4004/2 NOVEMBER 2019 SOLUTION GUIDE

QUESTION		SOLUTION	MARK	ADDITIONAL GUIDANCE
1	(a)	$4-\left(\frac{7}{4}+\frac{5}{3}\right)$ $=4-\left(\frac{21+20}{12}\right)$ $=4-\frac{41}{12}$ $=\frac{48-41}{12}$ $=\frac{7}{12}$ or 0,583	1 1	Rules of precedence or order of operation Simplify brackets first by adding the two fractions in the brackets. Remember the minus (–)sign.
	(b)	$=5{,}25\times4{,}15$ $=21{,}787$ $=21{,}79$	1 1	Appreciation of upper and lower limits (bounds) 1 decimal place means the 2nd decimal place is the key. yz (minimum) when both y and z take values on the lower limits.
	(c)(i)	$\frac{5}{8}\times2928$ 1 830	1 1	Application of ratio, 3:5
	(ii)	$\frac{66\,612-19\times1\,830}{2\,928-1\,830}$ \$29	1 1 1	Find number of Executive rooms. Find the total amount realised from General rooms and subtract it from \$66 612. Divide by the number of Executive rooms.

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2	(a)(i)	$3(x+2)-3\times14<7$ $3x+6-42<7$ $3-3\times14<7$ $3x<43$ $x<14\frac{1}{3}$ 14	1 1 1	Find the determinant of Matrix A, in terms of x. Given that this expression in less than 7, write an inequality, in x and solve it. On a number line what would be the largest integer value of x.
	(ii)	Det A = 48 – 42 = 6 $A^{-1}=\frac{1}{6}\begin{pmatrix}3 & -14\\ -3 & 16\end{pmatrix}$	2	Substitute 14 for χ and find the determinant of Matrix A, hence write down the inverse of Matrix A.
	(b)(i)	$\overrightarrow{QR}=6\boldsymbol{y}$	1	QW:WR is 1:5, therefore QR is 6 times QW.
	(ii)	$\overrightarrow{PR}=3\boldsymbol{x}+6\boldsymbol{y}$	1	$\overrightarrow{PR}=\overrightarrow{PQ}+\overrightarrow{QR}$. Use the triangle law of vector addition.
	(iii)	$\overrightarrow{PN}=\frac{2}{3}(3\boldsymbol{x}+6\boldsymbol{y})$ $=2\boldsymbol{x}+4\boldsymbol{y}$	1	Ratio PN:NR is 2:1 therefore PN is $\frac{2}{3}$ of PR.
	(iv)	$\overrightarrow{QN}=-3\boldsymbol{x}+2\boldsymbol{x}+4\boldsymbol{y}$ $=-\boldsymbol{x}+4\boldsymbol{y}$	1 1	$\overrightarrow{QN}=\overrightarrow{QP}+\overrightarrow{PN}$. Use the triangle law of vector addition.
3	(a)(i)	$S\hat{Q}P$	1	Equal arcs/chords subtend equal angles.
	(ii)	$P\hat{Q}S=90°-57°$ $=33°$	1	Angle in a semi-circle is equal to 90°. therefore

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				$S\hat{P}Q$ and $S\hat{Q}P$ are complimentary.
	(iii)	$Q\hat{R}S = 180° - 57°$ $= 123°$	1	PSRQ is a cyclic quadrilateral opposite angles are supplementary.
	(iv)	$Q\hat{S}R = 180° - (90° + 66°)$ $= 180° - 156°$ $= 24°$	1 1	In a cyclic quadrilateral, opposite angles add up to 180°.
	(b)(i)	PS $= \frac{3,7}{sin22,3°}$ $= 9,751$ cm	1 1	Angle QPS = $90° - 22,3° = 67,7°$ $cos67,7° = \frac{3,7}{PS}$ therefore PS $= \frac{3,7}{COS67,7°}$ or $\frac{PS}{sin134,6} = \frac{5,2}{SIN22,3}$ $PS = \frac{5,2Sin134,6}{Sin22,3} = 9,757cm$
	(ii)	$Q\hat{P}R = Cos^{-1}\left(\frac{3,7}{5,2}\right)$ $= 44,6°$	1 1	$CosQ\hat{P}R = \frac{3,7}{5,2}$ Find the angle QPR.
	(iii)	$S\hat{P}R = 67,7° - 44,6°$ $= 23,1°$	1 1	Subtract angle RPQ from angle SPQ getting angle SPR.
4	(a)(i)	V$= \frac{22}{7} \times \left(\frac{3,8}{2}\right)^2 \times 4,9$ $= 55,59$ cm^3	1 1	Application of the formula for volume of a cylinder = $\pi r^2 l$. Substitute for π, radius r and length l. Simplify.

