Photometric redshift estimation of galaxies in the Pan-STARRS 3π survey- I. Methodology

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ABSTRACT

We present a photometric redshift (photo-z) estimation technique for galaxies in the Pan-STARRS1 (PS1) 3π survey. Specifically, we train and test a regression and a classification Random-Forest (RF) models using photometric features (magnitudes, colors and moments of the radiation intensity) from the optical PS1 data release 2 (PS1-DR2) and from the AllWISE/unWISE infrared source catalogs. The classification RF model (RF_{clas}) has better performance in the local universe ($z \lesssim 0.1$), while the second one (RF_{reg}) is on average better for $0.1 \lesssim z \lesssim 1$. We adopt as labels the spectroscopic redshift of the galaxies from the Sloan Digital Sky Survey (SDSS) data release 16 (SDSS-DR16). We find that the combination of AllWISE/unWISE and PS1-DR2 features leads to an average bias of $\overline{\Delta z_{norm}} = 1 \times 10^{-3}$, a standard deviation $\sigma(\Delta z_{norm}) = 0.0225$, (where $\Delta z_{norm} \equiv (z_{phot} - z_{spec})/(1 + z_{spec})$), and an outlier rate of $P_0 = 1.48\%$ in the test set for the RF_{clas} model. In the low-redshift Universe ($z < 0.1$) that is of primary interest to many astronomical transient studies, our model produces an error estimate on the inferred magnitude of an object of ≤ 1 mag in 87% of the test sample.

Keywords: machine learning - galaxies - photo-z

1. INTRODUCTION

The photometric redshift (photo-z) provides estimates of the distance of astronomical objects, and is an essential tool in multiple areas of astronomy. The main benefit of photo-z is that distance estimates are obtained rather inexpensively for the sources detected in the images as opposed to spectroscopic redshift determination. The drawback is that photo-z estimates typically have lower accuracy when compared with their spectroscopic counterparts. Photo-z estimation is critical for a number of endeavours, including astronomical transient typing, constraining the presence dark energy (DE) with weak-lensing, and can also be employed for other DE probes like supernovae (SNe) type Ia, the mass function

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of galaxy clusters and galaxy clustering as well (e.g., Salvato et al. 2019). In this work we focus on photo-z for transient classification but we note this methodology can be extended to other areas of astronomy. Host galaxy redshift, as highlighted by Muthukrishna et al. (2019), is one of the most predictive contextual-information features for transient classification, because it enables a direct estimate of the intrinsic luminosity of the transient.

Astronomical transients are historically classified according to their spectral and photometric features. The wide field of view, high cadence and sensitivity of astronomical surveys is currently leading to the discovery of thousands of new transient events every night. This plethora of new transient discoveries is currently made possible thanks to surveys such the Zwicky Transient Facility (ZTF, Bellm et al. 2019), the Young Supernova Experiment (YSE, Jones et al. 2020) and the Asteroid Terrestrial-impact Last Alert System (ATLAS, Tonry et al. 2018). Forthcoming surveys such as the Legacy Survey of Space and Time (LSST, Ivezić et al. 2019)

carried out on Vera C. Rubin Observatory will further enhance the rate of discovery of new transients, which will make prompt spectroscopic classification of the majority of transients unfeasible. It is thus crucial to establish new paths for transient classification.

The two alternative classification methods consist of leveraging the transient photometry (e.g. Newling et al. 2011; Karpenka et al. 2013; Möller et al. 2016; Lochner et al. 2016; Sooknunan et al. 2018; Narayan et al. 2018; Pasquet et al. 2019; Muthukrishna et al. 2019; Ishida et al. 2019; Villar et al. 2019; Möller & de Boissière 2020) and the contextual information of the environments (e.g., redshift, host-galaxy morphology and star formation rate) where the transients happen (Foley & Mandel 2013; Baldeschi et al. 2020; Gagliano et al. 2020) by using machine learning (ML) algorithms. In this work we focus on the estimation of a primary contextual parameter, the host-galaxy redshift, using available optical and infra-red band photometry.

The methods used to infer galactic photometric redshifts fall into two broad categories: (i) methods based on physical modeling of the multi-band emission from galaxies, and (ii) supervised ML-based methods. An extensive recent review of the literature for both methods can be found in Salvato et al. (2019). Physical model-based algorithms (e.g. Benítez 2000; Arnouts et al. 2002; Ilbert et al. 2006; Beck et al. 2016) rely on the template-fitting approach where observed photometric data are compared to simulated photometry for a wide number of template galaxy spectra and redshifts. Supervised ML methods (e.g., Pasquet et al. 2019; Zhou et al. 2020; Tarrío & Zarattini 2020; Ansari et al. 2020; Schuldt et al. 2020), on the other hand, require large training sets of spectroscopic redshifts that are used to infer an intrinsic correlation with the photometric features (e.g., colors, magnitudes and shape parameters like moments of radiation intensity) in a data-driven fashion. Both physical model-based and ML-based algorithms are not yet competitive with the accuracy and precision of spectroscopic redshifts across the entire range of z where galaxies are detected and known to exist (i.e. in the local and in the distant Universe).

There are three main issues with the photo-z estimation using ML-based techniques: (i) the redshift of galaxies in the local Universe ($z \lesssim 0.1$) are typically overestimated (i.e. the inferred photo-$z \gtrsim 0.1$); (ii) the redshift of galaxies at larger z ($z \gtrsim 0.7$) are typically underestimated (i.e. the inferred photo-$z > 0.7$); (iii) presence of catastrophic outliers (i.e. sources for which the photo-z estimate significantly differ from the true spectroscopic redshift). In this work we attempt to mitigate the effects of the three issues above, with special focus on the use of photo-z by the astronomical transient community in the local Universe. Specifically, we train and test two Random-Forest (RF) models leveraging the PS1-DR2, AllWISE and the unWISE photometric features (magnitudes, colors and moments of the radiation intensity) and leveraging the SDSS labels (spectroscopic redshift of the galaxies). The combination of AllWISE, unWISE and PS1-DR2 photometry that covers the optical and infrared (IR) leads to improvements when compared to other studies that used the PS1-DR2 photometry alone (e.g., Pasquet et al. 2019; Tarrío & Zarattini 2020; Beck et al. 2020).

This work is the first of a series of two papers where we focus on the development of the ML model. In a forthcoming study, we will present a catalog with the photo-z estimate of PS1-DR2 galaxies. The paper is organized as follows. In §2 we describe the four datasets used (SDSS-DR16, PS1-DR2, AllWISE, unWISE). In §3 we create a training/testing set, we pre-process the data and we develop two RF models. In §4 we discuss our main results. Conclusions are drawn in §5.

