Gear Basics

李岱峰， 17机器人工程

2020.07.22

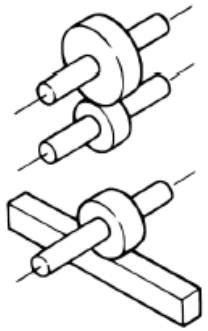
Updated by July 24,2020

Content

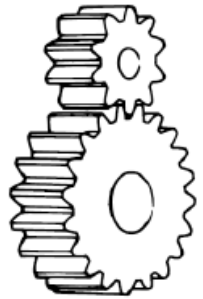
- Types of gear
- Calculate gear transmission
- 渐开线 Involute curves
- Nomenclature/Terminology

Types of Gears

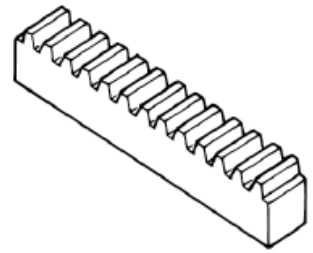
Parallel Axes



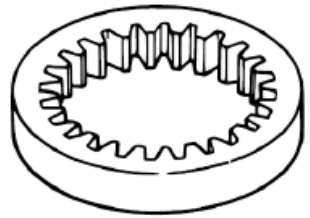
Spur Gear



Rack

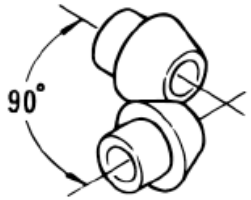


Internal Gear

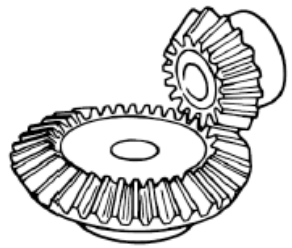


Helical Gear

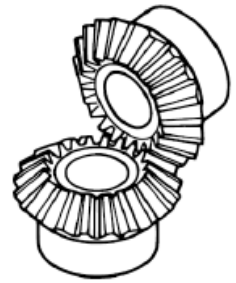
Intersecting Axes



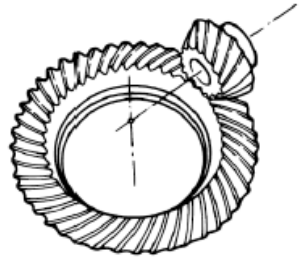
Bevel Gear



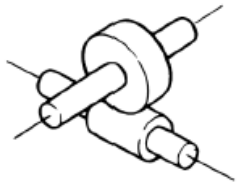
Miter Gear



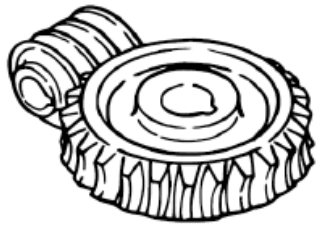
Spiral Bevel Gear