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	(ii)	$55{,}6 \times 0{,}63$ $= 35$ g	1 1	Use of formula for density to be used. $\left[density = \frac{mass}{volume}\right]$, hence mass = density × volume.
	(b)(i)	Area $= 15 \times 6 + \frac{1}{2} \times 20 \times 15$ $= 90 + 150$ $= 240$ cm^2	1 1 1	The arrow is made up of a rectangle measuring 15 by 6 and an isosceles triangle with a base of 20 and height 15. Alternatively find the area of the large rectangle 30 by 20. Subtract the areas of two smaller rectangles and two right angled triangles.
	(ii)	Perimeter $= 2 \times 7 + 2 \times 15 + 6 + 2\sqrt{15^2 + 10^2}$ $= 14 + 30 + 6 + 2\sqrt{325}$ $= 86{,}06$	1 + 1 1 1	Perimeter refers to distance along the edges, as such there is only one 6 cm edge. Application of Pythagoras Theorem Appreciation of composite shapes and finding their perimeters.

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				Divide finally by the common factor of x.
	(d)(i)	$p = kt^{-3}$ $\therefore k = \frac{p}{t^{-3}}$ $k = \frac{4}{(2)^{-3}}$ $k = 32$ $\therefore p = 32t^{-3}$ or $p = \frac{32}{t^3}$	1 1	Introduce a constant of proportional, k. Find the value of k by substitution.
	(ii)	$pt^3 = 32$ $2 \times \frac{1}{2} \times t^3 = 32 \times 2$ $t^3 = 64$ $t = 4$	1	Substitute for p Isolate t^3 and solve by finding the cube root of 64.
7	(a)	85% : \$55 $\therefore$ 100% : -- more $= \frac{\$55 \times 100}{85}$ = \$64,71	1 + 1 1	The selling price as a percentage is 85%. Original price as a percentage is 100%. Use simple proportion to find the original price.
	(b)(i)	$\$412 \times \frac{1}{100}$ \$4,12	1	Use of percentages

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6	(a)	$3^k = \frac{(3^4)^2 \times 3^5}{3^{11}}$ $3^k = \frac{3^8 \times 3^5}{3^{11}}$ $3^k = 3^{13-11}$ $3^k = 3^2$ $k = 2$	1 1	Express 81 as an exponent of 3. Appreciation of laws/rules of indices $[(x^a)^b = x^{ab}\ and\ x^a \div x^b = x^{a-b}]$ Equal bases implies equal powers.
	(b)(i)	$2(3y^2 - 5y + 2)$ $2(3y - 2)(y - 1)$	2	There is a common factor 2 and the quadratic factor that remains can be factorised using any method of quadratic expressions factorisation. Brackets are essential.
	(ii)	$ax + a + b + bx$ $= a(x + 1) + b(1 + x)$ $= (a + b)(x + 1)$	2	Regroup the terms. Factorise in two parts. Common bracket (factor) identified as $(x + 1)$
	(c)	$\frac{6}{x(2-x)} - \frac{3}{x}$ $= \frac{6-3(2-x)}{x(2-x)}$ $= \frac{6-6+3x}{x(2-x)}$ $= \frac{3x}{x(2-x)}$ $= \frac{3}{2-x}$	1 1 1	Factorise $2x - x^2$. Find the L.C.M of the denominators and express both fractions with that common denominator. The division line is essential. Pay particular attention to change of signs as brackets are removed.

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8	(a)(i)	$A = \frac{0{,}8(12 + 1{,}5)}{2}$ $= 0{,}4 \times 13{,}5$ $= 5{,}4$	1 1	Accurate substitution and evaluation to a numerical answer.
	(ii)	$h(12 + b) = 2A$ $h = \frac{2A}{12+b}$	1 1	Clear fractions by multiplying by the common denominator 2. Make h the subject by dividing both sides by $12 + b$.
	(b)(i)	$(2x)^2 = x^2 + 4^2 - 2 \times 4 \times x Cos120$ $4x^2 = x^2 + 4^2 - 8x \times \left(\frac{-1}{2}\right)$ $4x^2 = x^2 + 16 + 4x$ $3x^2 - 4x - 16 = 0$	1 1 1	Application of cosine rule. Remove the brackets on $(2x)^2$ $Cos120 = -Cos(180 - 120)$ $= -Cos60$ $-2 \times 4 \times x(-Cos60)$ simplying to $+ 4x$ Correct reduction with no wrong working seen.

