ZIMBABWE SCHOOL EXAMINATIONS COUNCIL
General Certificate of Education Advanced Level

MATHEMATICS 9164/1
PAPER 1 PURE MATHEMATICS

JUNE 2017 SESSION 3 hours

Additional materials:
Answer paper — Graph paper
List of Formulae — Non-programmable electronic calculator

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

Questions are printed in the order of their mark allocations.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed pages and 3 blank pages.

Copyright: Zimbabwe School Examinations Council, J2017.

©ZIMSEC J2017

Scanned with CamScanner

1 (a) On the same axis, sketch the graphs of $y = |x-1|$ and $y = x$. [2]

(b) Hence or otherwise solve the inequality $|x-1| < x$. [1]

2 If $f(x) = 3x^3 + x^2 - 8x + 4$, factorise $f(x)$ completely. [4]

3 (a) Find the product of $\left(x^{\frac{1}{3}} - 2\right)$ and $\left(x^{\frac{2}{3}} + 2x^{\frac{1}{3}} + 1\right)$. [2]

(b) Solve the equation $y^{\frac{2}{3}} - 5y^{\frac{1}{3}} + 6 = 0$. [4]

4 The radius of a spherical globe was increased from 11 cm to 11.02 cm.

Determine the percentage increase in the volume of the globe. [6]

[Volume of sphere = $\frac{4}{3}\pi r^3$]

5 (a) Expand $\frac{1}{\sqrt{1-2x}}$ up to and including the term in x^3. [3]

(b) State the values of x for which this expansion is valid. [1]

(c) Taking $x = \frac{-1}{6}$, use the expansion to find the value of $\sqrt{3}$, correct to 5 decimal places. [2]

6 The coordinates of P, Q and R are (1, 2, 1), (4, 7, 8) and (6, 4, 12) respectively.

(a) S is a point such that PQRS is a parallelogram.

Find the coordinates of S. [3]

(b) The points M and N are the midpoints of PQ and QR respectively.

Find the unit vector in the direction of $\overrightarrow{MN}$. [3]

7 It is given that $f(x) = x^2 - 8x + 10$.

(a) Express $f(x)$ in the form $(x+a)^2 + b$, where a and b are constants. [2]

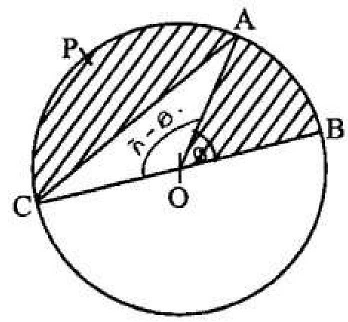
(b) Hence or otherwise, find the least value of the function, $f(x)$ and write down the equation of the line of symmetry of the graph $y = f(x)$. [2]

(c) State the geometrical transformations which map the graph of $y = x^2$ onto the graph of $y = x^2 - 8x + 10$. [3]

Scanned with CamScanner

8 **(a)** Solve the equation $2\cos^2 x + 3\sin x = 3$ for $0 < x < \pi$. [4]

(b)



In the diagram points A, B, C and P are on the circumference of a circle centre O and radius r. BOC is a diameter of the circle. $A\hat{O}B = \theta$ radians.

Show that the area of the shaded region is $\frac{1}{2}r^2(\pi - \sin\theta)$. [4]

9 It is given that $f(x) = \dfrac{x^3 + 2x^2 + x + 2}{x^2 + x}$.

(a) Express $f(x)$ in the form $P(x) + \dfrac{A}{x} + \dfrac{B}{x+1}$, where A and B are constants and P(x) is a function of x. [4]

(b) Hence find the exact value of $\int_1^2 f(x)dx$. [4]

10 The complex numbers z_1 and z_2 are such that $z_1 = 2 - 3i$ and $z_2 = 1 + 3i$.

(a) Find

(i) $\dfrac{z_1}{z_2}$ in the form $x + iy$ [3]

(ii) $\left|\dfrac{z_1}{z_2}\right|$ [2]

(iii) $\arg\left(\dfrac{z_1}{z_2}\right)$ [2]

(b) Hence represent $\dfrac{z_1}{z_2}$ on a clearly labelled Argand diagram. [2]

Scanned with CamScanner

11 (a) By sketching the graphs of $y = \ln x$ and $y = \frac{x}{x-1}$, on the same axes, show that the equation $\ln x = \frac{x}{x-1}$, has exactly two real roots. [3]

(b) Show by calculation that one of the roots, in **(a)** lies between 0.3 and 0.45. [3]

(c) Taking 0.45 as the first approximation to the root, use the Newton-Raphson method twice to find the root correct to 4 decimal places. [4]

12 The circle with equation $x^2 + y^2 - 4x + 6y = 12$ meets the line $3y = x + 4$ at points A and B.

Find the

(a) coordinates of the centre and radius of the circle. [2]

(b) length of the line AB, [4]

(c) length of the perpendicular of the line AB from the centre of the circle, [2]

(d) the area of the triangle ABC where C is the centre of the circle with equation $x^2 + y^2 - 4x + 6y = 12$. [2]

13 (a) (i) Write down the first 3 terms of the sequence U_r, where $U_r = 2\left(\frac{1}{3}\right)^r$, $r = 1;2;\ldots$

(ii) Find the value of n for which $\sum_{r=1}^{n} 2\left(\frac{1}{3}\right)^r = \frac{80}{81}$ [5]

(b) The sum of the first n terms of an arithmetic progression is $\frac{3}{2}n^2 - 2n$.

Find the

(i) first 3 terms of the progression,

(ii) *nth* term of the progression. [5]

Scanned with CamScanner

14 (a) Sketch the graph of $f(x) = x(x+1)(2-x)$, clearly showing the intercepts with the axes. [3]

(b) Hence solve the inequality $f(x) < 0$. [2]

(c) Find the

(i) area bounded by the x-axis, the curve $y = f(x)$ and the lines $x = -1$ and $x = 0$. [3]

(ii) exact volume generated when the area bounded by the curve, the x-axis, the lines $x = 0$ and $x = -1$ is rotated completely about the x-axis. [5]

15 At any time t at a college, the number of people infected with disease A is x and the number infected with disease B is y. The sum of the infected people with disease A and those infected with disease B is p. At any time t, the rate at which x is increasing is proportional to the product of x and y.

(a) Show that the situation can be modelled by the differential equation $\frac{dx}{dt} = cx(p-x)$ where c is a constant. [3]

(b) Solve the differential equation expressing x in terms of c, p and another constant k. [6]

(c) If $x = \frac{p}{10}$ at time $t = 0$, find, in terms of c and p, the time when $y = \frac{p}{10}$. [5]

Scanned with CamScanner