This is chapter 10 of the book, Exam Cheat Code: Maths 4. The book is designed to prepare students for Zimsec/Cambridge Exams. The book come with the following:

- Step by step worked examples.
- Detailed answers.
- Exam type Questions.
- Your Name, Nickname and Surname.
- A password only for you, derived from your birth date and birth month.

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10

Probability.

Probability is a measure of the likelihood of an event occurring. It's a number between 0 and 1 that shows the chance or probability of an event happening. A probability of 1 means that the event will definitely happen and a probability of 0 means that the event will not happen. The total probability of a sample size is 1.

Experimental probability

Experimental probability is the probability of an event occurring based on repeated trials or experiments. It's a way to estimate the probability of an event by conducting multiple trials and observing the outcomes.

Theoretical Probability.

Theoretical Probability is the probability of an event occurring based on the number of favourable outcomes and the total number of outcomes. It's a way to calculate the probability of an event without actually conducting experiments or trials.

Probability of a Single Event:

$$P(A) = \frac{number\ of\ favourable\ outcomes}{total\ number\ of\ outcomes}$$

Example 1

A bag contains 15 marbles, of which 6 are red, 5 are blue, and 4 are white. **(a)** Calculate the probability of picking a blue marble. **(b)** Calculate the probability of picking a red marble.

Solution:

(a) Calculate the probability of picking a blue marble:
Step 1: Find the total number of marbles.
Total number of marbles=6+5+4=15

Step 2: Find the probability.
Number of blue marbles = 5
Total number of marbles = 15

Probability of picking a blue marble = $\frac{Number\ of\ blue\ marbles}{Total\ numbre\ of\ marbles}$

$$= \frac{5}{15}$$

$$= \mathbf{\frac{1}{3}}$$

Therefore, the probability of picking a blue marble is $\mathbf{\frac{1}{3}}$

(b) Calculate the probability of picking a red marble:
Number of red marbles = 6
Total number of marbles = 15

Probability of picking a red marble = $\frac{Number\ of\ red\ marbles}{Total\ numbre\ of\ marbles}$

$$= \frac{6}{15}$$

$$= \mathbf{\frac{2}{5}}$$

Therefore, the probability of picking a red marble is $\mathbf{\frac{2}{5}}$

Example 2

A bag contains red, green and blue balls. The probability of picking a green ball is 0.4 and the probability of picking a blue ball is 0.25. Find the probability of picking a red ball.

Solution:

Let's use the fact that the sum of the probabilities of all possible outcomes is 1.

Probability of picking a green ball = 0.4
Probability of picking a blue ball = 0.25

Let x be the probability of picking a red ball.

We know that the sum of the probabilities of all possible outcomes is 1:

$0.4 + 0.25 + x = 1$

Combine the constants:

$0.65 + x = 1$

Subtract 0.65 from both sides:

$x = 1 - 0.65$
$x = 0.35$

Therefore, the probability of picking a red ball is **0.35.**

Example 3

It is given that set A={0; 3; 7; 0.5; 6; 11; 14; 8; 1; 0.3} , a number is chosen at random what is the probability that:

(a) It is a multiple of 2.

(b) It is a cube number.

c) It is a rational number.

(d) It is not an even number.

(Types on numbers are covered in book 3)

Solution.

(a) It is a multiple of 2.

Step 1: Identify the elements in set A

Set A = {0, 3, 7, 0.5, 6, 11, 14, 8, 1, 0.3}

Step 2: Identify the multiples of 2 in set A

Multiples of 2 = {6, 14, 8}

Step 3: Calculate the probability of choosing a multiple of 2

P (multiples of 2) = $\frac{number\ of\ favourable\ outcomes}{total\ number\ of\ outcomes}$

$= \frac{\mathbf{3}}{\mathbf{10}}$ **or 0.3**

(b) It is a cube number.

Step 1: Identify the cube numbers in set A

Cube numbers = {1; 8} (since $1^3 = 1; 2^3 = 8$)

Step 2: Calculate the probability of choosing a cube number

P (cube number) = $\frac{number\ of\ favourable\ outcomes}{total\ number\ of\ outcomes}$

$= \frac{2}{10}$

$= \frac{\mathbf{1}}{\mathbf{5}}$ **or 0.2**

c) It is a rational number.

Step 1: Identify the rational numbers in set A

Rational numbers = {0, 3, 7, 0.5, 6, 11, 14, 8, 1, 0.3} (since all numbers in set A are rational) [Note 0 also meets the definition of a rational number]

Step 2: Calculate the probability of choosing a rational number

P (rational number) = $\frac{number\ of\ favourable\ outcomes}{total\ number\ of\ outcomes}$

$= \frac{10}{10}$

$$= \mathbf{1}$$

(d) It is not an even number.

Step 1: Identify the numbers that are not even in set A
Not even numbers = {3, 7, 0.5, 11, 1, 0.3}

Step 2: Calculate the probability of choosing a number that is not even
P (not even numbers) = $\frac{number\ of\ favourable\ outcomes}{total\ number\ of\ outcomes}$

$$= \frac{6}{10}$$

$$= \frac{6 \div 2}{10 \div 2}$$

$$= \mathbf{\frac{3}{5}}$$

Coin

A coin has 2 possible outcomes: Heads (H) or Tails (T)

Die

A standard die has 6 possible outcomes: 1, 2, 3, 4, 5, or 6

Alphabet

The English alphabet has 26 letters
There are five vowels (A, E, I, O, or U) and 21 consonants.

Playing Cards

A standard deck of playing cards has 52 cards
There are four suits (Hearts, Diamonds, Clubs, or Spades)
There are 4 cards of a specific rank one from each suit for example there are 4 Queens one from each suit.

Exercise 10a [21 marks] [Answers]

1. A box contains 20 pens, of which 9 are black, 5 are blue, and 6 are red. **(a)** Calculate the probability of picking a black pen. [1] **(b)** Calculate the probability of picking a red pen. [1]
2. A box contains white, black, and grey marbles. The probability of picking a black marble is 0.35 and the probability of picking a grey marble is 0.2. Find the probability of picking a white marble. [2]

3. It is given that set B = {2, 5, 8, 1.2, 9, 11, 4, 6, 3, 0.8}, a number is chosen at random. What is the probability that:
(a) It is a prime number. [1]
(b) It is a decimal number. [1]
(c) It is an odd number. [1]
(d) It is a multiple of 3. [1]
4. A non-biased die is thrown find the probability of getting:
(a) a 3
(b) an even number. [1]
(c) a number greater than 2. [1]
5. If a random card is picked from a pack of playing cards find the probability of picking:
(a) a 7 [1]
(b) a 7 of hearts. [1]
(c) a black card [1]
(d) a black heart [1]
6. If a random letter is chosen from the alphabet, find the probability of picking:
(a) a vowel [1]
(b) an A or B [2]
(c) A letter from the word mathematics.
7. If a coin is tossed once what is the probability of getting
(a) a head. [1]
(b) a head or a tail. [1]
8. A bag contains yellow, orange, and purple sweets. The probability of picking an orange sweet is 0.3 and the probability of picking a purple sweet is 0.15. Find the probability of picking a yellow sweet. [2]

Mutually Exclusive Events.

Mutually Exclusive Events are events that cannot occur at the same time. In other words, if one event occurs, the other event cannot occur.
Example: Flipping a coin and getting either Heads or Tails. These two events are mutually exclusive because you can only get one outcome at a time.

