ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

PURE MATHEMATICS 6042/1

PAPER 1

NOVEMBER 2021 SESSION 3 hours

Additional materials:
Answer paper
Graph paper
List of Formulae MF7
Scientific calculator [non - programmable]

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of a non-programmable scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed pages and 3 blank pages.

[Turn over

1 Solve the equation,

$2e^{2x} - 7e^{x} + 6 = 0.$ [3]
Give the answer in exact form.

2 Prove the identity

$\frac{\sin 2\theta}{1+\cos 2\theta} = \tan \theta.$ [3]

3 Given that $y = ln[cos(x^2 + 1)]$, show that $\frac{dy}{dx} = -2x \tan (x^2 + 1)$. [3]

4 Given that M is inversely proportional to the cube root of $(n - 1)$ and that when $n = 9, M = 5$,
find

(a) a formula connecting M and n [3]

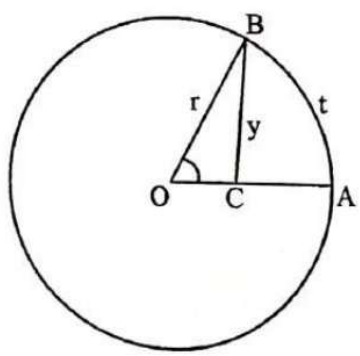
(b) the exact value of n when $M = 25$. [2]

5 (a) Express $2x^2 + 3x + 1$ in the form

$a(x + b)^2 + c$ where a, b and c are constants to be found. [3]

(b) Hence or otherwise solve the equation $2x^2 + 3x + 1 = 0$. [3]

6



In the diagram, A and B are two points on the circumference of a circle centre O and radius r. Angle AOB is θ radians and C is the mid-point of OA. The length of BC is y and the length of the arc BA is t.

(a) Express y^2 in terms of r and θ. [2]

(b) Hence or otherwise show that if θ is small then $y^2 \approx \frac{1}{4}r^2 + \frac{1}{2}t^2$. [4]

7 If $A = \begin{pmatrix} 3 & -1 \\ 4 & -1 \end{pmatrix}$, prove by induction that $A^n = \begin{pmatrix} 2n+1 & -n \\ 4n & 1-2n \end{pmatrix}$, where n is a positive integer. [7]

8 (a) Solve the equation $\frac{2^x+1}{2^x-1} = 5$

giving the answer correct to 3. significant figures. [3]

(b) Solve the inequality $|x-3| > 2|3x+1|$. [5]

9 Given that x is so small that x^4 and higher powers can be neglected,

(a) show that
$\frac{e^x}{1+x} \approx 1 + \frac{x^2}{2} - \frac{x^3}{3}$, [6]

(b) use the expansion to evaluate

$\frac{e^{0,01}}{1,01}$ correct to 7 decimal places. [2]

10 (a) Given that $f(x) = 2 - |x|$ and $g(x) = \frac{1}{3}x + 1$,
sketch on the same axes the graphs of $y = f(x)$ and $y = g(x)$, [3]

(b) Find the co-ordinates of the points of intersection of $f(x)$ and $g(x)$. [5]

11 (a) Express $\frac{2x}{(x+2)(x-2)(x-1)}$, in partial fractions. [4]

(b) Solve the inequality $\frac{2x}{(x+2)(x-2)(x-1)} < 0$. [4]

[Turn over

12 If $p = -4 + 3i$ and $q = -1 + \sqrt{3i}$,

(a) calculate the modulus of,

(i) p, [2]

(ii) q. [2]

(b) find

(i) the argument of q, [2]

(ii) pq^2 in the form $a + bi$, [3]

(iii) $\frac{p}{q}$ in the form $a + bi$. [3]

13 The curve $y = 10 - \frac{5}{x}$ and the line $y + x = 6$ intersect at two points P and Q.

Find the

(a) coordinates of P and Q, [5]

(b) coordinates of the midpoint of PQ, [2]

(c) equation of the perpendicular bisector of the line with P and Q as end points. [3]

14 The curve $y = x^2 + px + q$, where p and q are constants, has a turning point at $(-1; -5)$

Find the

(a) values of p and q, [5]

(b) equations of the tangent and the normal in the form $ay + bx + c = 0$, at the point where the curve cuts the y-axis. [5]

15 **(a)** A point $(1; 2)$ is mapped onto point $(7; 2)$ by a shear parallel to the x-axis. Find the shear factor. [3]

(b) A triangle with vertices A $(-3; 1)$, B $(3; 1)$ and C $(3; 5)$ is transformed by $M = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$.

(i) Find the area of triangle ABC. [2]

(ii) Find the co-ordinates of A_1 B_1 and C_1 the images of A, B and C under transformation M. [4]

(iii) Hence or otherwise find the area of the triangle $A_1B_1C_1$. [2]

16 The gradient function of a curve is directly proportional to $20 - y$. If $y = 0$, $x = 0$ and the gradient is 1

(a) Show that $\frac{dy}{dx} = 0{,}05\ (20 - y)$. [3]

(b) Solve the differential equation giving y in terms of x. [6]

(c) Find y when $x = 10$ giving the answer correct to 2 decimal places. [2]

(d) Briefly describe what happens to y as x becomes very large. [1]