4004/2 JUNE 2019 SOLUTION GUIDE

QUESTION		SOLUTION	MARK	ADDITIONAL GUIDANCE
1	(a)	$\frac{4+1}{6+1}=\frac{5}{7}$	1	Realise that both numerator and denominator is increasing by 1.
	(b)	$10=3+7$	1	Key is the concept of a prime number and these have to be less than 10.
	(c)(i)	$\$105\times\frac{112}{100}=\$117{,}60$	2	Finding 12% of 105 \$12,60 then adding to \$105.
	(ii)	Tendai gets $=\frac{4}{(4+3)}\times\$105$ $=\$60$ Chipo gets $=\frac{3}{(4+3)}\times\$105$ $=\$45$	1 1 1	Either calculating Chipo's share then subtracting it from \$105 so as to get Tendai's share or the other way round still acceptable.
2	(a)	$\theta=90-40\ and\ 180-50$ $=50 \qquad 130$	1 + 1	Alternatively $\text{Cos}\,40=0{,}7660$ Therefore $\text{Sin}^{-1}(0{,}7660)=50$ Recalling that Sine of an obtuse angle is also positive.
	(b)(i)	$\frac{AP}{9{,}4}=\text{Sin}\,37$ $\therefore AP=9{,}4\,\text{Sin}\,37$ $=5{,}657$	1 1	Identifying the correct trig ratio to use in the right angled triangle or 9,4 $\text{Cos}\,53$.(ii)
	(ii)	$\frac{AC}{\text{Sin}\,37}=\frac{9{,}4}{\text{Sin}\,48}$ $AC=\frac{9{,}4\ \text{Sin}\,37}{\text{Sin}\,48}$ $=7{,}612$	1 1	Realising that the shortest method would be using Sine rule. However using the calculated value of AP in triangle APC would be applicable giving $\frac{AP}{\text{Cos}\,42}\ or\ \frac{AP}{\text{Sin}\,48}$ ie $\frac{5{,}657}{\text{Cos}\,42}\ or\ \frac{5{,}657}{\text{Sin}\,48}$ Four significant figures should then be used.

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(c)	(i)	$360 - 3y$ or $2(4y + 4)$	1	Recall of angle property of a point as ab amount of turning of 360°. And that angle at the centre is twice that subtended on the circumference.
	(ii)	$2(4y + 4) = 360 - 3y$ $8y + 8 = 360 - 3y$ $11y = 360 - 8$ $11y = 352$ $y = 32$	1 1 1	Appreciation of circle geometry theorems. Formulation of the equation and solving it.
3	(a)(i)	$7: 91,70$ $\therefore 1: ?\ (less)$ $\frac{91,7}{7} = 13,1$ $\therefore 1: 13,10$	1	Appreciation of Ratio and expressing this as a fraction. \$/cent conversion and the importance of the zero.
	(ii)	$\$7 - 7\times 0,01 = \$6,93$ OR $7\times 0,99$	2	1% means 0,01 Even on the calculator $7 - 1\%$ one would get 6,93
	(b)	$84\%: \$210$ $\therefore 100\%: ?\ (more)$ $= \frac{\$210\times 100}{84}$ $= \$250$	3	Original price is always 100%, it is the selling price, as a percentage, that differs 16% means selling price as a percentage becomes 84%.
	(c)	$P = \frac{2010\times 100}{3\times 5}$ $= \$13\,400$	1	Recalling the formula $I = \frac{PRT}{100}$ Making P the subject $100I = PRT$ $\therefore \frac{100I}{RT} = P$

