THE UNIVERSITY OF MELBOURNE

Elasticity, Isostasy and Dynamic Topography

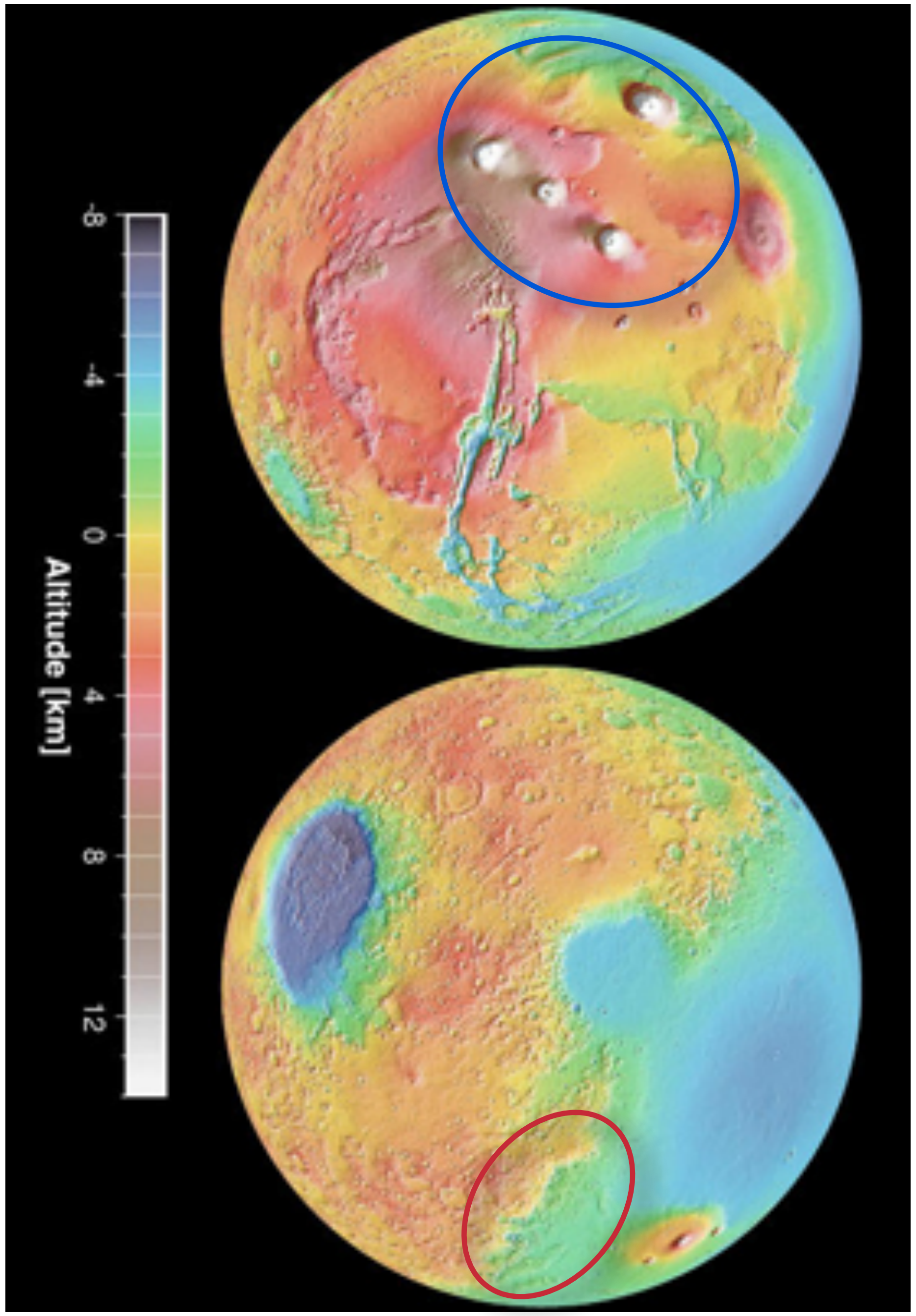
Louis Moresi

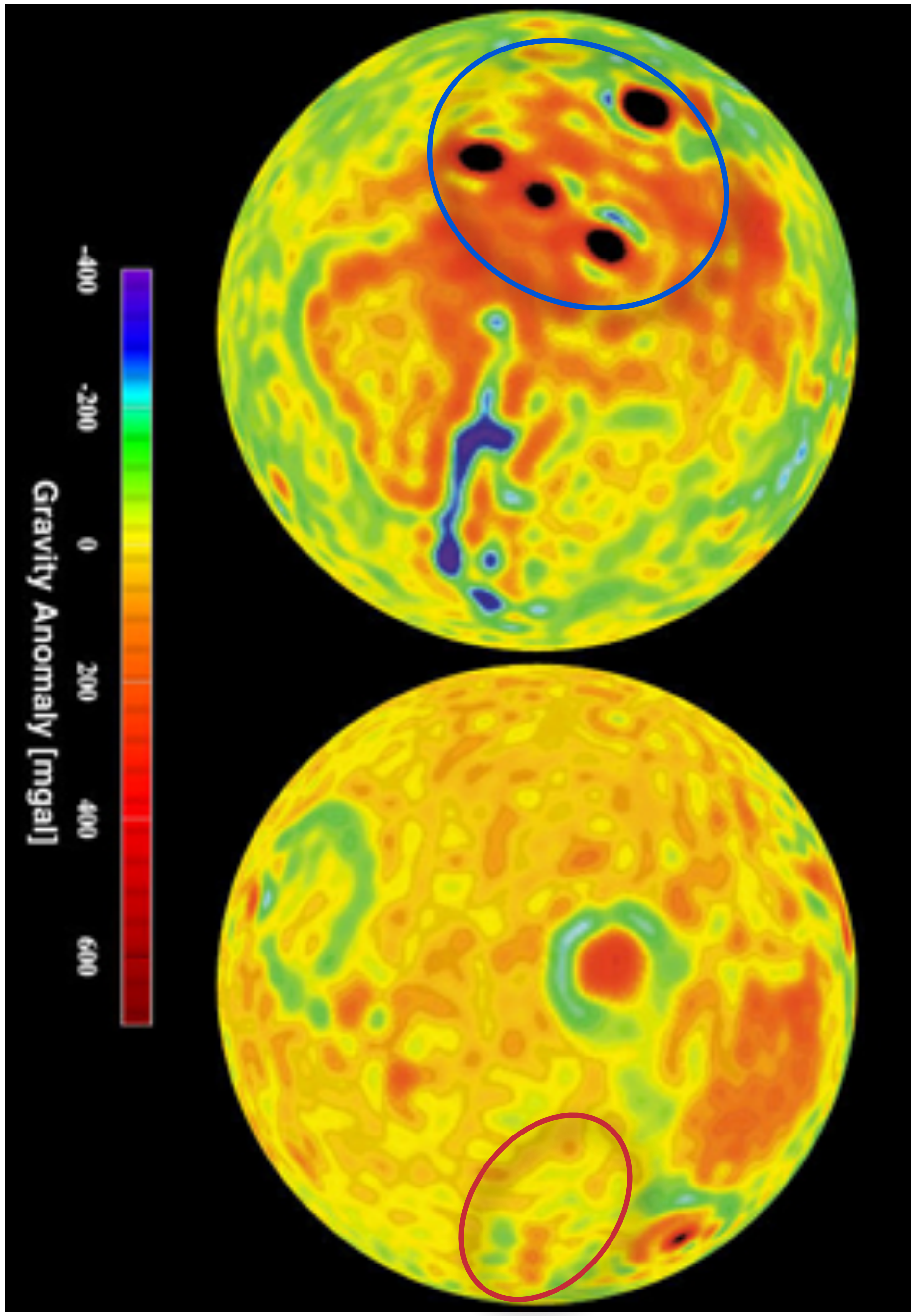
louis.moresi@unimelb.edu.au

Overview

- Recap
    - Gravity / topography
- Bending of beams / plates
- Elasticity and Flexure v. Isostasy
- Examples
    - Seamount loads
    - Plate bending at subduction zones
    - Passive margins
- Elastic thickness determination
- Post glacial rebound

Gravity & topography - elastic v. isostatic

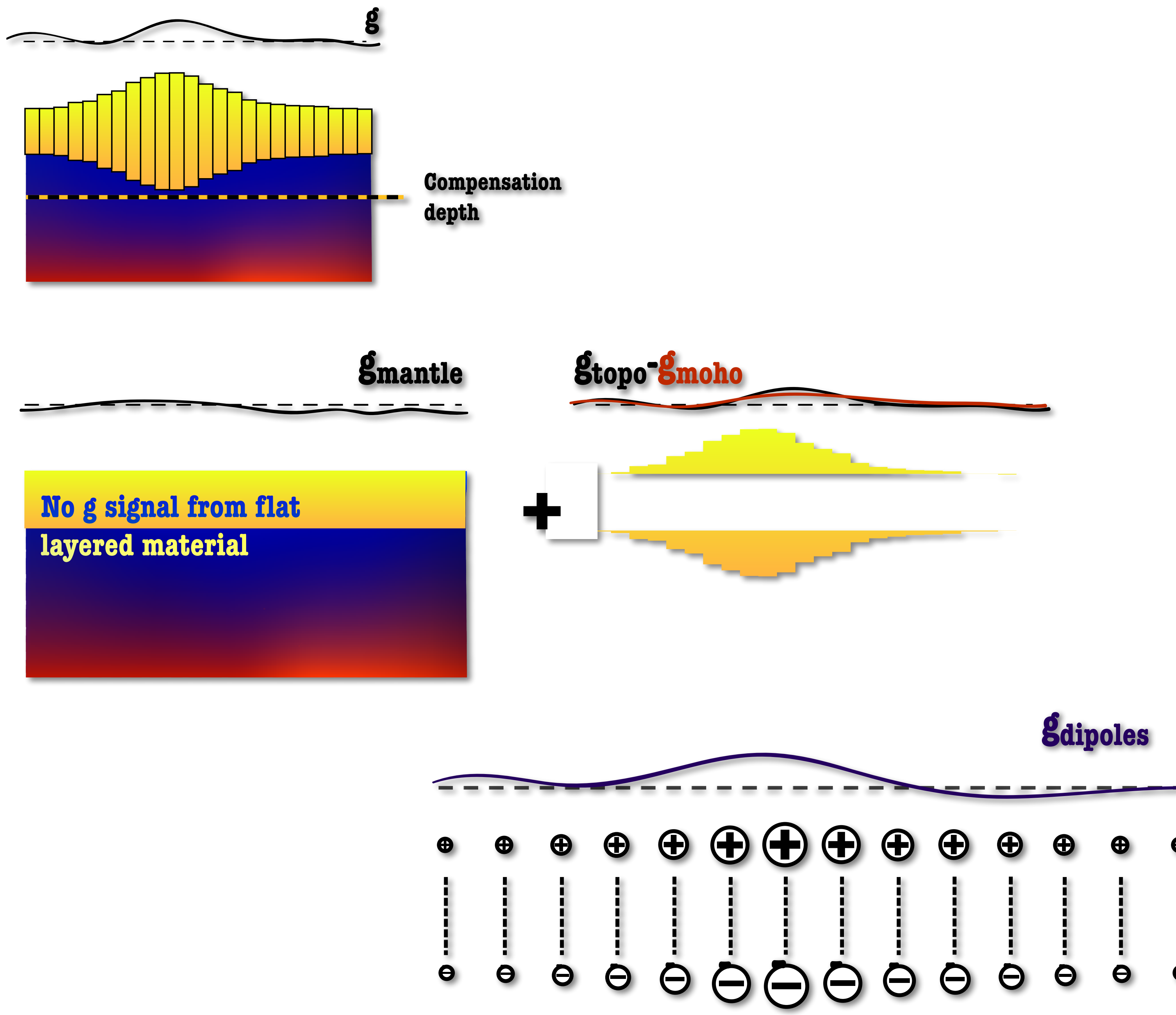




- crustal thickening,
- impact craters
- tharsis rise
- volcanic edifices.

Gravity anomaly and Isostasy

The lack of gravity (anomaly) signature associated with continental topography on Earth is explained simply by ISOSTASY — wherever there is a topographic high, there is a corresponding crustal root which produces a gravitational dipole



Equipotentials

gravity — the force of nature which manifests itself as an attraction between objects with mass.

The gravitational force between two objects is

$$\boxed{F = \frac{Gm_1m_2}{r^2}}$$

If one of the bodies is planet sized and spherical then the acceleration experienced by a nearby object is

$$\boxed{g = \frac{GM}{r^2}}$$

It takes energy to lift an object against the force of gravity This produces a gravitational potential energy:

$$\Delta E = GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Equipotentials

Energy required to take a test mass from some point above a spherical planet out to an infinite distance:

$$\boxed{U = \frac{GM}{r}}$$

Surfaces where U is constant are spherical. These surfaces are perpendicular to the direction of the gravitational force.

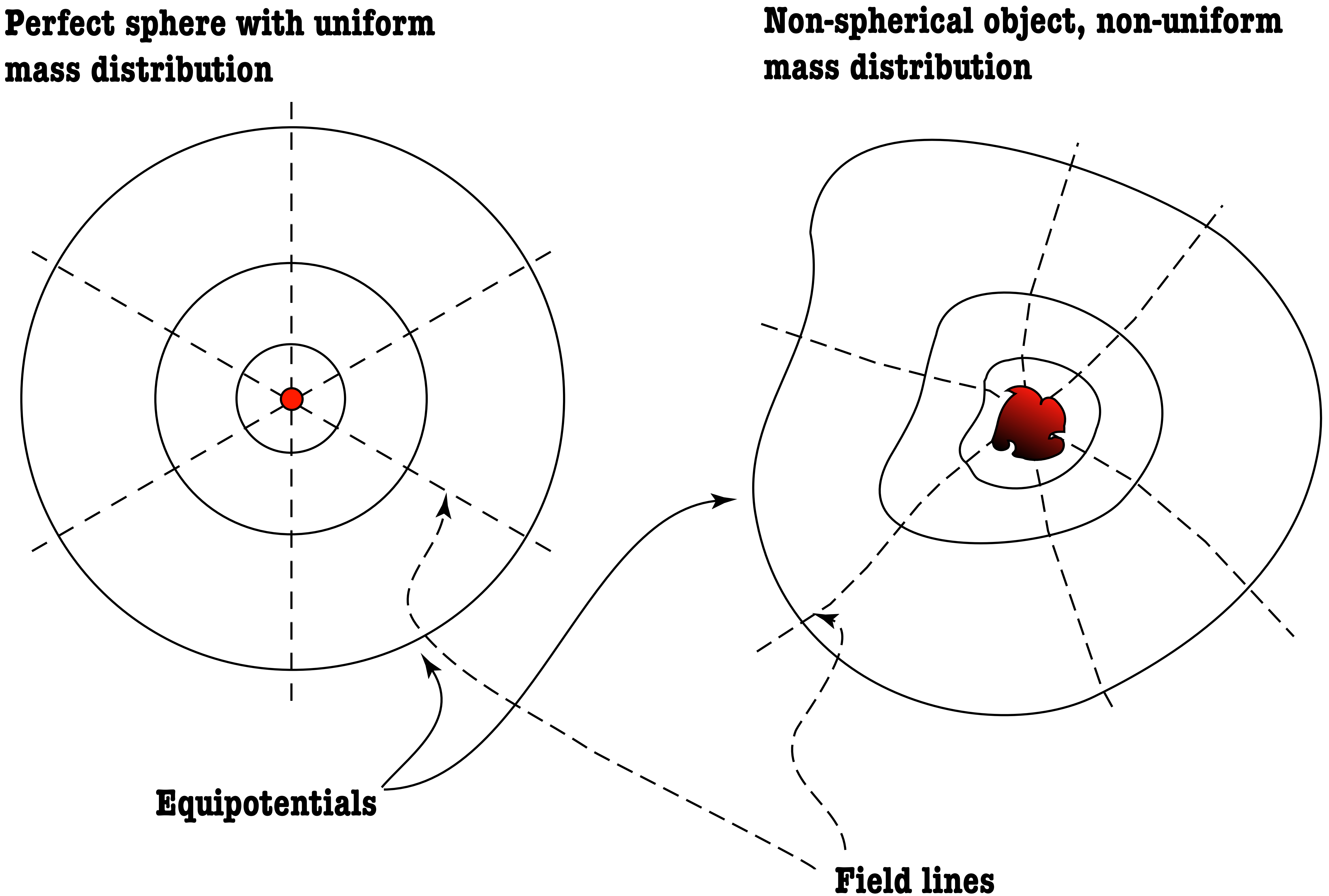
In the realistic, non-spherical planet, the **equipotentials** are not spherical and the field lines are not radial.

gradient vector

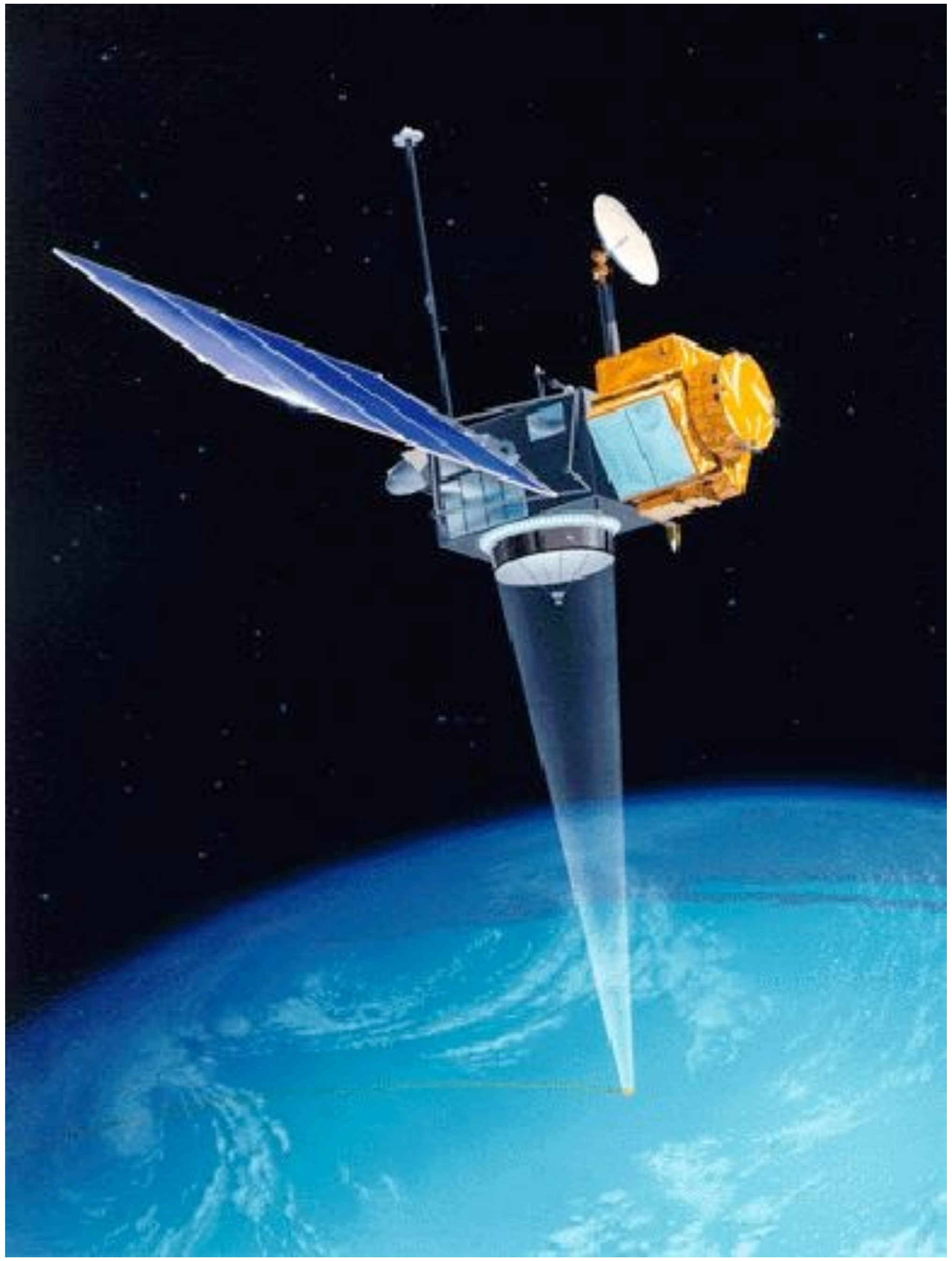
$$g = -\nabla U$$

U contains all the information about the gravitational field g plus a constant of integration.

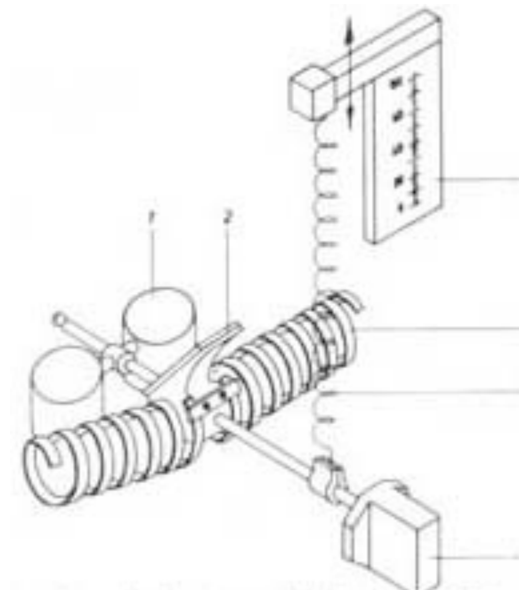
Equipotentials



Geoid/gravity measurement



Ocean — sea surface height

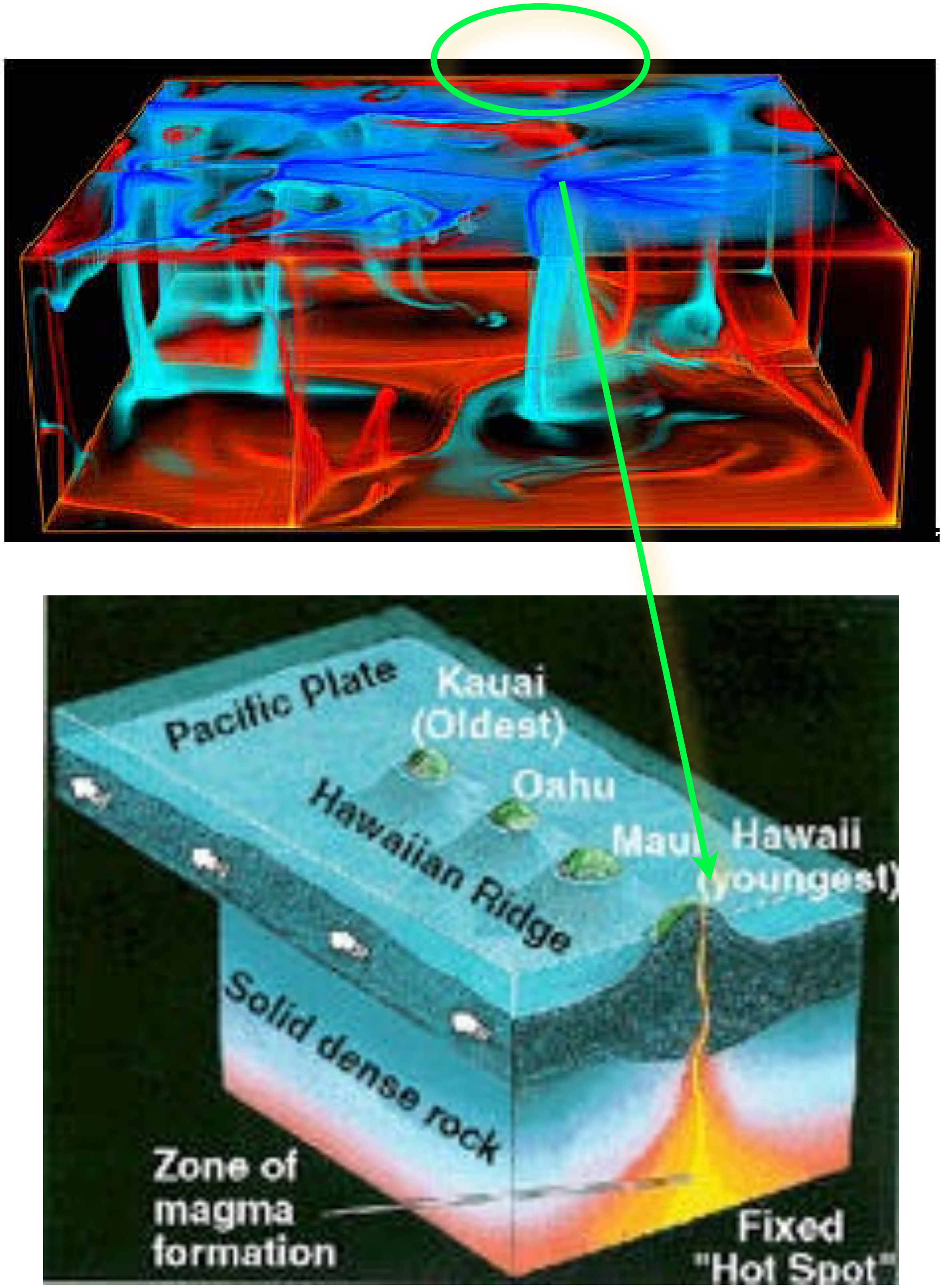


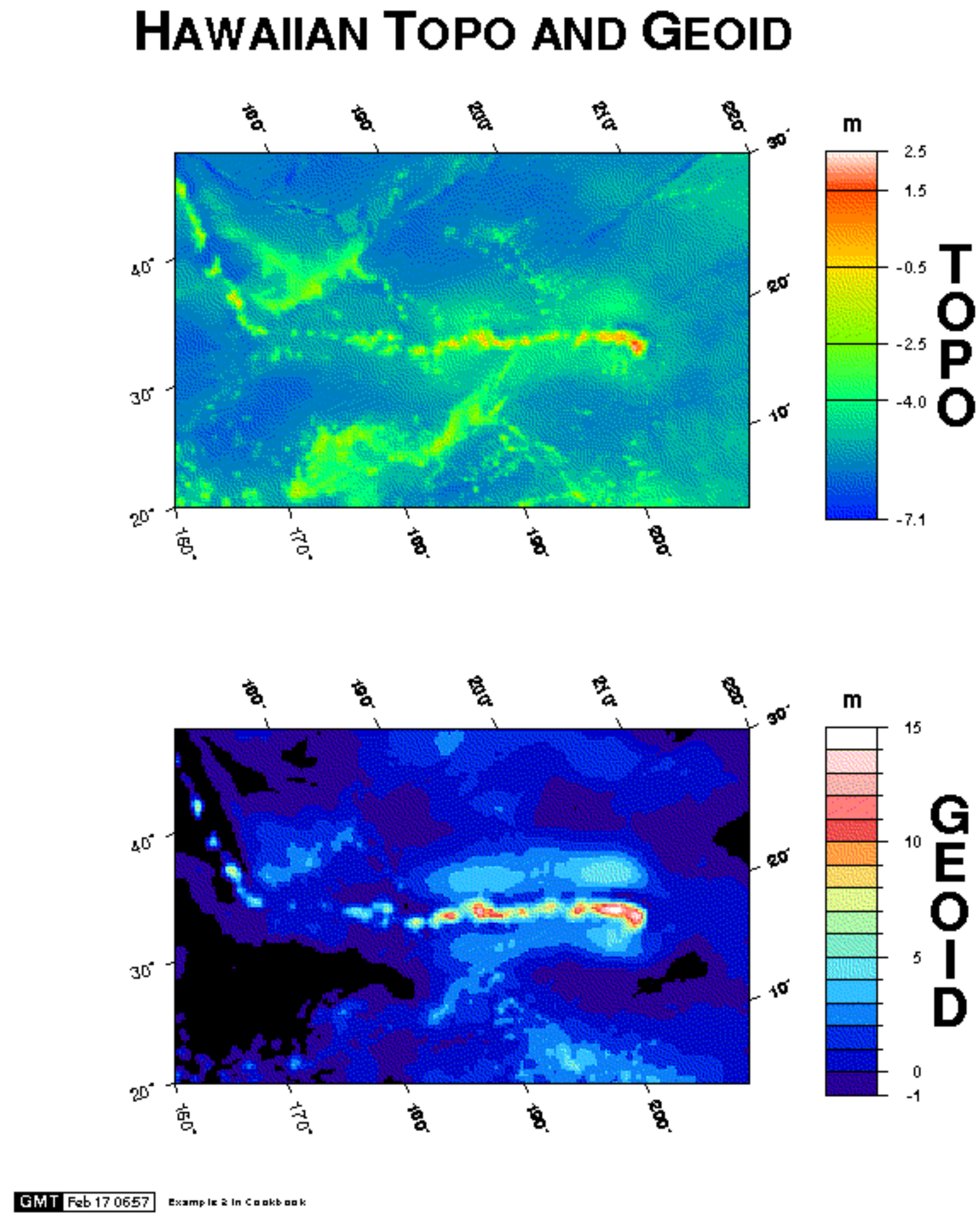
1. Capsul for barometric compensation
2. Mirror
3. Micrometric scale
4. Measuring spring
5. Main springs
6. Mass

