ADCHAKARA

VIDEO TUITION

REVISION QUESTIONS WITH SOLUTIONS

Circle Geometry

CIRCLE GEOMETRY

Revision E-book

By

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About the Author

Admire is an experienced and practicing teacher who offers online lessons through video lessons, daily exercises and weekly tests. Since 2014, Admire has offered the service to hundreds of both Zimsec and Cambridge candidates.

The revision guides, video lessons and exercises enable students to cover the syllabus and prepare for the final exam in just 6 months.

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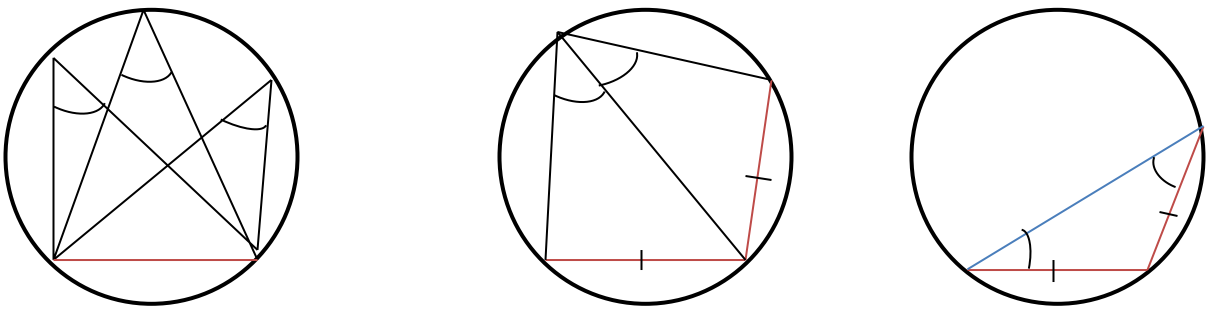
1. BASICS – angle sub tension
2. Cyclic quadrilateral – opposite sides, exterior angles,
3. Tangents
4. Similarity and congruency
5. Revision questions
6. Answers to Revision Questions

1. BASICS

In this lesson we are going to look at the basics of circle geometry

(a)Angle Subtension

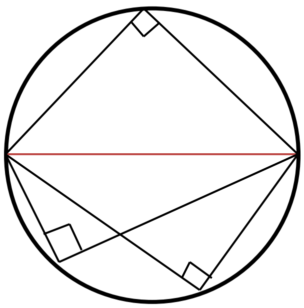
Angle subtended by the same arc or equal arcs



The diagrams above show angles subtended by same arc or arcs of same size.

The rule from this principle is **angles subtended by the same arc or arcs of same length are equal**

Angle subtended by the diameter

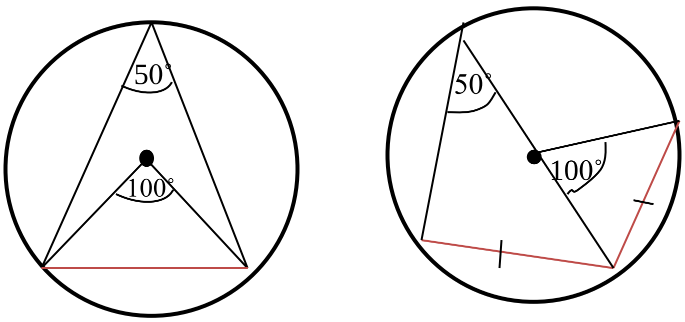


The diagram shows the diameter (in red) subtending three angles at the circumference.