2. DATASETS DESCRIPTION

In our analysis we use four catalogs: (i) the data release 16 of the the Sloan Digital Sky Survey (SDSS-DR16, Ahumada et al. 2020); (ii) the second Pan-STARRS1 (PS1, Chambers et al. 2016) data release of the 3π survey (PS1-DR2 hereafter); (iii) the AllWISE source catalog (Wright et al. 2010); (iv) the unWISE source catalog (Schlafly et al. 2019).

PS1 data have been collected with a 1.8 meter telescope to produce images of the sky through five filters (center wavelengths: y_{P1} [9633 Å], z_{P1} [8679 Å], i_{P1} [7545 Å], r_{P1} [6215 Å], g_{P1} [4866 Å]). Two surveys have been completed with PS1: the 3π survey (3πS) and the medium deep survey. Here, we utilize data from the 3πS, which covers the sky northern of declination $\delta = -30°$ and includes data collected between 2009-06-02 and 2014-03-31. The limiting magnitudes of the 3πS are ~21.5 and ~22.5 mag for the y_{P1} and z_{P1}, respectively, while it is ~23.5 mag for the i_{P1}, r_{P1} and g_{P1} filters. In this paper, we use the PS1-DR2 data from the "StackObjectAttributes" table[1] that contains photometric information (e.g., PSF-flux, Kron-flux) of the stacked data, estimated as presented in Magnier et al. (2013). Sources included in this table have been detected with a signal-to-noise, $S/N > 20$ for each individual exposure. The table also contains detections of the same source from consecutive exposures, which implies that there can be different photometric measurement in

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[1] StackObjectAttributes table link

the same band for a given source. In §4.1 we use the PS1-DR2 data to train our RF model. A detailed description of the meaningful features of this data set is provided in §3.2.

The Wide-Field Infrared Survey Explorer (WISE) mapped the sky at $\lambda =$ 22, 12, 4.6 and 3.4 μm (W4, W3, W2, and W1 bands) (Wright et al. 2010), with an angular resolution of 12.0″, 6.5″, 6.4″ and 6.1″ in the four bands, respectively. AllWISE includes the data acquired during the WISE full cryogenic mission phase, which was carried out between 7 January 2010 and 6 August 2010. The AllWISE data release consists of coadded and calibrated images and a catalog with photometric and positional information for ≈ 563 million sources found in the WISE images (Wright et al. 2010). The unWISE catalog includes the fluxes of two billion objects observed by WISE over the entire sky (Schlafly et al. 2019). The unWISE catalog has two advantages over AllWISE: (i) it is based on deeper imaging; (ii) it features a better modeling of crowded regions of the sky. However, a clear disadvantage of unWISE is that the fluxes are available at 3.4 and 4.6 μm, only.

The fourth catalog that we use is derived from the Sloan Digital Sky Survey (SDSS), which has been observing from the Apache Point Observatory (APO) since 1998 using a 2.5 m telescope (Gunn et al. 2006) and from Las-Campanas Observatory (LCO) since 2017 using a 2.5 m telescope. SDSS produces images of the sky through five filters (z_{S16} [9134 Å], i_{S16} [7625 Å], r_{S16} [6231 Å], g_{S16} [4770 Å] ,u_{S16} [3543 Å]). The SDSS-DR16 catalog (Ahumada et al. 2020) provides the spectroscopic redshift of the galaxies with $z \lesssim 1$ that we use as labels throughout the paper.

3. A MACHINE LEARNING APPROACH TO PHOTO-Z ESTIMATION

In this section, we analyze the PS1-DR2, AllWISE, unWISE and SDSS-DR16 catalogs. We cross-match the catalogs and for each common source we retain features that are meaningful to our subsequent analysis. We then introduce the RF classification and regression models for photo-z estimation. Finally, we present the metrics to rank the model.

3.1. *Training Sample Selection*

We analyzed the accuracy of the cross matching between SDSS-DR16 and PS1-DR2 by comparing the number of positional cross matches to PS1-DR2 sources using SDSS-DR16 positions and random angular offsets similar to approaches used by Stephen et al. (2006), D'Abrusco et al. (2013), and D'Abrusco et al. (2014). The SDSS-DR16 dataset contains both galaxies and
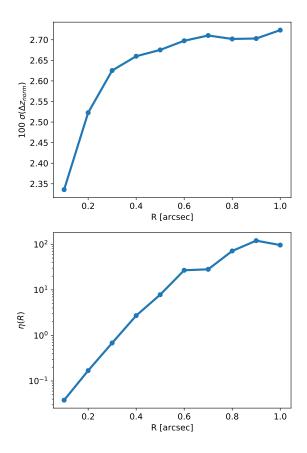

Figure 1. *Upper panel*: $\sigma(\Delta z_{norm})$ vs the crossmatching radius between the PS1-DR2 and the SDSS-DR16 sources. The $\sigma(\Delta z_{norm})$ has been estimated in tests set by leveraging a RF_{reg} model with 10 trees for each crossmatching radius. *Lower panel*: The contamination, $\eta(R) = \frac{\Delta N_{random}(R)}{\Delta N_{catalog}(R)}$, as a function of the search radius R between the PS1-DR2 and the SDSS-DR16 catalogs.

quasi-stellar object (QSO), which are labeled as `GALAXY` and `QSO`, respectively, in the `CLASS` field of the SDSS-DR16 `specObj` table. We selected 10^4 random SDSS-DR16 galaxies and QSOs, and we created a random source list consisting of 100 offset positions for each source in the catalog positions resulting in 10^6 random positions. Each random offset was chosen to be between $1'$ and $2'$ from the SDSS-DR16 position, and at a random position angle. The large offsets were chosen so that genuine cross matches would not contribute to the randomly generated cross matching. Random offsets and position angles were selected from separate uniform random distributions.
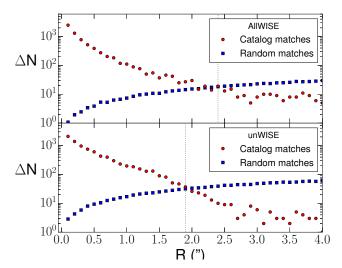
Figure 2. *Top panel*: ΔN, the number of additional cross-matches, between PS1-DR2 and AllWISE at $0.1''$ search radii intervals. The vertical dashed line at $R = 2.4''$ highlights the radius at which the average number of random cross matches to AllWISE is greater than the number of matches between PS1-DR2 and AllWISE. *Bottom panel*: ΔN for cross matches between PS1-DR2 and AllWISE at different search radii at 0.1" intervals. The vertical dashed line at $R = 1.9''$ highlights the radius at which the average number of random cross matches to unWISE is greater than the number of matches between PS1-DR2 and unWISE.