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	(ii)	412,00 - 4,12 $407,88	1	Understanding of Crediting and Debiting.
	(iii)	407,88 - 292,88 $115,00	1	Study the pattern. Add or subtract.
	(c)(i)	$\frac{600\times4\times3}{100}$ = $72	1 1	Formula for Simple Interest $\left[I = \frac{PRT}{100}\right]$ Substitute in the formula and simplify.
	(ii)	$600\times1,04^3 - 600$ $= 674,92 - 600$ = 74,92	1 1 1	Formula for Compound Interest as Amount (of investment) $=P\left(I + \frac{r}{100}\right)^t$ Then Interest = Amount – Principal OR Calculate interest for Y1 using the formula for simple interest then add that interest to Principal for Y_2 and so on until the end of three years. Getting $24; $24,96 and $25,96 Add to get $74,92.
	(iii)	$74,92 - 72,00 $2,92	1	Simple subtraction realising that interest from Option B is higher than interest from Option A.

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9	(a)	$x = 360 - (180 + 120)$ $= 360 - 300$ $= 60$	1	Angle property of a point as an amount of turning of 360°.
	(b)(i)	$P = \frac{80 \times 90}{120}$ $= 60$	1	Simple proportion.
	(ii)	$q = \frac{80}{2}$ $= 40$	1	Simple proportion.
	(iii)	$r = \frac{80 \times 90}{120}$	1	Simple proportion.
	(c)	Class centres: 1; 3; 5;7 Mean $= \frac{1 \times 80 + 3 \times 60 + 5 \times 40 + 7 \times 60}{240}$ $= \frac{80 + 180 + 200 + 420}{240}$ $= \frac{880}{240}$ $= 3{,}6$ or $3{,}67$ or $3\frac{2}{3}$ or 3 hours 40 minutes	1 1 1	Find class centres using the formula sum of class boundaries divided by two. Estimate mean: There is need to multiply the class centre of each class with its corresponding frequency. Add the products and divide by total frequency.
	(d)	$\frac{100}{240} \times \frac{99}{239}$ $= \frac{165}{956}$ or $0{,}1726$	1 1	Independent events apply the Product Law. Since there is no replacement both numerator and denominator should be less by 1. It is normal to give fraction in its lowest terms though answer as a decimal fraction is acceptable.

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	(ii)	$x = \frac{-(-4) \pm \sqrt{16 - 4\times3\times(-16)}}{2\times3}$ $= \frac{4 \pm \sqrt{16 + 192}}{6}$ $= \frac{4 \pm \sqrt{208}}{6}$ = 3,07 or – 1, 74	1 + 1 1 1 + 1	Correct substitution in the quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ Round off the answers to 3 significant figures. Completing the square. $\frac{3x^2}{3} - \frac{4x}{3} - \frac{16}{3} = \frac{0}{3}$ $x^2 - \frac{4x}{3} = \frac{16}{3}$ What must be added to make LHS a perfect square. $x^2 - \frac{4}{3}x + \frac{4}{9} = \frac{16}{3} + \frac{4}{9}$ $\left(x - \frac{2}{3}\right)^2 = \frac{48 + 4}{9}$ $\left(x - \frac{2}{3}\right)^2 = \frac{52}{9}$ $x = \frac{2}{3} \pm \sqrt{\frac{52}{9}}$ $x = \frac{2}{3} \pm \frac{\sqrt{52}}{3}$ $x = \frac{2}{3} \pm \frac{\sqrt{52}}{3}$ $x = \frac{2 \pm \sqrt{52}}{3}$

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	(b)	Triangle C correctly drawn with vertices at $\left(2;-2\frac{1}{2}\right)$, $(2;-7)$ and $\left(-1;-2\frac{1}{2}\right)$	1 + 1 + 1	Mark point $(2;-1)$, the centre of enlargement. Enlargement factor of $-1\frac{1}{2}$ means that the image is inverted and on the other side of the centre of enlargement. Join point on B to the centre of enlargement and produce that line. Use a ruler and short calculation to mark the three vertices of triangle C. Join the three vertices to have triangle C.
	(c)	$\begin{pmatrix}6\\-2\end{pmatrix}-\begin{pmatrix}-2\\2\end{pmatrix}$ $\begin{pmatrix}8\\-4\end{pmatrix}$	1	Translation vector got by: Final point – initial point expressed as column vectors not in coordinate form. Brackets are essential. On the graph join point $(-2;2)$ and $(6;-2)$ by a line segment. Express the line segment as a column vector.
	(d)	$\begin{pmatrix}1&0\\-2&1\end{pmatrix}\begin{pmatrix}-5&-2&-2\\2&2&4\end{pmatrix}$ $=\begin{pmatrix}2&-2&-2\\12&6&8\end{pmatrix}$ $(2;12),\ (-2;6),(-2;8)$		Express coordinates of A in matrix form. Pre-multiply by the operator. Express in coordinate system. Brackets essential.

