CS300
BRDF

Lighting Models

- Lighting models in computer graphics can be divided into two categories:
  - Empirical model
  - Physically based

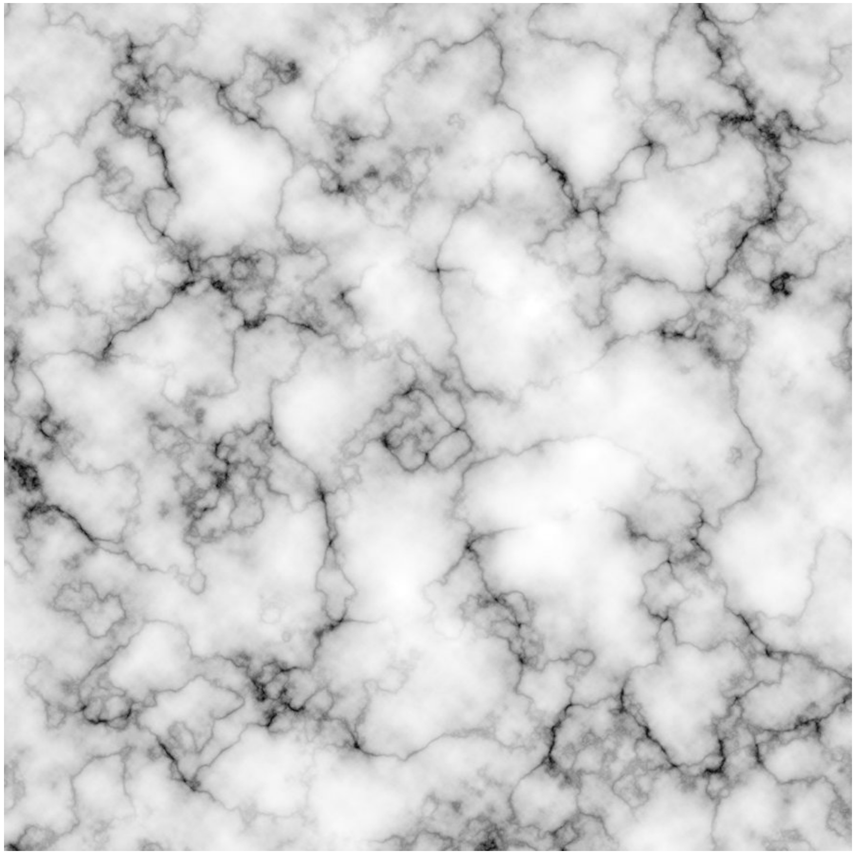
Empirical Lighting Models

- Based on observations and practical constraints, we approximate how light interacts with objects.
- Phong lighting model is one example:

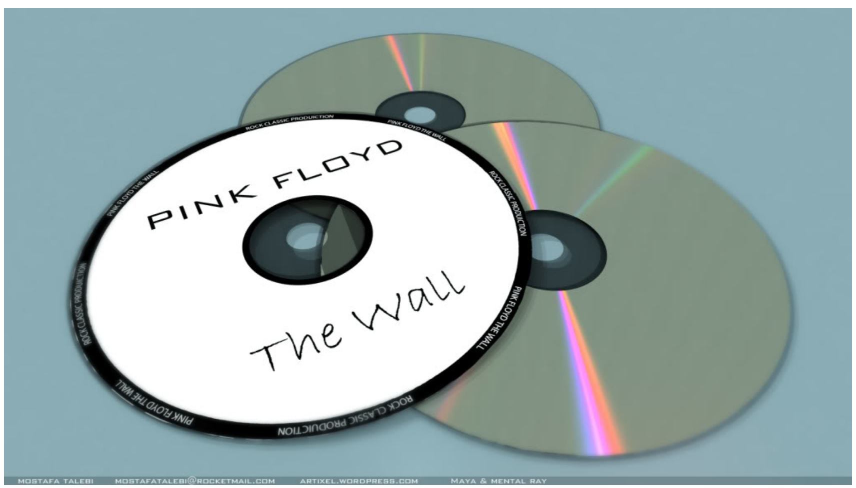
$$I_{tot} = I_{ambient} + I_{diffuse} + I_{specular} = I_a K_a + I_d K_d + I_s K_s$$

- The parameters in the equations (K_a, K_d, K_s) are not based on physical properties. For e.g.: what should we use for bronze/rubber/plastic/marble?

Complex Materials



Complex Surfaces



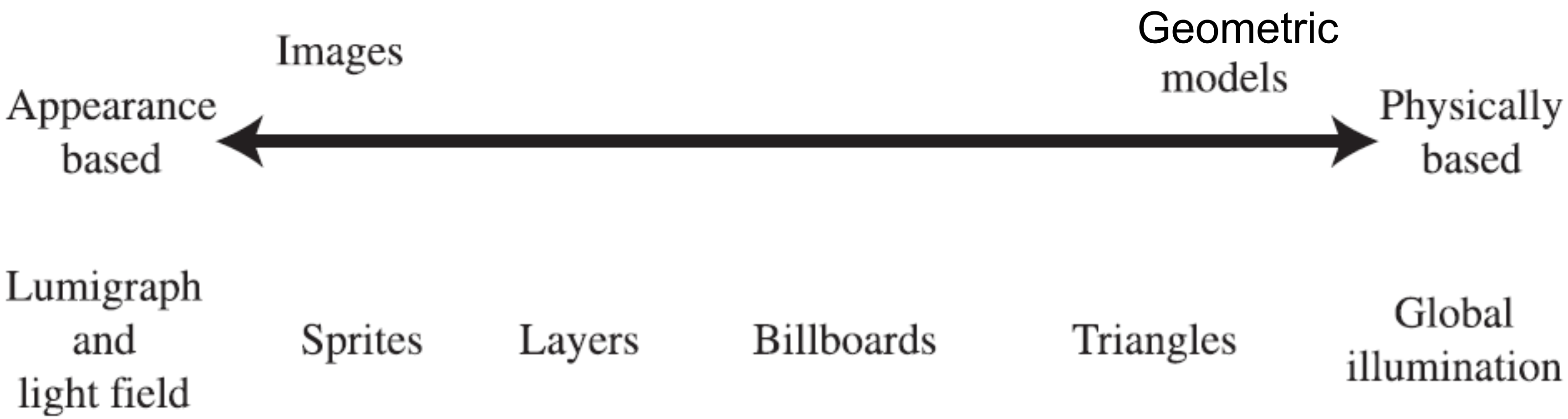
Physically Based Lighting Models

- Model that uses physical properties
- Each parameter has a physical meaning
- Tries to simulate light behavior in real scene
- Observes energy conservation
- Complicated and expensive

Lighting Models

- Example of lighting model used:
  - **Lambertian**– for perfectly diffuse surface.
  - **Phong**– model a plastic like specular.
  - **Torrance-Sparrow** – represent surfaces as distributions of perfectly-specular microfacets.
  - **Cook-Torrance** – similar to Torrance-Sparrow, but it accounts for wavelength, hence color shift.
  - **He,Torrance,Sillion,Greenberg (HTSG)** - a comprehensive physically-based model.

Lighting Model Spectrum



Environment Maps

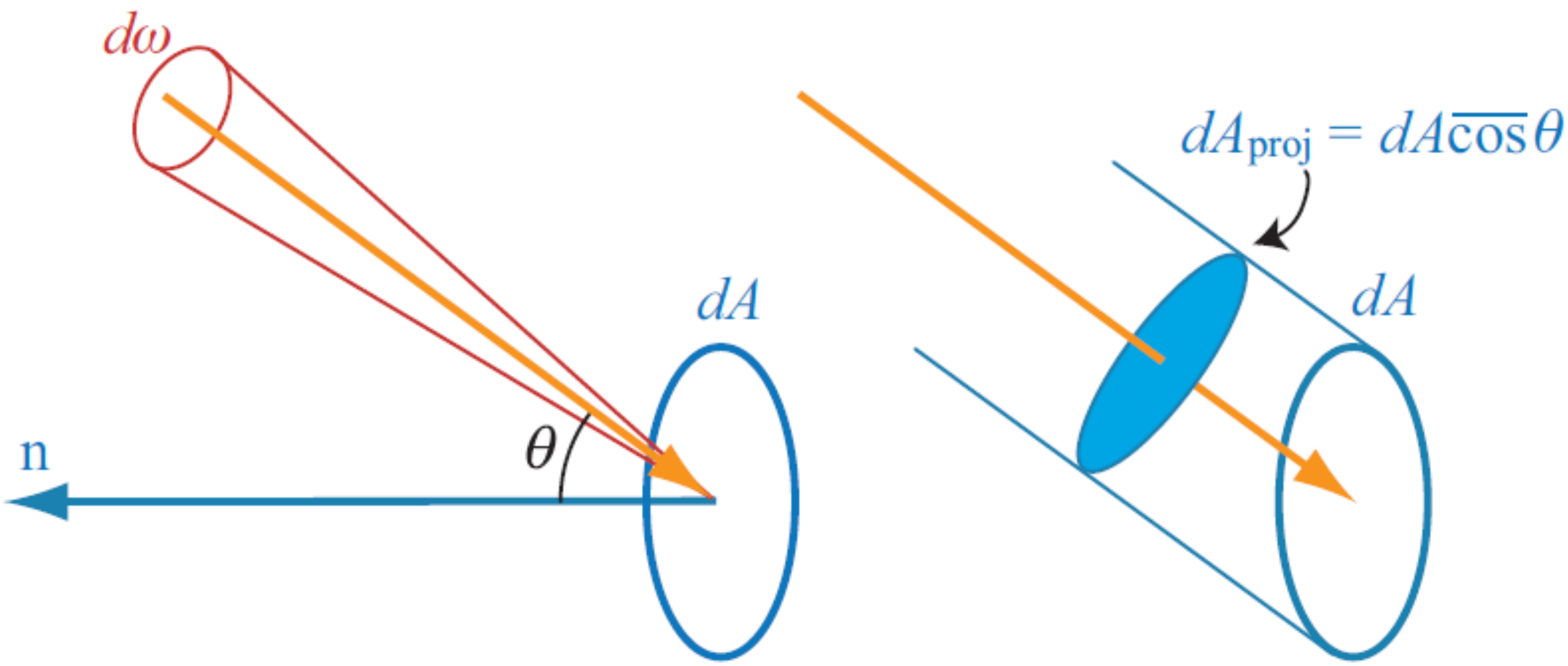
Some Terminologies

- *Flux* – the rate of light energy being emitted. Measured in **Watt (W)**.
- *Radiant Intensity* – the flux radiated into a solid angle in a particular direction. Measured in **W/sr**.
- *Radiance* – radiant intensity per unit foreshortened (projected) surface area. Measured in **W/(sr.m^2)**.
- *Irradiance (flux density)* – incident flux per (unforeshortened) unit area. Measured in **W/m^2**.

Some Terminologies

- *Foreshortened (projected) surface area*
  - The projection of the surface onto the plane perpendicular to the direction of radiation.
  - Calculated by multiplying the area with cos θ, where θ is the angle between the radiation direction and the surface normal.

