# DIRECTED NUMBERS

**Qsn.**

1. **Evaluate**
   a. $\frac{-3+5}{-2(7+-4)}$
   b. $(-4+-6)(3--2)$
   c. $3-+3(-2-1)$

**Sln**

a. $\frac{-3+5}{-2(7+-4)}$

$$= \frac{3}{-2(7-4)}$$
$$= \frac{3}{-2\times 3}$$
$$= \frac{3}{-6}$$
$$= -\frac{1}{2}$$

b. $(-4+-6)(3--2)$

$$= (-4-6)(3+2)$$
$$= -10\times 5$$
$$= -50$$

c. $3-+3(-2-1)$

$$= 3-3(-5)$$
$$= 3+15$$
$$= 18$$

**Qsn**

2. **Simplify as far as possible**

$$\frac{(-5+-4)-(2-+13)}{-2(3--1)}$$

**Ans**

$$\frac{(-5+-4)-(2-+13)}{-2(3--1)}$$

$$= \frac{(-5-4)-(2-13)}{-2(3+1)}$$

$$= \frac{(-9)-(-11)}{-2(4)}$$

$$= \frac{-9+11}{-8}$$

$$= \frac{2}{-8}$$

$$= -\frac{1}{4}$$

**Qsn**

3. **Evaluate** $\frac{(-3--5)-(-7--4)}{(-2--1)(-14--19)}$

**Ans**

$$\frac{(-3--5)-(-7--4)}{(-2--1)(-14--19)}$$

$$= \frac{(-3+5)-(-7+4)}{(-2+1)(-14+19)}$$

$$= \frac{2--3}{-1\times 5}$$

$$= \frac{5}{-5}$$

$$= -1$$

**Qsn**

4. **Simplify as far as possible**
   a) $-2(7-+12)(-3--5)$
   b) $\frac{(3--4)-(1-+2)+(-3-+1)}{-5-[(4-+7)+(-4--4)]}$

**Ans**

a) $-2(7--+12)(-3--5)$

$= -2(7-12)(-3+5)$

$= -2 \times -5 \times 2$

$= 20$

b) $\frac{(3--4)-(1--+2)+(-3--+1)}{-5-[(4--+7)+(-4--4)]}$

$= \frac{(3+4)-(1-2)+(-3-1)}{-5-[(4-7)+(-4+4)]}$

$= \frac{7--1+-4}{-5-[-3+0]}$

$= \frac{7+1-4}{-5+3}$

$= \frac{4}{-2}$

$= -2$

**Qsn**

5. $-2(9--+2)+3-(3--2)$

**Sln**

$-2(9--+2)+3-(3--2)$

$= -2(9-2)+3-(3+2)$

$= -2 \times 7+3-5$

$= -14+3-5$

$= -14-5+3$

$= -19+3$

$= -16$

Def:

- Even numbers-# *divisible by 2 and do not leave a remainder.*
- Odd numbers-# *divisible by 2 and leave a remainder 1*
- Prime numbers - *#divisible by 1 and itself.*
- Rational numbers – *fractions*
- Irrational numbers— *surds*
- Integers – *whole numbers*
- Natural numbers – *positive whole numbers.*

Ex:

1. $-3(4--8)$
2. $(3--+4) \times (-3--8)$
3. $\frac{-3(-8+2)}{-5--+4}$
4. $\frac{(-5--+3)\times(-11--7)}{-8+-4(10-+8)}$
5. *Given that $x$ is greater than $0,5$; state the least possible value of $x$ if $x$ is*
   a) *A prime number*
   b) *An even number*
   c) *An integer*
   d) *An odd numbers*
6. *Given the range $20 < x < 70$ list*
   a) *Prime numbers*
   b) *Perfect cubes*
   c) *Perfect squares*
7. $2-6 \times (3+2) \div 2+3$

# STANDARD FORM

Qsn

1. Express $\mathbf{0,000\ 0526}$ in standard form

J2008/1

Sln

$\mathbf{0,000\ 0526}$

$= \frac{\mathbf{5{,}26}}{\mathbf{100000}}$

$= \frac{\mathbf{5{,}26}}{\mathbf{10^5}}$

$= \mathbf{5,26 \times 10^{-5}}$

Qsn

2. Given that $\mathbf{m = 4 \times 10^6}$ and $\mathbf{n = 2,4 \times 10^3}$ giving each answer in standard form, calculate.
   a) $\mathbf{mn}$
   b) $\frac{\mathbf{n}}{\mathbf{m}}$

N2008/1

Sln

a) $\mathbf{mn}$

$= \mathbf{4 \times 10^6 \times 2,4 \times 10^3}$

$= \mathbf{4 \times 2,4 \times 10^6 \times 10^3}$

$= \mathbf{9,6 \times 10^{6+3}}$

$= \mathbf{9,6 \times 10^9}$

b) $\frac{\mathbf{n}}{\mathbf{m}}$

$= \mathbf{2,4 \times 10^3 \div 4 \times 10^6}$

$= \mathbf{2,4 \div 4 \times 10^3 \div 10^6}$

$= \mathbf{0,6 \times 10^{3-6}}$

$= \mathbf{6 \times 10^{-3-1}}$

$= \mathbf{6 \times 10^{-4}}$

Qsn

3. Sun earth mars

In the diagram, the sun, earth and mars are in a straight line. It is given that the earth is $\mathbf{1,496 \times 10^8 km}$ from the sun and mars are $\mathbf{2,279 \times 10^8 km}$ from the sun.

i. Write down $\mathbf{1,496 \times 10^8}$ in ordinary form.
ii. Find in standard form, the distance of mars from the earth.

N2008/2

Sln

i. $\mathbf{1,496 \times 10^8}$

$\mathbf{1496 \times 10^{8-3}}$

$\mathbf{149\ 600\ 000}$

ii. $\mathbf{2,279 \times 10^8 - 1,496 \times 10^8}$

$\mathbf{(2,279 - 1,496) \times 10^8}$

$\mathbf{0,783 \times 10^8}$

$\mathbf{7,83 \times 10^7}$

Qsn

4. Express $\mathbf{97,85}$ in standard form

J2009

Sln

$\mathbf{97,85}$

$\mathbf{9,785 \times 10^1}$

Qsn

5. Express $\mathbf{3 \times 10^4}$ as a common fraction of $\mathbf{5 \times 10^2}$ in its simplest form.

N2009

Sln

$= \frac{\mathbf{3 \times 10^2}}{\mathbf{5 \times 10^4}}$

$= \frac{\mathbf{300}}{\mathbf{50000}}$

$= \frac{\mathbf{3}}{\mathbf{500}}$

Qsn

6.
   a) Write down, in ordinary form, the value of $\mathbf{4,32 \times 10^4}$.
   b) Given that $\mathbf{M = 3,6 \times 10^2}$ and $\mathbf{N = 8 \times 10^{-1}}$, find in standard form, the value of
   i. $\mathbf{MN}$
   ii. $\mathbf{M + N}$

j2010

Sln

a) $\mathbf{4,32 \times 10^4}$

$\mathbf{= 43200}$

b)

i. $\mathbf{3,6 \times 10^2 \times 8 \times 10^{-1}}$

$\mathbf{3,6 \times 8 \times 10^2 \times 10^{-1}}$

$\mathbf{28,8 \times 10^{2+-1}}$

$\mathbf{2,88 \times 10^2}$

ii. $\mathbf{3,6 \times 10^2 + 8 \times 10^{-1}}$

$\mathbf{360 + 0,8}$

$\mathbf{360,8}$

$3{,}608 \times \mathbf{10^2}$

Qsn

7. Given that $\mathbf{m = 2 \times 10^3}$ and $\mathbf{n = 5 \times 10}$, evaluate, giving your answer in standard form.

a) $\mathbf{m+n}$

b) $\frac{\mathbf{n}}{\mathbf{m}}$

n2010

Sln

a) $\mathbf{m+n}$

$\mathbf{2\times10^3+5\times10}$

$\mathbf{2000+50}$

$\mathbf{2050}$

$\mathbf{2,05\times10^3}$

b) $\frac{\mathbf{n}}{\mathbf{m}}$

$=\frac{\mathbf{5\times10}}{\mathbf{2\times10^3}}$

$\mathbf{=5\div2\times10\div10^3}$

$\mathbf{=2,5\times10^{1-3}}$

$\mathbf{=2,5\times10^{-2}}$

Qsn

8. Express $\mathbf{0,008478}$ in standard form.

J1999/1

Sln

$\mathbf{0,008478}$

$=\frac{\mathbf{8,478}}{\mathbf{1000}}$

$=\frac{\mathbf{8,478}}{\mathbf{10^3}}=\mathbf{8,478\times10^{-3}}$

Qsn

9. Express $\mathbf{0,072}$ in standard form.

n1999/1

Sln

$\mathbf{0,072}$

$=\frac{\mathbf{7,2}}{\mathbf{100}}=\mathbf{7,2\times10^{-2}}$

Qsn

10. Simplify $\mathbf{(3,5\times10^5)\div(7\times10^2)}$, giving the answer in standard form.

J2000/1

Sln

$\mathbf{(3,5\times10^5)\div(7\times10^2)}$

$\mathbf{=3,5\div7\times10^5\div10^2}$

$\mathbf{=0,5\times10^{5-2}}$

$\mathbf{=5\times10^2}$

Qsn

11. A number in standard form is $\mathbf{6,714\times10^{23}}$

a) State the number of zeros in the number when it is written in ordinary form.

b) Express in standard form

i. Twice the number

ii. One-millionth of the number.

N2000/1

Sln

a) $\mathbf{6714\times10^{23-3}}$

$\mathbf{=20}$

b)

i. $\mathbf{2(6,714\times10^{23})}$

$\mathbf{=13,428\times10^{23}}$

$\mathbf{=1,3428\times10^{24}}$

ii. $\frac{\mathbf{6,714\times10^{23}}}{\mathbf{10^6}}$

$\mathbf{=6,714\times10^{17}}$

Qsn

12. Given that $\mathbf{m=4,8\times10^3}$ and $\mathbf{n=(1,2\times10^{-5})}$,

Find, giving your answer in standard form,

a) $\mathbf{5m}$

b) $\mathbf{n^2}$

c) $\frac{\mathbf{n}}{\mathbf{m}}$

J2001/1

Sln

a) $\mathbf{5m}$

$\mathbf{5(4,8\times10^3)}$

$\mathbf{24\times10^3}$

$\mathbf{2,4\times10^4}$

b) $\mathbf{n^2}$

$\mathbf{(1,2\times10^{-5})^2}$

$\mathbf{1,44\times10^{-10}}$

c) $\frac{\mathbf{n}}{\mathbf{m}}$

$=\frac{\mathbf{(1,2\times10^{-5})}}{\mathbf{(4,8\times10^3)}}$

$\mathbf{=1,2\div4,8\times10^{-5}\div10^3}$

$\mathbf{=0,25\times10^{-5-3}}$

$\mathbf{=2,5\times10^{-9}}$

Qsn

13. Express $\mathbf{37\ 100}$ in standard form.

N2001/1

Sln

$\mathbf{37\ 100}$

$\mathbf{=3,71\times10^4}$

Qsn

14. Given that $\mathbf{m = 4 \times 10^5}$ and $\mathbf{n = 5 \times 10^2}$, express in standard form.
    a) $\mathbf{m - n}$
    b) $\frac{\mathbf{m}}{\mathbf{n^2}}$

M1997/1

Sln

a) $\mathbf{4 \times 10^5 - 5 \times 10^2}$
$\mathbf{= 400\,000 - 500}$
$\mathbf{= 399\,500}$
$\mathbf{= 3,995 \times 10^5}$

b) $\frac{\mathbf{4 \times 10^5}}{\mathbf{(5 \times 10^2)^2}}$

$= \frac{\mathbf{4 \times 10^5}}{\mathbf{25 \times 10^4}}$

$\mathbf{= 4 \div 25 \times 10^5 \div 10^4}$
$\mathbf{= 0,16 \times 10^1}$
$\mathbf{= 1,6 \times 10^0}$
$\mathbf{= 1,6 \times 10^{-4+2}}$
$\mathbf{= 1,6 \times 10^{-2}}$

ii. $\mathbf{39\,000}$
$\mathbf{= 3,9 \times 10^4}$

Qsn

15. Evaluate
$$\mathbf{2 \times 10^3 - 8 \times 10^2}$$
**giving your answer in standard form**

N1996/1

Sln

$\mathbf{2 \times 10^3 - 8 \times 10^2}$
$\mathbf{= 2000 - 800}$
$\mathbf{= 1200}$
$\mathbf{= 1,2 \times 10^3}$

Qsn

16. Write down the value of
$$\mathbf{2 \times 10^2 + 4 \times 10 + 7 \times 10^{-1}}$$

m1998/1

Sln

$\mathbf{2 \times 10^2 + 4 \times 10 + 7 \times 10^{-1}}$
$\mathbf{= 200 + 40 + 0,7}$
$\mathbf{= 240,7}$
$\mathbf{= 2,407 \times 10^2}$

Qsn

17. Express
    i. $\mathbf{160 \times 10^{-4}}$
    ii. $\mathbf{39\,000}$

in standard form

m1998/7/1

Sln

i. $\mathbf{160 \times 10^{-4}}$

# NUMBER BASES.

Qsn

1.

a. Calculate the value of $1000_2 - 1_2$, leaving your answer in base two.

b. Write $5^3 + 5$ as a number in base 5.

c. Convert $34_5$ to a number in base two.

N1996/1

Sln

a.
$$\begin{array}{r} 1000_2 \\ -\quad 1_2 \\ \hline 111_2 \end{array} \qquad = 111_2$$

b. $5^3 + 5$

$= 125 + 5$

$= 130_{10}$

| | |
|---|---|
| 5 | 130 |
| 5 | 26 r 0 |
| 5 | 5 r 1 |
| 5 | 1 r 0 |
| | 0 r 1 |

$= 1010_5$

c. $34_5$

$= 3 \quad 4$

$5^1 \quad 5^0$

$= 3 \times 5 + 4 \times 1$

$= 19_{10}$

| | |
|---|---|
| 2 | 19 |
| 2 | 9 r 1 |
| 2 | 4 r 1 |
| 2 | 2 r 0 |
| 2 | 1 r 0 |
| | 0 r 1 |

$= 10011_2$

Qsn

2. Express $77_{10}$ as a number in base 5

M1997/1

Sln

| | |
|---|---|
| 5 | 77 |
| 5 | 15 r 2 |
| 5 | 3 r 0 |
| | 0 r 3 |

$= 302_5$

Qsn

3.

a. Evaluate $1100_2 - 111_2$ ,giving your answer in base 2.

b.

i. Express $1101_2$ in base 5.

ii. Evaluate $32_5 + 1101_2$ ,giving your answer in base 5.

N1997/1

Sln

a.
$$\begin{array}{r} 1100_2 \\ -\ 111_2 \\ \hline 101_2 \end{array} \qquad = 101_2$$

b.

i. $1101_2$

$= 1 \quad 1 \quad 0 \quad 1$

$2^3 2^2 \; 2^1 \; 2^0$

$= 1 \times 8 + 1 \times 4 + 1 \times 1$

$= 13_{10}$

| | |
|---|---|
| 5 | 13 |
| 5 | 2 r 3 |
| | 0 r 2 |

$= 23_5$

ii.
$$\begin{array}{r} 32_5 \\ -\ 23_5 \\ \hline 110_5 \end{array} \qquad = 110_5$$

Qsn

4.

a. Express $44_5$ as a number in base 2

b. Find $x$ such that $x_5 + 44_5 = 403_5$ .

J1998/1

Sln

a. $4 \quad 4$

$5^1 \quad 5^0$

$= 4 \times 5 + 4 \times 1$

$= 24_{10}$

| | | |
|---|---|---|
| 2 | 24 | |
| 2 | 12 | r 0 |
| 2 | 6 | r 0 |
| 2 | 3 | r 0 |
| 2 | 1 | r 1 |
| | 0 | r 1 |

$= \mathbf{11000_2}$

**b.** $\mathbf{x_5 = 403_5 - 44_5}$

$$\begin{array}{r} 403_5 \\ -\ 44_5 \\ \hline 304_5 \end{array}$$

$= \mathbf{304_5}$

Qsn

**5.**

a. Evaluate $\mathbf{432_5 + 414_5}$ , giving your answer in base five.

b. Express $\mathbf{2^4 + 2^3 + 2}$ as a number in base two

Sln

Qsn

6.

a. Find the value of the base **y** such that $\mathbf{6703_y - 725_y = 5756_y}$

b. Convert $\mathbf{134_5}$ to a number in base two.

J2008/1

Sln

**a.**

$$\begin{array}{r} 6703_y \\ 725_y \\ \hline 5756_y \end{array}$$

$\mathbf{3 + y - 5 = 6}$

$\mathbf{y = 6 + 5 - 3}$

$\mathbf{y = 8}$

b. $\mathbf{134_5}$

$$\begin{array}{ccc} 1 & 3 & 4 \\ \downarrow & \downarrow & \downarrow \\ 5^2 & 5^1 & 5^0 \end{array}$$

$\mathbf{1 \times 25 + 3 \times 5 + 4 \times 1}$

$\mathbf{25 + 15 + 4}$

$\mathbf{44_{10}}$

| | | |
|---|---|---|
| 2 | 44 | |
| 2 | 22 | r 0 |
| 2 | 11 | r 0 |
| 2 | 5 | r 1 |
| 2 | 2 | r 1 |
| 2 | 1 | r 0 |
| 2 | 0 | r 1 |

$= \mathbf{101100_2}$

Qsn

**7.**

a. Evaluate $\mathbf{54_6 + 305_6}$ giving your answer in base 6

b. Convert $\mathbf{10011_2}$ to a number in base 3.

n2008/1

Sln

**a.** $\mathbf{54_6 + 305_6}$

$$\begin{array}{r} 54_6 \\ +305_6 \\ \hline 403_6 \end{array}$$

**b.** $\mathbf{10011_2}$

$$\begin{array}{ccccc} 1 & 0 & 0 & 1 & 1 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

$\mathbf{1 \times 16 + 1 \times 2 + 1 \times 1}$

$\mathbf{16 + 2 + 1}$

$\mathbf{19_{10}}$

| | | |
|---|---|---|
| 3 | 19 | |
| 3 | 6 | r 1 |
| 3 | 2 | r 0 |
| | 0 | r 2 |

$= \mathbf{201_3}$

Qsn

**8. Giving your answer in base 9, evaluate $\mathbf{785_9 + 306_9}$**

n2009/1

Sln

$\mathbf{785_9 + 306_9}$

$$\begin{array}{r} 785_9 \\ +306_9 \\ \hline 1202_9 \end{array}$$

Qsn

9. **Express $1002_3$ as a number in base 7**

n2009/1

Sln

$$1002_3$$
$$1\ 0\ 0\ 2$$
$$3^3\ 3^2\ 3^1\ 3^0$$

$1 \times 27 + 3 \times 1$

$27 + 3$

$30_{10}$

| | |
|---|---|
| 7 | 30 |
| 7 | 4 r 2 ↑ |
| | 0 r 4 |

$= 42_7$

## FRACTIONS

Qsn

10. Calculate the exact value of

$$2\frac{1}{3} \times \frac{3}{14}$$

N1996/1

Sln

$$2\frac{1}{3} \times \frac{3}{14}$$

$$= \frac{7}{3} \times \frac{3}{14}$$

$$= \frac{1}{2}$$

Qsn

11. Evaluate $6\frac{1}{4} \times \frac{2}{5}$ giving your answer as an improper fraction.

M1997/1

Sln

$$6\frac{1}{4} \times \frac{2}{5}$$

$$= \frac{25}{4} \times \frac{2}{5}$$

$$= \frac{5}{2}$$

$$= 2\frac{1}{2}$$

Qsn

12. Giving your answer as a common fraction in its lowest terms, find the value of
    a. $\frac{1}{3} + \frac{3}{5}$
    b. $\frac{4}{5}$ **of** $\frac{15}{16}$

N1997/1

Sln

a. $\frac{1}{3} + \frac{3}{5}$

$= \frac{5+9}{15}$

$= \frac{14}{15}$

b. $\frac{4}{5}$ **of** $\frac{15}{16}$

$= \frac{4}{5} \times \frac{15}{16}$

$= \frac{3}{4}$

Qsn

13. Find the exact value of $2\frac{2}{5} \times 2\frac{1}{2}$.

J1998/1

Sln

$$2\frac{2}{5} \times 2\frac{1}{2}$$

$$= \frac{12}{5} \times \frac{5}{2}$$

$$= 6$$

Qsn

14. Giving your answer as a common fraction in its lowest terms, find the value of
    a. $\frac{3}{4} - \frac{2}{7}$

N1998/1

Sln

$$\frac{3}{4} - \frac{2}{7}$$

$$= \frac{21 - 8}{28}$$

$$= \frac{13}{28}$$

Qsn

15. Simplify $\frac{2}{3}-\frac{3}{4}$, giving your answer as a common fraction .

N2008/1

Sln

$$\frac{2}{3}-\frac{3}{4}$$

$$=\frac{8-9}{12}$$

$$=-\frac{1}{12}$$

Qsn

16. Express $3\frac{2}{5}-2\frac{13}{20}$ as a single fraction in its lowest terms.

N2008/2

Sln

$$3\frac{2}{5}-2\frac{13}{20}$$

$=\frac{17}{5}-\frac{53}{20}$

$$=\frac{68-53}{20}$$

$$=\frac{15}{20}=\frac{3}{4}$$

Qsn

17. Find the product of $\frac{4}{13}$ and $3,25$.

J2009/1

Sln

$\frac{4}{13}\times 3,25$

$=\frac{4}{13}\times\frac{325}{100}$

$$=1$$

Qsn

18. Find the value of $\frac{7}{8}-\frac{5}{6}$,giving your answer as a common fraction in its lowest terms.

N2009/1

Sln

$$\frac{7}{8}-\frac{5}{6}$$

$$=\frac{21-20}{24}$$

$$=\frac{1}{24}$$

Qsn

19. Express $0,096$ as a common fraction, giving your answer in its lowest terms.

J2010/1

Sln

$$\frac{96}{1000}$$

$$=\frac{24}{250}$$

Qsn

20. Find the value of $\frac{1}{3}+1\frac{7}{9}\div 2\frac{2}{3}$.

N2010/2

Sln

$$\frac{1}{3}+1\frac{7}{9}\div 2\frac{2}{3}$$

$$=\frac{1}{3}+\frac{16}{9}\div\frac{8}{3}$$

$$=\frac{1}{3}+\frac{16}{9}\times\frac{3}{8}$$

$$=\frac{1}{3}+\frac{2}{3}$$

$$=1$$

Qsn

21. Express 840m as a fraction of 2,1km, giving your answer in its simplest form.

J1999/1

Sln

$$\frac{840}{2100}$$

$$=\frac{2}{5}$$

Qsn

22. Evaluate $\frac{1}{2}-\frac{3}{5}\div\frac{2}{3}$.

J1999/2

Sln

$$2\frac{1}{2}-\frac{3}{5}\div\frac{2}{3}$$

$$=\frac{5}{2}-\frac{3}{5}\times\frac{3}{2}$$

$$=\frac{5}{2}-\frac{9}{10}$$

$$=\frac{25-9}{10}$$

$$= \frac{16}{10}$$

$$= 1\frac{3}{5}$$

Qsn

23. Express $\mathbf{0,072}$ as a fraction in its lowest terms.

N1999/1

Sln

$$\frac{72}{1000}$$

$$= \frac{9}{125}$$

Qsn

24. Evaluate $\frac{1}{3} \times \frac{3}{4} - \frac{1}{2}$.

N1999/2

Sln

$$\frac{16}{3} \times \frac{3}{4} - \frac{1}{2}$$

$$= 4 - \frac{1}{2}$$

$$= 3\frac{1}{2}$$

Qsn

25. Express 2,5mm as a fraction of 50cm in its lowest terms.

J2000/1

Sln

$$\frac{2,5}{500}$$

$$= \frac{25}{5000} = \frac{1}{200}$$

Qsn

26. Express as a fraction in its lowest terms,
    a. $\frac{3}{8} - \frac{2}{9}$
    b. $\frac{5}{16} \div \frac{7}{12}$

N2000/1

Sln

a. $\frac{3}{8} - \frac{2}{9}$

$$= \frac{27 - 16}{72}$$

$$= \frac{11}{72}$$

b. $\frac{5}{16} \div \frac{7}{12}$

$$= \frac{5}{16} \times \frac{12}{7}$$

$$= \frac{15}{28}$$

Qsn

27. Giving your answer as a common fraction in its lowest terms, find the value of,
    i. $7\frac{2}{5} - 6\frac{2}{3}$
    ii. $\mathbf{0,45\ of} \frac{5}{9}$

J2001/1

Sln

i. $\frac{37}{5} - \frac{20}{3}$

$$= \frac{111 - 100}{15}$$

$$= \frac{11}{15}$$

Qsn

28. Evaluate, giving your answers as common fractions in their lowest terms,
    a. $\frac{3}{4} + \frac{1}{5}$
    b. $\frac{5}{8} \times \frac{22}{45}$
    c. $\frac{1}{3} \div \frac{11}{24}$

N2001/1

Sln

a. $\frac{3}{4} + \frac{1}{5}$

$$= \frac{15 - 4}{20}$$

$$= \frac{11}{20}$$

b. $\frac{5}{8} \times \frac{22}{45}$

$$= \frac{11}{36}$$

c. $\frac{1}{3} \div \frac{11}{24}$

$$= \frac{1}{3} \times \frac{24}{11}$$

$$= \frac{8}{11}$$

# DECIMALS

Qsn

1.

a) Find the exact value of

i. $5,4 \times 0,06$

ii. $16,5 - 4,96$

b) Express $0,008478$ correct to two decimal places.

J1999/1

Ans

a)

i. $5,4$
$\times 0,06$
$324$
$= 0,324$

ii. $16,50$
$-4,96$
$11,54$

b) $0,008478$
$= 0,01$

Qsn

2. Find the value of

a) $5,08 + 0,946$

b) $\sqrt{0.0081}$

c) $5,6 - 7,5$

N1999/1

Ans

a) $5,08$
$+0,946$
$6,026$ $= 6,026$

b) $\sqrt{0.0081}$
$= \sqrt{\frac{81}{10000}}$
$= \frac{9}{100}$ $= 0,09$

c) $7,5$
$-5,6$
$1,9$ $= -1,9$

Qsn

3. Given that $\sqrt{7} = 2,646$ and $\sqrt{70} = 8,367$

a) $\sqrt{7000}$

b) $\sqrt{0,07}$

J2000/1

Ans

a) $\sqrt{70 \times 100}$
$= 8,367 \times 10$
$= 83,67$

*b*) $\sqrt{0,07}$
$= \sqrt{\frac{7}{100}}$
$= \frac{2,646}{10}$ $= 0,2646$

Qsn

4. Evaluate, giving your answer in decimal form, $1,549 - 0,07267$.

N2000/1

Ans

$1,54900$
$-0,07267$
$1,47633$ =1,47633

Qsn

5.

a) Express $\frac{15}{16}$ as a terminating decimal.

b) Express $158,697$

i. Correct to the nearest hundredth,

ii. Correct to two significant figures.