We calculated the cumulative number of cross matches between the SDSS-DR16 catalog position and PS1-DR2, $N_{catalog}$, at angular separations, R, at intervals $\Delta R = 0.1''$. We consider a PS1-DR2 source associated with a SDSS-DR16 source if it is within R of the SDSS-DR16 source. We then calculate the differential number of cross matches at a given angular separation $\Delta N_{catalog}(R) \equiv N_{catalog}(R) - N_{catalog}(R - \Delta R)$. Similarly, we calculated the differential number of cross matches for each random offset. Since we calculated 100 random offsets per SDSS-DR16 source, we normalized the number of randomized cross matches to the same sample size as $\Delta N_{catalog}(R)$ by dividing by 100. Thus, we defined $\Delta N_{random}(R)$ as the average number of cross matches per 10^4 positions. In the case of multiple cross matches between the two catalogs, we chose the pair with the closest angular separation. For $\Delta N_{random}(R)$, we only considered the radius at which the closest association was made, and thus multiple possible associations were only counted at the first possible association. The evolution of the contamination parameter, defined as $\eta(R) \equiv \frac{\Delta N_{random}(R)}{\Delta N_{catalog}(R)}$, as a function of the search radius R is shown in Figure 1. As expected the level of contamination increases with larger search radii. We ran the ML algorithm defined below using SDSS-

DR16 and PS1-DR2 cross match radii from $R = 0.1''$ to $R = 1.0''$, and found that $\sigma(\Delta z_{norm})$ significantly increases when $\eta \geq 0.1\%$, corresponding to SDSS-DR16 and PS1-DR2 cross matching radius $R \approx 0.1''$ (Figure 1). The $\sigma(\Delta z_{norm})$ value is a metric to evaluate the performance of the model and it will be defined in §3.3. Thus, we adopted an SDSS-DR16 and PS1-DR2 cross matching angular separation of $0.1''$, resulting in a sample size of 3,487,203 SDSS-DR16 sources with a counterpart in PS1-DR2.

The optimum cross matching angular separations between PS1-DR2 and AllWISE and unWISE catalogs were also examined similar to the SDSS-DR16 and PS1-DR2 method described above. In this case, we selected 10^4 random sources from the complete SDSS-DR16 and PS1-DR2 cross matched sample. The PS1-DR2 coordinates were cross matched against AllWISE and unWISE utilizing the identical 10^4 random sources. The PS1-DR2 positions were chosen so that the same selection could be adopted to the PS1-DR2 sample outside of the SDSS-DR16 footprint. We adopted the greatest radii at which the number of catalog matches is greater than the number of random cross matches, which corresponds to $R = 2.4''$ and $R = 1.9''$ for the PS1-DR2 to AllWISE and unWISE cross-matching radii, respectively, (Figure 2).

After retaining only those sources labeled as `GALAXY` in the `specObj` SDSS-DR16 table, the data set consists of 2,394,092 sources. In the PS1-DR2 catalog there are often multiple detections of the same source, which create multiple photometry sets for a given source. In case of multiple photometry sets for a given source, we retain the photometry associated with "primary-Detection" equal to 1 . The final SDSS-DR16/PS1-DR2/AllWISE/unWISE data set contains 1,251,249 unique galaxies.

3.2. *Feature Selection and Pre-processing*

For each galaxy, we identify the properties listed in PS1-DR2, AllWISE and unWISE that are meaningful to train a ML model, and associate the labels (redshift) from the SDSS-DR16 catalog. For each PS1 photometry filter, we identify the relevant features as the PSF-mag, the Kron-mag (Kron 1980), and the second moment of the radiation-intensity, defined as $< XY > = \int_{Sxy} uvI(u,v)\,du\,dv$ or $< X^2 > = \int_{Sx} u^2 I(u)\,du$, where I is the radiation-intensity. The moments of the radiation intensity are correlated with the distribution of light in a galaxy and provide information on the galaxy shape. These features are presented in the "StackObjectAttributes" table of PS1-

DR2 as `{g,r,i,z,y}PSFMag`, `{g,r,i,z,y}KronMag`, `momentYY`, `momentXY` and `momentXX`.

Furthermore, we use the AllWISE photometry: `w1mag`, `w2mag`, `w3mag`, `w4mag`, `w1mag_1`, `w2mag_1`, `w3mag_1`, `w4mag_1`, `w1mag_2`, `w2mag_2`, `w3mag_2`, `w4mag_2`, `w1mag_3`, `w2mag_3`, `w3mag_3`, `w4mag_3`, `w1mag_4`, `w2mag_4`, `w3mag_4`, `w4mag_4` as meaningful features. The `w1mag`, `w2mag`, `w3mag` `w4mag` features are the W1, W2, W3 and W4 magnitudes, respectively while the `w{1,2,3,4}mag_{1,2,3,4}` are the photometry taken with different extraction apertures.[2] We also use the unWISE photometry: `unwise_w1_mag_ab` and `unwise_w2_mag_ab` that are the $W1_{un}$ and $W2_{un}$ unWISE magnitudes. The observed magnitudes are distance-dependent quantities and therefore meaningful features for our ML model.

In addition to the observed mags, the observed colors of galaxies are important tracers of their distance in the universe (through their redshift dependence). For this reason, we add the colors as additional features in our ML model. We adopt the following set of colors as features: $g_{P1} - r_{P1}$, $r_{P1} - i_{P1}$, $i_{P1} - z_{P1}$, $z_{P1} - y_{P1}$ for both the Kron and PSF magnitudes. We also create the AllWISE colors as: e.g., W1-W2, W2-W3, W3-W4. We also add a mixture set of AllWISE/PS1-DR2 colors: e.g., $g_{P1} - W1$, $r_{P1} - W1$, $i_{P1} - W2$, $z_{P1} - W3$. Furthermore, we build the unWISE color $W2_{un} - W1_{un}$ and the unWISE/PS1-DR2 mixture colors: e.g., $g_{P1} - W1_{un}$, $r_{P1} - W1_{un}$, $i_{P1} - W2_{un}$, $z_{P1} - W3_{un}$. By adding other colors such as $g_{P1} - i_{P1}$, $g_{P1} - z_{P1}$, $g_{P1} - y_{P1}$, $r_{P1} - z_{P1}$, $r_{P1} - y_{P1}$, $i_{P1} - y_{P1}$ we do not find any improvement in our final results. In summary, we use as meaningful features: the PS1-DR2 PSF-colors (4 features), Kron-colors (4 features), PSF-magnitudes (5 features), Kron-magnitudes (5 features) and moments of the radiation intensity (15 features); the AllWISE magnitudes (4 features), magnitudes at different aperture radii (16 features), colors (3 features), PS1-DR2/AllWISE mixed colors (45 features), unWISE magnitudes (2 feature), PS1-DR2/unWISE mixed colors (5 features). We obtain 108 meaningful features in total.