The rule of the principle is:

Angles subtended by the diameter at the circumference are right angles

Angle at the center and at the circumference



Angles at the center is twice bigger than that at the circumference is subtended by the same arc or arcs of same length

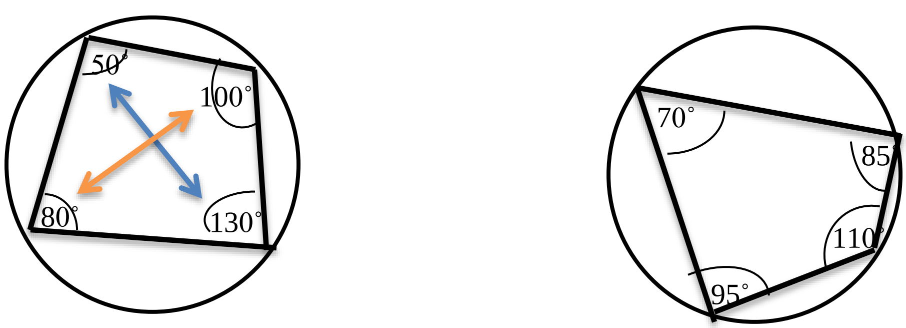
(b) CYCLIC QUADRILATERALS

Cyclic quad is another concept of circle geometry. Few things have to be noted.

- The sum of its angles remains 360˚
- All four corners must be on the circumference
- It must have four sides

(i) Opposite sides

Opposite sides of a cyclic quad are supplementary [they add up to 180˚]



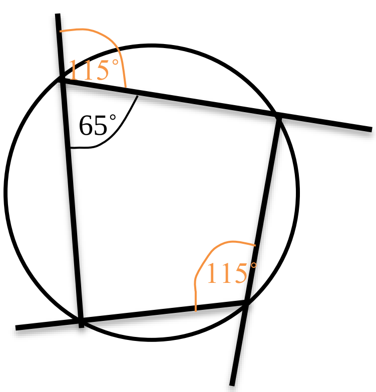
The angles a and c adds up to 180˚, this means a + c = 180 and b + d = 180.

On the other circle, you can see that the opposite angles add to 180˚

(ii) Exterior angles

Exterior angles of a cyclic quad are useful in calculating angles in circle geometry

Lets look at this

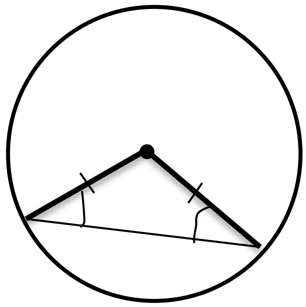


An exterior angle of a cyclic quad is always equal to the opposite angles

NB: interior and exterior angles are also supplementary

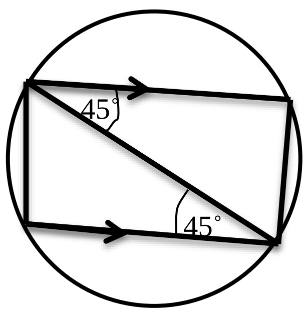
Important Facts in Circle Geometry

1. Radii are equal and form isosceles triangle when they meet at the center



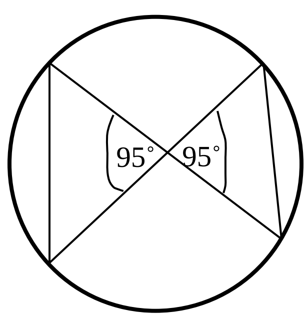
The diagram shows that radii are equal and form isosceles triangle with two equal base angles

2. Parallel lines always form alternative and corresponding angles



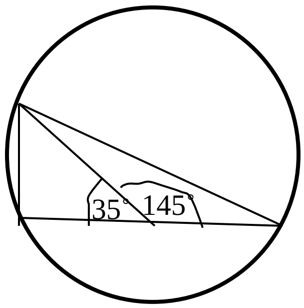
The diagram shows formation of alternate and corresponding angles in the cyclic quad

3. Crossing lines form equal opposite angles



The a diagram shows formation of equal opposite angles when two lines cross each other

4. Angles on a straight line add up to 180



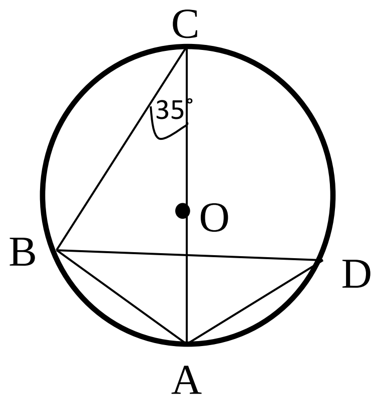
The diagram shows angles on a straight line and that they are supplementary

5. Sum of a Triangle angles remains 180

Whenever they is formation of a triangle in circle geometry, complete the angles using the sum of angles. Like in Fact 1, the isosceles triangle is filled to add up to 180°

Examples

1.

