ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

MATHEMATICS 9164/2

PAPER 2 PURE MATHEMATICS, MECHANICS, STATISTICS

NOVEMBER 2017 SESSION 3 hours

Additional materials:
Answer paper Graph paper
List of Formulae Non-programmable electronic calculator

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

If a numerical answer cannot be given exactly and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree and in other cases it should be given correct to 2 significant figures.

If a numerical value for g is necessary, take $g = 9.81\ ms^{-2}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

Within each section of the paper, questions are printed in the order of their mark allocations.

The use of a non-programmable electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 6 printed pages and 2 blank pages.

[Turn over

Section A: Pure Mathematics

1 Prove by induction that $\sum_{r=1}^{n}(r+1)2^r = n2^{n+1}$ for all $n \in Z^+$, where Z^+ is the set of positive integers. [8]

2 The circle defined by the equation $x^2 - 6x + y^2 - 4y = 0$ passes through the origin and crosses the y-axis at point **P**.

Find the

(i) centre and radius of the circle, [4]

(ii) coordinates of **P**, [2]

(iii) equation of the tangent to the circle at **P**. [4]

3 **(a)** Find the value of $\left(2+2\sqrt{3}i\right)^6$ using De Moivre's Theorem. [4]

(b) Express $\dfrac{\sin 6\theta}{4\sin\theta}$ in terms of $\cos\theta$. [6]

4 It is given that matrix $\mathbf{M} = \begin{pmatrix} 2 & 3 & 4 \\ -3 & 2 & 2 \\ 4 & -4 & 3 \end{pmatrix}$.

(a) Find the

(i) determinant of **M**, [2]

(ii) inverse of **M**. [5]

(b) Hence, or otherwise, solve the simultaneous equations:

$$\begin{aligned} 2x + 3y + 4z &= 1 \\ -3x + 2y + 2z &= 14 \\ 4x - 4y + 3z &= 22 \end{aligned}$$ [3]

5 **(a)** Given that $\ln y = \left(1+8e^{3x}\right)^{\frac{1}{2}}$, show that $\dfrac{dy}{dx} = \dfrac{12ye^{3x}}{\sqrt{1+8e^{3x}}}$. [3]

(b) Find the exact value of $\int_1^2 \dfrac{3}{x^2(3-x)}dx$. [8]

6 **(a)** **(i)** Sketch, on the same axis, the graphs of $f(x) = \cos x$ and $f(x) = \cos(x - 60°)$ for $0° \leq x \leq 360°$ showing clearly the intercepts with the axes. [3]

(ii) Hence state the geometrical relationship between the graph $f(x) = \cos(x - 60°)$ and the graph of $f(x) = \cos x$. [2]

(b) **(i)** Show that the equation $2\sin^2(x - 60°) = 1 + \cos(x - 60°)$ may be written as a quadratic equation in $\cos(x - 60°)$. [2]

(ii) Hence solve the equation in **(b)(i)** for $0° \leq x \leq 360°$. [4]

7 Relative to the origin **O**, the position vectors of points **P** and **Q** are $\begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ c \\ 2 \end{pmatrix}$ respectively.

(a) Determine whether the point **P** lies on line l whose equation is $\boldsymbol{r} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$ where λ is a parameter. [2]

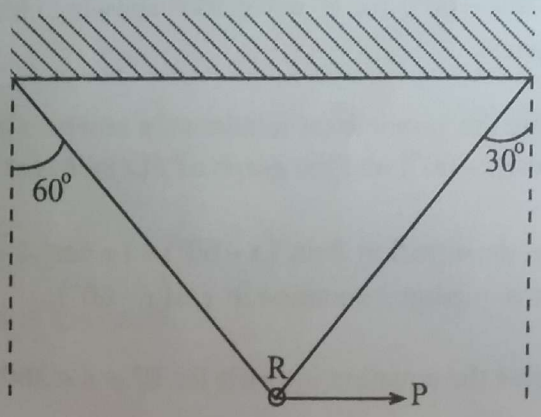
(b) **(i)** Given that line **PQ** intersects the line l, find the value of c. [3]

(ii) Hence, calculate the angle between line l and line **PQ**. [3]

(c) Find the position vector of the point **R** on line l such that line **PR** is perpendicular to line l. [4]

Section B: Mechanics

8



A light inextensible string is attached to two points at the same horizontal level. A smooth ring, R, of mass 2 kg can slide freely along the string. If a force of magnitude P newtons is applied horizontally to the ring and the string is inclined at 60° and 30° to the vertical, the ring rests in equilibrium (see diagram).

Find the

(i) tension in the string,

(ii) value of P.

[4]

9 A particle moves in a straight line with a constant velocity of 3 ms^{-1} for 3 seconds and then moves with a constant acceleration of -2 ms^{-2} for 8 seconds.

(i) Draw a velocity-time graph for the motion of the particle for the 11 seconds.

(ii) Find the displacement of the particle after the 11 seconds.

[5]

10 A particle of mass 12 kg rests on a smooth plane inclined at an angle of 45° to the horizontal. It is connected by a light inextensible string passing over a smooth pulley fixed at the top of the plane to a particle of mass 5 kg which is hanging freely.

Find the

(i) acceleration of the system and the tension in the string, [5]

(ii) force exerted by the string on the pulley. [2]

11 A particle is projected from a point O on a level ground with speed of 20 ms^{-1}. It passes through a point P, 10 m from O horizontally and 10 m vertically above O.

Find

(i) the **two** possible angles of projection, [4]

(ii) using the smaller angle of projection, the time taken by the particle to hit the ground. [2]

(iii) using the greater angle of projection, the direction of motion when the particle hits the ground. [2]

Section C: Statistics

12 A continuous random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} 2\left(\frac{a-x}{a^2}\right), & 0 \le x \le a, \\ 0 & \text{otherwise} \end{cases}$$

where a is a constant.

(a) Find E(X) in terms of a. [2]

(b) Show that the expression for the median reduces to $2m^2 - 4am + a^2 = 0$, where m is the median. [3]

13 The random variable X is Normally distributed with mean μ and standard deviation σ.

Given that $P(X > 3.6) = 0.5$ and $P(X > 2.8) = 0.6554$, find the value of μ and the value of σ. [5]

14 **A** and **B** play against each other in a game. Each game results in either a win for **A** or a win for **B**. The probability of **A** winning the first game is 0.6. If **A** wins a particular game, the probability of winning the next game is 0.7. If **A** loses a particular game the probability of winning the next game is 0.4.

Find the probability that

(i) **A** loses the second game, [2]

(ii) **A** wins the first game, given that **A** loses the second game. [3]

15 **(a)** State any **one** advantage and any **one** disadvantage of using a *stem and leaf diagram* as a method of representing data. [2]

(b) A group of 30 students had their heights measured correct to the nearest centimetre. The results are shown below:

167	174	156	180	162	169
177	154	165	174	160	184
169	179	151	163	173	148
171	168	158	158	167	166
149	153	171	162	182	162

(i) Using **five** stems, plot a stem and leaf diagram for the above information. [3]

(ii) Find the median. [1]

(iii) Find the interquartile range. [3]