ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

PURE MATHEMATICS

PAPER 1

6042/1

NOVEMBER 2018 SESSION

3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formula
- Scientific calculator

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

Questions are printed in the order of their mark allocations.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed pages and 3 blank pages.

1 The equation

$x^3 - 2x^2 + 4x - 8 = 0$ has $x = 2i$ as one of its roots.

Find the other roots. [3]

2 Solve the equation

$\text{Cos}(\theta - 75°) = \frac{\sqrt{6}-\sqrt{2}}{4}$, for $0° \leq \theta \leq 360°$. [3]

3 Solve the following simultaneous equations:

$e^{2x+y} = 1$

$4x - 3y = 10$ [3]

4 Solve the inequality

$\frac{21+13x}{2+3x^2} \geq 10.$ [4]

5 A curve has equation $y^2 = 3xy + x^2 - 3$.

Find the equation of the tangent to the curve at the point where

$y = 1$ and $x > 0$. [8]

6 The function $f(x) = -x^2 + 8x - 12$ is defined for $4 \leq x \leq 6$.

(i) Express $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [2]

(ii) State why the function of $f(x)$ has an inverse. [1]

(iii) Find $f^{-1}(x)$, the inverse of $f(x)$. [2]

(iv) Sketch the graphs of $f(x)$ and $f^{-1}(x)$ on the same axes and write down the domain of $f^{-1}(x)$. [3]

7 The complex number u is such that

$$\frac{-48 + 56i}{u} = 1 + 13i.$$

(a) Find u in the form $a + ib$, where a and b are real numbers. [4]

(b) (i) Write down u^*, the conjugate of u. [1]

(ii) Sketch u^* on an Argand diagram showing clearly the values of $|u^*|$ and arg. u^*. [3]

[3]

8 (a) Find $\int xe^{-2x}dx$.

(b) The area enclosed by the curve $y = 2x + e^{-2x}$, the $x-$axis, the lines $x = 0$ and $x = 1$ is rotated completely about the $x-$axis.

Find the volume generated, giving your answer correct to three significant figures. [6]

9 The functions $f(x) = \frac{15x^2+40x+2}{(4+x)(2+5x^2)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence or otherwise evaluate $\int_0^2 f(x)dx$, giving your answer as an exact single term. [4]

10 The points **A** and **B** have position vectors $-4\lambda\boldsymbol{i} - 3\lambda\boldsymbol{j} + 4\lambda\boldsymbol{k}$ and $3\boldsymbol{i} + 9\boldsymbol{j} - 6\boldsymbol{k}$, respectively relative to the origin O, where λ is a scalar. It is given that AB is perpendicular to OB.

(a) Calculate the

(i) value of λ,

(ii) angle AOB. [5]

(b) Calculate the

(i) coordinates of the centre of the circle passing through **O**, **A** and **B**,

(ii) radius of the circle passing through **O**, **A** and **B**. [4]

11 The expansion of $(1 + ax)^n$, where $n < 0$, has the coefficient of the terms in x and x^3 equal to -1 and $-\frac{14}{3}$ respectively.

(a) Show that n satisfies the equation

$$27n^2 + 3n - 2 = 0$$ [4]

(b) (i) Find the value of n.

(ii) Find the value of a.

(iii) Hence, state the values of x for which the expansion is valid. [5]

12 The function $f(x) = x^3 - 4e^{\frac{1}{2}x}$.

(i) Show that the equation $f(x) = 0$ has a root between 1.9 and 2.5. [3]

(ii) Show that the Newton – Raphson formula for finding the root of the equation in **(a)** above can be reduced to

$$X_{n+1} = \frac{2x_n^3 + 2e^{\frac{1}{2}x_n}(2 - x_n)}{3x_n^2 - 2e^{\frac{1}{2}x_n}}$$ [4]

(iii) Starting with $x_1 = 2$ use the formula to find x_2, correct to three decimal places. [2]

13 **(a)** The sum of the 5th and 10th terms of an arithmetic progression is 27. The third term is – 9.

Find the

(i) first term and the common difference, [4]

(ii) smallest value of n, for which $S_n > 0$, where S_n is the sum of the first n terms. [3]

(b) A geometric series has a common ratio $r < 0$, and its 4th and 6th terms are 2.56 and 1.6348, respectively.

Find the

(i) first terms and the common ratio, [3]

(ii) the sum to infinity of the series. [2]

14 A circle has centre $A(-3, k)$, where k is a constant. The tangent to the circle at $B(+3, 3k)$, has gradient $-\frac{3}{4}$.

(a) find the value of k. [2]

(b) Find the

(i) equation of the circle in the form

$$x^2 + y^2 + ax + by + c = 0,$$ [3]

(ii) values of x of the points where the circle crosses the x – axis, giving your answers correct to 2 decimal places, [3]

(iii) area of the minor segment enclosed by the circle and the x – axis. [5]

15 The function f(x) is give by $f(x) = \frac{1}{32}x + \frac{1}{2}(x-3)^{-1}, x \neq k$

(i) State the value of k. [1]

(ii) Find the coordinates of the turning points of the graph of $y = f(x)$, and investigate their nature. [9]

(iii) Show that the graph does not cross the x – axis. [3]