For Performance Measurement

ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

MATHEMATICS

9164/2

PAPER 2

JUNE 2014 SESSION

3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

If a numerical value for g is necessary, take $g = 9.81 \text{ ms}^{-2}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

Within each section of the paper, questions are printed in the order of their mark allocations and candidates are advised, within each section, to attempt questions sequentially.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 6 printed pages and 2 blank pages.

Section (a): Pure Mathematics

1 **(a)** The least value of the function $x^2 + px + q$ is 3 and this occurs when $x = -2$.

Find the values of p and q. [3]

(b) Find the values of k for which the equation $x^2 + (3k - 7)x + 2k + 6 = 0$ has real roots. [3]

2 Prove by induction that if $y = xe^x$, then $\frac{d^n y}{dx^n} = (x + n)e^x$ for all natural values of n. [7]

3 **(a)** If $y = ln\left(\tan\left(\frac{\pi}{2} - \frac{x}{2}\right)\right)$, show that $\frac{dy}{dx} = -\operatorname{cosec} x$. [4]

(b) Express $\frac{x^2 + 4}{x^2 - 4}$ in partial fractions. [2]

Hence find the exact value of $\int_3^4 \frac{x^2 + 4}{x^2 - 4} dx$. [3]

4 **(i)** Prove the identity $\frac{\sin 5\theta}{\sin\theta} - \frac{\cos 5\theta}{\cos\theta} \equiv 4\cos 2\theta$. [4]

(ii) By substituting $\theta = 18°$ into the identity in part **(i)** show that $4\sin 18° \cos 36° = 1$. [3]

(iii) Using the result of part **(ii)**, deduce that $\sin 18°$ is the root of the equation $4x - 8x^3 - 1 = 0$. [2]

5 **(a)** Express $\frac{e^{\frac{\pi}{2}i}}{e^{\frac{\pi}{3}i}}$ in the form $a + bi$. [2]

(b) Sketch on an Argand diagram the locus defined by $\arg\left(\frac{z+1}{1+\sqrt{3}i}\right) = \frac{\pi}{4}$. [4]

(c) Use de Moivre's theorem to show that $4\cos^3\theta = \cos 3\theta + 3\cos\theta$. [5]

6 **(a)** The triangle ABC of area 11 square units is mapped into triangle $A^{l}B^{l}C^{l}$ by $M = \begin{pmatrix} 3 & 2 \\ 1 & 5 \end{pmatrix}$.

Find the area of triangle $A^{1}B^{1}C^{1}$. [2]

(b) The point $\begin{pmatrix} x \\ y \end{pmatrix}$ is transformed by $\begin{pmatrix} x \\ y \end{pmatrix} : \mapsto \begin{pmatrix} -2 & 2 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$.

Find the equations of lines which are mapped onto themselves. [7]

(c) Given that $A^{-1} = \begin{pmatrix} -1 & -2 & 2 \\ 2 & 5 & -4 \\ 1 & 1 & -1 \end{pmatrix}$, $B^{-1} = \begin{pmatrix} 3 & -3 & 3 \\ -1 & 1 & 1 \\ 2 & 4 & -2 \end{pmatrix}$ and that

$AB\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 0 \\ 0 \end{pmatrix}$, find x, y and z. [5]

7 The point P has position vector $\boldsymbol{i} + 3\boldsymbol{j} + 3\boldsymbol{k}$, the line l has an equation $\boldsymbol{r} = \boldsymbol{i} + 5\boldsymbol{j} - \boldsymbol{k} + t(\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k})$ and the plane π has an equation $\boldsymbol{r} = 5\boldsymbol{i} + 5\boldsymbol{k} + \lambda(2\boldsymbol{i} + \boldsymbol{j}) + \mu(2\boldsymbol{i} + \boldsymbol{j} + 4\boldsymbol{k})$.

