Gradient Descent

Towards Neural Networks

Justin Stevens

Outline

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Ticket Prices	Availability	Interest	$\bar{x}$
Cheap	Yes	Yes	$(1, 1, 1)$
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Cheap	Yes	No	$(1, 1, 0)$	7
Cheap	No	Yes	$(1, 0, 1)$	4
Expensive	Yes	Yes	$(0, 1, 1)$	9
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We can now define my **activation threshold**, t, which will determine whether or not I go to the game, represented in binary.

Formula for Decision Making

The general formula for my decision to go to the Oilers game is

$$\text{output} = \begin{cases} 0 & \text{if } \bar{w} \cdot \bar{x} < t \\ 1 & \text{if } \bar{w} \cdot \bar{x} \geq t. \end{cases}$$

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For instance, if $t = 9$, we see I'll only go if I'm both available and interested. If $t = 7$, I'll also go if the tickets are cheap and I'm available:

Ticket Prices	Availability	Interest	$\bar{x}$	$\bar{x} \cdot \bar{w}$
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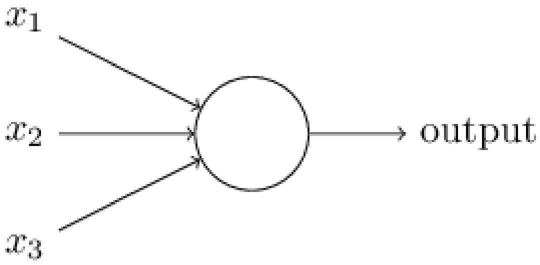

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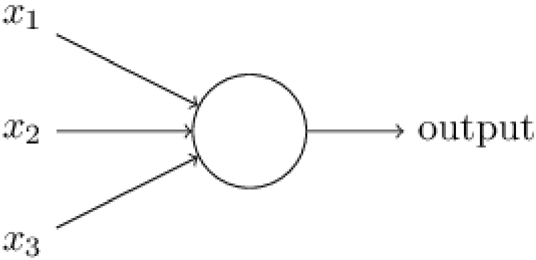

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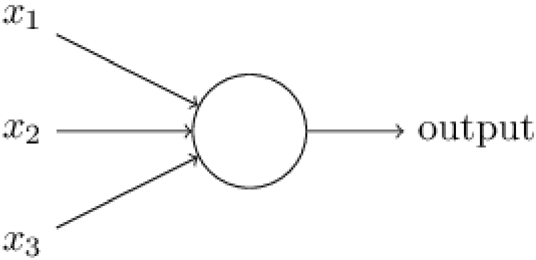

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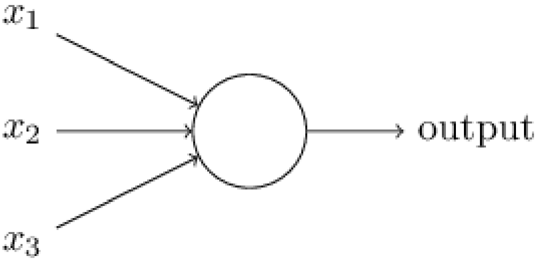

Each of these lines collect evidence and are weighted to produce an output. In practice, our inputs and outputs don't necessarily have to be binary; they can be real-valued. We therefore have to define a new activation function. First, we have to make a slight modification to our model.

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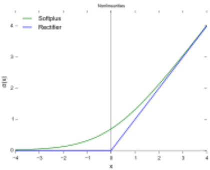

Figure 1: Rectifier, and a smooth approximation $\log(1 + e^x)$. (*Source: Wikipedia*).

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Example Images

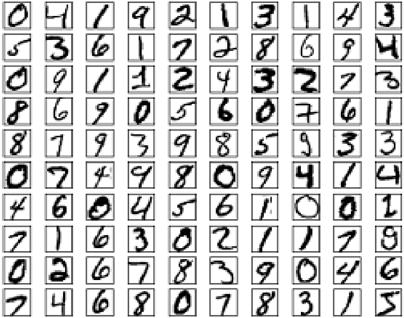

Figure 2: How would you devise a system for a **computer** to classify the digits? What assumptions do we have to make about the data set, known as MNIST?

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- The desired digit is represented as a number from 0 to 9.
- $60,000$ images are designated for training, and $10,000$ for testing.
- We'll build a model from the training images that will learn to classify digits!

What we're building towards

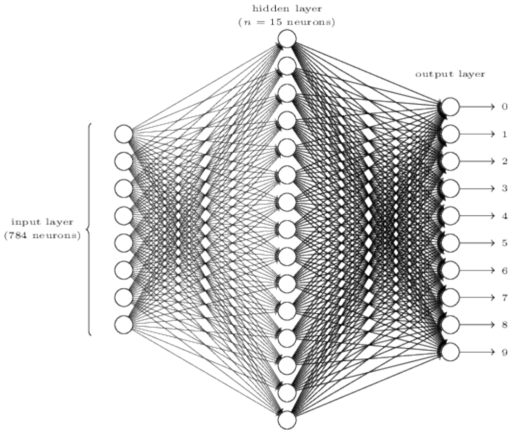

Figure 3: A simple neural network structure. The input vectors on the left hand side have $28 \times 28 = 784$ inputs for each pixel, and the output layer has 10 digits, one for each number from 0 to 9.

References

- Stewart Calculus: Early Transcedentals, 6th Edition
- Professor Leonard Calculus 3 (Full Length Videos)
- Paul's Online Math Notes, Calculus III