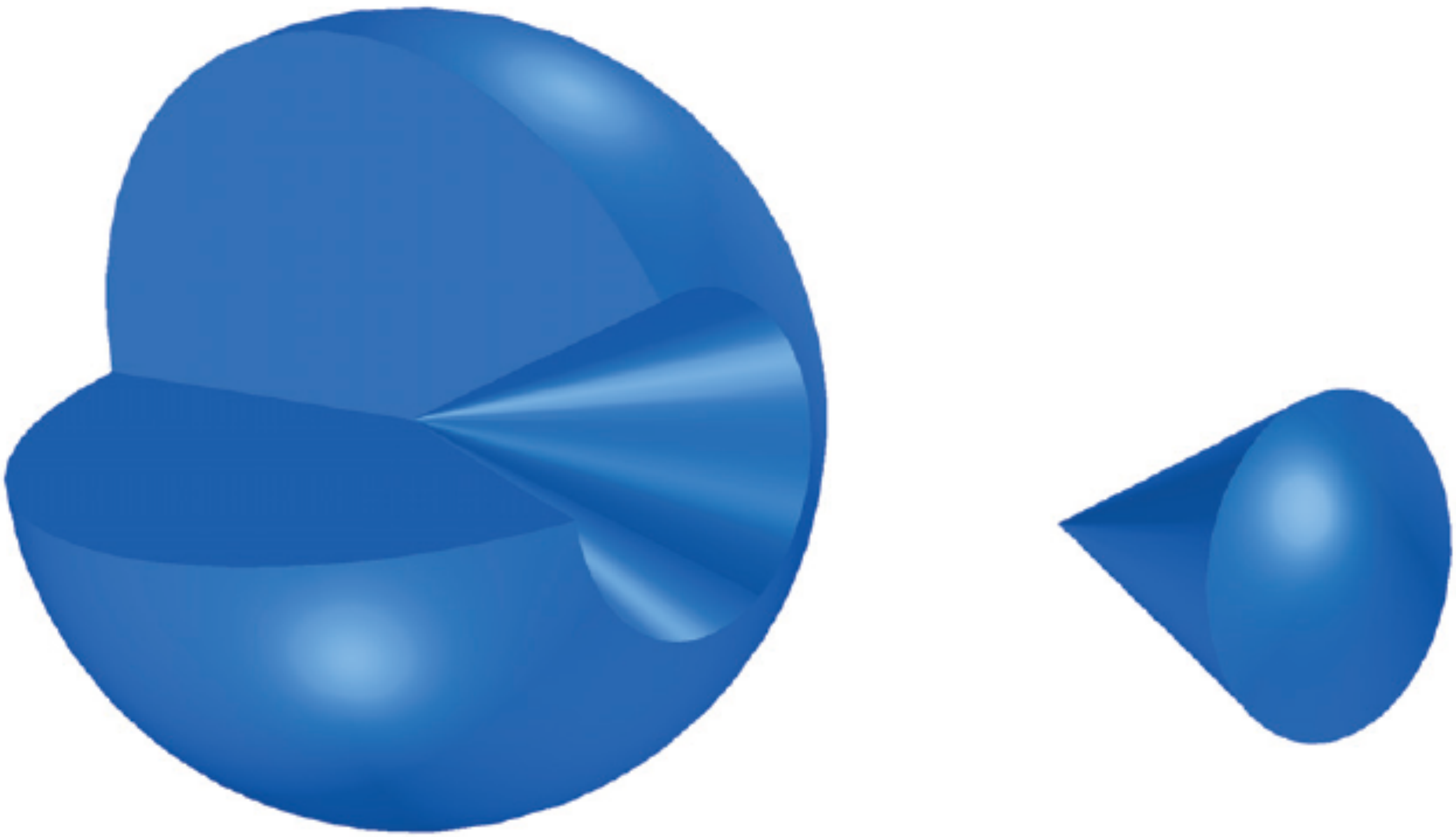
Projected Area



Some Terminologies

- *Solid Angle* – angle at the apex of a cone.
  - Measured in terms of area on a sphere intercepted by the cone whose apex is at the sphere’s center.
  - One **steradian (sr)** is the solid angle of such a cone that intercepts an area equal to the square of the sphere’s radius.

Solid Angle



Solid Angle

- In 2D, the angle in radians is related to the arc length of a circle that it cuts out.

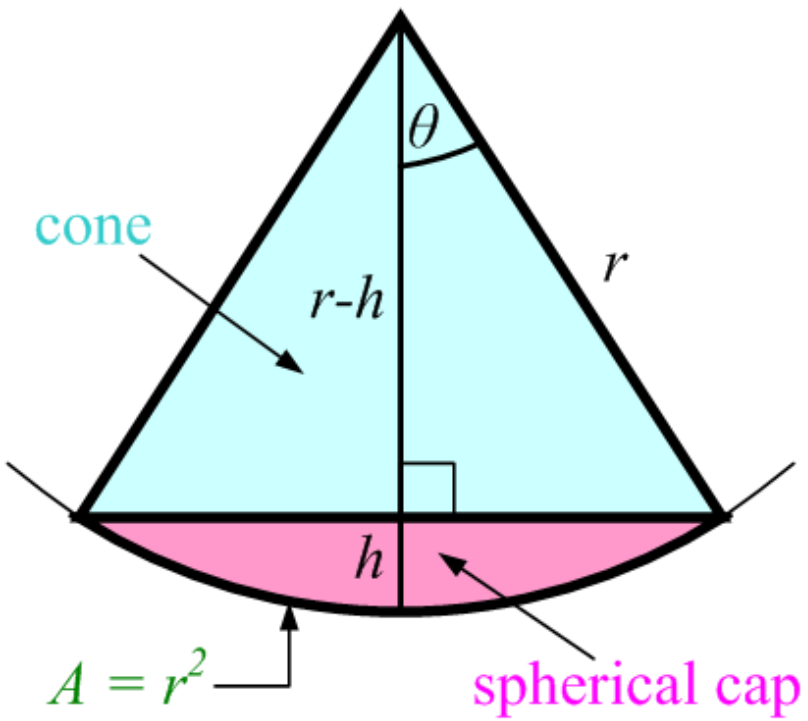
$$\theta = \frac{\text{arclength}}{r}$$

- In 3D, the solid angle in steradians is related to the area on the sphere that it cuts out.

$$\Omega = \frac{\text{surface area}}{r^2}$$

Solid Angle

- What is the angle (θ) at the apex of the cone when the solid angle is one steradian?



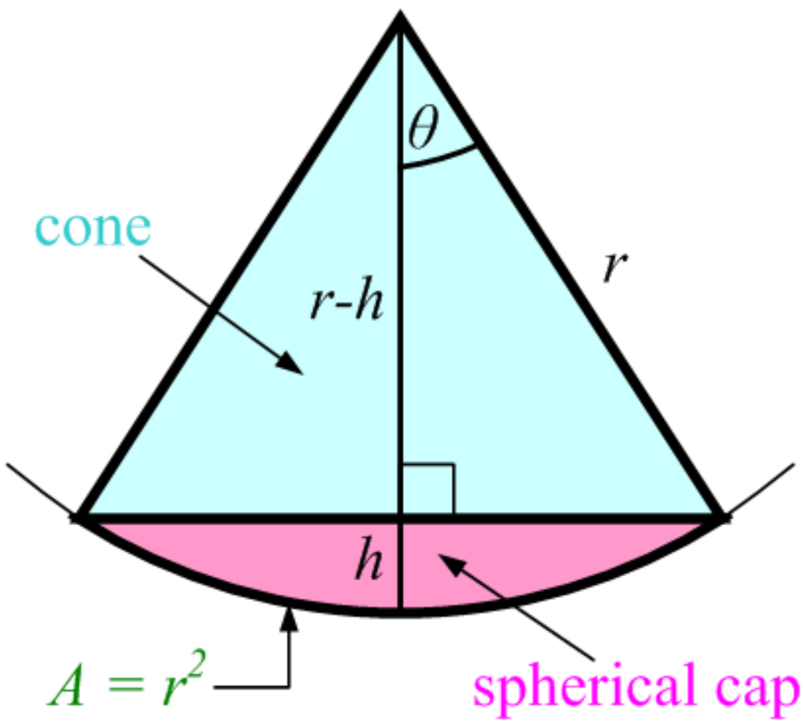
Solid Angle

- The surface area of the spherical cap:

$$A = 2\pi rh$$

- If the solid angle of the cone is one steradian, then:

$$\Omega = \frac{A}{r^2}, A = r^2$$



Solid Angle

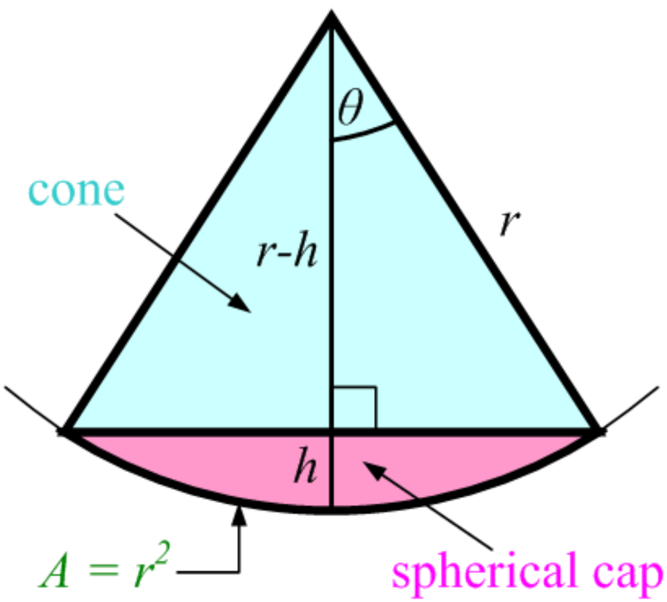
- To calculate for θ:

$$A = r^2 = 2\pi rh$$

$$r = 2\pi h$$

$$\theta = \cos^{-1}\left(\frac{r-h}{r}\right)$$

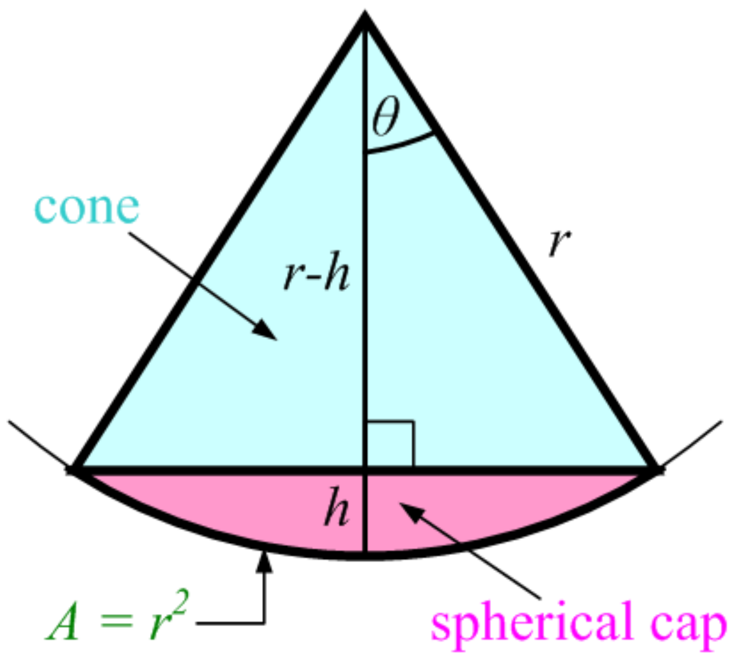
$$\theta = \cos^{-1}\left(1 - \frac{1}{2\pi}\right) \approx 0.572 \text{ rad or } 32.77°$$



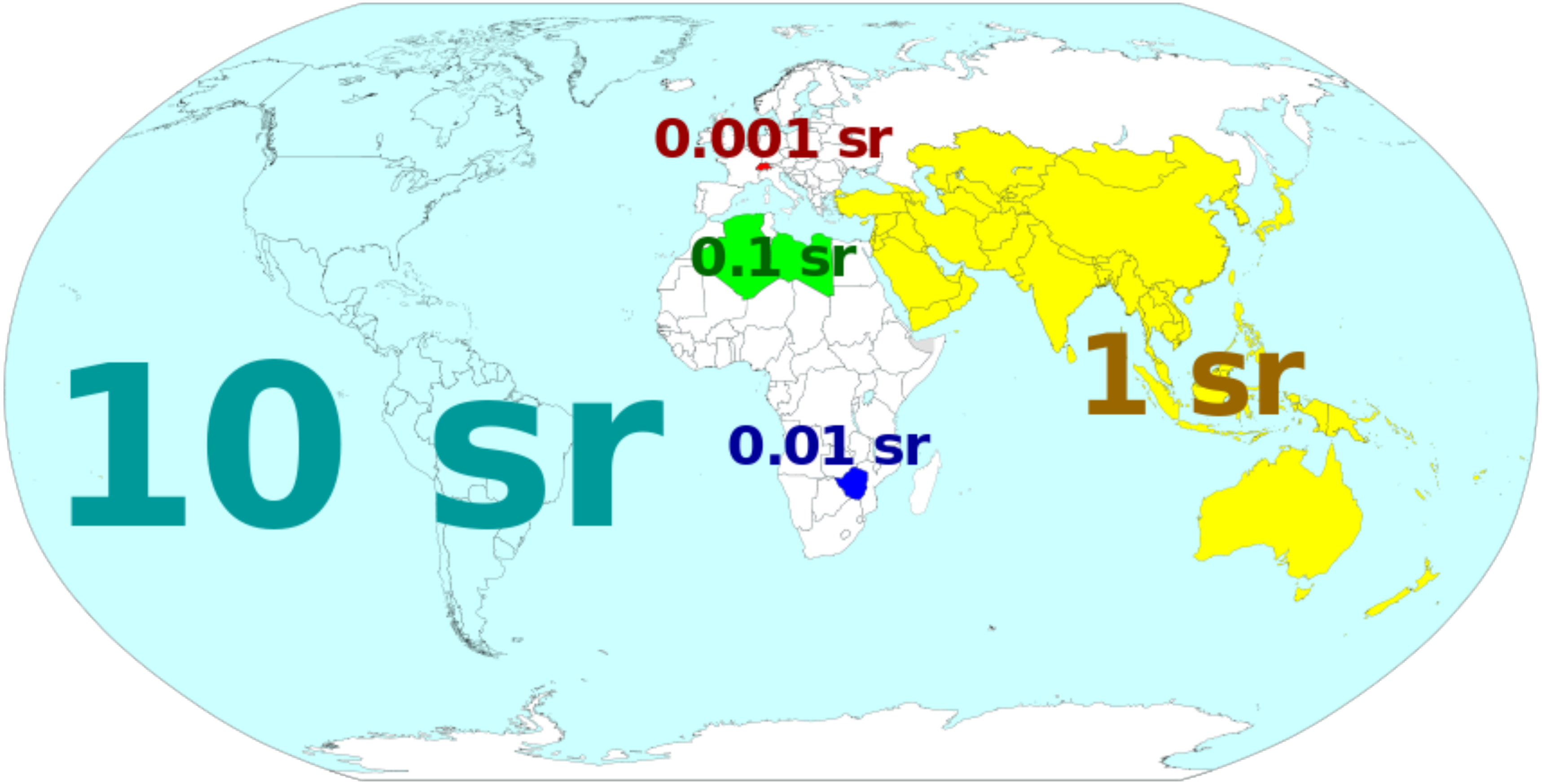
Solid Angle

- So:

$$2\theta = 1.144 \text{ rad or } 65.54°$$



Steradians in practice



BRDF

Bidirectional Reflectance Distribution Function

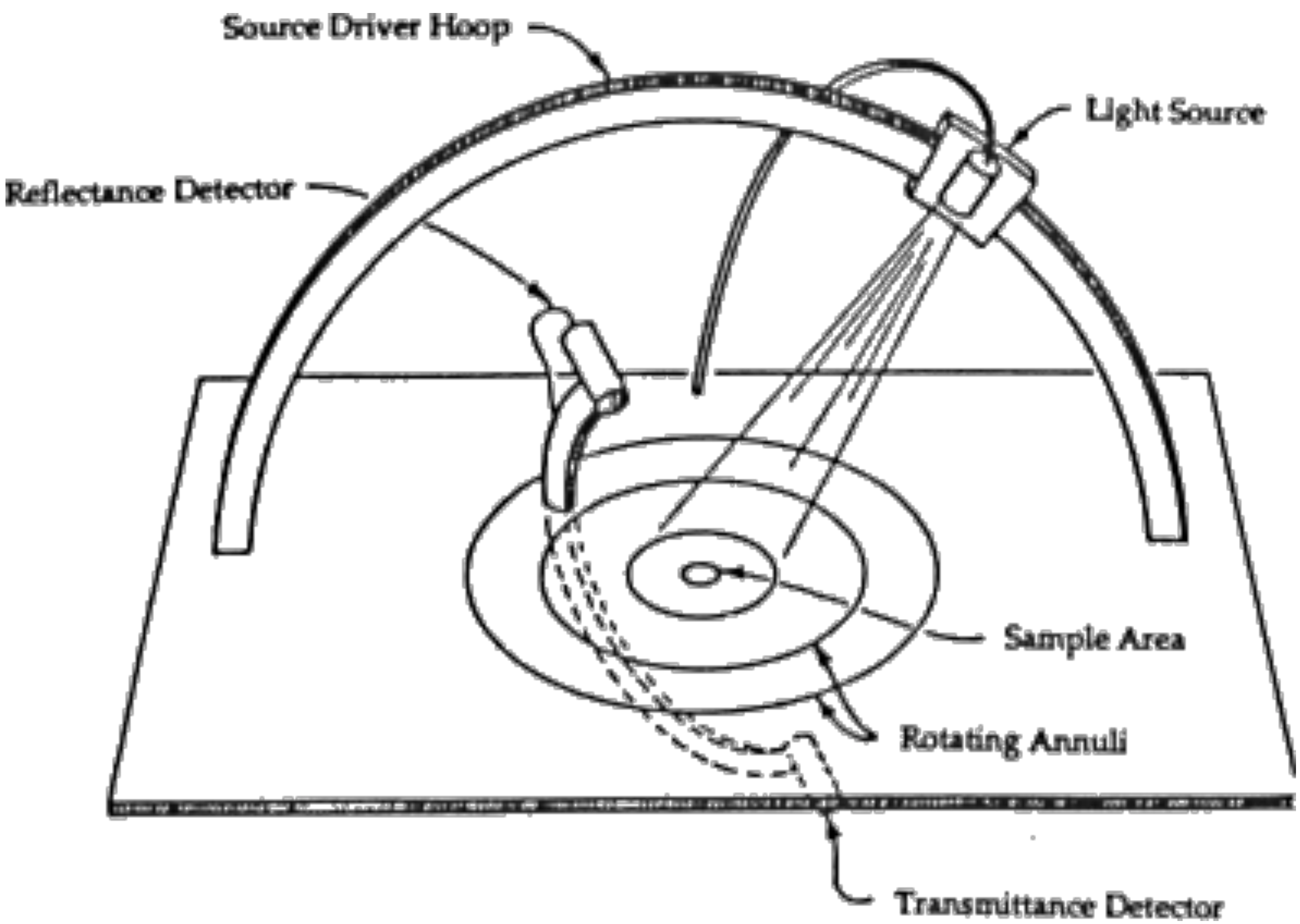
BRDF

- Bi-directional Reflectance Distribution Function
- A function that defines how light is reflected at a given point to a viewer.
- A BRDF is generally used on a solid opaque surface, where the light is reflected or absorbed.
- When modeling the transmitted light, one would use a BTDF (Bi-Directional Transmission Distribution Function)

BRDF

- BRDF depends on:
  - Direction of the incident and outgoing light relative to the surface.
  - The light wavelength. Different wavelength (color) will interact differently with the material.
  - The region on the surface where the light is reflected. Most material in the real world are not uniform. It is composed of different sub-materials which interact with the light differently.

Capturing BRDF



BRDF

- BRDF function:

$$\mathrm{BRDF}_{\lambda}\left(\theta_i, \phi_i, \theta_o, \phi_o, u, v\right)$$

- Where:
  - λ is the light wavelength.
  - (θ_i, ϕ_i) is the incoming light direction in spherical coordinate.
  - (θ_o, ϕ_o) is the outgoing light direction in spherical coordinate.
  - (u, v) is the position on the surface.

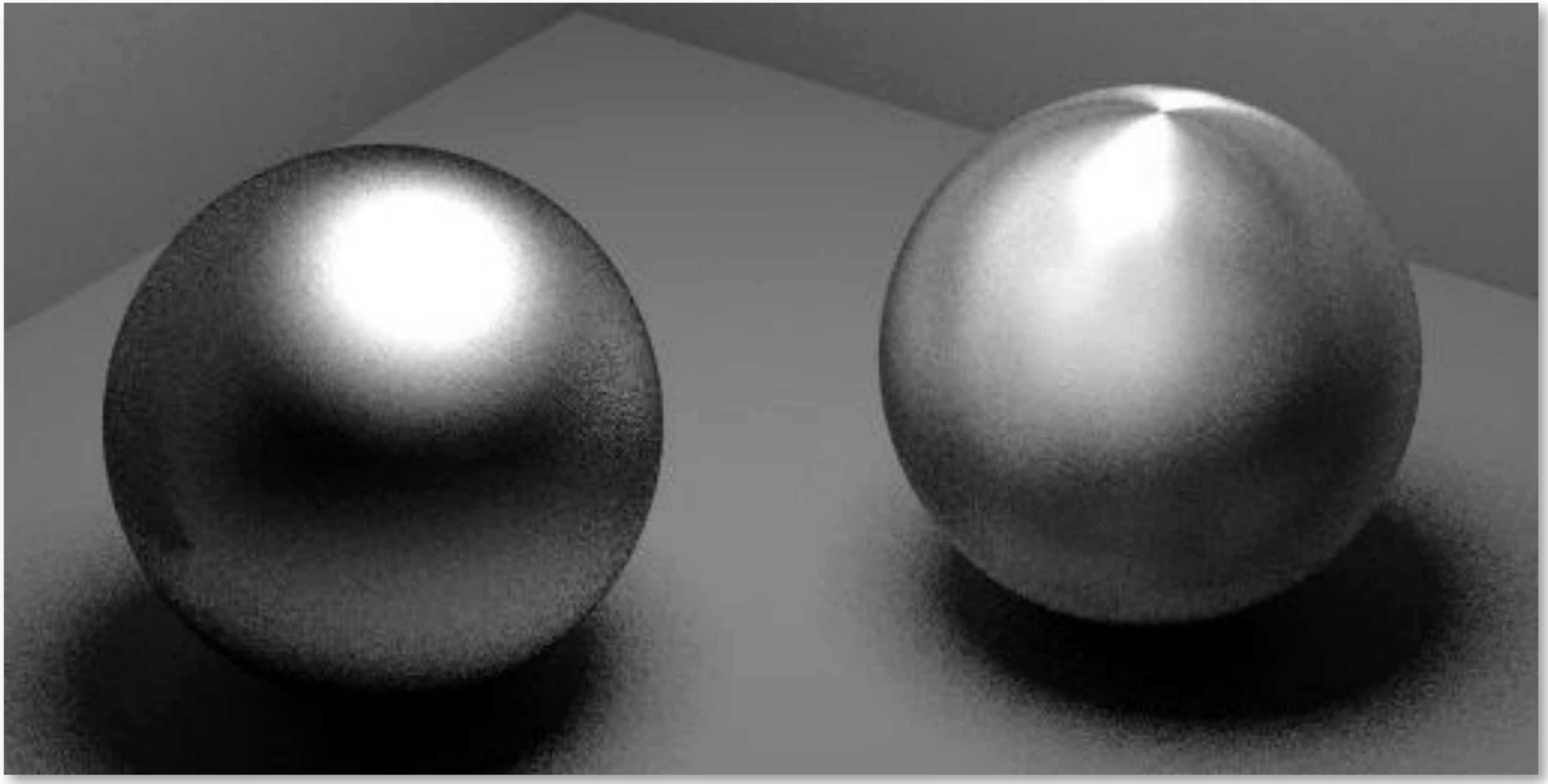
BRDF

- Typically, for convenience and simplicity, the λ in the formula is dropped. Just remember that BRDF **does** depend on the light wavelength.
- So, BRDF values must be determined for each color component (R, G, B). And the color calculations need to be done for each color component.