J2001/1

Ans

a) $\frac{15}{16}$
$= 0,9375$

b)

i. 158,70

ii. 160

Qsn

6. Calculate in decimal form the value of

a) $0,02 \times 0,6$

b) $\sqrt{0,000004}$

c) $5,6 - 7,3$

N2001/1

Ans

a)
$0,02$
$\times 0,6$
$0,012$
$= 0,012$

b) $\sqrt{0,000004}$
$= \sqrt{\frac{4}{1000000}} = \frac{2}{1000} = 0,002$

Qsn

7. Express 0, 0148 correct to one significant figure.

N2001/1

Ans

$$0,0148$$
$$= 0,01$$

Qsn

8.

a) Simplify $6,3 \times 1,1$, giving your answer as decimal

b) Given that

$$94 \times 152 = 14\,288$$

Write down the exact value of

i. $0,094 \times 1520$

ii. $0,14\,288 \div 0,0094$

N2008/1

Ans

a)

$$\begin{array}{r} 6,3 \\ \times\ 1,1 \\ \hline 630 \\ \underline{\ \ 63\ \ } \\ 693 \end{array}$$

b)

i. $\frac{94}{1000} \times 1520 = \frac{142880}{1000}$

$$= 142,88$$

ii. $0,14\,288 \div 0,0094$

$$= \frac{14288}{100000} \div \frac{94}{1000}$$

$$= \frac{14288}{100000} \times \frac{1000}{94}$$

$$= \frac{152}{100}$$

$$= 0,152$$

Qsn

9. Giving your answer as a decimal, evaluate

i. $15,915 + 24,09$

ii. $85,34 \div 17$

J2009/1

Ans

i.

$$\begin{array}{r} 15,915 \\ \underline{+24,09\ \ } \\ 40,005 \end{array} \quad = 40,005$$

ii. $85,34 \div 17$

$$= \frac{85,34}{17}$$

$$= 5,02$$

Qsn

10.

a) Find the value of $0,004 \div 0,8$, giving your answer as a decimal.

b) State the number of significant figures in the number $0,000320$.

N2009/1

Ans

a) $0,004 \div 0,8$

$$= \frac{0,004 \times 1000}{0,8 \times 1000}$$

$$= \frac{4}{800}$$

$$= \frac{1}{200}$$

$$= 0,005$$

b) 3

Qsn

11. Express $0,096$ correct to two decimal places.

J2010/1

Ans

$$0,096$$
$$= 0,10$$

Qsn

12.

a. Calculate the exact value of $14,9 - 2,15$

b. Express

i. $7,369$ correct to two significant figures,

ii. $\frac{5}{12}$ as a decimal, giving your answer correct to two decimal places.

N1996/1

Ans

a.

$$\begin{array}{r} 14,90 \\ -2,15 \\ \hline 12,75 \end{array}$$

$$= 12,75$$

b.

i. 7,369=7,4

ii. $\frac{5}{12} = 0,42$

# PERCENTAGES

Qsn

1. Express $\mathbf{0,072}$ as a percentage.

N1999/1

Ans

$$\mathbf{0,072}$$

$$\mathbf{= 0,072 \times 100}$$

$$\mathbf{= 7,2\%}$$

Qsn

2. During a sale the price of a radio was reduced by 28% to $\mathbf{\$486}$. Calculate the original price of the radio.

N1999/2

Ans

Original price =new price ÷ %new

$$\mathbf{= 486 \div \frac{72}{100}}$$

$$\mathbf{= 486 \times \frac{100}{72}}$$

$$\mathbf{= \$675}$$

Qsn

3. Express $\mathbf{0,84}$ as a percentage.

J2000/1

Ans

$$\mathbf{0,84}$$

$$\mathbf{= 0,84 \times 100}$$

$$\mathbf{= 84\%}$$

Qsn

4. A greengrocer bought 30kg of bananas at \$18,75 per kg. he sold 40% of them at \$20,50 per kg and $\mathbf{66\frac{2}{3}}$%of the remainder at \$19,50 per kg. if the rest could not be sold, find the percentage loss he made, giving your answer correct to 2 significant figures.

N2000/2

Ans

$\mathbf{18,75 \times 30 = \$562,50}$

$\mathbf{\frac{40}{100} \times 30 = 12 \gg 12 \times 20,50 = \$286}$

$\mathbf{\frac{200}{3}\% \times 18 = \frac{200}{300} \times 18 = 12 \gg 12 \times 19,5 = \$234}$

$\mathbf{loss = 562,50 - (234 + 246) = \$82,50}$

$$\mathbf{\%loss = \frac{82.5}{562,5} \times 100}$$

$$\mathbf{= 14,66666\%}$$

$\mathbf{= 15\%}$ to 2s.f

Qsn

5. Find 15% of \$270.

N2001/1

Ans

$$\mathbf{\frac{15}{100} \times 270}$$

$$\mathbf{= 35,5}$$

Qsn

6. A shop sells a packet of biscuits for $\mathbf{\$23,46}$ making a profit of 15% on the cost price. Calculate the cost price.

N2001/2

Ans

Cost price =selling price ÷ %selling

$$\mathbf{= 23,46 \div \frac{115}{100}}$$

$$\mathbf{= \$26,98}$$

7. The selling price of school shoes at a departmental store in January 2005 was \$750 000 per pair.
   a. Calculate the amount paid for one pair of school shoes if a 10% discount was allowed.
   b. In February 2005 the price rose by 15% . calculate the new selling price of one pair of school shoes .

J2008/1

Ans

a. Amount paid $= \mathbf{\frac{90}{100} \times 750\,000}$

$$\mathbf{= \$675\,000}$$

b. New selling price $= \mathbf{\frac{115}{100} \times 750\,000}$

$$\mathbf{= \$862\,500}$$

8. Find 5% of 130 meters.

N2008/1

Ans

$$\mathbf{\frac{5}{100} \times 130}$$

$$\mathbf{= 6.5\ m}$$

9. A man was awarded a 15% salary increase. If his previous salary was \$5 240, calculate his new salary.

Ans

New salary=%new × original

$$\mathbf{= \frac{115}{100} \times 5240}$$

$$\mathbf{= \$6026}$$

Qsn

10. In a certain town with a population of 840 000, 15% of the people are left-handed.
    a. Calculate the number of left-handed people in the town.
    b. The population of the town is 125% of what it was 20 years ago. Calculate the population of the town 20 years ago.

J2009/2

Ans

a. $15\% \ of \ 840000$
$= \frac{15}{100} \times 840000$
$= \$126\ 000$

b. Population 20 yrs ago=new ÷%new
$= 840\ 000 \div \frac{125}{100}$
$= 840\ 000 \times \frac{100}{125}$
$= 672\ 000$

Qsn

11. Express $0,085$ as a percentage.

N2009/1

Ans

$0,085$

$= 0,085 \times 100$
$= 8,5\%$

Qsn

12. Express 36 minutes as a percentage of two hours.

J2010/1

Ans

$\frac{36}{120} \times 100$

$= 30\%$

Qsn

13. During a sale, the price of a camera was reduced from \$160 to \$148,80. Calculate the percentage decrease in price.

Ans

Decrease $= 160 - 148,80 = 11,2$

$\%decrease = \frac{11.2}{160} \times 100$
$= 7\%$

Qsn

14. A shop-owner made a loss of 12% when he sold an item for \$264. What was the cost price of the item?

N1996/2

Ans

$cost\ price = selling\ price \div \%selling$

$= 264 \div \frac{88}{100}$

$= 264 \times \frac{100}{88}$

$= \$300$

Qsn

15. In 1994 the enrolment at a school was 500 pupils. Given that this enrolment had increased by 25% over that of 1993. Calculate the 1993 enrolment.

J1997/1

Ans

The 1993 enrolment =new ÷ %new

$= 500 \div \frac{125}{100}$

$= 500 \times \frac{100}{125}$

$= 400$

Qsn

16. Express $\frac{14}{25}$ as a percentage.

N1998/1

Ans

$\frac{14}{25} \times 100$
$= 56\%$

# RATIO, RATE AND PROPORTION

Qsn

1. A plan of a house is drawn to a scale of 1:50.
   a) On the plan, a room has a length of 8,4cm. Calculate the actual length of the room, giving your answer in meters.
   b) Calculate the area, in square meters, of the room represented on the plan by an area of 63 $cm^2$.

N1996/1

Ans

a) $1:50 = 8,4:x$

$$\frac{1}{50} = \frac{8,4}{x}$$

$$\therefore x = 50 \times 8,4$$

$= \frac{420}{100}$ to cm $= 4,2m$

b) $1:50$ length
$1:2500$ area

$$1:2500 = 63:x$$

$$\frac{1}{2500} = \frac{63}{x}$$

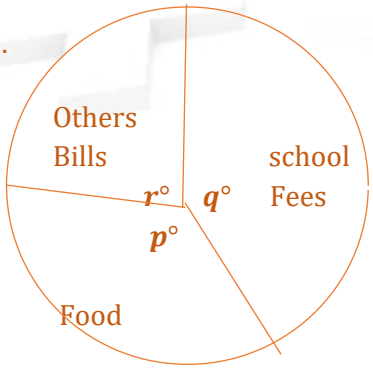
$$x = 157\,500cm^2$$

$[1m^2 = 10\,000cm^2]$

$$\therefore area = 15,75m^2$$

Qsn

2.




The pie chart represents how a household allocated its January income of food, school fees and other bills.

**Given that the ratio $p:q:r = 3:4:2$, find the value of q.**

J1997/1

Ans

[1rev = 360°]

$$p = \frac{4}{9} \times 360$$

$$p = 160°$$

Qsn

3. Chipo mixed lemon juice, sugar and water in the ratio $3:2:7$ respectively, by volume, to make lemon squash. In making the squash chipo used 35ml of water. Calculate the volume of sugar he used.

N1997/1

Ans

$$volume\ of\ sugar = \frac{2}{7} \times 35$$

$$= 10ml$$

Qsn

4. The plan of a house is made using a scale of $1:150$.
   Calculate
   i) The length of a line on the plan which represents a side of 12 metres on the house,
   ii) The volume, on cubic metres, of a pillar whose dimensions on the plan give volume of 20 cubic centimeters.

N1997/2

i) $1:150 = x:1200$

$$\frac{1}{150} = \frac{x}{1200}$$

$$150x = 1200$$

$$x = 8$$

**The length on the plan=8cm**

ii) $1:150\ length$
$1:3\,375\,000\ volume$

$$1:3\,375\,000 = 20:x$$

$$\frac{1}{3\,375\,000} = \frac{20}{x}$$

$$x = 67\ 500\ 000$$

$[1m^3 = 1\ 000\ 000\ cm^3]$

$$\therefore \textit{the volume} = 67,5m^3$$

Qsn

5. Given that $9p = 7q$, find the ratio $p:q$.

J1998/1

Ans

$$p:q = 7:9$$

Qsn

6. On a map, the distance from Gweru to Victoria Falls is 50 cm. the actual distance is 600 km.
   (a) Find the scale of the map, giving your answer in the form $1:n$.
   (b) Calculate the actual distance, in kilometers, between two towns which are 9cm apart on the map.

N1998/1

Ans

(a) $50cm:600km$

$50:60\ 000\ 000 = 1:n$

$= 1:1\ 200\ 000$

(b) $1:1\ 200\ 000 = 9:x$

$$\frac{1}{1200000} = \frac{9}{x}$$

$$x = 10\ 800\ 000$$

Actual distance $= 108\ km$

Qsn

7. All the length on a scale drawing are one eighth of their actual lengths.
   Calculate
   (a) The actual length of a line represented by a line 5,6 cm long on the scale drawing,
   (b) The area on the scale drawing which represents an actual area of 896 $cm^2$.

J1999/1

Ans

$$scale = 1:8$$

(a) $1:8 = 5,6:x$

$$\frac{1}{8} = \frac{5,6}{x}$$

$$x = 44,8$$

The actual length $= 44,8\ cm$

(b) $1:64 = x:896$

$$\frac{1}{64} = \frac{x}{896}$$

$$64x = 896$$

$$x = 14$$

The area on the scale $= 14cm^2$

Qsn

8. The exchange rate on a certain day was 3,8 dollars for 1 rand. Calculate the equivalent of
   (a) 150 rands in dollars,
   (b) 304 dollars in rands.

N1999/1

Ans

$$rate = 3,8:1$$

(a) $3,8:1 = x:150$

$$\frac{3,8}{1} = \frac{x}{150}$$

$$x = 570$$

$150\ rands = \$570$

(b) $3,8:1 = 304:x$

$$\frac{3,8}{1} = \frac{304}{x}$$

$$3,8x = 304$$

$$x = 80$$

$\$304 = 80\ rands$

Qsn

9. The scale on a map is such that 6cm on the map represents 1,5 km on the ground. Calculate
   (a) The length, in kilometres, of a road which measures 42 cm on the map.
   (b) The area on the map, in square centimeters, that represents a lake of area $8\ km^2$.

Ans

$$scale = 6cm : 1,5km$$

(a) $6cm: 1,5\ km = 42cm: x\ km$

$$\frac{6}{1,5} = \frac{42}{x}$$

$$6x = 63$$

$$x = 10,5$$

$$\text{The length} = 10,5km$$

(b) $36 : 2,25 = x : 8$

$$\frac{36}{2,25} = \frac{x}{8}$$

$$2,25x = 288$$

$$x = 128$$

$$\text{The area on the map} = 128cm^2$$

Qsn

10. The road from A to B is represented on a map, drawn to a scale of $1: 50\ 000$, by a line of length 12cm.
Find the length, in kilometres, of the road.

J2000/1

Ans

$$scale\ 1 : 50\ 000$$

$$1: 50\ 000 = 12: x$$

$$\frac{1}{50000} = \frac{12}{x}$$

$$x = 600\ 000$$

$$\text{The length} = 6km$$

11. A tourist from England visited Zimbabwe bringing with him $£2000$ which he exchanged for Zimbabwean dollars at the rate of $£1\ to\ Z\$30$.
(a) Calculate the amount he received in Zimbabwean dollars.
(b) After visiting various resort centres in Zimbabwe he had spent $Z\$40\ 500$.
He then exchanged the remainder of his money for pounds.
Find how many pounds he received.

Ans

$$rate\ 1: 30$$

(a) $1: 30 = 2000: x$

$$\frac{1}{30} = \frac{2000}{x}$$

$$x = 60\ 000$$

$$£2000 = Z\$60\ 000$$

(b) $60\ 000 - 40\ 500 = 19\ 500$

$$1: 30 = x: 19\ 500$$

$$\frac{1}{30} = \frac{x}{19500}$$

$$30x = 19500$$

$$x = 650$$

$$\text{He received } £650$$

Qsn

12. An orange syrup was diluted with water in the ratio $1: 5$ respectively. The volume of the orange syrup used was 350ml. calculate
(a) The volume of the diluted drink,
(b) The number of people who can be served, if each is given exactly 80ml of the diluted drink.

N2000/1

Ans

$$ratio\ 1: 5$$

(a) $diluted\ drink = \frac{6}{1} \times 350$

$$= 2100ml$$

(b) $1: 80 = x: 2100$

$$\frac{1}{80} = \frac{x}{2100}$$

$$80x = 2100$$

$$x = 26,25$$

$$\text{The number} = 26$$

Qsn

13. Two similar cylinders have their volumes in the ratio $1: 8$. Write down the ratio of the areas of their circular surfaces.

J2001/1

Ans

$$1: 8\ volume$$

$$1: 2\ length$$

Ratio of area $= (1: 2)^2 = 1: 4$

Qsn

14. If 18 200$m^2$ of land is shared in the ratio $2: 3: 8$, calculate the area of the smallest share.

Ans

$$\textbf{smallest share} = \frac{2}{13} \times 18\,200$$
$$= 2\,800m^2$$

Qsn

15. A bus has 80 passengers consisting of adults and school. Given that the ratio of the adults to school children is 2:3, calculate the number of adults on the bus.

N2001/1

Ans

$$\textbf{number of adults} = \frac{2}{5} \times 80$$
$$= 32$$

Qsn

16. At a certain party the ratio men: women: children was 5: 6: 4. If there were 80 children at the party, calculate the total number of people at the party.

J2008/1

Ans

$$\textbf{total number of people} = \frac{15}{4} \times 80$$
$$= 300$$

Qsn

17. Tapiwa and Netsai share some money in the ratio $2:5$. Given that Tapiwa's share is $\$620\,000$, calculate Netsai's share.

N2008/1

Ans

$$\text{Netsai's share} = \frac{5}{2} \times 620\,000$$
$$= \$1\,550\,000$$

Qsn

18. Two similar cones have their curved surface areas in the ratio $4:9$.
    (a) Write down the ratio of the height of the smaller cones to the height of the larger cone.
    (b) Given that the volume of the smaller cone is $40cm^3$, calculate the volume of the larger cone.

Ans

(a) $\sqrt{4:9}$
$= 2:3$ length

(b) $8:27 = 40:x$

$$\frac{8}{27} = \frac{40}{x}$$
$$8x = 1080$$
$$x = 135$$
$$\text{Volume} = 135cm^3$$

Qsn

19. In a certain town with population of 840 000, 15% of the people are left-handed.
    If among the left-handed people, the ratio of
    $\textbf{men}:\textbf{women}:\textbf{children}$ is 5: 11: 14, calculate the number of children who are left-handed.

J2009/2

Ans

Left-handed people $= \frac{15}{100} \times 840\,000$
$= 126\,000$

Number of children $= \frac{14}{30} \times 126\,000$

$= 58\,800$

Qsn

20.
    (a) Given that : $d = 3:7$, find the value of $d$.
    (b) A sum of money is divided in the ratio $2:3:7$. Given that the largest share is \$224 000, calculate the smallest share.

J2010/1

Ans

(a) $\frac{12}{d} = \frac{3}{7}$

$$3d = 84$$
$$d = 28$$

(b) $\textbf{smallest share} = \frac{2}{7} \times 224\,000$
$= 64\,000$

# ESTIMATIONS AND LIMITS

Qsn

1. Find the approximate value of $\frac{4,01\times5,98}{\sqrt{64,12}}$ giving your answer to the nearest whole number.

J1997/1

Ans

$$\frac{4,01\times5,98}{\sqrt{64,12}}$$

$$\approx \frac{4\times6}{\sqrt{64}}$$

$$\approx \frac{24}{8}$$

$$\approx 3$$

Qsn

2. A man estimates that each side of a square floor has a length of 4 metres, correct to the nearest metre.
   Find the difference between the largest and the smallest possible calculated values of the area of the floor.

J1999/1

Range of the estimated value is $3,5 \leq l < 4,5$

Area of a square$= s^2$

Smallest area $= 3,5^2$ largest area $= 4,4^2$

$= 12,25$ $= 19,36$

difference$= 19,36 - 12,25$

$= 7,11$

Qsn

3.

a. Measured correct to the nearest centimeter, the sides of a kite are 15cm and 12cm. find the smallest possible perimeter of the kite.

b. Estimate, correct to one significant figure, the value of $\frac{94,6}{0,0627}$.

N1999/1

a. Smallest possible perimeter $= 2(14,5 + 11,5)$

$= 52$

b. $\frac{94,6}{0,0627}$

$$= \frac{90}{0,06}$$

$$= \frac{9000}{6}$$

$= 1500$

Qsn

4. The length, $l$ cm, of a side of a square is given as 6cm, correct to one significant figure.

i. Write down the upper bound of $l$.

ii. The area of the square is $A\ cm^2$. Calculate the least possible value of $A$.

J2000/1

Ans

i. Upper bound $= 6,5$

ii. Least possible area $= 5,5^2$

$= 30,25$

Qsn

5.

a. Estimate, to the nearest whole number, the value of $\frac{7,9\times\sqrt{80,6}}{1,8\times3,1}$

b. The length of a square is given as 10cm correct to one significant figure. Find the smallest possible area of the square.

J2001/1

Ans

**a.** $\frac{7,9\times\sqrt{80,6}}{1,8\times3,1}$

$$\approx \frac{8\times\sqrt{81}}{2\times3}$$

$$\approx \frac{8\times9}{6}$$

$$\approx \frac{72}{6}$$

$$\approx 12$$

Qsn

6. The sides of a rectangle are 15cm and 7cm measured correct to the nearest centimeter. Find the smallest possible area of the rectangle.

N2001/1

Ans

Area of a rectangle$= \boldsymbol{l \times w}$

$$= \mathbf{14,5 \times 6,5}$$
$$= \mathbf{94,25}$$

Qsn

7. Estimate by first rounding off each number to one significant figure, the value of $\frac{3,04 \times 1,98 - 9,48}{2,91}$.

J2008/1

Ans

$$\frac{\mathbf{3,04 \times 1,98 - 9,48}}{\mathbf{2,91}}$$

$$\approx \frac{\mathbf{3 \times 2 - 9}}{\mathbf{3}}$$

$$\approx \frac{\mathbf{6 - 9}}{\mathbf{3}}$$

$$\approx -\frac{\mathbf{3}}{\mathbf{3}}$$

$$\approx \mathbf{-1}$$

Qsn

8. The radius, r cm, of a circle is given as 9cm, correct to the nearest whole number.
   a. Write down the limits between which r must lie,
   b. Calculate the least possible circumference of the circle, giving your answer in terms of $\boldsymbol{\pi}$.

J2008/1

Ans

a. Limits (range)$= \mathbf{8,5 \leq r < 9,5}$

b. Circumference of the circle $= \mathbf{2\pi r}$

$$= 2 \times \frac{22}{7} \times 8,5$$
$$= 53,43 \text{[2dp]}$$

Qsn

9. Estimate $\mathbf{4,63 \times 33,6}$ correct to 1 significant figure.

J2009/1

Ans

$$\mathbf{4,63 \times 33,6}$$
$$\mathbf{\approx 5 \times 30}$$
$$\mathbf{\approx 150}$$
$$\mathbf{\approx 200}$$

Qsn

10. A rectangle measures $\boldsymbol{(x + 2)cm}$ by $\boldsymbol{(2x - 1)cm}$. Given that $\boldsymbol{x = 3}$, correct to the nearest whole number, find the minimum possible area of the rectangle.

Ans

Area $= \mathbf{(2,5 + 2)(2 \times 2,5 - 1) = 4,5 \times 4 = 18}$

Qsn

11. By correcting each number to 1 significant figure, estimate the value of $\frac{371 \div 849}{\sqrt{668 - 643}}$.

Ans

$$\frac{\mathbf{371 \div 849}}{\sqrt{\mathbf{668 - 643}}}$$

$$\approx \frac{\mathbf{400 \div 800}}{\sqrt{\mathbf{700 - 600}}}$$

$$\approx \frac{\mathbf{0,5}}{\sqrt{\mathbf{100}}}$$

$$\approx \frac{\mathbf{0,5}}{\mathbf{10}}$$

$\approx \mathbf{0,05}$

Qsn

12. The length, $\boldsymbol{l\ cm}$, of the edge of a cube is given as $\mathbf{5,1}\ \boldsymbol{cm}$ correct to one decimal place.
   a. Give the limits for $\boldsymbol{l}$.
   b. Find the least possible surface area of the cube.

Ans

a. Limits (range) $= \mathbf{5,05 \leq l < 5,15}$

b. Surface area $= \mathbf{6s^2 = 6(5,05^2)}$
$$= \mathbf{153,015}$$

# LAWS OF INDICES

Qsn

1. Evaluate
   (a) $(-19)^0$
   (b) $3^2 \times 2^3$
   (c) $\left(\frac{1}{8}\right)^{\frac{2}{3}}$

N1996/1

Ans

(a) $(-19)^0 = 1$

(b) $3^2 \times 2^3 = 9 \times 8 = 72$

(c) $\left(\frac{1}{8}\right)^{\frac{2}{3}} = \left(\sqrt[3]{\frac{1}{8}}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Qsn

2. Simplify $x^{\frac{3}{2}} \times 5x^{-\frac{7}{2}}$. giving your answers with a positive index.

J1997/1

Ans

$$x^{\frac{3}{2}} \times 5x^{-\frac{7}{2}}$$

$$= 5x^{\frac{3}{2}+-\frac{7}{2}} = 5x^{\frac{3}{2}-\frac{7}{2}} = 5x^{-\frac{4}{2}} = 5x^{-2} = \frac{5}{x^2}$$

Qsn

3. Find the exact value of
   (i) $7^{\frac{1}{2}} \times 7^{1\frac{1}{2}} \times 7$
   (ii) $\left(\frac{1}{125}\right)^{-\frac{2}{3}}$

N1997/1

Ans

(i) $7^{\frac{1}{2}} \times 7^{1\frac{1}{2}} \times 7$
$= 7^{\frac{1}{2}+\frac{3}{2}+1} = 7^{\frac{4}{2}+1} = 7^3 =$

Qsn

4.
   (a) Evaluate
      (i) $2^4 + 2^0$
      (ii) $5^{\frac{1}{2}} \div 5^{\frac{3}{2}}$
   (b) Simplify $(8x^3)^{\frac{2}{3}}$

N1998/1

Ans

(a)
   (i) $2^4 + 2^0$
   $= 16 + 1$
   $= 17$

(ii) $5^{\frac{1}{2}} \div 5^{\frac{3}{2}} = 5^{\frac{1}{2}-\frac{3}{2}} = 5^{-\frac{2}{2}}$
$= 5^{-1} = \frac{1}{5}$

(b) $(8x^3)^{\frac{2}{3}} = \left(\sqrt[3]{8x^3}\right)^2 = (2x)^2 = 4x^2$

Qsn

5. Find the value of
   (a) $7^2 \div 4^0$
   (b) $\sqrt[3]{2^6 \times 27}$

N1999/1

Ans

(a) $7^2 \div 4^0$
$= 49 \div 1$
$= 49$

(b) $\sqrt[3]{2^6 \times 27} = 2^2 \times 3 = 4 \times 3 = 12$

Qsn

6. Evaluate
   a. $5^{\frac{2}{3}} \times 5^{\frac{1}{3}}$
   b. $16^{\frac{3}{4}}$
   c. $\left(-\frac{1}{3}\right)^{-2}$

J2000/1

Ans

a. $5^{\frac{2}{3}} \times 5^{\frac{1}{3}} = 5^{\frac{2}{3}+\frac{1}{3}} = 5$

b. $16^{\frac{3}{4}} = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$

c. $\left(-\frac{1}{3}\right)^{-2} = \left(-\frac{3}{1}\right)^2 = 9$

Qsn

7. Evaluate $-8^0$

N2000/1

Ans

$$-8^0 = -1$$

Qsn

8. Evaluate
   (a) $\left(\frac{1}{5}\right)^{-2}$
   (b) $-27^{\frac{2}{3}}$

J2001/1

Ans

(a) $\left(\frac{1}{5}\right)^{-2} = \left(\frac{5}{1}\right)^2 = 25$

(b) $-27^{\frac{2}{3}} = \left(\sqrt[3]{-27}\right)^2 = (-3)^2 = 9$

9. Evaluate
   i. $32^{\frac{3}{5}}$
   ii. $3^4 \times 3^0$

N2001/1

Ans

i. $32^{\frac{3}{5}} = \left(\sqrt[5]{32}\right)^3 = 2^3 = 8$

ii. $3^4 \times 3^0 = 3^{4+0} = 3^4 = 81$

Qsn

10. Evaluate
   (a) $(-32)^{\frac{3}{5}}$
   (b) $\frac{1}{5^{-3}}$

J2008/1

Ans

(a) $(-32)^{\frac{3}{5}} = \left(\sqrt[5]{-32}\right)^3 = (-2)^3 = -8$

(b) $\frac{1}{5^{-3}} = 5^3 = 125$

Qsn

11. Simplify $(0,2)^3 \times (0,2)^2$, giving your answer as a decimal.

N2008/1

Ans

$(0,2)^3 \times (0,2)^2$
$= (0,2)^{3+2}$
$= (0,2)^5$
$= 0,00032$

Qsn

12. Evaluate
   i. $9^2 \times 9^0$
   ii. $8^{\frac{2}{3}}$

J2009/1

Ans

i. $9^2 \times 9^0 = 81 \times 1 = 81$

ii. $8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$

Qsn

13. Simplify
   (a) $(2x)^3$
   (b) $\left(8^{-\frac{2}{3}}\right)^{\frac{1}{2}}$

N2009/1

Ans

(a) $(2x)^3 = 8x^3$

(b) $\left(8^{-\frac{2}{3}}\right)^{\frac{1}{2}} = 8^{-\frac{2}{3}\times\frac{1}{2}} = 8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}}$
$= \frac{1}{\sqrt[3]{8}} = \frac{1}{2}$

Qsn

14. Given that $m = \frac{1}{2}, n = 0$ and $r = 3$, evaluate
   a. $(mr)^n$
   b. $\left(2\frac{1}{4}\right)^m$
   c. $\sqrt[r]{-64}$

J2010/1

Ans

a. $(mr)^n = \left(\frac{1}{2} \times 3\right)^0 = 1$

b. $\left(2\frac{1}{4}\right)^m = \left(\frac{9}{4}\right)^m = \sqrt{\frac{9}{4}} = \frac{3}{2} = 1\frac{1}{2}$

Qsn

15. Evaluate
   (a) $\left(16^{\frac{1}{2}}\right)^{\frac{3}{2}}$
   (b) $7^{-\frac{1}{4}} \times 7^{\frac{5}{4}} \times 7$
   (c) $\left(\frac{1}{5}\right)^{-2}$

N2010/1

# LAWS OF LOGARITHMS

Qsn

1. Evaluate $\log_3 45 - \log_3 5$.

J1999/1

Ans

$$\log_3 45 - \log_3 5$$
$$= \log_3 \left(\frac{45}{5}\right)$$
$$= \log_3 9$$
$$= 2$$

Qsn

2. It is given that $\log_{10} 50 = 1,69897$ correct to five decimal places.
   (a) Write down $\log_{10} 50$correct to 4 decimal places.
   (b) Evaluate, giving each answer correct to 4 decimal places.
      (i) $\log_{10} 5$
      (ii) $\log_{10} 25$
   (c) Evaluate, giving the answer correct to 3 decimal places, $\log_{10} 2$.