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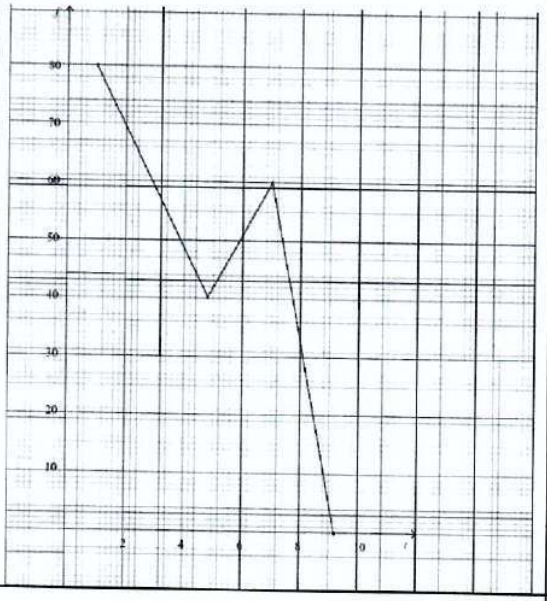
	(e)	Graph with points (1:80), (3:60) (5:40), (7:60) and (9:0). Correctly plotted and joined by straight lines. 	3	Find the class centres. Class centres used for drawing the frequency polygon using class centres and corresponding frequencies as the coordinates of the vertices. Time cannot be negative, hence the polygon cannot be closed on the negative time axis but on the positive at (9:0).
10	**(a)**	$p = 3^3 - 4(3)^2 + 4$ $= -5$	1	Substitute +3 for x in the function to find the value of p.

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	(e)	Rotation Centre $(-1;-1)$ Through 90° clockwise or 270 anticlockwise	1 1 1	Take any two points on triangle A and join them to their corresponding points on triangle B. If the two lines are not parallel it shows that the transformation is neither Reflection nor Translation, leaving it as Rotation. Find the centre of rotation using a ruler and compass. Establish the angle of rotation by joining a point on A to the centre of rotation and a corresponding point on B also to the centre of rotation. Then state the direction in terms of clockwise or anticlockwise direction.

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	(iii)	510 small squares:-- When 100 small square: 1 unit^2 $\frac{510\times1}{100}$ 5,1 unit^2	 1 1	The region should be correctly identified as evidenced by marking by straight lines without necessarily shading the region. Area should be found by counting squares and multiplying the number of squares by area of one square. The area of one square should be found using the scale. Using cm would result in having to multiply by 4 giving an answer of 20,4 cm^2 . Units of cm^2 essential.
	(iv)	$2 < x < 3{,}4$	1	The question is: For what range of values of x is the graph, curve above y $= -4$?
11	**(a)**	$x + y \leq 5$	1	Constraints or restrictions in formulating inequalities in linear programming. Key words are "not more than" and "at least"
	(b)	$2x + 4y \leq 16$ $x + 2y \leq 8$	1 1	What are the limitations? Express as an inequality.
	(c)	$x \geq 1$ $y \geq 1$	1 1	Appreciation of "at least" meaning "equal to" or "greater than".

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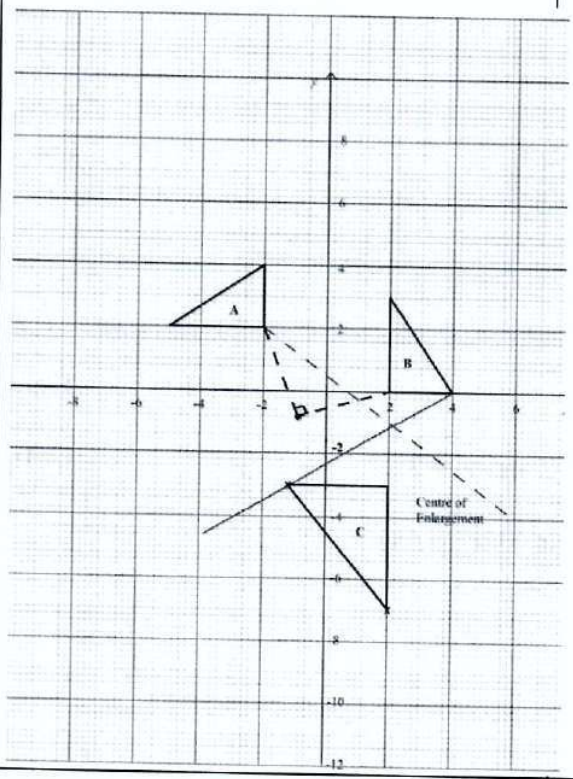
	(ii)	$Profit_{(max)} = 2\times30 + 3\times40$ $= 60 + 120$ $= 180$	1	Substitution in the equation Profit = $30\,x + 40y$
12				Use of correct scale for each axis is necessary so that all diagrams can fit on the given grid. Make sure all vertices are clearly marked and joined by straight lines.
	(a)(i)	Triangle A correctly drawn	1	Make sure all vertices are clearly marked and joined by straight lines to make it a complete triangle.
	(ii)	Triangle B correctly drawn	1	Make sure it is a complete triangle.