# MECH 421/423 Lab 4
# Op-Amp Circuits for Noisy Environments

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## Contents

# 1 Introduction

This lab investigates the design of an optical distance sensor built from discrete op amp stages. Each exercise reinforces a different portion of the signal chain—from generating the optical carrier, through transimpedance conversion and filtering, to rectification, digitization, and calibration. The write-up below follows the structure of the lab manual and includes design calculations and measured results taken from the handwritten notes.

# 2 Phase 1: Analog Front-End

## 2.1 Exercise 1: Optical Transmitter

### 2.1.1 Question 1: Assemble and Power the LED

The LED transmitter was mounted on the linear rail and connected to the Analog Discovery 2 (AD2) waveform generator. Vin was configured as a 5 V amplitude square wave with a 2.5 V DC offset so the LED could be pulsed at the desired current.

### 2.1.2 Question 2: Verify 1 Hz Modulation

With the generator at 1 Hz the LED visibly flashed, demonstrating that the wiring and mechanical alignment were correct and that the LED could traverse the slider smoothly.

### 2.1.3 Question 3: Verify 1 kHz Modulation

Increasing the modulation to 1 kHz made the LED appear continuously on, indicating that the chosen frequency exceeds the flicker fusion threshold and is suitable as the optical carrier for later exercises.

## 2.2 Exercise 2: Photodiode Amplifier and High-Pass Filter

### 2.2.1 Question 1: Choose $R_2$

Using the constraint $\Delta V_2 = 100\,\text{mV}$ for a $1\,\mu\text{A}$ photocurrent,

$$R_2 = \frac{\Delta V_2}{I_d} = \frac{0.1}{1 \times 10^{-6}} = 100\,\text{k}\Omega.$$

### 2.2.2 Question 2: Choose $R_3$ and $R_4$

Bias $V_1$ at $0.5\,\text{V}$ using a divider referenced to $5\,\text{V}$:

$$V_1 = \frac{R_4}{R_3 + R_4} V_{\text{ref}} = 0.5.$$

Selecting $R_3 = 200\,\text{k}\Omega$ and $R_4 = 22\,\text{k}\Omega$ yields

$$V_1 = \frac{22}{200 + 22} \times 5 \approx 0.495\,\text{V},$$

which is sufficiently close to the target bias.

### 2.2.3 Question 3: Choose $R_5$ and $C_1$

For the filter design, the transfer function amplitude is equal to $\frac{1}{\sqrt{2}}$ at the cutoff frequency:

$$\left| \frac{R}{R + \dfrac{1}{\omega C}} \right| = \frac{1}{\sqrt{2}}$$

$$\left| \frac{\omega RCj}{\omega RCj + 1} \right| = \frac{1}{\sqrt{2}}$$

$$\frac{(\omega RC)^2}{1 + (\omega RC)^2} = \frac{1}{2}$$

$$2(\omega RC)^2 = 1 + (\omega RC)^2$$

$$(\omega RC)^2 = 1$$

$$\omega RC = 1$$

$$RC = \frac{1}{\omega} = \frac{1}{2\pi \cdot 100}$$

$$RC \approx \boxed{1.59155 \times 10^{-3}\ \mathrm{s}}$$

$\mathrm{R}_5 = 1.6\ \mathrm{k}\Omega, \quad \mathrm{C}_1 = 1\ \mu\mathrm{F}$ are good enough

### 2.2.4 Question 4: Demonstrate Ambient-Light Impact


Figure 1: Waveform when photodiode is exposed to ambient light.

