4004/1 NOVEMBER 2018 SOLUTION GUIDE

QUESTION	SOLUTION	MARK	ADDITIONAL GUIDANCE
1 (a)	$\frac{2^3}{5^2}$ $= \frac{2\times2\times2}{5\times5}$ $= \frac{8}{25}$	1	Understanding that $2^3 = 2\times2\times2$ and $5^2 = 5\times5$ respectively is important. The division line is essential.
1 (b) (i)	$\frac{6}{25} = 0{,}24$	1	Evidence of correct division by 25 is required.
1 (b) (ii)	0,125 in standard form $= 1{,}25 \times 10^{-1}$	1	Knowledge of correct form of standard form is essential ($A \times 10^n$ where $1 \leq A < 10$ and n is an integer).
2 (a)	121	1	This question tests knowledge of types of numbers.
2 (b)	$\sqrt{\frac{3}{2}}$; π	2	This question tests knowledge of types of numbers.
3 (a)	$4\times5^3+3\times5^2+2$ $5^3 \quad 5^2 \quad 5^1 \quad 5^0$ 4 3 0 2 $4\,302_5$	1	Knowledge of expanded format of numbers in different bases, operations in the bases and the place value system of base five.
3 (b) (i)	$1\ 0111_2$ $+\quad 1010_2$ $1\ 0\ 0001_2$	1	General rules of addition of numbers in base two.

3 (b) (ii)	512_7 435_7 44_7	1	General rules of subtraction of numbers in base seven.
4 (a)	12.45 *a.m.*	1	The correct way of expressing time in 12 hour notation with only one dot between hours and minutes and showing whether it is before noon (a.m.) or afternoon (p.m.).
4 (b)	21 23 - 3 45 17 38 17 38 or 5.38*pm*	1	Knowledge of time zones and meaning of time ahead. Subtract time ahead from time given. The correct way of writing 24 hour notation with no dots in between and nothing written after.
4 (c)	1 hectare = $10\,000m^2$ 1 km^2 = 1 000m × 1 000m = $1\,000\,000m^2$ $\therefore$ 5 km^2 = $\frac{5\times 1\,000\,000}{10\,000}$ = 500 hectares.	1	Conversion of square kilometres to hectares being tested in this question. Recall that; 1 hectare = $10\,000m^2$ 1 km^2 =1 000m × $1000m$ = $1\,000\,000m^2$
5 (a)	$6{,}07\times10^4 = 6{,}07\times 10\,000$ = 60 700	1	Conversion from standard form to ordinary by recognizing that 10^4 = 10 000. Recognise that the digits move to the right number of places equivalent to the index of 10.
5 (b)	$2{,}53\times 10^1 + 6{,}1\times10^{-1}$ = 25,3 + 0,61 = 25,91 = $2{,}591\times10^1$	1	Convert both numbers to ordinary form, add and then convert the sum to standard form ($A\times10^n$,where $1\leq A < 10$ and n is an integer.)

6 (a)	$P\hat{R}Q = (180° - 96) \div 2$ $= 84 \div 2$ $= 42°$	 1	The question tests on understanding of types of angles between two parallel lines and their special relationships. The exterior angle in a triangle is equal to the sum of the two opposite interior angles and the fact that in an isosceles triangle two angles are equal.
6 (b)	$Q\hat{R}B = (180° - 84°) + 42°$ $= 96° + 42°$ $= 138°$	 1	Use of the theorem that allied or co-interior angles are supplementary.
6 (c)	$Q\hat{S}R = 90° - 42°$ $= 48°$	 1	Angles PQR and QRS are alternative hence equal. The candidate to recall that the sum of interior angles of a triangle is 180°.
7	$2x + 3y = 11$ (1) × 5 $3x - 5y = -12$ (2) × 3 $10x + 15y = 55$ $\underline{9x - 15y = -36}$ Add $19x = 19$ $x = 1$ Substitute 1 for x in (1) $2 \times 1 + 3y = 11$ $2 + 3y = 11$ $3y = 11 - 2$ $3y = 9$ $y = 3$	 1 1 1	The method of elimination consist of eliminating one of the variables with the same coefficient after multiplying and then substitute to get the other variable. Alternatively use the substitution, matrix and graphical methods. Matrix method $\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{19}\begin{pmatrix} -5 & -3 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 11 \\ -12 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{19}\begin{pmatrix} -55 + 36 \\ -33 - 24 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{19}\begin{pmatrix} -19 \\ -57 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $x = 1$ and $y = 3$.

