ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

MATHEMATICS 9164/2

PAPER 2 PURE MATHEMATICS, MECHANICS, STATISTICS

JUNE 2017 SESSION 3 hours

Additional materials:
Answer paper Graph paper
List of Formulae Non-programmable electronic calculator

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

If a numerical value for g is necessary, take $g = 9.81\ ms^{-2}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

Within each section of the paper, questions are printed in the order of their mark allocations.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 7 printed pages and 1 blank page.

Section A: Pure Mathematics

1 (i) Express $\frac{3x^2+6x+7}{x^2+2x+1}$ in the form $\frac{A}{(x+B)^2}+C$ where A, B and C are constants. [3]

(ii) Hence state the sequence of transformations by which the graph of $y=\frac{3x^2+6x+7}{x^2+2x+1}$ may be obtained from the graph of $y=\frac{1}{x^2}$. [3]

2 Find the equation of a circle which passes through the points (–1; 0), (1; 2) and (–5; 4). [6]

3 Prove by induction that $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$. [7]

4 (a) (i) Find the coordinates of the turning point of the curve $y=e^x+4e^{-2x}$. [4]

(ii) Determine the nature of the turning point in (i). [2]

(b) Find the equation of the normal to the curve $x^2-3xy+y^2=5$, at a point (1; 4). [5]

(c) Find the general solution of the differential equation

$$x\frac{d\theta}{dx}=\cos^2\theta.$$ [2]

5 (i) Matrix $M=\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & -1\\ -1 & -2 & 3\end{pmatrix}$.

Find the inverse of matrix M. [7]

(ii) Hence or otherwise solve the following simultaneous equations:

$$x+y+z=-2$$
$$x+y-z=2$$
$$-x-2y+3z=3$$ [3]

(iii) Given also that $MN=\begin{pmatrix}1 & 1 & 1\\ 2 & 0 & 1\\ 0 & 1 & -2\end{pmatrix}$,

find matrix N. [3]

6 The position vectors of points A and B are $3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $7\boldsymbol{i} - 9\boldsymbol{j}$; respectively.

Line l has equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ and plane P has equation $\boldsymbol{r} \bullet \begin{pmatrix} 2 \\ -4 \\ -1 \end{pmatrix} = 8$.

(a) Show that

(i) A lies in the plane P, [2]

(ii) $\overrightarrow{\mathbf{BA}}$ is perpendicular to the plane P. [3]

(b) Calculate $\mathrm{O\hat{B}A}$ where O is the origin. [3]

(c) Find the acute angle between the line l and plane P. [3]

(d) Find the perpendicular distance from point B to plane P. [2]

7 **(a)** Given that the complex numbers $W_1 = 1 + ix$ and $W_2 = x + iy$, where x and y are numbers, satisfy the equation $W_1 - W_2 = 3i$,

find the value of x and the value of y. [4]

(b) Indicate by shading on a single Argand diagram the region in which both of the following inequalities are satisfied:

$$\frac{\pi}{4} \leq \arg z \leq \frac{\pi}{2}$$

$$|z - 3i| \leq 3.$$ [3]

(c) Use De-moivres' theorem to

(i) find the value of $\left(\cos\frac{1}{4}\pi + i\sin\frac{1}{4}\pi\right)^{12}$, [2]

(ii) show that $\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$. [5]

Section B: Mechanics

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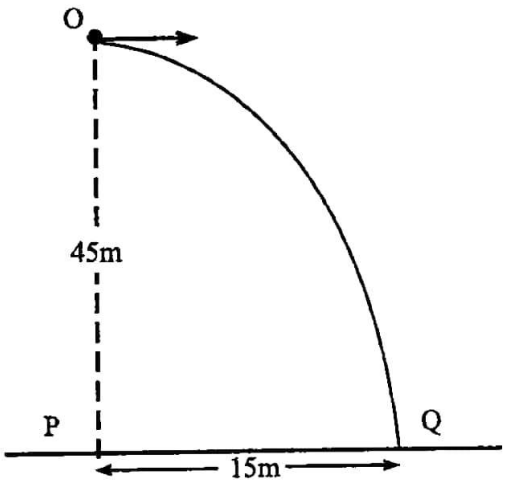

A particle is projected horizontally from a point O which is at a height of 45 m vertically above point P. Point P is on level ground.
The particle hits the level ground at point Q such that the distance PQ = 15 m (see diagram).

Calculate the

(i) time taken by the particle to reach point Q, [2]

(ii) speed of the particle at the point Q. [4]

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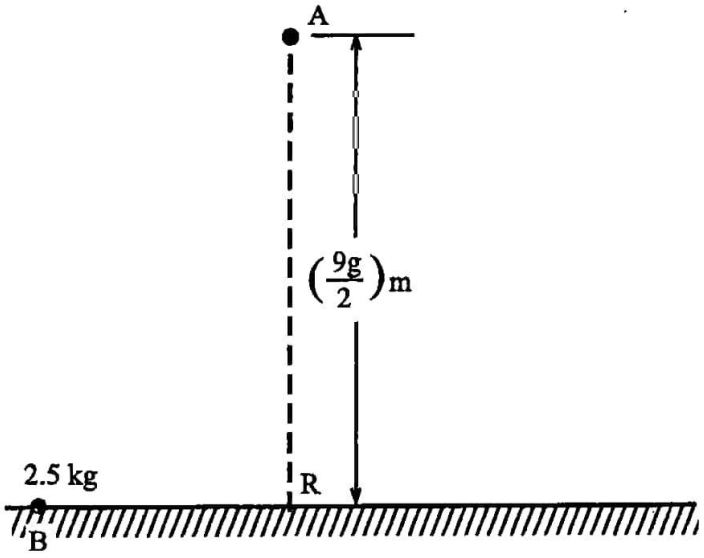

The diagram shows a particle, **A**, being dropped from a point $\left(\frac{9g}{2}\right)$ metres vertically above point R on a smooth horizontal surface. At the same instant particle **B** of mass 2.5 kg is accelerated from rest along the smooth horizontal surface at 4 ms^{-2} towards R.

The particles reach the point R at the same time.

(a) Find the value of the force acting on particle B. [1]

(b) Calculate the

(i) time taken by the particles to reach the point R, [2]

(ii) distance covered by the particle **B** to reach R. [2]

(c) Sketch the displacement-time graph for each particle on the same axes, labelling your graphs clearly. [3]

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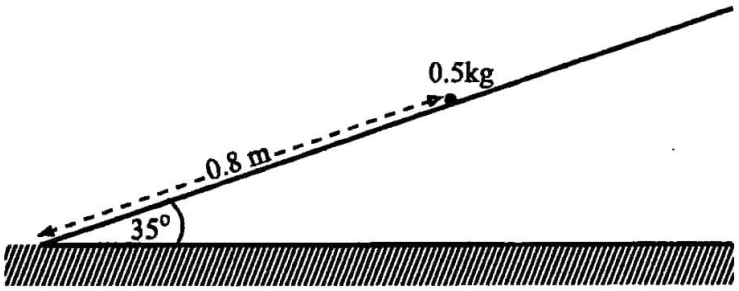

The diagram shows a particle of mass 0.5 kg resting in limiting equilibrium on a rough plane inclined to the horizontal at 35°. The particle is 0.8 m from the horizontal surface along the line of greatest slope.

(a) Calculate the coefficient of friction between the particle and the inclined plane correct to three decimal places. [3]

(b) The particle is held and the angle of inclination is increased to 65°. The particle is then released.

Calculate the

(i) time taken by the particle to reach the horizontal surface, [5]

(ii) velocity of the particle when it reaches the horizontal surface. [2]