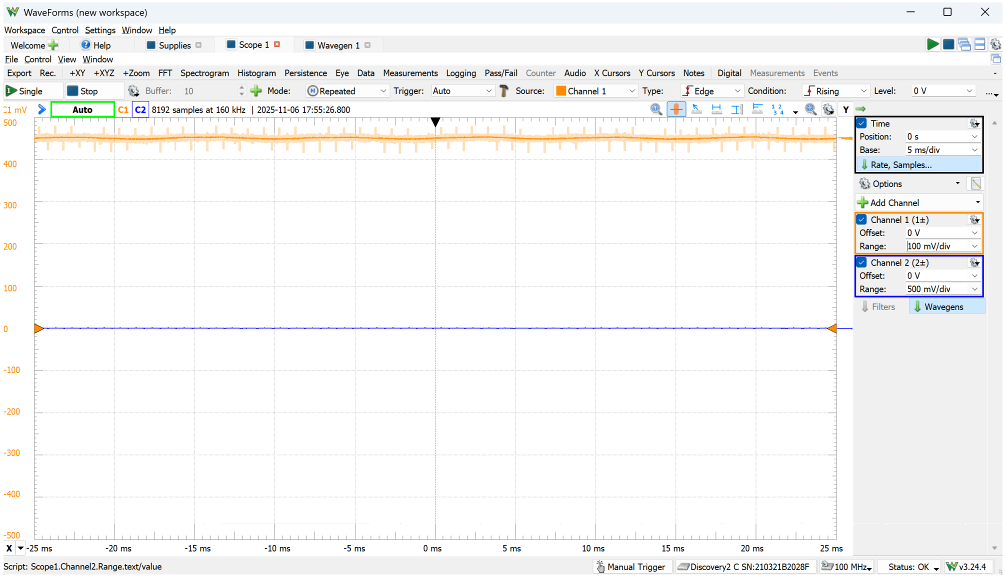

Figure 2: Waveform when photodiode is covered.

Figure 1 shows $V_2$ with the photodiode exposed to room light while Figure 2 shows the effect of covering it. As you can see, covering the photodiode reduces the DC offset, which is what we expected.

#### 2.2.5 Question 5: Detect the 1 kHz Carrier at $V_2$

First, setup the LED by using the AD2 waveform generator, connect the positive of the waveform to the anode of the LED and the negative to the cathode of the LED. Set the waveform generator to output a 2.5 V amplitude square wave with a 2.5 V DC offset at 1 kHz. Figure 3 shows the setup inside the WaveForm software.

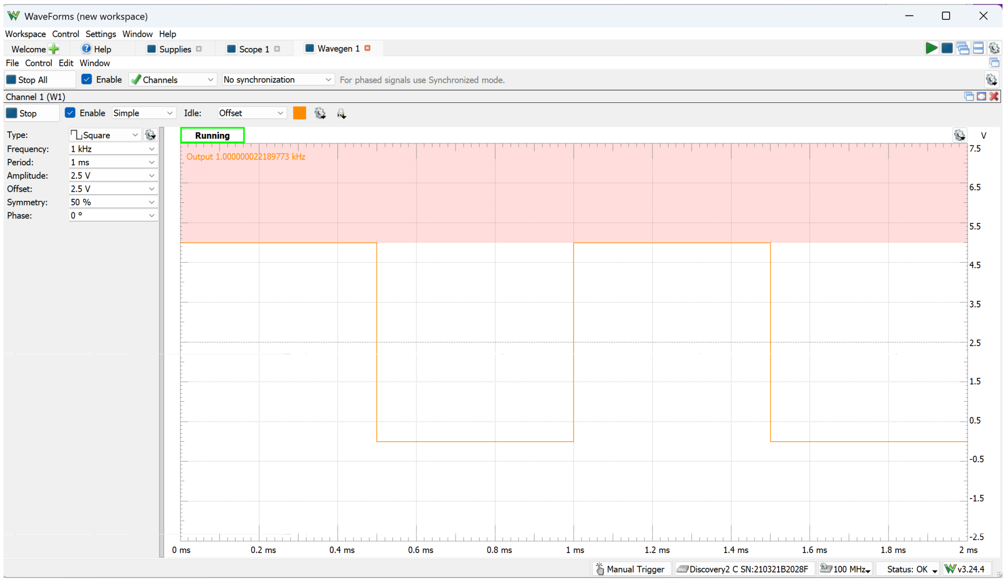


Figure 3: Waveform generator setup for LED modulation

After setting up the LED, bring the LED close to the photodiode, and observe the voltage using the AD2 oscilloscope at $V_2$.

