Sample Problem #1

$A = (-5, 0)$ $\quad K_a = 0.3$ $\quad I_a = (0, 0, 0)$

$L = (-8, 10)$ $\quad K_d = 0.3$ $\quad I_d = (255, 0, 0)$

$C = (3, 7)$ $\quad K_s = 0.3$ $\quad I_s = (255, 255, 255)$

$\bar{N} = (0, 1)$ $\quad ns = 10.0$

* Local Phong Model : No attenuation or fog

① Light Vector, $\bar{L} = L - A = \begin{bmatrix} -8 \\ 10 \end{bmatrix} - \begin{bmatrix} -5 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 10 \end{bmatrix}$

$|\bar{L}| = 10.440$

$\therefore \hat{L} = \begin{bmatrix} -0.287 \\ 0.957 \end{bmatrix}$

② View vector : $\bar{V} = C - A = \begin{bmatrix} 3 \\ 7 \end{bmatrix} - \begin{bmatrix} -5 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \end{bmatrix}$

$|\bar{V}| = 10.630 \quad \therefore \hat{V} = \begin{bmatrix} 0.753 \\ 0.659 \end{bmatrix}$

③ $\hat{N} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \hat{N} \cdot \hat{L} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -0.287 \\ 0.957 \end{bmatrix} = 0.957$

$\hat{R} = 2(\hat{N} \cdot \hat{L})\hat{N} - \hat{L} = 2(0.957)\begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} -0.287 \\ 0.957 \end{bmatrix}$

$\therefore \hat{R} = \begin{bmatrix} 0.287 \\ 0.957 \end{bmatrix}$

④ $I_{ambient} = I_a K_a$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \cdot 0.3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

⑤ $I_{diffuse} = I_d \cdot K_d \cdot \max(\hat{N} \cdot \hat{L}, 0)$

$$= \begin{bmatrix} 255 \\ 0 \\ 0 \end{bmatrix} \cdot (0.3)\ (0.957) = \begin{bmatrix} 73.211 \\ 0 \\ 0 \end{bmatrix}$$

⑥ $\hat{R} \cdot \hat{V} = \begin{bmatrix} 0.287 \\ 0.957 \end{bmatrix} \cdot \begin{bmatrix} 0.753 \\ 0.659 \end{bmatrix} = 0.847$

$$I_{specular} = I_s \cdot K_s \max(\hat{R} \cdot \hat{V}, 0)^{ns}$$

$$= \begin{bmatrix} 255 \\ 255 \\ 255 \end{bmatrix} (0.3)\ (0.847)^{10} = \begin{bmatrix} 14.499 \\ 14.499 \\ 14.499 \end{bmatrix}$$

$$\therefore I_{local} = I_{ambient} + I_{diffuse} + I_{specular}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 73.211 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 14.499 \\ 14.499 \\ 14.499 \end{bmatrix}$$

$$I_{local} = \begin{bmatrix} 87.710 \\ 14.499 \\ 14.499 \end{bmatrix} \rightarrow \begin{bmatrix} 88 \\ 15 \\ 15 \end{bmatrix}$$

□

Now let us add distance attenuation

Let $c_1 = 1$, $c_2 = 0.5$, $c_3 = 1.2$ (No fog)

$$Att = \min\left(\frac{1}{c_1 + c_2 d_c + c_3 d_c^2}, 1.0\right)$$

$$d_c = |\bar{C}| = 10.490$$

$$Att = \frac{1}{137.012} = 0.007$$

$$\therefore I_{local} = \begin{bmatrix} 87.710 \\ 14.499 \\ 14.499 \end{bmatrix} (0.007) = \begin{bmatrix} 0.640 \\ 0.101 \\ 0.101 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Now, fog included, no distance attenuation

$$I_{Fog} = \begin{bmatrix} 127 \\ 127 \\ 127 \end{bmatrix}, \quad z^{Near} = 2, \quad z^{Far} = 20$$

$$\therefore S = \frac{z^{Far} - |\bar{v}|}{z^{Far} - z^{Near}} = \frac{20 - 10.630}{20 - 2} = 0.521$$

$$\therefore I'_{local} = (0.521)\begin{bmatrix} 87.710 \\ 14.499 \\ 14.499 \end{bmatrix} + (0.479)\begin{bmatrix} 127 \\ 127 \\ 127 \end{bmatrix} = \begin{bmatrix} 106.53 \\ 68.387 \\ 68.387 \end{bmatrix}$$

$$\therefore I'_{local} = \begin{bmatrix} 107 \\ 68 \\ 68 \end{bmatrix}$$

Now consider both attenuation & fog.

$$I_{local} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{post-attenuation}$$

