The blackball sinker

ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

PURE MATHEMATICS 6042/2

PAPER 2

NOVEMBER 2020 SESSION 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae MF7
- Scientific calculator (Non-programmable)

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre Number and Candidate Number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions in **Section A** and any **five** questions from **Section B**.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

If a numerical value for g is necessary, take $g = 9.81\ \text{ms}^{-2}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of a non-programmable scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed pages and 3 blank pages.

Section A [40 marks]

Answer **all** questions in this section.

1 **(a)** A sequence is defined as $U_n = 1 + (-1)^n$.

(i) List the first four terms of the sequence. [1]

(ii) Describe the behaviour of the sequence. [1]

(b) The sum, S_n, of an arithmetic progression is given by $S_n = n(3n + 2)$.

Find the first term and the common difference. [4]

2 The complex number Z_1 and Z_2 are such that $Z_1 = -1 - \sqrt{3}i$ and
$Z_2 = 3 + 4i$.

(a) Express in the form $a + bi$

(i) $Z_1 + Z_2$, [2]

(ii) $\frac{Z_1}{Z_2}$. [2]

(b) Calculate

(i) $|Z_2|$, [1]

(ii) the argument of Z_1. [2]

3 Prove by induction $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$. [7]

4 A, B, C and D are the points with position vectors $2\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$, $\boldsymbol{i} + 4\boldsymbol{j} + 2\boldsymbol{k}$, $\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$ and $2\boldsymbol{i} + \lambda\boldsymbol{k}$, where λ is a constant, relative to the origin, O.

(a) Find the

(i) unit vector in the direction of $\overrightarrow{OA}$, [2]

(ii) position vector of the mid-point of A and B, [2]

(iii) position vector $\overrightarrow{OE}$ given that $4\overrightarrow{AB} = \overrightarrow{CE}$. [3]

(b) Given that the angle between $\overrightarrow{AD}$ and $\overrightarrow{BC}$ is 90°, find the value of λ. [3]

5 **(a)** The outcome table shows the entries of numbers modulo 4 under the operation of addition.

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	a	1	b

(i) Identify the entries a and b. [4]

(ii) Write down the identity element. [1]

(b) Find $\int ln(x+3)\,dx$. [5]

Section B [80 marks]

Answer any **five** questions from this section.

Each question carries 16 marks.

6 **(a)** **(i)** Given that $f(x) = e^{-x}sinx$, use Maclaurin's theorem to find the series expansion of $f(x)$ up to and including the term in x^3. [4]

(ii) Hence or otherwise evaluate $f(0.4)$ to 3 decimal places. [2]

(b) **(i)** Sketch on the same axes the graphs of $y = lnx$ and $y = 2 - x$. [3]

(ii) Verify that equation $lnx = 2 - x$ has one real root in the interval $1 < x < 2$. [3]

(iii) Starting with $x_0 = 1.8$ use the Newton-Raphson method twice to calculate the root of $lnx + x - 2 = 0$ giving the answer correct to 3 decimal places. [4]

7 **(a)** **(i)** Express $3 - 3\sqrt{3}i$ in exponential form, $re^{i\theta}$, where r is the modulus of the complex number and θ is the argument. [3]

(ii) Hence or otherwise find all the roots of the equation $Z^4 - 3 + 3\sqrt{3}i = 0$ in exponential form giving the answers correct to three significant figures. [6]

(b) Use de Moivre's theorem or otherwise to show that

$$tan5\theta \equiv \frac{5tan\theta - 10tan^3\theta + tan^5\theta}{1 - 10tan^2\theta + 5tan^4\theta}.$$ [7]

8 **(a)** Write down the matrices for the following transformations,

(i) a reflection in the y – axis, [1]

(ii) a stretch by a factor $\frac{5}{2}$ parallel to x-axis. [1]

(b) Solve the equation

$$\begin{vmatrix} 1 & 1 & 1 \\ y & y+1 & y-1 \\ y-1 & 2y & y+1 \end{vmatrix} = 0.$$ [3]

(c) **(i)** Find the inverse of the matrix $A = \begin{pmatrix} 2 & -3 & 4 \\ 3 & 2 & -1 \\ 1 & 1 & 1 \end{pmatrix}$. [7]

(ii) Hence or otherwise determine the solution of the equations:

$$2x - 3y + 4z = 22$$
$$3x + 2y - z = 10$$
$$x + y + z = 14$$ [4]

9 Given that $y = \frac{1}{3x+1}$,

(a) find $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ showing in each case that the derivatives can be written in the form $\frac{(-3)^n n!}{(3x+1)^{n+1}}$. [6]

(b) Prove the suggested n[th] derivative by induction. [10]

10 The points A, B and C have coordinates (1; 2; 3); (2; −1; 5) and (1; 3; −1 respectively relative to the origin O.

Line *l* passes through A and has equation $r = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$.

Line *m* passes through point B and has equation $r = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

(a) Show that $\overrightarrow{AB}$ is perpendicular to line *l*. [3]

(b) Find the coordinates of the point of intersection of line *l* and line *m*. [5]

(c) Find the angle between line *m* and line *l*. [4]

(d) Find the equation of the plane passing through points A, B and C. [4]

11 **(a)** Find $\int cotxdx$. [3]

(b) Find the exact value of $\int_0^2 \frac{2x+4}{x+3}dx$ in the form $a + ln\left(\frac{b}{c}\right)$ where a, b and c are constants. [6]

(c) Evaluate the exact value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} e^x cos3xdx$. [7]

12 **(a)** **(i)** Express $3cos2\theta - sin2\theta$ in the form $Rcos(2\theta + \alpha)$. [2]

(ii) Hence or otherwise solve the equation $3cos2\theta - sin2\theta = 2$ for $0 \le \theta \le 360°$. [4]

(b) Prove that $sin3x \equiv 3sinx - 4sin^3x$. [4]

(c) Hence or otherwise solve the equation $sin3x = sin^2x$ for $0 \le x \le 2\pi$ give the answers in terms of π. [6]