Sample Final - CS 330 Fall 2015

1 Framework

1.1 Prove that

1. $\frac{n+7}{2} \in O(n^2)$
2. $\frac{n(n+1)}{2} \in \Omega(n)$
3. $\frac{n^2(n+7)}{2} \in \Theta(n^3)$

2 Correctness

2.1 Prove that Sumcube(n)=$(\frac{n(n+1)}{2})^2$ (5 pts.)

```
Sumcube(n)
{
        if n==1 then return 1
        else return (Sumcube(n-1)+n*n*n)
}
```

2.2 Prove that ALG1 returns AB (10 pts.)

```
ALG1(A, B) // A,B are natural numbers
{
S = 0
I = 0
while (I < B)
{
        S = S + A
        I = I + 1
}
return S
}
```

3 Runningtime

3.1 Iterative Algorithm

What is the

(a) running time (exact formula) and

(b) complexity of the following algorithm and

(c) prove the complexity.

Assuming the cost of each "F()" function call is 2 units, the of each "G()" function call is 4 units, and ignoring the cost of the "++" and "<" operations in the for loops.

```
iterative(n)
{
        for (i = 0; i<n-5; i++)
        {
                G();
                for (j = 0; j < n*n; j++)
                {
                        F();
                        F();
                }
                F();
                G();
                for (i = 1; i=n; i++)
                {
                        F();
                        F();
                }
        }
        F();
        G();
}
```

3.2 Recursive Algorithm

Given is the following runtime equation: $F(n) = 3F(n/2) + n, F(0) = 1$

(a) Write an algorithm for the runtime equation. (Algorithm does not have to do anything useful.)

(b) Solve the recurrence using recursion tree.

(c) Use the Master Theorem to determine the order of growth.

4 Search

Shortly explain the following search algorithms. What is the best case and the worst case for them? Show the steps for both algorithms on the following array 7,3,9,3,1.

(a) Insertion Sort

(b) Selection Sort

(c) Bubble Sort

(d) Quicksort

(e) Merge Sort

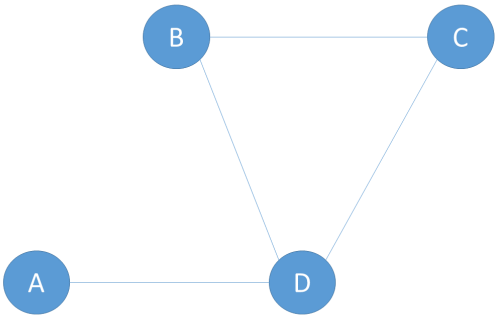
(f) Binary Search

(g) Heap Sort

5 Brute Force

5.1 Graph

Given is the following graph:



(a) Write down the adjacency matrix and adjacency lists specifying this graph.(Assume that the matrix rows and columns and vertices in the adjacency lists follow in the alphabetical order of the vertex labels.)

(b) Starting at vertex A and resolving ties by the vertex alphabetical order, traverse the graph by depth-first search and construct the corresponding depth-first search tree.

(c) Traverse the graph by breadth-first search and construct the corresponding breadth-first search tree. Start the traversal at vertex A and resolve ties by the vertex alphabetical order.

5.2 Combinatoric

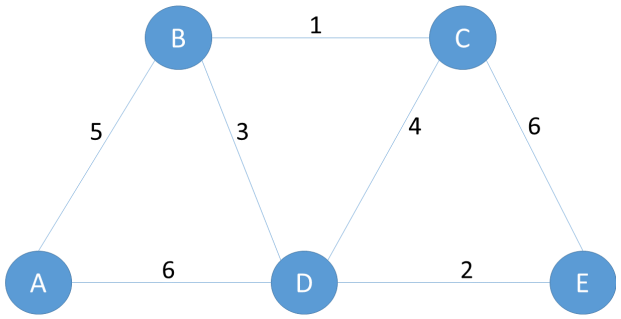
(a) Write down all permutations of {S,T,U,D}.

(b) Write down all subsets of {Y,C,S,I}.

(c) Write down all partitions of {S,F,U,N}

6 Greedy

Find the minimum spanning tree of the following graph. Use Kruskal's or Prim's algorithms to solve it. Point out which algorithm you use and show all steps to find the mst.



7 Dynamic Programming

Write down a dynamic programming algorithm to calculate the nth Fibonacci Number.