ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

MATHEMATICS

9164/2

PAPER 2 Pure, Mechanics and Statistics

JUNE 2018 SESSION   3 hours

Additional materials:
Answer paper
Graph paper
List of Formulae
Electronic calculator

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer *all* questions.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

If a numerical value for g is necessary, take $g = 9.81$ ms^{-2}.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

Within each section of the paper, questions are printed in the order of their mark allocations

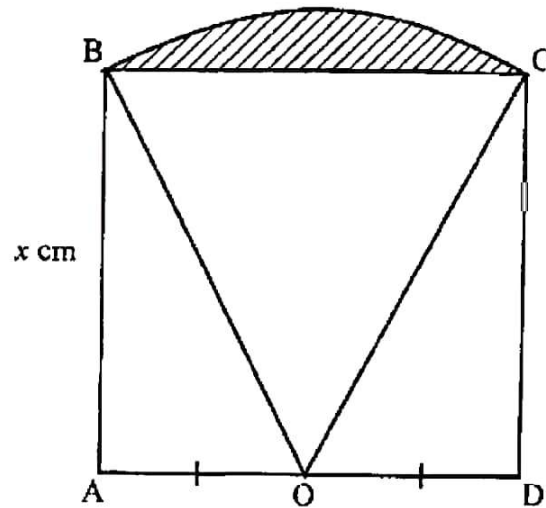The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

[Turn over

Section A : Pure Mathematics

1

ABCD is a square of side x cm. OBC is a sector of a circle centre O and radius OB.

Calculate, giving answers to 3 decimal places, the

(a)   angle AOB in radians,   [2]

(b)   area of the shaded region when $x = 5$.   [4]

2   Solve the simultaneous equations:

$$\log_2(8y) = x + 5$$

$$4^x = 3y$$ [7]

3   Find the turning points of the curve $y^3 + 3xy^2 - x^3 = 3$.   [7]

4   (a)   If matrix $A = \begin{pmatrix} 3 & 1 & 4 \\ 2 & 3 & 1 \\ 2 & 1 & 1 \end{pmatrix}$, find A^{-1}, the inverse of matrix A.   [5]

(b)   Hence, or otherwise, solve the simultaneous equations:

$$\begin{aligned} 3x + y + 4z &= 15 \\ 2x + 3y + z &= 12 \\ 2x + y + z &= 10 \end{aligned}$$

                                                                                                                 [4]

5 **(a)** Show that $(2r+1)^3 - (2r-1)^3 = (24r^2 + 2)$. [3]

(b) Prove by induction that a number $U_n = 2^{6n} + 3^{2n-2}$ is divisible by 5 for all positive integral values of n. [7]

6 **(a)** Find the real part of $\left(2 + \frac{1}{2}i\right)^8$, giving the answer in exact form. [6]

(b) It is given that $\left(x + 2\sqrt{2}\right)$ and $\left(x - 2\sqrt{2}\right)$ are factors of the polynomial

$$f(x) = x^4 - 6x^3 + ax^2 + bx - 104.$$

(i) Find the values of a and b. [5]

(ii) Hence, or otherwise, find the roots of the equation $f(x) = 0$. [5]

7 **(a)** A plane has vector equation $r \cdot \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = 1$ and the point P has coordinates $(-9; 17; -2)$.

(i) Find the coordinates of the foot of the perpendicular from P to the plane. [3]

(ii) Hence, or otherwise, find the coordinates of the image of P when reflected in the plane. [3]

(b) The plane π and the line l have equations

$$r \cdot \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = 6 \quad \text{and} \quad \frac{x-2}{1} = \frac{y-3}{2} = \frac{z+1}{-2} \quad \text{respectively.}$$

Find the

(i) coordinates of the point of intersection between π and l, [4]

(ii) angle between π and l, [3]

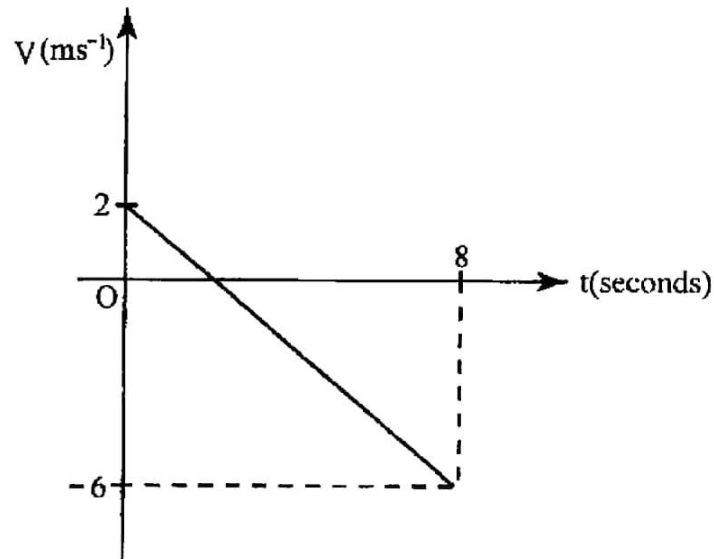**(iii)** shortest distance from the **origin** to l. [4]