p.267 "Tides of the Planet Earth" - Paul Melchior

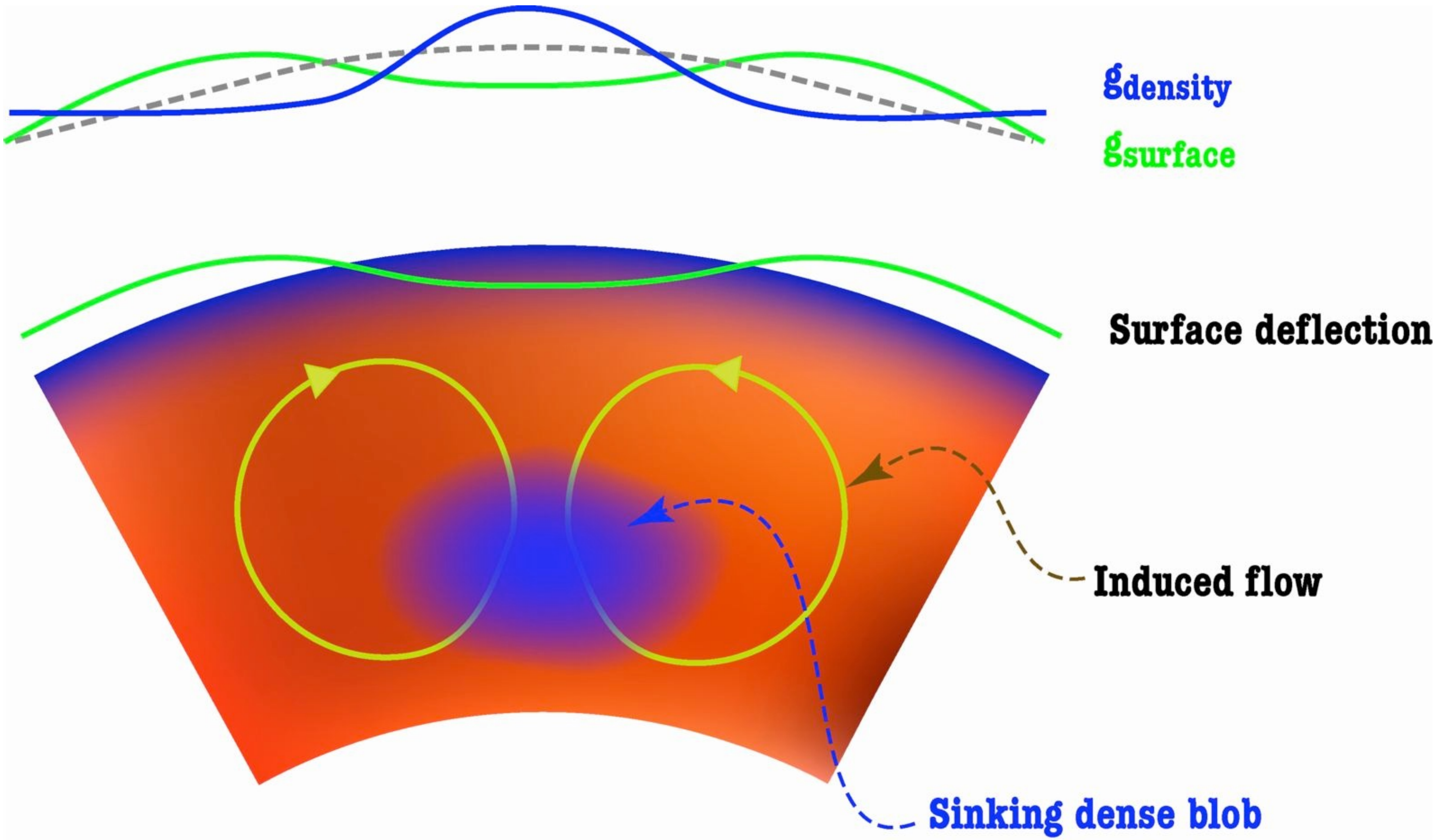
Land — strength of gravitational acceleration

Example: mantle plumes



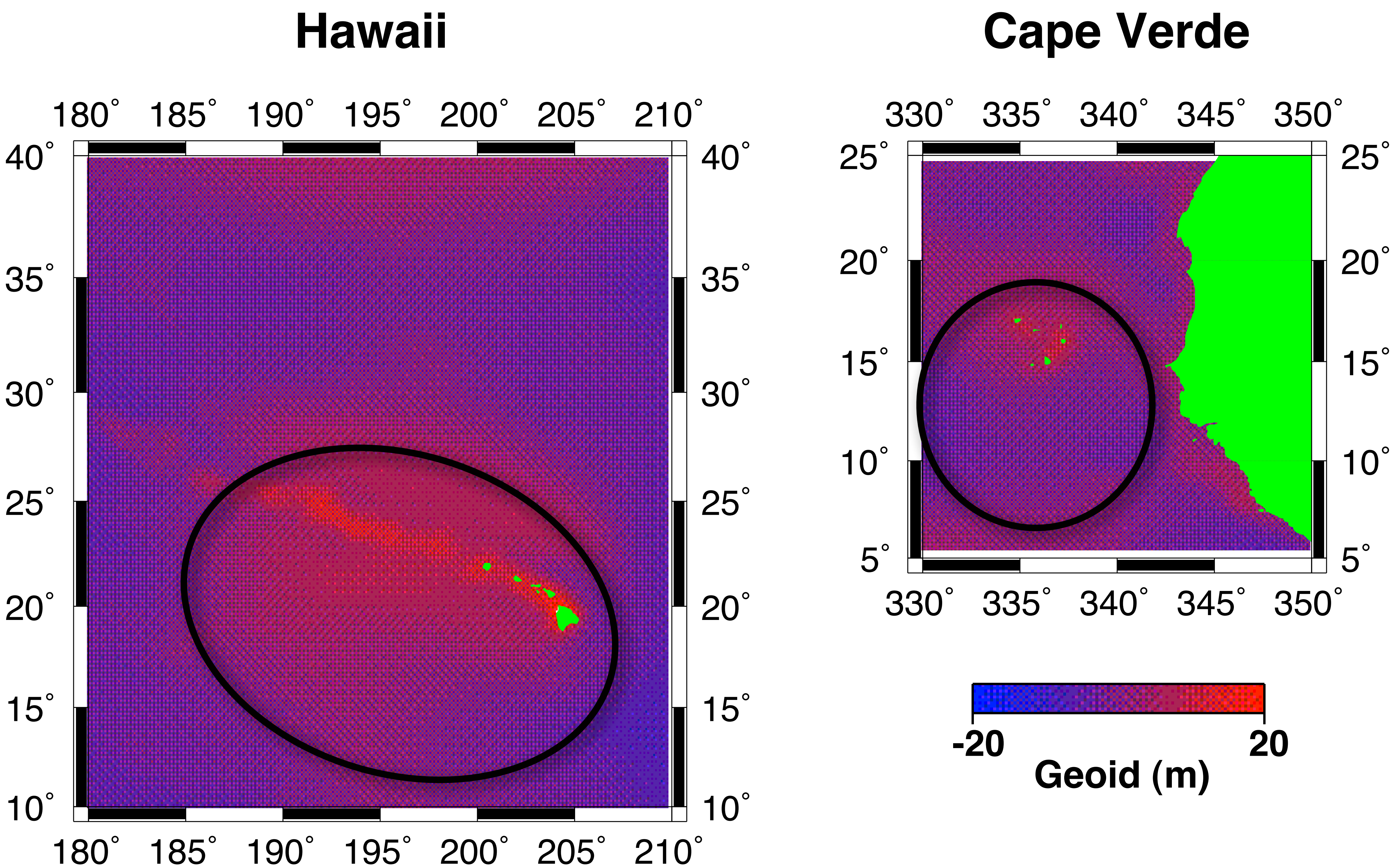


Dynamic topography / geoid



Dynamic topography (in the context of geodynamics) specifically means that topography which results from viscous flow in the interior of the Earth.

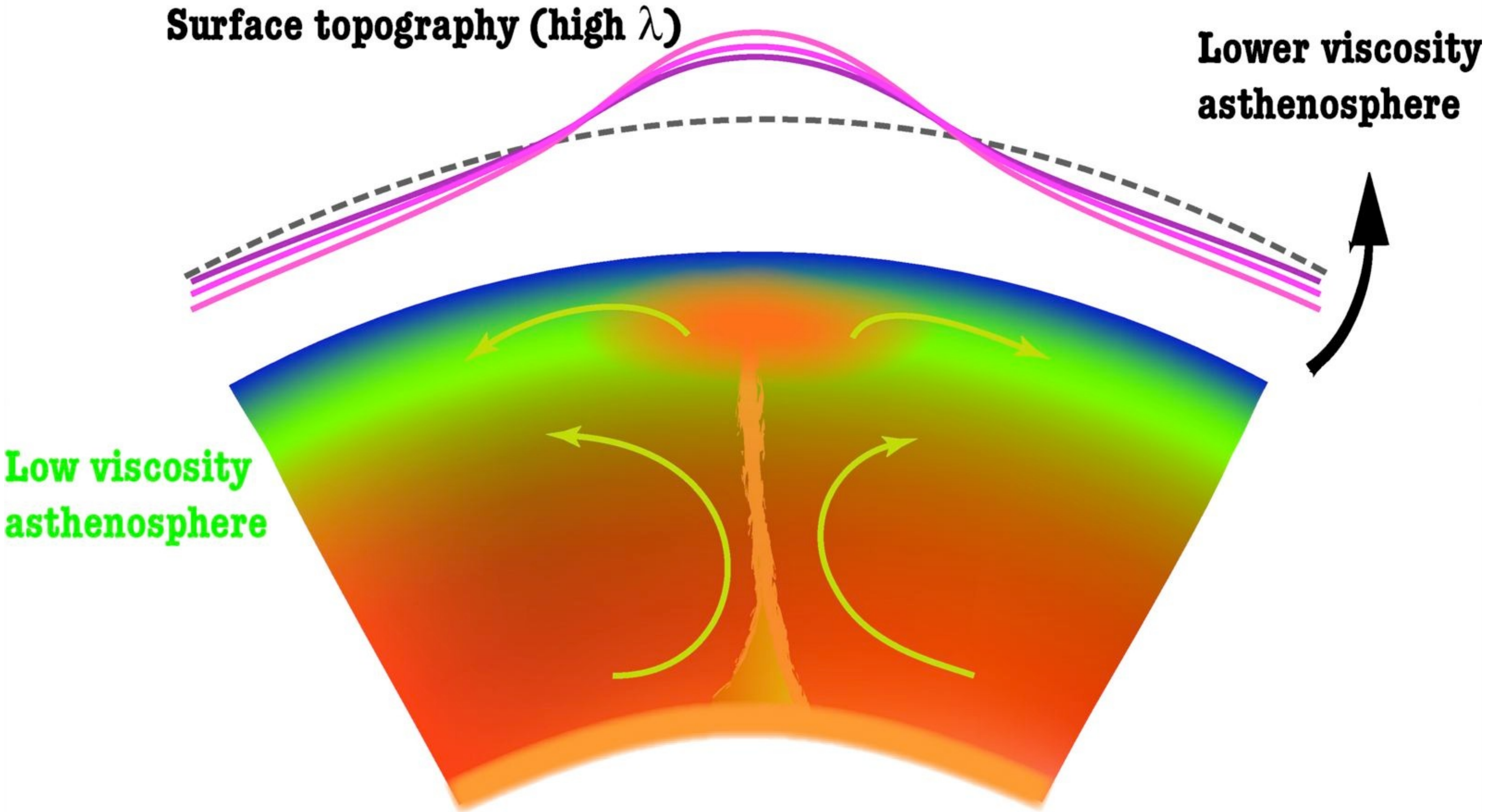
Geoid over a plume



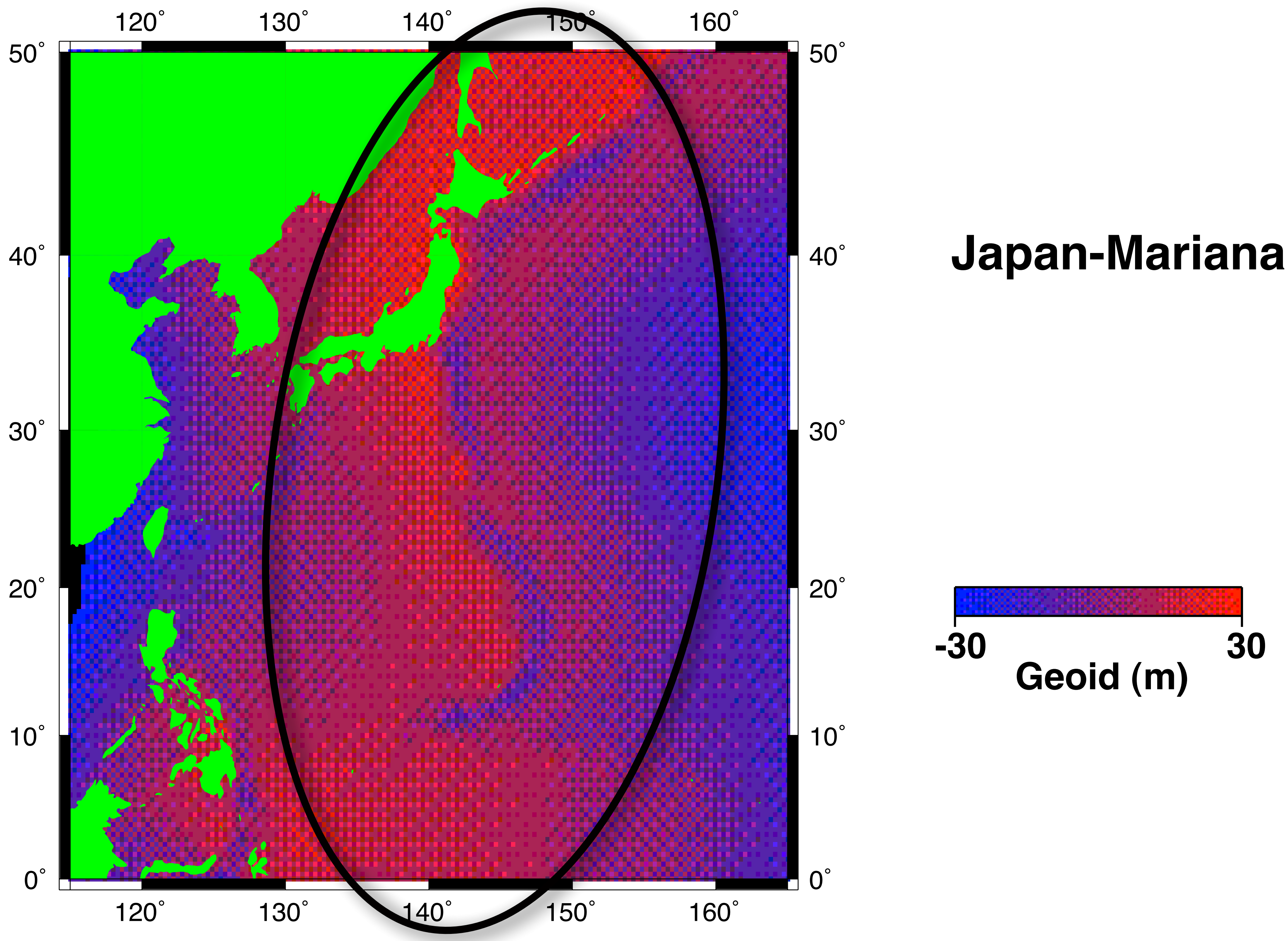
Broad scale geoid high over upwelling material

Dynamic topography / geoid of plume

Light material directly beneath the lithosphere causes significant uplift of surface

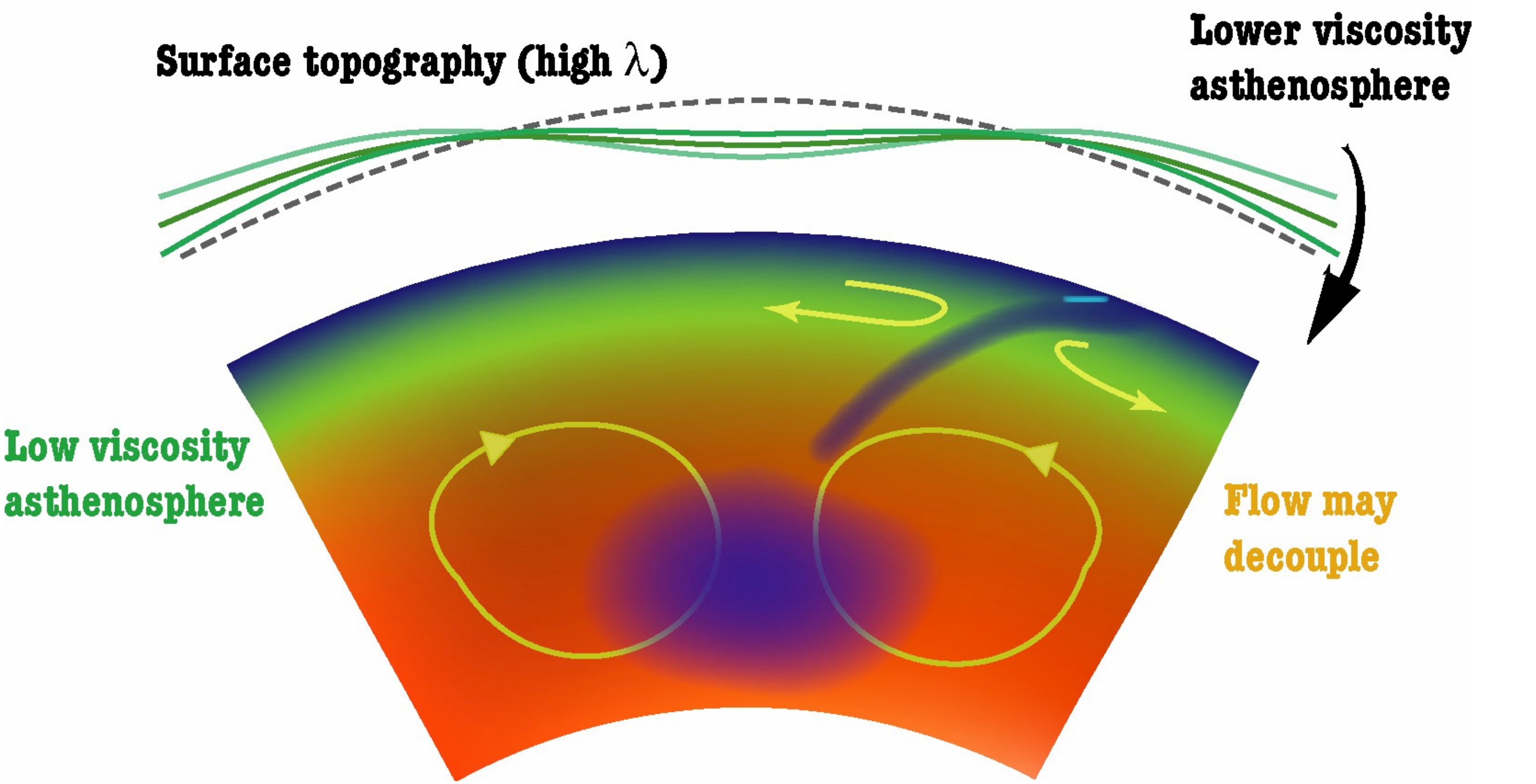


Geoid over subduction zone



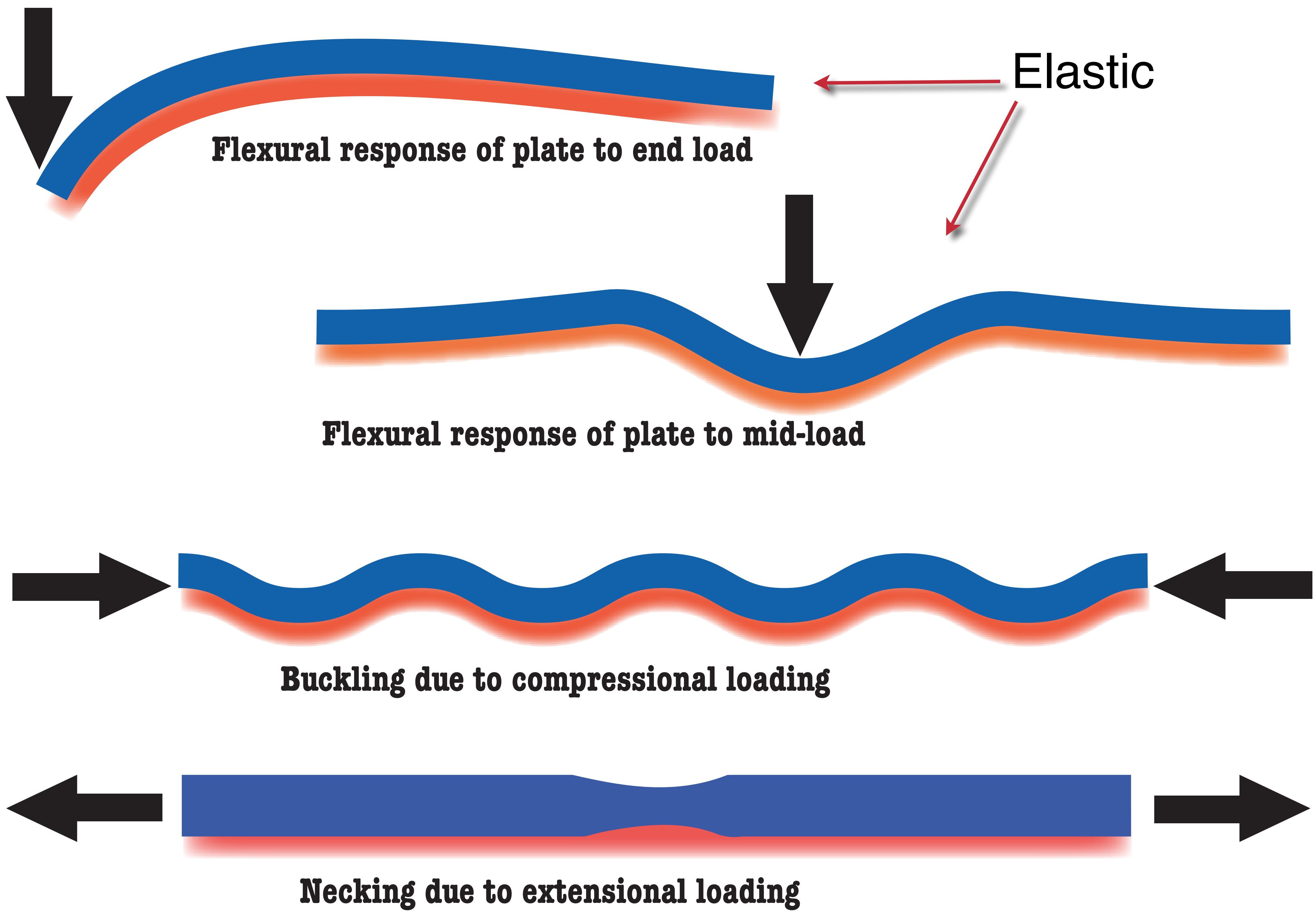
Broadscale geoid high over subducted material

Geoid over subduction zone

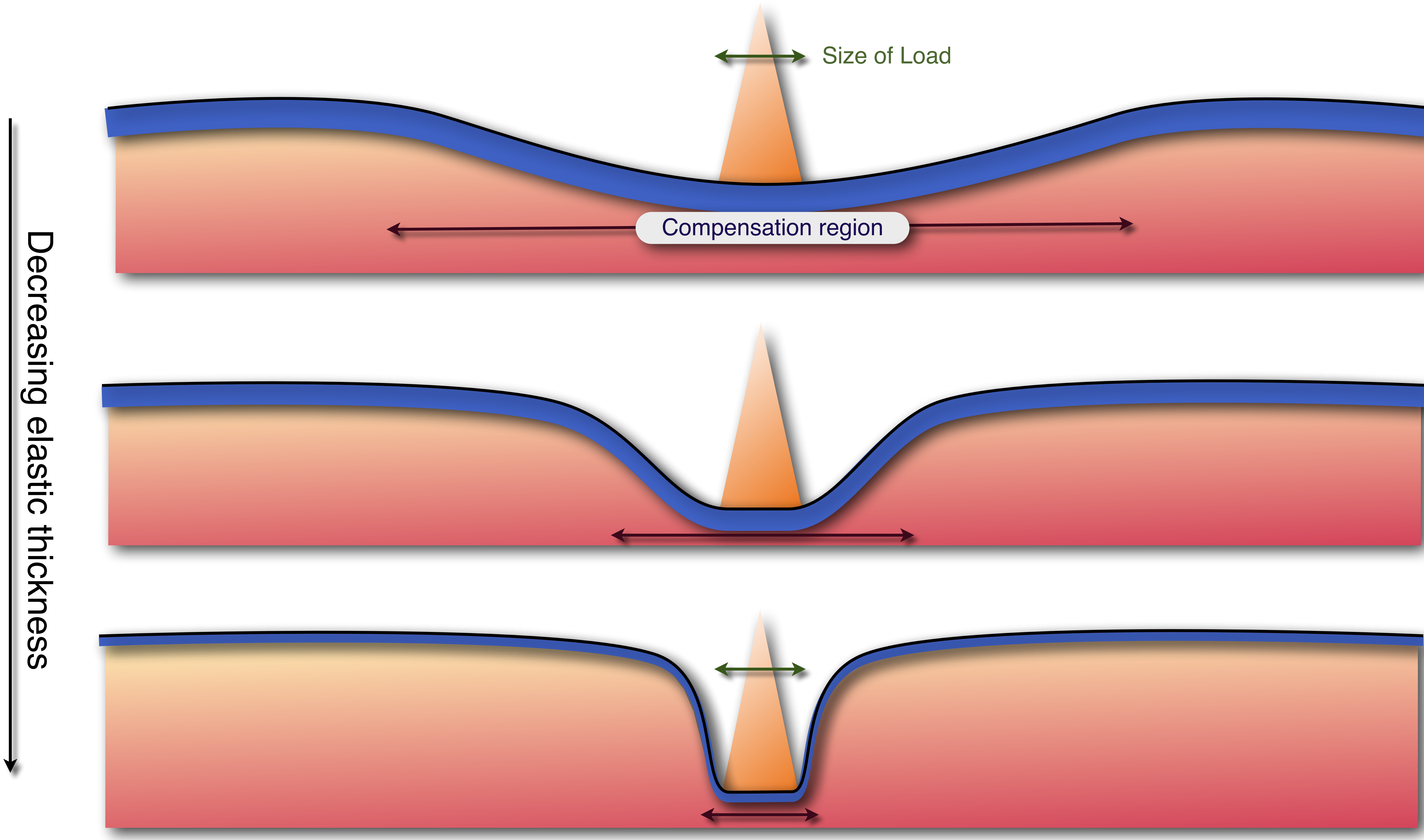


Dynamic topography driven weakly over a large area by deep mass, strongly local to the trench by the attached slab
Geoid contribution from the deep mass doesn't care about the details of the trench

Some deformation modes of the lithosphere

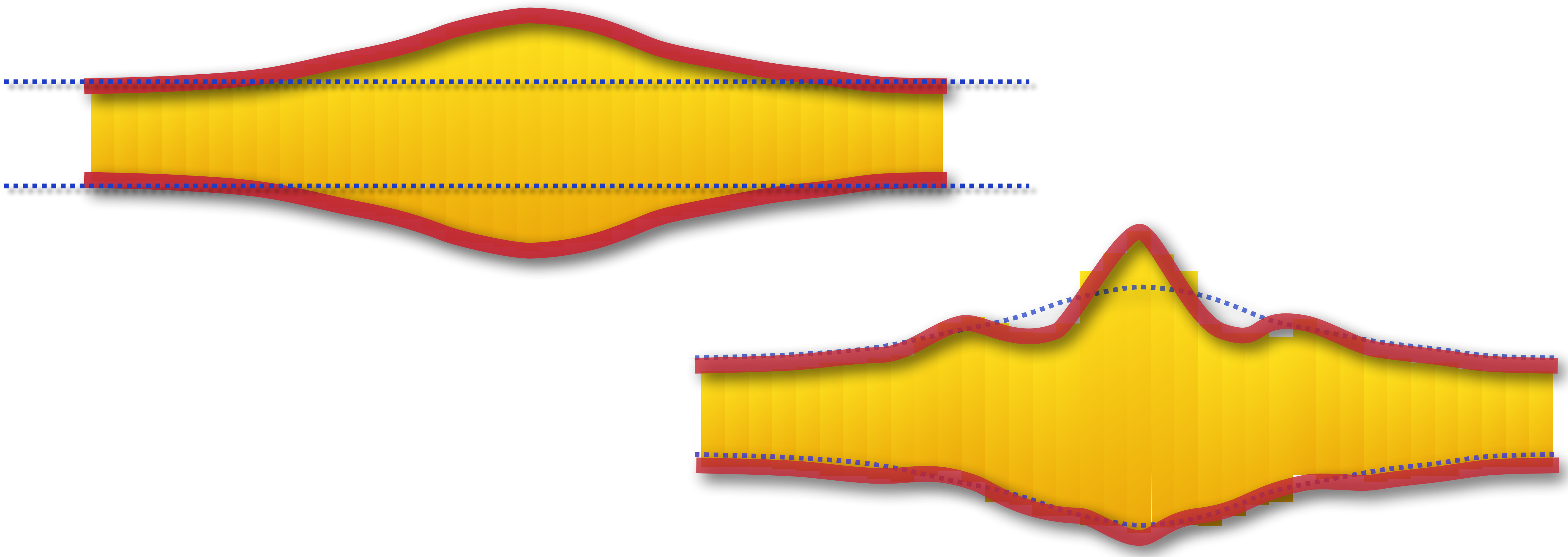


Flexure v. isostatic compensation



When the size of the load and the region compensating it become equal this becomes the Airy isostasy model; the load subsides until it is "floating"

Isostasy & elastic support



The wavelength-dependence of the elastic response means that short wavelength (~narrow) loads on thick lithosphere are more likely to be supported by elastic strength than long-wavelength (~wide) loads.