Nonparallel, Nonintersecting Axes



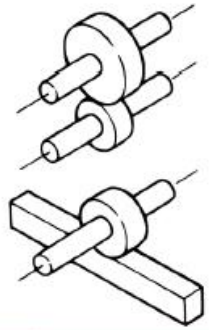
Worm & Worm Wheel



Screw Gear

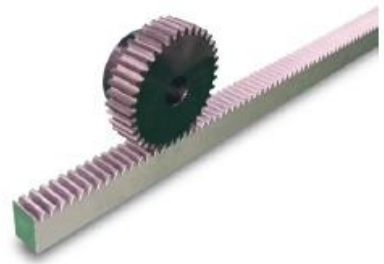
Types of Gears

Parallel Axes



Spur Gear

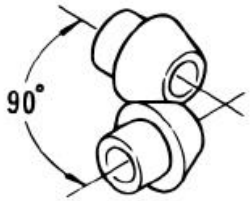
Rack



Internal Gear

Helical Gear

Intersecting Axes

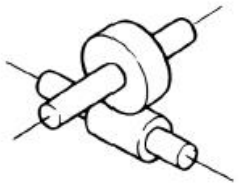


Bevel Gear

Miter Gear

Spiral Bevel Gear

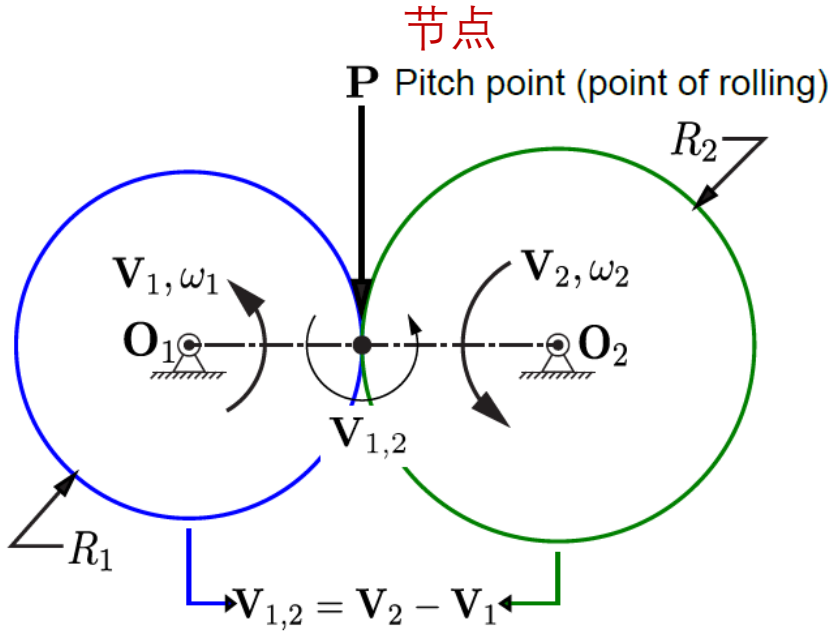
Nonparallel, Nonintersecting Axes



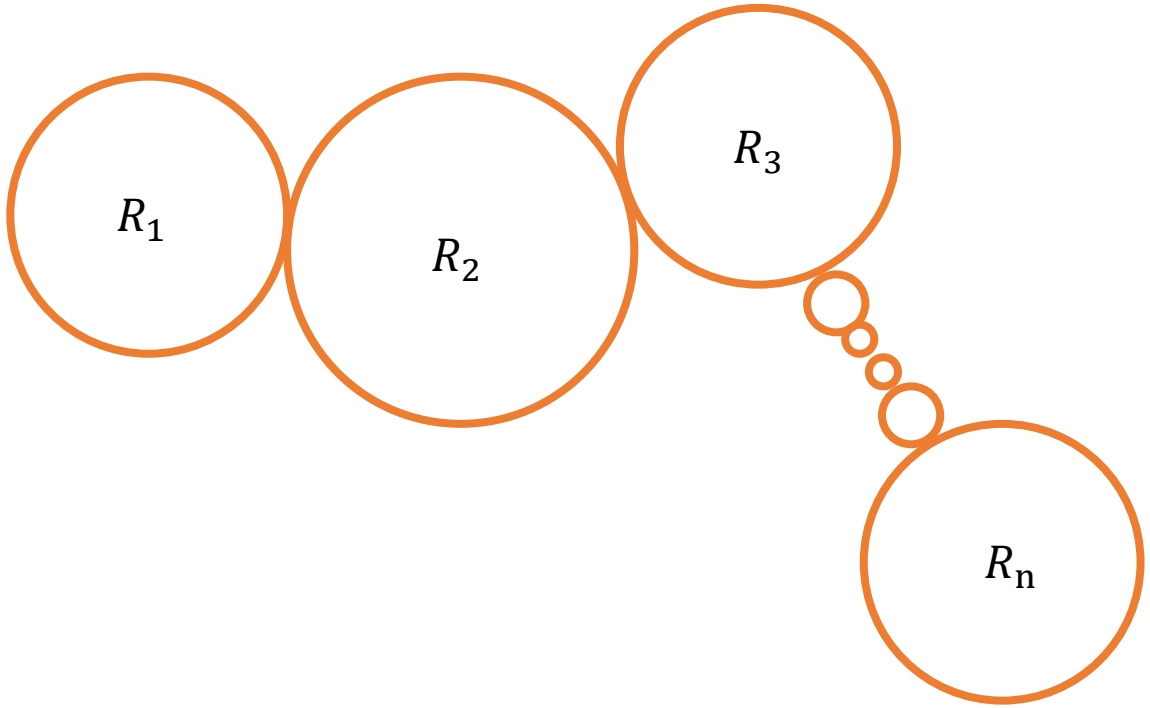
Worm & Worm Wheel

Screw Gear

Calculate gear transmission



$$\frac{\omega_2}{\omega_1} = -\frac{R_1}{R_2}$$

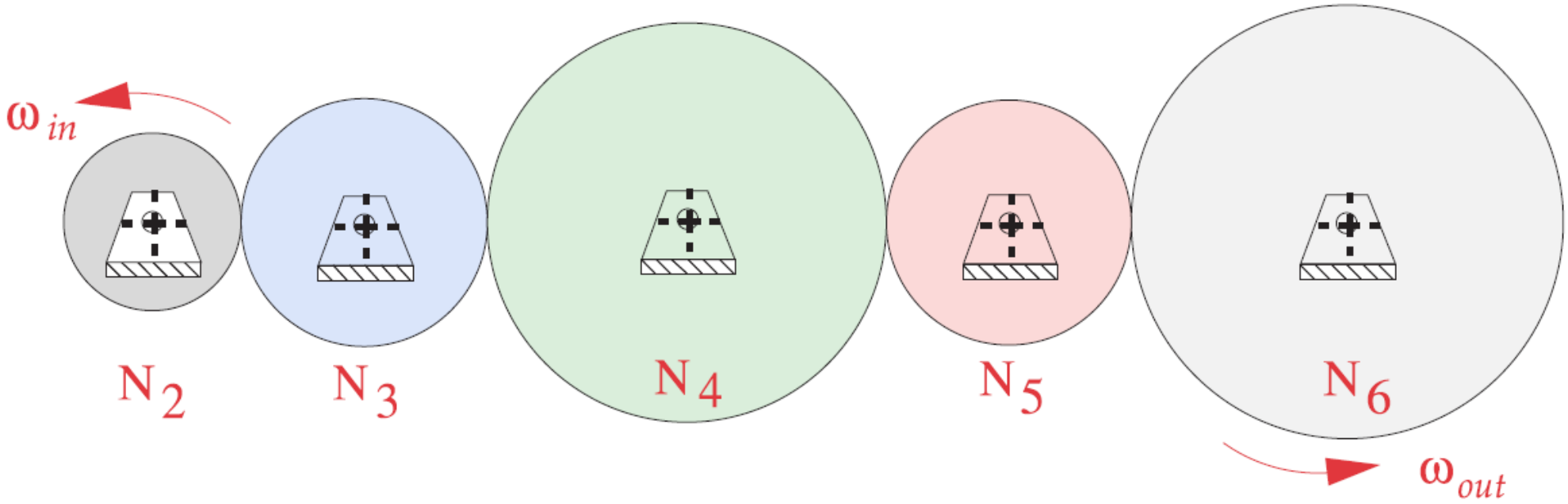


$$\frac{\omega_n}{\omega_1} = (-1)^{(n-1)}\frac{R_1}{R_n} = (-1)^{(n-1)}\frac{N_1}{N_n}$$