N1999/1

Ans

**(a)** $\log_{10} 50 = 1,6990$ [4dp]

**(b)**

(i) $\log_{10} 5$
$= \log_{10} \left(\frac{50}{10}\right)$
$= \log_{10} 50 - \log_{10} 10$
$= 1,69897 - 1$
$= 0,69897$
$=0,6990$ [4dp]

(ii) $\log_{10} 25$
$= \log_{10} 5^2$
$= 2\log_{10} 5$
$= 2(0,69897)$
$= 1,39794$
$= 1,3979$

**(c)** $\log_{10} 2$
$= \log_{10} \left(\frac{10}{5}\right)$
$= \log_{10} 10 - \log_{10} 5$
$= 1 - 0,69897$
$= 0,30103$
$= 0,301$

Qsn

3. Evaluate
   (a) $\log_{10} 16 \div \log_{10} 2$
   (b) $2\log_{10} 5 + \log_{10} 36 - \log_{10} 9$

J2000/1

Ans

**(a)** $\log_{10} 16 \div \log_{10} 2$
$= \frac{\log_{10} 16}{\log_{10} 2}$

$= \frac{\log_{10} 2^4}{\log_{10} 2} = \frac{4\log_{10} 2}{\log_{10} 2} = 4$

(b) $2\log_{10} 5 + \log_{10} 36 - \log_{10} 9$
$= \log_{10} 5^2 + \log_{10} \left(\frac{36}{9}\right)$
$= \log_{10}(25 \times 4)$
$= \log_{10} 100$
$= 2$

Qsn

4. Evaluate $\log_4 32 + \log_4 2$

N2000/1

Ans

$\log_4 32 + \log_4 2$
$= \log_4(32 \times 2)$
$= \log_4 64$
$= 3$

Qsn

5. Given that $\log 216 = 2,334$ and $\log 3 = 0,477$, find
   (a) $\log 2,16$
   (b) $\log 6$
   (c) $\log 72$

J2001/1

Ans

**(a)** $\log 2,16$
$= \log \left(\frac{216}{100}\right)$
$= \log 216 - \log 100$
$= 2,334 - 2$
$= 0,334$

**(b)** $\log 6$
$= \log \sqrt[3]{216}$
$= \log 216^{\frac{1}{3}}$
$= \frac{1}{3}\log 216$
$= \frac{2,334}{3}$
$= 0,778$

Qsn

6. If $\log 3 = m$ and $\log 8 = n$, express
   (i) $\log 9$ in terms of m
   (ii) $\log 2\frac{2}{3}$ in terms of m and

Ans

(i) $\log 9$
$= \log 3^2$
$= 2\log 3$
$= 2m$

(ii) $\log 2\frac{2}{3}$
$= \log\frac{8}{3}$
$= \log 8 - \log 3$
$= n - m$

7.
   (a) Evaluate
      (i) $\log_2 16$
      (ii) $\frac{\log_{10} 27}{\log_{10} 3}$
   (b) Given that $\log_{10} 2 = 0,301$ and $\log_{10} 7 = 0,845$, calculate the value of
      (i) $\log_{10} 3,5$
      (ii) $\log_{10} 40$

N1996/1

Ans

(a)
(i) $\log_2 16 = 4$
(ii) $\frac{\log_{10} 27}{\log_{10} 3} = \frac{\log_{10} 3^3}{\log_{10} 3} = \frac{3\log_{10} 3}{\log_{10} 3} = 3$

(b)
(i) $\log_{10} 3,5 = \log_{10}\left(\frac{7}{2}\right)$
$= \log_{10} 7 - \log_{10} 2$
$= 0,845 - 0301$
$= 0,544$

(ii) $\log_{10} 40 = \log_{10}(8 \times 5)$
$= \log_{10} 8 + \log_{10}\left(\frac{10}{2}\right)$
$= \log_{10} 2^3 + \log_{10} 10 - \log_{10} 2$
$= 3\log_{10} 2 + 1 - 0,301$
$= 3(0,301) + 0.699$
$= 0,903 + 0,699$
$= 1,602$

Qsn

8. Given that $\log 2 = m$ and $\log 3 = n$, express
   (a) $m + n$ as a logarithm of a single number.
   (b) $\log 9$ in terms of n
   (c) $\log 1\frac{1}{3}$ in terms of m and n.

J1997/1

Ans

(a) $m + n = \log 2 + \log 3 = \log(2 \times 3)$
$= \log 6$

(b) $\log 9 = \log 3^2 = 2\log 3 = 2n$

(c) $\log 1\frac{1}{3} = \log\frac{4}{3} = \log 4 - \log 3$
$= \log 2^2 - n$
$= 2m - n$

Qsn

9. Given that $\log m = -6$ and $\log n = 5$, evaluate
   (a) $\log mn$
   (b) $\log m^{\frac{1}{2}}$
   (c) $\log\frac{1}{n}$

N1997/1

Ans

(a) $\log mn = \log m + \log n = -6 + 5$
$= -1$

(b) $\log m^{\frac{1}{2}} = \frac{1}{2}\log m = \frac{1}{2}(-6) = -3$

(c) $\log\frac{1}{n} = \log 1 - \log n = 0 - 5 = -5$

Qsn

10. Given that $\log 3 = 0,477$ and $\log 5 = 0,699$, find
   (a) $\log 15$
   (b) $\log 225$
   (c) $\log 0,6$

J1998/1

Ans

(a) $\log 15 = \log(5 \times 3)$
$= \log 5 + \log 3$
$= 0,699 + 0,477$
$= 1,176$

(b) $\log 225 = \log 15^2$
$= 2\log 15$

$= 2(1,176) = 2,352$

(c) $\log 0,6 = \log\frac{3}{5}$
$= \log 3 - \log 5$
$= 0,477 - 0,699$
$= -0,222$

Qsn

11. Given that $\log_{10} 7 = 0,845$, evaluate
(a) $\log_{10} 49$
(b) $\log_{10} 70$
(c) $\log_{10}\frac{100}{7}$

N1998/1

Ans

(a) $\log_{10} 49$
$= \log_{10} 7^2$
$= 2\log_{10} 7$
$= 2(0,845)$
$= 1,690$

(b) $\log_{10} 70$
$= \log_{10}(7 \times 10)$
$= \log_{10} 7 + \log_{10} 10$
$= 0,845 + 1$
$= 1,845$

(c) $\log_{10}\frac{100}{7} = \log_{10} 100 - \log_{10} 7$
$= 2 - 0,845$
$= 1,176$

Qsn

12. Given that $\log 2 = 0,3010$ and $\log 3 = 0,4771$, calculate
(a) $\log 6$
(b) $\log 1,5$
(c) $\log\sqrt{2}$

J2008/1

Ans

(a) $\log 6 = \log(3 \times 2) =$
$\log 3 + \log 2$
$= 0,4771 + 0,3010$
$= 0,7781$

(b) $\log 1,5 = \log\frac{3}{2} = \log 3 - \log 2$
$= 0,4771 - 0,3010$
$= 0,1761$

(c) $\log\sqrt{2} = \log 2^{\frac{1}{2}} = \frac{1}{2}\log 2$
$= \frac{0,3010}{2}$
$= 0,1505$

Qsn

13. Given that $\log_5 2 = 0,431$ and $\log_5 3 = 0,683$, find the value of
(a) $\log_5 1\frac{1}{2}$
(b) $\log_5\sqrt{3}$

N2008/1

Ans

(a) $\log_5 1\frac{1}{2} = \log_5\frac{3}{2} =$
$\log_5 3 - \log_5 2$
$= 0,683 - 0,431$
$= 0,252$

(b) $\log_5\sqrt{3} = \log_5 3^{\frac{1}{2}} = \frac{1}{2}\log_5 3$
$= \frac{0,683}{2}$
$= 0,3415$

Qsn

14. Given that $\log 7 = 0,845$ and $\log 2 = 0,301$, calculate
(a) $\log 14$
(b) $\log\sqrt{2}$

N2009/1

Qsn

15. Simplify as far as possible
(a) $\log 9 \div \log 3$
(b) $4\log 2 + \log 20 - \log 3,2$.

J2010/1

Qsn

16. Evaluate $\frac{\log 3 + \log 9}{\log 405 - \log 5}$.

N2010/1

# SUBSTITUTION

Qsn

1. Given that $z = r\sqrt{n-1}$
   Find $z$ when $r = 0,3$ and $n = 50$.

J2008/2

Ans

$$z = r\sqrt{n-1}$$
$$= 0,3\sqrt{50-1}$$
$$= 0,3\sqrt{49}$$
$$= 0,3 \times 7$$
$$= 2,1$$

Qsn

2. Given the formula $A = 2\pi(r^2 - h)$. Calculate the value of $A$ when $= 3\frac{1}{7}, r = 1\frac{3}{4}$ and $h = 3\frac{1}{2}$.

J2009/2

Ans

$$A = 2\pi(r^2 - h)$$
$$= 2\left(\frac{22}{7}\right)\left(\left(\frac{7}{4}\right)^2 - \frac{7}{2}\right)$$
$$= \frac{44}{7}\left(\frac{49}{16} - \frac{7}{2}\right)$$
$$= \frac{44}{7}\left(\frac{21}{16}\right)$$
$$= \frac{33}{4} = 8\frac{1}{4}$$

Qsn

3. The volume, V, of material used to make a cylindrical tube of internal radius r, external radius R and length h is given by the formula
   $$V = \pi(R^2 - r^2)h$$
   Taking $\pi$ to be $\frac{22}{7}$, find the value of $V$ when $R = 4cm, r = 3cm$ and $h = 150cm$.

N2009/2

Ans

$$V = \pi(R^2 - r^2)h$$
$$= \frac{22}{7}(4^2 - 3^2)(150)$$
$$= \frac{22}{7} \times \frac{7}{1} \times \frac{150}{1}$$
$$= 22 \times 150$$
$$= 3300$$

4. It is given that $t = 2\pi\sqrt{\frac{d}{g}}$.
   Find $t$ when $\pi = \frac{22}{7}, d = 490$ and $g = 10$.

J2010/1

Ans

$$t = 2\pi\sqrt{\frac{d}{g}}$$
$$= 2\left(\frac{22}{7}\right)\sqrt{\frac{490}{10}}$$
$$= \frac{44}{7}\sqrt{49}$$
$$= \frac{44}{7} \times 7$$
$$= 44$$

5. It is given that $T = 2\pi\sqrt{\frac{l}{g}}$.
   Find $T$ when $l = 156,8; g = 9,8$ and $\pi = 3,1$.

N2010/1

Ans

$$T = 2\pi\sqrt{\frac{l}{g}}$$
$$= 2(3,1)\sqrt{\frac{156,8}{9,8}}$$
$$= 6,2\sqrt{16}$$
$$= 6,2 \times 4$$
$$= 24,8$$

6. It is given that $s = ut - \frac{1}{2}gt^2$.
   Find the value of $s$ if $g = 9,8; u = 20$ and $t = 2$.

N2010/2

Ans

$$s = ut - \frac{1}{2}gt^2$$
$$= (20)(2) - \frac{1}{2}(9,8)(2^2)$$
$$= 40 - 19,6$$
$$= 20,4$$

7. Given that $2p = \sqrt[3]{6q+5}$.
   Find the value of $p$ when $q = 20$.

J1999/2

Ans

$$2p = \sqrt[3]{6q+5}$$
$$2p = \sqrt[3]{6(20)+5}$$

$$2p = \sqrt[3]{125}$$
$$2p = 5$$
$$p = 2\frac{1}{2}$$

8. Given that $a = 3, b = -2$ *and* $c = 10$, calculate
   i. $ac^2$,
   ii. $a^b$.

J2001/2

Ans

i. $ac^2$
$= (3)(10)^2$
$= 3 \times 100$
$= 300$

ii. $a^b$
$= 3^{-2}$
$= \frac{1}{3^2}$
$= \frac{1}{9}$

Qsn

9. A formula that can be used for changing degrees Fahrenheit(F) to degrees centigrade(C) is

$$C = \frac{5F - 160}{9}$$

Calculate the value of C when F = 95.

N2001/2

Ans

$$C = \frac{5F - 160}{9}$$
$$= \frac{5(95) - 160}{9}$$
$$= \frac{315}{9}$$
$$= 35$$

Qsn

10. Given that $a = 2$ *and* $b = -3$. Calculate
    (i) $a^2 - b^2$
    (ii) $(a - b)^2$
    (iii) $\frac{1}{ab}$

J1996/2

Ans

(i) $a^2 - b^2$
$= 2^2 - (-3)^2$
$= 4 - 9$
$= -5$

(ii) $(a - b)^2 = (2 - -3)^2 = 5^2 = 25$

(iii) $\frac{1}{ab} = \frac{1}{(2)(-3)} = -\frac{1}{6}$

Qsn

11. It is given that $M = \frac{5+T}{5-T}$
Calculate giving your answer as a common fraction in its lowest terms, the value of M when $T = -20$.

N1996/1

Ans

$$M = \frac{5 + T}{5 - T}$$
$$= \frac{5 + -20}{5 - -20}$$
$$= \frac{5 - 20}{5 + 20}$$
$$= -\frac{15}{25} = -\frac{3}{4}$$

Qsn

12. Find the numerical value of
$p(1 - q^2)$
When $p = 4$ *and* $q = -3$.

N1996/2

Ans

$$p(1 - q^2)$$
$$= (4)(1 - (-3)^2)$$
$$= 4 \times -8$$
$$= -32$$

Qsn

13. Calculate the value of $xy - z$ given that $x = 4, y = -1$ *and* $z = -3$.

J1997/1

Ans

$$xy - z$$
$$= (4)(-1) - (-3)$$
$$= -4 - -3$$
$$= -4 + 3$$
$$= -1$$

Qsn

14. It is given that $q = \frac{5+4p}{6-3p}$
Giving your answer as a common fraction in its lowest terms, calculate the value of $q$ when $p = -2$.

N1997/1

Ans

$$q = \frac{5+4p}{6-3p}$$
$$= \frac{5+4(-2)}{6-3(-2)}$$
$$= \frac{5-8}{6+6}$$
$$= -\frac{3}{12} = -\frac{1}{4}$$

Qsn

15. The premium of a certain insurance policy is calculated using the formula

$$Q = \frac{(M-F)\times 10\,000}{R}$$

Where \$Q is the sum assured
\$M is the monthly premium
\$F is the policy fee
And R is a factor obtained from the insurance tables.
Given that $Q = 50\,000, F = 20$ *and* $R = 45$, find the value of M

N1997/2

Ans

$$Q = \frac{(M-F)\times 10\,000}{R}$$
$$50000 = \frac{(M-20)\times 10000}{45}$$
$$\frac{50000\times 45}{10000} = M - 20$$
$$225 + 20 = M$$
$$M = 245$$

Qsn

16. Given that $a = 3, b = -2$ *and* $c = -5$, find the value of
    (i) $2a + 7b - c$
    (ii) $b^2 - c^2$
    (iii) $\frac{a}{b} - \frac{c}{a}$ .

J1998/2

Ans

(i) $2a + 7b - c$
$= 2(3) + 7(-2) - (-5)$
$= 6 - 14 + 5$
$= -3$

(ii) $b^2 - c^2$
$= (-2)^2 - (-5)^2$
$= 4 - 25$
$= -21$

(iii) $\frac{a}{b} - \frac{c}{a}$
$= \frac{3}{-2} - -\frac{5}{3}$
$= -\frac{3}{2} + \frac{5}{3}$
$= \frac{1}{6}$

# EXPANSION

Qsn

1. Simplify $2m(3m+n)-5m^2$.

N1999/1

Ans

$$2m(3m+n)-5m^2$$
$$=6m^2+2mn-5m^2$$
$$=6m^2-5m^2+2mn$$
$$=m^2+2mn$$

Qsn

2. Expand and simplify
   (a) $(x-3)^2$
   (b) $5(4x-7)-6(3x-2)$

J2000/1

Asn

(a) $(x-3)^2$
$=(x-3)(x-3)$
$=x(x-3)-3(x-3)$
$=x^2-3x-3x+9$
$=x^2-6x+9$

(b) $5(4x-7)-6(3x-2)$
$=20x-35-18x+12$
$=20x-18x-35+12$
$=2x-23$

Qsn

3. Simplify $4rt^2(3r-t^3)$

N2000/2

Ans

$$4rt^2(3r-t^3)$$
$$=4r^2t^2-4rt^5$$

Qsn

4. Expand and simplify
   $(x+2)(5+2x-x^2)$

J2001/1

Ans

$$(x+2)(5+2x-x^2)$$
$$=x(5+2x-x^2)+2(5+2x-x^2)$$
$$=5x+2x^2-x^3+10+4x-2x^2$$
$$=5x+4x+2x^2-2x^2-x^3+10$$
$$=9x-x^3+10$$

Qsn

5. Expand $x^2y(x^4-xy^3)$

N2001/1

Ans

$$x^2y(x^4-xy^3)$$
$$=x^6y-x^3y^4$$

Qsn

6. Remove brackets and simplify
   $3(a+2c)-4(2a-c)$

N2008/2

Ans

$$3(a+2c)-4(2a-c)$$
$$=3a+6c-8a+4c$$
$$=3a-8a+6c+4c$$
$$=-5a+10c$$

Qsn

7. Giving your answer in ascending powers of $x$, expand $(x+2)^2$.

J2009/1

Ans

$$(x+2)^2$$
$$=(x+2)(x+2)$$
$$=x(x+2)+2(x+2)$$
$$=x^2+2x+2x+4$$
$$=4+4x+x^2$$

Qsn

8. Expand $(1-2x)(x+3)$.

N2009/2

Ans

$$(1-2x)(x+3)$$
$$=1(x+3)-2x(x+3)$$
$$=x+3-2x^2-6x$$
$$=3+x-6x-2x^2$$
$$=3-5x-2x^2$$

Qsn

9. Simplify $6x+12x\div 3$

J2010/1

Ans

$$6x+12x\div 3$$
$$=6x+4x$$
$$=10x$$

Qsn

10. Remove brackets and simplify the expression $3(5-x)-2x(x+3)$

J2010/2

Ans

$$3(5-x)-2x(x+3)$$
$$=15-3x-2x^2-6x$$
$$=15-3x-6x-2x^2$$
$$=15-9x-2x^2$$

Qsn

11. Simplify
    $6x-y+3z-(4x-2y+5z)$

# FACTORISATION

Qsn

1. Factorise completely
   $y(y+3)-2(y+3)$

J1999/1

Ans

$y(y+3)-2(y+3)$

$(y+3)(y-2)$

Qsn

2. Factorise completely
   a) $ax+x-3a-3$
   b) $2x^2-7x+3$

J1999/2

Sln

a) $ax+x-3a-3$

$x(a+1)-3(a+1)$

$(a+1)(x-3)$

b) $2x^2-7x+3$

$6x^2$- factors[-6 and -1]

$2x^2-6x-x+3$

$2x(x-3)-1(x-3)$

$(x-3)(2x-1)$

Qsn

3. Factorise completely
   a) $3a-ab-2b+6$
   b) $3x^2-147$

N1999/2

Sln

a) $3a-ab-2b+6$

$a(3-b)+2(-b+3)$

$(3-b)(a+2)$

b) $3x^2-147$

$3(x^2-49)$

$3(x-7)(x+7)$

Qsn

4. Factorise completely
   a) $8p^3-18p$
   b) $as+2at-3s-6t$.

J2000/2

Sln

a) $8p^3-18p$

$2p(4p^2-9)$

$2p(2p-3)(2p+3)$

b) $as+2at-3s-6t$

$a(s+2t)-3(s+2t)$

$(s+2t)(a-3)$

Qsn

5. Factorise completely
   a) $x^3+7x^2+12x$
   b) $ab-ad-bc+cd$

N2000/2

Sln

a) $x^3+7x^2+12x$

$12x^4$- factors [4 and 3]

$x^3+4x^2+3x^2+12x$

$x^2(x+4)+3(x+4)$

$(x+4)(x^2+3)$

b) $ab-ad-bc+cd$

$a(b-d)-c(b-d)$

$(b-d)(a-c)$

Qsn

6. Factorise completely
   $3x^2-6x-ax+2a$

J2001/1

Sln

$3x^2-6x-ax+2a$

$3x(x-2)-a(x-2)$

$(x-2)(3x-a)$

Qsn

7. Factorise completely
   $5r^2-5rt+qr-qt$

N2001/1

Sln

$5r^2-5rt+qr-qt$

$5r(r-t)+q(r-t)$

$(r-t)(5r+q)$

Qsn

8. Factorise completely
   a) $x^2-4$
   b) $x^2+4x+4$

N2001

Sln

a) $x^2-4$

$(x-2)(x+2)$

b) $x^2+4x+4$

$x^2+2x+2x+4$

$x(x+2)+2(x+2)$

$(x+2)(x+2)$

Qsn

9. Factorise completely
   a) $15m + 18 - 10mn - 12n$
   b) $27x^2 - 12y^2$

J2008/1

Sln

a) $15m + 18 - 10mn - 12n$
$3(5m + 6) - 2n(5m + 6)$
$(5m + 6)(3 - 2n)$

b) $27x^2 - 12y^2$
$3(9x^2 - 4y^2)$
$3(3x - 2y)(3x + 2y)$

Qsn

10. Factorise completely
$6y^2 - y - 12$

J2008/2

Sln

$6y^2 - y - 12$
$6y^2 - 9y + 8y - 12$
$3y(2y - 3) + 4(2y - 3)$
$(2y - 3)(3y + 4)$

Qsn

11. Factorise completely
$x^2 - y^2$

N2008/1

Sln

$x^2 - y^2$
$(x - y)(x - y)$

Qsn

12. Factorise completely
   a) $2x^2 + ax - 2bx - ab$
   b) $3 - 12y^2$.

N2008/2

Sln

a) $2x^2 + ax - 2bx - ab$
$x(2x + a) - b(2x + a)$
$(2x + a)(x - b)$

b) $3 - 12y^2$
$3(1 - 4y^2)$
$3(1 - 2y)(1 + 2y)$

Qsn

13. Factorise completely
$18x^2 + 15x + 3$

Sln

$18x^2 + 15x + 3$
$18x^2 + 9x + 6x + 3$
$9x(2x + 1) + 3(2x + 1)$
$(2x + 1)(9x + 3)$

Qsn

14. Factorise completely
   a) $x^2 - 12x + 27$
   b) $cx - dx - 4cy + 4dy$

N2009/1

Sln

a) $x^2 - 12x + 27$
$x^2 - 3x - 9x + 27$
$x(x - 3) - 9(x - 3)$
$(x - 3)(x - 9)$

b) $cx - dx - 4cy + 4dy$
$x(c - d) - 4y(c - d)$
$(c - d)(x - 4y)$

Qsn

15. Factorise completely
$3p^2 + 7p - 6$

N2009/2

Sln

$3p^2 + 7p - 6$
$3p^2 + 9p - 2p - 6$
$3p(p + 3) - 2(p + 3)$
$(p + 3)(3p - 2)$

Qsn

16. Factorise completely
$x^2 - 9y^2$

Sln

$x^2 - 9y^2$
$(x - 3y)(x + 3y)$

17. Factorise completely
   a) $xy^2 - x^2y$
   b) $98 - 2x^2$

Sln

a) $xy^2 - x^2y$
$xy(y - x)$

b) $98 - 2x^2$
$2(49 - x^2)$
$2(7 - x)(7 + x)$

# ALGEBRAIC FRACTIONS

Qsn

1. Simplify $\frac{2}{x-1}-\frac{3}{x+5}$.

J1999/2

Ans

$\frac{2}{x-1}-\frac{3}{x+5}$

$=\frac{2(x+5)-3(x-1)}{(x-1)(x+5)}=\frac{2x+10-3x+3}{(x-1)(x+5)}=\frac{2x-3x+10+3}{(x-1)(x+5)}$

$=\frac{13-x}{(x-1)(x+5)}$

Qsn

2. Express $\frac{5m}{8}-\frac{2m+3}{4}$ as a single fraction in its simplest form.

N1999/1

Ans

$\frac{5m}{8}-\frac{2m+3}{4}$

$=\frac{5m-2(2m+3)}{8}=\frac{5m-4m-6}{8}=\frac{m-6}{8}$

Qsn

3. Express $\frac{3}{x-2}-\frac{2}{x}$ as a single fraction.

J2000/1

Ans

$\frac{3}{x-2}-\frac{2}{x}$

$\frac{3x-2(x-2)}{x(x-2)}=\frac{3x-2x+4}{x(x-2)}=\frac{x+4}{x(x-2)}$

Qsn

4. Simplify $\frac{a^3b^4}{a^4b^2c}$.

N2000/1

Ans

$\frac{a^3b^4}{a^4b^2c}=\frac{b^2}{a^2c}$

Qsn

5. Express as a single fraction in its simplest form $\frac{y}{4y-1}+\frac{3}{5}$.

N2000/2

Ans

$\frac{y}{4y-1}+\frac{3}{5}$

$\frac{5y+3(4y-1)}{5(4y-1)}=\frac{5y+12y-3}{5(4y-1)}=\frac{17y-3}{5(4y-1)}$

Qsn

6. Simplify $\frac{a^2-b^2}{ab+a^2}\div\frac{ab-a^2}{2a^3}$.

J2001/1

Ans

$\frac{a^2-b^2}{ab+a^2}\div\frac{ab-a^2}{2a^3}$

$=\frac{a^2-b^2}{ab+a^2}\times\frac{2a^3}{ab-a^2}=\frac{(a-b)(a+b)}{a(b+a)}\times\frac{2a^3}{a(b-a)}$