The exact transition between elastic and isostatic support depends upon the elastic thickness of the lithosphere (and the time allowed for the loads to equilibrate)

Plate flexure

We can imagine the lithosphere to be an elastic skin floating on a low-viscosity mantle.

Engineers know very well how to deal with plates and beams.

This is the kind of thing that is used routinely in building structures and machines.

In 2D, the general equation for an (infinite) elastic plate which has horizontal (q) and vertical forces (P) on it is ...

$$\boxed{D\frac{d^4w}{dx^4} = q(x) - P\frac{d^2w}{dx^2}}$$

$$D = \frac{ET_e^3}{12(1-\nu^2)}$$

NB

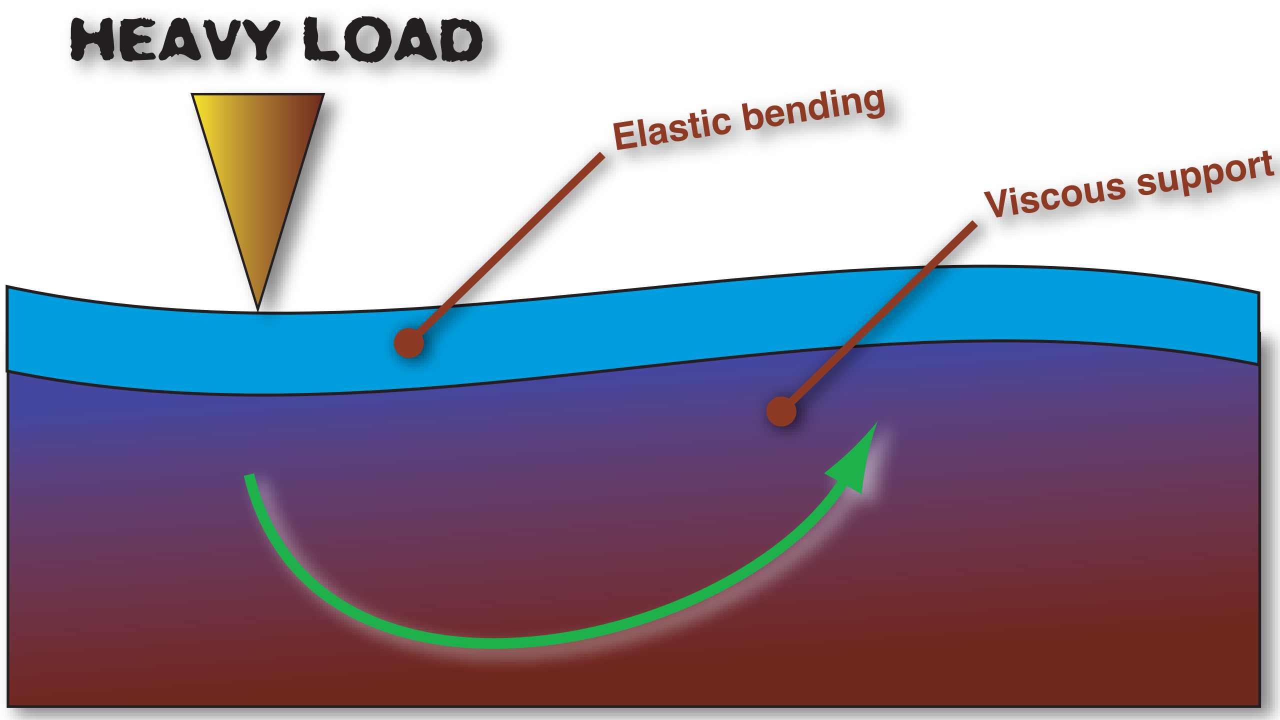
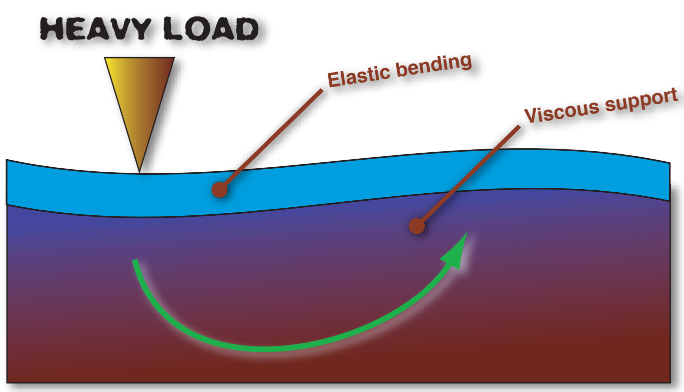


Plate flexure — solutions

A simple load (q) we could consider would be a sinusoidal variation with a single wavelength



$$q(x) = \rho_c g h_0 \sin kx$$

We can assume one or two things, **including the form of the solution**, and that $P = 0 \ldots$

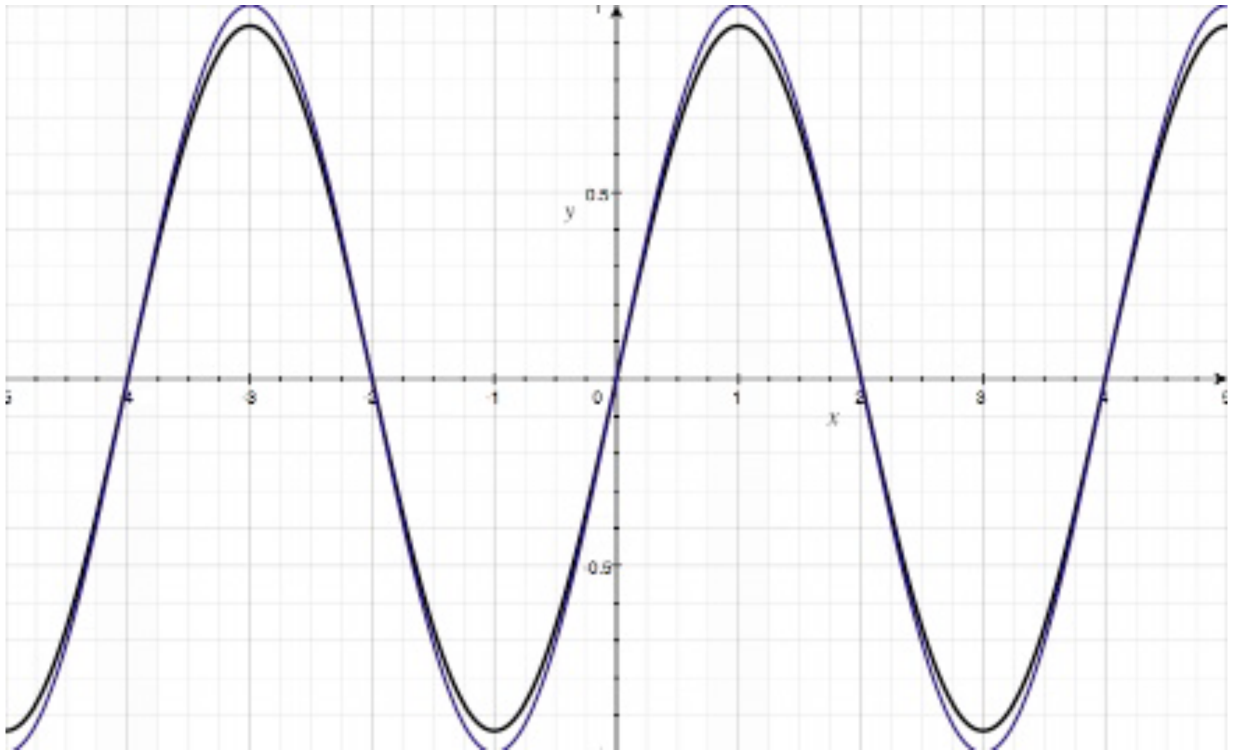
$$w = w_0 \sin kx$$

which makes everything a lot simpler - now we just subsitute into the equation.

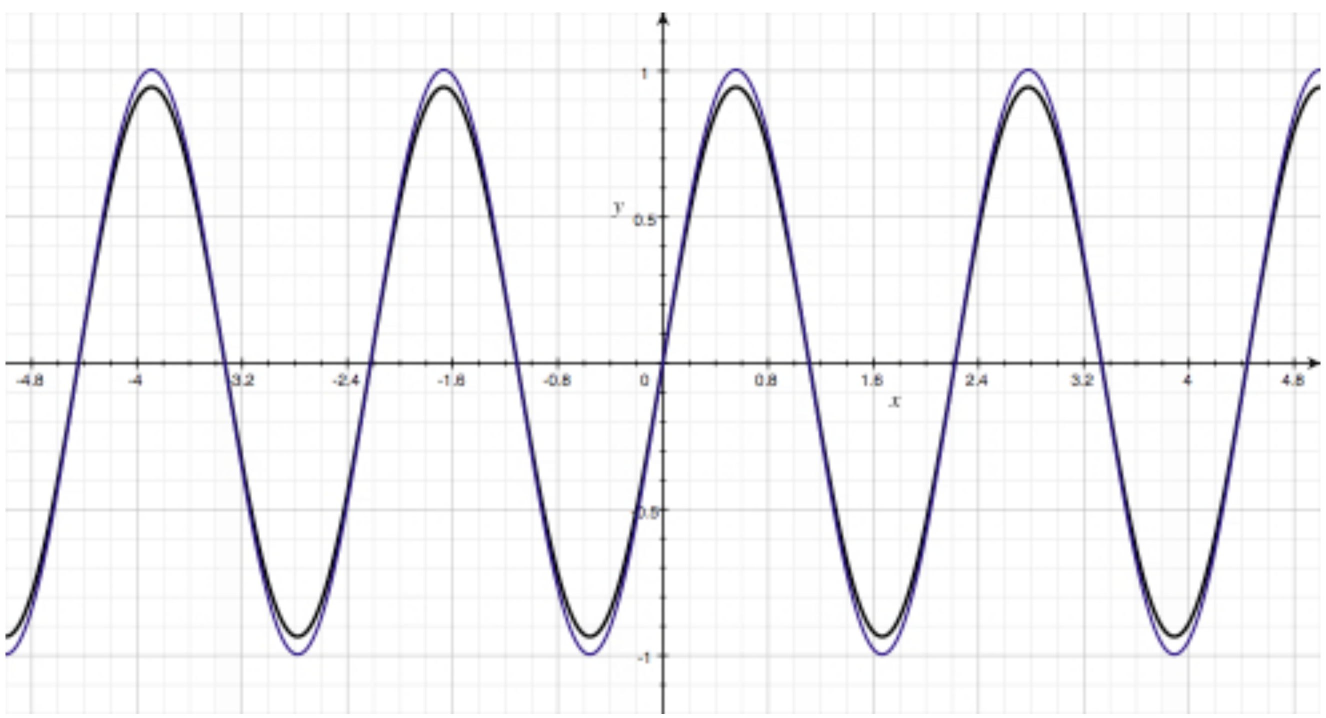
$$w_0 = \frac{h_0}{(\rho_m/\rho_c - 1) + \frac{D}{\rho_c g} k^4}$$

Short wavelength loads ($k \gg (D/(\rho_c g))^{1/4}$) produce almost no deflection of the lithosphere.

Plate flexure — solutions



Changing wavelength



Changing Rigidity

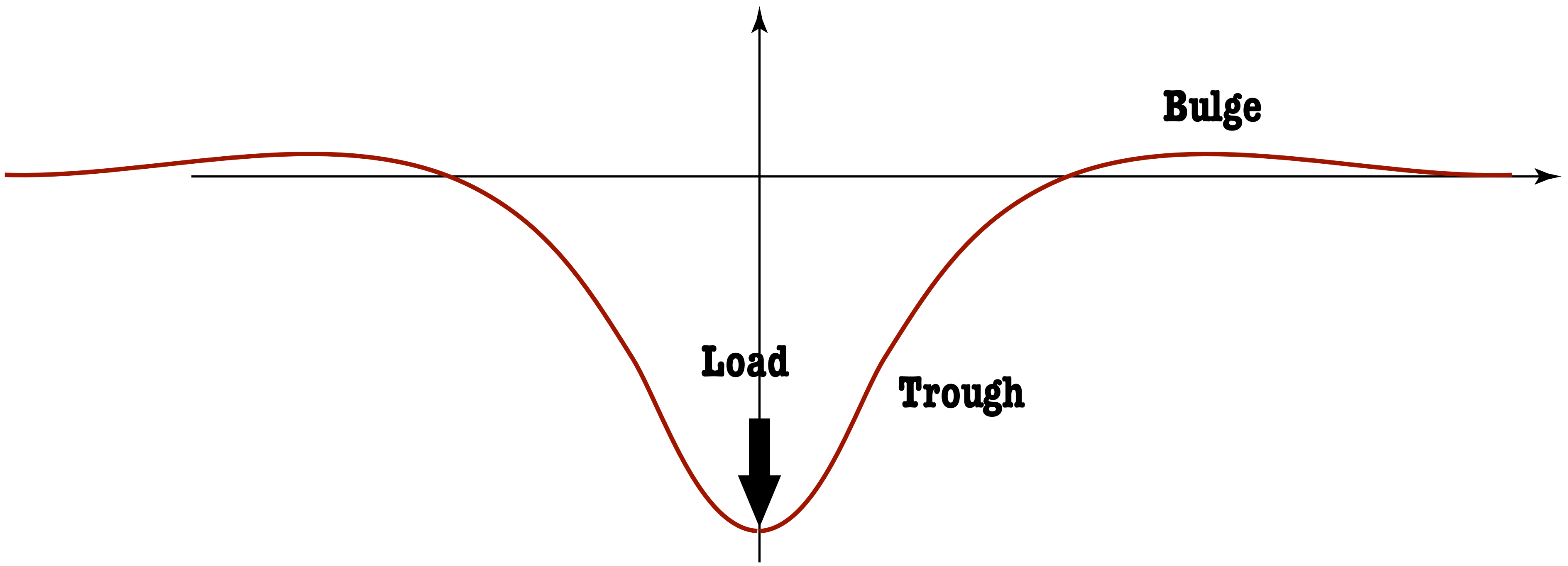
Example: Chain of seamounts

A chain of seamounts trailing behind a hotspot look like a line-load on the 3D plate. A cross section can be solved in 2D

$$w = \frac{V_0 \alpha^3}{8D} e^{-x/\alpha} \left(\cos x/\alpha + \sin x/\alpha \right) \quad (x \geq 0)$$

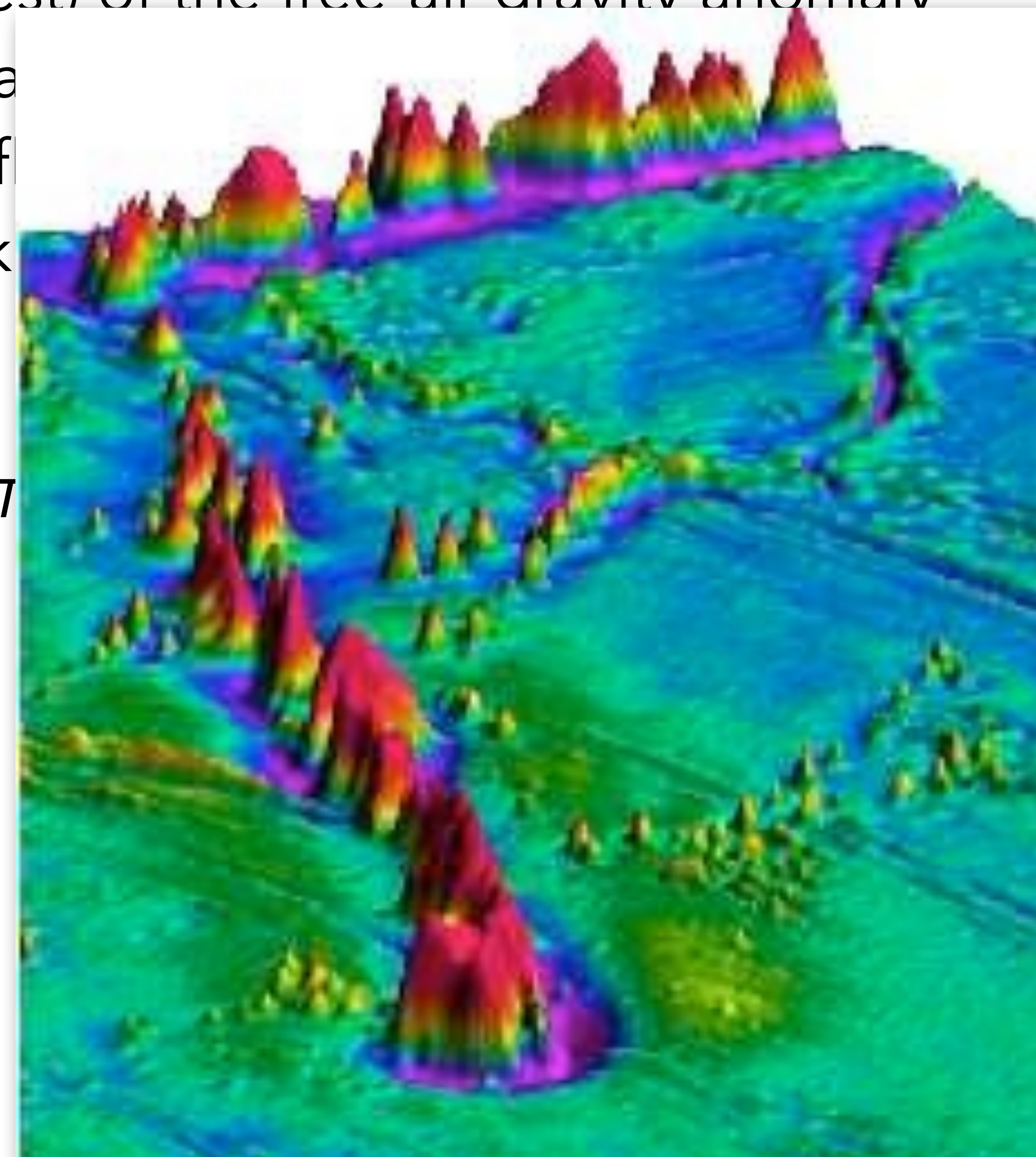
Where $\alpha = (4D/(\rho_m - \rho_w)g)^{1/4}$.

Which looks like this:



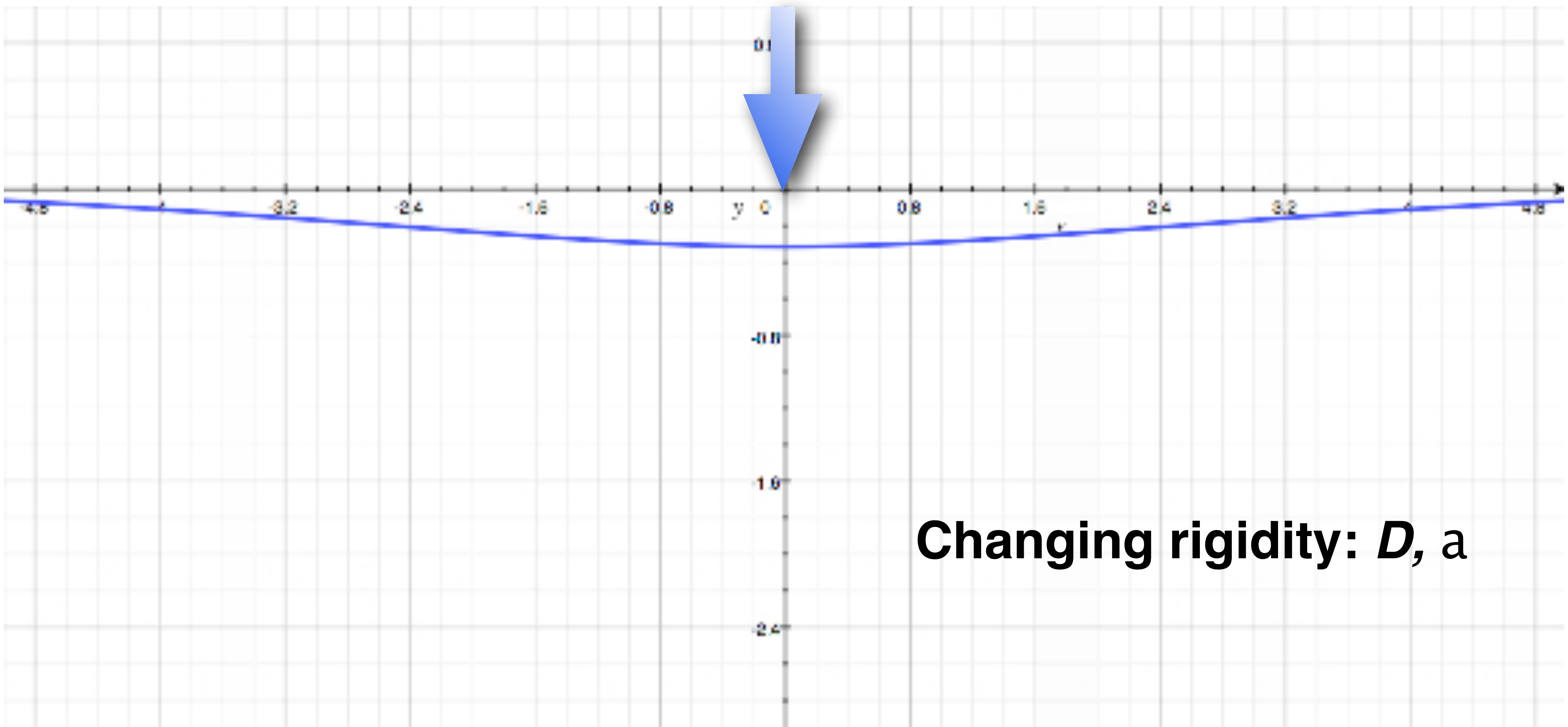
Example: Chain of seamounts

Perspective image (looking towards the west) of the free-air gravity anomaly along the Hawaiian-Emperor seamount cha foreground. The gravity expression of the f and bulges (yellow-green region) that flank chain are clearly visible.