Where N means the number of teeth

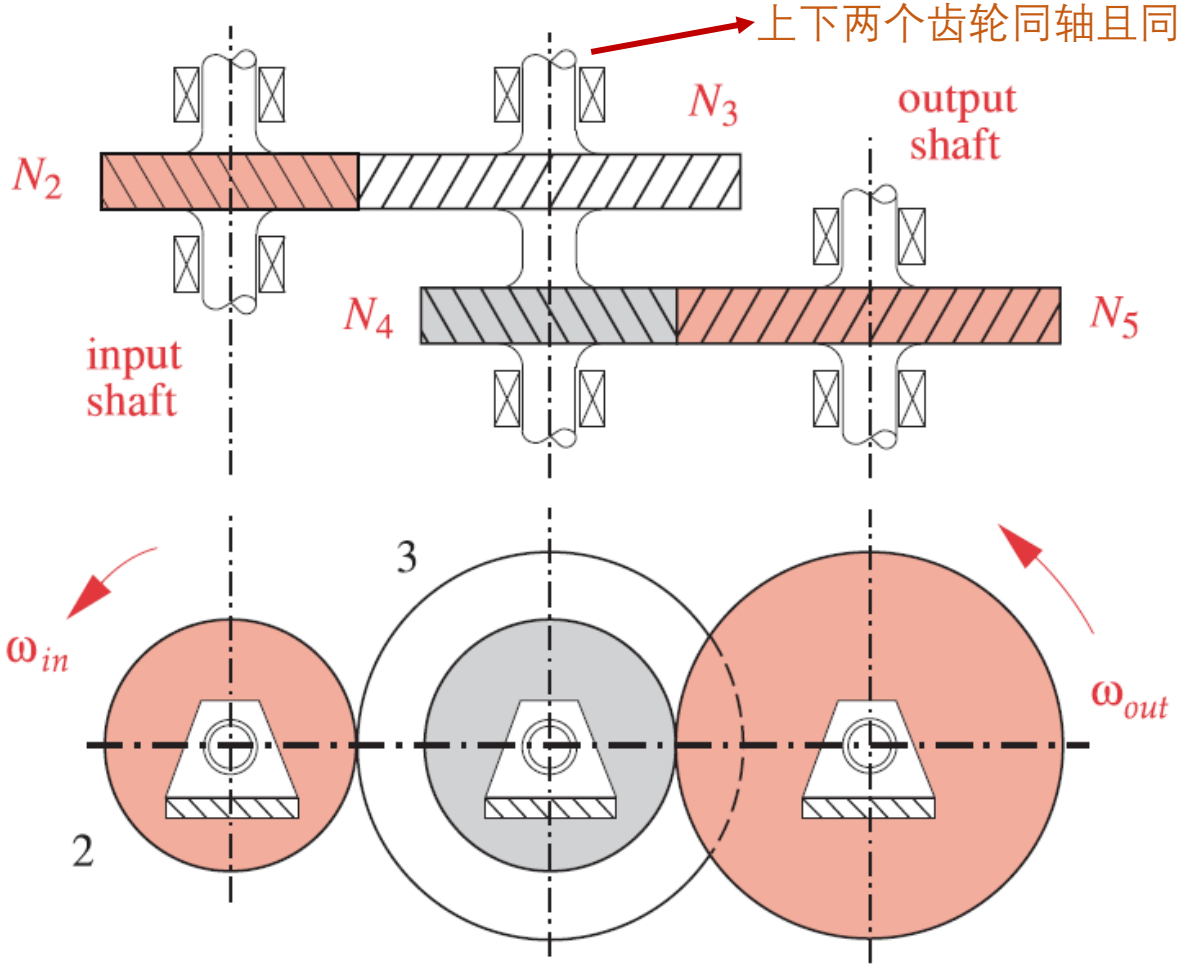
More explanation on the next slide -->

Calculate gear transmission



$$\frac{\omega_{out}}{\omega_{in}} = \left(-\frac{N_2}{N_3}\right)\left(-\frac{N_3}{N_4}\right)\left(-\frac{N_4}{N_5}\right)\left(-\frac{N_5}{N_6}\right) = +\frac{N_2}{N_6} = (-1)^n \frac{N_{in}}{N_{out}}$$

