# ZIMBABWE SCHOOL EXAMINATIONS COUNCIL
**General Certificate of Education Advanced Level**

# PURE MATHEMATICS 6042/1

PAPER 1

**JUNE 2020 SESSION** 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae MF7
- Non-programmable electronic scientific calculator

**TIME** 3 hours

## INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

## INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of a non-programmable electronic scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

**This question paper consists of 4 printed pages.**

1 By means of the substitution $y = x^{\frac{1}{3}}$, or otherwise, find the values of $x$ for which $x^{\frac{1}{3}} - 3x^{\frac{-1}{3}} = 2$. [4]

2 (a) Find the value of $k$ for which the line $kx + (k-2)y + 10 = 0$ is parallel to the line $3x + 2y - 16 = 0$. [3]

(b) Find the gradient of the line perpendicular to both lines. [1]

3 (a) Express $6x^2 - 24x - 25$ in the form $A(x + B)^2 + C$, giving the numerical values of A, B and C. [2]

(b) Hence, or otherwise, solve exactly the inequality $6x^2 - 24x - 25 > 0$. [3]

4 Solve the inequality $|2x + 2| > 1 - 4x$. [5]

5 (a) Expand $(p - x)^{-2}$ as a series of ascending powers of $x$ up to the term in $x^3$ where $p$ is a positive constant. [5]

(b) Given that the coefficient of $x^2$ in the expansion is $\frac{3}{16}$, find the value of $p$. [2]

(c) Hence state the set of values of $x$ for which the expansion is valid. [1]

6 The points A, B and C have position vectors $\boldsymbol{a} = \boldsymbol{i} - 2\boldsymbol{j} + p\boldsymbol{k}, \boldsymbol{b} = q\boldsymbol{i} + 5\boldsymbol{j} + 6\boldsymbol{k}$ and $\boldsymbol{c} = 5\boldsymbol{i} + 7\boldsymbol{j}$ respectively relative to the origin.

If $\overrightarrow{AB} = -5\boldsymbol{i} + 7\boldsymbol{j} + 3\boldsymbol{k}$ find the

(a) values of $p$ and $q$, [2]

(b) exact length of AC, [2]

(c) acute angle BAC. [3]

7 The population of a city is 500 000. The population grows at a rate of 5% every year.

Find

(a) in terms of $n$, the population at the end of the $n$th year. [2]

(b) the population to the nearest thousand after ten years. [2]

(c) the year in which the population first exceeds 1 000 000. [3]

8 The function $f(x) = 2 - \frac{1}{x}, x > 0$.

(a) Sketch the graph of $f(x)$ and state the range. [3]

(b) Find $f^{-1}(x)$, the inverse of $f(x)$. [2]

(c) Calculate the value of $x$ for which $f(x) = f^{-1}(x)$. [3]

9 A circle with centre $(2, -5)$ touches the line $x + 6y - 9 = 0$. Find the equation of the circle in the forms $(x - a)^2 + (y - b)^2 = r^2$ where $a$, $b$ and $r$ are constants. [8]

10 (a) Complex numbers $w$ and $v$ are such that $u = 1 + 2i$ and $v = -2 - i$. Find $w = \frac{5u}{v}$, leaving the answer in the form $a + ib$, where $a$ and $b$ are integers. [3]

(b) Hence, or otherwise, find

(i) $|w|$, [2]

(ii) $\arg w$.

[3]

11 The polynomial $2x^3 - 11x^2 + ax + b$ is exactly divisible by $(x - 2)$ and leaves a remainder of $-36$ when divided by $(x + 1)$.

(a) Find the values of the constants $a$ and $b$. [6]

(b) Hence factorise the polynomial completely. [3]

12 $f(x) = \frac{x^3}{x^2 - 5x + 6}$ $x \in \mathbb{R}$.

(a) Express $f(x)$ in the form

$Ax + B + \frac{C}{x-2} + \frac{D}{x-3}$, where A, B, C and D are constants. [5]

(b) Hence find $\int_4^6 f(x)dx$. Leave the answer in exact form. [4]



[Turn over

**13** **(a)** Express $2\cos x - 5\sin x$ in the form $R\cos(x+\theta)$, where $R > 0$ and $0° < \theta < 90°$. [2]

**(b)** Hence, or otherwise, solve the equation $2\cos 2x - 5\sin 2x = 2.5$ for $0° \leq x \leq 360°$. [4]

**(c)** Give values of $x$ between $0°$ and $360°$ at which the maximum and minimum values of $2\cos 2x - 5\sin 2x$ occur. [4]

**14** **(a)** Use the trapezium rule with 4 ordinates to evaluate $\int_0^{1.5} x^3 \sin^2 x\,dx$, giving the answer correct to 3 significant figures. [3]

**(b)** (i) Find the area of the region bounded by the curve $y^2 = 4x$ and the line $y = x$. [5]

(ii) The region in (i) is rotated through 360° about the $y$-axis. Find the volume generated giving the answer in terms of $\pi$. [5]

**15** **(a)** It is given that $y = \frac{1}{1+\cos x}$.

Find

(i) $\frac{d^2y}{dx^2}$ when $x = 0$, [5]

(ii) the Maclaurin's series of $y$ up to the term in $x^2$. [4]

**(b)** Variables $x$ and $y$ are related by the equation $y = ab^x$, where $a$ and $b$ are constants. The graph of $\ln y$ against $x$ is a straight line of gradient 0.7 and $\ln y$ intercept at 2.3.

Find the values of $a$ and $b$. [6]