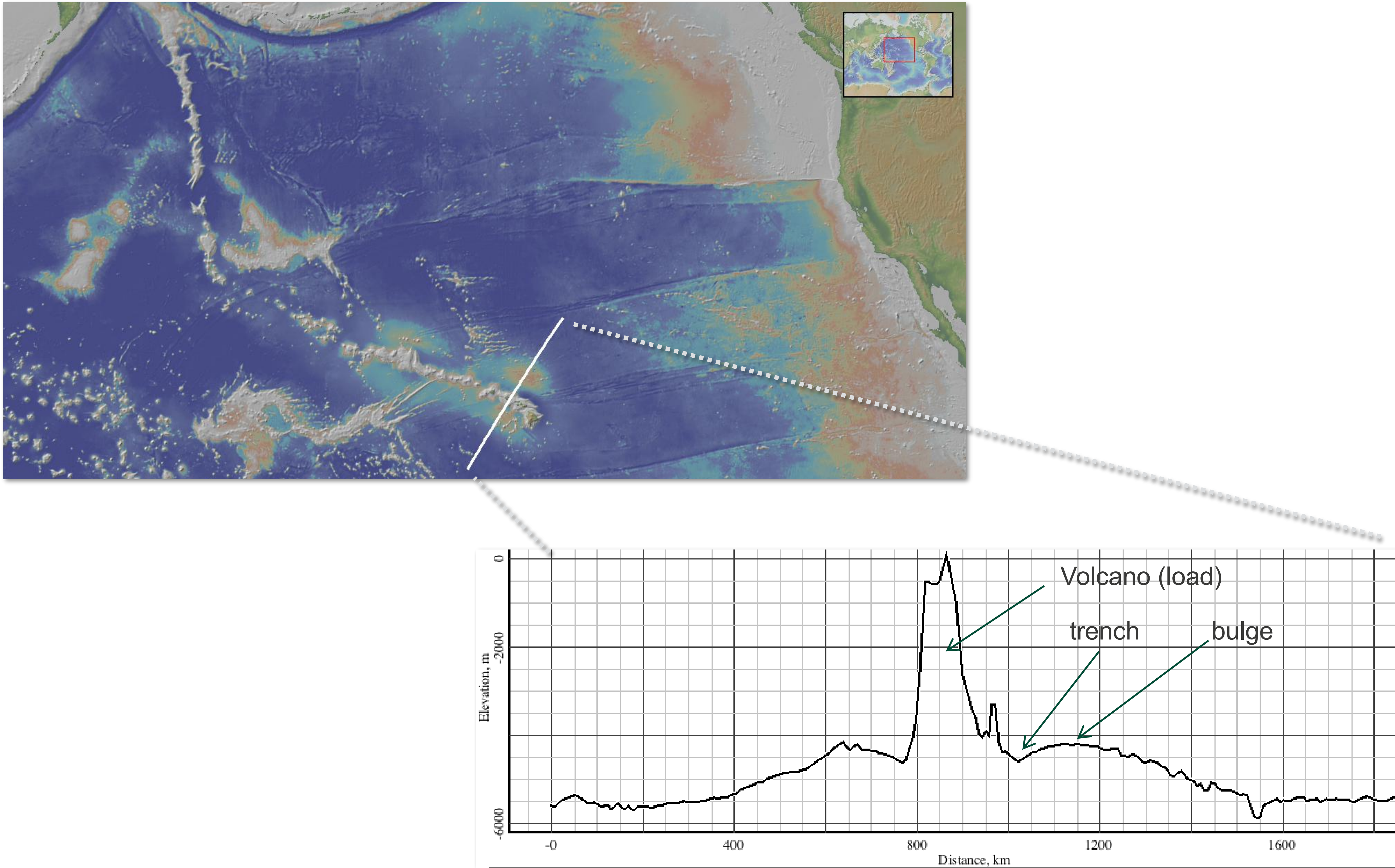
From Tony Watts using satellite-derived data of D. T and G. P. Woollard and colleagues (onshore)

Example: Chain of seamounts



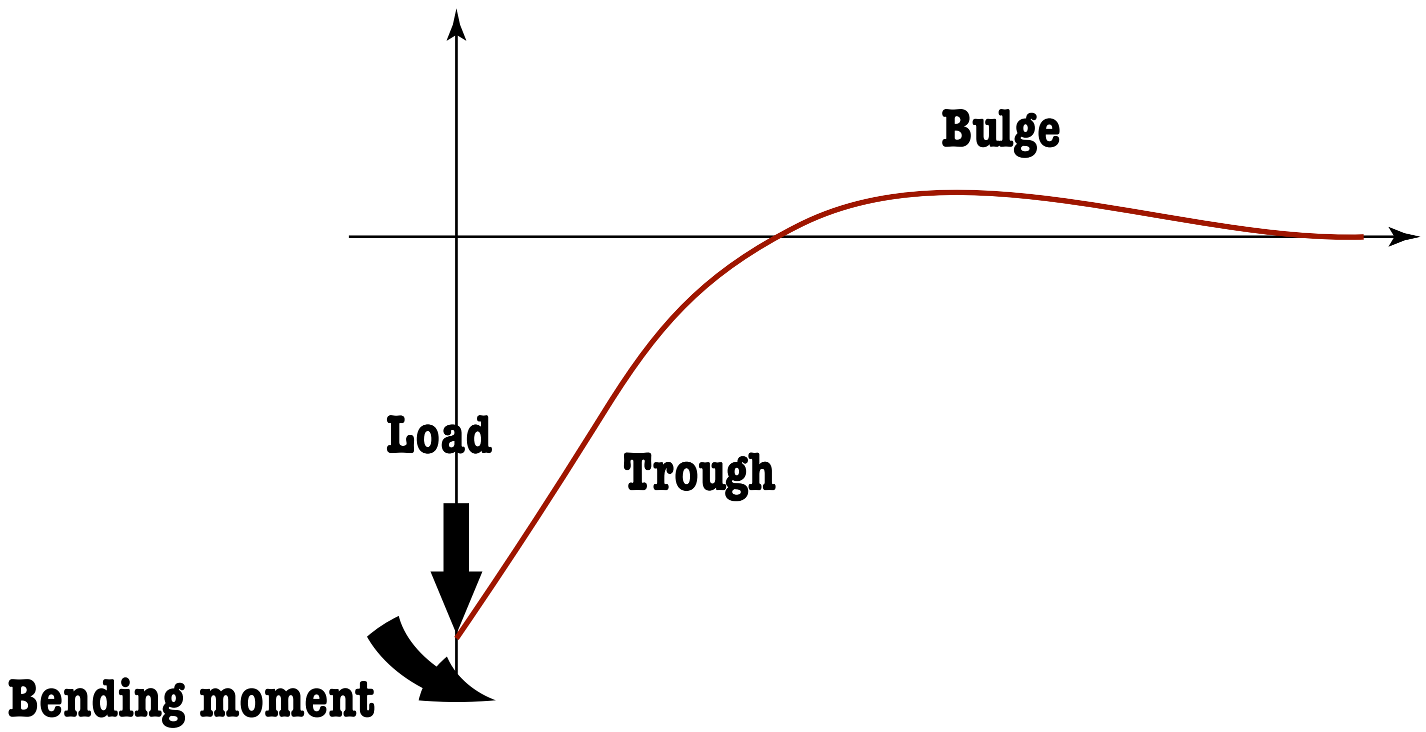
$$w = \frac{V_0 \alpha^3}{8D} e^{-x/\alpha} \left(\cos x/\alpha + \sin x/\alpha \right) \quad (x \geq 0)$$

Example: Chain of seamounts



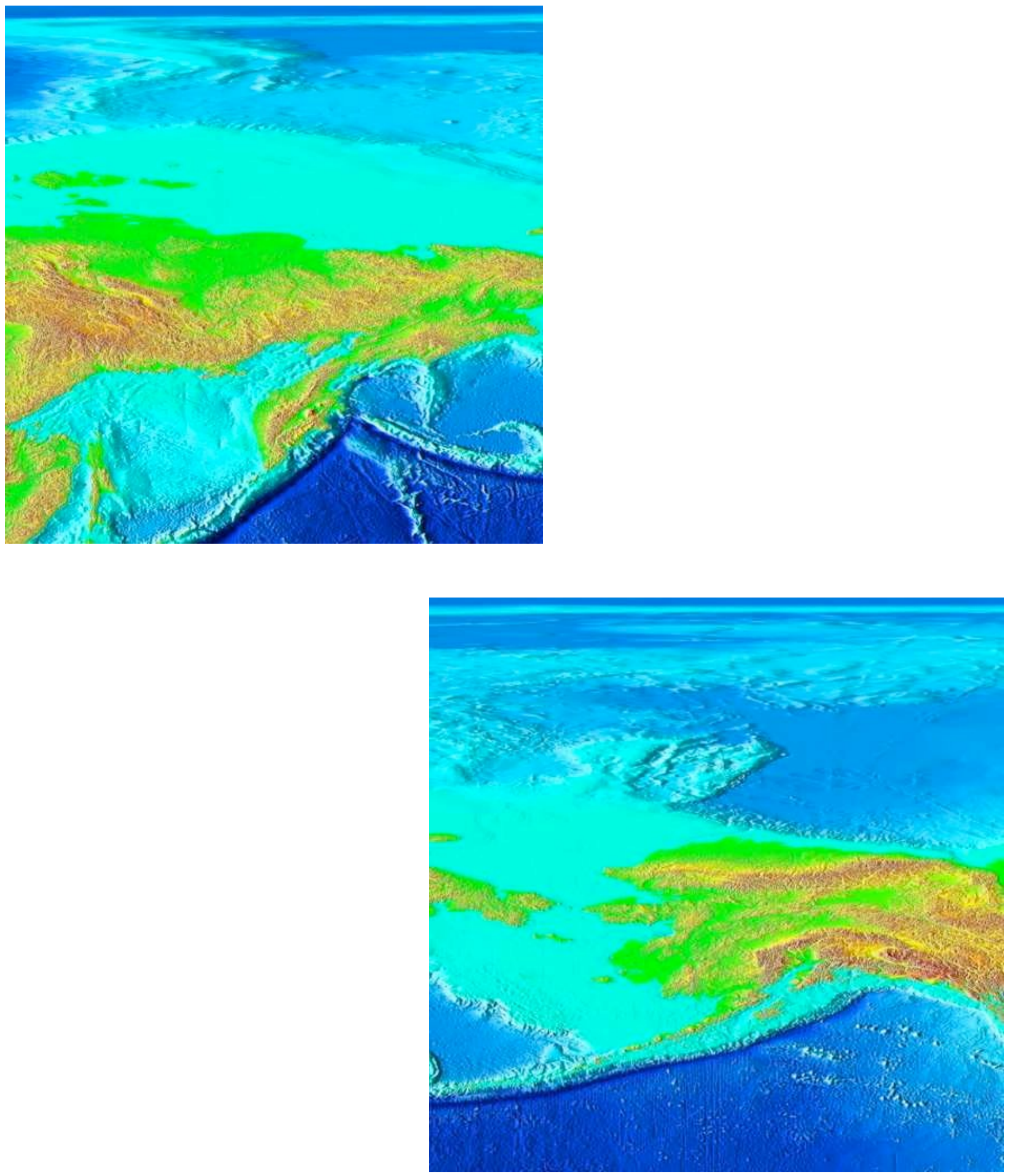
Example: End load on a plate

If the plate actually breaks under the load, or is physically flexed into the mantle, another, similar, solution is found.

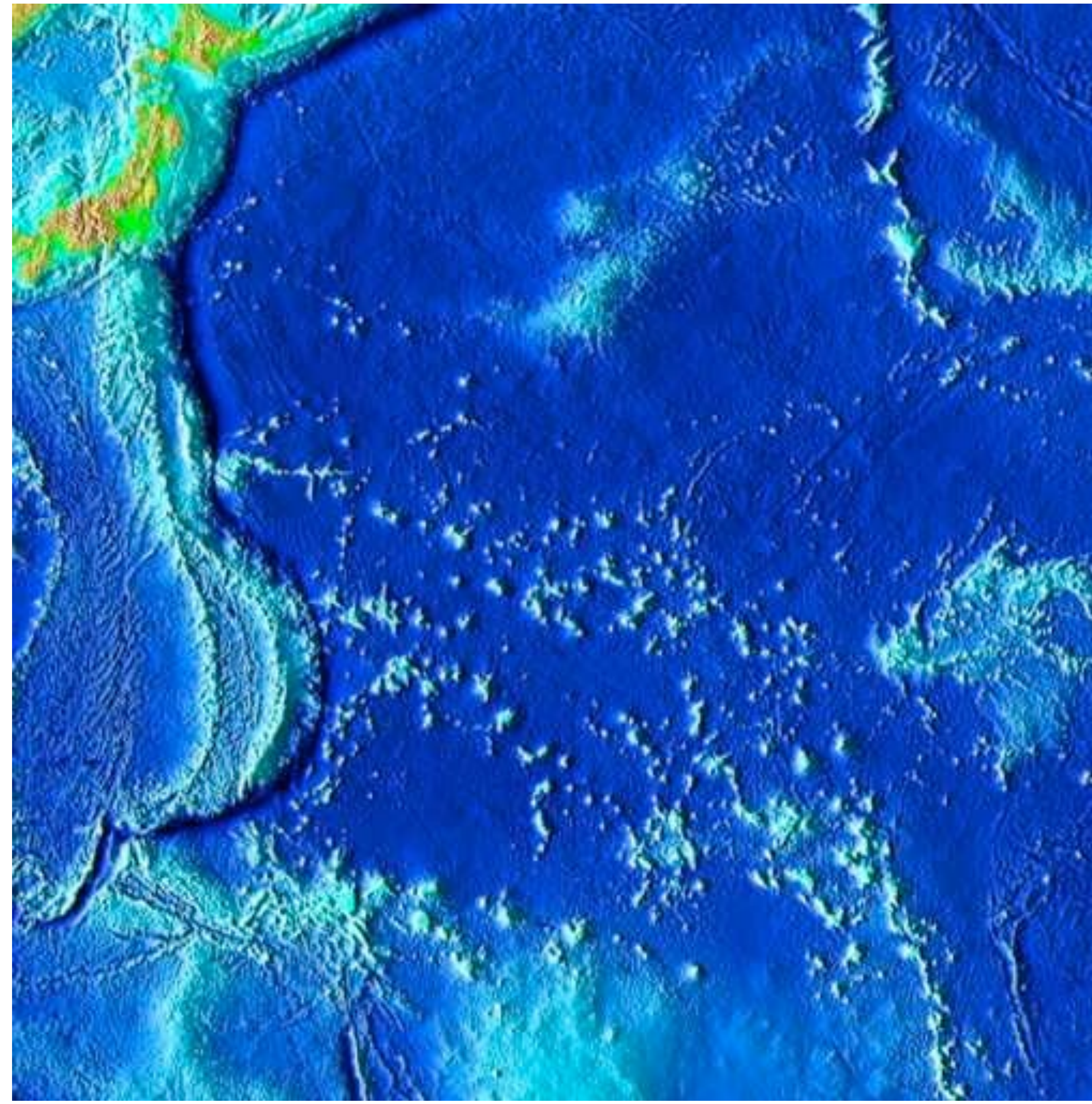


The solution is more complicated, but this does mean that we can tell by the shape of the flexed plate whether it has broken underneath the load

Example: End load on a plate

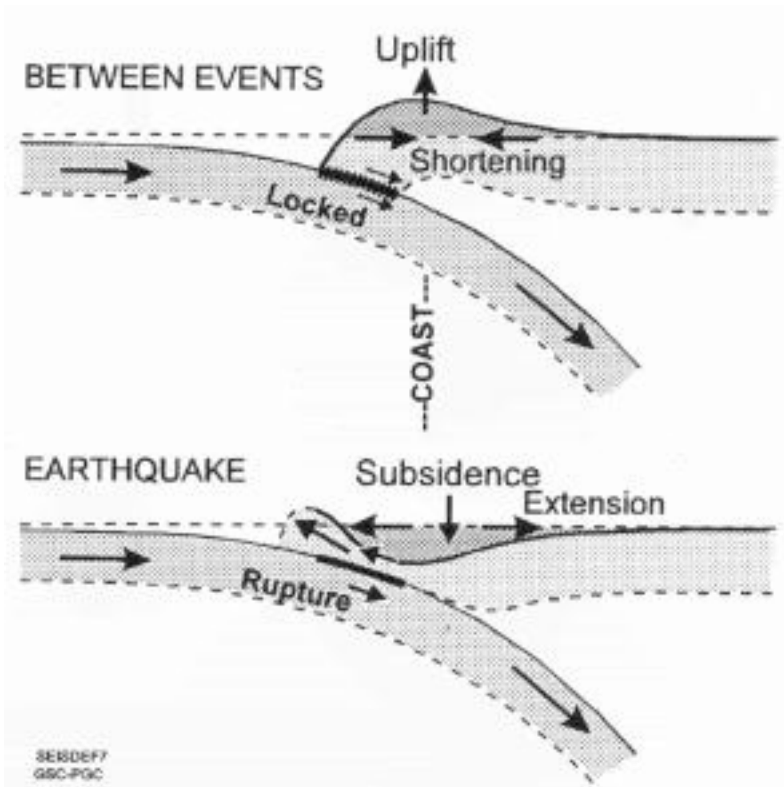


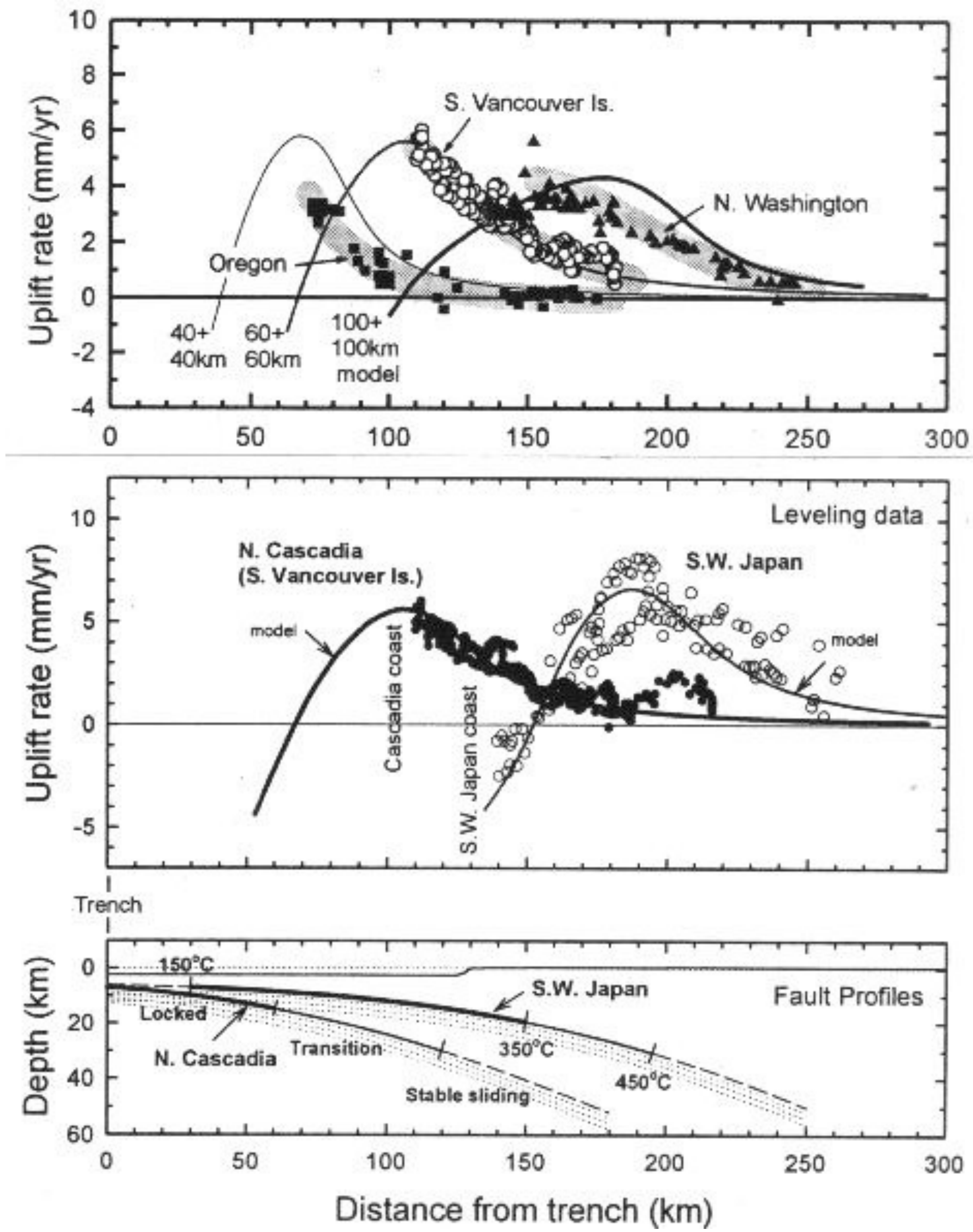
Aleutian trench



Topography in the Japan-Mariana area

Example: End load on a plate

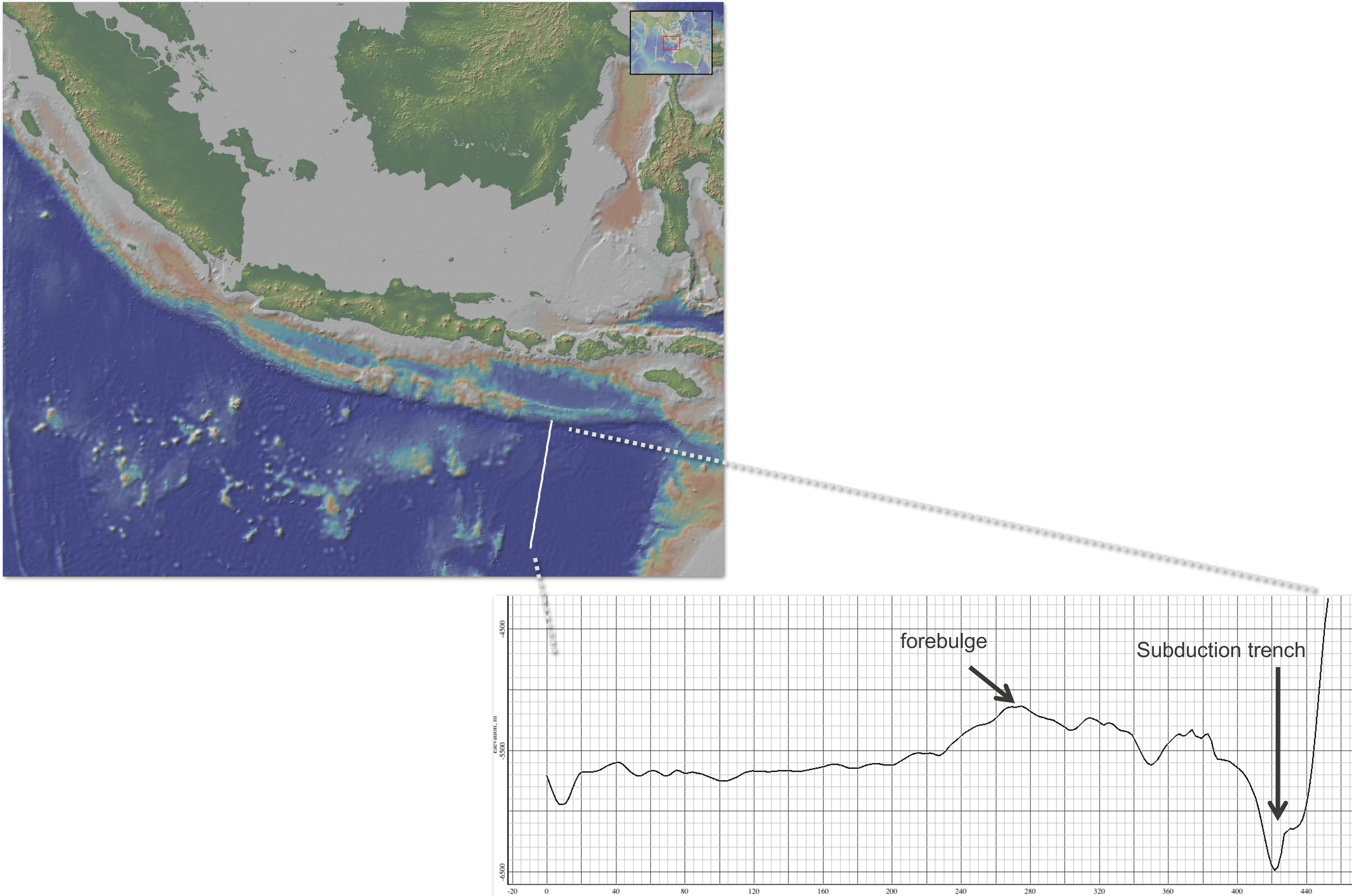




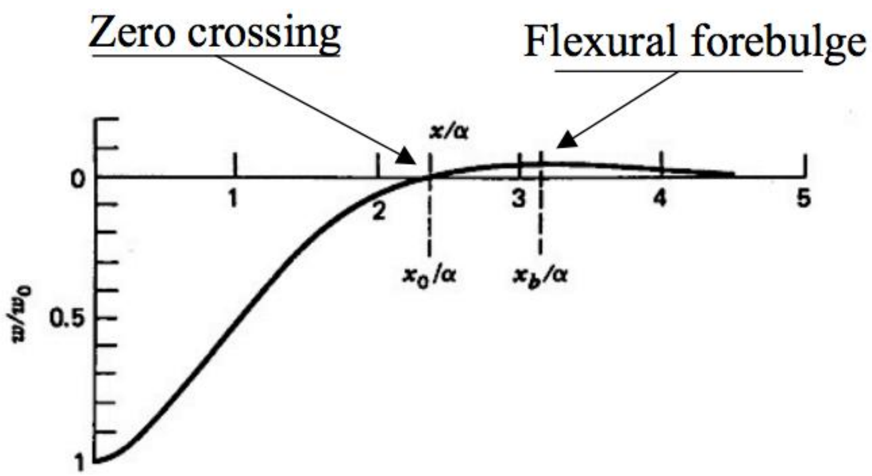
Two plates meet at a subduction zone. The upper plate may lock and the other plate then supplies a bending moment to the end.

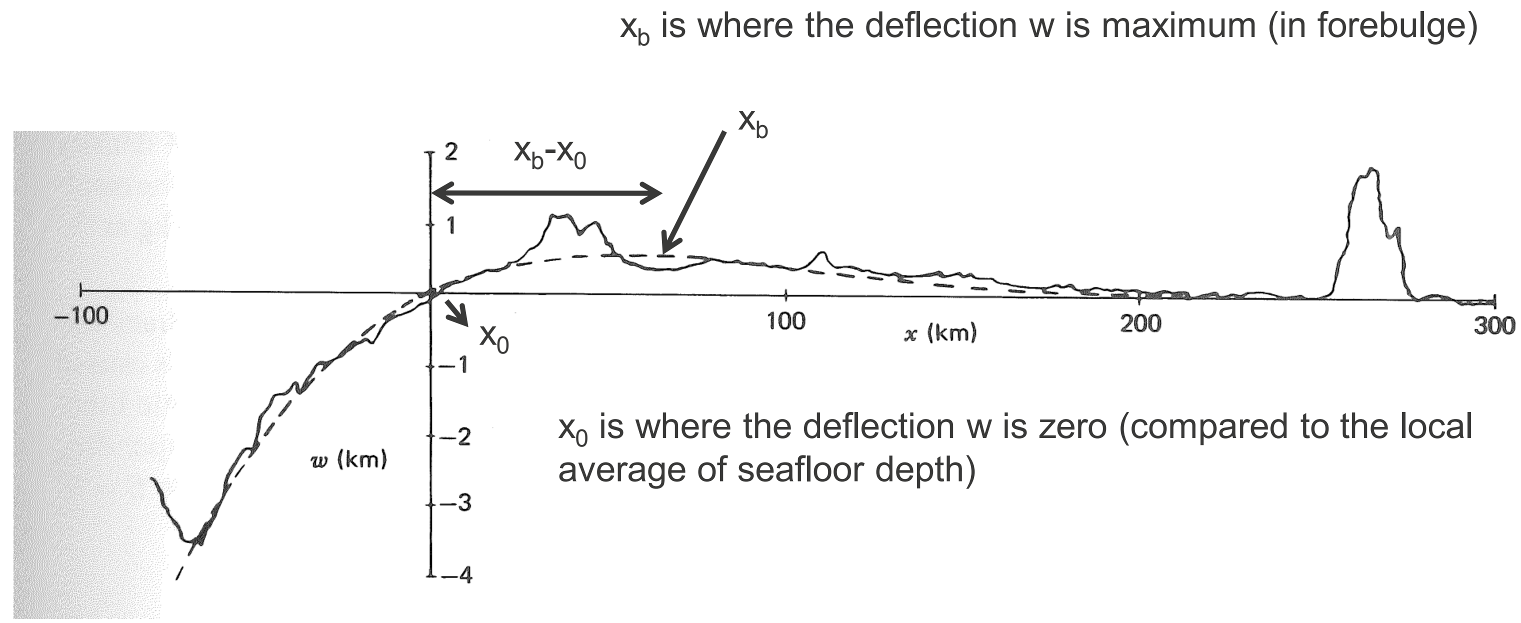
This can be detected by uplift measurements

Example: End load on a plate



Example (from David B)





$$x_b - x_0 = \frac{\pi}{4}\alpha$$

For the Mariana trench, the maximum deflection is ~ 0.5 km, and is located 55 km from x_0.