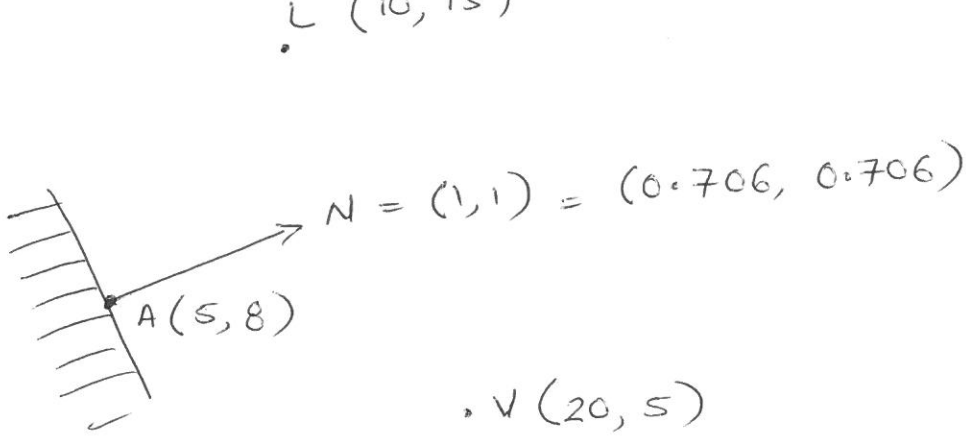
$$\therefore I'_{local} = (0.521)\begin{bmatrix} 0.640 \\ 0.101 \\ 0.101 \end{bmatrix} + (0.479)\begin{bmatrix} 127 \\ 127 \\ 127 \end{bmatrix}$$

$$= \begin{bmatrix} 61.166 \\ 60.886 \\ 60.886 \end{bmatrix} \rightarrow \begin{bmatrix} 61 \\ 61 \\ 61 \end{bmatrix}.$$

Sample Problems

Phong Illumination Model

(1)



Light parameters

$I_d = (255, 255, 255)$

$I_s = (255, 0, 255)$

$I_a = (100, 100, 100)$

$I_e = (0, 0, 0)$

Material Properties

$K_d = 0.5$

$K_s = 0.75$

$K_a = 0.25$

$n_s = 10.0$

near = 2

far = 10

$d_L = 0$ (No distance attenuation)

spotlight = 1

$I_{FOG} = \begin{bmatrix} 127 \\ 127 \\ 127 \end{bmatrix}$

① Light vector, $\hat{L} = \text{norm}(10-5, 15-8) = \text{norm}(5, 7)$

magnitude = 8.602

$\therefore \hat{L} = (0.581, 0.813)$

② View vector: $\hat{V} = \text{norm}(20-5, 5-8) = \text{norm}(15, -3)$

$\hat{V} = (0.980, 0.196)$

(3) Reflection Vector, $R = 2(\hat{N}\cdot\hat{L})\hat{N} - \hat{L}$

$\hat{N}\cdot\hat{L} = (1,1)\cdot(0.581, 0.813) =$

0.581
+0.813
1.394

$= (0.706, 0.706)\cdot(0.581, 0.813)$

$= 0.984$

$$R = 2(0.984)\begin{pmatrix}0.706\\0.706\end{pmatrix} - \begin{pmatrix}0.581\\0.813\end{pmatrix}$$

$$= \begin{pmatrix}1.389 - 0.581\\1.389 - 0.813\end{pmatrix} = \begin{pmatrix}0.808\\0.576\end{pmatrix} \qquad |R| = 0.984$$

$$\hat{R} = \begin{pmatrix}0.821\\0.585\end{pmatrix}$$

(4)

$I_{diffuse} = I_d K_d \max(\hat{N}\cdot\hat{L}, 0)$

$$= \begin{bmatrix}255\\255\\255\end{bmatrix}(0.5)(0.984) = \begin{bmatrix}125.460\\125.460\\125.460\end{bmatrix}$$

(5) $I_{ambient} = I_a K_a = \begin{bmatrix}100\\100\\100\end{bmatrix}(0.25) = \begin{bmatrix}25.00\\25.00\\25.00\end{bmatrix}$

