# CS330: Visualizing Recurrence Relations

Given a recursive function $T(n)$:

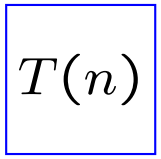
$T(n) = 2T\left(\frac{n}{2}\right) + n\ , T(1) = 1\ , n\ \epsilon\ 2\mathbb{Z}_+$ (n is even)

We can attempt to visualize $T(n)$ by unrolling the recurrence:

$$
\begin{aligned}
T(n) &= 2T\left(\frac{n}{2}\right) + n \\
&= 2\left(2T\left(\frac{n}{4}\right) + \frac{n}{2}\right) + n \\
&= 2\left(2\left(2T\left(\frac{n}{8}\right) + \frac{n}{4}\right) + \frac{n}{2}\right) + n \\
&\vdots \\
&= 2\left(2\left(\ldots 2\left(2T(1) + 2\right) + 4\right)\ldots + \frac{n}{2}\right) + n
\end{aligned}
$$

Alternatively we can use a tree to visualize the recurrence relation:

[0]



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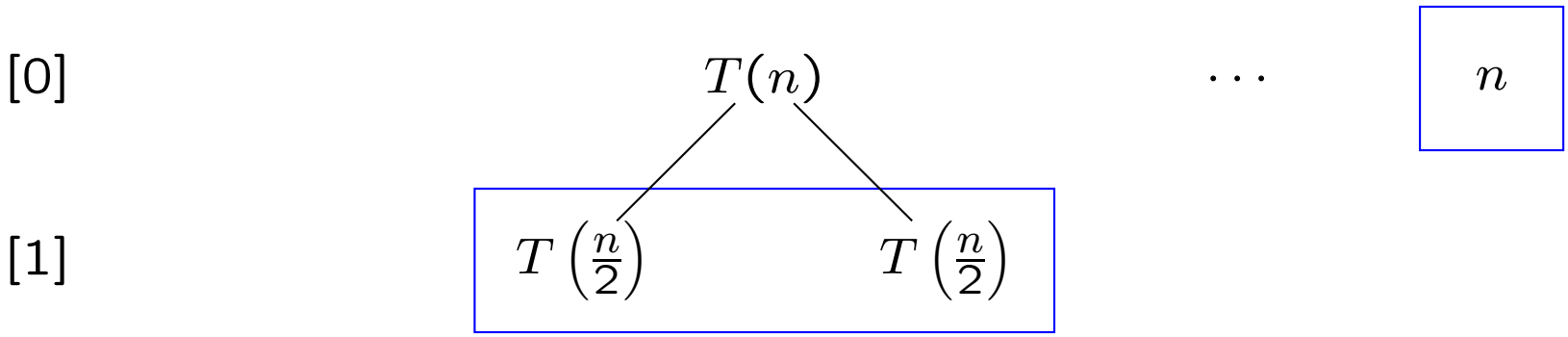

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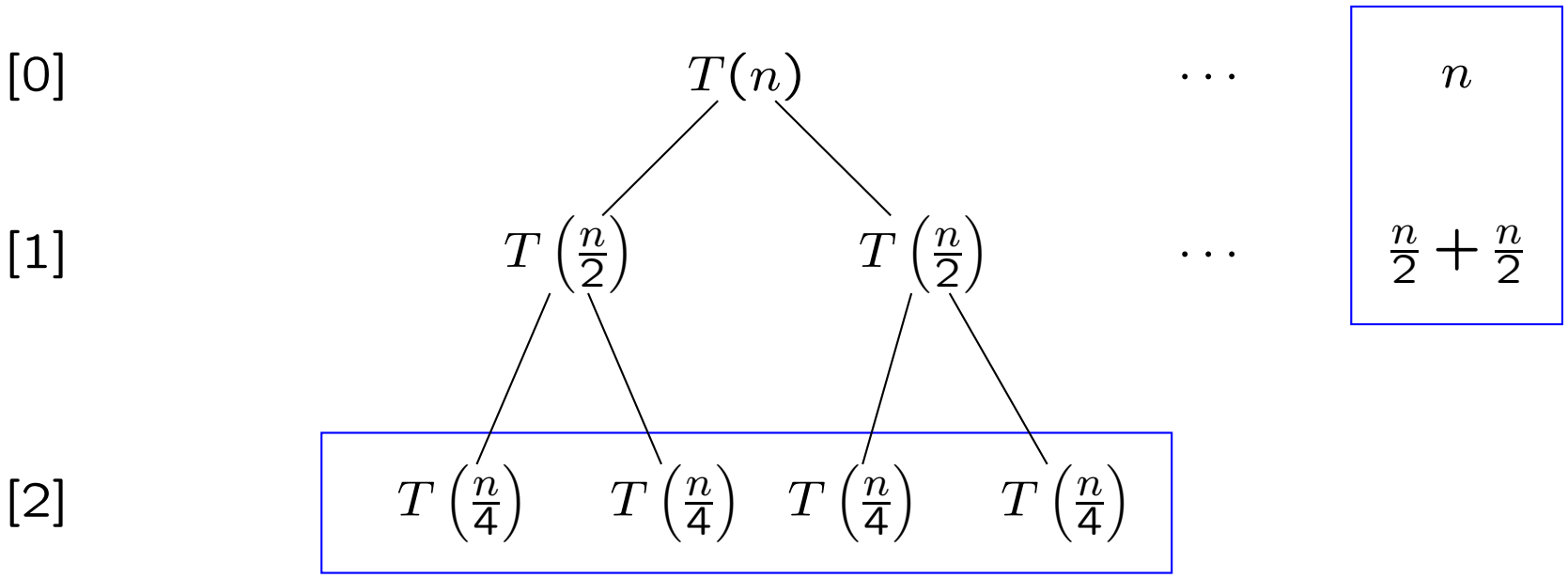