We deduce that α = 70 km

$$\alpha = \left[\frac{4D}{(\rho_m - \rho_w)g}\right]^{1/4}$$

With $\rho_m - \rho_w$ = 2300 kg.m^{-3} and g = 10 m.s^{-2}, we deduce that D = 1.4 x 10^{23} N.m

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

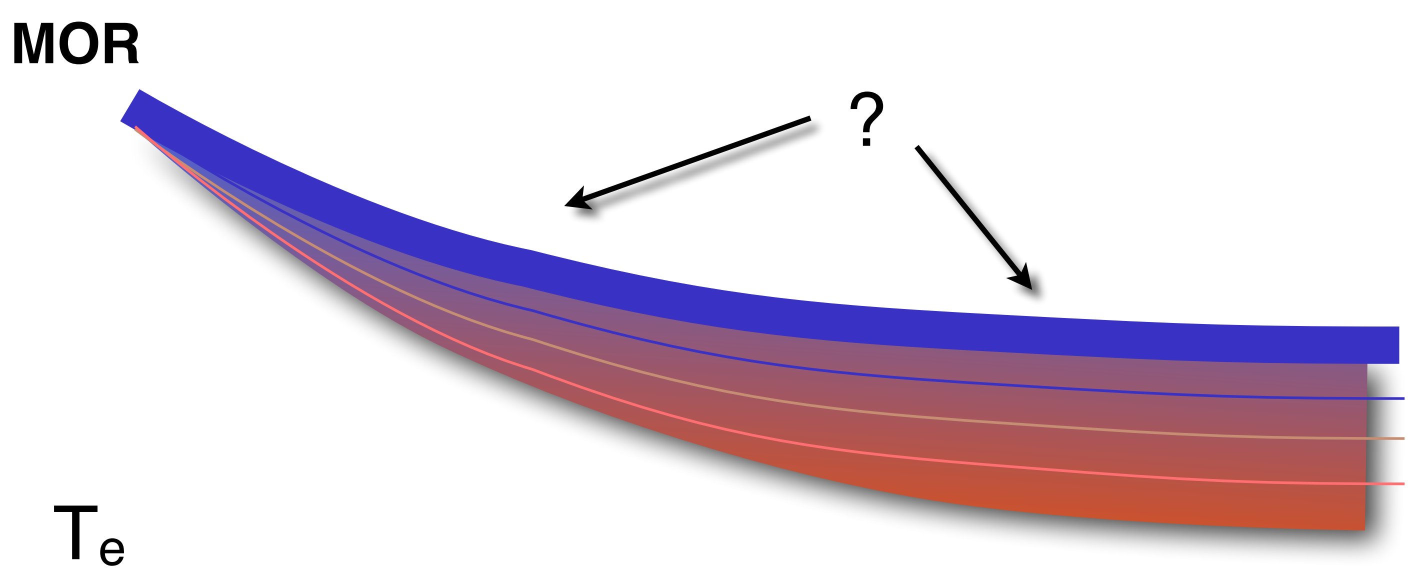
With E = 70 x 10^9 Pa and v = 0.25 we find that the elastic lithosphere thickness is **28** km

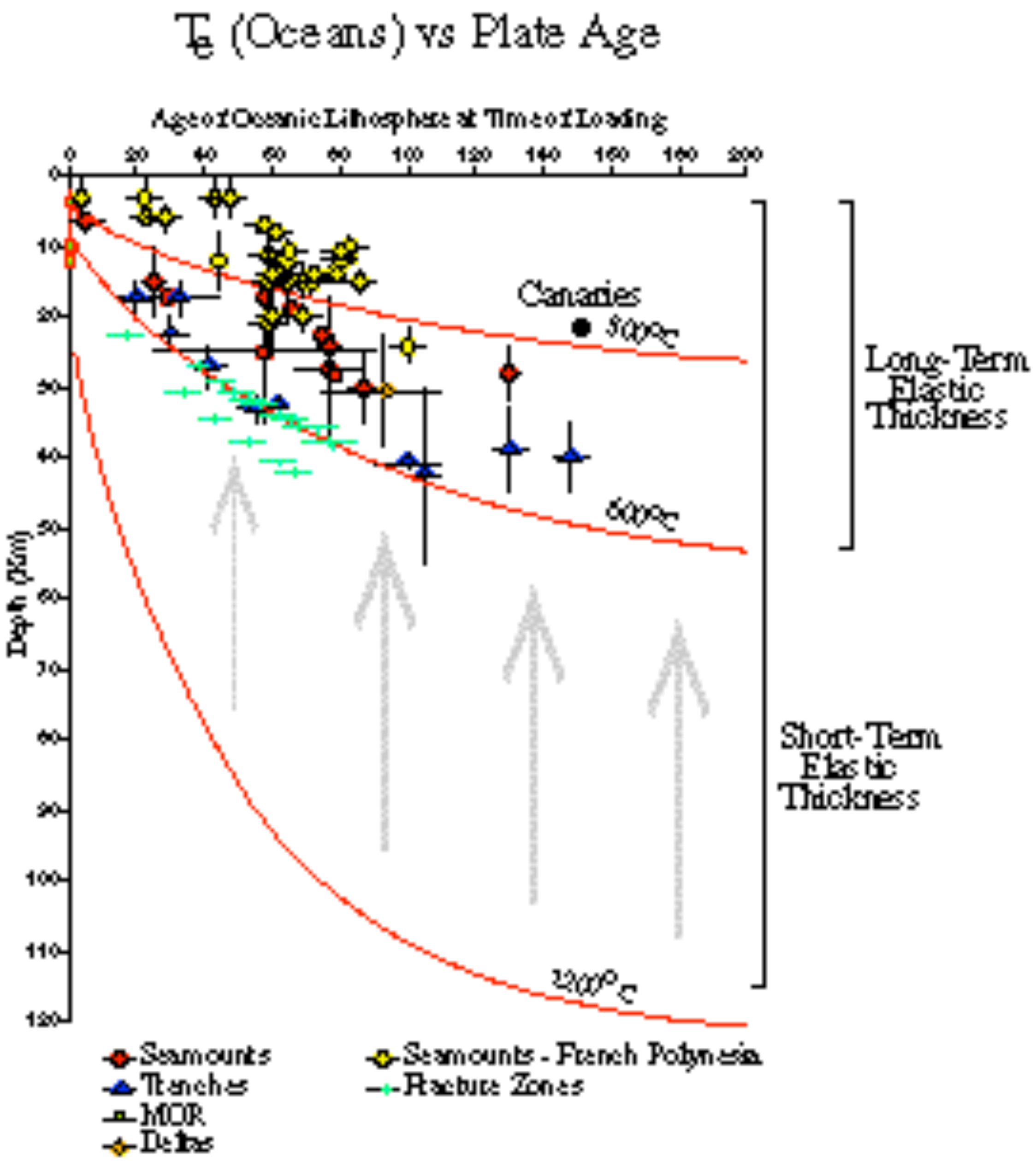
Elastic thickness in the oceans

- Oceanic lithosphere changes thickness with time as it cools

- "Thickness" is loosely defined in terms of the progression of a fuzzy cooling front

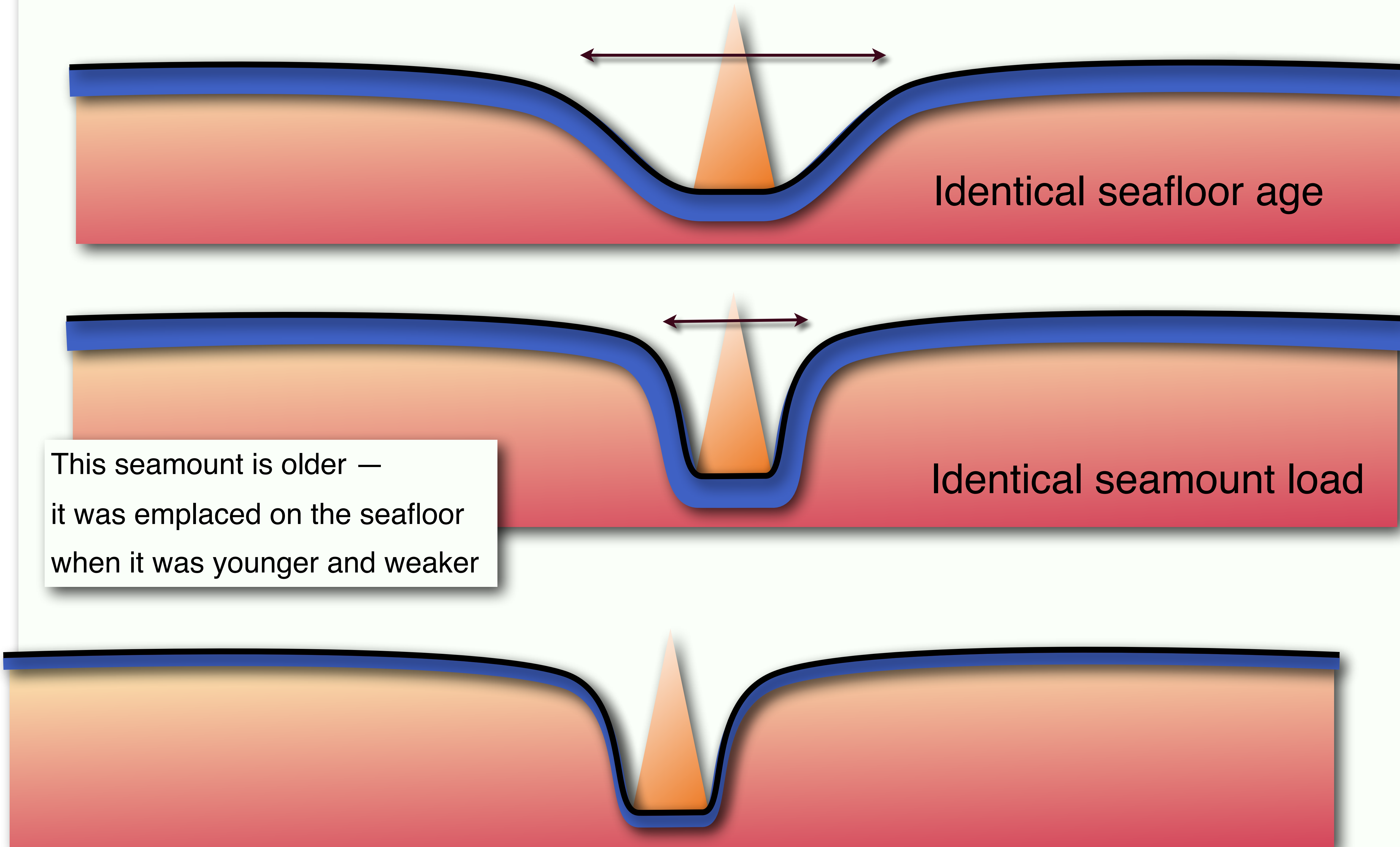
- Cooling of a plate with a pre-existing load ?





Strength v. age

Oceanic sea floor thickens as it cools; it simultaneously gains elastic strength. The pattern of flexure due to a load depends mostly on the elastic strength *when the load was emplaced.*



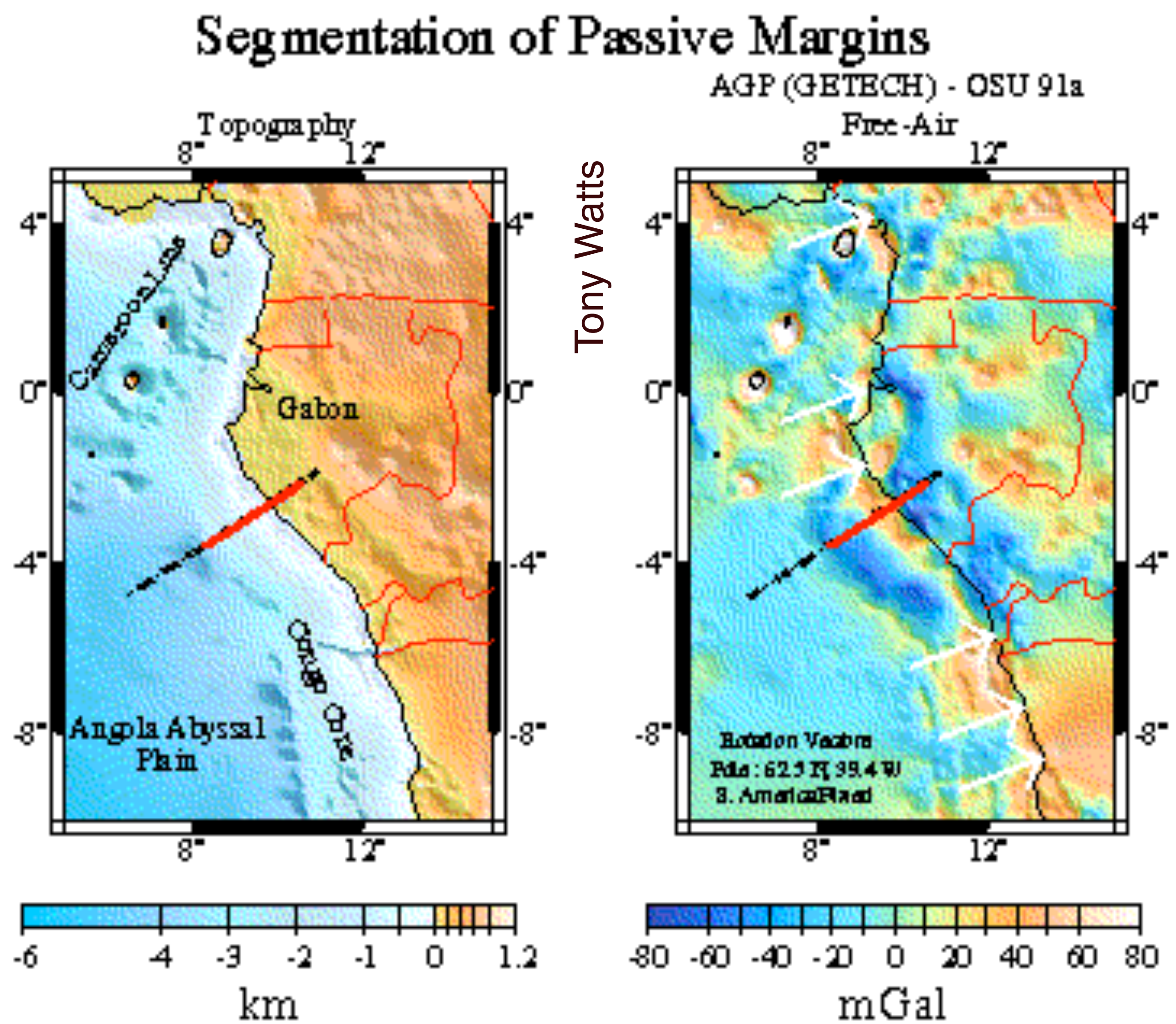
Strength of passive margins

Ultimate behaviour of passive margin depends on the history of stretching, thermal events, magmatism and loading.

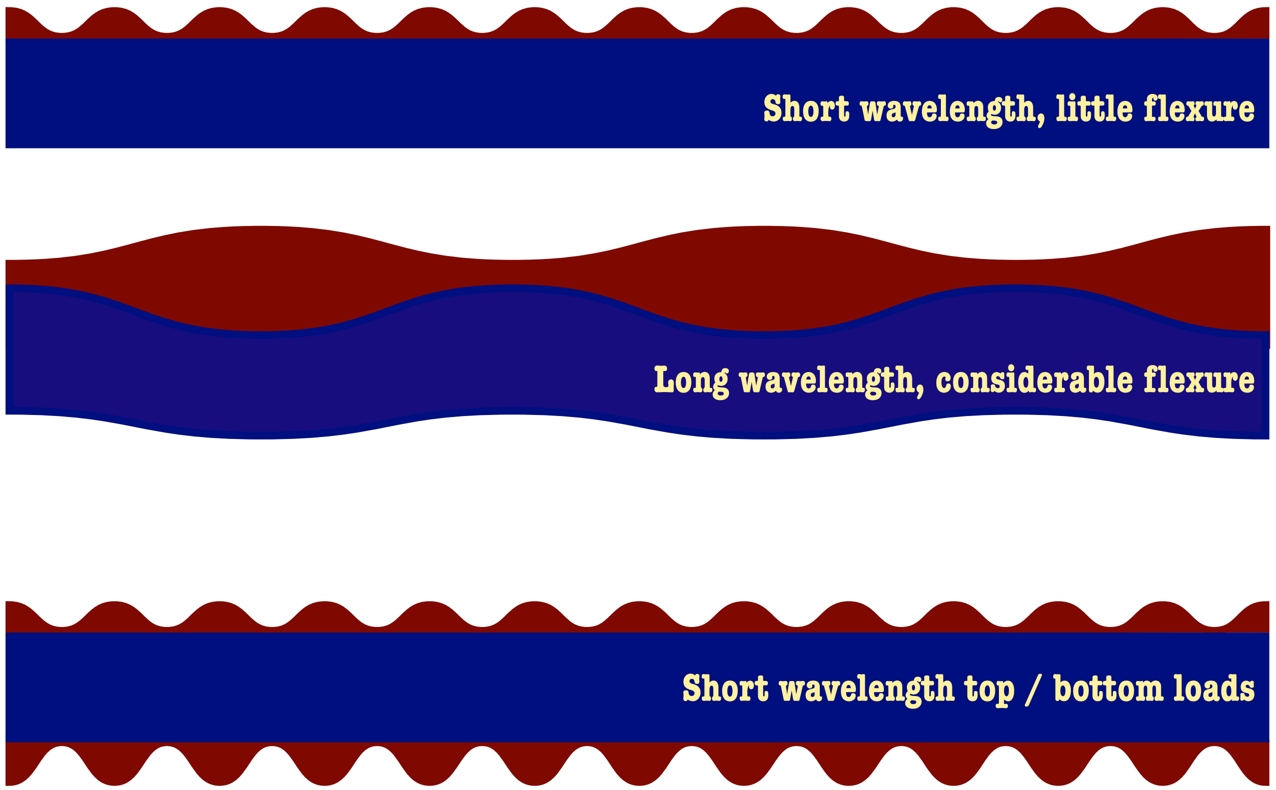
Example from W. Africa shows that the evolution is not unique even for a single margin ...

- Sedimentation
- Magmatism / underplating
- Flexure
- Isostasy

Elastic strength changes during the margin's evolution



Elastic Thickness determination

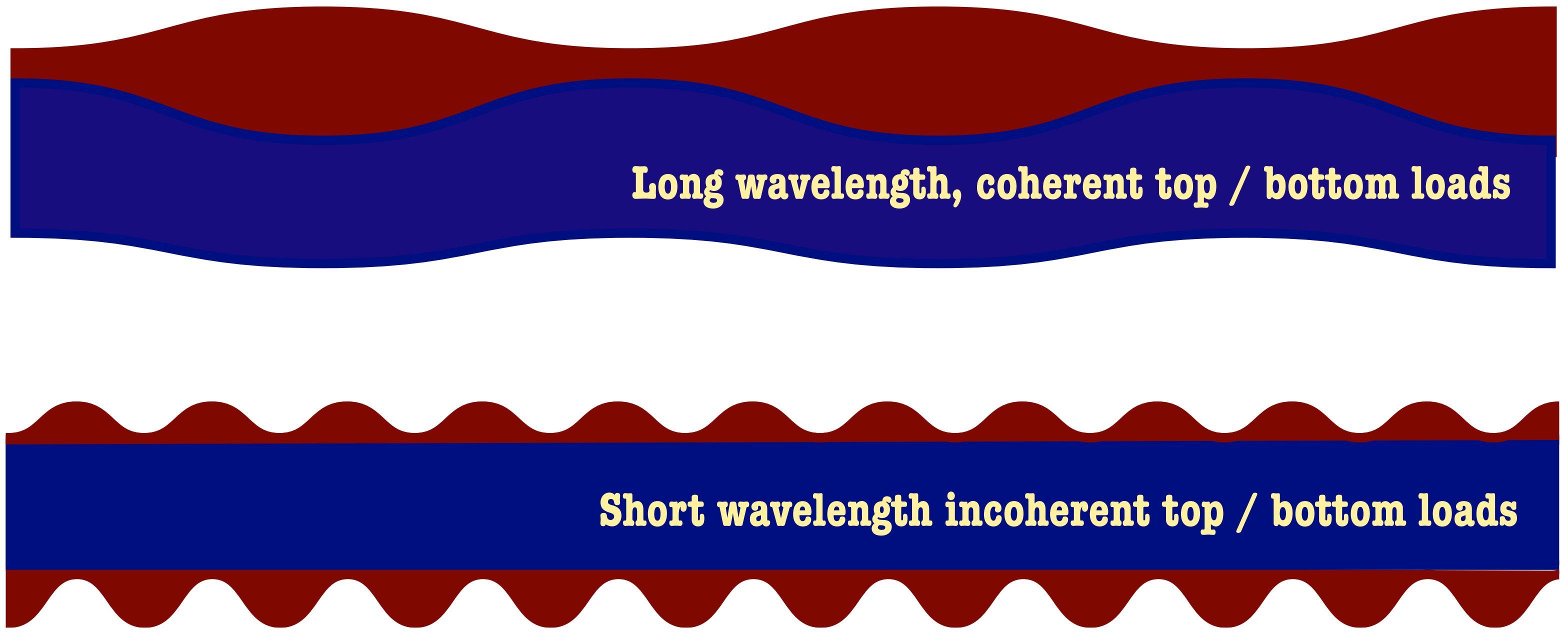


The gravity signal associated with a load dropped on a strong plate is quite distinct … ***no compensating masses at depth*** therefore strong gravity for amplitude of topography.

But the same plate is not able to support longer wavelength loads as well as it can short ones. ***Long wavelength loads are compensated*** and have weaker gravity signal

Therefore the gravity signal / topography signal should change from this can be predicted from the flexure equations to give an elastic thickness (considers amplitude of gravity / topography)

Elastic Thickness determination

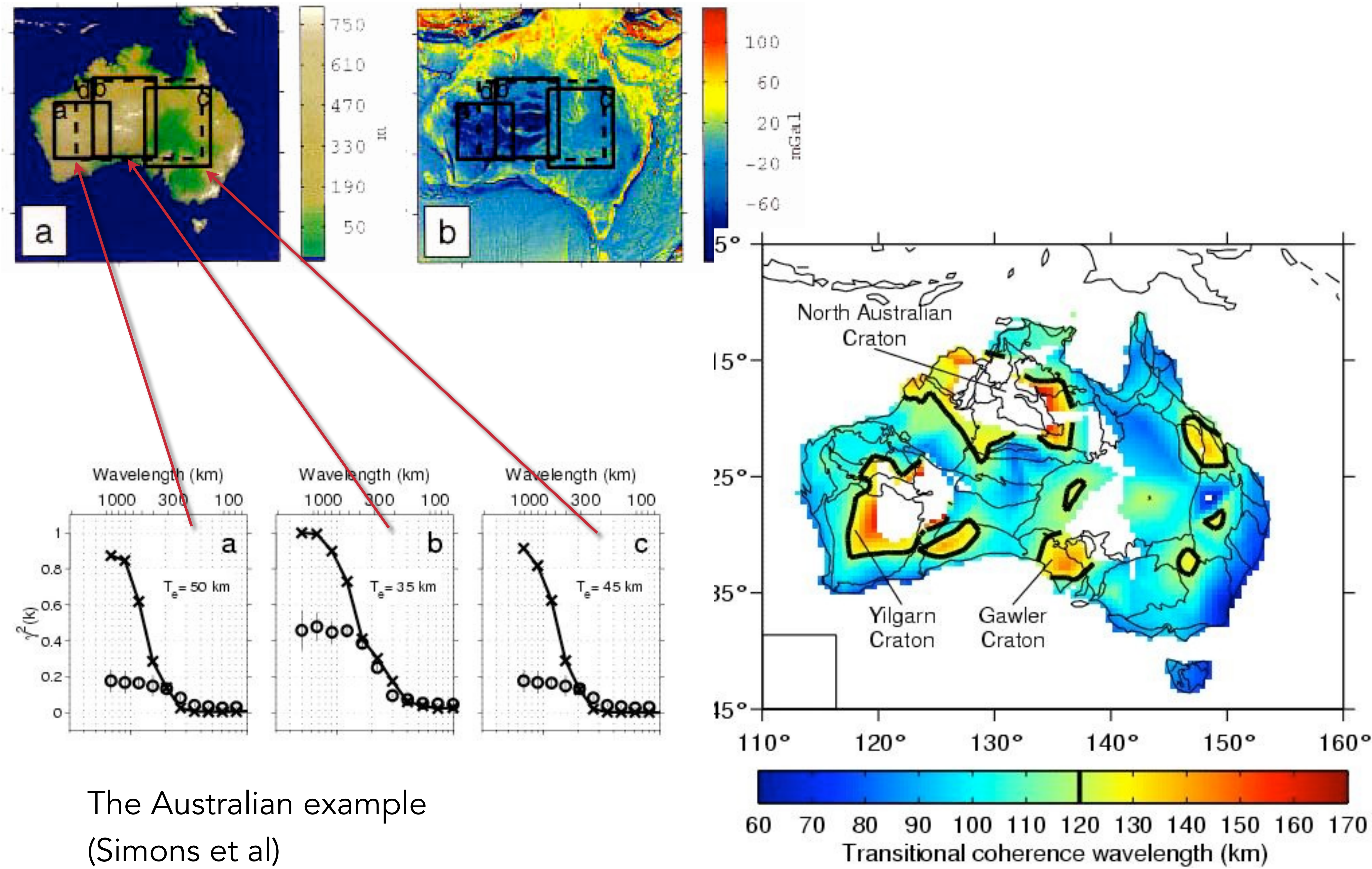


We can also assume that a strong plate has a random set of loads top and bottom which are not correlated with each other in anything like the same way as they are for a weak plate.

