Logic Arguments

Recall from class that, in these arguments, the black statements represent the **hypothesis** and the red statement is the **conclusion**. We decide if the conclusion follows from the hypothesis, and if it does, then we provide a proof using rules of inference.

1. **How's the weather?**
   It is not sunny today and it is colder than yesterday.
   We will go swimming only if it is sunny.
   If we do not go swimming, then we take a canoe trip.
   If we take a canoe trip, we will be home by sunset.
   We will be home by sunset.

   We translate the propositions as follows:

   $\neg$sunny $\wedge$ cold
   swim $\rightarrow$ sunny
   $\neg$swim $\rightarrow$ canoe
   canoe $\rightarrow$ home
   home

   This is a valid argument, they follow the rules of inference:

| | | |
|---|---|---|
| 1. | $\neg$sunny $\wedge$ cold | Hypothesis |
| 2. | $\neg$sunny | Simplification of (1) |
| 3. | swim $\rightarrow$ sunny | Hypothesis |
| 4. | $\neg$swim | Modus Tollens from (2)(3) |
| 5. | $\neg$swim $\rightarrow$ canoe | Hypothesis |
| 6. | canoe | Modus Ponens from (4)(5) |
| 7. | canoe $\rightarrow$ home | Hypothesis |
| 8. | home | Modus Ponens (6)(7) |

2. **Birds and honey**

   All hummingbirds are richly colored.
   No large birds live on honey.
   Birds that do not live on honey are dull in color.
   Hummingbirds are small.

   Let $x$ = birds, $H(x) = x$ is a hummingbird, $C(x) = x$ is richly colored, $L(x) = x$ is large, $S(x) = x$ lives on honey. We assume that birds that are not large are small, and birds that are not richly colored are dull in color. Then the statements can be written as follows, along with equivalent statements:

   $\forall x(H(x) \to C(x))$
   $\neg\exists x(L(x) \land S(x)) \equiv \forall x(\neg L(x) \lor \neg S(x)) \equiv \forall x(S(x) \to \neg L(x))$
   $\forall x(\neg S(x) \to \neg C(x)) \equiv \forall x(C(x) \to S(x))$
   $\forall x(H(x) \to \neg L(x))$

   Clearly the argument follows from a repeated application of Hypothetical Syllogism:

   $$\forall x\,((H(x) \to C(x)) \land (C(x) \to S(x)) \land (S(x) \to \neg L(x)))$$

   implies $\forall x(H(x) \to \neg L(x))$.

3. **Professors and vanity**

   No professors are ignorant.
   All ignorant people are vain.
   No professors are vain.

   This argument is not valid, since there might be professors who are vain, even if they are not ignorant.

4. **Lions and coffee**

   All lions are fierce.

   Some lions do not drink coffee.

   Some fierce creatures do not drink coffee.

   Let $x$ =creatures, $L(x) = x$ is a lion, $F(x) = x$ is fierce, $C(x) = x$ does not drink coffee. Then the statements can be written as follows:

   $\forall x(L(x) \to F(x))$

   $\exists x(L(x) \wedge C(x))$

   $\exists x(F(x) \wedge C(x))$

   The argument is correct, since there are some fierce creatures, namely some lions, who do not drink coffee. Let us prove it using rules of inference.

| | | |
|---|---|---|
| 1. | $\exists x(L(x) \wedge C(x))$ | Hypothesis |
| 2. | $L(s) \wedge C(s)$ (for some $s$) | Existential instantiation of (1) |
| 3. | $L(s)$ | Simplification of (2) |
| 4. | $\forall x(L(x) \to F(x))$ | Hypothesis |
| 5. | $L(s) \to F(s)$ | Universal instantiation of (4) |
| 6. | $F(s)$ | Modus Ponens from (3)(5) |
| 7. | $C(s)$ | Simplification of (2) |
| 8. | $F(s) \wedge C(s)$ | Conjunction of (6)(7) |
| 9. | $\exists x(F(x) \wedge C(x))$ | Existential Generalization of (8) |