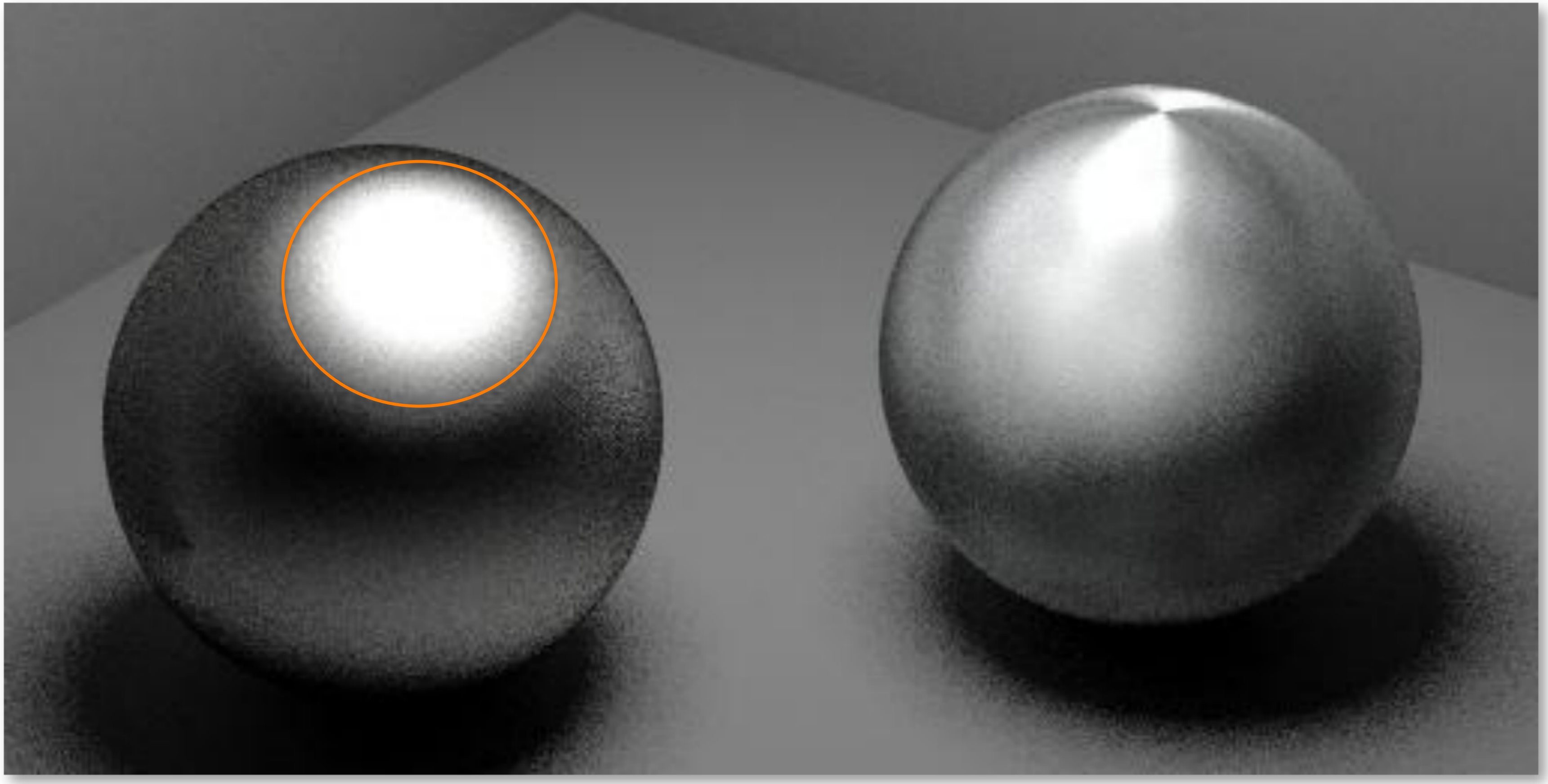
BRDF

- Position Variance and Invariance
  - A surface that interacts differently with light across its span uses a *position variant* BRDF. Most real-world materials have a degree of variation.
  - A surface that is uniform across its span uses a *position invariant* BRDF, where the u and v parameters are excluded from the formula. Many computerized models use an invariant BRDF.

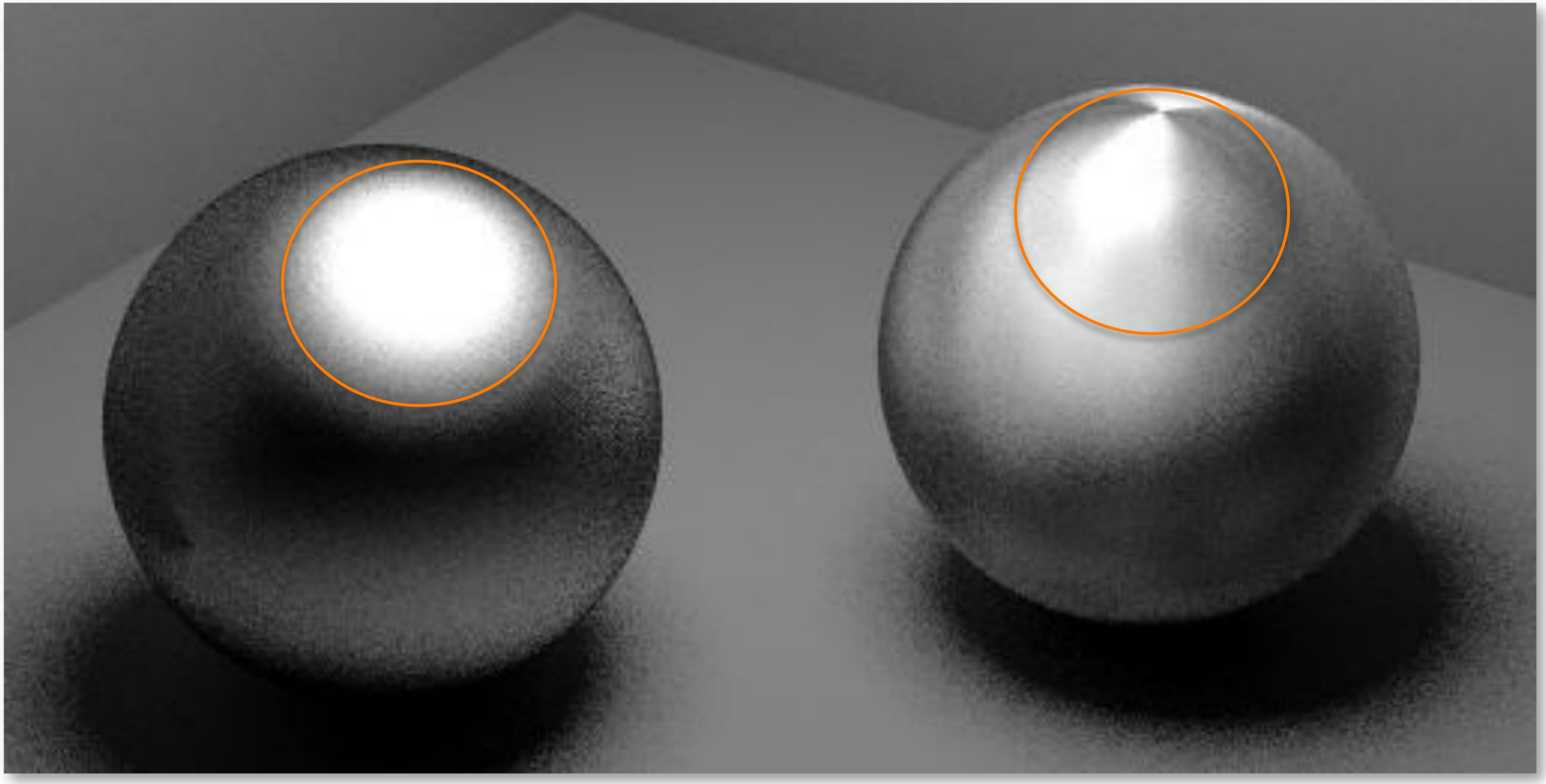
Example



Example

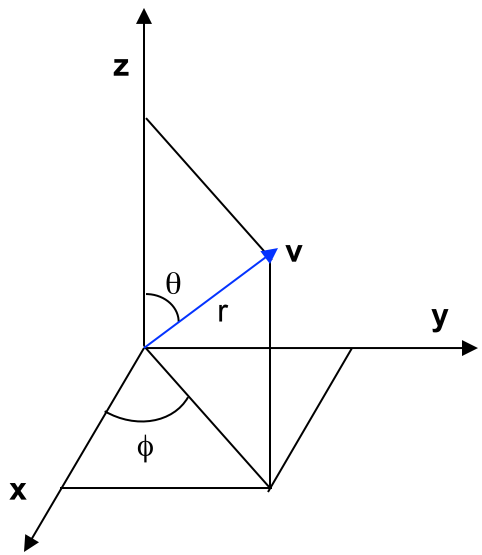


Example



BRDF

- Spherical Coordinates
  - Spherical coordinates are used to describe direction because a region of incoming light needs to be used rather then just a single point source.
  - Spherical coordinates use a vector of magnitude ‘r’ and a pair of angles θ and ϕ.



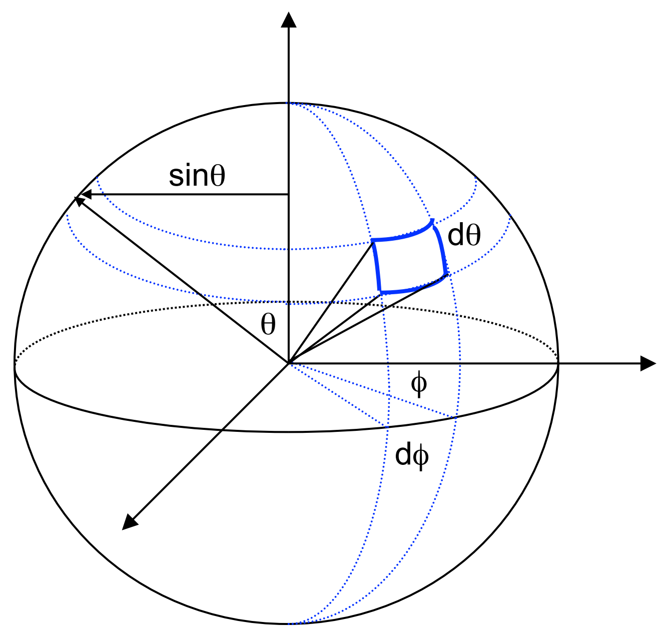
BRDF

- Differential Solid Angles
  - Light is measured in terms of **flow through an area**.
  - So that is why we refer to light in terms of being incident from a small region of directions, instead of from a single incoming direction.
  - We can obtain these regions by using a unit sphere. Where each small section of the surface is called a differential solid angle.

BRDF

- Differential Solid Angles
  - Given a direction in spherical coordinates, (θ,ϕ), and two small differential angular changes denoted as $d\theta$, $d\phi$, the differential solid angle would be:

$$d\omega = (\text{height})(\text{width})$$
$$d\omega = (d\theta)(\sin\theta * d\phi)$$
$$d\omega = \sin\theta * d\theta * d\phi$$

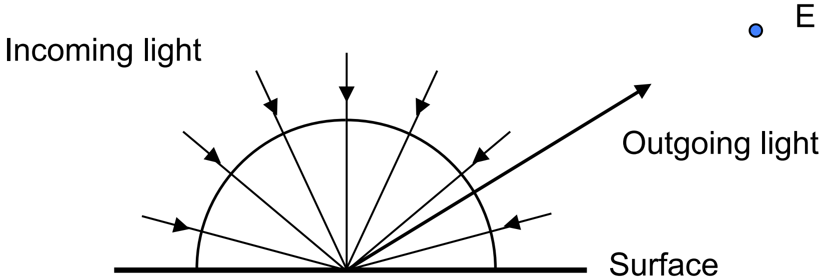


BRDF

- Differential Solid Angles
  - $d\omega$ represents a flat region on a unit sphere centered at the point of interest, in the direction (θ,ϕ).

BRDF

- Taking these differential solid angles into account, one can determine the amount of light coming in from the hemisphere above the surface, and thus can determine the amount of light reflected in a specific direction from a central point.



BRDF

- And so BRDF is:
  - the ratio of the reflected radiance (intensity) in one direction to the incident irradiance (flux density) responsible for it from another direction.
  - the ratio of the quantity of reflected light in outgoing direction to the amount of light that reaches the surface from incoming direction
  - a ratio of reflected light intensity to incident light intensity, divided by the projected solid angle.

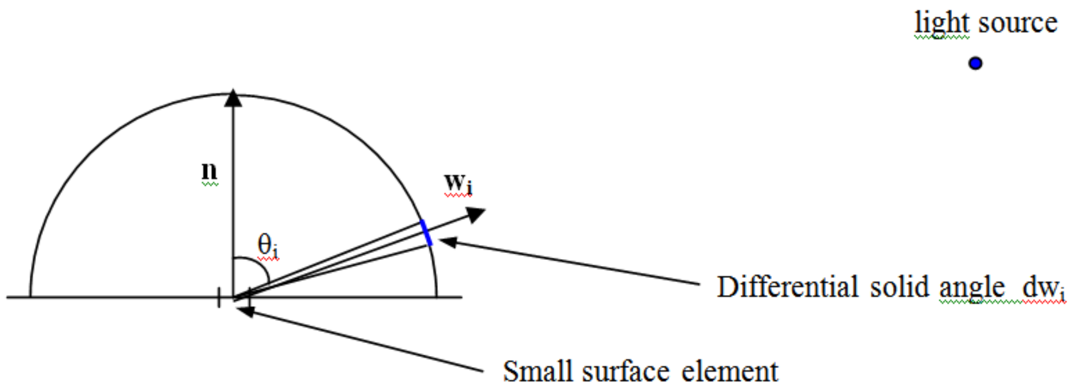
BRDF

$$BRDF = \frac{L_o}{E_i}$$

- Where:
  - L_o = the quantity of light reflected from the surface (the light radiance in $W/(sr.m^2)$)
  - E_i = the quantity of light arriving at the surface (the light irradiance in W/m^2)

- BRDF = (light out) / (light in)

BRDF



- Calculating E_i :
  - The incoming light intensity (radiance) is L_i
  - Since we are taking into account the region of incoming light, we use a solid angle $d\omega$.
  - The incoming light now becomes $L_i d\omega$.