8 (a)	$w \propto \frac{1}{f}$ $w = \frac{k}{f}$ $675 = \frac{k}{90}$ $k = 675 \times 90$ $k = 60\ 750$ $w = \frac{60\ 750}{f}$	 1 1	Recall the general form of inverse variation, in this question which is $w \propto \frac{1}{f}$ in order to find the equation. Use the given values of w and f to find the constant of variation or proportionality.
8 (b)	f when $w = 500$ $w = \frac{60\ 750}{f}$ $500 = \frac{60\ 750}{f}$ $500f = 60\ 750$ $f = \frac{60\ 750}{500}$ $f = \frac{1\ 215}{10}$ $f = 121{,}5$	 1	Substitution of given value in the equation found in (a) to the numerical value of f. Make f the subject of the equation.
9 (a)	45,3981 = 45,40.(correct to 4 s.g.f)	1	The zero at the end is very significant.
9 (b)	$7{,}5 \leq t < 8{,}5$	2	Knowledge of limits of accuracy needed.
10 (a)	$3x^2 - 15x$ $3x\ (x - 5)$.	1	Need to find the H.C.F of the two terms. Brackets are essential.
10 (b)	$8kl^2m = 2\times2\times2\times k\times l\times l\times m$ $= 2^3\ kl^2m$ $28k^2l^3m = 2\times2\times7\times k\times k\times l\times l\times l\times m$ $= 2^2\times7k^2l^3m$ $36l^2mn = 2\times2\times3\times3\times l\times l\times m\times n$ $= 2^2 \times 3^2l^2mn$	 1	There is need to express all terms as products of factors or as sets of factors so as to find the H.C.F.

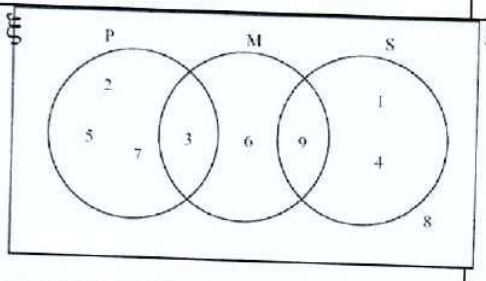
	H.C.F of $8kl^2m$, $28k^2l^3m$ and $36l^2mn$ is $2\times2\times l\times l\times m = 4lm^2$	1	
11(a)	A (6;2) and B (8;5) Gradient of line AB = $\frac{5-2}{8-6}$ $=\frac{3}{2.}$	 1	Use of formula for gradient of a given two points on the line. $\left(Gradient = \frac{increase\ in\ y}{increase\ in\ x}\right)$
(b)	$\frac{y-b}{x-a} = m$ $\frac{y-2}{x-6} = \frac{3}{2}$ $2(y-2) = 3(x-6)$ $2y - 4 = 3x - 18$ $2y = 3x - 18+4$ $2y = 3x - 14$ $y = \frac{3}{2}x - 7$	 1 1	Use of one of the points and the calculated gradient to find the equation. Make y the subject to find the equation in the required form. Alternatively use one of the points and the calculated gradient to find the value of c $\left(y = \frac{3}{2}x + c\right)$
12.	$\frac{2a+6}{a-3} \div \frac{a+3}{a-2a-3}$ $=\frac{2(a+3)}{a-3} \times \frac{(a+1)(a-3)}{a+3}$ $= 2(a+1)\ or\ 2a+2$	 1 + 1 1	Factorise the expression then inverts the divisor. Common factors can be divided to remain with the answer in that form where brackets are essential or expanded form with no brackets.
13 (a)	3,5kg: 800g 3 500: 800 35: 8	 1	Express the two masses in the same unit (grammes) and reduced to its simplest form. A simplified form with no common factor in the two terms.
13 (b)	120% is to 4,5t 100% is to? less $=\frac{100}{120} \times \frac{4,5}{1}$ $=\frac{7,5}{2}$ = 3,75 tonnes.	 1 1	Recall that 4,5 tonnes represent 120% hence there was need to find 100% that would represent the harvest in 2015.