Properties of Mutually Exclusive Events:

1. The probability of one event occurring does not affect the probability of the other event occurring.
2. The sum of the probabilities of two mutually exclusive events is less than or equal to 1.
3. The key word for mutually exclusive events is **or**.
Formula: If A and B are mutually exclusive events, then:

P(A or B) = P(A) + P(B)

You add individual probabilities.

Example 4

A bag contains 5 red balls, 3 blue balls, and 2 green balls. What is the probability of picking a red ball **or** a blue ball?

Solution:

Step 1: Find the individual probabilities.
Total number of balls=5+3+2=10.
P(red)= $\frac{5}{10}$ P(blue))= $\frac{3}{10}$

Step 2: Add the individual probabilities.
P(Red or Blue) = P(Red) + P(Blue)

$$= \frac{5}{10} + \frac{3}{10}$$

$$= \frac{5+3}{10}$$

$$= \frac{8}{10}$$

$$= \mathbf{\frac{4}{5}}$$ (reduce the fraction)

Example 5

Given that A={1; 3; 4; 7; 9} and B={2; 5; 6; 8}
(a) If a number is picked from A what is the probability that it is an even number.
(b) Find in A∪B the probability of picking a number that it is either an even number **or** a number less than 5.

Solution:

a) If a number is picked from A, what is the probability that it is an even number?
Step 1: Identify the even numbers in set A
Even numbers in A = {4}

Step 2: Calculate the probability of picking an even number
Number of even numbers = 1
Total number of outcomes = 5
Probability $= \frac{number\ of\ favourable\ outcomes}{total\ number\ of\ outcomes}$

$$= \mathbf{\frac{1}{5}}$$

(b) Find in A∪B the probability of picking a number that is either an even number or a number less than 5.

Step 1: Find the union of sets A and B
A∪B = {1, 2, 3, 4, 5, 6, 7, 8, 9}

Step 2: Identify the even numbers and numbers less than 5 in A∪B
Even numbers = {2, 4, 6, 8}
Numbers less than 5 = {1, 2, 3, 4}

Step 3: Find the individual probabilities.
Total number of outcomes=9
P (Even numbers) $=\frac{4}{9}$ P (numbers less than 5) = $\frac{4}{9}$

Step 4: Add the individual probabilities.
P (even number or number less than 5) = P (Even numbers)+ P (numbers less than 5)

$$=\frac{4}{9}+\frac{4}{9}$$
$$=\frac{4+4}{9}$$
$$= \mathbf{\frac{8}{9}}$$

Example 6

The probability that Allan passes a maths test is $\frac{1}{3}$ and that of Ben failing the same test is $\frac{3}{5}$. Find the probability that **a)** Ben passes the test **(b)** Either Allan or Ben passes the test.

Solution:

(a) Find the probability that Ben passes the test:

Given: Probability of Ben failing the test = $\frac{3}{5}$

To find the probability of Ben passing the test, we subtract the probability of failing from 1:

Probability of Ben passing = 1− Probability of Ben failing

$$= 1 - \frac{3}{5}$$
$$= \frac{5}{5} - \frac{3}{5}$$
$$= \frac{5-3}{5}$$

$$= \mathbf{\frac{2}{5}}$$

(b) Find the probability that either Allan or Ben passes the test:

Given: Probability of Allan passing $=\frac{1}{3}$

Probability of Ben passing = $\frac{2}{5}$

To find the probability of either Allan or Ben passing, we add the probabilities of each passing:

Probability of either Allan or Ben passing = Probability of Allan passing + Probability of Ben passing.

$= \frac{1}{3} + \frac{2}{5}$

To add fractions, we need a common denominator, which is 15:

$= \frac{1\times5}{3\times5} + \frac{2\times3}{5\times3}$

$= \frac{5}{15} + \frac{6}{15}$

$= \frac{5+6}{15}$

$= \mathbf{\frac{11}{15}}$

Exercise 10b [22 marks] [Answers]

1. A box contains 8 yellow pencils, 5 black pencils, and 2 red pencils. What is the probability of:

(a) picking a yellow pencil or a black pencil? [1]

(b) picking a green pencil [1]

2. A bag contains 4 white marbles, 6 black marbles, and 2 grey marbles. What is the probability of picking:

(a) a white marble or a grey marble? [1]

(b) a marble. [1]

3. Given that X = {2, 5, 8, 11, 13} and Y = {1, 3, 6, 9, 12},

(a) If a number is picked from X, what is the probability that it is an odd number. [1]

(b) Find in X∪Y the probability of picking a number that is either a multiple of 3 or a number greater than 10. [2]

4. The probability that Emily passes an English test is $\frac{2}{5}$ and that of David failing the same test is $\frac{9}{10}$. Find the probability that:

a) David passes the test [1]

b) Either Emily or David passes the test. [2]
c) State which one of them is more likely to pass. [2]
5. A fair sided die is thrown find the probability of getting:
(a) a 2 or a number divisible by 3. [2]
(b) a prime number or a perfect square. [2]
6. A card is picked from a pack of playing cards, what is the probability of getting:
(a) 4 or a queen of clubs. [2]
(b) a black card or a red 5 [2]
(c) an Ace or a 7 [2]

Independent Events.

Two events are said to be independent if the occurrence or non-occurrence of one event does not affect the probability of the occurrence of the other event.

Example:
Tossing a coin and rolling a die are independent events. The outcome of the coin toss does not affect the outcome of the die roll.

Properties of Independent Events:

$P(A \cap B) = P(A) \times P(B)$

The probability of both events A and B occurring is equal to the product of their individual probabilities.

Example 7

A coin is tossed and a die is thrown what is the probability of getting:
(a) a head and a 4
(b) a tail and a perfect square.

Solution:

(a) a head and a 4:

Step 1: Identify the probability of getting a head
$P(\text{Head}) = \frac{1}{2}$ (1 head and two sides)

Step 2: Identify the probability of getting a 4
$P(4) = \frac{1}{6}$ (one 4 and 6 faces)

Step 3: Calculate the probability of getting a head and a 4
$P(\text{Head and } 4) = P(\text{Head}) \times P(4)$

$$= \frac{1}{2} \times \frac{1}{6}$$

$$= \frac{1 \times 1}{2 \times 6}$$

$$= \mathbf{\frac{1}{12}}$$

(b) a tail and a perfect square.

Step 1: Identify the probability of getting a tail
P(Tail) = $\frac{1}{2}$ (1 tail and two sides)

Step 2: Identify the probability of getting a perfect square
Perfect squares on a die = {1, 4}
P(Perfect square) = $\frac{2}{6} = \frac{1}{3}$

Step 3: Calculate the probability of getting a tail and a perfect square
P(Tail and Perfect square) = P(Tail) × P(Perfect square)

$$= \frac{1}{2} \times \frac{1}{3}$$

$$= \frac{1 \times 1}{2 \times 3}$$

$$= \mathbf{\frac{1}{6}}$$

Example 8

A bag contains 5 red balls, 3 blue balls, and 2 green balls. Two balls are picked from the bag without replacement. What is the probability that the first ball is red and the second ball is blue?