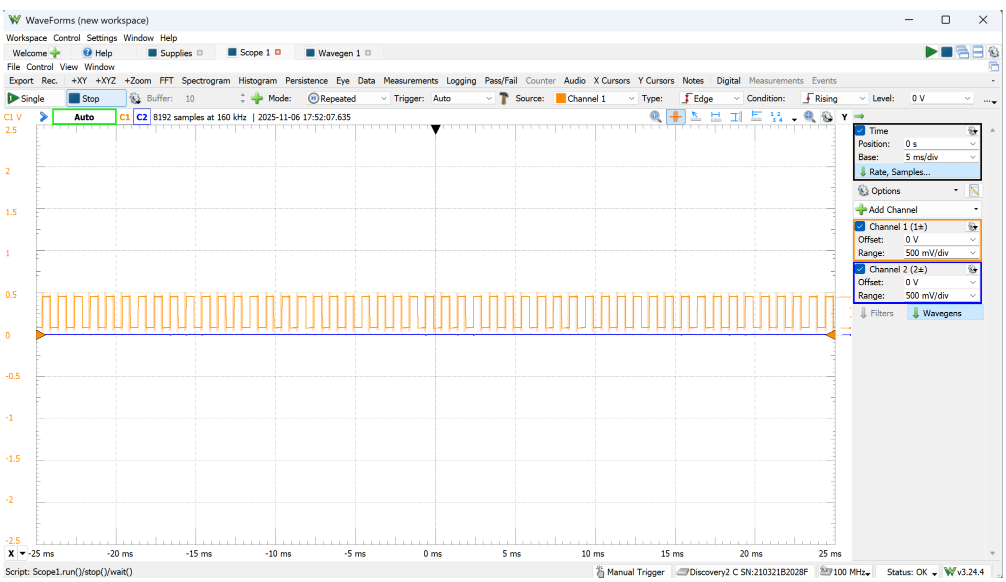


Figure 4: Waveform at $V_2$ with LED close to photodiode.

The resulting waveform is shown in Figure 4. Notice that this is a 1khz square wave signal (just count that there is 5 cycles in 5ms)

This is what we should expect, here is the math for our result:

Let the photodiode current be written as the sum of a DC component due to ambient light and an AC component due to the modulated LED,

$$i_{\text{ph}}(t) = I_{\text{amb}} + \Delta I\, q(t),$$

where $q(t)$ is a unit-amplitude, zero-mean square wave of frequency $f_0 = 1$ kHz (i.e. $q(t) = \pm 1$ with 50% duty cycle), and $\Delta I$ is the amplitude of the photocurrent variation when the LED is driven.

The op-amp is configured as a transimpedance amplifier referenced to $V_1$. Because of the virtual short between the inputs, the inverting node is held at

$$V_- \approx V_+ = V_1.$$

Applying KCL at the inverting node gives

$$\frac{V_2(t) - V_1}{R_2} = -i_{\text{ph}}(t),$$

so the output voltage of the transimpedance stage is

$$V_2(t) = V_1 - R_2\, i_{\text{ph}}(t) = \big(V_1 - R_2 I_{\text{amb}}\big) - R_2 \Delta I\, q(t).$$

Thus $V_2(t)$ consists of a DC offset

$$V_{2,\text{DC}} = V_1 - R_2 I_{\text{amb}}$$

plus a 1 kHz square-wave component of peak amplitude

$$V_{2,\text{AC}}(t) = -R_2 \Delta I\, q(t),$$

so the peak-to-peak value of the 1 kHz component at $V_2$ is

$$V_{2,\text{pp}} = 2 R_2 \Delta I.$$

As the LED–photodiode distance changes, $\Delta I$ changes, and the square-wave amplitude at $V_2$ scales linearly with the photocurrent variation.

### 2.2.6 Question 6: Observe the High-Pass Output

The result at $V_3$ is shown in Figure 5.

The node $V_2$ is now applied to the high-pass filter formed by $C_1$ and $R_5$. Taking $V_2(t)$ as the input and $V_3(t)$ as the output, the transfer function of this first-order high-pass filter is