$=\frac{2a^3(a-b)}{a^2(b-a)}=-2a$

Qsn

7. Express $\frac{4}{p}-\frac{3}{1-5p}$ as a single fraction in its simplest form.

J2001/2

Ans

$\frac{4}{p}-\frac{3}{1-5p}$

$=\frac{4(1-5p)-3p}{p(1-5p)}=\frac{4-20p-3p}{p(1-5p)}=\frac{4-23p}{p(1-5p)}$

Qsn

8. Express $\frac{1}{s}+\frac{1}{t}+\frac{1}{st}$ as a single fraction.

N2001/2

Ans

$\frac{1}{s}+\frac{1}{t}+\frac{1}{st}$

$=\frac{t+s+1}{st}$

Qsn

9. Express $\frac{b}{a^2-ab}+\frac{a}{b^2-ab}$ as a single fraction in its lowest term,

N2006/2

Ans

$\frac{b}{a^2-ab}+\frac{a}{b^2-ab}$

$=\frac{b}{a(a-b)}+\frac{a}{b(b-a)}=\frac{b}{a(a-b)}-\frac{a}{b(a-b)}=\frac{b^2-a^2}{ab(a-b)}$

$=\frac{(b-a)(b+a)}{ab(a-b)}=-\frac{b+a}{ab}$

Qsn

10. Express as a single fraction in its simplest form $+\frac{2n}{6n+5}$.

N2008/2

Ans

$n+\frac{2n}{6n+5}$

$=\frac{n(6n+5)+2n}{6n+5}=\frac{6n^2+5n+2n}{6n+5}$

$=\frac{6n^2+7n}{6n+5}$

Qsn

11. Express $\frac{1}{x-1}+\frac{2}{x+1}$ as a single fraction in its simplest form.

N2010/2

Ans

$\frac{1}{x-1}+\frac{2}{x+1}$

$=\frac{1(x+1)+2(x-1)}{(x-1)(x+1)}=\frac{x+1+2x-2}{(x-1)(x+1)}=\frac{3x-1}{(x-1)(x+1)}$

Qsn

12. Express $\frac{2}{2x-1}-\frac{3}{x}$ as a single fraction in its simplest form.

J2008/2

Ans

$\frac{2}{2x-1}-\frac{3}{x}$

$=\frac{2x-3(2x-1)}{x(2x-1)}=\frac{2x-6x+3}{x(2x-1)}=\frac{-4x+3}{x(2x-1)}$

13. Express $\frac{3}{b+1}-\frac{2}{b-2}$ as a single fraction in its simplest form.

J2009/1

Ans

$\frac{3}{b+1}-\frac{2}{b-2}$

$=\frac{3(b-2)-2(b+1)}{(b+1)(b-2)}=\frac{3b-6-2b-2}{(b+1)(b-2)}=\frac{b-8}{(b+1)(b-2)}$

Qsn

14. Simplify $\frac{n-3}{6}\div\frac{n^2-9}{4}$.

N2009/2

Ans

$\frac{n-3}{6}\div\frac{n^2-9}{4}$

$=\frac{n-3}{6}\div\frac{(n-3)(n+3)}{4}=\frac{\cancel{n-3}}{6}\times\frac{4}{\cancel{(n-3)}(n+3)}=\frac{2}{3(n+3)}$

Qsn

15. Express $\frac{3}{x-2}-\frac{4}{x+1}$ as a single fraction in its simplest form.

N2010/1

Ans

$\frac{3}{x-2}-\frac{4}{x+1}$

$=\frac{3(x+1)-4(x-2)}{(x-2)(x+1)}=\frac{3x+3-4x+8}{(x-2)(x+1)}=\frac{-x+11}{(x-2)(x+1)}$

Qsn

16. Express $\frac{r+2}{4r}-\frac{(2-3t)}{6t}$ as a single fraction in its simplest form.

N1996/2

Ans

$\frac{r+2}{4r}-\frac{(2-3t)}{6t}$

$=\frac{3t(r+2)-2r(2-3t)}{12rt}=\frac{3rt+6t-4r+6rt}{12rt}=\frac{9rt+6t-4r}{12rt}$

Qsn

17. Simplify $\frac{6a^2}{5}\div\frac{3a}{20}$.

J1997/1

Ans

$\frac{6a^2}{5}\div\frac{3a}{20}$

$=\frac{6a^2}{5}\times\frac{20}{3a}=\frac{2a\times4}{1}=8a$

Qsn

18. Express $\frac{1}{x^2-2x}+\frac{1}{x^2-6x+8}$ as a single fraction in its simplest form.

J1997/2

Ans

$\frac{1}{x^2-2x}+\frac{1}{x^2-6x+8}$

$=\frac{1}{x(x-2)}+\frac{1}{(x-4)(x-2)}=\frac{1(x-4)+1(x)}{x(x-2)(x-4)}$

$=\frac{x-4+x}{x(x-2)(x-4)}=\frac{2x-4}{x(x-2)(x-4)}=\frac{2(x-2)}{x(x-2)(x-4)}$

$=\frac{2}{x(x-4)}$

Qsn

19. Simplify $\frac{3}{(2x+1)(3-x)}-\frac{2}{x-3}$.

N1997/2

Qsn

20. Express as a single fraction $\frac{3}{x-2}+\frac{4}{5x+1}$

J1998/1

# LINEAR EQUATIONS

Qsn

1. Solve the equation
$\frac{2}{3}(x+4) = x-1$

J2008/2

Sln

$$\frac{2}{3}(x+4) = x-1$$
$$3\left[\frac{2}{3}(x+4) = x-1\right]$$
$$2(x+4) = 3(x-1)$$
$$2x+8 = 3x-3$$
$$2x-3x = -3-8$$
$$-x = -11$$
$$x = 11$$

Qsn

2. Solve the equation
$5x-2(x+3) = 9$

N2008/1

Sln

$$5x-2(x+3) = 9$$
$$5x-2x-6 = 9$$
$$3x = 9+6$$
$$3x = 15$$
$$x = 5$$

Qsn

3. Solve the equation
$\frac{4x-5}{7} = 1\frac{3}{4}$

N2008/2

Sln

$$\frac{4x-5}{7} = 1\frac{3}{4}$$
$$\frac{4x-5}{7} = \frac{7}{4}$$
$$28\left[\frac{4x-5}{7} = \frac{7}{4}\right]$$
$$4(4x-5) = 49$$
$$16x-20 = 49$$
$$16x = 49+20$$
$$16x = 69$$
$$x = 4\frac{5}{69}$$

Qsn

4. Solve the equation
   a. $4^x = 32$
   b. $\frac{2-x}{15} = \frac{2}{3}+\frac{x}{5}$

J2009/1

Sln

a. $4^x = 32$
$2^{2x} = 2^5$
$\therefore 2x = 5$
$$x = 2\frac{1}{2}$$

b. $\frac{2-x}{15} = \frac{2}{3}+\frac{x}{5}$
$$15\left[\frac{2-x}{15} = \frac{2}{3}+\frac{x}{5}\right]$$
$$2-x = 10+3x$$
$$-x-3x = 10-2$$
$$-4x = 8$$
$$x = -2$$

Qsn

5. Solve the equation
   a. $7(h+3)-2(h-4) = 4$
   b. $3^{(m+4)} = 9^{(m-1)}$

N2009/2

Sln

a. $7(h+3)-2(h-4) = 4$
$7h+21-2h+8 = 4$
$7h-2h = 4-21-8$
$5h = -25$
$h = -5$

b. $3^{(m+4)} = 9^{(m-1)}$
$3^{(m+4)} = 3^{2(m-1)}$
$\therefore m+4 = 2m-2$
$m-2m = -2-4$
$-m = -6$
$m = 6$

Qsn

6. Solve the equation
$\frac{3}{y} = \frac{12}{11}$

J2010/1

Sln

$$\frac{3}{y} = \frac{12}{11}$$
$$11y\left[\frac{3}{y} = \frac{12}{11}\right]$$
$$33 = 12y$$
$$y = 2\frac{3}{4}$$

Qsn

7. Solve the equation

$\frac{1}{x-1}+\frac{2}{x+1}=\frac{3}{x}$

N2010/2

Sln

$$\frac{1}{x-1}+\frac{2}{x+1}=\frac{3}{x}$$

$$\frac{1(x+1)+2(x-1)}{(x-1)(x+1)}=\frac{3}{x}$$

$$\frac{x+1+2x-2}{x^2-1}=\frac{3}{x}$$

$$\frac{3x-1}{x^2-1}=\frac{3}{x}$$

$$x(x^2-1)\left[\frac{3x-1}{x^2-1}=\frac{3}{x}\right]$$
$$x(3x-1)=3(x^2-1)$$
$$3x^2-x=3x^2-3$$
$$3x^2-3x^2-x=-3$$
$$-x=-3$$
$$x=3$$

Qsn

8. Solve the equation

$\frac{2}{x-1}+1=\frac{2}{3}$

J1996/2

Sln

$$\frac{2}{x-1}+1=\frac{2}{3}$$
$$3(x-1)\left[\frac{2}{x-1}+1=\frac{2}{3}\right]$$
$$6+3(x-1)=2(x-1)$$
$$6+3x-3=2x-2$$
$$3x-2x=-2+3-6$$
$$x=-5$$

Qsn

9. Solve the equation

$3-2w=5(w-2)$

J1997/2

Sln

$$3-2w=5(w-2)$$
$$3-2w=5w-10$$
$$-2w-5w=-10-3$$
$$-7w=-13$$
$$w=1\frac{6}{7}$$

Qsn

10. Solve the equation

$2-\frac{6}{x}=5$

N1997/1

Sln

$$2-\frac{6}{x}=5$$
$$x\left[2-\frac{6}{x}=5\right]$$
$$2x-6=5x$$
$$2x-5x=6$$
$$-3x=6$$
$$x=-2$$

Qsn

11. Solve the equation

$\frac{1}{5}(x-1)=\frac{2}{3}(x-2)$

N1997/2

Sln

$$\frac{1}{5}(x-1)=\frac{2}{3}(x-2)$$
$$15\left[\frac{1}{5}(x-1)=\frac{2}{3}(x-2)\right]$$
$$3(x-1)=10(x-2)$$
$$3x-3=10x-20$$
$$3x-10x=-20+3$$
$$-7x=-17$$
$$x=2\frac{3}{7}$$

Qsn

12. Solve the equation

$\frac{3d}{4}-\frac{d}{3}=2\frac{1}{2}$

J1998/1

Sln

$$\frac{3d}{4}-\frac{d}{3}=2\frac{1}{2}$$

$$\frac{3d}{4}-\frac{d}{3}=\frac{5}{2}$$
$$12\left[\frac{3d}{4}-\frac{d}{3}=\frac{5}{2}\right]$$
$$9d-4d=30$$
$$5d=30$$
$$d=6$$

# QUADRATIC EQUATIONS

Qsn

1. Solve the equation, $x^2 - 7x - 12 = 0$
   Giving your answers correct to 2dp.

N2010/2

Ans

$$x^2 - 7x - 12 = 0$$

$a = 1$
$b = -7$
$c = -12$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-12)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{49 + 48}}{2}$$

$$x = \frac{7 \pm \sqrt{97}}{2}$$

$x = \frac{7+\sqrt{97}}{2} = 8,42$

$x = \frac{7-\sqrt{97}}{2} = -1,42$

Qsn

2. Solve the equation $3x^2 - 5x - 8 = 0$.

N2010/1

Ans

$$3x^2 - 5x - 8 = 0$$

$$-24x^2$$

$$3x^2 - 8x + 3x - 8 = 0$$
$$x(3x - 8) + 1(3x - 8) = 0$$
$$(3x - 8)(x + 1) = 0$$
$$3x - 8 = 0 \ or \ x + 1 = 0$$
$$3x = 8 \quad or \quad x = -1$$
$$\therefore x = 2\frac{1}{3} \quad or \quad x = -1$$

Qsn

3. Solve the equation $x^2 + 5x = 24$.

J2010/2

Ans

$$x^2 + 5x = 24$$
$$x^2 + 5x - 24 = 0$$

$$-24x^2$$

$$x^2 + 8x - 3x - 24 = 0$$
$$x(x + 8) - 3(x + 8) = 0$$
$$(x + 8)(x - 3) = 0$$
$$x + 8 = 0 \ or \ x - 3 = 0$$
$$\therefore x = -8 \quad or \quad x = 3$$

Qsn

4. Solve the equation$x^2 - 2x - 3 = 0$.

J2010/2

Ans

$$x^2 - 2x - 3 = 0$$
$$x^2 - 3x + x - 3 = 0$$
$$x(x - 3) + 1(x - 3) = 0$$
$$(x - 3)(x + 1) = 0$$
$$x - 3 = 0 \ or \ x + 1 = 0$$
$$\therefore x = 3 \ or \ x = -1$$

Qsn

5. Solve the equation $4x^2 - 2x - 3 = 0$, giving your answer to 2 sig-fig.

J2008/2

Ans

$$4x^2 - 2x - 3 = 0$$

$a = 4$
$b = -2$
$c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(-3)}}{2(4)}$$

$$x = \frac{2 \pm \sqrt{4 + 48}}{8}$$

$$x = \frac{2 \pm \sqrt{52}}{8}$$

$x = \frac{2+\sqrt{52}}{8} = 1,2$

$x = \frac{2-\sqrt{52}}{8} = -0,65$

Qsn

6. Solve the equation $3x^2 + 8x - 44 = 0$

Giving your answers correct to 2dp.

N2008/2

Ans

$$3x^2 + 8x - 44 = 0$$

$a = 3$

Simplify

1

3 _ _2

5 C

1

4_ I _3

8 d

1

3_$b = 8$

$c = -44$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(8) \pm \sqrt{8^2 - 4(3)(-44)}}{2(3)}$$

$$x = \frac{-8 \pm \sqrt{64 + 528}}{6}$$

$$x = \frac{-8 \pm \sqrt{592}}{6}$$

$x = \frac{-8+\sqrt{592}}{6} = 2,72$

$x = \frac{-8-\sqrt{592}}{6} = -5,39$

Qsn

7. Solve the equation $(2x - 3)^2 = 9$.

N2008/1

Ans

$$(2x - 3)^2 = 9$$

$$\sqrt{(2x - 3)^2} = \sqrt{9}$$

$$2x - 3 = \pm 3$$

$$2x = \pm 3 + 3$$

$$x = \frac{\pm 3 + 3}{2}$$

$x = \frac{+3+3}{2} = \frac{6}{2} = 3$

or

$x = \frac{-3+3}{2} = \frac{0}{2} = 0$

Qsn

8. Solve the equation $x^2 - 4x + 3 = 0$.

N1997/1

Ans

$$x^2 - 4x + 3 = 0$$

$$x^2 - x - 3x + 3 = 0$$

$$x(x - 1) - 3(x - 1) = 0$$

$$(x - 1)(x - 3) = 0$$

$$x - 1 = 0 \ \ or \ \ x - 3 = 0$$

$$\therefore x = 1 \ \ or \ \ x = 3$$

Qsn

9. Solve the equation $x^2 + 6x - 2 = 0$

Giving your answers correct to 2dp.

J1998/2

Ans

$$x^2 + 6x - 2 = 0$$

$a = 1$

$b = 6$

$c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{36 + 8}}{2}$$

$$x = \frac{-6 \pm \sqrt{44}}{2}$$

$x = \frac{-6+\sqrt{44}}{2} = 0,32$

$x = \frac{-6-\sqrt{44}}{2} = -6,32$

# SIMULTANEOUS EQUATIONS

Qsn

1. Solve the simultaneous equations

$$x + 3y = 3,$$
$$x - 3y = 5$$

J1996/2

Ans [elimination]

$$x + 3y = 3 \text{.....(i)}$$
$$x - 3y = 5 \text{ .....(ii)}$$

$(i) - (ii)$

$$x - x + 3y - -3y = 3 - 5$$
$$6y = -2$$
$$y = -\frac{1}{3}$$

$sub\ y\ to\ (i)$

$$x + 3\left(-\frac{1}{3}\right) = 3$$
$$x - 1 = 3$$
$$x = 4$$

$$\therefore x = 4\ and\ y = -\frac{1}{3}$$

Qsn

2. Solve the simultaneous equations

$$8e + 3f + 1 = 0,$$
$$-5e - 2f + 2 = 0.$$

N1996/2

Ans [elimination]

$$8e + 3f + 1 = 0,$$
$$-5e - 2f + 2 = 0.$$

$$8e + 3f = -1 \text{.....(i)} \times 2$$
$$-5e - 2f = -2 \text{.....(ii)} \times 3$$

$$16e + 6f = -2 \text{ .....(iii)}$$
$$-15e - 6f = -6 \text{ .....(iv)}$$

$(iii) + (iv)$

$$16e + -15e + 6f + -6f = -2 + -6$$
$$e = -8$$

$sub\ e\ to\ (ii)$

$$-5(-8) - 2f = -2$$
$$40 - 2f = -2$$
$$-2f = -42$$
$$f = 21$$

$$\therefore e = -8 \quad and \quad f = 21$$

Qsn

3. Solve the simultaneous equations

$$3a - 2b = 7,$$
$$5a + 4b = 8.$$

J1997/2

Ans [matrix]

$$3a - 2b = 7,$$
$$5a + 4b = 8.$$

$$\begin{pmatrix} 3 & -2 \\ 5 & 4 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 7 \\ 8 \end{pmatrix}$$

$$det = (3 \times 4) - (-2 \times 5)$$
$$= 12 + 10 = 22$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{22}\begin{pmatrix} 4 & 2 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 7 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{22}\begin{pmatrix} 28 + 16 \\ -35 + 24 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{22}\begin{pmatrix} 44 \\ -11 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ -\frac{1}{2} \end{pmatrix}$$

$$\therefore a = 2 \quad and \quad b = -\frac{1}{2}$$

Qsn

4. Solve the simultaneous equations

$$3x + 4y = 5,$$
$$5x + 21 = 4y.$$

J1998/1

Ans [substitution]

$$3x + 4y = 5 \text{ ..... } (i)$$
$$5x + 21 = 4y \text{ ..... } (ii)$$

$$3x = 5 - 4y$$
$$x = \frac{5-4y}{3} \text{ .....(i)}$$

$sub\ (i)\ to\ (ii)$

$$5x + 21 = 4y$$
$$3\left[5\left(\frac{5-4y}{3}\right) + 21 = 4y\right]$$
$$5(5 - 4y) + 63 = 12y$$
$$25 - 20y + 63 = 12y$$
$$-20y - 12y = -25 - 63$$
$$-32y = -88$$

$$y = \frac{11}{4}$$

$$x = \frac{5 - 4\left(\frac{11}{4}\right)}{3}$$

$$x = \frac{5 - 11}{3}$$

$$x = -2$$

$\therefore x = -2 \ and \ y = 2\frac{3}{4}$

Qsn

5. Solve the simultaneous equations

$$y = 1 - 3x,$$
$$2x + y = 5.$$

N1998/1

Ans [substitution]

$$y = 1 - 3x \ldots\ldots\ldots\ldots (i)$$
$$2x + y = 5 \ldots\ldots\ldots\ldots (ii)$$

$sub\ (i)\ to\ (ii)$

$$2x + (1 - 3x) = 5$$
$$2x + 1 - 3x = 5$$
$$2x - 3x = 5 - 1$$
$$-x = 4$$
$$x = -4$$

$$y = 1 - 3(-4)$$
$$y = 1 + 12$$
$$y = 13$$

$\therefore x = -4 \ \ and \ \ y = 13$

Qsn

6. Use matrix method to solve the simultaneous equations

$$3x - y = 2,$$
$$x - 2y = -6$$

J2009/2

Ans [matrix]

$$3x - y = 2,$$
$$x - 2y = -6$$

$$\begin{pmatrix} 3 & -1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -6 \end{pmatrix}$$

$det = (3 \times -2) - (-1 \times 1)$
$= -6 - -1$
$= -5$

$$\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} -2 & 1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ -6 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} -4 - 6 \\ -2 - 18 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} -10 \\ -20 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$\therefore x = 2 \ \ and \ \ y = 4$

Qsn

7. Solve the simultaneous equations

$$3x + 2y = 0.$$
$$2x + y = -1$$

N2009/1

Ans [elimination]

$$3x + 2y = 0 \ldots\ldots\ldots\ldots (i) \times 1$$
$$2x + y = -1 \ldots\ldots\ldots\ldots (ii) \times 2$$

$$3x + 2y = 0 \ldots\ldots\ldots\ldots (iii)$$
$$4x + 2y = -2 \ldots\ldots\ldots\ldots (iv)$$

$(iii) - (iv)$

$$3x - 4x + 2y - 2y = 0 - -2$$
$$-x = 2$$
$$x = -2$$

$sub\ x\ to\ (ii)$

$$2(-2) + y = -1$$
$$-4 + y = -1$$
$$y = -1 + 4$$
$$y = 3$$

$\therefore x = -2 \ \ \ and \ \ \ y = 3$

Qsn

8. Solve the simultaneous equations

$$\frac{1}{2}x + 3y = 4,$$
$$3x + 2y = 8.$$

J2010/1

Ans [elimination]

$$2\left[\frac{1}{2}x + 3y = 4,\right]$$
$$3x + 2y = 8.$$

$$x + 6y = 8 \ldots\ldots\ldots (i) \times 3$$
$$3x + 2y = 8 \ldots\ldots\ldots\ldots (ii) \times 1$$

$$3x + 18y = 24 \ldots\ldots\ldots\ldots\ldots (iii)$$
$$3x + 2y = 8 \ldots\ldots\ldots\ldots (iv)$$
$$(iii) - (iv)$$
$$3x - 3x + 18y - 2y = 24 - 8$$
$$16y = 16$$
$$y = 1$$

Sub y to (i)

$$x + 6(1) = 8$$
$$x = 8 - 6$$
$$x = 2$$

$$\therefore x = 2 \quad and \quad y = 1$$

Qsn

9. Solve the simultaneous equations

$$0,4x + 3y = 2,6,$$
$$x - 2y = 4,6.$$

N2010/1

Ans

$$0,4x + 3y = 2,6 \ldots\ldots\ldots (i) \times 10$$
$$x - 2y = 4,6 \ldots\ldots\ldots (ii) \times 10$$

$$4x + 30y = 26 \ldots\ldots\ldots\ldots\ldots (i)$$
$$\times 5$$
$$10x - 20y = 46 \ldots\ldots\ldots\ldots\ldots (ii)$$
$$\times 2$$

$$20x + 150y = 130 \ldots\ldots\ldots\ldots (iii)$$
$$20x - 40y = 92 \ldots\ldots\ldots\ldots\ldots (iv)$$

$$(iii) - (iv)$$
$$20x - 20x + 150y - -40y = 130 - 92$$
$$190y = 222$$
$$y = \frac{111}{95}$$

Sub y to (ii)

$$10x - 20\left(\frac{111}{95}\right) = 46$$
$$10x = 46 + \frac{444}{19}$$
$$x = \frac{659}{95}$$

$$\therefore x = 6\frac{89}{95} \quad and \quad y = 1\frac{16}{95}$$

Qsn

10. Given that $x - y = 4$ and $x^2 - y^2 = 20$, find the value of $x$ and the value of $y$.

N2008/1

Ans

$$x - y = 4 \ldots\ldots\ldots\ldots (i)$$
$$(x - y)(x + y) = 20 \ldots\ldots\ldots\ldots\ldots (ii)$$

Sub (i) to (ii)

$$4(x + y) = 20$$
$$x + y = 5$$
$$x - y = 4 \ldots\ldots\ldots\ldots (i)$$
$$x + y = 5 \ldots\ldots\ldots (ii)$$
$$(i) + (ii)$$
$$x + x - y + y = 4 + 5$$
$$2x = 9$$
$$x = 4\frac{1}{2}$$

$$sub\ x\ to\ (i)$$
$$4\frac{1}{2} - y = 4$$
$$-y = 4 - 4\frac{1}{2}$$
$$-y = -\frac{1}{2}$$
$$y = \frac{1}{2}$$

$$\therefore x = 4\tfrac{1}{2} \quad and \quad y = \tfrac{1}{2}$$

Qsn

11. Solve the simultaneous equations

$$3x + 2y = -14,$$
$$3x - 5y = 56.$$

N1999/1

Qsn

12. Solve the simultaneous equations

$$2x + 3y + 5 = 0.$$
$$3x - 2y = 12$$

J2000/1

Qsn

13. Solve the simultaneous equations

$$2x - 3y = 13,$$
$$3x + 2y = 0$$

# CHANGE OF SUBJECT

Qsn

1. Given that $z = r\sqrt{n-1}$, make n the subject .

J2008/2

Ans

$$z = r\sqrt{n-1}$$
$$z^2 = \left(r\sqrt{n-1}\right)^2$$
$$z^2 = r^2(n-1)$$
$$\frac{z^2}{r^2} = n-1$$
$$\therefore n = \frac{z^2}{r^2} + 1$$

Qsn

2. Given the formula $A = 2\pi(r^2 - h)$.
   Express r in terms of $\pi, A \text{ and } h$.

J2009/2

Ans

$$A = 2\pi(r^2 - h)$$
$$A = 2\pi r^2 - 2\pi h$$
$$A + 2\pi h = 2\pi r^2$$
$$\frac{A + 2\pi h}{2\pi} = r^2$$
$$\therefore r = \sqrt{\frac{A + 2\pi h}{2\pi}}$$

Qsn

3. The volume, V, of material used to make a cylindrical tube of internal radius r, external radius R and length h is given by the formula
   $$V = \pi(R^2 - r^2)h$$
   Make R the subject.

N2009/2

Ans

$$V = \pi(R^2 - r^2)h$$
$$V = \pi h(R^2 - r^2)$$
$$V = \pi h R^2 - \pi h r^2$$
$$V + \pi h r^2 = \pi h R^2$$
$$\frac{V + \pi h r^2}{\pi h} = R^2$$
$$\therefore R = \sqrt{\frac{V + \pi h r^2}{\pi h}}$$

4. It is given that $t = 2\pi\sqrt{\frac{d}{g}}$.make d the subject.

J2010/1

Ans

$$t = 2\pi\sqrt{\frac{d}{g}}$$
$$t^2 = \left(2\pi\sqrt{\frac{d}{g}}\right)^2$$
$$t^2 = 4\pi^2\left(\frac{d}{g}\right)$$
$$g\left[t^2 = \frac{4\pi^2 d}{g}\right]$$
$$gt^2 = 4\pi^2 d$$
$$\therefore d = \frac{gt^2}{4\pi^2}$$

5. It is given that $T = 2\pi\sqrt{\frac{l}{g}}$. make g the subject.

N2010/1

Ans

$$T = 2\pi\sqrt{\frac{l}{g}}$$
$$T^2 = \left(2\pi\sqrt{\frac{l}{g}}\right)^2$$
$$T^2 = 4\pi^2\left(\frac{l}{g}\right)$$
$$g\left[T^2 = \frac{4\pi^2 l}{g}\right]$$
$$gT^2 = 4\pi^2 l$$
$$\therefore g = \frac{4\pi^2 l}{T^2}$$

6. It is given that $s = ut - \frac{1}{2}gt^2$.
   Make g the subject.

N2010/2

Ans

$$s = ut - \frac{1}{2}gt^2$$
$$2\left[s = ut - \frac{1}{2}gt^2\right]$$
$$2s = 2ut - gt^2$$
$$gt^2 = 2ut - 2s$$
$$\therefore g = \frac{2ut - 2s}{t^2}$$

7. Given that $2p = \sqrt[3]{6q + 5}$.
   express q in terms of p.

J1999/2

Ans

$$2p = \sqrt[3]{6q+5}$$
$$(2p)^3 = \left(\sqrt[3]{6q+5}\right)^3$$
$$8p^3 = 6q + 5$$
$$8p^3 - 5 = 6q$$
$$\therefore q = \frac{8p^3 - 5}{6}$$

Qsn

8. A formula that can be used for changing degrees Fahrenheit(F) to degrees centigrade(C) is

$$C = \frac{5F - 160}{9}$$

Express F in terms of C.

