ZIMBABWE SCHOOL EXAMINATIONS COUNCIL
General Certificate of Education Advanced Level

PURE MATHEMATICS 6042/1
PAPER 1

NOVEMBER 2019 SESSION 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae MF7
- Scientific calculator

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed pages and 3 blank pages.

1 Solve the equation $3^{x-1} + 3^{x-2} = 12$. [3]

2 Find the term independent of y in the expansion of $\left(y^2 - \frac{4}{y}\right)^6$. [3]

3 Given that $y = \frac{1}{3\sqrt{(4x-1)}}$,

find the approximate percentage change in y if x is increased by 10% when $x = 7$. [4]

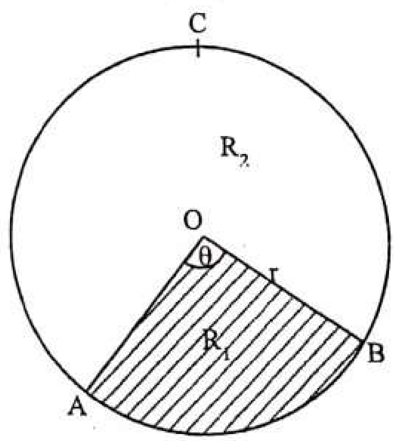
4 A complex number is given by $u = \frac{3+i}{2-i}$.

(a) Express u in the form $a + ib$ where a and b are real numbers. [2]

(b) Find the modulus and argument of u. [2]

(c) Show the complex number u on an Argand diagram. [1]

5



The diagram above shows a circle centre O and radius r, divided into two regions R_1 and R_2. $A\hat{O}B = \theta$ radians. The perimeter of the region R_1 is equal to the length of the major arc ACB.

(a) Show that $\theta = \pi - 1$. [2]

(b) Given that the area of region R_1 is 21 cm^2, find the area of region R_2 correct to 3 significant figures. [4]

6 (a) Express $3Cos\theta - 2Sin\theta$ in the form $RCos(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < 90°$. [2]

(b) Hence or otherwise solve the equation $3Cos\theta - 2Sin\theta = \frac{2}{3}$; for $-180° \leq \theta \leq 180°$. [3]

(c) Determine the maximum and minimum values of $\frac{1}{3Cos\theta - 2Sin\theta}$. [2]

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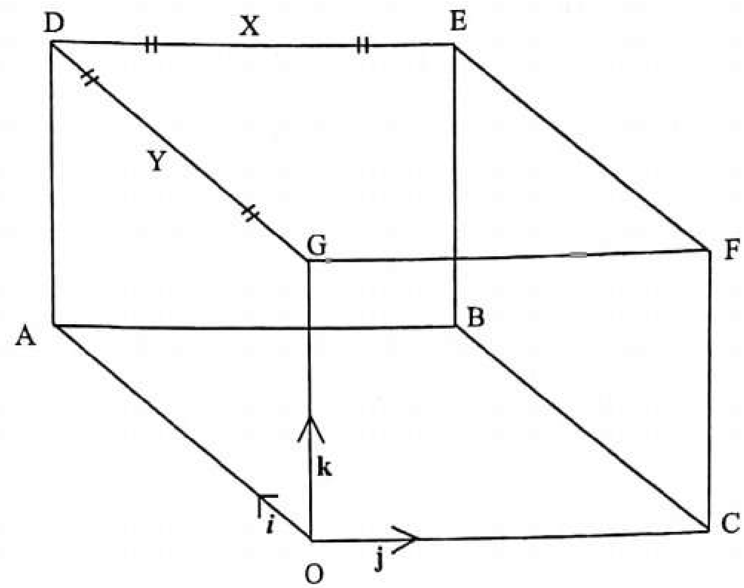

OABCDEFG is a perfect cube of edge 10 units. The unit vectors $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ are along OA, OC and OG, respectively. X and Y are the mid-points on ED and GD respectively.