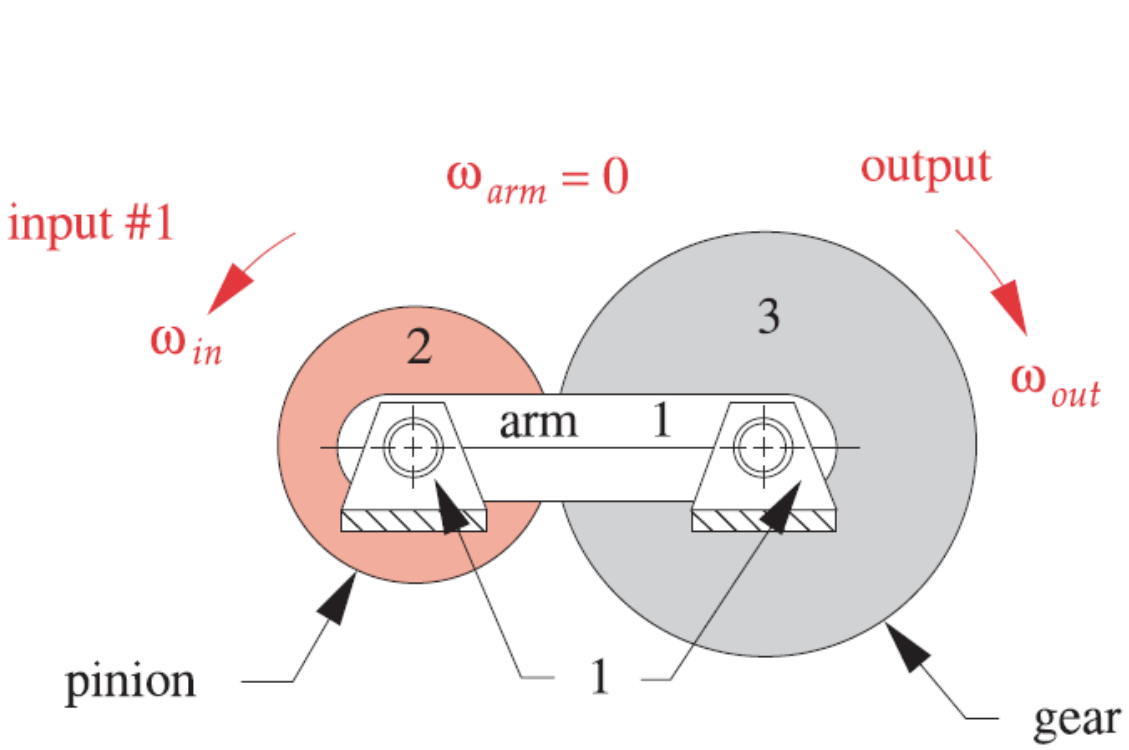
Calculate gear transmission-Compound Gear Trains



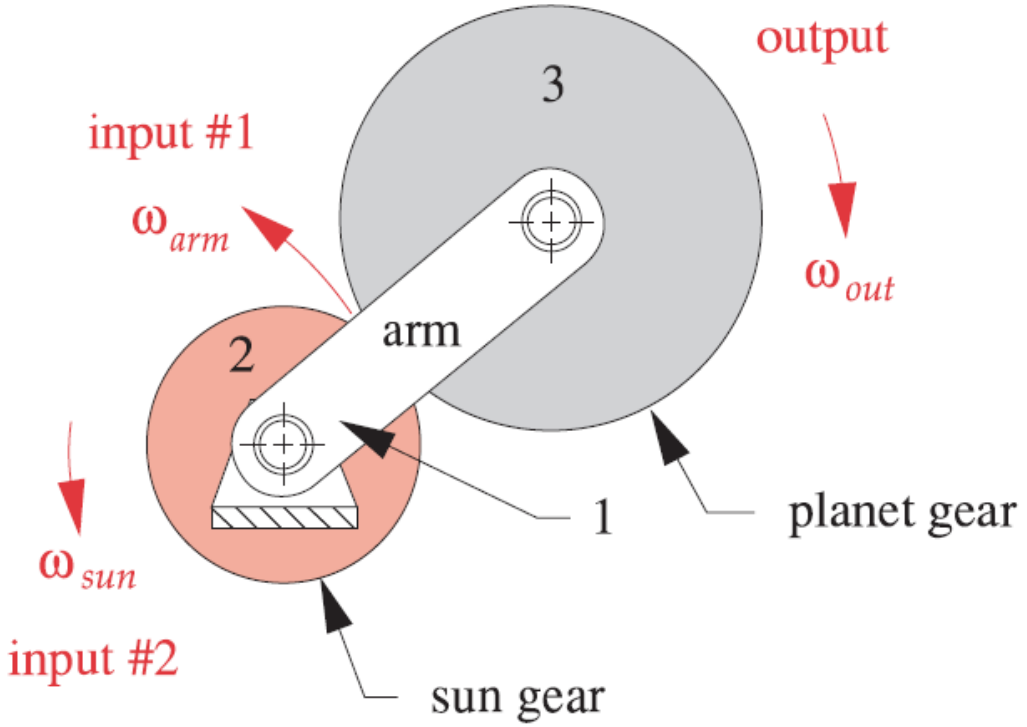
$$\frac{\omega_{out}}{\omega_{in}} = \left(-\frac{N_2}{N_3}\right)\left(-\frac{N_4}{N_5}\right) = \pm\frac{\text{prod. of } N_{drive}}{\text{prod. of } N_{driven}}$$

输出速度/输入速度=驱动齿轮齿数乘积/被驱动齿轮齿数乘积

Calculate gear transmission-Planetary Gear Set



(a) Conventional gearset



(b) Planetary or epicyclic gearset

参考系：站在arm 1上面观察

$$\frac{\omega_{out}}{\omega_{in}} = \frac{\omega_3}{\omega_2} = -\frac{N_2}{N_3}$$

