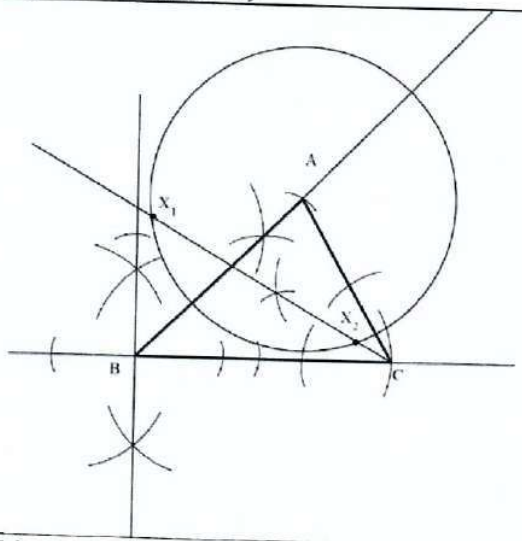
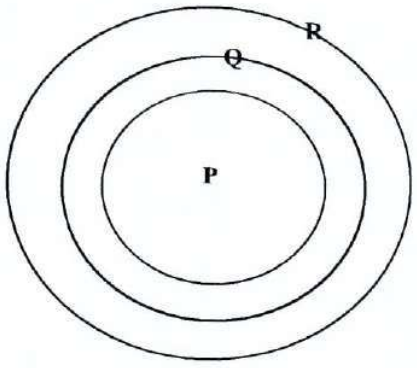
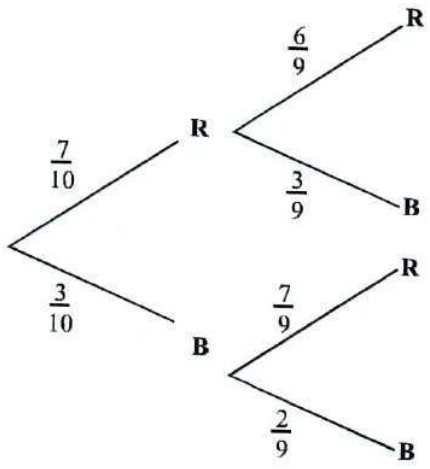
4004/2 NOVEMBER 2018 SOLUTION GUIDE

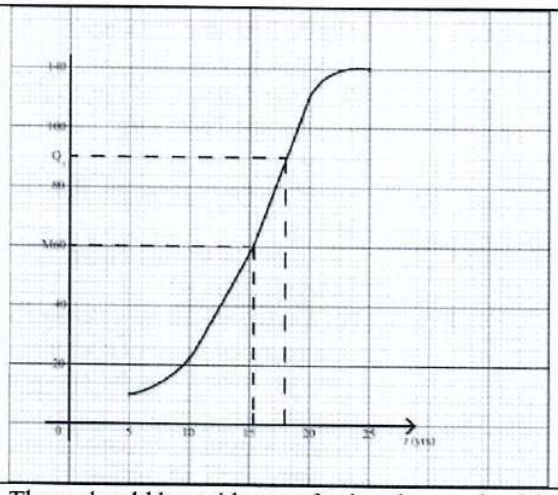
1(a) (i)	$ax - x(a-b) + 2bx$ $= ax - ax + bx + 2bx$ $= 3bx$	2	Evidence of correct removal of brackets and directed numbers $-x \times -b = +bx$ Simplifying by grouping like terms
(ii)	$(x-2)^2 - x^2$ $= x^2 - 4x + 4 - x^2$ $-4x + 4$	2	Expansion of the square bracket i.e. $(x-2)(x-2)$ Grouping and simplifying like terms
(b)	$P = \frac{1}{2}\left[\frac{1}{2} + 1\left(\frac{1}{2} + 1\right)\right]$ $= \frac{1}{2}\left(\frac{1}{2} + 1\frac{1}{2}\right)$ $= \frac{1}{2}(2)$ $= 1$	2	Correct substitution followed by correct removal of inner brackets and getting 2. Or multiplying $\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{3}{2}$ giving $\frac{1}{4} + \frac{3}{4} = 1$
(c)	$\frac{x-3}{x-2} - \frac{x+2}{x+3}$ $= \frac{(x-3)(x+3) - (x+2)(x-2)}{(x-2)(x+3)}$ $= \frac{x^2 + 3x - 3x - 9 - (x^2 - 2x + 2x - 4)}{(x-2)(x+3)}$ $= \frac{x^2 - 9 - (x^2 - 4)}{(x-2)(x+3)}$ $= \frac{x^2 - 9 - x^2 + 4}{(x-2)(x+3)}$ $\frac{-5}{(x-2)(x+3)}$	1 1 1	Evidence of correct denominator with all the essential brackets. Understanding of "difference of two squares" can be applied in expanding $(x-3)(x+3)$and $(x-2)(x+2)$. Brackets are very important on $-(x+2)(x-2)$ so that the pitfall of getting $-x^2 - 4$ is avoided.
2(a)(i)	$\frac{\$50}{5kg}$ $= \$10$	1 1	Understanding that 'per' means "divided by" dividing the total cost by 5 kg

