Tarakino . N-P.

ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

PURE MATHEMATICS — 6042/2

PAPER 2

NOVEMBER 2019 SESSION — 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae MF 7
- Scientific calculator

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions in Section A and any **five** questions from Section B.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 6 printed pages and 2 blank pages.

Section A (40 marks).

Answer **all** questions in this section.

1 **(a)** Prove that

$$Sin3\theta \equiv 3Sin\theta - 4Sin^3\theta.$$ [3]

(b) Hence or otherwise evaluate

$$\int_0^{\frac{\pi}{3}} Sin^3\theta d\theta.$$ [3]

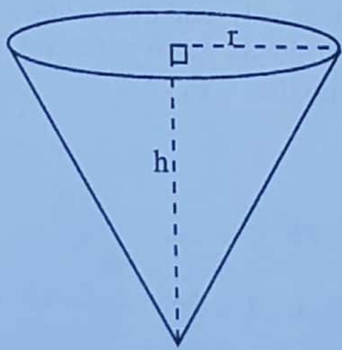
2 Find the two possible values of k for which the line $4y + 3x = k$ is a tangent to the curve $x^2 + y^2 - 4x - 21 = 0$. [7]

3 **(a)** Express $g(x) = \frac{2x-1}{x-3}$ in the form $g(x) = a + \frac{b}{x-3}$. [2]

(b) Hence state the correct sequence of transformation which transform the graph of $y = \frac{1}{x}$ onto the graph $y = g(x)$. [4]

(c) Sketch the graph of $y = g(x)$ showing clearly any asymptotes and intercepts with the axes. [3]

4



The diagram shows a container in the form of a right circular cone of height h and radius r. The height of the cone is 10 cm and the diameter is 2 cm. The container catches water from a tap leaking at a constant rate of 0.1 cm^3/s, the height and radius are always proportional.

Find the rate at which the top surface area of water is increasing when the water is halfway up the cone. [9]

[volume of a cone $= \frac{1}{3}\pi r^2 h$]

5 **(a)** By sketching a pair of two suitable graphs, determine the number of real roots of the equation

$$e^{-x} = 2x^2 - 1.$$ [3]

(b) Verify by calculation that one of the root lies between 0.5 and 1. [3]

(c) Use the iterative formula

$$x_{n+1} = \sqrt{\frac{1+e^{-x_n}}{2}},$$

taking $x_1 = 1$ to find the root of

$e^{-x} = 2x^2 - 1$, giving your answer to 3 decimal places. [3]

Section B (80 marks)

Answer any **five** questions from this section. Each question carries **16** marks.

6 (a) Evaluate exactly $\int_{-2}^{0} \frac{1+2x}{1-2x}\,dx$. [4]

(b) The rate of loss of temperature of a cooling body of temperature $\theta\ °C$ is proportional to the difference between the temperature of the body and that of its surroundings. The temperature of the surroundings at time t minutes, is $20°C$.

(i) Show that θ satisfies

$$\frac{d\theta}{dt} = -k(\theta - 20).$$

[2]

(ii) Given that the temperature of a body decreases from $60°C$ to $40°C$ in 5 minutes, solve the differential equation expressing θ in terms of t. [8]

(iii) Find the temperature of the body after 10 minutes. [2]

7 (a) If **S** is the set of all 2×2 matrices of the form $\begin{pmatrix} a & b \\ 2b & a \end{pmatrix}$, where a, b$\in \mathbb{R}$, show that **S** forms a group under addition of matrices. [7]

(b) Verify if the set $\{0; 1; 2; 3; 4\}$ forms a group under addition modulo 5. [9]

8 Two planes π_1 and π_2 have equations $x + 2y - 2z = 2$ and $2x - 3y + 6z = 3$.

(a) Show that the line l, with equation $\frac{x-2}{2} = y - 1 = \frac{z-1}{2}$ lies on π_1. [3]

(b) Find the

(i) equation of the line of intersection between the two planes, [6]

(ii) point of intersection of line l and π_2, [3]

(iii) angle between the line l and π_2. [4]

9 (a) Prove by induction the statement

$$a + ar + \ldots\ldots\ldots + ar^{n-1} = a\left(\frac{1-r^n}{1-r}\right) : r \neq 1$$

[7]

(b) (i) Express $\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}$ in the form $\frac{A}{r(r+1)(r+2)}$. [2]

(ii) Hence show that $\sum \frac{1}{r(r+1)(r+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$. [5]

(iii) Deduce the value of [2]

$$\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)}.$$

10 **(a)** The equation

$x^4 - 4x^3 + 3x^2 + 2x - 6 = 0$

has a root $1 - i$.

Find the other three roots. [6]

(b) The complex number z satisfies the inequalities $2 < |z| < 3$ and $\frac{\pi}{6} < \arg \boldsymbol{z} < \frac{\pi}{3}$.

Sketch and shade on an Argand diagram the region represented by the inequalities. [4]

(c) Solve the equation $Z^4 - 8\sqrt{3} + 8i = 0$ giving your answers in the form $a + ib$, correct to 2 decimal places. [6]

11 **(a)** A rhombus whose vertices are $(0; 0)$, $(1; 2)$, $(3; 3)$ and $(2; 1)$ is first reflected in the x – axis and sheared by the matrix $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. Find the vertices of the image of the rhombus. [4]

(b) It is given that the matrices $\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 1 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & -2 & -1 \\ -3 & 2 & 1 \\ 1 & 0 & -1 \end{pmatrix}$.

(i) Find **AB**. [2]

(ii) Hence or otherwise write down A^{-1}. [2]

(iii) Hence or otherwise solve the simultaneous equations:

$x + y = 3$
$x - z = 5$
$x + y + 2z = 7$

[4]

(c) Starting with $x_0 = 4$ use the Newton-Raphson method twice to find the root of $x^3 - 5x - 40 = 0$, correct to three significant figures. [4]

12 (a) (i) Find the Maclaurin series expansion for y up to and including the term in x^3 given that

$$\frac{dy}{dx} = y^3 + x^8 \text{ and } y = 1 \text{ when } x = 0.$$ [8]

(ii) Evaluate y when $x = 1.2$. [2]

(b) Use Taylor's theorem to obtain series expansion about $(x = 4)$ for $f(x) = e^{\sqrt{x}}$, up to and including the term in $(x-4)^2$. [6]

$$\left[\begin{array}{c} \textit{Tylor's series} \\ f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots = \\ \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \end{array}\right].$$