CS330 Midterm SAMPLE

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October 9, 2017

Framework

1. Prove that $6n + 12 \in O(n)$
2. Prove that $2n + 12 \in \Omega(n)$
3. Prove that $3n^2 + 48 \in \Theta(n^2)$

Analyse iterative Algorithms

What is the
(a) runningtime and
(b) complexity of the following algorithm
Assuming the cost of each "F()" function call is 3 units and the cost for each "G()" function call is 2 units, and ignoring the cost of the "++" and "<" operations in the for loops.

```
iterative(n)
{
        for (i = 0; i<n; i++)
        {
                for (j = 0; j <= n; j++)
                        F();
                        G();
        }
        for (i = 0; i<n; i++)
        {
                F();
                F();
                G();
                G();
        }
        F();
        G();
}
```

Analyse recursive Algorithms

1. Solve the following recurrence relation using a recursion tree
   $F(n) = 3F(n/2) + 4n$
   $F(1) = 1$

2. Write an algorithm for the following runtime equation
   $F(n) = 3F(n/2) + 4n$
   $F(1) = 1$ (Algorithm does not have to do anything useful.

Correctness of Algorithms

Prove correctness of the following algorithm:

```
Iterative(int A, int B) {
        R = A;
        I = 0;
        while (I < B) {
                R = R + 1;
                I = I + 1;
        }
        return R;
}
```

which calculates A + B.

Prove that $Sum(n) = \frac{n(n+1)}{2}$

```
Sum(n) {
        if n==1 then return 1;
        else return (Sum(n-1)+n);
}
```

Real World

Consider the following algorithm.

```
Mystery(n)
```

```
{
        S=0;
        for(i=1, i<=n, i++)
                S = S + i*i
        return S
}
```

a) What does this algorithm compute?

b) What is the basic operation?

c) How many times is the basic operation executed?

d) What is the efficiency class of this algorithm?

e) Suggest an improvement, or a better algorithm altogether, and indicate its efficiency class. If you cannot do it, try to prove that, in fact, it cannot be done.