N2001/2

Ans

$$C = \frac{5F - 160}{9}$$
$$9\left[C = \frac{5F - 160}{9}\right]$$
$$9C = 5F - 160$$
$$9C + 160 = 5F$$
$$\therefore F = \frac{9C + 160}{5}$$

Qsn

9. It is given that $M = \frac{5+T}{5-T}$, express T in terms of M.

N1996/1

Ans

$$M = \frac{5 + T}{5 - T}$$

$$5 - T\left[\ M = \frac{5 + T}{5 - T}\right]$$
$$M(5 - T) = 5 + T$$
$$5M - MT = 5 + T$$
$$5M - 5 = T + MT$$
$$5M - 5 = T(1 + M)$$
$$\therefore T = \frac{5M - 5}{1 + M}$$

Qsn

10. It is given that $q = \frac{5+4p}{6-3p}$, express p in terms of q.

N1997/1

Ans

$$q = \frac{5 + 4p}{6 - 3p}$$

$$6 - 3p\left[\ q = \frac{5 + 4p}{6 - 3p}\right]$$
$$q(6 - 3p) = 5 + 4p$$
$$6q - 3pq = 5 + 4p$$
$$6q - 5 = 4p + 3pq$$
$$6q - 5 = p(4 + 3q)$$
$$\therefore p = \frac{6q - 5}{4 + 3q}$$

# FUNCTIONAL NOTATION

Qsn

1. Given that $(x) = ax^2 + 5x - 6$, find the value of $a$ for which $f(-2) = 0$.

N1996/1

Ans

$f(x) = ax^2 + 5x - 6$

$f(-2) = a(-2)^2 + 5(-2) - 6$

$0 = 4a - 10 - 6$

$-4a = -16$

$\therefore a = 4$

Qsn

2. It is given that $f(x) = (x-3)^2 + 5$.
   (a) Find the value of $f(-2)$,
   (b) Solve the equation $f(x) = f(-2)$.

N1997/1

Ans

(a) $f(x) = (x-3)^2 + 5$

$f(-2) = (-2-3)^2 + 5$

$f(-2) = 25 + 5$

$f(-2) = 30$

(b) $f(x) = f(-2)$

$(x-3)^2 + 5 = 30$

$(x-3)^2 = 30 - 5$

$(x-3)^2 = 25$

$\sqrt{(x-3)^2} = \sqrt{25}$

$x - 3 = \pm 5$

$x = \pm 5 + 3$

$x = 5 + 3$ 0r $x = -5 + 3$

$x = 8$ $x = -2$

Qsn

3. Given that $f(x) = 5x^2 - kx + 6$ and $f(-3) = 15$. Find the value of $k$.

J1998/1

Ans

$f(x) = 5x^2 - kx + 6$

$f(-3) = 5(-3)^2 - k(-3) + 6$

$15 = 45 + 3k + 6$

$-3k = 45 + 6 - 15$

$-3k = 36$

$k = -12$

Qsn

4. If $f(x) = x^2 - 7x + 5$, find
   (a) $f(-1)$
   (b) The values of $x$ for which $f(x) = -7$.

N2008/1

Ans

a. $f(x) = x^2 - 7x + 5$

$f(-1) = (-1)^2 - 7(-1) + 5$

$f(-1) = 1 + 7 + 5$

$f(-1) = 13$

b. $f(x) = -7$

$x^2 - 7x + 5 = -7$

$x^2 - 7x + 5 + 7 = 0$

$x^2 - 7x + 12 = 0$

$x^2 - 4x - 3x + 12 = 0$

$x(x-4) - 3(x-4) = 0$

$(x-4)(x-3) = 0$

$x - 4 = 0$ or $x - 3 = 0$

$x = 4$ $x = 3$

Qsn

5. Given that $f(x) = x^2 - 4x + 3$, find all the values of $x$ for which $f(x) = 0$.

N2010/2

Ans

$f(x) = x^2 - 4x + 3$

$f(x) = 0$

$x^2 - 4x + 3 = 0$

$x^2 - 3x - x + 3 = 0$

$x(x-3) - 1(x-3) = 0$

$(x-3)(x-1) = 0$

$x - 3 = 0$ or $x - 1 = 0$

$x = 3$ $x = 1$

Qsn

6. If $f(x) = x^3 - 3x^2 + kx - 4$, find $k$ given that $f(3) = 11$.

N2006/2

Ans

$f(x) = x^3 - 3x^2 + kx - 4$

$f(3) = (3)^3 - 3(3)^2 + k(3) - 4$

$11 = 27 - 27 + 3k - 4$

$11 + 4 = 3k$

$3k = 15$

$k = 5$

Qsn

7. Given that $f(x) = x^2 + x$, find
   i. $f(3)$
   ii. The values of $x$ for which $f(x) = 0$.

J2011/1

Ans

i. $f(x) = x^2 + x$

$f(3) = (3)^2 + (3)$

$f(3) = 9 + 3$

$f(3) = 12$

ii. $f(x) = x^2 + x$

$f(x) = 0$

$x^2 + x = 0$

$x(x + 1) = 0$

$x = 0$ or $x + 1 = 0$

$x = -1$

Qsn

8. Given that $f(x) = 3 - 5x$, find
   (i) $f(4)$
   (ii) $f(7a)$.

N1999/1

Ans

(i) $f(x) = 3 - 5x$

$f(4) = 3 - 5(4)$

$f(4) = 3 - 20$

$f(4) = -17$

(ii) $f(x) = 3 - 5x$

$f(7a) = 3 - 5(7a)$

$f(7a) = 3 - 35a$

Qsn

9. Given that $f(x) = 2x^2 + 7x$, calculate
   (i) $f(3)$
   (ii) The values of $x$ for which $f(x) = 4$.

J2000/2

Ans

(i) $f(x) = 2x^2 + 7x$

$f(3) = 3(3)^2 + 7(3)$

$f(3) = 27 + 21$

$f(3) = 48$

(ii) $f(x) = 2x^2 + 7x$

$f(x) = 4$

$2x^2 + 7x = 4$

$2x^2 + 7x - 4 = 0$

$2x^2 + 8x - x - 4 = 0$

$2x(x + 4) - 1(x + 4) = 0$

$(2x - 1)(x + 4) = 0$

$2x - 1 = 0$ or $x + 4 = 0$

$2x = 1$ $x = -4$

$x = \frac{1}{2}$

Qsn

10. Given that $f(x) = x(x - 6)$ and $(1) = k$, find the value of $k$.

N2001/1

Ans

$f(x) = x(x - 6)$

$f(1) = 1(1 - 6)$

$k = -5$

# INEQUALITIES

Qsn

1. It is given that $-8 \leq m \leq 6$ and $-9 \leq n \leq 5$.Find
   (a) The greatest possible value of
      (i) $mn$
      (ii) $m - n$
   (b) The least value of $m^2 + n^2$.

N1996/1

Ans

(a)

(i) $mn$
$= (-8)(-9)$
$= 72$

(ii) $m - n$
$= 6 - -9$
$= 15$

(b) $m^2 + n^2$
$= 0^2 + 0^2$
$= 0$

Qsn

2. List the integer values of $x$ which satisfy all of the inequalities
   $1 + x \leq 19 - 2x < 12$
   and $11 < 2x + 3 < 19$

J1997/2

Ans

$1 + x \leq 19 - 2x < 12$

| | |
|---|---|
| $1 + x \leq 19 - 2x$ | $19 - 2x < 12$ |
| $x + 2x \leq 19 - 1$ | $-2x < 12 - 19$ |
| $3x \leq 18$ | $-2x < -7$ |
| $x \leq 6$ | $x > 3\frac{1}{2}$ |

$3\frac{1}{2} < x \leq 6$

And

$11 < 2x + 3 < 19$

| | |
|---|---|
| $11 < 2x + 3$ | $2x + 3 < 19$ |
| $11 - 3 < 2x$ | $2x < 19 - 3$ |
| $8 < 2x$ | $2x < 16$ |
| $4 < x$ | $x < 8$ |

$4 < x < 8$

$\therefore x = \{5 \text{ and } 6\}$

Qsn

3. Solve the inequality
   $-2 < 3n + 7 \leq 22$
   Hence write down the natural numbers which satisfy the inequality.

N1997/1

Ans

$-2 < 3n + 7 \leq 22$

| | |
|---|---|
| $-2 < 3n + 7$ | $3n + 7 \leq 22$ |
| $-2 - 7 < 3n$ | $3n \leq 22 - 7$ |
| $-9 < 3n$ | $3n \leq 15$ |
| $-3 < n$ | $n \leq 5$ |

$-3 < n \leq 5$

$natural\ numbers = \{0; 1; 2; 3; 4; 5\}$

Qsn

4. Given that $x$ is an integer and that $-13 < x \leq 35$,
   (a) Write down
      (i) The least value of $x$
      (ii) The greatest value of $x$, such that $x$ is a perfect cube;
   (b) Find two positive values of $x$ which have a remainder of 1 when divided by both 3 and 5.

J1998/1

Ans

(a)

(i) $-12$
(ii) $27$

(b) $16 \text{ and } 31$

Qsn

5. Solve the inequality
   $-21 < 8x - 5 \leq 75$
   and illustrate the solution set on a number line.

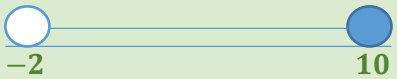
J1998/2

Ans

$-21 < 8x - 5 \leq 75$

| | |
|---|---|
| $-21 < 8x - 5$ | $8x - 5 \leq 75$ |
| $-21 + 5 < 8x$ | $8x \leq 75 + 5$ |
| $-16 < 8x$ | $8x \leq 80$ |
| $-2 < x$ | $x \leq 10$ |

$-2 < x \leq 10$




Qsn

6. Given that $4 \leq x \leq 6$ and $-2 \leq y \leq 7$, calculate
   (a) The greatest possible value of $x - y$
   (b) The least possible value of $x^2 + y^2$.

N1998/1

Ans

(a) $x - y$
$= 6 - -2$
$= 8$

(b) $x^2 + y^2$
$= 4^2 + 0^2$
$= 16$

Qsn

7. Solve the inequality $-7 < 2 - 3x \leq 5$
   Hence illustrate your answer on the number line.

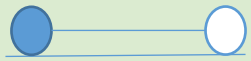
J1999/1

Ans

$$-7 < 2 - 3x \leq 5$$

| | |
|---|---|
| $-7 < 2 - 3x$ | $2 - 3x \leq 5$ |
| $-7 - 2 < -3x$ | $2 - 5 \leq 3x$ |
| $-9 < -3x$ | $-3 \leq 3x$ |
| $3 > x$ | $-1 \leq x$ |

$$-1 \leq x < 3$$




Qsn

8. Given that $-5 \leq x \leq 1$ and $6 \leq y \leq 17$, find
   (a) The greatest value of $y - x$
   (b) The least value of $x^3 - y$.

N2000/1

Ans

(a) $y - x$
$= 17 - -5$
$= 22$

(b) $x^3 - y$
$= 0^3 - 17$
$= -17$

Qsn

9. List the integer values of $x$ that satisfy the inequality

$$14 - x \leq 5x < 27$$

N2000/2

Ans

$$14 - x \leq 5x < 27$$

| | |
|---|---|
| $14 - x \leq 5x$ | $5x < 27$ |
| $-x - 5x \leq -14$ | $x < 5\frac{2}{5}$ |
| $-6x \leq -14$ | |
| $x \geq 2\frac{1}{3}$ | |

$$2\frac{1}{3} \leq x < 5\frac{2}{5}$$

$x = \{3; 4; 5\}$

Qsn

10. Solve the inequality $4 + 3x < 25$.

N2001/1

Ans

$$4 + 3x < 25$$
$$3x < 25 - 4$$
$$3x < 21$$
$$x < 7$$

Qsn

11. Solve the simultaneous inequality
    $2x - 6 < 5x + 3 \leq 3x - 11$
    Giving your answer in the form $a < x \leq b$ where $a$ and $b$ are integers.
    Write down the least possible value of $x$.

N2006/2

Ans

$$2x - 6 < 5x + 3 \leq 3x + 11$$

| | |
|---|---|
| $2x - 6 < 5x + 3$ | $5x + 3 \leq 3x + 11$ |
| $2x - 5x < 3 + 6$ | $5x - 3x \leq 11 - 3$ |
| $-3x < 9$ | $2x \leq 8$ |
| $x > -3$ | $x \leq 4$ |

$$-3 < x \leq 4$$

The least possible value $= -2$

Qsn

12. Solve the inequality $5y - 5 < 10$

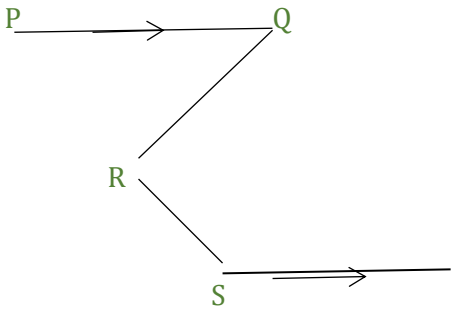
J2009/1

Qsn

13. Solve the inequality
    $8 - 2(2x + 3) \leq 3x$
    Illustrate your solution on a number line .

Qsn

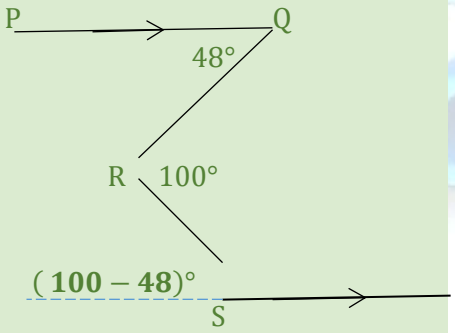
1.




In the diagram, PQ is parallel to ST. $\boldsymbol{P\hat{Q}R = 48°}$ and $\boldsymbol{Q\hat{R}S = 100°}$. Calculate $\boldsymbol{R\hat{S}T}$.

N1996/1

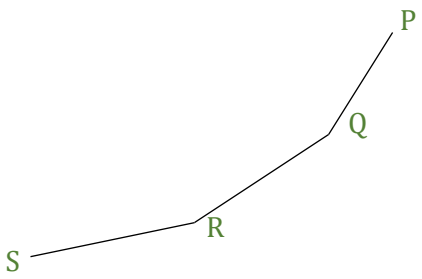
Ans




$$\boldsymbol{R\hat{S}T = 180 - [(100 - 48)°]}$$
$$\boldsymbol{= 180 - 52}$$
$$\boldsymbol{= 128°}$$

Qsn

2.



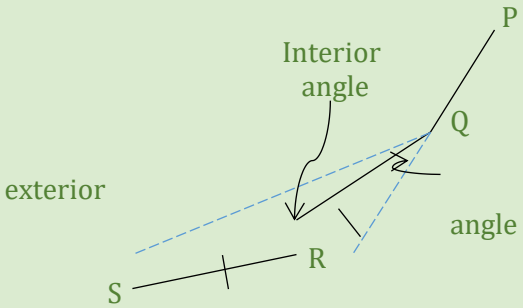

In the diagram, the points P, Q, R and S are consecutive vertices of a regular polygon with fifteen sides.
Calculate

(a) $\boldsymbol{P\hat{Q}R}$

(b) $\boldsymbol{R\hat{Q}S}$

J1997/1

Ans




(a) Exterior angle $= \frac{360}{n}$

$$= \frac{\boldsymbol{360}}{\boldsymbol{15}}$$
$$\boldsymbol{= 24°}$$

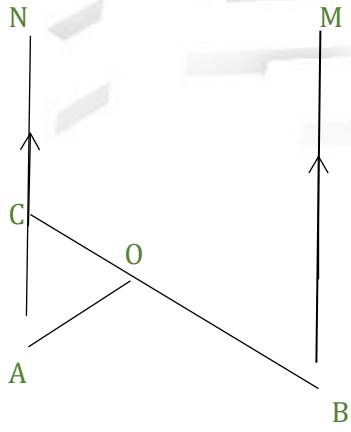
$$\boldsymbol{i + e = 180}$$
$$\boldsymbol{interior = 180 - e}$$
$$\boldsymbol{= 180 - 24}$$
$$\boldsymbol{= 156°}$$
$$\boldsymbol{\therefore P\hat{Q}R = 156°}$$

(b) SQR is isosceles

$$\boldsymbol{\therefore R\hat{Q}S = \frac{(180 - 156)}{2} = 24°}$$

Qsn

3.




In the diagram, AN is parallel to BM. $\boldsymbol{O\hat{A}N = 30°}$ , $\boldsymbol{A\hat{O}B = 95°}$ and $\boldsymbol{O\hat{B}M = x°}$. BO produced meets AN at C.
Calculate

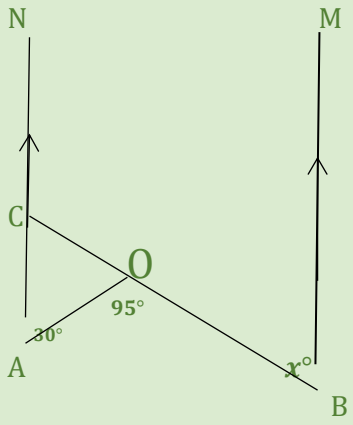
(a) The value of $\boldsymbol{x}$,

(b) $\boldsymbol{N\hat{C}B}$,

(c) Reflex $\boldsymbol{C\acute{A}O}$.

N1997/1

Ans

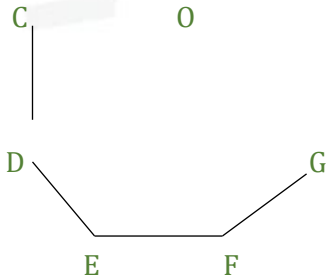



(a) $\boldsymbol{x + 30 = 95}$
$\boldsymbol{x = 95 - 30}$
$\boldsymbol{x = 65}$

(b) $\boldsymbol{N\acute{C}B = 180 - x}$
$\boldsymbol{= 180 - 65}$
$\boldsymbol{= 115}$

(c) $\boldsymbol{reflex\ C\acute{A}O = 360 - 30}$
$\boldsymbol{= 330°}$

Qsn 4.




The diagram is part of a regular decagon $\boldsymbol{ABCDEFGHIJ}$ with center $\boldsymbol{O}$.

(a) State the number of lines of symmetry of the decagon.

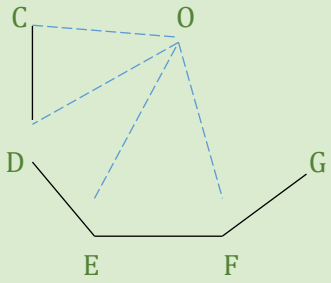
(b) Calculate

(i) The size of the interior angle of the decagon,

(ii) $\boldsymbol{C\acute{O}F}$.

J1998/1

Ans



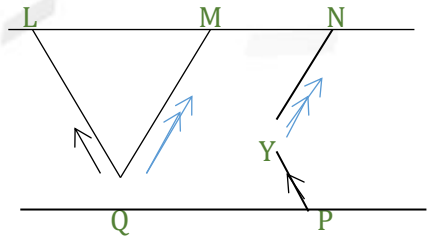

(a) Number of lines of symmetry of a regular polygon is equal to the sides.
$\boldsymbol{\therefore for\ a\ decagon =}$
$\boldsymbol{10\ lines\ of\ symmetry}$

(b)

(i) Exterior angle $= \frac{360}{n}$
$= \frac{360}{10}$
$\boldsymbol{= 36°}$
$\boldsymbol{i + e = 180}$
$\boldsymbol{i = 180 - 36}$
$\boldsymbol{\therefore interior\ angle\ = 144°}$

(ii) Angle at the center of a regular polygon is equal to the exterior angle.
$\boldsymbol{\therefore C\acute{O}F = 36 \times 3}$
$\boldsymbol{= 108°}$

Qsn 5.




In the diagram, LMQ is an isosceles triangle with QL = QM and $\boldsymbol{L\acute{Q}M = 50°}$. The straight line LMN is parallel to QP, YN is parallel to QM and PY is parallel to QL. Calculate

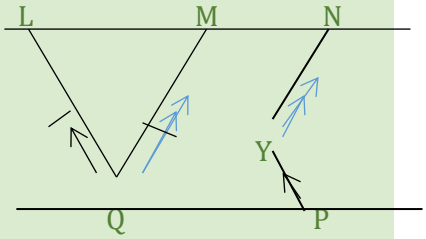
(i) $\boldsymbol{Q\acute{L}M}$

(ii) $\boldsymbol{M\acute{Q}P}$

(iii) $\boldsymbol{P\acute{Y}N}$

J1998/2

Ans



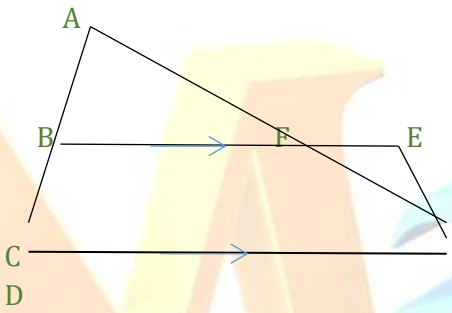

(i) $\boldsymbol{QLM = \frac{(180-50)}{2} = 65°}$

(ii) $\boldsymbol{MQP = 65°}$(alternate angles)

(iii) $\boldsymbol{PÝN = 130°}$(corresponding angles)

Qsn

6.



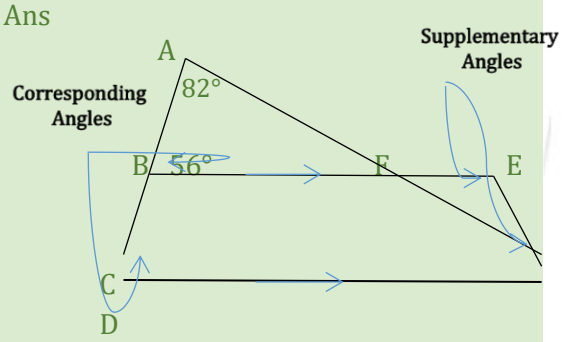

In the diagram, ABC, AFD and BFE are straight lines, BE is parallel to CD, $\boldsymbol{BÁF = 82°}$, $\boldsymbol{AB\acute{F} = 56°}$ and $\boldsymbol{CÐE = 75°}$.

(a) Write down the size of $\boldsymbol{AĆD}$,

(b) Calculate the size of $\boldsymbol{EÐF}$.

N1998/1

Ans




(a) $\boldsymbol{AĆD = 56°}$(corresponding angles)

(b) $\boldsymbol{EÐF = 180 - 75 = 105°}$(supplementary angles)

Qsn

7. Six angles of an octagon are 140° each. The remaining angles are equal. Find the size of each of the remaining angles.

J2008/2

Ans

$$\boldsymbol{an\ octagon = 8\ sides/angles}$$
$$\boldsymbol{sum = (n-2)180}$$
$$\boldsymbol{2x + 140(6) = (n-2)180}$$
$$\boldsymbol{2x = (8-2)180 - 840}$$
$$\boldsymbol{2x = 1080 - 840}$$
$$\boldsymbol{2x = 240}$$
$$\boldsymbol{\therefore the\ size\ of\ the\ remaining\ angles = 240°}$$

Qsn

8. The interior angle of a regular polygon is 162°. Find the number of sides of the polygon.

N2008/1

$$\boldsymbol{i + e = 180}$$
$$\boldsymbol{e = 180 - 162}$$
$$\boldsymbol{extrior\ angle = 18°}$$

And

$$\boldsymbol{exterior\ angle = \frac{360}{n}}$$
$$\boldsymbol{18 = \frac{360}{n}}$$
$$\boldsymbol{n = \frac{360}{18}}$$

$$\boldsymbol{\therefore the\ number\ of\ sides\ of\ the\ polygon = 20}$$

# MATRICES

Qsn

1.

(a) Find the value of $r$ for which the matrix $\begin{pmatrix} r & 24 \\ -4 & 3 \end{pmatrix}$ is singular.

(b) Given that $\begin{pmatrix} 3 & 2 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 16 \\ 7 \end{pmatrix}$, calculate the value of $x$ and $y$.

J1999/2

Ans

(a) $\begin{pmatrix} r & 24 \\ -4 & 3 \end{pmatrix}$ det=0

$$det = (3 \times r) - (24 \times -4)$$
$$0 = 3r + 96$$
$$-3r = 96$$
$$r = -24$$

(b) $\begin{pmatrix} 3 & 2 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 16 \\ 7 \end{pmatrix}$

$$:det=(3 \times -1) - (2 \times 1) = -5$$
$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} -1 & -2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 16 \\ 7 \end{pmatrix}$$
$$= -\frac{1}{5}\begin{pmatrix} -16 - 14 \\ -16 + 21 \end{pmatrix}$$
$$= -\frac{1}{5}\begin{pmatrix} -30 \\ 5 \end{pmatrix}$$
$$= \begin{pmatrix} 6 \\ -1 \end{pmatrix}$$
$$\therefore x = 6 \text{ and } y = -1$$

Qsn

2. The matrix $\begin{pmatrix} y^2 & 2 \\ 18 & 1 \end{pmatrix}$ is singular. Calculate the two possible values of $y$.

N1999/1

Ans

$$\begin{pmatrix} y^2 & 2 \\ 18 & 1 \end{pmatrix} det = 0$$
$$det = (y^2 \times 1) - (18 \times 2)$$
$$0 = y^2 - 36$$
$$y^2 = 36$$
$$y = \pm 6$$

Qsn

3. Given that the matrix $\begin{pmatrix} y+2 & 3 \\ y-1 & 4 \end{pmatrix}$ has determinant 4, find the value of $y$.

J2000/1

Ans

$$\begin{pmatrix} y+2 & 3 \\ y-1 & 4 \end{pmatrix} det = 4$$
$$det = 4(y+2) - 3(y-1)$$
$$4 = 4y + 8 - 3y + 3$$
$$4 - 11 = 4y - 3y$$
$$y = -7$$

Qsn

4. Given that $P = \begin{pmatrix} -4 & -7 \\ 1 & 3 \end{pmatrix}$ and $Q = \begin{pmatrix} 2 & -3 \\ 4 & -1 \end{pmatrix}$, calculate the matrices

(i) P+3Q,

(ii) $Q^2$,

(iii) The inverse of P.