Solution:

Step 1: Calculate the probability of the first ball being red
Total number of balls = 5 (red) + 3 (blue) + 2 (green) = 10
Probability of first ball being red = $\frac{number\ of\ red\ balls}{total\ number\ of\ balls}$

$$= \frac{5}{10}$$

$$= \frac{1}{2}$$

Step 2: Calculate the probability of the second ball being blue given that the first ball is red.
Since the first ball is red and not replaced, the total number of balls left = 10 − 1 = 9
Number of blue balls = 3 (remains the same)

Probability of second ball being blue $= \frac{number\ of\ blue\ balls}{total\ number\ of\ balls\ left}$

$$= \frac{3}{9}$$

$$= \frac{1}{3}$$

Step 3: Calculate the probability of both events occurring
Probability of first ball being red and second ball being blue = Probability of first ball being red × Probability of second ball being blue

$= \frac{1}{2} \times \frac{1}{3}$

$= \frac{1 \times 1}{2 \times 3}$

$= \mathbf{\frac{1}{6}}$

Example 9

Rutendo and Anesu are taking a mathematics exam. The probability that Rutendo will score an A grade is $\frac{2}{3}$ and the probability that Anesu will score an A grade is $\frac{3}{4}$. Calculate the probability that

(a) they both score an A grade,
(b) only one of them will score an A grade.

Solution:

(a) They both score an A grade:

Step 1: Identify the probability of Rutendo scoring an A grade
P(Rutendo scores A) = $\frac{2}{3}$

Step 2: Identify the probability of Anesu scoring an A grade
P(Anesu scores A) = $\frac{3}{4}$

Step 3: Calculate the probability of both Rutendo and Anesu scoring an A grade
P(Both score A) = P(Rutendo scores A) × P(Anesu scores A)

$= \frac{2}{3} \times \frac{3}{4}$

$= \frac{2 \times 3}{3 \times 4}$

$= \frac{6}{12}$

$= \mathbf{\frac{1}{2}}$

(b) Only one of them will score an A grade:

Step 1: Calculate the probability of Rutendo scoring an A grade and Anesu not scoring an A grade
P(Rutendo scores A and Anesu doesn't) = P(Rutendo scores A) × P(Anesu doesn't score A)

$= \frac{2}{3} \times \frac{1}{4}$

$= \frac{2\times1}{3\times4}$

$= \frac{2}{12}$

$= \frac{1}{6}$

Step 2: Calculate the probability of Anesu scoring an A grade and Rutendo not scoring an A grade
P(Anesu scores A and Rutendo doesn't) = P(Anesu scores A) × P(Rutendo doesn't score A)

$= \frac{3}{4} \times \frac{1}{3}$

$= \frac{3\times1}{4\times3}$

$= \frac{3}{12}$

$= \frac{1}{4}$

Step 3: Calculate the probability of only one of them scoring an A grade
P(Only one scores A) = P(Rutendo scores A and Anesu doesn't) + P(Anesu scores A and Rutendo doesn't)

$= \frac{1}{6} + \frac{1}{4}$ (common denominator is 12)

$= \frac{1\times2}{6\times2} + \frac{1\times3}{4\times3}$

$= \frac{2}{12} + \frac{3}{12}$

$= \frac{2+3}{12}$

$= \boldsymbol{\frac{5}{12}}$

Exercise 10c [33 marks] [Answers]

1. A bag contains 12 red marbles, 8 blue marbles, and 4 green marbles. Two marbles are picked from the bag without replacement. What is the probability of getting a red marble and then a blue marble. [2]

2. A machine produces two types of products, X and Y. The probability that product X is defective is $\frac{1}{4}$ and the probability that product Y is defective is $\frac{1}{5}$. Calculate the probability that

(a) both products are defective, [1]

(b) only one of the products is defective. [3]

3. A spinner has 8 equal sections numbered 1 to 8. A card is drawn from a pack of 52 cards. What is the probability of getting:

(a) a number greater than 4 on the spinner and a heart on the card, [2]

(b) an even number on the spinner and a diamond on the card. [2]

4. A jar contains 15 marbles, of which 5 are red, 4 are blue, and 6 are purple.

(a) Find the probability that a marble picked at random from the jar is

(i) blue, [1]

(ii) orange. [1]

(b) Two marbles are picked at random from the jar. Find the probability that they are

(i) both red, [2]

(ii) one red and one blue. [3]

5. Ten identical cards are numbered 1; 2; 3; 4; 5; 5; 7; 8; 9; 10. One of the cards is chosen at random.

(a) Write down the number whose probability of being chosen is $\frac{1}{5}$. [1]

(b) Two of the ten cards are taken at random. Find the probability that the sum of the two numbers is 19. [2]

6. Two cards were picked at random from a pack of 52 playing cards with replacement. Find the probability that one was a Heart and the other was a 7. [2]

7. The probability that Maria passes a mathematics exam is $\frac{2}{3}$ and that of David passing is $\frac{3}{y}$.

(a) If the probability that they both pass the exam is $\frac{1}{5}$, find y. [2]

(b) Calculate the probability that **(i)** David fails the exam,[1] **(ii)** either Maria or David passes the exam. [2]

8. The probability that a new restaurant will have more than 50 customers on a Friday night is $\frac{3}{8}$. The probability that it will have more than 50 customers on a Saturday night is $\frac{5}{8}$. Calculate the probability that on a given weekend

(a) both nights have more than 50 customers, [2]

(b) neither night has more than 50 customers, [2]

(c) only one of the nights has more than 50 customers. [3]

Outcome Tables

An outcome table is a table that shows all the possible outcomes of a probability experiment. It is used to organize and display the outcomes in a clear and systematic way.

Example 10

Two coins are tossed, find the probability of getting **(a)** two heads **(b)** a head and a tail.

Solution.

Step 1: Create an outcome table showing all the possible outcomes.

		first coin	
		T	H
second coin	T	TT	TH
	H	HT	HH

The black letters represent the face of a coin and the blue letters represent the outcomes from the second coin then first coin.

Step 2: find the probability of getting **(a)** two heads

		first coin	
		T	H
second coin	T	TT	TH
	H	HT	HH

The favourable outcome is circled in red, and there are 4 possible outcome.

Probability $=\frac{number\ of\ favourable\ outcomes}{total\ number\ of\ outcomes}$

$$=\frac{\mathbf{1}}{\mathbf{4}}$$

Step 3: find the probability of getting **(b)** a head and a tail.

first coin

second coin	T	H
T	TT	TH
H	HT	HH

The favourable outcomes are circled in red, and there are 4 possible outcomes.

Probability $= \dfrac{number\ of\ favourable\ outcomes}{total\ number\ of\ outcomes}$

$= \dfrac{2}{4}$

$= \dfrac{1}{2}$

Example 11

Two dice are thrown at the same time. Find the probability of getting **(a)** A total score divisible by 4. **(b)** a score which is a perfect square **(c)** at least one 4 on a die.

Solution.

Step 1: Create an outcome table showing all the possible outcomes.

second die						
6	7	8	9	10	11	12
5	6	7	8	9	10	11
4	5	6	7	8	9	10
3	4	5	6	7	8	9
2	3	4	5	6	7	8
1	2	3	4	5	6	7
	1	2	3	4	5	6

first die

The black letters represent the numbers on a face of a die and the blue letters represent the outcomes from the first die and second die.

Step 2: Find the probability of getting **(a)** A total score divisible by 4.

second die	6	7	8	9	10	11	12
	5	6	7	8	9	10	11
	4	5	6	7	8	9	10
	3	4	5	6	7	8	9
	2	3	4	5	6	7	8
	1	2	3	4	5	6	7
		1	2	3	4	5	6
				first die			

9 outcomes divisible by 4 are circled in red. There are 36 possible outcomes.