(ii)	115% : \$50 $\therefore$ 100% : ?(less) $\frac{\$50\times100}{115}$ = \$43,48 $\therefore$ VAT = \$50 – \$43,48 = \$6,52	 1 1	Any price i.e Cost Price is always 100%, so adding Vat means selling price becomes 115% 115% : \$50 $\therefore$ 15% (VAT) $\frac{\$50\times15\%}{115\%}$ giving \$6,52
(iii)	$\frac{10}{100}\times(2\times50+30+5\times28+28+3\times50)$ $=\frac{10}{100}\times(100+30+140+28+150)$ $=\frac{10}{100}\times(364)$ $=\frac{10}{100}\times364$ = \$36,40	 1 1 1	Evidence of calculating the total cost and then finding 10% of the total cost as the discount offered.
(b)	Amount = $\$400\left(1+\frac{3}{100}\right)^3$ = \$400 × 1,03^3 = \$437,09	 3	Recall and application of the formula for compound interest. $A = P\left(1+\frac{r}{100}\right)^t$ Alternative method would be to calculate yearly i.e. interest for year 1 and add it, becomes principal for year 2 and so on.
3(a)(i)	$4x-2\leq5x+2$ and $5x+2<2x+8$ $-2-2\leq5x-4x$ and $5x-2x<8-2$ $-4\leq x$ $\quad\quad$ $3x<6$ $x<2$ $-4\leq x<2$	 3	Ability to split the inequalities into two and working on each and combine the result.
(ii)			A number line with solution line closed $a+-4$ and open $a+\ +2$ Evidence of "open and closed intervals.

(b)	$R^2 = \frac{ax-P}{Q+bx}$ $R^2(Q + bx) = ax - p$ $QR^2 + bxR^2 = ax - p$ $QR^2 + p = ax - bxR^2$ $ax - bxR^2 = QR^2 + p$ $\frac{x(a-bR^2)}{a-bR^2} = \frac{QR^2+p}{a-bR^2}$ $\therefore x = \frac{p + QR^2}{a - bR^2}$	 1 1 1 1	Squarring both sides so as to remove the square root sign. Removal of fractions by multiplying both sides by Q + bx Collecting terms in x on one side and factorise to make x the subject. NB: All signs can change in the expression $\frac{p+QR^2}{a-bR^2}$
(c)	$m(2m^2n^2 + 3mn - 2)$ $= m[2mn(mn + 2) - 1(mn + 2)]$ $= m[(2mn - 1)(mn + 2)]$ $m(2mn - 1)(mn + 2)$	 1 1	Realising that there is a common factor of m. Factoring out m leaves a quadratic expression that can be factorised using any method like the ring number of $-4m^2n^2$
4			Show that ruler and compasses have been used through evidence of construction arcs and lines. If construction space is not enough or a mistake has been made which cannot be erased ask for plain paper and construct on the plain paper and attach it as additional material.
(a)	Triangle ABC with sides AB = 6 cm, BC = 6,5 cm and $A\hat{B}C = 45°$ with correct construction arcs.	4	First construct an angle of 90° then bisect the angle to get the 45° angle. The construction arcs are part of the solution so should be clear. Use a ruler for all straight lines.
(b)	A complete circle centre A with a radius of 4 cm	1	Circle to be clear and complete.