J2000/2

Ans

(i) P+3Q

$$= \begin{pmatrix} -4 & -7 \\ 1 & 3 \end{pmatrix} + 3\begin{pmatrix} 2 & -3 \\ 4 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} -4 & -7 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} 6 & -9 \\ 12 & -3 \end{pmatrix}$$
$$= \begin{pmatrix} 2 & -16 \\ 13 & 0 \end{pmatrix}$$

(ii) $Q^2 = \begin{pmatrix} 2 & -3 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 4 & -1 \end{pmatrix}$

$$= \begin{pmatrix} 4 - 12 & -6 + 3 \\ 8 - 4 & -12 + 1 \end{pmatrix}$$
$$= \begin{pmatrix} -8 & -3 \\ 4 & -11 \end{pmatrix}$$

(iii) $\begin{pmatrix} -4 & -7 \\ 1 & 3 \end{pmatrix}$

$$det = (-4 \times 3) - (-7 \times 1)$$
$$= -12 + 7 = -5$$

Inverse $= -\frac{1}{5}\begin{pmatrix} 3 & 7 \\ -1 & -4 \end{pmatrix}$

Qsn

5.

(a) Evaluate $(4 \quad 2)\begin{pmatrix} 5 \\ -17 \end{pmatrix}$.

(b) Express $3\begin{pmatrix} 2 & 6 \\ -1 & 4 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} -6 & 2 \\ 0 & 8 \end{pmatrix}$ as a single matrix.

N2000/1

Ans

(a) $(4 \quad 2)\begin{pmatrix} 5 \\ -17 \end{pmatrix}$

$$= 20 - 34$$
$$= -14$$

(b) $3\begin{pmatrix}2 & 6\\-1 & 4\end{pmatrix}-\frac{1}{2}\begin{pmatrix}-6 & 2\\0 & 8\end{pmatrix}$

$=\begin{pmatrix}6 & 18\\-3 & 12\end{pmatrix}-\begin{pmatrix}-3 & 1\\0 & 4\end{pmatrix}$

$=\begin{pmatrix}6+3 & 18-1\\-3-0 & 12-4\end{pmatrix}$

$=\begin{pmatrix}9 & 17\\-3 & 8\end{pmatrix}$

Qsn 6.

$$\boldsymbol{U}=\begin{pmatrix}4 & -1\\0 & 2\end{pmatrix},\boldsymbol{V}=\begin{pmatrix}2 & 3\\0 & t\end{pmatrix}$$

(i) Find the inverse of U

(ii) Given that $\boldsymbol{UV}=\boldsymbol{VU}$, calculate the value of t.

N2000/2

Ans

(i) $\begin{pmatrix}4 & -1\\0 & 2\end{pmatrix}$

$$\boldsymbol{det}=(4\times 2)-(-1\times 0)$$
$$=8$$
$$\boldsymbol{inverse}=\frac{1}{8}\begin{pmatrix}2 & 1\\0 & 4\end{pmatrix}$$

(ii) $\boldsymbol{UV}=\boldsymbol{VU}$

$$\begin{pmatrix}4 & -1\\0 & 2\end{pmatrix}\begin{pmatrix}2 & 3\\0 & t\end{pmatrix}=\begin{pmatrix}2 & 3\\0 & t\end{pmatrix}\begin{pmatrix}4 & -1\\0 & 2\end{pmatrix}$$
$$\begin{pmatrix}8 & 12-t\\0 & 2t\end{pmatrix}=\begin{pmatrix}8 & -2+6\\0 & 2t\end{pmatrix}$$
$$\therefore 12-t=4$$
$$t=12-4$$
$$t=8$$

Qsn 7.

$$\boldsymbol{A}=\begin{pmatrix}7 & -2\\4 & -1\end{pmatrix},\boldsymbol{B}=\begin{pmatrix}-1 & 2\\-4 & 7\end{pmatrix}$$
$$\boldsymbol{and}\ \boldsymbol{M}=\begin{pmatrix}-3 & -5\\2 & 6\end{pmatrix}.\ \text{Find}$$

(a) AB

(b) The inverse of M.

J2001/1

Ans

(a) $\boldsymbol{AB}$

$=\begin{pmatrix}7 & -2\\4 & -1\end{pmatrix}\begin{pmatrix}-1 & 2\\-4 & 7\end{pmatrix}$

$=\begin{pmatrix}-7+8 & 14-14\\-4+4 & 8-7\end{pmatrix}$

$=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

(b) $\begin{pmatrix}-3 & -5\\2 & 6\end{pmatrix}$

$\boldsymbol{det}=(-3\times 6)-(-5\times 2)=-8$

$$\boldsymbol{inverse}=-\frac{1}{8}\begin{pmatrix}6 & 5\\-2 & -3\end{pmatrix}$$

Qsn 8.

(a) M is a 2×2 matrix such that $\boldsymbol{M}-2\begin{pmatrix}1 & -1\\0 & 1\end{pmatrix}=3\begin{pmatrix}1 & 1\\-1 & 0\end{pmatrix}$. Find M.

(b) N is a 2×2 matrix such that $\boldsymbol{N}\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}-3\\1\end{pmatrix}$ and $\boldsymbol{N}\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}x\\2\end{pmatrix}$.

(i) Find the matrix N

(ii) Find the value of $\boldsymbol{x}$ if N has no inverse.

J2001/2

Ans

(a) $\boldsymbol{M}-2\begin{pmatrix}1 & -1\\0 & 1\end{pmatrix}=3\begin{pmatrix}1 & 1\\-1 & 0\end{pmatrix}$

$\boldsymbol{M}=\begin{pmatrix}2 & -2\\0 & 2\end{pmatrix}+\begin{pmatrix}3 & 3\\-3 & 0\end{pmatrix}$

$\boldsymbol{M}=\begin{pmatrix}5 & 1\\-3 & 2\end{pmatrix}$

(b)

(i) $\boldsymbol{N}\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}-3\\1\end{pmatrix}$

$\begin{pmatrix}a & b\\c & d\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}-3\\1\end{pmatrix}$

$\begin{pmatrix}a\\c\end{pmatrix}=\begin{pmatrix}-3\\1\end{pmatrix}$

$\boldsymbol{N}\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}x\\2\end{pmatrix}$

$\begin{pmatrix}a & b\\c & d\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}x\\2\end{pmatrix}$

$\begin{pmatrix}b\\d\end{pmatrix}=\begin{pmatrix}x\\2\end{pmatrix}$

$\therefore \boldsymbol{N}=\begin{pmatrix}-3 & x\\1 & 2\end{pmatrix}$

(ii) $\begin{pmatrix}-3 & x\\1 & 2\end{pmatrix}\ \boldsymbol{det}=0$

$\boldsymbol{det}=(-3\times 2)-(x\times 1)$

$0=-6-x$

$x=-6$

Qsn 9.

Given that $\boldsymbol{A}=\begin{pmatrix}3 & 1\\1 & -3\end{pmatrix}$ and $\boldsymbol{B}=\begin{pmatrix}0 & -2\\2 & -6\end{pmatrix}$, calculate

(i) $\boldsymbol{A}+\boldsymbol{B}$

(ii) $\boldsymbol{AB}$

(iii) The inverse of B

(iv) The value of K, given that $\boldsymbol{A^2} = \begin{pmatrix} \mathbf{10} & \mathbf{0} \\ \mathbf{0} & \mathbf{10} \end{pmatrix}$ and $\boldsymbol{A^6} = \boldsymbol{K}\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$.

N2001/2

Ans

(i) $\boldsymbol{A+B}$

$= \begin{pmatrix} 3 & 1 \\ 1 & -3 \end{pmatrix} + \begin{pmatrix} 0 & -2 \\ 2 & -6 \end{pmatrix}$

$= \begin{pmatrix} 3 & -1 \\ 3 & -9 \end{pmatrix}$

(ii) $\boldsymbol{AB}$

$= \begin{pmatrix} 3 & 1 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} 0 & -2 \\ 2 & -6 \end{pmatrix}$

$= \begin{pmatrix} 2 & -12 \\ -6 & 16 \end{pmatrix}$

(iii) $\begin{pmatrix} 0 & -2 \\ 2 & -6 \end{pmatrix}$

$\boldsymbol{det} = (0 \times -6) - (-2 \times 2)$

$= 4$

$\boldsymbol{inverse} = \frac{1}{4}\begin{pmatrix} -6 & 2 \\ -2 & 0 \end{pmatrix}$

(iv) K=1000

Qsn

10. It is given that $\boldsymbol{M} = \begin{pmatrix} 2x & x \\ x & 2x \end{pmatrix}$.

(a) Find $\boldsymbol{M^2}$ in terms of $\boldsymbol{x}$.

(b) Find $\boldsymbol{x}$ given also that $|\boldsymbol{M}| = \mathbf{48}$.

J2008/1

Ans

(a) $\begin{pmatrix} 2x & x \\ x & 2x \end{pmatrix}\begin{pmatrix} 2x & x \\ x & 2x \end{pmatrix}$

$= \begin{pmatrix} 4x^2 + x^2 & 2x^2 + 2x^2 \\ 2x^2 + 2x^2 & x^2 + 4x^2 \end{pmatrix}$

$= \begin{pmatrix} 5x^2 & 4x^2 \\ 4x^2 & 5x^2 \end{pmatrix}$

(b) $|\boldsymbol{M}| = \mathbf{48}$

$\boldsymbol{det} = \mathbf{48}$

$(5x^2 \times 5x^2) - (4x^2 \times 4x^2) = 48$

$25x^4 - 16x^4 = 48$

$9x^4 = 48$

Qsn

11. Given that

$(x \quad 2)\begin{pmatrix} 3 & 1 \\ 0 & y \end{pmatrix} = (15 \quad -7)$

Find the value of

(a) $\boldsymbol{x}$

(b) $\boldsymbol{y}$

J2008/2

Ans

$(x \quad 2)\begin{pmatrix} 3 & 1 \\ 0 & y \end{pmatrix} = (15 \quad -7)$

(a) $3x + 0 = 15$

$3x = 15$

$x = 5$

(b) $x + 2y = -7$

$5 + 2y = -7$

$2y = -7 - 5$

$2y = -12$

$y = -6$

Qsn

12. Given that $\boldsymbol{A} = \begin{pmatrix} 3 & 5 \\ -2 & 7 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 5 & y \\ y & 3 \end{pmatrix}$ find

(i) $\boldsymbol{A^2}$

(ii) The two possible values of $\boldsymbol{y}$ given that the determinant of the matrix B is $\mathbf{5y + 1}$.

N2008/2

Ans

(i) $\begin{pmatrix} 3 & 5 \\ -2 & 7 \end{pmatrix}\begin{pmatrix} 3 & 5 \\ -2 & 7 \end{pmatrix}$

$= \begin{pmatrix} 9 - 10 & 15 + 35 \\ -6 - 14 & -10 + 49 \end{pmatrix}$

$= \begin{pmatrix} -1 & 50 \\ -20 & 39 \end{pmatrix}$

(ii) $\begin{pmatrix} 5 & y \\ y & 3 \end{pmatrix}$ $\boldsymbol{det} = 5y + 1$

$\boldsymbol{det} = (5 \times 3) - (y \times y)$

$5y + 1 = 15 - y^2$

$y^2 + 5y - 14 = 0$

$y^2 + 7y - 2y - 14 = 0$

$y(y + 7) - 2(y + 7) = 0$

$(y + 7)(y - 2) = 0$

$\therefore y = -7 \; or \; y = 2$

Qsn

13.

(a) Write down the 2 by 2 identity matrix for multiplication.

(b) It is given that

$3\begin{pmatrix} p & 10 \\ 8 & 16 \end{pmatrix} - \begin{pmatrix} -4 & 14 \\ 0 & 4 \end{pmatrix} = 2\begin{pmatrix} -10 & 8 \\ 12 & q \end{pmatrix}$.

Find the value of $p$ and the value of $q$.
J2009/1

Qsn
14.
(a) Express as a single matrix
$$\begin{pmatrix} 8 & -4 \\ 5 & 3 \end{pmatrix} - 2\begin{pmatrix} 1 & 3 \\ 4 & 0 \end{pmatrix}$$
(b) If $M = \begin{pmatrix} 3 & -1 \\ 1 & -2 \end{pmatrix}$,
(i) Find the inverse of M.
(ii) Hence solve the following simultaneous equations
$$3x - y = 2$$
$$x - 2y = -6$$

J2009/2

Qsn
15. $X = \begin{pmatrix} 2 & 0 \\ 0 & 6 \end{pmatrix}$ and $Y = \begin{pmatrix} p & q \\ 0 & r \end{pmatrix}$.
Given that $XY = X + Y$, find
(a) p,
(b) q,
(c) r.
N2009/1

Qsn
16.
(a) Given that $A = (2 \quad 3)$ and $B = \begin{pmatrix} 4 & -1 \\ 5 & 6 \end{pmatrix}$,
Find
(i) $AB$
(ii) $B^{-1}$
(b) If $\begin{pmatrix} -2 & p \\ p+3 & -4p \end{pmatrix}$ is singular, find the two possible values of $p$.
N2009/2

Qsn
17. It is given that $D$ is a 2 by 2 matrix such that $D + \begin{pmatrix} -6 & -8 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$
(a) Find D,
(b) Write down the determinant of $\begin{pmatrix} -6 & -8 \\ 3 & 4 \end{pmatrix}$.
J2010/1

Qsn
18. Given that $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $N = \begin{pmatrix} 4 & -2 \\ 3 & 0 \end{pmatrix}$ and $3M + N = M$, find
(i) The matrix M,
(ii) $N^2$.
J2010/2

Qsn
19. M and N are 2 by 2 matrices such that $M = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$ and $MN = \begin{pmatrix} 4 & -2 \\ 0 & 7 \end{pmatrix}$ find
(a) $M^{-1}$.
(b) $N$
N2010/1

Qsn
20. Given that $M = \begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}$, $N = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$ and $R = (3 \quad -1)$, find
(i) MN
(ii) $M^{-1}$.
(iii) $RN$
N2010/2

# VARIATION

Qsn

1. The acceleration $a$ of a particle moving in a circle varies jointly as the square of the velocity $v$ and the inverse of the radius $r$ of the circle.
   (a) Write down an equation that expresses $a$ in terms of $v, r$ and a constant $k$.
   (b) Given that the acceleration of the particle is $20m/s^2$, when the radius is 1,8 m and the velocity is $6m/s$, calculate the value of the constant $k$.
   (c) Calculate the velocity of the particle when the acceleration is $30m/s^2$ and the radius is 2,7m.

N1996/1

Ans

(a) $a \alpha \frac{v^2}{r}$

$$a = \frac{kv^2}{r}$$

(b) $a = \frac{kv^2}{r}$

$1,8\left[20 = \frac{k6^2}{1,8}\right]$

$36 = 36k$

$k = 1$

(c) $a = \frac{v^2}{r}$

$2,7\left[30 = \frac{v^2}{2,7}\right]$

$81 = v^2$

$\therefore v = \pm 9$

Qsn

2. $p$ varies directly as $q$ and inversely as $r$.
   (a) Find the equation for $p$ in terms of $q, r$ and a constant $k$.
   (b) Calculate the value of $k$, given that $p = 2\frac{1}{2}$ when $q = 100$ and $r = 80$.
   (c) Find the value of $r$ when $p = 1\frac{1}{2}$ and $q = 180$.

N1997/1

Ans

(a) $p \alpha \frac{q}{r}$

$$p = \frac{kq}{r}$$

(b) $p = \frac{kq}{r}$

$2\frac{1}{2} = \frac{k100}{80}$

$80\left[\frac{5}{2} = \frac{100k}{80}\right]$

$200 = 100k$

$k = 2$

Qsn

3. Given that $f$ varies inversely as $w$,
   (a) Write down an expression for $f$ in terms of $w$ and a constant $k$.

   Given also that $f = 15$ when $w = 35$,
   (b) Calculate
      (i) The value of the constant $k$,
      (ii) $w$ when $f = 10,5$.

N1998/1

Ans

(a) $f \alpha \frac{1}{w}$

$$f = \frac{k}{w}$$

(b)

(i) $f = \frac{k}{w}$

$15 = \frac{k}{35}$

$k = 525$

(ii) $f = \frac{525}{w}$

$10,5 = \frac{525}{w}$

$w = \frac{525}{10,5}$

$w = 50$

Qsn

4. The strength , S, of a beam of metal varies jointly as the square of its depth, D, and the inverse of its length, L.
   (a) Write down a formula for S in terms of D, L and a constant $k$.
   (b) Given that $S = 18$ when $D = 3$ and $L = 5$, find the value of $k$.
   (c) Calculate the value of D when $S = 20$ and $L = 6$, leaving your answer in surd form.

J1999/1

Ans

(a) $S \alpha \frac{D}{L}$

$$S = \frac{kD}{L}$$

(b) $S = \frac{kD}{L}$

$$18 = \frac{3k}{5}$$

$$k = 30$$

(c) $S = \frac{30D}{L}$

$$20 = \frac{30D}{6}$$

$$D = 4$$

Qsn

5. It is given that $x$ varies directly as $y$ and inversely as the square of $z$.
   (i) Write down an equation that expresses $x$ in terms of $y, z$ and a constant $k$.
   (ii) Calculate the value of the constant $k$ when $x = 15, y = 27$ and $z = 3$.
   (iii) Calculate the value of $z$ when $x = 30$ and $y = 294$.

N1999/2

Ans

(i) $x \alpha \frac{y}{z^2}$

$$x = \frac{ky}{z^2}$$

(ii) $x = \frac{ky}{z^2}$

$$15 = \frac{27k}{3^2}$$

$$k = 5$$

(iii) $x = \frac{5y}{z^2}$

$$30 = \frac{5 \times 294}{z^2}$$

$$z^2 = 49$$

$$z = 7$$

Qsn

6. If $y$ varies directly as the cube of $x$ and $y = 40$ when $x = 2$, find the value of $y$ when $x = 3$.

J2000/2

Ans

$$y \alpha x^3$$
$$y = kx^3$$
$$40 = 2^3k$$
$$k = 5$$

$$y = 5x^3$$
$$y = 5(3^3)$$
$$y = 135$$

Qsn

7. It is given that $y$ is inversely proportional to $(x - 3)$ and that $y = 4$ when $x = 2$, express $y$ in terms of $x$.

N2000/2

Ans

$$y \alpha \frac{1}{x-3}$$
$$y = \frac{k}{x-3}$$
$$4 = \frac{k}{2-3}$$
$$k = -4$$

$\therefore y = \frac{-4}{x-3}$

Qsn

8. $m$ is directly proportional to $n^2$ and $m = 28$ when $n = 2$.
   (a) Write down an expression for $m$ in terms of $n$ and a constant $k$.
   (b) Calculate
      (i) The value of $k$,
      (ii) The values of $n$ when $m = 63$.

J2001/1

(a) $m \alpha n^2$

$$m = kn^2$$

(b)

(i) $m = kn^2$

$$28 = 2^2k$$

$$k = 7$$

(ii) $m = 7n^2$

$$63 = 7n^2$$

$$n^2 = 9$$

$$n = \pm 3$$

Qsn

9. It is given that $y$ varies inversely as $x + 4$.
   (a) Express $y$ in terms of $x$ and a constant $k$.
   (b) Given also that $x = 6$ when $y = 2$, find the value of $k$.
   (c) Find the value of $x$ when $y = 8$.

J2008/1

Ans

(a) $y \alpha \frac{1}{x+4}$

$$y = \frac{k}{x+4}$$

(b) $y = \frac{k}{x+4}$

$2 = \frac{k}{6+4}$

$k = 20$

(c) $y = \frac{20}{x+4}$

$8 = \frac{20}{x+4}$

$8(x+4) = 20$

$8x + 32 = 20$

$8x = -12$

$x = -1\frac{1}{2}$

Qsn

10. It is given that, $y^2 \alpha\, x$ and that $y = 15$ when $x = 25$. Find
    (a) $x$ when $y = 12$,
    (b) $y$ when $x = 4$.

J2009/1

Ans

$y^2 \alpha\, x$

$y^2 = kx$

$15^2 = 25k$

$225 = 25k$

$k = 9$

(a) $y^2 = 9x$

$12^2 = 9x$

$144 = 9x$

$x = 16$

(b) $y^2 = 9x$

$y^2 = 9 \times 4$

$y^2 = 36$

$y = \pm 6$

Qsn

11. A varies directly as the square of $r$.
    (a) Write down an equation connecting A, $r$ and a constant $k$.
    (b) Given that $A = 38\frac{1}{2}$ when $= 3\frac{1}{2}$, find the value of the constant $k$.

N2009/1

Ans

(a) $A \alpha\, r^2$

$A = kr^2$

(b) $A = kr^2$

$38\frac{1}{2} = \left(3\frac{1}{2}\right)^2 k$

$\frac{77}{2} = \frac{49k}{4}$

$k = \frac{22}{7}$

$k = 3\frac{1}{7}$

Qsn

12. Sibongile's weekly wage $W$ (in thousands of dollars), is partly constant and partly varies as the number of hours $N$ of overtime she works per week.
    (i) Express W in terms of N and constants $h$ and $k$.
    (ii) Given that when $W = 80$, $N = 10$ and when $W = 60$, $N = 6$, find the value of $h$ and the value of $k$.
    (iii) Sibongile's normal working time is 44 hours in a week. Find the total number of hours worked in a week in which she was paid \$90 thousand.

N2009/2

Qsn

13. D is proportional to the cube of n.
    (a) Express D in terms of $n$ and a constant $k$.
    (b) Given that $n = 3$ when $D = 21, 6$; find D when $n = 5$.

N2010/1

# VECTORS

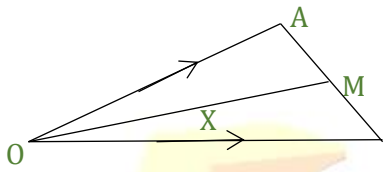
Qsn 1.

(a) Given that $\boldsymbol{c} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and $\boldsymbol{d} = \begin{pmatrix} 3 \\ -7 \end{pmatrix}$

(i) Express $2\boldsymbol{c} - 3\boldsymbol{d}$ as a column vector,

(ii) Find $|\boldsymbol{d}|$.

(b)




B

In the diagram, $\overrightarrow{OA} = \boldsymbol{p}$ and $\overrightarrow{OB} = \boldsymbol{q}$. M is the mid-point of AB and X is the mid-point of OM.
Express as simply as possible in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$

(i) $\overrightarrow{AB}$

(ii) $\overrightarrow{XM}$

J1996/2

Ans

(a)

(i) $2\boldsymbol{c} - 3\boldsymbol{d}$
$= 2\begin{pmatrix} -2 \\ 1 \end{pmatrix} - 3\begin{pmatrix} 3 \\ -7 \end{pmatrix}$
$= \begin{pmatrix} -4 \\ 2 \end{pmatrix} - \begin{pmatrix} 9 \\ -21 \end{pmatrix}$
$= \begin{pmatrix} -13 \\ 23 \end{pmatrix}$

(ii) $|\boldsymbol{d}|$
$= \sqrt{3^2 + (-7)^2}$
$= \sqrt{9 + 49}$
$= \sqrt{58}$
$= 7,62$ 2d.p

(b)

(i) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$
$= -\boldsymbol{p} + \boldsymbol{q}$
$= \boldsymbol{q} - \boldsymbol{p}$

(ii) $\overrightarrow{XM} = \frac{1}{2}\overrightarrow{OM}$
$= \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{AM})$
$= \frac{1}{2}\left(\boldsymbol{p} + \frac{1}{2}(\overrightarrow{AB})\right)$
$= \frac{1}{2}\left(\boldsymbol{p} + \frac{1}{2}(\boldsymbol{q} - \boldsymbol{p})\right)$
$= \frac{1}{2}\left(\boldsymbol{p} + \frac{1}{2}\boldsymbol{q} - \frac{1}{2}\boldsymbol{p}\right)$
$= \frac{1}{4}\boldsymbol{p} + \frac{1}{4}\boldsymbol{q}$

Qsn 2. It is given that $\boldsymbol{a} = \begin{pmatrix} 5 \\ -12 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

(i) Find $|\boldsymbol{a}|$,

(ii) Given that $\boldsymbol{C} = 3\boldsymbol{a} + 2\boldsymbol{b}$, find $\boldsymbol{C}$.

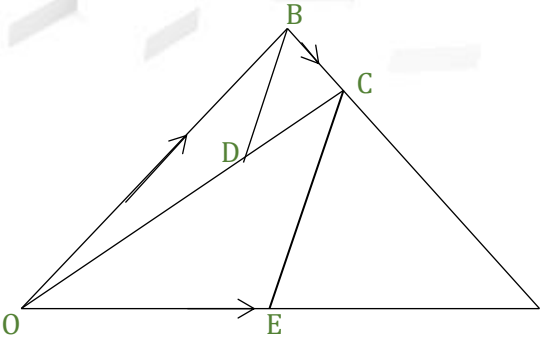
N1996/1

Ans

(i) $|\boldsymbol{a}|$
$= \sqrt{5^2 + (-12)^2}$
$= \sqrt{25 + 144}$
$= \sqrt{169}$
$= 13$

(ii) $\boldsymbol{C} = 3\boldsymbol{a} + 2\boldsymbol{b}$
$= 3\begin{pmatrix} 5 \\ -12 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 3 \end{pmatrix}$
$= \begin{pmatrix} 15 \\ -36 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \end{pmatrix}$
$= \begin{pmatrix} 19 \\ -30 \end{pmatrix}$

Qsn 3.




A

In the diagram, $\overrightarrow{OA} = 5\boldsymbol{a}, \overrightarrow{OB} = 5\boldsymbol{b}, \overrightarrow{BC} = \boldsymbol{a} - \boldsymbol{b}$ and $OD : DC = 3 : 1$.

(a) Express in terms of $\boldsymbol{a}$ and /or $\boldsymbol{b}$, the vectors

i. $\overrightarrow{BA}$

ii. $\overrightarrow{AC}$

iii. $\overrightarrow{OC}$

iv. $\overrightarrow{OD}$

v. $\overrightarrow{BD}$

(b) Given that E is the mid-point of OA find, in terms of $\boldsymbol{a}$ and/ or $\boldsymbol{b}$,

(i) $\overrightarrow{EA}$

(ii) $\overrightarrow{CE}$

(c)

i. What property is satisfied by the vectors $\overrightarrow{CE}$ and $\overrightarrow{BD}$? State the reason.

ii. Find the ratio $\boldsymbol{BD}:\boldsymbol{CE}$ in its simplest form.

N1996/2

Ans

(a)

(i) $\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA}$
$= -5b + 5a$

$= 5a - 5b$

(ii) $\overrightarrow{AC} = \overrightarrow{AB} - \overrightarrow{CB}$
$= -(5a - 5b) - (b - a)$
$= -5a + 5b - b + a$
$= 4b - 4a$

(iii) $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$
$= 5b + a - b$
$= 4b + a$

(iv) $\overrightarrow{OD} = \frac{3}{4}\overrightarrow{OC}$
$= \frac{3}{4}(4b + a)$
$= 3b + \frac{3}{4}a$

(v) $\overrightarrow{BD} = \overrightarrow{BO} + \overrightarrow{OD}$
$= -5b + 3b + \frac{3}{4}a$
$= \frac{3}{4}a - 2b$

(b)

(i) $\overrightarrow{EA} = \frac{1}{2}\overrightarrow{OA}$
$= \frac{1}{2}(5a)$
$= \frac{5}{2}a$

(ii) $\overrightarrow{CE} = \overrightarrow{CA} + \overrightarrow{AE}$
$= 4a - 4b - \frac{5}{2}a$
$= \frac{3}{2}a - 4b$

(c)

i. $\overrightarrow{CE} // \overrightarrow{BD}$
$\overrightarrow{CE} = \frac{1}{2}\overrightarrow{BD}$

ii. $BD:CE = 2:1$

Qsn

4. O is the origin, and A and B are the points (5, 12) and (1, 4) respectively.

(a) Find $\overrightarrow{AB}$ in column vector form,

(b) Given that $\overrightarrow{BP} = \overrightarrow{OA} + 2\overrightarrow{OB}$, find the coordinates of P.

