1 Correctness of recursive Algorithms

1.1 Sum of n

```
Sum(n)
{
        if n==1 then return 1
        else return (Sum(n-1)+n)
}
```

Prove that $Sum(n) = \frac{n(n+1)}{2}$

Claim

Sum(n) returns a value equals to $\frac{n(n+1)}{2}$

Base Case

Sum(n) = $\frac{n(n+1)}{2}$ if n=1
$1 = \frac{1(1+1)}{2}$
$1 = 1$
True

Induction Hypothesis

Sum(n) returns a value equals to $\frac{n(n+1)}{2}$

Induction Conclusion

$$Sum(n+1) = \frac{(n+1)((n+1)+1)}{2} \tag{1}$$
$$Sum((n+1)-1)+(n+1) = \frac{(n+1)(n+2)}{2} \tag{2}$$
$$\frac{n(n+1)}{2} + (n+1) = \frac{(n+1)(n+2)}{2} \tag{3}$$
$$\frac{n(n+1)+2(n+1)}{2} = \frac{(n+1)(n+2)}{2} \tag{4}$$
$$\frac{n^2+n+2n+2}{2} = \frac{(n+1)(n+2)}{2} \tag{5}$$
$$\frac{n^2+3n+2}{2} = \frac{(n+1)(n+2)}{2} \tag{6}$$
$$\frac{n^2+3n+2}{2} = \frac{n^2+2n+n+2}{2} \tag{7}$$
$$\frac{n^2+3n+2}{2} = \frac{n^2+3n+2}{2} \tag{8}$$

1.2 Sum of n^2 – DO IT YOURSELF

```
Sum-sq(n)
{
if n==1 then return 1
else return (Sum-sq(n-1)+n*n)
}
```

Prove that $Sum - sq(n) = \frac{2n^3+3n^2+n}{6}$

Claim

Base Case

Induction Hypothesis

Induction Conclusion