ZIMBABWE SCHOOL EXAMINATIONS COUNCIL
General Certificate of Education Advanced Level

MATHEMATICS 9164/2
PAPER 2 Pure, Mechanics and Statistics

JUNE 2015 SESSION 3 hours

Additional materials:
Answer paper
Graph paper
List of Formulae
Electronic calculator

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

If a numerical value for g is necessary, take $g = 9.81\ \text{ms}^{-2}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

Within each section of the paper, questions are printed in the order of their mark allocations and candidates are advised, within each section, to attempt questions sequentially.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 6 printed pages and 2 blank pages.

1 (i) If $y = e^x \sin x$, show that $\frac{d^2y}{dx^2} = 2\left(\frac{dy}{dx} - y\right)$. [3]

(ii) Find the Maclaurin expansion of the function $e^x \sin x$ as far as the term in x^3 by further differentiation of the result in part (i), [3]

2 (i) Find the equation of a circle which has the points $(-7;\ 3)$ and $(1;\ 9)$ as end points of its diameter. [3]

(ii) Hence, find the equation of the tangent to the circle which passes through the point $(-7;\ 3)$. [4]

3 Prove by induction that $\sum_{r=1}^{n} ap^r = ap\left[\frac{1-p^n}{1-p}\right]$ for all positive integral values of n where a and p are constants. [7]

4 The temperature of meat in an oven is θ. The rate of increase of the temperature of the meat in the oven is proportional to the difference in the temperature that exists between the meat and the oven. The oven is kept at a constant temperature of 100 °C at any instant.

(i) Show that $\frac{d\theta}{dt} = k(100 - \theta)$, where k is a constant. [1]

(ii) A piece of meat which was initially at 4°C is placed into the oven at time $t = 0$ seconds. After one minute, the temperature rose to 16°C.

Express θ in terms of t seconds in its simplified form. [8]

5 (a) A plane, π, contains the point X with position vector $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and the line, l, with vector equation

$\mathbf{r} = 2\mathbf{i} + \lambda(\mathbf{j} + \mathbf{k})$, where λ is a parameter.

Find (i) the vector equation of π,

(ii) the shortest distance from the origin to π. [7]

(b) Given two planes whose equations are

$$\mathbf{r} \bullet \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = 3 \text{ and } \mathbf{r} \bullet \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} = 4,$$

find (i) the equation of the line of intersection of the two planes, [4]

(ii) the angle between the two planes. [3]

6 (a) Given that $z = \dfrac{5+i}{2+3i}$, find the fifth roots of z in the form $re^{i\theta}$. [8]

(b) Given that $1+i$ is a root of the equation $z^3+pz^2+qz+6=0$ where p and q are constants,

find

1. the other two roots,

2. the values of p and q.

[6]

7 (a) Given that matrix $T = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$,

(i) interpret geometrically the transformation represented by T,

(ii) find the image of a line $y=2x+3$ under the transformation T, giving your answer in the form $ax+by+c=0$.

[5]

(b) (i) Given that matrix $N = \begin{pmatrix} 1 & -1 & 0 \\ 5 & -4 & 1 \\ 2 & 2 & 1 \end{pmatrix}$,

find N^{-1}.

(ii) Hence, or otherwise solve the simultaneous equations:

$$\begin{aligned} x-y &= -4 \\ 5x-4y+z &= -12 \\ 2x+2y+z &= 11 \end{aligned}$$

[10]

$1-i$

[Turn over

8 A particle of mass 4 kg resting on a rough horizontal table is acted on by two horizontal forces of magnitudes 20 N and 30 N. The angle between the two forces is 120°. The coefficient of friction between the particle and the table is $\frac{1}{4}$.

Find the acceleration of the particle. [5]

9 A stone is thrown vertically upwards with a speed of $15\,\text{ms}^{-1}$.

(a) Find

(i) the time, t, taken by the stone to reach the maximum height,

(ii) the distance, x, travelled by the stone after 3 seconds. [5]

(b) Sketch a $(t;\,x)$ graph for the motion of the stone in the first 3 seconds. [3]

10 A particle is projected from a point O which is 5 m above the horizontal ground. The initial velocity of the particle is $30\ \text{ms}^{-1}$ at an angle of elevation $\arctan\left(\frac{3}{4}\right)$.

(i) Calculate the speed and direction of the particle 4 seconds after projection. [5]

(ii) The particle hits the ground at point T.

Find the displacement OT. [6]

Section (c) : Statistics

11 The marks obtained by candidates in a mathematics examination were displayed as follows:

1	3
2	6
3	1
4	1 3
5	0 2 6 8
6	1 2 2 2 7
7	0 3 4 5 5 8 9
8	0 3 4 4 8
9	2 7 7 8

key 4|1 means 41 %

(a) (i) State the name given to this display,

(ii) Calculate the range of the marks.

[2]

(b) Comment on the skewness of the distribution. [1]

(c) State any **one** advantage of this type of display of information. [1]

12 A continuous random variable, X, has a probability density function defined as

$$f(x) = \begin{cases} 0.1x+k, & 4 \le x \le 6 \\ 0.3, & 6 \le x \le 8 \\ 0. & \text{otherwise.} \end{cases}$$

Find

(a) the value of constant k, [2]

(b) $P(5 \le X \le 7)$. [2]

13 The distribution table shows prizes corresponding to six values on a fair spinner used in a game. The spinner lands on only one of the six values.

value	1	2	3	4	5	6
prize in $	2	2	6	4	10	6

(a) Find the probability of the spinner landing on

(i) a prime value,

(ii) a value that gives a prize of not less than $4. [2]

(b) Calculate the expected prize for a single game. [2]

14 Two tetrahedral dice with faces marked 0, 1, 2, 3 are thrown and the number on which each lands on is noted. The score is the sum of the 2 numbers. By means of an outcome table or otherwise,

find the probability that

(a) the score is a prime number, [3]

(b) one die lands on a 3 given that the score is a prime number. [3]

15 The masses of letters posted by a certain school are normally distributed with mean 15 g. It is found that the masses of 92 % of the letters are within 10 g of the mean.

Find

(a) the standard deviation of the masses of the letters, [3]

(b) the probability that at least 2 out of a random sample of 8 letters have masses which are within 10 g of the mean. [3]

BY PROF. T. HWATI THE TROTTER

0774998145

SECRETS THAT WILL MAKE MATHEMATICS EASY