$$H(s) = \frac{V_3(s)}{V_2(s)} = \frac{s R_5 C_1}{1 + s R_5 C_1}.$$

For a sinusoidal component of angular frequency $\omega$,

$$H(j\omega) = \frac{j\omega R_5 C_1}{1 + j\omega R_5 C_1},$$

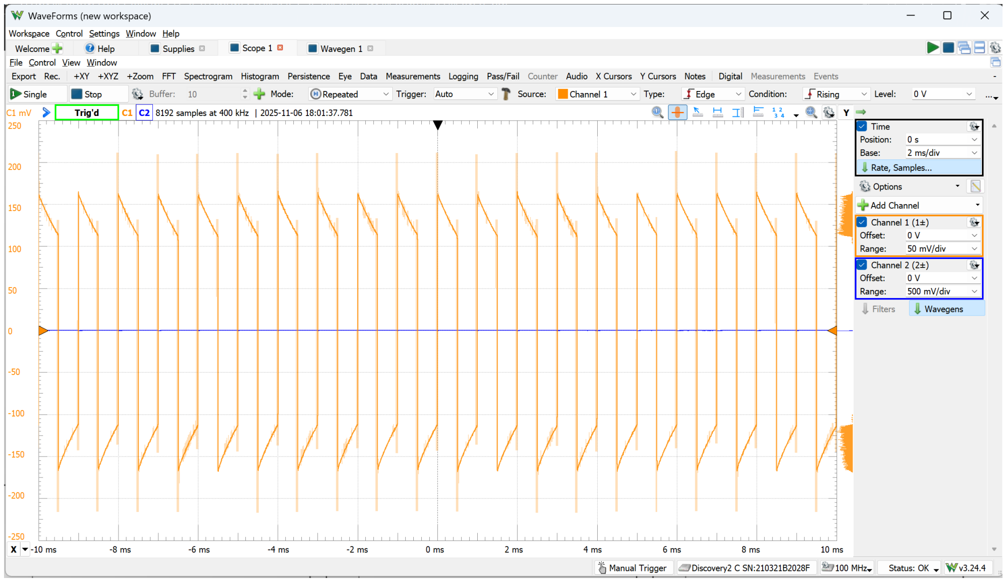


Figure 5: Waveform at $V_3$ with LED close to photodiode.

and its magnitude is

$$\left|H(j\omega)\right| = \frac{\omega R_5 C_1}{\sqrt{1 + (\omega R_5 C_1)^2}}.$$

The cut-off angular frequency is chosen as

$$\omega_c = \frac{1}{R_5 C_1} \approx 500 \text{ rad/s} \quad \text{(about 100 Hz)},$$

so at the LED modulation frequency $f_0 = 1$ kHz, $\omega_0 = 2\pi f_0$, we have

$$\omega_0 R_5 C_1 = \frac{\omega_0}{\omega_c} = \frac{2\pi \cdot 1000}{500} \approx 12.6,$$

and therefore

$$\left|H(j\omega_0)\right| = \frac{12.6}{\sqrt{1 + 12.6^2}} \approx 0.997 \approx 1.$$

The DC term $V_{2,\mathrm{DC}}$ is completely blocked by the high-pass filter (its gain at $\omega = 0$ is zero), while the 1 kHz square-wave component passes essentially unchanged in amplitude. Therefore the output at $V_3$ is approximately

$$V_3(t) \approx \left|H(j\omega_0)\right| V_{2,\mathrm{AC}}(t) \approx -R_2 \Delta I \, q(t),$$

with a peak-to-peak value

$$V_{3,\mathrm{pp}} \approx \left|H(j\omega_0)\right| V_{2,\mathrm{pp}} \approx V_{2,\mathrm{pp}} = 2R_2\Delta I.$$

Hence, at $V_3$ we observe a square wave at 1 kHz whose peak-to-peak amplitude is essentially the same as the AC component of $V_2$, but now centered around 0 V rather than around the bias voltage $V_1$. As the distance between the LED and photodiode is changed, $\Delta I$ changes, and the peak-to-peak amplitude at $V_3$ varies proportionally, which is what I found experimentally (and hopefully you too).

### 2.2.7 Question 7: Hardware Arrangement

Follow the instruction and you'll end up with the setup shown in the lab manual. Note that this is very crucial for later experiments. Because you need to align the photodiode and LED properly, make sure that the LED is facing the photodiode directly. I experienced a lot of issues because of misalignment, particularly I couldn't get the full range of distance measurement because the photodiode couldn't "see" the LED properly. So future mechatronics student, please pay attention to this. If you're not convinced, I have derivation once we build the full circuit.

## 2.3 Exercise 3: High-Pass Filter and AC Amplifier

### 2.3.1 Question 1: Bias Network

$$\frac{R_8}{R_8 + R_7} = \frac{1}{2} \tag{1}$$

$$2R_8 = R_8 + R_7 \tag{2}$$

$$R_8 = R_7 \tag{3}$$

We can choose, for example,

$$R_8 = R_7 = \boxed{10\ \text{k}\Omega}. \tag{4}$$

The input and feedback impedances of the op-amp stage are

$$Z_1 = R_5 + \frac{1}{sC_1} = \frac{1 + sR_5C_1}{sC_1}, \tag{5}$$

$$Z_2 = R_6 \parallel \frac{1}{sC_2} = \frac{R_6}{R_6 + \cfrac{1}{sC_2}} = \frac{R_6}{1 + sR_6C_2}. \tag{6}$$

