ZIMBABWE SCHOOL EXAMINATIONS COUNCIL
General Certificate of Education Advanced Level

PURE MATHEMATICS 6042/1
PAPER 1

JUNE 2019 SESSION 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae MF 7
- Scientific calculator

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

If a numerical answer cannot be given exactly and the accuracy required is not specified in the question then in the case of an angle it should be given to the nearest degree and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 6 printed pages and 2 blank pages.

[Turn over

9 (a) Evaluate $\int_1^4 x\ln x dx$. [4]

(b) (i) Use the trapezium rule with 4 ordinates to approximate the value of $\int_1^4 x\ln x dx$

(ii) Hence find the relative error when the trapezium rule is used to approximate the integral, $\int_1^4 x\ln x dx$. [5]

10 The function $f(x) = \frac{x^3-x-2}{(x-1)(x^2+1)}$.

(a) Express $f(x)$ in the form $A + \frac{B}{x-1} + \frac{Cx+D}{x^2+1}$ where A, B, C and D are constants. [5]

(b) Hence evaluate $\int_2^3 f(x)dx$. [4]

11 (a) By sketching suitable graphs, show that the equation $Sec\, x = 3 - x^2$ has exactly one root in the interval $0 < x < \frac{1}{2}\pi$. [3]

(b) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = Cos^{-1}\left(\frac{1}{3-x_n^2}\right)$$

converges, then it converges to a root of the equation given in part **(a)**. [2]

(c) Use the iterative formula in **(b)**, with the value $x_0 = 1{,}024627$ to determine the root of the equation in **(a)** in the interval $0 < x < \frac{1}{2}\pi$ correct to 2 decimal places, showing the result of each iteration. [4]

12 The rate at which a quantity x is decreasing is proportional to the product of x and $1 - x$.

(i) Form a differential equation which models this situation. [1]

(ii) Initially $x = 0{,}2$ and the rate of decrease is 1,6 units per unit time. Solve the differential equation. [8]

13 (a) (i) Express $2Sinx - 4Cosx$ in the form $RSin(x - \alpha)$ where $R > 0$ and α is acute. [2]

(ii) Hence solve the equation $2Sinx - 4Cosx = \sqrt{5}$, for $0° \le x \le 360°$. [4]

(b) (i) Obtain, in terms of the constant a, an expression for the first 3 simplified terms of the series expansion for $y = e^{a-x}$. [3]

(ii) If the coefficient of x in the series expansion in **(i)** is – 4, find the exact value of a. [2]

14 **(a)** A curve has parametric equations

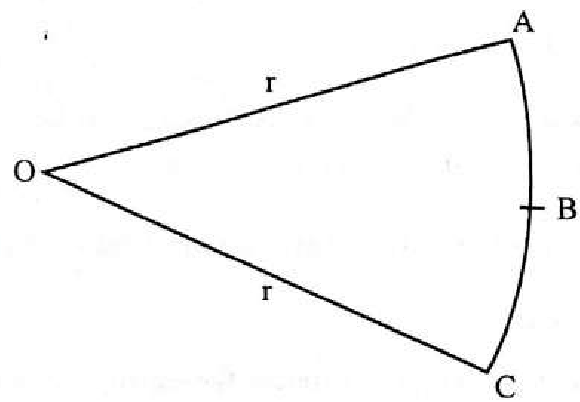
$$x = Cos^2\theta,$$
$$y = 1 - 2Sin^2\theta \text{ for } 0^0 \leq \theta \leq 2\pi.$$

(i) Find $\frac{dy}{dx}$. [4]

(ii) By writing $Cos3\theta$ as $Cos(2\theta + \theta)$ or otherwise, prove the identity

$$2Sin^2\theta Cos\theta \equiv \frac{1}{2}(Cos\theta - Cos3\theta).$$ [3]

(b) A dust pan is in the form of a sector ABCO of a circle of radius r (see diagram).