Strength is wavelength dependent so at the wavelength where the plate appears to become weak gives an indication of elastic thickness

The topography is 100% determined by the top loads, gravity is a mix of top and bottom loads. The **coherence** (measure alignment of peaks/troughs) of gravity / topography will suddenly decrease at "short" wavelength in a way which allows elastic thickness to be measured (considers phase of topography / gravity)

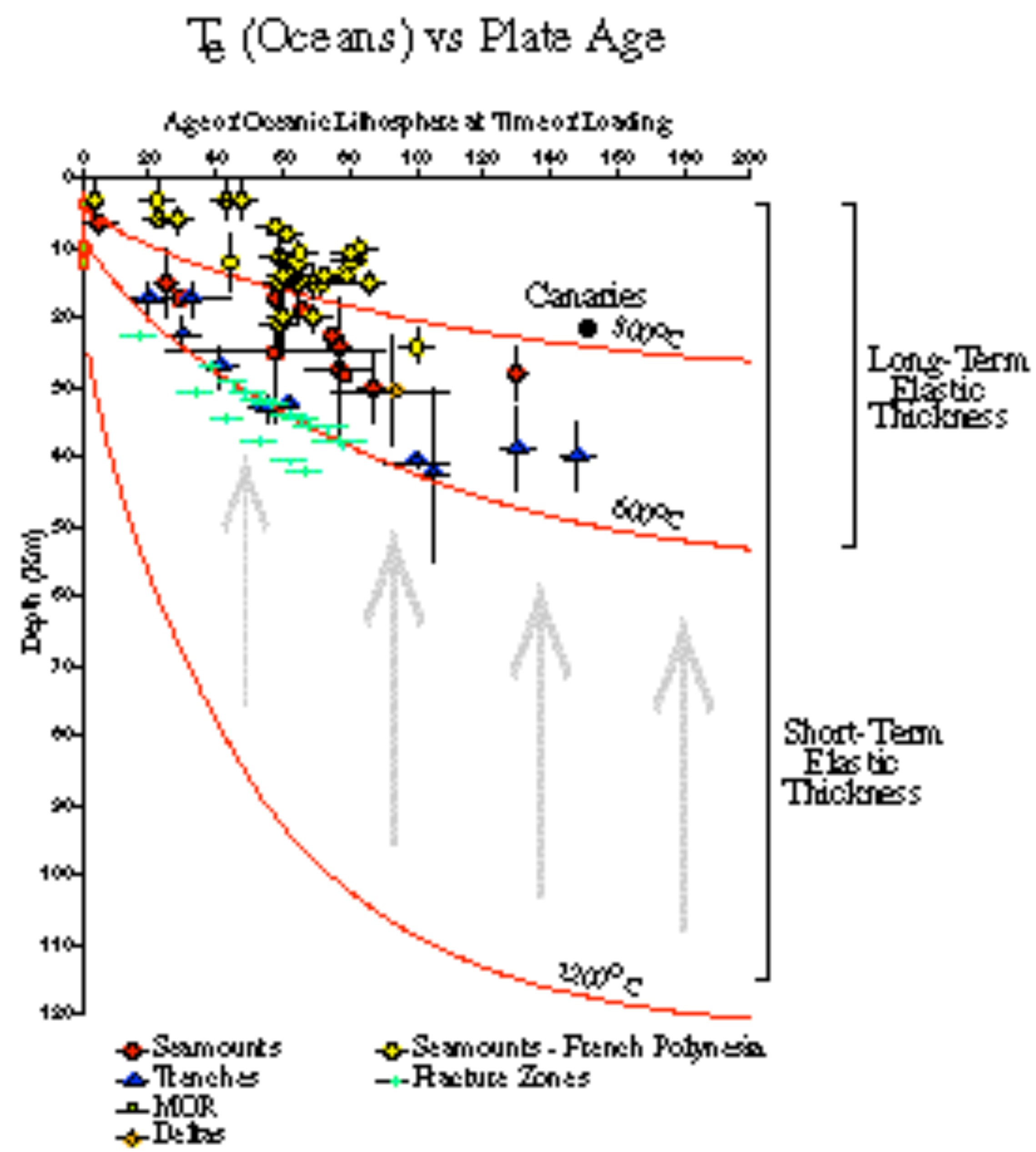
Elastic thickness determination — coherence



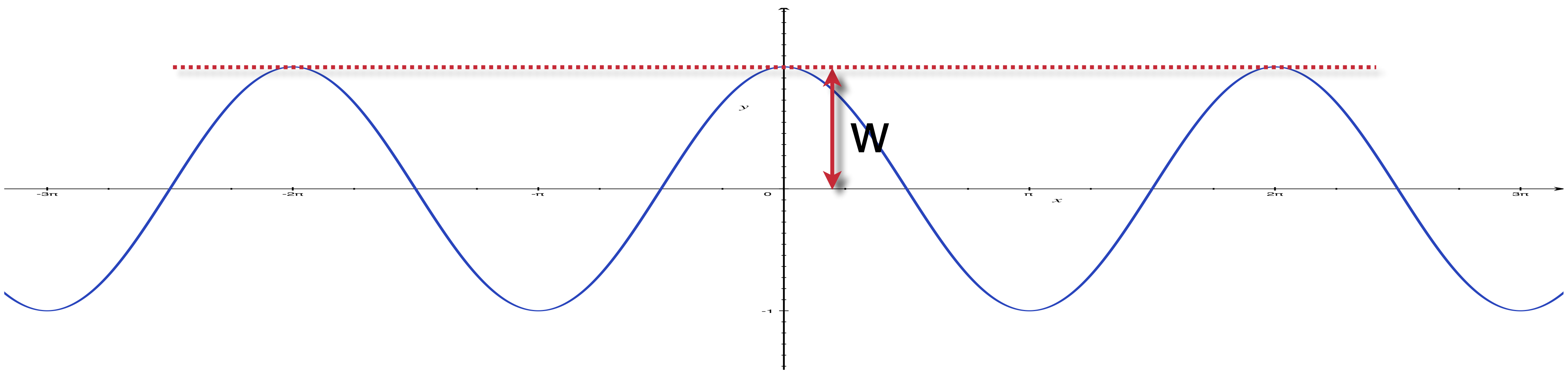
The Australian example
(Simons et al)

Elastic thickness

- Lithosphere response is very wavelength dependent
- Amplitude is also very dependent on lithospheric thickness
- Amplitude is also very dependent on lithospheric physical properties
- Flexure is like a filter that removes short wavelengths
- T_e is determined experimentally from the solutions to equation, not necessarily a unique physical property



The elastic and isostatic models say little about how long it takes for the lithosphere to deform once a load is emplaced or removed.

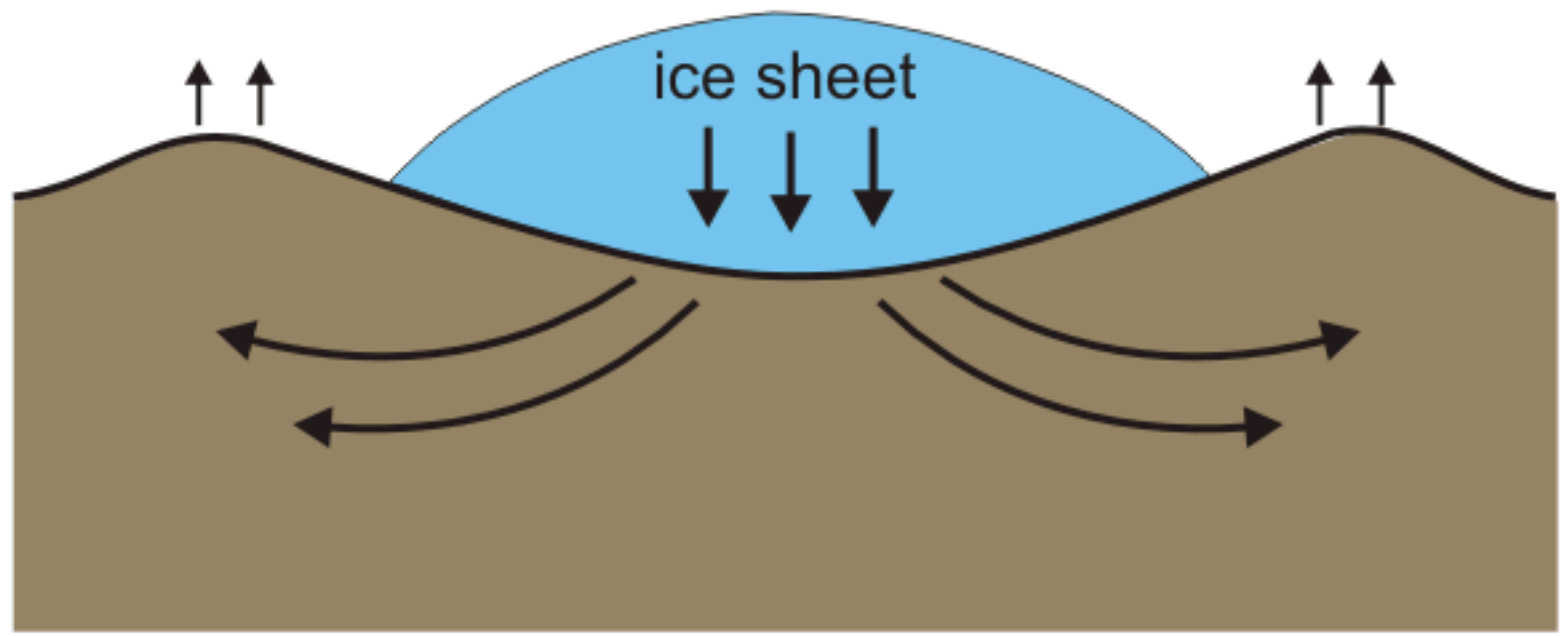


$$w = w_m \exp\left(-\frac{t}{\tau_r}\right) \qquad \tau_r = \frac{4\pi\eta}{\lambda\rho g}$$

If the loading is fast, then the time taken for the mantle to flow to the new shape of the lithosphere can be the rate-limiting step

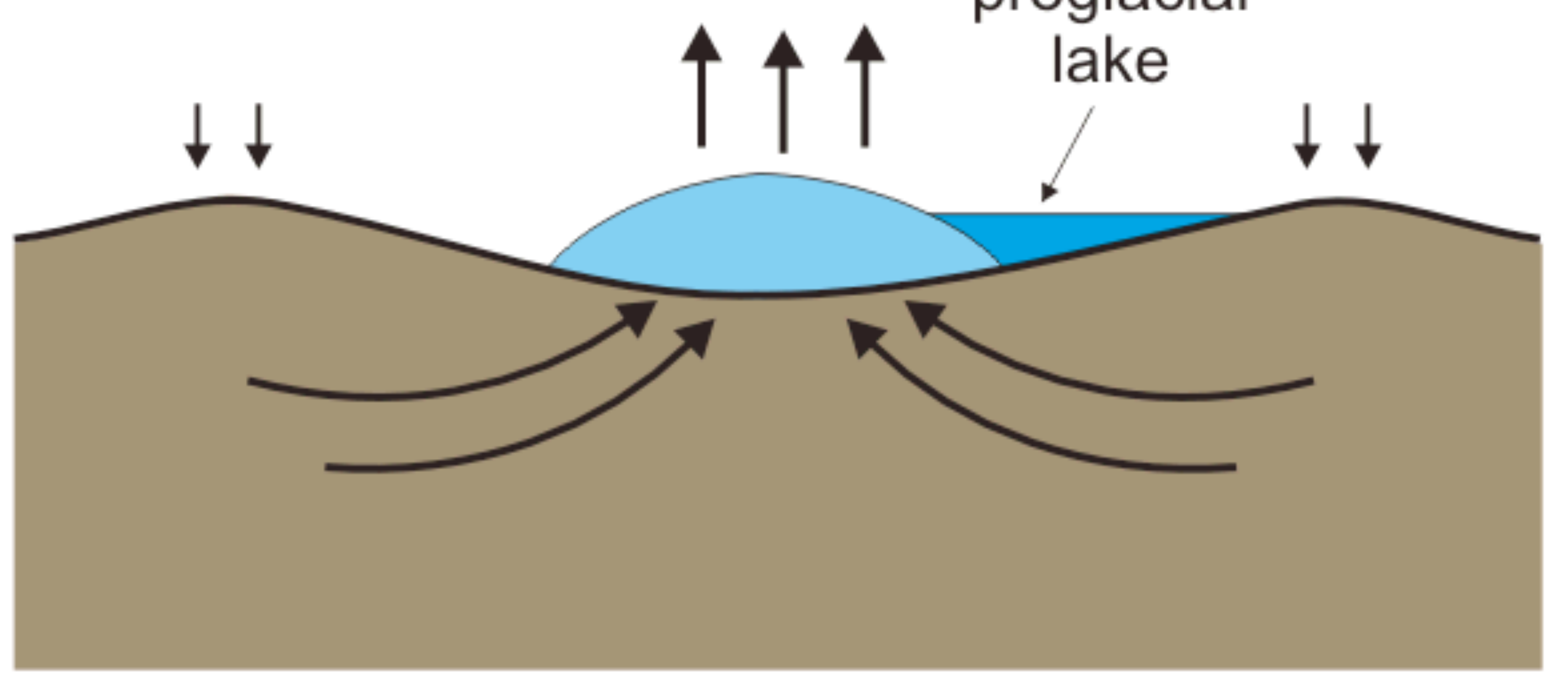
Ice loading and unloading

$$\tau_r = \frac{4\pi\eta}{\lambda\rho g}$$



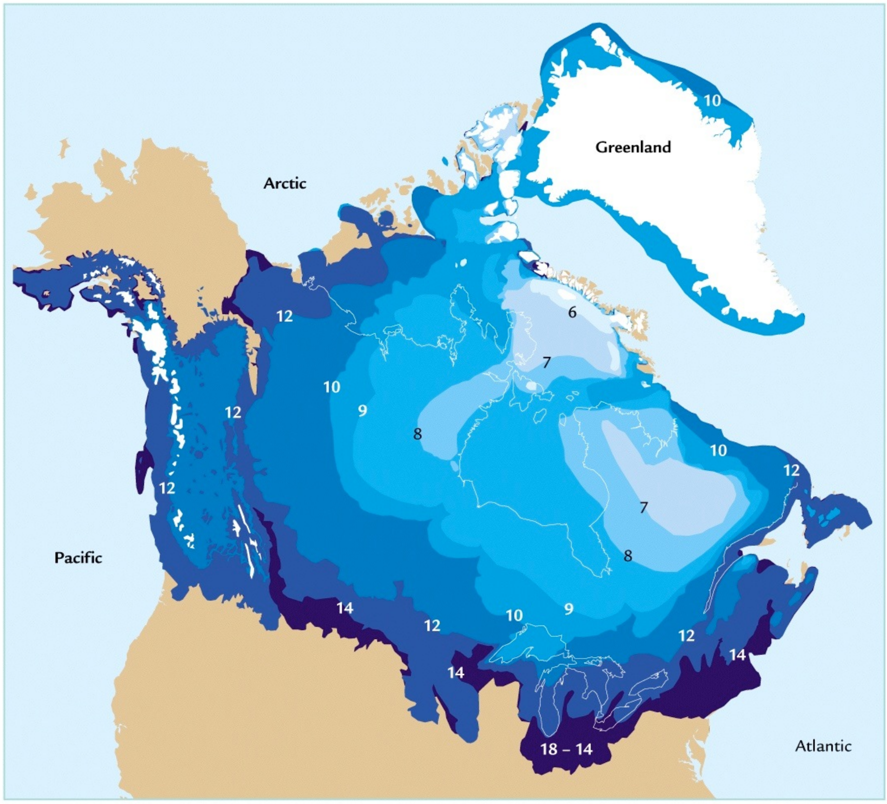
Ice loads come and go very quickly by geological standards.

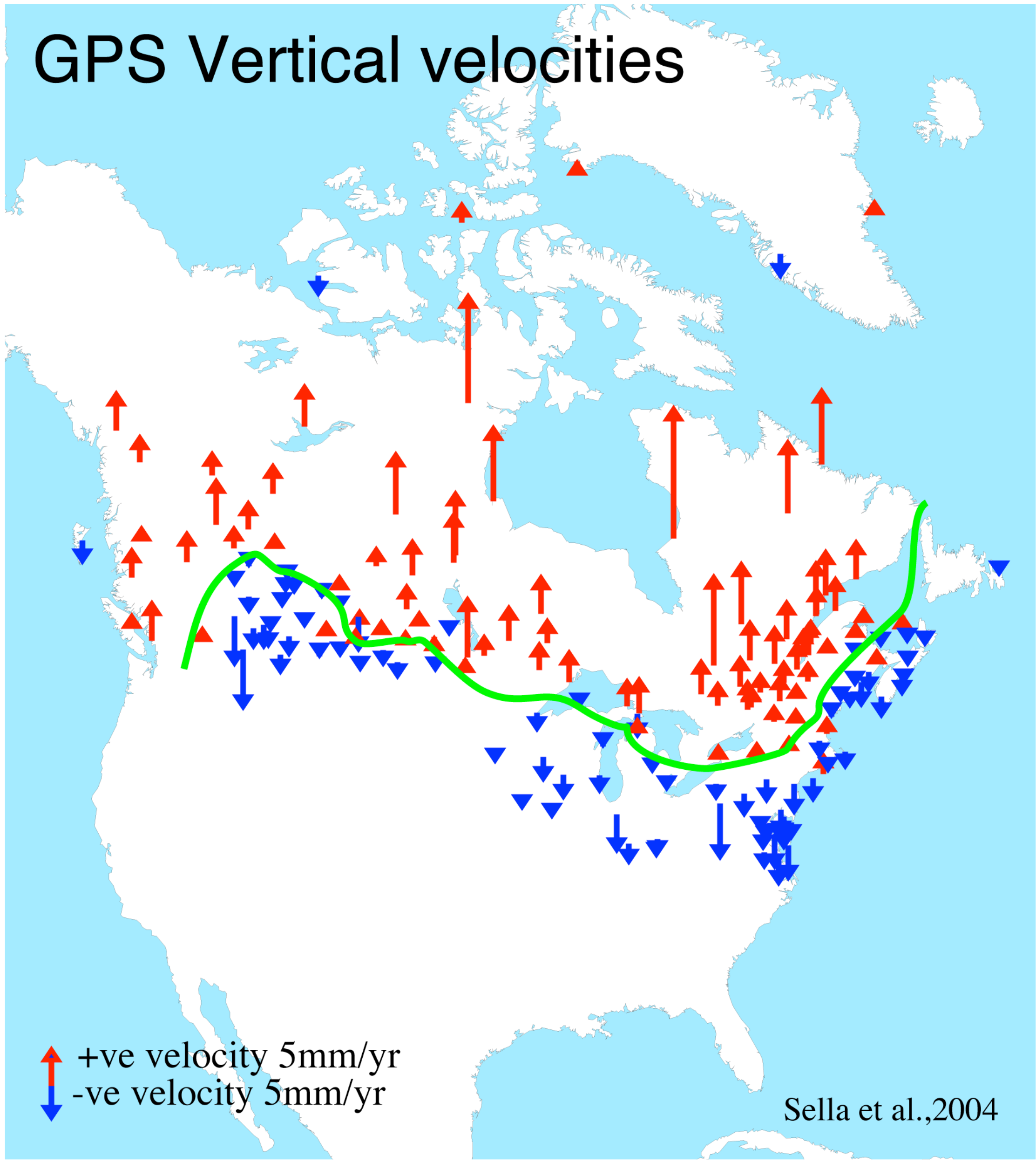
Relaxation rate is dependent upon the *absolute value of mantle viscosity* — one of the few observations sensitive to this value.

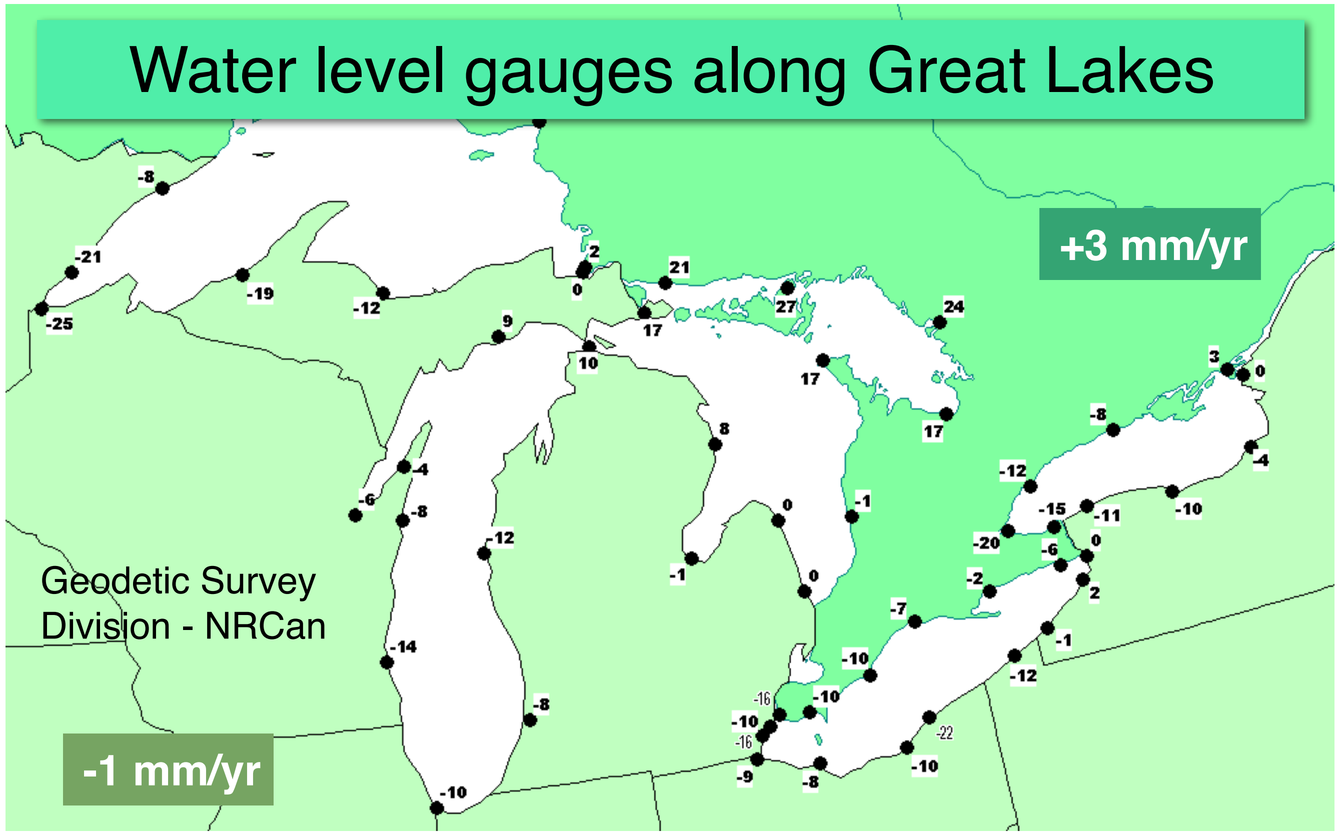


Post-glacial rebound - observations

Rebound is still evident in N. America, N. Europe after ice retreat from recent ice-age



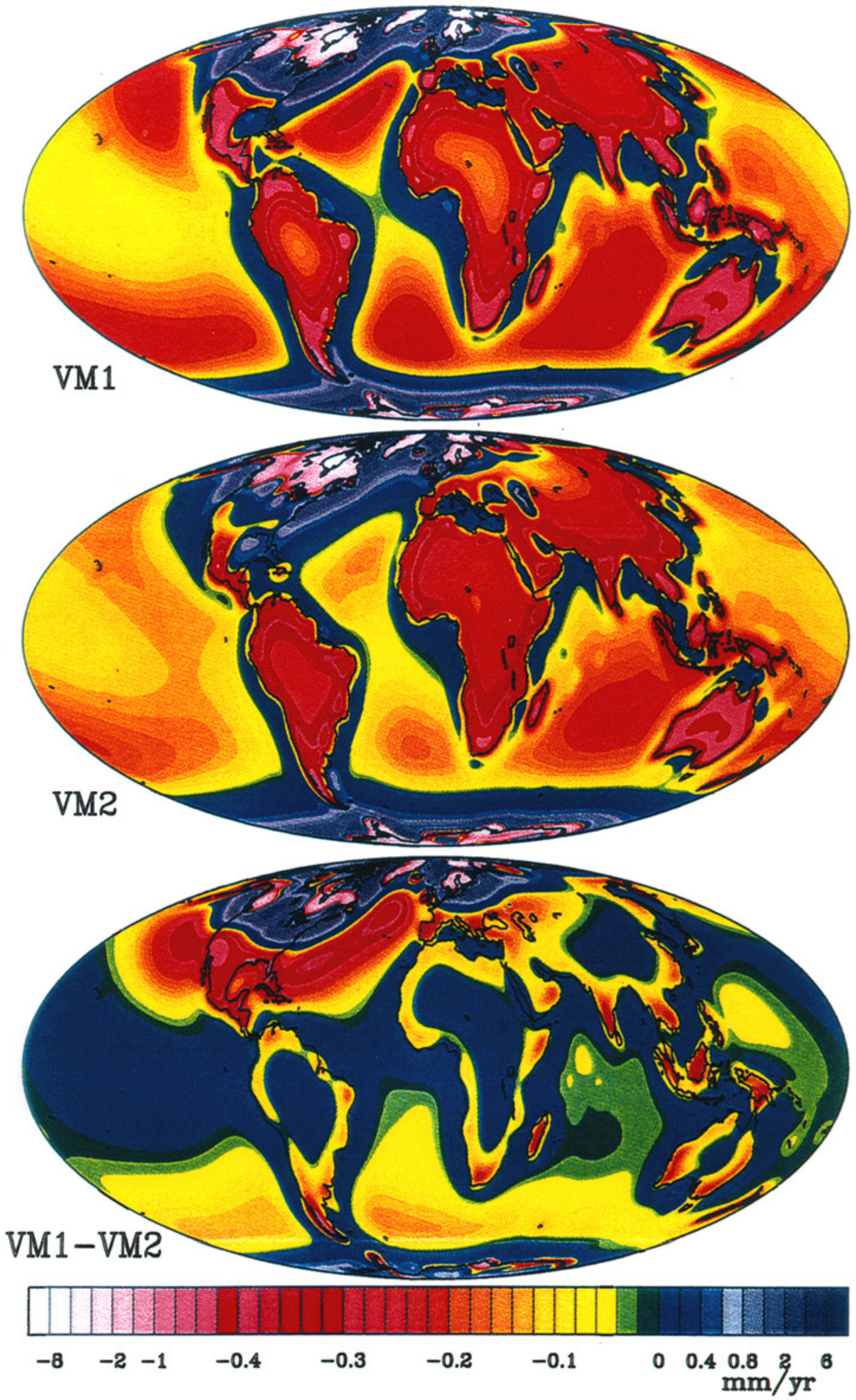




Viscosity layering & dynamics

This figure shows how sea level should be changing in response to the removal of the ice-age ice-load from the Earth.

The models are dependent upon the viscosity layering assumed for the Earth. These are used to constrain models of viscosity layering.



