ZIMBABWE SCHOOL EXAMINATIONS COUNCIL
General Certificate of Education Advanced Level

MATHEMATICS

9164/1

PAPER 1   Pure

JUNE 2018 SESSION

3 hours

Additional materials:
   Answer paper
   Graph paper
   List of Formulae
   Electronic calculator

TIME   3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer all questions.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

Questions are printed in the order of their mark allocations.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 6 printed pages and 2 blank pages.

1   Solve the equation $e^{2x} = 4e^{2-x}$, giving the answer in exact form.   [3]

2   (a)   Express $f(x) = 13 + 4x - 2x^2$ in the form $a + b(x + c)^2$.   [3]

    (b)   Hence, or otherwise, state the coordinates of the turning point of the curve $y = f(x)$.   [1]

3   Solve the inequality

$$|x + 3| > 2|x|.$$
    [4]

4   The function $f(x)$ is defined by $f(x) = ln(x - 2)$

    (a)   State the domain of $f(x)$.   [1]

    (b)   On the same axes, sketch the graphs of $y = f(-x)$ and

$$y = |f(x)|.$$
    [4]

5   Relative to the origin O, the position vectors of the points A, B and C are

$$6i + 2j + 6k, \quad 2i + 3j + k \quad \text{and} \quad 4i + 5k, \text{respectively.}$$

    (a)   Calculate angle ABC.   [3]

    (b)   Determine the exact value on the area of triangle ABC.   [3]

6   The complex number $w = -2 + (2\sqrt{3})i$.

    Find

    (a) $|w|$ the modulus of w,   [1]

    (b)   the argument of the conjugate of w,   [2]

    (c) $\dfrac{w+1}{w}$ in the form $x + iy$.   [3]

7   Express $\dfrac{x^3 - x^2 - x - 4}{x(x^2 + 2)}$ in partial fractions.   [6]

8   Given that $\dfrac{dy}{dx} = x + 2y$ and that $y = 1$ when $x = 0$,

   (a)   show that $\dfrac{d^2y}{dx^2} = 5$,   [3]

   (b)   obtain the Maclaurin's series for y in ascending powers of x as far
          as the term in x^3.   [3]

9   (a)   Show that $\cos 2A \equiv \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$.   [3]

   (b)   By letting $2A = 45°$, deduce that $\tan^2 22{,}5° = 3 - 2\sqrt{2}$.   [4]

10   The polynomials $f(x) = 2x^3 + 7x^2 + ax + b$ and $g(x) = x^3 + ax^2 - 5x + 2b$

   have a common factor $x + 3$.

   Find

   (a)   the value of a and the value of b,   [4]

   (b)   a second common factor.   [3]

11   Given.that $y = \dfrac{2-x}{x^2-3}$, find the coordinates of the turning points and determine

   the nature of each turning point of the curve.   [7]

12   (a)   The straight line L passes through the point $(4; -1)$ and is
          perpendicular to a line with equation $2x - 3y = 10$.

          Find the equation of line L, leaving the answer in the form
          $ax + by = c$.   [3]

[Turn over

(b)   The line $y + 2x = 0$ intersects the curve $2x^2 + y^2 + 4x - 3y = 4$ at the points P and Q.

Find the

(i)   coordinates of P and Q,   [6]

(ii) length of line PQ.   [2]

13   (a)   The salary scale of an employee begins at $315 a month and rises to a maximum of $765 by equal monthly increments of $25.

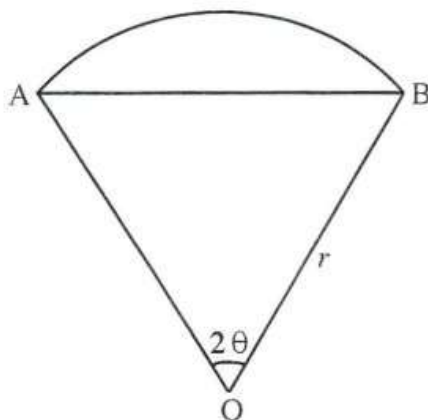Find the number of months it takes for the salary to reach the maximum.   [3]

(b)   Show that

$$\sum_{r=1}^{16} (\sqrt{3})^r = 9\,840 + 3\,280\,\sqrt{3}.$$

[4]

(c)   Find the term independent of x in the expansion of $\left(\dfrac{1}{x^2} - \dfrac{2x}{3}\right)^6$.   [4]


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In the diagram, OAB is a sector of circle centre O of radius r and $A\widehat{O}B = 2\theta$ radians.

(a)   Find in terms of r and θ expressions for the length of

(i)   arc AB,   [1]

(ii) chord AB.   [2]

(b) The length of the arc AB is 6 cm and the length of chord AB is 5 cm.

    (i) Show that the angle θ satisfies the equation $5\theta = 6 \sin \theta$.   [2]

    (ii) Verify that the equation $5\theta = 6 \sin \theta$ has a root, θ, between 0.9 and 1.1.   [2]

    (iii) Taking $\theta_1 = 1$ as the first approximation to the root, apply the Newton-Raphson method twice to find θ, giving the answer to 3 decimal places.   [4]

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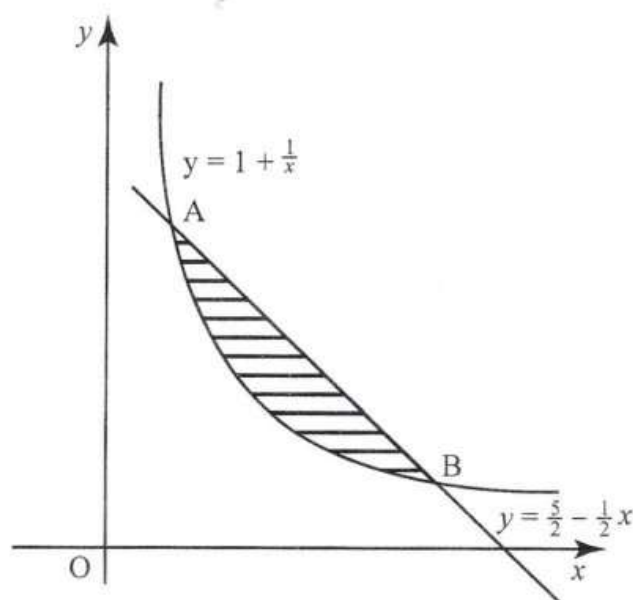
The diagram shows the shaded region enclosed by the line $y = \dfrac{5}{2} - \dfrac{1}{2}x$

and the curve $y = 1 + \dfrac{1}{x}$, $x > 0$. The curve and the line intersect at points A and B.

Find the

(a) coordinates of A and B,   [4]

(b) exact value of the area of the shaded region.   [3]

(c) volume of revolution generated when the shaded region is rotated about the x - axis through 2π radians, leaving the answer in exact form.   [6]

[Turn over

16 **(a)** Solve the differential equation $\frac{dy}{dx} = y \tan x$, given that $y = 1$ when $x = 0$, expressing y in terms of x. [4]

(b) A spray for cockroaches kills at a rate which is inversely proportional to the square root of the number, N, of live cockroaches at time, t minutes after spraying.

(i) Show that the information can be modelled by the differential equation

$$\frac{dN}{dt} = -\frac{k}{\sqrt{N}}$$

[2]

(ii) Solve the differential equation given that initially there were 144 live cockroaches and 36 minutes later, 81 live cockroaches remained. [5]

(iii) Find the number of live cockroaches left one hour after spraying. [2]

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MARKING SCHEME

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