Section B : Mechanics

8   A particle P is in equilibrium under the action of forces of magnitude
TN. TN, QN and 6N acting in the directions as shown in the diagram.
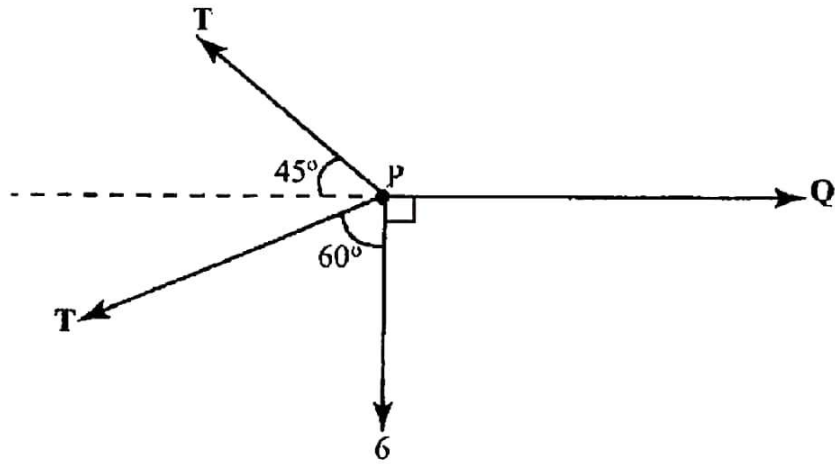
Find the exact values of **T** and **Q**.   [5]

9

The diagram is a velocity-time graph of a particle moving in a straight line.

(a)   Sketch the displacement-time graph.   [3]

(b)   Hence, or otherwise, find the average velocity of the particle.   [2]

10 A particle P of mass 8 kg rests on a rough plane of inclination $\tan^{-1}\left(\frac{3}{4}\right)$ to the horizontal. It is connected by a light inextensible string passing over a smooth pulley at the top of the plane to another particle Q of mass 2 kg which hangs freely.

The coefficient of friction between P and the plane is $\frac{1}{5}$.

Find the acceleration of the particles and the tension in the string. [6]

11 A particle is projected from the top of a vertical dam wall, 10 m above water level. The initial speed of the particle is 60 ms^{-1} at an angle of elevation of $\sin^{-1}\left(\frac{3}{5}\right)$.

Find the

(a) time the particle takes to reach the point whose horizontal displacement from the point of projection is 144m, [2]

(b) time taken by the particle to reach the water level, [3]

(c) maximum height attained by the particle above the water level. [3]

[Turn over

12 A die is weighted in such a way that the probability of each face coming up is proportional to the face value, x.

    (a) Construct the probability distribution function of X. [2]

    (b) Calculate $E(X)$. [2]

13 A and B played 12 games of chess of which 6 were won by A, 4 won by B and 2 ended in a draw. The two players agree to play a tournament of 3 games.

Find the probability that

    (a) two of the games end in a draw, [2]

    (b) B wins the tournament given that two of the games ended in a draw. [3]

14 The probability that an adult chosen at random from a particular group of adults is short sighted is $\frac{1}{5}$.

    (a) A committee comprising of 7 of the adults in the particular group, was chosen at random.

    Find the probability that at least one adult in the committee chosen was short sighted. [2]

    (b) A bus had 60 adult passengers from the same particular group of adults. Find the probability that at least 15 of the adult passengers were short sighted. [3]

15 Let X be a continuous random variable with a probability density function given by

$$f(x) = \begin{cases} ce^{-2x}, & \text{for } x > 0 \text{ where } c \text{ is a constant} \\ 0, & \text{otherwise} \end{cases}$$

    (a) Show that $c = 2$. [3]

    (b) Calculate the median. [2]

16 The sales (in US$) made by a vendor during the month of April are tabled below.

11	37	20	15	16	23
20	26	10	21	28	13
36	18	15	32	31	16
28	26	45	46	23	58
15	54	43	37	10	32

(a) Construct a stem and leaf diagram for the above data. [2]

(b) (i) Draw a box and whisker diagram.

    (ii) Comment on the nature of the distribution. [3]

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