ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

PURE MATHEMATICS 6042/2

PAPER 2

JUNE 2019 SESSION 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae MF7
- Scientific calculator

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions in Section A and any **five** questions from Section B

If a numerical answer cannot be given exactly and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed pages and 3 blank pages.

Section A (40 marks).

Answer **all** questions in this section.

1 Express $\frac{2x}{(x-1)(x+1)^2}$ in partial fractions. [5]

2 (i) Express $\sqrt{6}Cos\theta - \sqrt{2}Sin\theta$ in the form $RCos(\theta + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [2]

(ii) Hence or otherwise, solve the equation $\sqrt{6}Cos\theta - \sqrt{2}Sin\theta = 2$ for $0° \leq \theta \leq 360°$. [5]

3 Given that $(1 - ax)^n \simeq 1 - 15x + 90x^2 + bx^3$, find the values of a; b and n. [9]

4 The function f is defined by $f(x) = ln(2x - 3)$ for $x \in \mathbb{R}, x > \frac{3}{2}$.

(i) Find $f^{-1}(x)$ and state its domain. [3]

(ii) Solve the equation

$$f^{-1}(x) - 6f^{-1}(-x) + 5 = 0.$$ [6]

5 (i) Use the trapezium rule with 5 ordinates to find an approximate value of $\int_0^{\frac{\pi}{4}}[1 + Sin^2(2x)]\,dx$ correct to 8 decimal places. [4]

(ii) Find the exact value of $\int_0^{\frac{\pi}{4}}[1 + Sin^2(2x)]\,dx$. [4]

(iii) Hence, find the percentage error in using the trapezium rule as an approximation of $\int_0^{\frac{\pi}{4}}[1 + sin^2(2x)]dx$. [2]

Section B (80 marks)

Answer any **five** questions from this section. Each question carries **16** marks.

6 **(a)** Find the image of the line $y = 5x$ under the transformation matrix $\begin{pmatrix} 4 & -1 \\ 2 & 5 \end{pmatrix}$. [3]

(b) It is given that matrix $M = \begin{pmatrix} 2 & 3 & 4 \\ -3 & 2 & 2 \\ 4 & -4 & 3 \end{pmatrix}$.

(i) Evaluate M^2. [3]

(ii) Find M^{-1}, the inverse of matrix M. [6]

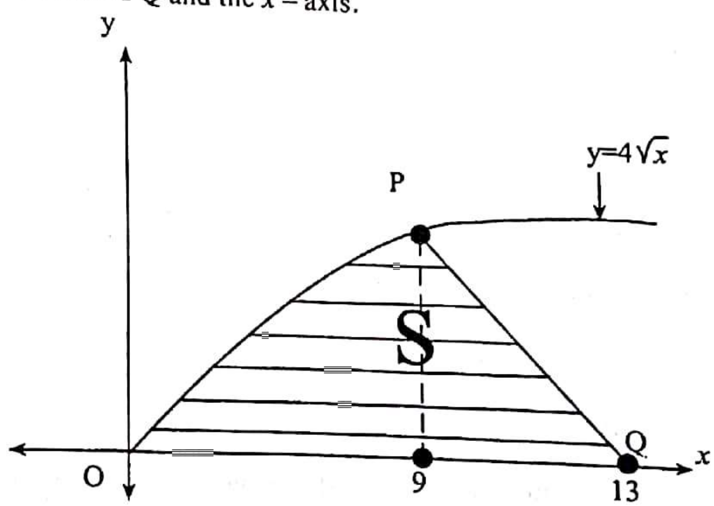
(iii) Hence, or otherwise solve the simultaneous equations:

$$2x + 3y + 4z = 1$$
$$-3x + 2y + 2z = 14$$
$$4x - 4y + 3z = 22$$

[4]

7 **(a)** Use the substitution $u^2 = x - 1$ to find the exact value of $\int_1^5 x\sqrt{(x-1)}\,dx$. [4]

(b) The diagram below shows the shaded region S bounded by the curve $y = 4\sqrt{x}$, line **PQ** and the x – axis.



Find the

(i) coordinates of **P**, [1]

(ii) equation of line **PQ**, [2]

(iii) area of region **S**, [4]

[Turn over

(iv) exact volume of the solid of revolution generated when region S is rotated completely about the x – axis. [5]

8 The rate at which a proportion x of a certain population is infected by cholera is directly proportional to the product of the proportion infected and that not yet infected. At time t days the proportion of the population infected is x and not infected is $(1-x)$.

(i) Write down a differential equation relating x and t. [2]

(ii) Initially 20% of the population was infected and 120 days later it rose to 80%. Solve the differential equation, expressing x in terms of t. [11]

(iii) Hence, find the proportion infected after 60 days. [2]

(iv) Describe what happens to the population as time increases. [1]

9 (a) The equation of a circle is given by $3x^2 + 3y^2 - 12x + 18y + 3k = 0$.

Find the value of k for which the circle has a radius 6 units. [4]

(b) Use Taylor's series expansion in ascending powers of $(x-1)$ to find $x^2 lnx$ up to and including the term in $(x-1)^4$ [5]

(c) (i) Show that the equation $x^5 - 10x - 5 = 0$ has a root between 1 and 2. [3]

(ii) Taking 1.7 as the first approximation, use the Newton-Raphson method twice to approximate the root of the equation $x^5 - 10x - 5 = 0$ to 2 decimal places. [4]

10 The plane π has equation $\boldsymbol{r}.\begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} = 1$. The points P and Q have coordinates $(-5; 1; 3)$ and $(2; 4; -1)$ respectively.

(i) Show that Q lies on plane π. [2]

(ii) Find the

(a) equation of the line perpendicular to π through P, [2]

(b) Cartesian equation of line PQ. [3]

(iii) Calculate the

(a) acute angle between π and the line PQ, [4]

(b) shortest distance from P to π. [5]

11 **(a)** On a single diagram shade the region defined by the inequalities $\frac{\pi}{6} \leq arg(z-4) \leq \frac{\pi}{4}$ and $|z-4| \leq 4$. [3]

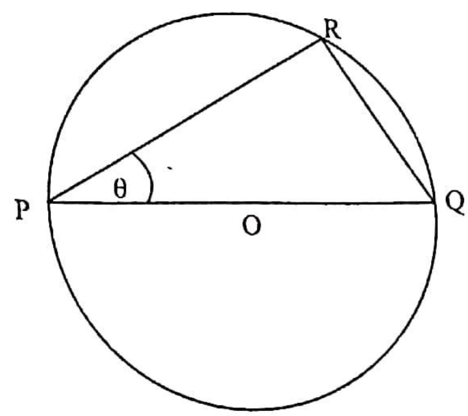
(b) Solve the equation.

$$Z^3 = -5 + 12i$$

[6]

(c) Use de Moivres theorem to show that $Sin\theta Sin5\theta = 16Sin^6\theta - 20Sin^4 + 5Sin^2\theta$. [7]

12 **(a)** The diagram shows a circle centre O, passing through the points P, Q and R. PQ = 4 cm and angle $Q\hat{P}R = \theta$. Given that θ is small, find the approximate area of triangle PQR in terms of θ. [4]



(b) Show that the set A = $\{1; i; -1; -i\}$ forms a group under multiplication. [6]

(c) Prove by induction that

$$\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$$

for all positive integers n. [6]