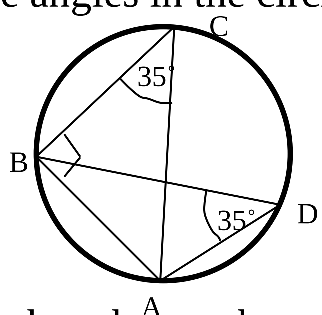


AC is a diameter to the circle. Given that $A\hat{C}B = 35^\circ$, calculate

(a) ADB (b) ABC [N96/P1]

Solution:

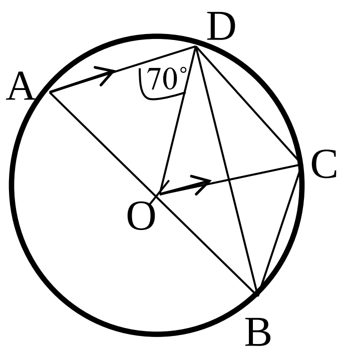
Tip: Fill all the angles in the circle to easily calculate the required



(a) Tip: look at the arc that subtends the angle and locate any other angle subtended by the same arc/chord
Therefore ADB = 35° [subtended with ACB at the same arc]

(b) Tip: look at the chord/arc that subtends the angle, you cab notice that it's a diameter.
Therefore ABC = 90°

2.



A, B, C and D are points on a circle, center O. AB is a diameter and AD is parallel to OC. Given that angle ADO = 70˚, calculate

(a) BOD (b) BDO (c) BCD

Solution:

(a) Tip: there are many ways to solve this. You can see that OA and OD are radii and that radii are equal, this means OAD = 70˚ and that AOD = 40˚ [180 – 140]
Since BOD is an adjascent angle of AOD, there are supplementary THEREFORE BOD = 140˚

(b) Since ADB is subtended by the diameter, it's a right angle 90˚. Therefore we are given ADO = 70˚, this makes BDO = 20˚

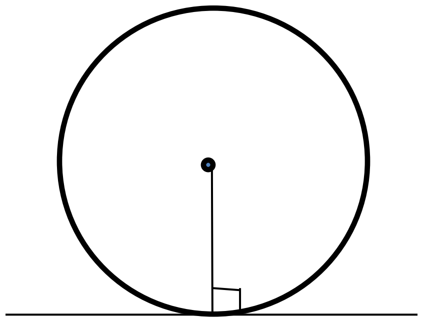
(c) ABCD is a cyclic quad and BCD is opposite to BAD = 70˚, this makes BCD = 110˚ [supplementary angles]

TANGENTS

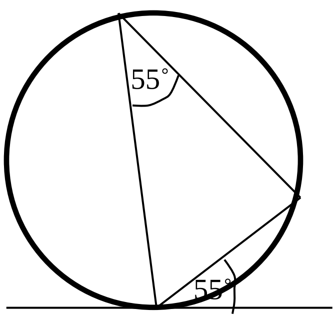
A tangent is a line that touches the circle. The point where it touches the circle is called , the point of tangency.

There are three principles to consider on tangents

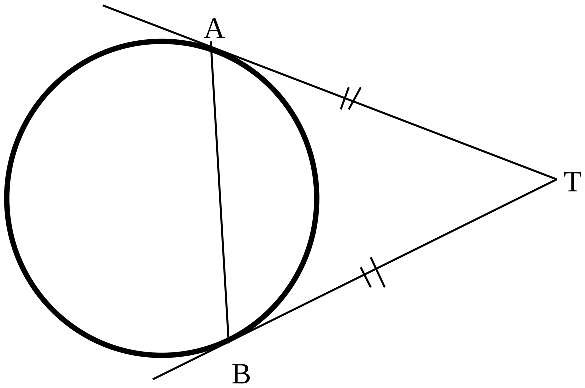
1. Tangent is perpendicular to radius/diameter angle)