(i) Find the point of intersection of l and π. [4]

(ii) Find the length of the perpendicular from P to π. [5]

(iii) Find the length of the perpendicular from P to l. [4]

(iv) Find the acute angle between l and π, correct to the nearest 0.1°. [3]

Section (b): Mechanics

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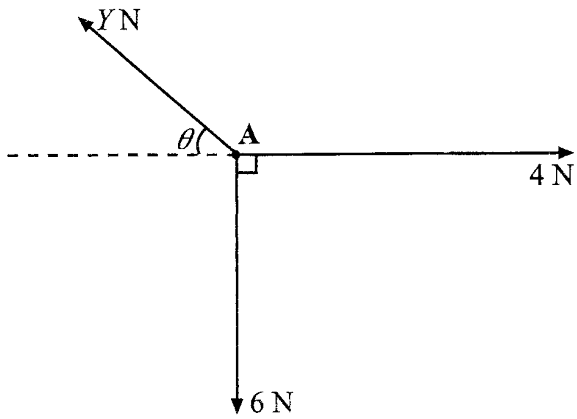

A particle **A** rests in equilibrium on a smooth horizontal table.

Forces of magnitude 4 N, 6 N and Y newtons act on **A** (see diagram).

(i) Find the values of Y and θ. [4]

(ii) State **one** assumption made in answering the question. [1]

9 Particles A and B of masses 5 kg and 8 kg respectively are connected by means of a light inextensible string which passes over a smooth pulley. Particle A lies on a smooth plane inclined at an angle $\tan^{-1}\left(\frac{4}{3}\right)$ to the horizontal and particle B is hanging freely.

When the system is released from rest, calculate

(i) the tension in the string, [3]

(ii) the acceleration of the particles for the part of the motion before B hits the ground. [2]

10 A particle decelerates uniformly from a velocity of 25 ms^{-1} to 15 ms^{-1} in 2 seconds. It maintains a velocity of 15 ms^{-1} for t_1 seconds and then accelerates uniformly to a velocity of 27 ms^{-1} in 3 seconds.

(i) Sketch a (t, v) graph for the whole journey. [3]

(ii) Given that it travels a total distance of 193 m, find the value of t_1. [3]

11 A particle is projected from a point O on a level ground with initial velocity of 10 ms^{-1} at an angle of elevation θ.

(i) Given that the particle passes through a point which is 3 m horizontally away from O and 2 m above the level of O, calculate the value of θ. [4]

(ii) Calculate the distance from the point of projection to the point where the particle is 8 m below the ground. [4]

Section (c): Statistics

12 A bag contains 24 counters of which 6 are red, 8 are green and 10 are yellow. Three counters are taken from the bag at random without replacement.

(i) Show that the probability that 2 of the counters taken are green is $\frac{56}{253}$. [2]

(ii) Given that 2 of the counters are green, find the probability that the first counter taken is red. [3]

13 The probability that a boy hits a target is 0.8. Assuming that shots are independent of each other and suppose that during each practice period, the boy fires shots until he hits the target.

(i) Find the mean and standard deviation of the number of shots fired per practice period. [3]

(ii) Find the probability that the boy will need to take at least five shots to hit the target. [2]

14 The following are television prices in dollars taken in 40 different shops.

40	130	170	240	360	520	170	130
240	360	520	120	220	170	330	480
160	290	200	120	480	160	210	330
70	140	180	260	370	90	150	200
280	450	80	140	190	420	270	120

(i) Construct a stem and leaf diagram for the data. [3]

(ii) Find the

1. median,

2. quartiles. [2]

(iii) Draw a box and whisker plot. [2]

15 The probability that a seed grown under specified conditions will germinate and produce a plant is 0.8. The minimum number of seeds, n, are to be planted under these conditions to ensure a probability of at least 0.9 that 60 or more seeds will germinate and produce a plant.

(i) Using a suitable approximation show that $n^2 - 149.2n + 5531.6 \geq 0$. [4]

(ii) 1. Solve the inequality in part **(i)**.

2. Hence or otherwise find the minimum value of n, the number of seeds to be planted.

[3]