For Performance Measurement

ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

PURE MATHEMATICS

PAPER 2

6042/2

NOVEMBER 2018 SESSION

3 hours

Additional materials:
Answer paper
Graph paper
List of Formula
Scientific calculator

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions in Section A and **any five** questions from Section B.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

Questions are printed in the order of their mark allocations.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 7 printed pages and 1 blank page.

[Turn over

Section A (40 marks)

Answer **all** questions in this section.

1 The polynomial $P(x) = ax^2 + bx + c$ is divisible by $x - 1$. When divided by $x + 1$ and $x - 2$ it leaves remainders of 2 and 8 respectively. Find the values of a, b, and c. [6]

2 **(a)** Show by calculation that the equation

$4 \text{ Cos } \frac{\pi}{2}x = 6 - x$ has a root between 3 and 4. [3]

(b) Use Newton Raphson method twice to find the root correct to two decimal places starting with $x_1 = 3.5$. [4]

3 **(a)** Use the substitution $x = \text{Cos}^2\theta$ to show that

$$\int \sqrt{\frac{x}{1-x}}\,dx = -\int 2\,\text{Cos}^2\theta\, d\,\theta.$$ [4]

(b) Hence evaluate $\int_0^{\frac{1}{2}} \sqrt{\frac{x}{1-x}}\,dx$ correct to 3 decimal places. [4]

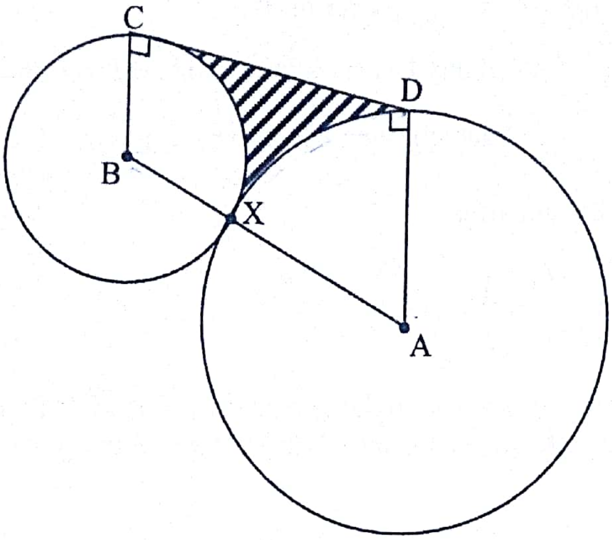
4 **(a)** Express $\frac{2x^2-7}{(x-3)\,(2x+5)}$ in the form $A + \frac{B}{x-3} + \frac{C}{2x+5}$. [5]

(b) Hence evaluate

$$\int_4^5 \frac{2x^2-7}{(x-3)\,(2x+5)}\,dx$$

correct to 3 decimal places. [4]

5



The diagram shows two circles touching at X. The bigger circle has centre A and radius 9 cm. the smaller circle has centre B and radius 3 cm. DC is a common tangent to both circles.

Find the

(i) perimeter of ABCD. [4]

(ii) angle BAD. [2]

(iii) exact area of the shaded region. [4]

Section B

Answer any **five** questions in this section.

Each question carries 16 marks.

6 **(a)** The line L_1 has equation

$$r = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

and the plane π_1 passes through the points A, B and C with coordinates $(2;\ -1;\ 3)$, $(4;\ 2;\ -5)$ and $(-1; 3;\ -2)$ respectively.

Find the

(i) Cartesian equation of π_1. [5]

(ii) acute angle between the plane π_1 and the line L_1. [3]

(b) The plane π_2 has equation

$$r \cdot \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = 6.$$

The line L_2 lies in the plane π_2 and is perpendicular to L_1. The line L_2 passes through the point $(4;\ 2;1)$.

Find the

(i) vector equation of L_2, [4]

(ii) vector equation of the line of intersection of the planes π_1 and π_2. [4]

7 **(a)** Solve the equation $z^2 + 3z + 5 = 0$, giving the answers in the form $x + iy$. [5]

(b) **(i)** On a sketch of an argand diagram show the locus representing the complex numbers satisfying the equation

$$|z| = |z - 3 - 5i|.$$ [2]

(ii) Find the

1. complex number represented by the point on the locus where $|z|$ is least. [1]

2. modulus and argument of the complex number in 1. giving the argument correct to 2 decimal places. [3]

(c) Use De Moivre's theorem to express $Cot\ 5\theta$ in terms of $Cot\ \theta$. [5]

8 (a) Prove by mathematical induction that for all positive integers, n, $10^n + 3\times4^{n+1} + 5$ is divisible by 3. [6]

(b) Under a 2×2 matrix M the images of A(1,0) and B(0,1) are $A_1(3,5)$ and B_1 (5,9) respectively.

Find the

(i) matrix M. [2]

(ii) image of point (2; -5) under matrix M. [2]

(c) Given the matrix

$$P = \begin{pmatrix} 2 & 6 & 1 \\ -1 & 2 & -1 \\ 4 & 3 & 1 \end{pmatrix},$$

find the inverse of matrix P. [6]

9 (a) (i) Show that the set of integers

$\{1, 3, 5, 7\}$ under multiplication modulus 8 forms a group. [7]

(ii) Write any **three** subgroups. [3]

(b) The set $\{f, g, h, k\}$ under the operation of composition of functions forms a group H, where

$$f: x \longrightarrow x$$

$$g: x \longrightarrow \frac{1}{x}$$

$$h: x \longrightarrow -x$$

$$k: x \longrightarrow -\frac{1}{x}$$

Show the operation table for H. [6]

10 (a) (i) Find

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)}.$$ [6]

(ii) Hence or otherwise deduce the value of

$$\sum_{r=1}^{\infty} \frac{2}{r(r+1)(r+2)}.$$ [2]

(b) Use the series expansion of Cos x to expand $(1+x)^2$ Cos x up to and including the term in x^3. [4]

(c) Given that $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 0$ where $y = 1$ at $x = 0$ and $\frac{dy}{dx} = 2$

at $x = 0$.

Use Taylors' series to find the first three terms of $(1+x)^2$ Cos x. [4]

11 (a) Solve the inequality

$$\frac{x-2}{x+1} < \frac{x-6}{x-2}.$$ [6]

(b) It is assumed that the length, l, in cm of a certain snake at time, t, months after birth increases at a rate proportional to $(10 - l)^{\frac{1}{2}}$.

When $t = 0$ $l = 1$ and $\frac{dl}{dt} = 0{,}3$.

(i) Show that l and t satisfy the differential equation

$$\frac{dl}{dt} = 0.1(10 - l)^{\frac{1}{2}}.$$ [2]

(ii) Solve the differential equation and obtain an expression for l in terms of t. [6]

(iii) Find the maximum length of the snake. [2]

12 (a) The equation $y = \sqrt{x}$ is used to approximate the square root of a number.

Find an approximation for $\sqrt{16.01}$. [4]

(b) The function f is defined by

$$f(x) = 1 + Cos\,x \qquad 0 \leq x < 2\pi.$$

Sketch on separate diagrams the graphs of the following functions showing clearly all intercepts and turning points.

(i) $y = f(x)$ [2]

(ii) $y = |f(x) - 3|$ [3]

(iii) $y = f\left(\frac{x}{2}\right)$ [2]

(iv) $y = f\left(x - \frac{\pi}{2}\right)$ [2]

(v) $y = -2f(x)$ [3]