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				Correct substitution and simplification.
	(d)	Amount $= \$600\times1{,}04^3$ $= \$674{,}918$ $= \$674{,}92$	3	Use of the formula Amount (Yield) $= P\left(1+\frac{r}{100}\right)^t$ followed by substitution and correct simplification. Alternatively one can calculate simple interest yearly remember that interest accrued is added onto the principal amount as one goes to the next year.
4	(a)	$\frac{3(x-1)-(x+2)}{x-1}$ $= \frac{3x-3-x-2}{x-1}$ $= \frac{2x-5}{x-1}$	1 1 1	Find the L.C.M of the denominators and express both fractions under that common denominator. The division line is important. Pay particular attention to change of signs as brackets are removed.
	(b)(i)	$x^2+3x-8=3x+1$ $x^2-8=1$ $x^2=9$ $x=\pm3$	1 1 1	Interpreting the functional notation. Formulating the equation. Solving the quadratic equation. Using any of the methods of solving quadratic equations.
	(ii)	$2^x=0{,}25$ $2^x=\frac{1}{4}$ $2^x=2^{-2}$ $x=-2$	 1 1 1	Expressing 0,25 as a common fraction. Appreciating that SAME BASE implies EQUAL POWERS.
	(c)	$ax+b=d^2$ $ax=d^2-b$ $x=\frac{d^2-b}{a}$	1 1 1	Removal of the square root sign by squaring both sides. Isolating the term in x on one side. Making x the subject by dividing both sides by a.

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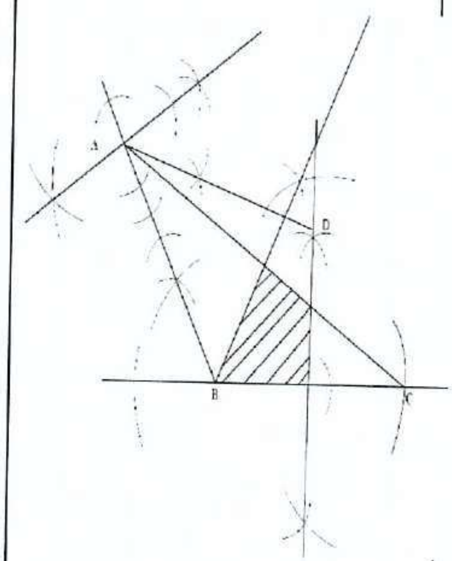
				Division line is essential and should cover $d^2 - b$.
5				Show that ruler and compasses have been used through evidence of construction arcs and lines. If construction space is not enough or a mistake has been made, ask for plain paper and construct on the plain paper and attach it as additional material. Remember to write your candidate details.
	(a)(i)	Triangle ABC with two sides 7cm and an angle of 120° with correct construction arcs.	3	Construction of 120° can be done by constructing an angle of 60 externally.
	(ii)	Bisector of ABC with correct and clear construction arcs.	2	The bisection arcs should show evidence of use of relevant mathematical instruments.
	(iii)	Perpendicular bisector of side BC with correct and clear construction arcs.	2	Points of intersection of the two sets of arcs must be clear.
5	**(b)(i)**	$B\hat{A}D$ correctly constructed with clear and correct construction arcs.	2	Emphasis is on construct of an angle of 45° at A. Could be done by first constructing an angle of 90° then bisect the angle to get 45°. OR realising that ABC is an isosceles triangle with angle at $A = 30°$, falling short of 15° to make it 45, therefore constructing 60° and bisecting it gives 30° angle which can be bisected also giving an angle of 15°.

