{3}{7} \cdot \frac{1}{7} \cdot \frac{5}{7} \cdot \frac{9}{7} \cdot \frac{13}{7} \cdot \frac{17}{7} \cdot 3.$$

$$\div -1 \cdot \frac{3}{7} \cdot \frac{1}{7} \cdot \frac{5}{7} \cdot \frac{12}{7} \cdot \frac{16}{7} \cdot \frac{23}{7} \cdot 3.$$

$$11. u = -\frac{1}{8} \quad a = \frac{2}{3} \quad n = 7$$

$$r = \frac{-\frac{1}{8} - \frac{2}{3}}{7 - 1} = \frac{-\frac{19}{24}}{6} = -\frac{19}{144}$$

$$\Rightarrow \frac{2}{3} - \frac{19}{144} = \frac{77}{144}$$

$$\frac{77}{144} - \frac{19}{144} = \frac{29}{72}$$

$$\frac{29}{72} - \frac{19}{144} = \frac{13}{48}$$

Continúa

## 11. Continuación

$$\frac{13}{48} - \frac{19}{144} = \frac{5}{36}$$

$$\frac{5}{36} - \frac{19}{144} = \frac{1}{144}$$

$$\text{Luego } \div \frac{2}{3} \cdot \frac{77}{144} \cdot \frac{29}{72} \cdot \frac{13}{48} \cdot \frac{5}{36} \cdot \frac{1}{144} \cdot -\frac{1}{8}.$$

$$12. u = -5 \quad a = -2 \quad m = 7$$

$$r = \frac{-5 + 2}{7 + 1} = -\frac{3}{8}$$

$$\Rightarrow -2 - \frac{3}{8} = -\frac{19}{8}$$

$$-\frac{19}{8} - \frac{3}{8} = -\frac{11}{4}$$

$$-\frac{11}{4} - \frac{3}{8} = -\frac{25}{8}$$

$$-\frac{25}{8} - \frac{3}{8} = -\frac{7}{2}$$

$$-\frac{7}{2} - \frac{3}{8} = -\frac{31}{8}$$

$$-\frac{31}{8} - \frac{3}{8} = -\frac{17}{4}$$

$$-\frac{17}{4} - \frac{3}{8} = -\frac{37}{8}$$

$$\text{Luego } \div -2 \cdot -2\frac{3}{8} \cdot -2\frac{3}{4} \cdot -3\frac{1}{8} \cdot -3\frac{1}{2} \cdot -3\frac{7}{8} \cdot -4\frac{1}{4} \cdot -4\frac{5}{8} \cdot -5.$$

$$13. u = -\frac{7}{10} \quad a = \frac{1}{2} \quad n = 10$$

$$r = \frac{-\frac{7}{10} - \frac{1}{2}}{10 - 1} = \frac{-\frac{6}{5}}{9} = -\frac{2}{15}$$

$$\Rightarrow \frac{1}{2} - \frac{2}{15} = \frac{11}{30}$$

$$\frac{11}{30} - \frac{2}{15} = \frac{7}{30}$$

$$\frac{7}{30} - \frac{2}{15} = \frac{1}{10}$$

$$\frac{1}{10} - \frac{2}{15} = -\frac{1}{30}$$

$$-\frac{1}{30} - \frac{2}{15} = -\frac{1}{6}$$

$$-\frac{1}{6} - \frac{2}{15} = -\frac{3}{10}$$

$$-\frac{3}{10} - \frac{2}{15} = -\frac{13}{30}$$

$$-\frac{13}{30} - \frac{2}{15} = -\frac{17}{30}$$

Luego

$$\div \frac{1}{2} \cdot \frac{11}{30} \cdot \frac{7}{30} \cdot \frac{1}{10} \cdot -\frac{1}{30} \cdot -\frac{1}{6} \cdot -\frac{3}{10} \cdot -\frac{13}{30} \cdot -\frac{17}{30} \cdot -\frac{7}{10}.$$

## EJERCICIO 290

1.  $a=7$   $n=20$   $r=7$

$$u=7+(19)(7)$$

$$u=7+133=140$$

$$s=\frac{(7+140)(20)}{2}$$

$$s=147 \cdot 10=1.470$$

2.  $a=5$   $n=80$   $r=5$

$$u=5+(79)(5)$$

$$u=5+395=400$$

$$s=\frac{(5+400)(80)}{2}$$

$$s=405 \cdot 40=16.200$$

3.  $a=9$   $n=43$   $r=10$

$$u=9+(42)(10)$$

$$u=9+420=429$$

$$s=\frac{(9+429)(43)}{2}$$

$$s=219 \cdot 43=9.417$$

4.  $a=2$   $n=100$   $r=2$

$$u=2+(99)(2)$$

$$u=2+198=200$$

$$s=\frac{(2+200)(100)}{2}$$

$$s=202 \cdot 50=10.100$$

5.  $a=9$   $n=100$   $r=2$

$$u=9+(99)(2)$$

$$u=9+198=207$$

$$s=\frac{(9+207)(100)}{2}$$

$$s=216 \cdot 50=10.800$$

6.  $a=8$   $n=50$   $r=3$

$$u=8+(49)(3)$$

$$u=8+147=155$$

$$s=\frac{(8+155)(50)}{2}$$

$$s=163 \cdot 25$$

$$s=4.075 \div 100$$

$$s=\$40,75$$

7.  $32 \rightarrow N^{\circ}$  normal de  
dientes en la boca  
de un adulto

$$a=10$$
  $n=32$   $r=2$

$$u=10+(31)(2)=72$$

$$s=\frac{(10+72)(32)}{2}$$

$$s=1.312 \div 10$$

$$s=\$131,20$$

8.  $a=77$   $n=72$   $r=11$

$$u=77+(71)(11)=858$$

$$s=\frac{(77+858)(72)}{2}$$

$$s=935 \cdot 36=33.660$$

9.  $n=5 \cdot 12=60$

$$a=2$$
  $r=3$

$$u=2+(59)(3)$$

$$u=2+177=179$$

$$s=\frac{(2+179)(60)}{2}$$

$$s=181 \cdot 30=5.430 \text{ bs.}$$

10.  $a=600$   $r=25$

$$\text{Si } n=8$$

$$u=600+(7)(25)$$

$$u=775 \div 100$$

$$\text{Avanzó } 7,75 \text{ m en}$$

$$\text{el } 8^{\circ} \text{ segundo}$$

$$s \rightarrow \text{Distancia}$$

$$\text{recorrida en } 8 \text{ seg.}$$

$$s=\frac{(600+775)(8)}{2}$$

$$s=1.375 \cdot 4$$

$$s=5.500 \div 100=55 \text{ m}$$

11.  $s=2.400$   $n=3$

$$x \rightarrow \text{Ahorro el } 2^{\circ} \text{ año}$$

$$\frac{x}{2} \rightarrow \text{Ahorro el } 1^{\circ} \text{ año}$$

$$r=x-\frac{x}{2}=\frac{x}{2}$$

$$\text{Luego el } 3^{\circ} \text{ año Ahorra:}$$

$$x+\frac{x}{2}=\frac{3x}{2}$$

Continúa

## 11. Continuación.

$$\Rightarrow \frac{x}{2} + x + \frac{3x}{2} = 2.400$$

$$x+2x+3x=4.800$$

$$6x=4.800$$

$$x=800$$

$$\text{El } 1^{\circ} \text{ año ahorro} \rightarrow \frac{x}{2} = \frac{800}{2} = 400 \text{ sucres}$$

$$\text{Para el } 2^{\circ} \text{ año}$$

$$a=400$$
  $n=2$   $r=\frac{x}{2}=\frac{800}{2}=400$

$$u=400+(1)(400)=800 \text{ sucres}$$

$$\text{Para el } 3^{\circ} \text{ año}$$

$$a=400$$
  $n=3$   $r=400$

$$u=400+(2)(400)=1.200 \text{ sucres}$$

12.  $2^{\circ}$  y  $4^{\circ}=22$

$$3^{\circ} \text{ y } 7^{\circ}=34$$

$$\text{El término medio entre}$$

$$\text{el } 2^{\circ} \text{ y el } 4^{\circ} \text{ es el } 3^{\circ}$$

$$\text{por tanto es una progresión}$$

$$\text{impar. Luego el duplo}$$

$$\text{del } 3^{\circ} \text{ término es } 22$$

$$x \rightarrow 3^{\text{er}} \text{ término}$$

$$2x=22$$

$$x=11$$

$$\text{Luego } 11 \rightarrow 3^{\text{er}} \text{ término}$$

$$\text{Como } 3^{\circ} + 7^{\circ} = 34$$

$$11+7^{\circ}=34$$

$$7^{\circ}=23$$

$$23 \rightarrow \text{es el } 7^{\circ} \text{ término}$$

$$\text{Interpolación para conocer}$$

$$\text{el } 4^{\circ} \text{ término}$$

$$\div 11 \dots 23$$

$$\text{Donde } n=5$$
  $a=11$   $u=23$

$$r=\frac{23-11}{5-1}=\frac{12}{4}=3$$

$$4^{\circ} \text{ término} \rightarrow 3^{\circ} + r = 11+3=14$$

$$2^{\circ} \text{ término} \rightarrow 22-4^{\circ} = 22-14=8$$

13.  $n=32$   $a=5$   $r=3$

$$u=5+(31)(3)$$

$$u=5+93=98$$

$$s=\frac{(5+98)(32)}{2}$$

$$s=103 \cdot 16=\$1.648$$

$$14. a=50 \quad r=-5\frac{1}{2} \quad n=8$$

$$u=50+(7)\left(-\frac{11}{2}\right)$$

$$u=\frac{100-77}{2}=\frac{23}{2}=11\frac{1}{2} \text{ Km}$$

$$s=\frac{\left(50+\frac{23}{2}\right)8}{2}=246 \text{ Km}$$

15. 3° y 10° términos son equidistantes a los extremos 1° y 12°, Luego la suma de estos extremos es igual a la suma de los términos equidistantes