We perform random sampling without replacement to split 90% of the sources in a training set and 10% in a test set. We obtain a training set of 1,126,124 galaxies and a test set of 125,125 galaxies. Finally, we normalize the features in the training set according to the formula $X_{st} = (X - \mu)/\sigma$, where X is the input feature, while σ

and μ and are the standard deviation and the mean of the column feature, respectively.

3.3. *Machine learning: a classification vs. regression approach*

The supervised ML goal consists of learning a mapping function between an input and an output based on example input-output pairs. ML models can perform regression and classification tasks. Classification models discriminate objects in two or more classes by "learning" a mapping function from a training set and then applying the mapping to unseen data. A regression model approximates a mapping function from input variables to a continuous output variable. A wide variety of ML models have been constructed, and several have been used in astronomy as well. In this paper we adopt the random forest (RF) model (e.g., Hastie et al. 2009) that is known to be one of the ML models that produces the lowest mean the lowest mean square error (MSE, Henghes et al. 2021).

The RF model is an ensemble ML method for regression and classification that works by creating multiple decision trees during training. The RF algorithm can be used for solving regression (RF_{reg}) and classification (RF_{clas}) problems (e.g., Hastie et al. 2009). The RF model depends on multiple hyperparameters: (i) the number of trees in the forest; (ii) the minimum number of samples needed to split a node; (iii) the maximum possible depth of a tree; (iv) the minimum number of samples needed to be a leaf node; (v) a metric measure of the quality of the split.

ML photometric redshift estimation can be considered a regression problem since given some features (magnitudes, colors and moments), we want to predict a positive real number (the redshift). In a RF model, each tree outputs a redshift value and the final output value is simply the mean value of the trees. This regression problem can be remapped to a classification problem as done by Pasquet et al. (2019). Pasquet et al. (2019) subdivided the redshift distribution in K equally spaced bins

$$z_{phot} = \sum_k z_k P(z_k), \qquad (1)$$

where $P(z_k)$ is the probability density function (PDF) of the redshift. The optimal K value should not be too large to ensure that each bin retains a sufficiently large number of sources, and should not be too small to sample a sufficiently large values of redhifts values. We explored several values of K for our subsequent analysis and we find that values of $K \lesssim 15$ lead to a larger Δz_{norm} and $\sigma(\Delta z_{norm})$ (see below for the definition). Values of $K \gtrsim 25$ lead to a larger computational

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[2] See description for a comprehensive description of the WISE photometric catalog.

-	$\overline{\Delta z_{norm}}$	$\sigma(\Delta z_{norm})$	P_0	$< \Delta z_{norm} >$	σ_{MAD}	$\overline{\Delta z_{norm}}'$	$\sigma(\Delta z_{norm})'$	O
This work RF_{cla}	1.0×10^{-3}	0.0225	1.48%	2.4×10^{-3}	0.01914	1.8×10^{-3}	0.0252	0.34%
This work RF_{reg}	2.8×10^{-4}	0.0209	1.42%	8.2×10^{-4}	0.01764	5.3×10^{-4}	0.0235	0.22%
Tarrío & Zarattini (2020)	-2.0×10^{-4}	0.0298	4.32%	$-$	$-$	$-$	$-$	$-$
Pasquet et al. (2019)	$-$	$-$	$-$	1.0×10^{-4}	0.00912	$-$	$-$	$-$
Beck et al. (2020)	$-$	$-$	$-$	$-$	$-$	5.0×10^{-4}	0.0322	1.89%

Table 1. Comparison between the results obtained in this work for both the RF_{cla} and RF_{reg} models with the findings of Tarrío & Zarattini (2020), Pasquet et al. (2019) and Beck et al. (2020). The evaluation metrics are defined in §3.3. Pasquet et al. (2019) achieved a lower σ_{MAD} compared to our work. However, our method has the advantage of not relying on SDSS photometry and can be scaled to the entire PS1-DR2 dataset to create a PS1-DR2 photo-z catalog.

-	$\overline{\Delta z_{norm}}$ (1)	$\sigma(\Delta z_{norm})$ (1)	P_0 (1)	$\overline{\Delta z_{norm}}$ (2)	$\sigma(\Delta z_{norm})$ (2)	P_0 (2)
This work, RF_{cla}	1.1×10^{-3}	0.0238	0.014	1.5×10^{-3}	0.0253	1.31%
This work, RF_{reg}	3.4×10^{-4}	0.0220	0.014	3.8×10^{-4}	0.0234	1.42%

Table 2. Comparison between the RF_{cla} and RF_{reg} models trained with just a subsample of meaningful features. (1) models trained with the PS1-DR2/AllWISE features. (2) models trained with the PS1-DR2 features only. The results obtained with the RF models trained with the PS1-DR2/AllWISE/unWISE features are presented in the firsts two lines of Table 1.

time without an improvement in the results in terms of Δz_{norm} and $\sigma(\Delta z_{norm})$. Hereafter, we use $K = 20$.

In this paper we adopt and compare both the RF-regression (RF_{reg}) and the RF-classification (RF_{clas}) models. We use RF models with 100 trees. In order to quantify the performance of the models we introduce several metrics that have been used in the photo-z literature that leverages the PS1 and SDSS photometry. In §4.1 we use these metrics to test our RF model and to compare our results with other photo-z efforts in literature.

- One common metric used by Beck et al. (2016) and Tarrío & Zarattini (2020) is the normalized redshift defined as: $\Delta z_{norm} \equiv (z_{phot} - z_{spec})/(1 + z_{spec})$, where z_{phot} and z_{spec} are the photometric and the spectroscopic redshift, respectively. Furthermore, after removing the outliers defined as $|\Delta z_{norm}| > 3\sigma(\Delta z_{norm})$ (following Beck et al. 2016 and Tarrío & Zarattini 2020) the average bias is defined as $\overline{\Delta z_{norm}}$. We also define the outliers rate (P_0) as the fraction of galaxies with $|\Delta z_{norm}| > 3\sigma(\Delta z_{norm})$.