J1997/1

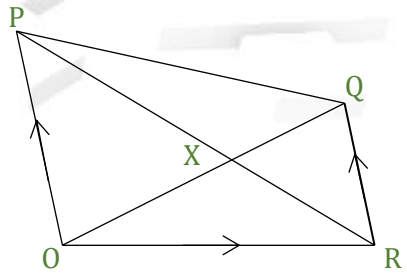
Ans

(a) $\overrightarrow{AB} = b - a$
$= \begin{pmatrix}1\\4\end{pmatrix} - \begin{pmatrix}5\\12\end{pmatrix}$
$= \begin{pmatrix}-4\\-8\end{pmatrix}$

(b) $\overrightarrow{BP} = \overrightarrow{OA} + 2\overrightarrow{OB}$
$= \begin{pmatrix}5\\12\end{pmatrix} + 2\begin{pmatrix}1\\4\end{pmatrix}$
$= \begin{pmatrix}7\\20\end{pmatrix}$

Qsn

5.




In the diagram, $\overrightarrow{OP} = 4a$, $\overrightarrow{OR} = 5b$ and $\overrightarrow{RQ} = 3a$.

(a) Express in terms of a and /or b

(i) $\overrightarrow{OQ}$

(ii) $\overrightarrow{RP}$

(iii) $\overrightarrow{PQ}$

(b) OQ and RP meet at X.
Given that $\overrightarrow{RX} = k\overrightarrow{RP}$, express $\overrightarrow{RX}$ in terms of a,b and k .

Hence show that $\overrightarrow{OX} = 5(1-k)b + 4ka$.

(c) Given also that $\overrightarrow{OX} = h\overrightarrow{OQ}$, express $\overrightarrow{OX}$ in terms of a, b and h.

(d) Using these two expressions for $\overrightarrow{OX}$, find the value of h and k.

(e) Find the numerical value of the ration $\frac{OX}{XQ}$

J1997/2

Ans

(a)

(i) $\overrightarrow{OQ} = \overrightarrow{OR} + \overrightarrow{RQ}$
$= 5b + 3a$

(ii) $\overrightarrow{RP} = \overrightarrow{RO} + \overrightarrow{OP}$
$= -5b + 4a$
$= 4a - 5b$

(iii) $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$
$= -4a + 5b + 3a$
$= 5b - 4a + 3a$
$= 5b - a$

(b) $\overrightarrow{RX} = k\overrightarrow{RP}$
$= k(4a - 5b)$
$= 4ka - 5kb$

$\therefore \overrightarrow{OX} = \overrightarrow{OR} + \overrightarrow{RX}$
$= 5b + 4ka - 5kb$
$= 5b - 5kb + 4ka$
$= 5(1-k)b + 4ka.$

(c) $\overrightarrow{OX} = h\overrightarrow{OQ}$
$= h(5b + 3a)$
$= 5hb + 3ha$

(d) $5(1-k)b + 4ka = 5hb + 3ha$

$5(1-k) = 5h$ .....(i)
$4k = 3h$ .....(ii)

$5h + 5k = 5 \rightarrow h + k = 1$
$\rightarrow h = 1 - k \rightarrow h = 1 - \frac{3}{7} \rightarrow h = \frac{4}{7}$

Sub (i) to (ii)
$4k = 3h$
$4k = 3(1-k)$
$7k = 3$
$k = \frac{3}{7}$

$\therefore h = \frac{4}{7}$ and $k = \frac{3}{7}$

(e) $\overrightarrow{OX} = \frac{4}{7}\overrightarrow{OQ}$

$\frac{OX}{XQ} = \frac{4}{3}$

Qsn

6. If $\overrightarrow{OA} = p + 5q, \overrightarrow{OB} = 7p + 3q$ and $\overrightarrow{AB} = 3hp + (h-k)q$. Find the values of h and k.

N1997/2

Ans

$\overrightarrow{OA} = A$

$\therefore \overrightarrow{AB} = B - A$
$= 7p + 3q - (p + 5q)$
$= 6p - 2q$

$3hp + (h-k)q = 6p - 2q$

$3h = 6$ $\quad$ $h - k = -2$
$h = 2$ $\quad$ $k = 4$

Qsn

7. $X$ and $Y$ are two points with coordinates $(4, 6)$ and $(8, 1)$ respectively.

(i) Find $\overrightarrow{XY}$ as a column vector,

(ii) Find $|\overrightarrow{XY}|$.

J1999/2

Ans

(i) $\overrightarrow{XY} = Y - X$
$= \begin{pmatrix} 8 \\ 1 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \end{pmatrix}$
$= \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

(ii) $|\overrightarrow{XY}| = \sqrt{4^2 + (-5)^2}$
$= \sqrt{16 + 25}$
$= \sqrt{41}$

8. It is given that $\overrightarrow{OP} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \overrightarrow{PQ} = \begin{pmatrix} 12 \\ a \end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix} 5b \\ -b \end{pmatrix}$.

(i) Find the value of $a$ and the value of $b$.

(ii) Calculate $|\overrightarrow{OP}|$.

N1999/2

Ans

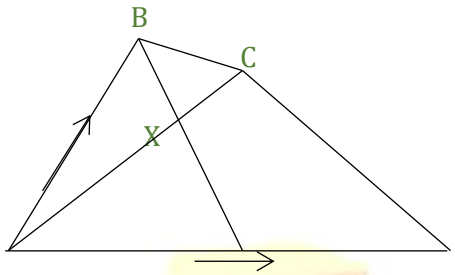
(i) $\overrightarrow{PQ} = Q - P$

$\begin{pmatrix} 12 \\ a \end{pmatrix} = \begin{pmatrix} 5b \\ -b \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$b = 3 \text{ and } a = -7$

(ii) $\left|\overrightarrow{OP}\right| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

9.




O

A

In the diagram, $\overrightarrow{OA} = a, \overrightarrow{OB} = b$.
The point C is such that $AC = 3CB$ and the point D is such that $OD = DA$.

(a) Express, in terms of $a$ and/ or $b$,the vectors

(i) $\overrightarrow{AB}$

(ii) $\overrightarrow{OD}$

(iii) $\overrightarrow{AC}$

(iv) $\overrightarrow{OC}$

(b) OC and BD meet at X. Given that $\overrightarrow{BX} = k\overrightarrow{BD}$, express $\overrightarrow{BX}$ in terms of a, b and $k$. Hence show that $\overrightarrow{OX} = \frac{1}{2}ka + (1 - k)b$.

(c) Given also that $\overrightarrow{OX} = h\overrightarrow{OC}$, express $\overrightarrow{OX}$ in terms of a, b and h.

(d) Using these two expression for $\overrightarrow{OX}$, find the value of $h$ and $k$.

(e) Find the numerical value of the ratio $BX : XD$.

J2000/2

10. P is the point (6; 3) and Q is the point (2; 6).
Calculate

(i) $\overrightarrow{PQ}$

(ii) $\left|\overrightarrow{PQ}\right|$

N2001/2

11. It is given that $\overrightarrow{OP} = 9a + 5b$.

(a) Express $5\overrightarrow{OP}$ in terms of $a$ and $b$.

(b) Given that $\frac{3h}{4}a + (h - k)b = \overrightarrow{OP}$, find the value of $h$ and the value of $k$.

N2001/1

12. It is given that $p = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$ and $q = \begin{pmatrix} -15 \\ y \end{pmatrix}$.

(a) Calculate $|p|$, leaving your answer in surd form.

(b) if $p$ is parallel to $q$, calculate the value of $y$.

J2008/1

13.

(a) It is given that $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} -8 \\ 6 \end{pmatrix}$, find

(i) $\overrightarrow{AC}$

(ii) $\overrightarrow{CX}$, given that $2\overrightarrow{CX} = \overrightarrow{BC}$.

(b) P is the point $(-3; 2)$ and $\overrightarrow{PQ} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$. Find the coordinates of point Q.

N2008/1

14. Given that $\overrightarrow{OA} = \begin{pmatrix} 10 \\ -2 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 1 \\ 10 \end{pmatrix}$ and X is the midpoint of OA,

(a) Express $\overrightarrow{OX}$ as a column vector,

(b) Find $\left|\overrightarrow{AB}\right|$.

N2009/1

# MENSURATION

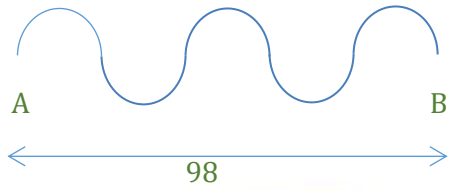
Qsn

1. Express
   (a) $\mathbf{5,3}\boldsymbol{km^2}$ in hectares
   (b) **20** litres in cubic centimeter
   (c) **408** hours in weeks and days.

J2008/1

Qsn

2.



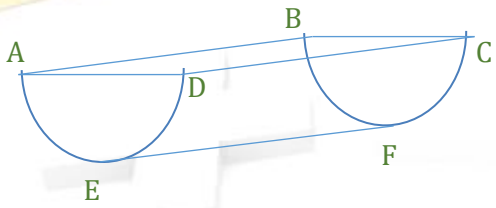

The diagram shows the pattern which is made up of identical semi-circular arcs. The shortest distance between the ends A and B is 98cm.
Calculate
   (a) The diameter of each semi-circle,
   (b) The distance from A to B along the curves.

[take $\boldsymbol{\pi} = \frac{22}{7}$]

J2008/1

Qsn

3.




In this diagram take $\boldsymbol{\pi}$ to be $\frac{22}{7}$.
The diagram ABCDEF represents a metal drinking trough made from a closed cylindrical drum that was bisected lengthwise.
The trough has a diameter of 56 cm and a capacity of 110 litres.

   (i) Calculate
      (a) The area of the cross-section ADE,
      (b) AB.
   (ii) The whole drum was brought for $\mathbf{\$25\ 500}$ and this represents a 70% increase in price of such a drum in the previous year. Calculate the price of such a drum in the previous year.

J2008/2

Qsn

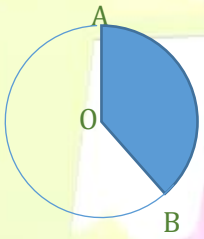
4. A hemispherical bowl is made of wood 2cm thick.
   (a) Given that the bowl has an internal diameter of 14cm, calculate the capacity of the bowl in litres.
   (b) Calculate the mass of the bowl given that the density of the wood is $\mathbf{0,8}\ \boldsymbol{g/cm^3}$.

[volume of a sphere=$\frac{4}{3}\boldsymbol{\pi r^3}$]

J2008/2

Qsn

5.



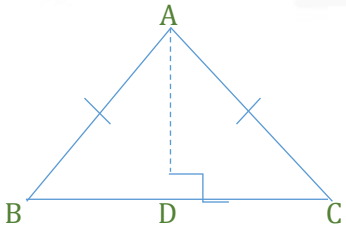

   (a) In the diagram, the shaded sector AOB is $\frac{7}{15}$ of the circle centre O. calculate $\boldsymbol{A\acute{O}B}$.
   (b) Calculate the radius of a circle whose area is $\boldsymbol{154cm^2}$.

[take $\boldsymbol{\pi}$ to be $\frac{22}{7}$]

N2008/1

Qsn

6.

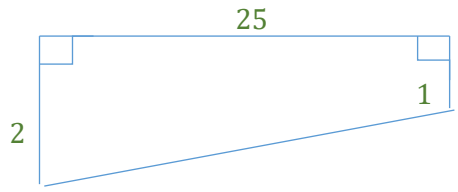



The diagram shows an isosceles triangle ABC with AB=AC, BC=24 cm and AD is perpendicular to BC.
Given that the area of the triangle is 108 $\boldsymbol{cm^2}$, find
   (a) AD
   (b) AC

N2008/1

Qsn

7.




The diagram shows the cross-section of a swimming pool, which is 25m long, 1m deep at the shallow end and 2m deep at the deep end.

(a) Calculate the area of the cross-section in $m^2$.

(b) Given that the swimming pool is 10m wide, calculate the volume of the pool in $m^3$.

N2008/1

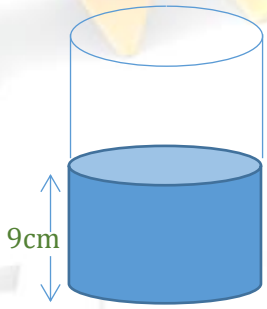
Qsn

8. Find the number of circular rings each of diameter 6,3cm which can be made from a wire 19,8m long.

N2008/2

Qsn

9.

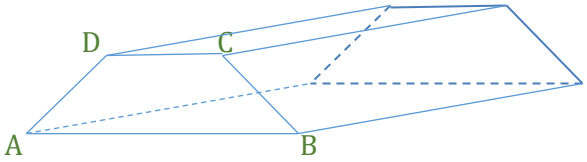



The diagram shows the cylindrical container with water up to the height of 9cm. the volume of the water in the container is $512cm^3$. A metal solid, of volume $217cm^3$, is lowered in to the container until the solid is completely immersed in water. Calculate the height by which the water level rises in the container. Give your answer correct to the nearest millimeter.

N2008/2

Qsn

10. The diagram shows a wooden block of length 150cm, whose cross-section, ABCD is a trapezium in which AB is parallel to DC. AB=65cm, AD=BC=32,5cm and the perpendicular height is 30cm.




(a) Calculate

(i) The length CD given that the area of the trapezium is $1\,575cm^2$

(ii) The volume of the block,

(iii) The mass of the block given that the density of the wood of which it is made is $0,72\ g/cm^3$,

(iv) The total surface area of the block.

(b) The block is to be vanished. One litre of vanish covers an area of $2\,000cm^2$ and is bought in 5-litre tins only. Calculate the number of tins of vanish that need to be bought to varnish the whole block.

N2008/2

Qsn

11. Express 2,6 $m^2$ in $cm^2$.

J2009/1

Qsn

12. A plot whose area is 60 000 $m^2$ is divided in to residential stands each of area 950 $m^2$.

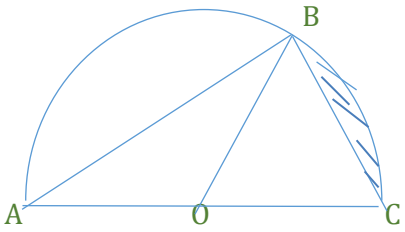
Find

(a) The number of stands obtained.

(b) The area of the plot left over.

Qsn

13. [in this question take $\pi$ to be $\frac{22}{7}$]




In the diagram, ABC is a semi-circle. OA, OB and OC are radii, AC=14 cm and $\boldsymbol{B\acute{A}O = 27°}$.

(a) Find $\boldsymbol{B\acute{O}C}$

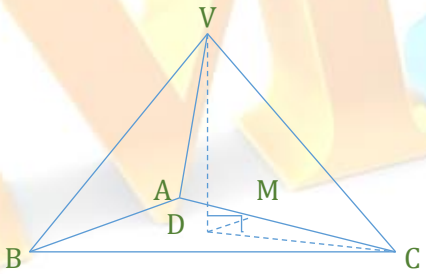
(b) Using as much of the information given below as is necessary, calculate the shaded area.

$\boldsymbol{[\sin 27° = 0,45 \ \cos 27° = 0,89 \ \tan 27 =}$
$\boldsymbol{[\sin 54° = 0,80 \ \cos 54° = 0,59 \ \tan 54° =}$

J2009/1

Qsn

14.




In the diagram, VABC is a right pyramid whose base ABC is an equilateral triangle of sides 8cm. the height of the pyramid is 10cm, D is the centre of the triangle ABC and M is the midpoint of AC.

(a) Show that DM=2,309 cm.

(b) Calculate

(i) VM

(ii) $\boldsymbol{V\acute{M}D}$

(iii) The volume of the pyramid

(iv) The area of one sloping face

$\boldsymbol{\left[volume\ of\ pyramid = \frac{1}{3} base\ area \times height\right]}$

J2009/2

Qsn

15. A bicycle wheel has a diameter of 70cm. taking $\pi$ to be $\frac{22}{7}$, calculate

(a) The circumference of the wheel

(b) The number of the revolutions made by the wheel in travelling 55km.

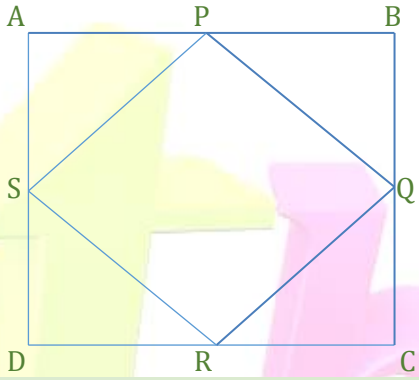
N2009/1

Qsn

16. The diagram shows two squares ABCD and PQRS. Given that AB=12cm, calculate

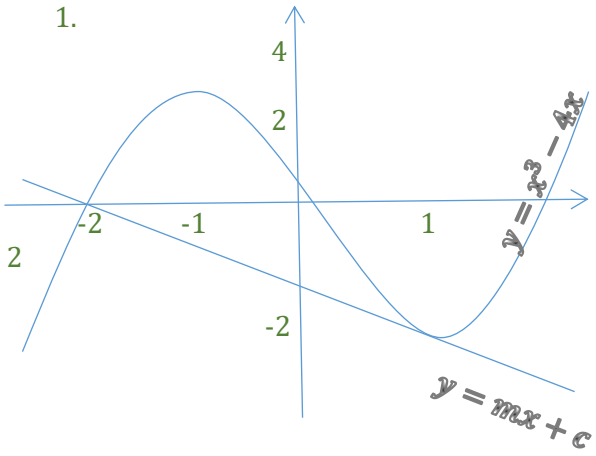
(i) The perimeter of PQRS

(ii) The area of triangle QRS



# FUNCTIONAL GRAPHS

1.




In the diagram the line $y = mx + c$ touches the curve $y = x^3 - 4x$ at (1; -3).
Write down

(a) The value of c
(b) The value of m
(c) The gradient of $y = x^3 - 4x$ when $x = 1$,
(d) The range of values of $x$ for which $mx + c > x^3 - 4x$.

J2008/1

2. Answer the whole of this question on a single sheet of graph paper.

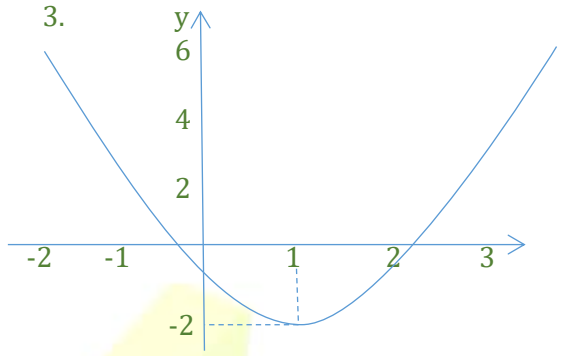
The following is an incomplete table of values for the graph of $= x^2 + \frac{1}{x}$.

| $x$ | 0,25 | 0,5 | 0,8 | 1 | 1,5 | 2 | 2,5 |
|---|---|---|---|---|---|---|---|
| $y$ | 4,1 | 2,3 | 1,9 | p | 2,9 | 4,5 | q |

(a) Calculate the value of $p$ and the value of $q$.
(b) Taking 4cm to represent 1 unit on the x-axis and 2cm to represent 1 unit on the y-axis, draw the graph of $y = x^2 + \frac{1}{x}$ for $0,25 \leq x \leq 3$.
(c) On the same axis draw the graph of $2y = 5x + 2$.
(d)
(i) Write the coordinates of their points of intersection of the graph of $y = x^2 + \frac{1}{x}$ and the graph $2y = 5x + 2$.
(ii) Estimate the area bounded by $y = x^2 + \frac{1}{x}$ and $2y = 5x + 2$ and the lines $x = 1$ and $x = 2$.

J2008/2

3.




The diagram shows the graph of the function $y = x^2 - 2x - 1$. Use the graph to find

(a) The roots of the equation $x^2 - 2x - 1 = 0$,
(b) The minimum value of $x^2 - 2x - 1$,
(c) The equation of the line of symmetry,
(d) The area enclosed by the curve, the x-axis, the y-axis and the line x=2.

N2008/1

4. Answer the whole of this question on a single sheet of graph paper.

A stone is thrown in to the air. Its height $h$ metres after $t$ seconds is given by the formula $h = 60 + 30t - 5t^2$.

Below is the table of values for $h = 60 + 30t - 5t^2$

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $h$ | 60 | 85 | 100 | $p$ | 100 | 85 | 60 | $q$ | -20 |

(a) Find the value of $p$ and the value of $q$.
(b) Using a horizontal scale of 2cm to represent 1 second and a vertical scale of 2cm to represent 20 meters, draw the graph of $h = 60 + 30t - 5t^2$ for $0 \leq t \leq 8$.
(c) Use the graph to find

(i) The maximum height reached by the stone,

(ii) The velocity of the stone when $t = 2$,

(iii) The time when the stone is at a height of 80m.

5. The straight line $y = 2x$ intersects with the curve $y = x^2 - 3$ at two points.
   (a) Form an equation in $x$ and show that it reduces to $x^2 - 2x - 3 = 0$.
   (b) Solve the equation $x^2 - 2x - 3 = 0$.
   (c) Find the coordinates of the two points of intersection.

J2009/1

6. Answer the whole of this question on a single sheet of graph paper.

The following table of values of the function $y = x^2 + 4x - 22$.

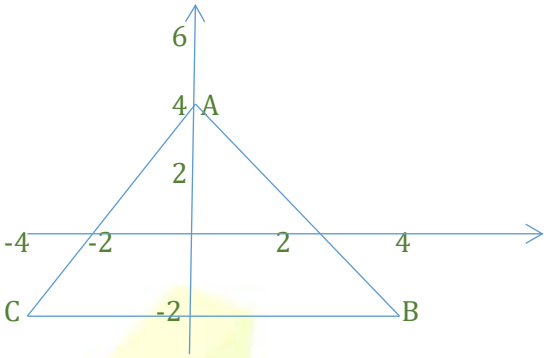
| $x$ | -8 | -7 | -6 | -4 | -2 | 0 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | 10 | -1 | -10 | -22 | -26 | -22 | -10 | -1 | 10 |

   (a) Using a scale of 2cm to represents 2 units on the $x$-axis and 2cm to represent 5 units on the y-axis, draw the graph of $y = x^2 + 4x - 22$ for $-8 \leq x \leq 4$ and $-30 \leq y \leq 15$.
   (b) Use your graph to
      (i) Write down the minimum value of $y$.
      (ii) State the range of values of $x$ for which the function is positive.
      (iii) Find the gradient of the curve at the point where $x = 0$.
   (c) By drawing a suitable straight line, solve the equation $x^2 + 4x - 22 = -x - 8$.

J2009/2

7. In the diagram below, triangle ABC has vertices A(0; 4), B(4; -2) and C(-4; -2).

   (a) Find the equation of the line AB.
   (b) Calculate
      (i) The length of the line AC leaving your answer in surd form,
      (ii) The area of the triangle ABC.




N2009/1

8. Answer the whole of this question on a single sheet of graph paper.

A boy playing on a swing has his velocity $V\ m/s$ at time $t$ seconds given by $V = t^2 - 4t + 4$.

The following is an complete table of values for $V = t^2 - 4t + 4$

| $t$ | 0 | 0,5 | 1 | 1,5 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| $v$ | 4 | $p$ | 1 | 0,25 | 0 | 1 | 4 |

   (a) Find the value of $p$.
   (b) Using a scale of 2cm to represent 0,5 seconds on the horizontal and 2cm to represent 1 m/s on the vertical axis, draw the graph of $V = t^2 - 4t + 4$ for $0 \leq t \leq 4$.
   (c) Use your graph to estimate
      (i) The time when the velocity was 1, 5 m/s.
      (ii) The acceleration at t=3
   (d) Using the same axis, draw the graph of $v = 10t$.
   (e) The function $v = 10t$ represents the velocity of a falling object.
      (i) Use your graph to find the time when the ball and the object had the same speed.
      (ii) Given that the boy and the object collided at the same time they had the same speed, use your graph to find the distance the object and travelled.

N2009/2

9. Answer the whole of this question on a single sheet of graph paper.
   (a) The following is an incomplete table of values for the function $\boldsymbol{y = \frac{3}{x+2}}$

| $x$ | -6 | -5 | -4 | -3 | -2,5 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | $-\frac{3}{4}$ | -1 | $p$ | -3 | -6 | 3 | $1\frac{1}{2}$ | 1 | $\frac{3}{4}$ |

   (b) On the same axes draw the graph of the function $\boldsymbol{y = 2x + 3}$ to intersect with the graph of $= \frac{3}{x+2}$.
   (c) Write down, in the form $\boldsymbol{ax^2 + bx + c = 0}$(where a, b and c are constants), the equation whose roots are the $\boldsymbol{x}$ coordinates of the points of intersection of the two graphs.
   (d) By drawing a suitable tangent, find the gradient of the graph $\boldsymbol{y = \frac{3}{x+2}}$ at the point (1; 1).

J2010/2

10.
   (a) Find the gradient of the line $\boldsymbol{l}$ which passes through (6; 8) and (0; 5).
   (b) Find the equation of the line which is parallel to line $\boldsymbol{l}$ in(a) and passes through (4; -5).

# STATISTICS

Finding the mean, the median and the mode.

Data representation

- Bar chart
- Pie chart
- Histogram
- Frequency polygon
- Cumulative frequency curve

**The syllabus requires you to be able to;**

- *collect, classify and tabulate statistical data;*
- *read, interpret, draw and make simple inferences from bar charts, pie charts, histograms and frequency tables/charts and frequency polygons;*
- *calculate the mean, mode, median from given data and distinguish between the purposes for which they are used;*
- *use an assumed mean where appropriate;*
- *read and interpret data presented in classes and determine the modal class;*
- *draw and use a cumulative frequency curve/orgive;*

Statistical data may be collected in places like schools, spots, examination and so on.

The **mean** of the data collected will be the average, calculated by finding the sum of the data dividing it by the number of data;

$$\boldsymbol{mean} = \frac{\sum \boldsymbol{x}}{\boldsymbol{n}} \quad \boldsymbol{or} \quad \boldsymbol{mean} = \frac{\sum \boldsymbol{fx}}{\sum \boldsymbol{x}}$$

Where $\sum$ means (summation) the sum of all the products of the frequency ($f$) and grade ($x$).

The **mode** is the grade / number with the highest frequency.

The number of times any particular number occurs is called the frequency.

E.g. for the #s 1 2 3 4 2 3 1 2 3 2. The mode is 2, because it has appeared most.