For the non-inverting configuration, with $V_4$ at the non-inverting input and $V_2$ feeding the inverting node through $Z_1$, the output is

$$V_0 = V_4\left(1 + \frac{Z_2}{Z_1}\right) - V_2\left(\frac{Z_2}{Z_1}\right) \tag{7}$$

$$= V_4 + \frac{Z_2}{Z_1}\left(V_4 - V_2\right). \tag{8}$$

We want the magnitude of the transfer ratio to be about 10:

$$\left|\frac{Z_2}{Z_1}\right| \approx 10. \tag{9}$$

Compute the ratio:

$$\frac{Z_2}{Z_1} = \frac{\dfrac{R_6}{1+sR_6C_2}}{R_5 + \dfrac{1}{sC_1}} \tag{10}$$

$$= \frac{R_6}{1+sR_6C_2} \cdot \frac{sC_1}{1+sR_5C_1} \tag{11}$$

$$= \frac{R_6}{R_5} \frac{sR_5C_1}{1+sR_5C_1} \cdot \frac{1}{1+sR_6C_2}. \tag{12}$$

In the midband of interest (where the frequency-dependent factors are near unity), the dominant term is $R_6/R_5$, so we choose

$$\frac{R_6}{R_5} \approx \boxed{10}. \tag{13}$$

Then we can choose our resistor, which in this case I choose to be:

$$R_5 = \boxed{1.6\ \text{k}\Omega}, \tag{14}$$

$$R_6 = \boxed{16\ \text{k}\Omega}. \tag{15}$$

### 2.3.2 Question 2: Choose $C_2$

The low-pass pole formed by $R_6$ and $C_2$ should be above 16 kHz:

$$\omega_{c,\text{LP}} = \frac{1}{R_6C_2} \geq 10^5\ \text{rad/s}.$$

With $R_6 = 16\,\text{k}\Omega$ this gives $C_2 \leq 0.622\,\text{nF}$. The closest available capacitor was 620 pF, yielding a calculated cutoff near 99.5 kHz.

### 2.3.3 Question 3: Verification

Applying a 100 mV sinusoid at 1 kHz produced the expected tenfold gain without saturation when the LED-photodiode spacing was swept between 3 cm and 25 cm. The high-pass chain maintained a detectable waveform across the entire travel range.

## 2.4 Exercise 4: High-Pass Filter, Rectifier, and Low-Pass Filter

### 2.4.1 Question 1: Duplicate High-Pass

This stage reused the Exercise 2 sizing so $R_9 = 1.6\,\text{k}\Omega$ and $C_3 = 1\,\mu\text{F}$, keeping the 100 Hz cutoff.

### 2.4.2 Question 2: Rectifier Gain

The feedback network $R_6$–$C_2$ implements a first–order low-pass term of the form

$$H_{\mathrm{LP}}(j\omega) = \frac{1}{1 + j\omega R_6 C_2}.$$

The magnitude is

$$|H_{\mathrm{LP}}(j\omega)| = \left|\frac{1}{1 + j\omega R_6 C_2}\right| = \frac{1}{\sqrt{1 + (\omega R_6 C_2)^2}}.$$

The $-3\,\mathrm{dB}$ cut-off occurs when $|H_{\mathrm{LP}}(j\omega_c)| = 1/\sqrt{2}$:

$$\begin{aligned}
\frac{1}{\sqrt{1 + (\omega_c R_6 C_2)^2}} &= \frac{1}{\sqrt{2}} \\
1 + (\omega_c R_6 C_2)^2 &= 2 \\
(\omega_c R_6 C_2)^2 &= 1 \\
\omega_c R_6 C_2 &= 1 \\
C_2 &= \frac{1}{\omega_c R_6}.
\end{aligned}$$

With $R_6 = 16\,\mathrm{k\Omega}$ and the desired cut-off $f_c = 16\,\mathrm{kHz}$ (so $\omega_c = 2\pi f_c \approx 1.01 \times 10^5$ rad/s),

$$\begin{aligned}
C_2 &= \frac{1}{(2\pi f_c) R_6} \\
&= \frac{1}{(2\pi)(16 \times 10^3)(16 \times 10^3)} \\
&\approx 6.22 \times 10^{-10} \text{ F} = 0.622 \text{ nF} \approx 620 \text{ pF}.
\end{aligned}$$

Thus the ideal design value is

$$\boxed{C_2 \approx 0.622\,\mathrm{nF} \approx 620\,\mathrm{pF}}.$$

In the lab, the largest available capacitor not exceeding this value was $C_2 = 100\,\mathrm{pF}$. Using this component shifts the pole to a higher frequency:

$$\begin{aligned}
\omega_{c,\text{actual}} &= \frac{1}{R_6 C_2} = \frac{1}{(16 \times 10^3)(100 \times 10^{-12})} = 6.25 \times 10^5 \text{ rad/s}, \\
f_{c,\text{actual}} &= \frac{\omega_{c,\text{actual}}}{2\pi} \approx 9.95 \times 10^4 \text{ Hz} \approx 100\,\mathrm{kHz}.
\end{aligned}$$

So the implemented filter still satisfies the specification ($\omega_c \geq 10^5$ rad/s) while providing even stronger attenuation of higher-frequency interference:

$$\boxed{C_2 = 100\,\mathrm{pF}, \quad f_{c,\text{actual}} \approx 100\,\mathrm{kHz}.}$$

so what is this circuit doing? below are the derivation to see what is happening.