- Pasquet et al. (2019) defined the average bias as $< \Delta z_{norm} >$ without removing the outliers and the standard deviation as $\sigma_{MAD} = 1.4826 \times MAD$, where MAD (median absolute deviation) is defined as $| \Delta z_{norm} - Median(\Delta z_{norm}) |$.

- Beck et al. (2020) defined the outliers (O) as the fraction of galaxies with $|\Delta z_{norm}| > 0.15$. These authors computed the average bias ($\overline{\Delta z_{norm}}'$) and
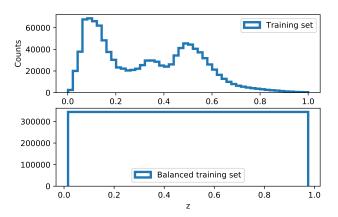

Figure 3. Upper panel: redshift distribution in the unbalanced training set. Lower panel: redshift distribution in the oversampled training set. Number of sources in the unbalanced training set: 1,126,124. Number of sources in the oversampled training set: 3,325,940. In this work we train an RF_{reg} in the unbalanced training set while we train a RF_{clas} in the oversampled training set.

the standard deviation ($\sigma(\Delta z_{norm})'$) by removing the outliers (O).

The values of these metrics for the photo-z works mentioned in this section are reported in Table 1.

3.4. Oversampling the training set

The redshift distribution of the training/testing set is not uniform as displayed in Figure 3. Since we subdivided the redshift distribution in 20 classes, the number of galaxies per class is highly unbalanced. ML classifi-

cation algorithms usually perform better on a balanced training set (i.e. same number of elements in each class). Oversampling algorithms balance the number of sources in each class by randomly duplicating the sources in the minority classes until the number of sources in the minority classes is equal to that in the majority class. The simplest oversampling technique consists of randomly sampling with replacement the minority classes but this method can result in overfitting. To minimize overfitting we employ the Synthetic Minority Oversampling Technique (SMOTE, Chawla et al. 2011).

SMOTE selects sources that are close in the feature-space, drawing a line between the examples in this space and drawing a new point along this line. Specifically, a random source from the minority class is first selected. Then a number J of the nearest neighbors for that source are found. A randomly selected neighbor (among the J options) is then selected and a synthetic example is created at a randomly selected point between the two examples in feature space. In this paper we oversample the minority classes with Borderline-SMOTE (Han et al. 2005) that is considered an improvement with respect to the standard SMOTE algorithm (Han et al. 2005). It is important to mention that we only oversample the training set and we do not oversample the test set. Figure 3 shows the unbalanced redshift distribution of galaxies in the training set (before oversampling, upper panel), and of the oversampled training set (i.e. balanced, lower panel). The oversampled training set contains 3,325,940 galaxies. In §4 we use the oversampled training set to train an RF classification algorithm and the unbalanced training set for the RF regression model.

4. ANALYSIS AND RESULTS

In this section we train and test the RF_{clas} and the RF_{reg} models (§4.1), and compare their performances. Since the RF_{clas} model performs better in the local universe $z \leq 0.1$, which is our primary interest, we focus our detailed analysis of the confidence intervals for the photo-z estimation (§4.1) and of the outliers (§4.2) on the RF_{clas} model. We compare our results with other works in the literature in §4.3. We display in Figures 4, 5, 6 and 9 the results relative to the RF_{clas} algorithm, and in Figure 7 we show a comparison between the RF_{clas} and RF_{reg}.

4.1. A photo-z ML model

We train the RF_{reg} and the RF_{clas} models in the training set and in the oversampled training set, respectively. We test both RF_{reg} and RF_{clas} in the test set. Our results for the RF_{clas} model are: $\overline{\Delta z_{norm}} = 1.0 \times 10^{-3}$, $\sigma(\Delta z_{norm}) = 0.0225$ and $P_0 = 0.0148$ in the
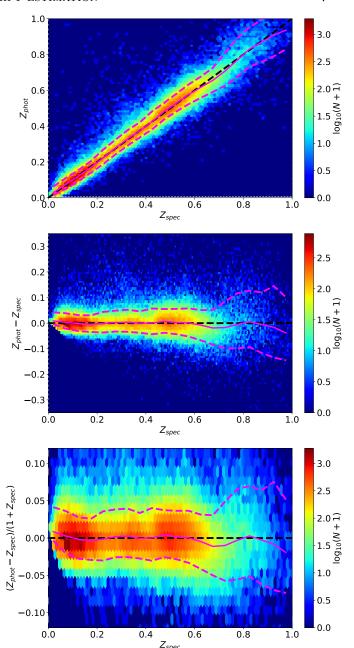

Figure 4. Comparison between the photometric redshift z_{phot} estimated in the test set with the spectroscopic SDSS redshift z_{spec}. *Upper Panel*: z_{phot} vs z_{spec}. *Central panel*: $z_{phot} - z_{spec}$ vs z_{spec}. *Lower Panel*: $(z_{phot} - z_{spec})/(1 + z_{spec})$ vs z_{spec}. The black dashed line corresponds to $z_{phot} = z_{spec}$. Magenta solid line: median value. The magenta dashed lines are the tenth and ninetieth percent quantile, respectively. This plot was obtained by applying the RF_{clas} model to the test set.