2. Tangent forms an alternate angle to a triangle inside a circle Opposite interior angle will be equal to opposite exterior angle formed by a tangent



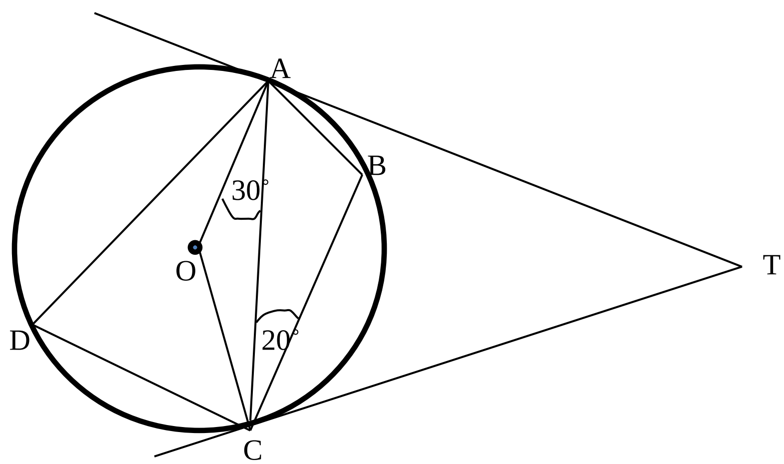
3. When tangents to the circle meet, they are equal from the point of tangency to the meeting point
   This, however forms an isosceles triangle with equal base angles



In the diagram, AT = BT.

Examples

1.



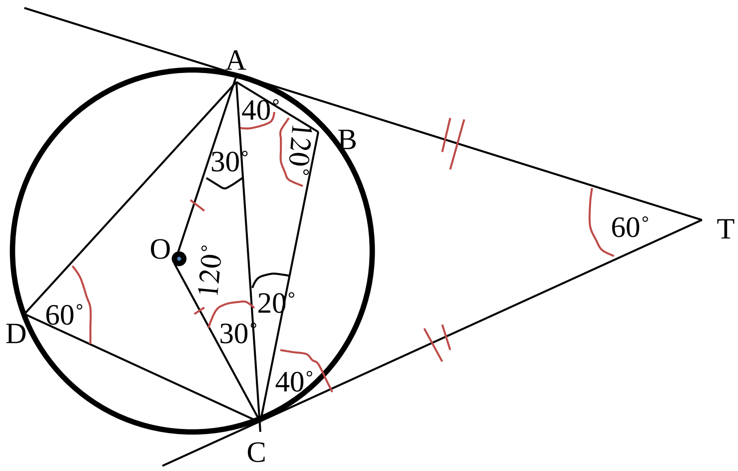
In the diagram, A,B, C and D are points on the circumference of a circle center O. AT and CT are tangents to the circle, $O\hat{A}C = 30^{\circ}$ and $A\hat{C}B = 20^{\circ}$. Find angle

(i) AOC
(ii) ADC
(iii) BCT

(iv) CAB
(v) ATC

Solution

Let's first fill all the angles and then answer the questions



(i) AOC = 120˚.
Explanation :
Using Triangle AOC, the radii are equal thereby an isosceles. The base angles are 30˚ each and then angle AOC = 120˚

(ii) ADC = 60˚. Angles subtended at the center is twice bigger than that at the circumference. AOC is at the center

(iii) BCT = 40˚. Since tangent is perpendicular to radius, ACO = 30˚ and ACB = 20˚, subtract them from 90˚

(iv) CAB = 40˚. Alternate to BCT

(v) ATC = 60˚ Since AT and CT are equal, base angles are equal, angle ACT = CAT = 60˚. From 180˚, you get 60˚

SIMILARITY AND CONGRUENCY

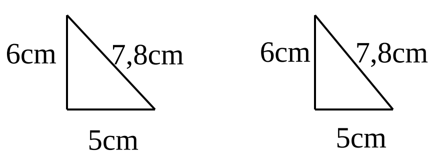
Similarity and congruency is now common in June examinations.