$$3^\circ + 10^\circ = 1^\circ + 12^\circ$$

$$\text{como } 1^\circ + 12^\circ = 53\frac{1}{2}$$

$$\Rightarrow 3^\circ + 10^\circ = 53\frac{1}{2}$$

$$16. n=11 \quad a=-2 \quad u=-52$$

$$r=\frac{-52-(-2)}{11-1}=-\frac{50}{10}=-5$$

$$2^\circ \text{ térm.} \rightarrow -2-5=-7$$

$$3^\circ \text{ térm.} \rightarrow -7-5=-12$$

$$4^\circ \text{ térm.} \rightarrow -12-5=-17$$

$$5^\circ \text{ térm.} \rightarrow -17-5=-22$$

$$6^\circ \text{ térm.} \rightarrow -22-5=-27$$

$$17. a=500 \quad u=1.900 \quad r=200$$

$$n=\frac{1.900-500+200}{200}$$

$$n=8 \text{ años}$$

$$18. n=11 \quad a=1.180 \quad u=6.180$$

$$r=\frac{6.180-1.180}{11-1}$$

$$r=\frac{5.000}{10}=\$500$$

$$19. n=5 \quad u=3.000 \quad r=-300$$

$$a=3.000-(4)(-300)$$

$$a=3.000+1.200=4.200 \text{ soles}$$

$$20. a=161 \quad r=322 \quad n=5$$

$$u=161+(4)(322)=1.449$$

$$s=\frac{(1.449+161)(5)}{2}$$

$$s=\frac{(1.610)(5)}{2}$$

$$s=\frac{8.050}{2}=4.025$$

$$=4.025 \div 10=402,5 \text{ pies}$$

$$21. a=51 \quad r=2 \quad u=813$$

$$n=\frac{813-51+2}{2}=\frac{764}{2}=382$$

$$s=\frac{(813+51)(382)}{2}$$

$$s=(864)(191)=165.024$$

$$22. 5^\circ \rightarrow 31 \quad 9^\circ \rightarrow 59$$

interpolando

$$n=5 \quad a=31 \quad u=59$$

$$r=\frac{59-31}{5-1}=\frac{28}{4}=7$$

$$10^\circ \text{ térm.} \rightarrow 59+7=66$$

$$11^\circ \text{ térm.} \rightarrow 66+7=73$$

$$12^\circ \text{ térm.} \rightarrow 73+7=80$$

$$23. n=3 \quad a=12.500 \quad u=20.500$$

$$a+u=12.500+20.500$$

$$a+u=33.000$$

$$\text{Luego } 2(2^\circ \text{ term.})=33.000$$

$$2^\circ \text{ term.}=16.500 \text{ colones}$$

## EJERCICIO 291

$$1. a=3 \quad n=7$$

$$r=6 \div 3=2$$

$$u=3(2^{7-1})$$

$$u=3(2^6)$$

$$u=3 \cdot 64=192$$

$$2. a=\frac{1}{3} \quad n=8$$

$$r=3 \div 1=3$$

$$u=\frac{1}{3}(3^{8-1})$$

$$u=3^6=729$$

$$3. a=8 \quad n=9$$

$$r=4 \div 8=\frac{1}{2}$$

$$u=8\left[\left(\frac{1}{2}\right)^{9-1}\right]$$

$$u=\frac{8}{2^8}=\frac{1}{2^5}=\frac{1}{32}$$

$$4. a=1 \quad n=6$$

$$r=\frac{4}{25} \cdot \frac{5}{2}=\frac{2}{5}$$

$$u=1\left[\left(\frac{2}{5}\right)^{6-1}\right]$$

$$u=\frac{2^5}{5^5}=\frac{32}{3.125}$$

$$5. a=3 \quad n=7$$

$$r=2 \div 3=\frac{2}{3}$$

$$u=3\left[\left(\frac{2}{3}\right)^{7-1}\right]$$

$$u=\frac{2^6}{3^5}=\frac{64}{243}$$

$$6. a=\frac{1}{2} \quad n=6$$

$$r=\frac{1}{5} \cdot 2=\frac{2}{5}$$

$$u=\frac{1}{2}\left[\left(\frac{2}{5}\right)^{6-1}\right]$$

$$u=\frac{2^4}{5^5}=\frac{16}{3.125}$$

$$7. a=\frac{9}{4} \quad n=8$$

$$r=3 \cdot \frac{4}{9}=\frac{4}{3}$$

$$u=\frac{9}{4}\left[\left(\frac{4}{3}\right)^{7-1}\right]$$

$$u=\frac{4^6}{3^5}=\frac{4.096}{243}=16\frac{208}{243}$$

$$8. a=-3 \quad n=6$$

$$r=6 \div (-3)=-2$$

$$u=-3[(-2)^5]$$

$$u=-3(-32)=96$$

$$9. a=3 \quad n=9$$

$$r=-1 \div 3=-\frac{1}{3}$$

$$u=3\left[\left(-\frac{1}{3}\right)^8\right]$$

$$u=\frac{1}{3^7}=\frac{1}{2.187}$$

$$10. a=\frac{5}{6} \quad n=5$$

$$r=\frac{1}{2} \cdot \frac{6}{5}=\frac{3}{5}$$

$$u=\frac{5}{6}\left[\left(\frac{3}{5}\right)^4\right]$$

$$u=\frac{3^3}{2 \cdot 5^3}=\frac{27}{250}$$

$$11. a=16 \quad n=8$$

$$r=1 \div (-4) = -\frac{1}{4}$$

$$u=16 \left[ \left( -\frac{1}{4} \right)^7 \right]$$

$$u = -\frac{1}{4^5} = -\frac{1}{1.024}$$

$$12. a=\frac{3}{4} \quad n=8$$

$$r=\frac{1}{3}(-2)=-\frac{2}{3}$$

$$u=\frac{3}{4} \left[ \left( -\frac{2}{3} \right)^7 \right]$$

$$u = -\frac{2^5}{3^6} = -\frac{32}{729}$$

$$13. a=-\frac{3}{5} \quad n=5$$

$$r=\frac{3}{2} \left( -\frac{5}{3} \right) = -\frac{5}{2}$$

$$u=-\frac{3}{5} \left[ \left( -\frac{5}{2} \right)^4 \right]$$

$$u = -\frac{3 \cdot 5^3}{2^4} = -\frac{375}{16} = -23\frac{7}{16}$$

$$14. a=-\frac{3}{4} \quad n=10$$

$$r=-\frac{1}{4} \left( -\frac{4}{3} \right) = \frac{1}{3}$$

$$u=-\frac{3}{4} \left[ \left( \frac{1}{3} \right)^9 \right]$$

$$u = -\frac{1}{4 \cdot 3^8} = -\frac{1}{26.244}$$

## EJERCICIO 292

$$1. r=\frac{1}{2} \quad u=\frac{1}{64} \quad n=7$$

$$a=\frac{\frac{1}{64}}{\left( \frac{1}{2} \right)^6} = \frac{\frac{1}{2^6}}{\frac{1}{2^6}} = \frac{2^6}{2^6} = 1$$

$$2. r=\frac{2}{3} \quad u=\frac{64}{2.187} \quad n=9$$

$$a=\frac{\frac{64}{2.187}}{\left( \frac{2}{3} \right)^8} = \frac{\frac{2^6}{2^8 \cdot 3^7}}{\frac{2^8}{3^8}} = \frac{2^6 \cdot 3^8}{2^8 \cdot 3^7} = \frac{3}{2^2} = \frac{3}{4}$$

$$3. r=\frac{32}{625} \cdot \frac{125}{16} = \frac{2}{5}$$

$$u=\frac{32}{625} \quad n=6$$

$$a=\frac{\frac{32}{625}}{\left( \frac{2}{5} \right)^5} = \frac{\frac{2^5}{5^4}}{\frac{2^5}{5^5}} = \frac{2^5 \cdot 5^5}{2^5 \cdot 5^4} = 5$$

$$4. a=2 \quad u=64 \quad n=6$$

$$r=\sqrt[5]{\frac{64}{2}} = \sqrt[5]{\frac{2^6}{2}} = \sqrt[5]{2^5} = 2$$

$$5. a=\frac{1}{3} \quad u=243 \quad n=7$$

$$r=\sqrt[6]{\frac{243}{\frac{1}{3}}} = \sqrt[6]{729} = \sqrt[6]{3^6} = \pm 3$$

$$6. a=-5 \quad u=640 \quad n=8$$

$$r=\sqrt[7]{\frac{640}{-5}}$$

$$r=\sqrt[7]{-128} = \sqrt[7]{-2^7} = -2$$

$$7. a=\frac{729}{2} \quad u=\frac{3}{2} \quad n=6$$

$$r=\sqrt[5]{\frac{\frac{3}{2}}{\frac{729}{2}}} = \sqrt[5]{\frac{1}{243}} = \frac{1}{3}$$

$$8. a=8 \quad u=\frac{1}{512} \quad n=7$$

$$r=\sqrt[6]{\frac{1}{\frac{512}{8}}} = \sqrt[6]{\frac{1}{4.096}} = \sqrt[6]{\frac{1}{4^6}} = \pm \frac{1}{4}$$

$$9. a=\frac{625}{16} \quad u=1 \quad n=5$$

$$r=\sqrt[4]{\frac{1}{\frac{625}{16}}} = \sqrt[4]{\frac{16}{625}} = \sqrt[4]{\frac{2^4}{5^4}} = \pm \frac{2}{5}$$

$$10. a=\frac{27}{64} \quad u=-\frac{2}{81} \quad n=8$$

$$r=\sqrt[7]{\frac{\frac{2}{81}}{\frac{27}{64}}}$$

$$r=\sqrt[7]{-\frac{2 \cdot 2^6}{3^4 \cdot 3^3}} = \sqrt[7]{-\frac{2^7}{3^7}} = -\frac{2}{3}$$