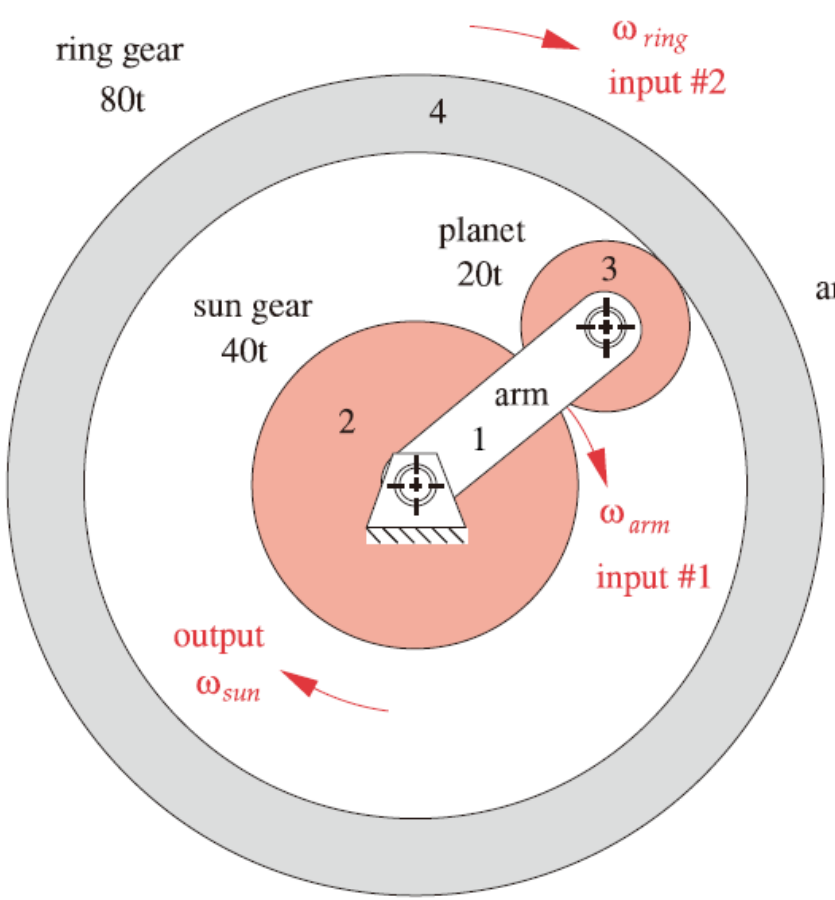
转换参考系

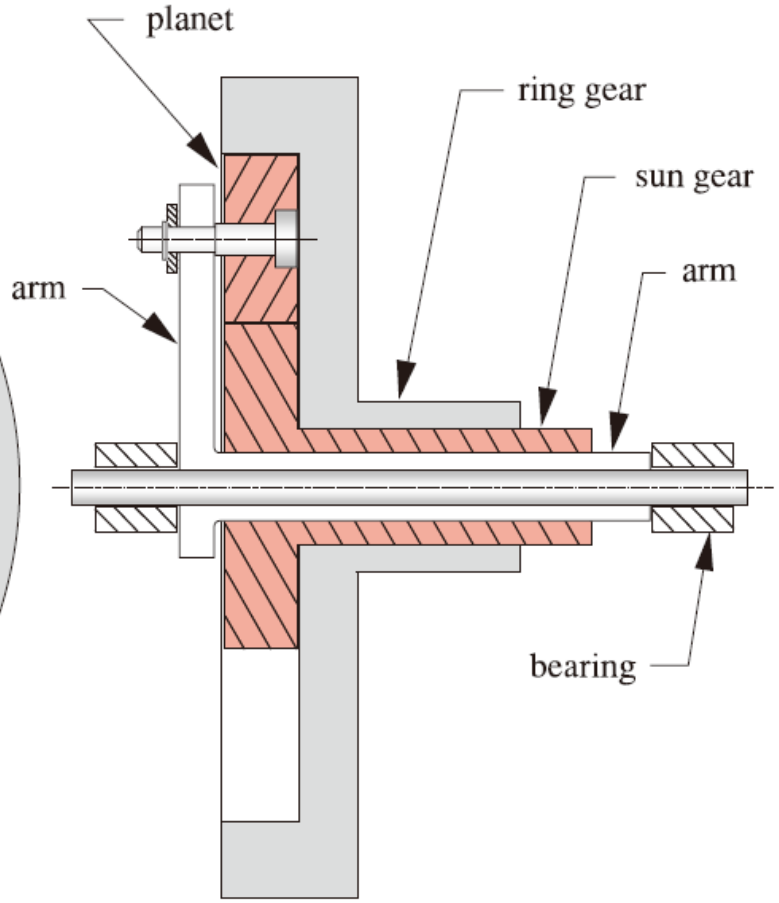
参考系：从arm 1上面跳下来

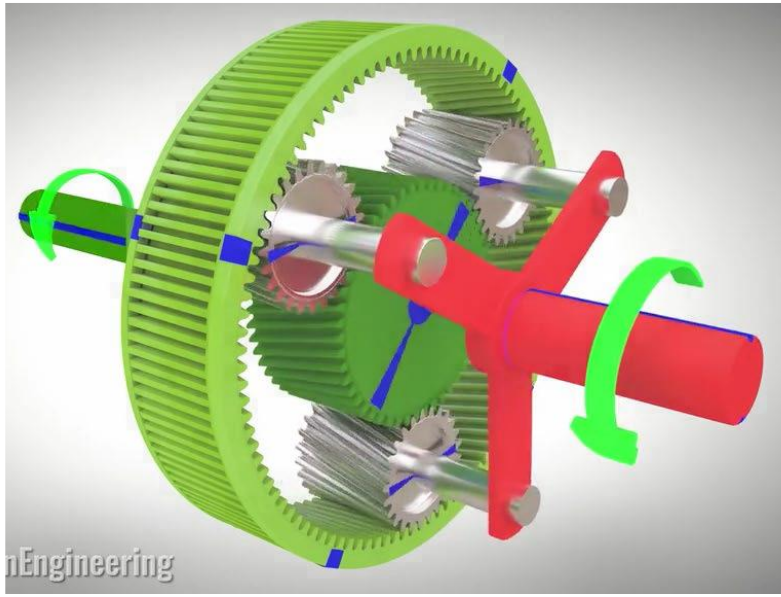
$$\frac{\omega_{1,3}}{\omega_{1,2}} = \frac{\omega_3 - \omega_1}{\omega_2 - \omega_1} = -\frac{N_2}{N_3}$$

ω_1, ω_2, ω_3 represent ω_{arm}, ω_{sun}, ω_{planet}. $\omega_{1,3}$ means velocity of 3 relative to 1.