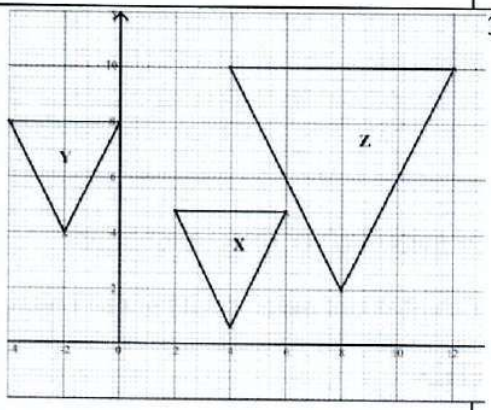
14 (a)	$4-5x< 2x+8$ $4-8 < 2x+5x$ $-4 < 7x$ $\frac{-4}{7} < x$	1 1	Solution of inequalities by grouping like terms and dividing both sides by 7.
14 (b)	The smallest integer that satisfies the inequality $4-5x < 2x+8$ *is* 0.	1	Understanding of real number line and position of numbers on the real number line. Recall that 0 is an integer.
15 (a)	$\log a = 3$ and $\log b = 7$ $\log ab = \log a + \log b$ $= 3+7$ $= 10$	1	The use of law of logarithms, $\log ab = \log a + \log b$.
15 (b)	$\log \frac{1}{b} = \log 1 - \log b$ $= 0-7$ $= -7.$	1	Use of law of logarithms, $\log \frac{a}{b} = \log a - \log b$. Recall that $\log 1 = 0$
15 (c)	$\text{Log } \sqrt[3]{a} = \log a^{\frac{1}{3}}$ $= \frac{1}{3} \times \frac{3}{1}$ $= 1$	1	There was need to express $\sqrt[3]{a} = a^{\frac{1}{3}}$ Hence $\log \sqrt[3]{a} = \log a^{\frac{1}{3}} = \frac{1}{3} log a.$
16 (a)	$f(x) = (x+4)(2x-1)$ $f(3) = (3+4)(2\times 3-1)$ $= 7\times 5$ $= 35$	1	Understanding of function notation where $f(3)$ means that substitute 3 for x in the function.
16 (b)	$\frac{3m}{4} - \frac{m}{3} = 2\frac{1}{2}$ $\frac{3m}{4} - \frac{m}{3} = \frac{5}{2}$ $\frac{12(3m)}{4} - \frac{12(m)}{3} = \frac{2(5)}{2}$		Knowledge of method for solving linear equations involving fractions by first multiplying every term by L.C.M. of the denominators to remove the denominators. The equation that remains is a linear equation in one variable.

	$9m - 4m = 30$ $5m = 30$ $m = 6$	1 1	
17 (a)	$\boldsymbol{p} - \boldsymbol{q} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} - \begin{pmatrix} x \\ 1 \end{pmatrix}$ $= \begin{pmatrix} -x \\ -4 \end{pmatrix}$	 1	Subtract corresponding elements. Vector brackets are essential and there is not supposed to be a division line.
17 (b)	$(-x)^2 + (-4)^2 = 5^2$ $x^2 + 16 = 25$ $x^2 = 25 - 16$ $x = \sqrt{9}$ $x = \pm 3$	1 1 1	Understanding of the modulus or magnitude sign and how to find the magnitude of a vector. Knowledge of formation of a quadratic equation and solving it is required.
18 (a)	Square	1	The clue is the word **regular** which means all sides are equal and all interior angles are also equal.
18 (b)	$115° + 89° + x + x + x + x =$ $(6 - 2) \times 180°$ $4x + 204° = 720°$ $4x = 720 - 204$ $4x = 516°$ $x = 129°$	1 1 1	The concept of sum of interior angles of a hexagon to be used to form an equation and solve it to find the value of x.
19 (a)	$\text{Cos } 45° = \frac{BC}{12}$ $\text{BC} = \text{Cos } 45° \times 12$ $= \frac{\sqrt{2}}{2} \times \frac{12}{1}$ $= 6\sqrt{2}$	 1 1	Knowledge of trigonometric ratios was required to answer this question. Use of cosine 45° in surd form to calculate the length of BC.

19 (b)	Sin $A\hat{C}D = \frac{6\sqrt{2}}{12}$ $= \frac{\sqrt{2}}{2}$	1	Knowledge of the fact that the sine of an obtuse angle is equal to sine of its supplement.
19 (c)	Tan $A\hat{C}D = -1$	2	Knowledge of the fact that the tangent of an obtuse angle is equal to the negative of the tangent of its supplement.
20 (a) (i)	Modal height $6< h \leq$	1	Modal class is the class with the highest frequency.
(ii)	Median height $6< h \leq 8$	1	The median is the entry in the middle of data when arranged in order. In this case the total frequency is 50 hence the median is the mean of the 25th and 26th entries. Both are found in the same class $6< h \leq 8$
20 (b)	$12\frac{\left(\frac{2+6}{2}\right)+16\left(\frac{6+8}{2}\right)+12\left(\frac{8+10}{2}\right)+10\left(\frac{10+12}{2}\right)}{50}$ $\frac{6(2+6)+8(6+8)+6(8+10)+5(10+12)}{50}$ $\frac{48+112+108+110}{50}$ $\frac{378}{50}$ $7\frac{28}{50}$	1 1 1	Calculate the class centres of each class by adding the lower and upper limits then divide by two. This class centre is then multiplied by the frequency and all the products of class centre and frequency are added and the sum divided by total frequency.
21 (a)	$\frac{1}{3}\times\frac{3}{4}$ $= \frac{2}{12}$	1 1	Application of the law of probability for both which is to multiplying the probabilities of both not scoring.
21 (b)	$\frac{2}{3}\times\frac{1}{4}$ $= \frac{2}{12}$	1 1	Applying the same law as above but now multiplying the probabilities that they do not score.