Similar shapes are proportionally different in dimensions and area. Congruent shapes have equal dimensions and area.

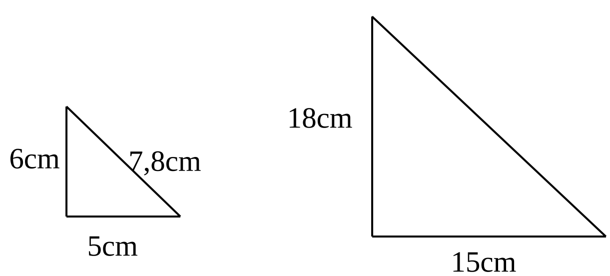
The most important thing in circle geometry is to identify

(a) Similar and congruent shapes
(b) Name similar and congruent shapes in correct order
(c) Find the similarity factor of shapes
(d) Calculate required length using similarity factor

Congruent shapes



Similar shapes

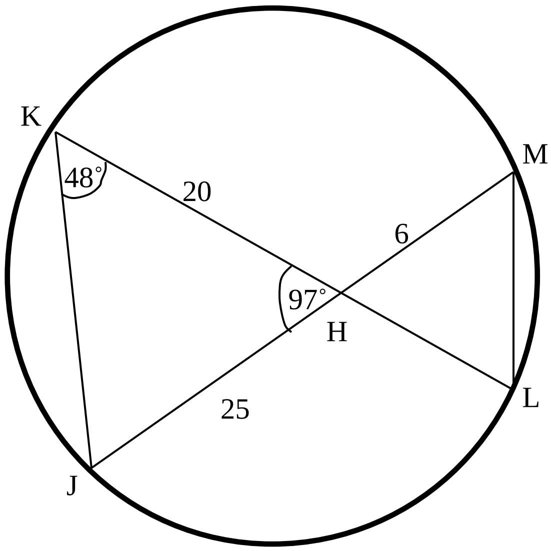


You can see that similar shapes have different dimension but proportional. Lets calculate the similarity factor

$\frac{18cm}{6cm}$ = 3 or $\frac{15cm}{5cm}$ = 3. This is used to calculate the missing side.

Missing side = 7,8cm × 3 = 23,4 cm

Examples



In the diagram, J, K, M and L lie on the circumference of a circle. JM and KL intersect at H. angle JHK = 97°, JKH = 48°, KH = 20cm , JH = 25cm and HM = 6cm

(a) Find $H\hat{L}M$.

(b) Name, in correct order, the triangle which is similar to triangle JKH

(c) Calculate

  (i) The length of JK

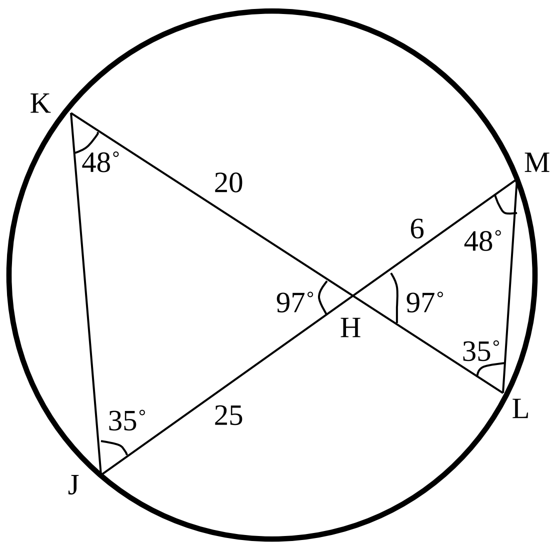
  (ii) The area of the triangle JKH

  (iii) The area of triangle HLM

[June 2016/2/4 Zimsec]

Solution:

Lets first fill all the angles and then answer the questions



Now that we have filled all the angles using the principles of ‘angles subtended by the same arc’ and ‘opposite equal angles’, lets answer the questions

(a) Angle HLM = 35˚

(b) To name in correct order, follow the sizes of the angle. Given triangle JKH, at J there is 35˚, K is 48˚ and H is 97˚. Similar positions are L (35˚), M(48˚) and H(97˚). Therefore JKH is similar to LMH

(c) (i) Length of JK is found by Sine Rule

$\frac{JK}{\sin 97^\circ} = \frac{25}{\sin 48^\circ}$, JK = $\frac{25 \sin 97}{\sin 48}$ = 33,4 to 3s.f.