Probability $=\dfrac{number\ of\ favourable\ outcomes}{total\ number\ of\ outcomes}$

$$=\frac{9}{36}$$

$$=\mathbf{\frac{1}{4}}$$

Step 3: Find the probability of getting **(b)** a perfect square.

second die	6	7	8	9	10	11	12
	5	6	7	8	9	10	11
	4	5	6	7	8	9	10
	3	4	5	6	7	8	9
	2	3	4	5	6	7	8
	1	2	3	4	5	6	7
		1	2	3	4	5	6
				first die			

7 possible outcomes which are perfect squares are circled in red.

Probability $=\dfrac{number\ of\ favourable\ outcomes}{total\ number\ of\ outcomes}$

$$=\mathbf{\frac{7}{36}}$$

Step 4: Find the probability of getting **(c)** at least one 4.

second die						
6	7	8	9	10	11	12
5	6	7	8	9	10	11
4	5	6	7	8	9	10
3	4	5	6	7	8	9
2	3	4	5	6	7	8
1	2	3	4	5	6	7
	1	2	3	4	5	6

first die

11 possible outcomes when die 1 and die 2 get an outcome of 4 are circled in red.

Probability $=\frac{number\ of\ favourable\ outcomes}{total\ number\ of\ outcomes}$

$$=\frac{\mathbf{11}}{\mathbf{36}}$$

Tree diagrams.

A tree diagram is a graphical representation of all the possible outcomes of a probability experiment. It is a diagram that shows the different stages of an experiment and the possible outcomes at each stage.

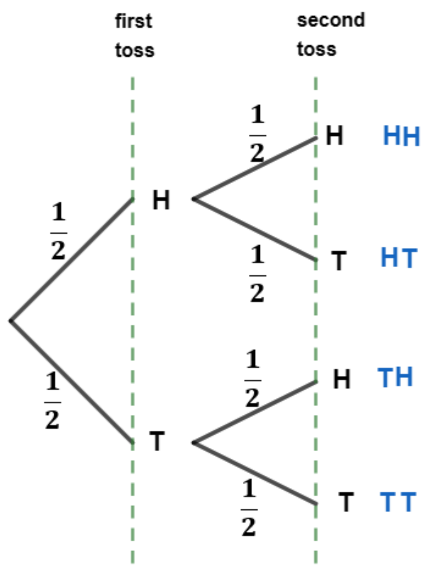
How to Draw a Tree Diagram

1. Start with a single branch to represent the first stage of the experiment.
2. Draw branches from the first stage to represent the possible outcomes at that stage.
3. From each of these branches, draw further branches to represent the possible outcomes at the next stage.
4. Continue this process until all the possible outcomes have been represented.

Example 12

A coin is tossed twice. **(a)** Draw a tree diagram to show the possible outcomes. **(b)** Using the tree diagram find the probability of getting **(i)** only one tail **(ii)** 2 heads (**iii)** at least one tail.

Solution:

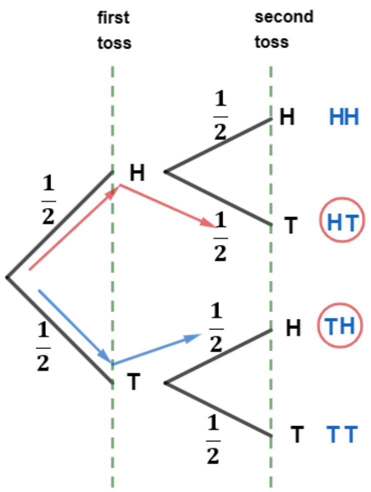


A coin has two faces heads and tails, so the probability of each face is:

P(tail)$= \frac{1}{2}$ P(head)$= \frac{1}{2}$

The blue letters are the outcomes from the first toss and the second toss.

(b) Using the tree diagram find the probability of getting **(i)** only one tail



The possible outcomes with only one tail (T) are circled in red, we find the probability for each outcome.

P(HT)$= \frac{1}{2} \times \frac{1}{2}$ (follow the red arrows for route)

$$= \frac{1 \times 1}{2 \times 2}$$

$$= \frac{1}{4}$$

P(TH)$= \frac{1}{2} \times \frac{1}{2}$ (follow the green arrows for route)

$$= \frac{1 \times 1}{2 \times 2}$$

$=\frac{1}{4}$

There are two possibilities that is HT **or** TH, we will add the individual probabilities.

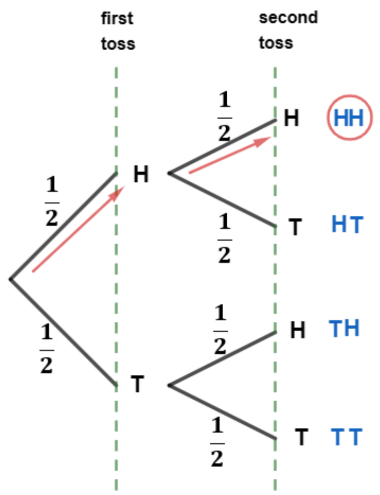
P(HT or TH)$=\frac{1}{4}+\frac{1}{4}$

$=\frac{1+1}{4}$

$=\frac{2}{4}$

$=\mathbf{\frac{1}{2}}$

(ii) 2 heads



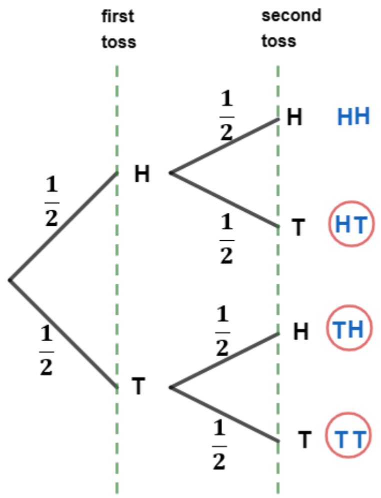
There is only one outcome for 2 heads circled in red.

P(HH)$=\frac{1}{2}\times\frac{1}{2}$ (follow the red arrows for route)

$=\frac{1\times1}{2\times2}$

$=\mathbf{\frac{1}{4}}$

(iii) at least one tail.



There are three possible outcomes circled in red. At least one T means one or more T(s).

P(HH)+P(HT)+P(TH)+P(TT)=1

P(HT)+P(TH)+P(TT)=1−P(HH)

P(at least one tail)=1−P(HH)

$$= 1 - \frac{1}{4} \qquad \left[P(HH) = \frac{1}{4}; from\ (i)\right]$$

$$= \frac{4}{4} - \frac{1}{4}$$

$$= \frac{4-1}{4}$$

$$= \mathbf{\frac{3}{4}}$$

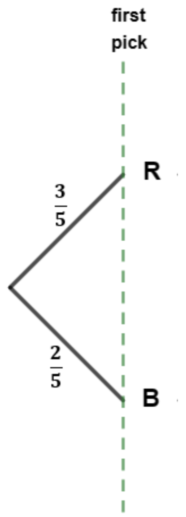
Example 13

A bag contains 3 red balls and 2 blue balls. A ball is drawn at random without replacing the first ball a second ball is drawn. **(a)** Draw a tree diagram to show the possible outcomes. Using the tree diagram to find the probability of picking **(i)** both red balls **(ii)** one is red and one is blue.

Solution:

a) Draw a tree diagram to show the possible outcomes.

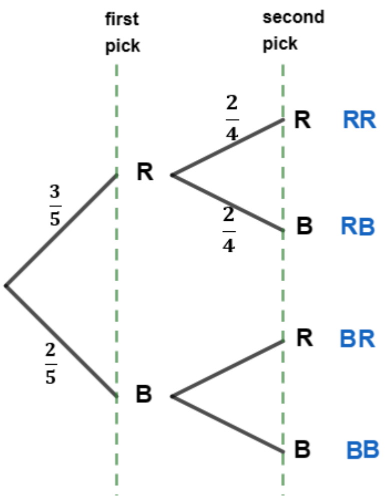
Step 1



No balls have been picked yet so there 3 red and 2 blue, which gives a total of 5 balls.

