ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

MATHEMATICS 9164/1

PAPER 1

JUNE 2016 SESSION 3 hours

Additional materials:
Answer paper
Graph paper
List of Formulae
Electronic calculator

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

Questions are printed in the order of their mark allocations and candidates are advised to attempt questions sequentially.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed pages and 3 blank pages.

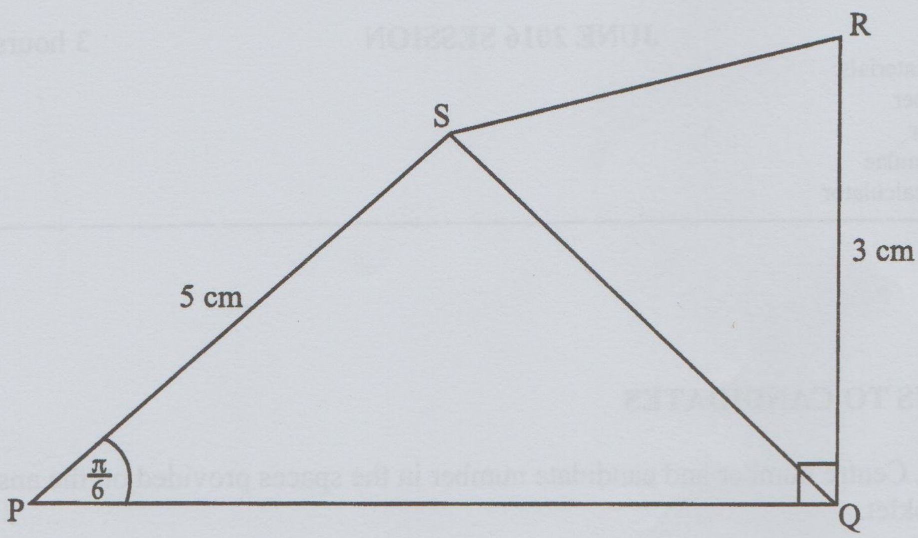
[Turn over

1 Find the inverse of $f(x) = ax + b$ where b and $a \neq 0$ are constants. [2]

2 Express $\dfrac{5x^3 - 3x^2 + 7x - 3}{\left(x^2+1\right)^2}$ in the form $\dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{\left(x^2+1\right)^2}$ where A, B, C and D are constants. [3]

3 Find the set of values of m such that the gradient of the line passing through the points $(m; 4)$ and $(1; 3 - 2m)$ is less than 5. [5]

4



In the diagram $Q\hat{P}S = \dfrac{\pi}{6}$, $P\hat{Q}R = \dfrac{\pi}{2}$, PS = 5 cm and QR = 3 cm.

Given that $\tan P\hat{Q}S = \dfrac{3}{4}$, find the exact length of RS. [6]

5 (i) Expand $(9 - 4x)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2. [4]

(ii) State the range of values of x for which the expansion is valid. [1]

(iii) By using $x = \dfrac{1}{9}$ in the expansion, find an approximation for $\sqrt{77}$ leaving your answer in the form $\dfrac{p}{q}$. [2]

6 (i) A geometric series has first term x and second term $\left(x^2 - x\right)$. If all the terms are positive, find the set of values of x for which the series converges. [3]

(ii) Given that $x = \dfrac{5}{3}$, find the smallest value of n for which the sum of the first n terms differs from the sum to infinity by less than 0,001. [4]

7 A closed cylindrical tin of radius r and height h has fixed volume V.

(i) Find the total surface area, S, of the tin in terms of π, r and V. [3]

(ii) Show that its total surface area is smallest when its diameter and height are equal. [4]

8 (i) Show by calculation that the equation $e^x - \cos x = 2$ has a root between $x = 0$ and $x = 1$. [3]

(ii) Use Newton-Raphson method twice, starting with $x_0 = 1$, to find the root of $e^x - \cos x - 2 = 0$, correct to 3 decimal places. [4]

9 Solve exactly the equations:

(a) $\frac{1}{2}\log_5(x-2) = 3\log_5 2 - \frac{3}{2}\log_5(x-2)$ [4]

(b) $2e^{2x} = 7e^x - 3$ [4]

10 (a) Prove the identity

$$\tan^2\theta - \sin^2\theta \equiv \tan^2\theta \sin^2\theta$$ [3]

(b) Solve the equation.

$$4\sin^2\theta\tan\theta - \tan\theta = 0 \text{ for } 0 \le \theta \le 2\pi.$$ [6]

11 (i) Express the complex number $w = 8 + \frac{4-i}{1+2i}$ in the form $x + iy$. [4]

(ii) Hence or otherwise, find

1. $|w|$ in the form $a\sqrt{b}$,
2. argument of w. [6]

12 The mass of an animal at time t is x grammes. The rate of increase of the mass is directly proportional to $(10\,000 - x^2)$ where $0 < x < 100$.

