

MAT 345 - Homework 3
Due Monday, October 8, 2018, in class

1. A sample of 2 items is selected at random from a box of 20 items, out of which 4 are defective. Let X count the number of defective items in the sample.
 - (a) Write the probability mass function of X . (Make sure probabilities add up to 1)
 - (b) Find the expectation $E[X]$.
 - (c) Find the variance $Var(X)$
2. If X has density function $f(x) = \frac{1}{2\sqrt{x}}$ for $0 < x < 1$ and 0 otherwise, find
 - (a) $P(1/3 < X < 2/3)$
 - (b) $E[X]$
 - (c) $Var(X)$
 - (d) The median, that is the value m so that $P(X \leq m) = 1/2$.
3. We want to test if a coin is fair. We decide to toss the coin 100 times and set up the decision rule:

"if we get between 40 and 60 heads (inclusive), the coin is fair."

 - (a) Find the probability of Type 1 error; that is, the probability that we get under 40 heads or over 60 heads, when the coin is indeed fair.
 - (b) Find the probability of Type 2 error; that is, find the probability that we get between 40 and 60 heads (inclusive), when the coin is biased
 - (i) with probability of heads being 0.6
 - (ii) with probability of heads being 0.7
4. An article in Fortune magazine claimed that 50% of engineering graduates continue their studies to get an advanced degree. However a study of 484 graduates revealed only 198 who were planning graduate study. Is the observation consistent with the claim? Test the hypothesis using a level of significance $\alpha = 0.05$.
5. A researcher claims that 10% of football helmets have manufacturing flaws that could potentially cause injury. A sample of 200 helmets revealed 13 were defective. Approximate the probability that at most 13 helmets in the sample are defective. Use hypothesis testing to decide if the claim should be rejected.

6. Part of the probability folklore is the idea that if you spin a penny, the result is heads about 30% of the time. To test this hypothesis, Sally Sievers from the 1999 probability class at Wells College spun 650 pennies and got 321 heads. What is the probability that we would get at least 321 heads if the probability of heads is indeed " $p=0.3$ ". Does this support the hypothesis?
7. It is commonly presumed that an unborn child has 50% chance of being female. But is this really the case? According to the Central Bureau of Statistics in the Netherlands, during a 3 year period, there were 585,609 children born of which 286,114 were girls. If p =probability that a newborn is female, let $H_0 : "p = 0.5"$ and $H_1 : "p \neq 0.5"$. Test the hypothesis H_0 with a level of significance $\alpha = 0.01$.
8. Weldon, an English biologist, was interested in the "pip effect" in dice – the idea that the spots, or "pips", which on some dice are produced by cutting small holes in the surface, make the sides with more spots lighter and more likely to turn up. Weldon threw 12 dice 26,306 times for a total of 315,672 throws and observed that a 5 or 6 came up 106,602. Let H_0 be the null hypothesis that the die is fair, with $H_0 : "p = 1/3"$, where p =probability of landing on 5 or 6. Use hypothesis testing with $H_1 : "p \neq 1/3"$ and level of significance $\alpha = 0.05$ to decide if H_0 should be rejected or not.