BRDF

- Since the differential solid angle ($d\omega$) is a small area, it can be considered to be uniformly lit by the incoming light.
- This light intensity (radiance) is next received by the projected surface area (foreshortening).
- So, the light intensity on the surface itself (irradiance) can be calculated by multiplying incoming radiance energy with cos $\theta_{i.}$

BRDF

- That gives:

$$E_i = L_i \cos\theta_i d\omega_i$$

- Where:
  - E_i is the light irradiance on the surface
  - L_i is the light radiance from the incoming direction.
  - θ_i is the angle between the normal and the incoming direction.
  - $d\omega$ is the differential solid angle.

BRDF

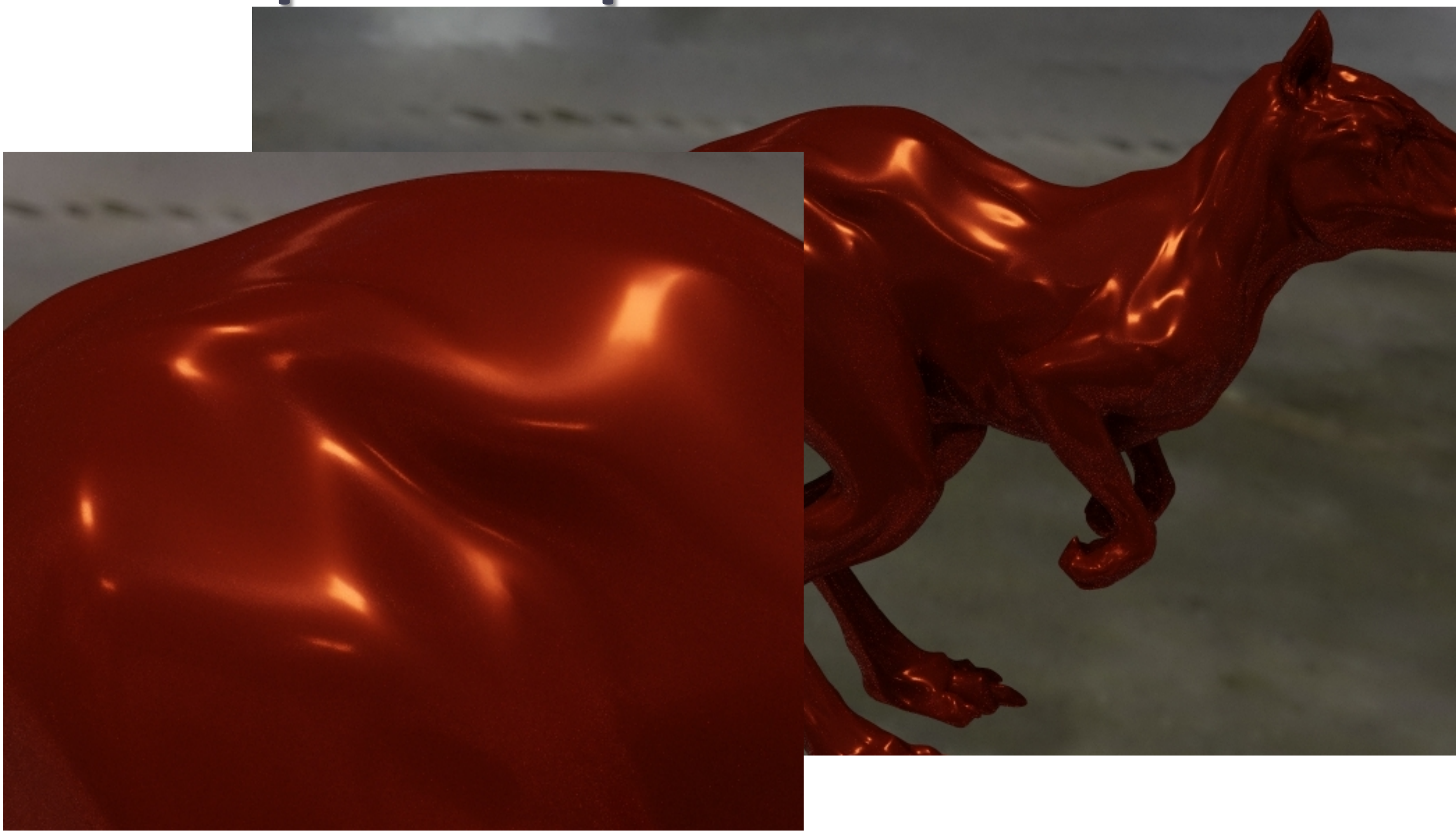
- And now the BRDF formula become:

$$BRDF = \frac{L_o}{L_i \cos\theta_i d\omega_i}$$

BRDF

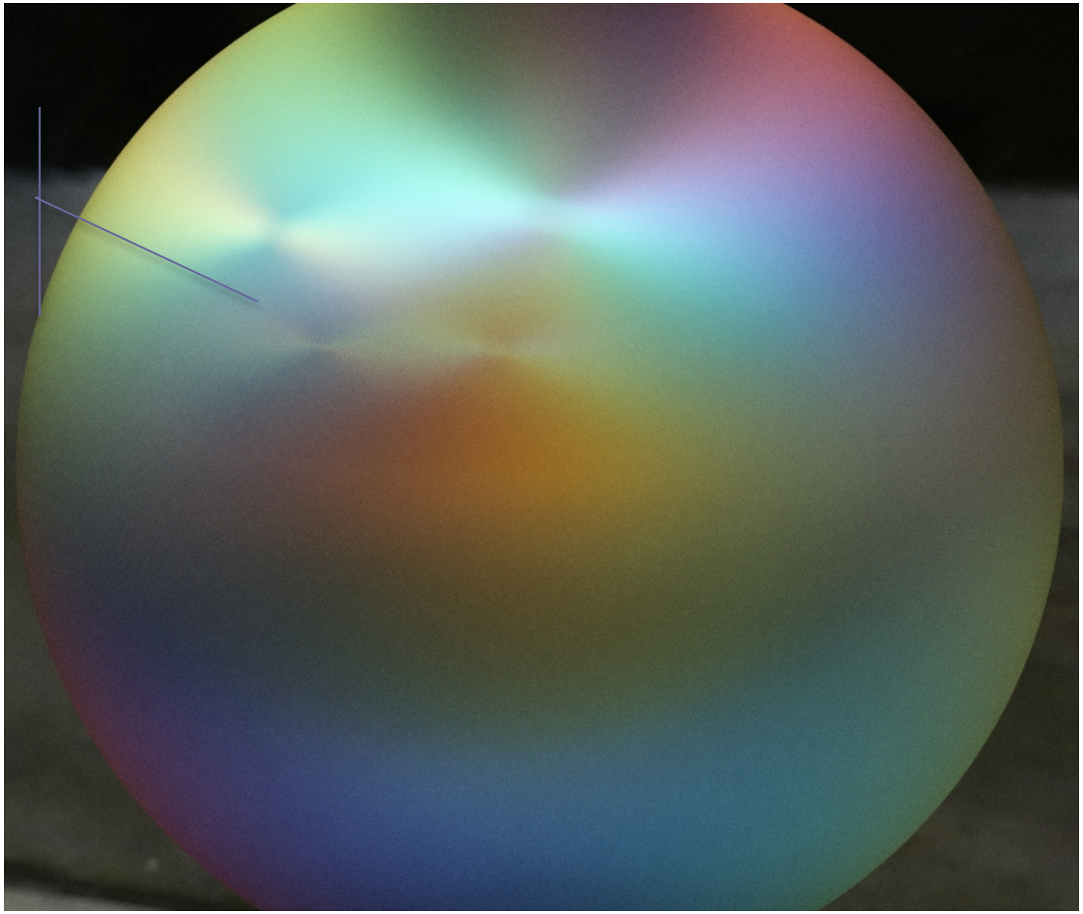
- Classes of BRDF
  - **Isotropic** – describes BRDFs that are invariant with respect to rotation of the surface around the surface normal vector. An example would be smooth plastics.
  - **Anisotropic** – describes BRDFs that do vary when rotated around their surface normal vector. An example would be stainless steel.

Isotropic Example

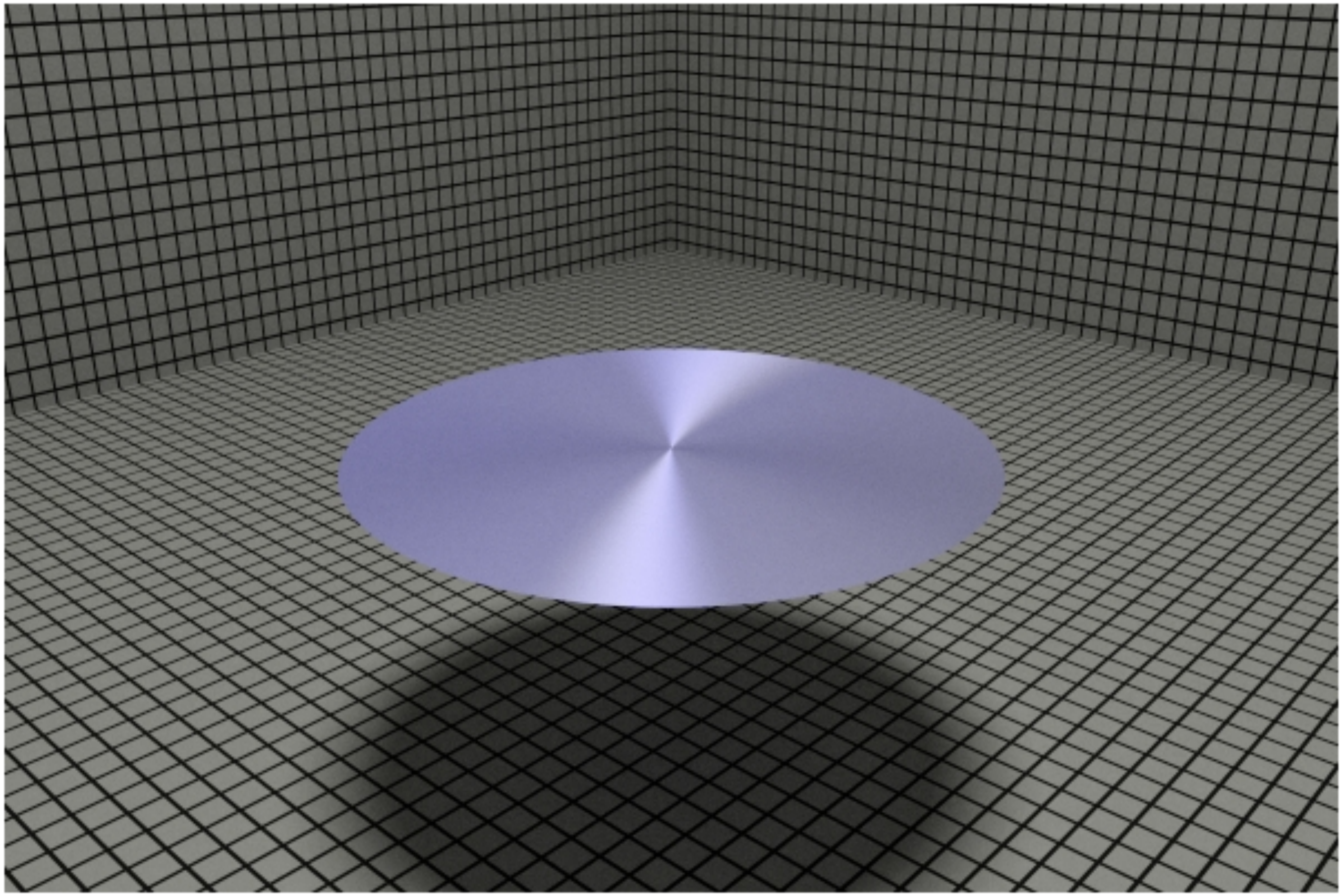


Anisotropic Example

Specularity changes with object rotation



More common example

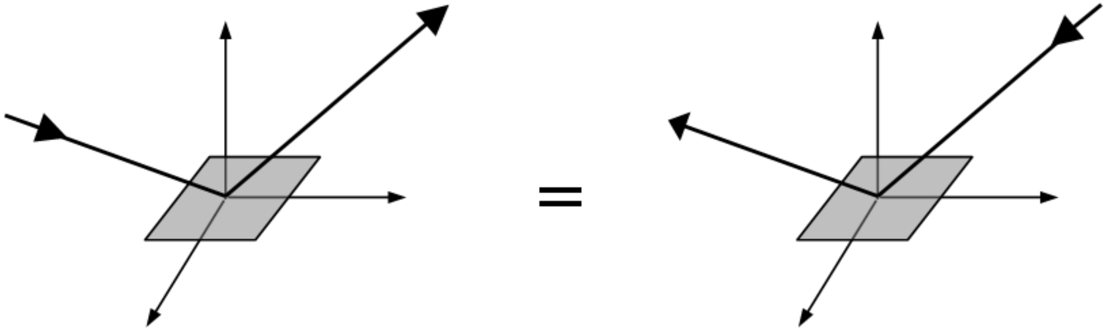


BRDF

- To be physically plausible in a real world environment, a BRDF must have two properties:
  - reciprocity
  - conservation of energy

BRDF - Reciprocity

- Reciprocity means that the incoming and outgoing light can be reversed and the BRDF will still hold true.



$$\mathrm{BRDF}(\theta_i, \phi_i, \theta_o, \phi_o) = \mathrm{BRDF}(\theta_o, \phi_o, \theta_i, \phi_i)$$

BRDF – Energy Conservation

- Conservation of energy means that the quantity of light reflected must be less than or equal to the quantity of incoming light.