For small AC signals, the bias node $V_4 \approx 2.5$ V is a DC level, so its AC value is effectively zero. Thus, the non-inverting input is at AC ground and the op-amp works as an *inverting* amplifier with:

$$V_2 \xrightarrow{C_1,\,R_5} \text{inverting node}, \qquad V_5 \xrightarrow{R_6 \| C_2} \text{feedback}.$$

Define

$$Z_1 = R_5 + \frac{1}{sC_1} \qquad \text{(series input impedance)},$$

$$Z_2 = R_6 \parallel \frac{1}{sC_2} = \frac{R_6}{1 + sR_6C_2} \qquad \text{(feedback impedance)}.$$

With the inverting-node voltage $\approx 0$ (ideal op-amp), Kirchhoff's current law gives

$$\frac{V_2(s) - 0}{Z_1} + \frac{V_5(s) - 0}{Z_2} = 0,$$

$$\Rightarrow \quad \boxed{\frac{V_5(s)}{V_2(s)} = -\frac{Z_2}{Z_1}}.$$

Substitute $Z_1$ and $Z_2$:

$$\frac{V_5(s)}{V_2(s)} = -\frac{\dfrac{R_6}{1 + sR_6C_2}}{R_5 + \dfrac{1}{sC_1}}$$

$$= -\frac{R_6}{1 + sR_6C_2} \cdot \frac{sC_1}{1 + sR_5C_1}$$

$$= \boxed{\frac{R_6}{R_5}\left(\frac{sR_5C_1}{1 + sR_5C_1}\right)\left(\frac{1}{1 + sR_6C_2}\right)}$$

This shows:

- $C_1$ and $R_5$ form a **high-pass** with cutoff $\omega_{\text{HP}} = \dfrac{1}{R_5C_1}$.
- $R_6$ and $C_2$ form a **low-pass** with cutoff $\omega_{\text{LP}} = \dfrac{1}{R_6C_2}$.
- In between these two cutoffs, the gain tends to the constant $\dfrac{R_6}{R_5}$ (inverting amplifier).

### 2.4.3 Question 3: Low-Pass Filter

Configure your waveform generator to 100 mV amplitude 1 kHz sine wave using the AD2 signal generator and connect it to V2. Then use your oscilloscope to measure the output at V5. You should see a sine wave at V5. Measure the amplitude of the output sine wave at V5. Vary the frequency to see it's effect on the output amplitude. Here are my result for 1khz, 100khz, and 1mhz:

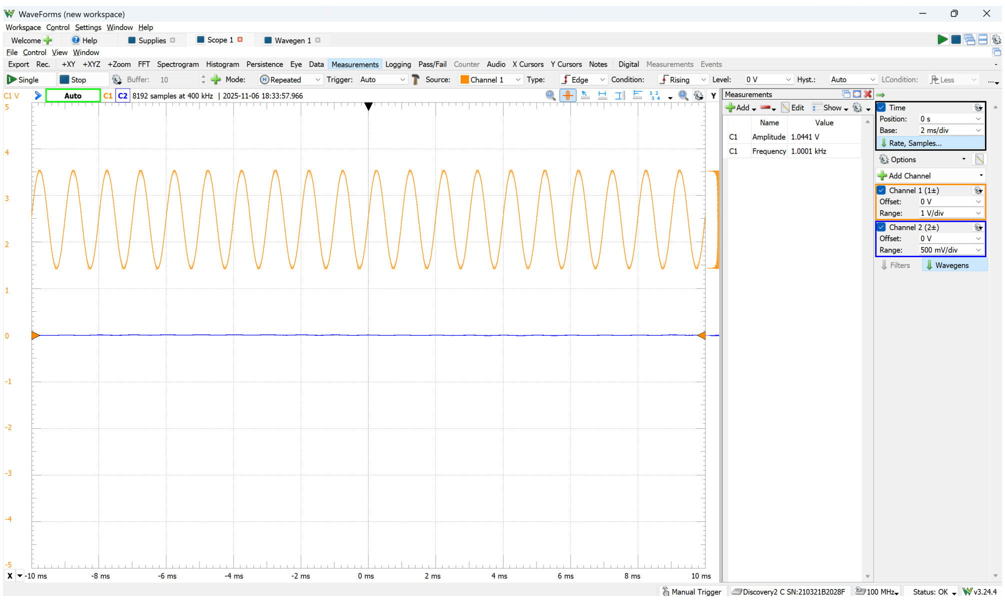


Figure 6: Waveform at $V_5$ with 1 kHz input.