(ii.) the area of triangle JKH is found by Sine Rule.

$\frac{1}{2}$ab Sin C = $\frac{1}{2}\times 20\times 25\times$Sin 97˚ = 248 cm^2

(iii.) Use the similarity ratio to find area of triangle HLM

Similarity ratio = $\frac{6}{20}$ (similar sides are HK and HM)and Area = $\left(\frac{6}{20}\right)^2$

Area of triangle HLM = $\left(\frac{6}{20}\right)^2 \times 248\ cm^2 = 22{,}32cm^2$

Or you can use the longest method of first finding side LH and then use sine rule to find area. LH is similar to JH

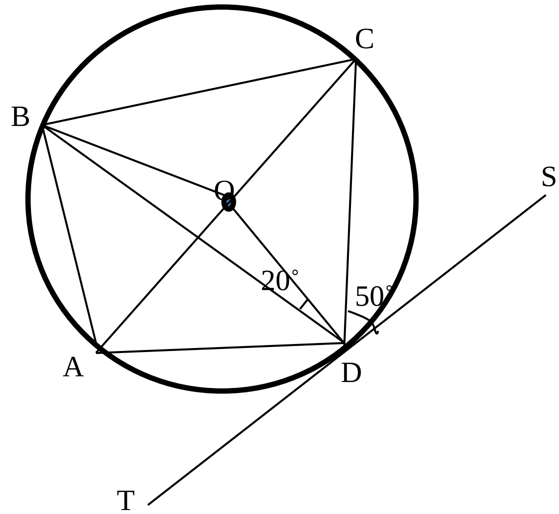
LH = $\frac{6}{20} \times 25cm = 7{,}5cm$

Therefore using Sine Rule

$\frac{1}{2} \times 6cm \times 7{,}5cm \times \text{Sin } 97 = 22{,}33\ cm^2$

REVISION QUESTIONS

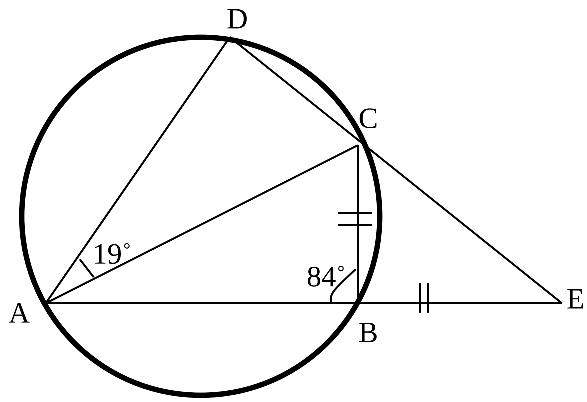
1.



In the diagram, points A,B,C and D are on the circumference of the circle with center O. angle CDS = 50˚ and BDO = 20˚. Line TS is tangent to the circle at D. Calculate

(a) DBC (b) DOC (c) ODC (d) ABD (e) DAC [5] [J2016/1/25]