Calculate gear transmission-Planetary Gear Set





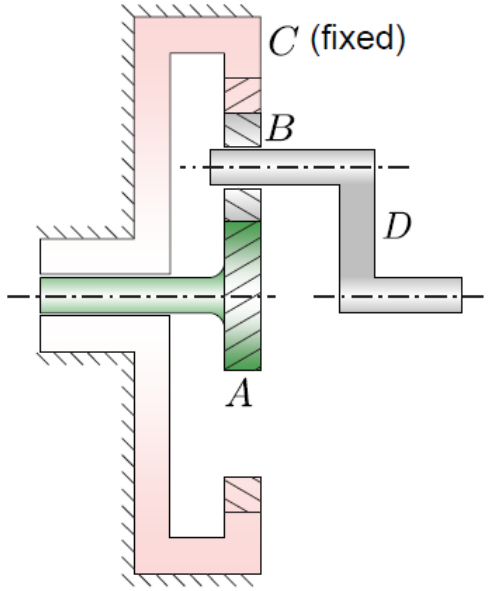


$$\frac{\omega_{1,4}}{\omega_{1,2}} = \frac{\omega_4 - \omega_1}{\omega_2 - \omega_1} = -\frac{N_2}{N_4}$$

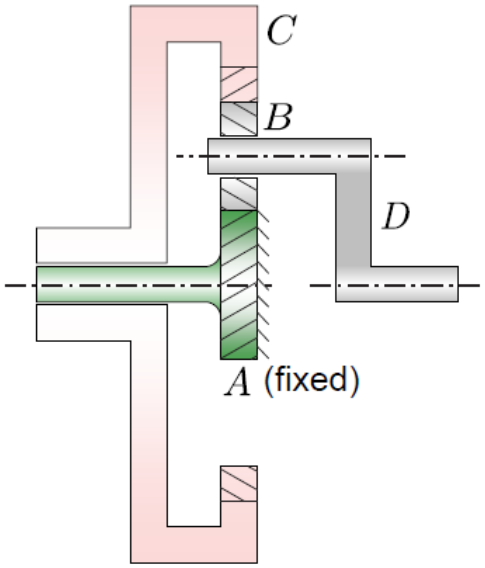
https://www.bilibili.com/video/BV1iW41187vZ/?spm_id_from=333.788.videocard.0

