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ZIMSEC
For Performance Measurement

ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

MATHEMATICS

PAPER 1 **9164/1**

NOVEMBER 2014 SESSION 3 hours

Additional materials:
Answer paper
Graph paper
List of Formulae

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.
Questions are printed in the order of their mark allocations and candidates are advised to attempt questions sequentially.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed pages and 1 blank page.

[Turn over

1 Express $\dfrac{6}{3+\sqrt{5}+\sqrt{14}}$ in the form $a+b\sqrt{c}+d\sqrt{e}$ where a, b, c, d and e are real numbers. [3]

2 Solve the inequality

$$\frac{10-2x}{x-2} > x+1.$$ [4]

3 Given that $F = kS^2$, where k is a constant, find the percentage change in F if S increases by 5%. [4]

4 (i) Use the trapezium rule with four trapezia of equal width to estimate the value of

$$\int_1^2 \frac{1}{x} e^{\frac{1}{3}x}\, dx,$$

giving your answer correct to 4 decimal places. [4]

(ii) Explain whether the trapezium rule has underestimated or overestimated the value of this integral. [1]

5 P is a fixed point on the circumference of a circle with centre O and radius 5 cm. Another point Q moves round the circumference of the same circle at a constant speed of 3 cms $^{-1}$.

The angle POQ is θ radians and the length of arc PQ is x cm.

Find the rate of increase of

(i) θ in radians per second, [3]

(ii) the area of sector POQ. [3]

6 Express $\dfrac{2x^3-17x-1}{(x-2)(x^2+5)}$ in partial fractions. [6]

7 The table shows the values of a variable p obtained experimentally from the values of q.

q	2	3	4	5
p	3.55	1.93	1.26	0.90

The variables p and q are related by the relation $\log p = b\log q + \log a$.

By plotting the graph of $\log p$ against $\log q$, determine the approximate values of a and b. [6]

8 (a) It is given that $f(x) = px^2 + 5x - 4$.

Find the set of values of p for which $f(x)$ is positive for all real values of x. [2]

(b) The remainder when $4x^4 - 5x^2 - 13x + 3$ is divided by $x + b$ is equal to the square of the remainder when $2x^2 - 3$ is divided by $x + b$.

Calculate the possible values of b, correct to 2 decimal places. [4]

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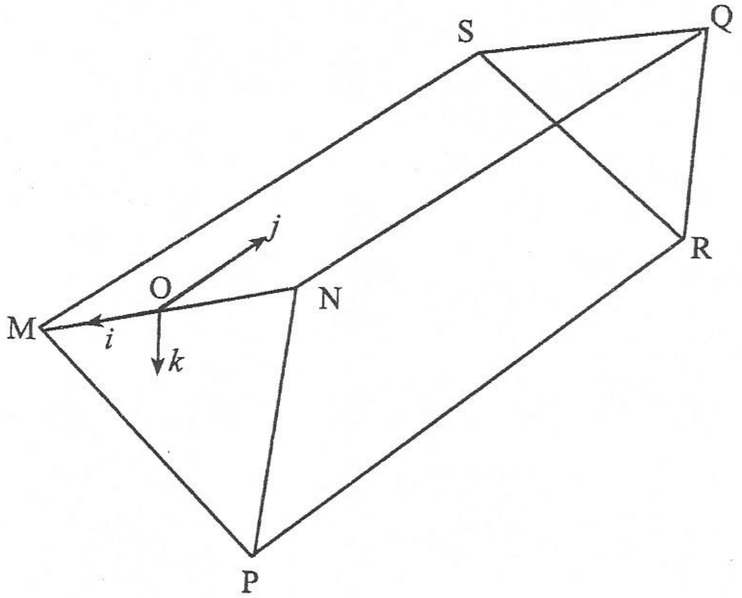

The diagram shows a triangular prism MNPRQS with MP = NP = 5 units. O is the midpoint of MN.

The unit vectors $\boldsymbol{i}$ and $\boldsymbol{k}$ are taken along $\overrightarrow{OM}$ and $\overrightarrow{OP}$ respectively, and $\boldsymbol{j}$ is taken parallel to MS.

Given that $\overrightarrow{OR} = 9\boldsymbol{j} + 4\boldsymbol{k}$,

find

(i) the unit vector in the direction $\overrightarrow{PS}$, [3]

(ii) the angle SPQ. [3]

[Turn over

10 The complex number z satisfies the equation

$$z + 2\bar{z} = \frac{13}{-2 + 3i}.$$

Find

(i) z in the form $x + iy$, [3]

(ii) the modulus and argument of $\frac{1}{z}$. [4]

11 In a geometric series, the ratio of the sum of the first four terms to the sum of the first two terms is 17: 16.

(i) Calculate the value of the common ratio r given that $r > 0$. [4]

(ii) Given that the third term is $2\frac{3}{4}$, find the value of the sum to infinity. [3]

12 It is given that $e^{-3y}(1 - 4x) = (7 + 3x)$.

(i) Show that $y = -\frac{1}{3}\left[ln(7 + 3x) - ln(1 - 4x)\right]$. [2]

(ii) Hence obtain the Maclaurin expansion of y up to and including the term in x^3. [5]

13 **(i)** Use the binomial expansion to simplify $(x + 2)^7 - (x - 2)^7$. [6]

(ii) Hence use your answer in **(i)** to find the exact value of $\left(\sqrt{5} + 2\right)^7 - \left(\sqrt{5} - 2\right)^7$. [2]

14 **(a)** Prove that $\sec 2\theta + \tan 2\theta = \frac{\sin\theta + \cos\theta}{\cos\theta - \sin\theta}$. [4]

(b) By using the substitution $U = \sin x$ or otherwise, find the exact value of $\int_0^{\frac{\pi}{2}} \frac{\cos x}{3 + \cos^2 x}dx$. [5]

15 (i) By sketching two appropriate graphs, show that the equation $\sin x + 1 - 4x^2 = 0$ where $-2\pi \le x \le 2\pi$, has two real roots. [2]

(ii) Verify by calculation that the positive root in **(i)** lies between $x = 0.6$ and $x = 0.7$. [3]

(iii) Taking $x_0 = 0.6$ as the first approximation of x in **(i)**, use the Newton-Raphson Method twice to find the root correct to 5 decimal places. [4]

16 (a) Find the perpendicular distance of the point P(−3; 2) from the line whose equation is $3x - 5y = 7$. [6]

(b) Find the equation of a circle which passes through the point (5; 4) and touches the y-axis at (0; 2). [6]

17 (a) The gradient function of a curve is given by $\frac{dy}{dx} = \frac{15}{(3x+4)^2}$ and

M(2; 6) is a point on the curve.

Find the equation of the curve. [4]

(b) The rate of increase in the number of people, x, who own a laptop is proportional to the product of x and $N - x$, where N is the total population.

Initially $x = \frac{1}{10}N$ and after one week $x = \frac{1}{4}N$.

(i) Form a differential equation from the above information and show that after t weeks, $\frac{x}{N-x} = 3^{t-2}$.

(ii) Find the time, t, in **(i)** when $x = \frac{3}{4}N$.

(iii) Sketch the graph of x against t for the equation in **(i)**.

[11]