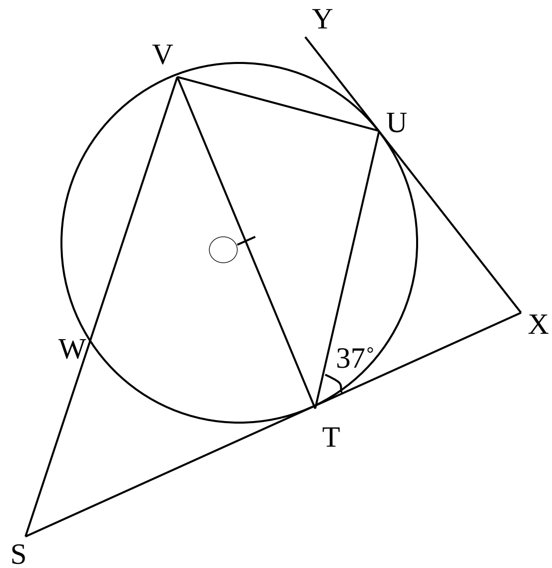
2.



ABCD is a cyclic quadrilateral. Given that BC = BE, ABC = 84˚ and DAC = 19˚, calculate angles

(a) BCE (b) ADC (c) CAB [N99/P1]

3.



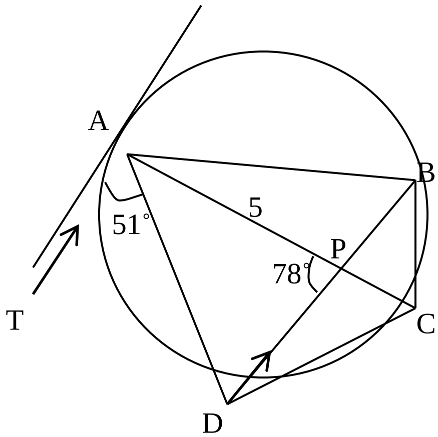
In the diagram, TUVW is a circle center O. TOV is a diameter. STX and XUT are tangents to the circle center at T and U respectively. SWV is parallel to TU and $U\hat{T}X = 37°$

(i) Find angles 1. TSV
2 SVT
3 UVT
4 TXU

(ii) Name the triangle that is similar to ▲UVT

(iii) Given that TU = 4,7 cm,
Calculate the radius of the circle [8 marks]
[N2013/2/5b]

4.



In the diagram, ABCD is a cyclic quadrilateral. TA is a tangent at A and is parallel to DB. AC and BD intersect at P such that AP = 5cm.

(a) Given that $T\hat{A}D = 51^\circ$ and $A\acute{P}D = 78^\circ$, calculate angles

(i) ACD

(ii) BAC

(iii) BCA [5]

(b) Write down the reason why ▲APD is isosceles [1]

(c) Calculate length of AD [2]

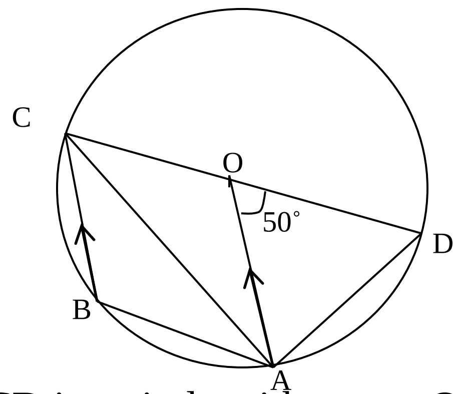
(d) Name in correct order, triangle which is

(i) Similar to ▲APD

(ii) Congruent to ▲ABP [2]

[J2014/2/5]

5.



In the diagram ABCD is a circle with center O. CD is a diameter of the circle, AOD = 50˚ and OA is parallel to CB.

(i) Find angle OCA

(ii) Find angle OAD

(iii) Find angle ABC

(iv) Find angle CAB [7] [N2015/2/5b]

ANSWERS TO REVISION QUESTIONS

1 (a) 50° (b) 100° (c) 40° (d) 40° (e) 50°

2 (a) 42° (b) 96° (c) 23°

3 (a) 1) 37° 2) 53° 3) 37° 4) 106°

(b) ▲TSV

(c) 1,42 cm

4 (a) (i) 51° (ii) 27° (iii) 51°

(b) Bounded by lines that forms equal angles

(c) 6,29cm

(d) (i) ▲BPC

(ii.) ▲DCP

5 (i) 25° (ii) 65° (iii) 115° (iv) 40°