ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

MATHEMATICS 9164/1

PAPER 1

NOVEMBER 2016 SESSION 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae
- Electronic calculator

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

Questions are printed in the order of their mark allocations and candidates are advised to attempt questions sequentially.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed pages and 3 blank pages.

1 The complex number $\dfrac{1}{-2+i}$ is denoted by u.

Find the

(i) modulus of u, [2]

(ii) argument of u. [2]

2 Find the series expansion of $(9+2x)^{\frac{-3}{2}}$ up to and including the term in x^2, simplifying the coefficients. [4]

3 The equation of a circle is $x^2 + y^2 = 4$ and the equation of its tangent is $y = 2mx + c$.

Show that $\boxed{16m^2 - c^2} + 4 = 0$. [4]

4 (i) Given that $f(x) = 2 - |x+3|$, sketch the graph of $y = |f(x)|$. [2]

(ii) Hence from **(i)** solve the inequality $|f(x)| > 1$. [2]

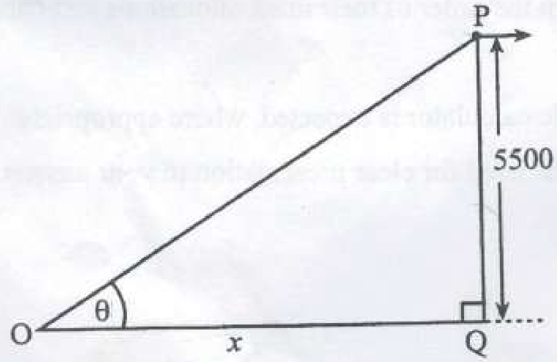
5 The variable quantities x and y are related by the rule $y = \dfrac{P}{\sqrt{x}}$, where P is a constant. Five pairs of values of x and y measured experimentally are shown in the table.

x	20	30	45	50	75
y	0.9	1.1	1.35	1.40	1.52

(i) Plot the graph of y against $\dfrac{1}{\sqrt{x}}$. [3]

(ii) Use the graph to find the value of P. [2]

6



The diagram shows the position of an aeroplane P flying in a straight line at a constant height of 5 500 metres. The angle of elevation of the plane from the observer O is θ° when the observer is x m from a point Q, on level ground, which is vertically below P. The speed of the aeroplane is 550 000 m/hr.
Find the rate of change of angle θ when θ is 25°. [5]

7 (i) Express $\dfrac{x-3}{x^2-1}$ in partial fractions. [2]

(ii) Hence from (i)find $\int_2^3 \dfrac{x-3}{x^2-1}dx$, giving the answer in terms of a single logarithm. [4]

8 (i) Show that the equation $\cos(45° + \theta) = 2\sin(45° - \theta)$ can be reduced to $\tan\theta = 1$. [4]

(ii) Hence or otherwise, solve the equation $\cos(45° + \theta) = 2\sin(45° - \theta)$ for $0° \leq \theta \leq 360°$. [2]

9 (a) The present population of a country is 27 million. The population is projected to be 38 million in 36 years time.

Calculate the annual growth rate correct to two decimal places, if this projection is based on a geometric sequence. [3]

(b) Find the sum, S, defined by $S = \sum_{n=1}^{15}\left(2n + \dfrac{1}{2}\right)$. [3]

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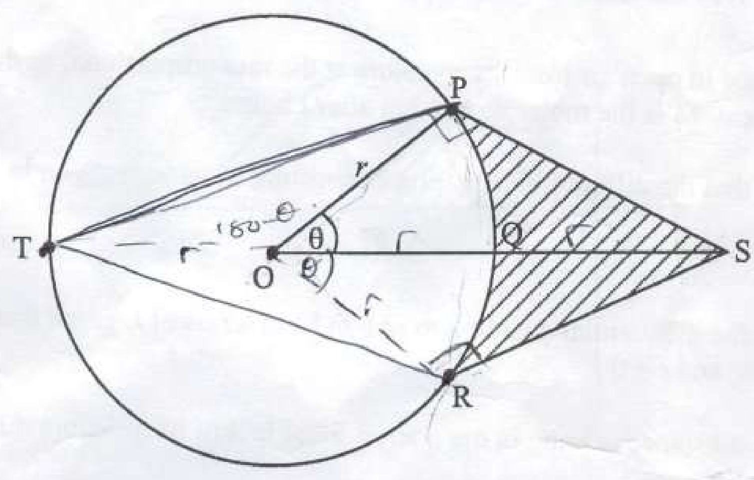

The points P, Q, R and T lie on the circle centre O and radius r. S is on OQ produced such that QS = r and PS = RS. Angle POS = θ radians.