Find

(a) $\overrightarrow{OX}$ in terms of i, j and k, [1]

(b) a unit vector parallel to $\overrightarrow{OE}$, [2]

(c) $X\hat{O}Y$. [4]

8 **(a)** Functions f and g are defined by:

$$f: x \rightarrow lnx;\ x > 0$$
$$g: x \rightarrow (x + 2);\ x > -2$$

Find

(i) $fg(x)$, [1]

(ii) the inverse of $fg(x)$. [2]

(b) Given that $h(\theta) = ksin\theta; k < 0$.

Sketch the graph of $y = h(\theta)$ for $0 \leq \theta \leq 2\pi$. [3]

(c) Describe the geometrical transformations which maps $f(x) = Sinx$ onto $y = 6Sin(x - \pi)$. [3]

[Turn over

9 (a) Find the sum to infinity of the geometric progression whose 4^{th} and 6^{th} terms are 64 and $\frac{256}{9}$ respectively, where the common ratio, $r > 0$. [5]

(b) In an arithmetic progression the first term is -20, the 13^{th} term is -5 and the last term is 15.

Find the sum of the terms of the progression. [6]

10 (a) Express $f(x) = \frac{4x+5}{(x+1)(2x+1)^2}$ in partial fractions. [5]

(b) Hence find the exact value of $\int_0^2 f(x)dx$. [4]

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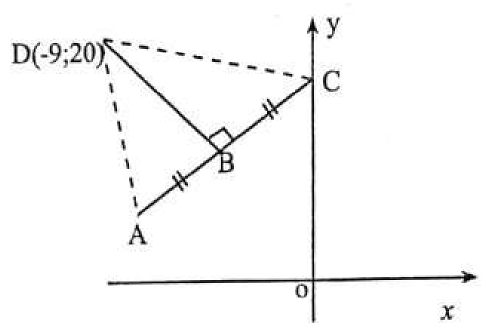

The diagram above shows points A, B and C lying on the line whose equation is $3y = x + 9$. The point C lies on the y-axis and AB = BC. The line from D$(-9; 20)$ to B is perpendicular to AC.

Calculate the

(a) co-ordinates of A and B, [6]

(b) exact area of triangle ADC. [3]

12 A curve C has equation $y = \sqrt{4x + 9}$.

(a) Find $\frac{dy}{dx}$, when $x = 4$. [3]

(b) A particle P moves along the curve C in such a way that the rate of increase of x is 0.20 units /second.

Find the rate of increase of y at the instant when $x = 4$. [2]

(c) Find the exact area enclosed by the curve, the y axis, the x – axis and the line $x = 4$. [4]

13 (a) The equation $x^4 + ax^3 + bx^2 - 2x - 4 = 0$ has roots -2 and 1.

(i) Find the values of a and b. [4]

(ii) Hence find the other roots. [4]

(b) (i) Express $4x^2 - 8x - 5$ in the form $a(x+b)^2 + c$ where a, b and c are constants. [2]

(ii) Find the co-ordinates of the stationary point of $4x^2 - 8x - 5$ and determine its nature. [2]

14 The height h metres of a tree at time t years after being planted assumes that the rate of increase of its height is directly proportional to $(10-h)^{\frac{1}{2}}$. When $t = 0$, $h = 1$ and $\frac{dh}{dt} = 0{,}4$.

(a) Show that this satisfies the differential equation $\frac{dh}{dt} = \frac{2}{15}(10-h)^{\frac{1}{2}}$. [3]

(b) Solve this differential equation to obtain an expression for h in terms of t. [6]

(c) Find the maximum height of the tree and the time taken to reach this height after planting. [2]

(d) Find the time taken to reach $\frac{2}{5}$ of its maximum height. [2]

15 (a) Find the exact value of $\int_1^3 x \ln x dx$. [4]

(b) (i) Show by sketching a suitable pair of graphs, that the equation $4 = x + \ln x$ has only one real root. [3]

(ii) Verify by calculation that this root lies between 2.9 and 3. [3]

(iii) Show also that the equation $4 = x + \ln x$ reduces to the equation $x_{n+1} = e^{4-x_n}$. [1]

(iv) Use the iteration $x_{n+1} = e^{4-x_n}$ twice with initial value $x_1 = 2.91$ to determine this root correct to 4 decimal places. [2]