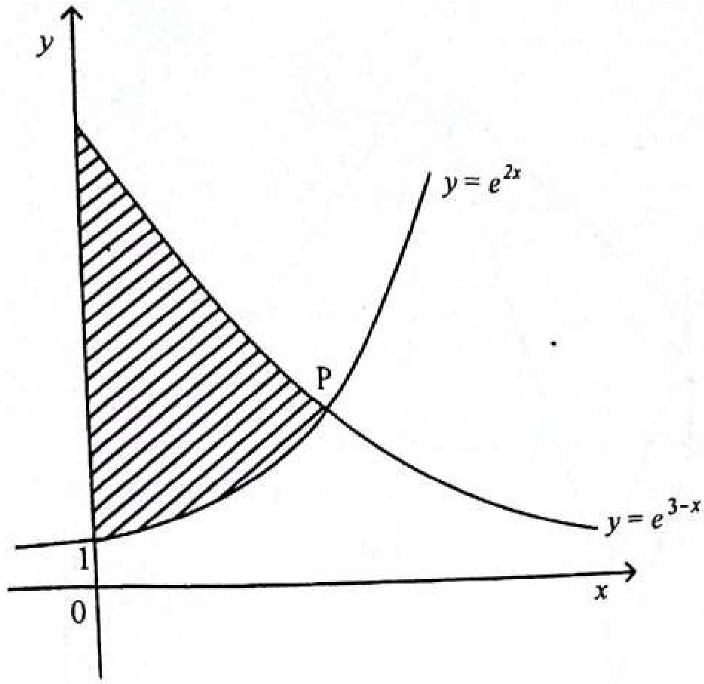


(i) If the length of arc AB is $\frac{1}{2}r$, find angle AOB in degrees. [2]

(ii) The segment ACB is cut off along straight line AC.

Find the percentage of the area cut off to the area of the remaining area of sector AOC. [2]

15



In the diagram, P is the point of intersection of the curves $y = e^{2x}$ and $y = e^{3-x}$.

(a) Find the x coordinate of P. [2]

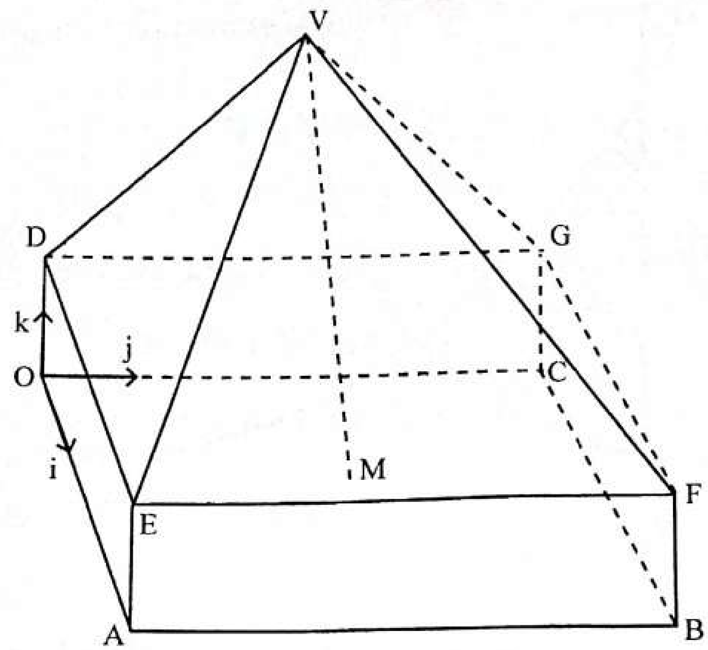
(b) Hence find the

(i) exact value of the area of the region bounded by the two curves and the y axis i.e (shaded region), [4]

(ii) volume of the solid of revolution formed when the shaded region is rotated completely about the x-axis through 360^0. [5]

[Turn over

16 **(a)**



The diagram shows a plan of a building whose floor is a rectangle OABC and whose roof is in the form of a pyramid DEFGV, with OA = 4 m, OC = 8 m, OD = 2 m and MV = 5 m. M is the point where the diagonals OB and AC intersect. Taking O as the origin and $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ unit vectors in the directions $\overrightarrow{OA}, \overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively,

(i) write the position vectors of B and V, [2]

(ii) find the exact value of the angle between $\overrightarrow{OV}$ and $\overrightarrow{OB}$. [4]

(b) The complex number $u = \frac{4-8i}{i}$.

(i) Express u in the form $x + iy$. [2]

(ii) Find the magnitude and argument of u. [3]

(iii) Sketch u on an Argand diagram. [2]