test set (Table 1). The RF_{reg} model yields: $\overline{\Delta z_{norm}} = 2.8 \times 10^{-4}$, $\sigma(\Delta z_{norm}) = 0.0209$ and $P_0 = 0.0142$ in the test set (Table 1). The RF_{reg} globally performs better than the RF_{clas} model. However, as we will show be-
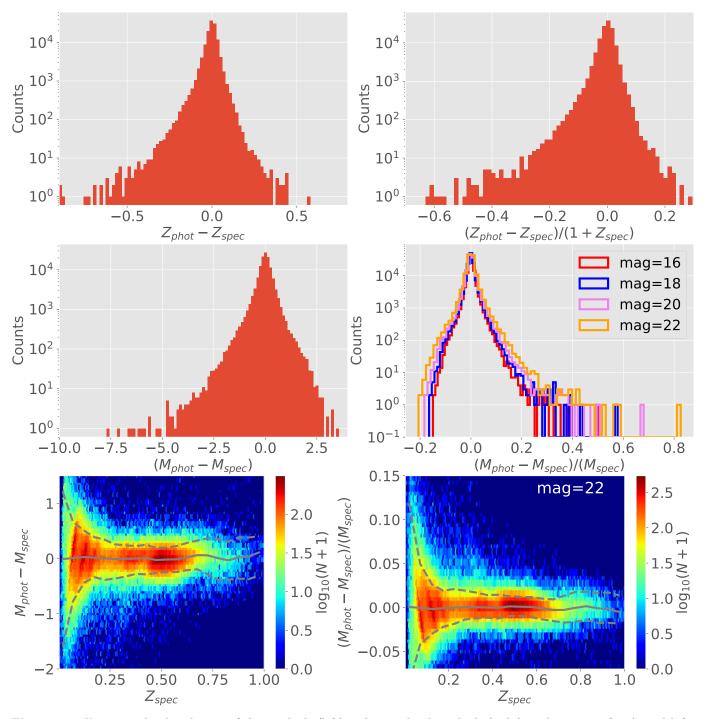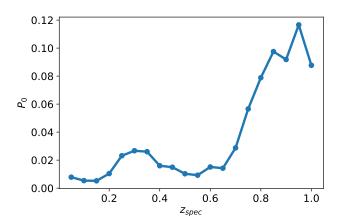
Figure 5. *Upper panels*: distribution of the residuals (left) and normalized residuals (right) in the test set for the redshift. *Central panels*: distribution of the residuals for the absolute magnitudes (left) and distribution of the normalized residuals (right) for the absolute magnitudes estimated at different apparent mags. *Lower panels*: (left) $M_{phot} - M_{spec}$ vs. z_{spec} and (right) $(M_{phot} - M_{spec})/(M_{spec})$ vs. z_{spec} assuming an apparent mag= 22. Grey solid line: median value. Dashed grey lines mark the 10th and 90th percent quantile, respectively. This plot was obtained by applying the RF_{clas} model to the test set.

Figure 6. Outliers fraction (P_0) as a function of z_{spec}. The outliers fraction increases for $z_{spec} > 0.7$. Outliers: sources with $|\Delta z_{norm}| > 3\sigma(\Delta z_{norm})$. This plot was obtained by applying the RF_{clas} model to the test set.

low, the RF_{clas} model performs better for $z < 0.1$ that is paramount for transient classification in the local Universe. For this reason we focus the subsequent analysis on the RF_{clas} model. Since the training/testing sets are built through random sampling, we repeat five times the same procedure described above to test for the stability of the model. We do not find any differences above 1% in $\sigma(\Delta z_{norm})$ in the different runs.

Figure 4 shows the comparison of the photometric redshift z_{phot} inferred with the RF_{clas} model, with the true spectroscopic redshift z_{spec} in the test set. The left panel of Figure 4 shows z_{phot} as a function of z_{spec} while the middle and right panels show $z_{phot} - z_{spec}$ and Δz_{norm} as a function of z_{spec}, respectively. Figure 4 reveals that the z_{phot} forecasting are overall compatible with z_{spec}. The most difficult range of redshift values to predict are those at $z \gtrsim 0.7$ (right panel of figure 4).

The photometric prediction error ($z_{phot} - z_{spec}$) will induce an absolute magnitude prediction error ($M_{phot} - M_{spec}$), where M_{phot} and M_{spec} are the absolute magnitude estimated at z_{phot} and z_{spec}, respectively. By definition, $M_{phot} - M_{spec} = 5\log_{10}(d_{spec}/d_{phot})$, where d_{spec} and d_{phot} are the luminosity distances[3] to a galaxy estimated at z_{spec} and z_{phot}, respectively. In the central left panel of Figure 5 we display the distribution of $M_{phot} - M_{spec}$ revealing that 97% of the galaxies have $|M_{phot} - M_{spec}| \leq 1$ mag and 73% of the galaxies have $|M_{phot} - M_{spec}| \leq 0.3$ mag. We also find that 87% of the galaxies with $z_{spec} < 0.1$ have $|M_{phot} - M_{spec}| \leq 1$ mag and 40% have $|M_{phot} - M_{spec}| \leq 0.3$ mag.

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[3] We estimate the distances by using the FlatLambdaCDM class from astropy based on Planck Collaboration et al. (2016)

In the lower left panel of Figure 5 we show the distribution of $M_{phot} - M_{spec}$ as a function of z_{spec} revealing that the median absolute value of the error is approximately 0 for all bins of z_{spec}. It is also important to explore the distribution of the normalized absolute magnitude error defined as ($M_{phot} - M_{spec})/M_{spec} = \log_{10}(d_{spec}/d_{phot})/(1 + 0.2\,mag - \log_{10}(d_{spec}/(pc))$, where mag is the apparent magnitude of the galaxy. In the central right panel of figure 5 we show the distribution of $(M_{phot} - M_{spec})/M_{spec}$ for different mag values revealing that 99% of the galaxies have a $(M_{phot} - M_{spec})/M_{spec}$ absolute value less than 0.1 for an apparent mag value of 22. Furthermore, in the lower panel of Figure 5 we display the $(M_{phot} - M_{spec})/M_{spec}$ distribution as a function of z_{spec} revealing that for $z_{spec} < 0.1$ we obtain the largest relative error value. In the upper panels of Figure 5 we also display for reference the distribution of $z_{phot} - z_{spec}$ and $\Delta z_{norm} = (z_{phot} - z_{spec})/(1 + z_{spec})$, respectively.

In the right panel of Figure 7 we show the median value of $z_{phot} - z_{spec}$ as a function of z_{spec} for both the RF_{reg} and the RF_{clas} models in the test set. The RF_{clas} model is better performing at $z_{spec} < 0.1$ and $z_{spec} > 0.6$ (in terms of median values) while it provides comparable results for $0.1 < z_{spec} < 0.6$. A similar result holds when we compare the median value of $M_{phot} - M_{spec}$ as a function of M_{spec} (see left panel of Figure 7) for the RF_{clas} and the RF_{reg} models. The RF_{clas} model is in median better performing than the RF_{reg} at $z_{spec} < 0.1$ and $z_{spec} > 0.6$ because the RF_{clas} was trained on an oversampled training set whereas the RF_{reg} in a training set with very few sources at very low and very high z (see figure 3). If we train the RF_{clas} in the standard (not oversampled) training set we obtain a result that is approximately identical to the one of the RF_{reg} model. Therefore, the differences between RF_{clas} and RF_{reg} are simply due to a different training set.