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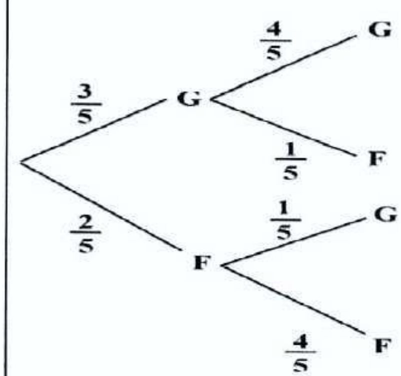
	(ii)	Point D clearly marked and labelled.	1	In form of small cross or clear dot.
	(iii)	Region correctly identified and clearly shaded.	2	Understanding of the intersection of the two Loci.
6	**(a)(i)**	$n(P \cap Q) = 0$	1	Note that $59 - (15 + 35 + 9) = 0$ Suggesting that set P and Q are disjoint sets.
	(ii)	$n(P \cup Q) = 35 + 15$ $= 50$	1	Following the conclusion that set P and Q are disjoint.
(b)	**(i)**		3	The key is the understanding that the two probabilities on the two attached branches should add up to 1, a CERTAINITY.
	(ii)	$\frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$ or 0,48	2	P(G and G) requires Product law.
	(iii)	$\frac{2}{5} \times \frac{4}{5} = \frac{8}{25}$ or 0,32	2	P(F and F) requires Product law.
	(iv)	$\frac{3}{5} \times \frac{1}{5} + \frac{2}{5} \times \frac{1}{5}$ $= \frac{1}{5}$ or 0,2	3	The use of the tree diagram will simplify the problem and where to apply addition law.
7	**(a)(i)**	$5\times -13 \leq x - b$ and $x - 6 < 9 + 4x$ $4x \leq 7$ and $-15 < 3x$	3	The inequality is to be split into two and solve the two separately then combine the results.

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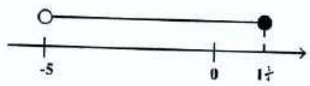
		$x \leq {}^{3}/_{4}$ and $-5 < x$ $-5 < x < 1\ {}^{3}/_{4}$		
	(ii)		1	A number line closed at $1\frac{3}{4}$ and open at -5.
	(iii)	$x = -4$	1	Directed numbers and positions on a number line and key is the term integer, a whole number positive or negative including zero.
(b)	(i)	$\text{Sin}\, A\hat{C}\, B = \dfrac{x+2}{2x+3}$	1	Recap on trig ratios in right angled triangles.
	(ii)	$\dfrac{x+2}{2x+3} = \dfrac{9}{16}$	1	Deductive reasoning.
	(iii)	$16(x+2) = 9(2x+3)$ $16x + 32 = 18x + 27$ $5 = 2x$ $2\frac{1}{2} = x$	1 1	Remove fractions by multiplying by $16(2x + 3)$ both sides or cross multiplying Collecting like terms Solving for x
	(iv)	$AC = 2\times\frac{5}{2} + 3$ $= 8cm$	1	Substitution for x.
	(v)	$BC = \sqrt{8^{2-}4{,}5^{2}}$ $= \sqrt{43{,}75}$ $= 6{,}614$	1 1	Application of Pythagoras theorem. Using the calculate to evaluate. $\sqrt{8^{2-}4{,}5^{2}}$
8	(a)(i)	$p = 1^{2}4(1)$		Substitute for x and simplify.

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		$= 1 - 4$ $= -3$	1	
	(ii)	$q = 5^2 - 4\times5$ $= 25 - 20$ $= 5$	1	Find the value of y when $x = 5$ in $y = x^2 - 4x$
	(b)(i)		4	Plots should be visible and the graph should be drawn using free hand, not grossly thick and passing through the correct points.