Light Incident with the surface = Light Transmitted + Light Reflected + Light Absorbed

BRDF – Energy Conservation

- So, the sum over all outgoing directions of the BRDF times the projected solid angle must be less than or equal to one in order for the ratio of the total amount of reflected light to the incident light to be less than one.

$$\sum_{out} \mathrm{BRDF}(\theta_i, \phi_i, \theta_o, \phi_o) \cos\theta_o \, d\omega_o \leq 1$$

BRDF – Energy Conservation

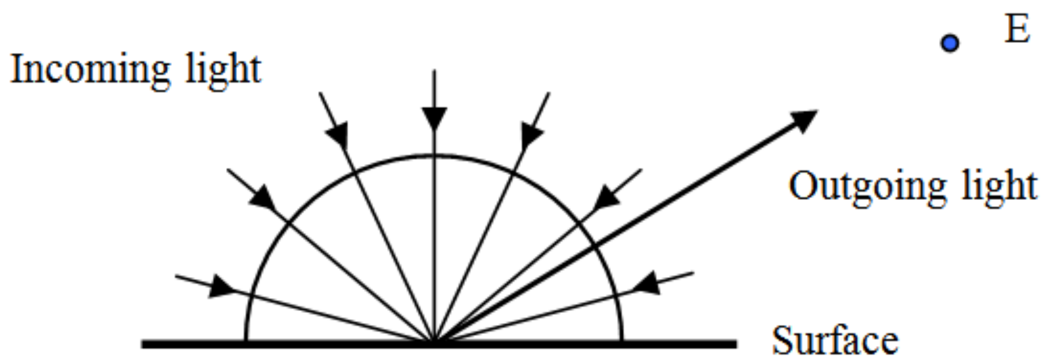
- When considering a continuous hemisphere, the integral of all directions must be less than or equal to one.

$$\int_{\Omega} BRDF(\theta_i, \phi_i, \theta_o, \phi_o) \cos\theta_o d\omega_o \leq 1$$

For all solid angles around the body

BRDF

- Light arriving from incoming directions contribute to the quantity of light reflected towards an observer.



- In continuous space that would look like:

$$L_o = \int_{\Omega} L_{o \text{ due to } i}(\omega_i, \omega_o) d\omega_i$$

BRDF

- To explain this discretely, we can take a finite number of incoming directions of light, and use this summation…

$$L_o = \sum_{in} L_{o \text{ due to } i}(\omega_i, \omega_o)$$

BRDF

- Looking back at this equation:

$$\mathrm{BRDF}(\theta_i, \phi_i, \theta_o, \phi_o) = \frac{Lo}{L_i \cos\theta_i d\omega_i}$$

- We see that:

$$Lo_{\text{ due to i}} = \mathrm{BRDF}(\theta_i, \phi_i, \theta_o, \phi_o) L_i \cos\theta_i d\omega_i$$

BRDF

- And so we obtain the general BRDF equation

$$L_o = \sum_{in} BRDF(\theta_i, \phi_i, \theta_o, \phi_o) L_i \cos\theta_i$$

BRDF

- Phong

$$L_o = L_i\left(k_d(\vec{L}\cdot\vec{N}) + k_s(\vec{R}\cdot\vec{V})^n\right)$$

BRDF

- Phong

$$L_o = L_i\left(k_d(\vec{L}\cdot\vec{N}) + k_s(\vec{R}\cdot\vec{V})^n\right)$$

$$L_o = L_i Refl(\vec{L},\vec{V})$$

BRDF

- Phong

$$L_o = L_i\left(k_d(\vec{L}\cdot\vec{N}) + k_s(\vec{R}\cdot\vec{V})^n\right)$$

$$L_o = L_i Refl\left(\vec{L},\vec{V}\right)$$

$$L_o = \frac{\cos\theta_i\, dw_i}{\cos\theta_i\, dw_i} Refl(\overrightarrow{w_i},\overrightarrow{w_o})L_i$$

BRDF

- Phong

$$L_o = L_i\left(k_d(\vec{L}\cdot\vec{N}) + k_s(\vec{R}\cdot\vec{V})^n\right)$$

$$L_o = L_i Refl\left(\vec{L},\vec{V}\right)$$

$$L_o = \frac{\cos\theta_i\, dw_i}{\cos\theta_i\, dw_i} Refl(\overrightarrow{w_\iota},\overrightarrow{w_o}) L_i$$

$$L_o = \frac{Refl(\overrightarrow{w_\iota},\overrightarrow{w_o})}{\cos\theta_i\, dw_i} L_i \cos\theta_i\, dw_i$$

BRDF

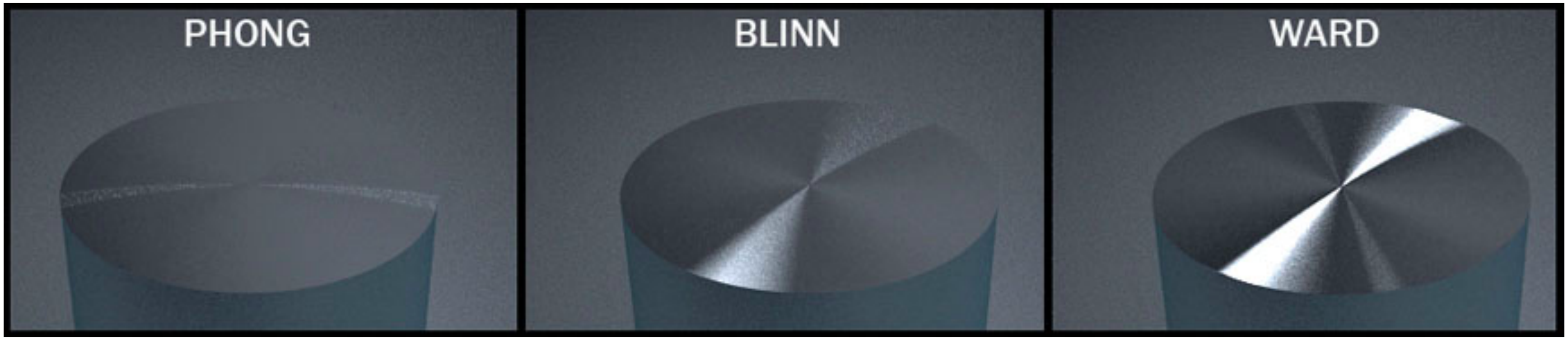
- Phong

$$BRDF(\theta_i, \phi_i, \theta_o, \phi_o) = \frac{Refl(\overrightarrow{w_\iota}, \overrightarrow{w_o})}{\cos\theta_i \, dw_i}$$

$$BRDF(\theta_i, \phi_i, \theta_o, \phi_o) = \frac{k_d(\overrightarrow{w_\iota} \cdot \vec{N}) + k_s(\vec{R} \cdot \overrightarrow{w_o})^n}{\cos\theta_i \, dw_i}$$

- Phong can be described as a particular type of BRDF. But the two physical properties of BRDF do not hold with Phong. No energy conservation nor reciprocity.

Shading Models as BRDF



BRDF

- To obtain the BRDF data, there are two common approaches:
  - Evaluate mathematically, functions derived from analytical models.
    - See Phong BRDF on previous slides
  - Resample BRDF data acquired from empirical measurements of real-world surfaces.

BRDF

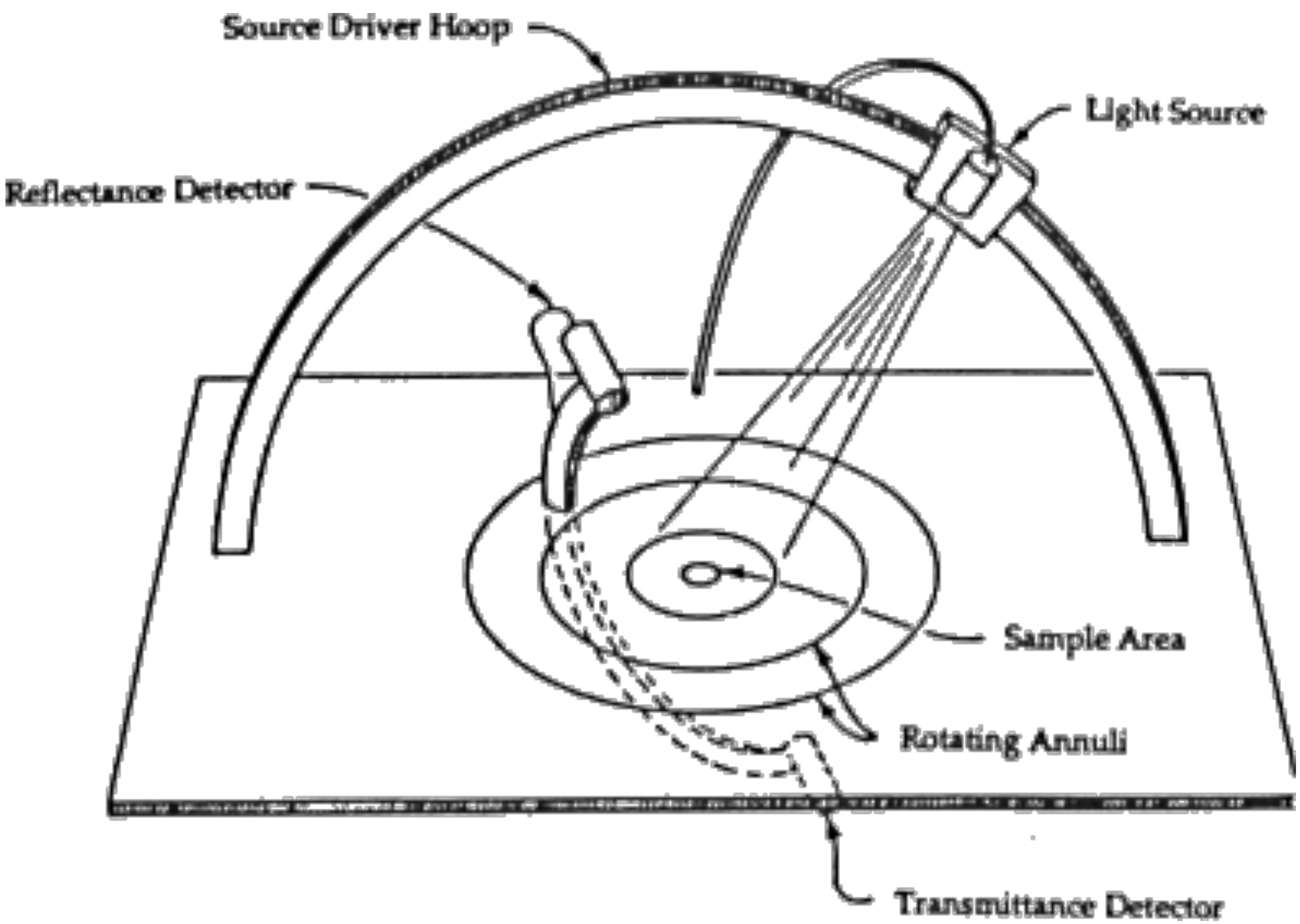
- The analytical models can be thought of as simple functions where you input the data required.
  - Direction vectors (θ_i, ϕ_i) and (θ_o, ϕ_o).
  - Other values that control reflectance properties

- The functions then compute the BRDF for different wavelengths based on that input.