Next we address the topic of the importance of the features in the models. We measure feature importance by considering a RF_{reg} and a RF_{clas} model with the PS1-DR2 features only and one model with the PS1-DR2/AllWISE features. We evaluate the goodness of the model by estimating $\overline{\Delta z_{norm}}$, $\sigma(\Delta z_{norm})$ and P_0 in the test set for the RF_{reg} and for the RF_{clas} models, respectively. Quantitative results are summarized in Table 2. This experiment shows that by adding the AllWISE and unWISE photometry we improve the results for both the RF_{reg} and RF_{clas}, respectively.

We conclude this section with a discussion of the confidence intervals for the photometric redshift estimation with the RF_{cla} model. We use the RF trees to build an empirical probability density function of the photo-
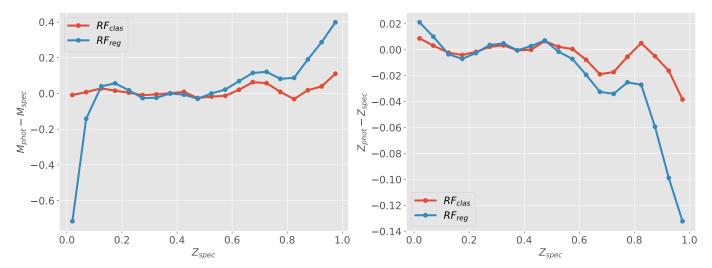
Figure 7. Comparison between the RF_{clas} and the RF_{reg} models. Right panel: median $z_{phot} - z_{spec}$ vs z_{spec} for both RF_{clas} and RF_{reg}. Left panel: median $M_{phot} - M_{spec}$ vs z_{spec} for both RF_{clas} and RF_{reg}. Median values were calculated by subdividing z_{spec} in 20 equally spaced bins and we then calculated the median $z_{phot} - z_{spec}$ and $M_{phot} - M_{spec}$ for each of those bins. The figure shows that RF_{clas} is better performing for $z < 0.1$ and $z > 0.6$ in terms of medians.

metric redshift for each galaxy. We define the 80% confidence interval as (z_{phot90}, z_{phot10}), where z_{phot90} and z_{phot10} are 90% and the 10% quantile, respectively. We find that $\approx 80\%$ of the spectroscopic redshift of the galaxies in the test sets are within the (z_{phot90}, z_{phot10}) confidence interval, for both RF_{cla} and RF_{reg}, indicating that this method yields realistic estimates of the true 80% confidence interval. In a future work we will release a catalog with the photometric redshfit estimates and with the confidence intervals for each of the sources in the PS1-DR2 catalog.

4.2. Outliers analysis

In §3 we defined the outliers as the galaxies with $|\Delta z_{norm}| > 3\sigma(\Delta z_{norm})$ following Tarrío & Zarattini (2020). In Figure 6 we display the fraction of outliers as a function of z_{spec} for the sources in the test set. The outliers fraction is larger for $z_{spec} > 0.7$.

It is meaningful to understand the properties of those outliers when compared to the entire population. In Figure 8 we show the normalized distribution of $g_{P1} - r_{P1}$, $r_{P1} - i_{P1}$, $i_{P1} - z_{P1}$ and $z_{P1} - y_{P1}$ in the test set for the entire sample and for the outliers, respectively. Figure 8 reveals that the distributions of colors of the outliers broadly overlap with, but is not identical to the color distributions of the entire test set. We perform a two-tail Kolmogorov-Smirnov test to quantify the distance between the outliers and the whole sample. We find that the probability that the two populations share the same parent population is $p_{value} < 10^{-10}$ for each of the four colors suggesting that the two populations are different.

The main difference between the entire distribution and the outliers distribution is in terms of the sample

standard deviation. The outliers appear more scattered than the whole distribution for each of the four colors. The ouliers and the whole distributions appear not particularly different in terms of median values. In Figure 8 we report the median values and the standard deviations of the distributions.

We repeat the same exercise with IR colors. Figure 9 displays the distribution of the AllWISE colors of the outliers and of the whole population. Figure 9 reveals that the outliers distribution is different from the whole distribution. A two-tail Kolmogorov-Smirnov test leads to a $p_{value} < 10^{-6}$ for each of the three colors, suggesting that the outliers and the whole distribution tend to have differen IR colors. The figure shows that the outliers are slightly redder than the whole population for the W1-W2 color, significantly redder for the W2-W3 color, while having comparable median values for the W3-W4 color.

Our dataset contains galaxies with missing features. The presence of missing feature often leads to poorer performances of a ML model. Thus, we compare the distribution of missing features in the entire population with the outlier sample in Figure 10. As expected, Figure 10 reveals that on average the outliers have a larger number of missing features than the whole population, 3.2 and 17.6 for whole population and outlier sample, respectively, which leads to a poorer performance of the ML algorithms for the outlier sample. In conclusion the outliers have on average (1) a higher number of missing features and (2) colors that are not well represented in the training set.

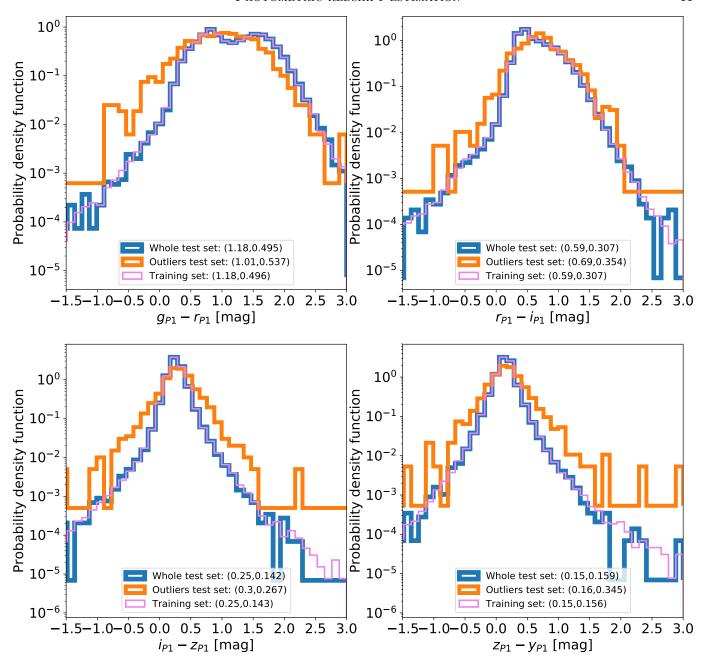### 4.3. Comparison with the literature

Figure 8. Normalized distribution of $g_{P1}-r_{P1}$, $r_{P1}-i_{P1}$, $i_{P1}-z_{P1}$ and $z_{P1}-y_{P1}$ colors for the galaxies in the test and training set. Blue: Entire test set. Orange: distribution of colors of outliers in the test set, i.e., sources with $|\Delta z_{norm}| > 3\sigma(\Delta z_{norm})$. Violet: training set. The numbers within the parenthesis in the legend are the median and the standard deviation of the data, respectively. The standard deviation of the outliers distribution is larger than that of one of the entire test set, indicating that sources that are outliers are more likely to span a wider range of colors than a typical source in the test set. This plot was obtained by applying the RF_{clas} model to the test set.