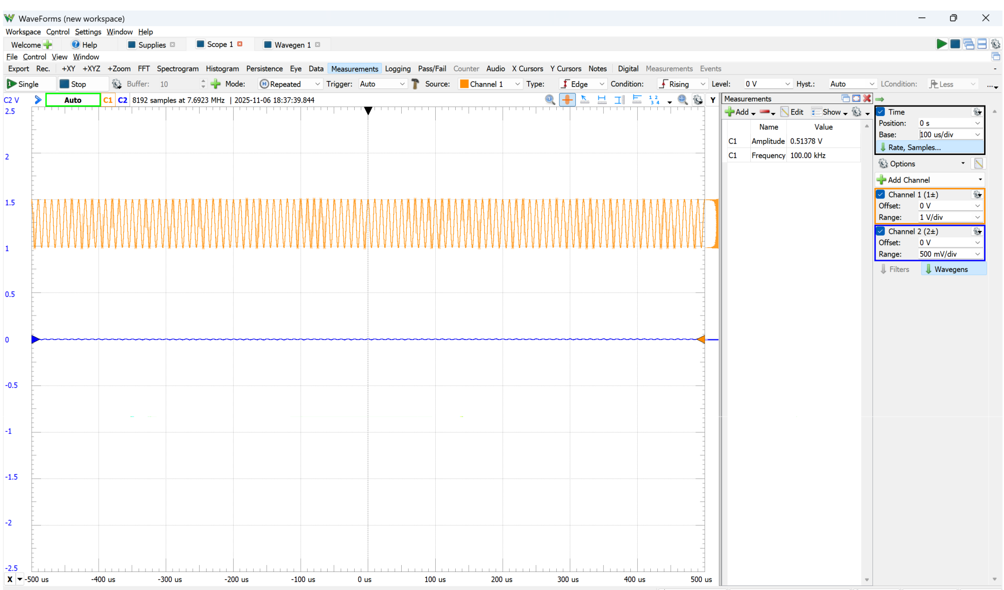


Figure 7: Waveform at $V_5$ with 100 kHz input.

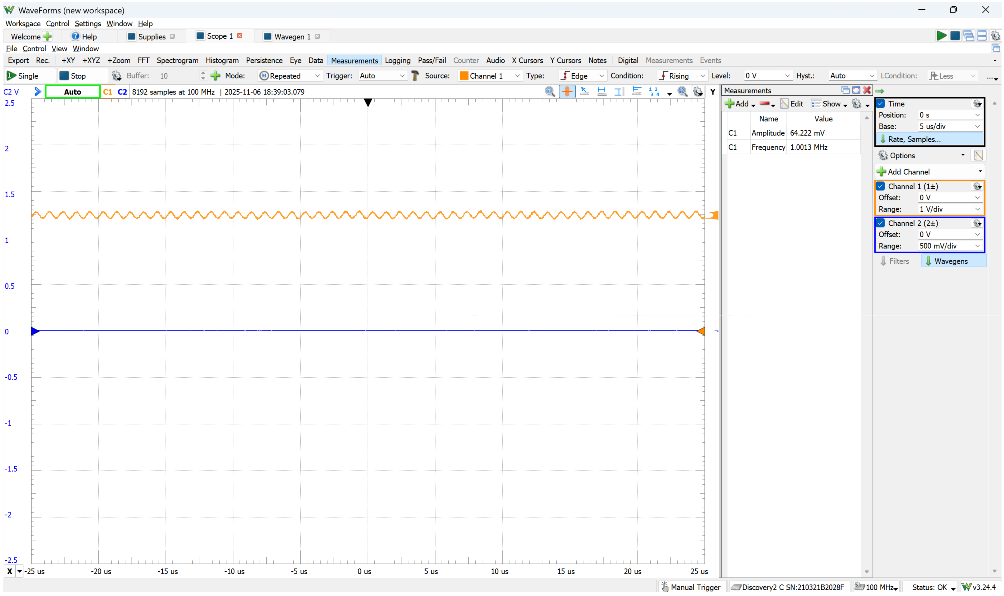

Figure 8: Waveform at $V_5$ with 1 MHz input.