Calculate gear transmission-Planetary Gear Set

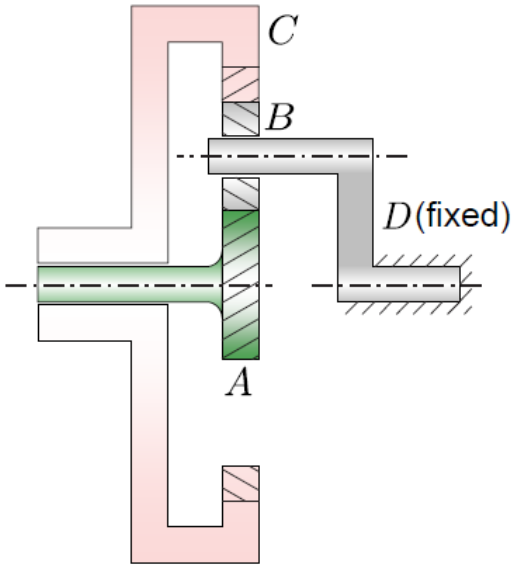
Three modes



(a) Planetary type



(b) Solar type



(b) Star type

$$\frac{\omega_C - \omega_D}{\omega_A - \omega_D} = -\frac{N_A}{N_C}$$

$$\frac{0 - \omega_D}{\omega_A - \omega_D} = -\frac{N_A}{N_C}$$

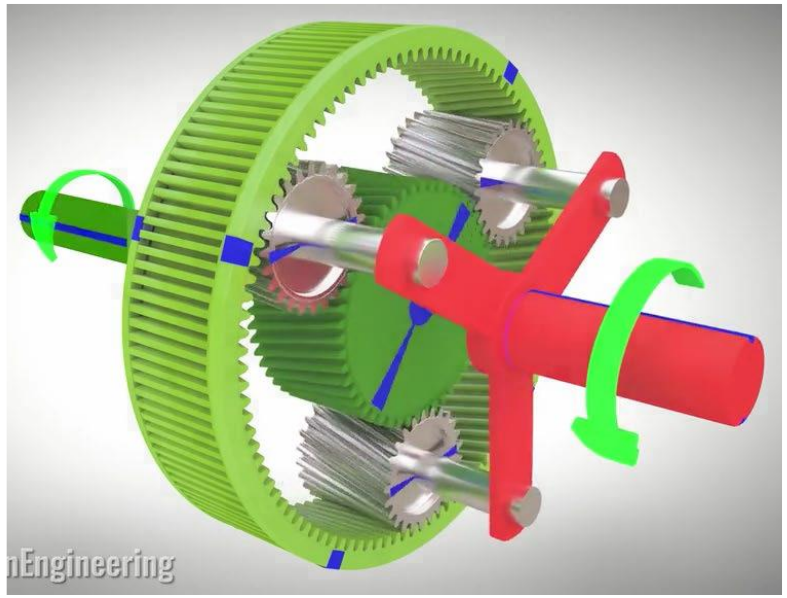
$$\boxed{\frac{\omega_D}{\omega_A} = \frac{N_A}{N_c + N_A}}$$

$$\frac{\omega_C - \omega_D}{0 - \omega_D} = -\frac{N_A}{N_C}$$

$$\boxed{\frac{\omega_C}{\omega_D} = 1 + \frac{N_A}{N_C}}$$

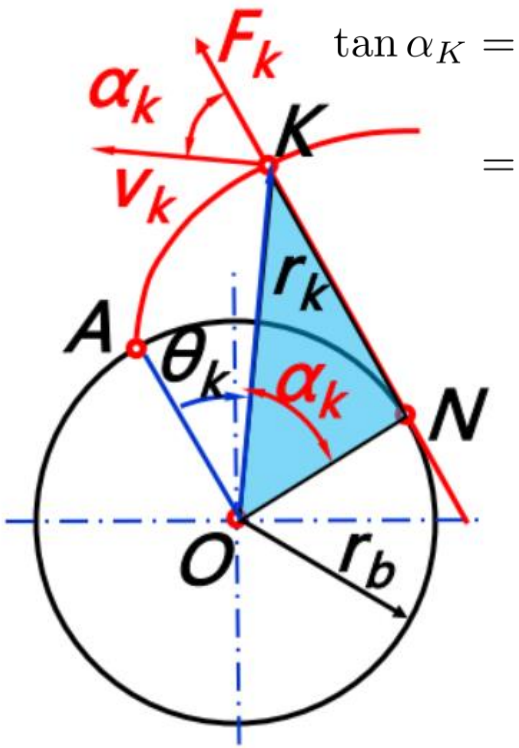
$$\frac{\omega_C - 0}{\omega_A - 0} = -\frac{N_A}{N_C}$$

$$\boxed{\frac{\omega_C}{\omega_A} = -\frac{N_A}{N_C}}$$



If the pitch point moves, then the angular speed of the driven gear changes

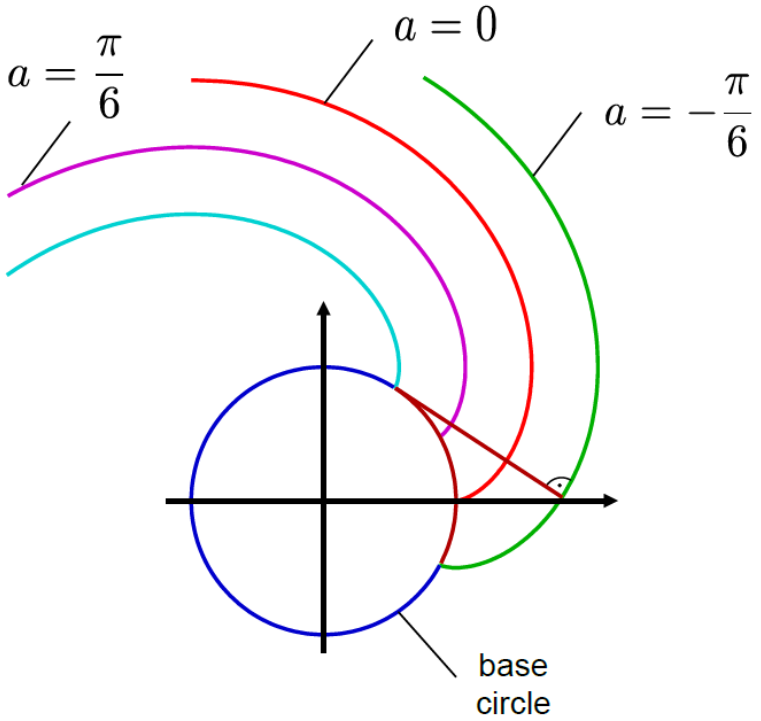
渐开线 Involute curves



$$\tan\alpha_K = \frac{\overline{KN}}{r_b} = \frac{\overline{AN}}{r_b} = \frac{r_b(\alpha_K + \theta_K)}{r_b} = \alpha_K + \theta_K$$

$$\begin{cases} \theta_K = \tan\alpha_K - \alpha_k = inv\alpha_K \\ r_K = \dfrac{r_b}{\cos\alpha_K} \end{cases}$$