(c)	Bisector of $B\hat{C}A$ with correct and clear construction arcs	2	The bisection arcs should be very clear to show ability to use relevant mathematical instruments.
(d)	The points X1 and X2 clearly marked on the intersection of the circle and bisector of $B\hat{C}A$.	2	Appreciation of the intersection of the two Loci. There should be marks to show the position of the two points with correct labelling.
(e)	Locus of points equidistant from sides CA and CB.	1	The key words are "points equidistant" which show locus language. Explanation that might not use these words is still acceptable provided the meaning in there.
5(a)(i)	A = {1; 4; 9}	2	The key is the ability to identify square numbers and knowing the cut off. Being able to use the set symbols and notation.
(ii)	A∩B = {4}	1	The intersection of set is the set of common elements in the given sets.
(iii)	n(A∪B) = 4	1	The 'n' refers to a number. The number of elements in the union of sets A and B should not be in set brackets.
(b)(i)		2	The correct Venn diagram should show that P is contained in Q and Q is contained in R. Any shapes e.g. rectangles or squares may be used. Correct labelling of sets is essential.
(ii)	P ⊂ R	1	The set symbol should show that set P is a subset of set R or R contains P.

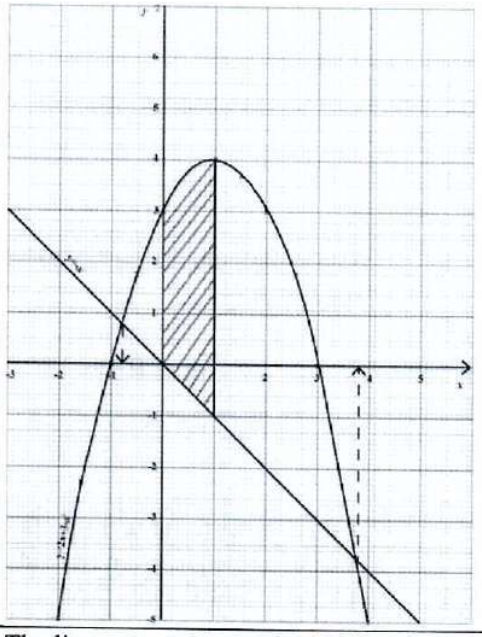
(c)(i)		2	The hint on completion of the tree diagrams is that the two probabilities on the two attached branches should add up to 1, a certainity. There is no replacement therefore subsequent probabilities denominators should decrease by 1.
(ii)	P (R and R)$= \frac{7}{10} \times \frac{6}{9}$ $= \frac{7}{15}$	1 1	The application of the tree diagram. Product law applies because these two events are independent.
(iii)	P (at least R) $= P(RR) or\ P(RB) or\ P(BR)$ $= \frac{7}{15} + \frac{7}{10} \times \frac{3}{9} + \frac{3}{10} \times \frac{7}{9}$ $= \frac{7}{15} + \frac{7}{30} + \frac{7}{30}$ $= \frac{28}{30}$ $= \frac{14}{15}$	1 1	Meaning of "at least one is Red" Or is translated to mean addition Alternatively 1 – p(Not R and R) = 1 – p(BB) $= 1 - \frac{3}{10} \times \frac{2}{7}$ $= \frac{14}{15}$
6(a)	Mid-point are 2,5; 7,5; 12,5; 17,5; 22,5. Mean $= (10 \times 2{,}5 + 12 \times 7{,}5 + 37 \times 12{,}5 + 51 \times 17{,}5 + 10 \times 22{,}5) \div (10 + 12 + 37 + 51 + 10)$ $= \frac{25+90+462{,}5+892{,}5+225}{120}$ $= \frac{1695}{120}$ $= 14, 125$ or $14\frac{1}{8}$	1 1 1	Understanding of the idea of mid-points or class centres in Grouped Data. Mid-point= $\frac{U.L.+L.L}{2}$ where U.L. is the Upper limit and L.L. is the Lower Limit of that class Finding Mean of grouped data i.e. Adding all frequency × class centre and divide by the total of all frequencies.
(b)(i)	$n = 22 + 37$ $= 59$	1	"Cumulative" means "adding on" This is got by adding frequencies up to the required class. [10 + 12 + 37]