Machine learning is playing an important role in photo-z estimation in astronomy. A detailed comparison between all the results of this work and other papers from the literature is not always possible as each study uses different data catalogs, features and redshift distributions for training and testing. Here we offer a quantitative comparison of our results and results from the literature that use the PS1 or the SDSS datasets us-

ing the metrics defined in §3.3 and summarized in Table 1. The most relevant metric to evaluate the performance of a photo-z model is the standard deviation that can be expressed as $\sigma(\Delta z_{norm})$, $\sigma(\Delta z_{norm})'$ and σ_{MAD}.

The study that is more directly comparable with ours is Tarrío & Zarattini (2020), where the authors used SDSS-DR16 and PS1-DR2. Tarrío & Zarattini (2020) used SDSS-DR16 spectroscopic labels as training labels

Figure 9. Normalized distribution of W1-W2, W2-W3 and W3-W4 colors for the galaxies in the test and training set. Blue: entire test set. Orange: outliers in the test set namely sources with $|\Delta z_{norm}| > 3\sigma(\Delta z_{norm})$. Violet: training set. The numbers within the parenthesis in the legend are the median and the standard deviation of the data, respectively. This plot was obtained by applying the RF_{clas} model to the test set.

and PS1-DR2 colors as as features. Tarrío & Zarattini (2020) adapted a linear regression method developed by Beck et al. (2016) for the photo-z estimation and obtained $\overline{\Delta z_{norm}} = -2.01 \times 10^{-4}$, a standard deviation $\sigma(\Delta z_{norm}) = 0.0298$ and an outlier rate of $P_0 = 4.32\%$ in the test set. We note that while Tarrío & Zarattini (2020) tested their model with galaxies without missing values (no missing PS1-DR2 colors), we include the galaxies with missing features when we calculated our metrics. In spite of this, we obtain a smaller $\sigma(\Delta z_{norm})$, a smaller P_0 and a slightly larger $\overline{\Delta z_{norm}}$ than Tarrío &

Zarattini (2020) with both the RF_{reg} and RF_{clas} models (see table 1).

Beck et al. (2020) leveraged the PS1-DR1 features and SDSS spectroscopic redshift for photo-z estimation. Beck et al. (2020) used a deep neural network for photo-z estimation obtaining $\overline{\Delta z_{norm}}' = 5 \times 10^{-4}$, $\sigma(\Delta z_{norm})' = 0.0322$ and $O = 1.89\%$. We obtain a smaller $\sigma(\Delta z_{norm})'$, O and $\overline{\Delta z_{norm}}$ than Beck et al. (2020) with both RF_{reg} and RF_{clas} models.

Pasquet et al. (2019) adopted a convolutional neural network approach on the SDSS-DR16 dataset for photo-
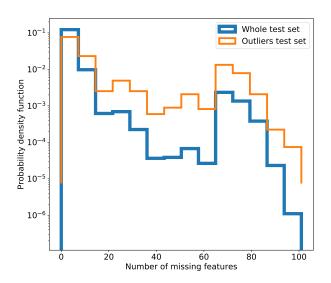
Figure 10. Normalized distribution of the number of missing features for the galaxies in the test set. Blue thick line: histogram for the entire population. Orange thick line: histogram of the outliers, namely sources with $|\Delta z_{norm}| > 3\sigma(\Delta z_{norm})$. Mean value for Orange: 17.6. Mean value for Blue: 3.2. Outliers galaxies have on average a larger number of missing features, as expected.

z estimation using the SDSS-DR16 spectroscopic redshift as labels and the DSS-DR16 images as features. Pasquet et al. (2019) obtained $< \Delta z_{norm} >= 0.0001$ and $\sigma_{MAD} = 0.00912$. Even though Pasquet et al. (2019) achieved a lower σ_{MAD} compared to our work, our method has the benefit to be directly applicable to the entire PS1-DR2 dataset.

5. SUMMARY AND CONCLUSIONS

We present two machine-learning models to compute photometric redshifts (photo-z) for galaxies. We use data from PS1-DR2, AllWISE/unWISE and SDSS-DR16. Our method relies on a random-forest regression (RF_{reg}) and on a random-forest classification (RF_{clas}) algorithm leveraging the PS1-DR2/AllWISE/unWISE features (colors, magnitudes and moments of the radiation intensity) and the SDSS-DR16 labels (spectroscopic redshift). The RF_{clas} was trained using an oversampled training set to equally weigh the underrepresented portion of the redshift distribution. We obtained $\overline{\Delta z_{norm}} = 1.0 \times 10^{-3}$, $\sigma(\Delta z_{norm}) = 0.0225$ and $P_0 = 1.48\%$ for the RF_{clas} model, and $\overline{\Delta z_{norm}} = 2.8 \times 10^{-4}$, $\sigma(\Delta z_{norm}) = 0.0209$ and $P_0 = 1.42\%$ for the RF_{reg} model, respectively.

We analyze the photo-z estimation as a function of the spectroscopic redshift finding that the largest photo-z

error ($z_{phot} - z_{spec}$) is for low redshifts ($z \lesssim 0.1$) and for high redshifts ($z \gtrsim 0.6$) for the RF_{reg} model. The RF_{clas} model performs better than the RF_{reg} model in the local Universe ($z \lesssim 0.1$) and at high redshift ($z \gtrsim 0.6$). This difference is mostly a consequence of the fact that the RF_{clas}, differently from the RF_{reg} model, was trained in a oversampled training set.

Furthermore, we explore how the photo-z error affects the absolute magnitude photometric error, defined as $M_{phot} - M_{spec}$. We find that 97% of the galaxies have $| M_{phot} - M_{spec} | \leq 1$ mag suggesting that our photo-z can be used for course transient classification. We also find that 87% of the galaxies with $z_{spec} < 0.1$ have $|M_{phot} - M_{spec}| \leq 1$ mag. In a follow-up paper we will release a catalog with the photo-z estimation of the entire PS1-DR2 dataset.

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