P(red)=$\frac{3}{5}$ P(blue)=$\frac{2}{5}$

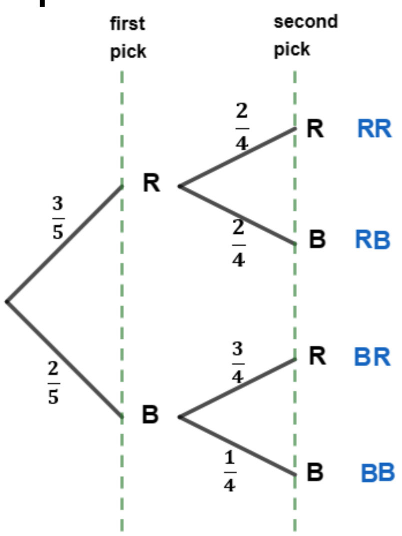
Step 2



If a red ball is picked it means 2 red ball remain, since blue has not been picked the number remains unchanged. The bag now contains 2 red and 2 blue. 4 balls remain altogether.

P(red)= $\frac{2}{4}$ P(blue))= $\frac{2}{4}$

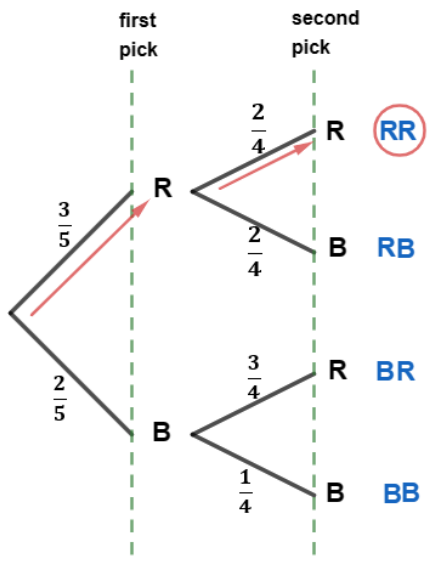
Step 3



If a blue ball is picked, one blue ball remains, three red balls remain since no red is picked, so there are three red balls and one blue ball. 4 balls remain altogether.

P(red)=$\frac{3}{4}$ P(blue)=$\frac{1}{4}$

(i) both red balls.



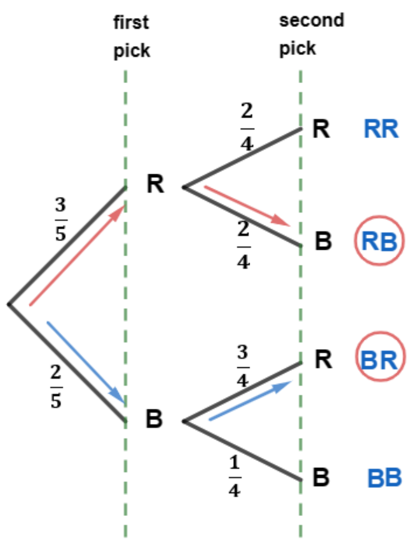
$$P(RR) = \frac{3}{5} \times \frac{2}{4}$$

$$= \frac{3 \times 2}{5 \times 4}$$

$$= \frac{6}{20}$$

$$= \mathbf{\frac{3}{10}}$$

(ii) one is red and one is blue.



$P(RB) = \frac{3}{5} \times \frac{2}{4}$ (red arrows)

$$= \frac{3 \times 2}{5 \times 4}$$

$$= \frac{6}{20}$$

$= \mathbf{\frac{3}{10}}$

P(BR)$= \frac{2}{5} \times \frac{3}{4}$ (Blue arrows)

$= \frac{2 \times 3}{5 \times 4}$

$= \frac{6}{20}$

$= \mathbf{\frac{3}{10}}$

P(one is red and one is blue)= P(RB)+ P(BR)

$= \frac{3}{10} + \frac{3}{10}$

$= \frac{3+3}{10}$

$= \frac{6}{10}$

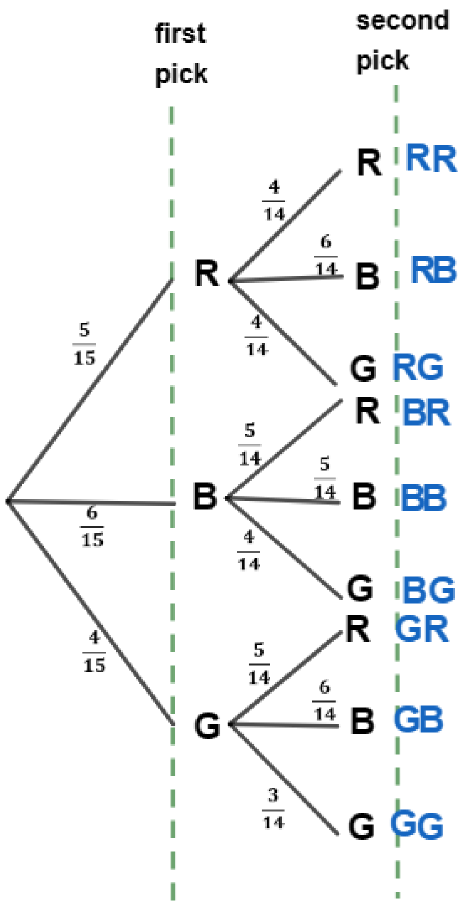
$= \mathbf{\frac{3}{5}}$

Example 14

A bag contains 15 marbles which are identical except for colour. Five of the marbles are red, six are blue and four are green. Two marbles are picked at random from the bag. Find the probability that they are **(a)** of the same colour **(b)** of different colours.

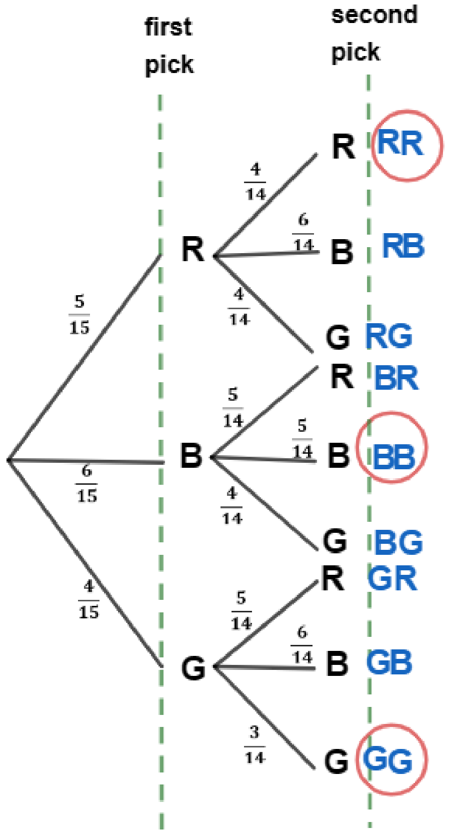
Solution:

Step 1: create a tree diagram.



Step 2: Answer the questions using the tree diagram.

