ZIMBABWE SCHOOL EXAMINATIONS COUNCIL
General Certificate of Education Advanced Level

PURE MATHEMATICS 6042/2

PAPER 2

JUNE 2020 SESSION 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae MF7
- Non-programmable electronic scientific calculator

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions in **Section A** and any **five** questions from **Section B**.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

If a numerical value for g is necessary, take $g = 9.81 \text{ ms}^{-2}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of a non-programmable electronic scientific calculator is expected, where appropriate.

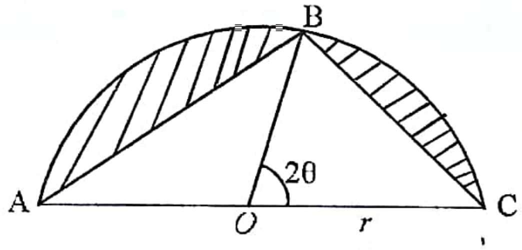
You are reminded of the need for clear presentation in your answers.

This question paper consists of 6 printed pages and 2 blank pages.

Section A [40 marks]

Answer **all** questions in this section.

1



In the diagram above, ABC is a semi-circle with centre O and radius r cm. $B\hat{O}C = 2\theta$ radians.

Find

(a) in terms of θ and r, the perimeter of the triangle ABC. [3]

(b) the exact area of the shaded region when $\theta = \frac{\pi}{6}$ rad and $r = \sqrt{2}$. [3]

2 **(a)** Prove the identity $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} \equiv \sec\theta - \tan\theta$. [3]

(b) **(i)** Express $\sin x - \sqrt{3}\cos x$ in the form $R\sin(x-\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [2]

(ii) Hence or otherwise, solve the equation

$$\sin x - \sqrt{3}\cos x = -1, \text{ for } 0 \leq x \leq 2\pi. \quad [3]$$

3 **(a)** It is given that $f(x) = \frac{\sin(x+30°)}{\cos(x+30°)}$.

If x is small enough to neglect terms in x^3 and higher powers, show that

$$f(x) = \frac{2+2\sqrt{3}x-x^2}{2\sqrt{3}-2x-\sqrt{3}x^2}. \quad [5]$$

(b) Hence find the percentage error in calculating tan30.5°, giving the answer correct to one decimal place. [3]

4 **(a)** A sequence U_n is defined for real numbers by

$$U_{n+1} = \frac{U_n}{2+U_n}.$$

When $U_1 = 3$, find

(i) U_2, U_3 and U_4. [2]

(ii) Hence describe the behaviour of the sequence as n increases. [1]

(b) The first three terms of an arithmetic progression have a sum of 21 and a product of 231.

Find the **two** possible values of the tenth term. [6]

5 The functions f and g are defined as

$$f(x) = 4 - x^2, x \in \mathrm{R},$$
$$g(x) = \sqrt{x}, x \geq 0.$$

(a) State the range of f. [1]

(b) Find the domain of $gf(x)$. [4]

(b) Sketch the graph of $y = |f(x)|$, indicating the intercepts with the axes. [2]

(d) State, with a reason, whether $f(x)$ is injective or not. [2]

Section B [80 marks]

Answer any **five** questions from this section. Each question carries 16 marks.

6 **(a)** The matrix $\begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$ maps points on the line $y = 4x + 8$ onto another line. Find the coordinates of the only invariant point. [5]

(b) Given the matrix $A = \begin{pmatrix} 5 & 2 & 1 \\ 2 & 8 & -1 \\ 3 & -1 & 3 \end{pmatrix}$,

(i) find the inverse of A, [6]

(ii) hence or otherwise, solve the simultaneous equations

$$5x + 2y + z = 5$$
$$2x + 8y - z = 11$$
$$3x - y + 3z = 18$$ [4]

(iii) give a geometrical interpretation of your answer in (ii) above. [1]

7 The equations of a line l and a plane π_1 are given by

$$l:\ \boldsymbol{r} = \begin{pmatrix} 3 \\ 5 \\ -3 \end{pmatrix} + t\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}, t \in \mathbb{R}$$

$$\pi_1;\ \boldsymbol{r}.\begin{pmatrix} 6 \\ m \\ 2 \end{pmatrix} = 5.$$

The point A has position vector $\begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix}$.

(a) Find the value of m if

(i) A lies on π_1. [2]

(ii) π_1 and l are parallel. [2]

(b) Calculate the shortest distance from A to l. [5]

(c) Obtain the Cartesian equation of the plane π_2 that contains both A and l. [5]

(d) Find the angle between π_2 and l. [2]

8 **(a)** Find the exact value of $\int_0^{\frac{\pi}{8}} cos^2\,(2x)dx$. [4]

(b) In a chemical reaction a substance A is continuously transformed into chemical B. The sum of the volumes of A and B is always constant and equal to H throughout the reaction. At time t hours after the start of the reaction, the volume of B is denoted by V. The rate of the formation of B is proportional to the volume of A present at any instant.

(i) Show that the above situation satisfies the differential equation $\frac{dV}{dt} = k(\mathrm{H} - \mathrm{V})$. [1]

(ii) Given that $\mathrm{V} = \frac{2}{3}\mathrm{H}$ when $t = ln9$, solve the differential equation, expressing V in terms of H and t. [8]

(iii) Calculate the value of t when V = 0.8 H. [2]

(iv) Sketch the graph of V against t. [1]

9 **(a)** Shade the region represented by the complex number z satisfying the inequalities $|2z - 1 - 4i| < 2$ and $\mathrm{Re}(z) \leq 0$, on an Argand diagram. [4]

(b) Given the polynomial $f(x) = x^3 - 5x^2 + 16x - 30$,

(i) show that $z = 1 + 3i$ is a root of the equation $f(x) = 0$, [3]

(ii) find the other two roots of $f(x) = 0$. [3]

(c) Use de Moivre's theorem to show that

$$Cos6\theta \equiv 32cos^6\theta - 48cos^4\theta + 18cos^2\theta - 1.$$ [6]

10 **(a)** Prove by induction that $6^n - 5n + 4$ is divisible by 5 for all positive integral values of n. [6]

(b) **(i)** By sketching a pair of suitable graphs, show that the equation $xln(x + 1) - 1$ has exactly two real roots. [3]

(ii) Verify by calculation that a root lies between 1.2 and 1.3. [3]

(iii) Using $x_1 - 1{,}25$ as the first approximation to the root, apply the Newton-Raphson method twice to find an approximate to the root. Give the answer to 3 decimal places. [4]

11 **(a)** A binary operation * is defined by

$$a * b = \frac{a+b}{4}.$$

Solve the equation $(3* x) * 1 = 1$. [3]

(b) Show that the set S = {0; 1; 2; 3} forms a group under addition modulo 4. [4]

(c) Prove that the set of integers B = {1; 3; 4; 5; 9} forms a group under multiplication modulo 11. [9]

12 **(a)** **(i)** Show that $r(r + 1)(r + 2) - (r - 1)(r)(r + 1) \equiv 3r(r + 1)$. [2]

(ii) Hence or otherwise show that

$$\sum_{r=1}^{k} r(r+1) = \frac{k}{3}(k+1)(k+2)$$ [6]

(b) **(i)** Use Taylor's theorem to obtain a series expansion for

$Cos\left(x + \frac{\pi}{4}\right)$ about $x = \frac{\pi}{4}$ up to and including the term in x^3. [6]

(ii) Hence evaluate cos46° leaving the answer in exact form. [2]

Taylor's series

$$\left[f(x) = f(a) + f\frac{'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n \right]$$