Or given a table showing the distribution of the data as;

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $f$ | 2 | 6 | 4 | 1 | 6 | 7 | 9 | 0 | 1 |

The modal class ($x$) is 6 because it has the highest frequency ($f$)

The **median** is the value or grade at the middle of the numbers;

By arranged in ascending order

1 2 3 4 2 3 1 2 3 3

$= 1\ 1\ 2\ 2\ 2\ 3\ 3\ 3\ 3\ 4$ *or* $1^2 2^3 3^4 4$

$= 1\ 2\ 2\ 2\ 3\ 3\ 3\ 3$ *or* $1^1 2^3 3^4$

$= 2\ 2\ 2\ 3\ 3\ 3$ *or* $2^3 3^3$

$= 2\ 2\ 3\ 3$ *or* $2^1 3^1$

$= 2\ 3$

$3 + 2 = \frac{5}{2} = 2{,}5$

By using the formula

$$Q_2 = \frac{1}{2}(n+1)^{th}\ term$$

Try these;

***On a farm the numbers of workers picking fruits on eleven days last march were 3, 8, 9, 12, 15, 12, 13, 10, 8, 4.find***

***(a) The median***

***(b) The mode of this distribution***

***(c) The mean***

### *Bar chart*

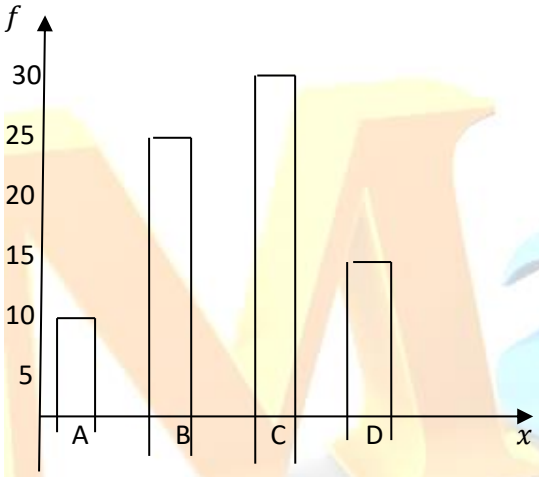
- A bar chart is statistical graph in which bars such that their lengths or heights are proportional to the quantity they represent.

- A bar will be showing the frequency of the grade.
- The bar will be vertically up right if the frequency is at the vertical axis.
- If the bar is horizontal, the frequency will also be at the horizontal axis.

For a given table as;

| Grade($x$) | A | B | C | D |
|---|---|---|---|---|
| Frequency($f$) | 10 | 25 | 30 | 15 |

The graph will be drawn as shown below




The bar's length is the one showing the frequency of the grade. Thus the grade with the greatest length will be the modal grade.

Try this;

***The table below shows the distribution of marks in a test.***

| Marks | 40 | 41 | 42 | 43 | 44 | 45 | 46 |
|---|---|---|---|---|---|---|---|
| frequency | 7 | 4 | 6 | 2 | 4 | 2 | 6 |

***a) Draw a bar chat to show this distribution.***

***b) How many people took the test?***

***c) Find the median mark.***

### *Pie chart*

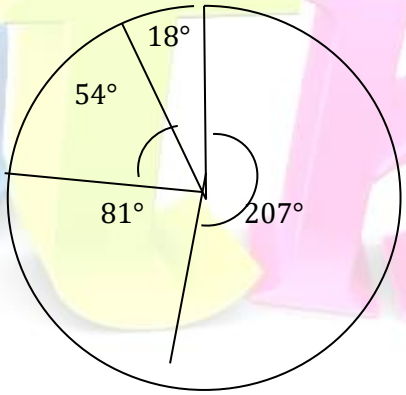
Is another way of representing data using sectors of a circle.

The angles of these sectors are found by finding the ratio of the frequency by 360°

*A communal farmer harvested 23 bags of maize, 9 bags of rapoko, 6 bags of ground nuts and two bags of other crops.*

*a) Draw a pie-chart to represent the farmer's yield.*

| ***No .of bags*** | ***Angle of sector*** |
|---|---|
| *23* | $\frac{23}{40} \times 360° = 207°$ |
| *9* | $\frac{9}{40} \times 360° = 81°$ |
| *6* | $\frac{6}{40} \times 360° = 54°$ |
| *2* | $\frac{2}{40} \times 360° = 18°$ |
| *40* | $360°$ |




Try this;

***The shoe sizes of a group of 30 students are;***

***6, 7, 6, 5, 4, 6, 7, 8, 7, 5, 5, 6, 7, 7, 6, 5, 4, 6, 6, 8, 7, 6, 6, 5, 6, 8, 6, 7, 6, 7.***

***For the data above, draw a pie chart to represent the given data.***

### *Histogram*

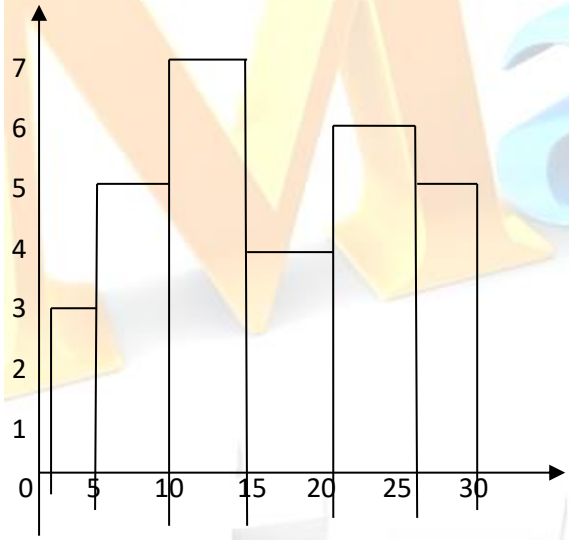
When a statistical data contain a large number of values, it is impractical to draw a bar chart and often difficult to calculate averages.

To overcome this problem, the data can be reduced to a **frequency distribution**.

A frequency distribution is a table in which the given values are divided in to **class intervals**.

A frequency distribution can be represented by a block graph called a **histogram.**

| Class interval | Frequency |
|---|---|
| $1-5$ | 3 |
| $5-10$ | 5 |
| $10-15$ | 7 |
| $15-20$ | 4 |
| $21-25$ | 6 |
| $26-30$ | 5 |




### *Frequency polygon*

The frequency polygon is obtained by considering the class center (mid-points of class intervals). Thus the mid-point of the graph are joined together.

### *Cumulative*

The cumulative frequency denotes the number of candidates who obtained a score equal to or less than the upper boundary of a given class.

The median score is the score obtained by the student who is in the middle after ranking the scores.

The median position is given by; $\frac{n+1}{2}$ where $n$ is the total number of scores. $(Q_2)$

The 1st quartile is that score which is such that $\frac{1}{4}$ of the candidates obtain that score or less. Similar the 3rd quartile is that score which is such that $\frac{3}{4}$ of the candidates obtain that score or less.

$Q_1 = \left(\frac{n+1}{4}\right)^{th}$ Term while $Q_3 = \left(\frac{3(n+1)}{4}\right)^{th}$term

# TRANSFORMATION

**The syllabus requires you to be able to;**

- *translate (T) simple plane figures;*
- *reflect(M) simple plane figures in the axes and in any line;*
- *rotate (R) about any point clockwise or anti-clockwise through 90° and 180°,*
- *enlarge(s) about any point using a rational scale factor;*
- *stretch (S); both one way and two way stretch using the axes as the invariant stretch lines and rational stretch factor,*
- *shear (H), using the axes as the invariant lines and rational shear factor.*
- *apply combinations of the above (e.g. if M(a)=b and R(b)=c then RM(a)=c);*
- *describe transformations fully;*
- *identify interpret and/or use matrices which represent the above transformations,*
- *describe transformations using coordinates and matrices (singular matrices are excluded);*

- A transformation is a change in position or shape or measurements (or both)
- The transformations are translation, **reflection, rotation, enlargement, shear** and **stretch**.

## *Translation*

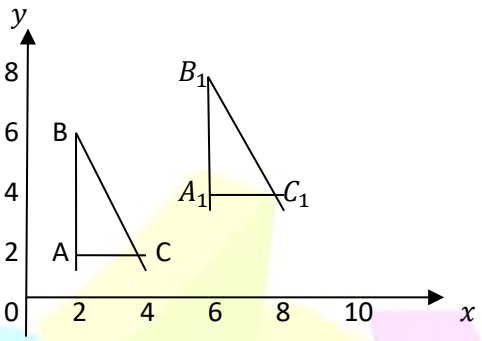
It is an asymmetrical transformation.

A translation vector (TV) is used to change the position of the shape but not changing the direction, size and shape.

$$original + translation\ vector = image$$

$$\boldsymbol{OR + TV = IM}$$

| Change | No change |
|---|---|
| position | Shape |
| | Size |
| | Direction |




Given that the translation vector is $\begin{pmatrix}4\\2\end{pmatrix}$

OR + TV = IM

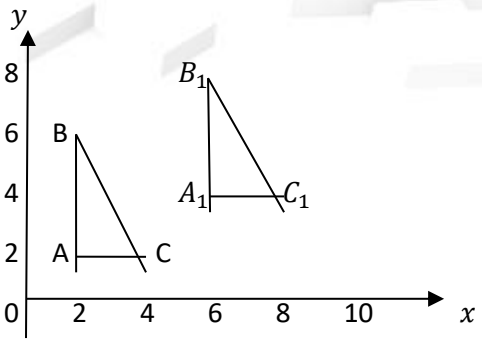
$A + TV = A_1$ ;$B + TV = B_1$

$\begin{pmatrix}2\\2\end{pmatrix} + \begin{pmatrix}4\\2\end{pmatrix} = \begin{pmatrix}6\\4\end{pmatrix}$ $\begin{pmatrix}2\\6\end{pmatrix} + \begin{pmatrix}4\\2\end{pmatrix} = \begin{pmatrix}6\\8\end{pmatrix}$

$C + TV = C_1$

$$\begin{pmatrix}4\\2\end{pmatrix} + \begin{pmatrix}4\\2\end{pmatrix} = \begin{pmatrix}8\\4\end{pmatrix}$$

If you are to describe the transformation taking place;




- First you identify what has been transformed, using the table of *change and no change.*
- If it is translation, position is the one that changes.

Description;

- *Translation*
- *Translation vector*

$$= image - \ original$$

$$= C_1 - C$$

$$= \begin{pmatrix} 8 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

***Try this; using a scale of 2cm to 1 unit on each axis and taking values of $x$ and $y$ from $-4$ to $5$ inclusive, draw triangle ABC with A(1; 2), B(2; 3) and C(3; 1). The matrix $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$ maps $\Delta ABC$ onto$\Delta A_1B_1C_1$. Draw and clearly label the vertices of this triangle.***

## Reflection

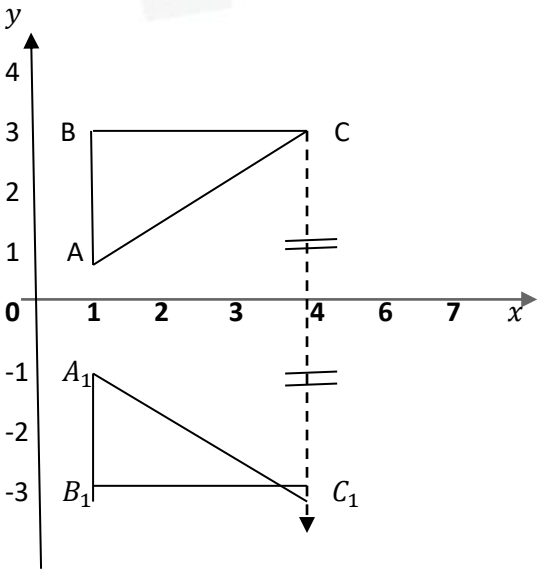
It is an asymmetrical transformation.

Shapes are being reflected using a mirror;

Mirrors:$x - axis$

$:y - axis$ *and lines with gradient*

*$x - axis$ as mirror*

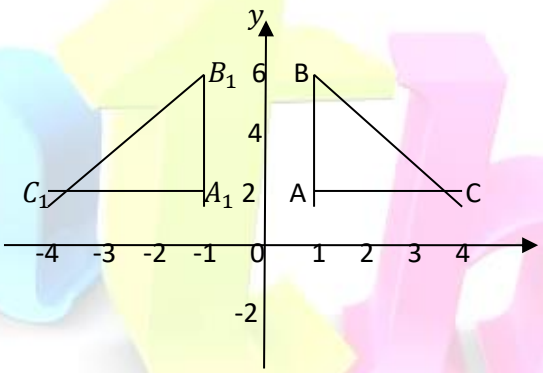



*When reflecting points*; the distance from the original point to the mirror=distance from the mirror line to the image point.

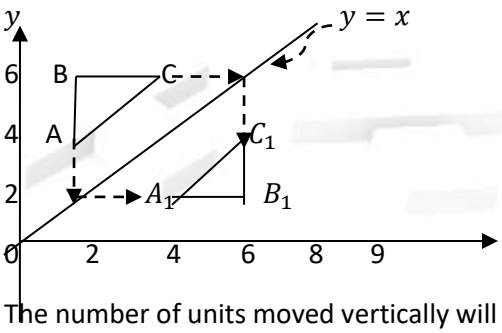
For the x-axis we use the vertical to vertical distances, being assisted by the scale of the graph.

The distance will be the number of units to the mirror line.

*$y - axis$ as mirror*




Lines with gradient




The number of units moved vertically will be equal to the number of units moved horizontally.

| Change | No change |
|---|---|
| Position | Size |
| direction | Shape |

These are the same with the changes when rotating. If it is reflection, the locus of points equidistant between the original and the image, will be the mirror line.

| Reflection in the | Matrix |
|---|---|

| | |
|---|---|
| $x-axis$ | $\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$ |
| $y-axis$ | $\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$ |
| Line $y=x$ | $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ |
| Line $y=-x$ | $\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}$ |

*Description*; reflection

; Equation of mirror line

## Rotation

It is an asymmetrical transformation.

- **For rotation about the origin only, matrices will be used;**

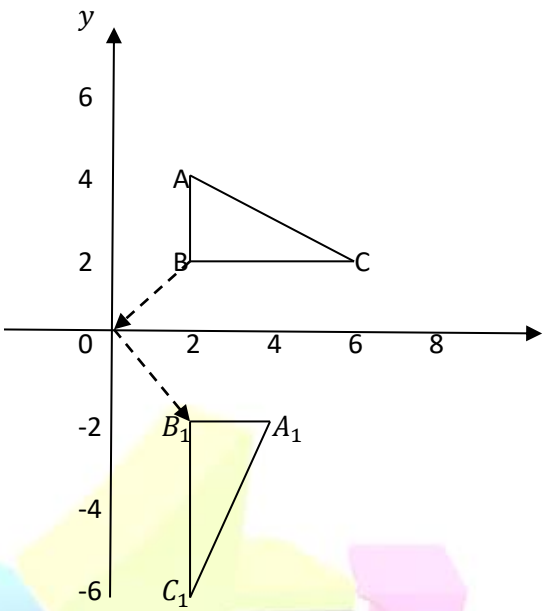
| Angle | matrix |
|---|---|
| 90° | $\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$ |
| 180° | $\begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}$ |
| 270° | $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$ |

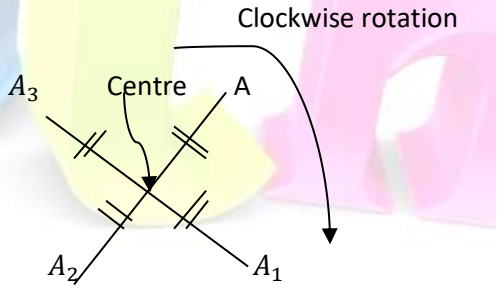
- For rotation about a certain point, compasses will be used to rotate.

Given the points of a triangle with coordinates A (2; 4), B (2; 2) and C (6; 2). If it is to be rotated 90° about the origin. The matrix will be used to multiply the coordinates of A, B and C.

$$\begin{matrix}&&A&B&C\\\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}&\begin{pmatrix}2 & 2 & 6\\4 & 2 & 2\end{pmatrix}\end{matrix}$$

$$\begin{matrix}&A_1&B_1&C_1\\=\begin{pmatrix}4 & 2 & 2\\-2 & -2 & -6\end{pmatrix}\end{matrix}$$




The distance from the centre of rotation to the points will be equal.

A compass is used to rotate a certain point by drawing an arc joining the two points.

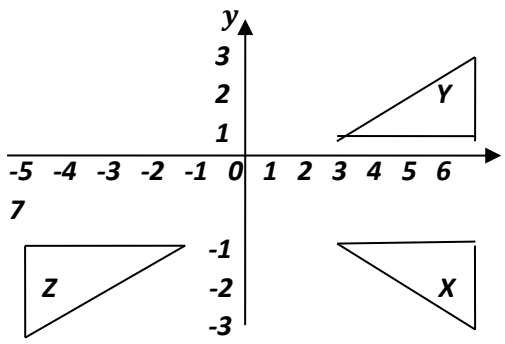
**Description**; *rotation*

*; Center of rotation*

*; Angle of rotation*

*Try this;*

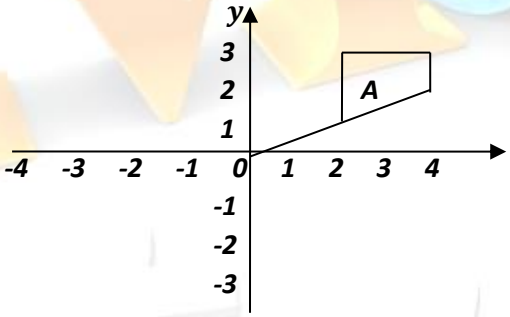
1.




a) ***Describe completely a single transformation which maps $\Delta Y$ on to$\Delta X$.***

b) ***Describe completely a single transformation which maps $\Delta Y$ onto $\Delta Z$.***

2.



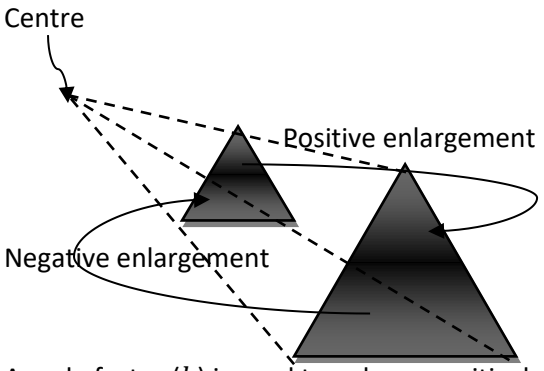

a) ***B, the image of A, under a reflection in the $x-axis$.***

b) ***C, the image of A, under an anti-clockwise rotation of 90° about O.***

## Enlargement

It is not an asymmetrical transformation.

There is positive and negative enlargement




A scale factor ($k$) is used to enlarge positively ($k > 1$) or negatively ($0 < k < 1$).

If the object is enlarged at the same side of the centre as given above the scale factor will be positive ($+k$).

If it is enlarged at opposite sides, the scale factor will be negative ($-k$).

$$Scale\ factor = \frac{image\ length}{original\ length}$$

**For origin as centre only;**

The matrix $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ can only be used. Where $k$ is a scale factor.

| change | No change |
|---|---|
| Size | Shape |
| Position | Direction(+k) |
| Direction(+k) | |

Description; *(positive or negative) Enlargement*

*; Scale factor*

*; Center of enlargement*

***Try these;***

1. ***Using a scale of 1cm to represent 1 unit on each axis, draw x- and y-axis for $-8 \leq x \leq 10$ and$-4 \leq y \leq 18$. Draw and label the triangle whose are A (1; 4), B (2; 4) and C (2; 1). The enlargement E has centre the origin and maps $\Delta ABC$ on to$\Delta A_1B_1C_1$. Given that $A_1$is the point (4; 16).***

***i)*** ***Draw and label the triangle*** $\Delta A_1B_1C_1$

***ii)*** ***Write down the scale factor of E***

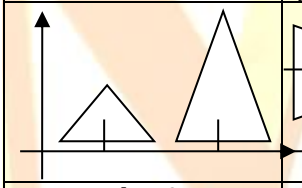
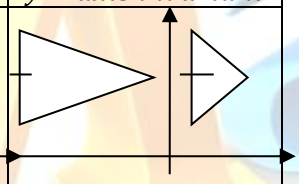
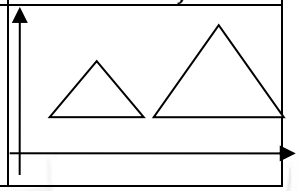
**2.** ***The coordinates of the vertices of a triangle are A(1; 2), B(-1; 1) and C(1;*** $-\frac{1}{2}$***)***

***If the triangle is enlarged with a scale factor 4 from the origin (0; 0) what are the coordinates of the image.***

## Stretch

There is one -way and two- way stretch;

For one- way there are invariant lines and for two- way there are invariant lines.

| $x-axis\ invariant$ | $y-axis\ invariant$ |
|---|---|
|  |  |
| Matrix $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$ $k-stretch\ factor$ | Matrix $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$ $k-stretch\ factor$ |
| $two-way\ stretch$ Matrix $\begin{pmatrix} k & 0 \\ 0 & h \end{pmatrix}$ |  |

$$stretch\ factor = \frac{image\ length}{original\ length}$$

| Change | No change |
|---|---|
| Size | Shape |
| Direction(-k) | Direction (+k) |
| Position | |

Description; two-way/one-way stretch

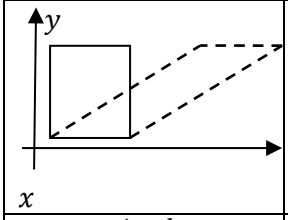
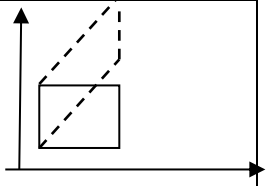
; Invariant line for one-way stretch

; Stretch factor/s

*; Matrix*

***Try this; draw*** $\Delta ABC$ ***with A (1; 1), B (2; 3) and C (3; 2). The transformation S is a one way stretch parallel to the y-axis with a scale factor 2. Draw*** $A_1B_1C_1$ ***which has the image of*** $\Delta ABC$***.***

## Shear

| $x-axis$ invarient | $y-axis$ invarient |
|---|---|
|  |  |
| Matrix $\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$ | matrix $\begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$ |
| $k-shear\ factor$ If the shear is parallel to the $x-axis$ in that direction the shear factor will be positive. If it is at the opposite direction the shear factor will be negative. | If the shear is parallel to the $y-axis$ in that direction the shear factor will be positive. If it is at the opposite direction the shear factor will be negative |

Try these;

1.

***i)*** ***On a graph paper draw*** $x-$ ***and*** $y-axis$ ***such that*** $-4 \leq x \leq 7$ ***and*** $-2 \leq y \leq 6$***.***

***ii)*** ***Plot the parallelogram EFGH whose vertices are at (0; 1), (-2; 5), (-4; 5) and (-2; 1).***

***iii)*** ***Plot the parallelogram KLMN whose vertices are at (3; 1), (7; 5), (5; 5) and (1; 1).***

***iv)*** ***Describe fully the single transformation which maps EFGH on to KLMN.***

# Travel graph

Kinematics consists of calculations in;

- Distance, speed and time
- Acceleration and retardation
- Area under the graph, for distance

Drawing graphs of;

- Distance-time
- Velocity-time

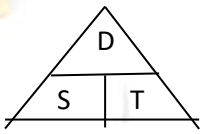
Conversions;

- Time
- Distance

**The syllabus requires you to be able to;**

- *Draw and interpret displacement-time and velocity-time graphs and solve problems involving acceleration, velocity and distance.*

- For calculations in; Distance, speed and time the use of the following would be necessary

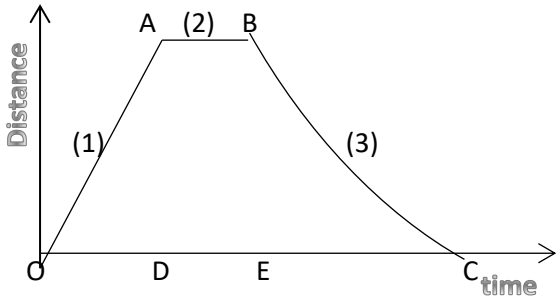



Speed =D/t

Time =D/s

Distance =s × t

Distance-time graphs




1. During the time interval OD the distance is changing uniformly with time.
   Now,
   $\frac{AD}{OD} = \frac{distance}{time} = gradient\ of\ OA.$
   However,
   $$\frac{distance}{time} = speed$$
   Thus the gradient is a measure of the rate of change of distance with time.
2. Over the interval DE there is zero distance. Hence DE gives the time of rest.
3. During the interval EC the distance is negative since the object is travelling in a direction opposite to the original direction.
   The rate of change of distance is non-uniform. However, the gradient of the tangent at a given point gives the speed of the object at that particular time.

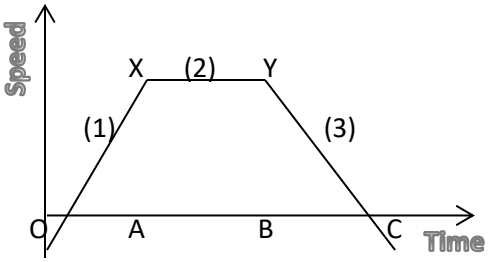
Try this

***Tendai left home at 0700 hours to visit his uncle, who stays 15 km away. He was running at an average speed of 12 km/h. he spent two hours at his uncle's place before deciding to walk back home. He walked at an average speed of 7 km/h. use a graph to find***

***i) The time tendai arrived home***

***ii) The time at which he was 6km away from home on the return journey.***

Speed-time graph

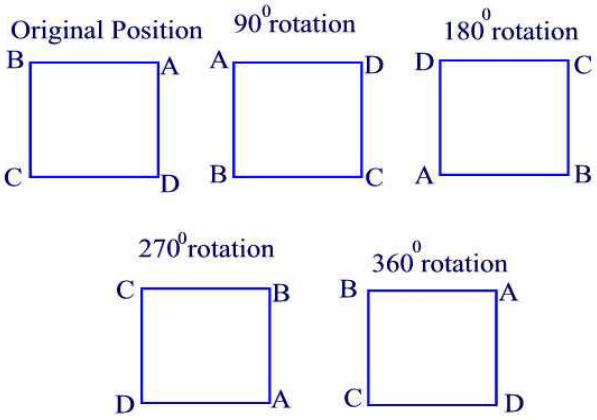



1. At the time interval OA the object will be accelerating at OX.

At O, thus the initial velocity and the final velocity will be at M.
The acceleration will be the gradient of that line.
Acceleration $= \frac{v-u}{t_{2-t_1}}$
Where; $v -$ final velocity
$;u -$ Initial velocity

2. At XY the object will be moving at constant velocity.
   Acceleration= 0
3. At the time interval BC the object will be at retardation/deceleration
   Deceleration = $-acceleration$
4. The distance travelled is equal to the area under the graph.

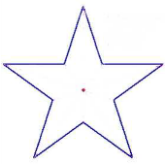
# SYMMETRY




## Rotational Symmetry:

A plane figure has rotational symmetry if and only if it can be rotated more than 0_ and less than or equal to 360_ about a fixed point called the center of rotation so that its image coincides with its original position.
Figure 39.2 shows the four different rotations of a square.
Figure 39.2
The following table list the number of turns, including a full turn, of some plane figures.

| Figure | Rotation |
|---|---|
| Rectangle | 2 rotations(180_ and 360_) |
| Square | 4 rotations(90_, 180_, 270_, 360_) |
| Rhombus | 2 rotations(180_ and 360_) |
| Parallelogram | 2 rotations(180_ and 360_) |
| Trapezoid | 1 rotation(360_) |
| Equilateral Triangle | 3 rotations(120_, 240_, and 360_) |



5 rotational symmetry.

In regular polygons, number of rotational symmetry = number of sides =lines of symmetry.