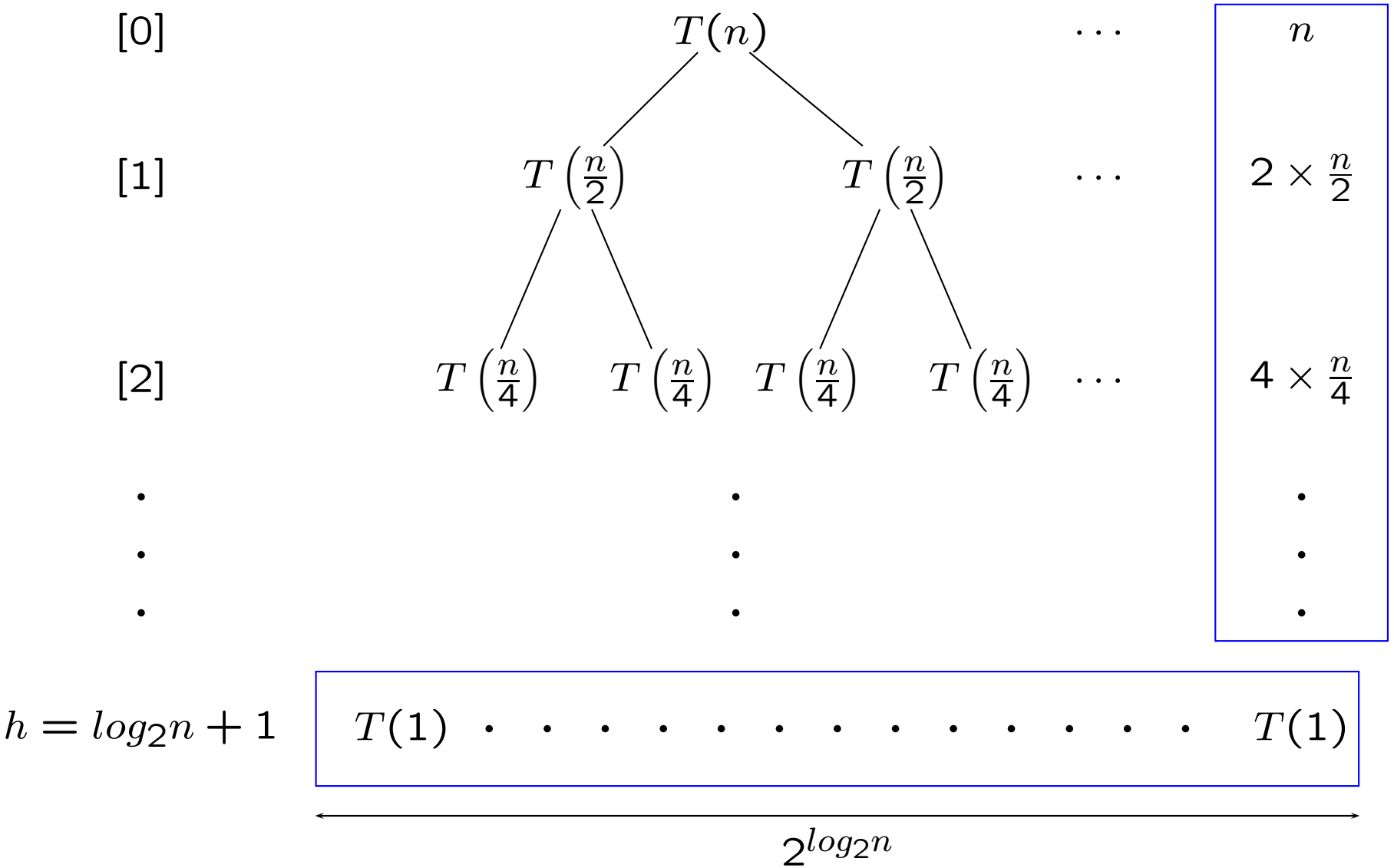
[0]
$T(n)$
$\cdots$
$n$
[1]
$T\left(\frac{n}{2}\right)$
$T\left(\frac{n}{2}\right)$
$\cdots$
$2 \times \frac{n}{2}$
[2]
$T\left(\frac{n}{4}\right)$
$T\left(\frac{n}{4}\right)$
$T\left(\frac{n}{4}\right)$
$T\left(\frac{n}{4}\right)$
$\cdots$
$4 \times \frac{n}{4}$
$h = log_2 n + 1$
$T(1)$ $T(1)$
$2^{log_2 n}$