(6) $I_{specular} = I_s K_s \max(\hat{R}\cdot\hat{V}, 0)^{ns}$

$$= \begin{bmatrix}255\\0\\255\end{bmatrix}(0.75)(0.919)^{10} = \begin{bmatrix}82.178\\0\\82.178\end{bmatrix}$$

$$\underline{I}_{total} = \underline{I}_{diffuse} + \underline{I}_{ambient} + \underline{I}_{specular}$$

$$= \begin{bmatrix} 125.460 \\ 125.460 \\ 125.460 \end{bmatrix} + \begin{bmatrix} 25.00 \\ 25.00 \\ 25.00 \end{bmatrix} + \begin{bmatrix} 82.178 \\ 0 \\ 82.178 \end{bmatrix} = \begin{bmatrix} 232.638 \\ 150.460 \\ 232.638 \end{bmatrix}$$

$$S = \frac{z - z_{near}}{z_{far} - z_{near}} = \frac{15.297 - 2}{10 - 2} = \frac{13.297}{8} = 1.66$$

Since $S > 1$, we clamp it to $\underline{1}$

$$\therefore \underline{I}_{final} = \begin{bmatrix} 127 \\ 127 \\ 127 \end{bmatrix}$$

Midterm Solutions

Phong Illumination I

A (-10, 0)

Light (-18, 10)

C (5, 7)

N (0, 1)

$K_a = 0.3$

$K_d = 0.3$

$K_s = 0.3$

$n_s = 10.0$

$I_e = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$I_d = \begin{bmatrix} 255 \\ 0 \\ 0 \end{bmatrix}$

$I_s = \begin{bmatrix} 255 \\ 255 \\ 255 \end{bmatrix}$

$I_a = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$Fog = \begin{bmatrix} 127 \\ 127 \\ 127 \end{bmatrix}$

$F_{near} = 2$

$F_{far} = 20$

$$\bar{L} = Light - A = \begin{bmatrix} -18 \\ 10 \end{bmatrix} - \begin{bmatrix} -10 \\ 0 \end{bmatrix} = \begin{bmatrix} -8 \\ 10 \end{bmatrix} \quad |\bar{L}| = 12.806$$

$$\therefore \hat{L} = \begin{bmatrix} -0.625 \\ 0.781 \end{bmatrix}$$

$$\bar{V} = C - A = \begin{bmatrix} 5 \\ 7 \end{bmatrix} - \begin{bmatrix} -10 \\ 0 \end{bmatrix} = \begin{bmatrix} 15 \\ 7 \end{bmatrix} \quad |\bar{V}| = 16.553$$

$$\therefore \hat{V} = \begin{bmatrix} 0.906 \\ 0.423 \end{bmatrix}$$

$$\hat{R} = 2(\hat{N}\cdot\hat{L})\hat{N} - \hat{L}$$

$$= 2\left(\begin{bmatrix}0\\1\end{bmatrix}\cdot\begin{bmatrix}-0.625\\0.781\end{bmatrix}\right)\begin{bmatrix}0\\1\end{bmatrix} - \begin{bmatrix}-0.625\\0.781\end{bmatrix}$$

$$= \begin{bmatrix}0.625\\0.781\end{bmatrix}$$

$$I_{ambient} = I_a K_a = \begin{bmatrix}0\\0\\0\end{bmatrix}(0.3) = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$I_{diffuse} = I_d \cdot K_d \cdot \max(\hat{N}\cdot\hat{L}, 0)$$

$$= \begin{bmatrix}255\\0\\0\end{bmatrix}(0.3)\left[\begin{bmatrix}0\\1\end{bmatrix}\cdot\begin{bmatrix}-0.625\\0.781\end{bmatrix}\right]$$

$$= \begin{bmatrix}59.747\\0\\0\end{bmatrix}$$

$$I_{specular} = I_s \cdot K_s \cdot \max(\hat{R}\cdot\hat{V}, 0)^{ns}$$

$$= \begin{bmatrix}255\\255\\255\end{bmatrix}(0.3)\left(\begin{bmatrix}0.625\\0.781\end{bmatrix}\begin{bmatrix}0.906\\0.423\end{bmatrix}\right)^{10}$$

$$= \begin{bmatrix}25.687\\25.687\\25.687\end{bmatrix}$$

$$I_{TOTAL} = I_{ambient} + I_{diffuse} + I_{specular} + I_{emissive}.$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 59.747 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 25.687 \\ 25.687 \\ 25.687 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 85.434 \\ 25.687 \\ 25.687 \end{bmatrix}$$

$$S = \frac{F_{far} - |\bar{v}|}{F_{far} - F_{near}} = \frac{20 - 16.553}{20 - 2} = 0.192$$

$$I_{final} = S \cdot I_{TOTAL} + (1 - S) I_{Fog}$$

$$= (0.192) \begin{bmatrix} 85.434 \\ 25.687 \\ 25.687 \end{bmatrix} + (0.809) \begin{bmatrix} 127 \\ 127 \\ 127 \end{bmatrix}$$

$$= \begin{bmatrix} 119.083 \\ 107.611 \\ 107.611 \end{bmatrix}$$