(a) of the same colour



$$P(RR) = \frac{5}{15} \times \frac{4}{14}$$

$$= \frac{5 \times 4}{15 \times 14}$$

$$= \frac{20}{210}$$

$$P(BB) = \frac{6}{15} \times \frac{5}{14}$$

$$= \frac{6 \times 5}{15 \times 14}$$

$$= \frac{30}{210}$$

$$P(GG) = \frac{4}{15} \times \frac{3}{14}$$

$$= \frac{4 \times 3}{15 \times 14}$$

$$= \frac{12}{210}$$

P(same colour)= P(RR)+ P(BB)+ P(GG)

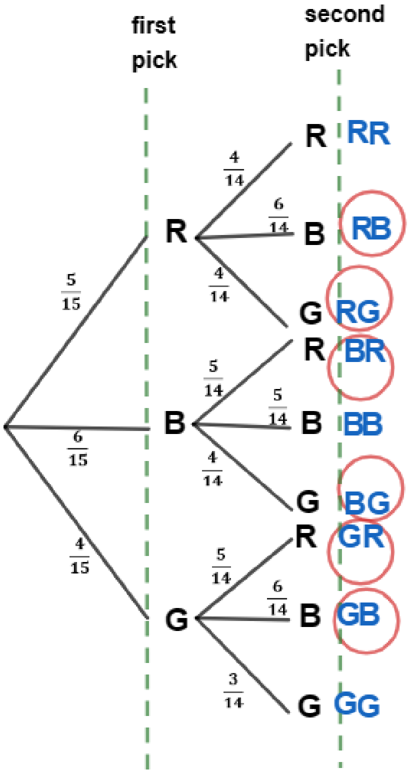
$$= \frac{20}{210} + \frac{30}{210} + \frac{12}{210}$$

$$=\frac{20+30+12}{210}$$

$$=\frac{62}{210}$$

$$=\mathbf{\frac{31}{105}}$$

(b) of different colours.



P(different colours)=1−P(same colours)

$$=1-\frac{31}{105}$$

$$=\frac{105}{105}-\frac{31}{105}$$

$$=\frac{105-31}{105}$$

$$=\mathbf{\frac{74}{105}}$$

Exercise 10d [43 marks] [Answers]

1. Two coins are tossed, find the probability of getting **(a)** two tails [2] **(b)** at least one head. [2]

2. Two dice are thrown at the same time. Find the probability of getting

(a) A total score divisible by 3. [2]

(b) a score which is a perfect cube [2]

(c) at least one 3 on a die. [2]

3. A jar contains 18 marbles which are identical except for colour. Six of the marbles are red, five are blue and seven are white. Two marbles are picked at random from the jar. Find the probability that they are

(a) of the same colour [3]

(b) of different colours. [2]

4. A toy store has 2 shelves of toys. Shelf A has 12 cars, 8 dolls, and 4 puzzles, while Shelf B has 10 cars and 6 dolls. Ava is asked to pick a toy from Shelf A and Liam from Shelf B. Find the probability that:

(a) Ava picked a car, [2]

(b) both Ava and Liam picked cars. [3]

(c) both Ava and Liam picked toys of the same type. [3]

5. Assuming there is an equal chance of a coin landing heads up or landing tails up, find the probability that:

(a) a coin lands heads up, [1]

(b) in three tosses of the coin, there are exactly two heads. [3]

6. Emily had three \$5 notes, two \$10 notes, and four \$20 notes in her wallet. She wanted to buy a book costing \$15. She randomly pulled out two notes, one after another, without first checking. Find the probability that she pulled out notes that

(a) were worth the same value, [3]

(b) was not enough to buy the book. [3]

c) was at least enough to buy the book. [2]

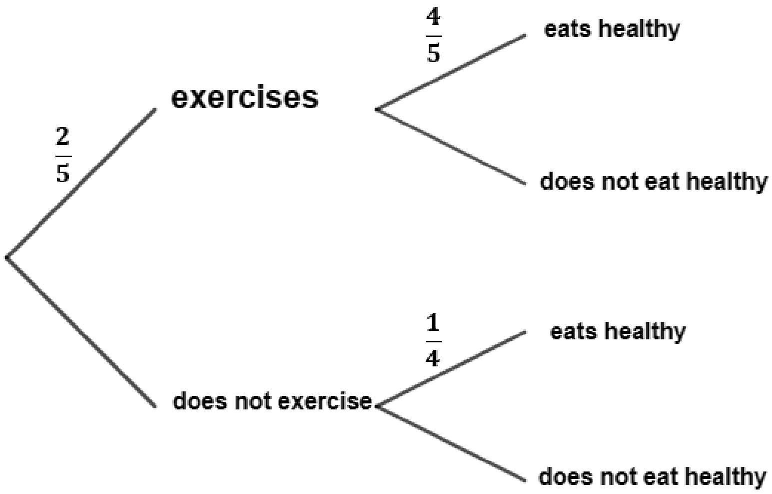
7. A survey found that the probability that a person who exercises regularly will also eat a healthy diet is $\frac{4}{5}$. If a person does not exercise regularly, the probability that they will eat a healthy diet is $\frac{1}{4}$. The probability that a person exercise is $\frac{2}{5}$

(a) Complete the probability tree diagram. [2]

(b) Hence or otherwise find the probability that a person who is selected at random, exercises regularly and eats a healthy diet. [2]

(c) Hence or otherwise find the probability that a person, who is selected at random, does not exercise regularly and does not eat a healthy diet. [2]

(d) Hence or otherwise find the probability that a person, who is selected at random, eats a healthy diet. [2]


$\frac{2}{5}$
exercises
$\frac{4}{5}$
eats healthy
does not eat healthy
does not exercise
$\frac{1}{4}$
eats healthy
does not eat healthy

Answers

Exercise 10a. back to [Probability]

1. (a) $\frac{9}{20}$

(b) $\frac{3}{10}$ $\left[\frac{6}{20}\right]$

2. 0,45 $[1-(0{,}35+0{,}2)]$

3. (a) $\frac{2}{5}$ $\{2;3;5;11\}$

(b) $\frac{1}{5}$ $\{1{,}2;0{,}8\}$

(c) $\frac{2}{5}$ $\{5;9;11;13\}$

(d) $\frac{3}{10}$ $\{3;6;9\}$

4. (a) $\frac{1}{6}$

(b) $\frac{1}{2}$ $\{2;4;6\}$

(c) $\frac{2}{3}$ $\{3;4;5;6\}$

5. (a) $\frac{1}{13}$ $\left[\frac{4}{52}\right]$

(b) $\frac{1}{52}$

(c) $\frac{1}{2}$ $\left[\frac{26}{52}\right]$

(d) 0 (hearts are red not black)

6. (a) $\frac{2}{3}$ $\{a;e;i;o;u\}$

(b) $\frac{1}{13}$ $\left[\frac{2}{26}\right]$

(c) $\frac{4}{13}$ $\left[\frac{8}{26}\ \textit{count all letters without repeating the repeated letters}\right]$

7. (a) $\frac{1}{2}$

(b) 1

8. 0,55 $[1-(0{,}15+0{,}3)]$

Exercise 10b. [Probability]

1. (a) $\frac{13}{15}$ $\left[\frac{8}{15}+\frac{5}{15}\right]$

(b) 0 [Since there are no green pencils in the box]

2. (a) $\frac{1}{2}$ $\left[\frac{4}{12}+\frac{2}{12}\right]$

(b) 1

[Since the question asks for the probability of picking "a marble", and there are only marbles in the bag]

3. (a) $\frac{3}{5}$ [numbers in X = {5, 11, 13}]

(b) $\frac{7}{10}$

[X∪Y = {1, 2, 3, 5, 6, 8, 9, 11, 12, 13}] [Multiples of 3 = {3, 6, 9, 12}] [Numbers greater than 10 = {11, 12, 13}