Viscosity layering in the Earth

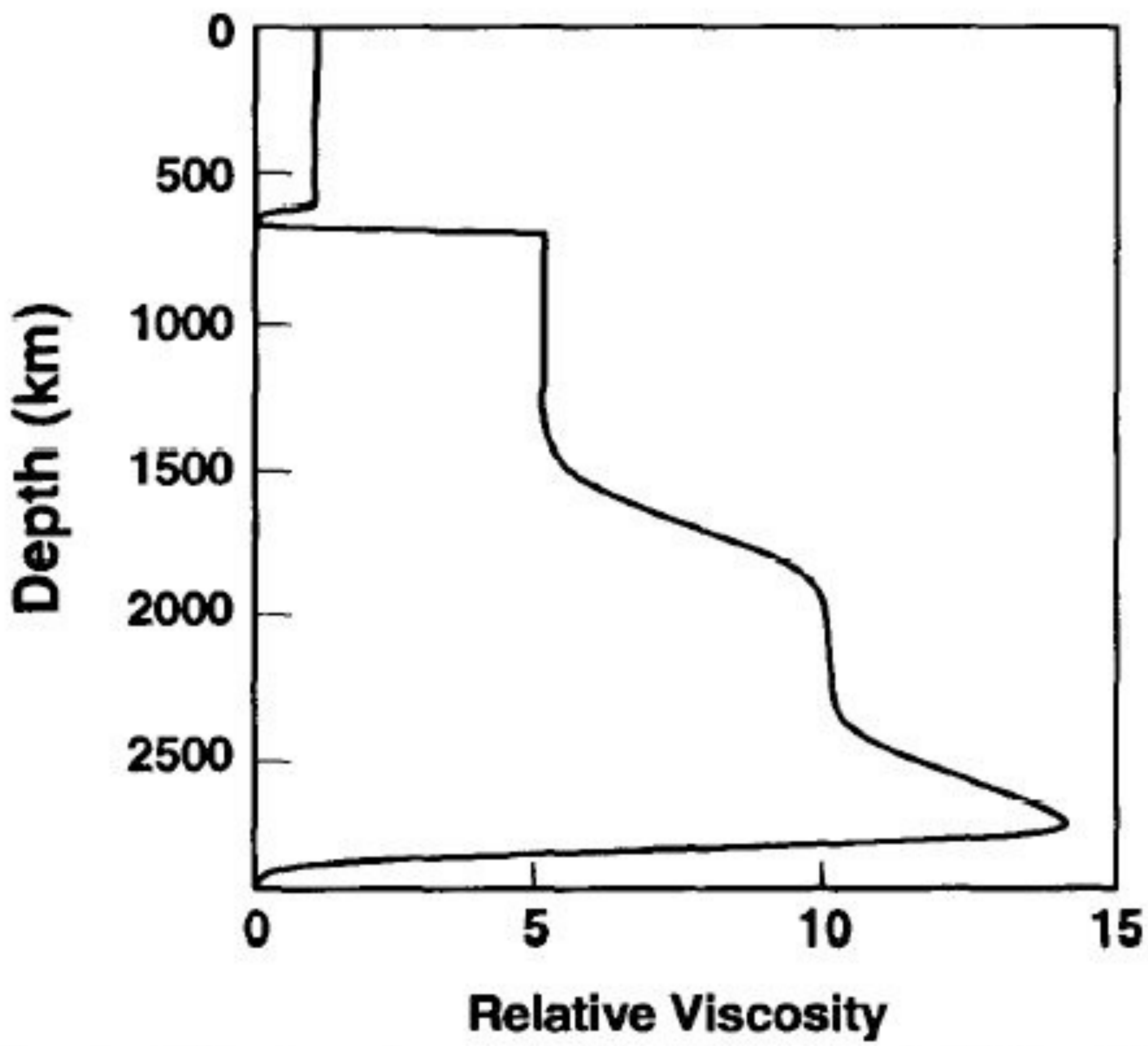
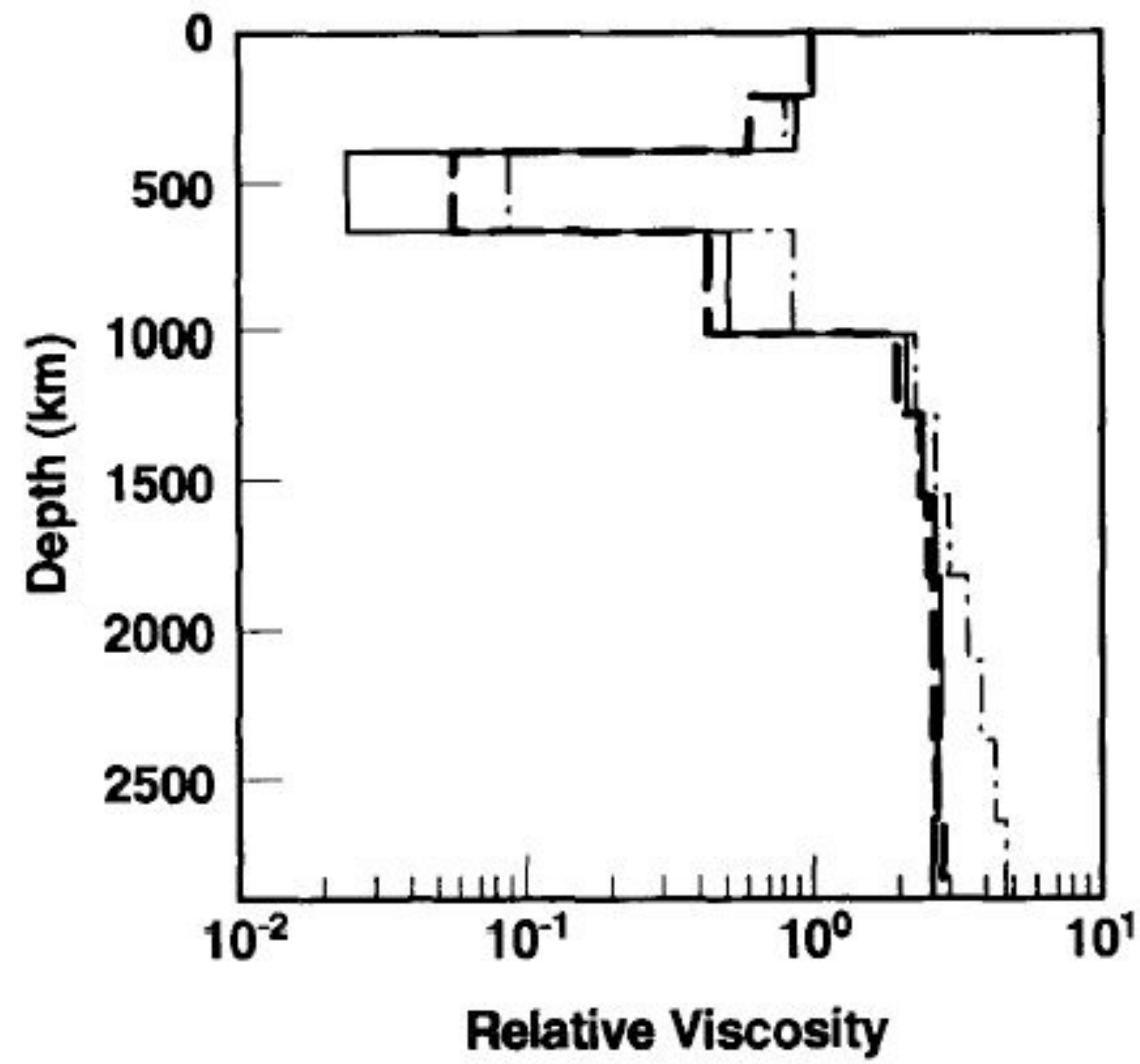


Fig. 6. 1-D viscosity model from Forte et al. [13]. This forward model provides good fits to geoid and plate velocities. It compares well with Figures 3, 4 and 5. The viscosities in this plot are scaled by a characteristic mantle viscosity ($\eta = 10^{21}$ Pa s).



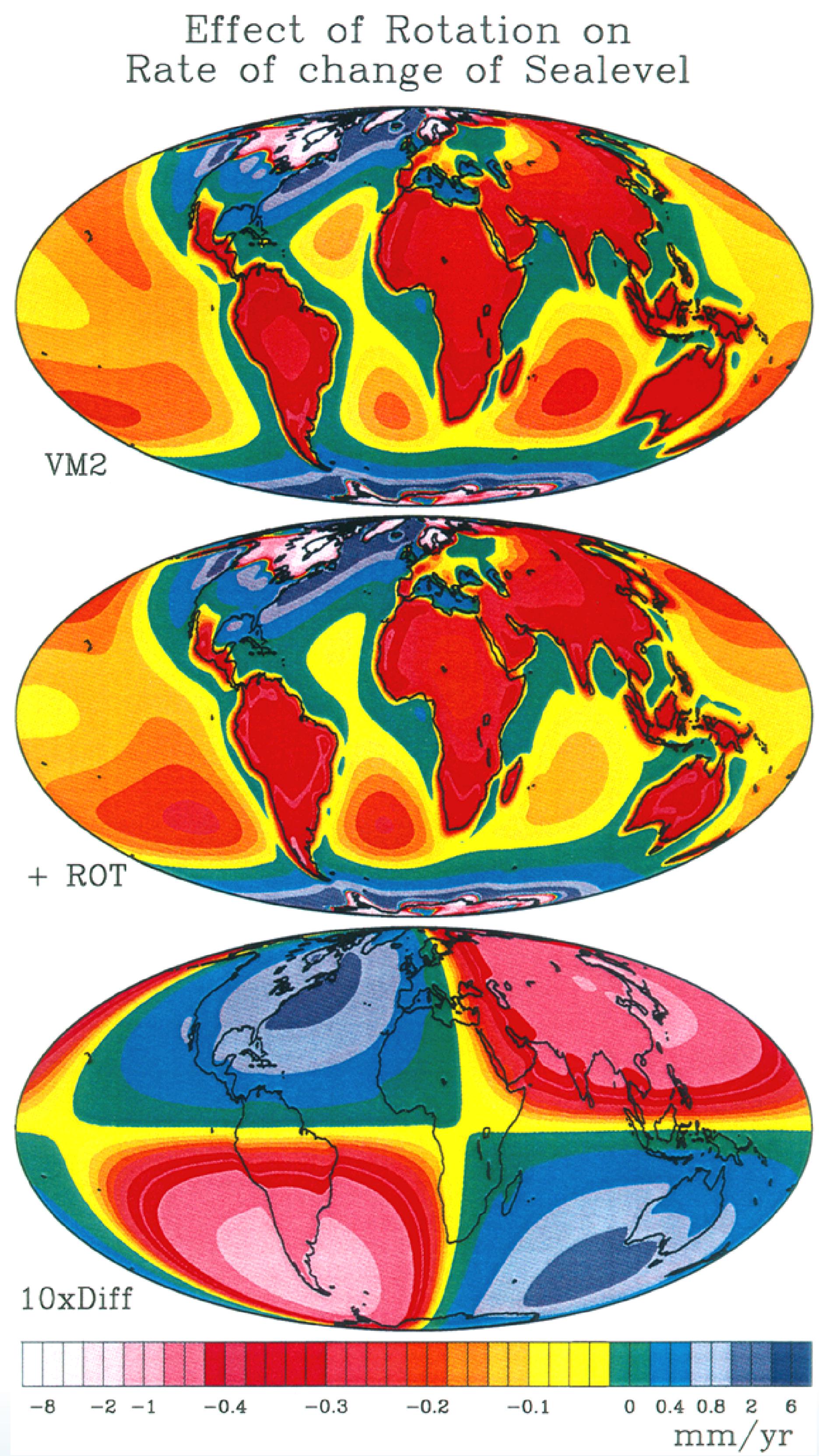
Radial viscosity structure — many possible models depending on data used but some things in common

Viscosity layering & dynamics

This plot shows the influence of the viscosity layering on the way rotation rate influences sea level change.

This indicates the strength of the response of the equatorial bulge of the solid Earth.

Internal strength variations influence how the Earth responds to changes in rotation — let us look in more detail at changes in rotation



Earth’s spin — detectable causes of wobble

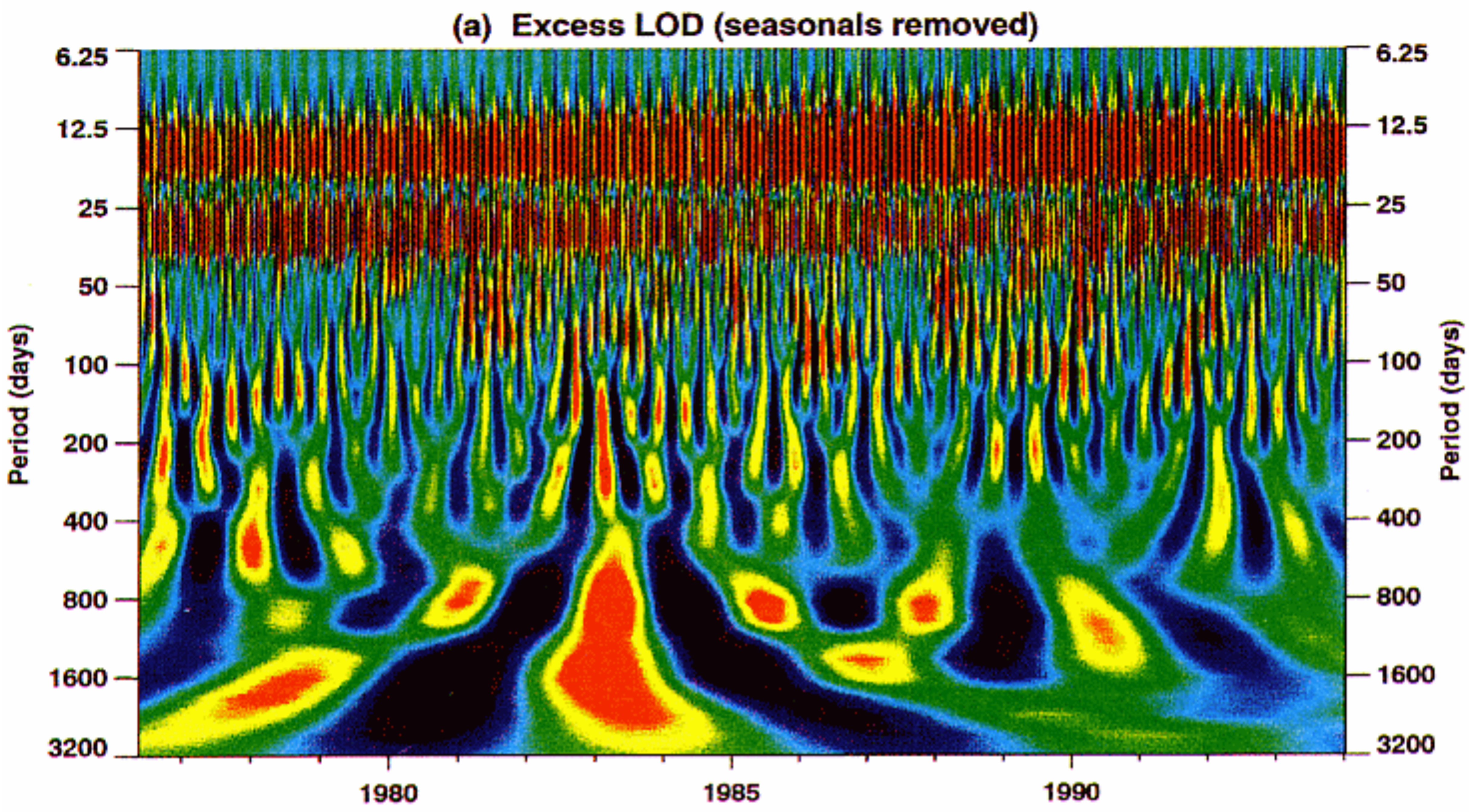
There are many components of the wobble that have been detected at the scale of 1/1000 of a second of arc.

- Seasonal, airmass, ice and water shifts — *annual period*
- Chandler Wobble: natural resonance excited by random impulses (unknown origin) that are damped by earth's elasticity — *14 months*
- Electromagnetic coupling of the core/mantle boundary — *decade*
- Variations in sea level — *century*
- Tidal friction — *thousand years*
- Continental Drift — *Million years*

Amplitudes run from 1/1000 of a second of arc to tens of degrees.

May have detectable signatures in climate patterns.

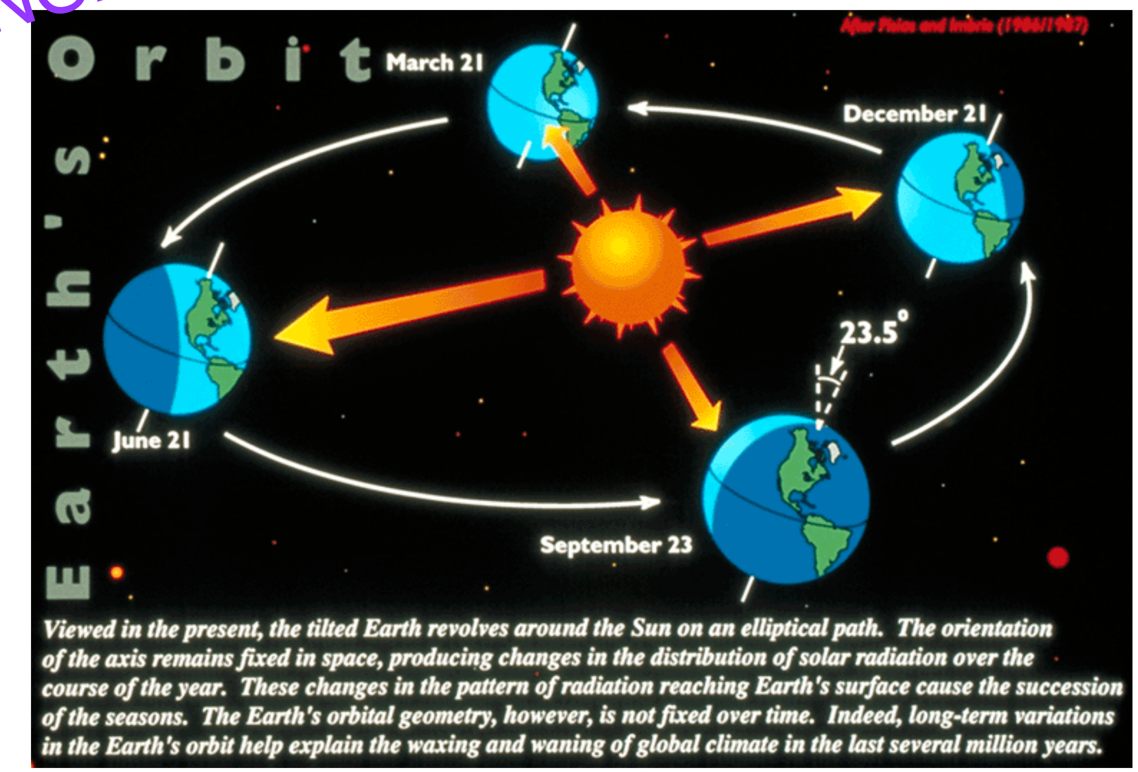
Earth's spin — length of day variations



Wavelet analysis of length of day variations - amplitude ~ 0.1ms

Precession of the equinoxes

NOAA



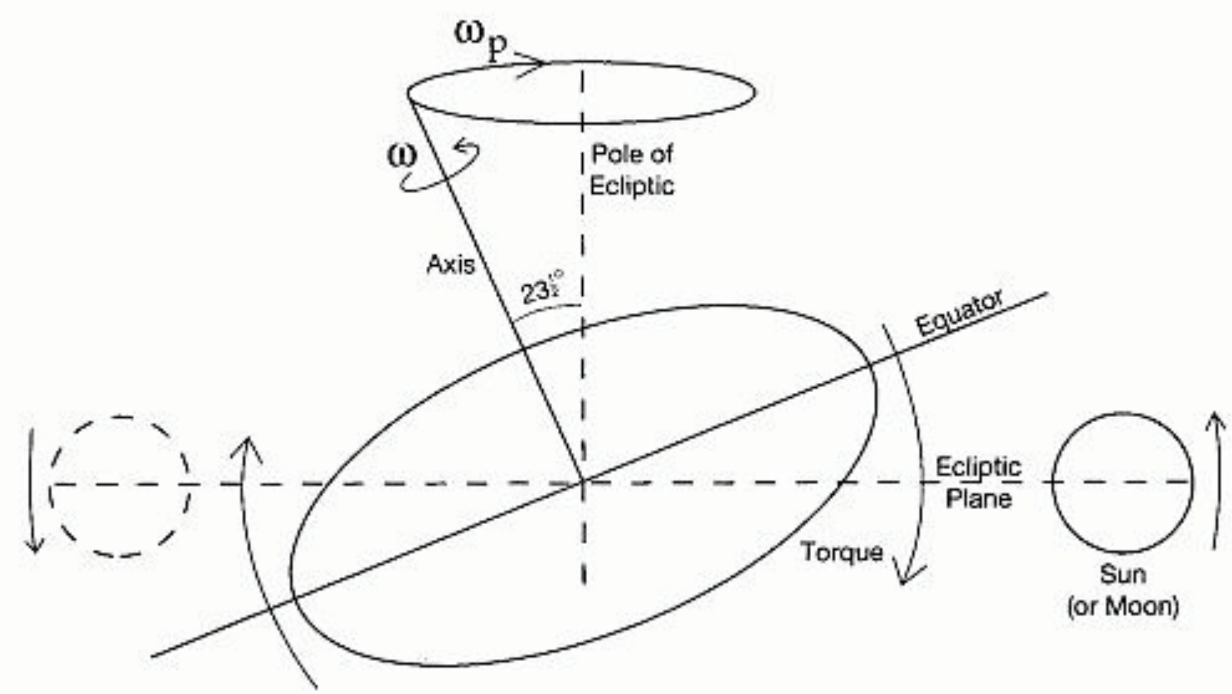


Figure 3.3. Origin of the precessional torque. The gravitational action of the Sun (and Moon) on the Earth's equatorial bulge exerts a torque that tends to pull the bulge into alignment with the instantaneous Earth-Sun (or Earth-Moon) axis. The torque vanishes when the Sun (or Moon) crosses the equatorial plane, but appears with the same sign for both halves of the orbit, causing a net average precessional torque.

The Earth's spin axis precesses in space due to solar / lunar drag on the equatorial bulge. This interacts with the (changing) eccentricity of the elliptical orbit to produces N/S variations in strength of seasons & ice ages.

- Changes in obliquity have a 42kyr period
- Changes in eccentricity have a 100 - 400 kyr period
- Precession has a 22 kyr period

Understanding the wobbling Earth

More subtle variations in day/year length and axis orientation are possible due to gravitational pulls on the non-tidal mass distribution.

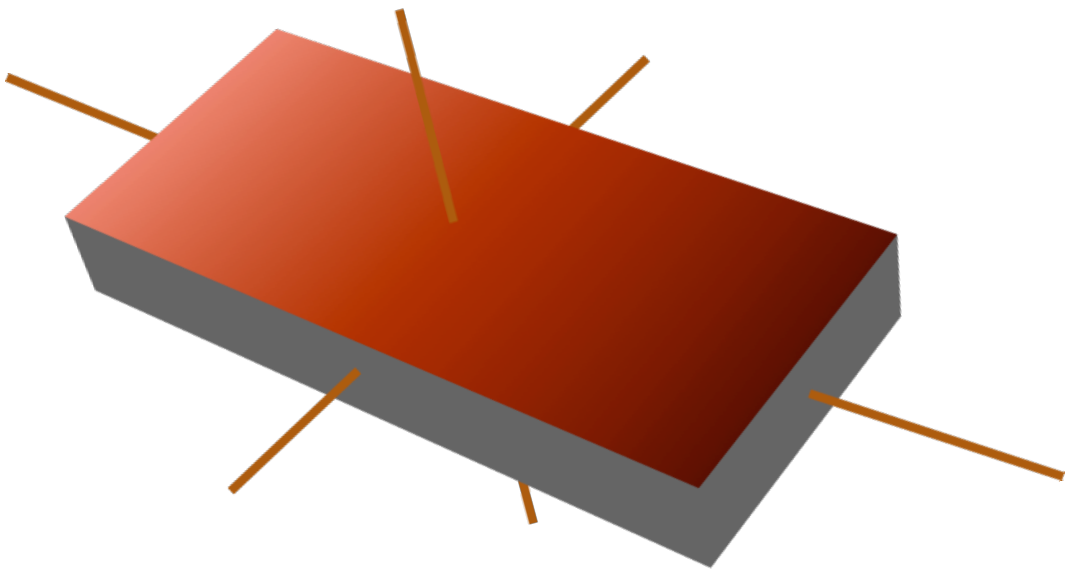
To understand how the Earth responds to orbital forcing, we need to understand how spinning planets behave

- Moment of inertia
- Distribution of mass
- Internal dynamics

The definition of the moment of inertia about a particular axis is

$$I = \int \rho r^2 dr$$

Obviously the value changes depending on the axis we choose.
In the cuboid there are 3 obvious principle axes / values (A,B,C)



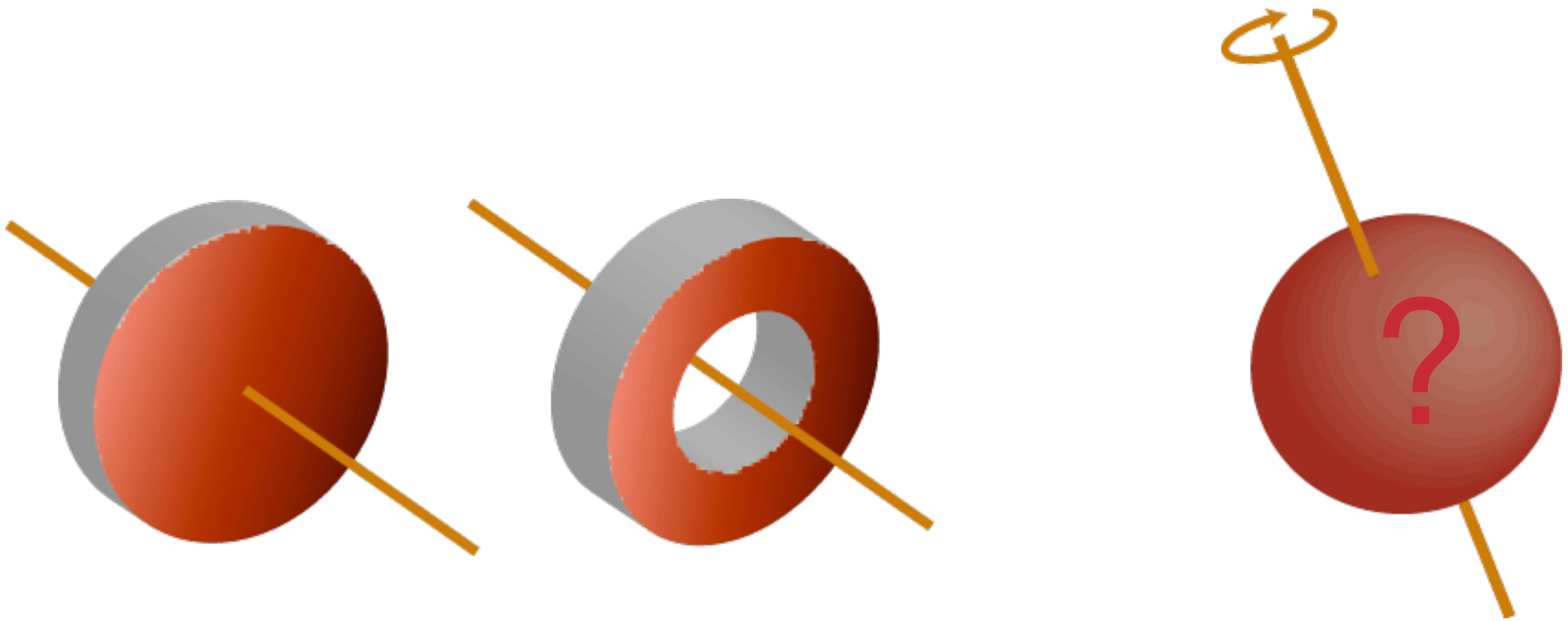
In general the inertia tensor is defined by the relationship between angular momentum and the rotation vector

$$L_i = I_{ij}\omega_j$$

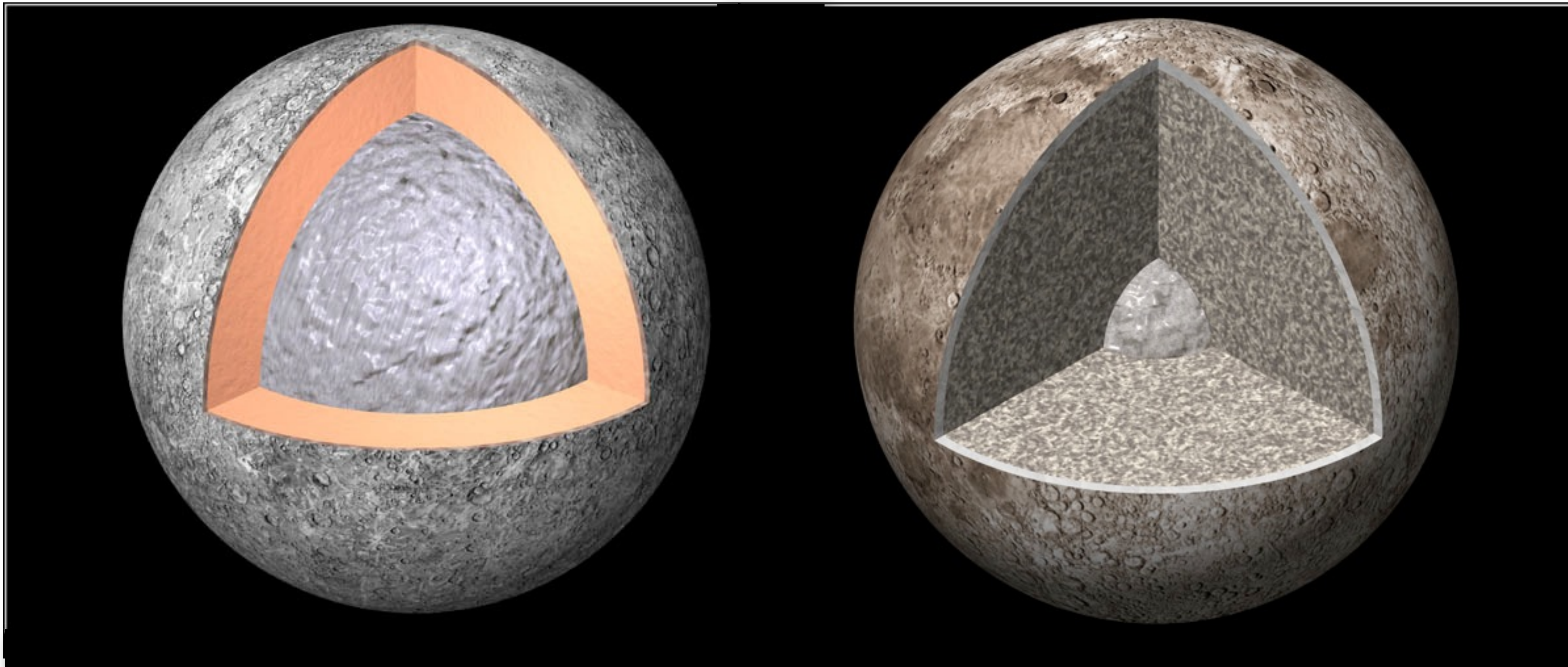
What does it tell us, how do we measure it ?