## EJERCICIO 293

$$1. a=6 \quad n=5$$

$$r=3 \div 6 = \frac{1}{2}$$

$$u=6 \left[ \left( \frac{1}{2} \right)^4 \right] = 6 \left( \frac{1}{16} \right) = \frac{3}{8}$$

$$s=\frac{\left( \frac{3}{8} \right) \left( \frac{1}{2} \right) - 6}{\frac{1}{2} - 1}$$

$$s = -\frac{93}{16}(-2) = \frac{93}{8} = 11\frac{5}{8}$$

$$2. a=4 \quad n=6$$

$$r=-8 \div 4 = -2$$

$$u=4 \left[ (-2)^5 \right]$$

$$u=4(-32) = -128$$

$$s=\frac{-128(-2)-4}{-2-1}$$

$$s=\frac{252}{-3} = -84$$

$$3. a=12 \quad n=7$$

$$r=4 \div 12 = \frac{1}{3}$$

$$u=12 \left[ \left( \frac{1}{3} \right)^6 \right] = 12 \left( \frac{1}{729} \right) = \frac{4}{243}$$

$$s=\frac{\left( \frac{4}{243} \right) \left( \frac{1}{3} \right) - 12}{\frac{1}{3} - 1}$$

$$s = -\frac{8.744}{729} \left( -\frac{3}{2} \right)$$

$$s = \frac{26.232}{1.458} = 17\frac{1.446}{1.458} = 17\frac{241}{243}$$

$$4. a=\frac{1}{4} \quad n=10$$

$$r=\frac{1}{2} \cdot 4 = 2$$

$$u=\frac{1}{4} \left[ 2^9 \right] = 128$$

$$s=\frac{128(2)-\frac{1}{4}}{2-1}$$

$$s=\frac{1.023}{4} = 255\frac{3}{4}$$

$$5. a = \frac{9}{4} \quad n = 8$$

$$r = \frac{3}{2} \cdot \frac{4}{9} = \frac{2}{3}$$

$$u = \frac{9}{4} \left[ \left( \frac{2}{3} \right)^7 \right] = \frac{2^5}{3^5}$$

$$s = \frac{\frac{2^5}{3^5} \left( \frac{2}{3} \right) - \frac{9}{4}}{\frac{2}{3} - 1}$$

$$s = \frac{2^8 - 3^8}{3^6 \cdot 2^2} (-3)$$

$$s = \frac{-6.305}{2.916} (-3)$$

$$s = \frac{18.915}{2.916} = 6 \frac{1.419}{2.916} = 6 \frac{473}{972}$$

$$6. a = -\frac{1}{10} \quad n = 7$$

$$r = \frac{1}{5} (-10) = -2$$

$$u = -\frac{1}{10} [(-2)^6] = -\frac{64}{10} = -\frac{32}{5}$$

$$s = \frac{-\frac{32}{5} (-2) + \frac{1}{10}}{-2 - 1}$$

$$s = \frac{\frac{129}{10}}{-3} = -\frac{129}{30} = -4 \frac{3}{10}$$

$$7. a = -6 \quad n = 10$$

$$r = -3 \div (-6) = \frac{1}{2}$$

$$u = -6 \left[ \left( \frac{1}{2} \right)^9 \right] = -\frac{3}{2^8}$$

$$s = \frac{-\frac{3}{2^8} \left( \frac{1}{2} \right) + 6}{\frac{1}{2} - 1}$$

$$s = \frac{\frac{3.069}{512}}{-\frac{1}{2}} = \frac{6.138}{-512} = -11 \frac{253}{256}$$

$$8. a = 2 \quad n = 8$$

$$r = -1 \div 2 = -\frac{1}{2}$$

$$u = 2 \left[ \left( -\frac{1}{2} \right)^7 \right] = -\frac{1}{2^6} = -\frac{1}{64}$$

$$s = \frac{-\frac{1}{64} \left( -\frac{1}{2} \right) - 2}{-\frac{1}{2} - 1}$$

$$s = \frac{\frac{255}{128}}{\frac{3}{2}} = \frac{510}{384} = 1 \frac{126}{384} = 1 \frac{21}{64}$$

$$9. a = \frac{3}{2} \quad n = 6$$

$$r = 1 \div \frac{3}{2} = \frac{2}{3}$$

$$u = \frac{3}{2} \left( \frac{2}{3} \right)^5 = \frac{2^4}{3^4} = \frac{16}{81}$$

$$s = \frac{\frac{16}{81} \left( \frac{2}{3} \right) - \frac{3}{2}}{\frac{2}{3} - 1}$$

$$s = \frac{-\frac{1 \frac{179}{486}}{\frac{1}{3}}}{-\frac{1}{3}} = 4 \frac{17}{162}$$

$$10. a = 9 \quad n = 6$$

$$r = 1 \div (-3) = -\frac{1}{3}$$

$$u = 9 \left( -\frac{1}{3} \right)^5 = -\frac{1}{3^3} = -\frac{1}{27}$$

$$s = \frac{-\frac{1}{27} \left( -\frac{1}{3} \right) - 9}{-\frac{1}{3} - 1}$$

$$s = \frac{-\frac{728}{81}}{-\frac{4}{3}}$$

$$s = \frac{2.184}{324} = 6 \frac{240}{324} = 6 \frac{20}{27}$$

## EJERCICIO 294

$$1. a = 5 \quad u = 3.125 \quad n = 5$$

$$r = \sqrt[4]{\frac{3.125}{5}} = \sqrt{25} \pm 5$$

$$5(\pm 5) = \pm 25$$

$$\pm 25(\pm 5) = 125$$

$$125(\pm 5) = \pm 625$$

$$+ \div 5: \pm 25: 125: \pm 625: 3.125$$

$$2. a = -7 \quad u = -224 \quad m = 4$$

$$r = \sqrt[5]{\frac{-224}{-7}} = \sqrt[5]{32} = 2$$

$$-7 \cdot 2 = -14$$

$$-14 \cdot 2 = -28$$

$$-28 \cdot 2 = -56$$

$$-56 \cdot 2 = -112$$

$$+ \div -7: -14: -28: -56: -112: -224$$

$$3. a = 128 \quad u = 2 \quad n = 7$$

$$r = \sqrt[6]{\frac{2}{128}} = \sqrt[6]{\frac{1}{2^6}} = \pm \frac{1}{2}$$

$$128 \left( \pm \frac{1}{2} \right) = \pm 64$$

$$\pm 64 \left( \pm \frac{1}{2} \right) = 32$$

$$32 \left( \pm \frac{1}{2} \right) = \pm 16$$

$$\pm 16 \left( \pm \frac{1}{2} \right) = 8$$

$$8 \left( \pm \frac{1}{2} \right) = \pm 4$$

$$+ \div 128: \pm 64: 32: \pm 16: 8: \pm 4: 2$$

$$4. a = \frac{9}{2} \quad u = \frac{16}{27} \quad n = 6$$

$$r = \sqrt[5]{\frac{\frac{16}{27}}{\frac{9}{2}}} = \sqrt[5]{\frac{2^5}{3^5}} = \frac{2}{3}$$

$$\frac{9}{2} \left( \frac{2}{3} \right) = 3$$

$$3 \left( \frac{2}{3} \right) = 2$$

$$2 \left( \frac{2}{3} \right) = \frac{4}{3}$$

$$\frac{4}{3} \left( \frac{2}{3} \right) = \frac{8}{9}$$

$$+ \div \frac{9}{2}: 3: 2: \frac{4}{3}: \frac{8}{9}: \frac{16}{27}$$

$$+ \div 4 \frac{1}{2}: 3: 2: 1 \frac{1}{3}: \frac{8}{9}: \frac{16}{27}$$



$$5. a=2 \quad u=\frac{2.187}{64} \quad m=6$$

$$r=\sqrt[7]{\frac{2.187}{64}}=\sqrt[7]{\frac{3^7}{2^7}}=\frac{3}{2}$$

$$2\left(\frac{3}{2}\right)=3$$

$$3\left(\frac{3}{2}\right)=\frac{9}{2}$$

$$\frac{9}{2}\left(\frac{3}{2}\right)=\frac{27}{4}$$

$$\frac{27}{4}\left(\frac{3}{2}\right)=\frac{81}{8}$$

$$\frac{81}{8}\left(\frac{3}{2}\right)=\frac{243}{16}$$

$$\frac{243}{16}\left(\frac{3}{2}\right)=\frac{729}{32}$$

$$\div \div 2: 3: 4\frac{1}{2}: 6\frac{3}{4}: 10\frac{1}{8}: 15\frac{3}{16}: 22\frac{25}{32}: 34\frac{11}{64}$$

$$6. a=\frac{4}{9} \quad u=\frac{27}{256} \quad n=6$$

$$r=\sqrt[5]{\frac{27}{\frac{4}{9}}}=\sqrt[5]{\frac{3^5}{4^5}}=\frac{3}{4}$$

$$\frac{4}{9}\left(\frac{3}{4}\right)=\frac{1}{3}$$

$$\frac{1}{3}\left(\frac{3}{4}\right)=\frac{1}{4}$$

$$\frac{1}{4}\left(\frac{3}{4}\right)=\frac{3}{16}$$

$$\frac{3}{16}\left(\frac{3}{4}\right)=\frac{9}{64}$$

$$\div \div \frac{4}{9}: \frac{1}{3}: \frac{1}{4}: \frac{3}{16}: \frac{9}{64}: \frac{27}{256}$$

$$7. a=8 \quad u=\frac{1}{32} \quad m=7$$

$$r=\sqrt[8]{\frac{1}{32}}=\sqrt[8]{\frac{1}{2^5}}=\pm\frac{1}{2}$$

$$8\left(\pm\frac{1}{2}\right)=\pm 4$$

$$\pm 4\left(\pm\frac{1}{2}\right)=2$$

$$2\left(\pm\frac{1}{2}\right)=\pm 1$$

$$\pm 1\left(\pm\frac{1}{2}\right)=\pm\frac{1}{2}$$

$$\frac{1}{2}\left(\pm\frac{1}{2}\right)=\pm\frac{1}{4}$$

$$\pm\frac{1}{4}\left(\pm\frac{1}{2}\right)=\pm\frac{1}{8}$$

$$\frac{1}{8}\left(\pm\frac{1}{2}\right)=\pm\frac{1}{16}$$

$$\div \div 8: \pm 4: 2: \pm 1: \frac{1}{2}: \pm\frac{1}{4}: \frac{1}{8}: \pm\frac{1}{16}: \frac{1}{32}$$

## EJERCICIO 295

$$1. a=2 \quad r=\frac{1}{2} \cdot \frac{1}{2}=\frac{1}{4}$$

$$s=\frac{2}{1-\frac{1}{4}}=\frac{2}{\frac{3}{4}}=\frac{8}{3}=2\frac{2}{3}$$

$$3. a=-5 \quad r=\frac{-2}{-5}=\frac{2}{5}$$

$$s=\frac{-5}{1-\frac{2}{5}}=\frac{-5}{\frac{3}{5}}=-\frac{25}{3}=-8\frac{1}{3}$$

$$5. a=\frac{3}{4} \quad r=\frac{1}{4}\left(\frac{4}{3}\right)=\frac{1}{3}$$

$$s=\frac{\frac{3}{4}}{1-\frac{1}{3}}=\frac{\frac{3}{4}}{\frac{2}{3}}=\frac{9}{8}=1\frac{1}{8}$$

$$7. a=2 \quad r=-\frac{2}{5} \cdot \frac{1}{2}=-\frac{1}{5}$$

$$s=\frac{2}{1+\frac{1}{5}}=\frac{2}{\frac{6}{5}}=\frac{10}{6}=1\frac{2}{3}$$

$$2. a=\frac{1}{2} \quad r=\frac{1}{6} \cdot 2=\frac{1}{3}$$

$$s=\frac{\frac{1}{2}}{1-\frac{1}{3}}=\frac{\frac{1}{2}}{\frac{2}{3}}=\frac{3}{4}$$

$$4. a=-4 \quad r=-\frac{8}{3}\left(-\frac{1}{4}\right)=\frac{2}{3}$$

$$s=\frac{-4}{1-\frac{2}{3}}=\frac{-4}{\frac{1}{3}}=-12$$

$$6. a=\frac{1}{6} \quad r=\frac{1}{7} \cdot 6=\frac{6}{7}$$

$$s=\frac{\frac{1}{6}}{1-\frac{6}{7}}=\frac{\frac{1}{6}}{\frac{1}{7}}=\frac{7}{6}=1\frac{1}{6}$$

$$8. a=-14 \quad r=\frac{-6}{-14}=\frac{3}{7}$$

$$s=\frac{-14}{1-\frac{3}{7}}=\frac{-14}{\frac{4}{7}}=-\frac{98}{4}=-24\frac{1}{2}$$