(i) Calculate the shaded area PQRS. [4]

(ii) Show that the **perimeter** of PTRS is given by $2r\left(\pi - \theta + \sqrt{5 - 4\cos\theta}\right)$. [5]

11 The polynomial $2x^3 + ax^2 - bx + 12$ where a and b are constants is denoted by $f(x)$. The result of differentiating $f(x)$ with respect to x is denoted by $f'(x)$. It is given that $(x - 1)$ is a factor of $f(x)$ and when $f'(x)$ is divided by $(x - 1)$ the remainder is -5.

(i) Find the values of a and b. [5]

(ii) When a and b have these values in **(i)**, solve the equation $f(x) = 0$. [4]

12 Relative to the origin O the position vectors of the points A and B are

$$\overrightarrow{OA} = 3\boldsymbol{i} + 4\boldsymbol{j} + (2 - b)\boldsymbol{k}$$
$$\overrightarrow{OB} = -\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$$

(i) Find the unit vector in the direction of $\overrightarrow{OB}$. [2]

(ii) Find the value of b, if $B\hat{O}A = 90^\circ$. [2]

(iii) Write down an expression for $|\overrightarrow{AB}|$. [3]

(iv) Hence find the value of b if $|\overrightarrow{AB}|^2$ is minimum. [3]

13 A wet substance in open air loses its moisture at the rate proportional to the moisture content. M is the moisture content after t hours.

(i) Show that the differential equation connecting M and t is given by $\dfrac{dM}{dt} = kM$, [2]

(ii) Solve the differential equation to obtain M in terms of t, given that initially $M = m_0$ and $t = 0$ [6]

(iii) If the substance is hung in open air, it loses half of its moisture during the first hour.

Find the time it will lose 95% moisture, in same weather conditions. [4]

14 **(i)** By sketching the graphs of $y = \tan x$ and $y = 4x$ on a single diagram for $0 \le x < \dfrac{\pi}{2}$, show that the equation $\tan x = 4x$ has exactly one positive root. [3]

(ii) Show, by calculation, that the root of the equation lies between 1 and 1.5. [3]

(iii) Show that the Newton-Raphson structure can be written in the form $\dfrac{x_n tan^2 x_n - tan x_n + x_n}{tan^2 x_n - 3}$. [3]

(iv) Use the structure in **(iii)** once starting with $x_1 = 1.4$, to estimate the root correct to 4 decimal places. [3]

15 The function f is defined by $f(x) = 2x - 3$, for $x \in \mathrm{R}$.

(a) Sketch on a single diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$ making clear the relationships between the graphs. [2]

(b) **(i)** The function g is defined by $g(x) = 3x^2 + 2x - 2$, for $x \in R$. Express $gf(x)$ in terms of x in its simplest form. [2]

(ii) Hence find the minimum value of $gf(x)$. [3]

(c) Given that $g(x) = 3x^2 + 2x - 2$ is defined for $x \geq \mathrm{k}$,

(i) state the minimum value of k, such that $g(x)$ can have an inverse, [3]

(ii) find, $g^{-1}(x)$. [2]

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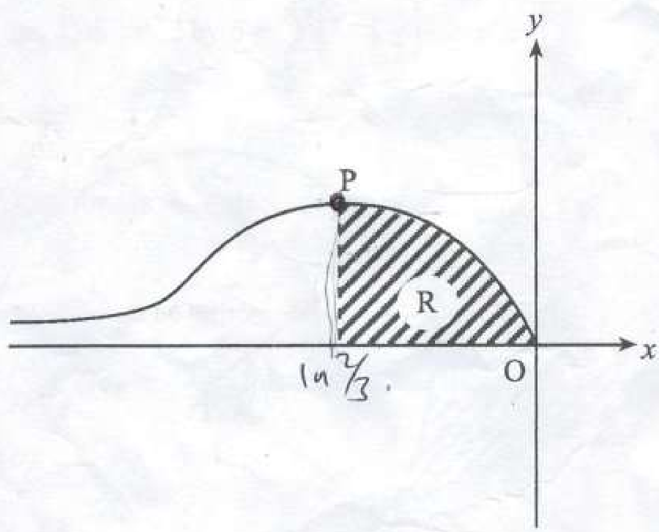

The diagram shows the graph of $y = e^{2x} - e^{3x}$ with shaded region R. Its maximum point is P.

(i) Show that the x - coordinate of P is $ln\frac{2}{3}$. [4]

(ii) Calculate the area of the shaded region R. [4]

(iii) Find the volume of the solid of revolution generated when the shaded region R is rotated completely about the x-axis giving the answer correct to 3 significant figures. [4]