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		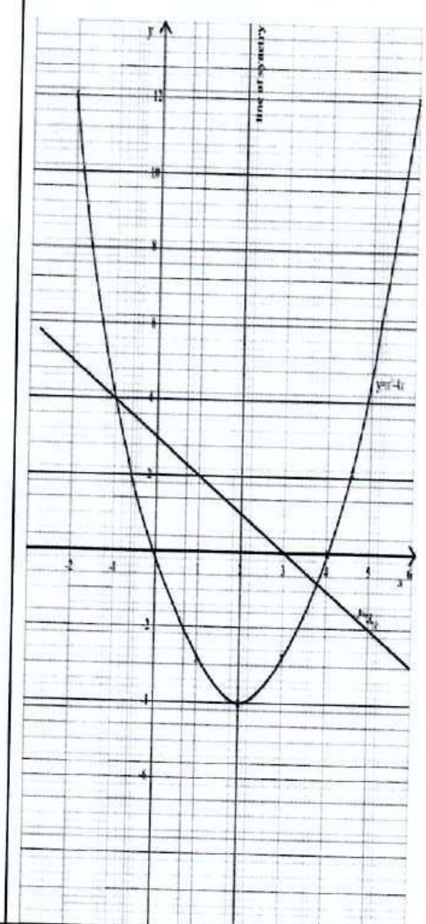		
	(ii)	The line $y = 3 - x$ correctly drawn to cut the curve in two places.	2	In general lines drawn should be at least 3cm unless stated otherwise.
	(c)(i)	$x = -0{,}8$ $x = 3{,}8$	1 1	These values are found where the straight line intersects the curve.
	(ii)	$x = 2$	2	Mirror line for bi-lateral symmetry. Parallel to Y axis ($x = 0$).
9	(a)	$\frac{1}{2} \times 26 \times 24 \times \text{Sin } 40$ $= 200{,}5cm^2$	1 1	The formula for finding area of a triangle given two sides and an included angle. $\left[\frac{1}{2} ab \ \text{Sin } C, where\ C\ is\ the\ included\ angle\right.$
	(b)	$AD^2 = 26^2 + 24^2 - 2 \times 26 \times 24 \ \text{Cos } 40$	1	Application of Cosine rule. Correct substitution in the formula

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		$AD^2 = 295{,}977$ $AD = \sqrt{295{,}977}$ $= 17{,}2cm$	1 1 1	$a^2 = b^2 + c^2 - 2bc\ \mathrm{Cos}\,A$ Use of the calculator or relevant mathematical tables.
	(c)	$\frac{BC}{\mathrm{Sin}\ 30} = \frac{24}{\mathrm{Sin}(110)}$ $BC = \frac{24\ \mathrm{Sin}\ 30}{\mathrm{Sin}\ 110}$ $= 12{,}77cm$	2 1 1	$B\hat{C}D = 180° - (30° + 40°)$ $= 110°$ Use of Sine rule.
	(d)	$\frac{d}{BC} = \mathrm{Sin}\ 40$ $d = 12{,}77\ \mathrm{Sin}\ 40$ $= 8{,}208cm$	 1 1	The shortest distance is one that makes a right angle with *BD* and passing through *C*. Use the trig ratio Sin 40° to find the shortest distance.
10	(a)(i)	$165 < h \leq 180$	1	That class interval with highest frequency. Stated as it is in the table.
	(ii)	$165 < h \leq 180$	1	Median at $\frac{1}{2}(42 + 1)^{th}$ $\frac{1}{2} \times 43^{th}\ i.e. 21{,}5^{th}$
	(iii)	$160 < h \leq 180$	1	Lower quartile at $\frac{1}{4}(43)^{th} = 10{,}75^{th}$
	(b)	$\left[5\frac{(150 + 160)}{2} + 9\frac{(160 + 165)}{2} + 18\frac{(165 + 180)}{2} + 10\frac{(180 + 190)}{2}\right]$ $\div 42$		Find the class centres for each class or interval. Multiply the class centre with its corresponding frequency. Add the products. Divide by total frequency.

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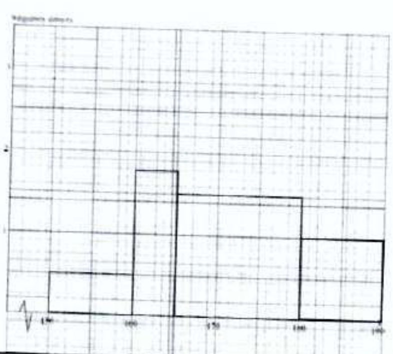
		$= (5\times155 + 9\times162{,}5 + 18\times172{,}5 + 10\times185) \div 42$ $= \frac{7192{,}5}{42}$ $= 171{,}25$	3	
	(c)	$= \frac{27}{42}\times\frac{26}{41}$ $= \frac{117}{287}$ or 0,4077	3	Since there is no replacement subsequent probabilities should be less by 1 for both numerator and denominator. Independent events hence apply the product law.
	(d)	Correct histogram drawn with frequency densities (heights) 0,5: 1,8: 1,2 and 1. 	3	For histogram, height is found by calculating frequency density, Using the formula: $f\cdot d = \frac{frequency}{class\ width}$
11	(a)	$2\times2{,}2 + \frac{1}{2}\times2\times0{,}6$ $= 4{,}4 + 0{,}6$ $= 5m^2$	3	Find area of ΔCDE Find area of rectangle $ABCE$ Add the two areas.
	(b)	$V = 5\times3$ $= 15m^3$	2	Volume = Cross-sectional Area × length
	(c)	$\frac{23m^2}{4{,}5m^2}$		Cannot buy a fraction of the tin therefore round up!

