Gradient Descent

Towards Neural Networks

Justin Stevens

Outline

Should I Stay or Should I Go?

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Cheap	No	No	$(1, 0, 0)$	1
Cheap	Yes	No	$(1, 1, 0)$	7
Cheap	No	Yes	$(1, 0, 1)$	4
Expensive	Yes	Yes	$(0, 1, 1)$	9
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We can now define my **activation threshold**, t, which will determine whether or not I go to the game, represented in binary.

Formula for Decision Making

The general formula for my decision to go to the Oilers game is

$$\text{output} = \begin{cases} 0 & \text{if } \bar{w} \cdot \bar{x} < t \\ 1 & \text{if } \bar{w} \cdot \bar{x} \geq t. \end{cases}$$

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For instance, if $t = 9$, we see I'll only go if I'm both available and interested. If $t = 7$, I'll also go if the tickets are cheap and I'm available:

Ticket Prices	Availability	Interest	$\bar{x}$	$\bar{x} \cdot \bar{w}$
Cheap	**Yes**	**Yes**	$(1, 1, 1)$	**10**
Cheap	No	No	$(1, 0, 0)$	1
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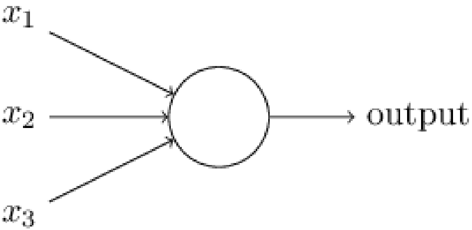

Figure 1: *Source: Nielsen*

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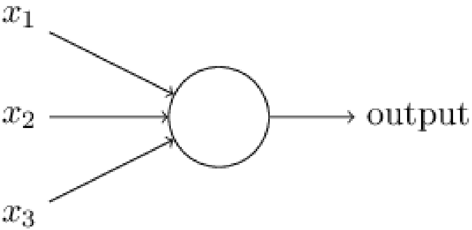

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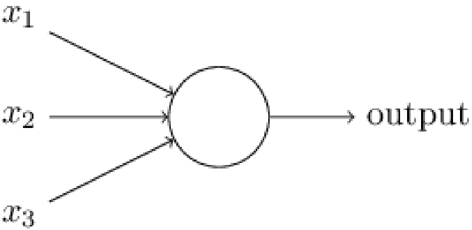

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Each of these lines collect evidence and are weighted to produce an output. In practice, our inputs and outputs don't necessarily have to be binary; they can be real-valued. We therefore have to define a new activation function.

Introducing the Bias

Instead of comparing our weighted sum to a threshold, we instead *add* a bias, b, to our weighted sum. We write this as $\bar{w} \cdot \bar{x} + b$ instead.

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This is known as the *heaviside step function*. In the next few slides, we'll see examples of other activation functions, but first we'll draw a picture.

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Extending our Model

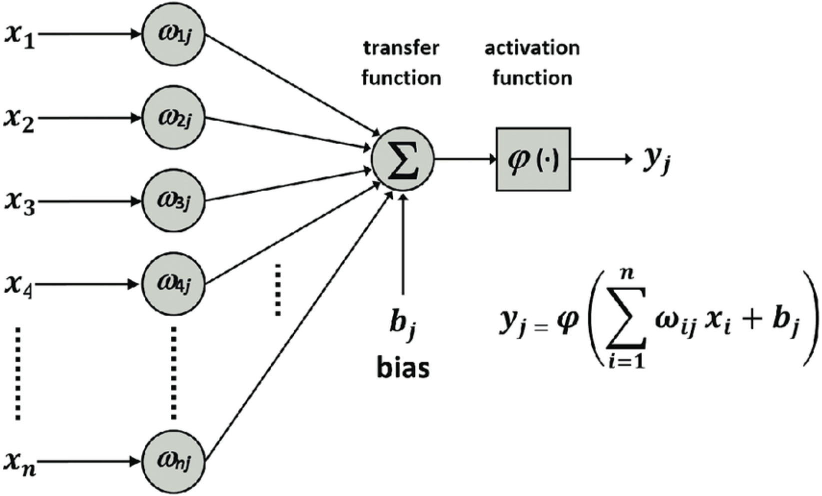

Figure 2: *Source: Daniel Alvarez, InTech*

Rectified Linear Unit

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Graphically, we can see:

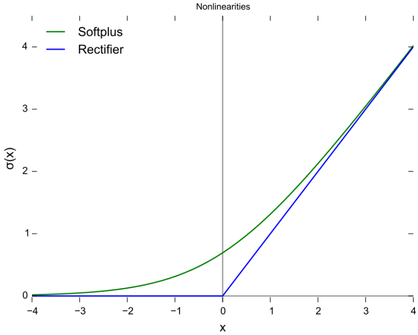

Figure 3: Rectifier, and a smooth approximation $\log(1 + e^x)$. (*Source: Wikipedia*).

Sigmoid Function

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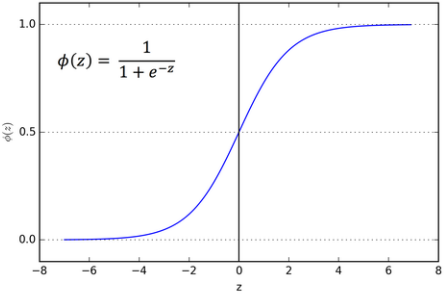

Figure 4: As $z \to \infty$, we see $\sigma(z) \to 1$. Alternatively, as $z \to -\infty$, $\sigma(z) \to 0$. (*Source: Towards Data Science*).

Outline

Example Images

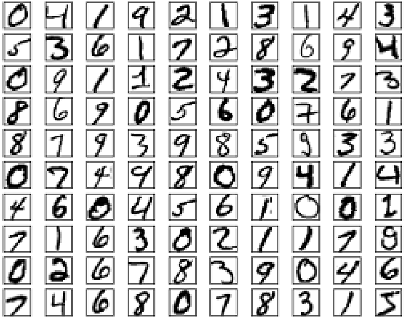

Figure 5: How would you devise a system for a **computer** to classify the digits? What assumptions do we have to make about the data set, known as MNIST?

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  - $60,000$ images are designated for training, and $10,000$ for testing:

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import tensorflow as tf
from tensorflow import keras
(x_train, y_train), (x_test, y_test) = keras.datasets.mnist.load_data()
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We'll build a model from the training images that will learn to classify digits!

What we're building towards

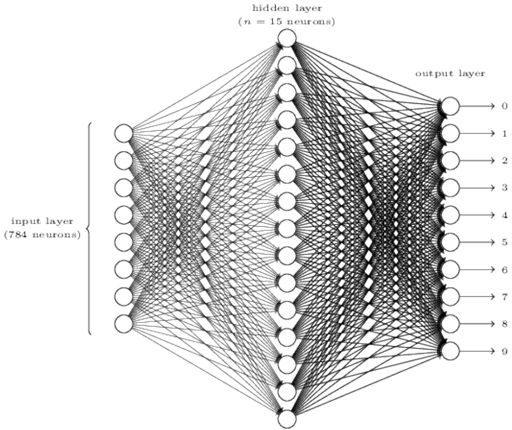

Figure 6: A simple neural network structure. The input vectors on the left hand side have $28 \times 28 = 784$ inputs for each pixel, and the output layer has 10 digits.

References

- Michael Nielsen: Using neural nets to recognize handwritten digits
- Towards Data Science: A Beginner's Guide to Neural Networks
- Paul's Online Math Notes, Calculus III