## EJERCICIO 296

$$1. \frac{6}{10} + \frac{6}{100} + \frac{6}{1.000} \dots$$

$$a=\frac{6}{10} \quad r=\frac{6}{100}\left(\frac{10}{6}\right)=\frac{1}{10}$$

$$s=\frac{\frac{6}{10}}{1-\frac{1}{10}}=\frac{\frac{6}{10}}{\frac{9}{10}}=\frac{6}{9}=\frac{2}{3}$$

$$2. \frac{12}{100} + \frac{12}{10.000} + \frac{12}{1.000.000} \dots$$

$$a=\frac{12}{100} \quad r=\frac{12}{10.000}\left(\frac{100}{12}\right)=\frac{1}{100}$$

$$s=\frac{\frac{12}{100}}{1-\frac{1}{100}}=\frac{\frac{12}{100}}{\frac{99}{100}}=\frac{12}{99}=\frac{4}{33}$$

$$3. \frac{159}{1.000} + \frac{159}{1.000.000} \dots$$

$$a=\frac{159}{10^3} \quad r=\frac{159}{10^6}\left(\frac{10^3}{159}\right)=\frac{1}{10^3}$$

$$s=\frac{\frac{159}{10^3}}{1-\frac{1}{10^3}}=\frac{\frac{159}{10^3}}{\frac{999}{10^3}}=\frac{159}{999}=\frac{53}{333}$$

$$4. \frac{32}{100} + \frac{32}{10.000} + \frac{32}{1.000.000} \dots$$

$$a = \frac{32}{100} \quad r = \frac{32}{10.000} \left( \frac{100}{32} \right) = \frac{1}{100}$$

$$s = \frac{\frac{32}{100}}{1 - \frac{1}{100}} = \frac{\frac{32}{100}}{\frac{99}{100}} = \frac{32}{99}$$

$$5. \frac{144}{10^3} + \frac{144}{10^6} + \frac{144}{10^9} \dots$$

$$a = \frac{144}{10^3} \quad r = \frac{144}{10^6} \left( \frac{10^3}{144} \right) = \frac{1}{10^3}$$

$$s = \frac{\frac{144}{10^3}}{1 - \frac{1}{10^3}} = \frac{\frac{144}{10^3}}{\frac{999}{10^3}} = \frac{144}{999} = \frac{16}{111}$$

$$6. \frac{3}{10} + \frac{5}{100} + \frac{5}{1.000} \dots$$

$$a = \frac{5}{100} \quad r = \frac{5}{1.000} \left( \frac{100}{5} \right) = \frac{1}{10}$$

$$s = \frac{\frac{5}{100}}{1 - \frac{1}{10}} = \frac{\frac{5}{100}}{\frac{9}{10}} = \frac{5}{90} = \frac{1}{18}$$

$$s = \frac{\frac{3}{10} + \frac{1}{18}}{1 - \frac{1}{18}} = \frac{\frac{64}{180}}{\frac{17}{180}} = \frac{16}{45}$$

$$7. \frac{18}{100} + \frac{1}{10^3} + \frac{1}{10^4} \dots$$

$$a = \frac{1}{10^3} \quad r = \frac{1}{10^4} \cdot 10^3 = \frac{1}{10}$$

$$s = \frac{\frac{1}{10^3}}{1 - \frac{1}{10}} = \frac{\frac{1}{10^3}}{\frac{9}{10}} = \frac{1}{900}$$

$$s = \frac{\frac{18}{100} + \frac{1}{900}}{1 - \frac{1}{900}} = \frac{\frac{163}{900}}{\frac{899}{900}} = \frac{163}{899}$$

$$8. \frac{3}{10} + \frac{18}{10^3} + \frac{18}{10^5} \dots$$

$$a = \frac{18}{10^3} \quad r = \frac{18}{10^5} \left( \frac{10^3}{18} \right) = \frac{1}{100}$$

$$s = \frac{\frac{18}{10^3}}{1 - \frac{1}{100}} = \frac{\frac{18}{10^3}}{\frac{99}{100}} = \frac{18}{99} = \frac{2}{11}$$

$$s = \frac{\frac{3}{10} + \frac{1}{55}}{1 - \frac{1}{55}} = \frac{\frac{175}{550}}{\frac{54}{550}} = \frac{7}{22}$$

$$9. 2 + \frac{18}{100} + \frac{18}{10^4} + \frac{18}{10^6} \dots$$

$$a = \frac{18}{100} \quad r = \frac{18}{10^4} \left( \frac{100}{18} \right) = \frac{1}{100}$$

$$s = \frac{\frac{18}{100}}{1 - \frac{1}{100}} = \frac{\frac{18}{100}}{\frac{99}{100}} = \frac{2}{11}$$

$$s = 2 + \frac{2}{11} = \frac{24}{11}$$

## EJERCICIO 297

$$1. a = 2 \quad r = 2$$

Para saber cuanto gano el sábado  $\rightarrow n = 6$

$$u = 2(2^5) = 64 \text{ Lempiras}$$

Para saber cuanto gané de lunes a sábado utilizo  $S$  para los 6 primeros términos de la progresión.

$$\div \div 2: 4: 8 \dots$$

$$\text{Donde } a = 2 \quad r = 2 \quad u = 64$$

$$s = \frac{64(2) - 2}{2 - 1} = 126 \text{ Lempiras}$$

$$2. n = 20 \quad r = 2 \quad a = 1$$

$$u = 1(2^{19}) = 524.288$$

$$s = \frac{524.288(2) - 1}{2 - 1}$$

$$s = 1.048.575 + 100$$

$$s = \$ 10.485,75$$

$$3. n = 8 \quad r = \frac{1}{3} \quad u = 1$$

$$a = \left( \frac{1}{3} \right)^7 = \frac{1}{2.187} = 2.187 \text{ balboas}$$

$$4. n = 9 \rightarrow \text{Impar}$$

Como es una sucesión impar los términos equidistan, luego la multiplicación del primer y último término es igual al producto entre el 3º y el 7º término.

Entonces la respuesta es:

$$= \frac{1}{216}$$

$$5. n = 5 \quad u = \frac{8}{81}$$

$$3^{\text{er}} \text{ término} = \sqrt{\frac{4}{81}} = \frac{2}{9}$$

Se encuentra la razón para  $n = 3$ , entre el 3º y 5º término.

$$r = \sqrt{\frac{\frac{8}{81}}{\frac{2}{9}}} = \sqrt{\frac{36}{81}} = \frac{2}{3}$$

$$\frac{2}{9} \cdot \frac{3}{2} = \frac{1}{3} \rightarrow 2^{\text{o}} \text{ término}$$

$$\frac{1}{3} \cdot \frac{3}{2} = \frac{1}{2} \rightarrow 1^{\text{o}} \text{ término}$$

$$6. a = \frac{1}{4} \quad u = \frac{1}{32} \quad n = 4$$

$$r = \sqrt[3]{\frac{\frac{1}{32}}{\frac{1}{4}}} = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$$

$$\frac{1}{32} \cdot 2 = \frac{1}{16} \rightarrow 6^{\text{o}} \text{ término}$$

$$7. r = \frac{2}{3} \quad u = 160 \quad n = 5$$

$$a = \frac{160}{\left( \frac{2}{3} \right)^4} = \frac{160}{\frac{16}{81}} = 810$$

$$s = \frac{160 \left( \frac{2}{3} \right) - 810}{\frac{2}{3} - 1}$$

$$s = \frac{2.110 - 810}{-\frac{1}{3}} = \$2.110$$

$$8. a = 59.049 \quad u = 100.000 \quad n = 6$$

$$r = \sqrt[5]{\frac{100.000}{59.049}} = \sqrt[5]{\frac{10^5}{9^5}} = \frac{10}{9}$$

$$9. r = \frac{1}{3} \quad a = 24.300 \quad n = 6$$

$$u = 24.300 \left( \frac{1}{3} \right)^5 = \frac{24.300}{243} = 100$$

$$s = \frac{100 \left( \frac{1}{3} \right) - 24.300}{\frac{1}{3} - 1} = \frac{-\frac{72.800}{3}}{-\frac{2}{3}} = \frac{72.800}{2} = 36.400 \text{ bs.}$$

$$10. n = 15 \quad a = 1 \quad r = 3$$

$$u = (3)^{14} = 4.782.969$$

$$s = \frac{4.782.969(3) - 1}{3 - 1} = \frac{14.348.906}{2} = \$7.174.453$$

## EJERCICIO 298

$$1. \text{Log}(532 \cdot 0,184)$$

$$= \text{Log } 532 + \text{Log } 0,184$$

$$= 2,72591 + \bar{1},26482$$

$$= 1,99073$$

$$\Rightarrow 10^{1,99073} = 97,888$$

$$2. \text{Log}191,7 + \text{Log}432$$

$$= 2,28262 + 2,63548$$

$$= 4,9181$$

$$\Rightarrow \text{Antilog } 4,9181 = 82.814,4$$

$$3. \text{Log}0,7 + \text{Log}0,013 + \text{Log}0,9$$

$$= \bar{1},84510 + \bar{2},11394 + \bar{1},95424$$

$$= -2,08672$$

$$\Rightarrow 10^{-2,08672} = 0,00819$$

$$4. \text{Log}7,5 + \text{Log}8,16 + \text{Log}0,35 + \text{Log}10,037$$

$$= 0,875061 + 0,911690 + \bar{1},544068 + 4,001603$$

$$= 4 - 1 + 2,332422$$

$$= 5,332422$$

$$\Rightarrow 10^{5,332422} = 214.991,85$$

$$\cong 214.992$$

$$5. \text{Log}3,2 + \text{Log}4,3 + \text{Log}7,8 + \text{Log}103,4 + \text{Log}0,019$$

