CS 330 Homework 1

Da. Show tn + 8 = O(n)

tn + 8 ≤ cn where n ≥ no · (>C-5)

4n + 8 ≤ 5n

4n+85cn :

where C=S \(\forall (n \(\frac{1}{2}\)\)

 $8 \leq n$ $\longrightarrow n_0 = 8$

6. Show 15 n2 - 50 = \text{\$\text{\$\text{\$O\$}}(n^2)\$}

C, n2 < 18n2-80 < C2 h2

left side

C, n2 5 15n2-50

→ c/= S

5n2 5 15n2 - 50

 $-10n^{2} \leq -50$

 $h^2 \geq 5$

 $n \geq \sqrt{5}$ $n_0 = 3$

right side

15n2 - SO & C, p2

G C2 = 20

15 p2 - SQ 5 2002

-- Sn2-SO50

· Sn2 + SO 20

 $n^2 \ge -10$

 $C_1 n^2 \le |S_n|^2 - 50 \le C_2 n^2$ where $C_1 = S_1$, $C_2 = 20$ $\forall (n \ge 3)$

c. Show n is not in O(Tn) $n \leq \sqrt{n}$ n/Vn = 1 √n ≤1 > this is not true $\forall (n > 1)$ hence, n is not in O(Vn) d. Show n2 = 0 (10") n2 5 (10" > C=1 n2 5 10 m $\log(n^2) \leq \log(10^n)$ 2 log(n) = n log(10) True for no = 1 log(n) = log (10) log (n)/n = 1/2 Y (n > no) 1 bow would one find the maximum of this? log(n)/nm

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② for (i=0; i < n; ++i) ② for (j=0; j \leq n; ++j) \{ \{ \{ \{ \{ \{ \{ \{ \{ \} \} \} \} \} \} \} \} \}
    g(); } 4
 a find concrete runtime T(n) g() = 2 unite
    The runtime of 1 iteration of scope 1 (\xi)
            +(n+1) + 2n + 6 \rightarrow This runtime
                                 is done n times
      Do the reentine of the code with fer loop is
         n (+ (n+1) + 2n+6) +4 <
                                          2 y () calls
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$$T(n) = n (4(n+1) + 2n+6) + 4$$

$$= n (4n+4+2n+6) + 4$$

$$= n (6n+10) + 4$$

$$T(n) = 6n^{2} + 10n + 4$$

6. Prove that the efficiency class is O(n2)

$$f(n) = 6n^{2} + 10n + 4$$

$$g(n) = n^{2}$$

Show that: $f(n) \leq cg(n) + f(n \geq n_0)$ C = 7

$$6n^{2} + 10n + 4 \le 7n^{2}$$

 $-n^{2} + 10n + 4 \le 0$ We should
 $n^{2} - 10n - 4 \ge 0$ try to

(-6,n+2)(n-2) resier

$$6n^2 - 12n + 2n - 4$$

$$6n^2 - 10n - 4$$
 \Rightarrow C = 12 makes
this possible

$$6n^{2} + 10n + 4 \leq 12n^{2}$$

$$-6n^{2} + 10n + 4 \leq 0$$

$$6n^{2} - 10n + 4 \geq 0$$

$$(6n + 2)(n - 2) \geq 0$$

$$(6n + 2)(n - 2) \geq 0$$

$$7;(n) \leq Cn^{2}$$
where $C = 12$, $N_{0} = 2$

$$Y(n \geq N_{0})$$

3 ALG(A)

$$Cur loop invariants are
$$C \neq R = 0$$

$$C \neq R = 2k$$
while $(d > 0)$ E
$$R = R + 2$$

$$C \neq R = 2k$$
while $(d > 0)$ E
$$R = R + 2$$

$$C \neq R = 2k$$
while $(d > 0)$ E
$$R = R + 2$$

$$C \neq R = 2k$$

$$C \neq R = 2k$$$$

Anduetive Step

Assume:
$$lk = A - k$$
, $Rk = 2k$
Show: $lk+1 = A - (k+1)$, $Rk+1 = 2(k+1)$

$$cl_{k-1} = A - (k+1)$$
 $R_{k} + 2 = 2(k+1)$
 $A - k - 1 = A - (k+1)$ $2k + 2 = 2(k+1)$
 $A - (k+1) = A - (k+1)$ $2(k+1) = 2(k+1)$

Proof of leop terminution

Since
$$cl$$
 decreases
$$\exists clt ((clt > 0) \land (clt) \leq 0)) \lor$$

Proof of algorithm correctness.

Whe know
$$dt + 1 = 0$$
 \Rightarrow when dk is 0

$$0 = t - k$$

$$k = A$$

ALG(A) R=O U=2 while (of = A) E $\mathbb{R} = \left[\frac{k+1}{2} \right] = 0$ if (I is even) R= R+1 d= d+1 ils is odd (Pa) = (da-1)/2 Uk is even RA = (dk-2)/2 Proof of Base Cass $R_0 = \left(\frac{O+1}{2} \right)$ do = 0+2 How our invariants change over an iteration. Ik is odd It is even RA+1 = RA+1 Ra+1 = Rk

K. A. I. S. L.

Unductive Step Clasume: 1/4 1 +2 Cle= k+2 Uf ilk is oder Ph=(dk-1)/2 If I is even R & = (Up - 2)/2 Show ! UR+1 = (k+1) +2 Uf detis odd Reti = (de+1 -1)/2
Uf detis even Reti = (de+1 -2)/2 Clp+1 = (k+1) +2 Clp11 proof de+1 = (k+1)+2 k+2+1= k+3 R+3= R+3 / (lk-1)/2 = (dk-1)/2 Rx+1 = (Ux+1-1)/2 = Rx+1 where Uk is even Re+1 = Ue/2 (Ue-2)/2 + 1 = Ue/2T da+1 is odd Uk/2-1+1 Uk/2 = Uk/2 /

Since it represents a phicky energy sequence of integers, it connot be bounded so $\exists \epsilon ((dt \leq A) \land (dt+1 \geq A))$		
Froof of algorithm consectness. (thin we near the loop terminate the 1, our algorithm should return $A/2$ So we look at the values when our wop terminates (this odd the is odd the is a +1. Re+1 = (the+1-1)/2 (t+2)+1 = A+1. Re+1 = (the+1-1)/2 t+3 = A+1. Re+1 = (t+2)/2 t+3 = A+1. Re+1	Proof of loop termination	
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Rt+1 = ($\frac{1}{2}$ + 1 - 1)/2		
Re+1 = $(t + 2)/2$ Re+1 = $A/2$ Ult+1 is even Re+1 = $(Ult+1 - 2)/2$ Re+1 = $((t + 2)+1-2)/2$ Re+1 = $(A-1)/2$ This makes sense Ult+1 = $A+1 \rightarrow let's seg A=3$ With = $A+1 \rightarrow let's seg A=3$		The Street
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