(i) Write down the differential equation connecting x and t. [2]

(ii) Solve the differential equation given that $x = 10$ when $t = 0$ and that $x = 50$ when $t = 10$, giving x in terms of t. [8]

13 The function f is defined by $f(x) = (x - 2)(x + 3)$, $x \in IR$.

Sketch on separate diagrams, showing clearly all the intercepts and turning points, the graphs of:

(i) $y = f(x)$ [4]

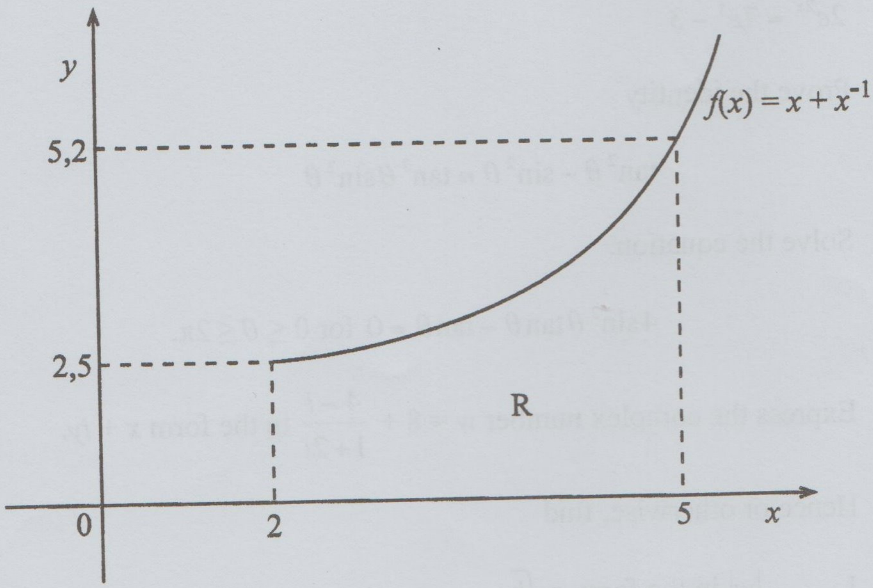
(ii) $y = |f(x)|$ [2]

(iii) $y = f(2x)$ [2]

(iv) $y = f(x-1)$ [2]

(v) $y = -2f(x)$ [2]

14



The diagram shows region R, bounded by the curve $f(x) = x + x^{-1}$, the x- axis, the lines $x = 2$ and $x = 5$.

(i) Calculate the exact area of the region R. [4]

(ii) Using the trapezium rule, with 4 ordinates, find the approximate area of region R correct to 3 decimal places. [4]

(iii) Calculate the exact volume generated when region R is rotated completely about the x-axis. [5]

15 (a) Relative to the origin O, the position vectors of P, Q and R are $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 1 \\ -2 \end{pmatrix}$ respectively.

Find $P\hat{Q}R$. [4]

(b) The vector $\mathbf{W} = \begin{pmatrix} m - \frac{1}{2} \\ m + \frac{1}{2} \\ m\sqrt{6} \end{pmatrix}$.

(i) If $\mathbf{W}$ is a unit vector, find the possible values of m. [2]

(ii) The vector $\mathbf{V} = \begin{pmatrix} m + 3 \\ 2m \\ -2\sqrt{6} \end{pmatrix}$.

If $\mathbf{W}$ is normal to $\mathbf{V}$, find the values of m. [4]

(iii) The vector $\mathbf{U} = \begin{pmatrix} m - \frac{1}{2} \\ n \\ 2\sqrt{6} \end{pmatrix}$.

Given that $\mathbf{W}$ is parallel to $\mathbf{U}$, find the values of m and n. [4]