4. (a) $\frac{1}{10}$ $\left[1-\frac{9}{10}\right]$

(b) $\frac{1}{2}$ $\left[\frac{2}{5}+\frac{1}{10}\right]$

c) Emily $\left[\frac{2}{5}>\frac{1}{10}\right]$

5. (a) $\frac{1}{2}$ $\left[\frac{1}{6}+\frac{2}{6}\right]$ numbers divisible by 3={3; 6}

(b) $\frac{5}{6}$ $\left[\frac{3}{6}+\frac{2}{6}\right]$ prime number={2; 3; 5} perfect square={1; 4}

6. (a) $\frac{5}{52}$

{4 of hearts, 4 of diamonds, 4 of clubs, 4 of spades, Queen of clubs} (4 cards with value 4, and 1 Queen of clubs)

(b) $\frac{7}{13}$

{26 black cards (13 spades and 13 clubs), 2 red 5s (one from hearts and one from diamonds)}

c) $\frac{2}{13}$ {4 Aces (one from each suit), 4 7s (one from each suit)}

Exercise 10c. [Probability]

1. (a) $\frac{4}{23}$ $\left[P(red)=\frac{12}{24}=\frac{1}{2}\right]\left[P(blue)=\frac{8}{23}\right]\left[P(red\ then\ blue)=\frac{1}{2}\times\frac{8}{23}\right]$

2. (a) $\frac{1}{20}$ $\left[\frac{1}{4}\times\frac{1}{5}\right]$

(b) $\frac{7}{20}$

$\left[P(X\ is\ defective\ and\ Y\ is\ not\ defective)\ =\frac{1}{4}\times\frac{4}{5}=\frac{4}{20}\right]\left[P(Y\ is\ defective\ and\ X\ is\ not\ defective)\ =\frac{1}{5}\times\frac{3}{4}=\frac{3}{20}\right]\left[\frac{4}{20}+\frac{3}{20}=\frac{4+3}{20}\right]$

3. $\frac{1}{8}$ $\left[\frac{1}{2}\times\frac{1}{4}\right]$[Number of sections greater than 4 = 4 (5, 6, 7, 8) ; 13 hearts of 52 cards]

4. (a))(i) $\frac{4}{15}$ **(ii)** 0 (since there are no orange marbles)

(b) (i) $\frac{2}{21}$ $\left[\frac{1}{3} \times \frac{2}{7}\right]$ (without replacement)

(ii) $\frac{4}{21}$

$\left[P(red\ first\ then\ blue) = \frac{1}{3} \times \frac{2}{7} = \frac{2}{21}\right]$ $\left[P(blue\ first\ then\ red) = \frac{4}{15} \times \frac{5}{14} = \frac{2}{21}\right]$ $\left[P(one\ is\ red\ and\ one\ is\ blue) = \frac{2}{21} + \frac{2}{21}\right]$

5. (a) 5 (occurs twice) $\left[\frac{2}{10} = \frac{1}{5}\right]$

(b) $\frac{1}{45}$

$\left[P(9\ then\ 10) = \frac{1}{10} \times \frac{1}{9} = \frac{1}{90}\right]$ $\left[P(10\ then\ 9) = \frac{1}{10} \times \frac{1}{9} = \frac{1}{90}\right]$ $\left[P(sum\ of\ the\ two\ numbers\ is\ 19\ ) = \frac{1}{90} + \frac{1}{90} = \frac{2}{90} = \frac{1}{45}\right]$

6. $\frac{1}{26}$

$\left[P(heart\ first\ then\ 7) = \frac{13}{52} \times \frac{4}{52} = \frac{1}{52}\right]$ $\left[P(7\ first\ then\ heart) = \frac{4}{52} \times \frac{13}{52} = \frac{1}{52}\right]$ $\left[P(\ a\ 7\ and\ a\ heart) = \frac{1}{52} + \frac{1}{52} = \frac{1}{26}\right]$

7. (a) 10 $\left[\frac{1}{5} = \frac{2}{3} \times \frac{3}{y}\right]$

(b) (i) $\frac{7}{10}$ $\left[1 - \frac{3}{10}\right]$

(ii) $\frac{17}{30}$ $\left[\frac{2}{3} \times \frac{7}{10} + \frac{1}{3} \times \frac{3}{10}\right]$

8. (a) $\mathbf{\frac{15}{64}}$ $\left[\frac{3}{8} \times \frac{5}{8}\right]$

(b) $\mathbf{\frac{15}{64}}$ $\left[\frac{3}{8} \times \frac{5}{8}\right]$

$\left[P(Friday\ \leq\ 50)\ =\ 1 - \ P(Friday\ >\ 50) = \ 1\ - \frac{3}{8}\ =\ \frac{5}{8}\right]$ $\left[P(Saturday\ \leq\ 50)\ =\ 1\ - P(Saturday\ >\ 50) = \ 1\ - \frac{5}{8} = \ \frac{3}{8}\right]$

c) $\mathbf{\frac{17}{32}}$

$\left[P(Friday\ >\ 50\ and\ Saturday\ \leq\ 50) = \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}\right]$ $\left[P(Friday\ \leq\ 50\ and\ Saturday\ >\ 50)\ = \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}\right]$ $\left[P(Only\ one\ night\ >\ 50) =\ \frac{9}{64} + \frac{25}{64} = \frac{34}{64} = \mathbf{\frac{17}{32}}\right]$

Exercise 10d. [Probability]

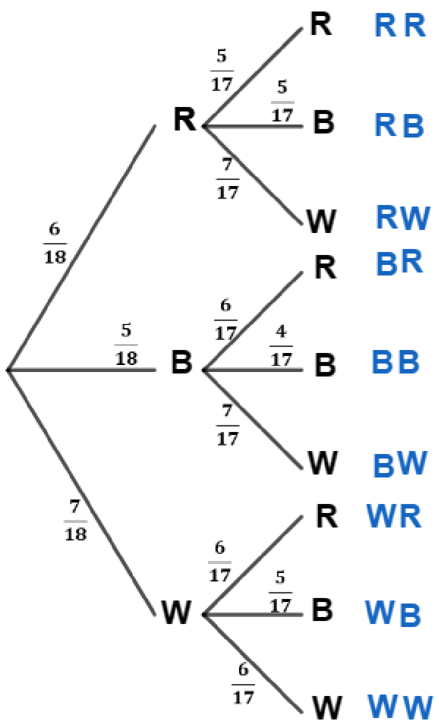
1. (a) $\frac{1}{4}$ **(b)** $\frac{3}{4}$

2. (a) $\frac{1}{3}$ $\left[\frac{12}{36}\right]$

(b) $\frac{5}{36}$ (8 is the only perfect cube)

c) $\frac{11}{36}$

3.

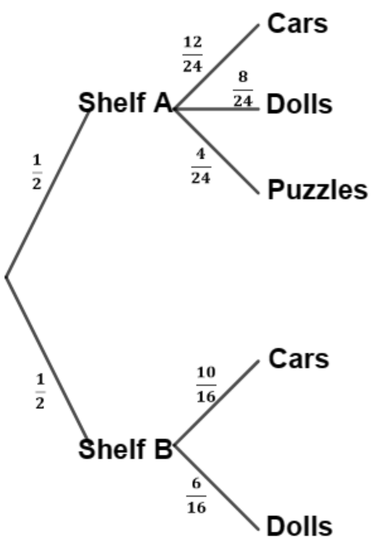


(a) $\frac{46}{153}$

$\left[P(RR) = \frac{6}{18} \times \frac{5}{17} = \frac{30}{306}\right]$ $\left[P(BB) = \frac{5}{18} \times \frac{4}{17} = \frac{20}{306}\right]$ $\left[\mathrm{P(WW)} = \frac{7}{18} \times \frac{6}{17} = \frac{42}{306}\right]$ [P(same colour)= P(RR)+ P(BB)+ P(WW)]

(b) $\frac{107}{153}$ $\left[1 - \frac{46}{153}\right]$

4.