(b)(ii)		4	Very smooth curve passing through the correct 5 points. Use of correct scale is a prerequisite as it shapes the curve. The principle used in graphs is that one large square is of length 2 cm. the points should be plotted with visible plots. The curve should be hand drawn and smooth.
(c)(i)	There should be evidence of using the graph with dash lines from cumulative frequency 60,5 point to intersect with curve and from this curve to intersect with the age (horizontal) axis. Median = 15,2	1 1	Median at $\frac{1}{2}(120+1)$ position on Cumulative Frequency axis.
(c)(ii)	A dashed line from cumulative frequency 90,75 point to intersect with the curve and from the curve intersection to the age (horizontal) axis. Upper quartile= 17,5	1 1	Upper quartile is at $\frac{3}{4}(120+1)$ position on the C.F. that is the 90,75 position
7(a)(i)	$B\hat{A}C = 140° - 75°$ $= 65°$	1	Bearing is with reference from the North and measured in the clockwise direction.
(a)(ii)	$\frac{AC}{Sin80°} = \frac{9}{Sin65°}$ $AC = \frac{9Sin80°}{Sin65°}$ $= 9.780\ km$	 1 1	Identify that this question requires use of "sine rule. Correct application of sine rule. Giving answer to the required degree of accuracy.
(a)(iii)	Let the shortest distance be $d\ km$ $\frac{d}{9} = Sin\ 35°$ $d = 9Sin35°$ $= 5{,}162\ km$	 1 1	Realising that the shortest distance is one that makes an angle of 90° with AC and passing through B. Use Sine trig ratio of angle ACB which is calculated as 180° – (80° +(65°) = 35°
7(b)(i)	$P\hat{S}Q = \frac{180° - 72°}{2}$ $= 54°$	1 1	Ability to see the isosceles triangle PQS. Angles opposite the equal sides being equal, "base angles".
(b)(ii)	$S\hat{R}P = 72°$	1	Angles subtended by same arc are equal.

