MAT 320 Homework 4
Fall 2018

Due date: Tuesday, Oct 16

- You can use SciLab, or write a program to help in calculations, for any part of this homework.
- Impulse response always refers to the output y_t of a filter given input $x_t = \delta_t$ where δ is the *Kronecker delta* $(1, 0, 0, \ldots, 0)$. You can also think of this as the first standard basis vector. You may also assume that unless otherwise stated, the values of a signal at negative sample indices are always zero.

1. Let $y_t = x_t - \frac{3}{2}x_{t-1} - x_{t-2}$ be a filter equation. Find the transfer function of this filter and factor it into two linear factors, then write the filter equations of these two new filters. Find the impulse response of the first filter, then confirm this by finding the impulse response of the cascade of the two new filters. (See page 70-71.)
2. Find the transfer function of the filter with equation $y_t = x_t - 2x_{t-1} + 2x_{t-2}$. Write this transfer function as a rational function and factor the numerator. Write the magnitude response function $|H(\omega)|$ and compute exact values of this function for $\omega = \pi$, $\omega = \pi/4$, and $\omega = \pi$ using square roots but no decimals. Sketch a graph of the magnitude response function. Find the impulse response for this filter.
3. Find the frequency response function $H(\omega)$ of the filter with equation $y_t = x_t - 2x_{t-1} + x_{t-2} - 2x_{t-3} + x_{t-4}$. Factor out a carefully chosen exponential and write this function in polar form, with magnitude and angle depending on ω. Sketch a graph of the magnitude response function. If a phasor with angular frequency $\omega = \pi/4$ is fed into this filter, what is the output phasor, and by how many samples is it delayed? Is this delay dependent on ω?
4. Define an inverse comb filter with equation $y_t = x_t - R^L x_{t-L}$ with $R = 0.999$ and $L = 6$. Find the transfer function $\mathcal{H}(z)$, the frequency response function $H(\omega)$. and the magnitude response function $|H(\omega)|$. Factor all of these functions. Find the maximum and minimum values of the magnitude response from $\omega = 0$ to $\omega = \pi$, and list them as points (a, b) with a in fractions of sample rate, and b in dB. Sketch a graph the magnitude response function with t axis in fractions of sample rate, and y axis in dB. Find the phase response $\theta(\omega)$ for each of the values at which the magnitude response is maximal or minimal.
5. Suppose the filter (1.1) of chapter 5 has a pole on the real axis at $z = 1.01$. This gives an unstable filter. For which t-value will this filter have impulse response that exceeds magnitude 10?
6. Let $y_t = x_t + \frac{1}{2}y_{t-1} + y_{t-2} - \frac{1}{2}y_{t-3}$. Find the transfer function and factor the denominator. Rewrite the transfer function using formula (2.4) with partial fractions. Solve for the unknown coefficients by clearing denominators and plugging in the zeros. Is this filter stable? Why or why not?