极坐标方程
其中：invα_k称为渐开线函数

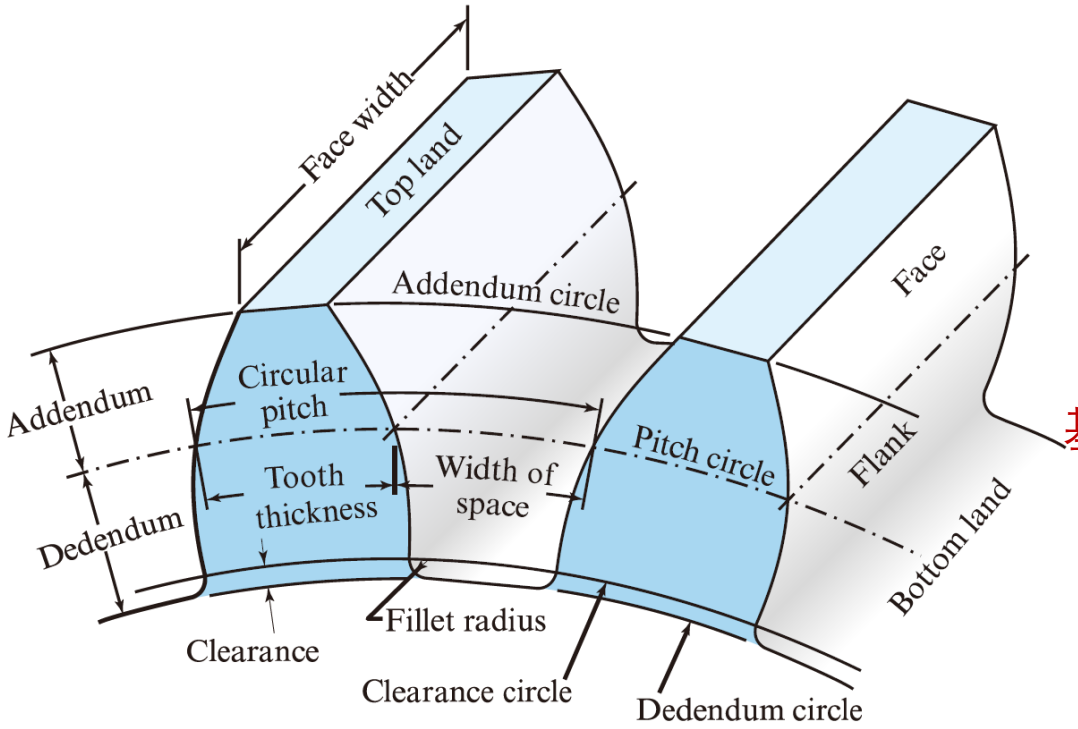


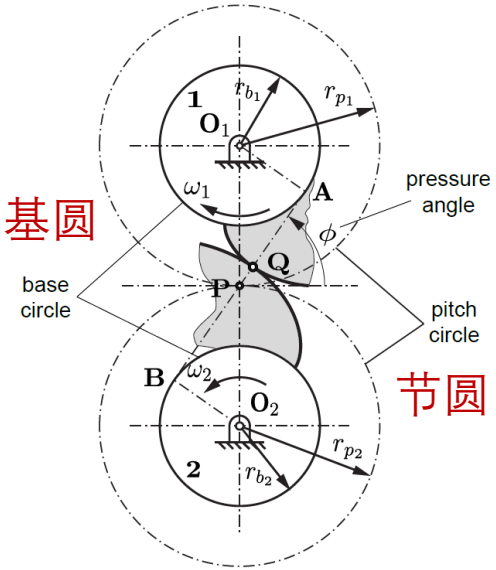
$$X_a(t) = r(\cos t + (t-a)\sin t)$$

$$Y_a(t) = r(\sin t + (t-a)\cos t)$$

笛卡尔坐标系方程
a代表起始位置

Nomenclature/Terminology





基圆

节圆

- **Circular pitch 齿距** p_c

$$p_c = \frac{2\pi r_{p_1}}{N_1} = \frac{2\pi r_{p_2}}{N_2} \Rightarrow \boxed{\frac{\omega_2}{\omega_1} = -\frac{r_{p_1}}{r_{p_2}} = -\frac{N_1}{N_2}}$$

N_1, N_2: teeth number of gear 1 and 2.

压力角 • **Pressure angle** ϕ (usually 14.5° or 20°)

基圆齿距 • **Base pitch** $p_b = p_c \cos\phi = 2\pi r_{b_1}/N_1 = 2\pi r_{b_2}/N_2$

模数 • **Module** $m = 2r_{p_1}/N_1 = 2r_{p_2}/N_2$

齿顶高 • **Addendum** a (usually chosen to be m or $0.8m$)

齿根高 • **Dedendum** d (usually chosen to be $1.25m$ or m)

空隙 • **Clearance** $d - a$

齿厚 • **Tooth thickness** (usually $0.5p_c$)

- **Width of space** (usually $0.5p_c$; always larger than tooth)
- **Backlash** = width of space − tooth thickness

Standard modules m (SI, mm/tooth; larger is bigger)

Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45

演示：SW Toolbox和今日制造插件

Reference

- 中国大学MOOC，西安交通大学，机械设计基础：https://www.icourse163.org/course/XJTU-1001595002?tid=1206706204
- SUSTech ME303 Introduction to Mechanical Design, Chaoyang Song: https://ancorasir.com/?page_id=2159
- SUSTech SDM232 Mechanical Design and Manufacturing, Yuanqing Wu.
- SW插件，今日制造下载：http://www.maidiyun.com/download/softinfo.aspx?id=1&tdsourcetag=s_pctim_aiomsg。相关介绍on bilibili：https://www.bilibili.com/video/BV1dT4y1J7eM?t=110&tdsourcetag=s_pctim_aiomsg