QUESTION		SOLUTION	MARK	ADDITIONAL GUIDANCE
		$= 5{,}111$ $= 6\ tins$	2	
	(d)(i)	$DE = \sqrt{1^2 + 0{,}6^2}$ $= 1{,}166$	2	Use of Pythagoras Theorem
	(ii)	$1{,}166 \times 3 \times 2$ $= 6{,}996 \times \$6{,}40$ $= \$44{,}77$	3	Area of the rectangular sides $DE \times 3m \times 2\ sides.$ Material costs \$6,40 for every square metre.
12	(a)(i)	$\binom{3}{9} - 3\binom{-3}{1}$ $= \binom{3}{9} - \binom{-9}{3}$ $= \binom{12}{6}$	2	Scalar multiplication means each entry is multiplied by the scalar. Addition / subtraction of vectors. Brackets are essential.
	(ii)	$\sqrt{12^2 + 6^2}$ $= \sqrt{180}$ $= \sqrt{13{,}42}$	1	Magnitude/modulus of the vector as size of the line segment representing vector $\binom{12}{6}$ Apply Pythagorus theorem
	(b)(i)	$2(\boldsymbol{p} + \boldsymbol{q})$ $= 2\boldsymbol{p} + 2\boldsymbol{q}$	1	Scalar multiplication of a vector.
	(ii)	$\boldsymbol{p} + \frac{1}{2}\boldsymbol{q}$	1	**OM = OA + AM** Use the triangle law of vector addition.
	(iii)	$\boldsymbol{q} + \boldsymbol{p} + \boldsymbol{q}$ $= \boldsymbol{p} + 2\boldsymbol{q}$	1	**AC = AB + BC** Use the triangle law of vector addition.

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	(iv)	$\overrightarrow{OT} = k(\boldsymbol{p} + \frac{1}{2}\boldsymbol{q})$ $= k\boldsymbol{p} + \frac{1}{2}k\boldsymbol{q}$	1	Scalar multiplication.
	(v)	$\overrightarrow{OT} = \boldsymbol{p} + h\square\, 2h\boldsymbol{q}$ $= (1 + h)\boldsymbol{p} + 2h\boldsymbol{q})$	1	$\boldsymbol{AT} = \boldsymbol{hAC}$ $= h(\boldsymbol{p} + 2\boldsymbol{q})$ $= h\boldsymbol{p} + 2h\boldsymbol{q}$ Therefore $\boldsymbol{OT} = \boldsymbol{OA} + \boldsymbol{AT}$ Use the triangle law of vector addition.
	(vi)	$(1 + h)\boldsymbol{p} + 2h\boldsymbol{q} = k\boldsymbol{p} + \frac{1}{2}k\boldsymbol{q}$ $\therefore 1 + h = k \ldots..(1)$ $2h = \frac{1}{2}k \ \ldots\ldots(2)$ $h = \frac{1}{3}$ $k = \frac{4}{3}\ or\ 1\frac{1}{3}$	1 1+1	Two equations are to be formed and solved simultaneously. Use any method of solving simultaneous equations.
	(vii)	$1{:}4\ or\ \frac{1}{4}$	1	Parallel vectors imply scalar multiple of each other.