Math 258 - Logic

Let p, q and r be propositions.

Logical Equivalences:

Rule	**Equivalence**
Identity Laws	$p \wedge T \equiv p$ $p \vee F \equiv p$
Domination Laws	$p \vee T \equiv T$ $p \wedge F \equiv F$
Idempotent Laws	$p \wedge p \equiv p$ $p \vee p \equiv p$
Double Negation	$\neg\neg p \equiv p$
Commutative Laws	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$
Associative Laws	$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$ $p \vee (q \vee r) \equiv (p \vee q) \vee r$
Distributive Laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
De Morgan Laws	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$
Other important laws	$p \to q \equiv \neg p \vee q$ $p \to q \equiv \neg q \to \neg p$
Quantifier laws	$\neg(\forall x P(x)) \equiv \exists x(\neg P(x))$ $\neg(\exists x P(x)) \equiv \forall x(\neg P(x))$

Inference Rules:

Inference Rule	**Tautology**
Addition	$p \to p \vee q$
Simplification	$p \wedge q \to p$
Conjunction	$[(p) \wedge (q)] \to p \wedge q$
Modus ponens	$[\, p \wedge (p \to q)\,] \to q$
Modus tollens	$[\, \neg q \wedge (p \to q)\,] \to \neg p$
Hypothetical syllogism	$[\,(p \to q) \wedge (q \to r)\,] \to (p \to r)$
Disjunctive syllogism	$[\,(p \vee q) \wedge \neg p\,] \to q$
Resolution	$[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r)$
Universal instantiation	$\forall x P(x) \to P(c)$
Universal generalization	$[\, P(c)$ for **arbitrary** c$\,] \to \forall x P(x)$
Existential instantiation	$\exists x P(x) \to [\, P(c)$ for **some** c$\,]$
Existential generalization	$[\, P(c)$ for **some** c$\,] \to \exists x P(x)$