21 (c)	$\frac{1}{3}\times\frac{1}{4}+\frac{3}{4}\times\frac{2}{3}$ $=\frac{1}{12}+\frac{6}{12}$ $=\frac{1+6}{12}$ $=\frac{7}{12.}$	1 1	This is the probability that one scores and the other does not score. There are two scenarios hence addition of two products.
22 (a)	$\frac{12}{1\,000}\div\frac{1}{60\times60}$ $=\frac{12}{1\,000}\times\frac{3\,600}{1}$ $=\frac{432}{10}$ = 43,2 km/h	 1 1	Conversion involves changing $12m$ to km by diving by a 1 000 and converting the seconds to hours by diving by 3 600.
22 (b) (i)	$12\times5+\frac{1}{2}\times3\times12$ = 60 + 80 = 78 metres	1 1	The area of the trapezium is the distance covered. The candidate can divide the shape into a triangle and a rectangle. Alternatively use the formula for calculating area of a trapezium. $\frac{1}{2}(8+5)\times12$ $=\frac{1}{2}\times13\times12$ = 78 metres.
(b) (ii)	$\frac{0-12}{8-5}$ $=\frac{-12}{3}$ $=-4\text{m/s}^2$	1 1	The acceleration is equal to the gradient of the graph from $t = 5$ to $t = 8$.
23 (a)	$A\hat{C}B=\frac{1}{2}\times60°$ $=30°$	1	Use of the theorem "angle at the centre is twice angle at the circumference".

23 (b)	$O\hat{A}B = \frac{1}{2}(180° - 60°$ $= 60°$	1	Angles in an isosceles triangle because sides OA and OB are equal being radii of the same circle. Since angle is 60° then the triangle is equilateral.
23 (c)	Length of minor arc AB $\frac{60°}{360°} \times \frac{2}{1} \times \frac{22}{7} \times \frac{7}{1}$ $= \frac{22}{3}$ $= 7\frac{1}{3}$ cm.	1 1	Knowledge of the formula for calculating length of an arc $\left(\frac{\theta}{360} \times 2\pi r\right)$ Substitute the values π and r simplified correctly.
23 (d)	Area of the minor sector AOB $\frac{60°}{360°} \times \frac{22}{7} \times \frac{7}{1} \times \frac{7}{1}$ $= \frac{77}{3}$ $= 25\frac{2}{3}$ cm^2	1 1	The candidate should know the for calculating area of a sector $\left(\frac{\theta}{360°} \times \pi r\right)$ Substitute the values and h simply correctly.
24 (a)	2;3;5;7	2	Clear understanding of prime numbers. Prime numbers are numbers with only two factors 1 and that number. **1 is not a prime number**.
24 (b)	P = {2;3;5;7} S = {1;4;9;} M = {3;6;9} $P \cap S \cap M$ *is an empty set* $n(P \cap S \cap M) = 0$	1	The three sets have no common elements hence their intersection is empty.
24 (c)		3	The Venn diagram should be completed with all elements as shown. No elements should be repeated. The format of the diagram shows that there is no intersection between the sets P and S.

25 (a)	Translation Translation vector $=\begin{pmatrix}-4\\8\end{pmatrix}-\begin{pmatrix}2\\5\end{pmatrix}$ $=\begin{pmatrix}-4-2\\8-5\end{pmatrix}$ $=\begin{pmatrix}-6\\3\end{pmatrix}$	1 2	The translation vector can be found by using corresponding vertices and calculating the movements in the x and y direction. Join the vertices on X to a corresponding vertices on Y. Observe parallel lines of same length and that helps to find the transformation.
25 (b) (i)	$\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}$	2	The matrix for an enlargement is the factor multiplying the identity matrix of a 2×2 matrix.
25 (b) (ii)	 $\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}\begin{pmatrix}2\\5\end{pmatrix}=\begin{pmatrix}4\\10\end{pmatrix}$ $\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}\begin{pmatrix}4\\1\end{pmatrix}=\begin{pmatrix}8\\2\end{pmatrix}$ $\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}\begin{pmatrix}6\\5\end{pmatrix}=\begin{pmatrix}12\\10\end{pmatrix}$ Vertices of triangle z are (4,10), (8;2) and (12;10).	3	To get vertices of triangle Z multiply the enlargement matrix by the vertices of triangle X