$$= 0,505149 + 0,633468 + 0,892094 + 2,014520 + \bar{2},278753$$

$$= 2,323988 \Rightarrow \text{Antilog } 2,323988 = 210,857$$

$$6. \text{Log}95,18 - \text{Log}7,23$$

$$= 1,978317 - 0,859138$$

$$= 1,119179 \Rightarrow \text{Antilog } 1,119179 = 13,15767031 \cong 13,1577$$

$$7. \text{Log}8,125 - \text{Log}0,9324$$

$$= \text{Log}8,125 + \text{Colog } 0,9324$$

$$= 0,909823 + 0,030397$$

$$= 0,940220 \Rightarrow 10^{0,940220} = 8,714063 \cong 8,7141$$

$$8. \text{Log}7.653,95 - \text{Log}12,354$$

$$= \text{log}7.653,95 + \text{Colog } 12,354$$

$$= 3,883885 + \bar{2},908192$$

$$= 2,792077 \Rightarrow \text{Antilog } 2,792077 = 619,55$$

$$9. \text{Log}0,72183 - \text{Log}0,0095$$

$$= \text{Log}0,72183 + \text{Colog } 0,0095$$

$$= \bar{1},858434 + 2,022276$$

$$= 1,88071 \Rightarrow 10^{1,88071} = 75,981873 \cong 75,982$$

$$10. \text{Log}9,114 - \text{Log}0,02$$

$$= \text{Log}9,114 + \text{Colog } 0,02$$

$$= 3,959709 + 1,698970$$

$$= 5,658679 \Rightarrow \text{Antilog } 5,658679 = 455.699,97 \cong 455.700$$

$$11. \text{Log } 2^{10} = 10(\text{log } 2)$$

$$= 10(0,301029)$$

$$= 3,010299$$

$$\text{Anti log } 3,010299 = 1.024$$

$$12. \text{Log } 0,15^3 = 3(\text{log } 0,15)$$

$$= 3(\bar{1},176091)$$

$$= 3(-1) + 3(0,176091)$$

$$= -2,471727$$

$$10^{-2,471727} = 0,00337499 \cong 0,003375$$

$$13. \text{Log } 18,65^4 = 4(\text{log } 18,65)$$

$$= 4(1,270678)$$

$$= 5,082715$$

$$\text{Anti log } 5,082715 = 120.980,4916$$

$$\cong 120.980,5$$

$$14. \text{Log } (2 \cdot 0,084)^2 = \text{log } (0,168)^2$$

$$= 2(\text{log } 0,168)$$

$$= 2(\bar{1},225309)$$

$$= 2(-1) + 2(0,225309)$$

$$= -1,549382 = \bar{2},450618$$

$$10^{-1,549382} = 0,028223963 \cong 0,028224$$

$$\text{Anti log } \bar{2},450618 = 0,028224$$

$$15. \text{Log } 7,2^6 = 6(\text{log } 7,2)$$

$$= 6(0,857332)$$

$$= 5,143994$$

$$\text{Anti log } 5,143994 = 139.313,75$$

$$16. \text{Log}\sqrt{3} = \frac{\log 3}{2} = \frac{0,477121}{2} = 0,238560$$

$$\text{Antilog } 0,238560 = 1,7320483 \approx 1,73205$$

$$17. \text{Log}\sqrt[3]{2} = \frac{\log 2}{3} = \frac{0,301029}{3} = 0,100343$$

$$10^{0,100343} = 1,259921$$

$$18. \text{Log}\sqrt[4]{5} = \frac{\log 5}{4} = \frac{0,698970}{4} = 0,174742$$

$$\text{Antilog } 0,174742 = 1,495348 \approx 1,49535$$

$$19. \text{Log}\sqrt[5]{63} = \frac{\log 63}{5} = \frac{1,799340}{5} = 0,359868$$

$$10^{0,359868} = 2,290172$$

$$20. \text{Log}\sqrt[7]{815} = \frac{\log 815}{7} = \frac{2,911157}{7} = 0,415879$$

$$\text{Antilog } 0,415879 = 2,605431$$

## EJERCICIO 299

$$1. \text{Log} 515 + \log 78,19 + \text{colog} 6,13$$

$$= 2,711807 + 1,893151 + \bar{1},212539$$

$$= 3,817497 \rightarrow \text{Antilog } 3,817497 = 6.568,96 \approx 6.569$$

$$2. \text{Log} 23,054 + \log 934,5 + \text{colog} 8.164$$

$$= 1,36275 + 2,97058 + \bar{4},0881$$

$$= 0,42143 \rightarrow \text{Antilog } 0,42143 = 2,63894$$

$$3. \text{Log} 8,14 + \log 9,73 + \text{colog} 0,6 + \text{colog} 7,8$$

$$= 0,91062 + 0,98811 + 0,22185 + \bar{1},10791$$

$$= 1,22849 \rightarrow \text{Antilog } 1,22849 = 16,923461 \approx 16,9235$$

$$4. \text{Log} 513,4 + \log 9,132 + \text{colog} 85,3 + \text{colog} 10,764$$

$$= 2,71046 + 0,96057 + \bar{2},06905 + \bar{2},96802$$

$$= 0,7081 \rightarrow \text{Antilog } 0,7081 = 5,10622$$

$$5. \text{Log} 53,245 + \log 4.325,6 + \text{colog} 32,815 + \text{colog} 91,79$$

$$= 1,72628 + 3,63605 + \bar{2},48392 + \bar{2},03721$$

$$= 1,88346 \rightarrow \text{Antilog } 1,88346 = 76,4645$$

$$6. \text{Log} 32,6 + \log 841,9 + \text{colog} 0,017 + \text{colog} 732,14$$

$$= 1,51322 + 2,92526 + 1,76955 + \bar{3},135406$$

$$= 3,343436 \rightarrow \text{Antilog } 3,343436 = 2.205,139151$$

Como existe un signo negativo en la operación entonces este afecta el resultado total, luego la respuesta final es  $\approx -2.205,14$

$$7. \text{Log} 95,36 + \log 0,14 + \text{colog} 83,7 + \text{colog} 2,936$$

$$= 1,97937 + \bar{1},14613 + \bar{2},07727 + \bar{1},53224$$

$$= -1,26499 \rightarrow \text{Antilog } -1,26499 = 0,054326$$

El signo negativo se anula puesto que se encuentra en el numerador y en el denominador luego la respuesta es positiva

$$8. \text{Log} 7,2 + \log 8,135 + \text{colog} 0,003 + \text{colog} 9.134,7$$

$$= 0,85733 + 0,91036 + 2,52288 + \bar{4},03930$$

$$= 0,32987 \rightarrow \text{Antilog } 0,32987 = -2,137333 \approx -2,13734$$

$$9. 3^5 \cdot 0,2^4 = 5 \log 3 + 4 \log 0,2$$

$$= 5(0,47712) + 4(\bar{1},30103)$$

$$= 2,38561 - 2,79588$$

$$= -0,41027 \rightarrow 10^{-0,41027} = 0,3888$$

\* En la tabla se busca  $\bar{1},58973$

$$\Rightarrow \text{Anti log } 0,58973 = 0,3888$$

$$10. \frac{1}{2} \log 5 + \frac{2}{3} \log 3$$

$$= \frac{1}{2}(0,69897) + \frac{2}{3}(0,47712)$$

$$= 0,349485 + 0,31808$$

$$= 0,66757$$

$$\rightarrow \text{Anti log } 0,66757 = 4,651199 \approx 4,6512$$

$$11. \frac{1}{5} \log 2 + \frac{1}{2} \log 3 + \frac{3}{4} \log 5$$

$$= \frac{1}{5}(0,30103) + \frac{1}{2}(0,47712) + \frac{3}{4}(0,69897)$$

$$= 0,060206 + 0,23856 + 0,5242275$$

$$= 0,82299$$

$$\rightarrow \text{Anti log } 0,82299 = 6,65257 \approx 6,6526$$

$$12. 8 \log 3 + 5 \text{colog } 5,6$$

$$= 8(0,47712) + 5(\bar{1},25181)$$

$$= 3,81696 - 3,74095$$

$$= 0,07601$$

$$\rightarrow \text{Anti log } 0,07601 = 1,19127 \approx 1,1913$$

$$13. 7 \log 0,53 + 3 \text{colog } 2,5$$

$$= 7(\bar{1},72428) + 3(\bar{1},60206)$$

$$= -1,93004 - 1,19382$$

$$= -3,12386 \rightarrow 10^{-3,12386} = 0,000751865$$

\* En la tabla se busca  $\bar{4},87614$

$$\begin{aligned}
 14. \quad & \frac{2}{5} \log 3 + \frac{5}{3} \operatorname{colog} 2 \\
 &= \frac{2}{5} (0,47712) + \frac{5}{3} (\bar{1},69897) \\
 &= 0,190848 - 0,501717 \\
 &= -0,310869 \rightarrow 10^{-0,310869} = 0,488799 \approx 0,4888 \\
 &* \text{ En la tabla se busca } \bar{1},68914
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & \frac{\operatorname{Log} 7,86 + \operatorname{Log} 8,14}{2} \\
 &= \frac{0,89542 + 0,91062}{2} \\
 &= \frac{1,80604}{2} \\
 &= 0,90302 \rightarrow \operatorname{Antilog} 0,90302 = 7,9987
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \frac{\operatorname{Log} 932,5 + \operatorname{Log} 813,6 + \operatorname{Log} 0,005}{2} \\
 &= \frac{2,96965 + 2,91041 + \bar{3},69897}{2} \\
 &= \frac{3,57903}{2} \\
 &= 1,78952 \quad \operatorname{Antilog} 1,78952 = 61,59138
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & \frac{\operatorname{Log} 93,7 + \operatorname{Log} 104,2 + \operatorname{colog} 8,35 + \operatorname{colog} 7,3}{2} \\
 &= \frac{1,97174 + 2,01787 + \bar{1},07831 + \bar{1},13668}{2} \\
 &= \frac{2,2046}{2} = 1,1023 \rightarrow \operatorname{Antilog} 1,1023 = 12,656174
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & \frac{\operatorname{Log} 23,725 + \operatorname{Log} 9,182 + \operatorname{Log} 7,184}{3} \\
 &= \frac{1,37521 + 0,96294 + 0,85637}{3} \\
 &= \frac{3,19452}{3} = 1,06484 \quad \operatorname{Antilog} 1,06484 = -11,610207
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & \frac{\operatorname{Log} 12,316 + \operatorname{Log} 0,25 + \operatorname{colog} 931,8 + \operatorname{colog} 0,07}{4} \\
 &= \frac{4,09047 + \bar{1},39794 + \bar{3},03068 + 1,15490}{4} \\
 &= \frac{1,67399}{4} = 0,41849 \rightarrow \operatorname{Antilog} 0,41849 = 2,621125
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & \frac{\operatorname{Log} 56,813 + \operatorname{colog} 22,117}{5} \\
 &= \frac{4,75446 + \bar{5},65527}{5} \\
 &= \frac{0,40973}{5} = 0,081946 \rightarrow \operatorname{Antilog} 0,081946 = 1,20766
 \end{aligned}$$