## Frequency response and output amplitudes

Let the input at $V_2$ be a sine wave

$$v_2(t) = \hat{V}_2 \sin(\omega t), \qquad \hat{V}_2 = 0.1 \text{ V (100 mV)}.$$

The complex gain at frequency $\omega$ is

$$A(j\omega) = \frac{V_5(j\omega)}{V_2(j\omega)} = \frac{R_6}{R_5} \left( \frac{sR_5C_1}{1 + sR_5C_1} \right) \left( \frac{1}{1 + sR_6C_2} \right).$$

The output amplitude is

$$\hat{V}_5 = |A(j\omega)|\, \hat{V}_2.$$

If we go through the calculations, these are the amplitude modulations at different frequencies for our circuit (assuming R6/R5 = 10):

$$\boxed{\hat{V}_5 \approx \begin{cases} 9.99999 \text{ V}, & f = 1 \text{ kHz}, \\ 5.3 \text{ V}, & f = 100 \text{ kHz}, \\ 0.62 \text{ V}, & f = 1 \text{ MHz}. \end{cases}}$$

Our result are similar to our calculation (1.044 V, 0.514 V, 64.2 mV) are of the same order, so everything seems to check out. I believe this is reasonable given tolerances in the actual resistor/capacitor values, and can also be caused by the bandwidth and slew-rate limits of the real op-amp.

### 2.4.4 Question 4: Response Checks

The next step is to check if your circuit works for Exericse 3 number 4, 5, 6. This is the perfect time for you to test if your circuit has the full 2.5V range. If the range is bad, please first try aligning your photodiode, because that might be the number one cause of this issue.

## 2.5 Exercise 5: Assemble Complete Circuit

### 2.5.1 Question 1: Integrate Stages

Finally, we are done with our circuit, the next step is to wire together the transimpedance amplifier (Exercise 2), AC amplifier (Exercise 3), and rectifier/low-pass chain (Exercise 4) to form the complete sensor. After this check all the connections again to make sure everything is correct. Once you are sure everything is correct, power up the circuit and test if it works as expected.

### 2.5.2 Question 2: Constrain Output Range

If everything is correct, you should be able to see the output voltage $V_{\text{out}}$ vary between 0 V and 2.5 V as you sweep the LED along the rail. It's okay if you don't get the full range, as long as it's close to 2.5V that's good enough.

# 3 Phase 3: Data Acquisition and Calibration

## 3.1 Exercise 6: Firmware and C# Acquisition

### 3.1.1 Question 1: MSP430 Firmware

The firmware on the MSP430FR5739 was written to sample the low-pass output with the on-chip 10-bit ADC referenced to the 3.3 V rail. Each conversion was formatted into a three-byte packet `[255, MS5B, LS5B]` where the most- and least-significant five bits occupy separate bytes to simplify parsing on the PC.

### 3.1.2 Question 2: C# Application

The companion C# program:

a) opened the appropriate serial port and maintained continuous communication,

b) recombined the MSBs and LSBs into a single 10-bit code,

c) displayed and plotted the live data stream while logging to disk, and

d) implemented a basic UI showing instantaneous voltage and providing controls for calibration capture.

## 3.2 Exercise 7: Calibration and Resolution

### 3.2.1 Question 1: Distance Sweep

Measurements were recorded at a minimum of five LED-photodiode separations across the full mechanical travel. Each point stored both the ADC code and the physical distance measured with a ruler on the extrusion.

### 3.2.2 Question 2: Curve Fit

The voltage-to-distance data were fitted with a smooth function (e.g., inverse power). Plotting both the raw scatter and the fitted curve in the report allowed visual confirmation that the model captured the sensor response with minimal residual error.

### 3.2.3 Question 3: Conversion to Position

The inverse of the fitted function was implemented in software so that each ADC code could be converted in real time to an estimated separation distance.

### 3.2.4 Question 4: UI Enhancements

The C# application was updated to display both raw ADC values and converted position simultaneously. Additional logic signaled when the sensor moved outside the characterized range, satisfying the lab requirement for an out-of-range indicator.

### 3.2.5 Question 5: Noise Characterization

With the slider set near mid-range, the converted position was logged for roughly 10 s and its standard deviation was interpreted as RMS noise. The experiment was repeated near both extremes of travel, and differences in noise level were attributed to the varying sensitivity (slope of the calibration curve) at those points.