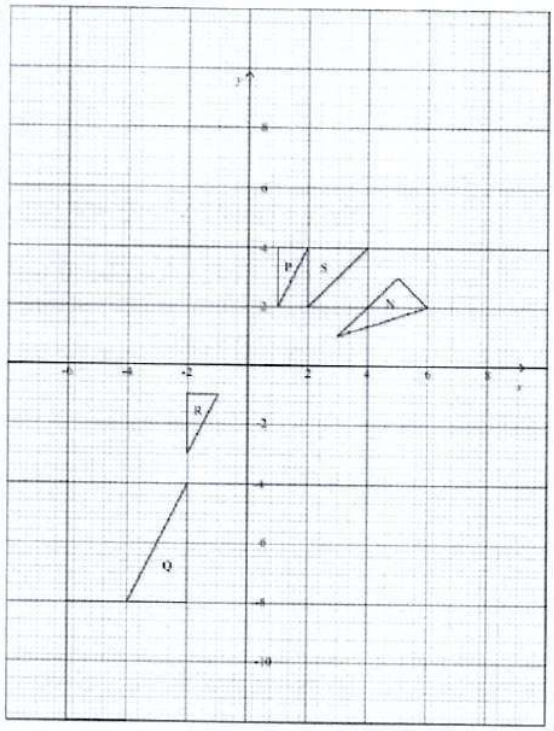
			Angles $S\hat{R}P$ and $P\hat{Q}S$ are subtended by arc PS.
(b)(iii)	$S\hat{P}R = 90° - 72°$ $= 18°$	1 1	PR is the diameter of the circle hence angle PSR is 90° The other two angles are complimentary meaning they add up to 90°
(b)(iv)	$P\hat{T}S = 180° - 2\times72°$ $= 180° - 144°$ $= 36°$	1 1	Use of the following circle theorems namely: The angle formed by a tangent and a chord at the point of contact is equal to the angle in the opposite segment Tangents from an external point to the same circle are equal
8 (a)(i)	$y \propto \frac{1}{\sqrt{x}} \leftrightarrow y = \frac{k}{\sqrt{x}}$ $\therefore k = y\sqrt{x}$ $k = 2\sqrt{9}$ $k = 6$ $y = \frac{6}{\sqrt{x}}$	1 1	Ability to express the statement as a general equation involving a constant of variation. Finding the particular equation/law from the general equation by substitution.
(ii)	$\frac{1}{2} = \frac{6}{\sqrt{x}}$ $(\sqrt{x})^2 = (12)^2$ $x = 144$	 1 1	Substitution for y Solving for x by removing the square root sign by squaring on both sides.
(b)(i)	Log(3x +1) + log(x-3) = log10. $\therefore log(3x+1) + log(x3) - log10 = 0$ $\therefore log_{10}\left[\frac{(3x+1)(x-3)}{10}\right] = 0$ $\therefore (10)^0 = \frac{(3x+1)(x-3)}{10}$ $1 = \frac{(3x+1)(x-3)}{10}$ $\therefore 10 = (3x+1)(x-3)$ $3x^2 - 8x - 13 = 0$	 1 1 1	Recall and apply the laws of Logarithms i.e. Log(MN) = log M+ log N, to do the correct reduction showing all stages clearly with no wrong working seen. 10^1 is based on the assumption that when the logarithm base is not given, the base is 10.

8 (b) (ii)	$3x^2 - 8x - 13 = 0$ $\Rightarrow a = 3,\ b = -8,\ c = -13$ But $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $= \frac{-(-8) \pm \sqrt{(-8)^2-4\times3\times(-13)}}{2(3)}$ $= \frac{8 \pm \sqrt{64+156}}{6}$ $= \frac{8 \pm \sqrt{220}}{6}$ $= -1{,}1$ or $3{,}8$	5	Application of the quadratic formula and giving answers to one decimal place. All stages to be clearly stated, however method of completing the square may still be used. $x^2 - \frac{8}{3}x = \frac{13}{3}$ What must be added to make $x^2 - \frac{8}{3}x$ a perfect square ? $\left(7 - \frac{4}{3}\right)^2 = \frac{13}{3} + \frac{16}{9}$ $\left(7 - \frac{4}{3}\right)^2 = \frac{55}{9}$ $x = \frac{4}{3} \pm \frac{\sqrt{55}}{3}$ $= \frac{4 \pm \sqrt{55}}{3}$ $= -1{,}1$ or $3{,}8$
9 (a) (i)	$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$ $= -\boldsymbol{a} + \boldsymbol{b}$	1	Appreciation of addition law on vectors. $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$
(ii)	$\overrightarrow{BN} = \frac{1}{3}\overrightarrow{BC}$ $= \frac{1}{3}(-\boldsymbol{a} + \boldsymbol{b})$ $= -\frac{1}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b}$		Ratio of $BN : NC$ $1 : 2$
(iii)	$\overrightarrow{AN} = \overrightarrow{AB} + \overrightarrow{BN}$ $= +\boldsymbol{a} + \left(-\frac{1}{3}\right)\boldsymbol{a} + \frac{1}{3}\boldsymbol{b}$ $= \frac{2}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b}$	*2*	*Use of then triangle law of vector addition.*
(iv)	$\overrightarrow{BM} = \overrightarrow{B\Delta} + \overrightarrow{\Delta M}$ $= -\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$	1	Triangle law of vector addition.