(a) $\frac{\mathbf{1}}{\mathbf{4}}$ $\left[\frac{1}{2} \times \frac{12}{24}\right]$

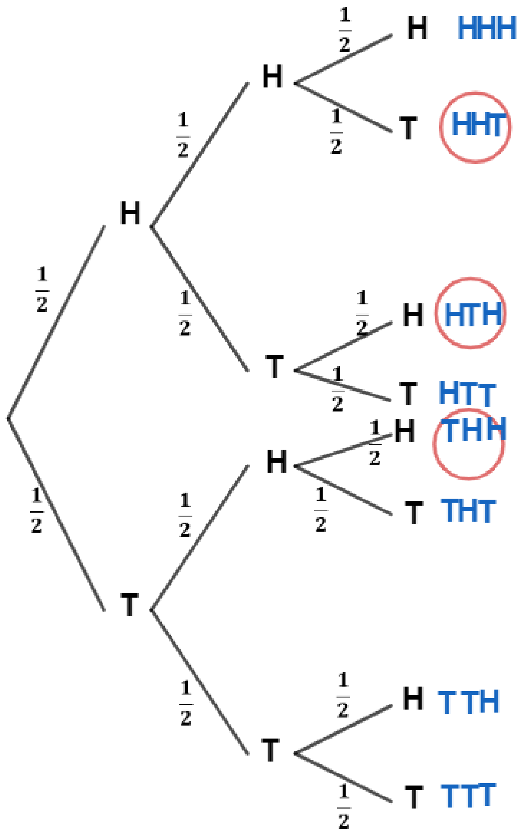
(b) $\frac{5}{64}$

$\left[P(Liam\ picking\ a\ car) = \frac{1}{2} \times \frac{10}{16} = \frac{5}{16}\right]\left[P(Ava\ picking\ a\ car) = \frac{1}{4}\right]\left[P(both\ picking\ a\ car) = \frac{1}{4} \times \frac{5}{16} = \frac{5}{64}\right]$

c) $\frac{41}{320}$

$\left[P(Ava\ picking\ a\ doll) = \frac{1}{2} \times \frac{8}{24} = \frac{1}{6}\right]\left[P(Liam\ picking\ a\ doll) = \frac{1}{2} \times \frac{6}{10} = \frac{3}{10}\right]\left[P(both\ pick\ a\ doll) = \frac{1}{6} \times \frac{3}{10} = \frac{1}{20}\right]\left[P(same\ type\ of\ toys) = P(both\ pick\ cars) + P(both\ pick\ dolls) = \frac{5}{64} \times \frac{1}{20} = \frac{41}{320}\right]$

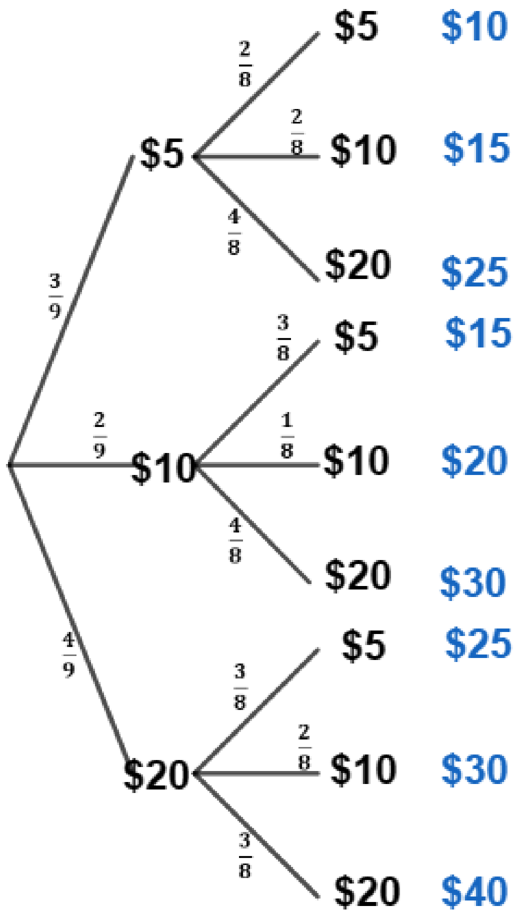
5.



(a) $\frac{1}{2}$

(b) $\frac{3}{8}$ $[P(exactly\ 2\ heads) = P(HHT) + P(HTH) + P(THH)]$

6.



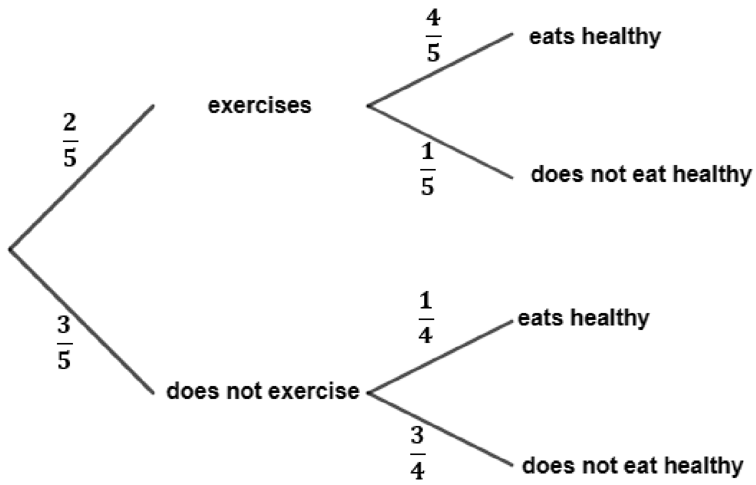
(a) $\frac{5}{18}$

$\left[P(\$5\ then\ \$5) = \frac{3}{9} \times \frac{2}{8} = \frac{1}{12}\right] \left[P(\$10\ then\ \$10) = \frac{2}{9} \times \frac{1}{8} = \frac{1}{36}\right] \left[P(\$20\ then\ \$20) = \frac{4}{9} \times \frac{3}{8} = \frac{1}{6}\right] \left[P(worth\ the\ same\ value) = \frac{1}{12} \times \frac{1}{36} \times \frac{1}{6} = \frac{5}{18}\right]$

(b) $\frac{1}{12}$ $\left[P(\$5\ then\ \$5) = \frac{3}{9} \times \frac{2}{8} = \frac{1}{12}\right]$

c) $\frac{11}{12}$ $\left[1 - \frac{1}{12}\right]$ (at least means $15 or more.)

7.(a)



(b) $\frac{8}{25}$ $\left[\frac{2}{5} \times \frac{4}{5}\right]$

c) $\frac{9}{20}$ $\left[\frac{3}{5} \times \frac{3}{4}\right]$

(d) $\frac{47}{100}$

$\left[P(does\ not\ exercise\ but\ eats\ healthy) = \frac{3}{5} \times \frac{1}{4} = \frac{3}{20}\right] \left[P(eats\ healthy) = \frac{8}{25} + \frac{3}{20}\right]$