$$\begin{aligned}
 21. \quad & \frac{3}{2} \log 0,0316 + \frac{3}{2} \operatorname{colog} 0,1615 \\
 &= \frac{3}{2} (\bar{2},49969 + 0,79183) \\
 &= \frac{3}{2} (-0,70848) = -1,06272 = \bar{2},93728 \\
 &10^{-1,06272} = 0,086552 \\
 &\operatorname{Anti} \log \bar{2},93728 = 0,08655625
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & \frac{3}{4} \log 3 + \frac{2}{3} \operatorname{colog} 5 \\
 &= \frac{3}{4} (0,47712) + \frac{2}{3} (\bar{1},30103) \\
 &= 0,35784 - 0,46598 \\
 &= -0,10814 = \bar{1},89186 \\
 &\operatorname{Anti} \log \bar{1},89186 = 0,77958
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & \frac{\operatorname{Log} 15 + \operatorname{Colog} 4}{7} \\
 &= \frac{1,17609 + \bar{1},39794}{7} \\
 &= \frac{0,57403}{7} = 0,082 \\
 &\operatorname{Antilog} 0,082 = 1,20782
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & \frac{\operatorname{Log} 5 + \operatorname{Colog} 3}{5} \\
 &= \frac{0,69897 + \bar{1},52288}{5} = 0,04437 \\
 &\operatorname{Antilog} 0,04437 = -1,10756
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & \frac{6}{5} (\operatorname{Log} 5 + \operatorname{Colog} 8) \\
 &= \frac{6}{5} (0,69897 + \bar{1},09691) \\
 &= \frac{6}{5} (-0,20412) = -0,24494 = \bar{1},75506 \\
 &\operatorname{Anti} \log \bar{1},75506 = 0,568926 \approx 0,56893
 \end{aligned}$$

$$\begin{aligned}
 26. \quad & \frac{\operatorname{Log} 3 + \operatorname{Colog} 5}{2} + \frac{\operatorname{Log} 5 + \operatorname{Colog} 7}{3} \\
 &= \frac{3(0,47712 + \bar{1},30103) + 2(0,69897 + \bar{1},15490)}{6} \\
 &= \frac{3(-0,22185) + 2(-0,14613)}{6} \\
 &= \frac{-0,6655 - 0,29226}{6} \\
 &= \frac{-0,95781}{6} = -0,159635 = \bar{1},840364 \\
 &10^{-0,159635} = 0,6924126 \\
 &\operatorname{Anti} \log \bar{1},840364 = 0,69241
 \end{aligned}$$

$$\begin{aligned}
 27. \quad & \frac{\log 2}{7} + \frac{\log 3}{5} + \frac{\log 0,2}{3} \\
 &= \frac{15(0,30103) + 21(0,47712) + 35(\bar{1},30103)}{105} \\
 &= \frac{4,51545 + 10,01952 - 24,46395}{105} \\
 &= \frac{-9,92898}{105} = -0,094562 = \bar{1},905438
 \end{aligned}$$

$$\text{Anti log } \bar{1},905438 = 0,8043369 \approx 0,80434$$

$$\begin{aligned}
 28. \quad & \frac{\log 32,14}{2} + \frac{\log 59,3}{3} + \frac{\log 317,6}{4} \\
 &= \frac{6(1,50705) + 4(1,77305) + 3(\bar{3},49812)}{12} \\
 &= \frac{9,0423 + 7,0922 - 7,50564}{12} \\
 &= \frac{8,62886}{12} = 0,71907
 \end{aligned}$$

$$\text{Anti log } 0,71907 = 5,2368483 \approx 5,23685$$

$$29. \quad \frac{2 \log 0,75 + \log 39,162 + \log 0,07 + \log 3,89}{2}$$

$$= \frac{2(\bar{1},87506) + 1,59286 + 1,15490 + \bar{1},41005}{2}$$

$$= \frac{1,90793}{2} = 0,953965 = 8,99425 \approx 8,9943$$

$$30. \quad \frac{3 \log 0,2 + 2 \log 0,3 + 4 \log 0,05 + \log 3,25}{2}$$

$$= \frac{3(\bar{1},30103) + 2(\bar{1},47712) + 4(1,30103) + \bar{1},48812}{2}$$

$$= \frac{-3 + 0,90309 - 2 + 0,95424 + 5,20412 - 1 + 0,48812}{2}$$

$$= \frac{1,54957}{2} = 0,774785$$

$$\text{Anti log } 0,774785 = 5,95366$$

## EJERCICIO 300

$$1. \quad 36 = 2^2 \cdot 3^2$$

$$\log 36 = 2 \log 2 + 2 \log 3$$

$$= 2(0,30103) + 2(0,47712)$$

$$= 0,60206 + 0,95424$$

$$\log 36 = 1,55630$$

$$2. \quad 75 = 3 \cdot 5^2$$

$$\log 75 = \log 3 + 2 \log 5$$

$$= 0,47712 + 2(0,69897)$$

$$= 0,47712 + 1,39794$$

$$\log 75 = 1,87506$$

$$3. \quad 30 = 2 \cdot 3 \cdot 5$$

$$\log 30 = \log 2 + \log 3 + \log 5$$

$$= 0,30103 + 0,47712 + 0,69897$$

$$\log 30 = 1,47712$$

$$4. \quad 48 = 2^4 \cdot 3$$

$$\log 48 = 4 \log 2 + \log 3$$

$$= 4(0,30103) + 0,47712$$

$$= 1,20412 + 0,47712$$

$$\log 48 = 1,68124$$

$$5. \quad 120 = 2^3 \cdot 3 \cdot 5$$

$$\log 120 = 3 \log 2 + \log 3 + \log 5$$

$$= 3(0,30103) + 0,47712 + 0,69897$$

$$= 0,90309 + 1,17609$$

$$\log 120 = 2,07918$$

$$6. \quad 98 = 7^2 \cdot 2$$

$$\log 98 = 2 \log 7 + \log 2$$

$$= 2(0,845098) + 0,30103$$

$$= 1,690196 + 0,30103$$

$$\log 98 = 1,991226$$

$$7. \quad 343 = 7^3$$

$$\log 343 = 3 \log 7$$

$$= 3(0,845098)$$

$$\log 343 = 2,535294$$

$$\log 0,343 = \bar{1},535294$$

$$8. \quad 225 = 5^2 \cdot 3^2$$

$$\log 225 = 2 \log 5 + 2 \log 3$$

$$= 2(0,69897) + 2(0,47712)$$

$$= 1,39794 + 0,95424$$

$$= 2,352182$$

$$\log 22,5 = 1,352182$$

$$9. \quad 196 = 2^2 \cdot 7^2$$

$$\log 196 = 2(\log 2 + \log 7)$$

$$= 2(0,30103 + 0,845098)$$

$$= 2(1,146128)$$

$$= 2,292256$$

$$\log 1,96 = 0,292256$$

10.  $875 = 5^3 \cdot 7$   
 $\text{Log } 875 = 3\log 5 + \log 7$   
 $= 3(0,69897) + 0,845098$   
 $= 2,09691 + 0,845098$   
 $= 2,942008$   
 $\text{Log } 0,875 = \bar{1},942008$
11.  $2.025 = 3^4 \cdot 5^2$   
 $\text{Log } 2.025 = 4\log 3 + 2\log 5$   
 $= 4(0,477121) + 2(0,69897)$   
 $= 1,908484 + 1,39794$   
 $= 3,306424$   
 $\text{Log } 202,5 = 2,306424$
12.  $448 = 2^6 \cdot 7$   
 $\text{Log } 448 = 6\log 2 + \log 7$   
 $= 6(0,30103) + 0,845098$   
 $= 1,80618 + 0,845098$   
 $= 2,651278$   
 $\text{Log } 44,8 = 1,651278$
13.  $2\frac{1}{2} = \frac{5}{2} = 2,5$   
 $25 = 5^2$   
 $\text{Log } 25 = 2(0,69897)$   
 $= 1,39794$   
 $\text{Log } 2\frac{1}{2} = 0,397940$
14.  $1\frac{1}{2} = \frac{3}{2} = 1,5$   
 $15 = 3 \cdot 5$   
 $\text{Log } 15 = \log 3 + \log 5$   
 $= 0,477121 + 0,698970$   
 $= 1,176091$   
 $\text{Log } 1\frac{1}{2} = 0,176091$
15.  $1\frac{2}{5} = \frac{7}{5} = 1,4$   
 $14 = 7 \cdot 2$   
 $\text{Log } 14 = \log 7 + \log 2$   
 $= 0,845098 + 0,30103$   
 $= 1,146128$   
 $\text{Log } 1\frac{2}{5} = 0,146128$
16.  $2\frac{1}{3} = \frac{7}{3}$   
 $\text{Log } 2\frac{1}{3} = \log \frac{7}{3}$   
 $= \log 7 + \text{colog } 3$   
 $= 0,845098 + \bar{1},522879$   
 $= 0,367977$
17.  $13 = \frac{143}{11}$   
 $\text{Log } 13 = \log \frac{143}{11}$   
 $= \log 143 + \text{colog } 11$   
 $= 2,155336 + \bar{2},958607$   
 $\text{Log } 13 = 1,113943$
18.  $25 = \frac{225}{9}$   
 $\text{Log } 25 = \log \frac{225}{9}$   
 $= \log 225 + \text{colog } 9$   
 $= 2,352183 + \bar{1},045757$   
 $\text{Log } 25 = 1,397940$