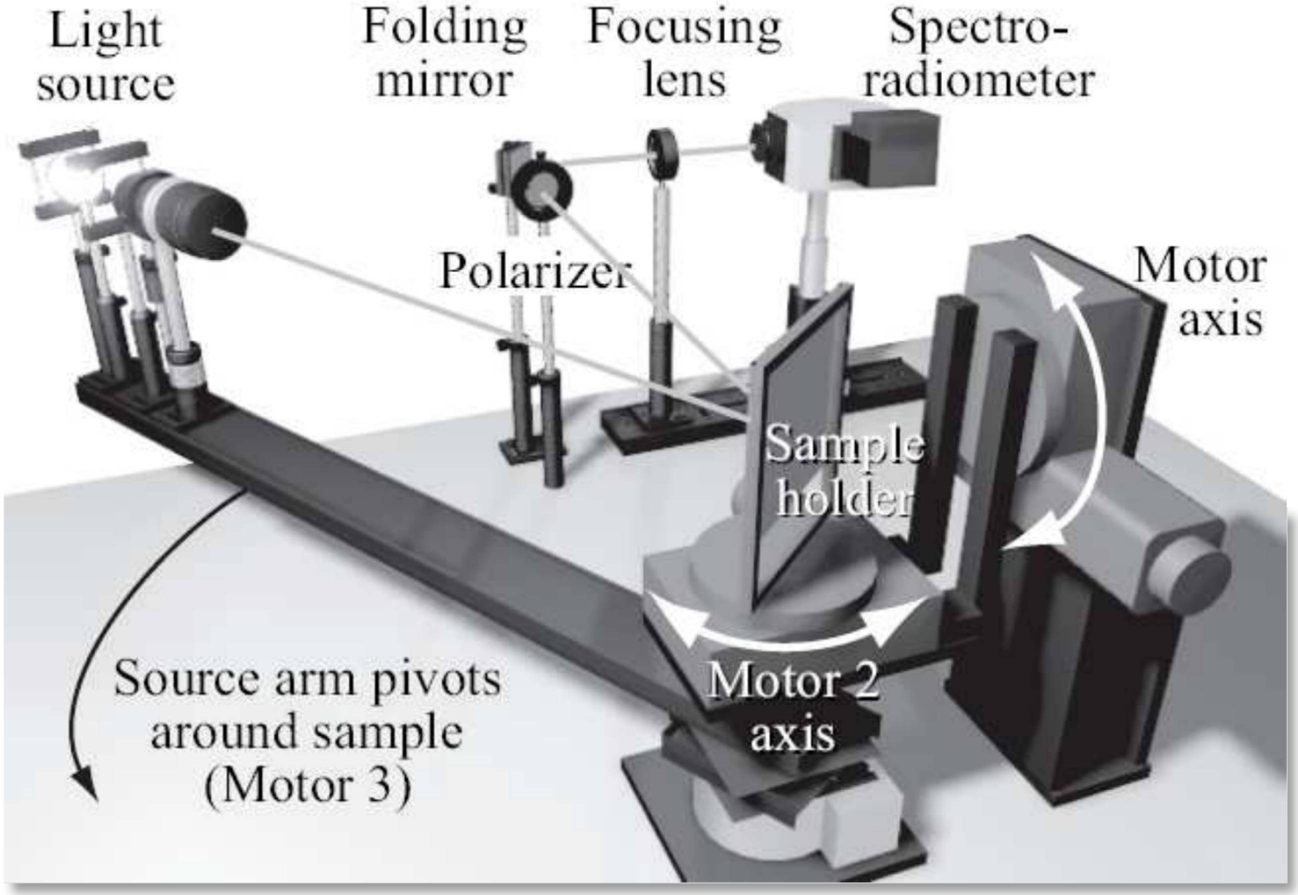
BRDF

- BRDF of real life surfaces can be examined and measured using a *gonioreflectometer* (a device that obtains the BRDF data of a surface).
- Data produced in this way is often referred to as acquired BRDF data.

Gonioreflectometer



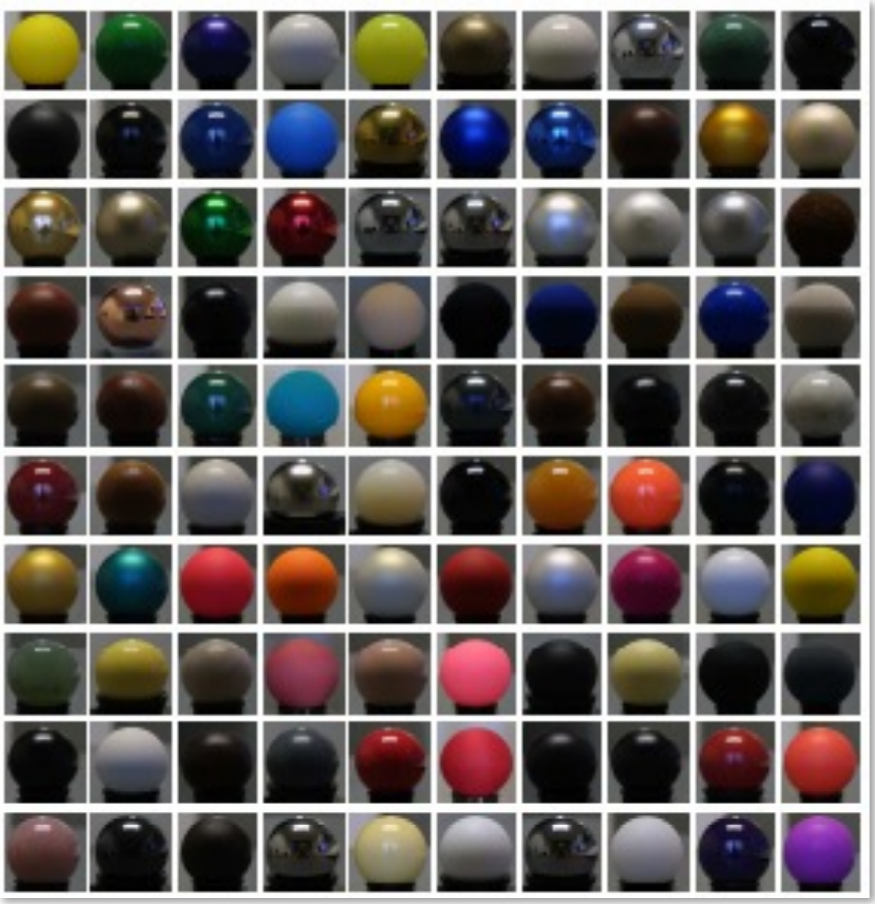
Cornell Set-up (www.graphics.cornell.edu/~westin/)



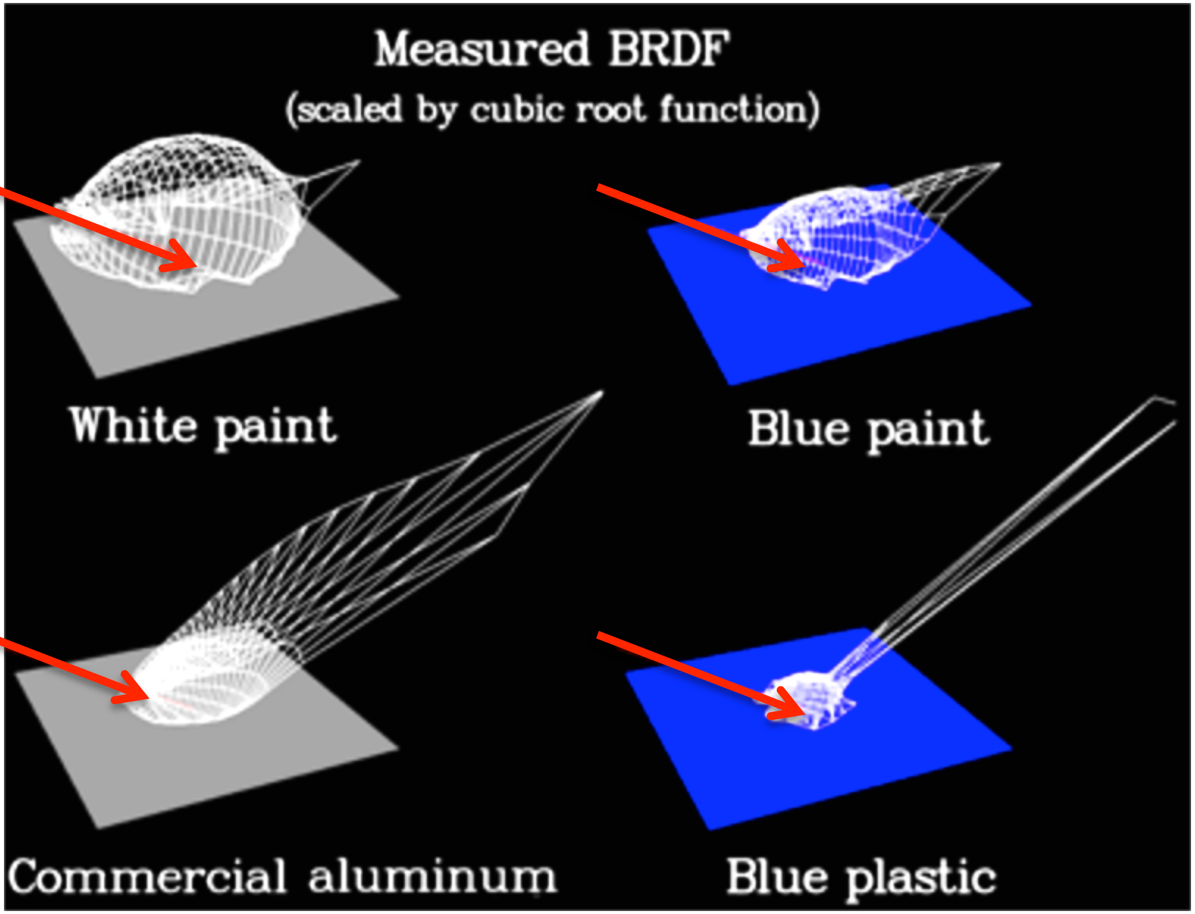
BRDF

- There is also the method of obtaining pre-calculated or computed BRDF data from others.
- Several academic institutions as well as a few commercial companies offer libraries of measured BRDF data at little or no cost.
- Additionally, some companies will actually measure specific data to meet individual customer needs.