(b)	$\overrightarrow{AX} = \overrightarrow{AB} + \overrightarrow{BX}$ $= \boldsymbol{a} + h(-\boldsymbol{a} + \frac{1}{2}\boldsymbol{b})$ $= -\boldsymbol{a} - h\boldsymbol{a} + \frac{1}{2}h\,\boldsymbol{b}$ $= (1-h)\boldsymbol{a} + \frac{1}{2}h\boldsymbol{b}$	2	Express $\overrightarrow{BX}$ in terms of scalar h. Express $\boldsymbol{a} - h\boldsymbol{b}$ in simple / factorised form
9 (c)	$\overrightarrow{AX} = k(\frac{2}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b})$ $= \frac{2}{3}k\boldsymbol{a} + \frac{1}{3}k\boldsymbol{b}$	1	Use the fact that $AX = K\overrightarrow{AN}$ Substitute for $\overrightarrow{AN}$.
9 (d)	$(l-h)\boldsymbol{a} + \frac{1}{2}h\boldsymbol{b} = \frac{2}{3}k\boldsymbol{a} + \frac{1}{3}k\boldsymbol{b}$ $\therefore l - h = \frac{2}{3}k \quad (1)$ $\frac{1}{2}h = \frac{1}{3}k \quad (1)$ $\therefore 1 - \frac{2}{3}k = \frac{2}{3}k$ $1 = \frac{4}{3}k$ $\frac{3}{4} = k, h = \frac{2}{3} \times \frac{3}{4}$ $h = \frac{1}{2}$	4	Equate the coefficients of the vectors, coming from $(1-h)\boldsymbol{a} = \frac{2}{3}k$. Solve the two equations simultaneously using any method of solving simultaneously equations though in this case substitution method method is the shortest.
10(a)	$p = 2(-1) + 3 - (-1)^2$ $p = -2 + 3 - 1$ $p = 0$ $q = 2(1) + 3 - 1^2$ $q = 2 + 3 - 1$ $q = 4$	1 1	Correct substitution of -1 and $+1$ for x in the function $y = 2x + 3 - x^2$

(b)(i)	Smooth curve passing through the correct 7 points. 	4	The use of correct scale will make the graph fit. Plots should be visible and the graph should be drawn using free hand, not grossly thick and passing through the correct points.
(ii)	The line $y = -x$ correctly drawn to cut the curve in two places.	2	In general lines drawn should be at least 3 cm long unless stated otherwise.
(c) (i)	$x = 0{,}8$ $x = 3{,}8$	1 1	These values are the x-coordinates where the straight line cuts the curve.
(ii)	Area = 4,2 units^2	3	There should be evidence of marking of the correct region. Area should be found by counting squares and multiplying the number of squares, by area of one square, in this case its 1 units^2 Where one decides to work out the area in cm^2, getting 4 cm^2 equivalent to 1 unit^2. In this case the units must be stated.

11(a)(i)	$V = \left[\pi\times2^2\times1\frac{1}{2} + \pi\times3^2\times1\frac{1}{2} + \pi\times4^2\times1\frac{1}{2}\right]$ $-\pi\times1^2\times3\times1\frac{1}{2}$ $= 6\pi + 13{,}5\pi + 24\pi - 4{,}5\pi$ $= 39\pi$ $= 122{,}5\ \text{cm}^3$	1 1 1 1	Realise that it's the volume of the three discs minus the volume of the hole.
(ii)	$\frac{2{,}8g}{cm^3}\times122{,}5\ \text{cm}^3$ ***343g***	1 1	Use of formula for density to be used. $D = \frac{m}{v}$ $\therefore m = D\times v$
(iii)	Cost = \$7,50 x 343 = \$2 572,50	2	
(b)	Area $= \frac{1}{2}\times400\times440\times Sin46°$ = 63 301,9 m^2 = 63 301.9 ÷ 10 000 = 6,330 ha	4	Recall of the formula for finding area of a triangle given two sides and an included angle then converting the area to hectares by dividing by 1 0000. For those who cannot recall the formula Area $= \frac{1}{2}abSinC$ Follow the longer route of first finding a perpendicular height, h $\frac{h}{400} = Sin46$ $h = 400Sin46$ = 287,7 Area $= \frac{1}{2}$base×height $= \frac{1}{2}\times440\times287{,}7$ = 63 294 = 63 300 ÷ 10 000 = 6,330 ha