Moment of inertia of equal mass objects with same surface morphology

- Is larger if mass in concentrated near the outside
- Can be detected by wobbles in spin



Different moment of inertia



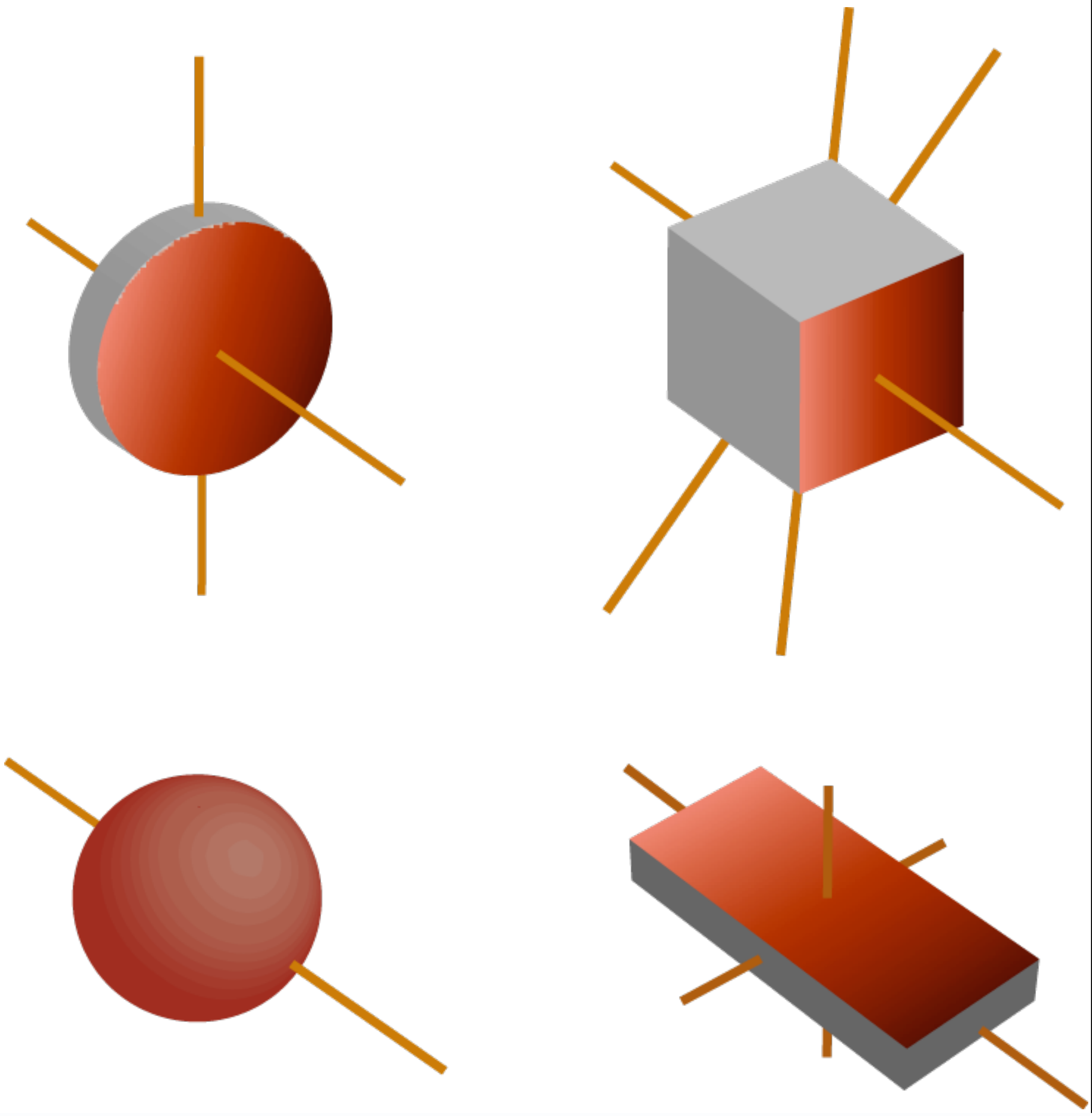
How do we know that planets have differentiated ?

How do we know the size of the core ?

Moment of inertia & symmetry

Higher symmetry —
more degeneracy in choice
of principle axes.

- *Sphere*: A=B=C
- *Disc/ellipsoid*: C > B = A
- *Cuboid*: C > B > A



For an ellipsoid rotation about axis with max moment of inertia is preferred.

(C-A)/A is an important ratio

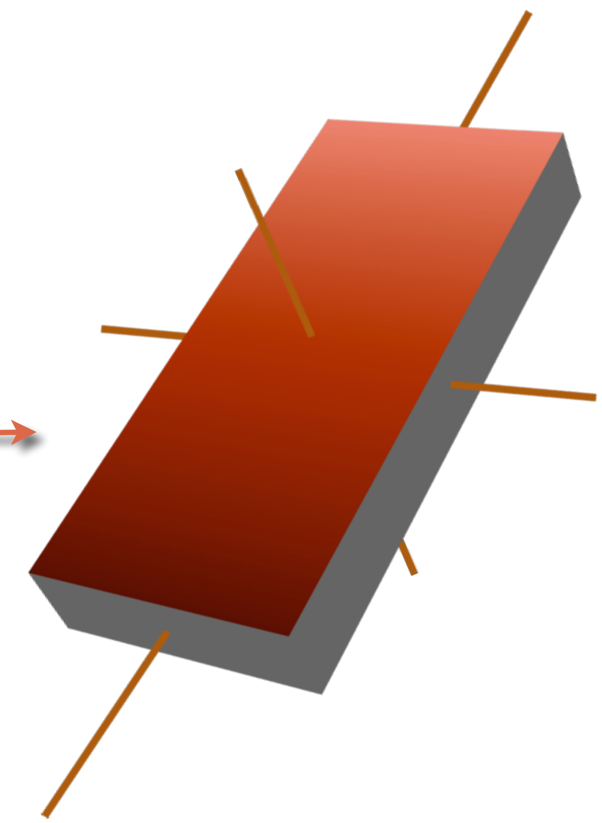
Moment of Inertia and stable spin

Bodies can spin in a stable state about the principle rotation axis with

- Maximum moment of inertia
- Minimum moment of inertia

But the spin about the axis with the intermediate moment of inertia is not stable — the body starts to spin about other axes too.

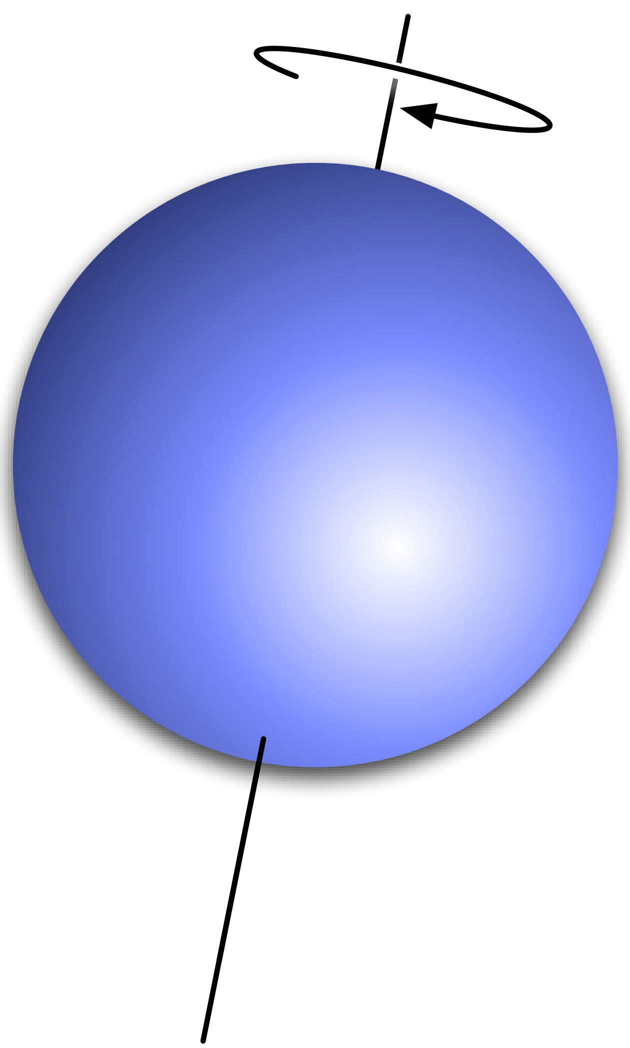
You can't spin this block
about this axis



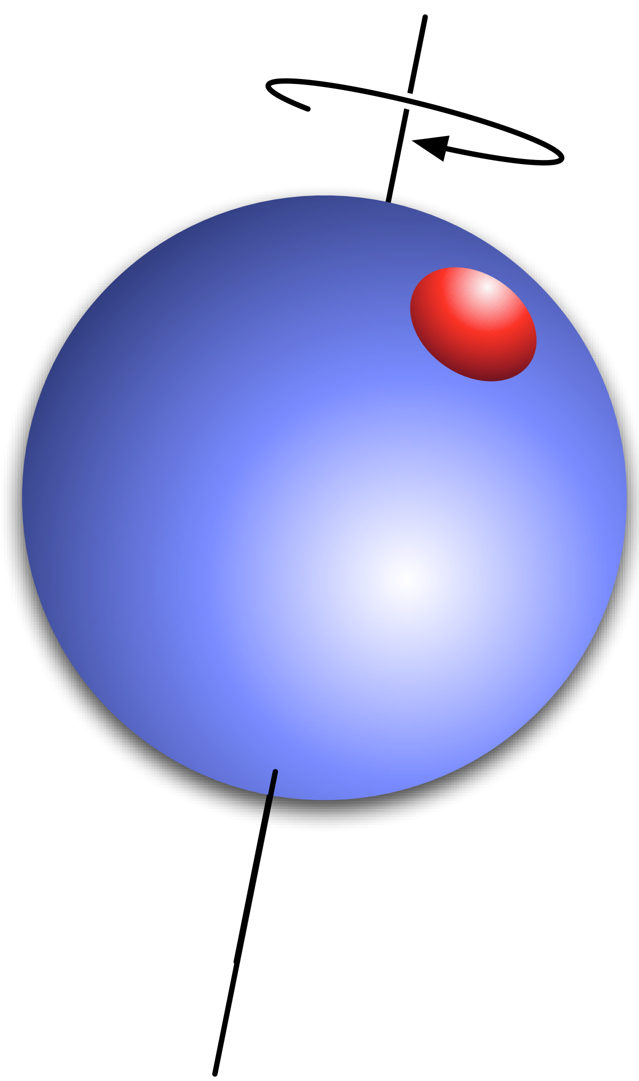
Unconvincing demonstration

Alright, see if you can do any better then !!

True polar wander



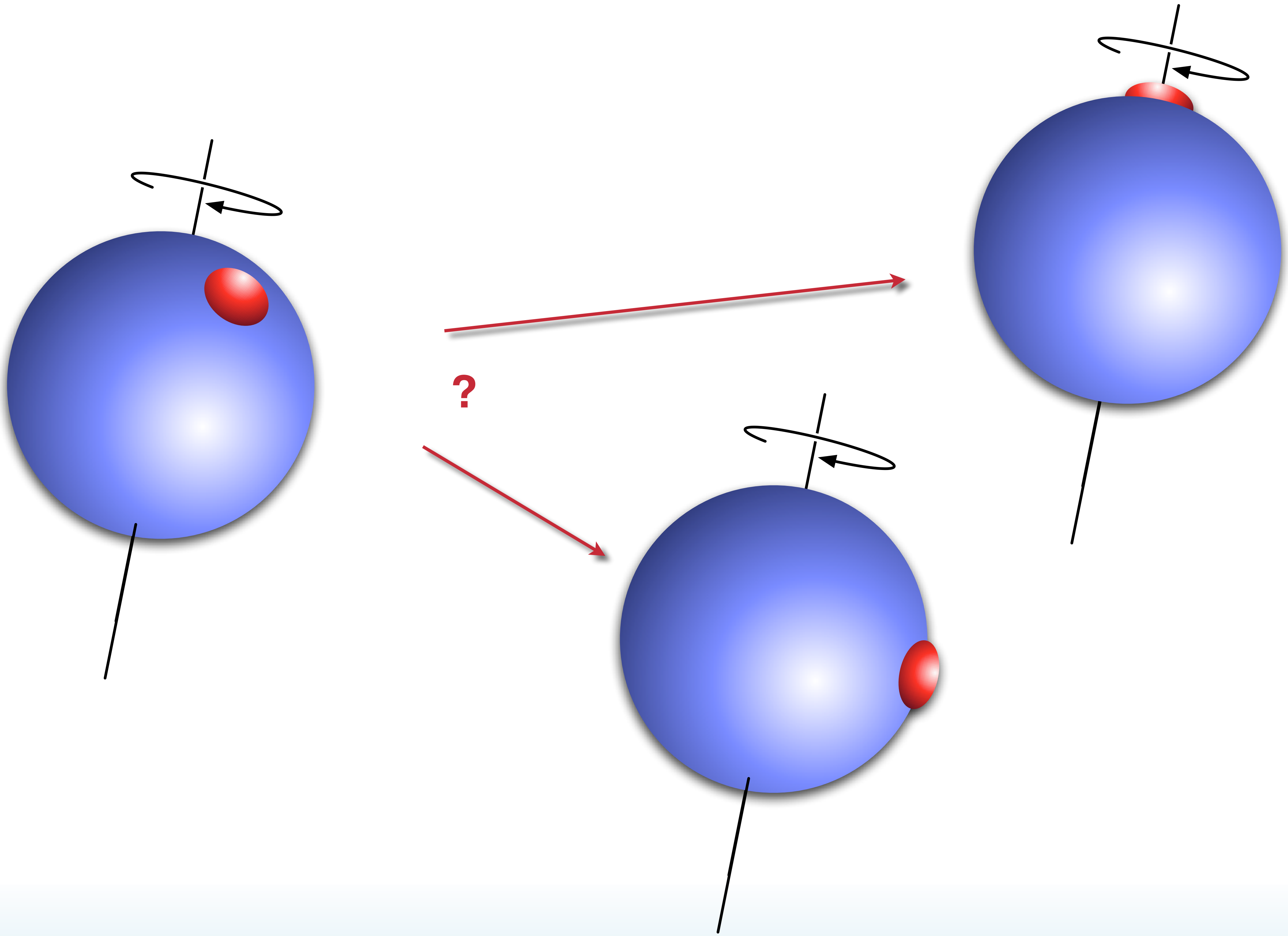
v



What does this big new volcano mean for a planet's spin ?

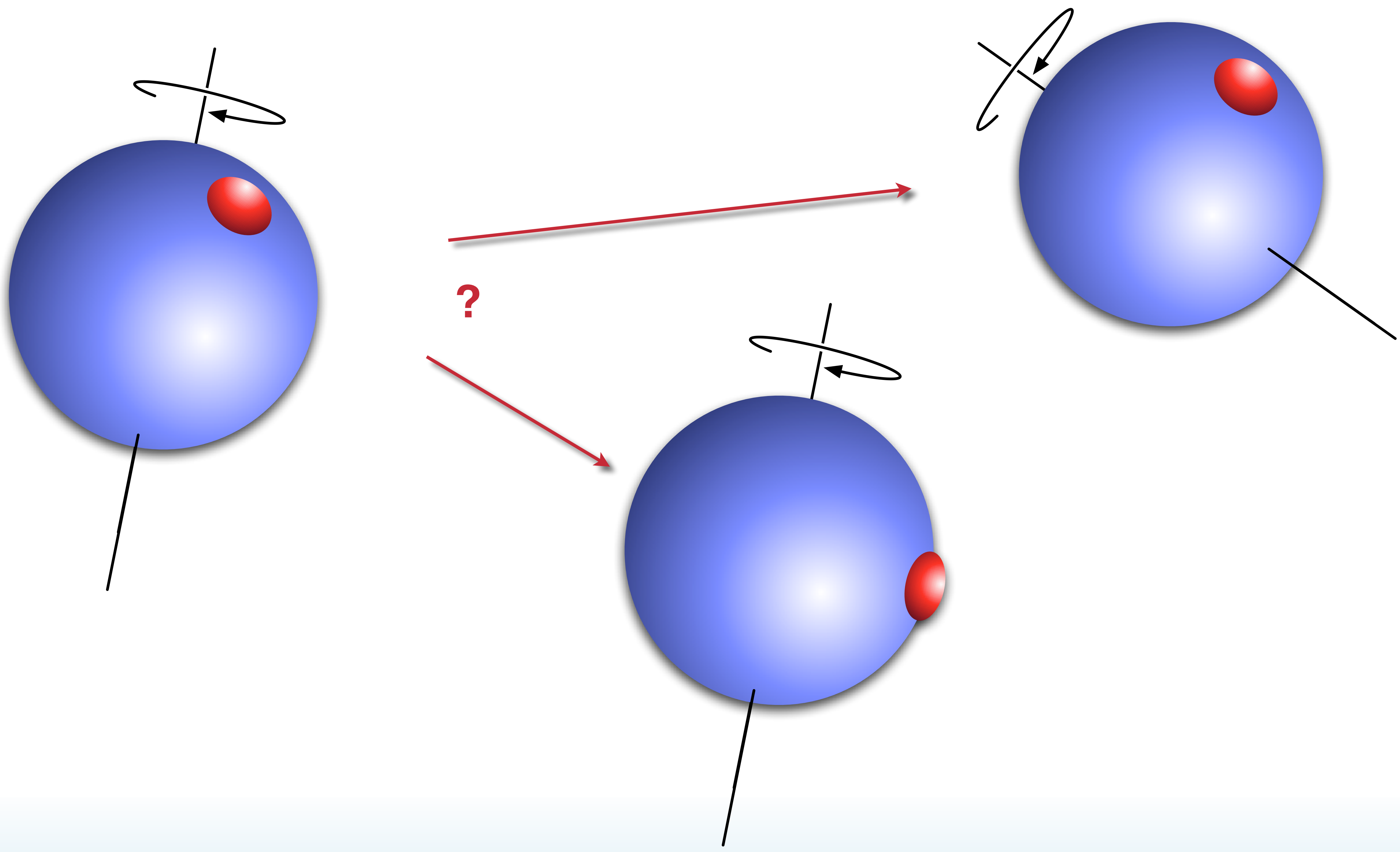
True polar wander

Which of these is more likely ?



True polar wander

Which of these possibilities do you expect to see ?



Rotation axis of the Earth

The value of (C-A) for the Earth is determined by the equatorial bulge, but the bulge is ***caused by rotation*** so the axis must actually be determined by some other mass anomalies.

The bulge can be calculated assuming that the Earth is a rotating fluid.

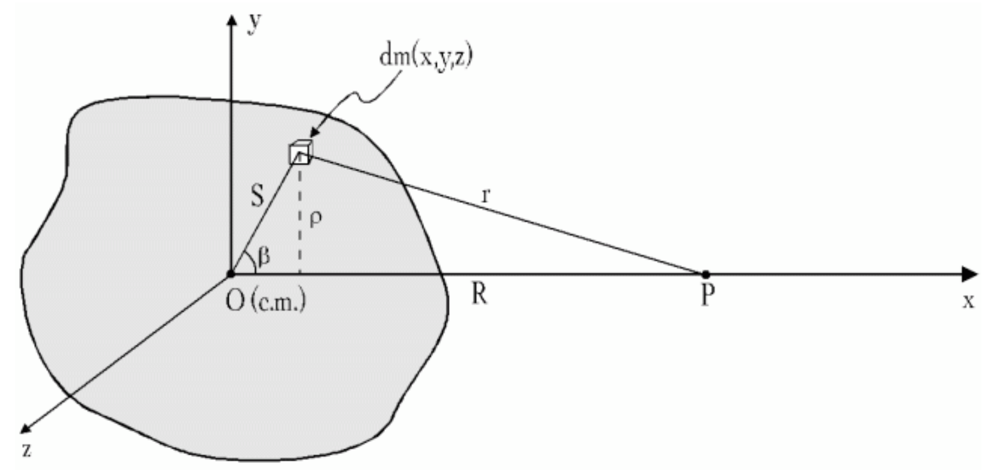
Once established, the bulge takes time to relax in response to changes in the rotation — this relaxation time stabilizes the mantle w.r.t. the rotation axis

but a significant change in the underlying density pattern will cause the mantle to migrate to a new orientation w.r.t. the rotation axis

Relationship to Geoid / Gravity

MacCullagh's formula relates geoid and moments of inertia

$$U(P) = \int \frac{dm}{r}$$



$$\boxed{U(P) = -\frac{GM}{R} - \frac{G}{2R^3}\left(A + B + C - 3I_{OP}\right) + O(S^3/R^3)}$$

Ellipsoid: $A = B < C$, β is colattitude

$$\boxed{U = -\frac{GM}{R} - \frac{G}{2R^3}(C - A)(3\sin^2\beta - 1)}$$

Relationship to Geoid / Gravity

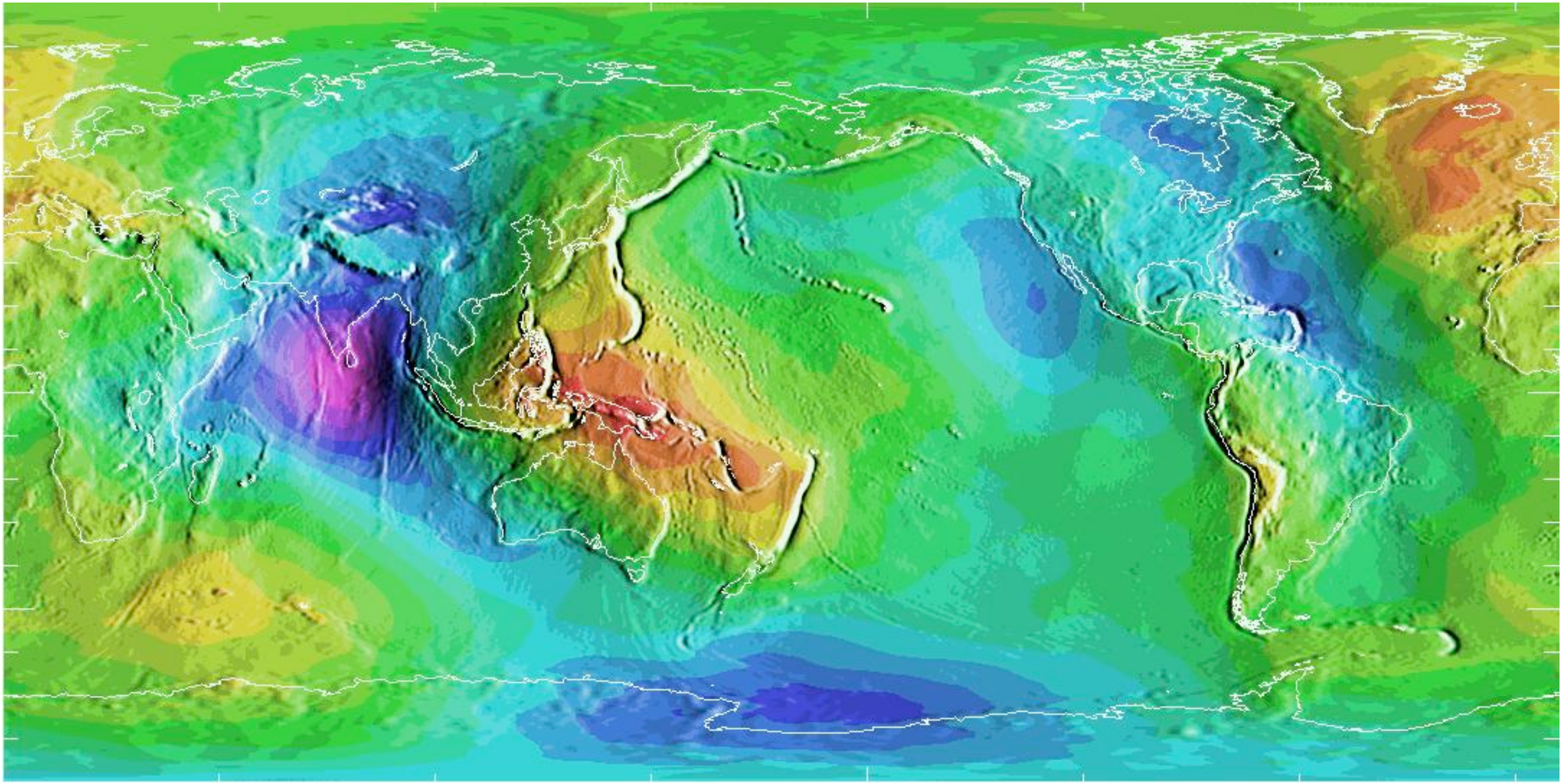
So, if we know the broad-scale geoid or gravity field of a planet, we can calculate principle moments of inertia.

- Radial distribution of mass
- Stable axes of spin

Alternatively, if we know how the planet's spin is changing we can tell how the mass distribution is shifting

- Satellite orbits are exquisitely sensitive to the gravitational potential and are measured with great precision through time

If we know the evolving density distribution we can predict the influence on spin / orbit characteristics



Non-hydrostatic geoid: equatorial bulge removed

We do know the present geoid very accurately. It is a combination of the effect of the equatorial bulge and the internal density distribution.

Density Distribution in the Earth

Magnitude of equatorial bulge, and hence the magnitude of (C-A) depends on layered structure of the Earth — density layering, strength layering, phase boundaries, size of the core etc.

Non-hydrostatic geoid shape is not dependent upon radial mass distributions, but Earth's layering is vital in determining this part of the geoid !

- Influences convection pattern
- Changes pattern of subduction / slab dynamics
- Different response to ice-loading