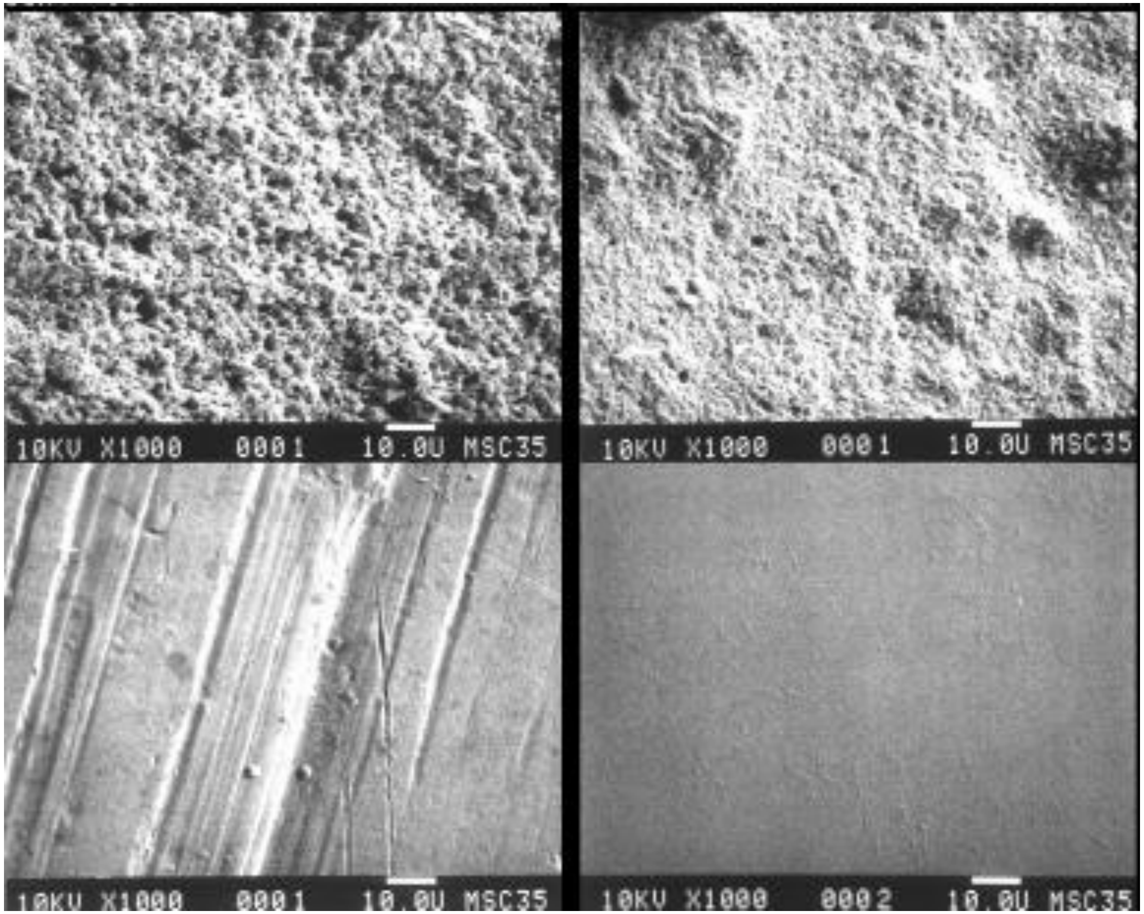
MERL Database



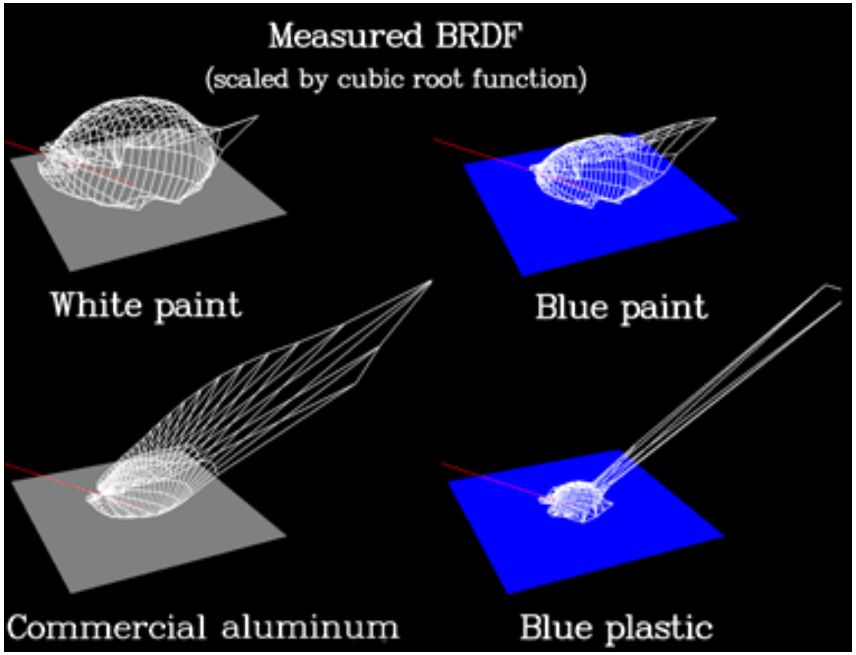
What does the BRDF look like?

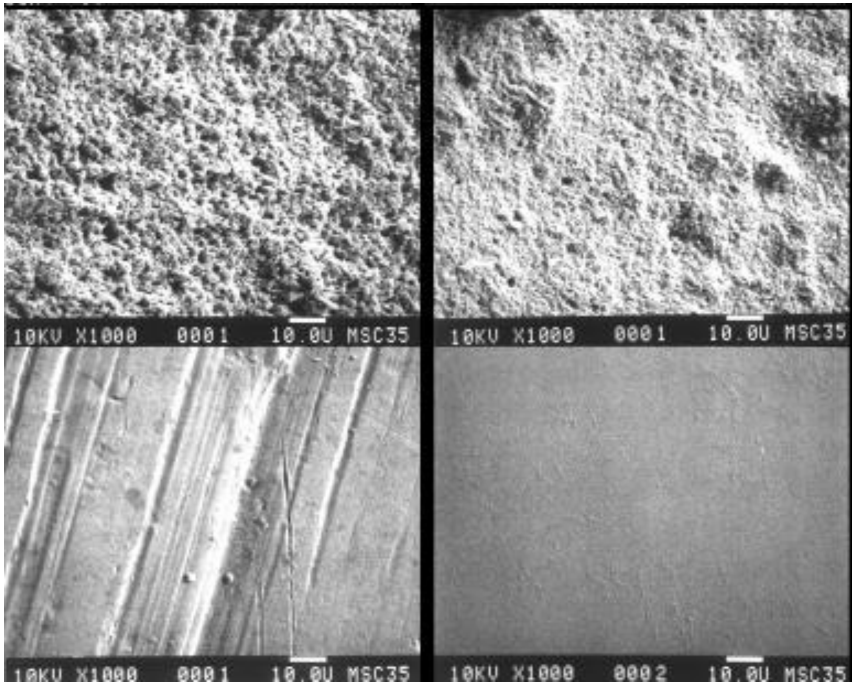


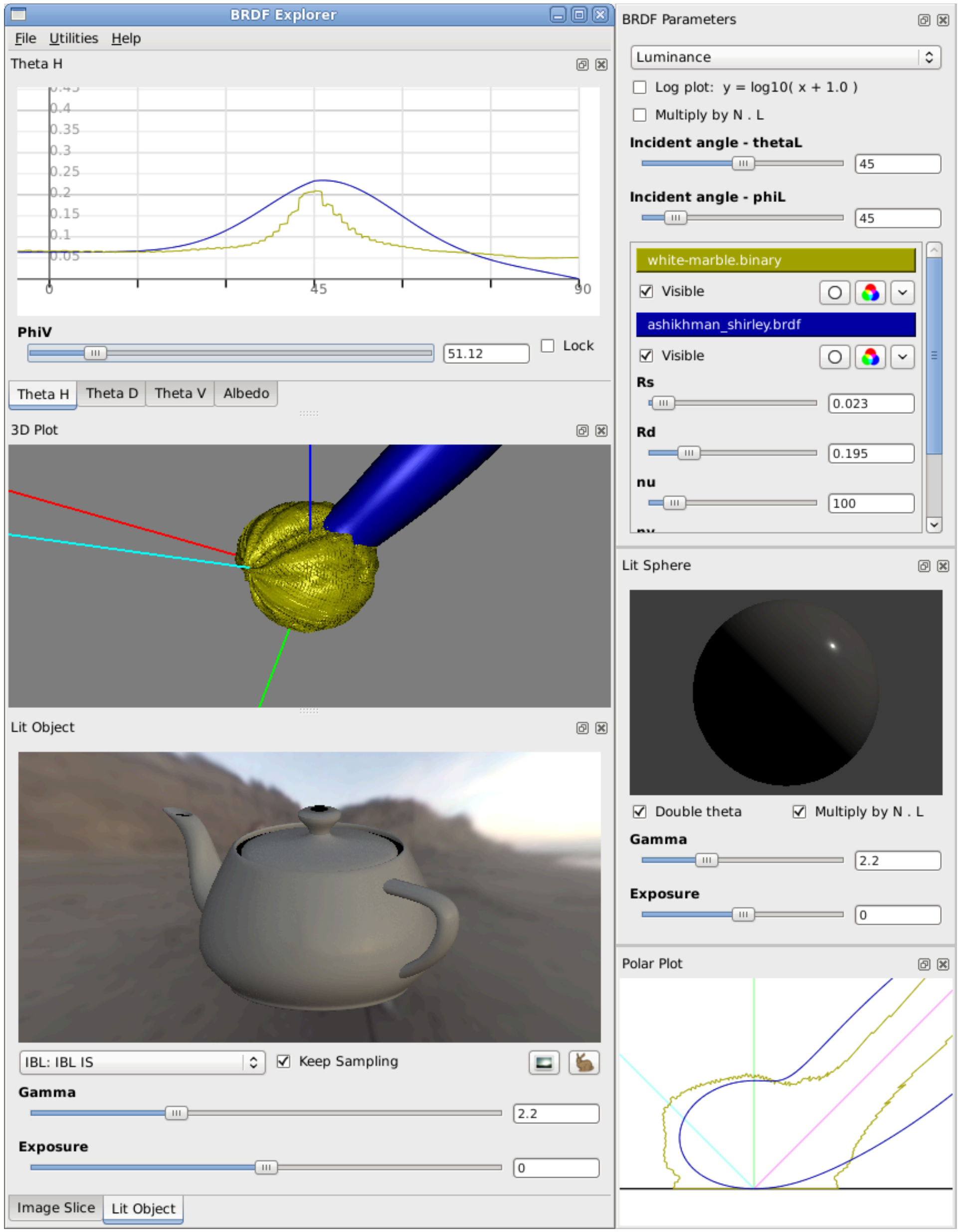
Actual surfaces …



Comparision







BRDF Editor

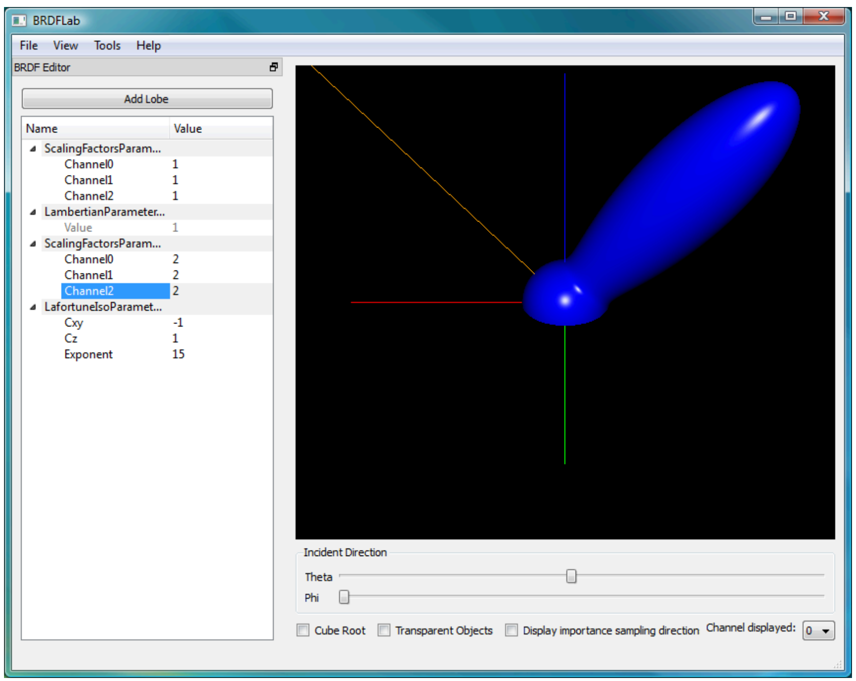
Disney Animation

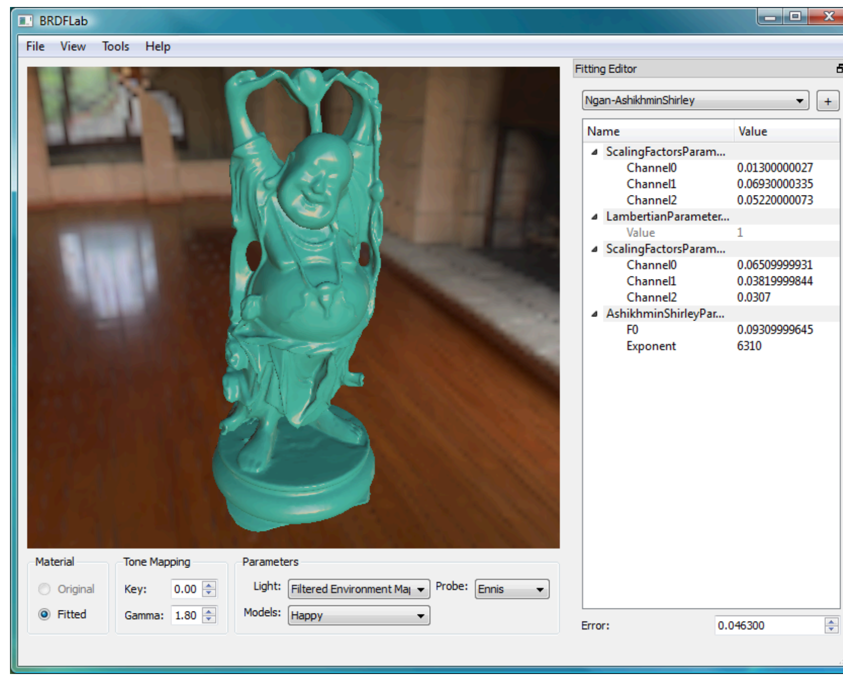
BRDF Explorer

(https://www.disneyanimation.com/technology/brdf.html)

BRDFLab (Open Source)

http://brdflab.sourceforge.net/





Generality of BRDFs

- BRDFs can simulate diffuse behavior for “rough” surfaces and specular behavior for “smooth” surfaces

GPU BRDF Implementation – GPU Gems (Chapter 18)

References

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