12(a)	Triangle P correctly drawn  	1	Make sure all vertices are clearly marked and joined by straight lines.
(b)	Triangle Q correctly drawn with vertices at $(-2; -4)$; $(-2; -8)$ and $(-4; -8)$	2	The coordinates of the vertices of the triangle can be found by multiplying by matrix for enlargement $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ i.e. $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & 4 \end{pmatrix}$ OR by drawing straight lines from each vertex passing through the centre of enlargement, in this case the origin and then measuring using the factor of 2, mindful that the image is inverted.
(c)	Triangle R correctly drawn at $(-2; -3), (-2; -1)$ and $(-1; -1)$	2	Translate each point by the given vector $\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} -3 \\ -5 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} -3 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -3 \\ -5 \end{pmatrix}$ OR using the graph to displace each point by $\begin{pmatrix} -3 \\ -5 \end{pmatrix}$ i.e. 3 units to the left followed by 5 units downwards.

(d)	$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ 2 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 5 & 6 \\ 1 & 3 & 2 \end{pmatrix}$ giving the points (3; 1), (5; 3) (6; 2)	3	Matrix multiplication
(i)	Triangle S drawn correctly.		Mark all points clearly and join them using a ruler
(ii)	One-way stretch y-axis ($x = 0$) invariant. Stretch factor 2	1 1 1	There is need to describe fully the single transformation by stating its name and properties.

ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Ordinary Level

MATHEMATICS 4004/1

PAPER 1

JUNE 2019 SESSION 2 hours 30 minutes

Additional materials:
Candidates answer on question paper
Geometrical instruments

Allow candidates 5 minutes to count pages before the examination.

TIME 2 hours 30 minutes

This booklet should not be punched or stapled, and pages should not be removed.

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces at the top of this page.
Write your centre number and candidate number in the box on the top right corner of every page of this paper.

Check that all the pages are in the booklet and ask the invigilator for a replacement if there are duplicate or missing pages.

Answer **all** questions.
Write your answers in the spaces provided on the question paper using **black** or **blue** pens.
If working is needed for any question, it must be shown in the space below that question.
Omission of essential working will result in loss of marks.
Decimal answers which are not exact should be given correct to three significant figures unless stated otherwise.
Answers in degrees should be given correct to one decimal place.

Mathematical tables or electronic calculators should not be brought in the examination room.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

Answer **all questions.**

NEITHER MATHEMATICAL TABLES NOR SLIDE RULES NOR CALCULATORS MAY BE USED IN THIS PAPER.

1. Express

a) $\frac{12}{25}$ as a decimal fraction,

Answer(a) ..... **[1]**

b) $\frac{2}{5}$ as a percentage,

Answer(b) ..... **[1]**

c) 0,0375 as a fraction in its lowest terms.

Answer(c) ..... **[1]**

2. Write down the next term in each of the following sequences.

a) 1 ; 4 ; 9 ; 16 ; 25 ; 36 ; _ _ _

Answer(a) ..... **[1]**

b) $\sqrt{2}$; $\sqrt{3}$; $\sqrt{5}$; $\sqrt{7}$; $\sqrt{11}$; _ _ _

Answer(b) ..... **[1]**

c) 16 ; 8 ; 4 ; 2 ; 1 ; _ _ _

Answer(c) ..... **[1]**

3. Three girls aged 12 years, 13 years and 15 years share \$100,00 in the ratio of their ages.

Calculate the amount of money that each girl receives.

Answer .....

.....

..... [3]

4. a) Convert

i) 434_5 to base ten,

Answer (a)(i) ..... [1]

ii) 75_{10} to base two.

Answer (a)(ii) ..... [1]

b) Evaluate $377_8 + 411_8$ leaving the answer in base 8.

Answer(b) ..... [1]