## EJERCICIO 301

1.  $5^x = 3$   
 $\text{Log } 5^x = \log 3$   
 $x \log 5 = \log 3$   
 $x(0,69897) = 0,477121$   
 $x = \frac{0,477121}{0,69897} = 0,6826$
2.  $7^x = 512$   
 $\text{Log } 7^x = \log 512$   
 $x \log 7 = \log 512$   
 $x = \frac{2,70927}{0,845098} = 3,205864$   
 $x \approx 3,2059$
3.  $0,2^x = 0,0016$   
 $\text{Log } 0,2^x = \log 0,0016$   
 $x \log 0,2 = \log 0,0016$   
 $x(\bar{1},30103) = \bar{3},20412$   
 $x = \frac{\bar{3},20412}{\bar{1},30103}$   
 $x = \frac{-3}{-0,69897} + \frac{0,20412}{-0,69897}$   
 $x = 4,29203 - 0,29203$   
 $x = 4$
4.  $9^x = 0,576$   
 $\text{Log } 9^x = \log 0,576$   
 $x \log 9 = \bar{1},76042$   
 $x(0,95424) = \bar{1},76042$   
 $x = \frac{\bar{1},76042}{0,95424}$   
 $x = \frac{-0,23958}{0,95424}$   
 $x = -0,2510689$   
 $x \approx -0,25107$
5.  $3^{x+1} = 729$   
 $\text{Log } 3^{x+1} = \log 729$   
 $(x+1)\log 3 = \log 729$   
 $(x+1)(0,477121) = 2,86273$   
 $x+1 = 6,00944$   
 $x = 6,00944 - 1$   
 $x = 5,00944 \Rightarrow x \approx 5$
6.  $5^{x-2} = 625$   
 $\text{Log } 5^{x-2} = \log 625$   
 $(x-2)\log 5 = 2,79588$   
 $x-2 = \frac{2,79588}{0,69897}$   
 $x-2 = 4 \Rightarrow x = 6$
7.  $2^{3x+1} = 128$   
 $\text{Log } 2^{3x+1} = \log 128$   
 $(3x+1)\log 2 = 2,10721$   
 $(3x+1)(0,30103) = 2,10721$   
 $3x+1 = 7$   
 $3x = 6$   
 $x = 2$
8.  $3^{2x-1} = 2.187$   
 $\text{Log } 3^{2x-1} = \log 2.187$   
 $(2x-1)\log 3 = 3,33985$   
 $(2x-1)(0,477121) = 3,33985$   
 $2x-1 = 7$   
 $2x = 8 \Rightarrow x = 4$
9.  $11^{2x} = 915$   
 $\text{Log } 11^{2x} = \log 915$   
 $2x \log 11 = 2,96142$   
 $2x = \frac{2,96142}{1,04139}$   
 $2x = 2,84372$   
 $x = 1,42186$

## EJERCICIO 302

1.  $a=3 \quad u=48 \quad r=6 \div 3=2$

$$n = \frac{\text{Log}48 + \text{Colog}3}{\text{Log}2} + 1$$

$$n = \frac{1,68124 + \bar{1},522879}{0,30103} + 1$$

$$n = \frac{1,204119}{0,30103} + 1 = 4 + 1 = 5$$

2.  $a=2 \quad u=\frac{243}{16} \quad r=\frac{3}{2}$

$$n = \frac{\text{Log}\frac{243}{16} + \text{Colog}2}{\text{Log}\frac{3}{2}} + 1$$

$$n = \frac{\text{Log}243 + \text{colog}16 + \text{colog}2}{\text{Log}1,5} + 1$$

$$n = \frac{2,38461 + \bar{2},79588 + \bar{1},69897}{0,176091} + 1$$

$$n = \frac{0,87946}{0,176091} + 1 = 4,994 + 1 \quad n=5,994 \quad n \approx 6$$

3.  $a=4 \quad u=512 \quad r=8 \div 4=2$

$$n = \frac{\text{Log}512 + \text{Colog}4}{\text{Log}2} + 1$$

$$n = \frac{2,70927 + \bar{1},39794}{0,30103} + 1$$

$$n = \frac{2,10721}{0,30103} + 1 = 7 + 1 = 8$$

4.  $a=6 \quad u=\frac{2.048}{81} \quad r=\frac{8}{6}=\frac{4}{3}$

$$n = \frac{\text{Log}\frac{2.048}{81} + \text{Colog}6}{\text{Log}\frac{4}{3}}$$

$$n = \frac{\text{Log}2.048 + \text{Colog}81 + \text{Colog}6}{\text{Log}4 + \text{Colog}3}$$

$$n = \frac{3,31133 + \bar{2},09151 + \bar{1},22184}{0,60206 + 1,522879} + 1$$

$$n = \frac{0,62468}{0,124939} + 1$$

$$n = 4,999879 + 1$$

$$n = 5,999879$$

$$n \approx 6$$

5.  $a=2 \quad u=\frac{625}{8} \quad r=\frac{5}{2}=2,5$

$$n = \frac{\text{Log}\frac{625}{8} + \text{Colog}2}{\text{Log}2,5} + 1$$

$$n = \frac{\text{Log}625 + \text{Colog}8 + \text{Colog}2}{\text{Log}2,5} + 1$$

$$n = \frac{2,79588 + \bar{1},09691 + \bar{1},69897}{0,39794} + 1$$

$$n = \frac{1,59176}{0,39794} + 1 = 4 + 1 = 5$$

## EJERCICIO 303

1.  $c=500 \quad r=0,06 \quad t=3$

$$C = 500(1+0,06)^3$$

$$C = 500(1,06)^3$$

$$\text{Log } C = \text{Log } 500(1,06)^3$$

$$\text{Log } C = \text{Log } 500 + 3 \log 1,06$$

$$\text{Log } C = 2,69897 + 3(0,02531)$$

$$\text{Log } C = 2,7749$$

$$\text{Anti } \log 2,7749 = \$595,51$$

2.  $c=3.500 \quad r=0,07 \quad t=5$

$$C = 3.500(1+0,07)^5$$

$$\text{Log } C = \text{Log } 3.500(1,07)^5$$

$$\text{Log } C = \text{Log } 3.500 + 5 \text{Log } 1,07$$

$$= 3,54407 + 5(0,02938)$$

$$= 3,54407 + 0,1469$$

$$\text{Log } C = 3,69097$$

$$\text{Anti } \log 3,69097 = 4.908,94 \text{ soles}$$

3.  $c=8.132 \quad r=0,09 \quad t=10$

$$C = 8.132(1+0,09)^{10}$$

$$\text{Log } C = \text{Log } 8.132(1,09)^{10}$$

$$= \text{Log } 8.132 + 10 \text{Log } 1,09$$

$$= 3,91020 + 10(0,03743)$$

$$= 3,91020 + 0,3743$$

$$\text{Log } C = 4,2845$$

$$\text{Anti } \log 4,2845 = 19.253,070 \text{ bs.}$$



$$4. c=930 \quad r=0,035 \quad t=7$$

$$C=930(1+0,035)^7$$

$$\text{Log } C = \text{Log } 930(1,035)^7$$

$$= \text{Log } 930 + 7 \text{Log } 1,035$$

$$= 2,9684829 + 7(0,0149403)$$

$$= 2,9684829 + 0,1045821$$

$$\text{Log } C = 3,073065 \quad \text{Anti log } 3,073065 = \$1.183,2186$$

$$5. C=12.318 \quad t=6$$

$$r=4\frac{1}{4}=\frac{17}{4}=4,25 \Rightarrow \frac{4,25}{100}=0,0425$$

$$C=12.318(1+0,0425)^6$$

$$\text{Log } C = \text{Log } 12.318(1,0425)^6$$

$$= \text{Log } 12.318 + 6 \text{Log } 1,0425$$

$$= 4,0905402 + 6(0,018076)$$

$$= 4,0905402 + 0,108456$$

$$\text{Log } C = 4,1989962$$

$$\text{Anti log } 4,1989962 = \$15.812,34204$$

$$6. C=24.186 \quad t=7$$

$$r=5\frac{1}{2}=\frac{11}{2}=5,5 \Rightarrow \frac{5,5}{100}=0,055$$

$$C=24.186(1+0,055)^7$$

$$\text{Log } C = \text{Log } 24.186(1,055)^7$$

$$= \text{Log } 24.186 + 7 \text{Log } 1,055$$

$$= 4,383564 + 7(0,0232524)$$

$$= 4,383564 + 0,1627668$$

$$\text{Log } C = 4,5463308$$

$$\text{Anti log } 4,5463308 = 35.182,83244 \text{ sucres}$$

$$7. C=54.293 \quad t=5$$

$$r=3\frac{3}{4}=\frac{15}{4}=3,75 \Rightarrow \frac{3,75}{100}=0,0375$$

$$\text{Log } C = \text{Log } 54.293(1,0375)^5$$

$$= \text{Log } 54.293 + 5 \text{Log } 1,0375$$

$$= 4,734743 + 5(0,015988)$$

$$= 4,734743 + 0,07994$$

$$= 4,814683$$

$$\text{Anti log } 4,814683 = \$65.265,399$$

$$8. c=800 \quad t=4 \quad r=\frac{0,03}{2}=0,015$$

$$C=800(1+0,015)^4$$

$$\text{Log } C = \text{Log } 800(1+0,015)^4$$

$$= \text{Log } 800 + 4 \text{Log } 1,015$$

$$= 2,90309 + 4(0,00647)$$

$$= 2,90309 + 0,02588$$

$$\text{Log } C = 2,92897 \quad \text{Anti log } C = \$849,09$$

$$9. c=900 \quad t=\frac{12}{3}=4 \quad r=\frac{0,04}{4}=0,01$$

$$C=900(1+0,01)^4$$

$$\text{Log } C = \text{Log } 900 + 4 \text{Log } 1,01$$

$$= 2,954242 + 4(0,004321)$$

$$= 2,954242 + 0,017284$$

$$= 2,971526$$

$$\text{Anti log } 2,971526 = \$936,54$$

$$10. r=0,05 \quad C=972,60 \quad t=4$$

$$c = \frac{972,60}{(1+0,05)^4}$$

$$\text{Log } c = \text{Log } 972,60 + 4 \text{Log } 1,05$$

$$= 2,987934 + 4(\bar{1},978811)$$

$$= 2,987934 - 0,084756$$

$$\text{Log } c = 2,903178$$

$$\text{Anti log } 2,903178 = \$800,16$$

$$11. C=1.893,5 \quad t=6$$

$$r=4\frac{1}{2}=\frac{9}{2}=4,5 \Rightarrow \frac{4,5}{100}=0,045$$

$$c = \frac{1.893,5}{(1+0,045)^6}$$

$$\text{Log } c = \text{Log } 1.893,5 + 6 \text{Log } 1,045$$

$$= 3,277265 + 6(\bar{1},980883)$$

$$= 3,277265 - 0,114702$$

$$= 3,162563$$

$$\text{Anti log } 3,162563 = \$1.454,02$$

$$12. C=54.198,16 \quad r=0,08 \quad t=7$$

$$c = \frac{54.198,16}{(1+0,08)^7}$$

$$\text{Log } c = \text{Log } 54.198,16 + 7 \text{Log } 1,08$$

$$= 4,733984 + 7(\bar{1},966576)$$

$$= 4,733984 - 0,233968$$

$$\text{Log } c = 4,500016$$

$$\text{Anti log } 4,500016 = 31.623,9 \approx 31.624 Q.$$

$$13. c=600 \quad r=0,03 \quad C=695,56$$

$$t = \frac{\text{Log} 695,56 - \text{Log} 600}{\text{Log} 1,03}$$

$$t = \frac{2,842334 - 2,778151}{0,012837}$$

$$t = \frac{0,064183}{0,012837} = 4,9998 \approx 5 \text{ años}$$

$$14. c=1.215 \quad C=1.709,61 \quad r=0,05$$

$$t = \frac{\text{Log} 1.709,61 - \text{log} 1.215}{\text{Log} (1+0,05)}$$

$$t = \frac{3,232897 - 3,084576}{0,021189}$$

$$t = \frac{0,148321}{0,021189} = 7 \text{ años}$$

$$15. c=800 \quad t=4 \quad C=1.048,63$$

$$\text{Log} (1+r) = \frac{\text{log} 1.048,63 - \text{log} 800}{4}$$

$$\text{Log} (1+r) = \frac{3,020622 - 2,903089}{4}$$

$$\text{Log} (1+r) = \frac{0,117533}{4} = 0,029384$$

$$\text{Anti log } 0,029384 = 1,07$$

$$1+r=1,07$$

$$r=0,07 \Rightarrow 7\%$$

$$16. c=6.354 \quad t=4 \quad C=7.151,46$$

$$\text{Log} (1+r) = \frac{\text{Log} 7.151,46 - \text{Log} 6.354}{4}$$

$$= \frac{3,854394 - 3,803047}{4}$$

$$\text{Log} (1+r) = 0,012836$$

$$\text{Anti log } 0,012836 = 1,03$$

$$1+r=1,03$$

$$r=0,03 \Rightarrow 3\%$$

$$17. c=900 \quad r=0,05 \quad t=2+\frac{1}{3}=2,33333$$

$$C=900(1+0,05)^{2,33333}$$

$$\text{Log } C = \text{Log } 900 + 2,33333 \text{log } 1,05$$

$$= 2,954242 + 2,33333(0,0211892)$$

$$= 2,954242 + 0,049441$$

$$= 3,003683$$

$$\text{Anti log } 3,003683 = 1.008,52$$

Los intereses producidos representan la diferencia entre la suma inicial y lo que se ha convertido

$$\Rightarrow \text{Intereses producidos} = 1.008,52 - 900 = \$108,52$$

## EJERCICIO 304

$$1. c=40.000 \quad r=0,05 \quad t=10$$

$$a = \frac{40.000(0,05)(1,05)^{10}}{(1,05)^{10} - 1}$$

$$\begin{aligned} \text{Log} (1,05)^{10} &= 10 \text{Log } 1,05 \\ &= 10(0,0211892) \\ &= 0,211892 \end{aligned}$$

$$\text{Anti log } 0,211892 = 1,62889$$

$$\text{Log } a = \text{log } 40.000 + \text{log } 0,05 + \text{log } 1,62889 + \text{co log } 0,62889$$

$$= 4,602059 + \overline{2},69897 + 0,211892 + 0,2014253$$

$$= 3,7143463$$

$$\text{Anti log } 3,7143463 = \$5.180,21$$

2.  $c=85.000 \quad r=0,03 \quad t=12$

$$a = \frac{85.000(0,03)(1+0,03)^{12}}{(1+0,03)^{12} - 1}$$

$$\begin{aligned} \text{Log}(1,03)^{12} &= 12 \log 1,03 \\ &= 12(0,012837) \\ &= 0,154044 \end{aligned}$$

$\text{Anti log } 0,154044 = 1,425761$

$$\begin{aligned} \text{Log } a &= \log 85.000 + \log 0,03 + \log 1,425761 + \text{co log } 0,425761 \\ &= 4,929418 + \bar{2},477121 + 0,154044 + 0,370834 \\ &= 3,931417 \end{aligned}$$

$\text{Anti log } 3,931417 = 8.539,2 \text{ soles}$

3.  $c=600.000 \quad r=0,05 \quad t=20$

$$a = \frac{600.000(0,05)(1+0,05)^{20}}{(1+0,05)^{20} - 1}$$

$$\begin{aligned} \text{Log}(1,05)^{20} &= 20 \log 1,05 \\ &= 20(0,0211892) \\ &= 0,423784 \end{aligned}$$

$\text{Anti log } 0,423784 = 2,65328$

$$\begin{aligned} \text{Log } a &= \log 600.000 + \log 0,05 + \log 2,65328 + \text{co log } 1,65328 \\ &= 5,778151 + \bar{2},69897 + 0,423784 + \bar{1},781653 \\ &= 4,682558 \end{aligned}$$

$\text{Anti log } 4,682558 = \$48.145,75$

4.  $c=5'000.000 \quad r=0,06 \quad t=30$

$$a = \frac{5'000.000(0,06)(1+0,06)^{30}}{(1+0,06)^{30} - 1}$$

$$\begin{aligned} \text{Log}(1,06)^{30} &= 30 \log 1,06 \\ &= 30(0,025306) \\ &= 0,75918 \end{aligned}$$

$\text{Anti log } 0,75918 = 5,7435$

$$\begin{aligned} \log a &= \log 5'000.000 + \log 0,06 + \log 5,7435 + \text{co log } 4,7435 \\ &= 6,69897 + \bar{2},778151 + 0,75918 + \bar{1},323901 \\ &= 5,5602 \end{aligned}$$

$\text{Anti log } 5,5602 = 363.245,2 \text{ bs.}$

5.  $c=3.000 \quad r=0,06 \quad t=5$

$a = 3.000 \cdot 0,237396$

$a = 712,188 \approx 712,19 \text{ bs.}$

$$a = \frac{3.000(0,06)(1,06)^5}{(1,06)^5 - 1}$$

$$a = \frac{180(1,338226)}{0,338226}$$

$$a = \frac{240,88068}{0,338226}$$

$a = 712,1885367 \approx 712,19 \text{ bs.}$

NOTA:

$0,237396 \rightarrow \text{Vr. que aparece en la tabla de interés compuesto decreciente para el } 6\% \text{ en } 5 \text{ años.}$

$1,338226 \rightarrow \text{Vr. que aparece en la tabla de interés compuesto para el } 6\% \text{ a } 5 \text{ años.}$

6.  $c=12.000 \quad r=0,07 \quad t=12$

$a = 12.000 \cdot 0,125902$

$a = 1.510,824 \text{ bs.}$

7.  $c=350.000 \quad r=4\frac{1}{2}\% \quad t=3$

$a = 350.000 \cdot 0,363773$

$a = 127.320,55 \text{ sucres}$

8.  $c=425.000 \quad r=6\% \quad t=10$

$a = 425.000 \cdot 0,135868$

$a = 57.743,9 \text{ soles}$

9.  $c=90.750 \quad r=5\% \quad t=25$

$a = 90.750 \cdot 0,070952$

$a = 6.438,894 \text{ bs.}$

10.  $c=73.550 \quad r=5\frac{1}{2}\% \quad t=30$

$a = 73.550 \cdot 0,068805$

$a = 5.060,60775 \approx 5.060,61 \text{ bs.}$

11.  $c=473.000 \quad r=3\frac{1}{2}\% \quad t=9$

$a = 473.000 \cdot 0,131446$

$a = 62.173,958 \approx 62.173,96 \text{ sucres}$

$$12. \quad c = 45.800 \quad r = 4\% \quad t = 30$$

$$a = 45.800 \cdot 0,057830$$

$$a = 2.648,614 \text{ soles}$$

$$a = \frac{45.800(0,04)(1,04)^{30}}{(1,04)^{30} - 1}$$

$$\text{Log}(1,04)^{30} = 30 \log 1,04$$

$$= 30(0,017033)$$

$$= 0,51099$$

$$\text{Anti log } 0,51099 = 3,243321$$

$$\text{Log } a = \text{Log } 45.800 + \log 0,04 + \log 3,243321 + \text{co log } 2,243321$$

$$= 4,660865 + \bar{2},60206 + 0,510989 + \bar{1},649108$$

$$= 3,423022$$

$$\text{Anti log } 3,423022 = 2.648,61 \text{ soles}$$

## EJERCICIO 305

$$1. \quad c = 30.000 \quad r = 0,06 \quad t = 9$$

$$i = \frac{30.000(0,06)}{(1+0,06)^{10} - (1+0,06)}$$

$$\text{Log}(1,06)^{10} = 10 \log 1,06$$

$$= 10(0,02531)$$

$$= 0,2531$$

$$\text{Anti log } 0,2531 = 1,791018$$

$$\text{Log } i = \text{Log } 30.000 + \log 0,06 + \text{co log } 0,731018$$

$$= 4,477121 + \bar{2},778151 + 0,136072$$

$$= 3,391344$$

$$\text{Anti log } 3,391344 = \$ 2.462,38$$

$$3. \quad c = 200.000 \quad r = 0,05 \quad t = 40$$

$$i = \frac{200.000(0,05)}{(1,05)^{41} - 1,05}$$

$$\text{Log}(1,05)^{41} = 41 \log 1,05$$

$$= 41(0,0211892)$$

$$= 0,868757$$

$$\text{Anti log } 0,868757 = 7,391915$$

$$\text{Log } i = \text{Log } 200.000 + \log 0,05 + \text{co log } 6,341915$$

$$= 5,30103 + \bar{2},69897 + \bar{1},19777$$

$$= 3,19777$$

$$\text{Anti log } 3,19777 = \$ 1.576,79$$

$$2. \quad c = 90.000 \quad r = 0,04 \quad t = 20$$

$$i = \frac{90.000(0,04)}{(1,04)^{21} - 1,04}$$

En la tabla de interés

$$\text{compuesto}(1,04)^{21} = 2,278768$$

$$\text{Log } i = \text{Log } 90.000 + \log 0,04 + \text{co log } 1,238768$$

$$= 4,95424 + \bar{2},60206 + \bar{1},90701$$

$$= 3,4633$$

$$\text{Anti log } 3,4633 = 2.906,03 \text{ sucres}$$

$$4. \quad c = 40.000 \quad r = 0,06 \quad t = 25$$

$$i = \frac{40.000(0,06)}{(1,06)^{26} - 1,06}$$

$$(1,06)^{26} \rightarrow 4,549383$$

$$\text{Log } i = \log 40.000 + \log 0,06 + \text{co log } 3,489383$$

$$= 4,60206 + \bar{2},778151 + \bar{1},457251$$

$$= 2,837462$$

$$\text{Anti log } 2,837462 = \$ 687,79$$